Computer Algebra
Independent Integration Tests

Summer 2024

4-Trig-functions/4.5-Secant/232-4.5.1.2

Nasser M. Abbasi

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**879**]. This is test number [232].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.89 (878)	0.11 (1)
Mathematica	99.09 (871)	0.91 (8)
Maple	83.39 (733)	16.61 (146)
Fricas	69.28 (609)	30.72 (270)
Giac	38.91 (342)	61.09 (537)
Mupad	36.75 (323)	63.25 (556)
Maxima	36.75 (323)	63.25 (556)
Reduce	28.56 (251)	71.44 (628)
Sympy	5.35 (47)	94.65 (832)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

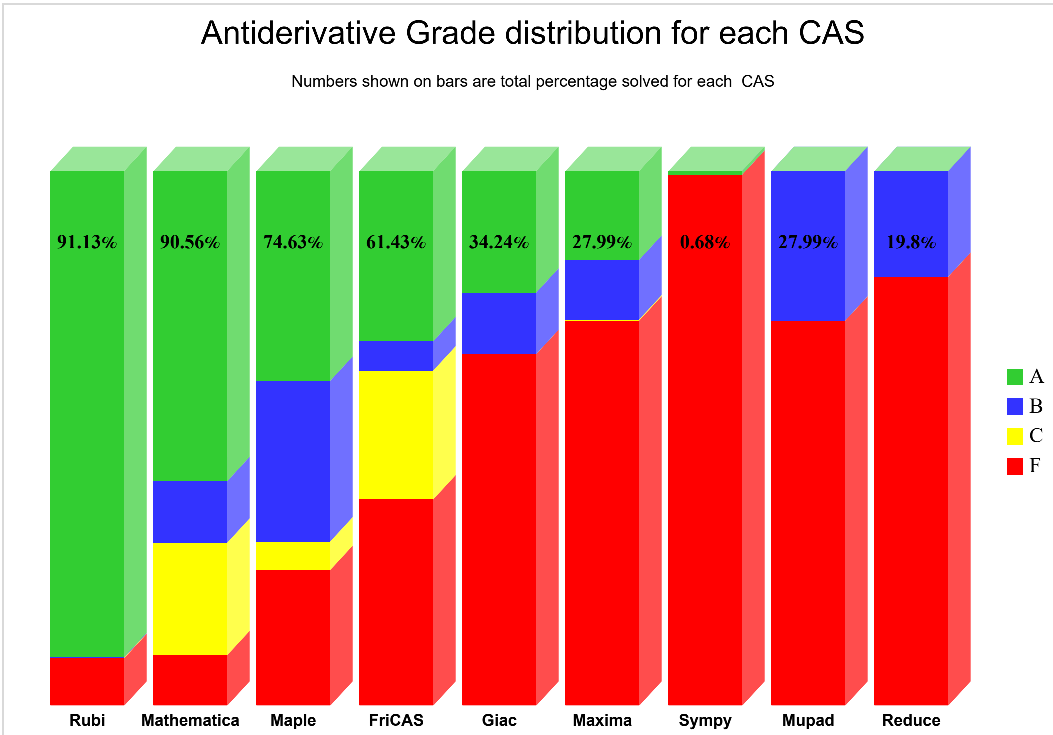
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

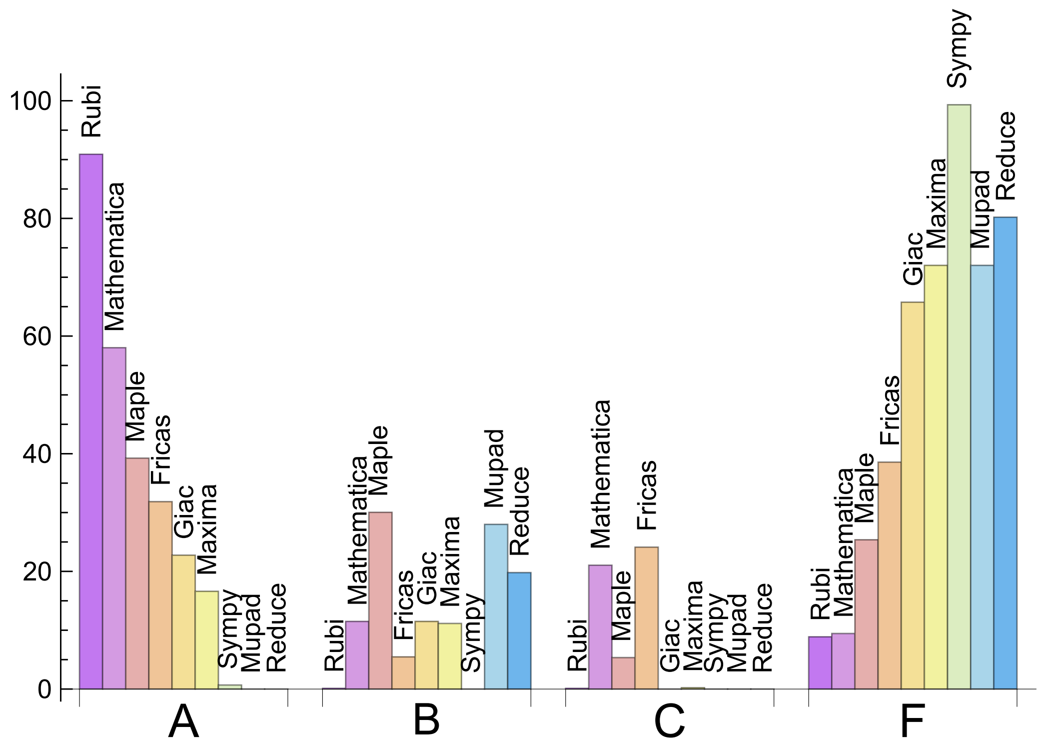
System	% A grade	% B grade	% C grade	% F grade
Rubi	90.899	0.114	0.114	8.874
Mathematica	58.020	11.490	21.047	9.443
Maple	39.249	30.034	5.347	25.370
Fricas	31.854	5.461	24.118	38.567
Giac	22.753	11.490	0.000	65.757
Maxima	16.610	11.149	0.228	72.014
Sympy	0.683	0.000	0.000	99.317
Mupad	0.000	27.986	0.000	72.014
Reduce	0.000	19.795	0.000	80.205

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	1	100.00	0.00	0.00
Mathematica	8	62.50	37.50	0.00
Maple	146	100.00	0.00	0.00
Fricas	270	66.30	33.70	0.00
Giac	537	89.76	6.70	3.54
Mupad	556	0.00	100.00	0.00
Maxima	556	82.91	9.89	7.19
Reduce	628	100.00	0.00	0.00
Sympy	832	66.95	33.05	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.17
Reduce	0.19
Maxima	0.48
Rubi	0.89
Giac	6.20
Mupad	11.43
Maple	11.61
Mathematica	12.40
Sympy	13.98

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	23.13	1.09	24.00	1.00
Giac	164.62	1.53	121.50	1.27
Rubi	166.03	1.07	140.00	1.02
Reduce	229.37	2.06	76.00	1.72
Fricas	247.11	1.93	175.00	1.63
Maple	352.80	1.94	179.00	1.47
Mupad	382.40	2.46	88.00	1.08
Mathematica	1258.82	6.22	136.00	1.08
Maxima	3913.36	22.64	104.00	1.21

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

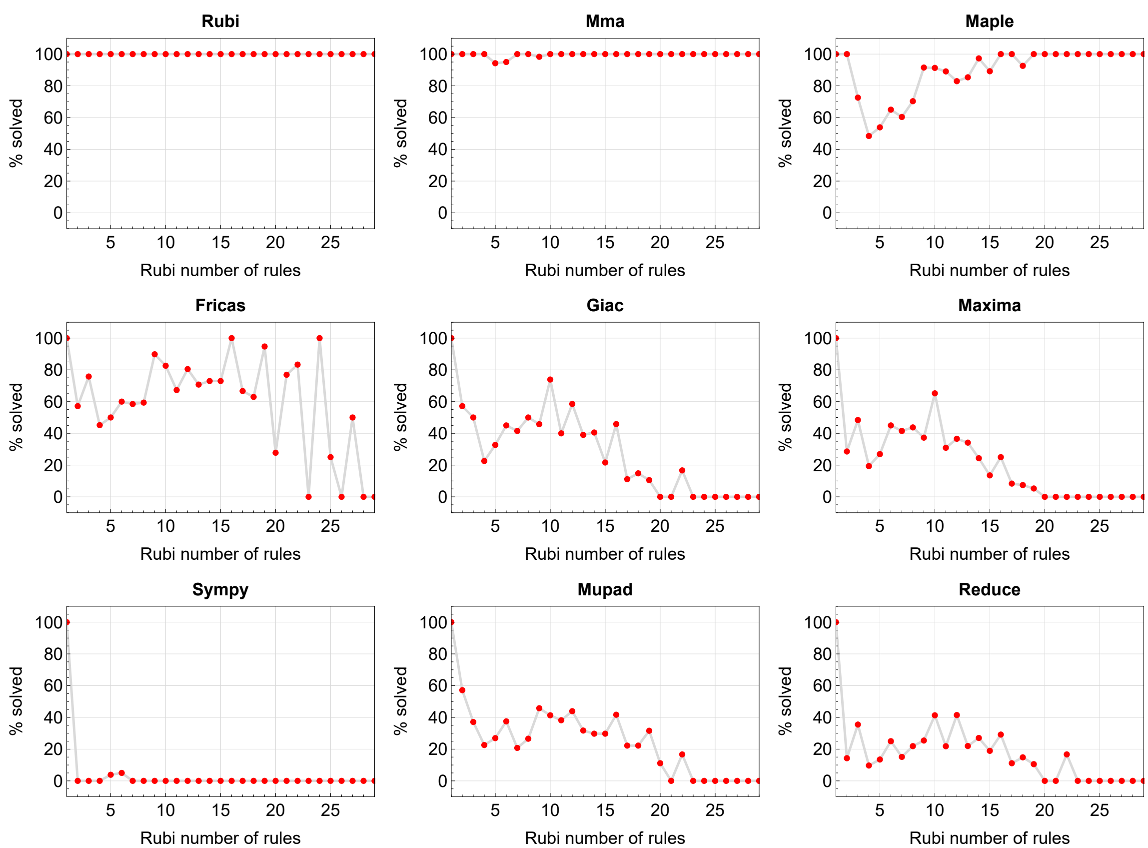


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

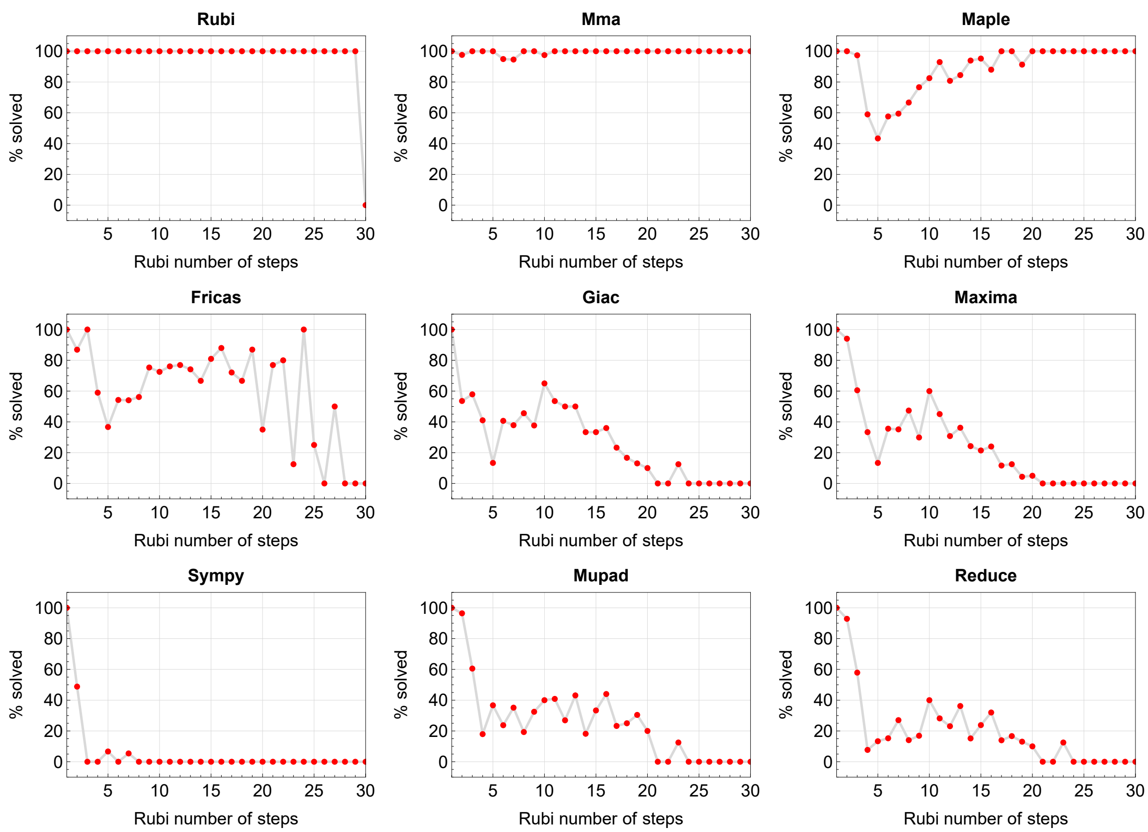


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

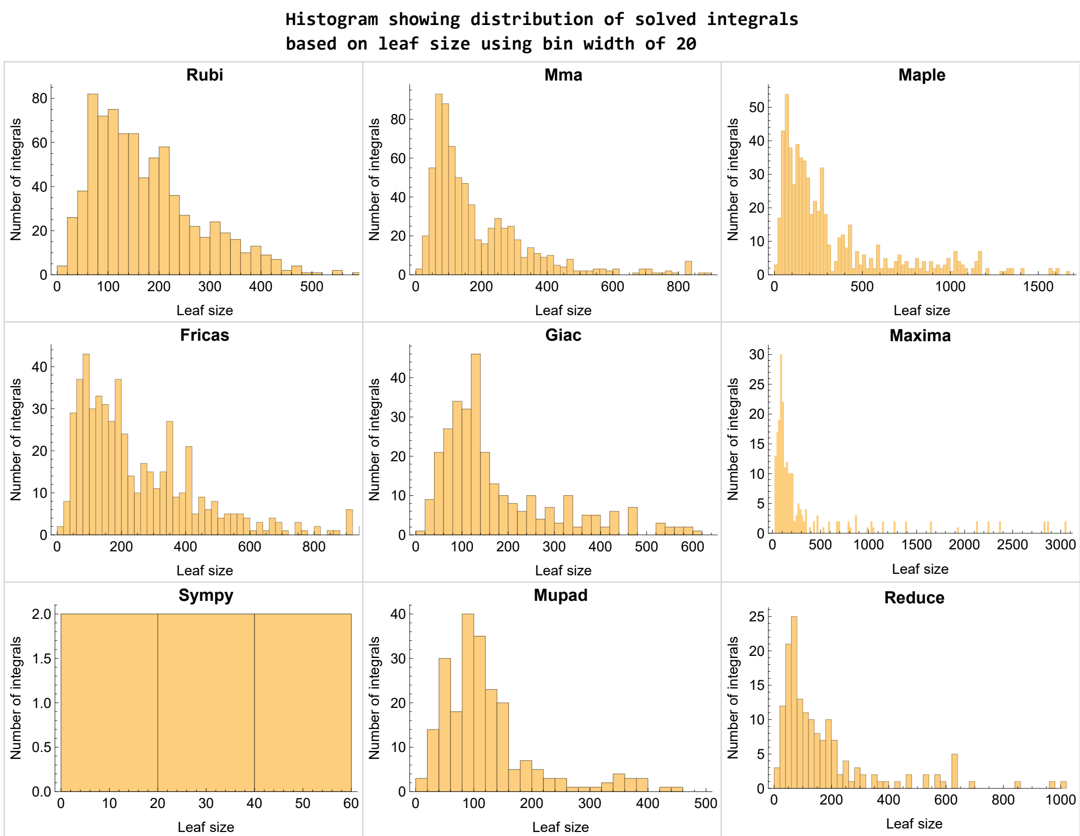


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

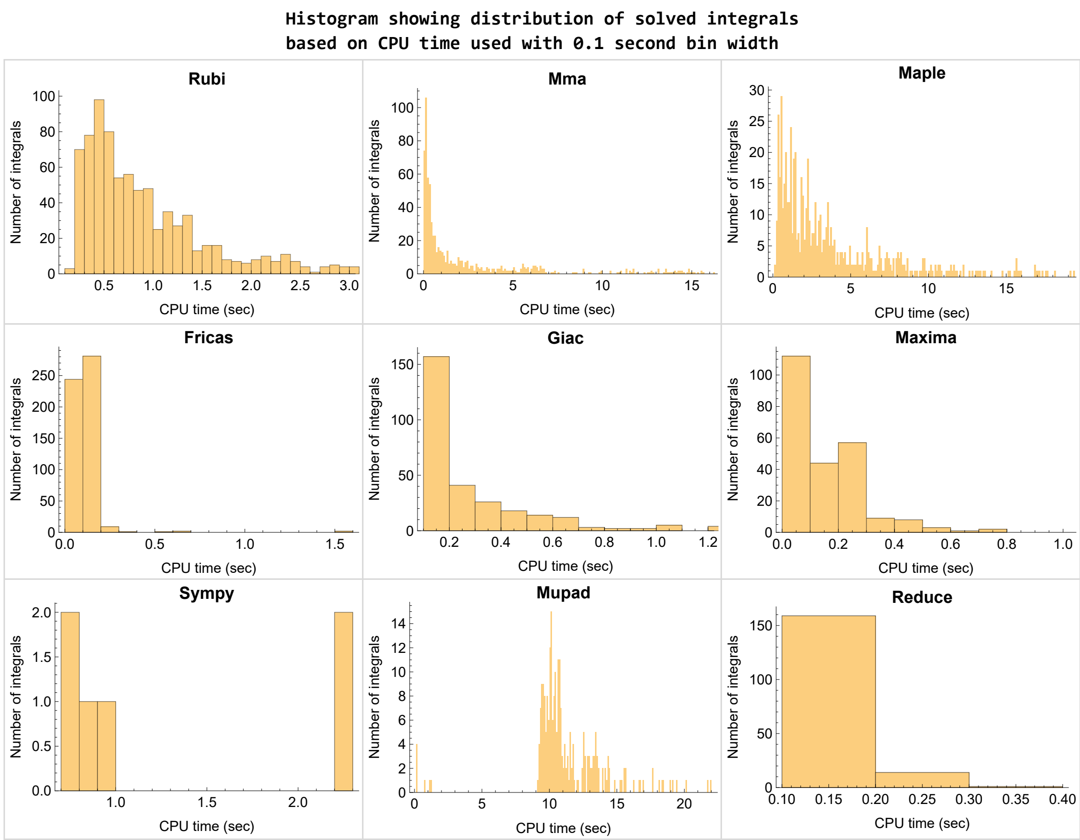


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

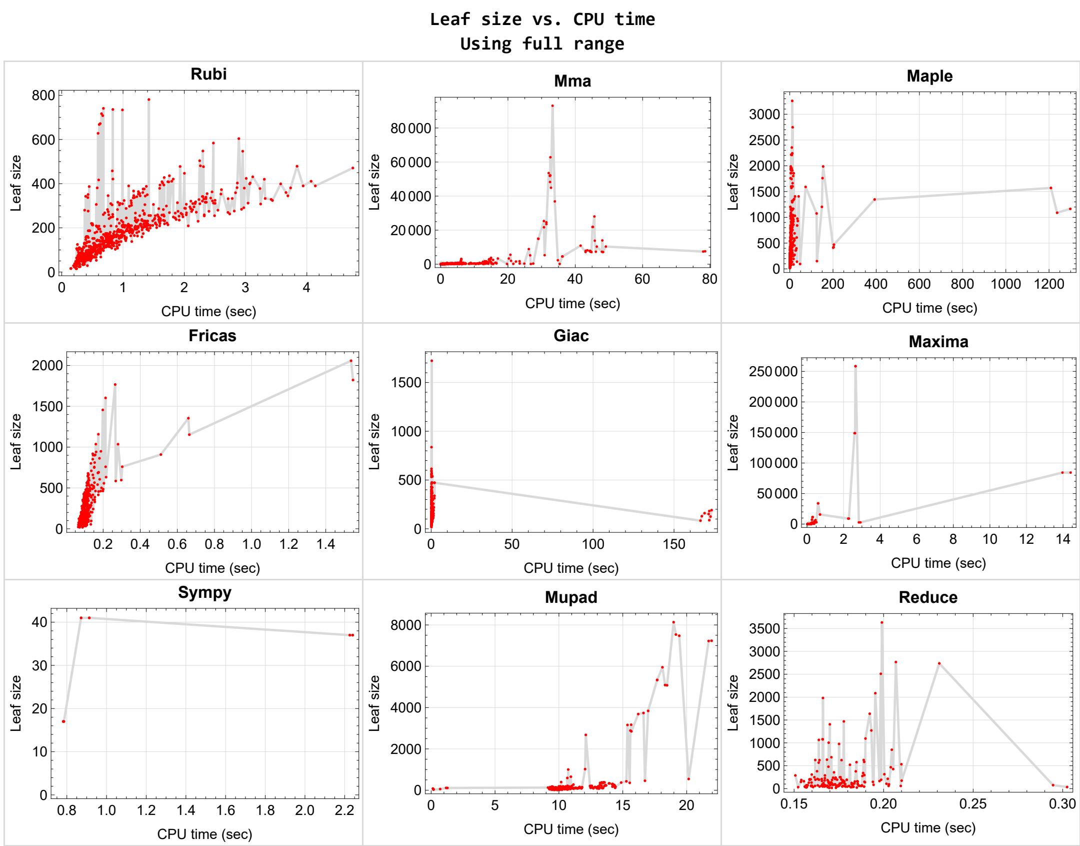


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{686, 691, 693, 698, 703, 705, 707, 712, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 782, 783, 784, 785, 786, 787, 791, 792, 793}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {147, 152, 159, 160, 161, 162, 163, 164, 275, 276, 277, 278, 279, 280, 281, 282, 283, 286, 302, 306, 332, 333, 335, 339, 340}

Mathematica {58, 68, 111, 123, 131, 139, 146, 147, 151, 152, 153, 157, 158, 163, 164, 188, 209, 218, 225, 226, 228, 233, 234, 235, 236, 237, 249, 255, 256, 257, 261, 263, 264, 269, 270, 280, 281, 282, 283, 284, 285, 286, 287, 294, 297, 298, 301, 302, 305, 306, 307, 310, 311, 314, 315, 318, 319, 320, 321, 325, 327, 329, 330, 331, 333, 334, 335, 337, 338, 339, 341, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 367, 368, 369, 395, 404, 417, 425, 427, 429, 430, 434, 435, 436, 437, 531, 533, 535, 537, 538, 539, 541, 543, 544, 545, 546, 547, 549, 550, 551, 552, 553, 554, 555, 556, 560, 562, 563, 564, 568, 570, 571, 572, 573, 576, 608, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 780, 781, 788, 789, 790, 817, 818, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 878, 879}

Maple {131, 132, 133, 139, 140, 237, 259, 263, 435}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
```

```
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

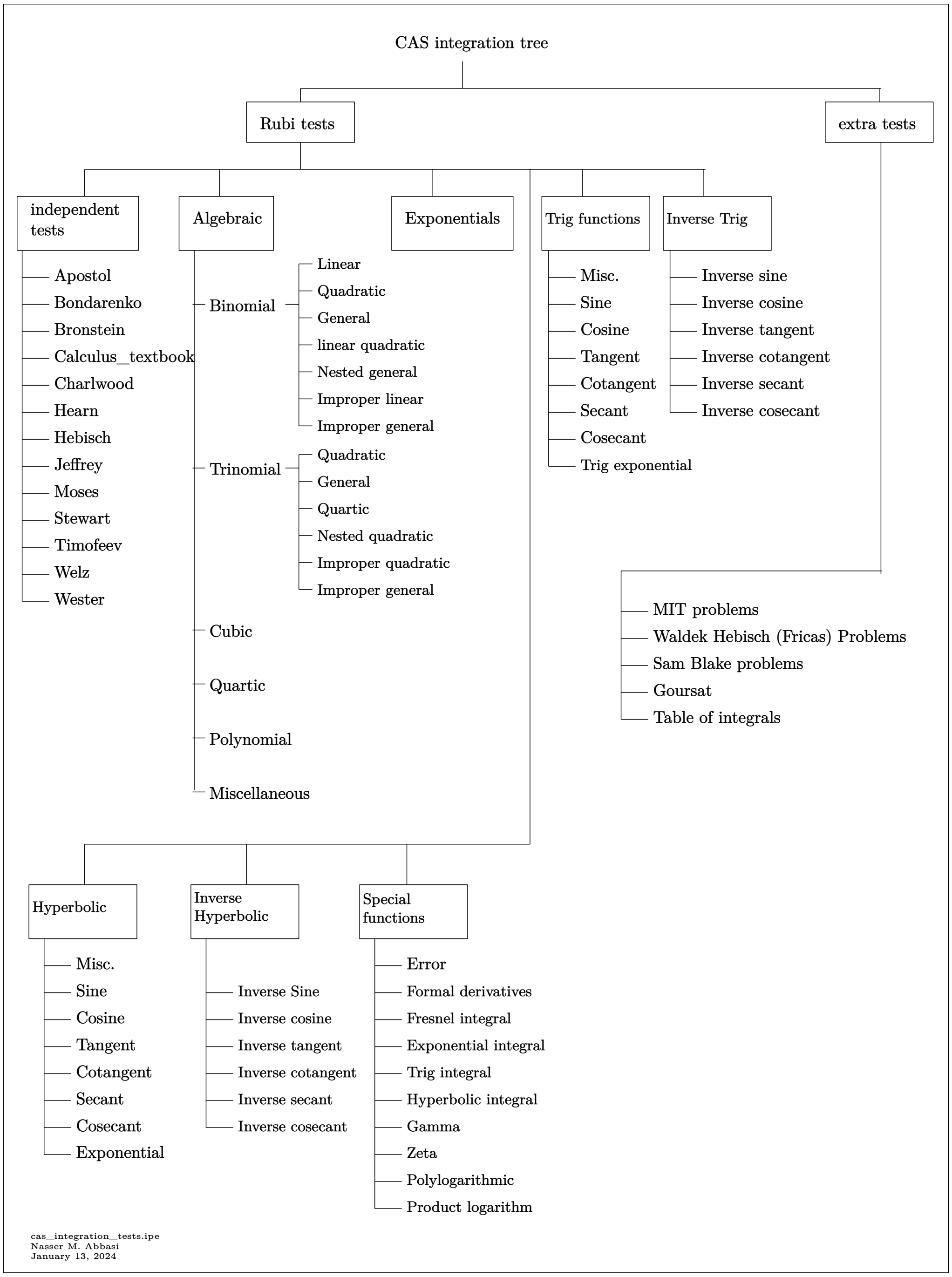
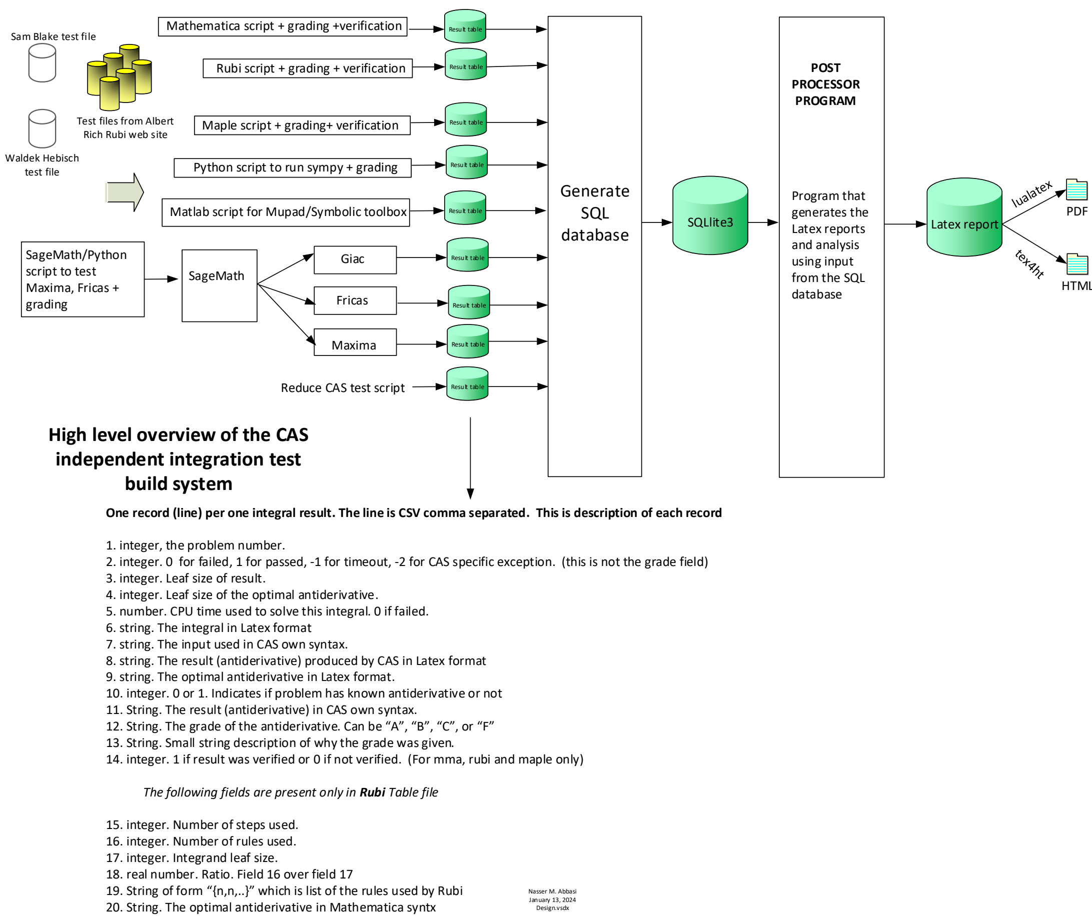


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 777, 778, 779, 780, 781, 788, 789, 790, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879 }

B grade { 819 }

C grade { 286 }

F normal fail { 854 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 79, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 116, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 139, 140, 141, 165, 166, 167, 168, 169, 170, 171, 175, 189, 204, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 254, 255, 256, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 286, 288, 289, 290, 291, 292, 293, 295, 296, 299, 300, 303, 304, 308, 309, 312, 313, 316, 317, 322, 326, 328, 341, 343, 344, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 542, 544, 545, 546, 547, 548, 549, 550, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 570, 571, 572, 573, 574, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623, 624, 625, 627, 629, 630, 631, 632, 633, 636, 637, 638, 639, 644, 645, 646, 649, 650, 651, 652, 653, 656, 657, 658, 659, 660, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 777, 778, 779, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 875, 876, 877 }

B grade { 34, 35, 48, 61, 70, 71, 77, 78, 80, 88, 146, 147, 151, 152, 157, 158, 163, 164, 253, 260, 280, 281, 282, 283, 284, 297, 298, 301, 302, 305, 306, 310, 311, 314, 315, 318, 319, 329, 330, 331, 333, 334, 335, 337, 338, 339, 345, 346, 347, 348, 431, 438, 480, 529, 530, 541, 543, 551, 552, 553, 567, 568, 569, 575, 576, 577, 578, 616, 626, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 780, 781, 788, 789, 790, 822, 878, 879 }

C grade { 96, 97, 98, 111, 113, 114, 115, 117, 133, 138, 142, 143, 144, 145, 148, 149, 150, 153, 154, 155, 156, 159, 160, 161, 162, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 272, 273, 274, 275, 276, 277, 278, 279, 285, 287, 294, 307, 320, 321, 325, 327, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 505, 628, 634, 635, 640, 641, 642, 643, 647, 648, 654, 655, 661, 662, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874 }

F normal fail { 323, 324, 332, 336, 340 }

F(-1) timedout fail { 342, 717, 746 }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 96, 97, 98, 99, 100, 101, 102, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 121, 123, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 139, 140, 167, 170, 178, 182, 184, 185, 190, 191, 192, 193, 195, 196, 197, 198, 199, 200, 203, 205, 206, 207, 208, 211, 212, 213, 214, 215, 216, 217, 218, 221, 222, 223, 224, 225, 226, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 269, 270, 271, 359, 367, 370, 374, 381, 389, 390, 391, 392, 393, 394, 395, 398, 399, 400, 401, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 534, 535, 542, 557, 558, 559, 560, 581, 589, 596, 612, 657, 865 }

B grade { 94, 95, 103, 104, 117, 122, 124, 132, 137, 138, 165, 166, 168, 169, 171, 172, 173, 174, 175, 176, 177, 179, 180, 181, 183, 186, 187, 188, 189, 194, 201, 202, 204, 209, 210, 219, 220, 227, 228, 243, 244, 245, 246, 247, 252, 265, 266, 267, 268, 351, 352, 353, 354, 355, 356, 357, 358, 360, 361, 362, 363, 364, 365, 366, 368, 369, 371, 372, 373, 375, 376, 377, 378, 379, 380, 382, 383, 384, 385, 386, 387, 388, 396, 397, 402, 531, 532, 533, 536, 537, 538, 539, 540, 541, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 582, 583, 584, 585, 586, 587, 588, 590, 591, 592, 593, 594, 595, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 630, 631, 632, 633, 637, 638, 639, 644, 645, 646, 650, 651, 652, 653, 656, 658, 659, 660, 663, 664, 665, 666, 667, 668, 670, 671, 678, 679, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 842, 843, 844, 848, 849, 850, 855, 856, 857, 858, 862, 863, 864, 866, 869, 870, 871, 872, 873 }

C grade { 20, 41, 82, 530, 628, 629, 634, 635, 636, 640, 641, 642, 643, 647, 648, 649, 654, 655, 661, 662, 669, 672, 673, 674, 675, 676, 677, 680, 681, 682, 683, 684, 840, 841, 845, 846, 847, 851, 852, 853, 854, 859, 860, 861, 867, 868, 874 }

F normal fail { 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 242, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 441, 442, 443, 444, 445, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 777, 778, 779, 780, 781, 788, 789, 790, 875, 876, 877, 878, 879 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 132, 133, 134, 135, 137, 140, 141, 142, 218, 221, 222, 223, 224, 225, 226, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 264, 270, 271, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 446, 447, 448, 451, 452, 453, 454, 455, 456, 457, 458, 459, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 490, 491, 492, 493, 494, 495, 496, 500, 501, 503, 504, 505, 508, 513, 523, 524, 525, 526, 527, 528, 529, 530, 670, 671, 678, 679 }

B grade { 4, 5, 14, 117, 118, 129, 130, 131, 136, 138, 139, 219, 220, 227, 228, 243, 244, 252, 259, 265, 266, 267, 268, 269, 449, 450, 460, 488, 489, 497, 498, 499, 502, 506, 507, 509, 510, 511, 512, 514, 515, 516, 517, 518, 519, 520, 521, 522 }

C grade { 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 630, 631, 632, 633, 637, 638, 639, 644, 645, 646, 650, 651, 652, 653, 656, 657, 658, 659, 660, 663, 664, 665, 666, 667, 668, 669, 672, 673, 674, 675, 676, 677, 680, 681, 682, 683, 684, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 837, 838, 839, 842, 843, 844, 848, 849, 850, 855, 856, 857, 858, 862, 863, 864, 865, 866, 869, 870, 871, 872, 873 }

F normal fail { 143, 144, 145, 148, 149, 150, 153, 154, 155, 156, 159, 160, 161, 162, 272, 273, 274, 275, 276, 277, 278, 279, 284, 286, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 441, 442, 443, 444, 445, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 578, 627, 636, 642, 643, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 777, 778, 779, 780, 781, 788, 789, 790, 819, 825, 830, 851, 875, 876, 877, 878, 879 }

F(-1) timedout fail { 146, 147, 151, 152, 157, 158, 163, 164, 280, 281, 282, 283, 285, 287, 551, 577, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 628, 629, 634, 635, 640, 641, 647, 648, 649, 654, 655, 661, 662, 686, 691, 693, 698, 703, 705, 707, 712, 713, 714, 715, 716, 817, 818, 820, 821, 822, 823, 824, 826, 827, 828, 829, 831, 832, 833, 834, 835, 836, 840, 841, 845, 846, 847, 852, 853, 854, 859, 860, 861, 867, 868, 874 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 43, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 221, 222, 229, 238, 247, 248, 269, 400, 401, 407, 414, 422, 423, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 523, 524, 525, 526, 527, 528, 529, 530 }

B grade { 30, 41, 42, 44, 48, 94, 95, 96, 97, 98, 103, 104, 111, 112, 117, 118, 218, 219, 220, 223, 224, 225, 226, 227, 228, 230, 231, 232, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 249, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 270, 271, 398, 399, 402, 403, 404, 405, 406, 408, 409, 410, 411, 412, 413, 415, 416, 417, 418, 419, 420, 421, 424, 425, 426, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440 }

C grade { 123, 142 }

F normal fail { 90, 91, 92, 93, 99, 100, 101, 102, 108, 109, 110, 116, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 203, 205, 206, 207, 208, 214, 215, 217, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 386, 389, 390, 391, 392, 441, 442, 443, 444, 445, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 564, 565, 566, 567, 568, 569, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 594, 595, 596, 597, 598, 599, 601, 602, 603, 604, 605, 608, 609, 610, 611, 612, 613, 618, 619, 620, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 709, 710, 711, 713, 714, 715, 716, 777, 778, 779, 780, 781, 788, 789, 790, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 830, 831, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879 }

F(-1) timedout fail { 105, 106, 107, 113, 114, 115, 159, 186, 201, 202, 204, 209, 210, 211, 212, 213, 385, 387, 388, 393, 394, 395, 396, 397, 544, 545, 562, 563, 570, 571, 572, 593, 600, 606, 607, 614, 615, 616, 617, 621, 622, 623, 624, 625, 626, 708, 816, 827, 828, 829, 832, 833, 834, 835, 836 }

F(-2) exception fail { 216, 257, 427, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 627 }

Giac

A grade { 1, 2, 3, 7, 8, 9, 10, 11, 12, 13, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 99, 100, 101, 102, 107, 108, 109, 110, 119, 120, 121, 122, 123, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 141, 142, 221, 224, 229, 230, 231, 232, 237, 238, 239, 240, 241, 248, 249, 250, 252, 255, 256, 259, 260, 263, 264, 269, 270, 271, 401, 405, 406, 407, 408, 412, 413, 414, 415, 420, 421, 422, 423, 424, 427, 428, 429, 430, 431, 432, 434, 435, 436, 437, 438, 439, 455, 470, 471, 472, 480, 481, 482, 488, 489, 491, 493, 494, 495, 497, 499, 500, 501, 502, 504, 505, 506, 507, 508, 512, 514, 516, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530 }

B grade { 4, 5, 6, 14, 15, 93, 94, 95, 96, 97, 98, 103, 104, 106, 111, 112, 113, 114, 116, 117, 118, 124, 125, 132, 133, 140, 218, 219, 220, 225, 227, 228, 234, 235, 236, 243, 244, 245, 247, 265, 266, 268, 402, 403, 404, 409, 411, 416, 417, 418, 425, 426, 446, 447, 448, 449, 450, 451, 452, 453, 454, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 473, 474, 475, 476, 477, 478, 479, 483, 484, 485, 486, 487, 490, 492, 496, 498, 503, 509, 510, 511, 513, 515, 517, 518, 519, 520 }

C grade { }

F normal fail { 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 222, 223, 242, 251, 258, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 441, 442, 443, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 777, 778, 779, 780, 781, 788, 789, 790, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879 }

F(-1) timedout fail { 261, 262, 732, 742, 743, 745, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776 }

F(-2) exception fail { 105, 115, 226, 233, 246, 253, 254, 257, 267, 292, 293, 410, 419, 433, 440, 444, 445, 744, 746 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 99, 100, 101, 102, 107, 108, 109, 110, 116, 221, 222, 223, 224, 229, 230, 231, 232, 238, 239, 240, 241, 242, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816 }

C grade { }

F normal fail { }

F(-1) timedout fail { 94, 95, 96, 97, 98, 103, 104, 105, 106, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 225, 226, 227, 228, 233, 234, 235, 236, 237, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 777, 778, 779, 780, 781, 788, 789, 790, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879 }

F(-2) exception fail { }

Sympy

A grade { 4, 5, 6, 449, 450, 451 }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 30, 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 109, 110, 111, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 150, 151, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 169, 170, 171, 174, 175, 176, 177, 178, 181, 182, 183, 184, 189, 190, 191, 195, 196, 197, 198, 199, 200, 203, 204, 205, 206, 207, 208, 212, 213, 214, 215, 216, 217, 219, 220, 221, 222, 223, 227, 228, 229, 230, 243, 244, 246, 247, 248, 249, 250, 253, 254, 255, 256, 261, 262, 263, 267, 268, 269, 270, 271, 273, 274, 277, 278, 279, 280, 281, 282, 283, 288, 289, 290, 291, 292, 293, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 354, 355, 356, 363, 364, 365, 371, 372, 373, 375, 376, 377, 378, 382, 383, 384, 385, 391, 392, 401, 402, 403, 409, 410, 421, 422, 423, 424, 428, 429, 430, 441, 442, 443, 444, 445, 446, 447, 448, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 476, 477, 478, 479, 480, 481, 482, 486, 487, 488, 489, 490, 491, 492, 493, 494, 496, 497, 498, 499, 500, 501, 502, 503, 504, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 546, 547, 548, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 580, 581, 582, 583, 584, 585, 588, 589, 590, 591, 592, 595, 596, 597, 598, 603, 604, 605, 609, 610, 611, 612, 613, 616, 617, 618, 619, 620, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 635, 636, 637, 638, 649, 650, 651, 652, 653, 656, 657, 658, 659, 664, 665, 666, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 687, 688, 689, 690, 692, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 777, 778, 779, 780, 781, 788, 789, 790, 798, 799, 800, 806, 807, 808, 814, 815, 816, 818, 819, 820, 821, 824, 825, 826, 827, 831, 832, 839, 840, 841, 846, 847, 856, 857, 858, 859, 863, 864, 865, 875, 876, 877, 878, 879 }

F(-1) timedout fail { 27, 28, 29, 37, 38, 39, 40, 50, 60, 97, 105, 106, 107, 108, 112, 113, 114, 115, 148, 149, 152, 165, 172, 173, 179, 180, 185, 186, 187, 188, 192, 193, 194, 201, 202, 209, 210, 211, 218, 224, 225, 226, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 245, 251, 252, 257, 258, 259, 260, 264, 265, 266, 272, 275, 276, 284, 285, 286, 287, 294, 307, 320, 347, 351, 352, 353, 357, 358, 359, 360, 361, 362, 366, 367, 368, 369, 370, 374, 379, 380, 381, 386, 387, 388, 389, 390, 393, 394, 395, 396, 397, 398, 399, 400, 404, 405, 406, 407, 408, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 425, 426, 427, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 473, 474, 475, 483, 484, 485, 495, 505, 543, 544, 545, 549, 550, 551, 552, 553, 579, 586, 587, 593, 594, 599, 600, 601, 602, 606, 607, 608, 614, 615, 621, 622, 633, 634, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 654, 655, 660, 661, 662, 663, 667, 668, 694, 695, 696, 717, 718, 719, 726, 727, 728, 729, 730, 731, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 756, 757, 758, 759, 765, 766, 767, 768, 769, 773, 774, 775, 776, 794, 795, 796, 797, 801, 802, 803, 804, 805, 809, 810, 811, 812, 813, 817, 822, 823, 828, 829, 830, 833, 834, 835, 836, 837, 838, 842, 843, 844, 845, 848, 849, 850, 851, 852, 853, 854, 855, 860, 861, 862, 866, 867, 868, 869, 870, 871, 872, 873, 874 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530 }

C grade { }

F normal fail { 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 777, 778, 779, 780, 781, 788, 789, 790, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	90	56	73	95	99	0	110	193	130
N.S.	1	1.06	0.66	0.86	1.12	1.16	0.00	1.29	2.27	1.53
time (sec)	N/A	0.488	0.142	1.046	0.037	0.103	0.000	0.172	0.199	12.658

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	64	60	60	70	88	0	96	155	102
N.S.	1	1.02	0.95	0.95	1.11	1.40	0.00	1.52	2.46	1.62
time (sec)	N/A	0.392	0.099	0.841	0.036	0.089	0.000	0.142	0.177	11.360

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	48	47	47	58	74	0	80	110	75
N.S.	1	1.02	1.00	1.00	1.23	1.57	0.00	1.70	2.34	1.60
time (sec)	N/A	0.373	0.009	0.765	0.037	0.080	0.000	0.132	0.175	10.256

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	30	29	60	37	63	59	47
N.S.	1	1.00	1.00	1.25	1.21	2.50	1.54	2.62	2.46	1.96
time (sec)	N/A	0.281	0.007	0.526	0.032	0.086	2.226	0.131	0.203	10.063

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	24	23	36	41	49	35	20
N.S.	1	1.00	1.00	1.50	1.44	2.25	2.56	3.06	2.19	1.25
time (sec)	N/A	0.147	0.001	0.165	0.032	0.098	0.872	0.137	0.161	9.885

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	26	16	20	17	17	39	15	15
N.S.	1	1.00	1.73	1.07	1.33	1.13	1.13	2.60	1.00	1.00
time (sec)	N/A	0.234	0.023	0.266	0.029	0.073	0.782	0.133	0.170	9.655

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	39	32	29	34	29	0	56	31	50
N.S.	1	1.03	0.84	0.76	0.89	0.76	0.00	1.47	0.82	1.32
time (sec)	N/A	0.281	0.062	0.372	0.034	0.075	0.000	0.142	0.169	10.457

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	55	57	40	46	42	0	72	43	55
N.S.	1	1.02	1.06	0.74	0.85	0.78	0.00	1.33	0.80	1.02
time (sec)	N/A	0.307	0.047	0.701	0.034	0.076	0.000	0.114	0.180	10.117

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	81	73	53	57	53	0	86	59	79
N.S.	1	1.07	0.96	0.70	0.75	0.70	0.00	1.13	0.78	1.04
time (sec)	N/A	0.386	0.099	1.017	0.028	0.104	0.000	0.120	0.159	13.150

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	121	68	111	133	124	0	138	249	170
N.S.	1	0.99	0.56	0.91	1.09	1.02	0.00	1.13	2.04	1.39
time (sec)	N/A	0.728	0.421	1.398	0.039	0.084	0.000	0.149	0.176	14.430

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	95	58	112	145	111	0	122	195	141
N.S.	1	0.99	0.60	1.17	1.51	1.16	0.00	1.27	2.03	1.47
time (sec)	N/A	0.553	0.289	1.251	0.036	0.085	0.000	0.167	0.164	12.867

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	80	74	75	85	96	0	106	157	112
N.S.	1	1.08	1.00	1.01	1.15	1.30	0.00	1.43	2.12	1.51
time (sec)	N/A	0.547	0.194	1.161	0.030	0.136	0.000	0.164	0.169	11.667

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	70	81	83	0	90	115	83
N.S.	1	1.00	1.00	1.30	1.50	1.54	0.00	1.67	2.13	1.54
time (sec)	N/A	0.397	0.122	0.854	0.030	0.087	0.000	0.161	0.171	10.380

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	42	41	76	0	79	71	56
N.S.	1	1.00	1.00	1.24	1.21	2.24	0.00	2.32	2.09	1.65
time (sec)	N/A	0.292	0.104	0.546	0.030	0.103	0.000	0.135	0.174	10.065

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	47	44	52	53	0	79	47	33
N.S.	1	1.00	1.38	1.29	1.53	1.56	0.00	2.32	1.38	0.97
time (sec)	N/A	0.341	0.027	0.369	0.030	0.088	0.000	0.134	0.174	9.784

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	34	31	48	36	0	64	34	57
N.S.	1	1.00	0.76	0.69	1.07	0.80	0.00	1.42	0.76	1.27
time (sec)	N/A	0.351	0.076	0.362	0.028	0.072	0.000	0.138	0.302	10.140

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	63	41	42	61	49	0	80	45	61
N.S.	1	1.11	0.72	0.74	1.07	0.86	0.00	1.40	0.79	1.07
time (sec)	N/A	0.459	0.071	0.679	0.030	0.080	0.000	0.131	0.167	9.803

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	86	53	55	83	63	0	96	61	89
N.S.	1	0.99	0.61	0.63	0.95	0.72	0.00	1.10	0.70	1.02
time (sec)	N/A	0.459	0.098	0.938	0.036	0.080	0.000	0.130	0.168	12.697

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	94	61	64	95	76	0	112	71	105
N.S.	1	0.91	0.59	0.62	0.92	0.74	0.00	1.09	0.69	1.02
time (sec)	N/A	0.597	0.094	1.317	0.037	0.079	0.000	0.142	0.188	13.430

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	122	145	179	124	0	138	249	170
N.S.	1	1.00	1.07	1.27	1.57	1.09	0.00	1.21	2.18	1.49
time (sec)	N/A	0.354	0.529	1.526	0.037	0.083	0.000	0.195	0.176	14.189

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	99	123	156	111	0	122	195	141
N.S.	1	1.00	1.06	1.32	1.68	1.19	0.00	1.31	2.10	1.52
time (sec)	N/A	0.330	0.311	1.309	0.036	0.091	0.000	0.156	0.174	12.866

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	86	94	104	98	0	106	157	112
N.S.	1	1.00	1.19	1.31	1.44	1.36	0.00	1.47	2.18	1.56
time (sec)	N/A	0.283	0.215	1.151	0.034	0.086	0.000	0.184	0.186	11.720

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	69	74	80	91	98	0	100	131	88
N.S.	1	1.05	1.12	1.21	1.38	1.48	0.00	1.52	1.98	1.33
time (sec)	N/A	0.425	0.167	0.870	0.029	0.085	0.000	0.127	0.169	10.050

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	33	55	64	91	0	80	94	57
N.S.	1	1.00	0.69	1.15	1.33	1.90	0.00	1.67	1.96	1.19
time (sec)	N/A	0.262	0.130	0.673	0.028	0.113	0.000	0.148	0.166	9.936

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	81	59	74	65	0	100	65	88
N.S.	1	1.00	1.37	1.00	1.25	1.10	0.00	1.69	1.10	1.49
time (sec)	N/A	0.272	0.189	0.498	0.036	0.093	0.000	0.151	0.160	10.030

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	44	42	71	50	0	80	45	63
N.S.	1	1.00	0.70	0.67	1.13	0.79	0.00	1.27	0.71	1.00
time (sec)	N/A	0.282	0.083	0.656	0.031	0.078	0.000	0.143	0.179	10.120

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	51	53	94	63	0	96	61	89
N.S.	1	1.00	0.60	0.62	1.11	0.74	0.00	1.13	0.72	1.05
time (sec)	N/A	0.314	0.071	0.930	0.037	0.090	0.000	0.179	0.167	13.084

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	63	66	117	76	0	112	71	105
N.S.	1	1.00	0.60	0.63	1.11	0.72	0.00	1.07	0.68	1.00
time (sec)	N/A	0.330	0.093	1.340	0.036	0.084	0.000	0.154	0.167	13.454

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	73	75	143	89	0	128	87	121
N.S.	1	1.00	0.57	0.58	1.11	0.69	0.00	0.99	0.67	0.94
time (sec)	N/A	0.367	0.126	1.697	0.036	0.084	0.000	0.159	0.172	12.474

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	146	171	270	137	0	154	275	199
N.S.	1	1.00	1.07	1.26	1.99	1.01	0.00	1.13	2.02	1.46
time (sec)	N/A	0.414	3.915	1.835	0.037	0.088	0.000	0.191	0.188	13.560

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	125	152	190	124	0	138	249	170
N.S.	1	1.00	1.13	1.37	1.71	1.12	0.00	1.24	2.24	1.53
time (sec)	N/A	0.363	1.645	1.717	0.036	0.085	0.000	0.210	0.161	14.354

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	110	133	175	111	0	122	195	141
N.S.	1	1.00	1.15	1.39	1.82	1.16	0.00	1.27	2.03	1.47
time (sec)	N/A	0.327	1.346	1.322	0.035	0.087	0.000	0.168	0.179	12.990

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	98	88	89	116	110	0	116	185	117
N.S.	1	1.08	0.97	0.98	1.27	1.21	0.00	1.27	2.03	1.29
time (sec)	N/A	0.594	1.319	1.139	0.033	0.091	0.000	0.165	0.169	10.236

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	272	91	110	111	0	129	155	115
N.S.	1	1.00	3.73	1.25	1.51	1.52	0.00	1.77	2.12	1.58
time (sec)	N/A	0.297	3.604	1.006	0.035	0.086	0.000	0.214	0.184	10.480

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	241	82	85	105	0	129	108	117
N.S.	1	1.00	3.30	1.12	1.16	1.44	0.00	1.77	1.48	1.60
time (sec)	N/A	0.305	3.411	0.822	0.034	0.096	0.000	0.204	0.183	10.110

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	91	70	97	80	0	116	76	93
N.S.	1	1.00	1.25	0.96	1.33	1.10	0.00	1.59	1.04	1.27
time (sec)	N/A	0.299	0.380	0.792	0.037	0.089	0.000	0.215	0.295	9.788

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	56	55	104	63	0	96	61	89
N.S.	1	1.00	0.64	0.63	1.20	0.72	0.00	1.10	0.70	1.02
time (sec)	N/A	0.314	0.117	1.004	0.034	0.081	0.000	0.232	0.157	12.986

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	63	66	128	76	0	112	71	105
N.S.	1	1.00	0.62	0.65	1.25	0.75	0.00	1.10	0.70	1.03
time (sec)	N/A	0.335	0.094	1.382	0.036	0.079	0.000	0.196	0.175	13.408

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	73	75	165	89	0	128	87	121
N.S.	1	1.00	0.57	0.59	1.30	0.70	0.00	1.01	0.69	0.95
time (sec)	N/A	0.370	0.135	1.747	0.035	0.090	0.000	0.205	0.170	12.510

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	83	88	187	102	0	144	97	137
N.S.	1	1.00	0.56	0.60	1.27	0.69	0.00	0.98	0.66	0.93
time (sec)	N/A	0.391	0.173	2.369	0.038	0.085	0.000	0.225	0.179	12.603

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	170	189	314	150	0	170	341	228
N.S.	1	1.00	1.09	1.21	2.01	0.96	0.00	1.09	2.19	1.46
time (sec)	N/A	0.433	5.041	2.208	0.063	0.087	0.000	0.240	0.189	13.694

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	102	83	134	205	124	0	114	190	96
N.S.	1	0.99	0.81	1.30	1.99	1.20	0.00	1.11	1.84	0.93
time (sec)	N/A	0.603	0.344	0.527	0.049	0.086	0.000	0.169	0.171	10.094

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	87	72	105	162	112	0	101	148	95
N.S.	1	1.02	0.85	1.24	1.91	1.32	0.00	1.19	1.74	1.12
time (sec)	N/A	0.560	0.211	0.503	0.047	0.083	0.000	0.199	0.194	9.890

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	42	74	119	97	0	84	92	67
N.S.	1	1.00	0.82	1.45	2.33	1.90	0.00	1.65	1.80	1.31
time (sec)	N/A	0.468	0.177	0.401	0.049	0.080	0.000	0.154	0.175	9.862

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	46	75	65	0	54	45	31
N.S.	1	1.00	0.71	1.21	1.97	1.71	0.00	1.42	1.18	0.82
time (sec)	N/A	0.354	0.095	0.313	0.052	0.079	0.000	0.161	0.177	9.921

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	17	17	23	22	0	16	16	16
N.S.	1	1.00	0.77	0.77	1.05	1.00	0.00	0.73	0.73	0.73
time (sec)	N/A	0.207	0.021	0.251	0.057	0.066	0.000	0.161	0.171	9.875

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	58	23	49	37	0	28	22	23
N.S.	1	1.00	2.00	0.79	1.69	1.28	0.00	0.97	0.76	0.79
time (sec)	N/A	0.192	0.198	0.269	0.142	0.075	0.000	0.138	0.185	9.848

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	45	89	28	92	46	0	58	43	66
N.S.	1	1.02	2.02	0.64	2.09	1.05	0.00	1.32	0.98	1.50
time (sec)	N/A	0.367	0.541	0.412	0.175	0.079	0.000	0.143	0.189	10.073

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	75	117	42	133	57	0	73	61	89
N.S.	1	1.01	1.58	0.57	1.80	0.77	0.00	0.99	0.82	1.20
time (sec)	N/A	0.443	0.631	0.452	0.132	0.079	0.000	0.157	0.175	9.883

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	92	143	54	176	70	0	88	72	70
N.S.	1	0.98	1.52	0.57	1.87	0.74	0.00	0.94	0.77	0.74
time (sec)	N/A	0.488	0.678	0.553	0.154	0.087	0.000	0.162	0.170	10.175

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	173	67	217	79	0	101	88	98
N.S.	1	1.00	1.47	0.57	1.84	0.67	0.00	0.86	0.75	0.83
time (sec)	N/A	0.579	0.718	0.665	0.139	0.079	0.000	0.158	0.172	11.286

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	129	74	120	190	162	0	122	215	122
N.S.	1	1.05	0.60	0.98	1.54	1.32	0.00	0.99	1.75	0.99
time (sec)	N/A	0.814	0.603	0.502	0.047	0.083	0.000	0.186	0.202	9.676

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	97	104	92	145	146	0	106	137	92
N.S.	1	1.09	1.17	1.03	1.63	1.64	0.00	1.19	1.54	1.03
time (sec)	N/A	0.679	0.312	0.447	0.068	0.083	0.000	0.162	0.177	9.629

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	70	84	62	98	114	0	77	61	43
N.S.	1	1.06	1.27	0.94	1.48	1.73	0.00	1.17	0.92	0.65
time (sec)	N/A	0.522	0.140	0.346	0.053	0.085	0.000	0.172	0.186	9.555

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	54	45	32	46	49	0	31	30	30
N.S.	1	0.98	0.82	0.58	0.84	0.89	0.00	0.56	0.55	0.55
time (sec)	N/A	0.326	0.053	0.281	0.051	0.067	0.000	0.167	0.182	9.449

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	54	60	32	47	51	0	31	32	30
N.S.	1	0.98	1.09	0.58	0.85	0.93	0.00	0.56	0.58	0.55
time (sec)	N/A	0.312	0.290	0.287	0.039	0.069	0.000	0.152	0.182	9.442

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	61	112	36	72	80	0	50	35	35
N.S.	1	1.07	1.96	0.63	1.26	1.40	0.00	0.88	0.61	0.61
time (sec)	N/A	0.411	0.317	0.293	0.137	0.083	0.000	0.122	0.169	9.603

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	80	97	55	118	90	0	79	80	91
N.S.	1	1.11	1.35	0.76	1.64	1.25	0.00	1.10	1.11	1.26
time (sec)	N/A	0.557	0.604	0.464	0.150	0.096	0.000	0.130	0.172	9.620

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	116	177	66	164	99	0	95	98	113
N.S.	1	1.05	1.61	0.60	1.49	0.90	0.00	0.86	0.89	1.03
time (sec)	N/A	0.687	1.399	0.566	0.138	0.091	0.000	0.165	0.182	9.631

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	136	199	77	207	108	0	108	112	135
N.S.	1	1.10	1.60	0.62	1.67	0.87	0.00	0.87	0.90	1.09
time (sec)	N/A	0.734	1.446	0.735	0.115	0.089	0.000	0.151	0.176	9.713

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	176	351	135	211	206	0	139	228	141
N.S.	1	1.09	2.17	0.83	1.30	1.27	0.00	0.86	1.41	0.87
time (sec)	N/A	1.152	1.866	0.595	0.039	0.084	0.000	0.182	0.167	9.559

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	138	116	105	165	190	0	122	150	111
N.S.	1	1.08	0.91	0.82	1.29	1.48	0.00	0.95	1.17	0.87
time (sec)	N/A	0.967	0.477	0.510	0.037	0.081	0.000	0.196	0.189	9.539

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	115	96	75	119	158	0	94	74	58
N.S.	1	1.10	0.91	0.71	1.13	1.50	0.00	0.90	0.70	0.55
time (sec)	N/A	0.770	0.201	0.368	0.038	0.090	0.000	0.191	0.157	9.667

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	85	57	45	67	75	0	46	45	45
N.S.	1	1.02	0.69	0.54	0.81	0.90	0.00	0.55	0.54	0.54
time (sec)	N/A	0.484	0.079	0.307	0.034	0.068	0.000	0.176	0.157	9.531

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	87	104	32	47	73	0	31	32	30
N.S.	1	1.05	1.25	0.39	0.57	0.88	0.00	0.37	0.39	0.36
time (sec)	N/A	0.449	0.413	0.351	0.036	0.068	0.000	0.193	0.152	9.519

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	87	86	45	67	75	0	46	45	45
N.S.	1	1.05	1.04	0.54	0.81	0.90	0.00	0.55	0.54	0.54
time (sec)	N/A	0.433	0.378	0.352	0.038	0.071	0.000	0.192	0.157	9.613

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	98	162	51	92	116	0	68	50	81
N.S.	1	1.11	1.84	0.58	1.05	1.32	0.00	0.77	0.57	0.92
time (sec)	N/A	0.587	0.382	0.316	0.114	0.074	0.000	0.166	0.160	9.531

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	114	107	66	137	126	0	96	91	113
N.S.	1	1.11	1.04	0.64	1.33	1.22	0.00	0.93	0.88	1.10
time (sec)	N/A	0.788	0.708	0.555	0.112	0.079	0.000	0.158	0.173	9.532

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	161	181	77	184	135	0	113	134	137
N.S.	1	1.10	1.23	0.52	1.25	0.92	0.00	0.77	0.91	0.93
time (sec)	N/A	0.956	1.716	0.580	0.111	0.102	0.000	0.160	0.169	9.793

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	211	403	148	231	250	0	155	241	160
N.S.	1	1.09	2.09	0.77	1.20	1.30	0.00	0.80	1.25	0.83
time (sec)	N/A	1.485	2.368	0.621	0.040	0.087	0.000	0.203	0.173	9.530

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	183	349	118	186	234	0	139	165	130
N.S.	1	1.15	2.19	0.74	1.17	1.47	0.00	0.87	1.04	0.82
time (sec)	N/A	1.258	2.117	0.523	0.038	0.094	0.000	0.178	0.198	9.406

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	151	193	88	139	202	0	110	87	83
N.S.	1	1.11	1.42	0.65	1.02	1.49	0.00	0.81	0.64	0.61
time (sec)	N/A	1.051	1.750	0.420	0.045	0.081	0.000	0.208	0.166	9.332

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	126	69	56	87	99	0	59	58	58
N.S.	1	1.05	0.58	0.47	0.72	0.82	0.00	0.49	0.48	0.48
time (sec)	N/A	0.667	0.144	0.317	0.037	0.076	0.000	0.194	0.168	9.256

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	118	87	57	87	99	0	59	58	58
N.S.	1	1.05	0.78	0.51	0.78	0.88	0.00	0.53	0.52	0.52
time (sec)	N/A	0.632	1.309	0.375	0.038	0.079	0.000	0.170	0.187	9.246

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	120	56	58	87	99	0	59	58	58
N.S.	1	1.07	0.50	0.52	0.78	0.88	0.00	0.53	0.52	0.52
time (sec)	N/A	0.575	1.093	0.376	0.040	0.073	0.000	0.178	0.177	9.471

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	120	112	57	87	99	0	59	58	58
N.S.	1	1.07	1.00	0.51	0.78	0.88	0.00	0.53	0.52	0.52
time (sec)	N/A	0.570	1.544	0.325	0.038	0.072	0.000	0.185	0.178	9.375

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	126	224	64	112	152	0	83	63	102
N.S.	1	1.14	2.02	0.58	1.01	1.37	0.00	0.75	0.57	0.92
time (sec)	N/A	0.763	1.581	0.329	0.112	0.076	0.000	0.152	0.165	9.250

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	150	263	77	158	162	0	112	106	137
N.S.	1	1.19	2.09	0.61	1.25	1.29	0.00	0.89	0.84	1.09
time (sec)	N/A	1.033	2.390	0.539	0.117	0.079	0.000	0.200	0.202	9.429

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	194	289	90	204	171	0	128	147	159
N.S.	1	1.10	1.64	0.51	1.16	0.97	0.00	0.73	0.84	0.90
time (sec)	N/A	1.225	2.697	0.591	0.118	0.099	0.000	0.181	0.154	9.456

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	232	401	131	206	278	0	155	178	149
N.S.	1	1.16	2.00	0.66	1.03	1.39	0.00	0.78	0.89	0.74
time (sec)	N/A	1.613	2.578	0.575	0.037	0.093	0.000	0.245	0.177	9.310

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	205	219	101	159	246	0	126	100	99
N.S.	1	1.16	1.24	0.57	0.90	1.39	0.00	0.71	0.56	0.56
time (sec)	N/A	1.368	2.447	0.395	0.036	0.088	0.000	0.213	0.186	9.251

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	167	97	69	107	123	0	72	71	127
N.S.	1	1.05	0.61	0.43	0.67	0.77	0.00	0.45	0.45	0.80
time (sec)	N/A	0.855	0.130	0.339	0.039	0.072	0.000	0.225	0.163	9.144

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	167	97	57	87	123	0	59	58	58
N.S.	1	1.05	0.61	0.36	0.55	0.77	0.00	0.37	0.36	0.36
time (sec)	N/A	0.851	1.329	0.404	0.039	0.110	0.000	0.233	0.181	9.388

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	147	110	45	67	123	0	46	45	45
N.S.	1	1.06	0.79	0.32	0.48	0.88	0.00	0.33	0.32	0.32
time (sec)	N/A	0.779	3.556	0.393	0.039	0.071	0.000	0.195	0.164	9.320

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	153	125	57	87	123	0	59	58	58
N.S.	1	1.07	0.87	0.40	0.61	0.86	0.00	0.41	0.41	0.41
time (sec)	N/A	0.756	4.050	0.377	0.039	0.070	0.000	0.232	0.210	9.393

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	153	138	71	107	123	0	72	71	127
N.S.	1	1.07	0.97	0.50	0.75	0.86	0.00	0.50	0.50	0.89
time (sec)	N/A	0.724	3.200	0.337	0.036	0.072	0.000	0.187	0.167	9.429

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	172	280	77	132	188	0	100	76	125
N.S.	1	1.19	1.94	0.53	0.92	1.31	0.00	0.69	0.53	0.87
time (sec)	N/A	0.989	2.953	0.365	0.109	0.085	0.000	0.182	0.175	9.311

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	191	319	90	178	198	0	129	119	159
N.S.	1	1.20	2.01	0.57	1.12	1.25	0.00	0.81	0.75	1.00
time (sec)	N/A	1.311	4.375	0.592	0.119	0.086	0.000	0.213	0.170	9.504

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	240	345	98	224	207	0	145	160	181
N.S.	1	1.12	1.60	0.46	1.04	0.96	0.00	0.67	0.74	0.84
time (sec)	N/A	1.611	6.545	0.642	0.117	0.090	0.000	0.235	0.185	9.460

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	138	58	72	0	82	0	120	23	331
N.S.	1	1.13	0.48	0.59	0.00	0.67	0.00	0.98	0.19	2.71
time (sec)	N/A	0.711	0.101	0.925	0.000	0.080	0.000	0.300	0.209	13.916

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	97	48	60	0	72	0	101	23	115
N.S.	1	1.13	0.56	0.70	0.00	0.84	0.00	1.17	0.27	1.34
time (sec)	N/A	0.540	0.079	0.829	0.000	0.079	0.000	0.307	0.183	13.335

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	36	46	0	60	0	82	23	108
N.S.	1	1.00	0.64	0.82	0.00	1.07	0.00	1.46	0.41	1.93
time (sec)	N/A	0.348	0.069	0.780	0.000	0.076	0.000	0.313	0.187	1.273

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	29	33	0	41	0	62	21	41
N.S.	1	1.00	1.12	1.27	0.00	1.58	0.00	2.38	0.81	1.58
time (sec)	N/A	0.220	0.050	0.741	0.000	0.079	0.000	0.338	0.190	0.176

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	60	88	146	133	0	130	14	0
N.S.	1	1.00	1.62	2.38	3.95	3.59	0.00	3.51	0.38	0.00
time (sec)	N/A	0.212	0.090	0.875	0.163	0.087	0.000	0.311	0.190	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	79	131	791	242	0	282	21	0
N.S.	1	1.00	1.27	2.11	12.76	3.90	0.00	4.55	0.34	0.00
time (sec)	N/A	0.330	0.150	2.573	0.208	0.088	0.000	0.259	0.203	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	101	47	136	1059	270	0	378	23	0
N.S.	1	0.99	0.46	1.33	10.38	2.65	0.00	3.71	0.23	0.00
time (sec)	N/A	0.490	0.074	2.546	0.221	0.093	0.000	0.337	0.193	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	142	47	147	1921	290	0	475	23	0
N.S.	1	1.03	0.34	1.07	13.92	2.10	0.00	3.44	0.17	0.00
time (sec)	N/A	0.668	0.067	2.713	0.347	0.099	0.000	0.366	0.194	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	183	47	156	6638	310	0	571	23	0
N.S.	1	1.05	0.27	0.90	38.15	1.78	0.00	3.28	0.13	0.00
time (sec)	N/A	0.859	0.063	2.770	0.464	0.098	0.000	0.352	0.184	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	182	70	83	0	98	0	180	45	429
N.S.	1	1.12	0.43	0.51	0.00	0.60	0.00	1.11	0.28	2.65
time (sec)	N/A	0.905	0.336	1.385	0.000	0.081	0.000	0.404	0.193	15.280

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	131	60	73	0	87	0	151	45	346
N.S.	1	1.13	0.52	0.63	0.00	0.75	0.00	1.30	0.39	2.98
time (sec)	N/A	0.679	0.121	1.248	0.000	0.084	0.000	0.429	0.217	13.280

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	91	48	61	0	74	0	121	45	116
N.S.	1	1.06	0.56	0.71	0.00	0.86	0.00	1.41	0.52	1.35
time (sec)	N/A	0.474	0.092	1.301	0.000	0.102	0.000	0.385	0.173	12.993

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	38	47	0	61	0	93	43	111
N.S.	1	1.00	0.64	0.80	0.00	1.03	0.00	1.58	0.73	1.88
time (sec)	N/A	0.336	0.063	1.140	0.000	0.078	0.000	0.358	0.189	1.171

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	75	124	997	235	0	195	34	0
N.S.	1	1.00	1.14	1.88	15.11	3.56	0.00	2.95	0.52	0.00
time (sec)	N/A	0.302	0.168	2.276	0.213	0.090	0.000	0.393	0.175	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	94	82	132	803	248	0	278	47	0
N.S.	1	1.45	1.26	2.03	12.35	3.82	0.00	4.28	0.72	0.00
time (sec)	N/A	0.471	0.150	2.980	0.238	0.091	0.000	0.388	0.214	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	104	108	137	0	278	0	0	51	0
N.S.	1	0.98	1.02	1.29	0.00	2.62	0.00	0.00	0.48	0.00
time (sec)	N/A	0.501	0.248	3.230	0.000	0.106	0.000	0.000	0.204	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	145	120	148	0	300	0	535	51	0
N.S.	1	1.01	0.83	1.03	0.00	2.08	0.00	3.72	0.35	0.00
time (sec)	N/A	0.668	0.360	3.240	0.000	0.097	0.000	0.659	0.242	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	226	80	95	0	121	0	209	69	542
N.S.	1	1.11	0.39	0.47	0.00	0.60	0.00	1.03	0.34	2.67
time (sec)	N/A	1.221	0.152	47.621	0.000	0.081	0.000	0.531	0.183	20.156

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	165	70	85	0	108	0	180	69	456
N.S.	1	1.13	0.48	0.58	0.00	0.74	0.00	1.23	0.47	3.12
time (sec)	N/A	0.846	0.313	17.394	0.000	0.090	0.000	0.622	0.213	16.734

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	125	60	75	0	95	0	151	69	349
N.S.	1	1.08	0.52	0.65	0.00	0.82	0.00	1.30	0.59	3.01
time (sec)	N/A	0.615	0.127	6.151	0.000	0.086	0.000	0.491	0.186	13.221

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	93	50	63	0	82	0	122	67	146
N.S.	1	1.04	0.56	0.71	0.00	0.92	0.00	1.37	0.75	1.64
time (sec)	N/A	0.455	0.068	2.600	0.000	0.079	0.000	0.560	0.214	13.149

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	102	360	135	1395	310	0	225	58	0
N.S.	1	1.04	3.67	1.38	14.23	3.16	0.00	2.30	0.59	0.00
time (sec)	N/A	0.542	8.928	2.796	0.234	0.097	0.000	0.481	0.201	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	97	82	137	1383	276	0	368	77	0
N.S.	1	1.03	0.87	1.46	14.71	2.94	0.00	3.91	0.82	0.00
time (sec)	N/A	0.513	0.227	10.618	0.241	0.097	0.000	0.673	0.222	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	108	150	139	0	294	0	364	83	0
N.S.	1	1.02	1.42	1.31	0.00	2.77	0.00	3.43	0.78	0.00
time (sec)	N/A	0.547	0.357	33.093	0.000	0.099	0.000	0.518	0.245	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	148	151	150	0	320	0	539	83	0
N.S.	1	1.03	1.05	1.04	0.00	2.22	0.00	3.74	0.58	0.00
time (sec)	N/A	0.761	0.535	125.655	0.000	0.099	0.000	0.735	0.223	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	189	92	160	0	346	0	0	83	0
N.S.	1	1.04	0.51	0.88	0.00	1.90	0.00	0.00	0.46	0.00
time (sec)	N/A	0.969	0.615	1.379	0.000	0.098	0.000	0.000	0.242	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	30	38	0	44	0	57	23	36
N.S.	1	1.00	1.11	1.41	0.00	1.63	0.00	2.11	0.85	1.33
time (sec)	N/A	0.225	0.120	0.951	0.000	0.074	0.000	0.194	0.177	9.442

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	85	81	146	182	0	65	16	0
N.S.	1	1.00	2.24	2.13	3.84	4.79	0.00	1.71	0.42	0.00
time (sec)	N/A	0.210	0.391	1.035	0.240	0.108	0.000	0.248	0.166	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	77	95	791	294	0	134	23	0
N.S.	1	1.00	1.18	1.46	12.17	4.52	0.00	2.06	0.35	0.00
time (sec)	N/A	0.332	0.204	2.616	0.293	0.113	0.000	0.197	0.217	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	154	106	131	0	347	0	150	36	0
N.S.	1	1.10	0.76	0.94	0.00	2.48	0.00	1.07	0.26	0.00
time (sec)	N/A	0.832	0.148	1.026	0.000	0.108	0.000	0.406	0.172	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	112	86	117	0	316	0	113	36	0
N.S.	1	1.08	0.83	1.12	0.00	3.04	0.00	1.09	0.35	0.00
time (sec)	N/A	0.533	0.104	0.988	0.000	0.104	0.000	0.395	0.199	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	83	95	0	262	0	108	36	0
N.S.	1	1.00	1.14	1.30	0.00	3.59	0.00	1.48	0.49	0.00
time (sec)	N/A	0.350	0.055	0.858	0.000	0.100	0.000	0.377	0.183	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	64	76	0	158	0	59	34	0
N.S.	1	1.00	1.39	1.65	0.00	3.43	0.00	1.28	0.74	0.00
time (sec)	N/A	0.232	0.037	0.868	0.000	0.129	0.000	0.208	0.196	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	130	131	698	294	0	69	28	0
N.S.	1	1.00	1.53	1.54	8.21	3.46	0.00	0.81	0.33	0.00
time (sec)	N/A	0.398	0.499	0.827	0.389	0.105	0.000	0.364	0.174	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	109	105	199	0	417	0	326	34	0
N.S.	1	1.01	0.97	1.84	0.00	3.86	0.00	3.02	0.31	0.00
time (sec)	N/A	0.534	0.093	2.476	0.000	0.113	0.000	0.396	0.197	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	159	118	208	0	446	0	423	36	0
N.S.	1	1.08	0.80	1.41	0.00	3.03	0.00	2.88	0.24	0.00
time (sec)	N/A	0.857	0.165	2.616	0.000	0.118	0.000	0.419	0.179	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	199	124	165	0	414	0	210	46	0
N.S.	1	1.09	0.68	0.90	0.00	2.26	0.00	1.15	0.25	0.00
time (sec)	N/A	1.073	0.272	1.670	0.000	0.118	0.000	0.463	0.187	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	154	114	149	0	387	0	179	46	0
N.S.	1	1.06	0.79	1.03	0.00	2.67	0.00	1.23	0.32	0.00
time (sec)	N/A	0.805	0.208	1.685	0.000	0.155	0.000	0.566	0.186	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	109	104	120	0	336	0	157	46	0
N.S.	1	1.04	0.99	1.14	0.00	3.20	0.00	1.50	0.44	0.00
time (sec)	N/A	0.546	0.210	1.465	0.000	0.117	0.000	0.472	0.205	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	94	115	0	329	0	108	46	0
N.S.	1	1.00	1.22	1.49	0.00	4.27	0.00	1.40	0.60	0.00
time (sec)	N/A	0.364	0.136	1.340	0.000	0.104	0.000	0.418	0.185	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	93	114	0	327	0	108	44	0
N.S.	1	1.00	1.21	1.48	0.00	4.25	0.00	1.40	0.57	0.00
time (sec)	N/A	0.353	0.077	1.290	0.000	0.108	0.000	0.324	0.180	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	114	120	181	169	0	491	0	47	38	0
N.S.	1	1.05	1.59	1.48	0.00	4.31	0.00	0.41	0.33	0.00
time (sec)	N/A	0.585	1.681	1.368	0.000	0.140	0.000	0.310	0.197	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	144	152	129	240	0	518	0	387	44	0
N.S.	1	1.06	0.90	1.67	0.00	3.60	0.00	2.69	0.31	0.00
time (sec)	N/A	0.825	0.590	3.127	0.000	0.150	0.000	0.511	0.173	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	185	200	177	250	0	536	0	473	46	0
N.S.	1	1.08	0.96	1.35	0.00	2.90	0.00	2.56	0.25	0.00
time (sec)	N/A	1.192	1.181	3.605	0.000	0.139	0.000	0.563	0.192	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	199	135	179	0	455	0	219	56	0
N.S.	1	1.09	0.74	0.98	0.00	2.49	0.00	1.20	0.31	0.00
time (sec)	N/A	1.115	0.918	1.594	0.000	0.112	0.000	0.481	0.193	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	155	125	142	0	404	0	188	56	0
N.S.	1	1.07	0.86	0.98	0.00	2.79	0.00	1.30	0.39	0.00
time (sec)	N/A	0.804	0.505	1.441	0.000	0.135	0.000	0.425	0.220	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	113	116	162	0	403	0	139	56	0
N.S.	1	1.06	1.08	1.51	0.00	3.77	0.00	1.30	0.52	0.00
time (sec)	N/A	0.558	0.454	1.405	0.000	0.113	0.000	0.545	0.212	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	112	115	192	0	399	0	139	56	0
N.S.	1	1.05	1.07	1.79	0.00	3.73	0.00	1.30	0.52	0.00
time (sec)	N/A	0.496	0.450	1.421	0.000	0.108	0.000	0.597	0.208	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	112	52	192	0	403	0	139	54	0
N.S.	1	1.05	0.49	1.79	0.00	3.77	0.00	1.30	0.50	0.00
time (sec)	N/A	0.479	0.047	1.473	0.000	0.112	0.000	0.438	0.203	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	144	156	211	212	0	585	0	78	48	0
N.S.	1	1.08	1.47	1.47	0.00	4.06	0.00	0.54	0.33	0.00
time (sec)	N/A	0.753	4.158	1.616	0.000	0.268	0.000	0.422	0.183	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	174	190	169	280	0	606	0	424	54	0
N.S.	1	1.09	0.97	1.61	0.00	3.48	0.00	2.44	0.31	0.00
time (sec)	N/A	1.114	1.305	3.463	0.000	0.181	0.000	0.639	0.207	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	62	0	0	161	0	67	37	0
N.S.	1	1.00	1.29	0.00	0.00	3.35	0.00	1.40	0.77	0.00
time (sec)	N/A	0.227	0.132	0.000	0.000	0.109	0.000	0.208	0.205	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	C	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	127	0	698	301	0	69	31	0
N.S.	1	1.00	1.46	0.00	8.02	3.46	0.00	0.79	0.36	0.00
time (sec)	N/A	0.405	0.490	0.000	0.382	0.096	0.000	0.264	0.181	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	383	421	105	0	0	0	0	0	25	0
N.S.	1	1.10	0.27	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.835	0.184	0.000	0.000	0.000	0.000	0.000	0.190	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	353	381	85	0	0	0	0	0	25	0
N.S.	1	1.08	0.24	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.578	0.093	0.000	0.000	0.000	0.000	0.000	0.216	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	326	351	66	0	0	0	0	0	23	0
N.S.	1	1.08	0.20	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.429	0.037	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	691	0	0	0	0	0	16	0
N.S.	1	1.00	8.97	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.344	3.531	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	2700	0	0	0	0	0	23	0
N.S.	1	1.00	35.06	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.418	14.499	0.000	0.000	0.000	0.000	0.000	0.224	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	413	458	96	0	0	0	0	0	47	0
N.S.	1	1.11	0.23	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.825	0.225	0.000	0.000	0.000	0.000	0.000	0.201	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	383	417	106	0	0	0	0	0	47	0
N.S.	1	1.09	0.28	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.602	0.310	0.000	0.000	0.000	0.000	0.000	0.221	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	356	387	66	0	0	0	0	0	45	0
N.S.	1	1.09	0.19	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.447	0.053	0.000	0.000	0.000	0.000	0.000	0.209	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	86	2694	0	0	0	0	0	36	0
N.S.	1	1.10	34.54	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.343	14.393	0.000	0.000	0.000	0.000	0.000	0.209	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	86	2700	0	0	0	0	0	49	0
N.S.	1	1.10	34.62	0.00	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	0.415	14.523	0.000	0.000	0.000	0.000	0.000	0.216	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	371	390	155	0	0	0	0	0	25	0
N.S.	1	1.05	0.42	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.092	0.262	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	336	346	95	0	0	0	0	0	25	0
N.S.	1	1.03	0.28	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.756	0.147	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	85	0	0	0	0	0	25	0
N.S.	1	1.00	0.28	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.564	0.077	0.000	0.000	0.000	0.000	0.000	0.189	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	65	0	0	0	0	0	23	0
N.S.	1	1.00	0.24	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.431	0.046	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	718	0	0	0	0	0	16	0
N.S.	1	1.00	9.57	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.338	3.094	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	240	0	0	0	0	0	23	0
N.S.	1	1.00	3.20	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.404	1.275	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-1)	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	766	781	111	0	0	0	0	0	45	0
N.S.	1	1.02	0.14	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.422	0.378	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	731	734	98	0	0	0	0	0	45	0
N.S.	1	1.00	0.13	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.991	0.185	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	731	736	90	0	0	0	0	0	45	0
N.S.	1	1.01	0.12	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.834	0.223	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	744	741	68	0	0	0	0	0	43	0
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.680	0.047	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	100	3007	0	0	0	0	0	36	0
N.S.	1	1.22	36.67	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.340	14.836	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	100	3011	0	0	0	0	0	43	0
N.S.	1	1.22	36.72	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.443	14.821	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	153	115	384	0	188	0	0	39	0
N.S.	1	1.01	0.76	2.54	0.00	1.25	0.00	0.00	0.26	0.00
time (sec)	N/A	0.650	0.268	3.989	0.000	0.100	0.000	0.000	0.185	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	125	83	368	0	167	0	0	37	0
N.S.	1	1.02	0.67	2.99	0.00	1.36	0.00	0.00	0.30	0.00
time (sec)	N/A	0.535	0.208	2.047	0.000	0.091	0.000	0.000	0.175	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	98	68	148	0	124	0	0	28	0
N.S.	1	1.01	0.70	1.53	0.00	1.28	0.00	0.00	0.29	0.00
time (sec)	N/A	0.516	0.151	1.312	0.000	0.089	0.000	0.000	0.174	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	49	150	0	107	0	0	30	0
N.S.	1	1.00	0.65	2.00	0.00	1.43	0.00	0.00	0.40	0.00
time (sec)	N/A	0.381	0.115	1.337	0.000	0.089	0.000	0.000	0.165	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	102	73	225	0	125	0	0	39	0
N.S.	1	1.01	0.72	2.23	0.00	1.24	0.00	0.00	0.39	0.00
time (sec)	N/A	0.491	0.161	3.584	0.000	0.099	0.000	0.000	0.177	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	129	93	219	0	145	0	0	39	0
N.S.	1	1.02	0.73	1.72	0.00	1.14	0.00	0.00	0.31	0.00
time (sec)	N/A	0.530	0.206	6.123	0.000	0.092	0.000	0.000	0.167	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	157	103	270	0	156	0	0	39	0
N.S.	1	1.04	0.68	1.79	0.00	1.03	0.00	0.00	0.26	0.00
time (sec)	N/A	0.653	0.261	9.613	0.000	0.096	0.000	0.000	0.167	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	287	439	0	215	0	0	61	0
N.S.	1	1.00	1.53	2.35	0.00	1.15	0.00	0.00	0.33	0.00
time (sec)	N/A	1.066	1.961	7.721	0.000	0.095	0.000	0.000	0.178	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	159	269	386	0	202	0	0	59	0
N.S.	1	0.99	1.67	2.40	0.00	1.25	0.00	0.00	0.37	0.00
time (sec)	N/A	0.890	1.629	3.542	0.000	0.141	0.000	0.000	0.182	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	264	371	0	179	0	0	50	0
N.S.	1	1.00	2.02	2.83	0.00	1.37	0.00	0.00	0.38	0.00
time (sec)	N/A	0.762	1.405	2.238	0.000	0.097	0.000	0.000	0.199	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	48	185	0	77	0	0	50	0
N.S.	1	1.00	0.75	2.89	0.00	1.20	0.00	0.00	0.78	0.00
time (sec)	N/A	0.448	0.684	2.265	0.000	0.089	0.000	0.000	0.171	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	156	228	0	134	0	0	52	0
N.S.	1	1.00	1.46	2.13	0.00	1.25	0.00	0.00	0.49	0.00
time (sec)	N/A	0.629	1.361	2.163	0.000	0.087	0.000	0.000	0.188	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	136	250	0	157	0	0	61	0
N.S.	1	1.00	1.01	1.85	0.00	1.16	0.00	0.00	0.45	0.00
time (sec)	N/A	0.772	1.646	6.296	0.000	0.093	0.000	0.000	0.188	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	163	149	272	0	170	0	0	61	0
N.S.	1	1.01	0.93	1.69	0.00	1.06	0.00	0.00	0.38	0.00
time (sec)	N/A	0.884	1.896	10.057	0.000	0.091	0.000	0.000	0.192	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	287	439	0	215	0	0	79	0
N.S.	1	1.00	1.53	2.35	0.00	1.15	0.00	0.00	0.42	0.00
time (sec)	N/A	0.450	2.879	6.077	0.000	0.093	0.000	0.000	0.191	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	267	386	0	200	0	0	70	0
N.S.	1	1.00	1.70	2.46	0.00	1.27	0.00	0.00	0.45	0.00
time (sec)	N/A	0.393	2.620	3.790	0.000	0.093	0.000	0.000	0.209	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	187	371	0	179	0	0	70	0
N.S.	1	1.00	1.43	2.83	0.00	1.37	0.00	0.00	0.53	0.00
time (sec)	N/A	0.375	2.307	3.234	0.000	0.094	0.000	0.000	0.184	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	169	172	0	148	0	0	70	0
N.S.	1	1.00	1.29	1.31	0.00	1.13	0.00	0.00	0.53	0.00
time (sec)	N/A	0.376	2.143	3.447	0.000	0.094	0.000	0.000	0.199	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	171	250	0	156	0	0	72	0
N.S.	1	1.00	1.31	1.91	0.00	1.19	0.00	0.00	0.55	0.00
time (sec)	N/A	0.369	1.887	4.017	0.000	0.088	0.000	0.000	0.183	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	146	272	0	170	0	0	81	0
N.S.	1	1.00	0.91	1.69	0.00	1.06	0.00	0.00	0.50	0.00
time (sec)	N/A	0.402	2.287	9.759	0.000	0.093	0.000	0.000	0.210	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	156	260	0	183	0	0	81	0
N.S.	1	1.00	0.83	1.39	0.00	0.98	0.00	0.00	0.43	0.00
time (sec)	N/A	0.432	2.688	15.688	0.000	0.114	0.000	0.000	0.192	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	289	492	0	228	0	0	99	0
N.S.	1	1.00	1.36	2.31	0.00	1.07	0.00	0.00	0.46	0.00
time (sec)	N/A	0.499	5.008	8.237	0.000	0.098	0.000	0.000	0.219	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	279	439	0	215	0	0	90	0
N.S.	1	1.00	1.49	2.35	0.00	1.15	0.00	0.00	0.48	0.00
time (sec)	N/A	0.446	4.168	6.104	0.000	0.106	0.000	0.000	0.201	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	286	386	0	202	0	0	90	0
N.S.	1	1.00	1.78	2.40	0.00	1.25	0.00	0.00	0.56	0.00
time (sec)	N/A	0.423	4.495	4.954	0.000	0.097	0.000	0.000	0.217	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	70	292	0	121	0	0	90	0
N.S.	1	1.00	0.59	2.47	0.00	1.03	0.00	0.00	0.76	0.00
time (sec)	N/A	0.398	2.193	4.161	0.000	0.096	0.000	0.000	0.193	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	184	194	0	162	0	0	90	0
N.S.	1	1.00	1.16	1.22	0.00	1.02	0.00	0.00	0.57	0.00
time (sec)	N/A	0.400	3.217	5.972	0.000	0.099	0.000	0.000	0.211	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	180	272	0	170	0	0	92	0
N.S.	1	1.00	1.12	1.69	0.00	1.06	0.00	0.00	0.57	0.00
time (sec)	N/A	0.412	3.144	6.996	0.000	0.102	0.000	0.000	0.188	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	156	260	0	183	0	0	101	0
N.S.	1	1.00	0.83	1.39	0.00	0.98	0.00	0.00	0.54	0.00
time (sec)	N/A	0.458	3.356	15.516	0.000	0.097	0.000	0.000	0.201	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	306	273	0	196	0	0	101	0
N.S.	1	1.00	1.44	1.28	0.00	0.92	0.00	0.00	0.47	0.00
time (sec)	N/A	0.507	3.981	22.154	0.000	0.100	0.000	0.000	0.196	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	168	291	413	0	248	0	0	32	0
N.S.	1	1.02	1.77	2.52	0.00	1.51	0.00	0.00	0.20	0.00
time (sec)	N/A	0.735	3.063	3.758	0.000	0.096	0.000	0.000	0.185	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	140	262	253	0	196	0	0	32	0
N.S.	1	1.03	1.93	1.86	0.00	1.44	0.00	0.00	0.24	0.00
time (sec)	N/A	0.695	2.056	2.676	0.000	0.105	0.000	0.000	0.174	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	116	201	200	0	184	0	0	30	0
N.S.	1	1.05	1.83	1.82	0.00	1.67	0.00	0.00	0.27	0.00
time (sec)	N/A	0.586	1.278	0.527	0.000	0.098	0.000	0.000	0.174	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	115	202	198	0	184	0	0	24	0
N.S.	1	1.05	1.84	1.80	0.00	1.67	0.00	0.00	0.22	0.00
time (sec)	N/A	0.564	1.017	1.309	0.000	0.088	0.000	0.000	0.184	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	116	317	199	0	186	0	0	31	0
N.S.	1	1.04	2.83	1.78	0.00	1.66	0.00	0.00	0.28	0.00
time (sec)	N/A	0.566	1.663	1.573	0.000	0.093	0.000	0.000	0.164	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	144	318	215	0	207	0	0	33	0
N.S.	1	1.03	2.27	1.54	0.00	1.48	0.00	0.00	0.24	0.00
time (sec)	N/A	0.721	3.436	2.267	0.000	0.090	0.000	0.000	0.186	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	172	347	229	0	217	0	0	33	0
N.S.	1	1.02	2.07	1.36	0.00	1.29	0.00	0.00	0.20	0.00
time (sec)	N/A	0.771	2.506	3.391	0.000	0.101	0.000	0.000	0.165	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	211	287	413	0	328	0	0	42	0
N.S.	1	1.04	1.42	2.04	0.00	1.62	0.00	0.00	0.21	0.00
time (sec)	N/A	1.042	3.671	8.135	0.000	0.095	0.000	0.000	0.187	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	182	252	405	0	278	0	0	42	0
N.S.	1	1.03	1.43	2.30	0.00	1.58	0.00	0.00	0.24	0.00
time (sec)	N/A	0.967	2.049	7.478	0.000	0.089	0.000	0.000	0.170	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	159	242	257	0	277	0	0	42	0
N.S.	1	1.07	1.62	1.72	0.00	1.86	0.00	0.00	0.28	0.00
time (sec)	N/A	0.821	1.979	6.037	0.000	0.101	0.000	0.000	0.182	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	98	188	0	150	0	0	40	0
N.S.	1	1.00	1.27	2.44	0.00	1.95	0.00	0.00	0.52	0.00
time (sec)	N/A	0.384	1.333	2.157	0.000	0.087	0.000	0.000	0.165	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	159	239	257	0	277	0	0	34	0
N.S.	1	1.07	1.60	1.72	0.00	1.86	0.00	0.00	0.23	0.00
time (sec)	N/A	0.823	2.123	1.650	0.000	0.085	0.000	0.000	0.163	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	158	260	257	0	277	0	0	41	0
N.S.	1	1.04	1.71	1.69	0.00	1.82	0.00	0.00	0.27	0.00
time (sec)	N/A	0.810	1.772	2.586	0.000	0.090	0.000	0.000	0.179	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	187	257	270	0	287	0	0	43	0
N.S.	1	1.05	1.44	1.52	0.00	1.61	0.00	0.00	0.24	0.00
time (sec)	N/A	0.977	2.367	2.979	0.000	0.125	0.000	0.000	0.166	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	214	271	283	0	297	0	0	43	0
N.S.	1	1.07	1.36	1.42	0.00	1.48	0.00	0.00	0.22	0.00
time (sec)	N/A	1.007	2.481	4.493	0.000	0.105	0.000	0.000	0.181	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	262	378	453	0	404	0	0	52	0
N.S.	1	1.06	1.53	1.83	0.00	1.64	0.00	0.00	0.21	0.00
time (sec)	N/A	1.344	4.507	21.631	0.000	0.104	0.000	0.000	0.165	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	234	371	555	0	354	0	0	52	0
N.S.	1	1.06	1.68	2.51	0.00	1.60	0.00	0.00	0.24	0.00
time (sec)	N/A	1.304	2.719	21.612	0.000	0.100	0.000	0.000	0.198	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-1)	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	210	274	268	0	353	0	0	52	0
N.S.	1	1.08	1.41	1.37	0.00	1.81	0.00	0.00	0.27	0.00
time (sec)	N/A	1.139	4.341	19.383	0.000	0.091	0.000	0.000	0.169	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-1)	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	210	371	270	0	353	0	0	52	0
N.S.	1	1.08	1.90	1.38	0.00	1.81	0.00	0.00	0.27	0.00
time (sec)	N/A	1.104	2.499	21.550	0.000	0.092	0.000	0.000	0.181	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-1)	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	210	371	270	0	353	0	0	50	0
N.S.	1	1.08	1.90	1.38	0.00	1.81	0.00	0.00	0.26	0.00
time (sec)	N/A	1.111	2.495	3.440	0.000	0.090	0.000	0.000	0.163	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	210	272	270	0	353	0	0	44	0
N.S.	1	1.08	1.39	1.38	0.00	1.81	0.00	0.00	0.23	0.00
time (sec)	N/A	1.107	4.634	3.217	0.000	0.090	0.000	0.000	0.160	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	210	386	270	0	353	0	0	51	0
N.S.	1	1.08	1.98	1.38	0.00	1.81	0.00	0.00	0.26	0.00
time (sec)	N/A	1.109	2.776	3.488	0.000	0.103	0.000	0.000	0.187	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	238	285	283	0	363	0	0	53	0
N.S.	1	1.08	1.29	1.28	0.00	1.64	0.00	0.00	0.24	0.00
time (sec)	N/A	1.307	2.974	4.630	0.000	0.099	0.000	0.000	0.169	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	266	297	296	0	373	0	0	53	0
N.S.	1	1.08	1.20	1.20	0.00	1.51	0.00	0.00	0.21	0.00
time (sec)	N/A	1.347	3.307	5.049	0.000	0.114	0.000	0.000	0.185	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	115	108	177	1264	353	0	335	30	0
N.S.	1	0.99	0.93	1.53	10.90	3.04	0.00	2.89	0.26	0.00
time (sec)	N/A	0.529	0.261	2.273	0.240	0.104	0.000	0.600	0.186	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	75	165	662	299	0	256	28	0
N.S.	1	1.00	1.04	2.29	9.19	4.15	0.00	3.56	0.39	0.00
time (sec)	N/A	0.373	0.145	2.237	0.218	0.099	0.000	0.549	0.198	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	54	122	241	186	0	120	22	0
N.S.	1	1.00	1.46	3.30	6.51	5.03	0.00	3.24	0.59	0.00
time (sec)	N/A	0.244	0.093	2.221	0.221	0.094	0.000	0.439	0.167	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	39	41	20	49	0	37	30	53
N.S.	1	1.00	1.08	1.14	0.56	1.36	0.00	1.03	0.83	1.47
time (sec)	N/A	0.240	0.066	1.164	0.199	0.075	0.000	0.311	0.176	10.603

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	49	54	113	66	0	0	30	69
N.S.	1	1.00	0.64	0.70	1.47	0.86	0.00	0.00	0.39	0.90
time (sec)	N/A	0.389	0.107	1.150	0.203	0.082	0.000	0.000	0.165	10.539

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	120	61	68	203	78	0	0	30	82
N.S.	1	1.04	0.53	0.59	1.77	0.68	0.00	0.00	0.26	0.71
time (sec)	N/A	0.530	0.131	1.188	0.207	0.087	0.000	0.000	0.166	10.687

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	163	71	72	293	88	0	99	30	93
N.S.	1	1.07	0.46	0.47	1.92	0.58	0.00	0.65	0.20	0.61
time (sec)	N/A	0.709	0.179	1.173	0.207	0.079	0.000	0.409	0.176	11.099

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	161	136	182	2361	397	0	472	59	0
N.S.	1	1.01	0.85	1.14	14.76	2.48	0.00	2.95	0.37	0.00
time (sec)	N/A	0.734	0.353	3.052	0.318	0.103	0.000	1.976	0.232	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	118	110	172	2244	367	0	0	57	0
N.S.	1	0.98	0.92	1.43	18.70	3.06	0.00	0.00	0.48	0.00
time (sec)	N/A	0.555	0.322	3.055	0.340	0.110	0.000	0.000	0.223	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	158	1143	307	0	258	49	0
N.S.	1	1.00	1.00	2.11	15.24	4.09	0.00	3.44	0.65	0.00
time (sec)	N/A	0.390	0.195	2.888	0.225	0.115	0.000	1.349	0.204	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	86	147	274	304	0	163	51	0
N.S.	1	1.00	1.13	1.93	3.61	4.00	0.00	2.14	0.67	0.00
time (sec)	N/A	0.399	0.234	2.905	0.218	0.102	0.000	1.217	0.171	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	50	55	38	69	0	73	59	70
N.S.	1	1.00	0.63	0.70	0.48	0.87	0.00	0.92	0.75	0.89
time (sec)	N/A	0.378	0.143	1.919	0.185	0.083	0.000	0.769	0.175	10.060

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	121	60	69	210	80	0	101	59	81
N.S.	1	1.04	0.52	0.59	1.81	0.69	0.00	0.87	0.51	0.70
time (sec)	N/A	0.550	0.186	1.926	0.203	0.080	0.000	0.677	0.174	10.380

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	166	72	73	303	92	0	130	59	94
N.S.	1	1.03	0.45	0.45	1.88	0.57	0.00	0.81	0.37	0.58
time (sec)	N/A	0.716	0.243	1.921	0.196	0.081	0.000	0.822	0.167	10.832

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	209	80	83	396	103	0	158	59	105
N.S.	1	1.04	0.40	0.41	1.97	0.51	0.00	0.79	0.29	0.52
time (sec)	N/A	0.984	0.363	1.889	0.213	0.082	0.000	1.007	0.185	11.510

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	207	162	194	3860	443	0	0	90	0
N.S.	1	1.04	0.81	0.97	19.30	2.22	0.00	0.00	0.45	0.00
time (sec)	N/A	1.004	0.469	17.625	0.465	0.110	0.000	0.000	0.253	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	164	136	184	3469	417	0	472	88	0
N.S.	1	1.02	0.85	1.15	21.68	2.61	0.00	2.95	0.55	0.00
time (sec)	N/A	0.803	0.347	2.944	0.381	0.105	0.000	2.109	0.265	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	122	106	171	2826	383	0	338	80	0
N.S.	1	1.02	0.88	1.42	23.55	3.19	0.00	2.82	0.67	0.00
time (sec)	N/A	0.597	0.304	3.023	2.916	0.099	0.000	2.047	0.220	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	119	91	176	11494	343	0	313	80	0
N.S.	1	1.06	0.81	1.57	102.62	3.06	0.00	2.79	0.71	0.00
time (sec)	N/A	0.596	0.441	3.006	0.318	0.100	0.000	1.968	0.218	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	118	122	103	178	593	361	0	194	82	0
N.S.	1	1.03	0.87	1.51	5.03	3.06	0.00	1.64	0.69	0.00
time (sec)	N/A	0.620	0.273	3.013	0.239	0.106	0.000	1.085	0.182	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	123	64	71	60	87	0	102	90	85
N.S.	1	1.03	0.54	0.60	0.50	0.73	0.00	0.86	0.76	0.71
time (sec)	N/A	0.551	0.197	1.769	0.172	0.076	0.000	0.971	0.199	10.601

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	165	94	75	323	100	0	130	90	96
N.S.	1	1.06	0.60	0.48	2.07	0.64	0.00	0.83	0.58	0.62
time (sec)	N/A	0.748	6.266	1.812	0.214	0.078	0.000	1.037	0.185	11.105

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	212	105	85	422	113	0	159	90	107
N.S.	1	1.05	0.52	0.42	2.10	0.56	0.00	0.79	0.45	0.53
time (sec)	N/A	0.990	6.271	1.914	0.209	0.088	0.000	1.207	0.201	11.744

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	255	117	95	521	126	0	187	90	356
N.S.	1	1.06	0.49	0.39	2.16	0.52	0.00	0.78	0.37	1.48
time (sec)	N/A	1.272	6.383	1.910	0.227	0.084	0.000	1.486	0.186	15.541

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	51	0	121	50	0	0	45	55
N.S.	1	1.00	1.34	0.00	3.18	1.32	0.00	0.00	1.18	1.45
time (sec)	N/A	0.245	0.321	0.000	0.140	0.087	0.000	0.000	0.183	0.704

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	54	122	241	186	0	120	22	0
N.S.	1	1.00	1.46	3.30	6.51	5.03	0.00	3.24	0.59	0.00
time (sec)	N/A	0.253	0.100	2.266	0.201	0.099	0.000	0.459	0.167	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	55	111	353	206	0	91	25	0
N.S.	1	1.00	1.45	2.92	9.29	5.42	0.00	2.39	0.66	0.00
time (sec)	N/A	0.262	0.144	2.426	0.226	0.100	0.000	0.176	0.168	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	137	125	221	876	478	0	299	43	0
N.S.	1	1.07	0.98	1.73	6.84	3.73	0.00	2.34	0.34	0.00
time (sec)	N/A	0.754	0.168	2.283	0.235	0.106	0.000	0.791	0.178	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	89	170	476	348	0	0	41	0
N.S.	1	1.00	0.94	1.79	5.01	3.66	0.00	0.00	0.43	0.00
time (sec)	N/A	0.501	0.077	2.271	0.244	0.103	0.000	0.000	0.164	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	75	96	90	160	0	96	35	0
N.S.	1	1.00	1.34	1.71	1.61	2.86	0.00	1.71	0.62	0.00
time (sec)	N/A	0.250	0.038	1.192	0.200	0.089	0.000	0.417	0.185	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	102	95	104	281	0	83	42	0
N.S.	1	1.00	1.10	1.02	1.12	3.02	0.00	0.89	0.45	0.00
time (sec)	N/A	0.403	0.134	1.130	0.198	0.090	0.000	0.624	0.177	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	139	120	116	282	318	0	88	44	0
N.S.	1	1.06	0.92	0.89	2.15	2.43	0.00	0.67	0.34	0.00
time (sec)	N/A	0.625	0.169	1.168	0.207	0.095	0.000	0.700	0.177	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	187	117	139	357	342	0	125	44	0
N.S.	1	1.11	0.69	0.82	2.11	2.02	0.00	0.74	0.26	0.00
time (sec)	N/A	0.910	0.740	1.184	0.217	0.096	0.000	0.674	0.188	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	182	252	270	4934	576	0	0	53	0
N.S.	1	1.05	1.45	1.55	28.36	3.31	0.00	0.00	0.30	0.00
time (sec)	N/A	1.104	0.406	3.000	0.430	0.116	0.000	0.000	0.173	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	140	220	222	2122	556	0	198	53	0
N.S.	1	1.04	1.64	1.66	15.84	4.15	0.00	1.48	0.40	0.00
time (sec)	N/A	0.747	0.339	2.966	0.270	0.117	0.000	1.276	0.198	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	220	130	15721	338	0	0	51	0
N.S.	1	1.00	2.27	1.34	162.07	3.48	0.00	0.00	0.53	0.00
time (sec)	N/A	0.402	0.332	1.845	0.706	0.093	0.000	0.000	0.175	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	120	130	1031	340	0	0	45	0
N.S.	1	1.00	1.24	1.34	10.63	3.51	0.00	0.00	0.46	0.00
time (sec)	N/A	0.411	0.109	1.868	0.211	0.091	0.000	0.000	0.175	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	139	145	121	7176	378	0	111	52	0
N.S.	1	1.01	1.06	0.88	52.38	2.76	0.00	0.81	0.38	0.00
time (sec)	N/A	0.650	0.320	1.868	0.271	0.096	0.000	1.040	0.189	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	187	150	145	33960	398	0	124	54	0
N.S.	1	1.06	0.85	0.82	191.86	2.25	0.00	0.70	0.31	0.00
time (sec)	N/A	0.928	0.621	1.872	0.600	0.097	0.000	1.041	0.184	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	235	163	163	0	418	0	0	54	0
N.S.	1	1.08	0.75	0.75	0.00	1.93	0.00	0.00	0.25	0.00
time (sec)	N/A	1.243	0.871	1.907	0.000	0.098	0.000	0.000	0.207	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	230	340	320	9048	664	0	0	63	0
N.S.	1	1.07	1.59	1.50	42.28	3.10	0.00	0.00	0.29	0.00
time (sec)	N/A	1.383	0.844	3.075	2.291	0.120	0.000	0.000	0.201	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	174	186	308	280	4988	662	0	220	63	0
N.S.	1	1.07	1.77	1.61	28.67	3.80	0.00	1.26	0.36	0.00
time (sec)	N/A	1.048	0.465	3.005	0.436	0.121	0.000	1.683	0.214	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	142	308	172	84332	426	0	112	63	0
N.S.	1	1.04	2.25	1.26	615.56	3.11	0.00	0.82	0.46	0.00
time (sec)	N/A	0.569	0.467	1.882	14.413	0.101	0.000	1.299	0.180	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	142	204	172	2875	422	0	0	61	0
N.S.	1	1.04	1.49	1.26	20.99	3.08	0.00	0.00	0.45	0.00
time (sec)	N/A	0.576	2.262	1.944	0.400	0.127	0.000	0.000	0.200	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	143	146	170	3049	426	0	0	55	0
N.S.	1	1.04	1.07	1.24	22.26	3.11	0.00	0.00	0.40	0.00
time (sec)	N/A	0.616	0.697	1.863	0.514	0.095	0.000	0.000	0.181	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	177	187	186	143	258456	446	0	133	62	0
N.S.	1	1.06	1.05	0.81	1460.20	2.52	0.00	0.75	0.35	0.00
time (sec)	N/A	0.951	0.911	1.863	2.647	0.101	0.000	1.437	0.193	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	235	165	175	148823	466	0	155	64	0
N.S.	1	1.08	0.76	0.81	685.82	2.15	0.00	0.71	0.29	0.00
time (sec)	N/A	1.231	1.566	1.901	2.596	0.122	0.000	1.450	0.183	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	132	140	225	1643	337	0	296	37	0
N.S.	1	1.05	1.11	1.79	13.04	2.67	0.00	2.35	0.29	0.00
time (sec)	N/A	0.858	0.260	2.148	0.264	0.096	0.000	0.983	0.164	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	91	111	213	873	297	0	233	37	0
N.S.	1	1.07	1.31	2.51	10.27	3.49	0.00	2.74	0.44	0.00
time (sec)	N/A	0.634	0.228	2.105	0.230	0.095	0.000	0.895	0.176	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	76	166	473	223	0	0	35	0
N.S.	1	1.00	1.41	3.07	8.76	4.13	0.00	0.00	0.65	0.00
time (sec)	N/A	0.426	0.077	2.053	0.242	0.091	0.000	0.000	0.172	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	40	92	87	88	0	93	29	0
N.S.	1	1.00	1.48	3.41	3.22	3.26	0.00	3.44	1.07	0.00
time (sec)	N/A	0.221	0.021	0.997	0.197	0.079	0.000	0.437	0.186	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	90	91	101	144	0	71	36	0
N.S.	1	1.00	1.45	1.47	1.63	2.32	0.00	1.15	0.58	0.00
time (sec)	N/A	0.367	0.182	0.975	0.223	0.090	0.000	0.583	0.169	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	102	118	111	279	163	0	76	38	0
N.S.	1	1.04	1.20	1.13	2.85	1.66	0.00	0.78	0.39	0.00
time (sec)	N/A	0.533	0.187	1.001	0.204	0.086	0.000	0.666	0.182	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	142	122	134	354	174	0	103	38	0
N.S.	1	1.06	0.91	1.00	2.64	1.30	0.00	0.77	0.28	0.00
time (sec)	N/A	0.752	0.138	1.027	0.223	0.083	0.000	0.677	0.167	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	325	337	71	0	0	0	0	0	26	0
N.S.	1	1.04	0.22	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.425	0.165	0.000	0.000	0.000	0.000	0.000	0.266	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	71	0	0	0	0	0	26	0
N.S.	1	1.00	0.25	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.373	0.094	0.000	0.000	0.000	0.000	0.000	0.201	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	326	343	71	0	0	0	0	0	26	0
N.S.	1	1.05	0.22	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.390	0.112	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	716	708	71	0	0	0	0	0	26	0
N.S.	1	0.99	0.10	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.669	0.150	0.000	0.000	0.000	0.000	0.000	0.257	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	673	668	71	0	0	0	0	0	26	0
N.S.	1	0.99	0.11	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.611	0.140	0.000	0.000	0.000	0.000	0.000	0.248	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	624	628	71	0	0	0	0	0	26	0
N.S.	1	1.01	0.11	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.593	0.094	0.000	0.000	0.000	0.000	0.000	0.198	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	662	672	71	0	0	0	0	0	26	0
N.S.	1	1.02	0.11	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.628	0.104	0.000	0.000	0.000	0.000	0.000	0.195	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	715	717	71	0	0	0	0	0	26	0
N.S.	1	1.00	0.10	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.649	0.127	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	760	0	0	0	0	0	39	0
N.S.	1	1.00	9.74	0.00	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.450	4.567	0.000	0.000	0.000	0.000	0.000	0.216	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	761	0	0	0	0	0	39	0
N.S.	1	1.00	10.01	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.463	6.795	0.000	0.000	0.000	0.000	0.000	0.240	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	64	3346	0	0	0	0	0	40	0
N.S.	1	0.84	44.03	0.00	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.492	17.006	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	72	585	0	0	0	0	0	46	0
N.S.	1	0.92	7.50	0.00	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.483	4.777	0.000	0.000	0.000	0.000	0.000	0.209	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	1982	0	0	0	0	0	25	0
N.S.	1	1.00	25.41	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.400	20.915	0.000	0.000	0.000	0.000	0.000	0.276	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	1566	0	0	0	0	0	25	0
N.S.	1	1.00	19.82	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.394	22.380	0.000	0.000	0.000	0.000	0.000	0.280	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	327	79	274	0	0	0	0	0	25	0
N.S.	1	0.24	0.84	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.398	6.853	0.000	0.000	0.000	0.000	0.000	0.278	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	2325	0	0	0	0	0	47	0
N.S.	1	1.00	29.06	0.00	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.408	34.828	0.000	0.000	0.000	0.000	0.000	0.213	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	304	305	208	0	0	0	0	0	95	0
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	1.357	3.561	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	230	239	165	0	0	0	0	0	74	0
N.S.	1	1.04	0.72	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.906	0.977	0.000	0.000	0.000	0.000	0.000	0.190	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	172	175	135	0	0	0	0	0	53	0
N.S.	1	1.02	0.78	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.740	0.493	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	106	0	0	0	0	0	30	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.405	0.094	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	183	140	0	0	0	0	0	25	0
N.S.	1	1.05	0.80	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.636	0.512	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	217	221	192	0	0	0	0	0	35	0
N.S.	1	1.02	0.88	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.939	1.217	0.000	0.000	0.000	0.000	0.000	0.190	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	157	398	0	0	0	0	0	77	0
N.S.	1	0.97	2.46	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.643	23.491	0.000	0.000	0.000	0.000	0.000	0.256	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	83	0	0	0	0	0	47	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.424	0.306	0.000	0.000	0.000	0.000	0.000	0.233	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	0	0	20	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.237	0.027	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	2938	0	0	0	0	0	30	0
N.S.	1	1.00	49.80	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.279	14.436	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	2990	0	0	0	0	0	40	0
N.S.	1	1.00	48.23	0.00	0.00	0.00	0.00	0.00	0.65	0.00
time (sec)	N/A	0.299	14.712	0.000	0.000	0.000	0.000	0.000	0.195	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	117	85	0	0	0	0	0	51	0
N.S.	1	0.98	0.71	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.446	0.309	0.000	0.000	0.000	0.000	0.000	0.218	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	64	67	0	0	0	0	0	24	0
N.S.	1	0.96	1.00	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.243	0.040	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	73	2951	0	0	0	0	0	34	0
N.S.	1	0.87	35.13	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.261	6.126	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	73	3003	0	0	0	0	0	44	0
N.S.	1	0.85	34.92	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.288	6.134	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	117	85	0	0	0	0	0	51	0
N.S.	1	0.98	0.71	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.467	0.277	0.000	0.000	0.000	0.000	0.000	0.221	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	64	67	0	0	0	0	0	24	0
N.S.	1	0.96	1.00	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.243	0.036	0.000	0.000	0.000	0.000	0.000	0.199	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	73	1021	0	0	0	0	0	34	0
N.S.	1	0.90	12.60	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.268	5.731	0.000	0.000	0.000	0.000	0.000	0.183	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	73	3003	0	0	0	0	0	44	0
N.S.	1	0.87	35.75	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.281	6.145	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	177	173	400	0	0	0	0	0	83	0
N.S.	1	0.98	2.26	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.780	2.198	0.000	0.000	0.000	0.000	0.000	0.282	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	86	0	0	0	0	0	51	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.489	0.282	0.000	0.000	0.000	0.000	0.000	0.238	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0	23	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.268	0.070	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	1023	0	0	0	0	0	36	0
N.S.	1	1.00	16.77	0.00	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.394	5.947	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	2992	0	0	0	0	0	46	0
N.S.	1	1.00	44.66	0.00	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.403	6.172	0.000	0.000	0.000	0.000	0.000	0.197	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	88	0	0	0	0	0	54	0
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.517	0.333	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	71	0	0	0	0	0	26	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.266	0.104	0.000	0.000	0.000	0.000	0.000	0.192	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	75	1025	0	0	0	0	0	39	0
N.S.	1	0.87	11.92	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.387	6.166	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	78	2030	0	0	0	0	0	49	0
N.S.	1	0.86	22.31	0.00	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.406	6.051	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	88	0	0	0	0	0	54	0
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.531	0.314	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	71	0	0	0	0	0	26	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.271	0.096	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	75	1026	0	0	0	0	0	39	0
N.S.	1	0.90	12.36	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.384	5.827	0.000	0.000	0.000	0.000	0.000	0.185	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	78	2030	0	0	0	0	0	49	0
N.S.	1	0.88	22.81	0.00	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.404	6.036	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	178	174	419	0	0	0	0	0	92	0
N.S.	1	0.98	2.35	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.844	5.742	0.000	0.000	0.000	0.000	0.000	0.267	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	316	0	0	0	0	0	60	0
N.S.	1	1.00	2.93	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.511	2.122	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	50	0	0	0	0	0	28	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	0.272	0.169	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	0	0	0	0	0	0	42	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	0.440	0.000	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0	51	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	0.446	0.000	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	302	0	0	0	0	0	57	0
N.S.	1	1.00	2.32	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.528	1.776	0.000	0.000	0.000	0.000	0.000	0.219	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	68	0	0	0	0	0	25	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.277	0.210	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	316	0	0	0	0	0	60	0
N.S.	1	1.00	2.43	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.529	0.979	0.000	0.000	0.000	0.000	0.000	0.216	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	70	0	0	0	0	0	28	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.288	0.211	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	821	0	0	0	0	0	21	0
N.S.	1	1.00	11.40	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.271	13.455	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	89	255	0	0	0	0	0	23	0
N.S.	1	0.96	2.74	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.273	2.016	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	823	0	0	0	0	0	23	0
N.S.	1	1.00	9.35	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.406	6.016	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	90	0	0	0	0	0	0	24	0
N.S.	1	0.82	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.411	0.000	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	85	823	0	0	0	0	0	25	0
N.S.	1	0.88	8.48	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.293	1.915	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	257	0	0	0	0	0	27	0
N.S.	1	1.00	3.67	0.00	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.292	0.389	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	87	825	0	0	0	0	0	27	0
N.S.	1	0.77	7.30	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.395	5.252	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	0	0	0	0	0	0	28	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.404	0.000	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	79	823	0	0	0	0	0	25	0
N.S.	1	0.81	8.48	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.273	5.055	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	79	257	0	0	0	0	0	27	0
N.S.	1	0.81	2.62	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.285	0.360	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	95	825	0	0	0	0	0	27	0
N.S.	1	0.84	7.30	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.395	5.127	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	96	0	0	0	0	0	0	28	0
N.S.	1	0.83	0.00	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.426	0.000	0.000	0.000	0.000	0.000	0.000	0.190	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	211	209	154	0	0	0	0	0	23	0
N.S.	1	0.99	0.73	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	1.068	0.914	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F(-1)	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	157	0	0	0	0	0	0	23	0
N.S.	1	1.01	0.00	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.727	0.000	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	95	0	0	0	0	0	23	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.477	0.215	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	0	0	0	0	21	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.343	0.069	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	83	711	0	0	0	0	0	14	0
N.S.	1	0.99	8.46	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.323	4.309	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	3781	0	0	0	0	0	21	0
N.S.	1	1.00	45.01	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.379	14.990	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	2529	0	0	0	0	0	32	0
N.S.	1	1.00	25.81	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.440	15.595	0.000	0.000	0.000	0.000	0.000	0.197	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	782	0	0	0	0	0	25	0
N.S.	1	1.00	8.15	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.409	8.913	0.000	0.000	0.000	0.000	0.000	0.192	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	820	0	0	0	0	0	36	0
N.S.	1	1.00	8.54	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.433	11.595	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	3349	0	0	0	0	0	36	0
N.S.	1	1.00	34.17	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.447	20.258	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	117	241	270	0	148	0	0	45	87
N.S.	1	1.05	2.17	2.43	0.00	1.33	0.00	0.00	0.41	0.78
time (sec)	N/A	0.567	4.774	6.085	0.000	0.104	0.000	0.000	0.204	10.188

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	89	232	219	0	137	0	0	45	80
N.S.	1	1.02	2.67	2.52	0.00	1.57	0.00	0.00	0.52	0.92
time (sec)	N/A	0.464	3.653	3.908	0.000	0.102	0.000	0.000	0.193	0.133

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	62	222	225	0	125	0	0	41	53
N.S.	1	1.02	3.64	3.69	0.00	2.05	0.00	0.00	0.67	0.87
time (sec)	N/A	0.445	3.175	2.299	0.000	0.091	0.000	0.000	0.207	9.868

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	155	150	0	107	0	0	28	27
N.S.	1	1.00	4.43	4.29	0.00	3.06	0.00	0.00	0.80	0.77
time (sec)	N/A	0.369	1.343	0.697	0.000	0.093	0.000	0.000	0.175	0.189

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	58	209	148	0	156	0	0	45	60
N.S.	1	1.02	3.67	2.60	0.00	2.74	0.00	0.00	0.79	1.05
time (sec)	N/A	0.478	3.286	1.124	0.000	0.090	0.000	0.000	0.186	10.050

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	85	247	368	0	175	0	0	45	87
N.S.	1	1.02	2.98	4.43	0.00	2.11	0.00	0.00	0.54	1.05
time (sec)	N/A	0.479	4.585	1.842	0.000	0.107	0.000	0.000	0.177	10.181

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	113	477	384	0	188	0	0	45	87
N.S.	1	1.02	4.30	3.46	0.00	1.69	0.00	0.00	0.41	0.78
time (sec)	N/A	0.560	6.285	3.015	0.000	0.105	0.000	0.000	0.186	10.413

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	141	294	437	0	199	0	0	45	87
N.S.	1	1.04	2.18	3.24	0.00	1.47	0.00	0.00	0.33	0.64
time (sec)	N/A	0.613	3.076	4.451	0.000	0.104	0.000	0.000	0.201	10.369

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	212	255	260	0	175	0	0	75	136
N.S.	1	1.44	1.73	1.77	0.00	1.19	0.00	0.00	0.51	0.93
time (sec)	N/A	1.382	5.894	9.870	0.000	0.102	0.000	0.000	0.227	10.446

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	184	245	272	0	162	0	0	75	129
N.S.	1	1.52	2.02	2.25	0.00	1.34	0.00	0.00	0.62	1.07
time (sec)	N/A	1.159	4.864	7.228	0.000	0.111	0.000	0.000	0.250	10.416

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	156	235	250	0	149	0	0	75	104
N.S.	1	1.64	2.47	2.63	0.00	1.57	0.00	0.00	0.79	1.09
time (sec)	N/A	1.033	4.111	5.719	0.000	0.092	0.000	0.000	0.232	10.297

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	128	224	228	0	134	0	0	69	59
N.S.	1	1.91	3.34	3.40	0.00	2.00	0.00	0.00	1.03	0.88
time (sec)	N/A	0.842	3.897	0.787	0.000	0.094	0.000	0.000	0.240	10.339

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	85	112	185	0	97	0	0	50	82
N.S.	1	1.93	2.55	4.20	0.00	2.20	0.00	0.00	1.14	1.86
time (sec)	N/A	0.632	0.254	1.499	0.000	0.108	0.000	0.000	0.188	10.465

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	152	111	371	0	187	0	0	75	109
N.S.	1	1.67	1.22	4.08	0.00	2.05	0.00	0.00	0.82	1.20
time (sec)	N/A	0.938	0.231	1.987	0.000	0.102	0.000	0.000	0.202	10.511

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	180	114	386	0	202	0	0	75	114
N.S.	1	1.49	0.94	3.19	0.00	1.67	0.00	0.00	0.62	0.94
time (sec)	N/A	1.156	0.302	3.262	0.000	0.107	0.000	0.000	0.186	10.488

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	208	114	439	0	215	0	0	75	114
N.S.	1	1.41	0.78	2.99	0.00	1.46	0.00	0.00	0.51	0.78
time (sec)	N/A	1.298	0.417	5.214	0.000	0.099	0.000	0.000	0.202	11.307

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	208	255	260	0	175	0	0	103	206
N.S.	1	1.41	1.73	1.77	0.00	1.19	0.00	0.00	0.70	1.40
time (sec)	N/A	0.607	6.570	19.177	0.000	0.112	0.000	0.000	0.258	11.002

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	182	245	272	0	162	0	0	103	143
N.S.	1	1.50	2.02	2.25	0.00	1.34	0.00	0.00	0.85	1.18
time (sec)	N/A	0.573	5.564	17.171	0.000	0.097	0.000	0.000	0.281	10.791

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	152	233	250	0	148	0	0	103	104
N.S.	1	1.67	2.56	2.75	0.00	1.63	0.00	0.00	1.13	1.14
time (sec)	N/A	0.529	5.157	14.001	0.000	0.103	0.000	0.000	0.258	10.186

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	152	146	172	0	180	0	0	95	104
N.S.	1	1.67	1.60	1.89	0.00	1.98	0.00	0.00	1.04	1.14
time (sec)	N/A	0.505	0.367	2.358	0.000	0.101	0.000	0.000	0.261	10.175

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	152	139	371	0	187	0	0	70	126
N.S.	1	1.67	1.53	4.08	0.00	2.05	0.00	0.00	0.77	1.38
time (sec)	N/A	0.523	0.345	2.619	0.000	0.103	0.000	0.000	0.204	10.717

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	178	138	386	0	200	0	0	103	154
N.S.	1	1.52	1.18	3.30	0.00	1.71	0.00	0.00	0.88	1.32
time (sec)	N/A	0.542	0.465	3.606	0.000	0.110	0.000	0.000	0.190	10.694

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	208	140	439	0	215	0	0	103	145
N.S.	1	1.41	0.95	2.99	0.00	1.46	0.00	0.00	0.70	0.99
time (sec)	N/A	0.570	0.647	5.726	0.000	0.102	0.000	0.000	0.188	10.743

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	193	314	229	0	208	0	0	32	0
N.S.	1	1.51	2.45	1.79	0.00	1.62	0.00	0.00	0.25	0.00
time (sec)	N/A	0.922	2.404	4.998	0.000	0.110	0.000	0.000	0.176	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	165	292	215	0	198	0	0	30	0
N.S.	1	1.65	2.92	2.15	0.00	1.98	0.00	0.00	0.30	0.00
time (sec)	N/A	0.884	2.323	3.609	0.000	0.109	0.000	0.000	0.200	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	137	270	199	0	186	0	0	24	0
N.S.	1	1.90	3.75	2.76	0.00	2.58	0.00	0.00	0.33	0.00
time (sec)	N/A	0.760	1.995	2.133	0.000	0.095	0.000	0.000	0.174	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	136	262	198	0	184	0	0	36	0
N.S.	1	1.94	3.74	2.83	0.00	2.63	0.00	0.00	0.51	0.00
time (sec)	N/A	0.736	1.362	1.481	0.000	0.098	0.000	0.000	0.174	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	137	263	200	0	184	0	0	40	0
N.S.	1	1.96	3.76	2.86	0.00	2.63	0.00	0.00	0.57	0.00
time (sec)	N/A	0.821	1.540	0.667	0.000	0.087	0.000	0.000	0.181	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	161	303	254	0	236	0	0	40	0
N.S.	1	1.68	3.16	2.65	0.00	2.46	0.00	0.00	0.42	0.00
time (sec)	N/A	0.942	1.978	1.476	0.000	0.105	0.000	0.000	0.177	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	189	338	413	0	258	0	0	40	0
N.S.	1	1.52	2.73	3.33	0.00	2.08	0.00	0.00	0.32	0.00
time (sec)	N/A	0.950	3.245	2.288	0.000	0.127	0.000	0.000	0.175	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	235	148	283	0	288	0	0	42	0
N.S.	1	1.47	0.92	1.77	0.00	1.80	0.00	0.00	0.26	0.00
time (sec)	N/A	1.244	0.989	6.922	0.000	0.121	0.000	0.000	0.198	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	208	136	270	0	278	0	0	40	0
N.S.	1	1.51	0.99	1.96	0.00	2.01	0.00	0.00	0.29	0.00
time (sec)	N/A	1.203	0.616	4.186	0.000	0.107	0.000	0.000	0.178	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	179	128	257	0	268	0	0	34	0
N.S.	1	1.60	1.14	2.29	0.00	2.39	0.00	0.00	0.30	0.00
time (sec)	N/A	1.018	0.506	3.579	0.000	0.131	0.000	0.000	0.194	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	180	114	257	0	268	0	0	52	0
N.S.	1	1.65	1.05	2.36	0.00	2.46	0.00	0.00	0.48	0.00
time (sec)	N/A	1.004	0.373	2.237	0.000	0.095	0.000	0.000	0.177	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	98	71	188	0	150	0	0	58	0
N.S.	1	1.72	1.25	3.30	0.00	2.63	0.00	0.00	1.02	0.00
time (sec)	N/A	0.520	0.286	2.310	0.000	0.106	0.000	0.000	0.181	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	180	312	257	0	268	0	0	58	0
N.S.	1	1.65	2.86	2.36	0.00	2.46	0.00	0.00	0.53	0.00
time (sec)	N/A	0.987	4.423	0.821	0.000	0.097	0.000	0.000	0.204	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	203	342	405	0	318	0	0	58	0
N.S.	1	1.49	2.51	2.98	0.00	2.34	0.00	0.00	0.43	0.00
time (sec)	N/A	1.193	6.384	2.245	0.000	0.104	0.000	0.000	0.188	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	232	372	413	0	338	0	0	58	0
N.S.	1	1.43	2.30	2.55	0.00	2.09	0.00	0.00	0.36	0.00
time (sec)	N/A	1.230	2.705	3.094	0.000	0.103	0.000	0.000	0.178	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	287	175	296	0	364	0	0	52	0
N.S.	1	1.39	0.85	1.43	0.00	1.76	0.00	0.00	0.25	0.00
time (sec)	N/A	1.583	2.156	11.204	0.000	0.118	0.000	0.000	0.177	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	259	166	283	0	354	0	0	50	0
N.S.	1	1.43	0.92	1.56	0.00	1.96	0.00	0.00	0.28	0.00
time (sec)	N/A	1.564	1.408	5.633	0.000	0.115	0.000	0.000	0.169	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	231	157	270	0	344	0	0	44	0
N.S.	1	1.49	1.01	1.74	0.00	2.22	0.00	0.00	0.28	0.00
time (sec)	N/A	1.348	1.318	4.312	0.000	0.121	0.000	0.000	0.190	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	231	146	270	0	344	0	0	68	0
N.S.	1	1.49	0.94	1.74	0.00	2.22	0.00	0.00	0.44	0.00
time (sec)	N/A	1.402	1.073	4.209	0.000	0.108	0.000	0.000	0.181	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-1)	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	231	146	270	0	344	0	0	76	0
N.S.	1	1.49	0.94	1.74	0.00	2.22	0.00	0.00	0.49	0.00
time (sec)	N/A	1.357	1.072	3.514	0.000	0.102	0.000	0.000	0.182	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-1)	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	231	139	270	0	344	0	0	76	0
N.S.	1	1.49	0.90	1.74	0.00	2.22	0.00	0.00	0.49	0.00
time (sec)	N/A	1.357	2.289	3.458	0.000	0.100	0.000	0.000	0.191	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-1)	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	231	603	268	0	344	0	0	76	0
N.S.	1	1.49	3.89	1.73	0.00	2.22	0.00	0.00	0.49	0.00
time (sec)	N/A	1.327	7.353	1.057	0.000	0.111	0.000	0.000	0.167	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	255	372	555	0	394	0	0	76	0
N.S.	1	1.41	2.06	3.07	0.00	2.18	0.00	0.00	0.42	0.00
time (sec)	N/A	1.525	2.712	3.503	0.000	0.104	0.000	0.000	0.168	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	283	402	453	0	414	0	0	76	0
N.S.	1	1.37	1.94	2.19	0.00	2.00	0.00	0.00	0.37	0.00
time (sec)	N/A	1.658	3.112	4.093	0.000	0.108	0.000	0.000	0.180	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	184	80	72	293	79	0	0	30	0
N.S.	1	1.20	0.52	0.47	1.92	0.52	0.00	0.00	0.20	0.00
time (sec)	N/A	0.906	0.143	1.480	0.194	0.087	0.000	0.000	0.187	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	141	61	62	203	69	0	0	30	0
N.S.	1	1.23	0.53	0.54	1.77	0.60	0.00	0.00	0.26	0.00
time (sec)	N/A	0.711	0.123	1.610	0.192	0.087	0.000	0.000	0.200	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	98	49	52	113	57	0	0	28	0
N.S.	1	1.27	0.64	0.68	1.47	0.74	0.00	0.00	0.36	0.00
time (sec)	N/A	0.521	0.083	1.387	0.209	0.078	0.000	0.000	0.175	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	39	41	20	49	0	37	22	0
N.S.	1	1.00	1.08	1.14	0.56	1.36	0.00	1.03	0.61	0.00
time (sec)	N/A	0.368	0.076	1.427	0.185	0.081	0.000	0.147	0.159	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	74	117	241	180	0	120	30	0
N.S.	1	1.00	1.30	2.05	4.23	3.16	0.00	2.11	0.53	0.00
time (sec)	N/A	0.403	0.118	2.785	0.200	0.101	0.000	0.199	0.176	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	93	95	157	662	325	0	256	30	0
N.S.	1	1.01	1.03	1.71	7.20	3.53	0.00	2.78	0.33	0.00
time (sec)	N/A	0.549	0.164	2.833	0.225	0.096	0.000	0.249	0.159	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	128	171	1264	355	0	335	30	0
N.S.	1	1.00	0.94	1.26	9.29	2.61	0.00	2.46	0.22	0.00
time (sec)	N/A	0.704	0.279	2.787	0.242	0.103	0.000	0.243	0.159	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	187	72	73	303	84	0	130	65	0
N.S.	1	1.16	0.45	0.45	1.88	0.52	0.00	0.81	0.40	0.00
time (sec)	N/A	0.926	0.217	2.228	0.209	0.089	0.000	0.232	0.247	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	142	60	61	210	72	0	101	65	0
N.S.	1	1.22	0.52	0.53	1.81	0.62	0.00	0.87	0.56	0.00
time (sec)	N/A	0.734	0.152	2.176	0.190	0.087	0.000	0.189	0.227	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	100	50	53	38	61	0	73	61	0
N.S.	1	1.27	0.63	0.67	0.48	0.77	0.00	0.92	0.77	0.00
time (sec)	N/A	0.511	0.128	2.158	0.173	0.084	0.000	0.188	0.232	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	97	81	146	274	298	0	163	49	0
N.S.	1	1.01	0.84	1.52	2.85	3.10	0.00	1.70	0.51	0.00
time (sec)	N/A	0.542	0.106	3.484	0.212	0.103	0.000	0.205	0.193	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	96	92	158	1143	337	0	258	65	0
N.S.	1	1.01	0.97	1.66	12.03	3.55	0.00	2.72	0.68	0.00
time (sec)	N/A	0.531	0.229	3.912	0.203	0.102	0.000	0.282	0.196	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	139	107	172	2244	369	0	0	65	0
N.S.	1	0.99	0.76	1.23	16.03	2.64	0.00	0.00	0.46	0.00
time (sec)	N/A	0.712	0.363	3.698	0.252	0.111	0.000	0.000	0.173	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	182	117	182	2361	391	0	472	65	0
N.S.	1	1.01	0.65	1.01	13.12	2.17	0.00	2.62	0.36	0.00
time (sec)	N/A	0.939	0.422	3.729	0.279	0.116	0.000	0.513	0.202	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	233	90	85	422	105	0	159	104	0
N.S.	1	1.16	0.45	0.42	2.10	0.52	0.00	0.79	0.52	0.00
time (sec)	N/A	1.201	5.567	1.497	0.212	0.095	0.000	0.291	0.308	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	186	84	75	323	92	0	130	104	0
N.S.	1	1.19	0.54	0.48	2.07	0.59	0.00	0.83	0.67	0.00
time (sec)	N/A	0.917	5.523	1.497	0.203	0.082	0.000	0.278	0.288	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	144	64	65	60	79	0	102	104	0
N.S.	1	1.21	0.54	0.55	0.50	0.66	0.00	0.86	0.87	0.00
time (sec)	N/A	0.730	5.465	1.415	0.177	0.089	0.000	0.240	0.288	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	143	93	179	593	339	0	194	98	0
N.S.	1	1.04	0.67	1.30	4.30	2.46	0.00	1.41	0.71	0.00
time (sec)	N/A	0.758	0.173	3.385	0.250	0.108	0.000	0.287	0.281	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	140	90	197	11494	373	0	313	80	0
N.S.	1	1.06	0.68	1.49	87.08	2.83	0.00	2.37	0.61	0.00
time (sec)	N/A	0.820	0.220	3.571	0.293	0.108	0.000	0.525	0.220	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	143	95	174	2826	385	0	338	104	0
N.S.	1	1.02	0.68	1.24	20.19	2.75	0.00	2.41	0.74	0.00
time (sec)	N/A	0.802	0.430	3.633	2.833	0.136	0.000	0.394	0.179	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	185	115	184	3469	411	0	472	104	0
N.S.	1	1.03	0.64	1.02	19.27	2.28	0.00	2.62	0.58	0.00
time (sec)	N/A	0.995	0.475	3.747	0.392	0.110	0.000	0.581	0.182	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	228	125	194	3860	437	0	0	104	0
N.S.	1	1.04	0.57	0.88	17.55	1.99	0.00	0.00	0.47	0.00
time (sec)	N/A	1.213	0.592	3.839	0.431	0.120	0.000	0.000	0.182	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	208	136	127	357	333	0	125	43	0
N.S.	1	1.10	0.72	0.67	1.89	1.76	0.00	0.66	0.23	0.00
time (sec)	N/A	1.096	0.211	1.523	0.202	0.101	0.000	172.559	0.175	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	160	116	117	282	309	0	88	41	0
N.S.	1	1.06	0.77	0.77	1.87	2.05	0.00	0.58	0.27	0.00
time (sec)	N/A	0.809	0.137	1.518	0.200	0.095	0.000	171.787	0.162	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	114	100	96	104	290	0	83	35	0
N.S.	1	1.01	0.88	0.85	0.92	2.57	0.00	0.73	0.31	0.00
time (sec)	N/A	0.585	0.072	1.427	0.192	0.094	0.000	166.353	0.169	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	76	95	91	90	169	0	54	47	0
N.S.	1	1.36	1.70	1.62	1.61	3.02	0.00	0.96	0.84	0.00
time (sec)	N/A	0.394	0.068	1.498	0.180	0.097	0.000	0.206	0.186	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	116	109	162	476	351	0	171	51	0
N.S.	1	0.86	0.81	1.20	3.53	2.60	0.00	1.27	0.38	0.00
time (sec)	N/A	0.686	0.074	2.736	0.240	0.112	0.000	0.283	0.169	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	158	145	207	876	529	0	299	51	0
N.S.	1	0.94	0.86	1.23	5.21	3.15	0.00	1.78	0.30	0.00
time (sec)	N/A	0.953	0.153	2.763	0.225	0.112	0.000	0.254	0.168	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	205	178	222	1646	559	0	380	51	0
N.S.	1	0.97	0.84	1.05	7.80	2.65	0.00	1.80	0.24	0.00
time (sec)	N/A	1.229	0.266	2.835	0.264	0.117	0.000	0.302	0.187	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	256	152	157	0	408	0	182	53	0
N.S.	1	1.08	0.64	0.66	0.00	1.72	0.00	0.77	0.22	0.00
time (sec)	N/A	1.421	0.677	2.297	0.000	0.103	0.000	171.786	0.168	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	208	133	147	33960	388	0	151	51	0
N.S.	1	1.06	0.68	0.75	172.39	1.97	0.00	0.77	0.26	0.00
time (sec)	N/A	1.104	0.450	2.003	0.611	0.114	0.000	171.417	0.179	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	160	138	122	7176	368	0	129	45	0
N.S.	1	1.02	0.88	0.78	45.71	2.34	0.00	0.82	0.29	0.00
time (sec)	N/A	0.788	0.638	2.029	0.269	0.100	0.000	167.158	0.192	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	118	131	124	1031	348	0	99	63	0
N.S.	1	1.01	1.12	1.06	8.81	2.97	0.00	0.85	0.54	0.00
time (sec)	N/A	0.558	0.282	2.153	0.226	0.096	0.000	0.274	0.168	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	118	248	123	15721	346	0	99	69	0
N.S.	1	1.01	2.12	1.05	134.37	2.96	0.00	0.85	0.59	0.00
time (sec)	N/A	0.551	0.607	2.125	0.714	0.097	0.000	0.334	0.167	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	161	248	214	2122	558	0	225	69	0
N.S.	1	0.93	1.43	1.23	12.20	3.21	0.00	1.29	0.40	0.00
time (sec)	N/A	0.932	0.544	3.329	0.256	0.121	0.000	0.590	0.190	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	203	272	262	4934	622	0	0	69	0
N.S.	1	0.95	1.27	1.22	23.06	2.91	0.00	0.00	0.32	0.00
time (sec)	N/A	1.234	0.463	3.503	0.445	0.148	0.000	0.000	0.181	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	256	144	177	148823	456	0	191	61	0
N.S.	1	1.08	0.61	0.75	627.95	1.92	0.00	0.81	0.26	0.00
time (sec)	N/A	1.449	0.787	2.359	2.627	0.121	0.000	173.230	0.179	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	197	208	141	144	258456	436	0	160	55	0
N.S.	1	1.06	0.72	0.73	1311.96	2.21	0.00	0.81	0.28	0.00
time (sec)	N/A	1.095	0.648	2.315	2.657	0.110	0.000	168.874	0.189	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	164	168	164	3049	416	0	130	79	0
N.S.	1	1.04	1.07	1.04	19.42	2.65	0.00	0.83	0.50	0.00
time (sec)	N/A	0.865	0.891	2.361	0.500	0.102	0.000	0.332	0.177	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	163	224	164	2875	416	0	130	87	0
N.S.	1	1.04	1.43	1.04	18.31	2.65	0.00	0.83	0.55	0.00
time (sec)	N/A	0.815	2.592	2.333	0.417	0.108	0.000	0.348	0.205	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	163	328	164	84332	416	0	130	87	0
N.S.	1	1.04	2.09	1.04	537.15	2.65	0.00	0.83	0.55	0.00
time (sec)	N/A	0.739	0.594	2.089	13.968	0.105	0.000	0.324	0.186	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	207	328	275	4988	646	0	256	87	0
N.S.	1	0.97	1.53	1.29	23.31	3.02	0.00	1.20	0.41	0.00
time (sec)	N/A	1.247	0.870	3.717	0.421	0.117	0.000	0.621	0.186	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	251	175	312	9048	710	0	0	87	0
N.S.	1	0.99	0.69	1.23	35.62	2.80	0.00	0.00	0.34	0.00
time (sec)	N/A	1.578	2.841	3.559	2.250	0.136	0.000	0.000	0.207	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	244	278	155	0	0	0	0	0	77	0
N.S.	1	1.14	0.64	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	1.325	0.849	0.000	0.000	0.000	0.000	0.000	0.190	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	179	203	125	0	0	0	0	0	56	0
N.S.	1	1.13	0.70	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	1.024	0.377	0.000	0.000	0.000	0.000	0.000	0.198	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	137	105	0	0	0	0	0	33	0
N.S.	1	1.04	0.80	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.452	0.102	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	178	207	165	0	0	0	0	0	28	0
N.S.	1	1.16	0.93	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.892	0.481	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	215	245	223	0	0	0	0	0	38	0
N.S.	1	1.14	1.04	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	1.200	0.854	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	90	59	73	95	99	0	164	202	152
N.S.	1	1.06	0.69	0.86	1.12	1.16	0.00	1.93	2.38	1.79
time (sec)	N/A	0.507	0.183	1.511	0.043	0.097	0.000	0.134	0.159	12.553

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	64	60	60	70	88	0	122	161	109
N.S.	1	1.02	0.95	0.95	1.11	1.40	0.00	1.94	2.56	1.73
time (sec)	N/A	0.413	0.116	1.253	0.030	0.095	0.000	0.144	0.155	11.798

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	48	47	47	58	74	0	107	116	85
N.S.	1	1.02	1.00	1.00	1.23	1.57	0.00	2.28	2.47	1.81
time (sec)	N/A	0.385	0.017	1.199	0.029	0.090	0.000	0.125	0.176	10.637

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	30	29	60	37	63	62	47
N.S.	1	1.00	1.00	1.25	1.21	2.50	1.54	2.62	2.58	1.96
time (sec)	N/A	0.298	0.012	0.731	0.038	0.098	2.241	0.127	0.159	9.995

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	24	23	36	41	49	38	57
N.S.	1	1.00	1.00	1.50	1.44	2.25	2.56	3.06	2.38	3.56
time (sec)	N/A	0.151	0.002	0.217	0.028	0.100	0.913	0.116	0.156	10.041

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	26	16	20	17	17	39	17	17
N.S.	1	1.00	1.73	1.07	1.33	1.13	1.13	2.60	1.13	1.13
time (sec)	N/A	0.242	0.021	0.295	0.039	0.089	0.785	0.115	0.173	10.043

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	39	35	32	34	29	0	82	33	31
N.S.	1	1.03	0.92	0.84	0.89	0.76	0.00	2.16	0.87	0.82
time (sec)	N/A	0.291	0.090	0.415	0.035	0.091	0.000	0.114	0.157	10.100

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	55	57	44	46	42	0	98	46	55
N.S.	1	1.02	1.06	0.81	0.85	0.78	0.00	1.81	0.85	1.02
time (sec)	N/A	0.332	0.072	0.889	0.031	0.088	0.000	0.125	0.156	10.445

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	81	73	57	57	53	0	140	63	75
N.S.	1	1.07	0.96	0.75	0.75	0.70	0.00	1.84	0.83	0.99
time (sec)	N/A	0.400	0.122	1.315	0.039	0.086	0.000	0.126	0.174	10.449

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	93	89	69	69	64	0	154	74	113
N.S.	1	1.01	0.97	0.75	0.75	0.70	0.00	1.67	0.80	1.23
time (sec)	N/A	0.405	0.134	1.474	0.034	0.090	0.000	0.123	0.158	14.400

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	128	90	110	132	136	0	272	308	221
N.S.	1	0.95	0.67	0.81	0.98	1.01	0.00	2.01	2.28	1.64
time (sec)	N/A	0.756	0.434	1.997	0.036	0.098	0.000	0.156	0.160	13.328

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	102	82	111	144	133	0	258	378	184
N.S.	1	0.93	0.75	1.01	1.31	1.21	0.00	2.35	3.44	1.67
time (sec)	N/A	0.593	0.202	1.882	0.039	0.096	0.000	0.157	0.184	13.069

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	87	71	74	84	100	0	178	194	141
N.S.	1	1.09	0.89	0.92	1.05	1.25	0.00	2.22	2.42	1.76
time (sec)	N/A	0.583	0.174	1.758	0.033	0.093	0.000	0.147	0.162	12.578

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	51	69	80	93	0	129	211	99
N.S.	1	1.00	0.86	1.17	1.36	1.58	0.00	2.19	3.58	1.68
time (sec)	N/A	0.436	0.133	1.216	0.033	0.090	0.000	0.143	0.163	11.556

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	41	40	74	0	77	79	181
N.S.	1	1.00	0.97	1.24	1.21	2.24	0.00	2.33	2.39	5.48
time (sec)	N/A	0.291	0.055	0.825	0.031	0.112	0.000	0.127	0.183	10.523

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	46	43	51	52	0	78	59	73
N.S.	1	1.00	1.39	1.30	1.55	1.58	0.00	2.36	1.79	2.21
time (sec)	N/A	0.353	0.029	0.427	0.028	0.102	0.000	0.145	0.162	10.754

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	46	43	47	40	0	96	45	42
N.S.	1	1.00	0.92	0.86	0.94	0.80	0.00	1.92	0.90	0.84
time (sec)	N/A	0.352	0.146	0.435	0.029	0.082	0.000	0.132	0.166	10.709

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	66	59	61	60	52	0	153	63	72
N.S.	1	1.14	1.02	1.05	1.03	0.90	0.00	2.64	1.09	1.24
time (sec)	N/A	0.473	0.256	0.893	0.037	0.093	0.000	0.138	0.188	10.846

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	93	86	75	82	77	0	224	95	93
N.S.	1	0.92	0.85	0.74	0.81	0.76	0.00	2.22	0.94	0.92
time (sec)	N/A	0.472	0.191	1.342	0.035	0.085	0.000	0.144	0.173	10.895

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	113	85	95	94	86	0	247	107	117
N.S.	1	1.02	0.77	0.86	0.85	0.77	0.00	2.23	0.96	1.05
time (sec)	N/A	0.632	0.164	1.872	0.044	0.090	0.000	0.135	0.186	10.675

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	201	120	148	181	170	0	367	533	258
N.S.	1	1.06	0.63	0.78	0.96	0.90	0.00	1.94	2.82	1.37
time (sec)	N/A	1.249	0.598	2.387	0.037	0.123	0.000	0.168	0.210	14.326

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	141	90	125	158	140	0	330	426	226
N.S.	1	1.08	0.69	0.96	1.22	1.08	0.00	2.54	3.28	1.74
time (sec)	N/A	0.892	0.310	2.123	0.043	0.092	0.000	0.166	0.205	14.361

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	105	75	96	106	126	0	205	314	157
N.S.	1	1.06	0.76	0.97	1.07	1.27	0.00	2.07	3.17	1.59
time (sec)	N/A	0.654	0.299	1.674	0.037	0.097	0.000	0.162	0.200	12.824

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	74	67	82	93	112	0	145	239	136
N.S.	1	1.01	0.92	1.12	1.27	1.53	0.00	1.99	3.27	1.86
time (sec)	N/A	0.267	0.091	1.134	0.036	0.102	0.000	0.130	0.172	10.584

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	88	57	66	94	0	131	114	97
N.S.	1	1.00	1.31	0.85	0.99	1.40	0.00	1.96	1.70	1.45
time (sec)	N/A	0.560	0.820	0.879	0.035	0.102	0.000	0.156	0.169	10.641

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	80	105	73	76	72	0	137	94	123
N.S.	1	1.01	1.33	0.92	0.96	0.91	0.00	1.73	1.19	1.56
time (sec)	N/A	0.575	0.323	0.540	0.040	0.110	0.000	0.159	0.203	10.504

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	102	80	67	73	66	0	170	75	77
N.S.	1	1.02	0.80	0.67	0.73	0.66	0.00	1.70	0.75	0.77
time (sec)	N/A	0.625	0.201	0.925	0.031	0.088	0.000	0.141	0.167	10.639

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	117	100	90	95	84	0	297	112	250
N.S.	1	0.95	0.81	0.73	0.77	0.68	0.00	2.41	0.91	2.03
time (sec)	N/A	0.788	0.358	1.345	0.036	0.094	0.000	0.142	0.184	13.209

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	144	130	113	119	110	0	332	139	287
N.S.	1	0.90	0.81	0.71	0.74	0.69	0.00	2.08	0.87	1.79
time (sec)	N/A	0.783	0.444	1.976	0.036	0.092	0.000	0.158	0.190	14.171

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	164	159	135	145	132	0	431	178	350
N.S.	1	0.89	0.86	0.73	0.78	0.71	0.00	2.33	0.96	1.89
time (sec)	N/A	1.021	0.471	2.503	0.039	0.101	0.000	0.160	0.167	13.447

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	258	154	209	275	217	0	592	850	370
N.S.	1	1.06	0.63	0.86	1.13	0.89	0.00	2.43	3.48	1.52
time (sec)	N/A	1.633	0.665	3.171	0.043	0.106	0.000	0.188	0.205	14.873

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	195	125	162	195	182	0	461	584	304
N.S.	1	1.09	0.70	0.91	1.09	1.02	0.00	2.58	3.26	1.70
time (sec)	N/A	1.196	0.531	2.797	0.053	0.109	0.000	0.189	0.189	14.204

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	157	106	147	180	163	0	360	578	245
N.S.	1	1.08	0.73	1.01	1.23	1.12	0.00	2.47	3.96	1.68
time (sec)	N/A	0.961	0.594	2.370	0.041	0.095	0.000	0.175	0.185	13.635

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	114	96	109	121	138	0	221	356	185
N.S.	1	1.07	0.90	1.02	1.13	1.29	0.00	2.07	3.33	1.73
time (sec)	N/A	0.446	0.181	1.806	0.038	0.109	0.000	0.144	0.172	10.322

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	103	280	96	115	130	0	179	295	152
N.S.	1	0.99	2.69	0.92	1.11	1.25	0.00	1.72	2.84	1.46
time (sec)	N/A	0.874	1.232	1.477	0.036	0.109	0.000	0.172	0.189	10.198

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	119	87	90	116	0	170	168	150
N.S.	1	1.00	1.10	0.81	0.83	1.07	0.00	1.57	1.56	1.39
time (sec)	N/A	0.816	0.904	1.118	0.035	0.098	0.000	0.161	0.166	10.258

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	116	128	98	102	98	0	212	126	158
N.S.	1	1.01	1.11	0.85	0.89	0.85	0.00	1.84	1.10	1.37
time (sec)	N/A	0.900	0.374	1.105	0.037	0.101	0.000	0.164	0.178	10.366

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	153	104	101	109	96	0	318	124	123
N.S.	1	1.06	0.72	0.70	0.75	0.66	0.00	2.19	0.86	0.85
time (sec)	N/A	0.935	0.356	1.411	0.037	0.105	0.000	0.148	0.182	10.180

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	164	133	125	133	121	0	425	156	330
N.S.	1	0.95	0.77	0.72	0.77	0.70	0.00	2.46	0.90	1.91
time (sec)	N/A	1.127	0.632	2.013	0.036	0.090	0.000	0.160	0.178	13.964

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	194	156	153	170	150	0	550	210	214
N.S.	1	0.91	0.73	0.72	0.80	0.70	0.00	2.58	0.99	1.00
time (sec)	N/A	1.142	0.709	2.533	0.037	0.096	0.000	0.166	0.173	10.675

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	170	147	160	198	183	0	380	625	274
N.S.	1	1.08	0.93	1.01	1.25	1.16	0.00	2.41	3.96	1.73
time (sec)	N/A	0.681	0.402	2.516	0.039	0.111	0.000	0.146	0.189	11.135

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	174	258	252	0	557	0	286	623	1021
N.S.	1	1.11	1.64	1.61	0.00	3.55	0.00	1.82	3.97	6.50
time (sec)	N/A	1.330	2.054	0.733	0.000	0.204	0.000	0.171	0.176	12.048

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	129	238	192	0	485	0	211	479	1002
N.S.	1	1.08	2.00	1.61	0.00	4.08	0.00	1.77	4.03	8.42
time (sec)	N/A	0.898	0.976	0.575	0.000	0.200	0.000	0.161	0.169	10.728

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	88	115	123	0	392	0	152	200	119
N.S.	1	1.04	1.35	1.45	0.00	4.61	0.00	1.79	2.35	1.40
time (sec)	N/A	0.612	0.489	0.467	0.000	0.135	0.000	0.162	0.176	9.993

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	102	83	0	290	0	120	136	186
N.S.	1	1.00	1.50	1.22	0.00	4.26	0.00	1.76	2.00	2.74
time (sec)	N/A	0.494	0.171	0.404	0.000	0.134	0.000	0.160	0.171	10.118

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	48	44	0	185	0	77	64	40
N.S.	1	1.00	0.98	0.90	0.00	3.78	0.00	1.57	1.31	0.82
time (sec)	N/A	0.289	0.037	0.290	0.000	0.104	0.000	0.147	0.169	9.756

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	60	65	0	230	0	218	83	186
N.S.	1	1.00	1.02	1.10	0.00	3.90	0.00	3.69	1.41	3.15
time (sec)	N/A	0.293	0.130	0.321	0.000	0.102	0.000	0.124	0.194	9.994

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	79	72	97	0	277	0	126	120	395
N.S.	1	1.04	0.95	1.28	0.00	3.64	0.00	1.66	1.58	5.20
time (sec)	N/A	0.430	0.302	0.490	0.000	0.114	0.000	0.144	0.166	10.174

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	121	97	138	0	334	0	178	174	592
N.S.	1	1.10	0.88	1.25	0.00	3.04	0.00	1.62	1.58	5.38
time (sec)	N/A	0.826	0.348	0.591	0.000	0.109	0.000	0.143	0.210	10.659

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	165	122	179	0	401	0	249	210	654
N.S.	1	1.11	0.82	1.21	0.00	2.71	0.00	1.68	1.42	4.42
time (sec)	N/A	1.185	0.395	0.764	0.000	0.123	0.000	0.141	0.165	10.937

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	218	153	256	0	482	0	393	289	2678
N.S.	1	1.13	0.79	1.33	0.00	2.50	0.00	2.04	1.50	13.88
time (sec)	N/A	1.677	0.608	0.997	0.000	0.128	0.000	0.153	0.167	12.104

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	234	285	277	0	909	0	299	1637	3685
N.S.	1	1.05	1.28	1.25	0.00	4.09	0.00	1.35	7.37	16.60
time (sec)	N/A	1.659	5.324	0.953	0.000	0.511	0.000	0.163	0.192	16.207

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	183	162	209	0	760	0	331	977	3159
N.S.	1	1.12	0.99	1.27	0.00	4.63	0.00	2.02	5.96	19.26
time (sec)	N/A	1.118	1.006	0.728	0.000	0.303	0.000	0.163	0.175	15.367

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	142	146	166	0	596	0	203	568	2848
N.S.	1	1.21	1.25	1.42	0.00	5.09	0.00	1.74	4.85	24.34
time (sec)	N/A	0.800	0.453	0.593	0.000	0.298	0.000	0.152	0.164	15.673

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	96	83	118	0	329	0	150	192	92
N.S.	1	1.13	0.98	1.39	0.00	3.87	0.00	1.76	2.26	1.08
time (sec)	N/A	0.477	0.189	0.392	0.000	0.155	0.000	0.146	0.179	10.364

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	97	83	118	0	332	0	150	192	92
N.S.	1	1.13	0.97	1.37	0.00	3.86	0.00	1.74	2.23	1.07
time (sec)	N/A	0.444	0.284	0.369	0.000	0.109	0.000	0.135	0.159	10.443

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	134	138	151	0	484	0	179	380	2886
N.S.	1	1.23	1.27	1.39	0.00	4.44	0.00	1.64	3.49	26.48
time (sec)	N/A	0.695	0.495	0.418	0.000	0.129	0.000	0.125	0.163	15.594

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	166	172	184	0	565	0	837	521	3169
N.S.	1	1.14	1.18	1.26	0.00	3.87	0.00	5.73	3.57	21.71
time (sec)	N/A	0.980	0.839	0.694	0.000	0.119	0.000	0.208	0.181	15.653

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	219	144	224	0	660	0	264	623	3738
N.S.	1	1.05	0.69	1.08	0.00	3.17	0.00	1.27	3.00	17.97
time (sec)	N/A	1.529	0.824	0.894	0.000	0.144	0.000	0.135	0.164	16.620

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	268	176	263	0	757	0	335	686	3839
N.S.	1	1.03	0.67	1.01	0.00	2.90	0.00	1.28	2.63	14.71
time (sec)	N/A	2.036	1.068	1.137	0.000	0.152	0.000	0.151	0.171	16.983

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	267	205	296	0	1354	0	383	2086	5332
N.S.	1	1.16	0.89	1.29	0.00	5.89	0.00	1.67	9.07	23.18
time (sec)	N/A	1.852	3.546	1.288	0.000	0.660	0.000	0.199	0.195	17.685

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	233	194	255	0	1153	0	347	1406	5078
N.S.	1	1.24	1.03	1.36	0.00	6.13	0.00	1.85	7.48	27.01
time (sec)	N/A	1.198	1.105	0.858	0.000	0.665	0.000	0.198	0.170	18.463

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	175	113	184	0	594	0	253	626	204
N.S.	1	1.17	0.76	1.23	0.00	3.99	0.00	1.70	4.20	1.37
time (sec)	N/A	0.717	0.347	0.568	0.000	0.125	0.000	0.189	0.162	12.503

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	159	115	195	0	565	0	277	467	210
N.S.	1	1.19	0.86	1.46	0.00	4.22	0.00	2.07	3.49	1.57
time (sec)	N/A	0.776	0.382	0.539	0.000	0.119	0.000	0.190	0.204	12.577

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	159	115	186	0	595	0	254	633	204
N.S.	1	1.20	0.86	1.40	0.00	4.47	0.00	1.91	4.76	1.53
time (sec)	N/A	0.805	0.579	0.535	0.000	0.121	0.000	0.185	0.168	12.433

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	217	205	237	0	919	0	322	1004	5090
N.S.	1	1.25	1.18	1.37	0.00	5.31	0.00	1.86	5.80	29.42
time (sec)	N/A	1.087	0.754	0.507	0.000	0.144	0.000	0.139	0.169	18.308

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	259	229	270	0	1037	0	357	1271	5338
N.S.	1	1.16	1.03	1.21	0.00	4.65	0.00	1.60	5.70	23.94
time (sec)	N/A	1.662	1.006	1.011	0.000	0.161	0.000	0.188	0.193	17.693

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	321	199	310	0	1158	0	1723	1469	5950
N.S.	1	1.08	0.67	1.05	0.00	3.91	0.00	5.82	4.96	20.10
time (sec)	N/A	2.295	1.779	1.267	0.000	0.174	0.000	0.369	0.178	18.102

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	373	416	425	0	2058	0	592	3632	7476
N.S.	1	1.18	1.32	1.34	0.00	6.51	0.00	1.87	11.49	23.66
time (sec)	N/A	2.606	6.645	1.409	0.000	1.536	0.000	0.203	0.199	19.422

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	314	250	383	0	1822	0	559	2767	7222
N.S.	1	1.21	0.97	1.48	0.00	7.03	0.00	2.16	10.68	27.88
time (sec)	N/A	1.888	3.806	1.368	0.000	1.547	0.000	0.218	0.207	21.718

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	267	158	285	0	903	0	403	1065	378
N.S.	1	1.20	0.71	1.28	0.00	4.07	0.00	1.82	4.80	1.70
time (sec)	N/A	1.202	0.737	0.836	0.000	0.154	0.000	0.187	0.164	13.507

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	245	165	294	0	902	0	431	1080	380
N.S.	1	1.19	0.80	1.43	0.00	4.38	0.00	2.09	5.24	1.84
time (sec)	N/A	1.053	0.932	0.799	0.000	0.146	0.000	0.197	0.166	13.570

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	233	164	297	0	901	0	431	1095	382
N.S.	1	1.21	0.85	1.55	0.00	4.69	0.00	2.24	5.70	1.99
time (sec)	N/A	0.974	1.138	0.750	0.000	0.154	0.000	0.210	0.190	13.494

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	226	163	284	0	905	0	403	1076	378
N.S.	1	1.23	0.89	1.54	0.00	4.92	0.00	2.19	5.85	2.05
time (sec)	N/A	0.998	1.147	0.714	0.000	0.157	0.000	0.189	0.166	13.349

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	306	268	365	0	1456	0	532	1981	7234
N.S.	1	1.26	1.11	1.51	0.00	6.02	0.00	2.20	8.19	29.89
time (sec)	N/A	1.496	1.373	0.731	0.000	0.198	0.000	0.152	0.166	21.953

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	355	293	399	0	1603	0	564	2510	7534
N.S.	1	1.19	0.98	1.33	0.00	5.36	0.00	1.89	8.39	25.20
time (sec)	N/A	2.372	1.641	1.454	0.000	0.213	0.000	0.210	0.198	19.154

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	431	263	439	0	1767	0	615	2737	8133
N.S.	1	1.11	0.68	1.13	0.00	4.57	0.00	1.59	7.07	21.02
time (sec)	N/A	3.117	5.659	2.242	0.000	0.265	0.000	0.208	0.231	18.977

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	41	30	32	47	33	0	30	24	21
N.S.	1	1.32	0.97	1.03	1.52	1.06	0.00	0.97	0.77	0.68
time (sec)	N/A	0.250	0.113	0.444	0.110	0.106	0.000	0.119	0.157	10.241

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	71	73	58	88	73	0	59	74	52
N.S.	1	1.27	1.30	1.04	1.57	1.30	0.00	1.05	1.32	0.93
time (sec)	N/A	0.464	0.261	0.530	0.114	0.102	0.000	0.117	0.186	10.288

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	101	108	74	131	116	0	75	132	79
N.S.	1	1.25	1.33	0.91	1.62	1.43	0.00	0.93	1.63	0.98
time (sec)	N/A	0.645	0.393	0.566	0.112	0.109	0.000	0.116	0.160	10.341

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	131	141	87	171	159	0	88	204	105
N.S.	1	1.24	1.33	0.82	1.61	1.50	0.00	0.83	1.92	0.99
time (sec)	N/A	0.810	0.607	0.805	0.118	0.126	0.000	0.129	0.159	10.878

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	28	69	39	70	52	0	43	38	21
N.S.	1	0.40	0.99	0.56	1.00	0.74	0.00	0.61	0.54	0.30
time (sec)	N/A	0.243	0.110	0.379	0.108	0.104	0.000	0.120	0.183	10.001

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	58	162	76	111	102	0	69	129	52
N.S.	1	0.61	1.71	0.80	1.17	1.07	0.00	0.73	1.36	0.55
time (sec)	N/A	0.461	0.256	0.516	0.115	0.096	0.000	0.123	0.158	10.126

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	88	241	106	155	155	0	85	188	78
N.S.	1	0.73	2.01	0.88	1.29	1.29	0.00	0.71	1.57	0.65
time (sec)	N/A	0.629	0.376	0.589	0.109	0.101	0.000	0.129	0.154	9.924

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	118	344	132	194	208	0	98	288	105
N.S.	1	0.81	2.37	0.91	1.34	1.43	0.00	0.68	1.99	0.72
time (sec)	N/A	0.850	0.594	0.674	0.116	0.096	0.000	0.134	0.151	10.094

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	298	366	860	0	0	0	0	22	0
N.S.	1	1.02	1.25	2.95	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.105	27.516	12.661	0.000	0.000	0.000	0.000	0.170	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	242	293	503	0	0	0	0	22	0
N.S.	1	1.00	1.22	2.09	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.773	15.584	10.725	0.000	0.000	0.000	0.000	0.165	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	208	463	0	0	0	0	20	0
N.S.	1	1.00	1.00	2.22	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.563	8.394	8.060	0.000	0.000	0.000	0.000	0.163	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	151	182	0	0	0	0	13	0
N.S.	1	1.00	1.21	1.46	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.256	1.845	5.754	0.000	0.000	0.000	0.000	0.177	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	337	373	471	0	0	0	0	20	0
N.S.	1	1.02	1.13	1.43	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.184	9.970	3.728	0.000	0.000	0.000	0.000	0.165	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	394	668	721	0	0	0	0	22	0
N.S.	1	0.99	1.69	1.82	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.603	13.807	6.041	0.000	0.000	0.000	0.000	0.163	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	421	550	1406	0	0	0	0	49	0
N.S.	1	1.04	1.36	3.47	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.820	14.032	24.372	0.000	0.000	0.000	0.000	0.210	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	350	471	1028	0	0	0	0	49	0
N.S.	1	1.02	1.38	3.01	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	1.396	11.648	21.095	0.000	0.000	0.000	0.000	0.175	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	288	408	852	0	0	0	0	49	0
N.S.	1	1.02	1.45	3.02	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	1.050	10.885	15.671	0.000	0.000	0.000	0.000	0.186	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	250	304	598	0	0	0	0	47	0
N.S.	1	1.00	1.22	2.40	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.833	7.515	12.270	0.000	0.000	0.000	0.000	0.207	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	684	635	0	0	0	0	38	0
N.S.	1	1.00	2.21	2.06	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.992	16.592	8.132	0.000	0.000	0.000	0.000	0.185	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	341	332	579	0	0	0	0	51	0
N.S.	1	1.02	0.99	1.73	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	1.223	8.496	8.385	0.000	0.000	0.000	0.000	0.216	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	390	394	891	819	0	0	0	0	55	0
N.S.	1	1.01	2.28	2.10	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	1.667	16.264	6.948	0.000	0.000	0.000	0.000	0.214	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	481	615	1590	0	0	0	0	79	0
N.S.	1	1.04	1.33	3.43	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	2.263	13.792	73.257	0.000	0.000	0.000	0.000	0.198	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	409	552	1406	0	0	0	0	79	0
N.S.	1	1.03	1.38	3.52	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	1.730	13.139	40.947	0.000	0.000	0.000	0.000	0.218	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	344	474	1029	0	0	0	0	79	0
N.S.	1	1.03	1.42	3.09	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	1.363	11.343	23.665	0.000	0.000	0.000	0.000	0.197	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	302	440	958	0	0	0	0	77	0
N.S.	1	1.02	1.49	3.24	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	1.139	12.792	16.921	0.000	0.000	0.000	0.000	0.219	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	353	421	808	0	0	0	0	68	0
N.S.	1	1.00	1.20	2.30	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	1.250	10.429	11.785	0.000	0.000	0.000	0.000	0.191	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	360	454	918	0	0	0	0	87	0
N.S.	1	1.02	1.29	2.60	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	1.322	16.580	16.806	0.000	0.000	0.000	0.000	0.254	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	408	463	929	0	0	0	0	93	0
N.S.	1	1.02	1.16	2.33	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	1.746	13.512	37.277	0.000	0.000	0.000	0.000	0.274	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	477	1018	1075	0	0	0	0	93	0
N.S.	1	1.04	2.21	2.34	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	2.312	15.019	124.135	0.000	0.000	0.000	0.000	0.256	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	530	547	1274	1346	0	0	0	0	93	0
N.S.	1	1.03	2.40	2.54	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	2.953	14.209	392.748	0.000	0.000	0.000	0.000	0.270	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	409	873	1166	0	0	0	0	98	0
N.S.	1	1.01	2.17	2.89	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	1.636	13.544	15.880	0.000	0.000	0.000	0.000	0.201	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	379	463	1031	0	0	0	0	34	0
N.S.	1	1.06	1.29	2.87	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.597	11.780	22.671	0.000	0.000	0.000	0.000	0.164	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	313	365	858	0	0	0	0	34	0
N.S.	1	1.04	1.21	2.85	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	1.136	9.379	16.872	0.000	0.000	0.000	0.000	0.184	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	248	341	501	0	0	0	0	34	0
N.S.	1	1.02	1.40	2.05	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.801	11.077	13.658	0.000	0.000	0.000	0.000	0.168	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	238	364	0	0	0	0	34	0
N.S.	1	1.00	1.17	1.78	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.579	13.129	8.394	0.000	0.000	0.000	0.000	0.197	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	93	114	0	0	0	0	32	0
N.S.	1	1.00	0.94	1.15	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.257	1.901	6.319	0.000	0.000	0.000	0.000	0.171	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	138	149	0	0	0	0	26	0
N.S.	1	1.00	1.30	1.41	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.243	2.034	4.620	0.000	0.000	0.000	0.000	0.173	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	346	239	376	0	0	0	0	32	0
N.S.	1	1.02	0.71	1.11	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.176	5.727	7.368	0.000	0.000	0.000	0.000	0.172	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	401	403	682	722	0	0	0	0	34	0
N.S.	1	1.00	1.70	1.80	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.646	12.341	8.506	0.000	0.000	0.000	0.000	0.172	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	411	455	1357	0	0	0	0	50	0
N.S.	1	1.03	1.14	3.40	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	1.795	10.985	20.224	0.000	0.000	0.000	0.000	0.169	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	345	470	983	0	0	0	0	50	0
N.S.	1	1.06	1.45	3.02	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	1.280	11.544	14.702	0.000	0.000	0.000	0.000	0.169	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	283	395	810	0	0	0	0	50	0
N.S.	1	1.10	1.54	3.15	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.909	9.856	11.647	0.000	0.000	0.000	0.000	0.175	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	261	249	490	0	0	0	0	50	0
N.S.	1	1.10	1.05	2.07	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.831	7.290	7.474	0.000	0.000	0.000	0.000	0.172	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	259	244	478	0	0	0	0	48	0
N.S.	1	1.10	1.03	2.03	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.768	7.313	3.598	0.000	0.000	0.000	0.000	0.167	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	376	751	654	0	0	0	0	42	0
N.S.	1	1.08	2.16	1.88	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.345	12.712	6.652	0.000	0.000	0.000	0.000	0.184	0.000

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	436	1069	949	0	0	0	0	48	0
N.S.	1	1.10	2.70	2.40	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.757	13.659	9.218	0.000	0.000	0.000	0.000	0.163	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	470	504	1330	1318	0	0	0	0	50	0
N.S.	1	1.07	2.83	2.80	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	2.250	12.893	10.474	0.000	0.000	0.000	0.000	0.175	0.000

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	427	447	578	1855	0	0	0	0	66	0
N.S.	1	1.05	1.35	4.34	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	2.002	13.929	18.115	0.000	0.000	0.000	0.000	0.192	0.000

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	388	556	1662	0	0	0	0	66	0
N.S.	1	1.07	1.54	4.59	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	1.380	14.044	13.088	0.000	0.000	0.000	0.000	0.174	0.000

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	356	503	1292	0	0	0	0	66	0
N.S.	1	1.06	1.49	3.83	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	1.272	11.717	11.128	0.000	0.000	0.000	0.000	0.176	0.000

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	336	486	1171	0	0	0	0	66	0
N.S.	1	1.06	1.53	3.69	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	1.166	11.065	7.149	0.000	0.000	0.000	0.000	0.184	0.000

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	323	360	925	0	0	0	0	64	0
N.S.	1	1.06	1.18	3.04	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	1.131	5.860	8.263	0.000	0.000	0.000	0.000	0.166	0.000

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	478	1382	1935	0	0	0	0	58	0
N.S.	1	1.07	3.08	4.32	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	1.931	12.559	8.914	0.000	0.000	0.000	0.000	0.164	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	510	548	1481	2242	0	0	0	0	64	0
N.S.	1	1.07	2.90	4.40	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	2.303	14.829	11.641	0.000	0.000	0.000	0.000	0.168	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	562	604	1730	2747	0	0	0	0	66	0
N.S.	1	1.07	3.08	4.89	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	2.890	13.588	13.270	0.000	0.000	0.000	0.000	0.167	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	535	584	1790	3259	0	0	0	0	74	0
N.S.	1	1.09	3.35	6.09	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	2.475	13.151	11.300	0.000	0.000	0.000	0.000	0.164	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	153	97	502	0	188	0	0	41	0
N.S.	1	1.01	0.64	3.32	0.00	1.25	0.00	0.00	0.27	0.00
time (sec)	N/A	0.670	0.403	6.854	0.000	0.112	0.000	0.000	0.180	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	125	85	396	0	167	0	0	39	0
N.S.	1	1.02	0.69	3.22	0.00	1.36	0.00	0.00	0.32	0.00
time (sec)	N/A	0.543	0.303	2.983	0.000	0.107	0.000	0.000	0.199	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	98	71	150	0	124	0	0	30	0
N.S.	1	1.01	0.73	1.55	0.00	1.28	0.00	0.00	0.31	0.00
time (sec)	N/A	0.475	0.202	1.867	0.000	0.116	0.000	0.000	0.177	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	52	152	0	107	0	0	32	0
N.S.	1	1.00	0.69	2.03	0.00	1.43	0.00	0.00	0.43	0.00
time (sec)	N/A	0.376	0.219	2.145	0.000	0.120	0.000	0.000	0.168	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	102	76	228	0	125	0	0	41	0
N.S.	1	1.01	0.75	2.26	0.00	1.24	0.00	0.00	0.41	0.00
time (sec)	N/A	0.486	0.242	4.404	0.000	0.101	0.000	0.000	0.197	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	129	88	262	0	145	0	0	41	0
N.S.	1	1.02	0.69	2.06	0.00	1.14	0.00	0.00	0.32	0.00
time (sec)	N/A	0.525	0.388	7.329	0.000	0.103	0.000	0.000	0.174	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	157	99	290	0	156	0	0	41	0
N.S.	1	1.04	0.66	1.92	0.00	1.03	0.00	0.00	0.27	0.00
time (sec)	N/A	0.655	0.561	10.829	0.000	0.110	0.000	0.000	0.178	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	194	139	689	0	235	0	0	67	0
N.S.	1	0.97	0.70	3.44	0.00	1.18	0.00	0.00	0.34	0.00
time (sec)	N/A	1.062	0.970	22.230	0.000	0.121	0.000	0.000	0.202	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	166	126	633	0	223	0	0	65	0
N.S.	1	0.95	0.72	3.62	0.00	1.27	0.00	0.00	0.37	0.00
time (sec)	N/A	0.917	1.163	6.143	0.000	0.105	0.000	0.000	0.185	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	136	93	513	0	190	0	0	56	0
N.S.	1	1.01	0.69	3.80	0.00	1.41	0.00	0.00	0.41	0.00
time (sec)	N/A	0.769	0.501	4.514	0.000	0.117	0.000	0.000	0.194	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	82	202	0	146	0	0	56	0
N.S.	1	1.00	0.76	1.87	0.00	1.35	0.00	0.00	0.52	0.00
time (sec)	N/A	0.634	0.743	4.394	0.000	0.111	0.000	0.000	0.198	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	87	283	0	147	0	0	58	0
N.S.	1	1.00	0.78	2.53	0.00	1.31	0.00	0.00	0.52	0.00
time (sec)	N/A	0.629	0.606	4.808	0.000	0.108	0.000	0.000	0.185	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	142	100	357	0	170	0	0	67	0
N.S.	1	1.01	0.71	2.53	0.00	1.21	0.00	0.00	0.48	0.00
time (sec)	N/A	0.777	0.846	7.788	0.000	0.112	0.000	0.000	0.183	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	170	120	362	0	191	0	0	67	0
N.S.	1	0.97	0.69	2.07	0.00	1.09	0.00	0.00	0.38	0.00
time (sec)	N/A	0.897	1.055	11.586	0.000	0.105	0.000	0.000	0.212	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	218	177	820	0	270	0	0	91	0
N.S.	1	0.93	0.76	3.50	0.00	1.15	0.00	0.00	0.39	0.00
time (sec)	N/A	1.290	2.929	8.431	0.000	0.120	0.000	0.000	0.223	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	181	134	711	0	244	0	0	82	0
N.S.	1	0.96	0.71	3.76	0.00	1.29	0.00	0.00	0.43	0.00
time (sec)	N/A	1.146	1.576	6.389	0.000	0.117	0.000	0.000	0.201	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	159	106	630	0	214	0	0	82	0
N.S.	1	1.01	0.67	3.99	0.00	1.35	0.00	0.00	0.52	0.00
time (sec)	N/A	0.981	1.013	5.553	0.000	0.116	0.000	0.000	0.184	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	167	108	303	0	182	0	0	82	0
N.S.	1	1.01	0.65	1.83	0.00	1.10	0.00	0.00	0.49	0.00
time (sec)	N/A	0.969	0.913	6.085	0.000	0.115	0.000	0.000	0.187	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	157	106	412	0	193	0	0	84	0
N.S.	1	1.01	0.68	2.64	0.00	1.24	0.00	0.00	0.54	0.00
time (sec)	N/A	0.972	0.812	7.988	0.000	0.107	0.000	0.000	0.209	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	194	132	421	0	216	0	0	93	0
N.S.	1	0.97	0.66	2.12	0.00	1.09	0.00	0.00	0.47	0.00
time (sec)	N/A	1.208	1.267	12.095	0.000	0.115	0.000	0.000	0.183	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	222	159	470	0	238	0	0	93	0
N.S.	1	0.95	0.68	2.01	0.00	1.02	0.00	0.00	0.40	0.00
time (sec)	N/A	1.380	1.535	15.171	0.000	0.113	0.000	0.000	0.194	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	269	256	1147	0	318	0	0	117	0
N.S.	1	0.94	0.89	4.00	0.00	1.11	0.00	0.00	0.41	0.00
time (sec)	N/A	1.757	2.628	10.618	0.000	0.138	0.000	0.000	0.230	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	243	168	898	0	289	0	0	108	0
N.S.	1	0.98	0.68	3.64	0.00	1.17	0.00	0.00	0.44	0.00
time (sec)	N/A	1.605	1.956	8.611	0.000	0.112	0.000	0.000	0.204	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	213	146	880	0	264	0	0	108	0
N.S.	1	1.02	0.70	4.21	0.00	1.26	0.00	0.00	0.52	0.00
time (sec)	N/A	1.404	2.484	7.384	0.000	0.115	0.000	0.000	0.199	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	207	130	777	0	242	0	0	108	0
N.S.	1	1.00	0.62	3.74	0.00	1.16	0.00	0.00	0.52	0.00
time (sec)	N/A	1.357	1.517	7.050	0.000	0.146	0.000	0.000	0.209	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	211	138	432	0	215	0	0	108	0
N.S.	1	1.02	0.67	2.09	0.00	1.04	0.00	0.00	0.52	0.00
time (sec)	N/A	1.395	1.121	8.698	0.000	0.111	0.000	0.000	0.194	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	219	142	476	0	234	0	0	110	0
N.S.	1	1.04	0.67	2.26	0.00	1.11	0.00	0.00	0.52	0.00
time (sec)	N/A	1.402	1.429	11.438	0.000	0.120	0.000	0.000	0.203	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	168	529	0	257	0	0	119	0
N.S.	1	1.00	0.69	2.16	0.00	1.05	0.00	0.00	0.49	0.00
time (sec)	N/A	1.591	1.851	17.096	0.000	0.119	0.000	0.000	0.190	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	275	199	586	0	286	0	0	119	0
N.S.	1	0.95	0.69	2.03	0.00	0.99	0.00	0.00	0.41	0.00
time (sec)	N/A	1.898	2.278	20.932	0.000	0.135	0.000	0.000	0.187	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	201	165	423	0	0	0	0	30	0
N.S.	1	1.07	0.88	2.25	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	1.549	35.369	10.103	0.000	0.000	0.000	0.000	0.162	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	116	83	353	0	0	0	0	30	0
N.S.	1	0.99	0.71	3.02	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.762	15.608	8.863	0.000	0.000	0.000	0.000	0.163	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	63	150	0	0	0	0	28	0
N.S.	1	1.00	1.29	3.06	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.327	0.287	0.841	0.000	0.000	0.000	0.000	0.169	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	74	47	187	0	0	0	0	22	0
N.S.	1	0.80	0.51	2.01	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.492	0.380	1.830	0.000	0.000	0.000	0.000	0.163	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	176	226	0	0	0	0	31	0
N.S.	1	1.00	1.30	1.67	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.908	19.729	3.555	0.000	0.000	0.000	0.000	0.189	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	178	194	552	0	0	0	0	33	0
N.S.	1	1.03	1.13	3.21	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	1.224	21.601	3.859	0.000	0.000	0.000	0.000	0.165	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	325	294	975	0	0	0	0	46	0
N.S.	1	0.95	0.86	2.85	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	2.502	5.094	35.183	0.000	0.000	0.000	0.000	0.165	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	266	351	841	0	0	0	0	46	0
N.S.	1	0.95	1.26	3.01	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	1.946	3.946	34.046	0.000	0.000	0.000	0.000	0.184	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-1)	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	212	582	608	0	0	0	0	46	0
N.S.	1	0.99	2.72	2.84	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	1.423	6.448	31.972	0.000	0.000	0.000	0.000	0.164	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	202	289	707	0	0	0	0	44	0
N.S.	1	0.97	1.39	3.40	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	1.334	5.645	4.148	0.000	0.000	0.000	0.000	0.161	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	216	251	788	0	0	0	0	38	0
N.S.	1	0.95	1.11	3.47	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	1.342	2.999	4.734	0.000	0.000	0.000	0.000	0.161	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	232	319	809	0	0	0	0	47	0
N.S.	1	0.95	1.31	3.32	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	1.484	4.246	6.049	0.000	0.000	0.000	0.000	0.200	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	289	279	1064	0	0	0	0	49	0
N.S.	1	0.95	0.92	3.50	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	2.006	4.926	6.003	0.000	0.000	0.000	0.000	0.168	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	382	532	1987	0	0	0	0	62	0
N.S.	1	0.98	1.37	5.12	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	2.861	6.540	154.424	0.000	0.000	0.000	0.000	0.169	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	318	335	1203	0	0	0	0	62	0
N.S.	1	1.01	1.06	3.82	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	2.185	5.597	148.674	0.000	0.000	0.000	0.000	0.202	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	310	428	1760	0	0	0	0	62	0
N.S.	1	0.99	1.37	5.62	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	2.139	5.640	151.488	0.000	0.000	0.000	0.000	0.170	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	307	429	1858	0	0	0	0	60	0
N.S.	1	1.00	1.40	6.07	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	2.033	5.439	6.530	0.000	0.000	0.000	0.000	0.165	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	321	286	1936	0	0	0	0	54	0
N.S.	1	0.99	0.89	5.99	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	2.107	4.822	6.883	0.000	0.000	0.000	0.000	0.187	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-1)	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	707	1957	0	0	0	0	63	0
N.S.	1	1.00	2.07	5.72	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	2.294	6.626	8.008	0.000	0.000	0.000	0.000	0.164	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	403	731	2216	0	0	0	0	65	0
N.S.	1	0.99	1.80	5.46	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	2.866	6.751	8.243	0.000	0.000	0.000	0.000	0.168	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	254	321	480	0	0	0	0	27	0
N.S.	1	1.07	1.35	2.03	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	2.326	24.865	6.886	0.000	0.000	0.000	0.000	0.183	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	96	251	0	0	0	0	21	0
N.S.	1	1.00	0.70	1.82	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	1.142	12.629	5.436	0.000	0.000	0.000	0.000	0.196	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	538	0	355	0	0	29	0
N.S.	1	1.00	1.00	8.03	0.00	5.30	0.00	0.00	0.43	0.00
time (sec)	N/A	0.400	0.180	3.353	0.000	0.154	0.000	0.000	0.168	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	193	156	572	0	415	0	0	29	0
N.S.	1	1.01	0.81	2.98	0.00	2.16	0.00	0.00	0.15	0.00
time (sec)	N/A	1.387	0.561	6.285	0.000	0.145	0.000	0.000	0.167	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	253	203	984	0	461	0	0	29	0
N.S.	1	1.04	0.83	4.03	0.00	1.89	0.00	0.00	0.12	0.00
time (sec)	N/A	1.914	0.819	8.434	0.000	0.187	0.000	0.000	0.193	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	322	237	1164	0	501	0	0	29	0
N.S.	1	1.06	0.78	3.82	0.00	1.64	0.00	0.00	0.10	0.00
time (sec)	N/A	2.459	1.086	10.733	0.000	0.191	0.000	0.000	0.169	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	308	411	1030	0	0	0	0	61	0
N.S.	1	1.03	1.37	3.44	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	3.246	4.217	10.431	0.000	0.000	0.000	0.000	0.233	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	255	394	711	0	0	0	0	53	0
N.S.	1	1.02	1.58	2.86	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	2.494	14.154	7.535	0.000	0.000	0.000	0.000	0.223	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	129	745	0	0	0	0	55	0
N.S.	1	1.00	0.62	3.56	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	2.065	26.794	7.490	0.000	0.000	0.000	0.000	0.187	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	188	156	680	0	415	0	0	63	0
N.S.	1	1.01	0.83	3.64	0.00	2.22	0.00	0.00	0.34	0.00
time (sec)	N/A	1.416	0.597	4.980	0.000	0.174	0.000	0.000	0.187	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	238	197	977	0	463	0	0	63	0
N.S.	1	0.99	0.82	4.07	0.00	1.93	0.00	0.00	0.26	0.00
time (sec)	N/A	1.867	0.952	4.973	0.000	0.199	0.000	0.000	0.215	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	311	237	1164	0	501	0	0	63	0
N.S.	1	1.03	0.78	3.84	0.00	1.65	0.00	0.00	0.21	0.00
time (sec)	N/A	2.501	1.293	6.218	0.000	0.129	0.000	0.000	0.190	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	390	602	1331	0	0	0	0	98	0
N.S.	1	1.06	1.63	3.61	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	4.138	6.532	15.947	0.000	0.000	0.000	0.000	0.276	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	324	560	1136	0	0	0	0	90	0
N.S.	1	1.03	1.78	3.62	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	3.439	6.435	7.703	0.000	0.000	0.000	0.000	0.238	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	270	421	1051	0	0	0	0	90	0
N.S.	1	1.03	1.60	4.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	2.712	5.521	6.416	0.000	0.000	0.000	0.000	0.214	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	263	409	914	0	0	0	0	92	0
N.S.	1	1.00	1.56	3.49	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	2.741	3.895	5.880	0.000	0.000	0.000	0.000	0.221	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	250	200	1089	0	462	0	0	100	0
N.S.	1	1.05	0.84	4.56	0.00	1.93	0.00	0.00	0.42	0.00
time (sec)	N/A	1.978	1.080	5.055	0.000	0.159	0.000	0.000	0.195	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	312	237	1164	0	501	0	0	100	0
N.S.	1	1.03	0.78	3.84	0.00	1.65	0.00	0.00	0.33	0.00
time (sec)	N/A	2.476	1.525	5.204	0.000	0.150	0.000	0.000	0.196	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	378	286	1570	0	541	0	0	100	0
N.S.	1	1.04	0.79	4.33	0.00	1.49	0.00	0.00	0.28	0.00
time (sec)	N/A	3.235	2.064	7.546	0.000	0.170	0.000	0.000	0.208	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	397	1043	0	0	0	0	41	0
N.S.	1	1.00	1.27	3.34	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	3.040	4.316	8.682	0.000	0.000	0.000	0.000	0.165	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	259	329	597	0	0	0	0	41	0
N.S.	1	1.05	1.34	2.43	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	2.326	15.384	5.423	0.000	0.000	0.000	0.000	0.163	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	203	0	0	0	0	39	0
N.S.	1	1.00	1.00	2.99	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.516	0.286	3.460	0.000	0.000	0.000	0.000	0.187	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	141	0	146	0	0	33	0
N.S.	1	1.00	1.00	2.10	0.00	2.18	0.00	0.00	0.49	0.00
time (sec)	N/A	0.412	0.170	2.977	0.000	0.122	0.000	0.000	0.165	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	96	437	0	355	0	0	42	0
N.S.	1	1.00	0.68	3.08	0.00	2.50	0.00	0.00	0.30	0.00
time (sec)	N/A	1.045	2.952	4.418	0.000	0.145	0.000	0.000	0.164	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	199	147	583	0	415	0	0	44	0
N.S.	1	1.02	0.75	2.99	0.00	2.13	0.00	0.00	0.23	0.00
time (sec)	N/A	1.443	0.650	5.442	0.000	0.128	0.000	0.000	0.182	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	261	193	984	0	464	0	0	44	0
N.S.	1	1.05	0.78	3.95	0.00	1.86	0.00	0.00	0.18	0.00
time (sec)	N/A	1.967	0.746	6.723	0.000	0.129	0.000	0.000	0.165	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	360	478	898	0	0	0	0	57	0
N.S.	1	1.04	1.39	2.60	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	3.659	2.981	8.115	0.000	0.000	0.000	0.000	0.173	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	214	434	661	0	0	0	0	57	0
N.S.	1	1.04	2.11	3.21	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	1.733	4.307	4.681	0.000	0.000	0.000	0.000	0.170	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	103	311	0	488	0	0	55	0
N.S.	1	1.00	0.82	2.47	0.00	3.87	0.00	0.00	0.44	0.00
time (sec)	N/A	0.634	0.294	0.308	0.000	0.142	0.000	0.000	0.189	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	211	156	317	0	525	0	0	49	0
N.S.	1	1.06	0.78	1.58	0.00	2.62	0.00	0.00	0.24	0.00
time (sec)	N/A	1.449	0.580	4.147	0.000	0.147	0.000	0.000	0.168	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	228	165	593	0	565	0	0	58	0
N.S.	1	1.07	0.77	2.77	0.00	2.64	0.00	0.00	0.27	0.00
time (sec)	N/A	1.591	0.659	5.467	0.000	0.153	0.000	0.000	0.169	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	295	203	739	0	632	0	0	60	0
N.S.	1	1.02	0.70	2.56	0.00	2.19	0.00	0.00	0.21	0.00
time (sec)	N/A	2.158	0.819	6.894	0.000	0.216	0.000	0.000	0.195	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	359	250	1045	0	692	0	0	60	0
N.S.	1	1.00	0.69	2.90	0.00	1.92	0.00	0.00	0.17	0.00
time (sec)	N/A	2.836	1.173	7.963	0.000	0.176	0.000	0.000	0.173	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	458	471	561	2354	0	0	0	0	73	0
N.S.	1	1.03	1.22	5.14	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	4.751	4.966	9.529	0.000	0.000	0.000	0.000	0.171	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	390	487	1916	0	0	0	0	73	0
N.S.	1	1.05	1.32	5.18	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	3.940	3.706	5.730	0.000	0.000	0.000	0.000	0.167	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	295	169	763	0	688	0	0	73	0
N.S.	1	1.06	0.61	2.75	0.00	2.48	0.00	0.00	0.26	0.00
time (sec)	N/A	2.091	0.834	3.975	0.000	0.174	0.000	0.000	0.197	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	288	178	1024	0	759	0	0	71	0
N.S.	1	1.02	0.63	3.64	0.00	2.70	0.00	0.00	0.25	0.00
time (sec)	N/A	2.019	0.904	2.769	0.000	0.213	0.000	0.000	0.172	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	314	196	1149	0	811	0	0	65	0
N.S.	1	1.04	0.65	3.80	0.00	2.69	0.00	0.00	0.22	0.00
time (sec)	N/A	2.176	1.136	5.303	0.000	0.154	0.000	0.000	0.170	0.000

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	332	208	1600	0	863	0	0	74	0
N.S.	1	1.05	0.66	5.05	0.00	2.72	0.00	0.00	0.23	0.00
time (sec)	N/A	2.353	1.138	6.784	0.000	0.180	0.000	0.000	0.217	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	399	257	1743	0	949	0	0	76	0
N.S.	1	1.02	0.66	4.46	0.00	2.43	0.00	0.00	0.19	0.00
time (sec)	N/A	3.066	1.439	8.218	0.000	0.188	0.000	0.000	0.182	0.000

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	474	479	292	2210	0	1036	0	0	76	0
N.S.	1	1.01	0.62	4.66	0.00	2.19	0.00	0.00	0.16	0.00
time (sec)	N/A	3.840	1.720	9.461	0.000	0.280	0.000	0.000	0.168	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	78	234	0	95	0	0	42	0
N.S.	1	1.00	0.64	1.92	0.00	0.78	0.00	0.00	0.34	0.00
time (sec)	N/A	0.688	0.304	3.850	0.000	0.119	0.000	0.000	0.200	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	68	217	0	92	0	0	42	0
N.S.	1	1.00	0.62	1.99	0.00	0.84	0.00	0.00	0.39	0.00
time (sec)	N/A	0.688	0.274	3.105	0.000	0.103	0.000	0.000	0.170	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	68	235	0	95	0	0	44	0
N.S.	1	1.00	0.63	2.18	0.00	0.88	0.00	0.00	0.41	0.00
time (sec)	N/A	0.911	0.158	2.998	0.000	0.119	0.000	0.000	0.173	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	78	226	0	87	0	0	44	0
N.S.	1	1.00	0.63	1.84	0.00	0.71	0.00	0.00	0.36	0.00
time (sec)	N/A	0.938	0.166	3.498	0.000	0.121	0.000	0.000	0.202	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	81	236	0	108	0	0	42	0
N.S.	1	1.00	0.64	1.86	0.00	0.85	0.00	0.00	0.33	0.00
time (sec)	N/A	0.682	0.302	3.805	0.000	0.113	0.000	0.000	0.173	0.000

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	72	223	0	108	0	0	44	0
N.S.	1	1.00	0.64	1.97	0.00	0.96	0.00	0.00	0.39	0.00
time (sec)	N/A	0.722	0.285	3.329	0.000	0.108	0.000	0.000	0.176	0.000

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	72	215	0	108	0	0	42	0
N.S.	1	1.00	0.56	1.67	0.00	0.84	0.00	0.00	0.33	0.00
time (sec)	N/A	0.675	0.152	2.980	0.000	0.106	0.000	0.000	0.176	0.000

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	81	228	0	108	0	0	44	0
N.S.	1	1.00	0.70	1.98	0.00	0.94	0.00	0.00	0.38	0.00
time (sec)	N/A	0.728	0.176	3.270	0.000	0.107	0.000	0.000	0.190	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	123	0	47	0	0	33	0
N.S.	1	1.00	1.00	2.02	0.00	0.77	0.00	0.00	0.54	0.00
time (sec)	N/A	0.314	0.174	1.127	0.000	0.112	0.000	0.000	0.188	0.000

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	115	0	44	0	0	33	0
N.S.	1	1.00	1.00	2.13	0.00	0.81	0.00	0.00	0.61	0.00
time (sec)	N/A	0.298	0.160	0.988	0.000	0.111	0.000	0.000	0.184	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	123	0	47	0	0	35	0
N.S.	1	1.00	1.00	2.28	0.00	0.87	0.00	0.00	0.65	0.00
time (sec)	N/A	0.363	0.122	1.004	0.000	0.102	0.000	0.000	0.207	0.000

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	119	0	44	0	0	35	0
N.S.	1	1.00	1.00	1.95	0.00	0.72	0.00	0.00	0.57	0.00
time (sec)	N/A	0.379	0.106	1.155	0.000	0.098	0.000	0.000	0.213	0.000

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	124	0	54	0	0	33	0
N.S.	1	1.00	1.00	2.03	0.00	0.89	0.00	0.00	0.54	0.00
time (sec)	N/A	0.296	0.161	1.151	0.000	0.134	0.000	0.000	0.156	0.000

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	117	0	54	0	0	35	0
N.S.	1	1.00	1.00	2.17	0.00	1.00	0.00	0.00	0.65	0.00
time (sec)	N/A	0.310	0.166	1.049	0.000	0.112	0.000	0.000	0.196	0.000

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	54	114	0	52	0	0	33	0
N.S.	1	1.00	0.87	1.84	0.00	0.84	0.00	0.00	0.53	0.00
time (sec)	N/A	0.294	0.108	0.985	0.000	0.108	0.000	0.000	0.167	0.000

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	61	120	0	52	0	0	35	0
N.S.	1	1.00	1.11	2.18	0.00	0.95	0.00	0.00	0.64	0.00
time (sec)	N/A	0.295	0.118	1.157	0.000	0.103	0.000	0.000	0.169	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	7160	0	0	0	0	0	21	0
N.S.	1	1.00	68.19	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.299	48.269	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	0	14	14	14	16
N.S.	1	1.00	1.14	0.86	1.00	0.00	1.00	1.00	1.00	1.14
time (sec)	N/A	0.185	17.190	0.075	0.390	0.000	0.492	0.198	0.162	10.053

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	362	376	21877	0	0	0	0	0	23	0
N.S.	1	1.04	60.43	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.327	45.109	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	305	311	7783	0	0	0	0	0	23	0
N.S.	1	1.02	25.52	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.951	44.279	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	10909	0	0	0	0	0	23	0
N.S.	1	1.00	41.96	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.678	41.583	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	7142	0	0	0	0	0	21	0
N.S.	1	1.00	68.02	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.272	44.805	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	0	14	14	14	16
N.S.	1	1.00	1.14	0.86	1.00	0.00	1.00	1.00	1.00	1.14
time (sec)	N/A	0.185	18.243	0.075	0.386	0.000	0.855	0.195	0.162	9.849

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	108	7313	0	0	0	0	0	49	0
N.S.	1	1.03	69.65	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.291	47.902	0.000	0.000	0.000	0.000	0.000	0.215	0.000

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	0	14	14	40	16
N.S.	1	1.00	1.14	0.86	1.00	0.00	1.00	1.00	2.86	1.14
time (sec)	N/A	0.186	99.329	0.073	0.395	0.000	13.310	0.617	0.183	10.637

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	412	428	28057	0	0	0	0	0	51	0
N.S.	1	1.04	68.10	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.672	45.678	0.000	0.000	0.000	0.000	0.000	0.209	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	356	364	21890	0	0	0	0	0	51	0
N.S.	1	1.02	61.49	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	1.251	45.258	0.000	0.000	0.000	0.000	0.000	0.236	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	299	304	7809	0	0	0	0	0	51	0
N.S.	1	1.02	26.12	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.941	44.431	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	108	7321	0	0	0	0	0	49	0
N.S.	1	1.03	69.72	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.278	44.486	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	0	14	14	40	16
N.S.	1	1.00	1.14	0.86	1.00	0.00	1.00	1.00	2.86	1.14
time (sec)	N/A	0.182	97.434	0.073	0.401	0.000	43.072	0.682	0.216	11.229

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	313	324	7796	0	0	0	0	0	23	0
N.S.	1	1.04	24.91	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.017	44.456	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	265	268	7195	0	0	0	0	0	23	0
N.S.	1	1.01	27.15	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.702	43.059	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	1748	0	0	0	0	0	23	0
N.S.	1	1.00	7.98	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.499	23.456	0.000	0.000	0.000	0.000	0.000	0.189	0.000

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	310	0	0	0	0	0	21	0
N.S.	1	1.00	2.95	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.280	14.220	0.000	0.000	0.000	0.000	0.000	0.154	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	0	15	14	14	16
N.S.	1	1.00	1.14	0.86	1.00	0.00	1.07	1.00	1.00	1.14
time (sec)	N/A	0.181	1.412	0.075	0.397	0.000	0.338	0.293	0.157	10.403

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	310	0	0	0	0	0	21	0
N.S.	1	1.00	2.95	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.285	15.107	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	0	15	14	14	16
N.S.	1	1.00	1.14	0.86	1.00	0.00	1.07	1.00	1.00	1.14
time (sec)	N/A	0.183	1.442	0.074	0.392	0.000	0.382	0.391	0.168	10.281

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	110	10343	0	0	0	0	0	46	0
N.S.	1	1.05	98.50	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.289	46.390	0.000	0.000	0.000	0.000	0.000	0.158	0.000

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	0	15	14	39	16
N.S.	1	1.00	1.14	0.86	1.00	0.00	1.07	1.00	2.79	1.14
time (sec)	N/A	0.179	88.933	0.073	0.398	0.000	1.259	0.553	0.158	11.926

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F(-1)	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	378	367	8160	0	0	0	0	0	48	0
N.S.	1	0.97	21.59	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	1.172	43.588	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	307	304	7918	0	0	0	0	0	48	0
N.S.	1	0.99	25.79	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.822	43.088	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	289	283	7325	0	0	0	0	0	48	0
N.S.	1	0.98	25.35	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.746	46.109	0.000	0.000	0.000	0.000	0.000	0.162	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	110	10363	0	0	0	0	0	46	0
N.S.	1	1.05	98.70	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.287	49.078	0.000	0.000	0.000	0.000	0.000	0.159	0.000

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	0	15	14	39	16
N.S.	1	1.00	1.14	0.86	1.00	0.00	1.07	1.00	2.79	1.14
time (sec)	N/A	0.185	84.392	0.075	0.386	0.000	2.202	0.723	0.181	11.610

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	195	4543	0	0	0	0	0	23	0
N.S.	1	1.12	26.11	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.623	36.232	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	195	4544	0	0	0	0	0	23	0
N.S.	1	1.12	26.11	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.601	36.113	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	195	7430	0	0	0	0	0	25	0
N.S.	1	1.12	42.70	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.613	77.989	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	195	7588	0	0	0	0	0	25	0
N.S.	1	1.12	43.61	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.602	78.571	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	0	21	23	23	0	23	22	27
N.S.	1	1.00	0.00	0.84	0.92	0.92	0.00	0.92	0.88	1.08
time (sec)	N/A	0.232	0.000	0.228	0.593	0.223	0.000	1.041	0.187	11.301

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	0	23	22	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.00	0.92	0.88	1.08
time (sec)	N/A	0.235	173.727	0.185	0.654	0.281	0.000	0.804	0.181	10.801

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	0	23	22	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.00	0.92	0.88	1.08
time (sec)	N/A	0.237	88.807	0.181	0.598	0.201	0.000	0.558	0.202	10.747

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	23	22	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.92	0.88	1.08
time (sec)	N/A	0.233	118.493	0.178	0.620	0.958	9.448	0.868	0.168	10.738

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	23	22	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.92	0.88	1.08
time (sec)	N/A	0.231	18.323	0.174	0.601	0.322	1.087	0.604	0.168	10.696

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	23	22	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.92	0.88	1.08
time (sec)	N/A	0.234	71.659	0.171	0.629	0.941	0.690	0.982	0.188	11.156

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	23	22	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.92	0.88	1.08
time (sec)	N/A	0.235	79.513	0.163	0.628	0.514	0.703	0.740	0.160	10.716

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	23	22	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.92	0.88	1.08
time (sec)	N/A	0.233	71.210	0.167	0.645	0.931	4.011	0.700	0.157	11.879

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	23	22	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.92	0.88	1.08
time (sec)	N/A	0.232	129.011	0.171	0.588	0.513	11.435	0.604	0.157	10.623

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	0	23	22	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.00	0.92	0.88	1.08
time (sec)	N/A	0.232	149.243	0.165	0.610	0.943	0.000	0.945	0.188	12.515

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	44	0	23	49	27
N.S.	1	1.00	1.08	0.84	0.92	1.76	0.00	0.92	1.96	1.08
time (sec)	N/A	0.241	149.686	0.174	0.594	0.183	0.000	2.281	0.346	12.063

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	42	0	23	49	27
N.S.	1	1.00	1.08	0.84	0.92	1.68	0.00	0.92	1.96	1.08
time (sec)	N/A	0.245	124.306	0.165	0.603	0.232	0.000	1.531	0.345	11.707

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	42	0	23	49	27
N.S.	1	1.00	1.08	0.84	0.92	1.68	0.00	0.92	1.96	1.08
time (sec)	N/A	0.240	91.858	0.158	0.676	0.168	0.000	1.234	0.323	11.653

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	0	23	49	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.00	0.92	1.96	1.08
time (sec)	N/A	0.241	119.669	0.161	0.595	0.975	0.000	1.673	0.290	11.468

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	0	23	49	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.00	0.92	1.96	1.08
time (sec)	N/A	0.238	84.017	0.165	0.691	0.521	0.000	1.182	0.272	11.598

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	0	49	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.00	1.96	1.08
time (sec)	N/A	0.243	151.758	0.162	0.714	0.947	22.374	0.000	0.194	12.078

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	23	49	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.92	1.96	1.08
time (sec)	N/A	0.244	82.298	0.166	0.592	0.494	13.923	154.883	0.197	10.285

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	23	49	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.92	1.96	1.08
time (sec)	N/A	0.242	70.667	0.161	0.625	0.931	33.341	175.001	0.200	11.503

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	23	49	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.92	1.96	1.08
time (sec)	N/A	0.238	147.243	0.162	0.622	0.534	82.106	145.380	0.182	10.172

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	0	23	49	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.00	0.92	1.96	1.08
time (sec)	N/A	0.238	172.105	0.164	0.666	0.920	0.000	146.381	0.224	11.841

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	60	0	23	79	27
N.S.	1	1.00	1.08	0.84	0.92	2.40	0.00	0.92	3.16	1.08
time (sec)	N/A	0.238	143.634	0.194	2.317	0.197	0.000	3.388	0.469	13.411

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	58	0	23	79	27
N.S.	1	1.00	1.08	0.84	0.92	2.32	0.00	0.92	3.16	1.08
time (sec)	N/A	0.241	118.237	0.164	2.362	0.211	0.000	2.677	0.430	12.446

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	58	0	23	79	27
N.S.	1	1.00	1.08	0.84	0.92	2.32	0.00	0.92	3.16	1.08
time (sec)	N/A	0.244	112.163	0.167	2.035	0.179	0.000	2.077	0.462	12.416

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	49	0	23	79	27
N.S.	1	1.00	1.08	0.84	0.92	1.96	0.00	0.92	3.16	1.08
time (sec)	N/A	0.241	113.638	0.167	2.044	0.932	0.000	2.698	0.368	12.222

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	49	0	23	79	27
N.S.	1	1.00	1.08	0.84	0.92	1.96	0.00	0.92	3.16	1.08
time (sec)	N/A	0.240	97.592	0.164	1.874	0.600	0.000	1.719	0.358	11.986

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	49	0	0	79	27
N.S.	1	1.00	1.08	0.84	0.92	1.96	0.00	0.00	3.16	1.08
time (sec)	N/A	0.237	158.434	0.168	1.854	0.958	0.000	0.000	0.208	12.776

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	49	0	0	79	27
N.S.	1	1.00	1.08	0.84	0.92	1.96	0.00	0.00	3.16	1.08
time (sec)	N/A	0.241	134.934	0.164	1.824	0.601	0.000	0.000	0.205	10.203

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	49	0	0	79	27
N.S.	1	1.00	1.08	0.84	0.92	1.96	0.00	0.00	3.16	1.08
time (sec)	N/A	0.244	149.097	0.162	2.066	0.943	0.000	0.000	0.206	12.720

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	49	0	0	79	27
N.S.	1	1.00	1.08	0.84	0.92	1.96	0.00	0.00	3.16	1.08
time (sec)	N/A	0.244	160.367	0.172	1.856	0.575	0.000	0.000	0.191	10.327

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	0	21	23	49	0	0	79	27
N.S.	1	1.00	0.00	0.84	0.92	1.96	0.00	0.00	3.16	1.08
time (sec)	N/A	0.247	0.000	0.168	1.819	0.968	0.000	0.000	0.239	12.699

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	0	0	34	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.00	0.00	1.36	1.08
time (sec)	N/A	0.240	94.799	0.177	0.553	0.158	0.000	0.000	0.175	11.354

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	0	0	34	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.00	0.00	1.36	1.08
time (sec)	N/A	0.244	111.059	0.177	0.642	0.232	0.000	0.000	0.170	11.561

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	0	0	34	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.00	0.00	1.36	1.08
time (sec)	N/A	0.238	16.687	0.167	0.594	0.147	0.000	0.000	0.191	11.386

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	0	34	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.00	1.36	1.08
time (sec)	N/A	0.237	16.662	0.165	0.588	1.100	3.143	0.000	0.178	11.296

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	0	34	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.00	1.36	1.08
time (sec)	N/A	0.233	1.366	0.168	0.593	0.312	1.182	0.000	0.183	11.249

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	44	26	0	37	27
N.S.	1	1.00	1.08	0.84	0.92	1.76	1.04	0.00	1.48	1.08
time (sec)	N/A	0.234	2.189	0.165	0.604	0.920	0.753	0.000	0.186	12.251

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	44	26	0	43	27
N.S.	1	1.00	1.08	0.84	0.92	1.76	1.04	0.00	1.72	1.08
time (sec)	N/A	0.235	93.297	0.170	0.607	0.546	1.644	0.000	0.159	10.396

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	46	26	0	37	27
N.S.	1	1.00	1.08	0.84	0.92	1.84	1.04	0.00	1.48	1.08
time (sec)	N/A	0.235	96.783	0.170	0.639	0.942	12.501	0.000	0.165	12.782

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	46	26	0	45	27
N.S.	1	1.00	1.08	0.84	0.92	1.84	1.04	0.00	1.80	1.08
time (sec)	N/A	0.239	112.157	0.174	1.353	0.586	29.682	0.000	0.169	10.489

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	46	0	0	37	27
N.S.	1	1.00	1.08	0.84	0.92	1.84	0.00	0.00	1.48	1.08
time (sec)	N/A	0.239	125.506	0.161	1.174	0.967	0.000	0.000	0.191	13.112

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	51	0	0	50	27
N.S.	1	1.00	1.08	0.84	0.92	2.04	0.00	0.00	2.00	1.08
time (sec)	N/A	0.242	121.983	0.181	0.620	0.189	0.000	0.000	0.175	12.447

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	51	0	0	50	27
N.S.	1	1.00	1.08	0.84	0.92	2.04	0.00	0.00	2.00	1.08
time (sec)	N/A	0.239	128.817	0.161	0.627	0.241	0.000	0.000	0.174	12.660

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	51	0	0	50	27
N.S.	1	1.00	1.08	0.84	0.92	2.04	0.00	0.00	2.00	1.08
time (sec)	N/A	0.246	112.170	0.157	0.586	0.200	0.000	0.000	0.170	12.768

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	51	24	0	50	27
N.S.	1	1.00	1.08	0.84	0.92	2.04	0.96	0.00	2.00	1.08
time (sec)	N/A	0.238	126.818	0.157	0.559	0.944	11.034	0.000	0.206	12.955

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	51	24	0	50	27
N.S.	1	1.00	1.08	0.84	0.92	2.04	0.96	0.00	2.00	1.08
time (sec)	N/A	0.242	109.034	0.158	0.595	0.614	4.145	0.000	0.181	12.929

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	60	26	0	53	27
N.S.	1	1.00	1.08	0.84	0.92	2.40	1.04	0.00	2.12	1.08
time (sec)	N/A	0.240	113.038	0.156	0.609	0.965	5.265	0.000	0.167	13.725

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	60	26	0	59	27
N.S.	1	1.00	1.08	0.84	0.92	2.40	1.04	0.00	2.36	1.08
time (sec)	N/A	0.242	115.974	0.165	1.109	0.731	13.164	0.000	0.186	10.940

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	62	26	0	53	27
N.S.	1	1.00	1.08	0.84	0.92	2.48	1.04	0.00	2.12	1.08
time (sec)	N/A	0.241	127.640	0.152	1.225	1.019	74.066	0.000	0.167	13.799

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	62	0	0	61	27
N.S.	1	1.00	1.08	0.84	0.92	2.48	0.00	0.00	2.44	1.08
time (sec)	N/A	0.241	103.938	0.165	1.170	0.843	0.000	0.000	0.167	10.708

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	62	0	0	53	27
N.S.	1	1.00	1.08	0.84	0.92	2.48	0.00	0.00	2.12	1.08
time (sec)	N/A	0.242	133.739	0.151	1.177	1.000	0.000	0.000	0.170	14.972

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	67	0	0	66	27
N.S.	1	1.00	1.08	0.84	0.92	2.68	0.00	0.00	2.64	1.08
time (sec)	N/A	0.241	118.310	0.168	0.566	0.316	0.000	0.000	0.191	13.988

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	67	0	0	66	27
N.S.	1	1.00	1.08	0.84	0.92	2.68	0.00	0.00	2.64	1.08
time (sec)	N/A	0.242	135.263	0.164	0.594	0.254	0.000	0.000	0.183	13.979

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	67	0	0	66	27
N.S.	1	1.00	1.08	0.84	0.92	2.68	0.00	0.00	2.64	1.08
time (sec)	N/A	0.241	115.630	0.155	0.657	0.279	0.000	0.000	0.166	14.118

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	67	24	0	66	27
N.S.	1	1.00	1.08	0.84	0.92	2.68	0.96	0.00	2.64	1.08
time (sec)	N/A	0.241	130.449	0.155	0.566	0.967	60.674	0.000	0.201	15.507

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	67	24	0	66	27
N.S.	1	1.00	1.08	0.84	0.92	2.68	0.96	0.00	2.64	1.08
time (sec)	N/A	0.241	118.745	0.171	1.097	1.056	25.333	0.000	0.181	15.848

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	76	26	0	69	27
N.S.	1	1.00	1.08	0.84	0.92	3.04	1.04	0.00	2.76	1.08
time (sec)	N/A	0.239	133.391	0.164	1.198	0.970	75.896	0.000	0.181	14.242

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	76	0	0	75	27
N.S.	1	1.00	1.08	0.84	0.92	3.04	0.00	0.00	3.00	1.08
time (sec)	N/A	0.236	122.877	0.166	1.168	1.461	0.000	0.000	0.185	11.069

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	78	0	0	69	27
N.S.	1	1.00	1.08	0.84	0.92	3.12	0.00	0.00	2.76	1.08
time (sec)	N/A	0.240	136.779	0.167	1.200	0.982	0.000	0.000	0.192	15.415

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	78	0	0	77	27
N.S.	1	1.00	1.08	0.84	0.92	3.12	0.00	0.00	3.08	1.08
time (sec)	N/A	0.238	108.240	0.169	2.011	1.989	0.000	0.000	0.172	11.200

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	78	0	0	69	27
N.S.	1	1.00	1.08	0.84	0.92	3.12	0.00	0.00	2.76	1.08
time (sec)	N/A	0.243	142.658	0.157	2.054	1.034	0.000	0.000	0.178	15.937

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	251	261	231	0	0	0	0	0	90	0
N.S.	1	1.04	0.92	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	1.350	0.586	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	181	180	171	0	0	0	0	0	63	0
N.S.	1	0.99	0.94	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.790	0.222	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	107	0	0	0	0	0	36	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.442	0.110	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	192	207	5280	0	0	0	0	0	27	0
N.S.	1	1.08	27.50	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.696	30.894	0.000	0.000	0.000	0.000	0.000	0.197	0.000

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	299	310	13816	0	0	0	0	0	43	0
N.S.	1	1.04	46.21	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.799	45.795	0.000	0.000	0.000	0.000	0.000	0.189	0.000

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	59	29
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	2.36	1.16
time (sec)	N/A	0.242	65.013	0.258	0.840	0.129	26.522	0.819	0.216	12.716

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	26	29
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.04	1.16
time (sec)	N/A	0.240	0.625	0.261	1.145	0.107	0.617	0.402	0.181	11.815

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	38	29
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.52	1.16
time (sec)	N/A	0.254	17.931	0.245	0.822	0.116	0.428	0.558	0.196	12.503

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	53	24	25	54	29
N.S.	1	1.00	1.08	0.92	1.00	2.12	0.96	1.00	2.16	1.16
time (sec)	N/A	0.259	17.343	0.250	0.929	0.133	1.697	0.912	0.185	14.050

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	23	20	23	23	27
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.10	1.29
time (sec)	N/A	0.220	5.407	0.243	1.134	0.121	6.450	0.343	0.189	11.842

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	22	25	27	29
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.96	1.09	1.17	1.26
time (sec)	N/A	0.226	0.811	0.265	1.184	0.123	6.037	0.404	0.215	11.676

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	274	273	8899	0	0	0	0	0	23	0
N.S.	1	1.00	32.48	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.759	26.198	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	221	220	5564	0	0	0	0	0	23	0
N.S.	1	1.00	25.18	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.484	20.733	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	2828	0	0	0	0	0	21	0
N.S.	1	1.00	27.46	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.285	14.361	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	12	14	14	16
N.S.	1	1.00	1.17	1.00	1.17	1.17	1.00	1.17	1.17	1.33
time (sec)	N/A	0.180	4.273	0.180	0.839	0.098	0.707	0.229	0.172	10.823

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	21	21	19	21	21	23
N.S.	1	1.00	1.11	1.00	1.11	1.11	1.00	1.11	1.11	1.21
time (sec)	N/A	0.211	9.381	0.235	1.258	0.097	8.031	0.323	0.176	10.928

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	23	20	23	23	25
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.10	1.19
time (sec)	N/A	0.220	11.305	0.320	1.940	0.106	39.024	0.317	0.177	11.716

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	145	90	318	0	159	0	0	47	87
N.S.	1	1.07	0.67	2.36	0.00	1.18	0.00	0.00	0.35	0.64
time (sec)	N/A	0.584	0.299	11.865	0.000	0.113	0.000	0.000	0.220	10.982

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	117	77	290	0	148	0	0	47	87
N.S.	1	1.05	0.69	2.61	0.00	1.33	0.00	0.00	0.42	0.78
time (sec)	N/A	0.561	0.393	7.263	0.000	0.113	0.000	0.000	0.201	10.896

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	89	66	262	0	137	0	0	47	80
N.S.	1	1.02	0.76	3.01	0.00	1.57	0.00	0.00	0.54	0.92
time (sec)	N/A	0.455	0.196	5.667	0.000	0.104	0.000	0.000	0.198	10.755

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	62	53	228	0	125	0	0	43	53
N.S.	1	1.02	0.87	3.74	0.00	2.05	0.00	0.00	0.70	0.87
time (sec)	N/A	0.439	0.112	2.446	0.000	0.109	0.000	0.000	0.220	0.171

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	152	0	107	0	0	30	33
N.S.	1	1.00	0.91	4.34	0.00	3.06	0.00	0.00	0.86	0.94
time (sec)	N/A	0.345	0.135	0.871	0.000	0.102	0.000	0.000	0.179	10.655

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	58	51	150	0	156	0	0	47	60
N.S.	1	1.02	0.89	2.63	0.00	2.74	0.00	0.00	0.82	1.05
time (sec)	N/A	0.430	0.149	1.109	0.000	0.096	0.000	0.000	0.174	10.897

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	85	65	396	0	175	0	0	47	87
N.S.	1	1.02	0.78	4.77	0.00	2.11	0.00	0.00	0.57	1.05
time (sec)	N/A	0.464	0.335	2.546	0.000	0.106	0.000	0.000	0.180	11.215

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	113	95	502	0	188	0	0	47	87
N.S.	1	1.02	0.86	4.52	0.00	1.69	0.00	0.00	0.42	0.78
time (sec)	N/A	0.561	0.260	3.920	0.000	0.123	0.000	0.000	0.208	11.766

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	219	113	398	0	195	0	0	81	135
N.S.	1	1.37	0.71	2.49	0.00	1.22	0.00	0.00	0.51	0.84
time (sec)	N/A	1.342	0.823	30.681	0.000	0.118	0.000	0.000	0.235	10.943

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	191	98	362	0	180	0	0	81	128
N.S.	1	1.41	0.73	2.68	0.00	1.33	0.00	0.00	0.60	0.95
time (sec)	N/A	1.137	0.641	29.035	0.000	0.116	0.000	0.000	0.231	10.767

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	163	79	357	0	162	0	0	81	102
N.S.	1	1.61	0.78	3.53	0.00	1.60	0.00	0.00	0.80	1.01
time (sec)	N/A	0.946	0.411	27.185	0.000	0.108	0.000	0.000	0.254	11.157

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	133	64	283	0	147	0	0	75	76
N.S.	1	1.85	0.89	3.93	0.00	2.04	0.00	0.00	1.04	1.06
time (sec)	N/A	0.794	0.782	2.657	0.000	0.103	0.000	0.000	0.228	11.130

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	129	62	202	0	178	0	0	56	81
N.S.	1	1.90	0.91	2.97	0.00	2.62	0.00	0.00	0.82	1.19
time (sec)	N/A	0.774	0.549	2.878	0.000	0.118	0.000	0.000	0.194	11.453

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	157	73	513	0	198	0	0	81	108
N.S.	1	1.65	0.77	5.40	0.00	2.08	0.00	0.00	0.85	1.14
time (sec)	N/A	0.945	0.682	3.588	0.000	0.103	0.000	0.000	0.219	11.537

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	187	124	633	0	223	0	0	81	113
N.S.	1	1.39	0.92	4.69	0.00	1.65	0.00	0.00	0.60	0.84
time (sec)	N/A	1.076	0.536	4.793	0.000	0.112	0.000	0.000	0.211	11.574

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	215	142	689	0	235	0	0	81	113
N.S.	1	1.34	0.89	4.31	0.00	1.47	0.00	0.00	0.51	0.71
time (sec)	N/A	1.278	0.728	6.894	0.000	0.118	0.000	0.000	0.216	11.333

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	243	137	470	0	227	0	0	115	178
N.S.	1	1.25	0.71	2.42	0.00	1.17	0.00	0.00	0.59	0.92
time (sec)	N/A	1.518	1.070	204.165	0.000	0.120	0.000	0.000	0.271	10.816

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	215	110	421	0	205	0	0	115	146
N.S.	1	1.35	0.69	2.65	0.00	1.29	0.00	0.00	0.72	0.92
time (sec)	N/A	1.336	0.811	202.751	0.000	0.133	0.000	0.000	0.267	10.762

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	178	84	412	0	185	0	0	115	125
N.S.	1	1.53	0.72	3.55	0.00	1.59	0.00	0.00	0.99	1.08
time (sec)	N/A	1.106	0.771	200.967	0.000	0.107	0.000	0.000	0.279	10.709

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	188	87	303	0	214	0	0	107	124
N.S.	1	1.49	0.69	2.40	0.00	1.70	0.00	0.00	0.85	0.98
time (sec)	N/A	1.165	0.783	3.803	0.000	0.108	0.000	0.000	0.237	10.747

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	180	84	630	0	222	0	0	82	128
N.S.	1	1.53	0.71	5.34	0.00	1.88	0.00	0.00	0.69	1.08
time (sec)	N/A	1.142	1.219	3.792	0.000	0.115	0.000	0.000	0.207	11.575

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	202	125	711	0	244	0	0	115	156
N.S.	1	1.36	0.84	4.77	0.00	1.64	0.00	0.00	0.77	1.05
time (sec)	N/A	1.309	1.007	4.693	0.000	0.113	0.000	0.000	0.207	11.615

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	239	177	820	0	270	0	0	115	147
N.S.	1	1.23	0.91	4.23	0.00	1.39	0.00	0.00	0.59	0.76
time (sec)	N/A	1.511	0.938	5.964	0.000	0.113	0.000	0.000	0.232	11.812

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	245	226	668	0	0	0	0	30	0
N.S.	1	1.61	1.49	4.39	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	1.850	11.339	4.930	0.000	0.000	0.000	0.000	0.161	0.000

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	199	158	552	0	0	0	0	28	0
N.S.	1	1.78	1.41	4.93	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	1.399	1.163	2.672	0.000	0.000	0.000	0.000	0.180	0.000

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	156	81	226	0	0	0	0	22	0
N.S.	1	2.08	1.08	3.01	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	1.055	10.446	2.201	0.000	0.000	0.000	0.000	0.165	0.000

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	48	187	0	0	0	0	35	0
N.S.	1	1.00	0.91	3.53	0.00	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	0.564	0.174	1.120	0.000	0.000	0.000	0.000	0.169	0.000

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	150	0	0	0	0	39	0
N.S.	1	1.00	1.00	5.17	0.00	0.00	0.00	0.00	1.34	0.00
time (sec)	N/A	0.419	0.166	0.112	0.000	0.000	0.000	0.000	0.178	0.000

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	137	195	353	0	0	0	0	39	0
N.S.	1	1.78	2.53	4.58	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.932	1.990	1.971	0.000	0.000	0.000	0.000	0.161	0.000

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	222	210	423	0	0	0	0	39	0
N.S.	1	1.73	1.64	3.30	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	1.717	2.712	2.880	0.000	0.000	0.000	0.000	0.164	0.000

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	310	266	1064	0	0	0	0	44	0
N.S.	1	1.27	1.09	4.36	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	2.176	1.370	5.144	0.000	0.000	0.000	0.000	0.172	0.000

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	253	252	809	0	0	0	0	38	0
N.S.	1	1.38	1.37	4.40	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	1.670	1.336	3.727	0.000	0.000	0.000	0.000	0.174	0.000

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	237	194	788	0	0	0	0	57	0
N.S.	1	1.42	1.16	4.72	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	1.581	2.500	3.147	0.000	0.000	0.000	0.000	0.182	0.000

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	223	229	707	0	0	0	0	63	0
N.S.	1	1.51	1.55	4.78	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	1.529	2.302	2.767	0.000	0.000	0.000	0.000	0.172	0.000

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	233	239	608	0	0	0	0	63	0
N.S.	1	1.51	1.55	3.95	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	1.595	2.378	1.951	0.000	0.000	0.000	0.000	0.190	0.000

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	287	278	841	0	0	0	0	63	0
N.S.	1	1.31	1.27	3.84	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	2.105	1.962	3.369	0.000	0.000	0.000	0.000	0.170	0.000

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	424	353	2216	0	0	0	0	60	0
N.S.	1	1.23	1.02	6.40	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	3.010	2.658	6.327	0.000	0.000	0.000	0.000	0.189	0.000

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	363	311	1957	0	0	0	0	54	0
N.S.	1	1.29	1.10	6.94	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	2.452	2.045	5.114	0.000	0.000	0.000	0.000	0.186	0.000

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	342	286	1936	0	0	0	0	79	0
N.S.	1	1.30	1.09	7.36	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	2.303	2.118	4.328	0.000	0.000	0.000	0.000	0.173	0.000

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	328	272	1858	0	0	0	0	87	0
N.S.	1	1.33	1.11	7.55	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	2.239	1.418	4.220	0.000	0.000	0.000	0.000	0.170	0.000

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	331	289	1760	0	0	0	0	87	0
N.S.	1	1.31	1.14	6.96	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	2.319	2.016	3.536	0.000	0.000	0.000	0.000	0.183	0.000

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	339	297	1203	0	0	0	0	87	0
N.S.	1	1.33	1.16	4.72	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	2.352	1.966	2.477	0.000	0.000	0.000	0.000	0.186	0.000

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	403	334	1987	0	0	0	0	87	0
N.S.	1	1.23	1.02	6.06	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	3.075	2.480	4.574	0.000	0.000	0.000	0.000	0.165	0.000

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	274	340	984	0	455	0	0	29	0
N.S.	1	1.12	1.39	4.03	0.00	1.86	0.00	0.00	0.12	0.00
time (sec)	N/A	2.119	6.625	8.049	0.000	0.109	0.000	0.000	0.171	0.000

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	214	273	591	0	417	0	0	27	0
N.S.	1	1.11	1.42	3.08	0.00	2.17	0.00	0.00	0.14	0.00
time (sec)	N/A	1.549	6.301	2.753	0.000	0.103	0.000	0.000	0.178	0.000

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	244	540	0	355	0	0	21	0
N.S.	1	1.00	3.64	8.06	0.00	5.30	0.00	0.00	0.31	0.00
time (sec)	N/A	0.510	4.300	0.686	0.000	0.095	0.000	0.000	0.193	0.000

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	159	14885	245	0	0	0	0	29	0
N.S.	1	1.15	107.86	1.78	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	1.309	29.061	4.322	0.000	0.000	0.000	0.000	0.167	0.000

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	275	23549	471	0	0	0	0	29	0
N.S.	1	1.16	99.36	1.99	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	2.402	31.421	6.037	0.000	0.000	0.000	0.000	0.201	0.000

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	332	383	1164	0	493	0	0	69	0
N.S.	1	1.10	1.26	3.84	0.00	1.63	0.00	0.00	0.23	0.00
time (sec)	N/A	2.715	7.139	7.675	0.000	0.112	0.000	0.000	0.248	0.000

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	259	344	977	0	455	0	0	69	0
N.S.	1	1.08	1.43	4.07	0.00	1.90	0.00	0.00	0.29	0.00
time (sec)	N/A	2.031	5.877	3.954	0.000	0.117	0.000	0.000	0.248	0.000

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	209	284	699	0	417	0	0	65	0
N.S.	1	1.12	1.52	3.74	0.00	2.23	0.00	0.00	0.35	0.00
time (sec)	N/A	1.597	4.651	2.753	0.000	0.107	0.000	0.000	0.224	0.000

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	230	25369	745	0	0	0	0	53	0
N.S.	1	1.10	121.38	3.56	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	2.225	30.680	4.590	0.000	0.000	0.000	0.000	0.194	0.000

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	276	24604	711	0	0	0	0	69	0
N.S.	1	1.11	98.81	2.86	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	2.815	31.416	5.371	0.000	0.000	0.000	0.000	0.193	0.000

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	329	51904	1030	0	0	0	0	69	0
N.S.	1	1.10	173.59	3.44	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	3.417	32.426	9.605	0.000	0.000	0.000	0.000	0.203	0.000

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	399	477	1570	0	533	0	0	114	0
N.S.	1	1.10	1.31	4.33	0.00	1.47	0.00	0.00	0.31	0.00
time (sec)	N/A	3.573	11.751	1208.687	0.000	0.123	0.000	0.000	0.296	0.000

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	333	386	1164	0	493	0	0	114	0
N.S.	1	1.10	1.27	3.84	0.00	1.63	0.00	0.00	0.38	0.00
time (sec)	N/A	2.782	9.246	1298.166	0.000	0.114	0.000	0.000	0.300	0.000

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	271	358	1089	0	456	0	0	114	0
N.S.	1	1.13	1.50	4.56	0.00	1.91	0.00	0.00	0.48	0.00
time (sec)	N/A	2.196	8.518	1237.281	0.000	0.112	0.000	0.000	0.317	0.000

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	284	36889	937	0	0	0	0	108	0
N.S.	1	1.08	140.80	3.58	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	2.934	33.918	10.144	0.000	0.000	0.000	0.000	0.279	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	291	44895	1135	0	0	0	0	90	0
N.S.	1	1.11	170.70	4.32	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	2.972	32.787	10.220	0.000	0.000	0.000	0.000	0.228	0.000

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	345	53538	1159	0	0	0	0	114	0
N.S.	1	1.10	170.50	3.69	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	3.691	32.100	13.556	0.000	0.000	0.000	0.000	0.193	0.000

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	0	62811	1339	0	0	0	0	114	0
N.S.	1	0.00	170.22	3.63	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.000	32.614	17.531	0.000	0.000	0.000	0.000	0.217	0.000

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	282	340	984	0	456	0	0	41	0
N.S.	1	1.13	1.37	3.95	0.00	1.83	0.00	0.00	0.16	0.00
time (sec)	N/A	2.135	6.316	12.287	0.000	0.175	0.000	0.000	0.164	0.000

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	220	265	594	0	417	0	0	39	0
N.S.	1	1.13	1.36	3.05	0.00	2.14	0.00	0.00	0.20	0.00
time (sec)	N/A	1.603	5.458	9.773	0.000	0.101	0.000	0.000	0.167	0.000

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	163	216	437	0	355	0	0	33	0
N.S.	1	1.15	1.52	3.08	0.00	2.50	0.00	0.00	0.23	0.00
time (sec)	N/A	1.236	3.057	7.201	0.000	0.096	0.000	0.000	0.165	0.000

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	102	135	0	147	0	0	46	0
N.S.	1	1.00	1.52	2.01	0.00	2.19	0.00	0.00	0.69	0.00
time (sec)	N/A	0.541	1.177	4.862	0.000	0.084	0.000	0.000	0.164	0.000

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	14986	195	0	0	0	0	50	0
N.S.	1	1.00	220.38	2.87	0.00	0.00	0.00	0.00	0.74	0.00
time (sec)	N/A	0.637	28.972	5.542	0.000	0.000	0.000	0.000	0.164	0.000

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	280	21698	589	0	0	0	0	50	0
N.S.	1	1.14	88.20	2.39	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	2.548	30.689	9.654	0.000	0.000	0.000	0.000	0.161	0.000

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	333	51912	1033	0	0	0	0	50	0
N.S.	1	1.07	166.38	3.31	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	3.320	32.495	15.629	0.000	0.000	0.000	0.000	0.161	0.000

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	380	419	1043	0	684	0	0	57	0
N.S.	1	1.06	1.16	2.90	0.00	1.90	0.00	0.00	0.16	0.00
time (sec)	N/A	2.979	8.955	15.263	0.000	0.172	0.000	0.000	0.164	0.000

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	316	382	756	0	622	0	0	55	0
N.S.	1	1.09	1.32	2.62	0.00	2.15	0.00	0.00	0.19	0.00
time (sec)	N/A	2.342	6.223	12.888	0.000	0.122	0.000	0.000	0.158	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	249	330	590	0	563	0	0	49	0
N.S.	1	1.16	1.54	2.76	0.00	2.63	0.00	0.00	0.23	0.00
time (sec)	N/A	1.821	6.505	9.724	0.000	0.112	0.000	0.000	0.176	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	232	245	311	0	527	0	0	68	0
N.S.	1	1.16	1.22	1.56	0.00	2.64	0.00	0.00	0.34	0.00
time (sec)	N/A	1.679	5.583	7.361	0.000	0.127	0.000	0.000	0.153	0.000

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	147	239	303	0	486	0	0	74	0
N.S.	1	1.17	1.90	2.40	0.00	3.86	0.00	0.00	0.59	0.00
time (sec)	N/A	0.810	5.603	2.477	0.000	0.119	0.000	0.000	0.161	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	235	48278	653	0	0	0	0	74	0
N.S.	1	1.14	234.36	3.17	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	1.917	32.496	9.101	0.000	0.000	0.000	0.000	0.164	0.000

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	381	52199	882	0	0	0	0	74	0
N.S.	1	1.10	151.30	2.56	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	3.735	32.565	13.311	0.000	0.000	0.000	0.000	0.163	0.000

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	420	527	1802	0	941	0	0	71	0
N.S.	1	1.07	1.35	4.61	0.00	2.41	0.00	0.00	0.18	0.00
time (sec)	N/A	3.306	11.993	15.721	0.000	0.162	0.000	0.000	0.190	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	353	507	1600	0	852	0	0	65	0
N.S.	1	1.11	1.60	5.05	0.00	2.69	0.00	0.00	0.21	0.00
time (sec)	N/A	2.598	11.385	13.157	0.000	0.144	0.000	0.000	0.163	0.000

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	335	398	1143	0	801	0	0	90	0
N.S.	1	1.11	1.32	3.78	0.00	2.65	0.00	0.00	0.30	0.00
time (sec)	N/A	2.431	6.248	10.586	0.000	0.134	0.000	0.000	0.159	0.000

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	309	447	1016	0	750	0	0	98	0
N.S.	1	1.10	1.59	3.62	0.00	2.67	0.00	0.00	0.35	0.00
time (sec)	N/A	2.254	9.965	7.999	0.000	0.152	0.000	0.000	0.165	0.000

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	316	311	755	0	679	0	0	98	0
N.S.	1	1.14	1.12	2.73	0.00	2.45	0.00	0.00	0.35	0.00
time (sec)	N/A	2.359	6.793	8.003	0.000	0.119	0.000	0.000	0.184	0.000

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	411	93062	1908	0	0	0	0	98	0
N.S.	1	1.11	251.52	5.16	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	4.070	33.250	11.490	0.000	0.000	0.000	0.000	0.168	0.000

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	266	299	222	0	0	0	0	0	90	0
N.S.	1	1.12	0.83	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	1.708	0.454	0.000	0.000	0.000	0.000	0.000	0.189	0.000

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	186	204	161	0	0	0	0	0	63	0
N.S.	1	1.10	0.87	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.994	0.257	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	137	106	0	0	0	0	0	36	0
N.S.	1	1.04	0.80	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.441	0.078	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	196	209	5216	0	0	0	0	0	27	0
N.S.	1	1.07	26.61	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.802	26.592	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	309	316	13974	0	0	0	0	0	43	0
N.S.	1	1.02	45.22	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.915	48.035	0.000	0.000	0.000	0.000	0.000	0.186	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [87] had the largest ratio of [1.33332999999999990]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	11	10	1.06	19	0.526
2	A	9	8	1.02	19	0.421
3	A	9	8	1.02	19	0.421
4	A	7	6	1.00	17	0.353
5	A	1	1	1.00	10	0.100
6	A	5	5	1.00	17	0.294
7	A	6	6	1.03	19	0.316
8	A	8	7	1.02	19	0.368
9	A	10	9	1.07	19	0.474
10	A	13	12	0.99	21	0.571
11	A	11	10	0.99	21	0.476
12	A	11	10	1.08	21	0.476
13	A	9	8	1.00	19	0.421
14	A	7	6	1.00	12	0.500
15	A	7	7	1.00	19	0.368
16	A	6	6	1.00	21	0.286
17	A	10	9	1.11	21	0.429
18	A	10	9	0.99	21	0.429
19	A	14	13	0.91	21	0.619
20	A	3	3	1.00	21	0.143
21	A	3	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	3	3	1.00	19	0.158
23	A	9	8	1.05	12	0.667
24	A	3	3	1.00	19	0.158
25	A	3	3	1.00	21	0.143
26	A	3	3	1.00	21	0.143
27	A	3	3	1.00	21	0.143
28	A	3	3	1.00	21	0.143
29	A	3	3	1.00	21	0.143
30	A	3	3	1.00	21	0.143
31	A	3	3	1.00	21	0.143
32	A	3	3	1.00	19	0.158
33	A	12	11	1.08	12	0.917
34	A	3	3	1.00	19	0.158
35	A	3	3	1.00	21	0.143
36	A	3	3	1.00	21	0.143
37	A	3	3	1.00	21	0.143
38	A	3	3	1.00	21	0.143
39	A	3	3	1.00	21	0.143
40	A	3	3	1.00	21	0.143
41	A	3	3	1.00	21	0.143
42	A	11	10	0.99	21	0.476
43	A	11	10	1.02	21	0.476
44	A	7	7	1.00	21	0.333
45	A	5	5	1.00	21	0.238
46	A	2	2	1.00	19	0.105
47	A	3	3	1.00	12	0.250
48	A	8	8	1.02	19	0.421
49	A	9	9	1.01	21	0.429
50	A	11	10	0.98	21	0.476
51	A	13	12	1.00	21	0.571
52	A	13	12	1.05	21	0.571
53	A	13	12	1.09	21	0.571

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	8	8	1.06	21	0.381
55	A	4	4	0.98	21	0.190
56	A	4	4	0.98	19	0.211
57	A	7	7	1.07	12	0.583
58	A	10	10	1.11	19	0.526
59	A	11	11	1.05	21	0.524
60	A	14	13	1.10	21	0.619
61	A	15	14	1.09	21	0.667
62	A	16	15	1.08	21	0.714
63	A	10	10	1.10	21	0.476
64	A	7	7	1.02	21	0.333
65	A	6	6	1.05	21	0.286
66	A	6	6	1.05	19	0.316
67	A	10	10	1.11	12	0.833
68	A	13	13	1.11	19	0.684
69	A	13	13	1.10	21	0.619
70	A	18	17	1.09	21	0.810
71	A	17	16	1.15	21	0.762
72	A	13	13	1.11	21	0.619
73	A	9	9	1.05	21	0.429
74	A	9	9	1.05	21	0.429
75	A	8	8	1.07	21	0.381
76	A	8	8	1.07	19	0.421
77	A	13	13	1.14	12	1.083
78	A	14	14	1.19	19	0.737
79	A	16	16	1.10	21	0.762
80	A	20	19	1.16	21	0.905
81	A	15	15	1.16	21	0.714
82	A	11	11	1.05	21	0.524
83	A	11	11	1.05	21	0.524
84	A	11	11	1.06	21	0.524
85	A	10	10	1.07	21	0.476

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	10	10	1.07	19	0.526
87	A	16	16	1.19	12	1.333
88	A	17	17	1.20	19	0.895
89	A	18	18	1.12	21	0.857
90	A	9	9	1.13	23	0.391
91	A	7	7	1.13	23	0.304
92	A	4	4	1.00	23	0.174
93	A	2	2	1.00	21	0.095
94	A	4	3	1.00	14	0.214
95	A	6	5	1.00	21	0.238
96	A	8	7	0.99	23	0.304
97	A	10	9	1.03	23	0.391
98	A	12	11	1.05	23	0.478
99	A	12	12	1.12	23	0.522
100	A	9	9	1.13	23	0.391
101	A	6	6	1.06	23	0.261
102	A	4	4	1.00	21	0.190
103	A	7	6	1.00	14	0.429
104	A	9	8	1.45	21	0.381
105	A	9	8	0.98	23	0.348
106	A	11	10	1.01	23	0.435
107	A	14	14	1.11	23	0.609
108	A	11	11	1.13	23	0.478
109	A	8	8	1.08	23	0.348
110	A	6	6	1.04	21	0.286
111	A	10	9	1.04	14	0.643
112	A	9	8	1.03	21	0.381
113	A	9	8	1.02	23	0.348
114	A	11	10	1.03	23	0.435
115	A	13	12	1.04	23	0.522
116	A	2	2	1.00	22	0.091
117	A	4	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	6	5	1.00	22	0.227
119	A	11	10	1.10	23	0.435
120	A	9	8	1.08	23	0.348
121	A	6	5	1.00	23	0.217
122	A	4	3	1.00	21	0.143
123	A	8	7	1.00	14	0.500
124	A	9	8	1.01	21	0.381
125	A	13	12	1.08	23	0.522
126	A	15	14	1.09	23	0.609
127	A	12	11	1.06	23	0.478
128	A	9	8	1.04	23	0.348
129	A	6	5	1.00	23	0.217
130	A	6	5	1.00	21	0.238
131	A	11	10	1.05	14	0.714
132	A	14	13	1.06	21	0.619
133	A	17	16	1.08	23	0.696
134	A	15	14	1.09	23	0.609
135	A	12	11	1.07	23	0.478
136	A	9	8	1.06	23	0.348
137	A	8	7	1.05	23	0.304
138	A	8	7	1.05	21	0.333
139	A	14	13	1.08	14	0.929
140	A	17	16	1.09	21	0.762
141	A	4	3	1.00	22	0.136
142	A	8	7	1.00	15	0.467
143	A	13	12	1.10	23	0.522
144	A	10	9	1.08	23	0.391
145	A	8	7	1.08	21	0.333
146	A	8	7	1.00	14	0.500
147	A	7	6	1.00	21	0.286
148	A	14	13	1.11	23	0.565
149	A	11	10	1.09	23	0.435

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	9	8	1.09	21	0.381
151	A	8	7	1.10	14	0.500
152	A	7	6	1.10	21	0.286
153	A	15	14	1.05	23	0.609
154	A	12	11	1.03	23	0.478
155	A	9	8	1.00	23	0.348
156	A	7	6	1.00	21	0.286
157	A	8	7	1.00	14	0.500
158	A	7	6	1.00	21	0.286
159	A	19	18	1.02	23	0.783
160	A	16	15	1.00	23	0.652
161	A	14	13	1.01	23	0.565
162	A	13	12	1.00	21	0.571
163	A	8	7	1.22	14	0.500
164	A	7	6	1.22	21	0.286
165	A	11	11	1.01	21	0.524
166	A	9	9	1.02	21	0.429
167	A	9	9	1.01	21	0.429
168	A	7	7	1.00	21	0.333
169	A	9	9	1.01	21	0.429
170	A	9	9	1.02	21	0.429
171	A	11	11	1.04	21	0.524
172	A	17	17	1.00	23	0.739
173	A	15	15	0.99	23	0.652
174	A	13	13	1.00	23	0.565
175	A	7	7	1.00	23	0.304
176	A	11	11	1.00	23	0.478
177	A	13	13	1.00	23	0.565
178	A	15	15	1.01	23	0.652
179	A	3	3	1.00	23	0.130
180	A	3	3	1.00	23	0.130
181	A	3	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	3	3	1.00	23	0.130
183	A	3	3	1.00	23	0.130
184	A	3	3	1.00	23	0.130
185	A	3	3	1.00	23	0.130
186	A	3	3	1.00	23	0.130
187	A	3	3	1.00	23	0.130
188	A	3	3	1.00	23	0.130
189	A	3	3	1.00	23	0.130
190	A	3	3	1.00	23	0.130
191	A	3	3	1.00	23	0.130
192	A	3	3	1.00	23	0.130
193	A	3	3	1.00	23	0.130
194	A	12	12	1.02	23	0.522
195	A	12	12	1.03	23	0.522
196	A	10	10	1.05	23	0.435
197	A	9	9	1.05	23	0.391
198	A	10	10	1.04	23	0.435
199	A	12	12	1.03	23	0.522
200	A	12	12	1.02	23	0.522
201	A	15	15	1.04	23	0.652
202	A	14	14	1.03	23	0.609
203	A	13	13	1.07	23	0.565
204	A	7	7	1.00	23	0.304
205	A	13	13	1.07	23	0.565
206	A	12	12	1.04	23	0.522
207	A	15	15	1.05	23	0.652
208	A	14	14	1.07	23	0.609
209	A	17	17	1.06	23	0.739
210	A	17	17	1.06	23	0.739
211	A	15	15	1.08	23	0.652
212	A	16	16	1.08	23	0.696
213	A	15	15	1.08	23	0.652

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	16	16	1.08	23	0.696
215	A	15	15	1.08	23	0.652
216	A	17	17	1.08	23	0.739
217	A	18	18	1.08	23	0.783
218	A	8	7	0.99	25	0.280
219	A	6	5	1.00	25	0.200
220	A	4	3	1.00	25	0.120
221	A	2	2	1.00	25	0.080
222	A	4	4	1.00	25	0.160
223	A	6	6	1.04	25	0.240
224	A	8	8	1.07	25	0.320
225	A	11	10	1.01	25	0.400
226	A	9	8	0.98	25	0.320
227	A	7	6	1.00	25	0.240
228	A	7	6	1.00	25	0.240
229	A	4	4	1.00	25	0.160
230	A	6	6	1.04	25	0.240
231	A	9	9	1.03	25	0.360
232	A	11	11	1.04	25	0.440
233	A	13	12	1.04	25	0.480
234	A	11	10	1.02	25	0.400
235	A	9	8	1.02	25	0.320
236	A	9	8	1.06	25	0.320
237	A	9	8	1.03	25	0.320
238	A	6	6	1.03	25	0.240
239	A	8	8	1.06	25	0.320
240	A	11	11	1.05	25	0.440
241	A	13	13	1.06	25	0.520
242	A	3	3	1.00	25	0.120
243	A	4	3	1.00	25	0.120
244	A	4	3	1.00	28	0.107
245	A	10	9	1.07	25	0.360

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	8	7	1.00	25	0.280
247	A	4	3	1.00	25	0.120
248	A	6	5	1.00	25	0.200
249	A	8	7	1.06	25	0.280
250	A	11	10	1.11	25	0.400
251	A	14	13	1.05	25	0.520
252	A	11	10	1.04	25	0.400
253	A	6	5	1.00	25	0.200
254	A	6	5	1.00	25	0.200
255	A	9	8	1.01	25	0.320
256	A	12	11	1.06	25	0.440
257	A	15	14	1.08	25	0.560
258	A	17	16	1.07	25	0.640
259	A	14	13	1.07	25	0.520
260	A	8	7	1.04	25	0.280
261	A	8	7	1.04	25	0.280
262	A	9	8	1.04	25	0.320
263	A	12	11	1.06	25	0.440
264	A	15	14	1.08	25	0.560
265	A	13	12	1.05	23	0.522
266	A	10	9	1.07	23	0.391
267	A	8	7	1.00	23	0.304
268	A	4	3	1.00	23	0.130
269	A	6	5	1.00	23	0.217
270	A	8	7	1.04	23	0.304
271	A	11	10	1.06	23	0.435
272	A	6	5	1.04	27	0.185
273	A	5	4	1.00	27	0.148
274	A	6	5	1.05	27	0.185
275	A	9	8	0.99	27	0.296
276	A	8	7	0.99	27	0.259
277	A	7	6	1.01	27	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	8	7	1.02	27	0.259
279	A	9	8	1.00	27	0.296
280	A	8	7	1.00	27	0.259
281	A	8	7	1.00	27	0.259
282	A	8	7	0.84	27	0.259
283	A	8	7	0.92	27	0.259
284	A	6	5	1.00	25	0.200
285	A	6	5	1.00	25	0.200
286	C	6	5	0.24	25	0.200
287	A	6	5	1.00	25	0.200
288	A	12	12	1.00	21	0.571
289	A	10	10	1.04	21	0.476
290	A	11	11	1.02	21	0.524
291	A	6	6	1.00	19	0.316
292	A	8	8	1.05	21	0.381
293	A	11	11	1.02	21	0.524
294	A	9	8	0.97	21	0.381
295	A	8	7	1.00	21	0.333
296	A	4	3	1.00	21	0.143
297	A	5	4	1.00	21	0.190
298	A	5	4	1.00	21	0.190
299	A	8	7	0.98	23	0.304
300	A	4	3	0.96	23	0.130
301	A	4	3	0.87	23	0.130
302	A	4	3	0.85	23	0.130
303	A	8	7	0.98	23	0.304
304	A	4	3	0.96	23	0.130
305	A	4	3	0.90	23	0.130
306	A	4	3	0.87	23	0.130
307	A	9	8	0.98	23	0.348
308	A	8	7	1.00	23	0.304
309	A	4	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	7	6	1.00	23	0.261
311	A	7	6	1.00	23	0.261
312	A	9	8	1.00	25	0.320
313	A	5	4	1.00	25	0.160
314	A	6	5	0.87	25	0.200
315	A	6	5	0.86	25	0.200
316	A	9	8	1.00	25	0.320
317	A	5	4	1.00	25	0.160
318	A	6	5	0.90	25	0.200
319	A	6	5	0.88	25	0.200
320	A	9	8	0.98	26	0.308
321	A	8	7	1.00	26	0.269
322	A	4	3	1.00	26	0.115
323	A	7	6	1.00	26	0.231
324	A	7	6	1.00	26	0.231
325	A	9	8	1.00	24	0.333
326	A	5	4	1.00	24	0.167
327	A	9	8	1.00	26	0.308
328	A	5	4	1.00	26	0.154
329	A	4	3	1.00	19	0.158
330	A	4	3	0.96	21	0.143
331	A	6	5	1.00	21	0.238
332	A	6	5	0.82	22	0.227
333	A	4	3	0.88	21	0.143
334	A	4	3	1.00	23	0.130
335	A	6	5	0.77	23	0.217
336	A	6	5	1.00	24	0.208
337	A	4	3	0.81	21	0.143
338	A	4	3	0.81	23	0.130
339	A	6	5	0.84	23	0.217
340	A	6	5	0.83	24	0.208
341	A	12	11	0.99	21	0.524

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	10	9	1.01	21	0.429
343	A	8	7	1.00	21	0.333
344	A	6	5	1.00	19	0.263
345	A	6	5	0.99	12	0.417
346	A	6	5	1.00	19	0.263
347	A	6	5	1.00	25	0.200
348	A	6	5	1.00	25	0.200
349	A	6	5	1.00	25	0.200
350	A	6	5	1.00	25	0.200
351	A	11	11	1.05	21	0.524
352	A	9	9	1.02	21	0.429
353	A	9	9	1.02	21	0.429
354	A	7	7	1.00	21	0.333
355	A	9	9	1.02	21	0.429
356	A	9	9	1.02	21	0.429
357	A	11	11	1.02	21	0.524
358	A	11	11	1.04	21	0.524
359	A	19	19	1.44	23	0.826
360	A	17	17	1.52	23	0.739
361	A	15	15	1.64	23	0.652
362	A	13	13	1.91	23	0.565
363	A	9	9	1.93	23	0.391
364	A	15	15	1.67	23	0.652
365	A	17	17	1.49	23	0.739
366	A	19	19	1.41	23	0.826
367	A	5	5	1.41	23	0.217
368	A	5	5	1.50	23	0.217
369	A	5	5	1.67	23	0.217
370	A	5	5	1.67	23	0.217
371	A	5	5	1.67	23	0.217
372	A	5	5	1.52	23	0.217
373	A	5	5	1.41	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
374	A	14	14	1.51	23	0.609
375	A	14	14	1.65	23	0.609
376	A	12	12	1.90	23	0.522
377	A	11	11	1.94	23	0.478
378	A	12	12	1.96	23	0.522
379	A	14	14	1.68	23	0.609
380	A	14	14	1.52	23	0.609
381	A	16	16	1.47	23	0.696
382	A	17	17	1.51	23	0.739
383	A	14	14	1.60	23	0.609
384	A	15	15	1.65	23	0.652
385	A	9	9	1.72	23	0.391
386	A	15	15	1.65	23	0.652
387	A	16	16	1.49	23	0.696
388	A	17	17	1.43	23	0.739
389	A	20	20	1.39	23	0.870
390	A	19	19	1.43	23	0.826
391	A	17	17	1.49	23	0.739
392	A	18	18	1.49	23	0.783
393	A	17	17	1.49	23	0.739
394	A	18	18	1.49	23	0.783
395	A	17	17	1.49	23	0.739
396	A	19	19	1.41	23	0.826
397	A	19	19	1.37	23	0.826
398	A	10	10	1.20	25	0.400
399	A	8	8	1.23	25	0.320
400	A	6	6	1.27	25	0.240
401	A	4	4	1.00	25	0.160
402	A	6	5	1.00	25	0.200
403	A	8	7	1.01	25	0.280
404	A	10	9	1.00	25	0.360
405	A	11	11	1.16	25	0.440

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
406	A	8	8	1.22	25	0.320
407	A	6	6	1.27	25	0.240
408	A	9	8	1.01	25	0.320
409	A	9	8	1.01	25	0.320
410	A	11	10	0.99	25	0.400
411	A	13	12	1.01	25	0.480
412	A	13	13	1.16	25	0.520
413	A	10	10	1.19	25	0.400
414	A	8	8	1.21	25	0.320
415	A	11	10	1.04	25	0.400
416	A	11	10	1.06	25	0.400
417	A	11	10	1.02	25	0.400
418	A	13	12	1.03	25	0.480
419	A	15	14	1.04	25	0.560
420	A	13	12	1.10	25	0.480
421	A	10	9	1.06	25	0.360
422	A	8	7	1.01	25	0.280
423	A	6	5	1.36	25	0.200
424	A	10	9	0.86	25	0.360
425	A	12	11	0.94	25	0.440
426	A	15	14	0.97	25	0.560
427	A	17	16	1.08	25	0.640
428	A	14	13	1.06	25	0.520
429	A	11	10	1.02	25	0.400
430	A	8	7	1.01	25	0.280
431	A	8	7	1.01	25	0.280
432	A	13	12	0.93	25	0.480
433	A	16	15	0.95	25	0.600
434	A	17	16	1.08	25	0.640
435	A	14	13	1.06	25	0.520
436	A	11	10	1.04	25	0.400
437	A	10	9	1.04	25	0.360

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
438	A	10	9	1.04	25	0.360
439	A	16	15	0.97	25	0.600
440	A	19	18	0.99	25	0.720
441	A	12	12	1.14	23	0.522
442	A	13	13	1.13	23	0.565
443	A	7	7	1.04	21	0.333
444	A	10	10	1.16	23	0.435
445	A	13	13	1.14	23	0.565
446	A	11	10	1.06	19	0.526
447	A	9	8	1.02	19	0.421
448	A	9	8	1.02	19	0.421
449	A	7	6	1.00	17	0.353
450	A	1	1	1.00	10	0.100
451	A	5	5	1.00	17	0.294
452	A	6	6	1.03	19	0.316
453	A	8	7	1.02	19	0.368
454	A	10	9	1.07	19	0.474
455	A	10	9	1.01	19	0.474
456	A	13	12	0.95	21	0.571
457	A	11	10	0.93	21	0.476
458	A	11	10	1.09	21	0.476
459	A	9	8	1.00	19	0.421
460	A	7	6	1.00	12	0.500
461	A	7	7	1.00	19	0.368
462	A	6	6	1.00	21	0.286
463	A	10	9	1.14	21	0.429
464	A	10	9	0.92	21	0.429
465	A	13	12	1.02	21	0.571
466	A	15	14	1.06	21	0.667
467	A	13	12	1.08	21	0.571
468	A	11	10	1.06	19	0.526
469	A	3	3	1.01	12	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
470	A	9	9	1.00	19	0.474
471	A	9	9	1.01	21	0.429
472	A	8	8	1.02	21	0.381
473	A	12	11	0.95	21	0.524
474	A	12	11	0.90	21	0.524
475	A	16	15	0.89	21	0.714
476	A	18	17	1.06	21	0.810
477	A	15	14	1.09	21	0.667
478	A	13	12	1.08	19	0.632
479	A	5	5	1.07	12	0.417
480	A	11	11	0.99	19	0.579
481	A	11	11	1.00	21	0.524
482	A	12	12	1.01	21	0.571
483	A	11	11	1.06	21	0.524
484	A	15	14	0.95	21	0.667
485	A	15	14	0.91	21	0.667
486	A	7	7	1.08	12	0.583
487	A	17	16	1.11	21	0.762
488	A	13	12	1.08	21	0.571
489	A	11	10	1.04	21	0.476
490	A	9	8	1.00	21	0.381
491	A	6	5	1.00	19	0.263
492	A	6	5	1.00	12	0.417
493	A	10	9	1.04	19	0.474
494	A	13	12	1.10	21	0.571
495	A	16	15	1.11	21	0.714
496	A	18	17	1.13	21	0.810
497	A	16	15	1.05	21	0.714
498	A	13	12	1.12	21	0.571
499	A	12	11	1.21	21	0.524
500	A	10	9	1.13	21	0.429
501	A	10	9	1.13	19	0.474

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
502	A	11	10	1.23	12	0.833
503	A	13	12	1.14	19	0.632
504	A	15	14	1.05	21	0.667
505	A	19	18	1.03	21	0.857
506	A	16	15	1.16	21	0.714
507	A	13	12	1.24	21	0.571
508	A	13	12	1.17	21	0.571
509	A	12	11	1.19	21	0.524
510	A	13	12	1.20	19	0.632
511	A	14	13	1.25	12	1.083
512	A	17	16	1.16	19	0.842
513	A	19	18	1.08	21	0.857
514	A	19	18	1.18	21	0.857
515	A	16	15	1.21	21	0.714
516	A	13	12	1.20	21	0.571
517	A	15	14	1.19	21	0.667
518	A	15	14	1.21	21	0.667
519	A	15	14	1.23	19	0.737
520	A	17	16	1.26	12	1.333
521	A	20	19	1.19	19	1.000
522	A	23	22	1.11	21	1.048
523	A	4	4	1.32	12	0.333
524	A	8	8	1.27	12	0.667
525	A	10	10	1.25	12	0.833
526	A	13	13	1.24	12	1.083
527	A	6	5	0.40	12	0.417
528	A	11	10	0.61	12	0.833
529	A	14	13	0.73	12	1.083
530	A	17	16	0.81	12	1.333
531	A	11	11	1.02	23	0.478
532	A	7	7	1.00	23	0.304
533	A	5	5	1.00	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
534	A	2	2	1.00	14	0.143
535	A	10	10	1.02	21	0.476
536	A	13	13	0.99	23	0.565
537	A	17	17	1.04	23	0.739
538	A	14	14	1.02	23	0.609
539	A	10	10	1.02	23	0.435
540	A	8	8	1.00	21	0.381
541	A	8	8	1.00	14	0.571
542	A	11	11	1.02	21	0.524
543	A	14	14	1.01	23	0.609
544	A	20	20	1.04	23	0.870
545	A	17	17	1.03	23	0.739
546	A	13	13	1.03	23	0.565
547	A	11	11	1.02	21	0.524
548	A	11	11	1.00	14	0.786
549	A	11	11	1.02	21	0.524
550	A	14	14	1.02	23	0.609
551	A	17	17	1.04	23	0.739
552	A	20	20	1.03	23	0.870
553	A	14	14	1.01	14	1.000
554	A	13	13	1.06	23	0.565
555	A	10	10	1.04	23	0.435
556	A	8	8	1.02	23	0.348
557	A	5	5	1.00	23	0.217
558	A	2	2	1.00	21	0.095
559	A	2	2	1.00	14	0.143
560	A	10	10	1.02	21	0.476
561	A	13	13	1.00	23	0.565
562	A	14	14	1.03	23	0.609
563	A	11	11	1.06	23	0.478
564	A	8	8	1.10	23	0.348
565	A	8	8	1.10	23	0.348

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
566	A	8	8	1.10	21	0.381
567	A	11	11	1.08	14	0.786
568	A	14	14	1.10	21	0.667
569	A	17	17	1.07	23	0.739
570	A	14	14	1.05	23	0.609
571	A	11	11	1.07	23	0.478
572	A	11	11	1.06	23	0.478
573	A	11	11	1.06	23	0.478
574	A	11	11	1.06	21	0.524
575	A	14	14	1.07	14	1.000
576	A	17	17	1.07	21	0.810
577	A	20	20	1.07	23	0.870
578	A	17	17	1.09	14	1.214
579	A	11	11	1.01	21	0.524
580	A	9	9	1.02	21	0.429
581	A	9	9	1.01	21	0.429
582	A	7	7	1.00	21	0.333
583	A	9	9	1.01	21	0.429
584	A	9	9	1.02	21	0.429
585	A	11	11	1.04	21	0.524
586	A	17	17	0.97	23	0.739
587	A	15	15	0.95	23	0.652
588	A	13	13	1.01	23	0.565
589	A	11	11	1.00	23	0.478
590	A	11	11	1.00	23	0.478
591	A	13	13	1.01	23	0.565
592	A	15	15	0.97	23	0.652
593	A	18	18	0.93	23	0.783
594	A	16	16	0.96	23	0.696
595	A	14	14	1.01	23	0.609
596	A	14	14	1.01	23	0.609
597	A	14	14	1.01	23	0.609

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
598	A	16	16	0.97	23	0.696
599	A	18	18	0.95	23	0.783
600	A	21	21	0.94	23	0.913
601	A	19	19	0.98	23	0.826
602	A	17	17	1.02	23	0.739
603	A	17	17	1.00	23	0.739
604	A	17	17	1.02	23	0.739
605	A	17	17	1.04	23	0.739
606	A	19	19	1.00	23	0.826
607	A	21	21	0.95	23	0.913
608	A	18	18	1.07	23	0.783
609	A	11	11	0.99	23	0.478
610	A	4	4	1.00	23	0.174
611	A	7	7	0.80	23	0.304
612	A	12	12	1.00	23	0.522
613	A	15	15	1.03	23	0.652
614	A	21	21	0.95	23	0.913
615	A	18	18	0.95	23	0.783
616	A	15	15	0.99	23	0.652
617	A	15	15	0.97	23	0.652
618	A	15	15	0.95	23	0.652
619	A	15	15	0.95	23	0.652
620	A	18	18	0.95	23	0.783
621	A	21	21	0.98	23	0.913
622	A	18	18	1.01	23	0.783
623	A	18	18	0.99	23	0.783
624	A	18	18	1.00	23	0.783
625	A	18	18	0.99	23	0.783
626	A	18	18	1.00	23	0.783
627	A	21	21	0.99	23	0.913
628	A	23	23	1.07	25	0.920
629	A	13	13	1.00	25	0.520

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
630	A	6	6	1.00	25	0.240
631	A	15	15	1.01	25	0.600
632	A	18	18	1.04	25	0.720
633	A	21	21	1.06	25	0.840
634	A	26	26	1.03	25	1.040
635	A	23	23	1.02	25	0.920
636	A	20	20	1.00	25	0.800
637	A	16	16	1.01	25	0.640
638	A	19	19	0.99	25	0.760
639	A	22	22	1.03	25	0.880
640	A	29	29	1.06	25	1.160
641	A	26	26	1.03	25	1.040
642	A	23	23	1.03	25	0.920
643	A	23	23	1.00	25	0.920
644	A	19	19	1.05	25	0.760
645	A	22	22	1.03	25	0.880
646	A	25	25	1.04	25	1.000
647	A	25	25	1.00	25	1.000
648	A	23	23	1.05	25	0.920
649	A	6	6	1.00	25	0.240
650	A	6	6	1.00	25	0.240
651	A	13	13	1.00	25	0.520
652	A	15	15	1.02	25	0.600
653	A	18	18	1.05	25	0.720
654	A	26	26	1.04	25	1.040
655	A	17	17	1.04	25	0.680
656	A	9	9	1.00	25	0.360
657	A	16	16	1.06	25	0.640
658	A	16	16	1.07	25	0.640
659	A	19	19	1.02	25	0.760
660	A	22	22	1.00	25	0.880
661	A	29	29	1.03	25	1.160

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
662	A	26	26	1.05	25	1.040
663	A	19	19	1.06	25	0.760
664	A	19	19	1.02	25	0.760
665	A	19	19	1.04	25	0.760
666	A	19	19	1.05	25	0.760
667	A	22	22	1.02	25	0.880
668	A	25	25	1.01	25	1.000
669	A	9	9	1.00	25	0.360
670	A	9	9	1.00	25	0.360
671	A	13	13	1.00	25	0.520
672	A	13	13	1.00	25	0.520
673	A	9	9	1.00	25	0.360
674	A	9	9	1.00	25	0.360
675	A	9	9	1.00	25	0.360
676	A	9	9	1.00	25	0.360
677	A	4	4	1.00	25	0.160
678	A	4	4	1.00	25	0.160
679	A	6	6	1.00	25	0.240
680	A	6	6	1.00	25	0.240
681	A	4	4	1.00	25	0.160
682	A	4	4	1.00	25	0.160
683	A	4	4	1.00	25	0.160
684	A	4	4	1.00	25	0.160
685	A	5	4	1.00	21	0.190
686	N/A	2	0	1.00	14	0.000
687	A	16	15	1.04	23	0.652
688	A	13	12	1.02	23	0.522
689	A	9	8	1.00	23	0.348
690	A	5	4	1.00	21	0.190
691	N/A	2	0	1.00	14	0.000
692	A	5	4	1.03	21	0.190
693	N/A	2	0	1.00	14	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
694	A	19	18	1.04	23	0.783
695	A	16	15	1.02	23	0.652
696	A	12	11	1.02	23	0.478
697	A	5	4	1.03	21	0.190
698	N/A	2	0	1.00	14	0.000
699	A	13	12	1.04	23	0.522
700	A	10	9	1.01	23	0.391
701	A	7	6	1.00	23	0.261
702	A	5	4	1.00	21	0.190
703	N/A	2	0	1.00	14	0.000
704	A	5	4	1.00	21	0.190
705	N/A	2	0	1.00	14	0.000
706	A	5	4	1.05	21	0.190
707	N/A	2	0	1.00	14	0.000
708	A	13	12	0.97	23	0.522
709	A	10	9	0.99	23	0.391
710	A	10	9	0.98	23	0.391
711	A	5	4	1.05	21	0.190
712	N/A	2	0	1.00	14	0.000
713	A	9	8	1.12	23	0.348
714	A	9	8	1.12	23	0.348
715	A	9	8	1.12	23	0.348
716	A	9	8	1.12	23	0.348
717	N/A	2	0	1.00	25	0.000
718	N/A	2	0	1.00	25	0.000
719	N/A	2	0	1.00	25	0.000
720	N/A	2	0	1.00	25	0.000
721	N/A	2	0	1.00	25	0.000
722	N/A	2	0	1.00	25	0.000
723	N/A	2	0	1.00	25	0.000
724	N/A	2	0	1.00	25	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
725	N/A	2	0	1.00	25	0.000
726	N/A	2	0	1.00	25	0.000
727	N/A	2	0	1.00	25	0.000
728	N/A	2	0	1.00	25	0.000
729	N/A	2	0	1.00	25	0.000
730	N/A	2	0	1.00	25	0.000
731	N/A	2	0	1.00	25	0.000
732	N/A	2	0	1.00	25	0.000
733	N/A	2	0	1.00	25	0.000
734	N/A	2	0	1.00	25	0.000
735	N/A	2	0	1.00	25	0.000
736	N/A	2	0	1.00	25	0.000
737	N/A	2	0	1.00	25	0.000
738	N/A	2	0	1.00	25	0.000
739	N/A	2	0	1.00	25	0.000
740	N/A	2	0	1.00	25	0.000
741	N/A	2	0	1.00	25	0.000
742	N/A	2	0	1.00	25	0.000
743	N/A	2	0	1.00	25	0.000
744	N/A	2	0	1.00	25	0.000
745	N/A	2	0	1.00	25	0.000
746	N/A	2	0	1.00	25	0.000
747	N/A	2	0	1.00	25	0.000
748	N/A	2	0	1.00	25	0.000
749	N/A	2	0	1.00	25	0.000
750	N/A	2	0	1.00	25	0.000
751	N/A	2	0	1.00	25	0.000
752	N/A	2	0	1.00	25	0.000
753	N/A	2	0	1.00	25	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
754	N/A	2	0	1.00	25	0.000
755	N/A	2	0	1.00	25	0.000
756	N/A	2	0	1.00	25	0.000
757	N/A	2	0	1.00	25	0.000
758	N/A	2	0	1.00	25	0.000
759	N/A	2	0	1.00	25	0.000
760	N/A	2	0	1.00	25	0.000
761	N/A	2	0	1.00	25	0.000
762	N/A	2	0	1.00	25	0.000
763	N/A	2	0	1.00	25	0.000
764	N/A	2	0	1.00	25	0.000
765	N/A	2	0	1.00	25	0.000
766	N/A	2	0	1.00	25	0.000
767	N/A	2	0	1.00	25	0.000
768	N/A	2	0	1.00	25	0.000
769	N/A	2	0	1.00	25	0.000
770	N/A	2	0	1.00	25	0.000
771	N/A	2	0	1.00	25	0.000
772	N/A	2	0	1.00	25	0.000
773	N/A	2	0	1.00	25	0.000
774	N/A	2	0	1.00	25	0.000
775	N/A	2	0	1.00	25	0.000
776	N/A	2	0	1.00	25	0.000
777	A	13	13	1.04	23	0.565
778	A	11	11	0.99	23	0.478
779	A	6	6	1.00	21	0.286
780	A	9	8	1.08	23	0.348
781	A	5	5	1.04	23	0.217
782	N/A	2	0	1.00	25	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
783	N/A	2	0	1.00	25	0.000
784	N/A	2	0	1.00	25	0.000
785	N/A	2	0	1.00	25	0.000
786	N/A	2	0	1.00	21	0.000
787	N/A	2	0	1.00	23	0.000
788	A	9	8	1.00	21	0.381
789	A	7	6	1.00	21	0.286
790	A	5	4	1.00	19	0.211
791	N/A	2	0	1.00	12	0.000
792	N/A	2	0	1.00	19	0.000
793	N/A	2	0	1.00	21	0.000
794	A	11	11	1.07	21	0.524
795	A	11	11	1.05	21	0.524
796	A	9	9	1.02	21	0.429
797	A	9	9	1.02	21	0.429
798	A	7	7	1.00	21	0.333
799	A	9	9	1.02	21	0.429
800	A	9	9	1.02	21	0.429
801	A	11	11	1.02	21	0.524
802	A	19	19	1.37	23	0.826
803	A	17	17	1.41	23	0.739
804	A	15	15	1.61	23	0.652
805	A	13	13	1.85	23	0.565
806	A	13	13	1.90	23	0.565
807	A	15	15	1.65	23	0.652
808	A	17	17	1.39	23	0.739
809	A	19	19	1.34	23	0.826
810	A	20	20	1.25	23	0.870
811	A	18	18	1.35	23	0.783
812	A	16	16	1.53	23	0.696
813	A	16	16	1.49	23	0.696

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
814	A	16	16	1.53	23	0.696
815	A	18	18	1.36	23	0.783
816	A	20	20	1.23	23	0.870
817	A	20	20	1.61	23	0.870
818	A	17	17	1.78	23	0.739
819	B	14	14	2.08	23	0.609
820	A	9	9	1.00	23	0.391
821	A	6	6	1.00	23	0.261
822	A	13	13	1.78	23	0.565
823	A	20	20	1.73	23	0.870
824	A	20	20	1.27	23	0.870
825	A	17	17	1.38	23	0.739
826	A	17	17	1.42	23	0.739
827	A	17	17	1.51	23	0.739
828	A	17	17	1.51	23	0.739
829	A	20	20	1.31	23	0.870
830	A	23	23	1.23	23	1.000
831	A	20	20	1.29	23	0.870
832	A	20	20	1.30	23	0.870
833	A	20	20	1.33	23	0.870
834	A	20	20	1.31	23	0.870
835	A	20	20	1.33	23	0.870
836	A	23	23	1.23	23	1.000
837	A	20	20	1.12	25	0.800
838	A	17	17	1.11	25	0.680
839	A	8	8	1.00	25	0.320
840	A	15	15	1.15	25	0.600
841	A	25	25	1.16	25	1.000
842	A	24	24	1.10	25	0.960
843	A	21	21	1.08	25	0.840
844	A	18	18	1.12	25	0.720
845	A	22	22	1.10	25	0.880

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
846	A	25	25	1.11	25	1.000
847	A	28	28	1.10	25	1.120
848	A	27	27	1.10	25	1.080
849	A	24	24	1.10	25	0.960
850	A	21	21	1.13	25	0.840
851	A	25	25	1.08	25	1.000
852	A	25	25	1.11	25	1.000
853	A	28	28	1.10	25	1.120
854	F	0	0	N/A	0.000	N/A
855	A	20	20	1.13	25	0.800
856	A	17	17	1.13	25	0.680
857	A	15	15	1.15	25	0.600
858	A	8	8	1.00	25	0.320
859	A	8	8	1.00	25	0.320
860	A	25	25	1.14	25	1.000
861	A	27	27	1.07	25	1.080
862	A	24	24	1.06	25	0.960
863	A	21	21	1.09	25	0.840
864	A	18	18	1.16	25	0.720
865	A	18	18	1.16	25	0.720
866	A	11	11	1.17	25	0.440
867	A	19	19	1.14	25	0.760
868	A	28	28	1.10	25	1.120
869	A	24	24	1.07	25	0.960
870	A	21	21	1.11	25	0.840
871	A	21	21	1.11	25	0.840
872	A	21	21	1.10	25	0.840
873	A	21	21	1.14	25	0.840
874	A	28	28	1.11	25	1.120
875	A	15	15	1.12	23	0.652
876	A	13	13	1.10	23	0.565
877	A	7	7	1.04	21	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
878	A	11	10	1.07	23	0.435
879	A	7	7	1.02	23	0.304

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \sec^4(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 85

$$\int \sec^4(c+dx)(a+a\sec(c+dx))\,dx = \frac{3a\text{arctanh}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)}{d}+\frac{3a\sec(c+dx)\tan(c+dx)}{8d}+\frac{a\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{a\tan^3(c+dx)}{3d}$$

output
```
3/8*a*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+3/8*a*sec(d*x+c)*tan(d*x+c)/d+1
/4*a*sec(d*x+c)^3*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.66

$$\int \sec^4(c+dx)(a+a\sec(c+dx))\,dx = \frac{a(9\text{arctanh}(\sin(c+dx))+\tan(c+dx)\,(9\sec(c+dx)+6\sec^3(c+dx)+8(3+\tan^2(c+dx))))}{24d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

output
```
(a*(9*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(9*Sec[c + d*x] + 6*Sec[c + d*x
]^3 + 8*(3 + Tan[c + d*x]^2))))/(24*d)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 4274, 3042, 4254, 2009, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \sec^5(c+dx)dx+a\int \sec^4(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx+a\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \text{4254}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx-\frac{a\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4257}$$

$$a\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

input
```
Int[Sec[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

output
```
-((a*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/d) + a*((Sec[c + d*x]^3*Tan[c + d
*x])/(4*d) + (3*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])
/(2*d)))/4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{-a\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+a\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
default	$\frac{-a\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+a\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
parts	$-\frac{a\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{a\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
norman	$\frac{\frac{13a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{31a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12d}+\frac{49a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{12d}-\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}-\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8d}+\frac{3a\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{8d}$
risch	$-\frac{ia\left(9\,e^{7i(dx+c)}+33\,e^{5i(dx+c)}-48\,e^{4i(dx+c)}-33\,e^{3i(dx+c)}-64\,e^{2i(dx+c)}-9\,e^{i(dx+c)}-16\right)}{12d\left(e^{2i(dx+c)}+1\right)^4}+\frac{3a\ln\left(e^{i(dx+c)}+i\right)}{8d}-3$
parallelrisch	$\frac{a\left(9(-3-\cos(4dx+4c)-4\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+9(3+\cos(4dx+4c)+4\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\right.}{24d(3+\cos(4dx+4c)+4\cos(2dx+2c))}$

input
```
int(sec(d*x+c)^4*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a*(-(-1/4*sec(d*x+c)^3-3/8*sec(
d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.16

$$\int \sec^4(c+dx)(a+a\sec(c+dx))\,dx = \frac{9\,a\cos(dx+c)^4\log(\sin(dx+c)+1) - 9\,a\cos(dx+c)^4\log(-\sin(dx+c)+1) + 2\left(16\,a\cos(dx+c)^3 \right.}{48\,d\cos(dx+c)^4}$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/48*(9*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 9*a*cos(d*x + c)^4*log(-s
in(d*x + c) + 1) + 2*(16*a*cos(d*x + c)^3 + 9*a*cos(d*x + c)^2 + 8*a*cos(d
*x + c) + 6*a)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]

$$\int \sec^4(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \sec^4(c+dx)\,dx + \int \sec^5(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**4*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(sec(c + d*x)**4, x) + Integral(sec(c + d*x)**5, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

$$\int \sec^4(c+dx)(a+a\sec(c+dx))\,dx$$

$$= \frac{16\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a-3\,a\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\mathrm{s}\right.\right.}{48\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a - 3*a*(2*(3*sin(d*x + c)^3 -
5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x +
c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.29

$$\int \sec^4(c+dx)(a+a\sec(c+dx))\,dx$$

$$= \frac{9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-49\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+31\,\ldots\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\ldots\right.}}{24\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/24*(9*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*a*log(abs(tan(1/2*d*x + 1
/2*c) - 1)) - 2*(9*a*tan(1/2*d*x + 1/2*c)^7 - 49*a*tan(1/2*d*x + 1/2*c)^5
+ 31*a*tan(1/2*d*x + 1/2*c)^3 - 39*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
1/2*c)^2 - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 12.66 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.53

$$\int \sec^4(c+dx)(a+a\sec(c+dx))\,dx = \frac{-\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{49\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}-\frac{31\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}+\frac{13\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)} + \frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$

input
```
int((a + a/cos(c + d*x))/cos(c + d*x)^4,x)
```

output
```
((13*a*tan(c/2 + (d*x)/2))/4 - (31*a*tan(c/2 + (d*x)/2)^3)/12 + (49*a*tan(
c/2 + (d*x)/2)^5)/12 - (3*a*tan(c/2 + (d*x)/2)^7)/4)/(d*(6*tan(c/2 + (d*x)
/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/
2)^8 + 1)) + (3*a*atanh(tan(c/2 + (d*x)/2)))/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.27

$$\int \sec^4(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\left(-16\cos(dx+c)\sin(dx+c)^3+24\cos(dx+c)\sin(dx+c)-9\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^4+\right.}{}$$

input
```
int(sec(d*x+c)^4*(a+a*sec(d*x+c)),x)
```

output
```
(a*( - 16*cos(c + d*x)*sin(c + d*x)**3 + 24*cos(c + d*x)*sin(c + d*x) - 9*
log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4 + 18*log(tan((c + d*x)/2) - 1)*s
in(c + d*x)**2 - 9*log(tan((c + d*x)/2) - 1) + 9*log(tan((c + d*x)/2) + 1)
*sin(c + d*x)**4 - 18*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2 + 9*log(ta
n((c + d*x)/2) + 1) - 9*sin(c + d*x)**3 + 15*sin(c + d*x)))/(24*d*(sin(c +
 d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.2 $\int \sec^3(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \sec^3(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)}{d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d} + \frac{a\tan^3(c+dx)}{3d}$$

output
```
1/2*a*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+1/2*a*sec(d*x+c)*tan(d*x+c)/d+1
/3*a*tan(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \sec^3(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{2d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d} + \frac{a\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

input
```
Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a
*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 4274, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \sec^4(c+dx)dx+a\int \sec^3(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{4254}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{a\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4257}$$

$$a\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

input
```
Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

output
```
a*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)) - (a*(
-Tan[c + d*x] - Tan[c + d*x]^3/3))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-a\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
default	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-a\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
parts	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}-\frac{a\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
risch	$-\frac{ia\left(3\,\mathrm{e}^{5i(dx+c)}-12\,\mathrm{e}^{2i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}-4\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2d}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2d}$
norman	$\frac{-\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{4a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$
parallelrisch	$\frac{a\left(9\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)-9\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(3dx+3c)-3\ln\left(\tan\right.\right.}{6d(\cos(3dx+3c)+3\cos(dx+c))}$

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-a*(-2/3-1
/3*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.40

$$\int \sec^3(c+dx)(a+a\sec(c+dx))\,dx$$
$$=\frac{3\,a\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,a\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\left(4\,a\cos\left(dx+c\right)^2+\right.}{12\,d\cos\left(dx+c\right)^3}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(3*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*cos(d*x + c)^3*log(-s
in(d*x + c) + 1) + 2*(4*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 2*a)*sin(d*x
 + c))/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \sec^3{(c+dx)}\,dx + \int \sec^4{(c+dx)}\,dx\right)$$

input
```
integrate(sec(d*x+c)**3*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(sec(c + d*x)**3, x) + Integral(sec(c + d*x)**4, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11

$$\int \sec^3(c+dx)(a+a\sec(c+dx))\,dx = \frac{4\left(\tan{(dx+c)}^3 + 3\tan{(dx+c)}\right)a - 3a\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log{(\sin{(dx+c)}+1)} + \log{(\sin{(dx+c)}-1)}\right)}{12\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a - 3*a*(2*sin(d*x + c)/(sin(d*x
 + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.52

$$\int \sec^3(c+dx)(a+a\sec(c+dx))\,dx$$
$$=\frac{3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{6\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(3*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*log(abs(tan(1/2*d*x + 1/
2*c) - 1)) - 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 4*a*tan(1/2*d*x + 1/2*c)^3 +
9*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 11.36 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.62

$$\int \sec^3(c+dx)(a+a\sec(c+dx))\,dx$$
$$=\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + a/cos(c + d*x))/cos(c + d*x)^3,x)
```

output
```
(a*atanh(tan(c/2 + (d*x)/2)))/d - (3*a*tan(c/2 + (d*x)/2) - (4*a*tan(c/2 +
 (d*x)/2)^3)/3 + a*tan(c/2 + (d*x)/2)^5)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*ta
n(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.46

$$\int \sec^3(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{a\left(-3\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2+3\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+3\cos(dx\right.}{}$$

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c)),x)
```

output
```
(a*( - 3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 + 3*cos(c
+ d*x)*log(tan((c + d*x)/2) - 1) + 3*cos(c + d*x)*log(tan((c + d*x)/2) + 1
)*sin(c + d*x)**2 - 3*cos(c + d*x)*log(tan((c + d*x)/2) + 1) - 3*cos(c + d
*x)*sin(c + d*x) + 4*sin(c + d*x)**3 - 6*sin(c + d*x)))/(6*cos(c + d*x)*d*
(sin(c + d*x)**2 - 1))
```

3.3 $\int \sec^2(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \sec^2(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)}{d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
1/2*a*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+1/2*a*sec(d*x+c)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)}{d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan
[c + d*x])/(2*d)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 4274, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \sec^3(c+dx)dx+a\int \sec^2(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{4254}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{a\int 1d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{24}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{a\tan(c+dx)}{d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a\tan(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a\tan(c+dx)}{d}$$

$$\Big\downarrow \text{4257}$$

$$a\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a\tan(c+dx)}{d}$$

input
```
Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

output
```
(a*Tan[c + d*x])/d + a*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c
+ d*x])/(2*d))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{a\tan(dx+c)+a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{a\tan(dx+c)+a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
parts	$\frac{a\tan(dx+c)}{d}+\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
norman	$\frac{\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$
parallelrisch	$\frac{\left((-1-\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2\sin(dx+c)+2\sin(2dx+2c)\right)a}{2d(1+\cos(2dx+2c))}$
risch	$-\frac{ia\left(e^{3i(dx+c)}-2\,e^{2i(dx+c)}-e^{i(dx+c)}-2\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{2d}$

input
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*tan(d*x+c)+a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c
))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.57

$$\int \sec^2(c+dx)(a+a\sec(c+dx))\,dx$$
$$=\frac{a\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-a\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\,a\cos\left(dx+c\right)+a\right)\mathrm{s}}{4\,d\cos\left(dx+c\right)^2}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - a*cos(d*x + c)^2*log(-sin(d*
x + c) + 1) + 2*(2*a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \sec^2(c+dx)\,dx + \int \sec^3(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**2*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(sec(c + d*x)**2, x) + Integral(sec(c + d*x)**3, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int \sec^2(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) - 4a\tan(dx+c)}{4d}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/4*(a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log
(sin(d*x + c) - 1)) - 4*a*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.70

$$\int \sec^2(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) - \frac{2\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 3a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2}}{2d}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c)
 - 1)) - 2*(a*tan(1/2*d*x + 1/2*c)^3 - 3*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*
d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 10.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

$$\int \sec^2(c+dx)(a+a\sec(c+dx))\,dx = \frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)} + \frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

input
```
int((a + a/cos(c + d*x))/cos(c + d*x)^2,x)
```

output
```
(3*a*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^3)/(d*(tan(c/2 + (d*x)/2)^4
 - 2*tan(c/2 + (d*x)/2)^2 + 1)) + (a*atanh(tan(c/2 + (d*x)/2)))/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.34

$$\int \sec^2(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\left(-2\cos(dx+c)\sin(dx+c)-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\log\left(\tan\right.\right.}{2d\left(\sin(dx+c)^2-1\right)}$$

input
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c)),x)
```

output
```
(a*( - 2*cos(c + d*x)*sin(c + d*x) - log(tan((c + d*x)/2) - 1)*sin(c + d*x
)**2 + log(tan((c + d*x)/2) - 1) + log(tan((c + d*x)/2) + 1)*sin(c + d*x)*
*2 - log(tan((c + d*x)/2) + 1) - sin(c + d*x)))/(2*d*(sin(c + d*x)**2 - 1)
)
```

3.4 $\int \sec(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 24

$$\int \sec(c+dx)(a+a\sec(c+dx))\,dx=\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{a\tan(c+dx)}{d}$$

output
```
a*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \sec(c+dx)(a+a\sec(c+dx))\,dx=\frac{a\coth^{-1}(\sin(c+dx))}{d}+\frac{a\tan(c+dx)}{d}$$

input
```
Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x]),x]
```

output
```
(a*ArcCoth[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\sec(c+dx)+a)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$
$$\Big\downarrow \text{4274}$$
$$a\int \sec^2(c+dx)dx+a\int \sec(c+dx)dx$$
$$\Big\downarrow \text{3042}$$
$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx$$
$$\Big\downarrow \text{4254}$$
$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{a\int 1d(-\tan(c+dx))}{d}$$
$$\Big\downarrow \text{24}$$
$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{a\tan(c+dx)}{d}$$
$$\Big\downarrow \text{4257}$$
$$\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{a\tan(c+dx)}{d}$$

input
```
Int[Sec[c + d*x]*(a + a*Sec[c + d*x]),x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\frac{a\ln(\sec(dx+c)+\tan(dx+c))+a\tan(dx+c)}{d}$	30
default	$\frac{a\ln(\sec(dx+c)+\tan(dx+c))+a\tan(dx+c)}{d}$	30
parts	$\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a\tan(dx+c)}{d}$	32
risch	$\frac{2ia}{d\left(e^{2i(dx+c)}+1\right)}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{d}$	59
parallelrisch	$\frac{\left(\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+\sin(dx+c)\right)a}{\cos(dx+c)d}$	60
norman	$-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	67

input
```
int(sec(d*x+c)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*ln(sec(d*x+c)+tan(d*x+c))+a*tan(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(24) = 48$.

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.50

$$\int \sec(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\cos(dx+c)\log(\sin(dx+c)+1) - a\cos(dx+c)\log(-\sin(dx+c)+1) + 2\,a\sin(dx+c)}{2\,d\cos(dx+c)}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x +
c) + 1) + 2*a*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [A] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \sec(c+dx)(a+a\sec(c+dx))\,dx = \begin{cases} \frac{a\log(\tan(c+dx)+\sec(c+dx))+a\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x(a\sec(c)+a)\sec(c) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c)),x)
```

output
```
Piecewise(((a*log(tan(c + d*x) + sec(c + d*x)) + a*tan(c + d*x))/d, Ne(d,
0)), (x*(a*sec(c) + a)*sec(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \sec(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+a\tan\left(dx+c\right)}{d}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
(a*log(sec(d*x + c) + tan(d*x + c)) + a*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(24) = 48$.

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int \sec(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}}{d}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
(a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1
)) - 2*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

Mupad [B] (verification not implemented)

Time = 10.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int \sec(c+dx)(a+a\sec(c+dx))\,dx = \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + a/cos(c + d*x))/cos(c + d*x),x)
```

output
```
(2*a*atanh(tan(c/2 + (d*x)/2)))/d - (2*a*tan(c/2 + (d*x)/2))/(d*(tan(c/2 +
 (d*x)/2)^2 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.46

$$\int \sec(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\left(-\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\sin\left(dx+c\right)\right)}{\cos\left(dx+c\right)d}$$

input
```
int(sec(d*x+c)*(a+a*sec(d*x+c)),x)
```

output
```
(a*( - cos(c + d*x)*log(tan((c + d*x)/2) - 1) + cos(c + d*x)*log(tan((c +
d*x)/2) + 1) + sin(c + d*x)))/(cos(c + d*x)*d)
```

3.5 $\int (a+a\sec (c+dx)) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 16

$$\int (a+a\sec (c+dx)) \, dx=ax+\frac{a\text{arctanh}(\sin (c+dx))}{d}$$

output
```
a*x+a*arctanh(sin(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a+a\sec (c+dx)) \, dx=ax+\frac{a\coth^{-1}(\sin (c+dx))}{d}$$

input
```
Integrate[a + a*Sec[c + d*x],x]
```

output
```
a*x + (a*ArcCoth[Sin[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)\,dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a\text{arctanh}(\sin(c+dx))}{d}+ax$$

input
```
Int[a + a*Sec[c + d*x],x]
```

output
```
a*x + (a*ArcTanh[Sin[c + d*x]])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

method	result	size
default	$ax+\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	24
parts	$ax+\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	24
derivativedivides	$\frac{(dx+c)a+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	29
parallelrisch	$\frac{a\left(\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\right)}{d}+ax$	37
norman	$ax+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	40
risch	$ax+\frac{a\ln(e^{i(dx+c)}+i)}{d}-\frac{a\ln(e^{i(dx+c)}-i)}{d}$	42

input
```
int(a+a*sec(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*x+a/d*ln(sec(d*x+c)+tan(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(16) = 32$.

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int (a+a\sec(c+dx))\,dx=\frac{2\,adx+a\log\left(\sin\left(dx+c\right)+1\right)-a\log\left(-\sin\left(dx+c\right)+1\right)}{2\,d}$$

input
```
integrate(a+a*sec(d*x+c),x, algorithm="fricas")
```

output
```
1/2*(2*a*d*x + a*log(sin(d*x + c) + 1) - a*log(-sin(d*x + c) + 1))/d
```

Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.56

$$\int (a + a\sec(c + dx))\,dx = ax + a\left(\begin{cases} \frac{\log(\tan(c+dx)+\sec(c+dx))}{d} & \text{for } d \neq 0 \\ \frac{x(\tan(c)\sec(c)+\sec^2(c))}{\tan(c)+\sec(c)} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+a*sec(d*x+c),x)
```

output
```
a*x + a*Piecewise((log(tan(c + d*x) + sec(c + d*x))/d, Ne(d, 0)), (x*(tan(
c)*sec(c) + sec(c)**2)/(tan(c) + sec(c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int (a + a\sec(c + dx))\,dx = ax + \frac{a\log(\sec(dx + c) + \tan(dx + c))}{d}$$

input
```
integrate(a+a*sec(d*x+c),x, algorithm="maxima")
```

output
```
a*x + a*log(sec(d*x + c) + tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(16) = 32.

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.06

$$\int (a + a\sec(c + dx))\,dx$$
$$= ax + \frac{a\left(\log\left(\left|\frac{1}{\sin(dx+c)} + \sin(dx + c) + 2\right|\right) - \log\left(\left|\frac{1}{\sin(dx+c)} + \sin(dx + c) - 2\right|\right)\right)}{4\,d}$$

input
```
integrate(a+a*sec(d*x+c),x, algorithm="giac")
```

output
```
a*x + 1/4*a*(log(abs(1/sin(d*x + c) + sin(d*x + c) + 2)) - log(abs(1/sin(d
*x + c) + sin(d*x + c) - 2)))/d
```

Mupad [B] (verification not implemented)

Time = 9.88 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int (a + a\sec(c + dx))\,dx = a\,x + \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\right)}{d}$$

input
```
int(a + a/cos(c + d*x),x)
```

output
```
a*x + (2*a*atanh(tan(c/2 + (d*x)/2)))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int (a + a\sec(c + dx))\,dx = \frac{a\left(-\log\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) + \log\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) + dx\right)}{d}$$

input
```
int(a+a*sec(d*x+c),x)
```

output
```
(a*( - log(tan((c + d*x)/2) - 1) + log(tan((c + d*x)/2) + 1) + d*x))/d
```

3.6 $\int \cos(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \cos(c+dx)(a+a\sec(c+dx))\,dx = ax + \frac{a\sin(c+dx)}{d}$$

output
```
a*x+a*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \cos(c+dx)(a+a\sec(c+dx))\,dx = ax + \frac{a\cos(dx)\sin(c)}{d} + \frac{a\cos(c)\sin(dx)}{d}$$

input
```
Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x]),x]
```

output
```
a*x + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 4274, 24, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \cos(c+dx)dx+a\int 1dx$$

$$\Big\downarrow \text{24}$$

$$a\int \cos(c+dx)dx+ax$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)dx+ax$$

$$\Big\downarrow \text{3117}$$

$$\frac{a\sin(c+dx)}{d}+ax$$

input
```
Int[Cos[c + d*x]*(a + a*Sec[c + d*x]),x]
```

output
```
a*x + (a*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
risch	$ax+\frac{a\sin(dx+c)}{d}$	16
parallelrisch	$\frac{a(dx+\sin(dx+c))}{d}$	16
derivativedivides	$\frac{a\sin(dx+c)+(dx+c)a}{d}$	21
default	$\frac{a\sin(dx+c)+(dx+c)a}{d}$	21
norman	$\frac{ax+ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}$	50

input
```
int(cos(d*x+c)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
a*x+a*sin(d*x+c)/d
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cos(c+dx)(a+a\sec(c+dx))\,dx = \frac{adx + a\sin(dx+c)}{d}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
(a*d*x + a*sin(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cos(c+dx)(a+a\sec(c+dx))\,dx = ax + a\left(\begin{cases} x\cos(c) & \text{for } d=0 \\ \frac{\sin(c+dx)}{d} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c)),x)
```

output
```
a*x + a*Piecewise((x*cos(c), Eq(d, 0)), (sin(c + d*x)/d, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cos(c+dx)(a+a\sec(c+dx))\,dx = \frac{(dx+c)a + a\sin(dx+c)}{d}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
((d*x + c)*a + a*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(15) = 30$.

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \cos(c+dx)(a+a\sec(c+dx))\,dx = \frac{(dx+c)a + \frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
((d*x + c)*a + 2*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1))/d
```

Mupad [B] (verification not implemented)

Time = 9.66 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(c+dx)(a+a\sec(c+dx))\,dx = a\,x + \frac{a\,\sin\left(c+d\,x\right)}{d}$$

input
```
int(cos(c + d*x)*(a + a/cos(c + d*x)),x)
```

output
```
a*x + (a*sin(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(c+dx)(a+a\sec(c+dx))\,dx = \frac{a(\sin\left(dx+c\right)+dx)}{d}$$

input
```
int(cos(d*x+c)*(a+a*sec(d*x+c)),x)
```

output
```
(a*(sin(c + d*x) + d*x))/d
```

3.7 $\int \cos^2(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 38

$$\int \cos^2(c+dx)(a+a\sec(c+dx))\,dx=\frac{ax}{2}+\frac{a\sin(c+dx)}{d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
1/2*a*x+a*sin(d*x+c)/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \cos^2(c+dx)(a+a\sec(c+dx))\,dx=\frac{a(2(c+dx)+4\sin(c+dx)+\sin(2(c+dx)))}{4d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

output
```
(a*(2*(c + d*x) + 4*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 4274, 3042, 3115, 24, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \cos^2(c+dx)dx+a\int \cos(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)dx+a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3115}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)dx+a\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{24}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)dx+a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)$$

$$\Big\downarrow \text{3117}$$

$$\frac{a\sin(c+dx)}{d}+a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)$$

input
```
Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

output
```
(a*Sin[c + d*x])/d + a*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

method	result	size
parallelrisch	$\frac{a(2dx+\sin(2dx+2c)+4\sin(dx+c))}{4d}$	29
risch	$\frac{ax}{2}+\frac{a\sin(dx+c)}{d}+\frac{a\sin(2dx+2c)}{4d}$	32
derivativedivides	$\frac{a\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a\sin(dx+c)}{d}$	38
default	$\frac{a\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a\sin(dx+c)}{d}$	38
norman	$\frac{\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{d}+ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\frac{ax}{2}+\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{2}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}}$	82

input
```
int(cos(d*x+c)^2*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*(2*d*x+sin(2*d*x+2*c)+4*sin(d*x+c))/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \cos^2(c+dx)(a+a\sec(c+dx))\,dx = \frac{adx+(a\cos(dx+c)+2a)\sin(dx+c)}{2d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(a*d*x + (a*cos(d*x + c) + 2*a)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^2(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \cos^2(c+dx)\sec(c+dx)\,dx + \int \cos^2(c+dx)\,dx\right)$$

input
```
integrate(cos(d*x+c)**2*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(cos(c + d*x)**2*sec(c + d*x), x) + Integral(cos(c + d*x)**2, x
))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \cos^2(c+dx)(a+a\sec(c+dx))\,dx = \frac{(2\,dx+2\,c+\sin{(2\,dx+2\,c)})a+4\,a\sin{(dx+c)}}{4\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a + 4*a*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int \cos^2(c+dx)(a+a\sec(c+dx))\,dx = \frac{(dx+c)a+\frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/2*((d*x + c)*a + 2*(a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 10.46 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \cos^2(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\,x}{2}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^2}$$

input
```
int(cos(c + d*x)^2*(a + a/cos(c + d*x)),x)
```

output
```
(a*x)/2 + (3*a*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^3)/(d*(tan(c/2 +
(d*x)/2)^2 + 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \cos^2(c+dx)(a+a\sec(c+dx))\,dx = \frac{a(\cos(dx+c)\sin(dx+c)+2\sin(dx+c)+dx)}{2d}$$

input
```
int(cos(d*x+c)^2*(a+a*sec(d*x+c)),x)
```

output
```
(a*(cos(c + d*x)*sin(c + d*x) + 2*sin(c + d*x) + d*x))/(2*d)
```

3.8 $\int \cos^3(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \cos^3(c+dx)(a+a\sec(c+dx))\,dx=\frac{ax}{2}+\frac{a\sin(c+dx)}{d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}-\frac{a\sin^3(c+dx)}{3d}$$

output
```
1/2*a*x+a*sin(d*x+c)/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int \cos^3(c+dx)(a+a\sec(c+dx))\,dx=\frac{a(c+dx)}{2d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(2(c+dx))}{4d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

output
```
(a*(c + d*x))/(2*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (a*S
in[2*(c + d*x)])/(4*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3042, 4274, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \cos^3(c+dx)dx+a\int \cos^2(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+a\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3113}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{a\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{24}$$

$$a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

input
```
Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

output
```
a*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (a*(-Sin[c + d*x] + Sin[c +
d*x]^3/3))/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$\frac{a(6dx+\sin(3dx+3c)+3\sin(2dx+2c)+9\sin(dx+c))}{12d}$	40
risch	$\frac{ax}{2}+\frac{3a\sin(dx+c)}{4d}+\frac{a\sin(3dx+3c)}{12d}+\frac{a\sin(2dx+2c)}{4d}$	48
derivativedivides	$\frac{\frac{a\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+a\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	49
default	$\frac{\frac{a\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+a\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	49
norman	$\frac{\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}+\frac{ax}{2}+\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{4a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{3ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2}+\frac{3ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2}+\frac{ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{2}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}$	115

input
```
int(cos(d*x+c)^3*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/12*a*(6*d*x+sin(3*d*x+3*c)+3*sin(2*d*x+2*c)+9*sin(d*x+c))/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \cos^3(c+dx)(a+a\sec(c+dx))\,dx = \frac{3\,adx+\left(2\,a\cos\left(dx+c\right)^2+3\,a\cos\left(dx+c\right)+4\,a\right)\sin\left(dx+c\right)}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(3*a*d*x + (2*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 4*a)*sin(d*x + c))
/d
```

Sympy [F]

$$\int \cos^3(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \cos^3(c+dx)\sec(c+dx)\,dx + \int \cos^3(c+dx)\,dx\right)$$

input
```
integrate(cos(d*x+c)**3*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(cos(c + d*x)**3*sec(c + d*x), x) + Integral(cos(c + d*x)**3, x
))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \cos^3(c+dx)(a+a\sec(c+dx))\,dx = -\frac{4\left(\sin(dx+c)^3-3\sin(dx+c)\right)a-3\left(2\,dx+2\,c+\sin(2\,dx+2\,c)\right)a}{12\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*a - 3*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*a)/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

$$\int \cos^3(c+dx)(a+a\sec(c+dx))\,dx = \frac{3\,(dx+c)a+\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(3*(d*x + c)*a + 2*(3*a*tan(1/2*d*x + 1/2*c)^5 + 4*a*tan(1/2*d*x + 1/2
*c)^3 + 9*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 10.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \cos^3(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\,x}{2}+\frac{2\,a\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}+\frac{a\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}$$

input
```
int(cos(c + d*x)^3*(a + a/cos(c + d*x)),x)
```

output
```
(a*x)/2 + (2*a*sin(c + d*x))/(3*d) + (a*cos(c + d*x)*sin(c + d*x))/(2*d) +
 (a*cos(c + d*x)^2*sin(c + d*x))/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \cos^3(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\left(3\cos\left(dx+c\right)\sin\left(dx+c\right)-2\sin\left(dx+c\right)^3+6\sin\left(dx+c\right)+3dx\right)}{6d}$$

input
```
int(cos(d*x+c)^3*(a+a*sec(d*x+c)),x)
```

output
```
(a*(3*cos(c + d*x)*sin(c + d*x) - 2*sin(c + d*x)**3 + 6*sin(c + d*x) + 3*d
*x))/(6*d)
```

3.9 $\int \cos^4(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \cos^4(c+dx)(a+a\sec(c+dx))\,dx=\frac{3ax}{8}+\frac{a\sin(c+dx)}{d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{a\sin^3(c+dx)}{3d}$$

output
```
3/8*a*x+a*sin(d*x+c)/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c)^3*si
n(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \cos^4(c+dx)(a+a\sec(c+dx))\,dx=\frac{3a(c+dx)}{8d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(2(c+dx))}{4d}+\frac{a\sin(4(c+dx))}{32d}$$

input
```
Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

output
```
(3*a*(c + d*x))/(8*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (a
*Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)])/(32*d)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {3042, 4274, 3042, 3113, 2009, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \cos^4(c+dx)dx+a\int \cos^3(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx+a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3113}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{a\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{24}$$

$$a\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

input
```
Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

output
```
-((a*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/d) + a*((Cos[c + d*x]^3*Sin[c + d
*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

method	result
parallelrisch	$\frac{a(36dx+72\sin(dx+c)+3\sin(4dx+4c)+8\sin(3dx+3c)+24\sin(2dx+2c))}{96d}$
derivativedivides	$\frac{a\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}}{d}$
default	$\frac{a\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}}{d}$
risch	$\frac{3ax}{8}+\frac{3a\sin(dx+c)}{4d}+\frac{a\sin(4dx+4c)}{32d}+\frac{a\sin(3dx+3c)}{12d}+\frac{a\sin(2dx+2c)}{4d}$
norman	$\frac{\frac{3ax}{8}+\frac{13a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{31a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12d}+\frac{49a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{12d}+\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}+\frac{3ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2}+\frac{9ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4}+3a}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}$

input
```
int(cos(d*x+c)^4*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/96*a*(36*d*x+72*sin(d*x+c)+3*sin(4*d*x+4*c)+8*sin(3*d*x+3*c)+24*sin(2*d*
x+2*c))/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

$$\int \cos^4(c+dx)(a+a\sec(c+dx))\,dx = \frac{9\,adx+\left(6\,a\cos\left(dx+c\right)^3+8\,a\cos\left(dx+c\right)^2+9\,a\cos\left(dx+c\right)+16\,a\right)\sin\left(dx+c\right)}{24\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/24*(9*a*d*x + (6*a*cos(d*x + c)^3 + 8*a*cos(d*x + c)^2 + 9*a*cos(d*x + c
) + 16*a)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^4(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \cos^4\left(c+dx\right)\sec\left(c+dx\right)dx + \int \cos^4\left(c+dx\right)dx\right)$$

input
```
integrate(cos(d*x+c)**4*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(cos(c + d*x)**4*sec(c + d*x), x) + Integral(cos(c + d*x)**4, x
))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

$$\int \cos^4(c+dx)(a+a\sec(c+dx))\,dx = -\frac{32\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a-3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a}{96\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*a - 3*(12*d*x + 12*c + sin(4*d
*x + 4*c) + 8*sin(2*d*x + 2*c))*a)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13

$$\int \cos^4(c+dx)(a+a\sec(c+dx))\,dx = \frac{9\,(dx+c)a + \frac{2\left(9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+49\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+31\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+39\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/24*(9*(d*x + c)*a + 2*(9*a*tan(1/2*d*x + 1/2*c)^7 + 49*a*tan(1/2*d*x + 1
/2*c)^5 + 31*a*tan(1/2*d*x + 1/2*c)^3 + 39*a*tan(1/2*d*x + 1/2*c))/(tan(1/
2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 13.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int \cos^4(c+dx)(a+a\sec(c+dx))\,dx = \frac{3\,a\,x}{8} + \frac{\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4} + \frac{49\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12} + \frac{31\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12} + \frac{13\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

input
```
int(cos(c + d*x)^4*(a + a/cos(c + d*x)),x)
```

output
```
(3*a*x)/8 + ((13*a*tan(c/2 + (d*x)/2))/4 + (31*a*tan(c/2 + (d*x)/2)^3)/12
+ (49*a*tan(c/2 + (d*x)/2)^5)/12 + (3*a*tan(c/2 + (d*x)/2)^7)/4)/(d*(tan(c
/2 + (d*x)/2)^2 + 1)^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.78

$$\int \cos^4(c+dx)(a+a\sec(c+dx))\,dx$$
$$=\frac{a\left(-6\cos(dx+c)\sin(dx+c)^3+15\cos(dx+c)\sin(dx+c)-8\sin(dx+c)^3+24\sin(dx+c)+9dx\right)}{24d}$$

input
```
int(cos(d*x+c)^4*(a+a*sec(d*x+c)),x)
```

output
```
(a*( - 6*cos(c + d*x)*sin(c + d*x)**3 + 15*cos(c + d*x)*sin(c + d*x) - 8*s
in(c + d*x)**3 + 24*sin(c + d*x) + 9*d*x))/(24*d)
```

3.10 $\int \sec^4(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 122

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{3a^2\text{arctanh}(\sin(c+dx))}{4d} + \frac{9a^2\tan(c+dx)}{5d} + \frac{3a^2\sec(c+dx)\tan(c+dx)}{4d} + \frac{a^2\sec^3(c+dx)\tan(c+dx)}{2d} + \frac{a^2\sec^4(c+dx)\tan(c+dx)}{5d} + \frac{3a^2\tan^3(c+dx)}{5d}$$

output

```
3/4*a^2*arctanh(sin(d*x+c))/d+9/5*a^2*tan(d*x+c)/d+3/4*a^2*sec(d*x+c)*tan(
d*x+c)/d+1/2*a^2*sec(d*x+c)^3*tan(d*x+c)/d+1/5*a^2*sec(d*x+c)^4*tan(d*x+c)
/d+3/5*a^2*tan(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.56

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2(15\text{arctanh}(\sin(c+dx))+\tan(c+dx)\,(15\sec(c+dx)+10\sec^3(c+dx)+4\sec^4(c+dx)+12(3+\text{ta}$$

input
```
Integrate[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*(15*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*Sec[c + d*x] + 10*Sec[c
+ d*x]^3 + 4*Sec[c + d*x]^4 + 12*(3 + Tan[c + d*x]^2))))/(20*d)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4275, 3042, 4255, 3042, 4255, 3042, 4257, 4534, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$2a^2\int \sec^5(c+dx)dx+\int \sec^4(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx+\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx$$

$$\Big\downarrow \text{4255}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+$$
$$2a^2\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+$$
$$2a^2\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{ 4255}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+$$
$$2a^2\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+$$
$$2a^2\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{ 4257}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+$$
$$2a^2\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{ 4534}$$

$$\frac{9}{5}a^2\int \sec^4(c+dx)dx+$$
$$2a^2\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+$$
$$\frac{a^2\tan(c+dx)\sec^4(c+dx)}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{9}{5}a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx+$$
$$2a^2\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+$$
$$\frac{a^2\tan(c+dx)\sec^4(c+dx)}{5d}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{9a^2\int\left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{5d}+2a^2\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{a^2\tan(c+dx)\sec^4(c+dx)}{5d}$$

$$\Big\downarrow \text{2009}$$

$$2a^2\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{9a^2\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{5d}+\frac{a^2\tan(c+dx)\sec^4(c+dx)}{5d}$$

input
```
Int[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) - (9*a^2*(-Tan[c + d*x] - Tan[c +
d*x]^3/3))/(5*d) + 2*a^2*((Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*(ArcTan
h[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{-a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+2a^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-a^2}{d}$
default	$\frac{-a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+2a^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-a^2}{d}$
parts	$-\frac{a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}-\frac{a^2\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}+\frac{2a^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\right.\right.}{}$
risch	$-\frac{ia^2\left(15\,e^{9i(dx+c)}+70\,e^{7i(dx+c)}-40\,e^{6i(dx+c)}-200\,e^{4i(dx+c)}-70\,e^{3i(dx+c)}-120\,e^{2i(dx+c)}-15\,e^{i(dx+c)}-24\right)}{10d\left(e^{2i(dx+c)}+1\right)^5}+\frac{3a^2}{}$
norman	$\frac{-\frac{13a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}+\frac{9a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}-\frac{72a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5d}+\frac{7a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}-\frac{3a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{2d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}-\frac{3a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{4d}$
parallelrisch	$\frac{6a^2\left(\left(-\frac{5\cos(dx+c)}{4}-\frac{5\cos(3dx+3c)}{8}-\frac{\cos(5dx+5c)}{8}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\left(\frac{5\cos(dx+c)}{4}+\frac{5\cos(3dx+3c)}{8}+\frac{\cos(5dx+5c)}{8}\right)\ln\right.}{d(\cos(5dx+5c)+5\cos(3dx+3c)+10\cos}$

input
```
int(sec(d*x+c)^4*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+2*a^2*(-(-1/4*sec(d*x+c)^3-3/
8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-a^2*(-8/15-1/5*sec
(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{15\,a^2\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\,a^2\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(24\,a^2\cos\left(dx\right.\right.}{40\,d\cos\left(dx+c\right)^5}$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/40*(15*a^2*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*a^2*cos(d*x + c)^5*
log(-sin(d*x + c) + 1) + 2*(24*a^2*cos(d*x + c)^4 + 15*a^2*cos(d*x + c)^3
+ 12*a^2*cos(d*x + c)^2 + 10*a^2*cos(d*x + c) + 4*a^2)*sin(d*x + c))/(d*co
s(d*x + c)^5)
```

Sympy [F]

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int \sec^4\left(c+dx\right)dx+\int 2\sec^5\left(c+dx\right)dx+\int \sec^6\left(c+dx\right)dx\right)$$

input
```
integrate(sec(d*x+c)**4*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(sec(c + d*x)**4, x) + Integral(2*sec(c + d*x)**5, x) + Inte
gral(sec(c + d*x)**6, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.09

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^2\,dx$$

$$=\frac{8\left(3\tan\left(dx+c\right)^5+10\tan\left(dx+c\right)^3+15\tan\left(dx+c\right)\right)a^2+40\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)a^2-15}{120\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/120*(8*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^2 + 40
*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 - 15*a^2*(2*(3*sin(d*x + c)^3 - 5*s
in(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
+ 1) + 3*log(sin(d*x + c) - 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.13

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^2\,dx$$

$$=\frac{15\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-70\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}}{20\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/20*(15*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) - 2*(15*a^2*tan(1/2*d*x + 1/2*c)^9 - 70*a^2*tan(1/2*d*x
+ 1/2*c)^7 + 144*a^2*tan(1/2*d*x + 1/2*c)^5 - 90*a^2*tan(1/2*d*x + 1/2*c)^
3 + 65*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 14.43 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.39

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{3\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d} - \frac{\frac{3\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{2}-7\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\frac{72\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{5}-9\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{13\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + a/cos(c + d*x))^2/cos(c + d*x)^4,x)
```

output
```
(3*a^2*atanh(tan(c/2 + (d*x)/2)))/(2*d) - ((72*a^2*tan(c/2 + (d*x)/2)^5)/5
 - 9*a^2*tan(c/2 + (d*x)/2)^3 - 7*a^2*tan(c/2 + (d*x)/2)^7 + (3*a^2*tan(c/
2 + (d*x)/2)^9)/2 + (13*a^2*tan(c/2 + (d*x)/2))/2)/(d*(5*tan(c/2 + (d*x)/2
)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x
)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.04

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2\left(-15\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^4+30\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx\right.}{}$$

input
```
int(sec(d*x+c)^4*(a+a*sec(d*x+c))^2,x)
```

output
```
(a**2*( - 15*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4 + 30*c
os(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 - 15*cos(c + d*x)*lo
g(tan((c + d*x)/2) - 1) + 15*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c
+ d*x)**4 - 30*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2 + 15
*cos(c + d*x)*log(tan((c + d*x)/2) + 1) - 15*cos(c + d*x)*sin(c + d*x)**3
+ 25*cos(c + d*x)*sin(c + d*x) + 24*sin(c + d*x)**5 - 60*sin(c + d*x)**3 +
 40*sin(c + d*x)))/(20*cos(c + d*x)*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2
 + 1))
```

3.11 $\int \sec^3(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 96

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{7a^2\text{arctanh}(\sin(c+dx))}{8d} + \frac{2a^2\tan(c+dx)}{d} + \frac{7a^2\sec(c+dx)\tan(c+dx)}{8d} + \frac{a^2\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{2a^2\tan^3(c+dx)}{3d}$$

output
```
7/8*a^2*arctanh(sin(d*x+c))/d+2*a^2*tan(d*x+c)/d+7/8*a^2*sec(d*x+c)*tan(d*
x+c)/d+1/4*a^2*sec(d*x+c)^3*tan(d*x+c)/d+2/3*a^2*tan(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.60

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2\left(21\text{arctanh}(\sin(c+dx)) + \tan(c+dx)\left(21\sec(c+dx) + 6\sec^3(c+dx) + 16(3+\tan^2(c+dx))\right)\right)}{24d}$$

input
```
Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*(21*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(21*Sec[c + d*x] + 6*Sec[c +
 d*x]^3 + 16*(3 + Tan[c + d*x]^2))))/(24*d)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4275, 3042, 4254, 2009, 4534, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$2a^2\int \sec^4(c+dx)dx+\int \sec^3(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx+\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx$$

$$\Big\downarrow \text{4254}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx-\frac{2a^2\int\left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx-\frac{2a^2\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4534}$$

$$\frac{7}{4}a^2\int \sec^3(c+dx)dx-\frac{2a^2\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}$$

$\big\downarrow$ 3042

$$\frac{7}{4}a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{2a^2\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}$$

$\big\downarrow$ 4255

$$\frac{7}{4}a^2\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{2a^2\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}$$

$\big\downarrow$ 3042

$$\frac{7}{4}a^2\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{2a^2\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}$$

$\big\downarrow$ 4257

$$\frac{7}{4}a^2\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{2a^2\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}$$

input
```
Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (7*a^2*(ArcTanh[Sin[c + d*x]]/(2
*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4 - (2*a^2*(-Tan[c + d*x] - Tan[
c + d*x]^3/3))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-2a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+a^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx}{8}\right.\right.}{d}$
default	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-2a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+a^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx}{8}\right.\right.}{d}$
parts	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{a^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+}{8}\right.}{d}$
norman	$\frac{\frac{25a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{83a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12d}+\frac{77a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{12d}-\frac{7a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}-\frac{7a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8d}+\frac{7a^2\ln\left(\mathrm{t}\right.}{}$
risch	$-\frac{ia^2(21\,\mathrm{e}^{7i(dx+c)}+45\,\mathrm{e}^{5i(dx+c)}-96\,\mathrm{e}^{4i(dx+c)}-45\,\mathrm{e}^{3i(dx+c)}-128\,\mathrm{e}^{2i(dx+c)}-21\,\mathrm{e}^{i(dx+c)}-32)}{12d(\mathrm{e}^{2i(dx+c)}+1)^4}+\frac{7a^2\ln(\mathrm{e}^{i(dx+c)}+i)}{8d}$
parallelrisch	$\frac{a^2\left(21(-3-\cos(4dx+4c)-4\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+21(3+\cos(4dx+4c)+4\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{24d(3+\cos(4dx+4c)+4\cos(2dx+2c))}$

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-2*a^2*(
-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))
*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.16

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$=\frac{21\,a^2\cos(dx+c)^4\log(\sin(dx+c)+1)-21\,a^2\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\left(32\,a^2\cos(dx\right.}{48\,d\cos(dx+c)^4}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/48*(21*a^2*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 21*a^2*cos(d*x + c)^4*
log(-sin(d*x + c) + 1) + 2*(32*a^2*cos(d*x + c)^3 + 21*a^2*cos(d*x + c)^2
+ 16*a^2*cos(d*x + c) + 6*a^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int \sec^3(c+dx)\,dx + \int 2\sec^4(c+dx)\,dx + \int \sec^5(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(sec(c + d*x)**3, x) + Integral(2*sec(c + d*x)**4, x) + Inte
gral(sec(c + d*x)**5, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.51

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{32\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^2-3a^2\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log\right.}{48d}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 - 3*a^2*(2*(3*sin(d*x + c)^
3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*
x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a^2*(2*sin(d*x + c)/(sin(d*x +
 c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.27

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{21\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-21\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(21\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-77\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}\right.}}{24\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/24*(21*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 21*a^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) - 2*(21*a^2*tan(1/2*d*x + 1/2*c)^7 - 77*a^2*tan(1/2*d*x
+ 1/2*c)^5 + 83*a^2*tan(1/2*d*x + 1/2*c)^3 - 75*a^2*tan(1/2*d*x + 1/2*c))/
(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 12.87 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.47

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{7\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d} - \frac{\frac{7\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}-\frac{77\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}+\frac{83\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}-\frac{25\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

input
```
int((a + a/cos(c + d*x))^2/cos(c + d*x)^3,x)
```

output
```
(7*a^2*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((83*a^2*tan(c/2 + (d*x)/2)^3)/1
2 - (77*a^2*tan(c/2 + (d*x)/2)^5)/12 + (7*a^2*tan(c/2 + (d*x)/2)^7)/4 - (2
5*a^2*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x
)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.03

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{a^2\left(-32\cos(dx+c)\sin(dx+c)^3+48\cos(dx+c)\sin(dx+c)-21\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^4\right.}{}$$

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^2,x)
```

output
```
(a**2*( - 32*cos(c + d*x)*sin(c + d*x)**3 + 48*cos(c + d*x)*sin(c + d*x) -
 21*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4 + 42*log(tan((c + d*x)/2) -
1)*sin(c + d*x)**2 - 21*log(tan((c + d*x)/2) - 1) + 21*log(tan((c + d*x)/2
) + 1)*sin(c + d*x)**4 - 42*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2 + 21
*log(tan((c + d*x)/2) + 1) - 21*sin(c + d*x)**3 + 27*sin(c + d*x)))/(24*d*
(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.12 $\int \sec^2(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 74

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{5a^2\tan(c+dx)}{3d}+\frac{a^2\sec(c+dx)\tan(c+dx)}{d}+\frac{a^2\sec^2(c+dx)\tan(c+dx)}{3d}$$

output
```
a^2*arctanh(sin(d*x+c))/d+5/3*a^2*tan(d*x+c)/d+a^2*sec(d*x+c)*tan(d*x+c)/d
+1/3*a^2*sec(d*x+c)^2*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{5a^2\tan(c+dx)}{3d}+\frac{a^2\sec(c+dx)\tan(c+dx)}{d}+\frac{a^2\sec^2(c+dx)\tan(c+dx)}{3d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*ArcTanh[Sin[c + d*x]])/d + (5*a^2*Tan[c + d*x])/(3*d) + (a^2*Sec[c +
d*x]*Tan[c + d*x])/d + (a^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4275, 3042, 4255, 3042, 4257, 4534, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$2a^2\int \sec^3(c+dx)dx+\int \sec^2(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx$$

$$\Big\downarrow \text{4255}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{4257}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{4534}$$

$$\frac{5}{3}a^2\int \sec^2(c+dx)dx+2a^2\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{3}a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+2a^2\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{5a^2\int 1d(-\tan(c+dx))}{3d}+2a^2\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d}$$

$$\Big\downarrow \text{24}$$

$$2a^2\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{5a^2\tan(c+dx)}{3d}+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d}$$

input
```
Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

output
```
(5*a^2*Tan[c + d*x])/(3*d) + (a^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + 2*a
^2*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{a^2\tan(dx+c)+2a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
default	$\frac{a^2\tan(dx+c)+2a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
parts	$\frac{a^2\tan(dx+c)}{d}-\frac{a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{a^2\sec(dx+c)\tan(dx+c)}{d}+\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$
risch	$-\frac{2ia^2(3\,e^{5i(dx+c)}-3\,e^{4i(dx+c)}-12\,e^{2i(dx+c)}-3\,e^{i(dx+c)}-5)}{3d(e^{2i(dx+c)}+1)^3}+\frac{a^2\ln(e^{i(dx+c)}+i)}{d}-\frac{a^2\ln(e^{i(dx+c)}-i)}{d}$
norman	$\frac{-\frac{6a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{16a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}+\frac{a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$
parallelrisch	$-\frac{3a^2\left(\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\left(-\cos(dx+c)-\frac{\cos(3dx+3c)}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\sin(dx+c\right.}{d(\cos(3dx+3c)+3\cos(dx+c))}$

input
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*tan(d*x+c)+2*a^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan
(d*x+c)))-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.30

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$=\frac{3\,a^2\cos{(dx+c)}^3\log{(\sin{(dx+c)}+1)}-3\,a^2\cos{(dx+c)}^3\log{(-\sin{(dx+c)}+1)}+2\left(5\,a^2\cos{(dx+c)}\right.}{6\,d\cos{(dx+c)}^3}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(3*a^2*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a^2*cos(d*x + c)^3*log
(-sin(d*x + c) + 1) + 2*(5*a^2*cos(d*x + c)^2 + 3*a^2*cos(d*x + c) + a^2)*
sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int \sec^2(c+dx)\,dx + \int 2\sec^3(c+dx)\,dx + \int \sec^4(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(sec(c + d*x)**2, x) + Integral(2*sec(c + d*x)**3, x) + Inte
gral(sec(c + d*x)**4, x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)a^2 - 3a^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right)}{6d}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/6*(2*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 - 3*a^2*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a^2*t
an(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.43

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-8\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{3\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/3*(3*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^2*log(abs(tan(1/2*d*x
+ 1/2*c) - 1)) - 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 8*a^2*tan(1/2*d*x + 1/2
*c)^3 + 9*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 11.67 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.51

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$
$$- \frac{2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{16\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+6\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + a/cos(c + d*x))^2/cos(c + d*x)^2,x)
```

output
```
(2*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (2*a^2*tan(c/2 + (d*x)/2)^5 - (16*a^
2*tan(c/2 + (d*x)/2)^3)/3 + 6*a^2*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2 + (d*x
)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.12

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{a^2\left(-3\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2+3\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+3\cos(dx\right.}{}$$

input
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c))^2,x)
```

output
```
(a**2*( - 3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 + 3*cos
(c + d*x)*log(tan((c + d*x)/2) - 1) + 3*cos(c + d*x)*log(tan((c + d*x)/2)
+ 1)*sin(c + d*x)**2 - 3*cos(c + d*x)*log(tan((c + d*x)/2) + 1) - 3*cos(c
+ d*x)*sin(c + d*x) + 5*sin(c + d*x)**3 - 6*sin(c + d*x)))/(3*cos(c + d*x)
*d*(sin(c + d*x)**2 - 1))
```

3.13 $\int \sec(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \sec(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{3a^2\text{arctanh}(\sin(c+dx))}{2d} + \frac{2a^2\tan(c+dx)}{d} + \frac{a^2\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
3/2*a^2*arctanh(sin(d*x+c))/d+2*a^2*tan(d*x+c)/d+1/2*a^2*sec(d*x+c)*tan(d*
x+c)/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \sec(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{3a^2\coth^{-1}(\sin(c+dx))}{2d} + \frac{2a^2\tan(c+dx)}{d} + \frac{a^2\sec(c+dx)\tan(c+dx)}{2d}$$

input
```
Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

output
```
(3*a^2*ArcCoth[Sin[c + d*x]])/(2*d) + (2*a^2*Tan[c + d*x])/d + (a^2*Sec[c
+ d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 4275, 3042, 4254, 24, 4534, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$2a^2\int \sec^2(c+dx)dx+\int \sec(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx$$

$$\Big\downarrow \text{4254}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx-\frac{2a^2\int 1d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{24}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+\frac{2a^2\tan(c+dx)}{d}$$

$$\Big\downarrow \text{4534}$$

$$\frac{3}{2}a^2\int \sec(c+dx)dx+\frac{2a^2\tan(c+dx)}{d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{2a^2\tan(c+dx)}{d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{3a^2\text{arctanh}(\sin(c+dx))}{2d}+\frac{2a^2\tan(c+dx)}{d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}$$

input
```
Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

output
```
(3*a^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a^2*Tan[c + d*x])/d + (a^2*Sec[c
+ d*x]*Tan[c + d*x])/(2*d)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{a^2 \ln(\sec(dx+c)+\tan(dx+c))+2a^2\tan(dx+c)+a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{a^2 \ln(\sec(dx+c)+\tan(dx+c))+2a^2\tan(dx+c)+a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
parts	$\frac{a^2 \ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{2a^2\tan(dx+c)}{d}$
norman	$\frac{\frac{5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{3a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-\frac{3a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{3a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$
risch	$-\frac{ia^2\left(e^{3i(dx+c)}-4e^{2i(dx+c)}-e^{i(dx+c)}-4\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{3a^2\ln\left(e^{i(dx+c)}-i\right)}{2d}+\frac{3a^2\ln\left(e^{i(dx+c)}+i\right)}{2d}$
parallelrisch	$\frac{a^2\left(3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(2dx+2c)-3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(2dx+2c)+4\sin(2dx+2c)+2\sin(dx+c)+3\ln\left(\tan\left(\right.\right.\right.}{2d(1+\cos(2dx+2c))}$

input
```
int(sec(d*x+c)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*ln(sec(d*x+c)+tan(d*x+c))+2*a^2*tan(d*x+c)+a^2*(1/2*sec(d*x+c)*ta
n(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.54

$$\int \sec(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$=\frac{3\,a^2\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-3\,a^2\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\left(4\,a^2\cos\left(dx+c\right)\right.}{4\,d\cos\left(dx+c\right)^2}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/4*(3*a^2*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 3*a^2*cos(d*x + c)^2*log
(-sin(d*x + c) + 1) + 2*(4*a^2*cos(d*x + c) + a^2)*sin(d*x + c))/(d*cos(d*
x + c)^2)
```

Sympy [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int \sec\left(c+dx\right)dx + \int 2\sec^2\left(c+dx\right)dx + \int \sec^3\left(c+dx\right)dx\right)$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(sec(c + d*x), x) + Integral(2*sec(c + d*x)**2, x) + Integra
l(sec(c + d*x)**3, x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.50

$$\int \sec(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) - 4a^2\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right.}{4d}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/4*(a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)) - 4*a^2*log(sec(d*x + c) + tan(d*x + c)) - 8*a^2*tan
(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.67

$$\int \sec(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(3*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^2*log(abs(tan(1/2*d*x
+ 1/2*c) - 1)) - 2*(3*a^2*tan(1/2*d*x + 1/2*c)^3 - 5*a^2*tan(1/2*d*x + 1/2
*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 10.38 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.54

$$\int \sec(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{3\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-5\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

input
```
int((a + a/cos(c + d*x))^2/cos(c + d*x),x)
```

output
```
(3*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (3*a^2*tan(c/2 + (d*x)/2)^3 - 5*a^2*
tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1)
)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.13

$$\int \sec(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{a^2\left(-4\cos\left(dx+c\right)\sin\left(dx+c\right)-3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2+3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+3\log}{2d\left(\sin\left(dx+c\right)^2-1\right)}$$

input
```
int(sec(d*x+c)*(a+a*sec(d*x+c))^2,x)
```

output
```
(a**2*( - 4*cos(c + d*x)*sin(c + d*x) - 3*log(tan((c + d*x)/2) - 1)*sin(c
+ d*x)**2 + 3*log(tan((c + d*x)/2) - 1) + 3*log(tan((c + d*x)/2) + 1)*sin(
c + d*x)**2 - 3*log(tan((c + d*x)/2) + 1) - sin(c + d*x)))/(2*d*(sin(c + d
*x)**2 - 1))
```

3.14 $\int (a + a\sec(c + dx))^2 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 34

$$\int (a + a\sec(c + dx))^2 \, dx = a^2x + \frac{2a^2\text{arctanh}(\sin(c + dx))}{d} + \frac{a^2\tan(c + dx)}{d}$$

output
```
a^2*x+2*a^2*arctanh(sin(d*x+c))/d+a^2*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int (a + a\sec(c + dx))^2 \, dx = a^2x + \frac{2a^2\coth^{-1}(\sin(c + dx))}{d} + \frac{a^2\tan(c + dx)}{d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2,x]
```

output
```
a^2*x + (2*a^2*ArcCoth[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4260, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^2dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{ 4260}$$

$$a^2\int \sec^2(c+dx)dx+2a^2\int \sec(c+dx)dx+a^2x$$

$$\Big\downarrow \text{ 3042}$$

$$2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+a^2x$$

$$\Big\downarrow \text{ 4254}$$

$$-\frac{a^2\int 1d(-\tan(c+dx))}{d}+2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+a^2x$$

$$\Big\downarrow \text{ 24}$$

$$2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{a^2\tan(c+dx)}{d}+a^2x$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{2a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)}{d}+a^2x$$

input
```
Int[(a + a*Sec[c + d*x])^2,x]
```

output
```
a^2*x + (2*a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4260
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Simp[2*a*b   Int[Csc[c + d*x], x], x] + Simp[b^2   Int[Csc[c + d*x]^2, x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

method	result	size
parts	$a^{2} x +\frac{a^{2} \tan \left(d x +c \right)}{d}+\frac{2 a^{2} \ln \left(\sec \left(d x +c \right)+\tan \left(d x +c \right)\right)}{d}$	42
derivativedivides	$\frac{a^{2} \left(d x +c \right)+2 a^{2} \ln \left(\sec \left(d x +c \right)+\tan \left(d x +c \right)\right)+a^{2} \tan \left(d x +c \right)}{d}$	44
default	$\frac{a^{2} \left(d x +c \right)+2 a^{2} \ln \left(\sec \left(d x +c \right)+\tan \left(d x +c \right)\right)+a^{2} \tan \left(d x +c \right)}{d}$	44
risch	$a^{2} x +\frac{2 i a^{2}}{d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)}+\frac{2 a^{2} \ln \left({\mathrm e}^{i \left(d x +c \right)}+i\right)}{d}-\frac{2 a^{2} \ln \left({\mathrm e}^{i \left(d x +c \right)}-i\right)}{d}$	71
parallelrisch	$\frac{\left(-2 \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right) \cos \left(d x +c \right)+2 \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right) \cos \left(d x +c \right)+d x \cos \left(d x +c \right)+\sin \left(d x +c \right)\right) a^{2}}{d \cos \left(d x +c \right)}$	72
norman	$\frac{a^{2} x \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}-a^{2} x -\frac{2 a^{2} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{d}}{-1+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}-\frac{2 a^{2} \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}{d}+\frac{2 a^{2} \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)}{d}$	98

input
```
int((a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+a^2*tan(d*x+c)/d+2*a^2/d*ln(sec(d*x+c)+tan(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(34) = 68$.

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.24

$$\int (a + a\sec(c + dx))^2 \, dx = \frac{a^2 dx \cos(dx + c) + a^2 \cos(dx + c) \log(\sin(dx + c) + 1) - a^2 \cos(dx + c) \log(-\sin(dx + c) + 1) + a^2 s}{d \cos(dx + c)}$$

input
```
integrate((a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
(a^2*d*x*cos(d*x + c) + a^2*cos(d*x + c)*log(sin(d*x + c) + 1) - a^2*cos(d
*x + c)*log(-sin(d*x + c) + 1) + a^2*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int (a + a\sec(c + dx))^2 \, dx = a^2 \left(\int 1 \, dx + \int 2 \sec(c + dx) \, dx + \int \sec^2(c + dx) \, dx \right)$$

input
```
integrate((a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(1, x) + Integral(2*sec(c + d*x), x) + Integral(sec(c + d*x)
**2, x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int (a + a\sec(c + dx))^2 \, dx = a^2x + \frac{2\,a^2 \log\left(\sec\left(dx + c\right) + \tan\left(dx + c\right)\right)}{d} + \frac{a^2 \tan\left(dx + c\right)}{d}$$

input
```
integrate((a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
a^2*x + 2*a^2*log(sec(d*x + c) + tan(d*x + c))/d + a^2*tan(d*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(34) = 68$.

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.32

$$\int (a + a\sec(c + dx))^2 \, dx = \frac{(dx + c)a^2 + 2\,a^2 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 2\,a^2 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\,a^2 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 1}}{d}$$

input
```
integrate((a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
((d*x + c)*a^2 + 2*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a^2*log(abs(
tan(1/2*d*x + 1/2*c) - 1)) - 2*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2
*c)^2 - 1))/d
```

Mupad [B] (verification not implemented)

Time = 10.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.65

$$\int (a+a\sec(c+dx))^2\,dx = a^2\,x+\frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

input
```
int((a + a/cos(c + d*x))^2,x)
```

output
```
a^2*x + (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (2*a^2*tan(c/2 + (d*x)/2))/(
d*(tan(c/2 + (d*x)/2)^2 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.09

$$\int (a+a\sec(c+dx))^2\,dx$$
$$=\frac{a^2\left(-2\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+2\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\cos\left(dx+c\right)dx+\sin}{\cos\left(dx+c\right)d}$$

input
```
int((a+a*sec(d*x+c))^2,x)
```

output
```
(a**2*( - 2*cos(c + d*x)*log(tan((c + d*x)/2) - 1) + 2*cos(c + d*x)*log(ta
n((c + d*x)/2) + 1) + cos(c + d*x)*d*x + sin(c + d*x)))/(cos(c + d*x)*d)
```

3.15 $\int \cos(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 34

$$\int \cos(c+dx)(a+a\sec(c+dx))^2\,dx=2a^2x+\frac{a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{a^2\sin(c+dx)}{d}$$

output
```
2*a^2*x+a^2*arctanh(sin(d*x+c))/d+a^2*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.38

$$\int \cos(c+dx)(a+a\sec(c+dx))^2\,dx=2a^2x+\frac{a^2\coth^{-1}(\sin(c+dx))}{d}+\frac{a^2\cos(dx)\sin(c)}{d}+\frac{a^2\cos(c)\sin(dx)}{d}$$

input
```
Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

output
```
2*a^2*x + (a^2*ArcCoth[Sin[c + d*x]])/d + (a^2*Cos[d*x]*Sin[c])/d + (a^2*C
os[c]*Sin[d*x])/d
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 4275, 24, 3042, 4533, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4275}$$

$$\int \cos(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx+2a^2\int 1dx$$

$$\Big\downarrow \text{24}$$

$$\int \cos(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx+2a^2x$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+2a^2x$$

$$\Big\downarrow \text{4533}$$

$$a^2\int \sec(c+dx)dx+\frac{a^2\sin(c+dx)}{d}+2a^2x$$

$$\Big\downarrow \text{3042}$$

$$a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{a^2\sin(c+dx)}{d}+2a^2x$$

$$\Big\downarrow \text{4257}$$

$$\frac{a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{a^2\sin(c+dx)}{d}+2a^2x$$

input
```
Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

output
```
2*a^2*x + (a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{a^2 \ln(\sec(dx+c)+\tan(dx+c))+2a^2(dx+c)+a^2\sin(dx+c)}{d}$
default	$\frac{a^2 \ln(\sec(dx+c)+\tan(dx+c))+2a^2(dx+c)+a^2\sin(dx+c)}{d}$
parallelrisch	$\frac{\left(\sin(dx+c)+2dx-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\right)a^2}{d}$
risch	$2a^2x-\frac{ia^2 e^{i(dx+c)}}{2d}+\frac{ia^2 e^{-i(dx+c)}}{2d}+\frac{a^2\ln(e^{i(dx+c)}+i)}{d}-\frac{a^2\ln(e^{i(dx+c)}-i)}{d}$
norman	$\frac{-2a^2x-\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}+2a^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{d}$

input
```
int(cos(d*x+c)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*ln(sec(d*x+c)+tan(d*x+c))+2*a^2*(d*x+c)+a^2*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int \cos(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{4\,a^2dx+a^2\log\left(\sin\left(dx+c\right)+1\right)-a^2\log\left(-\sin\left(dx+c\right)+1\right)+2\,a^2\sin\left(dx+c\right)}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(4*a^2*d*x + a^2*log(sin(d*x + c) + 1) - a^2*log(-sin(d*x + c) + 1) +
2*a^2*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\cos(c+dx)\sec(c+dx)\,dx + \int \cos(c+dx)\sec^2(c+dx)\,dx + \int \cos(c+dx)\,dx\right)$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*cos(c + d*x)*sec(c + d*x), x) + Integral(cos(c + d*x)*sec
(c + d*x)**2, x) + Integral(cos(c + d*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.53

$$\int \cos(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{4(dx+c)a^2 + a^2(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 2a^2\sin(dx+c)}{2d}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*(4*(d*x + c)*a^2 + a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
 + 2*a^2*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(34) = 68$.

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.32

$$\int \cos(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{2\,(dx+c)a^2 + a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
(2*(d*x + c)*a^2 + a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a^2*log(abs(ta
n(1/2*d*x + 1/2*c) - 1)) + 2*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c
)^2 + 1))/d
```

Mupad [B] (verification not implemented)

Time = 9.78 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \cos(c+dx)(a+a\sec(c+dx))^2\,dx = 2\,a^2\,x + \frac{a^2\,\left(2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+\sin\left(c+d\,x\right)\right)}{d}$$

input
```
int(cos(c + d*x)*(a + a/cos(c + d*x))^2,x)
```

output
```
2*a^2*x + (a^2*(2*atanh(tan(c/2 + (d*x)/2)) + sin(c + d*x)))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.38

$$\int \cos(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$=\frac{a^2\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\sin\left(dx+c\right)+2c+2dx\right)}{d}$$

input
```
int(cos(d*x+c)*(a+a*sec(d*x+c))^2,x)
```

output
```
(a**2*( - log(tan((c + d*x)/2) - 1) + log(tan((c + d*x)/2) + 1) + sin(c +
d*x) + 2*c + 2*d*x))/d
```

3.16 $\int \cos^2(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 45

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^2\,dx=\frac{3a^2x}{2}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
3/2*a^2*x+2*a^2*sin(d*x+c)/d+1/2*a^2*cos(d*x+c)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^2\,dx=\frac{a^2(6(c+dx)+8\sin(c+dx)+\sin(2(c+dx)))}{4d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*(6*(c + d*x) + 8*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4275, 3042, 3117, 4533, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4275}$$

$$2a^2\int \cos(c+dx)dx+\int \cos^2(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3117}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{2a^2\sin(c+dx)}{d}$$

$$\Big\downarrow \text{4533}$$

$$\frac{3a^2\int 1dx}{2}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}$$

$$\Big\downarrow \text{24}$$

$$\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a^2x}{2}$$

input
```
Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

output
```
(3*a^2*x)/2 + (2*a^2*Sin[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*
d)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{a^{2}(6dx+8\sin(dx+c)+\sin(2dx+2c))}{4d}$
risch	$\frac{3a^{2}x}{2}+\frac{2a^{2}\sin(dx+c)}{d}+\frac{a^{2}\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{a^{2}(dx+c)+2a^{2}\sin(dx+c)+a^{2}\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a^{2}(dx+c)+2a^{2}\sin(dx+c)+a^{2}\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
norman	$\frac{-\frac{3a^{2}x}{2}-\frac{5a^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{d}+\frac{3a^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{d}-\frac{3a^{2}x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2}+\frac{3a^{2}x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{2}+\frac{3a^{2}x\tan\left(\frac{dx}{2}+\right.}{2}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}$

input
```
int(cos(d*x+c)^2*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*(6*d*x+8*sin(d*x+c)+sin(2*d*x+2*c))/d
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \cos^{2}(c+dx)(a+a\sec(c+dx))^{2}\,dx=\frac{3\,a^{2}dx+\left(a^{2}\cos\left(dx+c\right)+4\,a^{2}\right)\sin\left(dx+c\right)}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(3*a^2*d*x + (a^2*cos(d*x + c) + 4*a^2)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\cos^2{(c+dx)}\sec{(c+dx)}\,dx + \int \cos^2{(c+dx)}\sec^2{(c+dx)}\,dx + \int \cos^2{(c+dx)}\,dx\right)$$

input
```
integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*cos(c + d*x)**2*sec(c + d*x), x) + Integral(cos(c + d*x)*
*2*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**2, x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{(2\,dx+2\,c+\sin{(2\,dx+2\,c)})a^2+4\,(dx+c)a^2+8\,a^2\sin{(dx+c)}}{4\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^2 + 4*(d*x + c)*a^2 + 8*a^2*sin(d*
x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.42

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{3\,(dx+c)a^2 + \frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(3*(d*x + c)*a^2 + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^3 + 5*a^2*tan(1/2*d*x
 + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 10.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{3\,a^2\,x}{2} + \frac{3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+5\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^2}$$

input
```
int(cos(c + d*x)^2*(a + a/cos(c + d*x))^2,x)
```

output
```
(3*a^2*x)/2 + (3*a^2*tan(c/2 + (d*x)/2)^3 + 5*a^2*tan(c/2 + (d*x)/2))/(d*(
tan(c/2 + (d*x)/2)^2 + 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2(\cos(dx+c)\sin(dx+c)+4\sin(dx+c)+3dx)}{2d}$$

input
```
int(cos(d*x+c)^2*(a+a*sec(d*x+c))^2,x)
```

output
```
(a**2*(cos(c + d*x)*sin(c + d*x) + 4*sin(c + d*x) + 3*d*x))/(2*d)
```

3.17 $\int \cos^3(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^2\,dx = a^2x+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{3d}$$

output
```
a^2*x+2*a^2*sin(d*x+c)/d+a^2*cos(d*x+c)*sin(d*x+c)/d-1/3*a^2*sin(d*x+c)^3/
d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2(12dx+21\sin(c+dx)+6\sin(2(c+dx))+\sin(3(c+dx)))}{12d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*(12*d*x + 21*Sin[c + d*x] + 6*Sin[2*(c + d*x)] + Sin[3*(c + d*x)]))/(
12*d)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4275, 3042, 3115, 24, 4532, 3042, 3492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{4275}$$

$$2a^2\int \cos^2(c+dx)dx+\int \cos^3(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3115}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+2a^2\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{24}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+2a^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)$$

$$\Big\downarrow \text{4532}$$

$$\int \cos(c+dx)\left(\cos^2(c+dx)a^2+a^2\right)dx+2a^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)$$

$$\Big\downarrow \text{3492}$$

$$2a^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{\int \left(2a^2-a^2\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$2a^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{\frac{1}{3}a^2\sin^3(c+dx)-2a^2\sin(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

output
```
2*a^2*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (-2*a^2*Sin[c + d*x] + (
a^2*Sin[c + d*x]^3)/3)/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4532
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{a^{2}\left(12 d x +21 \sin \left(d x +c \right)+\sin \left(3 d x +3 c \right)+6 \sin \left(2 d x +2 c \right)\right)}{12 d}$
risch	$a^{2} x +\frac{7 a^{2} \sin \left(d x +c \right)}{4 d}+\frac{a^{2} \sin \left(3 d x +3 c \right)}{12 d}+\frac{a^{2} \sin \left(2 d x +2 c \right)}{2 d}$
derivativedivides	$\frac{a^{2} \sin \left(d x +c \right)+2 a^{2} \left(\frac{\sin \left(d x +c \right) \cos \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)+\frac{a^{2} \left(2+\cos \left(d x +c \right)^{2}\right) \sin \left(d x +c \right)}{3}}{d}$
default	$\frac{a^{2} \sin \left(d x +c \right)+2 a^{2} \left(\frac{\sin \left(d x +c \right) \cos \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)+\frac{a^{2} \left(2+\cos \left(d x +c \right)^{2}\right) \sin \left(d x +c \right)}{3}}{d}$
norman	$\frac{a^{2} x \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{8}-a^{2} x -\frac{6 a^{2} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{d}+\frac{2 a^{2} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{3 d}+\frac{10 a^{2} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}{3 d}+\frac{2 a^{2} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{7}}{d}-2 a^{2} x \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{\left(1+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}\right)^{3} \left(-1+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}\right)}$

input
```
int(cos(d*x+c)^3*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*a^2*(12*d*x+21*sin(d*x+c)+sin(3*d*x+3*c)+6*sin(2*d*x+2*c))/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{3\,a^2dx + \left(a^2\cos\left(dx+c\right)^2 + 3\,a^2\cos\left(dx+c\right) + 5\,a^2\right)\sin\left(dx+c\right)}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3*(3*a^2*d*x + (a^2*cos(d*x + c)^2 + 3*a^2*cos(d*x + c) + 5*a^2)*sin(d*x
 + c))/d
```

Sympy [F]

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\cos^3\left(c+dx\right)\sec\left(c+dx\right)dx + \int \cos^3\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int \cos^3\left(c+dx\right)dx\right)$$

input
```
integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*cos(c + d*x)**3*sec(c + d*x), x) + Integral(cos(c + d*x)*
*3*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**3, x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{2\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^2-3\left(2\,dx+2\,c+\sin(2\,dx+2\,c)\right)a^2-6\,a^2\sin(dx+c)}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/6*(2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 - 3*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*a^2 - 6*a^2*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.40

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{3\,(dx+c)a^2+\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+8\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/3*(3*(d*x + c)*a^2 + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 8*a^2*tan(1/2*d*x
 + 1/2*c)^3 + 9*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/
d
```

Mupad [B] (verification not implemented)

Time = 9.80 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\,x+\frac{5\,a^2\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a^2\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{d}$$

input
```
int(cos(c + d*x)^3*(a + a/cos(c + d*x))^2,x)
```

output
```
a^2*x + (5*a^2*sin(c + d*x))/(3*d) + (a^2*cos(c + d*x)^2*sin(c + d*x))/(3*
d) + (a^2*cos(c + d*x)*sin(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2\left(3\cos\left(dx+c\right)\sin\left(dx+c\right)-\sin\left(dx+c\right)^3+6\sin\left(dx+c\right)+3dx\right)}{3d}$$

input
```
int(cos(d*x+c)^3*(a+a*sec(d*x+c))^2,x)
```

output
```
(a**2*(3*cos(c + d*x)*sin(c + d*x) - sin(c + d*x)**3 + 6*sin(c + d*x) + 3*
d*x))/(3*d)
```

3.18 $\int \cos^4(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{7a^2x}{8}+\frac{2a^2\sin(c+dx)}{d}+\frac{7a^2\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{2a^2\sin^3(c+dx)}{3d}$$

output
```
7/8*a^2*x+2*a^2*sin(d*x+c)/d+7/8*a^2*cos(d*x+c)*sin(d*x+c)/d+1/4*a^2*cos(d
*x+c)^3*sin(d*x+c)/d-2/3*a^2*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.61

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2(84dx+144\sin(c+dx)+48\sin(2(c+dx))+16\sin(3(c+dx))+3\sin(4(c+dx)))}{96d}$$

input
```
Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*(84*d*x + 144*Sin[c + d*x] + 48*Sin[2*(c + d*x)] + 16*Sin[3*(c + d*x)
] + 3*Sin[4*(c + d*x)]))/(96*d)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4275, 3042, 3113, 2009, 4533, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{4275}$$

$$2a^2\int \cos^3(c+dx)dx+\int \cos^4(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3113}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx-\frac{2a^2\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx-\frac{2a^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4533}$$

$$\frac{7}{4}a^2\int \cos^2(c+dx)dx-\frac{2a^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{2a^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{7}{4}a^2\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{2a^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}$$

$$\Big\downarrow \text{24}$$

$$-\frac{2a^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{7}{4}a^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)$$

input
```
Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (7*a^2*(x/2 + (Cos[c + d*x]*Sin[
c + d*x])/(2*d)))/4 - (2*a^2*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.63

method	result
parallelrisch	$\frac{a^2(84dx+3\sin(4dx+4c)+16\sin(3dx+3c)+48\sin(2dx+2c)+144\sin(dx+c))}{96d}$
risch	$\frac{7a^2x}{8}+\frac{3a^2\sin(dx+c)}{2d}+\frac{a^2\sin(4dx+4c)}{32d}+\frac{a^2\sin(3dx+3c)}{6d}+\frac{a^2\sin(2dx+2c)}{2d}$
derivativedivides	$\frac{a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{2a^2\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+a^2\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
default	$\frac{a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{2a^2\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+a^2\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
norman	$\frac{-\frac{7a^2x}{8}-\frac{25a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2d}+\frac{14a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{3d}+\frac{7a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{4d}-\frac{21a^2x\tan(\frac{dx}{2}+}{8}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4(-1+\tan(\frac{dx}{2}+}$

input
```
int(cos(d*x+c)^4*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/96*a^2*(84*d*x+3*sin(4*d*x+4*c)+16*sin(3*d*x+3*c)+48*sin(2*d*x+2*c)+144*
sin(d*x+c))/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{21\,a^2dx + \left(6\,a^2\cos\left(dx+c\right)^3 + 16\,a^2\cos\left(dx+c\right)^2 + 21\,a^2\cos\left(dx+c\right) + 32\,a^2\right)\sin\left(dx+c\right)}{24\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/24*(21*a^2*d*x + (6*a^2*cos(d*x + c)^3 + 16*a^2*cos(d*x + c)^2 + 21*a^2*
cos(d*x + c) + 32*a^2)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\cos^4\left(c+dx\right)\sec\left(c+dx\right)dx + \int \cos^4\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int \cos^4\left(c+dx\right)dx\right)$$

input
```
integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*cos(c + d*x)**4*sec(c + d*x), x) + Integral(cos(c + d*x)*
*4*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**4, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^2\,dx =$$

$$-\frac{64\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^2-3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^2-24\,(}{96\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/96*(64*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 - 3*(12*d*x + 12*c + sin(4
*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c)
)*a^2)/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.10

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^2\,dx$$

$$=\frac{21\,(dx+c)a^2+\frac{2\left(21\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+77\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+83\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+75\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/24*(21*(d*x + c)*a^2 + 2*(21*a^2*tan(1/2*d*x + 1/2*c)^7 + 77*a^2*tan(1/2
*d*x + 1/2*c)^5 + 83*a^2*tan(1/2*d*x + 1/2*c)^3 + 75*a^2*tan(1/2*d*x + 1/2
*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 12.70 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$=\frac{7\,a^2\,x}{8}+\frac{\frac{7\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{77\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}+\frac{83\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}+\frac{25\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

input
```
int(cos(c + d*x)^4*(a + a/cos(c + d*x))^2,x)
```

output
```
(7*a^2*x)/8 + ((83*a^2*tan(c/2 + (d*x)/2)^3)/12 + (77*a^2*tan(c/2 + (d*x)/
2)^5)/12 + (7*a^2*tan(c/2 + (d*x)/2)^7)/4 + (25*a^2*tan(c/2 + (d*x)/2))/4)
/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.70

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$=\frac{a^2(-6\cos(dx+c)\sin(dx+c)^3+27\cos(dx+c)\sin(dx+c)-16\sin(dx+c)^3+48\sin(dx+c)+21d}{24d}$$

input
```
int(cos(d*x+c)^4*(a+a*sec(d*x+c))^2,x)
```

output
```
(a**2*( - 6*cos(c + d*x)*sin(c + d*x)**3 + 27*cos(c + d*x)*sin(c + d*x) -
16*sin(c + d*x)**3 + 48*sin(c + d*x) + 21*d*x))/(24*d)
```

3.19 $\int \cos^5(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{3a^2x}{4} + \frac{2a^2\sin(c+dx)}{d} + \frac{3a^2\cos(c+dx)\sin(c+dx)}{4d} + \frac{a^2\cos^3(c+dx)\sin(c+dx)}{2d} - \frac{a^2\sin^3(c+dx)}{d} + \frac{a^2\sin^5(c+dx)}{5d}$$

output
```
3/4*a^2*x+2*a^2*sin(d*x+c)/d+3/4*a^2*cos(d*x+c)*sin(d*x+c)/d+1/2*a^2*cos(d
*x+c)^3*sin(d*x+c)/d-a^2*sin(d*x+c)^3/d+1/5*a^2*sin(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.59

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2(60dx+110\sin(c+dx)+40\sin(2(c+dx))+15\sin(3(c+dx))+5\sin(4(c+dx))+\sin(5(c+dx)))}{80d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*(60*d*x + 110*Sin[c + d*x] + 40*Sin[2*(c + d*x)] + 15*Sin[3*(c + d*x)
] + 5*Sin[4*(c + d*x)] + Sin[5*(c + d*x)]))/(80*d)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.91, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 4275, 3042, 3115, 3042, 3115, 24, 4532, 3042, 3492, 27, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{4275}$$

$$2a^2\int \cos^4(c+dx)dx+\int \cos^5(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3115}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+2a^2\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+2a^2\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{3115}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+2a^2\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{24}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+2a^2\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{4532}$$

$$\int \cos^3(c+dx)\left(\cos^2(c+dx)a^2+a^2\right)dx+2a^2\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(\sin\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2\right)dx+2a^2\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{3492}$$

$$2a^2\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{\int a^2\left(1-\sin^2(c+dx)\right)\left(2-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{27}$$

$$2a^2\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{a^2\int \left(1-\sin^2(c+dx)\right)\left(2-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{290}$$

$$2a^2\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{a^2\int \left(\sin^4(c+dx)-3\sin^2(c+dx)+2\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$2a^2\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{a^2\left(-\frac{1}{5}\sin^5(c+dx)+\sin^3(c+dx)-2\sin(c+dx)\right)}{d}$$

input
```
Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

output
```
-((a^2*(-2*Sin[c + d*x] + Sin[c + d*x]^3 - Sin[c + d*x]^5/5))/d) + 2*a^2*(
(Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x]
)/(2*d)))/4)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4532
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.62

method	result
parallelrisch	$\frac{a^{2}\left(60 d x+110 \sin \left(d x+c\right)+\sin \left(5 d x+5 c\right)+5 \sin \left(4 d x+4 c\right)+15 \sin \left(3 d x+3 c\right)+40 \sin \left(2 d x+2 c\right)\right)}{80 d}$
risch	$\frac{3 a^{2} x}{4}+\frac{11 a^{2} \sin \left(d x+c\right)}{8 d}+\frac{a^{2} \sin \left(5 d x+5 c\right)}{80 d}+\frac{a^{2} \sin \left(4 d x+4 c\right)}{16 d}+\frac{3 a^{2} \sin \left(3 d x+3 c\right)}{16 d}+\frac{a^{2} \sin \left(2 d x+2 c\right)}{2 d}$
derivativedivides	$\frac{\frac{a^{2}\left(2+\cos \left(d x+c\right)^{2}\right) \sin \left(d x+c\right)}{3}+2 a^{2}\left(\frac{\left(\cos \left(d x+c\right)^{3}+\frac{3 \cos \left(d x+c\right)}{2}\right) \sin \left(d x+c\right)}{4}+\frac{3 d x}{8}+\frac{3 c}{8}\right)+\frac{a^{2}\left(\frac{8}{3}+\cos \left(d x+c\right)^{4}+\frac{4 \cos \left(d x+c\right)^{2}}{3}\right) \cdots}{5}}{d}$
default	$\frac{\frac{a^{2}\left(2+\cos \left(d x+c\right)^{2}\right) \sin \left(d x+c\right)}{3}+2 a^{2}\left(\frac{\left(\cos \left(d x+c\right)^{3}+\frac{3 \cos \left(d x+c\right)}{2}\right) \sin \left(d x+c\right)}{4}+\frac{3 d x}{8}+\frac{3 c}{8}\right)+\frac{a^{2}\left(\frac{8}{3}+\cos \left(d x+c\right)^{4}+\frac{4 \cos \left(d x+c\right)^{2}}{3}\right) \cdots}{5}}{d}$
norman	$\frac{-\frac{3 a^{2} x}{4}-\frac{13 a^{2} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{2 d}-\frac{5 a^{2} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{2 d}-\frac{27 a^{2} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}{5 d}+\frac{37 a^{2} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{7}}{5 d}+\frac{11 a^{2} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{9}}{2 d}+\frac{3 a^{2} \tan \left(\frac{d x}{2}+ \cdots\right.}{2 d}}{\left(1+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}\right)^{5}\left(- \cdots\right.}$

input
```
int(cos(d*x+c)^5*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/80*a^2*(60*d*x+110*sin(d*x+c)+sin(5*d*x+5*c)+5*sin(4*d*x+4*c)+15*sin(3*d
*x+3*c)+40*sin(2*d*x+2*c))/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{15\,a^2dx + \left(4\,a^2\cos\left(dx+c\right)^4 + 10\,a^2\cos\left(dx+c\right)^3 + 12\,a^2\cos\left(dx+c\right)^2 + 15\,a^2\cos\left(dx+c\right) + 24\,a^2\right)\sin}{20\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/20*(15*a^2*d*x + (4*a^2*cos(d*x + c)^4 + 10*a^2*cos(d*x + c)^3 + 12*a^2*
cos(d*x + c)^2 + 15*a^2*cos(d*x + c) + 24*a^2)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\cos^5\left(c+dx\right)\sec\left(c+dx\right)dx + \int \cos^5\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int \cos^5\left(c+dx\right)dx\right)$$

input
```
integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*cos(c + d*x)**5*sec(c + d*x), x) + Integral(cos(c + d*x)*
*5*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**5, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.92

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$=\frac{16\left(3\sin\left(dx+c\right)^5-10\sin\left(dx+c\right)^3+15\sin\left(dx+c\right)\right)a^2-80\left(\sin\left(dx+c\right)^3-3\sin\left(dx+c\right)\right)a^2+15\,(}{240\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/240*(16*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^2 - 8
0*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 + 15*(12*d*x + 12*c + sin(4*d*x +
4*c) + 8*sin(2*d*x + 2*c))*a^2)/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.09

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$=\frac{15\,(dx+c)a^2+\frac{2\left(15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+70\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+144\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+90\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+65\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{20\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/20*(15*(d*x + c)*a^2 + 2*(15*a^2*tan(1/2*d*x + 1/2*c)^9 + 70*a^2*tan(1/2
*d*x + 1/2*c)^7 + 144*a^2*tan(1/2*d*x + 1/2*c)^5 + 90*a^2*tan(1/2*d*x + 1/
2*c)^3 + 65*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 13.43 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.02

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{3a^2x}{4} + \frac{\frac{3a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{2} + 7a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7 + \frac{72a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{5} + 9a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 + \frac{13a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^5}$$

input
```
int(cos(c + d*x)^5*(a + a/cos(c + d*x))^2,x)
```

output
```
(3*a^2*x)/4 + (9*a^2*tan(c/2 + (d*x)/2)^3 + (72*a^2*tan(c/2 + (d*x)/2)^5)/
5 + 7*a^2*tan(c/2 + (d*x)/2)^7 + (3*a^2*tan(c/2 + (d*x)/2)^9)/2 + (13*a^2*
tan(c/2 + (d*x)/2))/2)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.69

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2(-10\cos(dx+c)\sin(dx+c)^3 + 25\cos(dx+c)\sin(dx+c) + 4\sin(dx+c)^5 - 20\sin(dx+c)^3 + 40}{20d}$$

input
```
int(cos(d*x+c)^5*(a+a*sec(d*x+c))^2,x)
```

output
```
(a**2*( - 10*cos(c + d*x)*sin(c + d*x)**3 + 25*cos(c + d*x)*sin(c + d*x) +
 4*sin(c + d*x)**5 - 20*sin(c + d*x)**3 + 40*sin(c + d*x) + 15*d*x))/(20*d
)
```

3.20 $\int \sec^3(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 114

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{13a^3\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{4a^3\tan(c+dx)}{d}+\frac{13a^3\sec(c+dx)\tan(c+dx)}{8d}+\frac{3a^3\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{5a^3\tan^3(c+dx)}{3d}+\frac{a^3\tan^5(c+dx)}{5d}$$

output
```
13/8*a^3*arctanh(sin(d*x+c))/d+4*a^3*tan(d*x+c)/d+13/8*a^3*sec(d*x+c)*tan(
d*x+c)/d+3/4*a^3*sec(d*x+c)^3*tan(d*x+c)/d+5/3*a^3*tan(d*x+c)^3/d+1/5*a^3*
tan(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.07

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{13a^3\text{arctanh}(\sin(c+dx))}{8d} + \frac{19a^3\tan(c+dx)}{5d} + \frac{13a^3\sec(c+dx)\tan(c+dx)}{8d} + \frac{3a^3\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{a^3\sec^4(c+dx)\tan(c+dx)}{5d} + \frac{19a^3\tan^3(c+dx)}{15d}$$

input
```
Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

output
```
(13*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (19*a^3*Tan[c + d*x])/(5*d) + (13*a
^3*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (3*a^3*Sec[c + d*x]^3*Tan[c + d*x])/
(4*d) + (a^3*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + (19*a^3*Tan[c + d*x]^3)/
(15*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$

$$\Big\downarrow \text{ 4278}$$

$$\int \left(a^3\sec^6(c+dx)+3a^3\sec^5(c+dx)+3a^3\sec^4(c+dx)+a^3\sec^3(c+dx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{13a^3\text{arctanh}(\sin(c+dx))}{8d}+\frac{a^3\tan^5(c+dx)}{5d}+\frac{5a^3\tan^3(c+dx)}{3d}+\frac{4a^3\tan(c+dx)}{d}+\frac{3a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{13a^3\tan(c+dx)\sec(c+dx)}{8d}$$

input
```
Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

output
```
(13*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*Tan[c + d*x])/d + (13*a^3*Se
c[c + d*x]*Tan[c + d*x])/(8*d) + (3*a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
 + (5*a^3*Tan[c + d*x]^3)/(3*d) + (a^3*Tan[c + d*x]^5)/(5*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.53 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.27

method	result
risch	$-\frac{i a^3 (195\,\mathrm{e}^{9i(dx+c)}+750\,\mathrm{e}^{7i(dx+c)}-720\,\mathrm{e}^{6i(dx+c)}-2320\,\mathrm{e}^{4i(dx+c)}-750\,\mathrm{e}^{3i(dx+c)}-1520\,\mathrm{e}^{2i(dx+c)}-195\,\mathrm{e}^{i(dx+c)}-304}{60d(\mathrm{e}^{2i(dx+c)}+1)^5}$
derivativedivides	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-3a^3\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+3a^3\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(}{8}\right.\right.}{d}$
default	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-3a^3\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+3a^3\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(}{8}\right.\right.}{d}$
norman	$\frac{-\frac{51a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{133a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{6d}-\frac{416a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{15d}+\frac{91a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{6d}-\frac{13a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{4d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}-\frac{13a^3\ln\left(\tan\left(\right.\right.}{8}$
parts	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}-\frac{a^3\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}-\frac{3a^3\left(-\frac{2}{3}-\right.}{}$
parallelrisch	$-\frac{a^3\left(1950\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-1950\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+195\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(5dx+\right.}{}$

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/60*I*a^3*(195*exp(9*I*(d*x+c))+750*exp(7*I*(d*x+c))-720*exp(6*I*(d*x+c)
)-2320*exp(4*I*(d*x+c))-750*exp(3*I*(d*x+c))-1520*exp(2*I*(d*x+c))-195*exp
(I*(d*x+c))-304)/d/(exp(2*I*(d*x+c))+1)^5+13/8*a^3/d*ln(exp(I*(d*x+c))+I)-
13/8*a^3/d*ln(exp(I*(d*x+c))-I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.09

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{195\,a^3\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-195\,a^3\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+2\left(304\,a^3\cos\left(\right.\right.}{240\,d\cos\left(dx-\right.}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/240*(195*a^3*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 195*a^3*cos(d*x + c)
^5*log(-sin(d*x + c) + 1) + 2*(304*a^3*cos(d*x + c)^4 + 195*a^3*cos(d*x +
c)^3 + 152*a^3*cos(d*x + c)^2 + 90*a^3*cos(d*x + c) + 24*a^3)*sin(d*x + c)
)/(d*cos(d*x + c)^5)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int \sec^3(c+dx)\,dx + \int 3\sec^4(c+dx)\,dx + \int 3\sec^5(c+dx)\,dx + \int \sec^6(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**3,x)
```

output
```
a**3*(Integral(sec(c + d*x)**3, x) + Integral(3*sec(c + d*x)**4, x) + Inte
gral(3*sec(c + d*x)**5, x) + Integral(sec(c + d*x)**6, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.57

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{16\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)a^3+240\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^3 -}{}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3 + 2
40*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - 45*a^3*(2*(3*sin(d*x + c)^3 - 5
*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c
) + 1) + 3*log(sin(d*x + c) - 1)) - 60*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.21

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{195\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-195\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(195\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-910\,a^3\tan\left(\frac{1}{2}\right.}{}}{120\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/120*(195*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 195*a^3*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) - 2*(195*a^3*tan(1/2*d*x + 1/2*c)^9 - 910*a^3*tan(1/2
*d*x + 1/2*c)^7 + 1664*a^3*tan(1/2*d*x + 1/2*c)^5 - 1330*a^3*tan(1/2*d*x +
 1/2*c)^3 + 765*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/
d
```

Mupad [B] (verification not implemented)

Time = 14.19 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.49

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^3\,dx=\frac{13\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$
$$-\frac{\frac{13\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}-\frac{91\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{6}+\frac{416\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}-\frac{133\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6}+\frac{51\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + a/cos(c + d*x))^3/cos(c + d*x)^3,x)
```

output
```
(13*a^3*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((416*a^3*tan(c/2 + (d*x)/2)^5)
/15 - (133*a^3*tan(c/2 + (d*x)/2)^3)/6 - (91*a^3*tan(c/2 + (d*x)/2)^7)/6 +
 (13*a^3*tan(c/2 + (d*x)/2)^9)/4 + (51*a^3*tan(c/2 + (d*x)/2))/4)/(d*(5*ta
n(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5
*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.18

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^3\,dx$$

$$= \frac{a^3\left(-195\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^4+390\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(d\right.\right.}{}$$

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^3,x)
```

output
```
(a**3*( - 195*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4 + 390
*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 - 195*cos(c + d*x)
*log(tan((c + d*x)/2) - 1) + 195*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*si
n(c + d*x)**4 - 390*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2
 + 195*cos(c + d*x)*log(tan((c + d*x)/2) + 1) - 195*cos(c + d*x)*sin(c + d
*x)**3 + 285*cos(c + d*x)*sin(c + d*x) + 304*sin(c + d*x)**5 - 760*sin(c +
 d*x)**3 + 480*sin(c + d*x)))/(120*cos(c + d*x)*d*(sin(c + d*x)**4 - 2*sin
(c + d*x)**2 + 1))
```

3.21 $\int \sec^2(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 93

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{15a^3\operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{4a^3\tan(c+dx)}{d} + \frac{15a^3\sec(c+dx)\tan(c+dx)}{8d} + \frac{a^3\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{a^3\tan^3(c+dx)}{d}$$

output

```
15/8*a^3*arctanh(sin(d*x+c))/d+4*a^3*tan(d*x+c)/d+15/8*a^3*sec(d*x+c)*tan(
d*x+c)/d+1/4*a^3*sec(d*x+c)^3*tan(d*x+c)/d+a^3*tan(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.06

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{15a^3\text{arctanh}(\sin(c+dx))}{8d}+\frac{3a^3\tan(c+dx)}{d}+\frac{15a^3\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^3\sec^2(c+dx)\tan(c+dx)}{d}+\frac{a^3\sec^3(c+dx)\tan(c+dx)}{4d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

output
```
(15*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (3*a^3*Tan[c + d*x])/d + (15*a^3*Se
c[c + d*x]*Tan[c + d*x])/(8*d) + (a^3*Sec[c + d*x]^2*Tan[c + d*x])/d + (a^
3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^3\sec^5(c+dx)+3a^3\sec^4(c+dx)+3a^3\sec^3(c+dx)+a^3\sec^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{15a^3\text{arctanh}(\sin(c+dx))}{8d}+\frac{a^3\tan^3(c+dx)}{d}+\frac{4a^3\tan(c+dx)}{d}+\frac{a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{15a^3\tan(c+dx)\sec(c+dx)}{8d}$$

input
```
Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

output
```
(15*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*Tan[c + d*x])/d + (15*a^3*Se
c[c + d*x]*Tan[c + d*x])/(8*d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) +
 (a^3*Tan[c + d*x]^3)/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{a^3\tan(dx+c)+3a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-3a^3\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+a^3\left(-\left(-\frac{\sec}{}\right.\right.}{d}$
default	$\frac{a^3\tan(dx+c)+3a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-3a^3\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+a^3\left(-\left(-\frac{\sec}{}\right.\right.}{d}$
parts	$\frac{a^3\tan(dx+c)}{d}+\frac{a^3\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}+\frac{3a^3\left(\frac{\sec(dx+c)\tan(d}{2}\right.}{}$
norman	$\frac{\frac{49a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{73a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4d}+\frac{55a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4d}-\frac{15a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}-\frac{15a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8d}+\frac{15a^3\ln}{}$
risch	$-\frac{ia^3\left(15\,e^{7i(dx+c)}-8\,e^{6i(dx+c)}+23\,e^{5i(dx+c)}-72\,e^{4i(dx+c)}-23\,e^{3i(dx+c)}-88\,e^{2i(dx+c)}-15\,e^{i(dx+c)}-24\right)}{4d\left(e^{2i(dx+c)}+1\right)^4}-\frac{15a^3\ln}{}$
parallelrisch	$\frac{a^3\left(15(-3-\cos(4dx+4c)-4\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+15(3+\cos(4dx+4c)+4\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{8d(3+\cos(4dx+4c)+4\cos(2dx+2c))}$

input
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*tan(d*x+c)+3*a^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan
(d*x+c)))-3*a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a^3*(-(-1/4*sec(d*x+c)^
3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.19

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{15\,a^3\cos(dx+c)^4\log(\sin(dx+c)+1)-15\,a^3\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\left(24\,a^3\cos(dx\right.}{16\,d\cos(dx+c)^4}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/16*(15*a^3*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 15*a^3*cos(d*x + c)^4*
log(-sin(d*x + c) + 1) + 2*(24*a^3*cos(d*x + c)^3 + 15*a^3*cos(d*x + c)^2
+ 8*a^3*cos(d*x + c) + 2*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int \sec^2(c+dx)\,dx + \int 3\sec^3(c+dx)\,dx + \int 3\sec^4(c+dx)\,dx + \int \sec^5(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**3,x)
```

output
```
a**3*(Integral(sec(c + d*x)**2, x) + Integral(3*sec(c + d*x)**3, x) + Inte
gral(3*sec(c + d*x)**4, x) + Integral(sec(c + d*x)**5, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.68

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^3 - a^3\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1} - 3\log(\sin(dx+c)+1) + 3\log(s\ldots}{\ldots}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/16*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - a^3*(2*(3*sin(d*x + c)^3
- 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a^3*(2*sin(d*x + c)/(sin(d*x + c
)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 16*a^3*tan(d*x
 + c))/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.31

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-55\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\right.\right.}}{8\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/8*(15*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a^3*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) - 2*(15*a^3*tan(1/2*d*x + 1/2*c)^7 - 55*a^3*tan(1/2*d*x +
 1/2*c)^5 + 73*a^3*tan(1/2*d*x + 1/2*c)^3 - 49*a^3*tan(1/2*d*x + 1/2*c))/(
tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 12.87 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.52

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{15\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$
$$-\frac{\frac{15\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}-\frac{55\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\frac{73\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}-\frac{49\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

input
```
int((a + a/cos(c + d*x))^3/cos(c + d*x)^2,x)
```

output
```
(15*a^3*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((73*a^3*tan(c/2 + (d*x)/2)^3)/
4 - (55*a^3*tan(c/2 + (d*x)/2)^5)/4 + (15*a^3*tan(c/2 + (d*x)/2)^7)/4 - (4
9*a^3*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x
)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.10

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{a^3\left(-24\cos\left(dx+c\right)\sin\left(dx+c\right)^3+32\cos\left(dx+c\right)\sin\left(dx+c\right)-15\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^4\right.}{}$$

input
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c))^3,x)
```

output
```
(a**3*( - 24*cos(c + d*x)*sin(c + d*x)**3 + 32*cos(c + d*x)*sin(c + d*x) -
 15*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4 + 30*log(tan((c + d*x)/2) -
1)*sin(c + d*x)**2 - 15*log(tan((c + d*x)/2) - 1) + 15*log(tan((c + d*x)/2
) + 1)*sin(c + d*x)**4 - 30*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2 + 15
*log(tan((c + d*x)/2) + 1) - 15*sin(c + d*x)**3 + 17*sin(c + d*x)))/(8*d*(
sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.22 $\int \sec(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \sec(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{5a^3\text{arctanh}(\sin(c+dx))}{2d} + \frac{4a^3\tan(c+dx)}{d} + \frac{3a^3\sec(c+dx)\tan(c+dx)}{2d} + \frac{a^3\tan^3(c+dx)}{3d}$$

output
```
5/2*a^3*arctanh(sin(d*x+c))/d+4*a^3*tan(d*x+c)/d+3/2*a^3*sec(d*x+c)*tan(d*
x+c)/d+1/3*a^3*tan(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.19

$$\int \sec(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{a^3\coth^{-1}(\sin(c+dx))}{d} + \frac{3a^3\text{arctanh}(\sin(c+dx))}{2d} + \frac{4a^3\tan(c+dx)}{d} + \frac{3a^3\sec(c+dx)\tan(c+dx)}{2d} + \frac{a^3\tan^3(c+dx)}{3d}$$

input
```
Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

output
```
(a^3*ArcCoth[Sin[c + d*x]])/d + (3*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (4*a
^3*Tan[c + d*x])/d + (3*a^3*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a^3*Tan[c
+ d*x]^3)/(3*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$

$$\Big\downarrow \text{ 4278}$$

$$\int \left(a^3\sec^4(c+dx)+3a^3\sec^3(c+dx)+3a^3\sec^2(c+dx)+a^3\sec(c+dx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{5a^3\text{arctanh}(\sin(c+dx))}{2d}+\frac{a^3\tan^3(c+dx)}{3d}+\frac{4a^3\tan(c+dx)}{d}+\frac{3a^3\tan(c+dx)\sec(c+dx)}{2d}$$

input
```
Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

output
```
(5*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (4*a^3*Tan[c + d*x])/d + (3*a^3*Sec[
c + d*x]*Tan[c + d*x])/(2*d) + (a^3*Tan[c + d*x]^3)/(3*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))+3a^3\tan(dx+c)+3a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-a^3\left(-\frac{2}{3}-\frac{\sec(dx+}{3}\right.}{d}$
default	$\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))+3a^3\tan(dx+c)+3a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-a^3\left(-\frac{2}{3}-\frac{\sec(dx+}{3}\right.}{d}$
parts	$\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))}{d}-\frac{a^3\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{3a^3\tan(dx+c)}{d}+\frac{3a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\right.}{d}$
risch	$-\frac{ia^3\left(9\,e^{5i(dx+c)}-18\,e^{4i(dx+c)}-48\,e^{2i(dx+c)}-9\,e^{i(dx+c)}-22\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}+\frac{5a^3\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{5a^3\ln\left(e^{i(dx+c)}-i\right)}{2d}$
norman	$\frac{-\frac{11a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{40a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{5a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}-\frac{5a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{5a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$
parallelrisch	$\frac{a^3\left(15\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(3dx+3c)-15\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(3dx+3c)+45\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)-4\right.}{6d(\cos(3dx+3c)+3\cos(dx+c))}$

input
```
int(sec(d*x+c)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^3*tan(d*x+c)+3*a^3*(1/2*sec(d*x+c)*
tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(
d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.36

$$\int \sec(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{15\,a^3\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-15\,a^3\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(22\,a^3\cos\left(dx-\right.\right.}{12\,d\cos\left(dx+c\right)^3}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/12*(15*a^3*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 15*a^3*cos(d*x + c)^3*
log(-sin(d*x + c) + 1) + 2*(22*a^3*cos(d*x + c)^2 + 9*a^3*cos(d*x + c) + 2
*a^3)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^3\,dx=a^3\left(\int \sec\left(c+dx\right)dx+\int 3\sec^2\left(c+dx\right)dx+\int 3\sec^3\left(c+dx\right)dx+\int \sec^4\left(c+dx\right)dx\right)$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))**3,x)
```

output
```
a**3*(Integral(sec(c + d*x), x) + Integral(3*sec(c + d*x)**2, x) + Integra
l(3*sec(c + d*x)**3, x) + Integral(sec(c + d*x)**4, x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.44

$$\int \sec(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{4\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^3-9\,a^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{12\,d}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - 9*a^3*(2*sin(d*x + c)/(sin
(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^3
*log(sec(d*x + c) + tan(d*x + c)) + 36*a^3*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.47

$$\int \sec(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-40\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\right.}}{6\,d}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/6*(15*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a^3*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) - 2*(15*a^3*tan(1/2*d*x + 1/2*c)^5 - 40*a^3*tan(1/2*d*x +
 1/2*c)^3 + 33*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 11.72 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.56

$$\int \sec(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{5\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{5\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{40\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+11\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

input
```
int((a + a/cos(c + d*x))^3/cos(c + d*x),x)
```

output
```
(5*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (5*a^3*tan(c/2 + (d*x)/2)^5 - (40*a^
3*tan(c/2 + (d*x)/2)^3)/3 + 11*a^3*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2 + (d*
x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.18

$$\int \sec(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{a^3\left(-15\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2+15\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+15\cos\right.}{}$$

input
```
int(sec(d*x+c)*(a+a*sec(d*x+c))^3,x)
```

output
```
(a**3*( - 15*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 + 15*c
os(c + d*x)*log(tan((c + d*x)/2) - 1) + 15*cos(c + d*x)*log(tan((c + d*x)/
2) + 1)*sin(c + d*x)**2 - 15*cos(c + d*x)*log(tan((c + d*x)/2) + 1) - 9*co
s(c + d*x)*sin(c + d*x) + 22*sin(c + d*x)**3 - 24*sin(c + d*x)))/(6*cos(c
+ d*x)*d*(sin(c + d*x)**2 - 1))
```

3.23 $\int (a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 66

$$\int (a+a\sec(c+dx))^3\,dx = a^3x+\frac{7a^3\text{arctanh}(\sin(c+dx))}{2d}+\frac{5a^3\tan(c+dx)}{2d}+\frac{(a^3+a^3\sec(c+dx))\tan(c+dx)}{2d}$$

output
```
a^3*x+7/2*a^3*arctanh(sin(d*x+c))/d+5/2*a^3*tan(d*x+c)/d+1/2*(a^3+a^3*sec(
d*x+c))*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.12

$$\int (a+a\sec(c+dx))^3\,dx = a^3x+\frac{3a^3\coth^{-1}(\sin(c+dx))}{d}+\frac{a^3\text{arctanh}(\sin(c+dx))}{2d}+\frac{3a^3\tan(c+dx)}{d}+\frac{a^3\sec(c+dx)\tan(c+dx)}{2d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^3,x]
```

output
```
a^3*x + (3*a^3*ArcCoth[Sin[c + d*x]])/d + (a^3*ArcTanh[Sin[c + d*x]])/(2*d
) + (3*a^3*Tan[c + d*x])/d + (a^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4262, 3042, 4402, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$

$$\Big\downarrow \text{4262}$$

$$\frac{1}{2}a\int (\sec(c+dx)a+a)(5\sec(c+dx)a+2a)dx+\frac{\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\int \left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(5\csc\left(c+dx+\frac{\pi}{2}\right)a+2a\right)dx+\frac{\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{2d}$$

$$\Big\downarrow \text{4402}$$

$$\frac{1}{2}a\left(5a^2\int \sec^2(c+dx)dx+7a^2\int \sec(c+dx)dx+2a^2x\right)+\frac{\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\left(7a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+5a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+2a^2x\right)+\frac{\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{2d}$$

$$\Big\downarrow \text{4254}$$

$$\frac{1}{2}a\left(-\frac{5a^2\int 1d(-\tan(c+dx))}{d}+7a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+2a^2x\right)+\frac{\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{2d}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{1}{2}a\left(7a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{5a^2\tan(c+dx)}{d}+2a^2x\right)+\frac{\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{2d}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{2d}+\frac{1}{2}a\left(\frac{7a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{5a^2\tan(c+dx)}{d}+2a^2x\right)$$

input
```
Int[(a + a*Sec[c + d*x])^3,x]
```

output
```
((a^3 + a^3*Sec[c + d*x])*Tan[c + d*x])/(2*d) + (a*(2*a^2*x + (7*a^2*ArcTa
nh[Sin[c + d*x]])/d + (5*a^2*Tan[c + d*x])/d))/2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)    Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4262
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[a/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Inte
gerQ[2*n]
```

rule 4402
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Simp[b*d   Int[Csc[e + f*x]^2, x], x
] + Simp[(b*c + a*d)   Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{a^3(dx+c)+3a^3\ln(\sec(dx+c)+\tan(dx+c))+3a^3\tan(dx+c)+a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{a^3(dx+c)+3a^3\ln(\sec(dx+c)+\tan(dx+c))+3a^3\tan(dx+c)+a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
parts	$a^3x+\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{3a^3\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{3a^3\tan(dx+c)}{d}$
risch	$a^3x-\frac{ia^3(e^{3i(dx+c)}-6\,e^{2i(dx+c)}-e^{i(dx+c)}-6)}{d(e^{2i(dx+c)}+1)^2}+\frac{7a^3\ln(e^{i(dx+c)}+i)}{2d}-\frac{7a^3\ln(e^{i(dx+c)}-i)}{2d}$
norman	$\frac{a^3x+a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\frac{7a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{5a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}-2a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-\frac{7a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{7a}{}$
parallelrisch	$\frac{a^3\left(2dx\cos(2dx+2c)+7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(2dx+2c)-7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(2dx+2c)+2dx+6\sin(2dx+2c)+\right.}{2d(1+\cos(2dx+2c))}$

input
```
int((a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(d*x+c)+3*a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^3*tan(d*x+c)+a^3*(1/2
*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.48

$$\int (a+a\sec(c+dx))^3\,dx$$
$$= \frac{4\,a^3dx\cos\left(dx+c\right)^2+7\,a^3\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-7\,a^3\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right.}{4\,d\cos\left(dx+c\right)^2}$$

input
```
integrate((a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4*(4*a^3*d*x*cos(d*x + c)^2 + 7*a^3*cos(d*x + c)^2*log(sin(d*x + c) + 1)
 - 7*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(6*a^3*cos(d*x + c) + a
^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int (a+a\sec(c+dx))^3\,dx = a^3\left(\int 1\,dx+\int 3\sec\left(c+dx\right)dx+\int 3\sec^2\left(c+dx\right)dx + \int \sec^3\left(c+dx\right)dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**3,x)
```

output
```
a**3*(Integral(1, x) + Integral(3*sec(c + d*x), x) + Integral(3*sec(c + d*
x)**2, x) + Integral(sec(c + d*x)**3, x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.38

$$\int (a + a\sec(c + dx))^3 \, dx$$
$$= a^3x - \frac{a^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right)}{4\,d} + \frac{3\,a^3\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d} + \frac{3\,a^3\tan\left(dx+c\right)}{d}$$

input
```
integrate((a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
a^3*x - 1/4*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) +
1) + log(sin(d*x + c) - 1))/d + 3*a^3*log(sec(d*x + c) + tan(d*x + c))/d +
 3*a^3*tan(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.52

$$\int (a + a\sec(c + dx))^3 \, dx$$
$$= \frac{2\,(dx+c)a^3 + 7\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 7\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}}{2\,d}$$

input
```
integrate((a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(2*(d*x + c)*a^3 + 7*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 7*a^3*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*a^3
*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 10.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.33

$$\int (a+a\sec(c+dx))^3\,dx = a^3\,x + \frac{7\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{5\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 - 7\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 - 2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + 1\right)}$$

input
```
int((a + a/cos(c + d*x))^3,x)
```

output
```
a^3*x + (7*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (5*a^3*tan(c/2 + (d*x)/2)^3
- 7*a^3*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2
)^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.98

$$\int (a+a\sec(c+dx))^3\,dx = \frac{a^3\left(-6\cos(dx+c)\sin(dx+c) - 7\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2 + 7\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right) + 7\log\cdots\right.}{2d\left(\sin(dx \cdots\right.}$$

input
```
int((a+a*sec(d*x+c))^3,x)
```

output
```
(a**3*( - 6*cos(c + d*x)*sin(c + d*x) - 7*log(tan((c + d*x)/2) - 1)*sin(c
+ d*x)**2 + 7*log(tan((c + d*x)/2) - 1) + 7*log(tan((c + d*x)/2) + 1)*sin(
c + d*x)**2 - 7*log(tan((c + d*x)/2) + 1) + 2*sin(c + d*x)**2*d*x - sin(c
+ d*x) - 2*d*x))/(2*d*(sin(c + d*x)**2 - 1))
```

3.24 $\int \cos(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int \cos(c+dx)(a+a\sec(c+dx))^3\,dx = 3a^3x+\frac{3a^3\text{arctanh}(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}$$

output
```
3*a^3*x+3*a^3*arctanh(sin(d*x+c))/d+a^3*sin(d*x+c)/d+a^3*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.69

$$\int \cos(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{a^3\left(3dx+3\coth^{-1}(\sin(c+dx))+\sin(c+dx)+\tan(c+dx)\right)}{d}$$

input
```
Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

output
```
(a^3*(3*d*x + 3*ArcCoth[Sin[c + d*x]] + Sin[c + d*x] + Tan[c + d*x]))/d
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^3\cos(c+dx)+a^3\sec^2(c+dx)+3a^3\sec(c+dx)+3a^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3a^3\text{arctanh}(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}+3a^3x$$

input
```
Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

output
```
3*a^3*x + (3*a^3*ArcTanh[Sin[c + d*x]])/d + (a^3*Sin[c + d*x])/d + (a^3*Ta
n[c + d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{a^3\tan(dx+c)+3a^3\ln(\sec(dx+c)+\tan(dx+c))+3a^3(dx+c)+a^3\sin(dx+c)}{d}$
default	$\frac{a^3\tan(dx+c)+3a^3\ln(\sec(dx+c)+\tan(dx+c))+3a^3(dx+c)+a^3\sin(dx+c)}{d}$
parallelrisch	$\frac{a^3\left(6dx\cos(dx+c)-6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+\sin(2dx+2c)+2\sin(dx+c)\right)}{2d\cos(dx+c)}$
risch	$3a^3x-\frac{ia^3e^{i(dx+c)}}{2d}+\frac{ia^3e^{-i(dx+c)}}{2d}+\frac{2ia^3}{d(e^{2i(dx+c)}+1)}-\frac{3a^3\ln(e^{i(dx+c)}-i)}{d}+\frac{3a^3\ln(e^{i(dx+c)}+i)}{d}$
norman	$\frac{3a^3x+\frac{4a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{4a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}-3a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-3a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+3a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-\frac{3a^3\ln(t}{}$

input
```
int(cos(d*x+c)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*tan(d*x+c)+3*a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^3*(d*x+c)+a^3*sin(
d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.90

$$\int\cos(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{6\,a^3dx\cos(dx+c)+3\,a^3\cos(dx+c)\log(\sin(dx+c)+1)-3\,a^3\cos(dx+c)\log(-\sin(dx+c)+1)+}{2\,d\cos(dx+c)}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(6*a^3*d*x*cos(d*x + c) + 3*a^3*cos(d*x + c)*log(sin(d*x + c) + 1) - 3
*a^3*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(a^3*cos(d*x + c) + a^3)*sin(
d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \cos(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int 3\cos(c+dx)\sec(c+dx)\,dx + \int 3\cos(c+dx)\sec^2(c+dx)\,dx + \int \cos(c+dx)\sec^3(c+dx)\,dx + \int \cos(c+dx)\,dx\right)$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))**3,x)
```

output
```
a**3*(Integral(3*cos(c + d*x)*sec(c + d*x), x) + Integral(3*cos(c + d*x)*s
ec(c + d*x)**2, x) + Integral(cos(c + d*x)*sec(c + d*x)**3, x) + Integral(
cos(c + d*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.33

$$\int \cos(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{6\,(dx+c)a^3 + 3\,a^3(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 2\,a^3\sin(dx+c) + 2\,a^3\tan(dx+c)}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(6*(d*x + c)*a^3 + 3*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
)) + 2*a^3*sin(d*x + c) + 2*a^3*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.67

$$\int \cos(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{3\,(dx+c)a^3+3\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{4\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1}}{d}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
(3*(d*x + c)*a^3 + 3*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^3*log(ab
s(tan(1/2*d*x + 1/2*c) - 1)) - 4*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1
/2*c)^4 - 1))/d
```

Mupad [B] (verification not implemented)

Time = 9.94 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \cos(c+dx)(a+a\sec(c+dx))^3\,dx=3\,a^3\,x+\frac{6\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-1\right)}$$

input
```
int(cos(c + d*x)*(a + a/cos(c + d*x))^3,x)
```

output
```
3*a^3*x + (6*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (4*a^3*tan(c/2 + (d*x)/2))
/(d*(tan(c/2 + (d*x)/2)^4 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.96

$$\int \cos(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{a^3\left(-3\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+3\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\cos(dx+c)\sin(dx+\right.}{\cos(dx+c)\,d}$$

input
```
int(cos(d*x+c)*(a+a*sec(d*x+c))^3,x)
```

output
```
(a**3*( - 3*cos(c + d*x)*log(tan((c + d*x)/2) - 1) + 3*cos(c + d*x)*log(ta
n((c + d*x)/2) + 1) + cos(c + d*x)*sin(c + d*x) + 3*cos(c + d*x)*c + 3*cos
(c + d*x)*d*x + sin(c + d*x)))/(cos(c + d*x)*d)
```

3.25 $\int \cos^2(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{7a^3x}{2}+\frac{a^3\text{arctanh}(\sin(c+dx))}{d}+\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
7/2*a^3*x+a^3*arctanh(sin(d*x+c))/d+3*a^3*sin(d*x+c)/d+1/2*a^3*cos(d*x+c)*
sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.37

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{a^3\left(14dx-4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+12\sin\right.}{4d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

output
```
(a^3*(14*d*x - 4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]] + 12*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\sec(c+dx)+a)^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$
$$\Big\downarrow \text{4278}$$
$$\int \left(a^3\cos^2(c+dx)+3a^3\cos(c+dx)+a^3\sec(c+dx)+3a^3\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{a^3\text{arctanh}(\sin(c+dx))}{d}+\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^3x}{2}$$

input
```
Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

output
```
(7*a^3*x)/2 + (a^3*ArcTanh[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x])/d + (a^
3*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

method	result
parallelrisch	$\frac{a^3\left(14dx+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+12\sin(dx+c)+\sin(2dx+2c)\right)}{4d}$
derivativedivides	$\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))+3a^3(dx+c)+3a^3\sin(dx+c)+a^3\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))+3a^3(dx+c)+3a^3\sin(dx+c)+a^3\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
risch	$\frac{7a^3x}{2}-\frac{3ia^3e^{i(dx+c)}}{2d}+\frac{3ia^3e^{-i(dx+c)}}{2d}+\frac{a^3\ln(e^{i(dx+c)}+i)}{d}-\frac{a^3\ln(e^{i(dx+c)}-i)}{d}+\frac{a^3\sin(2dx+2c)}{4d}$
norman	$\frac{\frac{7a^3x}{2}+\frac{7a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{9a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}-\frac{3a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}+\frac{5a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}-7a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\frac{7a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{2}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$

input
```
int(cos(d*x+c)^2*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^3*(14*d*x+4*ln(tan(1/2*d*x+1/2*c)+1)-4*ln(tan(1/2*d*x+1/2*c)-1)+12*s
in(d*x+c)+sin(2*d*x+2*c))/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{7\,a^3dx + a^3\log\left(\sin\left(dx+c\right)+1\right) - a^3\log\left(-\sin\left(dx+c\right)+1\right) + \left(a^3\cos\left(dx+c\right)+6\,a^3\right)\sin\left(dx+c\right)}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(7*a^3*d*x + a^3*log(sin(d*x + c) + 1) - a^3*log(-sin(d*x + c) + 1) +
(a^3*cos(d*x + c) + 6*a^3)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int 3\cos^2\left(c+dx\right)\sec\left(c+dx\right)dx + \int 3\cos^2\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int \cos^2\left(c+dx\right)\sec^3\left(c+dx\right)dx + \int \cos^2\left(c+dx\right)dx\right)$$

input
```
integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**3,x)
```

output
```
a**3*(Integral(3*cos(c + d*x)**2*sec(c + d*x), x) + Integral(3*cos(c + d*x
)**2*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**2*sec(c + d*x)**3, x) +
Integral(cos(c + d*x)**2, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.25

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^3+12\,(dx+c)a^3+2\,a^3(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+}{4\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 + 12*(d*x + c)*a^3 + 2*a^3*(log(
sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*a^3*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.69

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{7\,(dx+c)a^3+2\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+7}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(7*(d*x + c)*a^3 + 2*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a^3*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(5*a^3*tan(1/2*d*x + 1/2*c)^3 + 7*a^3
*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 10.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.49

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{7\,a^3\,x}{2} + \frac{2\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} + \frac{5\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 + 7\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + 1\right)}$$

input
```
int(cos(c + d*x)^2*(a + a/cos(c + d*x))^3,x)
```

output
```
(7*a^3*x)/2 + (2*a^3*atanh(tan(c/2 + (d*x)/2)))/d + (5*a^3*tan(c/2 + (d*x)
/2)^3 + 7*a^3*tan(c/2 + (d*x)/2))/(d*(2*tan(c/2 + (d*x)/2)^2 + tan(c/2 + (
d*x)/2)^4 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{a^3\left(\cos(dx+c)\sin(dx+c) - 2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right) + 2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right) + 6\sin(dx+c) + 7c + \right.}{2d}$$

input
```
int(cos(d*x+c)^2*(a+a*sec(d*x+c))^3,x)
```

output
```
(a**3*(cos(c + d*x)*sin(c + d*x) - 2*log(tan((c + d*x)/2) - 1) + 2*log(tan
((c + d*x)/2) + 1) + 6*sin(c + d*x) + 7*c + 7*d*x))/(2*d)
```

3.26 $\int \cos^3(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{5a^3x}{2}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}-\frac{a^3\sin^3(c+dx)}{3d}$$

output
```
5/2*a^3*x+4*a^3*sin(d*x+c)/d+3/2*a^3*cos(d*x+c)*sin(d*x+c)/d-1/3*a^3*sin(d
*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{a^3(30c+30dx+45\sin(c+dx)+9\sin(2(c+dx))+\sin(3(c+dx)))}{12d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

output
```
(a^3*(30*c + 30*d*x + 45*Sin[c + d*x] + 9*Sin[2*(c + d*x)] + Sin[3*(c + d*
x)]))/(12*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{ 4278}$$

$$\int \left(a^3\cos^3(c+dx)+3a^3\cos^2(c+dx)+3a^3\cos(c+dx)+a^3\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{5a^3x}{2}$$

input
```
Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

output
```
(5*a^3*x)/2 + (4*a^3*Sin[c + d*x])/d + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(
2*d) - (a^3*Sin[c + d*x]^3)/(3*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.67

method	result
parallelrisch	$\frac{a^{3}\left(30 d x +45 \sin \left(d x +c \right)+\sin \left(3 d x +3 c \right)+9 \sin \left(2 d x +2 c \right)\right)}{12 d}$
risch	$\frac{5 a^{3} x}{2}+\frac{15 a^{3} \sin \left(d x +c \right)}{4 d}+\frac{a^{3} \sin \left(3 d x +3 c \right)}{12 d}+\frac{3 a^{3} \sin \left(2 d x +2 c \right)}{4 d}$
derivativedivides	$\frac{a^{3} \left(d x +c \right)+3 a^{3} \sin \left(d x +c \right)+3 a^{3} \left(\frac{\sin \left(d x +c \right) \cos \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)+\frac{a^{3} \left(2+\cos \left(d x +c \right)^{2}\right) \sin \left(d x +c \right)}{3}}{d}$
default	$\frac{a^{3} \left(d x +c \right)+3 a^{3} \sin \left(d x +c \right)+3 a^{3} \left(\frac{\sin \left(d x +c \right) \cos \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)+\frac{a^{3} \left(2+\cos \left(d x +c \right)^{2}\right) \sin \left(d x +c \right)}{3}}{d}$
norman	$\frac{\frac{5 a^{3} x}{2}+\frac{11 a^{3} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{d}-\frac{26 a^{3} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{3 d}-\frac{32 a^{3} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}{3 d}+\frac{10 a^{3} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{7}}{3 d}+\frac{5 a^{3} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{9}}{d}+\frac{5 a^{3} x \tan \left(\frac{d x}{2}+\right.}{2}}{\left(1+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}\right)^{3} \left(-1+\tan \left(\frac{d x}{2}+\right.\right.}$

input
```
int(cos(d*x+c)^3*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*a^3*(30*d*x+45*sin(d*x+c)+sin(3*d*x+3*c)+9*sin(2*d*x+2*c))/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{15\,a^3dx+\left(2\,a^3\cos\left(dx+c\right)^2+9\,a^3\cos\left(dx+c\right)+22\,a^3\right)\sin\left(dx+c\right)}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/6*(15*a^3*d*x + (2*a^3*cos(d*x + c)^2 + 9*a^3*cos(d*x + c) + 22*a^3)*sin
(d*x + c))/d
```

Sympy [F]

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int 3\cos^3\left(c+dx\right)\sec\left(c+dx\right)dx + \int 3\cos^3\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int \cos^3\left(c+dx\right)\sec^3\left(c+dx\right)dx + \int \cos^3\left(c+dx\right)dx\right)$$

input
```
integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**3,x)
```

output
```
a**3*(Integral(3*cos(c + d*x)**3*sec(c + d*x), x) + Integral(3*cos(c + d*x
)**3*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**3*sec(c + d*x)**3, x) +
Integral(cos(c + d*x)**3, x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.13

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^3\,dx =$$
$$-\frac{4\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^3-9\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^3-12\left(dx+c\right)a^3-36\,a^3\sin\left(d\right.}{12\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 - 9*(2*d*x + 2*c + sin(2*d*
x + 2*c))*a^3 - 12*(d*x + c)*a^3 - 36*a^3*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{15\left(dx+c\right)a^3+\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+40\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+33\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/6*(15*(d*x + c)*a^3 + 2*(15*a^3*tan(1/2*d*x + 1/2*c)^5 + 40*a^3*tan(1/2*
d*x + 1/2*c)^3 + 33*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)
^3)/d
```

Mupad [B] (verification not implemented)

Time = 10.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{5\,a^3\,x}{2}+\frac{11\,a^3\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a^3\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}+\frac{3\,a^3\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}$$

input
```
int(cos(c + d*x)^3*(a + a/cos(c + d*x))^3,x)
```

output
```
(5*a^3*x)/2 + (11*a^3*sin(c + d*x))/(3*d) + (a^3*cos(c + d*x)^2*sin(c + d*
x))/(3*d) + (3*a^3*cos(c + d*x)*sin(c + d*x))/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.71

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{a^3\left(9\cos\left(dx+c\right)\sin\left(dx+c\right)-2\sin\left(dx+c\right)^3+24\sin\left(dx+c\right)+15dx\right)}{6d}$$

input
```
int(cos(d*x+c)^3*(a+a*sec(d*x+c))^3,x)
```

output
```
(a**3*(9*cos(c + d*x)*sin(c + d*x) - 2*sin(c + d*x)**3 + 24*sin(c + d*x) +
 15*d*x))/(6*d)
```

3.27 $\int \cos^4(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 85

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{15a^3x}{8}+\frac{4a^3\sin(c+dx)}{d}+\frac{15a^3\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^3\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{a^3\sin^3(c+dx)}{d}$$

output
```
15/8*a^3*x+4*a^3*sin(d*x+c)/d+15/8*a^3*cos(d*x+c)*sin(d*x+c)/d+1/4*a^3*cos
(d*x+c)^3*sin(d*x+c)/d-a^3*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.60

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{a^3(60dx+104\sin(c+dx)+32\sin(2(c+dx))+8\sin(3(c+dx))+\sin(4(c+dx)))}{32d}$$

input
```
Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```

output
```
(a^3*(60*d*x + 104*Sin[c + d*x] + 32*Sin[2*(c + d*x)] + 8*Sin[3*(c + d*x)]
 + Sin[4*(c + d*x)]))/(32*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^3\cos^4(c+dx)+3a^3\cos^3(c+dx)+3a^3\cos^2(c+dx)+a^3\cos(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3\sin^3(c+dx)}{d}+\frac{4a^3\sin(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{15a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{15a^3x}{8}$$

input
```
Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```

output
```
(15*a^3*x)/8 + (4*a^3*Sin[c + d*x])/d + (15*a^3*Cos[c + d*x]*Sin[c + d*x])
/(8*d) + (a^3*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a^3*Sin[c + d*x]^3)/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.62

method	result
parallelrisch	$\frac{a^{3}\left(60dx+104\sin\left(dx+c\right)+\sin\left(4dx+4c\right)+8\sin\left(3dx+3c\right)+32\sin\left(2dx+2c\right)\right)}{32d}$
risch	$\frac{15a^{3}x}{8}+\frac{13a^{3}\sin\left(dx+c\right)}{4d}+\frac{a^{3}\sin\left(4dx+4c\right)}{32d}+\frac{a^{3}\sin\left(3dx+3c\right)}{4d}+\frac{a^{3}\sin\left(2dx+2c\right)}{d}$
derivativedivides	$\frac{a^{3}\sin\left(dx+c\right)+3a^{3}\left(\frac{\sin\left(dx+c\right)\cos\left(dx+c\right)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^{3}\left(2+\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)+a^{3}\left(\frac{\left(\cos\left(dx+c\right)^{3}+\frac{3\cos\left(dx+c\right)}{2}\right)\sin(\cdots}{4}\cdots\right.}{d}$
default	$\frac{a^{3}\sin\left(dx+c\right)+3a^{3}\left(\frac{\sin\left(dx+c\right)\cos\left(dx+c\right)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^{3}\left(2+\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)+a^{3}\left(\frac{\left(\cos\left(dx+c\right)^{3}+\frac{3\cos\left(dx+c\right)}{2}\right)\sin(\cdots}{4}\cdots\right.}{d}$
norman	$\frac{\frac{15a^{3}x}{8}+\frac{49a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{25a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{4d}-\frac{21a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{2d}-\frac{11a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{2d}+\frac{25a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{4d}+\frac{15a^{3}\tan\left(\frac{dx}{2}\cdots\right.}{4d}\cdots}{\left(1+\tan\left(\frac{dx}{2}+\cdots\right.\right.}$

input
```
int(cos(d*x+c)^4*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/32*a^3*(60*d*x+104*sin(d*x+c)+sin(4*d*x+4*c)+8*sin(3*d*x+3*c)+32*sin(2*d
*x+2*c))/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.74

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{15\,a^3dx+\left(2\,a^3\cos\left(dx+c\right)^3+8\,a^3\cos\left(dx+c\right)^2+15\,a^3\cos\left(dx+c\right)+24\,a^3\right)\sin\left(dx+c\right)}{8\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/8*(15*a^3*d*x + (2*a^3*cos(d*x + c)^3 + 8*a^3*cos(d*x + c)^2 + 15*a^3*co
s(d*x + c) + 24*a^3)*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{32\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^3-\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^3-24\,(2}{32\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/32*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 - (12*d*x + 12*c + sin(4*d
*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*
a^3 - 32*a^3*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.13

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{15\,(dx+c)a^3+\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+55\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+73\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+49\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{8\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/8*(15*(d*x + c)*a^3 + 2*(15*a^3*tan(1/2*d*x + 1/2*c)^7 + 55*a^3*tan(1/2*
d*x + 1/2*c)^5 + 73*a^3*tan(1/2*d*x + 1/2*c)^3 + 49*a^3*tan(1/2*d*x + 1/2*
c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 13.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{15\,a^3\,x}{8}+\frac{\frac{15\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{55\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\frac{73\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{49\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

input
```
int(cos(c + d*x)^4*(a + a/cos(c + d*x))^3,x)
```

output
```
(15*a^3*x)/8 + ((73*a^3*tan(c/2 + (d*x)/2)^3)/4 + (55*a^3*tan(c/2 + (d*x)/
2)^5)/4 + (15*a^3*tan(c/2 + (d*x)/2)^7)/4 + (49*a^3*tan(c/2 + (d*x)/2))/4)
/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.72

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{a^3\left(-2\cos\left(dx+c\right)\sin\left(dx+c\right)^3+17\cos\left(dx+c\right)\sin\left(dx+c\right)-8\sin\left(dx+c\right)^3+32\sin\left(dx+c\right)+15dx\right.}{8d}$$

input
```
int(cos(d*x+c)^4*(a+a*sec(d*x+c))^3,x)
```

output
```
(a**3*( - 2*cos(c + d*x)*sin(c + d*x)**3 + 17*cos(c + d*x)*sin(c + d*x) -
8*sin(c + d*x)**3 + 32*sin(c + d*x) + 15*d*x))/(8*d)
```

3.28 $\int \cos^5(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{13a^3x}{8} + \frac{4a^3\sin(c+dx)}{d} + \frac{13a^3\cos(c+dx)\sin(c+dx)}{8d} + \frac{3a^3\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{5a^3\sin^3(c+dx)}{3d} + \frac{a^3\sin^5(c+dx)}{5d}$$

output
```
13/8*a^3*x+4*a^3*sin(d*x+c)/d+13/8*a^3*cos(d*x+c)*sin(d*x+c)/d+3/4*a^3*cos
(d*x+c)^3*sin(d*x+c)/d-5/3*a^3*sin(d*x+c)^3/d+1/5*a^3*sin(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.60

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{a^3(780dx + 1380\sin(c+dx) + 480\sin(2(c+dx)) + 170\sin(3(c+dx)) + 45\sin(4(c+dx)) + 6\sin(5(c+$$
$$480d$$

input
```
Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
```

output
```
(a^3*(780*d*x + 1380*Sin[c + d*x] + 480*Sin[2*(c + d*x)] + 170*Sin[3*(c +
d*x)] + 45*Sin[4*(c + d*x)] + 6*Sin[5*(c + d*x)]))/(480*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^3\cos^5(c+dx)+3a^3\cos^4(c+dx)+3a^3\cos^3(c+dx)+a^3\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\sin^5(c+dx)}{5d}-\frac{5a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{13a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{13a^3x}{8}$$

input
```
Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
```

output
```
(13*a^3*x)/8 + (4*a^3*Sin[c + d*x])/d + (13*a^3*Cos[c + d*x]*Sin[c + d*x])
/(8*d) + (3*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (5*a^3*Sin[c + d*x]^3
)/(3*d) + (a^3*Sin[c + d*x]^5)/(5*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

method	result
parallelrisch	$\frac{a^{3}\left(780 d x+1380 \sin \left(d x+c\right)+6 \sin \left(5 d x+5 c\right)+45 \sin \left(4 d x+4 c\right)+170 \sin \left(3 d x+3 c\right)+480 \sin \left(2 d x+2 c\right)\right)}{480 d}$
risch	$\frac{13 a^{3} x}{8}+\frac{23 a^{3} \sin \left(d x+c\right)}{8 d}+\frac{a^{3} \sin \left(5 d x+5 c\right)}{80 d}+\frac{3 a^{3} \sin \left(4 d x+4 c\right)}{32 d}+\frac{17 a^{3} \sin \left(3 d x+3 c\right)}{48 d}+\frac{a^{3} \sin \left(2 d x+2 c\right)}{d}$
derivativedivides	$\frac{a^{3}\left(\frac{\sin \left(d x+c\right) \cos \left(d x+c\right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)+a^{3}\left(2+\cos \left(d x+c\right)^{2}\right) \sin \left(d x+c\right)+3 a^{3}\left(\frac{\left(\cos \left(d x+c\right)^{3}+\frac{3 \cos \left(d x+c\right)}{2}\right) \sin \left(d x+c\right)}{4}+\frac{3 d x}{8}+\frac{3c}{8}\right.}{d}$
default	$\frac{a^{3}\left(\frac{\sin \left(d x+c\right) \cos \left(d x+c\right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)+a^{3}\left(2+\cos \left(d x+c\right)^{2}\right) \sin \left(d x+c\right)+3 a^{3}\left(\frac{\left(\cos \left(d x+c\right)^{3}+\frac{3 \cos \left(d x+c\right)}{2}\right) \sin \left(d x+c\right)}{4}+\frac{3 d x}{8}+\frac{3c}{8}\right.}{d}$
norman	$\frac{\frac{13 a^{3} x}{8}+\frac{51 a^{3} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{4 d}-\frac{10 a^{3} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{3 d}-\frac{77 a^{3} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}{20 d}-\frac{272 a^{3} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{7}}{15 d}+\frac{13 a^{3} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{9}}{20 d}+\frac{26 a^{3} \tan \left(\frac{d}{}\right.}{3 d}}{}$

input
```
int(cos(d*x+c)^5*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/480*a^3*(780*d*x+1380*sin(d*x+c)+6*sin(5*d*x+5*c)+45*sin(4*d*x+4*c)+170*
sin(3*d*x+3*c)+480*sin(2*d*x+2*c))/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.72

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{195\,a^3dx+\left(24\,a^3\cos\left(dx+c\right)^4+90\,a^3\cos\left(dx+c\right)^3+152\,a^3\cos\left(dx+c\right)^2+195\,a^3\cos\left(dx+c\right)+304\,a}{120\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/120*(195*a^3*d*x + (24*a^3*cos(d*x + c)^4 + 90*a^3*cos(d*x + c)^3 + 152*
a^3*cos(d*x + c)^2 + 195*a^3*cos(d*x + c) + 304*a^3)*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.11

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{32\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\sin\left(dx+c\right)\right)a^3-480\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^3+45}{480\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^3 - 4
80*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 + 45*(12*d*x + 12*c + sin(4*d*x +
 4*c) + 8*sin(2*d*x + 2*c))*a^3 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3
)/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.07

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{195\,(dx+c)a^3 + \frac{2\left(195\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+910\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1664\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1330\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+765\,a^3\tan\left(\frac{1}{2}\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{120\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/120*(195*(d*x + c)*a^3 + 2*(195*a^3*tan(1/2*d*x + 1/2*c)^9 + 910*a^3*tan
(1/2*d*x + 1/2*c)^7 + 1664*a^3*tan(1/2*d*x + 1/2*c)^5 + 1330*a^3*tan(1/2*d
*x + 1/2*c)^3 + 765*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)
^5)/d
```

Mupad [B] (verification not implemented)

Time = 13.45 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{13\,a^3\,x}{8} + \frac{\frac{13\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+\frac{91\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{6}+\frac{416\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}+\frac{133\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6}+\frac{51\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^5}$$

input
```
int(cos(c + d*x)^5*(a + a/cos(c + d*x))^3,x)
```

output
```
(13*a^3*x)/8 + ((133*a^3*tan(c/2 + (d*x)/2)^3)/6 + (416*a^3*tan(c/2 + (d*x
)/2)^5)/15 + (91*a^3*tan(c/2 + (d*x)/2)^7)/6 + (13*a^3*tan(c/2 + (d*x)/2)^
9)/4 + (51*a^3*tan(c/2 + (d*x)/2))/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.68

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{a^3\left(-90\cos\left(dx+c\right)\sin\left(dx+c\right)^3+285\cos\left(dx+c\right)\sin\left(dx+c\right)+24\sin\left(dx+c\right)^5-200\sin\left(dx+c\right)^3+\right.}{120d}$$

input
```
int(cos(d*x+c)^5*(a+a*sec(d*x+c))^3,x)
```

output
```
(a**3*( - 90*cos(c + d*x)*sin(c + d*x)**3 + 285*cos(c + d*x)*sin(c + d*x)
+ 24*sin(c + d*x)**5 - 200*sin(c + d*x)**3 + 480*sin(c + d*x) + 195*d*x))/
(120*d)
```

3.29 $\int \cos^6(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 129

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{23a^3x}{16}+\frac{4a^3\sin(c+dx)}{d}+\frac{23a^3\cos(c+dx)\sin(c+dx)}{16d}+\frac{23a^3\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{a^3\cos^5(c+dx)\sin(c+dx)}{6d}-\frac{7a^3\sin^3(c+dx)}{3d}+\frac{3a^3\sin^5(c+dx)}{5d}$$

output
```
23/16*a^3*x+4*a^3*sin(d*x+c)/d+23/16*a^3*cos(d*x+c)*sin(d*x+c)/d+23/24*a^3
*cos(d*x+c)^3*sin(d*x+c)/d+1/6*a^3*cos(d*x+c)^5*sin(d*x+c)/d-7/3*a^3*sin(d
*x+c)^3/d+3/5*a^3*sin(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.57

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{a^3(1380dx+2520\sin(c+dx)+945\sin(2(c+dx))+380\sin(3(c+dx))+135\sin(4(c+dx))+36\sin(5}{960d}$$

input
```
Integrate[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```

output
```
(a^3*(1380*d*x + 2520*Sin[c + d*x] + 945*Sin[2*(c + d*x)] + 380*Sin[3*(c +
 d*x)] + 135*Sin[4*(c + d*x)] + 36*Sin[5*(c + d*x)] + 5*Sin[6*(c + d*x)]))
/(960*d)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^3\cos^6(c+dx)+3a^3\cos^5(c+dx)+3a^3\cos^4(c+dx)+a^3\cos^3(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3a^3\sin^5(c+dx)}{5d}-\frac{7a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{23a^3\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{23a^3\sin(c+dx)\cos(c+dx)}{16d}+\frac{23a^3x}{16}$$

input
```
Int[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```

output
```
(23*a^3*x)/16 + (4*a^3*Sin[c + d*x])/d + (23*a^3*Cos[c + d*x]*Sin[c + d*x]
)/(16*d) + (23*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a^3*Cos[c + d*x]
^5*Sin[c + d*x])/(6*d) - (7*a^3*Sin[c + d*x]^3)/(3*d) + (3*a^3*Sin[c + d*x
]^5)/(5*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.58

method	result
parallelrisch	$\frac{\left(276dx+\sin(6dx+6c)+504\sin(dx+c)+189\sin(2dx+2c)+76\sin(3dx+3c)+27\sin(4dx+4c)+\frac{36\sin(5dx+5c)}{5}\right)a^3}{192d}$
risch	$\frac{23a^3x}{16}+\frac{21a^3\sin(dx+c)}{8d}+\frac{a^3\sin(6dx+6c)}{192d}+\frac{3a^3\sin(5dx+5c)}{80d}+\frac{9a^3\sin(4dx+4c)}{64d}+\frac{19a^3\sin(3dx+3c)}{48d}+63$
derivativedivides	$\frac{\frac{a^3(2+\cos(dx+c)^2)\sin(dx+c)}{3}+3a^3\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{3a^3\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right.}{5}}{d}$
default	$\frac{\frac{a^3(2+\cos(dx+c)^2)\sin(dx+c)}{3}+3a^3\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{3a^3\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right.}{5}}{d}$
norman	$\frac{23a^3x}{16}+\frac{105a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8d}+\frac{353a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{40d}-\frac{1303a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{40d}-\frac{1339a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{120d}+\frac{989a^3\tan}{}$

input
```
int(cos(d*x+c)^6*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/192*(276*d*x+sin(6*d*x+6*c)+504*sin(d*x+c)+189*sin(2*d*x+2*c)+76*sin(3*d
*x+3*c)+27*sin(4*d*x+4*c)+36/5*sin(5*d*x+5*c))*a^3/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.69

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{345\,a^3dx+\left(40\,a^3\cos\left(dx+c\right)^5+144\,a^3\cos\left(dx+c\right)^4+230\,a^3\cos\left(dx+c\right)^3+272\,a^3\cos\left(dx+c\right)^2+345\right.}{240\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/240*(345*a^3*d*x + (40*a^3*cos(d*x + c)^5 + 144*a^3*cos(d*x + c)^4 + 230
*a^3*cos(d*x + c)^3 + 272*a^3*cos(d*x + c)^2 + 345*a^3*cos(d*x + c) + 544*
a^3)*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**6*(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.11

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{192\left(3\sin\left(dx+c\right)^5 - 10\sin\left(dx+c\right)^3 + 15\sin\left(dx+c\right)\right)a^3 - 5\left(4\sin\left(2dx+2c\right)^3 - 60dx - 60c - 9\text{ s}\right.}{}$$

input
```
integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/960*(192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^3 -
5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*
x + 2*c))*a^3 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 + 90*(12*d*x + 1
2*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3)/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.99

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{345\,(dx+c)a^3 + \frac{2\left(345\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+1955\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+4554\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+5814\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+3165\,a^3\tan\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^6}}{240\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/240*(345*(d*x + c)*a^3 + 2*(345*a^3*tan(1/2*d*x + 1/2*c)^11 + 1955*a^3*t
an(1/2*d*x + 1/2*c)^9 + 4554*a^3*tan(1/2*d*x + 1/2*c)^7 + 5814*a^3*tan(1/2
*d*x + 1/2*c)^5 + 3165*a^3*tan(1/2*d*x + 1/2*c)^3 + 1575*a^3*tan(1/2*d*x +
 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

Mupad [B] (verification not implemented)

Time = 12.47 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.94

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{23\,a^3\,x}{16} + \frac{\frac{23\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{391\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+\frac{759\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20}+\frac{969\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20}+\frac{211\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{8}+\frac{105\,a^3\tan\left(\right.}{8}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^6}$$

input
```
int(cos(c + d*x)^6*(a + a/cos(c + d*x))^3,x)
```

output
```
(23*a^3*x)/16 + ((211*a^3*tan(c/2 + (d*x)/2)^3)/8 + (969*a^3*tan(c/2 + (d*
x)/2)^5)/20 + (759*a^3*tan(c/2 + (d*x)/2)^7)/20 + (391*a^3*tan(c/2 + (d*x)
/2)^9)/24 + (23*a^3*tan(c/2 + (d*x)/2)^11)/8 + (105*a^3*tan(c/2 + (d*x)/2)
)/8)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.67

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{a^3\left(40\cos(dx+c)\sin(dx+c)^5-310\cos(dx+c)\sin(dx+c)^3+615\cos(dx+c)\sin(dx+c)+144\sin\right.}{240d}$$

input
```
int(cos(d*x+c)^6*(a+a*sec(d*x+c))^3,x)
```

output
```
(a**3*(40*cos(c + d*x)*sin(c + d*x)**5 - 310*cos(c + d*x)*sin(c + d*x)**3
+ 615*cos(c + d*x)*sin(c + d*x) + 144*sin(c + d*x)**5 - 560*sin(c + d*x)**
3 + 960*sin(c + d*x) + 345*d*x))/(240*d)
```

3.30 $\int \sec^3(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 136

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{49a^4\text{arctanh}(\sin(c+dx))}{16d} + \frac{8a^4\tan(c+dx)}{d} + \frac{49a^4\sec(c+dx)\tan(c+dx)}{16d} + \frac{41a^4\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{a^4\sec^5(c+dx)\tan(c+dx)}{6d} + \frac{4a^4\tan^3(c+dx)}{d} + \frac{4a^4\tan^5(c+dx)}{5d}$$

output
```
49/16*a^4*arctanh(sin(d*x+c))/d+8*a^4*tan(d*x+c)/d+49/16*a^4*sec(d*x+c)*ta
n(d*x+c)/d+41/24*a^4*sec(d*x+c)^3*tan(d*x+c)/d+1/6*a^4*sec(d*x+c)^5*tan(d*
x+c)/d+4*a^4*tan(d*x+c)^3/d+4/5*a^4*tan(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 3.91 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.07

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{49a^4\text{arctanh}(\sin(c+dx))}{16d}+\frac{36a^4\tan(c+dx)}{5d}+\frac{49a^4\sec(c+dx)\tan(c+dx)}{16d} + \frac{41a^4\sec^3(c+dx)\tan(c+dx)}{24d}+\frac{4a^4\sec^4(c+dx)\tan(c+dx)}{5d} + \frac{a^4\sec^5(c+dx)\tan(c+dx)}{6d}+\frac{12a^4\tan^3(c+dx)}{5d}$$

input
```
Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^4,x]
```

output
```
(49*a^4*ArcTanh[Sin[c + d*x]])/(16*d) + (36*a^4*Tan[c + d*x])/(5*d) + (49*
a^4*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (41*a^4*Sec[c + d*x]^3*Tan[c + d*x
])/(24*d) + (4*a^4*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + (a^4*Sec[c + d*x]^
5*Tan[c + d*x])/(6*d) + (12*a^4*Tan[c + d*x]^3)/(5*d)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\sec(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^4 \sec^7(c+dx) + 4a^4 \sec^6(c+dx) + 6a^4 \sec^5(c+dx) + 4a^4 \sec^4(c+dx) + a^4 \sec^3(c+dx)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{49a^4 \text{arctanh}(\sin(c+dx))}{16d} + \frac{4a^4 \tan^5(c+dx)}{5d} + \frac{4a^4 \tan^3(c+dx)}{d} + \frac{8a^4 \tan(c+dx)}{d} + \frac{a^4 \tan(c+dx) \sec^5(c+dx)}{6d} + \frac{41a^4 \tan(c+dx) \sec^3(c+dx)}{24d} + \frac{49a^4 \tan(c+dx) \sec(c+dx)}{16d}$$

input
```
Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^4,x]
```

output
```
(49*a^4*ArcTanh[Sin[c + d*x]])/(16*d) + (8*a^4*Tan[c + d*x])/d + (49*a^4*S
ec[c + d*x]*Tan[c + d*x])/(16*d) + (41*a^4*Sec[c + d*x]^3*Tan[c + d*x])/(2
4*d) + (a^4*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + (4*a^4*Tan[c + d*x]^3)/d
+ (4*a^4*Tan[c + d*x]^5)/(5*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.26

method	result
norman	$\frac{\frac{207a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}-\frac{1471a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{24d}+\frac{1967a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{20d}-\frac{1617a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{20d}+\frac{833a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{24d}-\frac{49a^4\tan\left(\frac{dx}{2}\right.}{8d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^6}$
risch	$-\frac{ia^4\left(735\,e^{11i(dx+c)}+3845\,e^{9i(dx+c)}-1920\,e^{8i(dx+c)}+3750\,e^{7i(dx+c)}-11520\,e^{6i(dx+c)}-3750\,e^{5i(dx+c)}-15360\,e^{4i(dx\right.}{120d\left(e^{2i(dx+c)}+1\right)^6}$
derivativedivides	$\frac{a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-4a^4\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+6a^4\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(c}{8}\right.\right.}{}$
default	$\frac{a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-4a^4\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+6a^4\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(c}{8}\right.\right.}{}$
parallelrisch	$\frac{a^4\left(735(-10-\cos(6dx+6c)-6\cos(4dx+4c)-15\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+735(\cos(6dx+6c)+6\cos(4dx+4c)\right.}{240d(c}$
parts	$\frac{a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{a^4\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\right.}{d}$

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
(207/8*a^4/d*tan(1/2*d*x+1/2*c)-1471/24*a^4/d*tan(1/2*d*x+1/2*c)^3+1967/20
*a^4/d*tan(1/2*d*x+1/2*c)^5-1617/20*a^4/d*tan(1/2*d*x+1/2*c)^7+833/24*a^4/
d*tan(1/2*d*x+1/2*c)^9-49/8*a^4/d*tan(1/2*d*x+1/2*c)^11)/(-1+tan(1/2*d*x+1
/2*c)^2)^6-49/16*a^4/d*ln(tan(1/2*d*x+1/2*c)-1)+49/16*a^4/d*ln(tan(1/2*d*x
+1/2*c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$=\frac{735\,a^4\cos(dx+c)^6\log(\sin(dx+c)+1)-735\,a^4\cos(dx+c)^6\log(-\sin(dx+c)+1)+2\left(1152\,a^4\cos\right.}{48}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/480*(735*a^4*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 735*a^4*cos(d*x + c)
^6*log(-sin(d*x + c) + 1) + 2*(1152*a^4*cos(d*x + c)^5 + 735*a^4*cos(d*x +
 c)^4 + 576*a^4*cos(d*x + c)^3 + 410*a^4*cos(d*x + c)^2 + 192*a^4*cos(d*x
+ c) + 40*a^4)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^4\,dx = a^4\left(\int \sec^3(c+dx)\,dx + \int 4\sec^4(c+dx)\,dx + \int 6\sec^5(c+dx)\,dx + \int 4\sec^6(c+dx)\,dx + \int \sec^7(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**4,x)
```

output
```
a**4*(Integral(sec(c + d*x)**3, x) + Integral(4*sec(c + d*x)**4, x) + Inte
gral(6*sec(c + d*x)**5, x) + Integral(4*sec(c + d*x)**6, x) + Integral(sec
(c + d*x)**7, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(126) = 252.

Time = 0.04 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.99

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$= \frac{128\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)a^4+640\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^4 -}{}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/480*(128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^4 +
640*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 - 5*a^4*(2*(15*sin(d*x + c)^5 -
40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 +
3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1
)) - 180*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*si
n(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 1
20*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(
sin(d*x + c) - 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.13

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^4\,dx$$

$$= \frac{735\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-735\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(735\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-4165\,a^4\tan(}{}}{240\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/240*(735*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 735*a^4*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) - 2*(735*a^4*tan(1/2*d*x + 1/2*c)^11 - 4165*a^4*tan(1
/2*d*x + 1/2*c)^9 + 9702*a^4*tan(1/2*d*x + 1/2*c)^7 - 11802*a^4*tan(1/2*d*
x + 1/2*c)^5 + 7355*a^4*tan(1/2*d*x + 1/2*c)^3 - 3105*a^4*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

Mupad [B] (verification not implemented)

Time = 13.56 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.46

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{49\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}$$

$$-\frac{\frac{49\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}-\frac{833\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+\frac{1617\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20}-\frac{1967\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20}+\frac{1471\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24}-2}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-6\,t\right.}$$

input
```
int((a + a/cos(c + d*x))^4/cos(c + d*x)^3,x)
```

output
```
(49*a^4*atanh(tan(c/2 + (d*x)/2)))/(8*d) - ((1471*a^4*tan(c/2 + (d*x)/2)^3
)/24 - (1967*a^4*tan(c/2 + (d*x)/2)^5)/20 + (1617*a^4*tan(c/2 + (d*x)/2)^7
)/20 - (833*a^4*tan(c/2 + (d*x)/2)^9)/24 + (49*a^4*tan(c/2 + (d*x)/2)^11)/
8 - (207*a^4*tan(c/2 + (d*x)/2))/8)/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/
2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan
(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.02

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$= \frac{a^4\left(-1152\cos\left(dx+c\right)\sin\left(dx+c\right)^5+2880\cos\left(dx+c\right)\sin\left(dx+c\right)^3-1920\cos\left(dx+c\right)\sin\left(dx+c\right)-7\right.}{}$$

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^4,x)
```

output
```
(a**4*( - 1152*cos(c + d*x)*sin(c + d*x)**5 + 2880*cos(c + d*x)*sin(c + d*
x)**3 - 1920*cos(c + d*x)*sin(c + d*x) - 735*log(tan((c + d*x)/2) - 1)*sin
(c + d*x)**6 + 2205*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4 - 2205*log(t
an((c + d*x)/2) - 1)*sin(c + d*x)**2 + 735*log(tan((c + d*x)/2) - 1) + 735
*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**6 - 2205*log(tan((c + d*x)/2) + 1
)*sin(c + d*x)**4 + 2205*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2 - 735*l
og(tan((c + d*x)/2) + 1) - 735*sin(c + d*x)**5 + 1880*sin(c + d*x)**3 - 11
85*sin(c + d*x)))/(240*d*(sin(c + d*x)**6 - 3*sin(c + d*x)**4 + 3*sin(c +
d*x)**2 - 1))
```

3.31 $\int \sec^2(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{7a^4\text{arctanh}(\sin(c+dx))}{2d} + \frac{8a^4\tan(c+dx)}{d} + \frac{7a^4\sec(c+dx)\tan(c+dx)}{2d} + \frac{a^4\sec^3(c+dx)\tan(c+dx)}{d} + \frac{8a^4\tan^3(c+dx)}{3d} + \frac{a^4\tan^5(c+dx)}{5d}$$

output

```
7/2*a^4*arctanh(sin(d*x+c))/d+8*a^4*tan(d*x+c)/d+7/2*a^4*sec(d*x+c)*tan(d*
x+c)/d+a^4*sec(d*x+c)^3*tan(d*x+c)/d+8/3*a^4*tan(d*x+c)^3/d+1/5*a^4*tan(d*
x+c)^5/d
```

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.13

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{7a^4\text{arctanh}(\sin(c+dx))}{2d} + \frac{83a^4\tan(c+dx)}{15d} + \frac{7a^4\sec(c+dx)\tan(c+dx)}{2d} + \frac{34a^4\sec^2(c+dx)\tan(c+dx)}{15d} + \frac{a^4\sec^3(c+dx)\tan(c+dx)}{d} + \frac{a^4\sec^4(c+dx)\tan(c+dx)}{5d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^4,x]
```

output
```
(7*a^4*ArcTanh[Sin[c + d*x]])/(2*d) + (83*a^4*Tan[c + d*x])/(15*d) + (7*a^
4*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (34*a^4*Sec[c + d*x]^2*Tan[c + d*x])/
(15*d) + (a^4*Sec[c + d*x]^3*Tan[c + d*x])/d + (a^4*Sec[c + d*x]^4*Tan[c +
 d*x])/(5*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\sec(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4dx$$

$$\Big\downarrow \text{ 4278}$$

$$\int \left(a^4 \sec^6(c+dx) + 4a^4 \sec^5(c+dx) + 6a^4 \sec^4(c+dx) + 4a^4 \sec^3(c+dx) + a^4 \sec^2(c+dx)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{7a^4 \text{arctanh}(\sin(c+dx))}{2d} + \frac{a^4 \tan^5(c+dx)}{5d} + \frac{8a^4 \tan^3(c+dx)}{3d} + \frac{8a^4 \tan(c+dx)}{d} + \frac{a^4 \tan(c+dx) \sec^3(c+dx)}{d} + \frac{7a^4 \tan(c+dx) \sec(c+dx)}{2d}$$

input
```
Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^4,x]
```

output
```
(7*a^4*ArcTanh[Sin[c + d*x]])/(2*d) + (8*a^4*Tan[c + d*x])/d + (7*a^4*Sec[
c + d*x]*Tan[c + d*x])/(2*d) + (a^4*Sec[c + d*x]^3*Tan[c + d*x])/d + (8*a^
4*Tan[c + d*x]^3)/(3*d) + (a^4*Tan[c + d*x]^5)/(5*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.37

method	result
norman	$\frac{-\frac{25a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{158a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{896a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{15d}+\frac{98a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{3d}-\frac{7a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}-\frac{7a^4\ln\left(\tan\left(\frac{d}{2}\right.\right.}{2d}$
risch	$-\frac{ia^4\left(105\,\mathrm{e}^{9i(dx+c)}-30\,\mathrm{e}^{8i(dx+c)}+330\,\mathrm{e}^{7i(dx+c)}-480\,\mathrm{e}^{6i(dx+c)}-1180\,\mathrm{e}^{4i(dx+c)}-330\,\mathrm{e}^{3i(dx+c)}-800\,\mathrm{e}^{2i(dx+c)}-105\,\mathrm{e}\right.}{15d\left(\mathrm{e}^{2i(dx+c)}+1\right)^5}$
derivativedivides	$\frac{a^4\tan(dx+c)+4a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-6a^4\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+4a^4\left(-\left(-\frac{\mathrm{se}}{}\right.\right.}{d}$
default	$\frac{a^4\tan(dx+c)+4a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-6a^4\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+4a^4\left(-\left(-\frac{\mathrm{se}}{}\right.\right.}{d}$
parts	$\frac{a^4\tan(dx+c)}{d}-\frac{a^4\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}+\frac{2a^4\sec(dx+c)\tan(dx+c)}{d}+\frac{2a^4\ln(\sec(dx+c}{d}$
parallelrisch	$\frac{77a^4\left(\left(-\frac{15\cos(dx+c)}{11}-\frac{15\cos(3dx+3c)}{22}-\frac{3\cos(5dx+5c)}{22}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\left(\frac{15\cos(dx+c)}{11}+\frac{15\cos(3dx+3c)}{22}+\frac{3\cos(5dx-}{22}\right.\right.}{3d(\cos(5dx+5c)+5\cos(3dx+3c)}$

input
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
(-25*a^4/d*tan(1/2*d*x+1/2*c)+158/3*a^4/d*tan(1/2*d*x+1/2*c)^3-896/15*a^4/
d*tan(1/2*d*x+1/2*c)^5+98/3*a^4/d*tan(1/2*d*x+1/2*c)^7-7*a^4/d*tan(1/2*d*x
+1/2*c)^9)/(-1+tan(1/2*d*x+1/2*c)^2)^5-7/2*a^4/d*ln(tan(1/2*d*x+1/2*c)-1)+
7/2*a^4/d*ln(tan(1/2*d*x+1/2*c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.12

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$=\frac{105\,a^4\cos(dx+c)^5\log(\sin(dx+c)+1)-105\,a^4\cos(dx+c)^5\log(-\sin(dx+c)+1)+2\,(166\,a^4\cos(}{60\,d\cos(dx+}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/60*(105*a^4*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 105*a^4*cos(d*x + c)^
5*log(-sin(d*x + c) + 1) + 2*(166*a^4*cos(d*x + c)^4 + 105*a^4*cos(d*x + c
)^3 + 68*a^4*cos(d*x + c)^2 + 30*a^4*cos(d*x + c) + 6*a^4)*sin(d*x + c))/(
d*cos(d*x + c)^5)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^4\,dx = a^4\left(\int \sec^2(c+dx)\,dx + \int 4\sec^3(c+dx)\,dx + \int 6\sec^4(c+dx)\,dx + \int 4\sec^5(c+dx)\,dx + \int \sec^6(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**4,x)
```

output
```
a**4*(Integral(sec(c + d*x)**2, x) + Integral(4*sec(c + d*x)**3, x) + Inte
gral(6*sec(c + d*x)**4, x) + Integral(4*sec(c + d*x)**5, x) + Integral(sec
(c + d*x)**6, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.71

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{4\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)a^4+120\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^4-1}{}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/60*(4*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^4 + 120
*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 - 15*a^4*(2*(3*sin(d*x + c)^3 - 5*s
in(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
+ 1) + 3*log(sin(d*x + c) - 1)) - 60*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 -
 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 60*a^4*tan(d*x + c)
)/d
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.24

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^4\,dx$$

$$= \frac{105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-490\,a^4\tan\left(\frac{1}{2}\right.\right.}{}}{30\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/30*(105*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*a^4*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 2*(105*a^4*tan(1/2*d*x + 1/2*c)^9 - 490*a^4*tan(1/2*
d*x + 1/2*c)^7 + 896*a^4*tan(1/2*d*x + 1/2*c)^5 - 790*a^4*tan(1/2*d*x + 1/
2*c)^3 + 375*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 14.35 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.53

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{7\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{7\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-\frac{98\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}+\frac{896\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}-\frac{158\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+25\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + a/cos(c + d*x))^4/cos(c + d*x)^2,x)
```

output
```
(7*a^4*atanh(tan(c/2 + (d*x)/2)))/d - ((896*a^4*tan(c/2 + (d*x)/2)^5)/15 -
 (158*a^4*tan(c/2 + (d*x)/2)^3)/3 - (98*a^4*tan(c/2 + (d*x)/2)^7)/3 + 7*a^
4*tan(c/2 + (d*x)/2)^9 + 25*a^4*tan(c/2 + (d*x)/2))/(d*(5*tan(c/2 + (d*x)/
2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*
x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.24

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$= \frac{a^4\left(-105\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^4+210\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(d\right.\right.}{}$$

input
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c))^4,x)
```

output
```
(a**4*( - 105*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4 + 210
*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 - 105*cos(c + d*x)
*log(tan((c + d*x)/2) - 1) + 105*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*si
n(c + d*x)**4 - 210*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2
 + 105*cos(c + d*x)*log(tan((c + d*x)/2) + 1) - 105*cos(c + d*x)*sin(c + d
*x)**3 + 135*cos(c + d*x)*sin(c + d*x) + 166*sin(c + d*x)**5 - 400*sin(c +
 d*x)**3 + 240*sin(c + d*x)))/(30*cos(c + d*x)*d*(sin(c + d*x)**4 - 2*sin(
c + d*x)**2 + 1))
```

3.32 $\int \sec(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal result

Integrand size = 19, antiderivative size = 96

$$\int \sec(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{35a^4\operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{8a^4\tan(c+dx)}{d} + \frac{27a^4\sec(c+dx)\tan(c+dx)}{8d} + \frac{a^4\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{4a^4\tan^3(c+dx)}{3d}$$

output
```
35/8*a^4*arctanh(sin(d*x+c))/d+8*a^4*tan(d*x+c)/d+27/8*a^4*sec(d*x+c)*tan(
d*x+c)/d+1/4*a^4*sec(d*x+c)^3*tan(d*x+c)/d+4/3*a^4*tan(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.15

$$\int \sec(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{a^4\coth^{-1}(\sin(c+dx))}{d} + \frac{27a^4\text{arctanh}(\sin(c+dx))}{8d} + \frac{8a^4\tan(c+dx)}{d} + \frac{27a^4\sec(c+dx)\tan(c+dx)}{8d} + \frac{a^4\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{4a^4\tan^3(c+dx)}{3d}$$

input
```
Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^4,x]
```

output
```
(a^4*ArcCoth[Sin[c + d*x]])/d + (27*a^4*ArcTanh[Sin[c + d*x]])/(8*d) + (8*
a^4*Tan[c + d*x])/d + (27*a^4*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^4*Sec[
c + d*x]^3*Tan[c + d*x])/(4*d) + (4*a^4*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\sec(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^4 \sec^5(c+dx) + 4a^4 \sec^4(c+dx) + 6a^4 \sec^3(c+dx) + 4a^4 \sec^2(c+dx) + a^4 \sec(c+dx)\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{35a^4\text{arctanh}(\sin(c+dx))}{8d} + \frac{4a^4\tan^3(c+dx)}{3d} + \frac{8a^4\tan(c+dx)}{d} + \frac{a^4\tan(c+dx)\sec^3(c+dx)}{4d} + \frac{27a^4\tan(c+dx)\sec(c+dx)}{8d}$$

input
```
Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^4,x]
```

output
```
(35*a^4*ArcTanh[Sin[c + d*x]])/(8*d) + (8*a^4*Tan[c + d*x])/d + (27*a^4*Se
c[c + d*x]*Tan[c + d*x])/(8*d) + (a^4*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) +
 (4*a^4*Tan[c + d*x]^3)/(3*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.39

method	result
norman	$\frac{\frac{93a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{511a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12d}+\frac{385a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{12d}-\frac{35a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}-\frac{35a^4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8d}+\frac{35a^4}{}$
parts	$\frac{4a^4\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^4\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}+4a$
derivativedivides	$\frac{a^4\ln(\sec(dx+c)+\tan(dx+c))+4a^4\tan(dx+c)+6a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-4a^4\left(-\frac{2}{3}-\frac{\sec(dx}{3}\right.}{d}$
default	$\frac{a^4\ln(\sec(dx+c)+\tan(dx+c))+4a^4\tan(dx+c)+6a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-4a^4\left(-\frac{2}{3}-\frac{\sec(dx}{3}\right.}{d}$
risch	$-\frac{ia^4\left(81\,e^{7i(dx+c)}-96\,e^{6i(dx+c)}+105\,e^{5i(dx+c)}-480\,e^{4i(dx+c)}-105\,e^{3i(dx+c)}-544\,e^{2i(dx+c)}-81\,e^{i(dx+c)}-160\right)}{12d\left(e^{2i(dx+c)}+1\right)^4}+$
parallelrisch	$\frac{a^4\left(105(-3-\cos(4dx+4c)-4\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+105(3+\cos(4dx+4c)+4\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{24d(3+\cos(4dx+4c)+4\cos(2dx+2c))}$

input
```
int(sec(d*x+c)*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
(93/4*a^4/d*tan(1/2*d*x+1/2*c)-511/12*a^4/d*tan(1/2*d*x+1/2*c)^3+385/12*a^
4/d*tan(1/2*d*x+1/2*c)^5-35/4*a^4/d*tan(1/2*d*x+1/2*c)^7)/(-1+tan(1/2*d*x+
1/2*c)^2)^4-35/8*a^4/d*ln(tan(1/2*d*x+1/2*c)-1)+35/8*a^4/d*ln(tan(1/2*d*x+
1/2*c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.16

$$\int \sec(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$=\frac{105\,a^4\cos(dx+c)^4\log(\sin(dx+c)+1)-105\,a^4\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\,(160\,a^4\cos(}{48\,d\cos(dx+c)^4}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/48*(105*a^4*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 105*a^4*cos(d*x + c)^
4*log(-sin(d*x + c) + 1) + 2*(160*a^4*cos(d*x + c)^3 + 81*a^4*cos(d*x + c)
^2 + 32*a^4*cos(d*x + c) + 6*a^4)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^4\,dx = a^4\left(\int \sec(c+dx)\,dx + \int 4\sec^2(c+dx)\,dx + \int 6\sec^3(c+dx)\,dx + \int 4\sec^4(c+dx)\,dx + \int \sec^5(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))**4,x)
```

output
```
a**4*(Integral(sec(c + d*x), x) + Integral(4*sec(c + d*x)**2, x) + Integra
l(6*sec(c + d*x)**3, x) + Integral(4*sec(c + d*x)**4, x) + Integral(sec(c
+ d*x)**5, x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.82

$$\int \sec(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{64\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^4 - 3a^4\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1} - 3\log(\sin(dx+c)+1) + 3\log\right.}{}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/48*(64*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 - 3*a^4*(2*(3*sin(d*x + c)^
3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*
x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 72*a^4*(2*sin(d*x + c)/(sin(d*x +
 c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*a^4*log(s
ec(d*x + c) + tan(d*x + c)) + 192*a^4*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.27

$$\int \sec(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-385\,a^4\tan\left(\frac{1}{2}\right.\right.}{\left(\mathrm{t}\right.}}{24\,d}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/24*(105*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*a^4*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 2*(105*a^4*tan(1/2*d*x + 1/2*c)^7 - 385*a^4*tan(1/2*
d*x + 1/2*c)^5 + 511*a^4*tan(1/2*d*x + 1/2*c)^3 - 279*a^4*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 12.99 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.47

$$\int \sec(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{35\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d} - \frac{\frac{35\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}-\frac{385\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}+\frac{511\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}-\frac{93\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

input
```
int((a + a/cos(c + d*x))^4/cos(c + d*x),x)
```

output
```
(35*a^4*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((511*a^4*tan(c/2 + (d*x)/2)^3)
/12 - (385*a^4*tan(c/2 + (d*x)/2)^5)/12 + (35*a^4*tan(c/2 + (d*x)/2)^7)/4
- (93*a^4*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 +
(d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.03

$$\int \sec(c+dx)(a+a\sec(c+dx))^4\,dx$$

$$= \frac{a^4\left(-160\cos\left(dx+c\right)\sin\left(dx+c\right)^3+192\cos\left(dx+c\right)\sin\left(dx+c\right)-105\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+\right.\right.}{}$$

input
```
int(sec(d*x+c)*(a+a*sec(d*x+c))^4,x)
```

output
```
(a**4*( - 160*cos(c + d*x)*sin(c + d*x)**3 + 192*cos(c + d*x)*sin(c + d*x)
 - 105*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4 + 210*log(tan((c + d*x)/2
) - 1)*sin(c + d*x)**2 - 105*log(tan((c + d*x)/2) - 1) + 105*log(tan((c +
d*x)/2) + 1)*sin(c + d*x)**4 - 210*log(tan((c + d*x)/2) + 1)*sin(c + d*x)*
*2 + 105*log(tan((c + d*x)/2) + 1) - 81*sin(c + d*x)**3 + 87*sin(c + d*x))
)/(24*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.33 $\int (a+a\sec(c+dx))^4\,dx$

Optimal result

Integrand size = 12, antiderivative size = 91

$$\int (a+a\sec(c+dx))^4\,dx = a^4x+\frac{6a^4\text{arctanh}(\sin(c+dx))}{d}+\frac{5a^4\tan(c+dx)}{d}+\frac{(a^2+a^2\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{4(a^4+a^4\sec(c+dx))\tan(c+dx)}{3d}$$

output
```
a^4*x+6*a^4*arctanh(sin(d*x+c))/d+5*a^4*tan(d*x+c)/d+1/3*(a^2+a^2*sec(d*x+
c))^2*tan(d*x+c)/d+4/3*(a^4+a^4*sec(d*x+c))*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97

$$\int (a+a\sec(c+dx))^4\,dx = a^4x+\frac{4a^4\coth^{-1}(\sin(c+dx))}{d}+\frac{2a^4\text{arctanh}(\sin(c+dx))}{d}+\frac{7a^4\tan(c+dx)}{d}+\frac{2a^4\sec(c+dx)\tan(c+dx)}{d}+\frac{a^4\tan^3(c+dx)}{3d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^4,x]
```

output
```
a^4*x + (4*a^4*ArcCoth[Sin[c + d*x]])/d + (2*a^4*ArcTanh[Sin[c + d*x]])/d
+ (7*a^4*Tan[c + d*x])/d + (2*a^4*Sec[c + d*x]*Tan[c + d*x])/d + (a^4*Tan[
c + d*x]^3)/(3*d)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {3042, 4262, 3042, 4405, 27, 3042, 4402, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^4dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4dx$$

$$\Big\downarrow \text{4262}$$

$$\frac{1}{3}a\int (\sec(c+dx)a+a)^2(8\sec(c+dx)a+3a)dx+\frac{\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}a\int \left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left(8\csc\left(c+dx+\frac{\pi}{2}\right)a+3a\right)dx+\frac{\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{3d}$$

$$\Big\downarrow \text{4405}$$

$$\frac{1}{3}a\left(\frac{1}{2}\int 6(\sec(c+dx)a+a)\left(5\sec(c+dx)a^2+a^2\right)dx+\frac{4\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{d}\right)+\frac{\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{3d}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}a\left(3\int (\sec(c+dx)a+a)\left(5\sec(c+dx)a^2+a^2\right)dx+\frac{4\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{d}\right)+\frac{\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{3d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{3}a\left(3\int \left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(5\csc\left(c+dx+\frac{\pi}{2}\right)a^2+a^2\right)dx+\frac{4\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{d}\right)+\frac{\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{3d}$$

$$\Big\downarrow \ 4402$$

$$\frac{1}{3}a\left(3\left(5a^3\int \sec^2(c+dx)dx+6a^3\int \sec(c+dx)dx+a^3x\right)+\frac{4\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{d}\right)+\frac{\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{3d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{3}a\left(3\left(6a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+5a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+a^3x\right)+\frac{4\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{d}\right)+\frac{\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{3d}$$

$$\Big\downarrow \ 4254$$

$$\frac{1}{3}a\left(3\left(-\frac{5a^3\int 1d(-\tan(c+dx))}{d}+6a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+a^3x\right)+\frac{4\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{d}\right)+\frac{\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{3d}$$

$$\Big\downarrow \ 24$$

$$\frac{1}{3}a\left(3\left(6a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{5a^3\tan(c+dx)}{d}+a^3x\right)+\frac{4\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{d}\right)+\frac{\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{3d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{3}a\left(3\left(\frac{6a^3\text{arctanh}(\sin(c+dx))}{d}+\frac{5a^3\tan(c+dx)}{d}+a^3x\right)+\frac{4\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{d}\right)+\frac{\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{3d}$$

input
```
Int[(a + a*Sec[c + d*x])^4,x]
```

output
```
((a^2 + a^2*Sec[c + d*x])^2*Tan[c + d*x])/(3*d) + (a*((4*(a^3 + a^3*Sec[c
+ d*x])*Tan[c + d*x])/d + 3*(a^3*x + (6*a^3*ArcTanh[Sin[c + d*x]])/d + (5*
a^3*Tan[c + d*x])/d)))/3
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4262
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[a/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Inte
gerQ[2*n]
```

rule 4402
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Simp[b*d   Int[Csc[e + f*x]^2, x], x
] + Simp[(b*c + a*d)   Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4405
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Simp[1/m   Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m +
 (b*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2
*m]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98

method	result
parts	$a^4x-\frac{a^4\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{6a^4\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{6a^4\tan(dx+c)}{d}+\frac{2a^4\sec(dx+c)\,\mathrm{ta}}{d}$
derivativedivides	$\frac{a^4(dx+c)+4a^4\ln(\sec(dx+c)+\tan(dx+c))+6a^4\tan(dx+c)+4a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-a^4\big(}{d}$
default	$\frac{a^4(dx+c)+4a^4\ln(\sec(dx+c)+\tan(dx+c))+6a^4\tan(dx+c)+4a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-a^4\big(}{d}$
risch	$a^4x-\frac{4ia^4\left(3\,\mathrm{e}^{5i(dx+c)}-9\,\mathrm{e}^{4i(dx+c)}-21\,\mathrm{e}^{2i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}-10\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}-\frac{6a^4\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}+\frac{6a^4\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}$
parallelrisch	$-\frac{18\left(\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\left(-\cos(dx+c)-\frac{\cos(3dx+3c)}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{dx\cos(dx+}{6}\right.}{d(\cos(3dx+3c)+3\cos(dx+c))}$
norman	$\frac{a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6-a^4x-\frac{18a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{76a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{10a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}+3a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-3a^4x\tan\left(\frac{dx}{2}\right.}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}$

input
```
int((a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
a^4*x-a^4/d*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+6*a^4/d*ln(sec(d*x+c)+tan(d
*x+c))+6*a^4*tan(d*x+c)/d+2*a^4*sec(d*x+c)*tan(d*x+c)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.21

$$\int (a + a\sec(c + dx))^4 \, dx = \frac{3\,a^4 dx \cos\left(dx + c\right)^3 + 9\,a^4 \cos\left(dx + c\right)^3 \log\left(\sin\left(dx + c\right) + 1\right) - 9\,a^4 \cos\left(dx + c\right)^3 \log\left(-\sin\left(dx + c\right) + 1}{3\,d \cos\left(dx + c\right)^3}$$

input
```
integrate((a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/3*(3*a^4*d*x*cos(d*x + c)^3 + 9*a^4*cos(d*x + c)^3*log(sin(d*x + c) + 1)
 - 9*a^4*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + (20*a^4*cos(d*x + c)^2 +
6*a^4*cos(d*x + c) + a^4)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int (a + a\sec(c + dx))^4 \, dx = a^4 \left(\int 1 \, dx + \int 4\sec\left(c + dx\right) dx + \int 6\sec^2\left(c + dx\right) dx + \int 4\sec^3\left(c + dx\right) dx + \int \sec^4\left(c + dx\right) dx \right)$$

input
```
integrate((a+a*sec(d*x+c))**4,x)
```

output
```
a**4*(Integral(1, x) + Integral(4*sec(c + d*x), x) + Integral(6*sec(c + d*
x)**2, x) + Integral(4*sec(c + d*x)**3, x) + Integral(sec(c + d*x)**4, x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27

$$\int (a + a\sec(c + dx))^4 \, dx$$
$$= a^4x + \frac{\left(\tan\left(dx + c\right)^3 + 3\,\tan\left(dx + c\right)\right)a^4}{3\,d} - \frac{a^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx + c\right) + 1\right) + \log\left(\sin\left(dx + c\right) - 1\right)\right)}{d} + \frac{4\,a^4\log\left(\sec\left(dx + c\right) + \tan\left(dx + c\right)\right)}{d} + \frac{6\,a^4\tan\left(dx + c\right)}{d}$$

input
```
integrate((a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
a^4*x + 1/3*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4/d - a^4*(2*sin(d*x + c)/
(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1))/d +
4*a^4*log(sec(d*x + c) + tan(d*x + c))/d + 6*a^4*tan(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27

$$\int (a + a\sec(c + dx))^4 \, dx$$
$$= \frac{3\left(dx + c\right)a^4 + 18\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 18\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{}}{3\,d}$$

input
```
integrate((a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(3*(d*x + c)*a^4 + 18*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 18*a^4*
log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(15*a^4*tan(1/2*d*x + 1/2*c)^5 - 38
*a^4*tan(1/2*d*x + 1/2*c)^3 + 27*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
1/2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 10.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.29

$$\int (a + a\sec(c + dx))^4 \, dx$$
$$= a^4 x + \frac{12 a^4 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{d x}{2}\right)\right)}{d} - \frac{10 a^4 \tan\left(\frac{c}{2} + \frac{d x}{2}\right)^5 - \frac{76 a^4 \tan\left(\frac{c}{2} + \frac{d x}{2}\right)^3}{3} + 18 a^4 \tan\left(\frac{c}{2} + \frac{d x}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{d x}{2}\right)^6 - 3 \tan\left(\frac{c}{2} + \frac{d x}{2}\right)^4 + 3 \tan\left(\frac{c}{2} + \frac{d x}{2}\right)^2 - 1\right)}$$

input
```
int((a + a/cos(c + d*x))^4,x)
```

output
```
a^4*x + (12*a^4*atanh(tan(c/2 + (d*x)/2)))/d - (10*a^4*tan(c/2 + (d*x)/2)^
5 - (76*a^4*tan(c/2 + (d*x)/2)^3)/3 + 18*a^4*tan(c/2 + (d*x)/2))/(d*(3*tan
(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.03

$$\int (a + a\sec(c + dx))^4 \, dx$$
$$= \frac{a^4 \left(-18 \cos(dx+c) \log\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) \sin(dx+c)^2 + 18 \cos(dx+c) \log\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) + 18 \cos\right.}{}$$

input
```
int((a+a*sec(d*x+c))^4,x)
```

output
```
(a**4*( - 18*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 + 18*c
os(c + d*x)*log(tan((c + d*x)/2) - 1) + 18*cos(c + d*x)*log(tan((c + d*x)/
2) + 1)*sin(c + d*x)**2 - 18*cos(c + d*x)*log(tan((c + d*x)/2) + 1) + 3*co
s(c + d*x)*sin(c + d*x)**2*d*x - 6*cos(c + d*x)*sin(c + d*x) - 3*cos(c + d
*x)*d*x + 20*sin(c + d*x)**3 - 21*sin(c + d*x)))/(3*cos(c + d*x)*d*(sin(c
+ d*x)**2 - 1))
```

3.34 $\int \cos(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int \cos(c+dx)(a+a\sec(c+dx))^4\,dx = 4a^4x + \frac{13a^4\operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{a^4\sin(c+dx)}{d} + \frac{4a^4\tan(c+dx)}{d} + \frac{a^4\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
4*a^4*x+13/2*a^4*arctanh(sin(d*x+c))/d+a^4*sin(d*x+c)/d+4*a^4*tan(d*x+c)/d
+1/2*a^4*sec(d*x+c)*tan(d*x+c)/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 272 vs. $2(73) = 146$.

Time = 3.60 (sec) , antiderivative size = 272, normalized size of antiderivative = 3.73

$$\int \cos(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{1}{64}a^4(1+\cos(c+dx))^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(16x - \frac{26\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{26\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{4\cos(dx)\sin(c)}{d} + \frac{4\cos(c)\sin(dx)}{d} + \frac{1}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{16\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} - \frac{1}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{16\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)$$

input
```
Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^4,x]
```

output
```
(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(16*x - (26*Log[Cos[(c + d*x)
/2] - Sin[(c + d*x)/2]])/d + (26*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])
/d + (4*Cos[d*x]*Sin[c])/d + (4*Cos[c]*Sin[d*x])/d + 1/(d*(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])^2) + (16*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos
[(c + d*x)/2] - Sin[(c + d*x)/2])) - 1/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^2) + (16*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]))))/64
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\sec(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^4\cos(c+dx)+a^4\sec^3(c+dx)+4a^4\sec^2(c+dx)+6a^4\sec(c+dx)+4a^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{13a^4\text{arctanh}(\sin(c+dx))}{2d}+\frac{a^4\sin(c+dx)}{d}+\frac{4a^4\tan(c+dx)}{d}+\frac{a^4\tan(c+dx)\sec(c+dx)}{2d}+4a^4x$$

input
```
Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^4,x]
```

output
```
4*a^4*x + (13*a^4*ArcTanh[Sin[c + d*x]])/(2*d) + (a^4*Sin[c + d*x])/d + (4
*a^4*Tan[c + d*x])/d + (a^4*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+4a^4\tan(dx+c)+6a^4\ln(\sec(dx+c)+\tan(dx+c))+4a^4(dx+c)+a^4 \mathrm{s}}{d}$
default	$\frac{a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+4a^4\tan(dx+c)+6a^4\ln(\sec(dx+c)+\tan(dx+c))+4a^4(dx+c)+a^4 \mathrm{s}}{d}$
parallelrisch	$\frac{a^4\left(-13(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+8dx\cos(2dx+2c)+13\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(2dx+2c)+8dx+13\ln\left(\right.\right.}{2d(1+\cos(2dx+2c))}$
risch	$4a^4x-\frac{ia^4\mathrm{e}^{i(dx+c)}}{2d}+\frac{ia^4\mathrm{e}^{-i(dx+c)}}{2d}-\frac{ia^4\left(\mathrm{e}^{3i(dx+c)}-8\,\mathrm{e}^{2i(dx+c)}-\mathrm{e}^{i(dx+c)}-8\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}+\frac{13a^4\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2d}-\frac{13}{}$
norman	$\frac{-4a^4x-\frac{11a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{13a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}+\frac{3a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}-\frac{5a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}+8a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-8a^4x\tan\left(\frac{d}{2}\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}$

input
```
int(cos(d*x+c)*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+4*a^4*t
an(d*x+c)+6*a^4*ln(sec(d*x+c)+tan(d*x+c))+4*a^4*(d*x+c)+a^4*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.52

$$\int \cos(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$=\frac{16\,a^4dx\cos\left(dx+c\right)^2+13\,a^4\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-13\,a^4\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)\right.}{4\,d\cos\left(dx+c\right)^2}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/4*(16*a^4*d*x*cos(d*x + c)^2 + 13*a^4*cos(d*x + c)^2*log(sin(d*x + c) +
1) - 13*a^4*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*a^4*cos(d*x + c)^
2 + 8*a^4*cos(d*x + c) + a^4)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \cos(c+dx)(a+a\sec(c+dx))^4\,dx = a^4\left(\int 4\cos(c+dx)\sec(c+dx)\,dx + \int 6\cos(c+dx)\sec^2(c+dx)\,dx + \int 4\cos(c+dx)\sec^3(c+dx)\,dx + \int \cos(c+dx)\sec^4(c+dx)\,dx + \int \cos(c+dx)\,dx\right)$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))**4,x)
```

output
```
a**4*(Integral(4*cos(c + d*x)*sec(c + d*x), x) + Integral(6*cos(c + d*x)*s
ec(c + d*x)**2, x) + Integral(4*cos(c + d*x)*sec(c + d*x)**3, x) + Integra
l(cos(c + d*x)*sec(c + d*x)**4, x) + Integral(cos(c + d*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.51

$$\int \cos(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{16\,(dx+c)a^4 - a^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + 12\,a^4(\log(\sin(dx+c}{4\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/4*(16*(d*x + c)*a^4 - a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin
(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^4*(log(sin(d*x + c) + 1) -
log(sin(d*x + c) - 1)) + 4*a^4*sin(d*x + c) + 16*a^4*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.77

$$\int \cos(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{8\,(dx+c)a^4 + 13\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 13\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1} -}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/2*(8*(d*x + c)*a^4 + 13*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 13*a^4*
log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d
*x + 1/2*c)^2 + 1) - 2*(7*a^4*tan(1/2*d*x + 1/2*c)^3 - 9*a^4*tan(1/2*d*x +
 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 10.48 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.58

$$\begin{aligned}\int \cos(c+dx)(a+a\sec(c+dx))^4\,dx &= 4\,a^4\,x + \frac{13\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} \\ &\quad + \frac{5\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 + 2\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 - 11\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}\end{aligned}$$

input
```
int(cos(c + d*x)*(a + a/cos(c + d*x))^4,x)
```

output
```
4*a^4*x + (13*a^4*atanh(tan(c/2 + (d*x)/2)))/d + (2*a^4*tan(c/2 + (d*x)/2)
^3 + 5*a^4*tan(c/2 + (d*x)/2)^5 - 11*a^4*tan(c/2 + (d*x)/2))/(d*(tan(c/2 +
 (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.12

$$\int \cos(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$= \frac{a^4\left(-8\cos\left(dx+c\right)\sin\left(dx+c\right)-13\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2+13\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+13\right.}{}$$

input
```
int(cos(d*x+c)*(a+a*sec(d*x+c))^4,x)
```

output
```
(a**4*( - 8*cos(c + d*x)*sin(c + d*x) - 13*log(tan((c + d*x)/2) - 1)*sin(c
 + d*x)**2 + 13*log(tan((c + d*x)/2) - 1) + 13*log(tan((c + d*x)/2) + 1)*s
in(c + d*x)**2 - 13*log(tan((c + d*x)/2) + 1) + 2*sin(c + d*x)**3 + 8*sin(
c + d*x)**2*c + 8*sin(c + d*x)**2*d*x - 3*sin(c + d*x) - 8*c - 8*d*x))/(2*
d*(sin(c + d*x)**2 - 1))
```

3.35 $\int \cos^2(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{13a^4x}{2}+\frac{4a^4\text{arctanh}(\sin(c+dx))}{d}+\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\cos(c+dx)\sin(c+dx)}{2d}+\frac{a^4\tan(c+dx)}{d}$$

output
```
13/2*a^4*x+4*a^4*arctanh(sin(d*x+c))/d+4*a^4*sin(d*x+c)/d+1/2*a^4*cos(d*x+
c)*sin(d*x+c)/d+a^4*tan(d*x+c)/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 241 vs. $2(73) = 146$.

Time = 3.41 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.30

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{1}{64}a^4(1+\cos(c+dx))^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(26x - \frac{16\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{16\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{16\cos(dx)\sin(c)}{d} + \frac{\cos(2dx)\sin(2c)}{d} + \frac{16\cos(c)\sin(dx)}{d} + \frac{\cos(2c)\sin(2dx)}{d} + \frac{4\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{4\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)$$

input

```
Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^4,x]
```

output

```
(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(26*x - (16*Log[Cos[(c + d*x)
/2] - Sin[(c + d*x)/2]])/d + (16*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])
/d + (16*Cos[d*x]*Sin[c])/d + (Cos[2*d*x]*Sin[2*c])/d + (16*Cos[c]*Sin[d*x
])/d + (Cos[2*c]*Sin[2*d*x])/d + (4*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])
*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (4*Sin[(d*x)/2])/(d*(Cos[c/2] +
Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/64
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\sec(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^4\cos^2(c+dx)+4a^4\cos(c+dx)+a^4\sec^2(c+dx)+4a^4\sec(c+dx)+6a^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4a^4\text{arctanh}(\sin(c+dx))}{d}+\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\tan(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos(c+dx)}{2d}+\frac{13a^4x}{2}$$

input
```
Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^4,x]
```

output
```
(13*a^4*x)/2 + (4*a^4*ArcTanh[Sin[c + d*x]])/d + (4*a^4*Sin[c + d*x])/d +
(a^4*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a^4*Tan[c + d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{a^4\tan(dx+c)+4a^4\ln(\sec(dx+c)+\tan(dx+c))+6a^4(dx+c)+4a^4\sin(dx+c)+a^4\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a^4\tan(dx+c)+4a^4\ln(\sec(dx+c)+\tan(dx+c))+6a^4(dx+c)+4a^4\sin(dx+c)+a^4\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
parallelrisch	$-\frac{\left(-16\sin(2dx+2c)-9\sin(dx+c)-\sin(3dx+3c)-52dx\cos(dx+c)+32\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-32\ln\left(\tan\left(\frac{dx}{2} \cdots\right.\right.\right.}{8\cos(dx+c)d}$
risch	$\frac{13a^4x}{2}-\frac{ia^4e^{2i(dx+c)}}{8d}-\frac{2ia^4e^{i(dx+c)}}{d}+\frac{2ia^4e^{-i(dx+c)}}{d}+\frac{ia^4e^{-2i(dx+c)}}{8d}+\frac{2ia^4}{d(e^{2i(dx+c)}+1)}+\frac{4a^4\ln(e^{i(dx+c)} \cdots}{d}$
norman	$\frac{-\frac{13a^4x}{2}-\frac{11a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{20a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}-\frac{2a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}-\frac{12a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}+\frac{5a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{d}+\frac{13a^4x\tan\left(\frac{dx}{2} \cdots\right.}{2}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(-1+\tan\left(\cdots\right.\right.}$

input
```
int(cos(d*x+c)^2*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*tan(d*x+c)+4*a^4*ln(sec(d*x+c)+tan(d*x+c))+6*a^4*(d*x+c)+4*a^4*si
n(d*x+c)+a^4*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.44

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$=\frac{13\,a^4dx\cos(dx+c)+4\,a^4\cos(dx+c)\log(\sin(dx+c)+1)-4\,a^4\cos(dx+c)\log(-\sin(dx+c)+1)}{2\,d\cos(dx+c)}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/2*(13*a^4*d*x*cos(d*x + c) + 4*a^4*cos(d*x + c)*log(sin(d*x + c) + 1) -
4*a^4*cos(d*x + c)*log(-sin(d*x + c) + 1) + (a^4*cos(d*x + c)^2 + 8*a^4*co
s(d*x + c) + 2*a^4)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^4\,dx=a^4\left(\int 4\cos^2(c+dx)\sec(c+dx)\,dx+\int 6\cos^2(c+dx)\sec^2(c+dx)\,dx+\int 4\cos^2(c+dx)\sec^3(c+dx)\,dx+\int \cos^2(c+dx)\sec^4(c+dx)\,dx+\int \cos^2(c+dx)\,dx\right)$$

input
```
integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**4,x)
```

output
```
a**4*(Integral(4*cos(c + d*x)**2*sec(c + d*x), x) + Integral(6*cos(c + d*x
)**2*sec(c + d*x)**2, x) + Integral(4*cos(c + d*x)**2*sec(c + d*x)**3, x)
+ Integral(cos(c + d*x)**2*sec(c + d*x)**4, x) + Integral(cos(c + d*x)**2,
 x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.16

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^4+24\,(dx+c)a^4+8\,a^4(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+}{4\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 24*(d*x + c)*a^4 + 8*a^4*(log(
sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 16*a^4*sin(d*x + c) + 4*a^4*t
an(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.77

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{13\,(dx+c)a^4+8\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-8\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/2*(13*(d*x + c)*a^4 + 8*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 8*a^4*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) - 4*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*
x + 1/2*c)^2 - 1) + 2*(7*a^4*tan(1/2*d*x + 1/2*c)^3 + 9*a^4*tan(1/2*d*x +
1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 10.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.60

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{13\,a^4\,x}{2}+\frac{8\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{-5\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+2\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+11\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

input
```
int(cos(c + d*x)^2*(a + a/cos(c + d*x))^4,x)
```

output
```
(13*a^4*x)/2 + (8*a^4*atanh(tan(c/2 + (d*x)/2)))/d + (2*a^4*tan(c/2 + (d*x
)/2)^3 - 5*a^4*tan(c/2 + (d*x)/2)^5 + 11*a^4*tan(c/2 + (d*x)/2))/(d*(tan(c
/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.48

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{a^4\left(-8\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+8\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+8\cos(dx+c)\sin(dx}{2\cos(dx+c)\,d}$$

input
```
int(cos(d*x+c)^2*(a+a*sec(d*x+c))^4,x)
```

output
```
(a**4*( - 8*cos(c + d*x)*log(tan((c + d*x)/2) - 1) + 8*cos(c + d*x)*log(ta
n((c + d*x)/2) + 1) + 8*cos(c + d*x)*sin(c + d*x) + 13*cos(c + d*x)*c + 13
*cos(c + d*x)*d*x - sin(c + d*x)**3 + 3*sin(c + d*x)))/(2*cos(c + d*x)*d)
```

3.36 $\int \cos^3(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^4\,dx=6a^4x+\frac{a^4\text{arctanh}(\sin(c+dx))}{d}+\frac{7a^4\sin(c+dx)}{d}+\frac{2a^4\cos(c+dx)\sin(c+dx)}{d}-\frac{a^4\sin^3(c+dx)}{3d}$$

output
```
6*a^4*x+a^4*arctanh(sin(d*x+c))/d+7*a^4*sin(d*x+c)/d+2*a^4*cos(d*x+c)*sin(
d*x+c)/d-1/3*a^4*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$=\frac{a^4\left(72dx-12\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+81}{12d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^4,x]
```

output
```
(a^4*(72*d*x - 12*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]] + 81*Sin[c + d*x] + 12*Sin[2*(c + d*x)] + S
in[3*(c + d*x)]))/(12*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a\sec(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^4\cos^3(c+dx)+4a^4\cos^2(c+dx)+6a^4\cos(c+dx)+a^4\sec(c+dx)+4a^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4\text{arctanh}(\sin(c+dx))}{d}-\frac{a^4\sin^3(c+dx)}{3d}+\frac{7a^4\sin(c+dx)}{d}+\frac{2a^4\sin(c+dx)\cos(c+dx)}{d}+6a^4x$$

input
```
Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^4,x]
```

output
```
6*a^4*x + (a^4*ArcTanh[Sin[c + d*x]])/d + (7*a^4*Sin[c + d*x])/d + (2*a^4*
Cos[c + d*x]*Sin[c + d*x])/d - (a^4*Sin[c + d*x]^3)/(3*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

method	result
parallelrisch	$\frac{a^4\left(72dx+12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+81\sin(dx+c)+\sin(3dx+3c)+12\sin(2dx+2c)\right)}{12d}$
derivativedivides	$\frac{a^4\ln(\sec(dx+c)+\tan(dx+c))+4a^4(dx+c)+6a^4\sin(dx+c)+4a^4\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{a^4\left(2+\cos(dx+c)^2\right)\sin(\cdots}{3}}{d}$
default	$\frac{a^4\ln(\sec(dx+c)+\tan(dx+c))+4a^4(dx+c)+6a^4\sin(dx+c)+4a^4\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{a^4\left(2+\cos(dx+c)^2\right)\sin(\cdots}{3}}{d}$
risch	$6a^4x-\frac{27ia^4e^{i(dx+c)}}{8d}+\frac{27ia^4e^{-i(dx+c)}}{8d}+\frac{a^4\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{a^4\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{a^4\sin(3dx+3c)}{12d}+\frac{a^4\cdots}{}$
norman	$\frac{-6a^4x-\frac{18a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{86a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{12a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}-\frac{28a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}-\frac{14a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{3d}+\frac{10a^4\tan\left(\frac{d\cdots}{2}\right)}{d}\cdots}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}$

input
```
int(cos(d*x+c)^3*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/12*a^4*(72*d*x+12*ln(tan(1/2*d*x+1/2*c)+1)-12*ln(tan(1/2*d*x+1/2*c)-1)+8
1*sin(d*x+c)+sin(3*d*x+3*c)+12*sin(2*d*x+2*c))/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{36\,a^4dx + 3\,a^4\log\left(\sin\left(dx+c\right)+1\right) - 3\,a^4\log\left(-\sin\left(dx+c\right)+1\right) + 2\,\left(a^4\cos\left(dx+c\right)^2 + 6\,a^4\cos\left(dx+\right.\right.}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/6*(36*a^4*d*x + 3*a^4*log(sin(d*x + c) + 1) - 3*a^4*log(-sin(d*x + c) +
1) + 2*(a^4*cos(d*x + c)^2 + 6*a^4*cos(d*x + c) + 20*a^4)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^4\,dx = a^4\left(\int 4\cos^3\left(c+dx\right)\sec\left(c+dx\right)dx + \int 6\cos^3\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int 4\cos^3\left(c+dx\right)\sec^3\left(c+dx\right)dx + \int \cos^3\left(c+dx\right)\sec^4\left(c+dx\right)dx + \int \cos^3\left(c+dx\right)dx\right)$$

input
```
integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**4,x)
```

output
```
a**4*(Integral(4*cos(c + d*x)**3*sec(c + d*x), x) + Integral(6*cos(c + d*x
)**3*sec(c + d*x)**2, x) + Integral(4*cos(c + d*x)**3*sec(c + d*x)**3, x)
+ Integral(cos(c + d*x)**3*sec(c + d*x)**4, x) + Integral(cos(c + d*x)**3,
 x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.33

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^4\,dx =$$
$$-\frac{2\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^4-6\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^4-24\left(dx+c\right)a^4-3\,a^4(\log\left(\mathrm{si}\right.}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/6*(2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 - 6*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*a^4 - 24*(d*x + c)*a^4 - 3*a^4*(log(sin(d*x + c) + 1) - log(sin(d
*x + c) - 1)) - 36*a^4*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.59

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$=\frac{18\left(dx+c\right)a^4+3\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5\right.}{}}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(18*(d*x + c)*a^4 + 3*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^4*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*a^4*tan(1/2*d*x + 1/2*c)^5 + 38*
a^4*tan(1/2*d*x + 1/2*c)^3 + 27*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1
/2*c)^2 + 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 9.79 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.27

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^4\,dx = 6\,a^4\,x+\frac{20\,a^4\,\sin\left(c+d\,x\right)}{3\,d}+\frac{2\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^4\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}+\frac{2\,a^4\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{d}$$

input
```
int(cos(c + d*x)^3*(a + a/cos(c + d*x))^4,x)
```

output
```
6*a^4*x + (20*a^4*sin(c + d*x))/(3*d) + (2*a^4*atanh(sin(c/2 + (d*x)/2)/co
s(c/2 + (d*x)/2)))/d + (a^4*cos(c + d*x)^2*sin(c + d*x))/(3*d) + (2*a^4*co
s(c + d*x)*sin(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{a^4\left(6\cos\left(dx+c\right)\sin\left(dx+c\right)-3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\sin\left(dx+c\right)^3+21\,\mathrm{s}\right.}{3d}$$

input
```
int(cos(d*x+c)^3*(a+a*sec(d*x+c))^4,x)
```

output
```
(a**4*(6*cos(c + d*x)*sin(c + d*x) - 3*log(tan((c + d*x)/2) - 1) + 3*log(t
an((c + d*x)/2) + 1) - sin(c + d*x)**3 + 21*sin(c + d*x) + 18*c + 18*d*x))
/(3*d)
```

3.37 $\int \cos^4(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{35a^4x}{8}+\frac{8a^4\sin(c+dx)}{d}+\frac{27a^4\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^4\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{4a^4\sin^3(c+dx)}{3d}$$

output
```
35/8*a^4*x+8*a^4*sin(d*x+c)/d+27/8*a^4*cos(d*x+c)*sin(d*x+c)/d+1/4*a^4*cos
(d*x+c)^3*sin(d*x+c)/d-4/3*a^4*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{a^4(420c+420dx+672\sin(c+dx)+168\sin(2(c+dx))+32\sin(3(c+dx))+3\sin(4(c+dx)))}{96d}$$

input
```
Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^4,x]
```

output
```
(a^4*(420*c + 420*d*x + 672*Sin[c + d*x] + 168*Sin[2*(c + d*x)] + 32*Sin[3
*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(96*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a\sec(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^4\cos^4(c+dx)+4a^4\cos^3(c+dx)+6a^4\cos^2(c+dx)+4a^4\cos(c+dx)+a^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{27a^4\sin(c+dx)\cos(c+dx)}{8d}+\frac{35a^4x}{8}$$

input
```
Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^4,x]
```

output
```
(35*a^4*x)/8 + (8*a^4*Sin[c + d*x])/d + (27*a^4*Cos[c + d*x]*Sin[c + d*x])
/(8*d) + (a^4*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (4*a^4*Sin[c + d*x]^3)/
(3*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.63

method	result
parallelrisch	$\frac{a^4(420dx+3\sin(4dx+4c)+32\sin(3dx+3c)+168\sin(2dx+2c)+672\sin(dx+c))}{96d}$
risch	$\frac{35a^4x}{8}+\frac{7a^4\sin(dx+c)}{d}+\frac{a^4\sin(4dx+4c)}{32d}+\frac{a^4\sin(3dx+3c)}{3d}+\frac{7a^4\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{a^4(dx+c)+4a^4\sin(dx+c)+6a^4\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{4a^4\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+a^4\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(d}{2}\right.}{4}\right.}{d}$
default	$\frac{a^4(dx+c)+4a^4\sin(dx+c)+6a^4\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{4a^4\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+a^4\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(d}{2}\right.}{4}\right.}{d}$
norman	$-\frac{35a^4x}{8}-\frac{93a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{163a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{6d}+\frac{311a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{12d}-\frac{17a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}-\frac{329a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{12d}+\frac{35a^4\text{ta}}{}$

input
```
int(cos(d*x+c)^4*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/96*a^4*(420*d*x+3*sin(4*d*x+4*c)+32*sin(3*d*x+3*c)+168*sin(2*d*x+2*c)+67
2*sin(d*x+c))/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$= \frac{105\,a^4dx + \left(6\,a^4\cos\left(dx+c\right)^3 + 32\,a^4\cos\left(dx+c\right)^2 + 81\,a^4\cos\left(dx+c\right) + 160\,a^4\right)\sin\left(dx+c\right)}{24\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/24*(105*a^4*d*x + (6*a^4*cos(d*x + c)^3 + 32*a^4*cos(d*x + c)^2 + 81*a^4
*cos(d*x + c) + 160*a^4)*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^4\,dx =$$
$$-\frac{128\left(\sin\left(dx+c\right)^3 - 3\,\sin\left(dx+c\right)\right)a^4 - 3\left(12\,dx + 12\,c + \sin\left(4\,dx+4\,c\right) + 8\,\sin\left(2\,dx+2\,c\right)\right)a^4 - 14}{96\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/96*(128*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 - 3*(12*d*x + 12*c + sin(
4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4 - 144*(2*d*x + 2*c + sin(2*d*x + 2*
c))*a^4 - 96*(d*x + c)*a^4 - 384*a^4*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.10

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{105\,(dx+c)a^4 + \frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+385\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+511\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+279\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/24*(105*(d*x + c)*a^4 + 2*(105*a^4*tan(1/2*d*x + 1/2*c)^7 + 385*a^4*tan(
1/2*d*x + 1/2*c)^5 + 511*a^4*tan(1/2*d*x + 1/2*c)^3 + 279*a^4*tan(1/2*d*x
+ 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 12.99 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{35\,a^4\,x}{8} + \frac{\frac{35\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{385\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}+\frac{511\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}+\frac{93\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

input
```
int(cos(c + d*x)^4*(a + a/cos(c + d*x))^4,x)
```

output
```
(35*a^4*x)/8 + ((511*a^4*tan(c/2 + (d*x)/2)^3)/12 + (385*a^4*tan(c/2 + (d*
x)/2)^5)/12 + (35*a^4*tan(c/2 + (d*x)/2)^7)/4 + (93*a^4*tan(c/2 + (d*x)/2)
)/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.70

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$=\frac{a^4\left(-6\cos\left(dx+c\right)\sin\left(dx+c\right)^3+87\cos\left(dx+c\right)\sin\left(dx+c\right)-32\sin\left(dx+c\right)^3+192\sin\left(dx+c\right)+10\right.}{24d}$$

input
```
int(cos(d*x+c)^4*(a+a*sec(d*x+c))^4,x)
```

output
```
(a**4*( - 6*cos(c + d*x)*sin(c + d*x)**3 + 87*cos(c + d*x)*sin(c + d*x) -
32*sin(c + d*x)**3 + 192*sin(c + d*x) + 105*d*x))/(24*d)
```

3.38 $\int \cos^5(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{7a^4x}{2} + \frac{8a^4\sin(c+dx)}{d} + \frac{7a^4\cos(c+dx)\sin(c+dx)}{2d} + \frac{a^4\cos^3(c+dx)\sin(c+dx)}{d} - \frac{8a^4\sin^3(c+dx)}{3d} + \frac{a^4\sin^5(c+dx)}{5d}$$

output
```
7/2*a^4*x+8*a^4*sin(d*x+c)/d+7/2*a^4*cos(d*x+c)*sin(d*x+c)/d+a^4*cos(d*x+c
)^3*sin(d*x+c)/d-8/3*a^4*sin(d*x+c)^3/d+1/5*a^4*sin(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{a^4(840dx + 1470\sin(c+dx) + 480\sin(2(c+dx)) + 145\sin(3(c+dx)) + 30\sin(4(c+dx)) + 3\sin(5(c+}{240d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^4,x]
```

output
```
(a^4*(840*d*x + 1470*Sin[c + d*x] + 480*Sin[2*(c + d*x)] + 145*Sin[3*(c +
d*x)] + 30*Sin[4*(c + d*x)] + 3*Sin[5*(c + d*x)]))/(240*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a\sec(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^4\cos^5(c+dx)+4a^4\cos^4(c+dx)+6a^4\cos^3(c+dx)+4a^4\cos^2(c+dx)+a^4\cos(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4\sin^5(c+dx)}{5d}-\frac{8a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^3(c+dx)}{d}+\frac{7a^4\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^4x}{2}$$

input
```
Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^4,x]
```

output
```
(7*a^4*x)/2 + (8*a^4*Sin[c + d*x])/d + (7*a^4*Cos[c + d*x]*Sin[c + d*x])/(
2*d) + (a^4*Cos[c + d*x]^3*Sin[c + d*x])/d - (8*a^4*Sin[c + d*x]^3)/(3*d)
+ (a^4*Sin[c + d*x]^5)/(5*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.65

method	result
parallelrisch	$\frac{a^4(840dx+1470\sin(dx+c)+3\sin(5dx+5c)+30\sin(4dx+4c)+145\sin(3dx+3c)+480\sin(2dx+2c))}{240d}$
risch	$\frac{7a^4x}{2}+\frac{49a^4\sin(dx+c)}{8d}+\frac{a^4\sin(5dx+5c)}{80d}+\frac{a^4\sin(4dx+4c)}{8d}+\frac{29a^4\sin(3dx+3c)}{48d}+\frac{2a^4\sin(2dx+2c)}{d}$
derivativedivides	$\frac{a^4\sin(dx+c)+4a^4\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2a^4\left(2+\cos(dx+c)^2\right)\sin(dx+c)+4a^4\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\text{si}}{4}\right.}{d}$
default	$\frac{a^4\sin(dx+c)+4a^4\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2a^4\left(2+\cos(dx+c)^2\right)\sin(dx+c)+4a^4\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\text{si}}{4}\right.}{d}$
norman	$\frac{-\frac{7a^4x}{2}-\frac{25a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{67a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{349a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{15d}+\frac{203a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{15d}-\frac{533a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{15d}-\frac{259a^4\text{ta}}{}}{}$

input
```
int(cos(d*x+c)^5*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/240*a^4*(840*d*x+1470*sin(d*x+c)+3*sin(5*d*x+5*c)+30*sin(4*d*x+4*c)+145*
sin(3*d*x+3*c)+480*sin(2*d*x+2*c))/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.75

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{105\,a^4dx + \left(6\,a^4\cos\left(dx+c\right)^4 + 30\,a^4\cos\left(dx+c\right)^3 + 68\,a^4\cos\left(dx+c\right)^2 + 105\,a^4\cos\left(dx+c\right) + 166\,a^4\right)}{30\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/30*(105*a^4*d*x + (6*a^4*cos(d*x + c)^4 + 30*a^4*cos(d*x + c)^3 + 68*a^4
*cos(d*x + c)^2 + 105*a^4*cos(d*x + c) + 166*a^4)*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.25

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{8\left(3\sin\left(dx+c\right)^5 - 10\sin\left(dx+c\right)^3 + 15\sin\left(dx+c\right)\right)a^4 - 240\left(\sin\left(dx+c\right)^3 - 3\sin\left(dx+c\right)\right)a^4 + 15\,(}{}$$

input
```
integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/120*(8*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4 - 24
0*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 + 15*(12*d*x + 12*c + sin(4*d*x +
4*c) + 8*sin(2*d*x + 2*c))*a^4 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^4
+ 120*a^4*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.10

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{105\,(dx+c)a^4 + \frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+490\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+896\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+790\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+375\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{30\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/30*(105*(d*x + c)*a^4 + 2*(105*a^4*tan(1/2*d*x + 1/2*c)^9 + 490*a^4*tan(
1/2*d*x + 1/2*c)^7 + 896*a^4*tan(1/2*d*x + 1/2*c)^5 + 790*a^4*tan(1/2*d*x
+ 1/2*c)^3 + 375*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)
/d
```

Mupad [B] (verification not implemented)

Time = 13.41 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.03

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{7\,a^4\,x}{2} + \frac{7\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\frac{98\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}+\frac{896\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}+\frac{158\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+25\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^5}$$

input
```
int(cos(c + d*x)^5*(a + a/cos(c + d*x))^4,x)
```

output
```
(7*a^4*x)/2 + ((158*a^4*tan(c/2 + (d*x)/2)^3)/3 + (896*a^4*tan(c/2 + (d*x)
/2)^5)/15 + (98*a^4*tan(c/2 + (d*x)/2)^7)/3 + 7*a^4*tan(c/2 + (d*x)/2)^9 +
 25*a^4*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.70

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{a^4\left(-30\cos\left(dx+c\right)\sin\left(dx+c\right)^3+135\cos\left(dx+c\right)\sin\left(dx+c\right)+6\sin\left(dx+c\right)^5-80\sin\left(dx+c\right)^3+24\right.}{30d}$$

input
```
int(cos(d*x+c)^5*(a+a*sec(d*x+c))^4,x)
```

output
```
(a**4*( - 30*cos(c + d*x)*sin(c + d*x)**3 + 135*cos(c + d*x)*sin(c + d*x)
+ 6*sin(c + d*x)**5 - 80*sin(c + d*x)**3 + 240*sin(c + d*x) + 105*d*x))/(3
0*d)
```

3.39 $\int \cos^6(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 127

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{49a^4x}{16}+\frac{8a^4\sin(c+dx)}{d}+\frac{49a^4\cos(c+dx)\sin(c+dx)}{16d}+\frac{41a^4\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{a^4\cos^5(c+dx)\sin(c+dx)}{6d}-\frac{4a^4\sin^3(c+dx)}{d}+\frac{4a^4\sin^5(c+dx)}{5d}$$

output
```
49/16*a^4*x+8*a^4*sin(d*x+c)/d+49/16*a^4*cos(d*x+c)*sin(d*x+c)/d+41/24*a^4
*cos(d*x+c)^3*sin(d*x+c)/d+1/6*a^4*cos(d*x+c)^5*sin(d*x+c)/d-4*a^4*sin(d*x
+c)^3/d+4/5*a^4*sin(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.57

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{a^4(2940dx+5280\sin(c+dx)+1905\sin(2(c+dx))+720\sin(3(c+dx))+225\sin(4(c+dx))+48\sin(}{960d}$$

input
```
Integrate[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^4,x]
```

output
```
(a^4*(2940*d*x + 5280*Sin[c + d*x] + 1905*Sin[2*(c + d*x)] + 720*Sin[3*(c
+ d*x)] + 225*Sin[4*(c + d*x)] + 48*Sin[5*(c + d*x)] + 5*Sin[6*(c + d*x)])
)/(960*d)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a\sec(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^4\cos^6(c+dx)+4a^4\cos^5(c+dx)+6a^4\cos^4(c+dx)+4a^4\cos^3(c+dx)+a^4\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4a^4\sin^5(c+dx)}{5d}-\frac{4a^4\sin^3(c+dx)}{d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{41a^4\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{49a^4\sin(c+dx)\cos(c+dx)}{16d}+\frac{49a^4x}{16}$$

input
```
Int[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^4,x]
```

output
```
(49*a^4*x)/16 + (8*a^4*Sin[c + d*x])/d + (49*a^4*Cos[c + d*x]*Sin[c + d*x]
)/(16*d) + (41*a^4*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a^4*Cos[c + d*x]
^5*Sin[c + d*x])/(6*d) - (4*a^4*Sin[c + d*x]^3)/d + (4*a^4*Sin[c + d*x]^5)
/(5*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.59

method	result
parallelrisch	$\frac{\left(588 d x+\sin \left(6 d x+6 c\right)+1056 \sin \left(d x+c\right)+381 \sin \left(2 d x+2 c\right)+144 \sin \left(3 d x+3 c\right)+45 \sin \left(4 d x+4 c\right)+\frac{48 \sin \left(5 d x+5 c\right)}{5}\right) a^{4}}{192 d}$
risch	$\frac{49 a^{4} x}{16}+\frac{11 a^{4} \sin \left(d x+c\right)}{2 d}+\frac{a^{4} \sin \left(6 d x+6 c\right)}{192 d}+\frac{a^{4} \sin \left(5 d x+5 c\right)}{20 d}+\frac{15 a^{4} \sin \left(4 d x+4 c\right)}{64 d}+\frac{3 a^{4} \sin \left(3 d x+3 c\right)}{4 d}+\frac{127}{}$
derivativedivides	$\frac{a^{4}\left(\frac{\sin \left(d x+c\right) \cos \left(d x+c\right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)+\frac{4 a^{4}\left(2+\cos \left(d x+c\right)^{2}\right) \sin \left(d x+c\right)}{3}+6 a^{4}\left(\frac{\left(\cos \left(d x+c\right)^{3}+\frac{3 \cos \left(d x+c\right)}{2}\right) \sin \left(d x+c\right)}{4}+\frac{3 d x}{8}+\frac{3 c}{8}\right)}{d}$
default	$\frac{a^{4}\left(\frac{\sin \left(d x+c\right) \cos \left(d x+c\right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)+\frac{4 a^{4}\left(2+\cos \left(d x+c\right)^{2}\right) \sin \left(d x+c\right)}{3}+6 a^{4}\left(\frac{\left(\cos \left(d x+c\right)^{3}+\frac{3 \cos \left(d x+c\right)}{2}\right) \sin \left(d x+c\right)}{4}+\frac{3 d x}{8}+\frac{3 c}{8}\right)}{d}$

input
```
int(cos(d*x+c)^6*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/192*(588*d*x+sin(6*d*x+6*c)+1056*sin(d*x+c)+381*sin(2*d*x+2*c)+144*sin(3
*d*x+3*c)+45*sin(4*d*x+4*c)+48/5*sin(5*d*x+5*c))*a^4/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.70

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$=\frac{735\,a^4dx+\left(40\,a^4\cos\left(dx+c\right)^5+192\,a^4\cos\left(dx+c\right)^4+410\,a^4\cos\left(dx+c\right)^3+576\,a^4\cos\left(dx+c\right)^2+735\right.}{240\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/240*(735*a^4*d*x + (40*a^4*cos(d*x + c)^5 + 192*a^4*cos(d*x + c)^4 + 410
*a^4*cos(d*x + c)^3 + 576*a^4*cos(d*x + c)^2 + 735*a^4*cos(d*x + c) + 1152
*a^4)*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**6*(a+a*sec(d*x+c))**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.30

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{256\left(3\sin(dx+c)^5 - 10\sin(dx+c)^3 + 15\sin(dx+c)\right)a^4 - 5\left(4\sin(2dx+2c)^3 - 60dx - 60c - 9\,\mathrm{s}\right.}{}$$

input
```
integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/960*(256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4 -
5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*
x + 2*c))*a^4 - 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 + 180*(12*d*x +
 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4 + 240*(2*d*x + 2*c + si
n(2*d*x + 2*c))*a^4)/d
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.01

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{735\,(dx+c)a^4 + \frac{2\left(735\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+4165\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+9702\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+11802\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+7355\,a^4\tan\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^6}}{240\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/240*(735*(d*x + c)*a^4 + 2*(735*a^4*tan(1/2*d*x + 1/2*c)^11 + 4165*a^4*t
an(1/2*d*x + 1/2*c)^9 + 9702*a^4*tan(1/2*d*x + 1/2*c)^7 + 11802*a^4*tan(1/
2*d*x + 1/2*c)^5 + 7355*a^4*tan(1/2*d*x + 1/2*c)^3 + 3105*a^4*tan(1/2*d*x
+ 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

Mupad [B] (verification not implemented)

Time = 12.51 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.95

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{49\,a^4\,x}{16} + \frac{\frac{49\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{833\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+\frac{1617\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20}+\frac{1967\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20}+\frac{1471\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24}+\frac{207\,a^4\,\mathrm{ta}}{}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^6}$$

input
```
int(cos(c + d*x)^6*(a + a/cos(c + d*x))^4,x)
```

output
```
(49*a^4*x)/16 + ((1471*a^4*tan(c/2 + (d*x)/2)^3)/24 + (1967*a^4*tan(c/2 +
(d*x)/2)^5)/20 + (1617*a^4*tan(c/2 + (d*x)/2)^7)/20 + (833*a^4*tan(c/2 + (
d*x)/2)^9)/24 + (49*a^4*tan(c/2 + (d*x)/2)^11)/8 + (207*a^4*tan(c/2 + (d*x
)/2))/8)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.69

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{a^4\left(40\cos(dx+c)\sin(dx+c)^5-490\cos(dx+c)\sin(dx+c)^3+1185\cos(dx+c)\sin(dx+c)+192\,\mathrm{si}\right.}{240d}$$

input
```
int(cos(d*x+c)^6*(a+a*sec(d*x+c))^4,x)
```

output
```
(a**4*(40*cos(c + d*x)*sin(c + d*x)**5 - 490*cos(c + d*x)*sin(c + d*x)**3
+ 1185*cos(c + d*x)*sin(c + d*x) + 192*sin(c + d*x)**5 - 960*sin(c + d*x)*
*3 + 1920*sin(c + d*x) + 735*d*x))/(240*d)
```

3.40 $\int \cos^7(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 147

$$\int \cos^7(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{11a^4x}{4}+\frac{8a^4\sin(c+dx)}{d}+\frac{11a^4\cos(c+dx)\sin(c+dx)}{4d}+\frac{11a^4\cos^3(c+dx)\sin(c+dx)}{6d}+\frac{2a^4\cos^5(c+dx)\sin(c+dx)}{3d}-\frac{16a^4\sin^3(c+dx)}{3d}+\frac{9a^4\sin^5(c+dx)}{5d}-\frac{a^4\sin^7(c+dx)}{7d}$$

output
```
11/4*a^4*x+8*a^4*sin(d*x+c)/d+11/4*a^4*cos(d*x+c)*sin(d*x+c)/d+11/6*a^4*co
s(d*x+c)^3*sin(d*x+c)/d+2/3*a^4*cos(d*x+c)^5*sin(d*x+c)/d-16/3*a^4*sin(d*x
+c)^3/d+9/5*a^4*sin(d*x+c)^5/d-1/7*a^4*sin(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.56

$$\int \cos^7(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$=\frac{a^4(18480dx+33915\sin(c+dx)+13020\sin(2(c+dx))+5495\sin(3(c+dx))+2100\sin(4(c+dx))+6}{6720d}$$

input
```
Integrate[Cos[c + d*x]^7*(a + a*Sec[c + d*x])^4,x]
```

output
```
(a^4*(18480*d*x + 33915*Sin[c + d*x] + 13020*Sin[2*(c + d*x)] + 5495*Sin[3
*(c + d*x)] + 2100*Sin[4*(c + d*x)] + 651*Sin[5*(c + d*x)] + 140*Sin[6*(c
+ d*x)] + 15*Sin[7*(c + d*x)]))/(6720*d)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(c+dx)(a\sec(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^7}dx$$

$$\Big\downarrow \text{ 4278}$$

$$\int \left(a^4\cos^7(c+dx)+4a^4\cos^6(c+dx)+6a^4\cos^5(c+dx)+4a^4\cos^4(c+dx)+a^4\cos^3(c+dx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^4\sin^7(c+dx)}{7d}+\frac{9a^4\sin^5(c+dx)}{5d}-\frac{16a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{2a^4\sin(c+dx)\cos^5(c+dx)}{3d}+\frac{11a^4\sin(c+dx)\cos^3(c+dx)}{6d}+\frac{11a^4\sin(c+dx)\cos(c+dx)}{4d}+\frac{11a^4x}{4}$$

input
```
Int[Cos[c + d*x]^7*(a + a*Sec[c + d*x])^4,x]
```

output
```
(11*a^4*x)/4 + (8*a^4*Sin[c + d*x])/d + (11*a^4*Cos[c + d*x]*Sin[c + d*x])
/(4*d) + (11*a^4*Cos[c + d*x]^3*Sin[c + d*x])/(6*d) + (2*a^4*Cos[c + d*x]^
5*Sin[c + d*x])/(3*d) - (16*a^4*Sin[c + d*x]^3)/(3*d) + (9*a^4*Sin[c + d*x
]^5)/(5*d) - (a^4*Sin[c + d*x]^7)/(7*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.60

method	result
parallelrisch	$\frac{a^4(18480dx+33915\sin(dx+c)+15\sin(7dx+7c)+140\sin(6dx+6c)+651\sin(5dx+5c)+2100\sin(4dx+4c)+5495\sin(3dx}{6720d}$
risch	$\frac{11a^4x}{4}+\frac{323a^4\sin(dx+c)}{64d}+\frac{a^4\sin(7dx+7c)}{448d}+\frac{a^4\sin(6dx+6c)}{48d}+\frac{31a^4\sin(5dx+5c)}{320d}+\frac{5a^4\sin(4dx+4c)}{16d}+15$
derivativedivides	$\frac{\frac{a^4(2+\cos(dx+c)^2)\sin(dx+c)}{3}+4a^4\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{6a^4\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right.}{5}}{}$
default	$\frac{\frac{a^4(2+\cos(dx+c)^2)\sin(dx+c)}{3}+4a^4\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{6a^4\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right.}{5}}{}$

input
```
int(cos(d*x+c)^7*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/6720*a^4*(18480*d*x+33915*sin(d*x+c)+15*sin(7*d*x+7*c)+140*sin(6*d*x+6*c
)+651*sin(5*d*x+5*c)+2100*sin(4*d*x+4*c)+5495*sin(3*d*x+3*c)+13020*sin(2*d
*x+2*c))/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.69

$$\int \cos^7(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$=\frac{1155\,a^4dx+\left(60\,a^4\cos\left(dx+c\right)^6+280\,a^4\cos\left(dx+c\right)^5+576\,a^4\cos\left(dx+c\right)^4+770\,a^4\cos\left(dx+c\right)^3+90}{420\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/420*(1155*a^4*d*x + (60*a^4*cos(d*x + c)^6 + 280*a^4*cos(d*x + c)^5 + 57
6*a^4*cos(d*x + c)^4 + 770*a^4*cos(d*x + c)^3 + 908*a^4*cos(d*x + c)^2 + 1
155*a^4*cos(d*x + c) + 1816*a^4)*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^7(c+dx)(a+a\sec(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**7*(a+a*sec(d*x+c))**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.27

$$\int \cos^7(c+dx)(a+a\sec(c+dx))^4\,dx = -\frac{48\left(5\sin(dx+c)^7-21\sin(dx+c)^5+35\sin(dx+c)^3-35\sin(dx+c)\right)a^4-672\left(3\sin(dx+c)^5-\right.}{}$$

input
```
integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/1680*(48*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35
*sin(d*x + c))*a^4 - 672*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*
x + c))*a^4 + 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c
) - 48*sin(2*d*x + 2*c))*a^4 + 560*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 -
 210*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4)/d
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.98

$$\int \cos^7(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{1155\,(dx+c)a^4+\frac{2\left(1155\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+7700\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+21791\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+33792\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+31521\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{7}}}{420\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/420*(1155*(d*x + c)*a^4 + 2*(1155*a^4*tan(1/2*d*x + 1/2*c)^13 + 7700*a^4
*tan(1/2*d*x + 1/2*c)^11 + 21791*a^4*tan(1/2*d*x + 1/2*c)^9 + 33792*a^4*ta
n(1/2*d*x + 1/2*c)^7 + 31521*a^4*tan(1/2*d*x + 1/2*c)^5 + 14700*a^4*tan(1/
2*d*x + 1/2*c)^3 + 5565*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
+ 1)^7)/d
```

Mupad [B] (verification not implemented)

Time = 12.60 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.93

$$\int \cos^7(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{11\,a^4\,x}{4} + \frac{\frac{11\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{2}+\frac{110\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{3}+\frac{3113\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{30}+\frac{5632\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{35}+\frac{1501\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{10}+70\,a^4\cdots}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^7}$$

input
```
int(cos(c + d*x)^7*(a + a/cos(c + d*x))^4,x)
```

output
```
(11*a^4*x)/4 + (70*a^4*tan(c/2 + (d*x)/2)^3 + (1501*a^4*tan(c/2 + (d*x)/2)
^5)/10 + (5632*a^4*tan(c/2 + (d*x)/2)^7)/35 + (3113*a^4*tan(c/2 + (d*x)/2)
^9)/30 + (110*a^4*tan(c/2 + (d*x)/2)^11)/3 + (11*a^4*tan(c/2 + (d*x)/2)^13
)/2 + (53*a^4*tan(c/2 + (d*x)/2))/2)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.66

$$\int \cos^7(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{a^4\left(280\cos(dx+c)\sin(dx+c)^5-1330\cos(dx+c)\sin(dx+c)^3+2205\cos(dx+c)\sin(dx+c)-60\,\mathrm{s}\cdots\right.}{420d}$$

input
```
int(cos(d*x+c)^7*(a+a*sec(d*x+c))^4,x)
```

output
```
(a**4*(280*cos(c + d*x)*sin(c + d*x)**5 - 1330*cos(c + d*x)*sin(c + d*x)**
3 + 2205*cos(c + d*x)*sin(c + d*x) - 60*sin(c + d*x)**7 + 756*sin(c + d*x)
**5 - 2240*sin(c + d*x)**3 + 3360*sin(c + d*x) + 1155*d*x))/(420*d)
```

3.41 $\int \sec^3(c+dx)(a+a\sec(c+dx))^5\,dx$

Optimal result

Integrand size = 21, antiderivative size = 156

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^5\,dx = \frac{93a^5\text{arctanh}(\sin(c+dx))}{16d} + \frac{16a^5\tan(c+dx)}{d} + \frac{93a^5\sec(c+dx)\tan(c+dx)}{16d} + \frac{85a^5\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{5a^5\sec^5(c+dx)\tan(c+dx)}{6d} + \frac{28a^5\tan^3(c+dx)}{3d} + \frac{13a^5\tan^5(c+dx)}{5d} + \frac{a^5\tan^7(c+dx)}{7d}$$

output
```
93/16*a^5*arctanh(sin(d*x+c))/d+16*a^5*tan(d*x+c)/d+93/16*a^5*sec(d*x+c)*t
an(d*x+c)/d+85/24*a^5*sec(d*x+c)^3*tan(d*x+c)/d+5/6*a^5*sec(d*x+c)^5*tan(d
*x+c)/d+28/3*a^5*tan(d*x+c)^3/d+13/5*a^5*tan(d*x+c)^5/d+1/7*a^5*tan(d*x+c)
^7/d
```

Mathematica [A] (verified)

Time = 5.04 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.09

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^5\,dx = \frac{93a^5\text{arctanh}(\sin(c+dx))}{16d} + \frac{479a^5\tan(c+dx)}{35d} + \frac{93a^5\sec(c+dx)\tan(c+dx)}{16d} + \frac{85a^5\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{76a^5\sec^4(c+dx)\tan(c+dx)}{35d} + \frac{5a^5\sec^5(c+dx)\tan(c+dx)}{6d} + \frac{a^5\sec^6(c+dx)\tan(c+dx)}{7d} + \frac{479a^5\tan^3(c+dx)}{105d}$$

input
```
Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^5,x]
```

output
```
(93*a^5*ArcTanh[Sin[c + d*x]])/(16*d) + (479*a^5*Tan[c + d*x])/(35*d) + (9
3*a^5*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (85*a^5*Sec[c + d*x]^3*Tan[c + d
*x])/(24*d) + (76*a^5*Sec[c + d*x]^4*Tan[c + d*x])/(35*d) + (5*a^5*Sec[c +
 d*x]^5*Tan[c + d*x])/(6*d) + (a^5*Sec[c + d*x]^6*Tan[c + d*x])/(7*d) + (4
79*a^5*Tan[c + d*x]^3)/(105*d)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\sec(c+dx)+a)^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3 \left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^5 dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^5\sec^8(c+dx)+5a^5\sec^7(c+dx)+10a^5\sec^6(c+dx)+10a^5\sec^5(c+dx)+5a^5\sec^4(c+dx)+a^5\sec^3(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{93a^5\text{arctanh}(\sin(c+dx))}{16d}+\frac{a^5\tan^7(c+dx)}{7d}+\frac{13a^5\tan^5(c+dx)}{5d}+\frac{28a^5\tan^3(c+dx)}{3d}+\frac{16a^5\tan(c+dx)}{d}+\frac{5a^5\tan(c+dx)\sec^5(c+dx)}{6d}+\frac{85a^5\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{93a^5\tan(c+dx)\sec(c+dx)}{16d}$$

input
```
Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^5,x]
```

output
```
(93*a^5*ArcTanh[Sin[c + d*x]])/(16*d) + (16*a^5*Tan[c + d*x])/d + (93*a^5*
Sec[c + d*x]*Tan[c + d*x])/(16*d) + (85*a^5*Sec[c + d*x]^3*Tan[c + d*x])/(
24*d) + (5*a^5*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + (28*a^5*Tan[c + d*x]^3
)/(3*d) + (13*a^5*Tan[c + d*x]^5)/(5*d) + (a^5*Tan[c + d*x]^7)/(7*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.21 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.21

method	result
risch	$-\frac{i a^5 (9765\, e^{13 i(dx+c)}+62860\, e^{11 i(dx+c)}-16800\, e^{10 i(dx+c)}+118825\, e^{9 i(dx+c)}-162400\, e^{8 i(dx+c)}-374080\, e^{6 i(dx+c)}-}{840 d (e^{2 i(dx+c)}+1)^7}$
norman	$\frac{-\frac{419 a^5 \tan(\frac{dx}{2}+\frac{c}{2})}{8d}+\frac{943 a^5 \tan(\frac{dx}{2}+\frac{c}{2})^3}{6d}-\frac{37169 a^5 \tan(\frac{dx}{2}+\frac{c}{2})^5}{120d}+\frac{11904 a^5 \tan(\frac{dx}{2}+\frac{c}{2})^7}{35d}-\frac{8773 a^5 \tan(\frac{dx}{2}+\frac{c}{2})^9}{40d}+\frac{155 a^5 \mathrm{ta}}{}}{\left(-1+\tan(\frac{dx}{2}+\frac{c}{2})^2\right)^7}$
parallelrisch	$\frac{3395 a^5 \left(\frac{279\left(-\cos(dx+c)-\frac{3\cos(3dx+3c)}{5}-\frac{\cos(5dx+5c)}{5}-\frac{\cos(7dx+7c)}{35}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{194}+\frac{279\left(\frac{\cos(7dx+7c)}{35}+\frac{\cos(5dx+5c}{}\right.}{}\right.}{24 d (\cos(}$
derivativedivides	$\frac{a^5\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-5a^5\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+10a^5\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(}{}\right.\right.}{}$
default	$\frac{a^5\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-5a^5\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+10a^5\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(}{}\right.\right.}{}$
parts	$\frac{a^5\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}-\frac{a^5\left(-\frac{16}{35}-\frac{\sec(dx+c)^6}{7}-\frac{6\sec(dx+c)^4}{35}-\frac{8\sec(dx+c)^2}{35}\right)\tan(dx+c)}{d}$

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
-1/840*I*a^5*(9765*exp(13*I*(d*x+c))+62860*exp(11*I*(d*x+c))-16800*exp(10*
I*(d*x+c))+118825*exp(9*I*(d*x+c))-162400*exp(8*I*(d*x+c))-374080*exp(6*I*
(d*x+c))-118825*exp(5*I*(d*x+c))-305088*exp(4*I*(d*x+c))-62860*exp(3*I*(d*
x+c))-107296*exp(2*I*(d*x+c))-9765*exp(I*(d*x+c))-15328)/d/(exp(2*I*(d*x+c
))+1)^7-93/16*a^5/d*ln(exp(I*(d*x+c))-I)+93/16*a^5/d*ln(exp(I*(d*x+c))+I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.96

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^5\,dx$$
$$=\frac{9765\,a^5\cos(dx+c)^7\log(\sin(dx+c)+1)-9765\,a^5\cos(dx+c)^7\log(-\sin(dx+c)+1)+2\,(15328\,a^5}{}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/3360*(9765*a^5*cos(d*x + c)^7*log(sin(d*x + c) + 1) - 9765*a^5*cos(d*x +
 c)^7*log(-sin(d*x + c) + 1) + 2*(15328*a^5*cos(d*x + c)^6 + 9765*a^5*cos(
d*x + c)^5 + 7664*a^5*cos(d*x + c)^4 + 5950*a^5*cos(d*x + c)^3 + 3648*a^5*
cos(d*x + c)^2 + 1400*a^5*cos(d*x + c) + 240*a^5)*sin(d*x + c))/(d*cos(d*x
 + c)^7)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^5\,dx = a^5\left(\int \sec^3(c+dx)\,dx + \int 5\sec^4(c+dx)\,dx + \int 10\sec^5(c+dx)\,dx + \int 10\sec^6(c+dx)\,dx + \int 5\sec^7(c+dx)\,dx + \int \sec^8(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**5,x)
```

output
```
a**5*(Integral(sec(c + d*x)**3, x) + Integral(5*sec(c + d*x)**4, x) + Inte
gral(10*sec(c + d*x)**5, x) + Integral(10*sec(c + d*x)**6, x) + Integral(5
*sec(c + d*x)**7, x) + Integral(sec(c + d*x)**8, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(142) = 284$.

Time = 0.06 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.01

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^5\,dx = \frac{96\left(5\tan(dx+c)^7 + 21\tan(dx+c)^5 + 35\tan(dx+c)^3 + 35\tan(dx+c)\right)a^5 + 2240\left(3\tan(dx+c)^5\right.}{}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/3360*(96*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*
tan(d*x + c))*a^5 + 2240*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*
x + c))*a^5 + 5600*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^5 - 175*a^5*(2*(15*
sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*
sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log
(sin(d*x + c) - 1)) - 2100*a^5*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin
(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(
d*x + c) - 1)) - 840*a^5*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*
x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.09

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^5\,dx$$

$$= \frac{9765\,a^5\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9765\,a^5\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(9765\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-65100\,a\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

output
```
1/1680*(9765*a^5*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9765*a^5*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) - 2*(9765*a^5*tan(1/2*d*x + 1/2*c)^13 - 65100*a^5*
tan(1/2*d*x + 1/2*c)^11 + 184233*a^5*tan(1/2*d*x + 1/2*c)^9 - 285696*a^5*t
an(1/2*d*x + 1/2*c)^7 + 260183*a^5*tan(1/2*d*x + 1/2*c)^5 - 132020*a^5*tan
(1/2*d*x + 1/2*c)^3 + 43995*a^5*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c
)^2 - 1)^7)/d
```

Mupad [B] (verification not implemented)

Time = 13.69 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.46

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^5\,dx = \frac{93\,a^5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d} - \frac{\frac{93\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{8} - \frac{155\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{2} + \frac{8773\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{40} - \frac{11904\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{35} + \frac{37169\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{120} -}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14} - 7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} + 21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} - 35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8} + 35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6} - 2\right.}$$

input
```
int((a + a/cos(c + d*x))^5/cos(c + d*x)^3,x)
```

output
```
(93*a^5*atanh(tan(c/2 + (d*x)/2)))/(8*d) - ((37169*a^5*tan(c/2 + (d*x)/2)^
5)/120 - (943*a^5*tan(c/2 + (d*x)/2)^3)/6 - (11904*a^5*tan(c/2 + (d*x)/2)^
7)/35 + (8773*a^5*tan(c/2 + (d*x)/2)^9)/40 - (155*a^5*tan(c/2 + (d*x)/2)^1
1)/2 + (93*a^5*tan(c/2 + (d*x)/2)^13)/8 + (419*a^5*tan(c/2 + (d*x)/2))/8)/
(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/
2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 - 7*tan(c/2 + (d
*x)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.19

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^5\,dx = \frac{a^5\left(-9765\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^6 + 29295\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\right.}{}$$

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^5,x)
```

output
```
(a**5*( - 9765*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**6 + 29
295*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4 - 29295*cos(c +
 d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 + 9765*cos(c + d*x)*log(ta
n((c + d*x)/2) - 1) + 9765*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c +
d*x)**6 - 29295*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4 + 2
9295*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2 - 9765*cos(c +
 d*x)*log(tan((c + d*x)/2) + 1) - 9765*cos(c + d*x)*sin(c + d*x)**5 + 2548
0*cos(c + d*x)*sin(c + d*x)**3 - 17115*cos(c + d*x)*sin(c + d*x) + 15328*s
in(c + d*x)**7 - 53648*sin(c + d*x)**5 + 64960*sin(c + d*x)**3 - 26880*sin
(c + d*x)))/(1680*cos(c + d*x)*d*(sin(c + d*x)**6 - 3*sin(c + d*x)**4 + 3*
sin(c + d*x)**2 - 1))
```

3.42 $\int \frac{\sec^5(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \frac{\sec^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{3\text{arctanh}(\sin(c+dx))}{2ad}+\frac{4\tan(c+dx)}{ad} -\frac{3\sec(c+dx)\tan(c+dx)}{2ad} -\frac{\sec^3(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))}+\frac{4\tan^3(c+dx)}{3ad}$$

output
```
-3/2*arctanh(sin(d*x+c))/a/d+4*tan(d*x+c)/a/d-3/2*sec(d*x+c)*tan(d*x+c)/a/
d-sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))+4/3*tan(d*x+c)^3/a/d
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.81

$$\int \frac{\sec^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{-18\text{arctanh}(\sin(c+dx))(1+\sec(c+dx))+(11+22\cos(c+dx)+7\cos(2(c+dx))+8\cos(3(c+dx))}{12ad(1+\sec(c+dx))}$$

input
```
Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

output
```
(-18*ArcTanh[Sin[c + d*x]]*(1 + Sec[c + d*x]) + (11 + 22*Cos[c + d*x] + 7*
Cos[2*(c + d*x)] + 8*Cos[3*(c + d*x)])*Sec[c + d*x]^3*Tan[c + d*x])/(12*a*
d*(1 + Sec[c + d*x]))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4305, 3042, 4274, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4305}$$

$$-\frac{\int \sec^3(c+dx)(3a-4a\sec(c+dx))dx}{a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(3a-4a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4274}$$

$$-\frac{3a\int \sec^3(c+dx)dx-4a\int \sec^4(c+dx)dx}{a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-4a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx}{a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\frac{4a\int (\tan^2(c+dx)+1)d(-\tan(c+dx))}{d}+3a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{4a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}}{a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4255}$$

$$-\frac{3a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{4a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}}{a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{4a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}}{a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{3a\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{4a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}}{a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}$$

input
```
Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

output
```
-((Sec[c + d*x]^3*Tan[c + d*x])/(d*(a + a*Sec[c + d*x]))) - (3*a*(ArcTanh[
Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)) + (4*a*(-Tan[c +
d*x] - Tan[c + d*x]^3/3))/d)/a^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4305
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Simp[d^2/(a*b)   Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
 - 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ
[a^2 - b^2, 0] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{5}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-}{da}$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{5}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-}{da}$
parallelrisch	$\frac{(27\cos(dx+c)+9\cos(3dx+3c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(-27\cos(dx+c)-9\cos(3dx+3c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+44(\cos(}{6ad(\cos(3dx+3c)+3\cos(dx+c))}$
risch	$\frac{i(9e^{6i(dx+c)}+9e^{5i(dx+c)}+24e^{4i(dx+c)}+24e^{3i(dx+c)}+39e^{2i(dx+c)}+7e^{i(dx+c)}+16)}{3da(e^{2i(dx+c)}+1)^{3}(e^{i(dx+c)}+1)}-\frac{3\ln(e^{i(dx+c)}+i)}{2ad}+\frac{3\ln(e^{i(}}{2}$
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{ad}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{37\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3ad}+\frac{49\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{3ad}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{4}}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2ad}-\frac{3\ln(t}{}$

input
```
int(sec(d*x+c)^5/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(tan(1/2*d*x+1/2*c)-1/3/(tan(1/2*d*x+1/2*c)+1)^3+1/(tan(1/2*d*x+1/2*
c)+1)^2-5/2/(tan(1/2*d*x+1/2*c)+1)-3/2*ln(tan(1/2*d*x+1/2*c)+1)-1/3/(tan(1
/2*d*x+1/2*c)-1)^3-1/(tan(1/2*d*x+1/2*c)-1)^2-5/2/(tan(1/2*d*x+1/2*c)-1)+3
/2*ln(tan(1/2*d*x+1/2*c)-1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.20

$$\int \frac{\sec^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{9\left(\cos\left(dx+c\right)^4+\cos\left(dx+c\right)^3\right)\log\left(\sin\left(dx+c\right)+1\right)-9\left(\cos\left(dx+c\right)^4+\cos\left(dx+c\right)^3\right)\log\left(-\sin\left(\right.\right.}{12\left(ad\cos\left(dx+c\right)^4+ad\cos\right.}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/12*(9*(cos(d*x + c)^4 + cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 9*(cos(
d*x + c)^4 + cos(d*x + c)^3)*log(-sin(d*x + c) + 1) - 2*(16*cos(d*x + c)^3
 + 7*cos(d*x + c)^2 - cos(d*x + c) + 2)*sin(d*x + c))/(a*d*cos(d*x + c)^4
 + a*d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sec^5(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**5/(a+a*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**5/(sec(c + d*x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(97) = 194$.

Time = 0.05 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.99

$$\int \frac{\sec^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a-\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}} - \frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a} + \frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a} + \frac{6\sin(dx+c)}{a(\cos(dx+c)+1)}}{6d}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/6*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 16*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a - 3*a*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6) - 9*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1
)/a + 9*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 6*sin(d*x + c)/(a*(co
s(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.11

$$\int \frac{\sec^5(c+dx)}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{\frac{9\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{9\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}+\frac{2\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-16\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+9\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3a}}{6d}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/6*(9*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 9*log(abs(tan(1/2*d*x + 1/2
*c) - 1))/a - 6*tan(1/2*d*x + 1/2*c)/a + 2*(15*tan(1/2*d*x + 1/2*c)^5 - 16
*tan(1/2*d*x + 1/2*c)^3 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2
 - 1)^3*a))/d
```

Mupad [B] (verification not implemented)

Time = 10.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.93

$$\int \frac{\sec^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a\,d}-\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a\,d}-\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-\frac{16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)^3}$$

input
```
int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))),x)
```

output
```
tan(c/2 + (d*x)/2)/(a*d) - (3*atanh(tan(c/2 + (d*x)/2)))/(a*d) - (3*tan(c/
2 + (d*x)/2) - (16*tan(c/2 + (d*x)/2)^3)/3 + 5*tan(c/2 + (d*x)/2)^5)/(a*d*
(tan(c/2 + (d*x)/2)^2 - 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.84

$$\int \frac{\sec^5(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{9\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^3 - 9\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c) - 9}{}$$

input
```
int(sec(d*x+c)^5/(a+a*sec(d*x+c)),x)
```

output
```
(9*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**3 - 9*cos(c + d*x)
*log(tan((c + d*x)/2) - 1)*sin(c + d*x) - 9*cos(c + d*x)*log(tan((c + d*x)
/2) + 1)*sin(c + d*x)**3 + 9*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c
+ d*x) + 9*cos(c + d*x)*sin(c + d*x)**2 - 6*cos(c + d*x) + 16*sin(c + d*x)
**4 - 24*sin(c + d*x)**2 + 6)/(6*cos(c + d*x)*sin(c + d*x)*a*d*(sin(c + d*
x)**2 - 1))
```

3.43 $\int \frac{\sec^4(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 85

$$\int \frac{\sec^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{3\text{arctanh}(\sin(c+dx))}{2ad} - \frac{2\tan(c+dx)}{ad} + \frac{3\sec(c+dx)\tan(c+dx)}{2ad} - \frac{\sec^2(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))}$$

output
```
3/2*arctanh(sin(d*x+c))/a/d-2*tan(d*x+c)/a/d+3/2*sec(d*x+c)*tan(d*x+c)/a/d
-sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int \frac{\sec^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{3\text{arctanh}(\sin(c+dx))(1+\sec(c+dx)) - (1+\cos(c+dx)+2\cos(2(c+dx)))\sec^2(c+dx)\tan(c+dx)}{2ad(1+\sec(c+dx))}$$

input
```
Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

output
```
(3*ArcTanh[Sin[c + d*x]]*(1 + Sec[c + d*x]) - (1 + Cos[c + d*x] + 2*Cos[2*
(c + d*x)])*Sec[c + d*x]^2*Tan[c + d*x])/(2*a*d*(1 + Sec[c + d*x]))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {3042, 4305, 3042, 4274, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4305}$$

$$-\frac{\int \sec^2(c+dx)(2a-3a\sec(c+dx))dx}{a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(2a-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4274}$$

$$-\frac{2a\int \sec^2(c+dx)dx-3a\int \sec^3(c+dx)dx}{a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx-3a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{-\frac{2a\int 1d(-\tan(c+dx))}{d}-3a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow 24$$

$$-\frac{\frac{2a\tan(c+dx)}{d}-3a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow 4255$$

$$-\frac{\frac{2a\tan(c+dx)}{d}-3a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow 3042$$

$$-\frac{\frac{2a\tan(c+dx)}{d}-3a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow 4257$$

$$-\frac{\frac{2a\tan(c+dx)}{d}-3a\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}$$

input
```
Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

output
```
-((Sec[c + d*x]^2*Tan[c + d*x])/(d*(a + a*Sec[c + d*x]))) - ((2*a*Tan[c +
d*x])/d - 3*a*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(
2*d)))/a^2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4305
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Simp[d^2/(a*b)   Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
 - 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ
[a^2 - b^2, 0] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.24

method	result
parallelrisch	$\frac{(-3\cos(2dx+2c)-3)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(3\cos(2dx+2c)+3)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-2(1+2\cos(2dx+2c)+\cos(dx+c)}{2da(1+\cos(2dx+2c))}$
derivativedivides	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right.}}{da}$
default	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right.}}{da}$
risch	$-\frac{i\left(3\,e^{4i(dx+c)}+3\,e^{3i(dx+c)}+5\,e^{2i(dx+c)}+e^{i(dx+c)}+4\right)}{da\left(e^{2i(dx+c)}+1\right)^2\left(e^{i(dx+c)}+1\right)}+\frac{3\ln\left(e^{i(dx+c)}+i\right)}{2ad}-\frac{3\ln\left(e^{i(dx+c)}-i\right)}{2ad}$
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{ad}+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2ad}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2ad}$

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*((-3*cos(2*d*x+2*c)-3)*ln(tan(1/2*d*x+1/2*c)-1)+(3*cos(2*d*x+2*c)+3)*l
n(tan(1/2*d*x+1/2*c)+1)-2*(1+2*cos(2*d*x+2*c)+cos(d*x+c))*tan(1/2*d*x+1/2*
c))/d/a/(1+cos(2*d*x+2*c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.32

$$\int \frac{\sec^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{3\left(\cos(dx+c)^3+\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-3\left(\cos(dx+c)^3+\cos(dx+c)^2\right)\log(-\sin(dx}{4\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2\right)}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(3*(cos(d*x + c)^3 + cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 3*(cos(d*
x + c)^3 + cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(4*cos(d*x + c)^2 +
cos(d*x + c) - 1)*sin(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sec^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**4/(a+a*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.91

$$\int \frac{\sec^4(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= -\frac{\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{2\sin(dx+c)}{a(\cos(dx+c)+1)}}{2d}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a + 3*log
(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(cos(d*x + c)
+ 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.19

$$\int \frac{\sec^4(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{\frac{3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}+\frac{2\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a}}{2d}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(3*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 3*log(abs(tan(1/2*d*x + 1/2*
c) - 1))/a - 2*tan(1/2*d*x + 1/2*c)/a + 2*(3*tan(1/2*d*x + 1/2*c)^3 - tan(
1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a))/d
```

Mupad [B] (verification not implemented)

Time = 9.89 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

$$\int \frac{\sec^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a\right)}$$

input
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))),x)
```

output
```
(3*atanh(tan(c/2 + (d*x)/2)))/(a*d) - tan(c/2 + (d*x)/2)/(a*d) - (tan(c/2
+ (d*x)/2) - 3*tan(c/2 + (d*x)/2)^3)/(d*(a - 2*a*tan(c/2 + (d*x)/2)^2 + a*
tan(c/2 + (d*x)/2)^4))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.74

$$\int \frac{\sec^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{4\cos(dx+c)\sin(dx+c)^2-2\cos(dx+c)-3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^3+3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{2\sin(dx+}$$

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c)),x)
```

output
```
(4*cos(c + d*x)*sin(c + d*x)**2 - 2*cos(c + d*x) - 3*log(tan((c + d*x)/2)
- 1)*sin(c + d*x)**3 + 3*log(tan((c + d*x)/2) - 1)*sin(c + d*x) + 3*log(ta
n((c + d*x)/2) + 1)*sin(c + d*x)**3 - 3*log(tan((c + d*x)/2) + 1)*sin(c +
d*x) - 3*sin(c + d*x)**2 + 2)/(2*sin(c + d*x)*a*d*(sin(c + d*x)**2 - 1))
```

3.44 $\int \frac{\sec^3(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \frac{\sec^3(c+dx)}{a+a\sec(c+dx)}\,dx=-\frac{\operatorname{arctanh}(\sin(c+dx))}{ad}+\frac{\tan(c+dx)}{ad}+\frac{\tan(c+dx)}{d(a+a\sec(c+dx))}$$

output
```
-arctanh(sin(d*x+c))/a/d+tan(d*x+c)/a/d+tan(d*x+c)/d/(a+a*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{\sec^3(c+dx)}{a+a\sec(c+dx)}\,dx=\frac{-\operatorname{arctanh}(\sin(c+dx))+\frac{(2+\sec(c+dx))\tan(c+dx)}{1+\sec(c+dx)}}{ad}$$

input
```
Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

output
```
(-ArcTanh[Sin[c + d*x]] + ((2 + Sec[c + d*x])*Tan[c + d*x])/(1 + Sec[c + d
*x]))/(a*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4277, 3042, 4276, 3042, 4257, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{ 4277}$$

$$\frac{\tan(c+dx)}{ad}-\int \frac{\sec^2(c+dx)}{\sec(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan(c+dx)}{ad}-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{ 4276}$$

$$-\frac{\int \sec(c+dx)dx}{a}+\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx+\frac{\tan(c+dx)}{ad}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{\tan(c+dx)}{ad}$$

$$\Big\downarrow \text{ 4257}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{\text{arctanh}(\sin(c+dx))}{ad}+\frac{\tan(c+dx)}{ad}$$

$$\Big\downarrow \text{ 4281}$$

$$-\frac{\text{arctanh}(\sin(c+dx))}{ad}+\frac{\tan(c+dx)}{ad}+\frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

input
```
Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

output
```
-(ArcTanh[Sin[c + d*x]]/(a*d)) + Tan[c + d*x]/(a*d) + Tan[c + d*x]/(d*(a +
 a*Sec[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4276
```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[1/b   Int[Csc[e + f*x], x], x] - Simp[a/b   Int[Csc[e + f*x]/(
a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 4277
```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[-Cot[e + f*x]/(b*f), x] - Simp[a/b   Int[Csc[e + f*x]^2/(a + b
*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.45

method	result	size
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da}$	74
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da}$	74
parallelrisch	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\cos(dx+c)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad\cos(dx+c)}$	82
risch	$\frac{2i\left(e^{2i(dx+c)}+e^{i(dx+c)}+2\right)}{da\left(e^{i(dx+c)}+1\right)\left(e^{2i(dx+c)}+1\right)}+\frac{\ln\left(e^{i(dx+c)}-i\right)}{ad}-\frac{\ln\left(e^{i(dx+c)}+i\right)}{ad}$	98
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}$	112

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(tan(1/2*d*x+1/2*c)-1/(tan(1/2*d*x+1/2*c)+1)-ln(tan(1/2*d*x+1/2*c)+1
)-1/(tan(1/2*d*x+1/2*c)-1)+ln(tan(1/2*d*x+1/2*c)-1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.90

$$\int \frac{\sec^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\left(\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right.\right.}{2\left(ad\cos\left(dx+c\right)^2+ad\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*((cos(d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c) + 1) - (cos(d*x + c
)^2 + cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*cos(d*x + c) + 1)*sin(d*
x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sec^3(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**3/(a+a*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**3/(sec(c + d*x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(51) = 102.

Time = 0.05 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.33

$$\int \frac{\sec^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a} - \frac{2\sin(dx+c)}{\left(a-\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x +
 c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.65

$$\int \frac{\sec^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}+\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a}}{d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-(log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - log(abs(tan(1/2*d*x + 1/2*c) - 1)
)/a - tan(1/2*d*x + 1/2*c)/a + 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*
c)^2 - 1)*a))/d
```

Mupad [B] (verification not implemented)

Time = 9.86 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.31

$$\int \frac{\sec^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}$$

input
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))),x)
```

output
```
(2*tan(c/2 + (d*x)/2))/(d*(a - a*tan(c/2 + (d*x)/2)^2)) - (2*atanh(tan(c/2
 + (d*x)/2)))/(a*d) + tan(c/2 + (d*x)/2)/(a*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.80

$$\int \frac{\sec^3(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)-\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\sin(dx+c)+\cos(c}{\cos(dx+c)\sin(dx+c)\,ad}$$

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c)),x)
```

output
```
(cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x) - cos(c + d*x)*log(ta
n((c + d*x)/2) + 1)*sin(c + d*x) + cos(c + d*x) + 2*sin(c + d*x)**2 - 1)/(
cos(c + d*x)*sin(c + d*x)*a*d)
```

3.45 $\int \frac{\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 38

$$\int \frac{\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{ad} - \frac{\tan(c+dx)}{d(a+a\sec(c+dx))}$$

output
```
arctanh(sin(d*x+c))/a/d-tan(d*x+c)/d/(a+a*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\coth^{-1}(\sin(c+dx)) - \tan\left(\frac{1}{2}(c+dx)\right)}{ad}$$

input
```
Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

output
```
(ArcCoth[Sin[c + d*x]] - Tan[(c + d*x)/2])/(a*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4276, 3042, 4257, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{ 4276}$$

$$\frac{\int \sec(c+dx)dx}{a}-\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\text{arctanh}(\sin(c+dx))}{ad}-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{ 4281}$$

$$\frac{\text{arctanh}(\sin(c+dx))}{ad}-\frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

input
```
Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

output
```
ArcTanh[Sin[c + d*x]]/(a*d) - Tan[c + d*x]/(d*(a + a*Sec[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4276
```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[1/b   Int[Csc[e + f*x], x], x] - Simp[a/b   Int[Csc[e + f*x]/(
a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

method	result	size
derivativedivides	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	46
default	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	46
parallelrisch	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	46
risch	$-\frac{2i}{da\left(e^{i(dx+c)}+1\right)}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{ad}-\frac{\ln\left(e^{i(dx+c)}-i\right)}{ad}$	65
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{ad}}{-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}$	93

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(-tan(1/2*d*x+1/2*c)-ln(tan(1/2*d*x+1/2*c)-1)+ln(tan(1/2*d*x+1/2*c)+
1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \frac{\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{(\cos(dx+c)+1)\log(\sin(dx+c)+1) - (\cos(dx+c)+1)\log(-\sin(dx+c)+1) - 2\sin(dx+c)}{2\,(ad\cos(dx+c)+ad)}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - (cos(d*x + c) + 1)*log(-si
n(d*x + c) + 1) - 2*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sec^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**2/(a+a*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.97

$$\int \frac{\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x +
c) + 1) - 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int \frac{\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a} - \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a} - \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}}{d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
(log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - log(abs(tan(1/2*d*x + 1/2*c) - 1))
/a - tan(1/2*d*x + 1/2*c)/a)/d
```

Mupad [B] (verification not implemented)

Time = 9.92 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right) - \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}$$

input
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))),x)
```

output
```
(2*atanh(tan(c/2 + (d*x)/2)) - tan(c/2 + (d*x)/2))/(a*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int \frac{\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}$$

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c)),x)
```

output
```
( - log(tan((c + d*x)/2) - 1) + log(tan((c + d*x)/2) + 1) - tan((c + d*x)/
2))/(a*d)
```

3.46 $\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\tan(c+dx)}{d(a+a\sec(c+dx))}$$

output
```
tan(d*x+c)/d/(a+a*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\tan\left(\frac{1}{2}(c+dx)\right)}{ad}$$

input
```
Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x]),x]
```

output
```
Tan[(c + d*x)/2]/(a*d)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{ 4281}$$

$$\frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

input
```
Int[Sec[c + d*x]/(a + a*Sec[c + d*x]),x]
```

output
```
Tan[c + d*x]/(d*(a + a*Sec[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}$	17
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}$	17
norman	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}$	17
parallelrisch	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}$	17
risch	$\frac{2i}{da\left(e^{i(dx+c)}+1\right)}$	23

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/a/d*tan(1/2*d*x+1/2*c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\sin(dx+c)}{ad\cos(dx+c)+ad}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
sin(d*x + c)/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sec(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)/(sec(c + d*x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\sin(dx+c)}{ad(\cos(dx+c)+1)}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
sin(d*x + c)/(a*d*(cos(d*x + c) + 1))
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{ad}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
tan(1/2*d*x + 1/2*c)/(a*d)
```

Mupad [B] (verification not implemented)

Time = 9.87 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}$$

input
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))),x)
```

output
```
tan(c/2 + (d*x)/2)/(a*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}$$

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c)),x)
```

output
```
tan((c + d*x)/2)/(a*d)
```

3.47 $\int \frac{1}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 29

$$\int \frac{1}{a+a\sec(c+dx)}\,dx=\frac{x}{a}-\frac{\tan(c+dx)}{d(a+a\sec(c+dx))}$$

output
```
x/a-tan(d*x+c)/d/(a+a*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.00

$$\int \frac{1}{a+a\sec(c+dx)}\,dx = \frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(dx\cos\left(\frac{dx}{2}\right)+dx\cos\left(c+\frac{dx}{2}\right)-2\sin\left(\frac{dx}{2}\right)\right)}{2ad}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(-1),x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]*(d*x*Cos[(d*x)/2] + d*x*Cos[c + (d*x)/2] - 2*Si
n[(d*x)/2]))/(2*a*d)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4264, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{ 4264}$$

$$-\frac{\int -adx}{a^2}-\frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{x}{a}-\frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

input
```
Int[(a + a*Sec[c + d*x])^(-1),x]
```

output
```
x/a - Tan[c + d*x]/(d*(a + a*Sec[c + d*x]))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4264
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Simp[1/(a^2*(2*n + 1))
   Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Int
egerQ[2*n]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

method	result	size
parallelrisch	$\frac{dx-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}$	23
norman	$\frac{x}{a}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}$	24
risch	$\frac{x}{a}-\frac{2i}{da\left(\mathrm{e}^{i(dx+c)}+1\right)}$	29
derivativedivides	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	32
default	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	32

input
```
int(1/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(d*x-tan(1/2*d*x+1/2*c))/a/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int \frac{1}{a+a\sec(c+dx)}\,dx=\frac{dx\cos(dx+c)+dx-\sin(dx+c)}{ad\cos(dx+c)+ad}$$

input
```
integrate(1/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
(d*x*cos(d*x + c) + d*x - sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{1}{a + a\sec(c + dx)}\,dx = \frac{\int \frac{1}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(1/(a+a*sec(d*x+c)),x)
```

output
```
Integral(1/(sec(c + d*x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int \frac{1}{a + a\sec(c + dx)}\,dx = \frac{\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

input
```
integrate(1/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x +
c) + 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{1}{a + a\sec(c + dx)}\,dx = \frac{\frac{dx+c}{a} - \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}}{d}$$

input
```
integrate(1/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
((d*x + c)/a - tan(1/2*d*x + 1/2*c)/a)/d
```

Mupad [B] (verification not implemented)

Time = 9.85 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1}{a + a\sec(c + dx)}\,dx = \frac{x}{a} - \frac{\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{a\,d}$$

input
```
int(1/(a + a/cos(c + d*x)),x)
```

output
```
x/a - tan(c/2 + (d*x)/2)/(a*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{1}{a + a\sec(c + dx)}\,dx = \frac{-\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + dx}{ad}$$

input
```
int(1/(a+a*sec(d*x+c)),x)
```

output
```
( - tan((c + d*x)/2) + d*x)/(a*d)
```

3.48 $\int \frac{\cos(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \frac{\cos(c+dx)}{a+a\sec(c+dx)}\,dx=-\frac{x}{a}+\frac{2\sin(c+dx)}{ad}-\frac{\sin(c+dx)}{d(a+a\sec(c+dx))}$$

output
```
-x/a+2*sin(d*x+c)/a/d-sin(d*x+c)/d/(a+a*sec(d*x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 89 vs. 2(44) = 88.

Time = 0.54 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.02

$$\int \frac{\cos(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$=\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(-2dx\cos\left(\frac{dx}{2}\right)-2dx\cos\left(c+\frac{dx}{2}\right)+5\sin\left(\frac{dx}{2}\right)+\sin\left(c+\frac{dx}{2}\right)+\sin\left(c+\frac{3dx}{2}\right)+s\right.}{4ad}$$

input
```
Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x]),x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]*(-2*d*x*Cos[(d*x)/2] - 2*d*x*Cos[c + (d*x)/2] +
 5*Sin[(d*x)/2] + Sin[c + (d*x)/2] + Sin[c + (3*d*x)/2] + Sin[2*c + (3*d*x
)/2]))/(4*a*d)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 4306, 25, 3042, 4274, 24, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{4306}$$

$$-\frac{\int -\cos(c+dx)(2a-a\sec(c+dx))dx}{a^2}-\frac{\sin(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \cos(c+dx)(2a-a\sec(c+dx))dx}{a^2}-\frac{\sin(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{\sin(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4274}$$

$$\frac{2a\int \cos(c+dx)dx-a\int 1dx}{a^2}-\frac{\sin(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{24}$$

$$\frac{2a\int \cos(c+dx)dx-ax}{a^2}-\frac{\sin(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \ 3042$$

$$\frac{2a \int \sin \left(c + dx + \frac{\pi}{2}\right) dx - ax}{a^2} - \frac{\sin(c + dx)}{d(a \sec(c + dx) + a)}$$

$$\Big\downarrow \ 3117$$

$$\frac{\frac{2a \sin(c+dx)}{d} - ax}{a^2} - \frac{\sin(c + dx)}{d(a \sec(c + dx) + a)}$$

input
```
Int[Cos[c + d*x]/(a + a*Sec[c + d*x]),x]
```

output
```
-(Sin[c + d*x]/(d*(a + a*Sec[c + d*x]))) + (-(a*x) + (2*a*Sin[c + d*x])/d)
/a^2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

method	result	size
parallelrisch	$\frac{\sin(dx+c)-dx+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}$	28
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	56
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	56
risch	$-\frac{x}{a}-\frac{i e^{i(dx+c)}}{2ad}+\frac{i e^{-i(dx+c)}}{2ad}+\frac{2i}{da\left(e^{i(dx+c)}+1\right)}$	66
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{ad}-\frac{x}{a}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a}}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}$	76

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/a/d*(sin(d*x+c)-d*x+tan(1/2*d*x+1/2*c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{\cos(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{dx\cos(dx+c)+dx-(\cos(dx+c)+2)\sin(dx+c)}{ad\cos(dx+c)+ad}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-(d*x*cos(d*x + c) + d*x - (cos(d*x + c) + 2)*sin(d*x + c))/(a*d*cos(d*x +
 c) + a*d)
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\cos(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)/(sec(c + d*x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(44) = 88$.

Time $= 0.17$ (sec) , antiderivative size $= 92$, normalized size of antiderivative $= 2.09$

$$\int \frac{\cos(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{2\sin(dx+c)}{\left(a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 2*sin(d*x + c)/((a + a*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(c
os(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.32

$$\int \frac{\cos(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{dx+c}{a} - \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a} - \frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a}}{d}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-((d*x + c)/a - tan(1/2*d*x + 1/2*c)/a - 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*
d*x + 1/2*c)^2 + 1)*a))/d
```

Mupad [B] (verification not implemented)

Time = 10.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.50

$$\int \frac{\cos(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+(-c-d\,x)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

input
```
int(cos(c + d*x)/(a + a/cos(c + d*x)),x)
```

output
```
(sin(c/2 + (d*x)/2) - cos(c/2 + (d*x)/2)*(c + d*x) + 2*cos(c/2 + (d*x)/2)^
2*sin(c/2 + (d*x)/2))/(a*d*cos(c/2 + (d*x)/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{\cos(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{-\cos(dx+c)+\sin(dx+c)^2-\sin(dx+c)\,dx+1}{\sin(dx+c)\,ad}$$

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c)),x)
```

output
```
( - cos(c + d*x) + sin(c + d*x)**2 - sin(c + d*x)*d*x + 1)/(sin(c + d*x)*a
*d)
```

3.49 $\int \frac{\cos^2(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 74

$$\int \frac{\cos^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{3x}{2a}-\frac{2\sin(c+dx)}{ad}+\frac{3\cos(c+dx)\sin(c+dx)}{2ad} -\frac{\cos(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}$$

output
```
3/2*x/a-2*sin(d*x+c)/a/d+3/2*cos(d*x+c)*sin(d*x+c)/a/d-cos(d*x+c)*sin(d*x+
c)/d/(a+a*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.58

$$\int \frac{\cos^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(12dx\cos\left(\frac{dx}{2}\right)+12dx\cos\left(c+\frac{dx}{2}\right)-20\sin\left(\frac{dx}{2}\right)-4\sin\left(c+\frac{dx}{2}\right)-3\sin\left(c+\frac{3dx}{2}\right.\right.}{16ad}$$

input
```
Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]*(12*d*x*Cos[(d*x)/2] + 12*d*x*Cos[c + (d*x)/2]
- 20*Sin[(d*x)/2] - 4*Sin[c + (d*x)/2] - 3*Sin[c + (3*d*x)/2] - 3*Sin[2*c
+ (3*d*x)/2] + Sin[2*c + (5*d*x)/2] + Sin[3*c + (5*d*x)/2]))/(16*a*d)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4306, 25, 3042, 4274, 3042, 3115, 24, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{ 4306}$$

$$-\frac{\int -\cos^2(c+dx)(3a-2a\sec(c+dx))dx}{a^2}-\frac{\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \cos^2(c+dx)(3a-2a\sec(c+dx))dx}{a^2}-\frac{\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{3a-2a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx}{a^2}-\frac{\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{ 4274}$$

$$\frac{3a\int \cos^2(c+dx)dx-2a\int \cos(c+dx)dx}{a^2}-\frac{\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-2a\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{3a\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-2a\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{24}$$

$$\frac{3a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-2a\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3117}$$

$$\frac{3a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{2a\sin(c+dx)}{d}}{a^2}-\frac{\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}$$

input
```
Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

output
```
-((Cos[c + d*x]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))) + ((-2*a*Sin[c + d
*x])/d + 3*a*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/a^2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.57

method	result	size
parallelrisch	$\frac{6dx-4\sin(dx+c)+\sin(2dx+2c)-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}$	42
derivativedivides	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}}+3\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	74
default	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}}+3\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	74
risch	$\frac{3x}{2a}+\frac{i\,e^{i(dx+c)}}{2ad}-\frac{i\,e^{-i(dx+c)}}{2ad}-\frac{2i}{da\left(e^{i(dx+c)}+1\right)}+\frac{\sin(2dx+2c)}{4ad}$	83
norman	$\frac{\frac{3x}{2a}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{ad}+\frac{3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{a}+\frac{3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{2a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}}$	113

input
```
int(cos(d*x+c)^2/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/4*(6*d*x-4*sin(d*x+c)+sin(2*d*x+2*c)-4*tan(1/2*d*x+1/2*c))/a/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \frac{\cos^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{3\,dx\cos(dx+c)+3\,dx+\left(\cos(dx+c)^2-\cos(dx+c)-4\right)\sin(dx+c)}{2\left(ad\cos(dx+c)+ad\right)}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(3*d*x*cos(d*x + c) + 3*d*x + (cos(d*x + c)^2 - cos(d*x + c) - 4)*sin(
d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\cos^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(cos(d*x+c)**2/(a+a*sec(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.80

$$\int \frac{\cos^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-((sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
)/(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)
/(a*(cos(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{\cos^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{3\,(dx+c)}{a} - \frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a} - \frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(3*(d*x + c)/a - 2*tan(1/2*d*x + 1/2*c)/a - 2*(3*tan(1/2*d*x + 1/2*c)^
3 + tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a))/d
```

Mupad [B] (verification not implemented)

Time = 9.88 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.20

$$\int \frac{\cos^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right) - \frac{3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(c+d\,x)}{2} + 3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right) - 2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

input
```
int(cos(c + d*x)^2/(a + a/cos(c + d*x)),x)
```

output
```
-(sin(c/2 + (d*x)/2) - (3*cos(c/2 + (d*x)/2)*(c + d*x))/2 + 3*cos(c/2 + (d
*x)/2)^2*sin(c/2 + (d*x)/2) - 2*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/(
a*d*cos(c/2 + (d*x)/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

$$\int \frac{\cos^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cos(dx+c)\sin(dx+c)^2+2\cos(dx+c)-2\sin(dx+c)^2+3\sin(dx+c)\,dx-2}{2\sin(dx+c)\,ad}$$

input
```
int(cos(d*x+c)^2/(a+a*sec(d*x+c)),x)
```

output
```
(cos(c + d*x)*sin(c + d*x)**2 + 2*cos(c + d*x) - 2*sin(c + d*x)**2 + 3*sin
(c + d*x)*d*x - 2)/(2*sin(c + d*x)*a*d)
```

3.50 $\int \frac{\cos^3(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 94

$$\int \frac{\cos^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{3x}{2a}+\frac{4\sin(c+dx)}{ad}-\frac{3\cos(c+dx)\sin(c+dx)}{2ad} -\frac{\cos^2(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{4\sin^3(c+dx)}{3ad}$$

output
```
-3/2*x/a+4*sin(d*x+c)/a/d-3/2*cos(d*x+c)*sin(d*x+c)/a/d-cos(d*x+c)^2*sin(d
*x+c)/d/(a+a*sec(d*x+c))-4/3*sin(d*x+c)^3/a/d
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.52

$$\int \frac{\cos^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(-36dx\cos\left(\frac{dx}{2}\right)-36dx\cos\left(c+\frac{dx}{2}\right)+69\sin\left(\frac{dx}{2}\right)+21\sin\left(c+\frac{dx}{2}\right)+18\sin\left(c+\right.\right.}{48ad}$$

input
```
Integrate[Cos[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]*(-36*d*x*Cos[(d*x)/2] - 36*d*x*Cos[c + (d*x)/2]
 + 69*Sin[(d*x)/2] + 21*Sin[c + (d*x)/2] + 18*Sin[c + (3*d*x)/2] + 18*Sin[
2*c + (3*d*x)/2] - 2*Sin[2*c + (5*d*x)/2] - 2*Sin[3*c + (5*d*x)/2] + Sin[3
*c + (7*d*x)/2] + Sin[4*c + (7*d*x)/2]))/(48*a*d)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4306, 25, 3042, 4274, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{4306}$$

$$-\frac{\int -\cos^3(c+dx)(4a-3a\sec(c+dx))dx}{a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \cos^3(c+dx)(4a-3a\sec(c+dx))dx}{a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{4a-3a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx}{a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4274}$$

$$\frac{4a\int \cos^3(c+dx)dx-3a\int \cos^2(c+dx)dx}{a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4a\int \sin\left(c+dx+\frac{\pi}{2}\right)^3 dx - 3a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2 dx}{a^2} - \frac{\sin(c+dx)\cos^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \; 3113$$

$$\frac{-\frac{4a\int (1-\sin^2(c+dx))d(-\sin(c+dx))}{d} - 3a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2 dx}{a^2} - \frac{\sin(c+dx)\cos^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \; 2009$$

$$\frac{-3a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2 dx - \frac{4a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2} - \frac{\sin(c+dx)\cos^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \; 3115$$

$$\frac{-3a\left(\frac{\int 1dx}{2} + \frac{\sin(c+dx)\cos(c+dx)}{2d}\right) - \frac{4a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2} - \frac{\sin(c+dx)\cos^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \; 24$$

$$\frac{-\frac{4a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d} - 3a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d} + \frac{x}{2}\right)}{a^2} - \frac{\sin(c+dx)\cos^2(c+dx)}{d(a\sec(c+dx)+a)}$$

input
```
Int[Cos[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

output
```
-((Cos[c + d*x]^2*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))) + (-3*a*(x/2 + (
Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (4*a*(-Sin[c + d*x] + Sin[c + d*x]^3/3
))/d)/a^2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.57

method	result
parallelrisch	$\frac{-18dx+(31+\cos(3dx+3c)-\cos(2dx+2c)+17\cos(dx+c))\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{12da}$
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8\left(-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{8}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}}-3\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8\left(-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{8}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}}-3\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$
risch	$-\frac{3x}{2a}-\frac{7i\mathrm{e}^{i(dx+c)}}{8ad}+\frac{7i\mathrm{e}^{-i(dx+c)}}{8ad}+\frac{2i}{da\left(\mathrm{e}^{i(dx+c)}+1\right)}+\frac{\sin(3dx+3c)}{12ad}-\frac{\sin(2dx+2c)}{4ad}$
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{ad}-\frac{3x}{2a}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3ad}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{ad}-\frac{9x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2a}-\frac{9x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{2a}-\frac{3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{2a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}}$

input
```
int(cos(d*x+c)^3/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/12*(-18*d*x+(31+cos(3*d*x+3*c)-cos(2*d*x+2*c)+17*cos(d*x+c))*tan(1/2*d*x
+1/2*c))/d/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int \frac{\cos^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{9\,dx\cos(dx+c)+9\,dx-\left(2\,\cos(dx+c)^{3}-\cos(dx+c)^{2}+7\,\cos(dx+c)+16\right)\sin(dx+c)}{6\,(ad\cos(dx+c)+ad)}$$

input
```
integrate(cos(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/6*(9*d*x*cos(d*x + c) + 9*d*x - (2*cos(d*x + c)^3 - cos(d*x + c)^2 + 7*
cos(d*x + c) + 16)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.87

$$\int \frac{\cos^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a+\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{3\sin(dx+c)}{a(\cos(dx+c)+1)}}{3d}$$

input
```
integrate(cos(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/3*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a + 3*a*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6) - 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
 + 3*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\int \frac{\cos^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{9\,(dx+c)}{a} - \frac{6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a} - \frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+16\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3 a}}{6\,d}$$

input
```
integrate(cos(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/6*(9*(d*x + c)/a - 6*tan(1/2*d*x + 1/2*c)/a - 2*(15*tan(1/2*d*x + 1/2*c
)^5 + 16*tan(1/2*d*x + 1/2*c)^3 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x +
1/2*c)^2 + 1)^3*a))/d
```

Mupad [B] (verification not implemented)

Time = 10.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int \frac{\cos^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{15\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{3\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{4}-\frac{\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{12}+\frac{\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{24}}{a\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{3\,x}{2\,a}$$

input
```
int(cos(c + d*x)^3/(a + a/cos(c + d*x)),x)
```

output
```
((15*sin(c/2 + (d*x)/2))/8 + (3*sin((3*c)/2 + (3*d*x)/2))/4 - sin((5*c)/2
+ (5*d*x)/2)/12 + sin((7*c)/2 + (7*d*x)/2)/24)/(a*d*cos(c/2 + (d*x)/2)) -
(3*x)/(2*a)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.77

$$\int \frac{\cos^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{-3\cos(dx+c)\sin(dx+c)^2-6\cos(dx+c)-2\sin(dx+c)^4+12\sin(dx+c)^2-9\sin(dx+c)\,dx+6}{6\sin(dx+c)\,ad}$$

input
```
int(cos(d*x+c)^3/(a+a*sec(d*x+c)),x)
```

output
```
( - 3*cos(c + d*x)*sin(c + d*x)**2 - 6*cos(c + d*x) - 2*sin(c + d*x)**4 +
12*sin(c + d*x)**2 - 9*sin(c + d*x)*d*x + 6)/(6*sin(c + d*x)*a*d)
```

3.51 $\int \frac{\cos^4(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 118

$$\int \frac{\cos^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{15x}{8a} - \frac{4\sin(c+dx)}{ad} + \frac{15\cos(c+dx)\sin(c+dx)}{8ad} + \frac{5\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{\cos^3(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{4\sin^3(c+dx)}{3ad}$$

output
```
15/8*x/a-4*sin(d*x+c)/a/d+15/8*cos(d*x+c)*sin(d*x+c)/a/d+5/4*cos(d*x+c)^3*
sin(d*x+c)/a/d-cos(d*x+c)^3*sin(d*x+c)/d/(a+a*sec(d*x+c))+4/3*sin(d*x+c)^3
/a/d
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.47

$$\int \frac{\cos^4(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(360dx\cos\left(\frac{dx}{2}\right)+360dx\cos\left(c+\frac{dx}{2}\right)-552\sin\left(\frac{dx}{2}\right)-168\sin\left(c+\frac{dx}{2}\right)-120\sin\right.}{}$$

input
```
Integrate[Cos[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]*(360*d*x*Cos[(d*x)/2] + 360*d*x*Cos[c + (d*x)/2
] - 552*Sin[(d*x)/2] - 168*Sin[c + (d*x)/2] - 120*Sin[c + (3*d*x)/2] - 120
*Sin[2*c + (3*d*x)/2] + 40*Sin[2*c + (5*d*x)/2] + 40*Sin[3*c + (5*d*x)/2]
- 5*Sin[3*c + (7*d*x)/2] - 5*Sin[4*c + (7*d*x)/2] + 3*Sin[4*c + (9*d*x)/2]
 + 3*Sin[5*c + (9*d*x)/2]))/(384*a*d)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4306, 25, 3042, 4274, 3042, 3113, 2009, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{4306}$$

$$-\frac{\int -\cos^4(c+dx)(5a-4a\sec(c+dx))dx}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \cos^4(c+dx)(5a-4a\sec(c+dx))dx}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{5a-4a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4274}$$

$$\frac{5a\int \cos^4(c+dx)dx-4a\int \cos^3(c+dx)dx}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-4a\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{ 3113}$$

$$\frac{\frac{4a\int (1-\sin^2(c+dx))d(-\sin(c+dx))}{d}+5a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{5a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx+\frac{4a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{5a\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{4a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5a\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{4a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{5a\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{4a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\frac{4a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+5a\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\sec(c+dx)+a)}$$

input
```
Int[Cos[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

output
```
-((Cos[c + d*x]^3*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))) + ((4*a*(-Sin[c
+ d*x] + Sin[c + d*x]^3/3))/d + 5*a*((Cos[c + d*x]^3*Sin[c + d*x])/(4*d) +
 (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/a^2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.57

method	result
parallelrisch	$\frac{180dx+(-221+3\cos(4dx+4c)-2\cos(3dx+3c)+38\cos(2dx+2c)-82\cos(dx+c))\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{96ad}$
derivativedivides	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{4}-\frac{115\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{12}-\frac{109\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{12}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{4}}+\frac{15\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}}{da}$
default	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{4}-\frac{115\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{12}-\frac{109\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{12}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{4}}+\frac{15\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}}{da}$
risch	$\frac{15x}{8a}+\frac{7i e^{i(dx+c)}}{8ad}-\frac{7i e^{-i(dx+c)}}{8ad}-\frac{2i}{da\left(e^{i(dx+c)}+1\right)}+\frac{\sin(4dx+4c)}{32ad}-\frac{\sin(3dx+3c)}{12ad}+\frac{\sin(2dx+2c)}{2ad}$
norman	$\frac{\frac{15x}{8a}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{157\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{12ad}-\frac{187\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{12ad}-\frac{41\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{4ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{ad}+\frac{15x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2a}+\frac{45x\tan}{}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{4}}$

input
```
int(cos(d*x+c)^4/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/96*(180*d*x+(-221+3*cos(4*d*x+4*c)-2*cos(3*d*x+3*c)+38*cos(2*d*x+2*c)-82
*cos(d*x+c))*tan(1/2*d*x+1/2*c))/a/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.67

$$\int \frac{\cos^4(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{45\,dx\cos(dx+c)+45\,dx+\left(6\,\cos(dx+c)^4-2\,\cos(dx+c)^3+13\,\cos(dx+c)^2-19\,\cos(dx+c)-6\right.}{24\,(ad\cos(dx+c)+ad)}$$

input
```
integrate(cos(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/24*(45*d*x*cos(d*x + c) + 45*d*x + (6*cos(d*x + c)^4 - 2*cos(d*x + c)^3
+ 13*cos(d*x + c)^2 - 19*cos(d*x + c) - 64)*sin(d*x + c))/(a*d*cos(d*x + c
) + a*d)
```

Sympy [F]

$$\int \frac{\cos^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\cos^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(cos(d*x+c)**4/(a+a*sec(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.84

$$\int \frac{\cos^4(c+dx)}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{\frac{\frac{21\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{109\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{115\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{75\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a+\frac{4\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\,a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\,a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{45\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{12\,\sin(dx+c)}{a(\cos(dx+c)+1)}}{12\,d}$$

input
```
integrate(cos(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/12*((21*sin(d*x + c)/(cos(d*x + c) + 1) + 109*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 115*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 75*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7)/(a + 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 45*arctan(sin(d*x + c)/(cos(d
*x + c) + 1))/a + 12*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

$$\int \frac{\cos^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{45\,(dx+c)}{a} - \frac{24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a} - \frac{2\left(75\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+115\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+109\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4 a}}{24\,d}$$

input
```
integrate(cos(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/24*(45*(d*x + c)/a - 24*tan(1/2*d*x + 1/2*c)/a - 2*(75*tan(1/2*d*x + 1/2
*c)^7 + 115*tan(1/2*d*x + 1/2*c)^5 + 109*tan(1/2*d*x + 1/2*c)^3 + 21*tan(1
/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a))/d
```

Mupad [B] (verification not implemented)

Time = 11.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.83

$$\int \frac{\cos^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{15\,x}{8\,a} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d} - \frac{\frac{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{115\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}+\frac{109\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}+\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

input
```
int(cos(c + d*x)^4/(a + a/cos(c + d*x)),x)
```

output
```
(15*x)/(8*a) - tan(c/2 + (d*x)/2)/(a*d) - ((7*tan(c/2 + (d*x)/2))/4 + (109
*tan(c/2 + (d*x)/2)^3)/12 + (115*tan(c/2 + (d*x)/2)^5)/12 + (25*tan(c/2 +
(d*x)/2)^7)/4)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.75

$$\int \frac{\cos^4(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{-6\cos(dx+c)\sin(dx+c)^4+27\cos(dx+c)\sin(dx+c)^2+24\cos(dx+c)+8\sin(dx+c)^4-48\sin(}{24\sin(dx+c)\,ad}$$

input
```
int(cos(d*x+c)^4/(a+a*sec(d*x+c)),x)
```

output
```
( - 6*cos(c + d*x)*sin(c + d*x)**4 + 27*cos(c + d*x)*sin(c + d*x)**2 + 24*
cos(c + d*x) + 8*sin(c + d*x)**4 - 48*sin(c + d*x)**2 + 45*sin(c + d*x)*d*
x - 24)/(24*sin(c + d*x)*a*d)
```

3.52 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\begin{aligned}\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \frac{7\text{arctanh}(\sin(c+dx))}{2a^2d} - \frac{16\tan(c+dx)}{3a^2d} \\ &\quad + \frac{7\sec(c+dx)\tan(c+dx)}{2a^2d} \\ &\quad - \frac{8\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^3(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}\end{aligned}$$

output
```
7/2*arctanh(sin(d*x+c))/a^2/d-16/3*tan(d*x+c)/a^2/d+7/2*sec(d*x+c)*tan(d*x
+c)/a^2/d-8/3*sec(d*x+c)^2*tan(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*sec(d*x+c)^
3*tan(d*x+c)/d/(a+a*sec(d*x+c))^2
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.60

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{42\text{arctanh}(\sin(c+dx)) - \frac{(37+60\cos(c+dx)+43\cos(2(c+dx))+16\cos(3(c+dx)))\sec(c+dx)\tan(c+dx)}{(1+\cos(c+dx))^2}}{12a^2d}$$

input
```
Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

output
```
(42*ArcTanh[Sin[c + d*x]] - ((37 + 60*Cos[c + d*x] + 43*Cos[2*(c + d*x)] +
 16*Cos[3*(c + d*x)])*Sec[c + d*x]*Tan[c + d*x])/(1 + Cos[c + d*x])^2)/(12
*a^2*d)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4303, 3042, 4507, 3042, 4274, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{\sec^3(c+dx)(3a-5a\sec(c+dx))}{\sec(c+dx)a+a}dx}{3a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(3a-5a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{\frac{\int \sec^2(c+dx)\left(16a^2-21a^2\sec(c+dx)\right)dx}{a^2}+\frac{8\tan(c+dx)\sec^2(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(16a^2-21a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{8\tan(c+dx)\sec^2(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4274}$$

$$-\frac{\frac{16a^2\int \sec^2(c+dx)dx-21a^2\int \sec^3(c+dx)dx}{a^2}+\frac{8\tan(c+dx)\sec^2(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{16a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx-21a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}+\frac{8\tan(c+dx)\sec^2(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\frac{-\frac{16a^2\int 1d(-\tan(c+dx))}{d}-21a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}+\frac{8\tan(c+dx)\sec^2(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\frac{\frac{16a^2\tan(c+dx)}{d}-21a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}+\frac{8\tan(c+dx)\sec^2(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4255}$$

$$-\frac{\frac{\frac{16a^2\tan(c+dx)}{d}-21a^2\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}+\frac{8\tan(c+dx)\sec^2(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{16a^2\tan(c+dx)}{d}-21a^2\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}+\frac{8\tan(c+dx)\sec^2(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\frac{\frac{16a^2\tan(c+dx)}{d}-21a^2\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}+\frac{8\tan(c+dx)\sec^2(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/3*(Sec[c + d*x]^3*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^2) - ((8*Sec[c
+ d*x]^2*Tan[c + d*x])/(d*(1 + Sec[c + d*x])) + ((16*a^2*Tan[c + d*x])/d -
 21*a^2*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))
/a^2)/(3*a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{5}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{\cdots}{\tan(\cdots}}{2d\,a^2}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{5}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{\cdots}{\tan(\cdots}}{2d\,a^2}$
parallelrisch	$\frac{(-42\cos(2dx+2c)-42)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(42\cos(2dx+2c)+42)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-60\left(\cos(dx+c)+\frac{43\cos(2dx+\cdots}{60}\cdots\right.}{12a^2d(1+\cos(2dx+2c))}$
risch	$-\frac{i\left(21\,e^{6i(dx+c)}+63\,e^{5i(dx+c)}+98\,e^{4i(dx+c)}+126\,e^{3i(dx+c)}+97\,e^{2i(dx+c)}+75\,e^{i(dx+c)}+32\right)}{3d\,a^2\left(e^{2i(dx+c)}+1\right)^2\left(e^{i(dx+c)}+1\right)^3}-\frac{7\ln\left(e^{i(dx+c)}-i\right)}{2a^2d}+\cdots$
norman	$\frac{-\frac{13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{149\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{6ad}-\frac{100\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{3ad}+\frac{18\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{ad}-\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{6ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{6ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4a}-\frac{7\ln(\tan\cdots}{\cdots}$

input
```
int(sec(d*x+c)^5/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/d/a^2*(-1/3*tan(1/2*d*x+1/2*c)^3-7*tan(1/2*d*x+1/2*c)-1/(tan(1/2*d*x+1
/2*c)+1)^2+5/(tan(1/2*d*x+1/2*c)+1)+7*ln(tan(1/2*d*x+1/2*c)+1)+1/(tan(1/2*
d*x+1/2*c)-1)^2+5/(tan(1/2*d*x+1/2*c)-1)-7*ln(tan(1/2*d*x+1/2*c)-1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.32

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{21\left(\cos(dx+c)^4+2\cos(dx+c)^3+\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-21\left(\cos(dx+c)^4+2\cos(dx\right.}{12\left(a^2d\cos(dx+c)^4+\right.}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/12*(21*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*log(sin(d*x
+ c) + 1) - 21*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*log(-s
in(d*x + c) + 1) - 2*(32*cos(d*x + c)^3 + 43*cos(d*x + c)^2 + 6*cos(d*x +
c) - 3)*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2
*d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sec^5(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**5/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.54

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2-\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{6d}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/6*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3)/(a^2 - 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 21*log(sin(d*x + c)/(cos(d*x + c) + 1
) + 1)/a^2 + 21*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.99

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{21\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{21\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}+\frac{6\left(5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^2}-\frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+21a^4\tan(}{a^6}}{6d}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/6*(21*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 21*log(abs(tan(1/2*d*x +
1/2*c) - 1))/a^2 + 6*(5*tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c))/(
(tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2) - (a^4*tan(1/2*d*x + 1/2*c)^3 + 21*a^4
*tan(1/2*d*x + 1/2*c))/a^6)/d
```

Mupad [B] (verification not implemented)

Time = 9.68 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.99

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{7\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6\,a^2\,d} - \frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^2\right)} - \frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^2\,d}$$

input
```
int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^2),x)
```

output
```
(7*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - tan(c/2 + (d*x)/2)^3/(6*a^2*d) - (
3*tan(c/2 + (d*x)/2) - 5*tan(c/2 + (d*x)/2)^3)/(d*(a^2*tan(c/2 + (d*x)/2)^
4 - 2*a^2*tan(c/2 + (d*x)/2)^2 + a^2)) - (7*tan(c/2 + (d*x)/2))/(2*a^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.75

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{-21\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+42\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-21\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{}$$

input
```
int(sec(d*x+c)^5/(a+a*sec(d*x+c))^2,x)
```

output
```
( - 21*log(tan((c + d*x)/2) - 1)*tan((c + d*x)/2)**4 + 42*log(tan((c + d*x
)/2) - 1)*tan((c + d*x)/2)**2 - 21*log(tan((c + d*x)/2) - 1) + 21*log(tan(
(c + d*x)/2) + 1)*tan((c + d*x)/2)**4 - 42*log(tan((c + d*x)/2) + 1)*tan((
c + d*x)/2)**2 + 21*log(tan((c + d*x)/2) + 1) - tan((c + d*x)/2)**7 - 19*t
an((c + d*x)/2)**5 + 71*tan((c + d*x)/2)**3 - 39*tan((c + d*x)/2))/(6*a**2
*d*(tan((c + d*x)/2)**4 - 2*tan((c + d*x)/2)**2 + 1))
```

3.53 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{2\text{arctanh}(\sin(c+dx))}{a^2d}+\frac{4\tan(c+dx)}{3a^2d}+\frac{2\tan(c+dx)}{a^2d(1+\sec(c+dx))}-\frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}$$

output
```
-2*arctanh(sin(d*x+c))/a^2/d+4/3*tan(d*x+c)/a^2/d+2*tan(d*x+c)/a^2/d/(1+se
c(d*x+c))-1/3*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^2
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.17

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(-24\text{arctanh}(\sin(c+dx))\cos^3\left(\frac{1}{2}(c+dx)\right)+\sec(c+dx)\left(2\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.}{3a^2d(1+\sec(c+dx))^2}$$

input
```
Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^2*(-24*ArcTanh[Sin[c + d*x]]*Cos[(c + d*x)/
2]^3 + Sec[c + d*x]*(2*Sin[(c + d*x)/2] + 9*Sin[(3*(c + d*x))/2] + 5*Sin[(
5*(c + d*x))/2])))/(3*a^2*d*(1 + Sec[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4303, 27, 3042, 4496, 25, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{2\sec^2(c+dx)(a-2a\sec(c+dx))}{\sec(c+dx)a+a}dx}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\int \frac{\sec^2(c+dx)(a-2a\sec(c+dx))}{\sec(c+dx)a+a}dx}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4496}$$

$$-\frac{2\left(-\frac{\int -\sec(c+dx)\left(3a^2-2a^2\sec(c+dx)\right)dx}{a^2}-\frac{3\tan(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\left(\frac{\int \sec(c+dx)\left(3a^2-2a^2\sec(c+dx)\right)dx}{a^2}-\frac{3\tan(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$-\frac{2\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(3a^2-2a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}-\frac{3\tan(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 4274

$$-\frac{2\left(\frac{3a^2\int \sec(c+dx)dx-2a^2\int \sec^2(c+dx)dx}{a^2}-\frac{3\tan(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$-\frac{2\left(\frac{3a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2}-\frac{3\tan(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 4254

$$-\frac{2\left(\frac{\frac{2a^2\int 1d(-\tan(c+dx))}{d}+3a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{3\tan(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 24

$$-\frac{2\left(\frac{3a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{2a^2\tan(c+dx)}{d}}{a^2}-\frac{3\tan(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 4257

$$-\frac{2\left(\frac{\frac{3a^2\text{arctanh}(\sin(c+dx))}{d}-\frac{2a^2\tan(c+dx)}{d}}{a^2}-\frac{3\tan(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/3*(Sec[c + d*x]^2*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^2) - (2*((-3*Ta
n[c + d*x])/(d*(1 + Sec[c + d*x])) + ((3*a^2*ArcTanh[Sin[c + d*x]])/d - (2
*a^2*Tan[c + d*x])/d)/a^2))/(3*a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4496
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(b^2*(2*m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b
*B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Ne
Q[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d\,a^{2}}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d\,a^{2}}$
parallelrisch	$\frac{6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+7\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\left(\cos(dx+c)+\frac{5\cos(2dx+2c)}{14}+\frac{4}{7}\right)\mathrm{t}}{3a^{2}d\cos(dx+c)}$
risch	$\frac{4i\left(3\,e^{4i(dx+c)}+9\,e^{3i(dx+c)}+11\,e^{2i(dx+c)}+12\,e^{i(dx+c)}+5\right)}{3d\,a^{2}\left(e^{i(dx+c)}+1\right)^{3}\left(e^{2i(dx+c)}+1\right)}-\frac{2\ln\left(e^{i(dx+c)}+i\right)}{a^{2}d}+\frac{2\ln\left(e^{i(dx+c)}-i\right)}{a^{2}d}$
norman	$\frac{-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{34\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3ad}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{6ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}a}+\frac{2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^{2}d}-2\ln($

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/d/a^2*(1/3*tan(1/2*d*x+1/2*c)^3+5*tan(1/2*d*x+1/2*c)-2/(tan(1/2*d*x+1/
2*c)+1)-4*ln(tan(1/2*d*x+1/2*c)+1)-2/(tan(1/2*d*x+1/2*c)-1)+4*ln(tan(1/2*d
*x+1/2*c)-1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.64

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{3\left(\cos\left(dx+c\right)^3+2\,\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(\cos\left(dx+c\right)^3+2\,\cos\left(dx\right.\right.}{3\left(a^2d\cos\left(dx+c\right)^3+2\,a^2d\cos\right.}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/3*(3*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c
) + 1) - 3*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*log(-sin(d*x
 + c) + 1) - (10*cos(d*x + c)^2 + 14*cos(d*x + c) + 3)*sin(d*x + c))/(a^2*
d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sec^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.63

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}+\frac{12\sin(dx+c)}{\left(a^2-\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}}{6\,d}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/6*((15*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) +
1)^3)/a^2 - 12*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(d
*x + c)/(cos(d*x + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.19

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{12\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{12\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}+\frac{12\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^2}-\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/6*(12*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 12*log(abs(tan(1/2*d*x +
 1/2*c) - 1))/a^2 + 12*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*
a^2) - (a^4*tan(1/2*d*x + 1/2*c)^3 + 15*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

Mupad [B] (verification not implemented)

Time = 9.63 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{6\,a^2\,d}-\frac{4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^2\,d} - \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-a^2\right)}+\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a^2\,d}$$

input
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^2),x)
```

output
```
tan(c/2 + (d*x)/2)^3/(6*a^2*d) - (4*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - (
2*tan(c/2 + (d*x)/2))/(d*(a^2*tan(c/2 + (d*x)/2)^2 - a^2)) + (5*tan(c/2 +
(d*x)/2))/(2*a^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.54

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{12\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-12\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-12\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\tan\left(\frac{dx}{2}+\right.}{6a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\right.}$$

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^2,x)
```

output
```
(12*log(tan((c + d*x)/2) - 1)*tan((c + d*x)/2)**2 - 12*log(tan((c + d*x)/2
) - 1) - 12*log(tan((c + d*x)/2) + 1)*tan((c + d*x)/2)**2 + 12*log(tan((c
+ d*x)/2) + 1) + tan((c + d*x)/2)**5 + 14*tan((c + d*x)/2)**3 - 27*tan((c
+ d*x)/2))/(6*a**2*d*(tan((c + d*x)/2)**2 - 1))
```

3.54 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 66

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{a^2d} - \frac{5\tan(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}$$

output
```
arctanh(sin(d*x+c))/a^2/d-5/3*tan(d*x+c)/a^2/d/(1+sec(d*x+c))+1/3*tan(d*x+
c)/d/(a+a*sec(d*x+c))^2
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.27

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(6\operatorname{arctanh}(\sin(c+dx))\cos^3\left(\frac{1}{2}(c+dx)\right) - 3\sin\left(\frac{1}{2}(c+dx)\right) - 2\sin\left(\frac{3}{2}(c+\right.\right.}{3a^2d(1+\sec(c+dx))^2}$$

input
```
Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

output
```
(2*Cos[(c + d*x)/2]*Sec[c + d*x]^2*(6*ArcTanh[Sin[c + d*x]]*Cos[(c + d*x)/
2]^3 - 3*Sin[(c + d*x)/2] - 2*Sin[(3*(c + d*x))/2]))/(3*a^2*d*(1 + Sec[c +
 d*x])^2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4286, 25, 3042, 4486, 3042, 4257, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{ 4286}$$

$$\frac{\int -\frac{\sec(c+dx)(2a-3a\sec(c+dx))}{\sec(c+dx)a+a}dx}{3a^2}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{\int \frac{\sec(c+dx)(2a-3a\sec(c+dx))}{\sec(c+dx)a+a}dx}{3a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2a-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}$$

$$\Big\downarrow \text{ 4486}$$

$$\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{5a\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx-3\int \sec(c+dx)dx}{3a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{5a\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-3\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{3a^2}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{5a\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{3\text{arctanh}(\sin(c+dx))}{d}}{3a^2}$$

$$\Big\downarrow \text{4281}$$

$$\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{5a\tan(c+dx)}{d(a\sec(c+dx)+a)}-\frac{3\text{arctanh}(\sin(c+dx))}{d}}{3a^2}$$

input
```
Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

output
```
Tan[c + d*x]/(3*d*(a + a*Sec[c + d*x])^2) - ((-3*ArcTanh[Sin[c + d*x]])/d
+ (5*a*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4286
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Simp[1/(a^2*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[
a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d\,a^{2}}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d\,a^{2}}$
parallelrisch	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{6a^{2}d}$
risch	$-\frac{2i\left(3\,{\mathrm e}^{2i(dx+c)}+9\,{\mathrm e}^{i(dx+c)}+4\right)}{3d\,a^{2}\left({\mathrm e}^{i(dx+c)}+1\right)^{3}}-\frac{\ln\left({\mathrm e}^{i(dx+c)}-i\right)}{a^{2}d}+\frac{\ln\left({\mathrm e}^{i(dx+c)}+i\right)}{a^{2}d}$
norman	$\frac{-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{6ad}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{6ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{6ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}a}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^{2}d}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^{2}d}$

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/d/a^2*(-1/3*tan(1/2*d*x+1/2*c)^3-3*tan(1/2*d*x+1/2*c)-2*ln(tan(1/2*d*x
+1/2*c)-1)+2*ln(tan(1/2*d*x+1/2*c)+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.73

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{3\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\log(}{6\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(3*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - 3*(co
s(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-sin(d*x + c) + 1) - 2*(4*cos(d*x +
 c) + 5)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*
d)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sec^3\left(c+dx\right)}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.48

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= -\frac{\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{6\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{6\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{6\,d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/6*((9*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) +
1)^3)/a^2 - 6*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*x
 + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.17

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{6\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2} - \frac{6\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2} - \frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/6*(6*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*log(abs(tan(1/2*d*x + 1/
2*c) - 1))/a^2 - (a^4*tan(1/2*d*x + 1/2*c)^3 + 9*a^4*tan(1/2*d*x + 1/2*c))
/a^6)/d
```

Mupad [B] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.65

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-12\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6\,a^2\,d}$$

input
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^2),x)
```

output
```
-(9*tan(c/2 + (d*x)/2) - 12*atanh(tan(c/2 + (d*x)/2)) + tan(c/2 + (d*x)/2)
^3)/(6*a^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{-6\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+6\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{6a^2d}$$

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^2,x)
```

output
```
( - 6*log(tan((c + d*x)/2) - 1) + 6*log(tan((c + d*x)/2) + 1) - tan((c + d
*x)/2)**3 - 9*tan((c + d*x)/2))/(6*a**2*d)
```

3.55 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 55

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx=-\frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{2\tan(c+dx)}{3d\left(a^2+a^2\sec(c+dx)\right)}$$

output
```
-1/3*tan(d*x+c)/d/(a+a*sec(d*x+c))^2+2/3*tan(d*x+c)/d/(a^2+a^2*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx=\frac{\sec^3\left(\frac{1}{2}(c+dx)\right)\left(3\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)}{12a^2d}$$

input
```
Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Sec[(c + d*x)/2]^3*(3*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(12*a^2*d
)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4284, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{ 4284}$$

$$\frac{2\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a}-\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}-\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 4281}$$

$$\frac{2\tan(c+dx)}{3ad(a\sec(c+dx)+a)}-\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/3*Tan[c + d*x]/(d*(a + a*Sec[c + d*x])^2) + (2*Tan[c + d*x])/(3*a*d*(a
+ a*Sec[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4284
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Simp[m/(b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.58

method	result	size
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^{2}}$	32
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^{2}}$	32
parallelrisch	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{6a^{2}d}$	32
risch	$\frac{2i\left(1+3\,e^{i(dx+c)}\right)}{3d\,a^{2}\left(e^{i(dx+c)}+1\right)^{3}}$	36
norman	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{6ad}}{a\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}$	76

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/d/a^2*(1/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{(\cos(dx+c)+2)\sin(dx+c)}{3\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3*(cos(d*x + c) + 2)*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*
x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sec^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{6\,a^2d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)
^3)/(a^2*d)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{6\,a^2d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/6*(tan(1/2*d*x + 1/2*c)^3 + 3*tan(1/2*d*x + 1/2*c))/(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 9.45 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.55

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+3\right)}{6\,a^2\,d}$$

input
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^2),x)
```

output
```
(tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^2 + 3))/(6*a^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.55

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+3\right)}{6a^2d}$$

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^2,x)
```

output
```
(tan((c + d*x)/2)*(tan((c + d*x)/2)**2 + 3))/(6*a**2*d)
```

3.56 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 55

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2} dx = \frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\tan(c+dx)}{3d\,(a^2+a^2\sec(c+dx))}$$

output
```
1/3*tan(d*x+c)/d/(a+a*sec(d*x+c))^2+1/3*tan(d*x+c)/d/(a^2+a^2*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2} dx = \frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(3\sin\left(\frac{dx}{2}\right)-3\sin\left(c+\frac{dx}{2}\right)+2\sin\left(c+\frac{3dx}{2}\right)\right)}{12a^2d}$$

input
```
Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^3*(3*Sin[(d*x)/2] - 3*Sin[c + (d*x)/2] + 2*Sin[
c + (3*d*x)/2]))/(12*a^2*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4283, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4283}$$

$$\frac{\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4281}$$

$$\frac{\tan(c+dx)}{3ad(a\sec(c+dx)+a)}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

output
```
Tan[c + d*x]/(3*d*(a + a*Sec[c + d*x])^2) + Tan[c + d*x]/(3*a*d*(a + a*Sec
[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.58

method	result	size
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^{2}}$	32
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^{2}}$	32
parallelrisch	$-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{6a^{2}d}$	32
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{6ad}}{a}$	42
risch	$\frac{2i\left(3\,\mathrm{e}^{2i(dx+c)}+3\,\mathrm{e}^{i(dx+c)}+2\right)}{3d\,a^{2}\left(\mathrm{e}^{i(dx+c)}+1\right)^{3}}$	47

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/d/a^2*(-1/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{(2\cos(dx+c)+1)\sin(dx+c)}{3\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3*(2*cos(d*x + c) + 1)*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(
d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sec(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{6\,a^2d}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)
^3)/(a^2*d)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{6\,a^2d}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/6*(tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c))/(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.55

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-3\right)}{6\,a^2\,d}$$

input
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^2),x)
```

output
```
-(tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^2 - 3))/(6*a^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.58

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+3\right)}{6a^2d}$$

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^2,x)
```

output
```
(tan((c + d*x)/2)*( - tan((c + d*x)/2)**2 + 3))/(6*a**2*d)
```

3.57 $\int \frac{1}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 57

$$\int \frac{1}{(a+a\sec(c+dx))^2}\,dx = \frac{x}{a^2} - \frac{4\tan(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}$$

output
```
x/a^2-4/3*tan(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*tan(d*x+c)/d/(a+a*sec(d*x+c)
)^2
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.96

$$\int \frac{1}{(a+a\sec(c+dx))^2}\,dx = \frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(9dx\cos\left(\frac{dx}{2}\right)+9dx\cos\left(c+\frac{dx}{2}\right)+3dx\cos\left(c+\frac{3dx}{2}\right)+3dx\cos\left(2c+\frac{3dx}{2}\right)-18\text{s}\right.}{24a^2d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(-2),x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^3*(9*d*x*Cos[(d*x)/2] + 9*d*x*Cos[c + (d*x)/2]
+ 3*d*x*Cos[c + (3*d*x)/2] + 3*d*x*Cos[2*c + (3*d*x)/2] - 18*Sin[(d*x)/2]
+ 12*Sin[c + (d*x)/2] - 10*Sin[c + (3*d*x)/2]))/(24*a^2*d)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 4264, 25, 3042, 4407, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4264}$$

$$-\frac{\int -\frac{3a-a\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3a-a\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4407}$$

$$\frac{3x-4a\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3x-4a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4281}$$

$$\frac{3x-\frac{4a\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[(a + a*Sec[c + d*x])^(-2),x]
```

output
```
-1/3*Tan[c + d*x]/(d*(a + a*Sec[c + d*x])^2) + (3*x - (4*a*Tan[c + d*x])/(
d*(a + a*Sec[c + d*x])))/(3*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4264
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Simp[1/(a^2*(2*n + 1))
   Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Int
egerQ[2*n]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.63

method	result	size
parallelrisch	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+6dx-9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{6a^{2}d}$	36
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^{2}}$	46
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^{2}}$	46
norman	$\frac{\frac{x}{a}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{6ad}}{a}$	47
risch	$\frac{x}{a^{2}}-\frac{2i\left(6\,\mathrm{e}^{2i(dx+c)}+9\,\mathrm{e}^{i(dx+c)}+5\right)}{3d\,a^{2}\left(\mathrm{e}^{i(dx+c)}+1\right)^{3}}$	53

input
```
int(1/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*(tan(1/2*d*x+1/2*c)^3+6*d*x-9*tan(1/2*d*x+1/2*c))/a^2/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.40

$$\int \frac{1}{(a+a\sec(c+dx))^2}\,dx = \frac{3\,dx\cos\left(dx+c\right)^{2}+6\,dx\cos\left(dx+c\right)+3\,dx-\left(5\,\cos\left(dx+c\right)+4\right)\sin\left(dx+c\right)}{3\,\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

input
```
integrate(1/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3*(3*d*x*cos(d*x + c)^2 + 6*d*x*cos(d*x + c) + 3*d*x - (5*cos(d*x + c) +
 4)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(c+dx))^2}\,dx=\frac{\int \frac{1}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(1/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(1/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.26

$$\int \frac{1}{(a+a\sec(c+dx))^2}\,dx=-\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{6\,d}$$

input
```
integrate(1/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/6*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) +
1)^3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{6\,(dx+c)}{a^2} + \frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

input
```
integrate(1/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/6*(6*(d*x + c)/a^2 + (a^4*tan(1/2*d*x + 1/2*c)^3 - 9*a^4*tan(1/2*d*x + 1
/2*c))/a^6)/d
```

Mupad [B] (verification not implemented)

Time = 9.60 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.61

$$\int \frac{1}{(a+a\sec(c+dx))^2}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6\,d\,x}{6\,a^2\,d}$$

input
```
int(1/(a + a/cos(c + d*x))^2,x)
```

output
```
(tan(c/2 + (d*x)/2)^3 - 9*tan(c/2 + (d*x)/2) + 6*d*x)/(6*a^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.61

$$\int \frac{1}{(a+a\sec(c+dx))^2}\,dx = \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+6dx}{6a^2d}$$

input
```
int(1/(a+a*sec(d*x+c))^2,x)
```

output
```
(tan((c + d*x)/2)**3 - 9*tan((c + d*x)/2) + 6*d*x)/(6*a**2*d)
```

3.58 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{2x}{a^2}+\frac{10\sin(c+dx)}{3a^2d}-\frac{2\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

output
```
-2*x/a^2+10/3*sin(d*x+c)/a^2/d-2*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*sin(d
*x+c)/d/(a+a*sec(d*x+c))^2
```

Mathematica [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.35

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\sin(c+dx)\left(24\arcsin(\cos(c+dx))\cos^4\left(\frac{1}{2}(c+dx)\right)+\left(10+14\cos(c+dx)+3\cos^2(c+dx)\right)\sqrt{\sin^2(c+}\right.}{3a^2d\sqrt{1-\cos(c+dx)}(1+\cos(c+dx))^{5/2}}$$

input
```
Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Sin[c + d*x]*(24*ArcSin[Cos[c + d*x]]*Cos[(c + d*x)/2]^4 + (10 + 14*Cos[c
 + d*x] + 3*Cos[c + d*x]^2)*Sqrt[Sin[c + d*x]^2]))/(3*a^2*d*Sqrt[1 - Cos[c
 + d*x]]*(1 + Cos[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 4304, 27, 3042, 4508, 3042, 4274, 24, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{2\cos(c+dx)(2a-a\sec(c+dx))}{\sec(c+dx)a+a}dx}{3a^2}-\frac{\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{\cos(c+dx)(2a-a\sec(c+dx))}{\sec(c+dx)a+a}dx}{3a^2}-\frac{\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{2a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4508}$$

$$\frac{2\left(\frac{\int \cos(c+dx)\left(5a^2-3a^2\sec(c+dx)\right)dx}{a^2}-\frac{3\sin(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{\int \frac{5a^2-3a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{3\sin(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 4274}$$

$$\frac{2\left(\frac{5a^2\int\cos(c+dx)dx-3a^2\int 1dx}{a^2}-\frac{3\sin(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{2\left(\frac{5a^2\int\cos(c+dx)dx-3a^2x}{a^2}-\frac{3\sin(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\left(\frac{5a^2\int\sin\left(c+dx+\frac{\pi}{2}\right)dx-3a^2x}{a^2}-\frac{3\sin(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3117}$$

$$\frac{2\left(\frac{\frac{5a^2\sin(c+dx)}{d}-3a^2x}{a^2}-\frac{3\sin(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/3*Sin[c + d*x]/(d*(a + a*Sec[c + d*x])^2) + (2*((-3*Sin[c + d*x])/(d*(1
 + Sec[c + d*x])) + (-3*a^2*x + (5*a^2*Sin[c + d*x])/d)/a^2))/(3*a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

method	result	size
parallelrisch	$\frac{-12dx+6\sin(dx+c)+16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{6a^{2}d}$	55
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}-8\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^{2}}$	72
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}-8\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^{2}}$	72
risch	$-\frac{2x}{a^{2}}-\frac{i e^{i(dx+c)}}{2a^{2}d}+\frac{i e^{-i(dx+c)}}{2a^{2}d}+\frac{2i\left(9\,e^{2i(dx+c)}+15\,e^{i(dx+c)}+8\right)}{3d\,a^{2}\left(e^{i(dx+c)}+1\right)^{3}}$	90
norman	$\frac{-\frac{2x}{a}+\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{6ad}-\frac{2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)a}$	99

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*(-12*d*x+6*sin(d*x+c)+16*tan(1/2*d*x+1/2*c)-tan(1/2*d*x+1/2*c)*sec(1/2
*d*x+1/2*c)^2)/a^2/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{6\,dx\cos\left(dx+c\right)^{2}+12\,dx\cos\left(dx+c\right)+6\,dx-\left(3\,\cos\left(dx+c\right)^{2}+14\,\cos\left(dx+c\right)+10\right)\sin\left(dx+c\right)}{3\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/3*(6*d*x*cos(d*x + c)^2 + 12*d*x*cos(d*x + c) + 6*d*x - (3*cos(d*x + c)
^2 + 14*cos(d*x + c) + 10)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*c
os(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\cos(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(cos(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.64

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{12\sin(dx+c)}{\left(a^2+\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}}{6\,d}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/6*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) +
1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*sin(d*x +
c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.10

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{12\,(dx+c)}{a^2}-\frac{12\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^2}+\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/6*(12*(d*x + c)/a^2 - 12*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2
+ 1)*a^2) + (a^4*tan(1/2*d*x + 1/2*c)^3 - 15*a^4*tan(1/2*d*x + 1/2*c))/a^6
)/d
```

Mupad [B] (verification not implemented)

Time = 9.62 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-16\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-12\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+12\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3(c+}{6\,a^2\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3}$$

input
```
int(cos(c + d*x)/(a + a/cos(c + d*x))^2,x)
```

output
```
-(sin(c/2 + (d*x)/2) - 16*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) - 12*cos
(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 12*cos(c/2 + (d*x)/2)^3*(c + d*x))/
(6*a^2*d*cos(c/2 + (d*x)/2)^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-12\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2dx+27\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-12dx}{6a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)}$$

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^2,x)
```

output
```
( - tan((c + d*x)/2)**5 + 14*tan((c + d*x)/2)**3 - 12*tan((c + d*x)/2)**2*
d*x + 27*tan((c + d*x)/2) - 12*d*x)/(6*a**2*d*(tan((c + d*x)/2)**2 + 1))
```

3.59 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{7x}{2a^2} - \frac{16\sin(c+dx)}{3a^2d} + \frac{7\cos(c+dx)\sin(c+dx)}{2a^2d} - \frac{8\cos(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\cos(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

output
```
7/2*x/a^2-16/3*sin(d*x+c)/a^2/d+7/2*cos(d*x+c)*sin(d*x+c)/a^2/d-8/3*cos(d*
x+c)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(
d*x+c))^2
```

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.61

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(252dx\cos\left(\frac{dx}{2}\right)+252dx\cos\left(c+\frac{dx}{2}\right)+84dx\cos\left(c+\frac{3dx}{2}\right)+84dx\cos\left(2c+\frac{3dx}{2}\right)\right.}{}$$

input
```
Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^3*(252*d*x*Cos[(d*x)/2] + 252*d*x*Cos[c + (d*x)
/2] + 84*d*x*Cos[c + (3*d*x)/2] + 84*d*x*Cos[2*c + (3*d*x)/2] - 381*Sin[(d
*x)/2] + 147*Sin[c + (d*x)/2] - 239*Sin[c + (3*d*x)/2] - 63*Sin[2*c + (3*d
*x)/2] - 15*Sin[2*c + (5*d*x)/2] - 15*Sin[3*c + (5*d*x)/2] + 3*Sin[3*c + (
7*d*x)/2] + 3*Sin[4*c + (7*d*x)/2]))/(192*a^2*d)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4304, 25, 3042, 4508, 3042, 4274, 3042, 3115, 24, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{\cos^2(c+dx)(5a-3a\sec(c+dx))}{\sec(c+dx)a+a}dx}{3a^2}-\frac{\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\cos^2(c+dx)(5a-3a\sec(c+dx))}{\sec(c+dx)a+a}dx}{3a^2}-\frac{\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{5a-3a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4508}$$

$$\frac{\frac{\int \cos^2(c+dx)\left(21a^2-16a^2\sec(c+dx)\right)dx}{a^2}-\frac{8\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{21a^2-16a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx}{a^2}-\frac{8\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4274}$$

$$\frac{\frac{21a^2\int\cos^2(c+dx)dx-16a^2\int\cos(c+dx)dx}{a^2}-\frac{8\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{21a^2\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx-16a^2\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{8\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{21a^2\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-16a^2\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{8\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{21a^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-16a^2\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{8\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3117}$$

$$\frac{\frac{21a^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{16a^2\sin(c+dx)}{d}}{a^2}-\frac{8\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/3*(Cos[c + d*x]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^2) + ((-8*Cos[c +
 d*x]*Sin[c + d*x])/(d*(1 + Sec[c + d*x])) + ((-16*a^2*Sin[c + d*x])/d + 2
1*a^2*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/a^2)/(3*a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.60

method	result	size
parallelrisch	$\frac{-163\left(\cos(dx+c)+\frac{12\cos(2dx+2c)}{163}-\frac{3\cos(3dx+3c)}{163}+\frac{140}{163}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+168dx}{48a^{2}d}$	66
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}}+14\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^{2}}$	88
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}}+14\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^{2}}$	88
risch	$\frac{7x}{2a^{2}}-\frac{i e^{2i(dx+c)}}{8a^{2}d}+\frac{i e^{i(dx+c)}}{a^{2}d}-\frac{i e^{-i(dx+c)}}{a^{2}d}+\frac{i e^{-2i(dx+c)}}{8a^{2}d}-\frac{2i\left(12\,e^{2i(dx+c)}+21\,e^{i(dx+c)}+11\right)}{3d\,a^{2}\left(e^{i(dx+c)}+1\right)^{3}}$	126
norman	$\frac{\frac{7x}{2a}-\frac{13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{71\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{6ad}-\frac{19\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{6ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{6ad}+\frac{7x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{a}+\frac{7x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{2a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}a}$	135

input
```
int(cos(d*x+c)^2/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/48*(-163*(cos(d*x+c)+12/163*cos(2*d*x+2*c)-3/163*cos(3*d*x+3*c)+140/163)
*tan(1/2*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^2+168*d*x)/a^2/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.90

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{21\,dx\cos\left(dx+c\right)^2+42\,dx\cos\left(dx+c\right)+21\,dx+\left(3\,\cos\left(dx+c\right)^3-6\,\cos\left(dx+c\right)^2-43\,\cos\left(dx+c\right)-\right.}{6\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(21*d*x*cos(d*x + c)^2 + 42*d*x*cos(d*x + c) + 21*d*x + (3*cos(d*x + c
)^3 - 6*cos(d*x + c)^2 - 43*cos(d*x + c) - 32)*sin(d*x + c))/(a^2*d*cos(d*
x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\cos^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(cos(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.49

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{42\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{6\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/6*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)/(cos(d*x + c)
+ 1))/a^2)/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{21\,(dx+c)}{a^2} - \frac{6\left(5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a^2} + \frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-21\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/6*(21*(d*x + c)/a^2 - 6*(5*tan(1/2*d*x + 1/2*c)^3 + 3*tan(1/2*d*x + 1/2*
c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + (a^4*tan(1/2*d*x + 1/2*c)^3 - 2
1*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

Mupad [B] (verification not implemented)

Time = 9.63 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.03

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right) - 22\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right) - 30\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right) + 12\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}\right.}{6\,a^2\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3}$$

input
```
int(cos(c + d*x)^2/(a + a/cos(c + d*x))^2,x)
```

output
```
(sin(c/2 + (d*x)/2) - 22*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) - 30*cos(
c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 12*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d
*x)/2) + 21*cos(c/2 + (d*x)/2)^3*(c + d*x))/(6*a^2*d*cos(c/2 + (d*x)/2)^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.89

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{-9\cos(dx+c)\sin(dx+c)^2+21\cos(dx+c)\sin(dx+c)\,dx-2\cos(dx+c)-3\sin(dx+c)^4-31\sin}{6\sin(dx+c)\,a^2d\,(\cos(dx+c)+1)}$$

input
```
int(cos(d*x+c)^2/(a+a*sec(d*x+c))^2,x)
```

output
```
( - 9*cos(c + d*x)*sin(c + d*x)**2 + 21*cos(c + d*x)*sin(c + d*x)*d*x - 2*
cos(c + d*x) - 3*sin(c + d*x)**4 - 31*sin(c + d*x)**2 + 21*sin(c + d*x)*d*
x + 2)/(6*sin(c + d*x)*a**2*d*(cos(c + d*x) + 1))
```

3.60 $\int \frac{\cos^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 124

$$\int \frac{\cos^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{5x}{a^2} + \frac{12\sin(c+dx)}{a^2d} - \frac{5\cos(c+dx)\sin(c+dx)}{a^2d} - \frac{10\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\cos^2(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{4\sin^3(c+dx)}{a^2d}$$

output
```
-5*x/a^2+12*sin(d*x+c)/a^2/d-5*cos(d*x+c)*sin(d*x+c)/a^2/d-10/3*cos(d*x+c)
^2*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*sec(
d*x+c))^2-4*sin(d*x+c)^3/a^2/d
```

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.60

$$\int \frac{\cos^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(-360dx\cos\left(\frac{dx}{2}\right) - 360dx\cos\left(c+\frac{dx}{2}\right) - 120dx\cos\left(c+\frac{3dx}{2}\right) - 120dx\cos\left(2c+\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^3*(-360*d*x*Cos[(d*x)/2] - 360*d*x*Cos[c + (d*x
)/2] - 120*d*x*Cos[c + (3*d*x)/2] - 120*d*x*Cos[2*c + (3*d*x)/2] + 516*Sin
[(d*x)/2] - 156*Sin[c + (d*x)/2] + 342*Sin[c + (3*d*x)/2] + 118*Sin[2*c +
(3*d*x)/2] + 30*Sin[2*c + (5*d*x)/2] + 30*Sin[3*c + (5*d*x)/2] - 3*Sin[3*c
 + (7*d*x)/2] - 3*Sin[4*c + (7*d*x)/2] + Sin[4*c + (9*d*x)/2] + Sin[5*c +
(9*d*x)/2]))/(192*a^2*d)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 4304, 27, 3042, 4508, 27, 3042, 4274, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{2\cos^3(c+dx)(3a-2a\sec(c+dx))}{\sec(c+dx)a+a}dx}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{\cos^3(c+dx)(3a-2a\sec(c+dx))}{\sec(c+dx)a+a}dx}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{3a-2a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 4508$$

$$\frac{2\left(\frac{\int 3\cos^3(c+dx)\left(6a^2-5a^2\sec(c+dx)\right)dx}{a^2}-\frac{5\sin(c+dx)\cos^2(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(\frac{3\int \cos^3(c+dx)\left(6a^2-5a^2\sec(c+dx)\right)dx}{a^2}-\frac{5\sin(c+dx)\cos^2(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\left(\frac{3\int \frac{6a^2-5a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx}{a^2}-\frac{5\sin(c+dx)\cos^2(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 4274$$

$$\frac{2\left(\frac{3\left(6a^2\int \cos^3(c+dx)dx-5a^2\int \cos^2(c+dx)dx\right)}{a^2}-\frac{5\sin(c+dx)\cos^2(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\left(\frac{3\left(6a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx-5a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx\right)}{a^2}-\frac{5\sin(c+dx)\cos^2(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3113$$

$$\frac{2\left(\frac{3\left(-\frac{6a^2\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}-5a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx\right)}{a^2}-\frac{5\sin(c+dx)\cos^2(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{2\left(\frac{3\left(-5a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{6a^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{a^2}-\frac{5\sin(c+dx)\cos^2(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \ 3115$$

$$\frac{2\left(\frac{3\left(-5a^2\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{6a^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{a^2}-\frac{5\sin(c+dx)\cos^2(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \ 24$$

$$\frac{2\left(\frac{3\left(-\frac{6a^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}-5a^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{a^2}-\frac{5\sin(c+dx)\cos^2(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input

```
Int[Cos[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

output

```
-1/3*(Cos[c + d*x]^2*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^2) + (2*((-5*Co
s[c + d*x]^2*Sin[c + d*x])/(d*(1 + Sec[c + d*x])) + (3*(-5*a^2*(x/2 + (Cos
[c + d*x]*Sin[c + d*x])/(2*d)) - (6*a^2*(-Sin[c + d*x] + Sin[c + d*x]^3/3)
)/d))/a^2))/(3*a^2)
```

Defintions of rubi rules used

rule 24

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.62

method	result
parallelrisch	$\frac{43\left(\cos(dx+c)+\frac{14\cos(2dx+2c)}{129}-\frac{\cos(3dx+3c)}{129}+\frac{\cos(4dx+4c)}{258}+\frac{73}{86}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-40dx}{8a^{2}d}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8\left(-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{2}-\frac{10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}}-20\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^{2}}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8\left(-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{2}-\frac{10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}}-20\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^{2}}$
risch	$-\frac{5x}{a^{2}}+\frac{i e^{2i(dx+c)}}{4a^{2}d}-\frac{15i e^{i(dx+c)}}{8a^{2}d}+\frac{15i e^{-i(dx+c)}}{8a^{2}d}-\frac{i e^{-2i(dx+c)}}{4a^{2}d}+\frac{2i\left(15\,e^{2i(dx+c)}+27\,e^{i(dx+c)}+14\right)}{3d\,a^{2}\left(e^{i(dx+c)}+1\right)^{3}}+\frac{\sin(3}{1}$
norman	$\frac{-\frac{5x}{a}+\frac{21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{80\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3ad}+\frac{23\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{ad}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{6ad}-\frac{15x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{a}-\frac{15x\tan\left(\frac{d}{}\right.}{a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}a}$

input
```
int(cos(d*x+c)^3/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*(43*(cos(d*x+c)+14/129*cos(2*d*x+2*c)-1/129*cos(3*d*x+3*c)+1/258*cos(4
*d*x+4*c)+73/86)*tan(1/2*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^2-40*d*x)/a^2/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.87

$$\int \frac{\cos^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{15\,dx\cos\left(dx+c\right)^{2}+30\,dx\cos\left(dx+c\right)+15\,dx-\left(\cos\left(dx+c\right)^{4}-\cos\left(dx+c\right)^{3}+6\,\cos\left(dx+c\right)^{2}+3\right.}{3\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

input
```
integrate(cos(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/3*(15*d*x*cos(d*x + c)^2 + 30*d*x*cos(d*x + c) + 15*d*x - (cos(d*x + c)
^4 - cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 33*cos(d*x + c) + 24)*sin(d*x + c
))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.67

$$\int \frac{\cos^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{4\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{27\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{60\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{6d}$$

input
```
integrate(cos(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/6*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 + 3*a^2*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^
2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) +
1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 60*arctan(sin(d*x + c)/(co
s(d*x + c) + 1))/a^2)/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.87

$$\int \frac{\cos^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{30\,(dx+c)}{a^2} - \frac{4\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+20\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3 a^2} + \frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-27\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

input
```
integrate(cos(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/6*(30*(d*x + c)/a^2 - 4*(15*tan(1/2*d*x + 1/2*c)^5 + 20*tan(1/2*d*x + 1
/2*c)^3 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^2) + (
a^4*tan(1/2*d*x + 1/2*c)^3 - 27*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

Mupad [B] (verification not implemented)

Time = 9.71 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.09

$$\int \frac{\cos^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-28\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-60\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+40\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin}{6\,a^2\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}$$

input
```
int(cos(c + d*x)^3/(a + a/cos(c + d*x))^2,x)
```

output
```
-(sin(c/2 + (d*x)/2) - 28*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) - 60*cos
(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 40*cos(c/2 + (d*x)/2)^6*sin(c/2 + (
d*x)/2) - 16*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 30*cos(c/2 + (d*x)/
2)^3*(c + d*x))/(6*a^2*d*cos(c/2 + (d*x)/2)^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

$$\int \frac{\cos^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{-\cos(dx+c)\sin(dx+c)^4+9\cos(dx+c)\sin(dx+c)^2-15\cos(dx+c)\sin(dx+c)\,dx+\cos(dx+c)}{3\sin(dx+c)\,a^2d\,(\cos(dx+c)+1)}$$

input
```
int(cos(d*x+c)^3/(a+a*sec(d*x+c))^2,x)
```

output
```
( - cos(c + d*x)*sin(c + d*x)**4 + 9*cos(c + d*x)*sin(c + d*x)**2 - 15*cos
(c + d*x)*sin(c + d*x)*d*x + cos(c + d*x) + 2*sin(c + d*x)**4 + 23*sin(c +
 d*x)**2 - 15*sin(c + d*x)*d*x - 1)/(3*sin(c + d*x)*a**2*d*(cos(c + d*x) +
 1))
```

3.61 $\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 162

$$\begin{aligned}\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= \frac{13\text{arctanh}(\sin(c+dx))}{2a^3d} - \frac{152\tan(c+dx)}{15a^3d} \\ &+ \frac{13\sec(c+dx)\tan(c+dx)}{2a^3d} - \frac{\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} \\ &- \frac{11\sec^3(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} \\ &- \frac{76\sec^2(c+dx)\tan(c+dx)}{15d\,(a^3+a^3\sec(c+dx))}\end{aligned}$$

output
```
13/2*arctanh(sin(d*x+c))/a^3/d-152/15*tan(d*x+c)/a^3/d+13/2*sec(d*x+c)*tan
(d*x+c)/a^3/d-1/5*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^3-11/15*sec(d
*x+c)^3*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^2-76/15*sec(d*x+c)^2*tan(d*x+c)/d/
(a^3+a^3*sec(d*x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 351 vs. $2(162) = 324$.

Time = 1.87 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.17

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(24960\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cdots\right.\right.\right.}{\cdots}$$

input
```
Integrate[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/480*(Cos[(c + d*x)/2]*Sec[c + d*x]^3*(24960*Cos[(c + d*x)/2]^5*(Log[Cos
[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]]) + Sec[c/2]*Sec[c]*Sec[c + d*x]^2*(-1235*Sin[(d*x)/2] + 3805*Sin[(3*d*x
)/2] - 4329*Sin[c - (d*x)/2] + 1989*Sin[c + (d*x)/2] - 3575*Sin[2*c + (d*x
)/2] - 475*Sin[c + (3*d*x)/2] + 2005*Sin[2*c + (3*d*x)/2] - 2275*Sin[3*c +
 (3*d*x)/2] + 2673*Sin[c + (5*d*x)/2] + 105*Sin[2*c + (5*d*x)/2] + 1593*Si
n[3*c + (5*d*x)/2] - 975*Sin[4*c + (5*d*x)/2] + 1325*Sin[2*c + (7*d*x)/2]
+ 255*Sin[3*c + (7*d*x)/2] + 875*Sin[4*c + (7*d*x)/2] - 195*Sin[5*c + (7*d
*x)/2] + 304*Sin[3*c + (9*d*x)/2] + 90*Sin[4*c + (9*d*x)/2] + 214*Sin[5*c
+ (9*d*x)/2])))/(a^3*d*(1 + Sec[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4303, 3042, 4507, 3042, 4507, 3042, 4274, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^6}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{\sec^4(c+dx)(4a-7a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4\left(4a-7a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{\frac{\int \frac{\sec^3(c+dx)\left(33a^2-43a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}+\frac{11a\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(33a^2-43a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{11a\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{\frac{\frac{\int \sec^2(c+dx)\left(152a^3-195a^3\sec(c+dx)\right)dx}{a^2}+\frac{76a^2\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{11a\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(152a^3-195a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{76a^2\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{11a\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4274}$$

$$-\frac{\frac{\frac{152a^3\int \sec^2(c+dx)dx-195a^3\int \sec^3(c+dx)dx}{a^2}+\frac{76a^2\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{11a\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{152a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx-195a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}+\frac{76a^2\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{11a\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\frac{\frac{-\frac{152a^3\int 1d(-\tan(c+dx))}{d}-195a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}+\frac{76a^2\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{11a\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\frac{\frac{\frac{152a^3\tan(c+dx)}{d}-195a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}+\frac{76a^2\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{11a\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4255}$$

$$-\frac{\frac{\frac{\frac{152a^3\tan(c+dx)}{d}-195a^3\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}+\frac{76a^2\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{11a\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{\frac{152a^3\tan(c+dx)}{d}-195a^3\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}+\frac{76a^2\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{11a\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{-\frac{\frac{76a^2\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}+\frac{\frac{152a^3\tan(c+dx)}{d}-195a^3\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}}{3a^2}+\frac{11a\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/5*(Sec[c + d*x]^4*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^3) - ((11*a*Sec
[c + d*x]^3*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + ((76*a^2*Sec[c +
d*x]^2*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])) + ((152*a^3*Tan[c + d*x])/d
- 195*a^3*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)
))/a^2)/(3*a^2))/(5*a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{14}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-26\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{\cdots}{(\tan(\cdots}}{4da^{3}}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{14}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-26\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{\cdots}{(\tan(\cdots}}{4da^{3}}$
parallelrisch	$\frac{(-1560\cos(2dx+2c)-1560)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(1560\cos(2dx+2c)+1560)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-2331\left(\cos(dx+c)+\cdots\right.}{240da^{3}(1+\cos(2dx+2c))}$
risch	$-\frac{i\left(195\,e^{8i(dx+c)}+975\,e^{7i(dx+c)}+2275\,e^{6i(dx+c)}+3575\,e^{5i(dx+c)}+4329\,e^{4i(dx+c)}+3805\,e^{3i(dx+c)}+2673\,e^{2i(dx+c)}+1\cdots\right.}{15da^{3}\left(e^{2i(dx+c)}+1\right)^{2}\left(e^{i(dx+c)}+1\right)^{5}}$
norman	$\frac{\frac{51\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{721\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{12ad}+\frac{6613\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{60ad}-\frac{1165\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{12ad}+\frac{475\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{12ad}-\frac{59\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{12ad}-\frac{5\tan(\cdots}{1\cdots}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{5}a^{2}}$

input
```
int(sec(d*x+c)^6/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^3*(-1/5*tan(1/2*d*x+1/2*c)^5-8/3*tan(1/2*d*x+1/2*c)^3-31*tan(1/2*d
*x+1/2*c)+2/(tan(1/2*d*x+1/2*c)-1)^2+14/(tan(1/2*d*x+1/2*c)-1)-26*ln(tan(1
/2*d*x+1/2*c)-1)-2/(tan(1/2*d*x+1/2*c)+1)^2+14/(tan(1/2*d*x+1/2*c)+1)+26*l
n(tan(1/2*d*x+1/2*c)+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.27

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{195\left(\cos(dx+c)^5+3\cos(dx+c)^4+3\cos(dx+c)^3+\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-195\,(\cos\,(}{6}$$

input
```
integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/60*(195*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 3*cos(d*x + c)^3 + cos(d*x
+ c)^2)*log(sin(d*x + c) + 1) - 195*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 3
*cos(d*x + c)^3 + cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(304*cos(d*x
+ c)^4 + 717*cos(d*x + c)^3 + 479*cos(d*x + c)^2 + 45*cos(d*x + c) - 15)*s
in(d*x + c))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(
d*x + c)^3 + a^3*d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sec^6(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**6/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**6/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.30

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3-\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{390\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{390\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right.}{a^3}}{60\,d}$$

input
```
integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/60*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 7*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3)/(a^3 - 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) + 40*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^
5)/a^3 - 390*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 390*log(sin(d*
x + c)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.86

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{390\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{390\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}+\frac{60\left(7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^3}-\frac{3a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+40a}{}}{60\,d}$$

input
```
integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/60*(390*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 390*log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/a^3 + 60*(7*tan(1/2*d*x + 1/2*c)^3 - 5*tan(1/2*d*x + 1/2*c
))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3) - (3*a^12*tan(1/2*d*x + 1/2*c)^5 +
 40*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

Mupad [B] (verification not implemented)

Time = 9.56 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.87

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{13\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^3\,d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{20\,a^3\,d} - \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3\,a^3\,d} - \frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a^3\right)} - \frac{31\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4\,a^3\,d}$$

input
```
int(1/(cos(c + d*x)^6*(a + a/cos(c + d*x))^3),x)
```

output
```
(13*atanh(tan(c/2 + (d*x)/2)))/(a^3*d) - tan(c/2 + (d*x)/2)^5/(20*a^3*d) -
 (2*tan(c/2 + (d*x)/2)^3)/(3*a^3*d) - (5*tan(c/2 + (d*x)/2) - 7*tan(c/2 +
(d*x)/2)^3)/(d*(a^3*tan(c/2 + (d*x)/2)^4 - 2*a^3*tan(c/2 + (d*x)/2)^2 + a^
3)) - (31*tan(c/2 + (d*x)/2))/(4*a^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.41

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{-390\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+780\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-390\log\left(\tan\left(\frac{dx}{2}\right.\right.}{}$$

input
```
int(sec(d*x+c)^6/(a+a*sec(d*x+c))^3,x)
```

output
```
( - 390*log(tan((c + d*x)/2) - 1)*tan((c + d*x)/2)**4 + 780*log(tan((c + d
*x)/2) - 1)*tan((c + d*x)/2)**2 - 390*log(tan((c + d*x)/2) - 1) + 390*log(
tan((c + d*x)/2) + 1)*tan((c + d*x)/2)**4 - 780*log(tan((c + d*x)/2) + 1)*
tan((c + d*x)/2)**2 + 390*log(tan((c + d*x)/2) + 1) - 3*tan((c + d*x)/2)**
9 - 34*tan((c + d*x)/2)**7 - 388*tan((c + d*x)/2)**5 + 1310*tan((c + d*x)/
2)**3 - 765*tan((c + d*x)/2))/(60*a**3*d*(tan((c + d*x)/2)**4 - 2*tan((c +
 d*x)/2)**2 + 1))
```

3.62 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 128

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{3\text{arctanh}(\sin(c+dx))}{a^3d}+\frac{9\tan(c+dx)}{5a^3d} - \frac{\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{3\sec^2(c+dx)\tan(c+dx)}{5ad(a+a\sec(c+dx))^2}+\frac{3\tan(c+dx)}{d\,(a^3+a^3\sec(c+dx))}$$

output
```
-3*arctanh(sin(d*x+c))/a^3/d+9/5*tan(d*x+c)/a^3/d-1/5*sec(d*x+c)^3*tan(d*x
+c)/d/(a+a*sec(d*x+c))^3-3/5*sec(d*x+c)^2*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^
2+3*tan(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.91

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(-240\text{arctanh}(\sin(c+dx))\cos^5\left(\frac{1}{2}(c+dx)\right)+\sec(c+dx)\left(20\sin\left(\frac{1}{2}(c+d\right.\right.\right.}{10a^3d(1+\sec(c+dx))^3}$$

input
```
Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^3*(-240*ArcTanh[Sin[c + d*x]]*Cos[(c + d*x)
/2]^5 + Sec[c + d*x]*(20*Sin[(c + d*x)/2] + 57*Sin[(3*(c + d*x))/2] + 45*S
in[(5*(c + d*x))/2] + 12*Sin[(7*(c + d*x))/2])))/(10*a^3*d*(1 + Sec[c + d*
x])^3)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4303, 27, 3042, 4507, 27, 3042, 4496, 25, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{3\sec^3(c+dx)(a-2a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\int \frac{\sec^3(c+dx)(a-2a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{3\left(\frac{\int \frac{3\sec^2(c+dx)\left(2a^2-3a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}+\frac{a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{3\left(\frac{\int \frac{\sec^2(c+dx)\left(2a^2-3a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{a^2}+\frac{a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{3\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(2a^2-3a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{a^2}+\frac{a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \textbf{4496}$$

$$-\frac{3\left(\frac{-\frac{\int -\sec(c+dx)\left(5a^3-3a^3\sec(c+dx)\right)dx}{a^2}-\frac{5a^2\tan(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}+\frac{a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \textbf{25}$$

$$-\frac{3\left(\frac{\frac{\int \sec(c+dx)\left(5a^3-3a^3\sec(c+dx)\right)dx}{a^2}-\frac{5a^2\tan(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}+\frac{a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{3\left(\frac{\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(5a^3-3a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}-\frac{5a^2\tan(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}+\frac{a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \textbf{4274}$$

$$-\frac{3\left(\frac{\frac{5a^3\int \sec(c+dx)dx-3a^3\int \sec^2(c+dx)dx}{a^2}-\frac{5a^2\tan(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}+\frac{a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$-\frac{3\left(\frac{\frac{5a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-3a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2}-\frac{5a^2\tan(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}+\frac{a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4254

$$-\frac{3\left(\frac{\frac{\frac{3a^3\int 1d(-\tan(c+dx))}{d}+5a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{5a^2\tan(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}+\frac{a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 24

$$-\frac{3\left(\frac{\frac{5a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{3a^3\tan(c+dx)}{d}}{a^2}-\frac{5a^2\tan(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}+\frac{a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4257

$$-\frac{3\left(\frac{\frac{\frac{5a^3\text{arctanh}(\sin(c+dx))}{d}-\frac{3a^3\tan(c+dx)}{d}}{a^2}-\frac{5a^2\tan(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}+\frac{a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/5*(Sec[c + d*x]^3*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^3) - (3*((a*Sec
[c + d*x]^2*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^2) + ((-5*a^2*Tan[c + d*
x])/(d*(a + a*Sec[c + d*x])) + ((5*a^3*ArcTanh[Sin[c + d*x]])/d - (3*a^3*T
an[c + d*x])/d)/a^2)/a^2))/(5*a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4496
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(b^2*(2*m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b
*B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Ne
Q[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+12\ln\left(\mathrm{ta}\right.}{4d\,a^3}$
default	$\frac{-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+12\ln\left(\mathrm{ta}\right.}{4d\,a^3}$
parallelrisch	$\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+\frac{57\left(\cos(dx+c)+\frac{\cos(2dx+2c)}{2}+\frac{2\cos(3dx+3c)}{19}+\frac{67}{114}\right)}{20}}{\cos(dx+c)a^3d}$
risch	$\frac{2i\left(15\,\mathrm{e}^{6i(dx+c)}+75\,\mathrm{e}^{5i(dx+c)}+160\,\mathrm{e}^{4i(dx+c)}+200\,\mathrm{e}^{3i(dx+c)}+189\,\mathrm{e}^{2i(dx+c)}+105\,\mathrm{e}^{i(dx+c)}+24\right)}{5d\,a^3\left(\mathrm{e}^{i(dx+c)}+1\right)^5\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{3\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d\,a^3}+$
norman	$\frac{\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{45\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2ad}+\frac{591\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{20ad}-\frac{81\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{5ad}+\frac{51\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{20ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{10ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{1}}{20ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4a^2}$

input
```
int(sec(d*x+c)^5/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^3*(-4/(tan(1/2*d*x+1/2*c)+1)-12*ln(tan(1/2*d*x+1/2*c)+1)+1/5*tan(1
/2*d*x+1/2*c)^5+2*tan(1/2*d*x+1/2*c)^3+17*tan(1/2*d*x+1/2*c)-4/(tan(1/2*d*
x+1/2*c)-1)+12*ln(tan(1/2*d*x+1/2*c)-1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.48

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{15\left(\cos(dx+c)^4+3\cos(dx+c)^3+3\cos(dx+c)^2+\cos(dx+c)\right)\log(\sin(dx+c)+1)-15\left(\cos(\right.}{10\left(a^3d\cos(\right.}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/10*(15*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x
+ c))*log(sin(d*x + c) + 1) - 15*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*co
s(d*x + c)^2 + cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(24*cos(d*x + c)^3
 + 57*cos(d*x + c)^2 + 39*cos(d*x + c) + 5)*sin(d*x + c))/(a^3*d*cos(d*x +
 c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x +
c))
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sec^5(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.29

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{40\sin(dx+c)}{\left(a^3-\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} + \frac{\frac{85\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3} - \frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3} + \frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\right.}{a^3}}{20\,d}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/20*(40*sin(d*x + c)/((a^3 - a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(co
s(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3
/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(
sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x +
c) + 1) - 1)/a^3)/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$
$$-\frac{\frac{60\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{60\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}+\frac{40\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^3}-\frac{a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+10\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}{a^{15}}}{20\,d}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/20*(60*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/a^3 + 40*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)
*a^3) - (a^12*tan(1/2*d*x + 1/2*c)^5 + 10*a^12*tan(1/2*d*x + 1/2*c)^3 + 85
*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

Mupad [B] (verification not implemented)

Time = 9.54 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.87

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2\,a^3\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{20\,a^3\,d}-\frac{6\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d} - \frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^3\right)}+\frac{17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^3\,d}$$

input
```
int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^3),x)
```

output
```
tan(c/2 + (d*x)/2)^3/(2*a^3*d) + tan(c/2 + (d*x)/2)^5/(20*a^3*d) - (6*atan
h(tan(c/2 + (d*x)/2)))/(a^3*d) - (2*tan(c/2 + (d*x)/2))/(d*(a^3*tan(c/2 +
(d*x)/2)^2 - a^3)) + (17*tan(c/2 + (d*x)/2))/(4*a^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.17

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{60\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-60\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-60\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\tan\left(\frac{dx}{2}+\right.}{20a^3d\left(\tan\left(\right.\right.}$$

input
```
int(sec(d*x+c)^5/(a+a*sec(d*x+c))^3,x)
```

output
```
(60*log(tan((c + d*x)/2) - 1)*tan((c + d*x)/2)**2 - 60*log(tan((c + d*x)/2
) - 1) - 60*log(tan((c + d*x)/2) + 1)*tan((c + d*x)/2)**2 + 60*log(tan((c
+ d*x)/2) + 1) + tan((c + d*x)/2)**7 + 9*tan((c + d*x)/2)**5 + 75*tan((c +
 d*x)/2)**3 - 125*tan((c + d*x)/2))/(20*a**3*d*(tan((c + d*x)/2)**2 - 1))
```

3.63 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{a^3d} - \frac{\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{7\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{29\tan(c+dx)}{15d\,(a^3+a^3\sec(c+dx))}$$

output
```
arctanh(sin(d*x+c))/a^3/d-1/5*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^3
+7/15*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^2-29/15*tan(d*x+c)/d/(a^3+a^3*sec(d*
x+c))
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(120\operatorname{arctanh}(\sin(c+dx))\cos^5\left(\frac{1}{2}(c+dx)\right) - 35\sin\left(\frac{1}{2}(c+dx)\right) - 40\sin\left(\frac{3}{2}\right.\right.}{15a^3d(1+\sec(c+dx))^3}$$

input
```
Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^3*(120*ArcTanh[Sin[c + d*x]]*Cos[(c + d*x)/
2]^5 - 35*Sin[(c + d*x)/2] - 40*Sin[(3*(c + d*x))/2] - 11*Sin[(5*(c + d*x)
)/2]))/(15*a^3*d*(1 + Sec[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4303, 3042, 4496, 25, 3042, 4486, 3042, 4257, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{ 4303}$$

$$-\frac{\int \frac{\sec^2(c+dx)(2a-5a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(2a-5a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 4496}$$

$$-\frac{-\frac{\int -\frac{\sec(c+dx)\left(14a^2-15a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{7a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{\int \frac{\sec(c+dx)\left(14a^2-15a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{7a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(14a^2-15a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{7a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4486}$$

$$-\frac{\frac{29a^2\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx-15a\int \sec(c+dx)dx}{3a^2}-\frac{7a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{29a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-15a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{3a^2}-\frac{7a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\frac{29a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{15a\,\text{arctanh}(\sin(c+dx))}{d}}{3a^2}-\frac{7a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4281}$$

$$-\frac{\frac{\frac{29a^2\tan(c+dx)}{d(a\sec(c+dx)+a)}-\frac{15a\,\text{arctanh}(\sin(c+dx))}{d}}{3a^2}-\frac{7a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/5*(Sec[c + d*x]^2*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^3) - ((-7*a*Tan
[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + ((-15*a*ArcTanh[Sin[c + d*x]])/d
 + (29*a^2*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3*a^2))/(5*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

rule 4496
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(b^2*(2*m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b
*B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Ne
Q[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4d\,a^{3}}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4d\,a^{3}}$
parallelrisch	$\frac{-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-20\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-60\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+60\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-105\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{60d\,a^{3}}$
risch	$-\frac{2i\left(15\,{\mathrm e}^{4i(dx+c)}+75\,{\mathrm e}^{3i(dx+c)}+145\,{\mathrm e}^{2i(dx+c)}+95\,{\mathrm e}^{i(dx+c)}+22\right)}{15d\,a^{3}\left({\mathrm e}^{i(dx+c)}+1\right)^{5}}-\frac{\ln\left({\mathrm e}^{i(dx+c)}-i\right)}{d\,a^{3}}+\frac{\ln\left({\mathrm e}^{i(dx+c)}+i\right)}{d\,a^{3}}$
norman	$\frac{\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{59\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{12ad}+\frac{43\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{10ad}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{10ad}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{60ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{20ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}a^{2}}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{d\,a^{3}}$

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^3*(-1/5*tan(1/2*d*x+1/2*c)^5-4/3*tan(1/2*d*x+1/2*c)^3-7*tan(1/2*d*
x+1/2*c)-4*ln(tan(1/2*d*x+1/2*c)-1)+4*ln(tan(1/2*d*x+1/2*c)+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.50

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$=\frac{15\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\log\left(\sin\left(dx+c\right)+1\right)-15\left(\cos\left(dx+c\right)^{3}+3\,c\right.}{30\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\right.}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/30*(15*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*log(sin(
d*x + c) + 1) - 15*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*log(-sin(d*x + c) + 1) - 2*(22*cos(d*x + c)^2 + 51*cos(d*x + c) + 32)*si
n(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d
*x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sec^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.13

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3} - \frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3} + \frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{60\,d}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/60*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x +
 c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) + 1) -
 1)/a^3)/d
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{60\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3} - \frac{60\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3} - \frac{3\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+20\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+105\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/60*(60*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*log(abs(tan(1/2*d*x +
 1/2*c) - 1))/a^3 - (3*a^12*tan(1/2*d*x + 1/2*c)^5 + 20*a^12*tan(1/2*d*x +
 1/2*c)^3 + 105*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

Mupad [B] (verification not implemented)

Time = 9.67 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.55

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{105\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-120\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{60\,a^3\,d}$$

input
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^3),x)
```

output
```
-(105*tan(c/2 + (d*x)/2) - 120*atanh(tan(c/2 + (d*x)/2)) + 20*tan(c/2 + (d
*x)/2)^3 + 3*tan(c/2 + (d*x)/2)^5)/(60*a^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.70

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{-60\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+60\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5-20\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-105\tan\left(\right.}{60a^3d}$$

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^3,x)
```

output
```
( - 60*log(tan((c + d*x)/2) - 1) + 60*log(tan((c + d*x)/2) + 1) - 3*tan((c
 + d*x)/2)**5 - 20*tan((c + d*x)/2)**3 - 105*tan((c + d*x)/2))/(60*a**3*d)
```

3.64 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{7\tan(c+dx)}{15d\,(a^3+a^3\sec(c+dx))}$$

output
```
1/5*tan(d*x+c)/d/(a+a*sec(d*x+c))^3-8/15*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^2
+7/15*tan(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.69

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\sec^5\left(\frac{1}{2}(c+dx)\right)\left(10\sin\left(\frac{1}{2}(c+dx)\right)+5\sin\left(\frac{3}{2}(c+dx)\right)+\sin\left(\frac{5}{2}(c+dx)\right)\right)}{120a^3d}$$

input
```
Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Sec[(c + d*x)/2]^5*(10*Sin[(c + d*x)/2] + 5*Sin[(3*(c + d*x))/2] + Sin[(5
*(c + d*x))/2]))/(120*a^3*d)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4286, 25, 3042, 4488, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4286}$$

$$\frac{\int -\frac{\sec(c+dx)(3a-5a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\int \frac{\sec(c+dx)(3a-5a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3a-5a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}$$

$$\Big\downarrow \text{4488}$$

$$\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{8a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{7}{3}\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{5a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{8a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{7}{3}\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{5a^2}$$

$$\Big\downarrow \text{4281}$$

$$\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{8a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{7\tan(c+dx)}{3d(a\sec(c+dx)+a)}}{5a^2}$$

input
```
Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

output
```
Tan[c + d*x]/(5*d*(a + a*Sec[c + d*x])^3) - ((8*a*Tan[c + d*x])/(3*d*(a +
a*Sec[c + d*x])^2) - (7*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])))/(5*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4286
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Simp[1/(a^2*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[
a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4488
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
 /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

method	result	size
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^{3}}$	45
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^{3}}$	45
risch	$\frac{4i\left(10\,{\mathrm e}^{2i(dx+c)}+5\,{\mathrm e}^{i(dx+c)}+1\right)}{15d\,a^{3}\left({\mathrm e}^{i(dx+c)}+1\right)^{5}}$	47
parallelrisch	$\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{60d\,a^{3}}$	47
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{30ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{15ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{20ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}a^{2}}$	114

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5+2/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1
/2*c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\left(2\cos\left(dx+c\right)^2+6\cos\left(dx+c\right)+7\right)\sin\left(dx+c\right)}{15\left(a^3d\cos\left(dx+c\right)^3+3a^3d\cos\left(dx+c\right)^2+3a^3d\cos\left(dx+c\right)+a^3d\right)}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/15*(2*cos(d*x + c)^2 + 6*cos(d*x + c) + 7)*sin(d*x + c)/(a^3*d*cos(d*x +
 c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sec^3(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**3/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{60a^3d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/60*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3*d)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.55

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+10\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{60\,a^3d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/60*(3*tan(1/2*d*x + 1/2*c)^5 + 10*tan(1/2*d*x + 1/2*c)^3 + 15*tan(1/2*d*
x + 1/2*c))/(a^3*d)
```

Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+15\right)}{60\,a^3\,d}$$

input
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^3),x)
```

output
```
(tan(c/2 + (d*x)/2)*(10*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + 15
))/(60*a^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+15\right)}{60a^3d}$$

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^3,x)
```

output
```
(tan((c + d*x)/2)*(3*tan((c + d*x)/2)**4 + 10*tan((c + d*x)/2)**2 + 15))/(
60*a**3*d)
```

3.65 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\tan(c+dx)}{5ad(a+a\sec(c+dx))^2} + \frac{\tan(c+dx)}{5d\,(a^3+a^3\sec(c+dx))}$$

output
```
-1/5*tan(d*x+c)/d/(a+a*sec(d*x+c))^3+1/5*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^2
+1/5*tan(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.25

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{8\coth^{-1}(\sin(c+dx))\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx) - 8\text{arctanh}(\sin(c+dx))\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+d}{5a^3d(1+\sec(c+dx))^3}$$

input
```
Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

output
```
(8*ArcCoth[Sin[c + d*x]]*Cos[(c + d*x)/2]^6*Sec[c + d*x]^3 - 8*ArcTanh[Sin
[c + d*x]]*Cos[(c + d*x)/2]^6*Sec[c + d*x]^3 + (1 + 3*Sec[c + d*x] + Sec[c
 + d*x]^2)*Tan[c + d*x])/(5*a^3*d*(1 + Sec[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4284, 3042, 4283, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{ 4284}$$

$$\frac{3\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^2}dx}{5a}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 4283}$$

$$\frac{3\left(\frac{\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4281}$$

$$\frac{3\left(\frac{\tan(c+dx)}{3ad(a\sec(c+dx)+a)}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/5*Tan[c + d*x]/(d*(a + a*Sec[c + d*x])^3) + (3*(Tan[c + d*x]/(3*d*(a +
a*Sec[c + d*x])^2) + Tan[c + d*x]/(3*a*d*(a + a*Sec[c + d*x]))))/(5*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

rule 4284
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Simp[m/(b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.39

method	result	size
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^{3}}$	32
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^{3}}$	32
parallelrisch	$-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{20d\,a^{3}}$	32
risch	$\frac{2i\left(5\,\mathrm{e}^{3i(dx+c)}+5\,\mathrm{e}^{2i(dx+c)}+5\,\mathrm{e}^{i(dx+c)}+1\right)}{5d\,a^{3}\left(\mathrm{e}^{i(dx+c)}+1\right)^{5}}$	58
norman	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{4ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{20ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{20ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)a^{2}}$	95

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^3*(-1/5*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+1/2*c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\left(\cos\left(dx+c\right)^2+3\,\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)}{5\,\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/5*(cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sin(d*x + c)/(a^3*d*cos(d*x + c)
^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sec^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.57

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{5\sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{20\,a^3d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/20*(5*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1
)^5)/(a^3*d)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.37

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{20\,a^3d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/20*(tan(1/2*d*x + 1/2*c)^5 - 5*tan(1/2*d*x + 1/2*c))/(a^3*d)
```

Mupad [B] (verification not implemented)

Time = 9.52 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.36

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-5\right)}{20\,a^3\,d}$$

input
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^3),x)
```

output
```
-(tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^4 - 5))/(20*a^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.39

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+5\right)}{20a^3d}$$

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^3,x)
```

output
```
(tan((c + d*x)/2)*( - tan((c + d*x)/2)**4 + 5))/(20*a**3*d)
```

3.66 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 83

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{2\tan(c+dx)}{15d\left(a^3+a^3\sec(c+dx)\right)}$$

output
```
1/5*tan(d*x+c)/d/(a+a*sec(d*x+c))^3+2/15*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^2
+2/15*tan(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(40\sin\left(\frac{dx}{2}\right)-30\sin\left(c+\frac{dx}{2}\right)+20\sin\left(c+\frac{3dx}{2}\right)-15\sin\left(2c+\frac{3dx}{2}\right)+7\sin\left(2c+\right.\right.}{240a^3d}$$

input
```
Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^5*(40*Sin[(d*x)/2] - 30*Sin[c + (d*x)/2] + 20*S
in[c + (3*d*x)/2] - 15*Sin[2*c + (3*d*x)/2] + 7*Sin[2*c + (5*d*x)/2]))/(24
0*a^3*d)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 4283, 3042, 4283, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4283}$$

$$\frac{2\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^2}dx}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4283}$$

$$\frac{2\left(\frac{\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4281}$$

$$\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{2\left(\frac{\tan(c+dx)}{3ad(a\sec(c+dx)+a)}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}$$

input
```
Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

output
```
Tan[c + d*x]/(5*d*(a + a*Sec[c + d*x])^3) + (2*(Tan[c + d*x]/(3*d*(a + a*S
ec[c + d*x])^2) + Tan[c + d*x]/(3*a*d*(a + a*Sec[c + d*x]))))/(5*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

method	result	size
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}$	45
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}$	45
parallelrisch	$\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5-10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{60d\,a^3}$	47
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{6ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{20ad}}{a^2}$	61
risch	$\frac{2i\left(15\,e^{4i(dx+c)}+30\,e^{3i(dx+c)}+40\,e^{2i(dx+c)}+20\,e^{i(dx+c)}+7\right)}{15d\,a^3\left(e^{i(dx+c)}+1\right)^5}$	69

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5-2/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1
/2*c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\left(7\cos\left(dx+c\right)^2+6\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)}{15\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/15*(7*cos(d*x + c)^2 + 6*cos(d*x + c) + 2)*sin(d*x + c)/(a^3*d*cos(d*x +
 c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sec(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x
) + 1), x)/a**3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1} - \frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{60\,a^3d}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/60*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3*d)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.55

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 10\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 15\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{60\,a^3d}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/60*(3*tan(1/2*d*x + 1/2*c)^5 - 10*tan(1/2*d*x + 1/2*c)^3 + 15*tan(1/2*d*
x + 1/2*c))/(a^3*d)
```

Mupad [B] (verification not implemented)

Time = 9.61 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+15\right)}{60\,a^3\,d}$$

input
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^3),x)
```

output
```
(tan(c/2 + (d*x)/2)*(3*tan(c/2 + (d*x)/2)^4 - 10*tan(c/2 + (d*x)/2)^2 + 15
))/(60*a^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4-10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+15\right)}{60a^3d}$$

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^3,x)
```

output
```
(tan((c + d*x)/2)*(3*tan((c + d*x)/2)**4 - 10*tan((c + d*x)/2)**2 + 15))/(
60*a**3*d)
```

3.67 $\int \frac{1}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 88

$$\int \frac{1}{(a+a\sec(c+dx))^3}\,dx = \frac{x}{a^3}-\frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{7\tan(c+dx)}{15ad(a+a\sec(c+dx))^2}-\frac{22\tan(c+dx)}{15d\,(a^3+a^3\sec(c+dx))}$$

output
```
x/a^3-1/5*tan(d*x+c)/d/(a+a*sec(d*x+c))^3-7/15*tan(d*x+c)/a/d/(a+a*sec(d*x
+c))^2-22/15*tan(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.84

$$\int \frac{1}{(a+a\sec(c+dx))^3}\,dx = \frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(60dx\cos^5\left(\frac{1}{2}(c+dx)\right)-3\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+26\cos^2\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\text{s}\right.}{15a^3d(1+\text{s}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(-3),x]
```

output
```
(2*Cos[(c + d*x)/2]*Sec[c + d*x]^3*(60*d*x*Cos[(c + d*x)/2]^5 - 3*Sec[c/2]
*Sin[(d*x)/2] + 26*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] - 128*Cos[(c +
 d*x)/2]^4*Sec[c/2]*Sin[(d*x)/2] - 3*Cos[(c + d*x)/2]*Tan[c/2] + 26*Cos[(c
 + d*x)/2]^3*Tan[c/2]))/(15*a^3*d*(1 + Sec[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {3042, 4264, 25, 3042, 4410, 25, 3042, 4407, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4264}$$

$$-\frac{\int -\frac{5a-2a\sec(c+dx)}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5a-2a\sec(c+dx)}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{5a-2a\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4410}$$

$$\frac{-\frac{\int -\frac{15a^2-7a^2\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{7a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{15a^2-7a^2\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{7a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{15a^2-7a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{7a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4407

$$\frac{\frac{15ax-22a^2\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{7a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{15ax-22a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{7a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4281

$$\frac{\frac{15ax-\frac{22a^2\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{7a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[(a + a*Sec[c + d*x])^(-3),x]
```

output
```
-1/5*Tan[c + d*x]/(d*(a + a*Sec[c + d*x])^3) + ((-7*a*Tan[c + d*x])/(3*d*(
a + a*Sec[c + d*x])^2) + (15*a*x - (22*a^2*Tan[c + d*x])/(d*(a + a*Sec[c +
 d*x])))/(3*a^2))/(5*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4264
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Simp[1/(a^2*(2*n + 1))
   Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Int
egerQ[2*n]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4410
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e +
f*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&
EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.58

method	result	size
parallelrisch	$\frac{-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+20\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+60dx-105\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{60d\,a^{3}}$	51
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+8\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^{3}}$	59
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+8\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^{3}}$	59
norman	$\frac{\frac{x}{a}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{20ad}}{a^{2}}$	66
risch	$\frac{x}{a^{3}}-\frac{2i\left(45\,\mathrm{e}^{4i(dx+c)}+135\,\mathrm{e}^{3i(dx+c)}+185\,\mathrm{e}^{2i(dx+c)}+115\,\mathrm{e}^{i(dx+c)}+32\right)}{15d\,a^{3}\left(\mathrm{e}^{i(dx+c)}+1\right)^{5}}$	75

input
```
int(1/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/60*(-3*tan(1/2*d*x+1/2*c)^5+20*tan(1/2*d*x+1/2*c)^3+60*d*x-105*tan(1/2*d
*x+1/2*c))/d/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.32

$$\int \frac{1}{(a+a\sec(c+dx))^3}\,dx$$
$$=\frac{15\,dx\cos\left(dx+c\right)^{3}+45\,dx\cos\left(dx+c\right)^{2}+45\,dx\cos\left(dx+c\right)+15\,dx-\left(32\,\cos\left(dx+c\right)^{2}+51\,\cos\left(dx+\right.\right.}{15\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

input
```
integrate(1/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/15*(15*d*x*cos(d*x + c)^3 + 45*d*x*cos(d*x + c)^2 + 45*d*x*cos(d*x + c)
+ 15*d*x - (32*cos(d*x + c)^2 + 51*cos(d*x + c) + 22)*sin(d*x + c))/(a^3*d
*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{1}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(1/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(1/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x)/
a**3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.05

$$\int \frac{1}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{60\,d}$$

input
```
integrate(1/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/60*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d
*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.77

$$\int \frac{1}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{60\,(dx+c)}{a^3}-\frac{3\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-20\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+105\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

input
```
integrate(1/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/60*(60*(d*x + c)/a^3 - (3*a^12*tan(1/2*d*x + 1/2*c)^5 - 20*a^12*tan(1/2*
d*x + 1/2*c)^3 + 105*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.92

$$\int \frac{1}{(a+a\sec(c+dx))^3}\,dx = \frac{x}{a^3} - \frac{\frac{32\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{15} - \frac{13\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{30} + \frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{20}}{a^3\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}$$

input
```
int(1/(a + a/cos(c + d*x))^3,x)
```

output
```
x/a^3 - (sin(c/2 + (d*x)/2)/20 - (13*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/
2))/30 + (32*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/15)/(a^3*d*cos(c/2 +
 (d*x)/2)^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.57

$$\int \frac{1}{(a+a\sec(c+dx))^3}\,dx = \frac{-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5 + 20\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 - 105\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + 60dx}{60a^3d}$$

input
```
int(1/(a+a*sec(d*x+c))^3,x)
```

output
```
( - 3*tan((c + d*x)/2)**5 + 20*tan((c + d*x)/2)**3 - 105*tan((c + d*x)/2)
+ 60*d*x)/(60*a**3*d)
```

3.68 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 103

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{3x}{a^3}+\frac{24\sin(c+dx)}{5a^3d}-\frac{\sin(c+dx)}{5d(a+a\sec(c+dx))^3} -\frac{3\sin(c+dx)}{5ad(a+a\sec(c+dx))^2}-\frac{3\sin(c+dx)}{d\left(a^3+a^3\sec(c+dx)\right)}$$

output
```
-3*x/a^3+24/5*sin(d*x+c)/a^3/d-1/5*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-3/5*sin
(d*x+c)/a/d/(a+a*sec(d*x+c))^2-3*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 0.71 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\sin(c+dx)\left(120\arcsin(\cos(c+dx))\cos^6\left(\frac{1}{2}(c+dx)\right)+(24+57\cos(c+dx)+39\cos^2(c+dx)+5\cos^3(\cdots\right.}{5a^3d\sqrt{1-\cos(c+dx)}(1+\cos(c+dx))^{7/2}}$$

input
```
Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Sin[c + d*x]*(120*ArcSin[Cos[c + d*x]]*Cos[(c + d*x)/2]^6 + (24 + 57*Cos[
c + d*x] + 39*Cos[c + d*x]^2 + 5*Cos[c + d*x]^3)*Sqrt[Sin[c + d*x]^2]))/(5
*a^3*d*Sqrt[1 - Cos[c + d*x]]*(1 + Cos[c + d*x])^(7/2))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {3042, 4304, 27, 3042, 4508, 27, 3042, 4508, 3042, 4274, 24, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{3\cos(c+dx)(2a-a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{\cos(c+dx)(2a-a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{2a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4508}$$

$$\frac{3\left(\frac{\int \frac{3\cos(c+dx)\left(3a^2-2a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{a\sin(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(\frac{\int \frac{\cos(c+dx)\left(3a^2-2a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{a^2}-\frac{a\sin(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{3\left(\frac{\int \frac{3a^2-2a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{a^2}-\frac{a\sin(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4508

$$\frac{3\left(\frac{\frac{\int \cos(c+dx)\left(8a^3-5a^3\sec(c+dx)\right)dx}{a^2}-\frac{5a^2\sin(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}-\frac{a\sin(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{3\left(\frac{\frac{\int \frac{8a^3-5a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{5a^2\sin(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}-\frac{a\sin(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4274

$$\frac{3\left(\frac{\frac{8a^3\int \cos(c+dx)dx-5a^3\int 1dx}{a^2}-\frac{5a^2\sin(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}-\frac{a\sin(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 24

$$\frac{3\left(\frac{\frac{8a^3\int \cos(c+dx)dx-5a^3x}{a^2}-\frac{5a^2\sin(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}-\frac{a\sin(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{3\left(\frac{\frac{8a^3\int \sin\left(c+dx+\frac{\pi}{2}\right)dx-5a^3x}{a^2}-\frac{5a^2\sin(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}-\frac{a\sin(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3117

$$\frac{3\left(\frac{\frac{\frac{8a^3\sin(c+dx)}{d}-5a^3x}{a^2}-\frac{5a^2\sin(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}-\frac{a\sin(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/5*Sin[c + d*x]/(d*(a + a*Sec[c + d*x])^3) + (3*(-((a*Sin[c + d*x])/(d*(
a + a*Sec[c + d*x])^2)) + ((-5*a^2*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
+ (-5*a^3*x + (8*a^3*Sin[c + d*x])/d)/a^2)/a^2))/(5*a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.64

method	result	size
parallelrisch	$\frac{\frac{243\left(\cos(dx+c)+\frac{26\cos(2dx+2c)}{81}+\frac{5\cos(3dx+3c)}{243}+\frac{58}{81}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{80}-3dx}{d\,a^{3}}$	66
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}-24\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^{3}}$	85
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}-24\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^{3}}$	85
risch	$-\frac{3x}{a^{3}}-\frac{i\,e^{i(dx+c)}}{2d\,a^{3}}+\frac{i\,e^{-i(dx+c)}}{2d\,a^{3}}+\frac{4i\left(15\,e^{4i(dx+c)}+50\,e^{3i(dx+c)}+70\,e^{2i(dx+c)}+45\,e^{i(dx+c)}+12\right)}{5d\,a^{3}\left(e^{i(dx+c)}+1\right)^{5}}$	112
norman	$\frac{-\frac{3x}{a}+\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{4ad}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{20ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{20ad}-\frac{3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)a^{2}}$	118

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
3/80*(81*(cos(d*x+c)+26/81*cos(2*d*x+2*c)+5/243*cos(3*d*x+3*c)+58/81)*tan(
1/2*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^4-80*d*x)/d/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.22

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{15\,dx\cos{(dx+c)}^3+45\,dx\cos{(dx+c)}^2+45\,dx\cos{(dx+c)}+15\,dx-\left(5\,\cos{(dx+c)}^3+39\,\cos{(dx}}{5\,\left(a^3d\cos{(dx+c)}^3+3\,a^3d\cos{(dx+c)}^2+3\,a^3d\cos{(dx+c)}+\right.}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/5*(15*d*x*cos(d*x + c)^3 + 45*d*x*cos(d*x + c)^2 + 45*d*x*cos(d*x + c)
+ 15*d*x - (5*cos(d*x + c)^3 + 39*cos(d*x + c)^2 + 57*cos(d*x + c) + 24)*s
in(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(
d*x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\cos{(c+dx)}}{\sec^3{(c+dx)}+3\sec^2{(c+dx)}+3\sec{(c+dx)}+1}\,dx}{a^3}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(cos(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x
) + 1), x)/a**3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.33

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{40\sin(dx+c)}{\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{20\,d}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/20*(40*sin(d*x + c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(co
s(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3
/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arc
tan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.93

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{60\,(dx+c)}{a^3}-\frac{40\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^3}-\frac{a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-10\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+85\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{20\,d}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/20*(60*(d*x + c)/a^3 - 40*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2
 + 1)*a^3) - (a^12*tan(1/2*d*x + 1/2*c)^5 - 10*a^12*tan(1/2*d*x + 1/2*c)^3
 + 85*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.10

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-12\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+96\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+40\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}\right.}{20\,a^3\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5}$$

input
```
int(cos(c + d*x)/(a + a/cos(c + d*x))^3,x)
```

output
```
(sin(c/2 + (d*x)/2) - 12*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 96*cos(
c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 40*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d
*x)/2) - 60*cos(c/2 + (d*x)/2)^5*(c + d*x))/(20*a^3*d*cos(c/2 + (d*x)/2)^5
)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7-9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+75\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-60\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2dx+125\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-60dx}{20a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)}$$

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^3,x)
```

output
```
(tan((c + d*x)/2)**7 - 9*tan((c + d*x)/2)**5 + 75*tan((c + d*x)/2)**3 - 60
*tan((c + d*x)/2)**2*d*x + 125*tan((c + d*x)/2) - 60*d*x)/(20*a**3*d*(tan(
(c + d*x)/2)**2 + 1))
```

3.69 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 147

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{13x}{2a^3}-\frac{152\sin(c+dx)}{15a^3d}+\frac{13\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{11\cos(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}-\frac{76\cos(c+dx)\sin(c+dx)}{15d\left(a^3+a^3\sec(c+dx)\right)}$$

output
```
13/2*x/a^3-152/15*sin(d*x+c)/a^3/d+13/2*cos(d*x+c)*sin(d*x+c)/a^3/d-1/5*co
s(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-11/15*cos(d*x+c)*sin(d*x+c)/a/d/(
a+a*sec(d*x+c))^2-76/15*cos(d*x+c)*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.23

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(-3\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+46\cos^2\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)-508\cos^4\left(\frac{1}{2}(c\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

output
```
(2*Cos[(c + d*x)/2]*Sec[c + d*x]^3*(-3*Sec[c/2]*Sin[(d*x)/2] + 46*Cos[(c +
 d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] - 508*Cos[(c + d*x)/2]^4*Sec[c/2]*Sin[(d*
x)/2] + 15*Cos[(c + d*x)/2]^5*(26*d*x - 12*Sin[c + d*x] + Sin[2*(c + d*x)]
) - 3*Cos[(c + d*x)/2]*Tan[c/2] + 46*Cos[(c + d*x)/2]^3*Tan[c/2]))/(15*a^3
*d*(1 + Sec[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 4304, 25, 3042, 4508, 3042, 4508, 3042, 4274, 3042, 3115, 24, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{\cos^2(c+dx)(7a-4a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\cos^2(c+dx)(7a-4a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{7a-4a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4508}$$

$$\frac{\frac{\int \frac{\cos^2(c+dx)\left(43a^2-33a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{11a\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{43a^2-33a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{11a\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4508

$$\frac{\frac{\frac{\int \cos^2(c+dx)\left(195a^3-152a^3\sec(c+dx)\right)dx}{a^2}-\frac{76a^2\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{11a\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{195a^3-152a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx}{a^2}-\frac{76a^2\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{11a\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4274

$$\frac{\frac{\frac{195a^3\int \cos^2(c+dx)dx-152a^3\int \cos(c+dx)dx}{a^2}-\frac{76a^2\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{11a\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{195a^3\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-152a^3\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{76a^2\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{11a\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3115

$$\frac{\frac{\frac{195a^3\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-152a^3\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{76a^2\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{11a\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{\frac{195a^3\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-152a^3\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{76a^2\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{11a\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3117}$$

$$\frac{\frac{\frac{195a^3\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{152a^3\sin(c+dx)}{d}}{a^2}-\frac{76a^2\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{11a\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/5*(Cos[c + d*x]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^3) + ((-11*a*Cos[
c + d*x]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + ((-76*a^2*Cos[c + d*
x]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])) + ((-152*a^3*Sin[c + d*x])/d + 1
95*a^3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/a^2)/(3*a^2))/(5*a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.52

method	result
parallelrisch	$\frac{-\frac{1001\left(\cos(dx+c)+\frac{928\cos(2dx+2c)}{3003}+\frac{15\cos(3dx+3c)}{1001}-\frac{5\cos(4dx+4c)}{2002}+\frac{331}{462}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{160}+\frac{13dx}{2}}{d\,a^3}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-28\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-20\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+52\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^3}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-28\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-20\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+52\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^3}$
risch	$\frac{13x}{2a^3}-\frac{ie^{2i(dx+c)}}{8d\,a^3}+\frac{3ie^{i(dx+c)}}{2d\,a^3}-\frac{3ie^{-i(dx+c)}}{2d\,a^3}+\frac{ie^{-2i(dx+c)}}{8d\,a^3}-\frac{2i\left(150\,e^{4i(dx+c)}+525\,e^{3i(dx+c)}+745\,e^{2i(dx+c)}\right.}{15d\,a^3\left(e^{i(dx+c)}+1\right)^5}$
norman	$\frac{\frac{13x}{2a}-\frac{51\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{131\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{6ad}-\frac{97\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{15ad}+\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{30ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{20ad}+\frac{13x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a}+\frac{13x\tan\left(\right.}{2}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2a^2}$

input
```
int(cos(d*x+c)^2/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
13/160*(-77*(cos(d*x+c)+928/3003*cos(2*d*x+2*c)+15/1001*cos(3*d*x+3*c)-5/2
002*cos(4*d*x+4*c)+331/462)*tan(1/2*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^4+80*d*x
)/d/a^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.92

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{195\,dx\cos{(dx+c)}^3+585\,dx\cos{(dx+c)}^2+585\,dx\cos{(dx+c)}+195\,dx+\left(15\,\cos{(dx+c)}^4-45\,\cos\right.}{30\left(a^3d\cos{(dx+c)}^3+3\,a^3d\cos{(dx+c)}^2+3\,a^3d\cos\right.}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/30*(195*d*x*cos(d*x + c)^3 + 585*d*x*cos(d*x + c)^2 + 585*d*x*cos(d*x +
c) + 195*d*x + (15*cos(d*x + c)^4 - 45*cos(d*x + c)^3 - 479*cos(d*x + c)^2
 - 717*cos(d*x + c) - 304)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*c
os(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\cos^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(cos(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.25

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3+\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}-\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{780\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{60\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/60*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) - 40*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^
5)/a^3 - 780*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.77

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{390\,(dx+c)}{a^3} - \frac{60\,\left(7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a^3} - \frac{3\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-40\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+465\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/60*(390*(d*x + c)/a^3 - 60*(7*tan(1/2*d*x + 1/2*c)^3 + 5*tan(1/2*d*x + 1
/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) - (3*a^12*tan(1/2*d*x + 1/2*c)
^5 - 40*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*a^12*tan(1/2*d*x + 1/2*c))/a^15)
/d
```

Mupad [B] (verification not implemented)

Time = 9.79 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.93

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{3\sin\left(\frac{c}{2}+\frac{dx}{2}\right) - 46\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right) + 508\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right) + 420\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{60\,a^3\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5}$$

input
```
int(cos(c + d*x)^2/(a + a/cos(c + d*x))^3,x)
```

output
```
-(3*sin(c/2 + (d*x)/2) - 46*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 508*
cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 420*cos(c/2 + (d*x)/2)^6*sin(c/2
 + (d*x)/2) - 120*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) - 390*cos(c/2 +
(d*x)/2)^5*(c + d*x))/(60*a^3*d*cos(c/2 + (d*x)/2)^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.91

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9+34\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7-388\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+390\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4dx-1310\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+}{60a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)}$$

input
```
int(cos(d*x+c)^2/(a+a*sec(d*x+c))^3,x)
```

output
```
( - 3*tan((c + d*x)/2)**9 + 34*tan((c + d*x)/2)**7 - 388*tan((c + d*x)/2)*
*5 + 390*tan((c + d*x)/2)**4*d*x - 1310*tan((c + d*x)/2)**3 + 780*tan((c +
 d*x)/2)**2*d*x - 765*tan((c + d*x)/2) + 390*d*x)/(60*a**3*d*(tan((c + d*x
)/2)**4 + 2*tan((c + d*x)/2)**2 + 1))
```

3.70 $\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 193

$$\begin{aligned}\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^4}\,dx &= \frac{21\text{arctanh}(\sin(c+dx))}{2a^4d} - \frac{576\tan(c+dx)}{35a^4d} \\ &+ \frac{21\sec(c+dx)\tan(c+dx)}{2a^4d} - \frac{43\sec^3(c+dx)\tan(c+dx)}{35a^4d(1+\sec(c+dx))^2} \\ &- \frac{288\sec^2(c+dx)\tan(c+dx)}{35a^4d(1+\sec(c+dx))} \\ &- \frac{\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\sec^4(c+dx)\tan(c+dx)}{5ad(a+a\sec(c+dx))^3}\end{aligned}$$

output
```
21/2*arctanh(sin(d*x+c))/a^4/d-576/35*tan(d*x+c)/a^4/d+21/2*sec(d*x+c)*tan
(d*x+c)/a^4/d-43/35*sec(d*x+c)^3*tan(d*x+c)/a^4/d/(1+sec(d*x+c))^2-288/35*
sec(d*x+c)^2*tan(d*x+c)/a^4/d/(1+sec(d*x+c))-1/7*sec(d*x+c)^5*tan(d*x+c)/d
/(a+a*sec(d*x+c))^4-2/5*sec(d*x+c)^4*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 403 vs. $2(193) = 386$.

Time = 2.37 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.09

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(376320\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^7/(a + a*Sec[c + d*x])^4,x]
```

output
```
-1/2240*(Cos[(c + d*x)/2]*Sec[c + d*x]^4*(376320*Cos[(c + d*x)/2]^7*(Log[C
os[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]]) + Sec[c/2]*Sec[c]*Sec[c + d*x]^2*(-24402*Sin[(d*x)/2] + 55556*Sin[(3
*d*x)/2] - 61054*Sin[c - (d*x)/2] + 33614*Sin[c + (d*x)/2] - 51842*Sin[2*c
 + (d*x)/2] - 12460*Sin[c + (3*d*x)/2] + 33716*Sin[2*c + (3*d*x)/2] - 3430
0*Sin[3*c + (3*d*x)/2] + 39788*Sin[c + (5*d*x)/2] - 2940*Sin[2*c + (5*d*x)
/2] + 26068*Sin[3*c + (5*d*x)/2] - 16660*Sin[4*c + (5*d*x)/2] + 21351*Sin[
2*c + (7*d*x)/2] + 1295*Sin[3*c + (7*d*x)/2] + 14911*Sin[4*c + (7*d*x)/2]
- 5145*Sin[5*c + (7*d*x)/2] + 7329*Sin[3*c + (9*d*x)/2] + 1225*Sin[4*c + (
9*d*x)/2] + 5369*Sin[5*c + (9*d*x)/2] - 735*Sin[6*c + (9*d*x)/2] + 1152*Si
n[4*c + (11*d*x)/2] + 280*Sin[5*c + (11*d*x)/2] + 872*Sin[6*c + (11*d*x)/2
]))/(a^4*d*(1 + Sec[c + d*x])^4)
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.09, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$, Rules used = {3042, 4303, 3042, 4507, 3042, 4507, 27, 3042, 4507, 3042, 4274, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{(a\sec(c+dx)+a)^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^7}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{\sec^5(c+dx)(5a-9a\sec(c+dx))}{(\sec(c+dx)a+a)^3}dx}{7a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5\left(5a-9a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{\frac{\int \frac{\sec^4(c+dx)\left(56a^2-73a^2\sec(c+dx)\right)}{(\sec(c+dx)a+a)^2}dx}{5a^2}+\frac{14a\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4\left(56a^2-73a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}+\frac{14a\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{\frac{\frac{\int \frac{9\sec^3(c+dx)\left(43a^3-53a^3\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}+\frac{43\tan(c+dx)\sec^3(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}+\frac{14a\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\frac{3\int \frac{\sec^3(c+dx)\left(43a^3-53a^3\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{a^2}+\frac{43\tan(c+dx)\sec^3(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}+\frac{14a\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(43a^3-53a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{a^2}+\frac{43\tan(c+dx)\sec^3(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}+\frac{14a\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{\frac{\frac{3\left(\frac{\int \sec^2(c+dx)\left(192a^4-245a^4\sec(c+dx)\right)dx}{a^2}+\frac{96a^3\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}+\frac{43\tan(c+dx)\sec^3(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}+\frac{14a\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{3\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(192a^4-245a^4\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{96a^3\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}+\frac{43\tan(c+dx)\sec^3(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}+\frac{14a\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4274}$$

$$-\frac{\frac{\frac{3\left(\frac{192a^4\int \sec^2(c+dx)dx-245a^4\int \sec^3(c+dx)dx}{a^2}+\frac{96a^3\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}+\frac{43\tan(c+dx)\sec^3(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}+\frac{14a\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{3\left(\frac{192a^4\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx-245a^4\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}+\frac{96a^3\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}+\frac{43\tan(c+dx)\sec^3(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}+\frac{14a\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\frac{\frac{3\left(\frac{-\frac{192a^4\int 1d(-\tan(c+dx))}{d}-245a^4\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}+\frac{96a^3\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}+\frac{43\tan(c+dx)\sec^3(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}+\frac{14a\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 24

$$-\frac{\frac{\frac{3\left(\frac{\frac{192a^4\tan(c+dx)}{d}-245a^4\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}+\frac{96a^3\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}+\frac{43\tan(c+dx)\sec^3(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}+\frac{14a\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 4255

$$-\frac{\frac{\frac{3\left(\frac{\frac{192a^4\tan(c+dx)}{d}-245a^4\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}+\frac{96a^3\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}+\frac{43\tan(c+dx)\sec^3(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}+\frac{14a\tan(c+dx)\sec^4}{5d(a\sec(c+dx)+}}{7a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{3\left(\frac{\frac{192a^4\tan(c+dx)}{d}-245a^4\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}+\frac{96a^3\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}+\frac{43\tan(c+dx)\sec^3(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}+\frac{14a\tan(c+dx)}{5d(a\sec(c+}}{7a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 4257

$$-\frac{\frac{\frac{3\left(\frac{96a^3\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}+\frac{\frac{192a^4\tan(c+dx)}{d}-245a^4\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}\right)}{a^2}+\frac{43\tan(c+dx)\sec^3(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}+\frac{14a\tan(c+d}{5d(a\sec(c}}{7a^2}$$

$$\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

input
```
Int[Sec[c + d*x]^7/(a + a*Sec[c + d*x])^4,x]
```

output
```
-1/7*(Sec[c + d*x]^5*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^4) - ((14*a*Sec
[c + d*x]^4*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((43*Sec[c + d*x]
^3*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^2) + (3*((96*a^3*Sec[c + d*x]^2*Tan
[c + d*x])/(d*(a + a*Sec[c + d*x])) + ((192*a^4*Tan[c + d*x])/d - 245*a^4*
(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/a^2))/a
^2)/(5*a^2))/(7*a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-111\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{36}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+84\ln\left(\tan\left(\right.\right.}{8d\,a^4}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-111\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{36}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+84\ln\left(\tan\left(\right.\right.}{8d\,a^4}$
parallelrisch	$\frac{(-23520\cos(2dx+2c)-23520)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(23520\cos(2dx+2c)+23520)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-34168\tan\left(\right.}{2240d\,a^4(1+\cos(}$
risch	$-\frac{i(735\,\mathrm{e}^{10i(dx+c)}+5145\,\mathrm{e}^{9i(dx+c)}+16660\,\mathrm{e}^{8i(dx+c)}+34300\,\mathrm{e}^{7i(dx+c)}+51842\,\mathrm{e}^{6i(dx+c)}+61054\,\mathrm{e}^{5i(dx+c)}+55556\,\mathrm{e}^{4i(d}}{35d\,a^4\left(\mathrm{e}^{2i(dx+c)}+1\right)^2\left(\mathrm{e}^{i(dx+c)}+1\right)^7}$

input
```
int(sec(d*x+c)^7/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/8/d/a^4*(-1/7*tan(1/2*d*x+1/2*c)^7-9/5*tan(1/2*d*x+1/2*c)^5-13*tan(1/2*d
*x+1/2*c)^3-111*tan(1/2*d*x+1/2*c)-4/(tan(1/2*d*x+1/2*c)+1)^2+36/(tan(1/2*
d*x+1/2*c)+1)+84*ln(tan(1/2*d*x+1/2*c)+1)+4/(tan(1/2*d*x+1/2*c)-1)^2+36/(t
an(1/2*d*x+1/2*c)-1)-84*ln(tan(1/2*d*x+1/2*c)-1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.30

$$\int\frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^4}\,dx$$
$$=\frac{735\left(\cos\left(dx+c\right)^6+4\,\cos\left(dx+c\right)^5+6\,\cos\left(dx+c\right)^4+4\,\cos\left(dx+c\right)^3+\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/140*(735*(cos(d*x + c)^6 + 4*cos(d*x + c)^5 + 6*cos(d*x + c)^4 + 4*cos(d
*x + c)^3 + cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 735*(cos(d*x + c)^6 +
4*cos(d*x + c)^5 + 6*cos(d*x + c)^4 + 4*cos(d*x + c)^3 + cos(d*x + c)^2)*l
og(-sin(d*x + c) + 1) - 2*(1152*cos(d*x + c)^5 + 3873*cos(d*x + c)^4 + 454
8*cos(d*x + c)^3 + 2012*cos(d*x + c)^2 + 140*cos(d*x + c) - 35)*sin(d*x +
c))/(a^4*d*cos(d*x + c)^6 + 4*a^4*d*cos(d*x + c)^5 + 6*a^4*d*cos(d*x + c)^
4 + 4*a^4*d*cos(d*x + c)^3 + a^4*d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\int \frac{\sec^7(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**7/(a+a*sec(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**7/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c +
d*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.20

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\frac{280\left(\frac{7\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4-\frac{2a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\sin(dx+c)}{\cos(dx+c)+1}+\frac{455\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{2940\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}}{280\,d}$$

input
```
integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/280*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^4 - 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1) +
455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c)
+ 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 2940*log(sin(d*x + c
)/(cos(d*x + c) + 1) + 1)/a^4 + 2940*log(sin(d*x + c)/(cos(d*x + c) + 1) -
 1)/a^4)/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.80

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^4}\,dx$$
$$= \frac{\frac{2940\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4} - \frac{2940\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4} + \frac{280\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^4} - \frac{5\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+}{}}{280\,d}$$

input
```
integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/280*(2940*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 2940*log(abs(tan(1/2*
d*x + 1/2*c) - 1))/a^4 + 280*(9*tan(1/2*d*x + 1/2*c)^3 - 7*tan(1/2*d*x + 1
/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^4) - (5*a^24*tan(1/2*d*x + 1/2*c)
^7 + 63*a^24*tan(1/2*d*x + 1/2*c)^5 + 455*a^24*tan(1/2*d*x + 1/2*c)^3 + 38
85*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.83

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{21\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^4\,d} - \frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{40\,a^4\,d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{56\,a^4\,d} - \frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8\,a^4\,d} - \frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\,\left(a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^4\right)} - \frac{111\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^4\,d}$$

input
```
int(1/(cos(c + d*x)^7*(a + a/cos(c + d*x))^4),x)
```

output
```
(21*atanh(tan(c/2 + (d*x)/2)))/(a^4*d) - (9*tan(c/2 + (d*x)/2)^5)/(40*a^4*
d) - tan(c/2 + (d*x)/2)^7/(56*a^4*d) - (13*tan(c/2 + (d*x)/2)^3)/(8*a^4*d)
 - (7*tan(c/2 + (d*x)/2) - 9*tan(c/2 + (d*x)/2)^3)/(d*(a^4*tan(c/2 + (d*x)
/2)^4 - 2*a^4*tan(c/2 + (d*x)/2)^2 + a^4)) - (111*tan(c/2 + (d*x)/2))/(8*a
^4*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.25

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{-2940\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+5880\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2940\log\left(\tan\left(\right.\right.}{}$$

input
```
int(sec(d*x+c)^7/(a+a*sec(d*x+c))^4,x)
```

output
```
( - 2940*log(tan((c + d*x)/2) - 1)*tan((c + d*x)/2)**4 + 5880*log(tan((c +
 d*x)/2) - 1)*tan((c + d*x)/2)**2 - 2940*log(tan((c + d*x)/2) - 1) + 2940*
log(tan((c + d*x)/2) + 1)*tan((c + d*x)/2)**4 - 5880*log(tan((c + d*x)/2)
+ 1)*tan((c + d*x)/2)**2 + 2940*log(tan((c + d*x)/2) + 1) - 5*tan((c + d*x
)/2)**11 - 53*tan((c + d*x)/2)**9 - 334*tan((c + d*x)/2)**7 - 3038*tan((c
+ d*x)/2)**5 + 9835*tan((c + d*x)/2)**3 - 5845*tan((c + d*x)/2))/(280*a**4
*d*(tan((c + d*x)/2)**4 - 2*tan((c + d*x)/2)**2 + 1))
```

3.71 $\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 159

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{4\operatorname{arctanh}(\sin(c+dx))}{a^4d}+\frac{244\tan(c+dx)}{105a^4d} - \frac{88\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2}+\frac{4\tan(c+dx)}{a^4d(1+\sec(c+dx))} - \frac{\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}-\frac{12\sec^3(c+dx)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3}$$

output
```
-4*arctanh(sin(d*x+c))/a^4/d+244/105*tan(d*x+c)/a^4/d-88/105*sec(d*x+c)^2*
tan(d*x+c)/a^4/d/(1+sec(d*x+c))^2+4*tan(d*x+c)/a^4/d/(1+sec(d*x+c))-1/7*se
c(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^4-12/35*sec(d*x+c)^3*tan(d*x+c)/a
/d/(a+a*sec(d*x+c))^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 349 vs. $2(159) = 318$.

Time = 2.12 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.19

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^4}\,dx$$
$$= \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(107520\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\mathrm{co}\right.\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^4,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^4*(107520*Cos[(c + d*x)/2]^7*(Log[Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) +
Sec[c/2]*Sec[c]*Sec[c + d*x]*(-10780*Sin[(d*x)/2] + 18788*Sin[(3*d*x)/2] -
 20524*Sin[c - (d*x)/2] + 14644*Sin[c + (d*x)/2] - 16660*Sin[2*c + (d*x)/2
] - 4690*Sin[c + (3*d*x)/2] + 14378*Sin[2*c + (3*d*x)/2] - 9100*Sin[3*c +
(3*d*x)/2] + 11668*Sin[c + (5*d*x)/2] - 630*Sin[2*c + (5*d*x)/2] + 9358*Si
n[3*c + (5*d*x)/2] - 2940*Sin[4*c + (5*d*x)/2] + 4228*Sin[2*c + (7*d*x)/2]
 + 315*Sin[3*c + (7*d*x)/2] + 3493*Sin[4*c + (7*d*x)/2] - 420*Sin[5*c + (7
*d*x)/2] + 664*Sin[3*c + (9*d*x)/2] + 105*Sin[4*c + (9*d*x)/2] + 559*Sin[5
*c + (9*d*x)/2])))/(1680*a^4*d*(1 + Sec[c + d*x])^4)
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.15, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {3042, 4303, 27, 3042, 4507, 3042, 4507, 3042, 4496, 25, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a\sec(c+dx)+a)^4}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^6}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{4\sec^4(c+dx)(a-2a\sec(c+dx))}{(\sec(c+dx)a+a)^3}dx}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4\int \frac{\sec^4(c+dx)(a-2a\sec(c+dx))}{(\sec(c+dx)a+a)^3}dx}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{4\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{4\left(\frac{\int \frac{\sec^3(c+dx)\left(9a^2-13a^2\sec(c+dx)\right)}{(\sec(c+dx)a+a)^2}dx}{5a^2}+\frac{3a\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{4\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(9a^2-13a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}+\frac{3a\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{4\left(\frac{\frac{\int \frac{\sec^2(c+dx)\left(44a^3-61a^3\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}+\frac{22\tan(c+dx)\sec^2(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}+\frac{3a\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{4\left(\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(44a^3-61a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{22\tan(c+dx)\sec^2(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}+\frac{3a\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 4496

$$-\frac{4\left(\frac{\frac{-\frac{\int -\sec(c+dx)\left(105a^4-61a^4\sec(c+dx)\right)dx}{a^2}-\frac{105a^3\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{22\tan(c+dx)\sec^2(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}+\frac{3a\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 25

$$-\frac{4\left(\frac{\frac{\frac{\int \sec(c+dx)\left(105a^4-61a^4\sec(c+dx)\right)dx}{a^2}-\frac{105a^3\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{22\tan(c+dx)\sec^2(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}+\frac{3a\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 3042

$$-\frac{4\left(\frac{\frac{\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(105a^4-61a^4\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}-\frac{105a^3\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{22\tan(c+dx)\sec^2(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}+\frac{3a\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 4274

$$-\frac{4\left(\frac{\frac{\frac{105a^4\int \sec(c+dx)dx-61a^4\int \sec^2(c+dx)dx}{a^2}-\frac{105a^3\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{22\tan(c+dx)\sec^2(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}+\frac{3a\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 3042

$$-\frac{4\left(\frac{\frac{\frac{105a^4\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-61a^4\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2}-\frac{105a^3\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{22\tan(c+dx)\sec^2(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}+\frac{3a\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 4254

$$-\frac{4\left(\frac{\frac{\frac{\frac{61a^4\int 1d(-\tan(c+dx))}{d}+105a^4\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{105a^3\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{22\tan(c+dx)\sec^2(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}+\frac{3a\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 24

$$-\frac{4\left(\frac{\frac{\frac{105a^4\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{61a^4\tan(c+dx)}{d}}{a^2}-\frac{105a^3\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{22\tan(c+dx)\sec^2(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}+\frac{3a\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 4257

$$-\frac{4\left(\frac{\frac{\frac{\frac{105a^4\text{arctanh}(\sin(c+dx))}{d}-\frac{61a^4\tan(c+dx)}{d}}{a^2}-\frac{105a^3\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{22\tan(c+dx)\sec^2(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}+\frac{3a\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

input
```
Int[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^4,x]
```

output
```
-1/7*(Sec[c + d*x]^4*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^4) - (4*((3*a*S
ec[c + d*x]^3*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((22*Sec[c + d*
x]^2*Tan[c + d*x])/(3*d*(1 + Sec[c + d*x])^2) + ((-105*a^3*Tan[c + d*x])/(
d*(a + a*Sec[c + d*x])) + ((105*a^4*ArcTanh[Sin[c + d*x]])/d - (61*a^4*Tan
[c + d*x])/d)/a^2)/(3*a^2))/(5*a^2)))/(7*a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4496
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(b^2*(2*m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b
*B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Ne
Q[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{23\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+49\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-32\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\ldots}}{8d\,a^{4}}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{23\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+49\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-32\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{8}{\tan\left(\frac{dx}{2}+\ldots\right)\ldots}}{8d\,a^{4}}$
parallelrisch	$\frac{3360\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-3360\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+2861\left(\cos(dx+c)+\frac{1650\cos(2dx+2c)}{2861}+\frac{559\ldots}{\ldots}\ldots\right.}{840d\,a^{4}\cos(dx+c)}$
risch	$\frac{8i\left(105\,e^{8i(dx+c)}+735\,e^{7i(dx+c)}+2275\,e^{6i(dx+c)}+4165\,e^{5i(dx+c)}+5131\,e^{4i(dx+c)}+4697\,e^{3i(dx+c)}+2917\,e^{2i(dx+c)}+105\ldots\right.}{105d\,a^{4}\left(e^{i(dx+c)}+1\right)^{7}\left(e^{2i(dx+c)}+1\right)}$

input
```
int(sec(d*x+c)^6/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7+7/5*tan(1/2*d*x+1/2*c)^5+23/3*tan(1/2*
d*x+1/2*c)^3+49*tan(1/2*d*x+1/2*c)-8/(tan(1/2*d*x+1/2*c)+1)-32*ln(tan(1/2*
d*x+1/2*c)+1)-8/(tan(1/2*d*x+1/2*c)-1)+32*ln(tan(1/2*d*x+1/2*c)-1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.47

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{210\left(\cos(dx+c)^5+4\cos(dx+c)^4+6\cos(dx+c)^3+4\cos(dx+c)^2+\cos(dx+c)\right)\log(\sin(dx+\ldots}{\ldots}$$

input
```
integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/105*(210*(cos(d*x + c)^5 + 4*cos(d*x + c)^4 + 6*cos(d*x + c)^3 + 4*cos(
d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c) + 1) - 210*(cos(d*x + c)^5 + 4
*cos(d*x + c)^4 + 6*cos(d*x + c)^3 + 4*cos(d*x + c)^2 + cos(d*x + c))*log(
-sin(d*x + c) + 1) - (664*cos(d*x + c)^4 + 2236*cos(d*x + c)^3 + 2636*cos(
d*x + c)^2 + 1184*cos(d*x + c) + 105)*sin(d*x + c))/(a^4*d*cos(d*x + c)^5
+ 4*a^4*d*cos(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2
 + a^4*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\int \frac{\sec^6(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**6/(a+a*sec(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**6/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c +
d*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.17

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\frac{1680\sin(dx+c)}{\left(a^4-\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} + \frac{\frac{5145\sin(dx+c)}{\cos(dx+c)+1}+\frac{805\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4} - \frac{3360\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4} +}{840\,d}$$

input
```
integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/840*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*
(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4)/d
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.87

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\frac{3360\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4} - \frac{3360\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^4} + \frac{1680\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^4} - \frac{15a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+147a^{24}\tan\left(\frac{1}{2}c\right.}{}}{840d}$$

input
```
integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/840*(3360*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 3360*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a^4 + 1680*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)
^2 - 1)*a^4) - (15*a^24*tan(1/2*d*x + 1/2*c)^7 + 147*a^24*tan(1/2*d*x + 1/
2*c)^5 + 805*a^24*tan(1/2*d*x + 1/2*c)^3 + 5145*a^24*tan(1/2*d*x + 1/2*c))
/a^28)/d
```

Mupad [B] (verification not implemented)

Time = 9.41 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.82

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{23\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24a^4d} + \frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{40a^4d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{56a^4d} - \frac{8\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^4d} - \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-a^4\right)} + \frac{49\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d}$$

input
```
int(1/(cos(c + d*x)^6*(a + a/cos(c + d*x))^4),x)
```

output
```
(23*tan(c/2 + (d*x)/2)^3)/(24*a^4*d) + (7*tan(c/2 + (d*x)/2)^5)/(40*a^4*d)
 + tan(c/2 + (d*x)/2)^7/(56*a^4*d) - (8*atanh(tan(c/2 + (d*x)/2)))/(a^4*d)
 - (2*tan(c/2 + (d*x)/2))/(d*(a^4*tan(c/2 + (d*x)/2)^2 - a^4)) + (49*tan(c
/2 + (d*x)/2))/(8*a^4*d)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.04

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^4}\,dx$$
$$= \frac{3360\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-3360\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-3360\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\mathrm{ta}}{}$$

input
```
int(sec(d*x+c)^6/(a+a*sec(d*x+c))^4,x)
```

output
```
(3360*log(tan((c + d*x)/2) - 1)*tan((c + d*x)/2)**2 - 3360*log(tan((c + d*
x)/2) - 1) - 3360*log(tan((c + d*x)/2) + 1)*tan((c + d*x)/2)**2 + 3360*log
(tan((c + d*x)/2) + 1) + 15*tan((c + d*x)/2)**9 + 132*tan((c + d*x)/2)**7
+ 658*tan((c + d*x)/2)**5 + 4340*tan((c + d*x)/2)**3 - 6825*tan((c + d*x)/
2))/(840*a**4*d*(tan((c + d*x)/2)**2 - 1))
```

3.72 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 136

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{a^4d} + \frac{11\tan(c+dx)}{21a^4d(1+\sec(c+dx))^2} - \frac{43\tan(c+dx)}{21a^4d(1+\sec(c+dx))} - \frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\sec^2(c+dx)\tan(c+dx)}{7ad(a+a\sec(c+dx))^3}$$

output
```
arctanh(sin(d*x+c))/a^4/d+11/21*tan(d*x+c)/a^4/d/(1+sec(d*x+c))^2-43/21*ta
n(d*x+c)/a^4/d/(1+sec(d*x+c))-1/7*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c
))^4-2/7*sec(d*x+c)^2*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^3
```

Mathematica [A] (verified)

Time = 1.75 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.42

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(1344\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\text{co}\right.\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^4,x]
```

output
```
-1/84*(Cos[(c + d*x)/2]*Sec[c + d*x]^4*(1344*Cos[(c + d*x)/2]^7*(Log[Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
) + Sec[c/2]*(686*Sin[(d*x)/2] - 434*Sin[c + (d*x)/2] + 525*Sin[c + (3*d*x
)/2] - 147*Sin[2*c + (3*d*x)/2] + 203*Sin[2*c + (5*d*x)/2] - 21*Sin[3*c +
(5*d*x)/2] + 32*Sin[3*c + (7*d*x)/2])))/(a^4*d*(1 + Sec[c + d*x])^4)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 4303, 3042, 4507, 27, 3042, 4496, 25, 3042, 4486, 3042, 4257, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a\sec(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{\sec^3(c+dx)(3a-7a\sec(c+dx))}{(\sec(c+dx)a+a)^3}dx}{7a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(3a-7a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{\frac{\int \frac{5\sec^2(c+dx)\left(4a^2-7a^2\sec(c+dx)\right)}{(\sec(c+dx)a+a)^2}dx}{5a^2}+\frac{2a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\int \frac{\sec^2(c+dx)\left(4a^2-7a^2\sec(c+dx)\right)}{(\sec(c+dx)a+a)^2}dx}{a^2}+\frac{2a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(4a^2-7a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{a^2}+\frac{2a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 4496$$

$$-\frac{\frac{-\frac{\int -\frac{\sec(c+dx)\left(22a^3-21a^3\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{11\tan(c+dx)}{3d(\sec(c+dx)+1)^2}}{a^2}+\frac{2a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\frac{\int \frac{\sec(c+dx)\left(22a^3-21a^3\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{11\tan(c+dx)}{3d(\sec(c+dx)+1)^2}}{a^2}+\frac{2a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(22a^3-21a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{11\tan(c+dx)}{3d(\sec(c+dx)+1)^2}}{a^2}+\frac{2a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 4486$$

$$-\frac{\frac{\frac{43a^3\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx-21a^2\int \sec(c+dx)dx}{3a^2}-\frac{11\tan(c+dx)}{3d(\sec(c+dx)+1)^2}}{a^2}+\frac{2a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{43a^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-21a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{3a^2}-\frac{11\tan(c+dx)}{3d(\sec(c+dx)+1)^2}}{a^2}+\frac{2a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{ 4257}$$

$$-\frac{\frac{\frac{43a^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{21a^2\text{arctanh}(\sin(c+dx))}{d}}{3a^2}-\frac{11\tan(c+dx)}{3d(\sec(c+dx)+1)^2}}{a^2}+\frac{2a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{ 4281}$$

$$-\frac{\frac{\frac{\frac{43a^3\tan(c+dx)}{d(a\sec(c+dx)+a)}-\frac{21a^2\text{arctanh}(\sin(c+dx))}{d}}{3a^2}-\frac{11\tan(c+dx)}{3d(\sec(c+dx)+1)^2}}{a^2}+\frac{2a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

input
```
Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^4,x]
```

output
```
-1/7*(Sec[c + d*x]^3*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^4) - ((2*a*Sec[
c + d*x]^2*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^3) + ((-11*Tan[c + d*x])/
(3*d*(1 + Sec[c + d*x])^2) + ((-21*a^2*ArcTanh[Sin[c + d*x]])/d + (43*a^3*
Tan[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3*a^2))/a^2)/(7*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

rule 4496
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(b^2*(2*m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b
*B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Ne
Q[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.65

method	result
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{8d\,a^{4}}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{8d\,a^{4}}$
parallelrisch	$\frac{-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}-21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-77\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-168\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+168\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-315\tan(}{168d\,a^{4}}$
risch	$-\frac{2i\left(21\,e^{6i(dx+c)}+147\,e^{5i(dx+c)}+434\,e^{4i(dx+c)}+686\,e^{3i(dx+c)}+525\,e^{2i(dx+c)}+203\,e^{i(dx+c)}+32\right)}{21d\,a^{4}\left(e^{i(dx+c)}+1\right)^{7}}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{d\,a^{4}}$
norman	$\frac{-\frac{15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{169\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{24ad}-\frac{229\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{24ad}+\frac{293\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{56ad}-\frac{121\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{168ad}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{168ad}-\frac{3\tan(}{5}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{4}a^{3}}$

input
```
int(sec(d*x+c)^5/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/8/d/a^4*(-1/7*tan(1/2*d*x+1/2*c)^7-tan(1/2*d*x+1/2*c)^5-11/3*tan(1/2*d*x
+1/2*c)^3-15*tan(1/2*d*x+1/2*c)-8*ln(tan(1/2*d*x+1/2*c)-1)+8*ln(tan(1/2*d*
x+1/2*c)+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.49

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^4}\,dx$$
$$= \frac{21\left(\cos\left(dx+c\right)^4+4\,\cos\left(dx+c\right)^3+6\,\cos\left(dx+c\right)^2+4\,\cos\left(dx+c\right)+1\right)\log\left(\sin\left(dx+c\right)+1\right)-21\,(c}{42\,\left(a^4d\cos\right.}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/42*(21*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4*cos(d*x
 + c) + 1)*log(sin(d*x + c) + 1) - 21*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 +
 6*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*log(-sin(d*x + c) + 1) - 2*(32*cos
(d*x + c)^3 + 107*cos(d*x + c)^2 + 124*cos(d*x + c) + 52)*sin(d*x + c))/(a
^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*
a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\int \frac{\sec^5(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**5/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c +
d*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.02

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{77\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{168\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{168\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}}{168\,d}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/168*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^4
 + 168*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4)/d
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.81

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\frac{168\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{168\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}-\frac{3\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+21\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+77\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+31}{a^{28}}}{168\,d}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/168*(168*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 168*log(abs(tan(1/2*d*
x + 1/2*c) - 1))/a^4 - (3*a^24*tan(1/2*d*x + 1/2*c)^7 + 21*a^24*tan(1/2*d*
x + 1/2*c)^5 + 77*a^24*tan(1/2*d*x + 1/2*c)^3 + 315*a^24*tan(1/2*d*x + 1/2
*c))/a^28)/d
```

Mupad [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.61

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\frac{11\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24\,a^4}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{8\,a^4}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{56\,a^4}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^4}+\frac{15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8\,a^4}}{d}$$

input
```
int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^4),x)
```

output
```
-((11*tan(c/2 + (d*x)/2)^3)/(24*a^4) + tan(c/2 + (d*x)/2)^5/(8*a^4) + tan(
c/2 + (d*x)/2)^7/(56*a^4) - (2*atanh(tan(c/2 + (d*x)/2)))/a^4 + (15*tan(c/
2 + (d*x)/2))/(8*a^4))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.64

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{-168\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+168\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7-21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5-77\tan}{168a^4d}$$

input
```
int(sec(d*x+c)^5/(a+a*sec(d*x+c))^4,x)
```

output
```
( - 168*log(tan((c + d*x)/2) - 1) + 168*log(tan((c + d*x)/2) + 1) - 3*tan(
(c + d*x)/2)**7 - 21*tan((c + d*x)/2)**5 - 77*tan((c + d*x)/2)**3 - 315*ta
n((c + d*x)/2))/(168*a**4*d)
```

3.73 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4} dx$

Optimal result

Integrand size = 21, antiderivative size = 120

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4} dx = \frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{3\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} - \frac{8\tan(c+dx)}{35d\left(a^2+a^2\sec(c+dx)\right)^2} + \frac{\tan(c+dx)}{5d\left(a^4+a^4\sec(c+dx)\right)}$$

output
```
1/7*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^4+3/35*tan(d*x+c)/a/d/(a+a*
sec(d*x+c))^3-8/35*tan(d*x+c)/d/(a^2+a^2*sec(d*x+c))^2+1/5*tan(d*x+c)/d/(a
^4+a^4*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.58

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4} dx = \frac{\sec^7\left(\frac{1}{2}(c+dx)\right)\left(35\sin\left(\frac{1}{2}(c+dx)\right)+21\sin\left(\frac{3}{2}(c+dx)\right)+7\sin\left(\frac{5}{2}(c+dx)\right)+\sin\left(\frac{7}{2}(c+dx)\right)\right)}{1120a^4d}$$

input
```
Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^4,x]
```

output
```
(Sec[(c + d*x)/2]^7*(35*Sin[(c + d*x)/2] + 21*Sin[(3*(c + d*x))/2] + 7*Sin
[(5*(c + d*x))/2] + Sin[(7*(c + d*x))/2]))/(1120*a^4*d)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4297, 3042, 4286, 25, 3042, 4488, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a\sec(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{4297}$$

$$\frac{3\int \frac{\sec^3(c+dx)}{(\sec(c+dx)a+a)^3}dx}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4286}$$

$$\frac{3\left(\frac{\int -\frac{\sec(c+dx)(3a-5a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\int \frac{\sec(c+dx)(3a-5a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3a-5a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4488}$$

$$\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{8a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{7}{3}\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{8a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{7}{3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4281}$$

$$\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{8a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{7\tan(c+dx)}{3d(a\sec(c+dx)+a)}}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

input
```
Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^4,x]
```

output
```
(Sec[c + d*x]^3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + (3*(Tan[c + d
*x]/(5*d*(a + a*Sec[c + d*x])^3) - ((8*a*Tan[c + d*x])/(3*d*(a + a*Sec[c +
 d*x])^2) - (7*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])))/(5*a^2)))/(7*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4286
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Simp[1/(a^2*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[
a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4297
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[d*((m + 1)/(b*(2*m + 1)))
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ
[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m,
-2^(-1)] && IntegerQ[2*m]
```

rule 4488
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
 /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.47

method	result	s
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$	5
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$	5
risch	$\frac{4i\left(35\,{e}^{3i(dx+c)}+21\,{e}^{2i(dx+c)}+7\,{e}^{i(dx+c)}+1\right)}{35d\,a^{4}\left({e}^{i(dx+c)}+1\right)^{7}}$	5
parallelrisch	$\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{280d\,a^{4}}$	6
norman	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{4ad}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{40ad}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{70ad}-\frac{13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{280ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{140ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{56ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}a^{3}}$	1

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7+3/5*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+1
/2*c)^3+tan(1/2*d*x+1/2*c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.82

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\left(2\cos\left(dx+c\right)^{3}+8\cos\left(dx+c\right)^{2}+13\cos\left(dx+c\right)+12\right)\sin\left(dx+c\right)}{35\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\left(dx+c\right)^{2}+4\,a^{4}d\cos\left(dx+c\right)+a^{4}d\right)}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/35*(2*cos(d*x + c)^3 + 8*cos(d*x + c)^2 + 13*cos(d*x + c) + 12)*sin(d*x
+ c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)
^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\int \frac{\sec^4(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**4/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c +
d*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.72

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\frac{35\sin(dx+c)}{\cos(dx+c)+1} + \frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{280\,a^4d}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/280*(35*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7)/(a^4*d)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.49

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 + 21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{280\,a^4d}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/280*(5*tan(1/2*d*x + 1/2*c)^7 + 21*tan(1/2*d*x + 1/2*c)^5 + 35*tan(1/2*d
*x + 1/2*c)^3 + 35*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.48

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+21\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+35\right)}{280\,a^4\,d}$$

input
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^4),x)
```

output
```
(tan(c/2 + (d*x)/2)*(35*tan(c/2 + (d*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^4 + 5
*tan(c/2 + (d*x)/2)^6 + 35))/(280*a^4*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.48

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+35\right)}{280a^4d}$$

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^4,x)
```

output
```
(tan((c + d*x)/2)*(5*tan((c + d*x)/2)**6 + 21*tan((c + d*x)/2)**4 + 35*tan
((c + d*x)/2)**2 + 35))/(280*a**4*d)
```

3.74 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 112

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{11\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{13\tan(c+dx)}{105d\,(a^2+a^2\sec(c+dx))^2} + \frac{13\tan(c+dx)}{105d\,(a^4+a^4\sec(c+dx))}$$

output
```
1/7*tan(d*x+c)/d/(a+a*sec(d*x+c))^4-11/35*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^
3+13/105*tan(d*x+c)/d/(a^2+a^2*sec(d*x+c))^2+13/105*tan(d*x+c)/d/(a^4+a^4*
sec(d*x+c))
```

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(35\sin\left(\frac{dx}{2}\right)-35\sin\left(c+\frac{dx}{2}\right)+2\left(21\sin\left(c+\frac{3dx}{2}\right)+7\sin\left(2c+\frac{5dx}{2}\right)+\sin\left(3c+\right.\right.\right.}{1680a^4d}$$

input
```
Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^4,x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^7*(35*Sin[(d*x)/2] - 35*Sin[c + (d*x)/2] + 2*(2
1*Sin[c + (3*d*x)/2] + 7*Sin[2*c + (5*d*x)/2] + Sin[3*c + (7*d*x)/2])))/(1
680*a^4*d)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4286, 25, 3042, 4488, 3042, 4283, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a\sec(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{4286}$$

$$\frac{\int -\frac{\sec(c+dx)(4a-7a\sec(c+dx))}{(\sec(c+dx)a+a)^3}dx}{7a^2}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}-\frac{\int \frac{\sec(c+dx)(4a-7a\sec(c+dx))}{(\sec(c+dx)a+a)^3}dx}{7a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(4a-7a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}$$

$$\Big\downarrow \text{4488}$$

$$\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}-\frac{\frac{11a\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{13}{5}\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^2}dx}{7a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}-\frac{\frac{11a\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{13}{5}\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{7a^2}$$

$$\Big\downarrow \text{ 4283}$$

$$\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}-\frac{\frac{11a\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{13}{5}\left(\frac{\int\frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{7a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}-\frac{\frac{11a\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{13}{5}\left(\frac{\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{7a^2}$$

$$\Big\downarrow \text{ 4281}$$

$$\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}-\frac{\frac{11a\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{13}{5}\left(\frac{\tan(c+dx)}{3ad(a\sec(c+dx)+a)}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{7a^2}$$

input
```
Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^4,x]
```

output
```
Tan[c + d*x]/(7*d*(a + a*Sec[c + d*x])^4) - ((11*a*Tan[c + d*x])/(5*d*(a +
 a*Sec[c + d*x])^3) - (13*(Tan[c + d*x]/(3*d*(a + a*Sec[c + d*x])^2) + Tan
[c + d*x]/(3*a*d*(a + a*Sec[c + d*x]))))/5)/(7*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4283

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

rule 4286

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Simp[1/(a^2*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[
a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4488

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
 /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.51

method	result	size
parallelrisch	$-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{5}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{3}-7\right)}{56d\,a^{4}}$	57
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$	58
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$	58
risch	$\frac{8i\left(35\,{e}^{4i(dx+c)}+35\,{e}^{3i(dx+c)}+42\,{e}^{2i(dx+c)}+14\,{e}^{i(dx+c)}+2\right)}{105d\,a^{4}\left({e}^{i(dx+c)}+1\right)^{7}}$	69
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{24ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{60ad}+\frac{31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{420ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{280ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{56ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}a^{3}}$	133

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/56*tan(1/2*d*x+1/2*c)*(tan(1/2*d*x+1/2*c)^6+7/5*tan(1/2*d*x+1/2*c)^4-7/
3*tan(1/2*d*x+1/2*c)^2-7)/d/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\left(8\,\cos\left(dx+c\right)^3+32\,\cos\left(dx+c\right)^2+52\,\cos\left(dx+c\right)+13\right)\sin\left(dx+c\right)}{105\,\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\cos\left(dx+c\right)^3+6\,a^4d\cos\left(dx+c\right)^2+4\,a^4d\cos\left(dx+c\right)+a^4d\right)}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/105*(8*cos(d*x + c)^3 + 32*cos(d*x + c)^2 + 52*cos(d*x + c) + 13)*sin(d*
x + c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x +
c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\int \frac{\sec^3(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**3/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c +
d*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1} + \frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{840\,a^4d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/840*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7)/(a^4*d)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.53

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{840\,a^4d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/840*(15*tan(1/2*d*x + 1/2*c)^7 + 21*tan(1/2*d*x + 1/2*c)^5 - 35*tan(1/2
*d*x + 1/2*c)^3 - 105*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(-15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+105\right)}{840\,a^4\,d}$$

input
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^4),x)
```

output
```
(tan(c/2 + (d*x)/2)*(35*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 - 1
5*tan(c/2 + (d*x)/2)^6 + 105))/(840*a^4*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6-21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+105\right)}{840a^4d}$$

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^4,x)
```

output
```
(tan((c + d*x)/2)*( - 15*tan((c + d*x)/2)**6 - 21*tan((c + d*x)/2)**4 + 35
*tan((c + d*x)/2)**2 + 105))/(840*a**4*d)
```

3.75 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 112

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4}+\frac{4\tan(c+dx)}{35ad(a+a\sec(c+dx))^3}+\frac{8\tan(c+dx)}{105d\,(a^2+a^2\sec(c+dx))^2}+\frac{8\tan(c+dx)}{105d\,(a^4+a^4\sec(c+dx))}$$

output
```
-1/7*tan(d*x+c)/d/(a+a*sec(d*x+c))^4+4/35*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^
3+8/105*tan(d*x+c)/d/(a^2+a^2*sec(d*x+c))^2+8/105*tan(d*x+c)/d/(a^4+a^4*se
c(d*x+c))
```

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.50

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{(13+52\sec(c+dx)+32\sec^2(c+dx)+8\sec^3(c+dx))\tan(c+dx)}{105a^4d(1+\sec(c+dx))^4}$$

input
```
Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^4,x]
```

output
```
((13 + 52*Sec[c + d*x] + 32*Sec[c + d*x]^2 + 8*Sec[c + d*x]^3)*Tan[c + d*x
])/(105*a^4*d*(1 + Sec[c + d*x])^4)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4284, 3042, 4283, 3042, 4283, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a\sec(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{4284}$$

$$\frac{4\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^3}dx}{7a}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4283}$$

$$\frac{4\left(\frac{2\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^2}dx}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\left(\frac{2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4283}$$

$$\frac{4\left(\frac{2\left(\frac{\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\left(\frac{2\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4281}$$

$$\frac{4\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{2\left(\frac{\tan(c+dx)}{3ad(a\sec(c+dx)+a)}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}\right)}{7a}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

input
```
Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^4,x]
```

output
```
-1/7*Tan[c + d*x]/(d*(a + a*Sec[c + d*x])^4) + (4*(Tan[c + d*x]/(5*d*(a +
a*Sec[c + d*x])^3) + (2*(Tan[c + d*x]/(3*d*(a + a*Sec[c + d*x])^2) + Tan[c
 + d*x]/(3*a*d*(a + a*Sec[c + d*x]))))/(5*a)))/(7*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

rule 4284
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Simp[m/(b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

method	result	size
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}$	58
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}$	58
parallelrisch	$\frac{15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7-21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5-35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+105\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{840d\,a^4}$	60
risch	$\frac{2i\left(105\,{\rm e}^{5i(dx+c)}+175\,{\rm e}^{4i(dx+c)}+280\,{\rm e}^{3i(dx+c)}+168\,{\rm e}^{2i(dx+c)}+91\,{\rm e}^{i(dx+c)}+13\right)}{105d\,a^4\left({\rm e}^{i(dx+c)}+1\right)^7}$	80
norman	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{6ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{60ad}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{70ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{56ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)a^3}$	114

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7-1/5*tan(1/2*d*x+1/2*c)^5-1/3*tan(1/2*d
*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\left(13\cos(dx+c)^3+52\cos(dx+c)^2+32\cos(dx+c)+8\right)\sin(dx+c)}{105\left(a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+6a^4d\cos(dx+c)^2+4a^4d\cos(dx+c)+a^4d\right)}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/105*(13*cos(d*x + c)^3 + 52*cos(d*x + c)^2 + 32*cos(d*x + c) + 8)*sin(d*
x + c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x +
c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\int \frac{\sec^2(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**2/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c +
d*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{840\,a^4d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/840*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7)/(a^4*d)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.53

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{840\,a^4d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/840*(15*tan(1/2*d*x + 1/2*c)^7 - 21*tan(1/2*d*x + 1/2*c)^5 - 35*tan(1/2*
d*x + 1/2*c)^3 + 105*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

Mupad [B] (verification not implemented)

Time = 9.47 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(-15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-105\right)}{840\,a^4\,d}$$

input
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^4),x)
```

output
```
-(tan(c/2 + (d*x)/2)*(35*tan(c/2 + (d*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^4 -
15*tan(c/2 + (d*x)/2)^6 - 105))/(840*a^4*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx$$
$$= \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6-21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4-35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+105\right)}{840a^4d}$$

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^4,x)
```

output
```
(tan((c + d*x)/2)*(15*tan((c + d*x)/2)**6 - 21*tan((c + d*x)/2)**4 - 35*ta
n((c + d*x)/2)**2 + 105))/(840*a**4*d)
```

3.76 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 112

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{3\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{2\tan(c+dx)}{35d\left(a^2+a^2\sec(c+dx)\right)^2} + \frac{2\tan(c+dx)}{35d\left(a^4+a^4\sec(c+dx)\right)}$$

output
```
1/7*tan(d*x+c)/d/(a+a*sec(d*x+c))^4+3/35*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^3
+2/35*tan(d*x+c)/d/(a^2+a^2*sec(d*x+c))^2+2/35*tan(d*x+c)/d/(a^4+a^4*sec(d
*x+c))
```

Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(210\sin\left(\frac{dx}{2}\right)-210\sin\left(c+\frac{dx}{2}\right)+147\sin\left(c+\frac{3dx}{2}\right)-105\sin\left(2c+\frac{3dx}{2}\right)+49\sin\right.}{2240a^4d}$$

input
```
Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^4,x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^7*(210*Sin[(d*x)/2] - 210*Sin[c + (d*x)/2] + 14
7*Sin[c + (3*d*x)/2] - 105*Sin[2*c + (3*d*x)/2] + 49*Sin[2*c + (5*d*x)/2]
- 35*Sin[3*c + (5*d*x)/2] + 12*Sin[3*c + (7*d*x)/2]))/(2240*a^4*d)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 4283, 3042, 4283, 3042, 4283, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a\sec(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{4283}$$

$$\frac{3\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^3}dx}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4283}$$

$$\frac{3\left(\frac{2\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^2}dx}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4283}$$

$$\frac{3\left(\frac{2\left(\frac{\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{2\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4281}$$

$$\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}+\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{2\left(\frac{\tan(c+dx)}{3ad(a\sec(c+dx)+a)}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}\right)}{7a}$$

input
```
Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^4,x]
```

output
```
Tan[c + d*x]/(7*d*(a + a*Sec[c + d*x])^4) + (3*(Tan[c + d*x]/(5*d*(a + a*S
ec[c + d*x])^3) + (2*(Tan[c + d*x]/(3*d*(a + a*Sec[c + d*x])^2) + Tan[c +
d*x]/(3*a*d*(a + a*Sec[c + d*x]))))/(5*a)))/(7*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.51

method	result	size
parallelrisch	$-\frac{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-\frac{21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{5}+7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-7\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{56d\,a^{4}}$	57
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$	58
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$	58
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{8ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{40ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{56ad}}{a^{3}}$	80
risch	$\frac{2i\left(35\,e^{6i(dx+c)}+105\,e^{5i(dx+c)}+210\,e^{4i(dx+c)}+210\,e^{3i(dx+c)}+147\,e^{2i(dx+c)}+49\,e^{i(dx+c)}+12\right)}{35d\,a^{4}\left(e^{i(dx+c)}+1\right)^{7}}$	91

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/56*(tan(1/2*d*x+1/2*c)^6-21/5*tan(1/2*d*x+1/2*c)^4+7*tan(1/2*d*x+1/2*c)
^2-7)*tan(1/2*d*x+1/2*c)/d/a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\left(12\cos\left(dx+c\right)^{3}+13\cos\left(dx+c\right)^{2}+8\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)}{35\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\left(dx+c\right)^{2}+4\,a^{4}d\cos\left(dx+c\right)+a^{4}d\right)}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/35*(12*cos(d*x + c)^3 + 13*cos(d*x + c)^2 + 8*cos(d*x + c) + 2)*sin(d*x
+ c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)
^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\int \frac{\sec(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x
)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\frac{35\sin(dx+c)}{\cos(dx+c)+1} - \frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{280\,a^4d}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/280*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7)/(a^4*d)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.53

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{280\,a^4d}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/280*(5*tan(1/2*d*x + 1/2*c)^7 - 21*tan(1/2*d*x + 1/2*c)^5 + 35*tan(1/2*
d*x + 1/2*c)^3 - 35*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

Mupad [B] (verification not implemented)

Time = 9.37 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-35\right)}{280\,a^4\,d}$$

input
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^4),x)
```

output
```
-(tan(c/2 + (d*x)/2)*(35*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 +
5*tan(c/2 + (d*x)/2)^6 - 35))/(280*a^4*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4-35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+35\right)}{280a^4d}$$

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^4,x)
```

output
```
(tan((c + d*x)/2)*( - 5*tan((c + d*x)/2)**6 + 21*tan((c + d*x)/2)**4 - 35*
tan((c + d*x)/2)**2 + 35))/(280*a**4*d)
```

3.77 $\int \frac{1}{(a+a\sec(c+dx))^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 111

$$\int \frac{1}{(a+a\sec(c+dx))^4}\,dx = \frac{x}{a^4} - \frac{11\tan(c+dx)}{21a^4d(1+\sec(c+dx))^2} - \frac{32\tan(c+dx)}{21a^4d(1+\sec(c+dx))} - \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\tan(c+dx)}{7ad(a+a\sec(c+dx))^3}$$

output
```
x/a^4-11/21*tan(d*x+c)/a^4/d/(1+sec(d*x+c))^2-32/21*tan(d*x+c)/a^4/d/(1+se
c(d*x+c))-1/7*tan(d*x+c)/d/(a+a*sec(d*x+c))^4-2/7*tan(d*x+c)/a/d/(a+a*sec(
d*x+c))^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 224 vs. $2(111) = 222$.

Time = 1.58 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.02

$$\int \frac{1}{(a+a\sec(c+dx))^4}\,dx = \frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(735dx\cos\left(\frac{dx}{2}\right)+735dx\cos\left(c+\frac{dx}{2}\right)+441dx\cos\left(c+\frac{3dx}{2}\right)+441dx\cos\left(2c+\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(-4),x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^7*(735*d*x*Cos[(d*x)/2] + 735*d*x*Cos[c + (d*x)
/2] + 441*d*x*Cos[c + (3*d*x)/2] + 441*d*x*Cos[2*c + (3*d*x)/2] + 147*d*x*
Cos[2*c + (5*d*x)/2] + 147*d*x*Cos[3*c + (5*d*x)/2] + 21*d*x*Cos[3*c + (7*
d*x)/2] + 21*d*x*Cos[4*c + (7*d*x)/2] - 1988*Sin[(d*x)/2] + 1652*Sin[c + (
d*x)/2] - 1428*Sin[c + (3*d*x)/2] + 756*Sin[2*c + (3*d*x)/2] - 560*Sin[2*c
 + (5*d*x)/2] + 168*Sin[3*c + (5*d*x)/2] - 104*Sin[3*c + (7*d*x)/2]))/(268
8*a^4*d)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 4264, 25, 3042, 4410, 27, 3042, 4410, 25, 3042, 4407, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{4264}$$

$$-\frac{\int -\frac{7a-3a\sec(c+dx)}{(\sec(c+dx)a+a)^3}dx}{7a^2}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{7a-3a\sec(c+dx)}{(\sec(c+dx)a+a)^3}dx}{7a^2}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{7a-3a\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4410}$$

$$\frac{-\frac{\int -\frac{5\left(7a^2-4a^2\sec(c+dx)\right)}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{7a^2-4a^2\sec(c+dx)}{(\sec(c+dx)a+a)^2}dx}{a^2}-\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{7a^2-4a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{a^2}-\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 4410

$$\frac{\frac{-\frac{\int -\frac{21a^3-11a^3\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{11\tan(c+dx)}{3d(\sec(c+dx)+1)^2}}{a^2}-\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{21a^3-11a^3\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{11\tan(c+dx)}{3d(\sec(c+dx)+1)^2}}{a^2}-\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{21a^3-11a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{11\tan(c+dx)}{3d(\sec(c+dx)+1)^2}}{a^2}-\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 4407

$$\frac{\frac{\frac{21a^2x-32a^3\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{11\tan(c+dx)}{3d(\sec(c+dx)+1)^2}}{a^2}-\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{21a^2x-32a^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{11\tan(c+dx)}{3d(\sec(c+dx)+1)^2}}{a^2}-\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4281}$$

$$\frac{\frac{\frac{21a^2x-\frac{32a^3\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{11\tan(c+dx)}{3d(\sec(c+dx)+1)^2}}{a^2}-\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

input
```
Int[(a + a*Sec[c + d*x])^(-4),x]
```

output
```
-1/7*Tan[c + d*x]/(d*(a + a*Sec[c + d*x])^4) + ((-2*a*Tan[c + d*x])/(d*(a
+ a*Sec[c + d*x])^3) + ((-11*Tan[c + d*x])/(3*d*(1 + Sec[c + d*x])^2) + (2
1*a^2*x - (32*a^3*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3*a^2))/a^2)/(7
*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4264
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Simp[1/(a^2*(2*n + 1))
   Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Int
egerQ[2*n]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4410
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e +
f*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&
EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.58

method	result	size
parallelrisch	$\frac{3 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}-21 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+77 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+168 dx-315 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{168 d\, a^{4}}$	64
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+\frac{11 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-15 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16 \arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8 d\, a^{4}}$	72
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+\frac{11 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-15 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16 \arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8 d\, a^{4}}$	72
norman	$\frac{\frac{x}{a}-\frac{15 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8 a d}+\frac{11 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{24 a d}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{8 a d}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{56 a d}}{a^{3}}$	85
risch	$\frac{x}{a^{4}}-\frac{4 i \left(42 \,{\mathrm e}^{6 i (dx+c)}+189 \,{\mathrm e}^{5 i (dx+c)}+413 \,{\mathrm e}^{4 i (dx+c)}+497 \,{\mathrm e}^{3 i (dx+c)}+357 \,{\mathrm e}^{2 i (dx+c)}+140 \,{\mathrm e}^{i (dx+c)}+26\right)}{21 d\, a^{4} \left({\mathrm e}^{i (dx+c)}+1\right)^{7}}$	97

input
```
int(1/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/168*(3*tan(1/2*d*x+1/2*c)^7-21*tan(1/2*d*x+1/2*c)^5+77*tan(1/2*d*x+1/2*c
)^3+168*d*x-315*tan(1/2*d*x+1/2*c))/d/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.37

$$\int \frac{1}{(a+a\sec(c+dx))^4}\,dx$$
$$=\frac{21\,dx\cos\left(dx+c\right)^4+84\,dx\cos\left(dx+c\right)^3+126\,dx\cos\left(dx+c\right)^2+84\,dx\cos\left(dx+c\right)+21\,dx-\left(52\cos\left(\right.\right.}{21\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\cos\left(dx+c\right)^3+6\,a^4d\cos\left(dx+c\right)^2\right.}$$

input
```
integrate(1/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/21*(21*d*x*cos(d*x + c)^4 + 84*d*x*cos(d*x + c)^3 + 126*d*x*cos(d*x + c)
^2 + 84*d*x*cos(d*x + c) + 21*d*x - (52*cos(d*x + c)^3 + 124*cos(d*x + c)^
2 + 107*cos(d*x + c) + 32)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*c
os(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(c+dx))^4}\,dx=\frac{\int \frac{1}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(1/(a+a*sec(d*x+c))**4,x)
```

output
```
Integral(1/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*se
c(c + d*x) + 1), x)/a**4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.01

$$\int \frac{1}{(a+a\sec(c+dx))^4}\,dx = -\frac{\frac{\frac{315\sin(dx+c)}{\cos(dx+c)+1}-\frac{77\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{336\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}}{168\,d}$$

input
```
integrate(1/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/168*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 77*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^4)
/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.75

$$\int \frac{1}{(a+a\sec(c+dx))^4}\,dx = \frac{\frac{168(dx+c)}{a^4}+\frac{3a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-21a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+77a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-315a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{28}}}{168\,d}$$

input
```
integrate(1/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/168*(168*(d*x + c)/a^4 + (3*a^24*tan(1/2*d*x + 1/2*c)^7 - 21*a^24*tan(1/
2*d*x + 1/2*c)^5 + 77*a^24*tan(1/2*d*x + 1/2*c)^3 - 315*a^24*tan(1/2*d*x +
 1/2*c))/a^28)/d
```

Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.92

$$\int \frac{1}{(a+a\sec(c+dx))^4}\,dx = \frac{x}{a^4} + \frac{-\frac{52\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{21}+\frac{16\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{21}-\frac{5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{28}+\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{56}}{a^4\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}$$

input
```
int(1/(a + a/cos(c + d*x))^4,x)
```

output
```
x/a^4 + (sin(c/2 + (d*x)/2)/56 - (5*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2
))/28 + (16*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/21 - (52*cos(c/2 + (d
*x)/2)^6*sin(c/2 + (d*x)/2))/21)/(a^4*d*cos(c/2 + (d*x)/2)^7)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.57

$$\int \frac{1}{(a+a\sec(c+dx))^4}\,dx = \frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7-21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+77\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-315\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+168dx}{168a^4d}$$

input
```
int(1/(a+a*sec(d*x+c))^4,x)
```

output
```
(3*tan((c + d*x)/2)**7 - 21*tan((c + d*x)/2)**5 + 77*tan((c + d*x)/2)**3 -
 315*tan((c + d*x)/2) + 168*d*x)/(168*a**4*d)
```

3.78 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 126

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{4x}{a^4} + \frac{664\sin(c+dx)}{105a^4d} - \frac{88\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{4\sin(c+dx)}{a^4d(1+\sec(c+dx))} - \frac{\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{12\sin(c+dx)}{35ad(a+a\sec(c+dx))^3}$$

output
```
-4*x/a^4+664/105*sin(d*x+c)/a^4/d-88/105*sin(d*x+c)/a^4/d/(1+sec(d*x+c))^2
-4*sin(d*x+c)/a^4/d/(1+sec(d*x+c))-1/7*sin(d*x+c)/d/(a+a*sec(d*x+c))^4-12/
35*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 263 vs. 2(126) = 252.

Time = 2.39 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.09

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^4}\,dx =$$
$$-\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(29400dx\cos\left(\frac{dx}{2}\right)+29400dx\cos\left(c+\frac{dx}{2}\right)+17640dx\cos\left(c+\frac{3dx}{2}\right)+17640dx\ldots\right.}{\ldots}$$

input
```
Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^4,x]
```

output
```
-1/26880*(Sec[c/2]*Sec[(c + d*x)/2]^7*(29400*d*x*Cos[(d*x)/2] + 29400*d*x*
Cos[c + (d*x)/2] + 17640*d*x*Cos[c + (3*d*x)/2] + 17640*d*x*Cos[2*c + (3*d
*x)/2] + 5880*d*x*Cos[2*c + (5*d*x)/2] + 5880*d*x*Cos[3*c + (5*d*x)/2] + 8
40*d*x*Cos[3*c + (7*d*x)/2] + 840*d*x*Cos[4*c + (7*d*x)/2] - 60830*Sin[(d*
x)/2] + 46130*Sin[c + (d*x)/2] - 46116*Sin[c + (3*d*x)/2] + 18060*Sin[2*c
+ (3*d*x)/2] - 19292*Sin[2*c + (5*d*x)/2] + 2100*Sin[3*c + (5*d*x)/2] - 37
91*Sin[3*c + (7*d*x)/2] - 735*Sin[4*c + (7*d*x)/2] - 105*Sin[4*c + (9*d*x)
/2] - 105*Sin[5*c + (9*d*x)/2]))/(a^4*d)
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.19, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {3042, 4304, 27, 3042, 4508, 3042, 4508, 3042, 4508, 3042, 4274, 24, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a\sec(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{4\cos(c+dx)(2a-a\sec(c+dx))}{(\sec(c+dx)a+a)^3}dx}{7a^2}-\frac{\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int \frac{\cos(c+dx)(2a-a\sec(c+dx))}{(\sec(c+dx)a+a)^3}dx}{7a^2}-\frac{\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \frac{2a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\Big\downarrow$ 4508

$$\frac{4\left(\frac{\int \frac{\cos(c+dx)\left(13a^2-9a^2\sec(c+dx)\right)}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{3a\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\Big\downarrow$ 3042

$$\frac{4\left(\frac{\int \frac{13a^2-9a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{3a\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\Big\downarrow$ 4508

$$\frac{4\left(\frac{\frac{\int \frac{\cos(c+dx)\left(61a^3-44a^3\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{22\sin(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}-\frac{3a\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\Big\downarrow$ 3042

$$\frac{4\left(\frac{\frac{\int \frac{61a^3-44a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{22\sin(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}-\frac{3a\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\Big\downarrow$ 4508

$$\frac{4\left(\frac{\frac{\frac{\int \cos(c+dx)\left(166a^4-105a^4\sec(c+dx)\right)dx}{a^2}-\frac{105a^3\sin(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{22\sin(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}-\frac{3a\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\Big\downarrow$ 3042

$$\frac{4\left(\frac{\frac{\frac{\int \frac{166a^4-105a^4\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{105a^3\sin(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{22\sin(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}-\frac{3a\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \textbf{4274}$$

$$\frac{4\left(\frac{\frac{\frac{166a^4\int\cos(c+dx)dx-105a^4\int 1dx}{a^2}-\frac{105a^3\sin(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{22\sin(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}-\frac{3a\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \textbf{24}$$

$$\frac{4\left(\frac{\frac{\frac{166a^4\int\cos(c+dx)dx-105a^4x}{a^2}-\frac{105a^3\sin(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{22\sin(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}-\frac{3a\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{4\left(\frac{\frac{\frac{166a^4\int\sin\left(c+dx+\frac{\pi}{2}\right)dx-105a^4x}{a^2}-\frac{105a^3\sin(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{22\sin(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}-\frac{3a\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \textbf{3117}$$

$$\frac{4\left(\frac{\frac{\frac{\frac{166a^4\sin(c+dx)}{d}-105a^4x}{a^2}-\frac{105a^3\sin(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{22\sin(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}-\frac{3a\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

input
```
Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^4,x]
```

output
```
-1/7*Sin[c + d*x]/(d*(a + a*Sec[c + d*x])^4) + (4*((-3*a*Sin[c + d*x])/(5*
d*(a + a*Sec[c + d*x])^3) + ((-22*Sin[c + d*x])/(3*d*(1 + Sec[c + d*x])^2)
 + ((-105*a^3*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])) + (-105*a^4*x + (166*
a^4*Sin[c + d*x])/d)/a^2)/(3*a^2))/(5*a^2)))/(7*a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.61

method	result
parallelrisch	$\frac{781\left(\cos(dx+c)+\frac{2741\cos(2dx+2c)}{6248}+\frac{74\cos(3dx+3c)}{781}+\frac{105\cos(4dx+4c)}{24992}+\frac{16171}{24992}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-840dx}{210d\,a^{4}}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{23\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+49\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}-64\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^{4}}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{23\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+49\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}-64\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^{4}}$
risch	$-\frac{4x}{a^{4}}-\frac{i\mathrm{e}^{i(dx+c)}}{2d\,a^{4}}+\frac{i\mathrm{e}^{-i(dx+c)}}{2d\,a^{4}}+\frac{4i\left(525\,\mathrm{e}^{6i(dx+c)}+2625\,\mathrm{e}^{5i(dx+c)}+5950\,\mathrm{e}^{4i(dx+c)}+7420\,\mathrm{e}^{3i(dx+c)}+5397\,\mathrm{e}^{2i(dx-}\right.}{105d\,a^{4}\left(\mathrm{e}^{i(dx+c)}+1\right)^{7}}$
norman	$\frac{-\frac{4x}{a}+\frac{65\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{6ad}-\frac{47\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{60ad}+\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{70ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{56ad}-\frac{4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)a^{3}}$

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/210*(781*(cos(d*x+c)+2741/6248*cos(2*d*x+2*c)+74/781*cos(3*d*x+3*c)+105/
24992*cos(4*d*x+4*c)+16171/24992)*tan(1/2*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^6-
840*d*x)/d/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.29

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{420\,dx\cos{(dx+c)}^4+1680\,dx\cos{(dx+c)}^3+2520\,dx\cos{(dx+c)}^2+1680\,dx\cos{(dx+c)}+420\,dx -}{105\left(a^4d\cos{(dx+c)}^4+4\,a^4d\cos{(dx+c)}^3+\right.}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/105*(420*d*x*cos(d*x + c)^4 + 1680*d*x*cos(d*x + c)^3 + 2520*d*x*cos(d*
x + c)^2 + 1680*d*x*cos(d*x + c) + 420*d*x - (105*cos(d*x + c)^4 + 1184*co
s(d*x + c)^3 + 2636*cos(d*x + c)^2 + 2236*cos(d*x + c) + 664)*sin(d*x + c)
)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2
+ 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\int \frac{\cos{(c+dx)}}{\sec^4{(c+dx)}+4\sec^3{(c+dx)}+6\sec^2{(c+dx)}+4\sec{(c+dx)}+1}\,dx}{a^4}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))**4,x)
```

output
```
Integral(cos(c + d*x)/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x
)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.25

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\frac{1680\,\sin(dx+c)}{\left(a^4+\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} + \frac{\frac{5145\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{805\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4} - \frac{6720\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}}{840\,d}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/840*(1680*sin(d*x + c)/((a^4 + a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*
(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) - 805*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 6720*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a^4)/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.89

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\frac{3360\,(dx+c)}{a^4} - \frac{1680\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^4} + \frac{15\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-147\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+805\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5145\,a^{24}\tan\left(\frac{1}{2}\,d\right.}{a^{28}}}{840\,d}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/840*(3360*(d*x + c)/a^4 - 1680*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2
*c)^2 + 1)*a^4) + (15*a^24*tan(1/2*d*x + 1/2*c)^7 - 147*a^24*tan(1/2*d*x +
 1/2*c)^5 + 805*a^24*tan(1/2*d*x + 1/2*c)^3 - 5145*a^24*tan(1/2*d*x + 1/2*
c))/a^28)/d
```

Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.09

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{15\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-192\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+1144\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-6112\cos\left(\frac{c}{2}+\right.}{840\,a^4\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right.}$$

input
```
int(cos(c + d*x)/(a + a/cos(c + d*x))^4,x)
```

output
```
-(15*sin(c/2 + (d*x)/2) - 192*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 11
44*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) - 6112*cos(c/2 + (d*x)/2)^6*sin
(c/2 + (d*x)/2) - 1680*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 3360*cos(
c/2 + (d*x)/2)^7*(c + d*x))/(840*a^4*d*cos(c/2 + (d*x)/2)^7)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.84

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9+132\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7-658\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+4340\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-3360\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 d}{840a^4d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)}$$

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^4,x)
```

output
```
( - 15*tan((c + d*x)/2)**9 + 132*tan((c + d*x)/2)**7 - 658*tan((c + d*x)/2
)**5 + 4340*tan((c + d*x)/2)**3 - 3360*tan((c + d*x)/2)**2*d*x + 6825*tan(
(c + d*x)/2) - 3360*d*x)/(840*a**4*d*(tan((c + d*x)/2)**2 + 1))
```

3.79 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 176

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx &= \frac{21x}{2a^4}-\frac{576\sin(c+dx)}{35a^4d}+\frac{21\cos(c+dx)\sin(c+dx)}{2a^4d}\\ &\quad-\frac{43\cos(c+dx)\sin(c+dx)}{35a^4d(1+\sec(c+dx))^2}-\frac{288\cos(c+dx)\sin(c+dx)}{35a^4d(1+\sec(c+dx))}\\ &\quad-\frac{\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4}-\frac{2\cos(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^3}\end{aligned}$$

output
```
21/2*x/a^4-576/35*sin(d*x+c)/a^4/d+21/2*cos(d*x+c)*sin(d*x+c)/a^4/d-43/35*
cos(d*x+c)*sin(d*x+c)/a^4/d/(1+sec(d*x+c))^2-288/35*cos(d*x+c)*sin(d*x+c)/
a^4/d/(1+sec(d*x+c))-1/7*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^4-2/5*co
s(d*x+c)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^3
```

Mathematica [A] (verified)

Time = 2.70 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.64

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx$$
$$= \frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(102900dx\cos\left(\frac{dx}{2}\right)+102900dx\cos\left(c+\frac{dx}{2}\right)+61740dx\cos\left(c+\frac{3dx}{2}\right)+61740dx\right.}{}$$

input
```
Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^4,x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^7*(102900*d*x*Cos[(d*x)/2] + 102900*d*x*Cos[c +
 (d*x)/2] + 61740*d*x*Cos[c + (3*d*x)/2] + 61740*d*x*Cos[2*c + (3*d*x)/2]
+ 20580*d*x*Cos[2*c + (5*d*x)/2] + 20580*d*x*Cos[3*c + (5*d*x)/2] + 2940*d
*x*Cos[3*c + (7*d*x)/2] + 2940*d*x*Cos[4*c + (7*d*x)/2] - 179830*Sin[(d*x)
/2] + 128730*Sin[c + (d*x)/2] - 140826*Sin[c + (3*d*x)/2] + 44310*Sin[2*c
+ (3*d*x)/2] - 60487*Sin[2*c + (5*d*x)/2] + 1225*Sin[3*c + (5*d*x)/2] - 12
001*Sin[3*c + (7*d*x)/2] - 3185*Sin[4*c + (7*d*x)/2] - 315*Sin[4*c + (9*d*
x)/2] - 315*Sin[5*c + (9*d*x)/2] + 35*Sin[5*c + (11*d*x)/2] + 35*Sin[6*c +
 (11*d*x)/2]))/(35840*a^4*d)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.10, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {3042, 4304, 25, 3042, 4508, 3042, 4508, 27, 3042, 4508, 3042, 4274, 3042, 3115, 24, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a\sec(c+dx)+a)^4}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{\cos^2(c+dx)(9a-5a\sec(c+dx))}{(\sec(c+dx)a+a)^3}dx}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\cos^2(c+dx)(9a-5a\sec(c+dx))}{(\sec(c+dx)a+a)^3}dx}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{9a-5a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4508}$$

$$\frac{\frac{\int \frac{\cos^2(c+dx)\left(73a^2-56a^2\sec(c+dx)\right)}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{14a\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{73a^2-56a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{14a\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4508}$$

$$\frac{\frac{\frac{\int \frac{9\cos^2(c+dx)\left(53a^3-43a^3\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{43\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}-\frac{14a\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3\int \frac{\cos^2(c+dx)\left(53a^3-43a^3\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{a^2}-\frac{43\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}-\frac{14a\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{3\int \frac{53a^3-43a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{a^2}-\frac{43\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}-\frac{14a\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\Big\downarrow$ 4508

$$\frac{\frac{\frac{3\left(\frac{\int \cos^2(c+dx)\left(245a^4-192a^4\sec(c+dx)\right)dx}{a^2}-\frac{96a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}-\frac{43\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}-\frac{14a\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{3\left(\frac{\int \frac{245a^4-192a^4\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx}{a^2}-\frac{96a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}-\frac{43\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}-\frac{14a\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\Big\downarrow$ 4274

$$\frac{\frac{\frac{3\left(\frac{245a^4\int \cos^2(c+dx)dx-192a^4\int \cos(c+dx)dx}{a^2}-\frac{96a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}-\frac{43\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}-\frac{14a\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{3\left(\frac{245a^4\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-192a^4\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{96a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}-\frac{43\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}-\frac{14a\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\Big\downarrow$ 3115

$$\frac{\frac{\frac{3\left(\frac{245a^4\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-192a^4\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{96a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}-\frac{43\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}-\frac{14a\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\frac{\frac{3\left(\frac{245a^4\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-192a^4\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{96a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}-\frac{43\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}-\frac{14a\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{ 3117}$$

$$\frac{\frac{\frac{3\left(\frac{245a^4\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{192a^4\sin(c+dx)}{d}}{a^2}-\frac{96a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}-\frac{43\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}-\frac{14a\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

input
```
Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^4,x]
```

output
```
-1/7*(Cos[c + d*x]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^4) + ((-14*a*Cos[
c + d*x]*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((-43*Cos[c + d*x]*S
in[c + d*x])/(d*(1 + Sec[c + d*x])^2) + (3*((-96*a^3*Cos[c + d*x]*Sin[c +
d*x])/(d*(a + a*Sec[c + d*x])) + ((-192*a^4*Sin[c + d*x])/d + 245*a^4*(x/2
 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/a^2))/a^2)/(5*a^2))/(7*a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.51

method	result
parallelrisch	$\frac{94080dx-(55656-35\cos(5dx+5c)+280\cos(4dx+4c)+7873\cos(3dx+3c)+37504\cos(2dx+2c)+85762\cos(dx+c))\tan}{8960d\,a^4}$
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-111\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-72\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-56\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+168\arctan\left(\tan\right.}{8d\,a^4}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-111\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-72\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-56\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+168\arctan\left(\tan\right.}{8d\,a^4}$
risch	$\frac{21x}{2a^4}-\frac{i e^{2i(dx+c)}}{8d\,a^4}+\frac{2i e^{i(dx+c)}}{d\,a^4}-\frac{2i e^{-i(dx+c)}}{d\,a^4}+\frac{i e^{-2i(dx+c)}}{8d\,a^4}-\frac{2i\left(700\,e^{6i(dx+c)}+3675\,e^{5i(dx+c)}+8505\,e^{4i(dx+}\right.}{35d}$
norman	$\frac{\frac{21x}{2a}-\frac{167\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{281\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8ad}-\frac{217\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{20ad}+\frac{167\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{140ad}-\frac{53\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{280ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{56ad}+\frac{21x\,\mathrm{ta}}{}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2a^3}$

input
```
int(cos(d*x+c)^2/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/8960*(94080*d*x-(55656-35*cos(5*d*x+5*c)+280*cos(4*d*x+4*c)+7873*cos(3*d
*x+3*c)+37504*cos(2*d*x+2*c)+85762*cos(d*x+c))*tan(1/2*d*x+1/2*c)*sec(1/2*
d*x+1/2*c)^6)/d/a^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.97

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx$$
$$= \frac{735\,dx\cos{(dx+c)}^4+2940\,dx\cos{(dx+c)}^3+4410\,dx\cos{(dx+c)}^2+2940\,dx\cos{(dx+c)}+735\,dx+(\cdots}{70\left(a^4d\cos{(dx+c)}^4+4\,a^4d\cos{(dx\cdots}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/70*(735*d*x*cos(d*x + c)^4 + 2940*d*x*cos(d*x + c)^3 + 4410*d*x*cos(d*x
+ c)^2 + 2940*d*x*cos(d*x + c) + 735*d*x + (35*cos(d*x + c)^5 - 140*cos(d*
x + c)^4 - 2012*cos(d*x + c)^3 - 4548*cos(d*x + c)^2 - 3873*cos(d*x + c) -
 1152)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^
4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\int \frac{\cos^2{(c+dx)}}{\sec^4{(c+dx)}+4\sec^3{(c+dx)}+6\sec^2{(c+dx)}+4\sec{(c+dx)}+1}\,dx}{a^4}$$

input
```
integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**4,x)
```

output
```
Integral(cos(c + d*x)**2/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c +
d*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.16

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\frac{280\left(\frac{7\sin(dx+c)}{\cos(dx+c)+1}+\frac{9\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4+\frac{2a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\sin(dx+c)}{\cos(dx+c)+1}-\frac{455\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{5880\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}}{280\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/280*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) + 9*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^4 + 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1) -
455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c)
+ 1)^5 - 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 5880*arctan(sin(d*x
+ c)/(cos(d*x + c) + 1))/a^4)/d
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.73

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\frac{2940\,(dx+c)}{a^4}-\frac{280\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2a^4}+\frac{5a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-63a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+455a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-388}{a^{28}}}{280\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/280*(2940*(d*x + c)/a^4 - 280*(9*tan(1/2*d*x + 1/2*c)^3 + 7*tan(1/2*d*x
+ 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^4) + (5*a^24*tan(1/2*d*x + 1/2
*c)^7 - 63*a^24*tan(1/2*d*x + 1/2*c)^5 + 455*a^24*tan(1/2*d*x + 1/2*c)^3 -
 3885*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

Mupad [B] (verification not implemented)

Time = 9.46 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.90

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-78\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+596\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-4408\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\mathrm{s}}{280\,a^4\,}$$

input
```
int(cos(c + d*x)^2/(a + a/cos(c + d*x))^4,x)
```

output
```
(5*sin(c/2 + (d*x)/2) - 78*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 596*c
os(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) - 4408*cos(c/2 + (d*x)/2)^6*sin(c/2
 + (d*x)/2) - 2520*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 560*cos(c/2 +
 (d*x)/2)^10*sin(c/2 + (d*x)/2) + 2940*cos(c/2 + (d*x)/2)^7*(c + d*x))/(28
0*a^4*d*cos(c/2 + (d*x)/2)^7)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.84

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}-53\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9+334\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7-3038\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+2940\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4dx-}{280a^4d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tan\left(\frac{dx}{2}+\right.\right.}$$

input
```
int(cos(d*x+c)^2/(a+a*sec(d*x+c))^4,x)
```

output
```
(5*tan((c + d*x)/2)**11 - 53*tan((c + d*x)/2)**9 + 334*tan((c + d*x)/2)**7
 - 3038*tan((c + d*x)/2)**5 + 2940*tan((c + d*x)/2)**4*d*x - 9835*tan((c +
 d*x)/2)**3 + 5880*tan((c + d*x)/2)**2*d*x - 5845*tan((c + d*x)/2) + 2940*
d*x)/(280*a**4*d*(tan((c + d*x)/2)**4 + 2*tan((c + d*x)/2)**2 + 1))
```

3.80 $\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 200

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{5\text{arctanh}(\sin(c+dx))}{a^5d}+\frac{181\tan(c+dx)}{63a^5d} - \frac{\sec^5(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5}-\frac{5\sec^4(c+dx)\tan(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{29\sec^3(c+dx)\tan(c+dx)}{63a^2d(a+a\sec(c+dx))^3} - \frac{67\sec^2(c+dx)\tan(c+dx)}{63a^3d(a+a\sec(c+dx))^2}+\frac{5\tan(c+dx)}{d\,(a^5+a^5\sec(c+dx))}$$

output
```
-5*arctanh(sin(d*x+c))/a^5/d+181/63*tan(d*x+c)/a^5/d-1/9*sec(d*x+c)^5*tan(
d*x+c)/d/(a+a*sec(d*x+c))^5-5/21*sec(d*x+c)^4*tan(d*x+c)/a/d/(a+a*sec(d*x+
c))^4-29/63*sec(d*x+c)^3*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3-67/63*sec(d*x
+c)^2*tan(d*x+c)/a^3/d/(a+a*sec(d*x+c))^2+5*tan(d*x+c)/d/(a^5+a^5*sec(d*x+
c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 401 vs. 2(200) = 400.

Time = 2.58 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.00

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx$$
$$= \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(322560\cos^9\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\mathrm{co}\right.\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^7/(a + a*Sec[c + d*x])^5,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^5*(322560*Cos[(c + d*x)/2]^9*(Log[Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) +
Sec[c/2]*Sec[c]*Sec[c + d*x]*(-33978*Sin[(d*x)/2] + 52002*Sin[(3*d*x)/2] -
 56952*Sin[c - (d*x)/2] + 43722*Sin[c + (d*x)/2] - 47208*Sin[2*c + (d*x)/2
] - 18144*Sin[c + (3*d*x)/2] + 41796*Sin[2*c + (3*d*x)/2] - 28350*Sin[3*c
+ (3*d*x)/2] + 34578*Sin[c + (5*d*x)/2] - 5691*Sin[2*c + (5*d*x)/2] + 2871
9*Sin[3*c + (5*d*x)/2] - 11550*Sin[4*c + (5*d*x)/2] + 15517*Sin[2*c + (7*d
*x)/2] - 504*Sin[3*c + (7*d*x)/2] + 13186*Sin[4*c + (7*d*x)/2] - 2835*Sin[
5*c + (7*d*x)/2] + 4149*Sin[3*c + (9*d*x)/2] + 252*Sin[4*c + (9*d*x)/2] +
3582*Sin[5*c + (9*d*x)/2] - 315*Sin[6*c + (9*d*x)/2] + 496*Sin[4*c + (11*d
*x)/2] + 63*Sin[5*c + (11*d*x)/2] + 433*Sin[6*c + (11*d*x)/2])))/(2016*a^5
*d*(1 + Sec[c + d*x])^5)
```

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.16, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.905$, Rules used = {3042, 4303, 27, 3042, 4507, 3042, 4507, 27, 3042, 4507, 3042, 4496, 25, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{(a\sec(c+dx)+a)^5}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^7}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{5\sec^5(c+dx)(a-2a\sec(c+dx))}{(\sec(c+dx)a+a)^4}dx}{9a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{5\int \frac{\sec^5(c+dx)(a-2a\sec(c+dx))}{(\sec(c+dx)a+a)^4}dx}{9a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5\left(a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{5\left(\frac{\int \frac{\sec^4(c+dx)\left(12a^2-17a^2\sec(c+dx)\right)}{(\sec(c+dx)a+a)^3}dx}{7a^2}+\frac{3a\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4\left(12a^2-17a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}+\frac{3a\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{5\left(\frac{\frac{\int \frac{3\sec^3(c+dx)\left(29a^3-38a^3\sec(c+dx)\right)}{(\sec(c+dx)a+a)^2}dx}{5a^2}+\frac{29a^2\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}+\frac{3a\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{5\left(\frac{\frac{3\int \frac{\sec^3(c+dx)\left(29a^3-38a^3\sec(c+dx)\right)}{(\sec(c+dx)a+a)^2}dx}{5a^2}+\frac{29a^2\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}+\frac{3a\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5\left(\frac{\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(29a^3-38a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}+\frac{29a^2\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}+\frac{3a\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 4507$$

$$-\frac{5\left(\frac{\frac{3\left(\frac{\int \frac{\sec^2(c+dx)\left(134a^4-181a^4\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}+\frac{67a^3\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}+\frac{29a^2\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}+\frac{3a\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5\left(\frac{\frac{3\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(134a^4-181a^4\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{67a^3\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}+\frac{29a^2\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}+\frac{3a\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 4496$$

$$-\frac{5\left(\frac{3\left(\frac{-\frac{\int -\sec(c+dx)\left(315a^5-181a^5\sec(c+dx)\right)dx}{a^2}-\frac{315a^4\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{67a^3\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}+\frac{29a^2\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}+\frac{3a\tan(c+dx)\sec \cdots}{7d(a\sec(c+ \cdots}\right.}{9a^2}$$

$$\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 25$$

$$-\frac{5\left(\frac{3\left(\frac{\frac{\int \sec(c+dx)\left(315a^5-181a^5\sec(c+dx)\right)dx}{a^2}-\frac{315a^4\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{67a^3\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}+\frac{29a^2\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}+\frac{3a\tan(c+dx)\sec^4(\cdots}{7d(a\sec(c+dx) \cdots}\right.}{9a^2}$$

$$\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5\left(\frac{3\left(\frac{\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(315a^5-181a^5\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}-\frac{315a^4\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{67a^3\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}+\frac{29a^2\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}+\frac{3a\tan(c+ \cdots}{7d(a\sec \cdots}\right.}{9a^2}$$

$$\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 4274$$

$$-\frac{5\left(\frac{\frac{3\left(\frac{\frac{315a^5\int \sec(c+dx)dx-181a^5\int \sec^2(c+dx)dx}{a^2}-\frac{315a^4\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{67a^3\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}+\frac{29a^2\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}+\frac{3a\tan(c+dx)\,\text{se}\cdots}{7d(a\sec(c+d\cdots}\right.}{9a^2}$$

$$\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 3042

$$-\frac{5\left(\frac{\frac{3\left(\frac{\frac{315a^5\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-181a^5\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2}-\frac{315a^4\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{67a^3\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}+\frac{29a^2\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}+\frac{3a\tan(\cdots}{7d(a\cdots}\right.}{9a^2}$$

$$\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 4254

$$-\frac{5\left(\frac{\frac{3\left(\frac{\frac{181a^5\int 1d(-\tan(c+dx))}{d}+315a^5\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{315a^4\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{67a^3\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}+\frac{29a^2\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}+\frac{3a\tan(c\cdots}{7d(a\,s\cdots}\right.}{9a^2}$$

$$\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 24

$$-\frac{5\left(\frac{\frac{3\left(\frac{\frac{315a^5\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{181a^5\tan(c+dx)}{d}}{a^2}}{3a^2}-\frac{315a^4\tan(c+dx)}{d(a\sec(c+dx)+a)}+\frac{67a^3\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}+\frac{29a^2\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}+\frac{3a\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 4257

$$-\frac{5\left(\frac{\frac{29a^2\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{3\left(\frac{67a^3\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}+\frac{\frac{\frac{315a^5\text{arctanh}(\sin(c+dx))}{d}-\frac{181a^5\tan(c+dx)}{d}}{a^2}}{3a^2}-\frac{315a^4\tan(c+dx)}{d(a\sec(c+dx)+a)}\right)}{5a^2}}{7a^2}+\frac{3a\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

input
```
Int[Sec[c + d*x]^7/(a + a*Sec[c + d*x])^5,x]
```

output
```
-1/9*(Sec[c + d*x]^5*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^5) - (5*((3*a*S
ec[c + d*x]^4*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + ((29*a^2*Sec[c
+ d*x]^3*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + (3*((67*a^3*Sec[c +
d*x]^2*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + ((-315*a^4*Tan[c + d*x
])/(d*(a + a*Sec[c + d*x])) + ((315*a^5*ArcTanh[Sin[c + d*x]])/d - (181*a^
5*Tan[c + d*x])/d)/a^2)/(3*a^2)))/(5*a^2))/(7*a^2)))/(9*a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4496
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(b^2*(2*m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b
*B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Ne
Q[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+24\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+129\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{16}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-80\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{16d\,a^{5}}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+24\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+129\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{16}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-80\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{16d\,a^{5}}$
parallelrisch	$\frac{40320\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-40320\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+31846\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\cos(dx+c)+\frac{10010}{}\right.}{8064d\,a^{5}\cos(dx+c)}$
risch	$\frac{2i\left(315\,e^{10i(dx+c)}+2835\,e^{9i(dx+c)}+11550\,e^{8i(dx+c)}+28350\,e^{7i(dx+c)}+47208\,e^{6i(dx+c)}+56952\,e^{5i(dx+c)}+52002\,e^{4i(dx\right.}{63d\,a^{5}\left(e^{i(dx+c)}+1\right)^{9}\left(e^{2i(dx+c)}+1\right)}$

input
```
int(sec(d*x+c)^7/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9+8/7*tan(1/2*d*x+1/2*c)^7+6*tan(1/2*d*
x+1/2*c)^5+24*tan(1/2*d*x+1/2*c)^3+129*tan(1/2*d*x+1/2*c)-16/(tan(1/2*d*x+
1/2*c)+1)-80*ln(tan(1/2*d*x+1/2*c)+1)-16/(tan(1/2*d*x+1/2*c)-1)+80*ln(tan(
1/2*d*x+1/2*c)-1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.39

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{315\left(\cos\left(dx+c\right)^6+5\,\cos\left(dx+c\right)^5+10\,\cos\left(dx+c\right)^4+10\,\cos\left(dx+c\right)^3+5\,\cos\left(dx+c\right)^2+\cos\left(dx\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

output
```
-1/126*(315*(cos(d*x + c)^6 + 5*cos(d*x + c)^5 + 10*cos(d*x + c)^4 + 10*co
s(d*x + c)^3 + 5*cos(d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c) + 1) - 31
5*(cos(d*x + c)^6 + 5*cos(d*x + c)^5 + 10*cos(d*x + c)^4 + 10*cos(d*x + c)
^3 + 5*cos(d*x + c)^2 + cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(496*cos(
d*x + c)^5 + 2165*cos(d*x + c)^4 + 3633*cos(d*x + c)^3 + 2840*cos(d*x + c)
^2 + 946*cos(d*x + c) + 63)*sin(d*x + c))/(a^5*d*cos(d*x + c)^6 + 5*a^5*d*
cos(d*x + c)^5 + 10*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 5*a^5
*d*cos(d*x + c)^2 + a^5*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\int \frac{\sec^7(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

input
```
integrate(sec(d*x+c)**7/(a+a*sec(d*x+c))**5,x)
```

output
```
Integral(sec(c + d*x)**7/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
 d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.03

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\frac{2016\sin(dx+c)}{\left(a^5-\frac{a^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{8127\sin(dx+c)}{\cos(dx+c)+1}+\frac{1512\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{378\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{72\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{7\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-\frac{5040\log\left(\frac{\sin}{\cos(}\right.}{a}}{1008\,d}$$

input
```
integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/1008*(2016*sin(d*x + c)/((a^5 - a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1)) + (8127*sin(d*x + c)/(cos(d*x + c) + 1) + 1512*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
72*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 7*sin(d*x + c)^9/(cos(d*x + c) +
1)^9)/a^5 - 5040*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^5 + 5040*log(s
in(d*x + c)/(cos(d*x + c) + 1) - 1)/a^5)/d
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.78

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\frac{5040\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^5}-\frac{5040\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^5}+\frac{2016\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^5}-\frac{7a^{40}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+72a^{40}\tan\left(\frac{1}{2}dx\right.}{}}{1008\,d}$$

input
```
integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/1008*(5040*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 5040*log(abs(tan(1/
2*d*x + 1/2*c) - 1))/a^5 + 2016*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c
)^2 - 1)*a^5) - (7*a^40*tan(1/2*d*x + 1/2*c)^9 + 72*a^40*tan(1/2*d*x + 1/2
*c)^7 + 378*a^40*tan(1/2*d*x + 1/2*c)^5 + 1512*a^40*tan(1/2*d*x + 1/2*c)^3
 + 8127*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

Mupad [B] (verification not implemented)

Time = 9.31 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.74

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{2\,a^5\,d} + \frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{8\,a^5\,d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{14\,a^5\,d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{144\,a^5\,d} - \frac{10\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^5\,d} - \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,\left(a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-a^5\right)} + \frac{129\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16\,a^5\,d}$$

input
```
int(1/(cos(c + d*x)^7*(a + a/cos(c + d*x))^5),x)
```

output
```
(3*tan(c/2 + (d*x)/2)^3)/(2*a^5*d) + (3*tan(c/2 + (d*x)/2)^5)/(8*a^5*d) +
tan(c/2 + (d*x)/2)^7/(14*a^5*d) + tan(c/2 + (d*x)/2)^9/(144*a^5*d) - (10*a
tanh(tan(c/2 + (d*x)/2)))/(a^5*d) - (2*tan(c/2 + (d*x)/2))/(d*(a^5*tan(c/2
 + (d*x)/2)^2 - a^5)) + (129*tan(c/2 + (d*x)/2))/(16*a^5*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.89

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{5040\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 - 5040\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right) - 5040\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\mathrm{ta}}{}$$

input
```
int(sec(d*x+c)^7/(a+a*sec(d*x+c))^5,x)
```

output
```
(5040*log(tan((c + d*x)/2) - 1)*tan((c + d*x)/2)**2 - 5040*log(tan((c + d*
x)/2) - 1) - 5040*log(tan((c + d*x)/2) + 1)*tan((c + d*x)/2)**2 + 5040*log
(tan((c + d*x)/2) + 1) + 7*tan((c + d*x)/2)**11 + 65*tan((c + d*x)/2)**9 +
 306*tan((c + d*x)/2)**7 + 1134*tan((c + d*x)/2)**5 + 6615*tan((c + d*x)/2
)**3 - 10143*tan((c + d*x)/2))/(1008*a**5*d*(tan((c + d*x)/2)**2 - 1))
```

3.81 $\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 177

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{a^5d} - \frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{13\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{34\sec^2(c+dx)\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3} + \frac{173\tan(c+dx)}{315a^3d(a+a\sec(c+dx))^2} - \frac{661\tan(c+dx)}{315d\,(a^5+a^5\sec(c+dx))}$$

output

```
arctanh(sin(d*x+c))/a^5/d-1/9*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^5
-13/63*sec(d*x+c)^3*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^4-34/105*sec(d*x+c)^2*
tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3+173/315*tan(d*x+c)/a^3/d/(a+a*sec(d*x+
c))^2-661/315*tan(d*x+c)/d/(a^5+a^5*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 2.45 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.24

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^5}\,dx =$$
$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(80640\cos^9\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\right.\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^5,x]
```

output
```
-1/2520*(Cos[(c + d*x)/2]*Sec[c + d*x]^5*(80640*Cos[(c + d*x)/2]^9*(Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]]) + Sec[c/2]*(35973*Sin[(d*x)/2] - 25515*Sin[c + (d*x)/2] + 29757*Sin[c
 + (3*d*x)/2] - 11235*Sin[2*c + (3*d*x)/2] + 14733*Sin[2*c + (5*d*x)/2] -
2835*Sin[3*c + (5*d*x)/2] + 4077*Sin[3*c + (7*d*x)/2] - 315*Sin[4*c + (7*d
*x)/2] + 488*Sin[4*c + (9*d*x)/2])))/(a^5*d*(1 + Sec[c + d*x])^5)
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.16, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4303, 3042, 4507, 27, 3042, 4507, 3042, 4496, 25, 3042, 4486, 3042, 4257, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a\sec(c+dx)+a)^5}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^6}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$

$$\Big\downarrow \text{ 4303}$$

$$-\frac{\int \frac{\sec^4(c+dx)(4a-9a\sec(c+dx))}{(\sec(c+dx)a+a)^4}dx}{9a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 3042

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4\left(4a-9a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 4507

$$-\frac{\frac{\int \frac{3\sec^3(c+dx)\left(13a^2-21a^2\sec(c+dx)\right)}{(\sec(c+dx)a+a)^3}dx}{7a^2}+\frac{13a\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 27

$$-\frac{\frac{3\int \frac{\sec^3(c+dx)\left(13a^2-21a^2\sec(c+dx)\right)}{(\sec(c+dx)a+a)^3}dx}{7a^2}+\frac{13a\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(13a^2-21a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}+\frac{13a\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 4507

$$-\frac{\frac{3\left(\frac{\int \frac{\sec^2(c+dx)\left(68a^3-105a^3\sec(c+dx)\right)}{(\sec(c+dx)a+a)^2}dx}{5a^2}+\frac{34a^2\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}+\frac{13a\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{3\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(68a^3-105a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}+\frac{34a^2\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}+\frac{13a\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 4496

$$-\frac{\frac{3\left(\frac{-\frac{\int -\frac{\sec(c+dx)\left(346a^4-315a^4\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{173a^3\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}+\frac{34a^2\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}+\frac{13a\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 25

$$-\frac{\frac{3\left(\frac{\frac{\int \frac{\sec(c+dx)\left(346a^4-315a^4\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{173a^3\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}+\frac{34a^2\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}+\frac{13a\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{3\left(\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(346a^4-315a^4\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{173a^3\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}+\frac{34a^2\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}+\frac{13a\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 4486

$$-\frac{\frac{3\left(\frac{\frac{661a^4\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx-315a^3\int \sec(c+dx)dx}{3a^2}-\frac{173a^3\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}+\frac{34a^2\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}+\frac{13a\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{3\left(\frac{\frac{661a^4\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-315a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{3a^2}-\frac{173a^3\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}+\frac{34a^2\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}+\frac{13a\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\frac{3\left(\frac{\frac{661a^4\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{315a^3\text{arctanh}(\sin(c+dx))}{d}}{3a^2}-\frac{173a^3\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}+\frac{34a^2\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}+\frac{13a\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{4281}$$

$$-\frac{\frac{3\left(\frac{34a^2\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{\frac{\frac{661a^4\tan(c+dx)}{d(a\sec(c+dx)+a)}-\frac{315a^3\text{arctanh}(\sin(c+dx))}{d}}{3a^2}-\frac{173a^3\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}\right)}{7a^2}+\frac{13a\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

input

```
Int[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^5,x]
```

output

```
-1/9*(Sec[c + d*x]^4*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^5) - ((13*a*Sec
[c + d*x]^3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + (3*((34*a^2*Sec[c
 + d*x]^2*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((-173*a^3*Tan[c +
d*x])/(3*d*(a + a*Sec[c + d*x])^2) + ((-315*a^3*ArcTanh[Sin[c + d*x]])/d +
 (661*a^4*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3*a^2))/(5*a^2)))/(7*a^
2))/(9*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

rule 4496
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(b^2*(2*m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b
*B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Ne
Q[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.57

method	result
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{9}-\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}-\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{26\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-16\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+16\ln(\mathrm{t}}{16 d a^5}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{9}-\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}-\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{26\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-16\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+16\ln(\mathrm{t}}{16 d a^5}$
parallelrisch	$\frac{-35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9-270\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7-1008\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5-2730\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-5040\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+5040\ln(}{5040 d a^5}$
risch	$-\frac{2i\left(315\,\mathrm{e}^{8i(dx+c)}+2835\,\mathrm{e}^{7i(dx+c)}+11235\,\mathrm{e}^{6i(dx+c)}+25515\,\mathrm{e}^{5i(dx+c)}+35973\,\mathrm{e}^{4i(dx+c)}+29757\,\mathrm{e}^{3i(dx+c)}+14733\,\mathrm{e}^{2i(d}{315 d a^5\left(\mathrm{e}^{i(dx+c)}+1\right)^9}$

input
```
int(sec(d*x+c)^6/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/16/d/a^5*(-1/9*tan(1/2*d*x+1/2*c)^9-6/7*tan(1/2*d*x+1/2*c)^7-16/5*tan(1/
2*d*x+1/2*c)^5-26/3*tan(1/2*d*x+1/2*c)^3-31*tan(1/2*d*x+1/2*c)-16*ln(tan(1
/2*d*x+1/2*c)-1)+16*ln(tan(1/2*d*x+1/2*c)+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.39

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{315\left(\cos\left(dx+c\right)^5+5\,\cos\left(dx+c\right)^4+10\,\cos\left(dx+c\right)^3+10\,\cos\left(dx+c\right)^2+5\,\cos\left(dx+c\right)+1\right)\log\left(\sin\right.}{}$$

input
```
integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/630*(315*(cos(d*x + c)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 10*cos
(d*x + c)^2 + 5*cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - 315*(cos(d*x + c
)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 10*cos(d*x + c)^2 + 5*cos(d*x
 + c) + 1)*log(-sin(d*x + c) + 1) - 2*(488*cos(d*x + c)^4 + 2125*cos(d*x +
 c)^3 + 3549*cos(d*x + c)^2 + 2740*cos(d*x + c) + 863)*sin(d*x + c))/(a^5*
d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a
^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\int \frac{\sec^6(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

input
```
integrate(sec(d*x+c)**6/(a+a*sec(d*x+c))**5,x)
```

output
```
Integral(sec(c + d*x)**6/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
 d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.90

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\frac{\frac{9765\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{2730\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1008\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{270\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-\frac{5040\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^5}+\frac{5040\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+}\right.}{a^5}}{5040\,d}$$

input
```
integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/5040*((9765*sin(d*x + c)/(cos(d*x + c) + 1) + 2730*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 1008*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 270*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^5 -
 5040*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^5 + 5040*log(sin(d*x + c)
/(cos(d*x + c) + 1) - 1)/a^5)/d
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.71

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\frac{5040\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^5}-\frac{5040\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^5}-\frac{35\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+270\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1008\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{a^{45}}}{5040\,d}$$

input
```
integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

output
```
1/5040*(5040*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 5040*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a^5 - (35*a^40*tan(1/2*d*x + 1/2*c)^9 + 270*a^40*tan(1
/2*d*x + 1/2*c)^7 + 1008*a^40*tan(1/2*d*x + 1/2*c)^5 + 2730*a^40*tan(1/2*d
*x + 1/2*c)^3 + 9765*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.56

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^5}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{5\,a^5}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{56\,a^5}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{144\,a^5}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^5}+\frac{31\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,a^5}}{d}$$

input
```
int(1/(cos(c + d*x)^6*(a + a/cos(c + d*x))^5),x)
```

output
```
-((13*tan(c/2 + (d*x)/2)^3)/(24*a^5) + tan(c/2 + (d*x)/2)^5/(5*a^5) + (3*t
an(c/2 + (d*x)/2)^7)/(56*a^5) + tan(c/2 + (d*x)/2)^9/(144*a^5) - (2*atanh(
tan(c/2 + (d*x)/2)))/a^5 + (31*tan(c/2 + (d*x)/2))/(16*a^5))/d
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.56

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{-5040\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+5040\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9-270\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7-10}{5040a^5d}$$

input
```
int(sec(d*x+c)^6/(a+a*sec(d*x+c))^5,x)
```

output
```
( - 5040*log(tan((c + d*x)/2) - 1) + 5040*log(tan((c + d*x)/2) + 1) - 35*t
an((c + d*x)/2)**9 - 270*tan((c + d*x)/2)**7 - 1008*tan((c + d*x)/2)**5 -
2730*tan((c + d*x)/2)**3 - 9765*tan((c + d*x)/2))/(5040*a**5*d)
```

3.82 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 159

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{4\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{4\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3} - \frac{32\tan(c+dx)}{315ad\left(a^2+a^2\sec(c+dx)\right)^2} + \frac{4\tan(c+dx)}{45d\left(a^5+a^5\sec(c+dx)\right)}$$

output
```
1/9*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^5+4/63*sec(d*x+c)^3*tan(d*x
+c)/a/d/(a+a*sec(d*x+c))^4+4/105*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3-32/31
5*tan(d*x+c)/a/d/(a^2+a^2*sec(d*x+c))^2+4/45*tan(d*x+c)/d/(a^5+a^5*sec(d*x
+c))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.61

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(126\sin\left(\frac{1}{2}(c+dx)\right)+84\sin\left(\frac{3}{2}(c+dx)\right)+36\sin\left(\frac{5}{2}(c+dx)\right)+9\sin\left(\frac{7}{2}(c+\right.\right.}{315a^5d(1+\sec(c+dx))^5}$$

input
```
Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^5,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^5*(126*Sin[(c + d*x)/2] + 84*Sin[(3*(c + d*
x))/2] + 36*Sin[(5*(c + d*x))/2] + 9*Sin[(7*(c + d*x))/2] + Sin[(9*(c + d*
x))/2]))/(315*a^5*d*(1 + Sec[c + d*x])^5)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4297, 3042, 4297, 3042, 4286, 25, 3042, 4488, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a\sec(c+dx)+a)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$

$$\Big\downarrow \text{4297}$$

$$\frac{4\int \frac{\sec^4(c+dx)}{(\sec(c+dx)a+a)^4}dx}{9a}+\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a}+\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 4297

$$\frac{4\left(\frac{3\int \frac{\sec^3(c+dx)}{(\sec(c+dx)a+a)^3}dx}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}+\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 3042

$$\frac{4\left(\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}+\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 4286

$$\frac{4\left(\frac{3\left(\frac{\int -\frac{\sec(c+dx)(3a-5a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}+\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 25

$$\frac{4\left(\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\int \frac{\sec(c+dx)(3a-5a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}+\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 3042

$$\frac{4\left(\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3a-5a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}+\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{4488}$$

$$\frac{4\left(\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{8a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{7}{3}\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}+\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\left(\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{8a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{7}{3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}+\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{4281}$$

$$\frac{4\left(\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{8a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{7\tan(c+dx)}{3d(a\sec(c+dx)+a)}}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}+\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

input
```
Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^5,x]
```

output
```
(Sec[c + d*x]^4*Tan[c + d*x])/(9*d*(a + a*Sec[c + d*x])^5) + (4*((Sec[c +
d*x]^3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + (3*(Tan[c + d*x]/(5*d*
(a + a*Sec[c + d*x])^3) - ((8*a*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
 - (7*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])))/(5*a^2)))/(7*a)))/(9*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4286
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Simp[1/(a^2*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[
a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4297
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[d*((m + 1)/(b*(2*m + 1)))
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ
[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m,
-2^(-1)] && IntegerQ[2*m]
```

rule 4488
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
 /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.43

method	result	size
risch	$\frac{16i\left(126\,\mathrm{e}^{4i(dx+c)}+84\,\mathrm{e}^{3i(dx+c)}+36\,\mathrm{e}^{2i(dx+c)}+9\,\mathrm{e}^{i(dx+c)}+1\right)}{315d\,a^{5}\left(\mathrm{e}^{i(dx+c)}+1\right)^{9}}$	69
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$	71
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$	71
parallelrisch	$\frac{35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}+180\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+378\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+420\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+315\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{5040d\,a^{5}}$	73

input
```
int(sec(d*x+c)^5/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
16/315*I*(126*exp(4*I*(d*x+c))+84*exp(3*I*(d*x+c))+36*exp(2*I*(d*x+c))+9*e
xp(I*(d*x+c))+1)/d/a^5/(exp(I*(d*x+c))+1)^9
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.77

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^5}\,dx$$
$$= \frac{\left(8\cos(dx+c)^4+40\cos(dx+c)^3+84\cos(dx+c)^2+100\cos(dx+c)+83\right)\sin(dx+c)}{315\left(a^5d\cos(dx+c)^5+5a^5d\cos(dx+c)^4+10a^5d\cos(dx+c)^3+10a^5d\cos(dx+c)^2+5a^5d\cos(dx\right.}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/315*(8*cos(d*x + c)^4 + 40*cos(d*x + c)^3 + 84*cos(d*x + c)^2 + 100*cos(
d*x + c) + 83)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4
 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c
) + a^5*d)
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\int \frac{\sec^5(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

input
```
integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**5,x)
```

output
```
Integral(sec(c + d*x)**5/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
 d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.67

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1} + \frac{420\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{378\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{180\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{35\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{5040\,a^5d}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/5040*(315*sin(d*x + c)/(cos(d*x + c) + 1) + 420*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 180*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.45

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+180\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+378\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+420\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+315\,\tan}{5040\,a^5d}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

output
```
1/5040*(35*tan(1/2*d*x + 1/2*c)^9 + 180*tan(1/2*d*x + 1/2*c)^7 + 378*tan(1
/2*d*x + 1/2*c)^5 + 420*tan(1/2*d*x + 1/2*c)^3 + 315*tan(1/2*d*x + 1/2*c))
/(a^5*d)
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.80

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(315\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+420\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+378\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\right.}{5040\,a^5\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}$$

input
```
int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^5),x)
```

output
```
(sin(c/2 + (d*x)/2)*(315*cos(c/2 + (d*x)/2)^8 + 35*sin(c/2 + (d*x)/2)^8 +
180*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^6 + 378*cos(c/2 + (d*x)/2)^4*s
in(c/2 + (d*x)/2)^4 + 420*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2))/(504
0*a^5*d*cos(c/2 + (d*x)/2)^9)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.45

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+180\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+378\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+420\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+315\right)}{5040a^5d}$$

input
```
int(sec(d*x+c)^5/(a+a*sec(d*x+c))^5,x)
```

output
```
(tan((c + d*x)/2)*(35*tan((c + d*x)/2)**8 + 180*tan((c + d*x)/2)**6 + 378*
tan((c + d*x)/2)**4 + 420*tan((c + d*x)/2)**2 + 315))/(5040*a**5*d)
```

3.83 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 159

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{5\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}+\frac{\tan(c+dx)}{21a^2d(a+a\sec(c+dx))^3}-\frac{8\tan(c+dx)}{63ad\left(a^2+a^2\sec(c+dx)\right)^2}+\frac{\tan(c+dx)}{9d\left(a^5+a^5\sec(c+dx)\right)}$$

output
```
-1/9*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^5+5/63*sec(d*x+c)^3*tan(d*
x+c)/a/d/(a+a*sec(d*x+c))^4+1/21*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3-8/63*
tan(d*x+c)/a/d/(a^2+a^2*sec(d*x+c))^2+1/9*tan(d*x+c)/d/(a^5+a^5*sec(d*x+c)
)
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.61

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(63\sin\left(\frac{dx}{2}\right)-63\sin\left(c+\frac{dx}{2}\right)+84\sin\left(c+\frac{3dx}{2}\right)+36\sin\left(2c+\frac{5dx}{2}\right)+9\sin\left(3c+\right.\right.}{8064a^5d}$$

input
```
Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^5,x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^9*(63*Sin[(d*x)/2] - 63*Sin[c + (d*x)/2] + 84*S
in[c + (3*d*x)/2] + 36*Sin[2*c + (5*d*x)/2] + 9*Sin[3*c + (7*d*x)/2] + Sin
[4*c + (9*d*x)/2]))/(8064*a^5*d)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4298, 3042, 4297, 3042, 4286, 25, 3042, 4488, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a\sec(c+dx)+a)^5}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$

$$\Big\downarrow \text{ 4298}$$

$$\frac{5\int \frac{\sec^4(c+dx)}{(\sec(c+dx)a+a)^4}dx}{9a}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{4297}$$

$$\frac{5\left(\frac{3\int \frac{\sec^3(c+dx)}{(\sec(c+dx)a+a)^3}dx}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{4286}$$

$$\frac{5\left(\frac{3\left(\frac{\int -\frac{\sec(c+dx)(3a-5a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{5\left(\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\int \frac{\sec(c+dx)(3a-5a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3a-5a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{4488}$$

$$\frac{5\left(\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{8a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{7}{3}\int\frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{8a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{7}{3}\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{4281}$$

$$\frac{5\left(\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{8a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{7\tan(c+dx)}{3d(a\sec(c+dx)+a)}}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

input
```
Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^5,x]
```

output
```
-1/9*(Sec[c + d*x]^4*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^5) + (5*((Sec[c
 + d*x]^3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + (3*(Tan[c + d*x]/(5
*d*(a + a*Sec[c + d*x])^3) - ((8*a*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])
^2) - (7*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])))/(5*a^2)))/(7*a)))/(9*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4286
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Simp[1/(a^2*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[
a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4297
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[d*((m + 1)/(b*(2*m + 1)))
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ
[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m,
-2^(-1)] && IntegerQ[2*m]
```

rule 4298
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[m/(a*(2*m + 1))   Int[(a + b*Csc[e +
 f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LtQ[m, -2^(-1)]
```

rule 4488
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
 /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.36

method	result
parallelrisch	$-\frac{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+\frac{18\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{7}-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-9\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{144d\,a^{5}}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$
risch	$\frac{4i\left(63\,e^{5i(dx+c)}+63\,e^{4i(dx+c)}+84\,e^{3i(dx+c)}+36\,e^{2i(dx+c)}+9\,e^{i(dx+c)}+1\right)}{63d\,a^{5}\left(e^{i(dx+c)}+1\right)^{9}}$
norman	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{48ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{16ad}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{112ad}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{1008ad}+\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{336ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{336ad}-\tan}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}a^{4}}$

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
-1/144*(tan(1/2*d*x+1/2*c)^8+18/7*tan(1/2*d*x+1/2*c)^6-6*tan(1/2*d*x+1/2*c
)^2-9)*tan(1/2*d*x+1/2*c)/d/a^5
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.77

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{(2\cos(dx+c)^4+10\cos(dx+c)^3+21\cos(dx+c)^2+25\cos(dx+c)+5)\sin(dx+c)}{63\,(a^5d\cos(dx+c)^5+5\,a^5d\cos(dx+c)^4+10\,a^5d\cos(dx+c)^3+10\,a^5d\cos(dx+c)^2+5\,a^5d\cos(dx+$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/63*(2*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 21*cos(d*x + c)^2 + 25*cos(d*
x + c) + 5)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 +
10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) +
 a^5*d)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\int \frac{\sec^4(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

input
```
integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**5,x)
```

output
```
Integral(sec(c + d*x)**4/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
 d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.55

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\frac{63\,\sin(dx+c)}{\cos(dx+c)+1} + \frac{42\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{18\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{7\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{1008\,a^5d}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/1008*(63*sin(d*x + c)/(cos(d*x + c) + 1) + 42*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 18*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 7*sin(d*x + c)^9/(cos
(d*x + c) + 1)^9)/(a^5*d)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.37

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 + 18\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 42\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 63\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{1008\,a^5d}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/1008*(7*tan(1/2*d*x + 1/2*c)^9 + 18*tan(1/2*d*x + 1/2*c)^7 - 42*tan(1/2
*d*x + 1/2*c)^3 - 63*tan(1/2*d*x + 1/2*c))/(a^5*d)
```

Mupad [B] (verification not implemented)

Time = 9.39 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.36

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(-7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-18\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+42\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+63\right)}{1008\,a^5\,d}$$

input
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^5),x)
```

output
```
(tan(c/2 + (d*x)/2)*(42*tan(c/2 + (d*x)/2)^2 - 18*tan(c/2 + (d*x)/2)^6 - 7
*tan(c/2 + (d*x)/2)^8 + 63))/(1008*a^5*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.36

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8-18\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+42\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+63\right)}{1008a^5d}$$

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^5,x)
```

output
```
(tan((c + d*x)/2)*( - 7*tan((c + d*x)/2)**8 - 18*tan((c + d*x)/2)**6 + 42*
tan((c + d*x)/2)**2 + 63))/(1008*a**5*d)
```

3.84 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 139

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{2\tan(c+dx)}{9ad(a+a\sec(c+dx))^4} + \frac{\tan(c+dx)}{15a^2d(a+a\sec(c+dx))^3} + \frac{2\tan(c+dx)}{45a^3d(a+a\sec(c+dx))^2} + \frac{2\tan(c+dx)}{45d\,(a^5+a^5\sec(c+dx))}$$

output
```
1/9*tan(d*x+c)/d/(a+a*sec(d*x+c))^5-2/9*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^4+
1/15*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3+2/45*tan(d*x+c)/a^3/d/(a+a*sec(d*
x+c))^2+2/45*tan(d*x+c)/d/(a^5+a^5*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 3.56 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.79

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(81\sin\left(\frac{dx}{2}\right)-45\sin\left(c+\frac{dx}{2}\right)+54\sin\left(c+\frac{3dx}{2}\right)-30\sin\left(2c+\frac{3dx}{2}\right)+36\sin\left(2c\right.\right.}{5760a^5d}$$

input
```
Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^5,x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^9*(81*Sin[(d*x)/2] - 45*Sin[c + (d*x)/2] + 54*S
in[c + (3*d*x)/2] - 30*Sin[2*c + (3*d*x)/2] + 36*Sin[2*c + (5*d*x)/2] + 9*
Sin[3*c + (7*d*x)/2] + Sin[4*c + (9*d*x)/2]))/(5760*a^5*d)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4286, 25, 3042, 4488, 3042, 4283, 3042, 4283, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a\sec(c+dx)+a)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$

$$\Big\downarrow \text{4286}$$

$$\frac{\int -\frac{\sec(c+dx)(5a-9a\sec(c+dx))}{(\sec(c+dx)a+a)^4}dx}{9a^2}+\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}-\frac{\int \frac{\sec(c+dx)(5a-9a\sec(c+dx))}{(\sec(c+dx)a+a)^4}dx}{9a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(5a-9a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a^2}$$

$$\Big\downarrow \text{4488}$$

$$\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}-\frac{\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^4}-3\int\frac{\sec(c+dx)}{(\sec(c+dx)a+a)^3}dx}{9a^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}-\frac{\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^4}-3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{9a^2}$$

$$\Big\downarrow \ 4283$$

$$\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}-\frac{\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^4}-3\left(\frac{2\int\frac{\sec(c+dx)}{(\sec(c+dx)a+a)^2}dx}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{9a^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}-\frac{\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^4}-3\left(\frac{2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{9a^2}$$

$$\Big\downarrow \ 4283$$

$$\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}-\frac{\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^4}-3\left(\frac{2\left(\frac{\int\frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{9a^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}-\frac{\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^4}-3\left(\frac{2\left(\frac{\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{9a^2}$$

$$\Big\downarrow \ 4281$$

$$\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}-\frac{\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^4}-3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{2\left(\frac{\tan(c+dx)}{3ad(a\sec(c+dx)+a)}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}\right)}{9a^2}$$

input
```
Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^5,x]
```

output
```
Tan[c + d*x]/(9*d*(a + a*Sec[c + d*x])^5) - ((2*a*Tan[c + d*x])/(d*(a + a*
Sec[c + d*x])^4) - 3*(Tan[c + d*x]/(5*d*(a + a*Sec[c + d*x])^3) + (2*(Tan[
c + d*x]/(3*d*(a + a*Sec[c + d*x])^2) + Tan[c + d*x]/(3*a*d*(a + a*Sec[c +
 d*x]))))/(5*a)))/(9*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

rule 4286
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Simp[1/(a^2*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[
a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4488
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
 /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.32

method	result	size
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$	45
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$	45
parallelrisch	$\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}-18\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+45\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{720d\,a^{5}}$	47
risch	$\frac{4i\left(30\,\mathrm{e}^{6i(dx+c)}+45\,\mathrm{e}^{5i(dx+c)}+81\,\mathrm{e}^{4i(dx+c)}+54\,\mathrm{e}^{3i(dx+c)}+36\,\mathrm{e}^{2i(dx+c)}+9\,\mathrm{e}^{i(dx+c)}+1\right)}{45d\,a^{5}\left(\mathrm{e}^{i(dx+c)}+1\right)^{9}}$	91
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{8ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{80ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{20ad}-\frac{13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{720ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{72ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{144ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}a^{4}}$	152

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9-2/5*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+
1/2*c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.88

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^5}\,dx$$
$$= \frac{\left(2\,\cos\left(dx+c\right)^{4}+10\,\cos\left(dx+c\right)^{3}+21\,\cos\left(dx+c\right)^{2}+10\,\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)}{45\,\left(a^{5}d\cos\left(dx+c\right)^{5}+5\,a^{5}d\cos\left(dx+c\right)^{4}+10\,a^{5}d\cos\left(dx+c\right)^{3}+10\,a^{5}d\cos\left(dx+c\right)^{2}+5\,a^{5}d\cos\left(dx+\right.\right.}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/45*(2*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 21*cos(d*x + c)^2 + 10*cos(d*
x + c) + 2)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 +
10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) +
 a^5*d)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^5}\,dx$$
$$= \frac{\int \frac{\sec^{3}(c+dx)}{\sec^{5}(c+dx)+5\sec^{4}(c+dx)+10\sec^{3}(c+dx)+10\sec^{2}(c+dx)+5\sec(c+dx)+1}\,dx}{a^{5}}$$

input
```
integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**5,x)
```

output
```
Integral(sec(c + d*x)**3/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
 d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.48

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\frac{45\sin(dx+c)}{\cos(dx+c)+1} - \frac{18\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{5\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{720\,a^5d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/720*(45*sin(d*x + c)/(cos(d*x + c) + 1) - 18*sin(d*x + c)^5/(cos(d*x + c
) + 1)^5 + 5*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.33

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 18\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 45\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{720\,a^5d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

output
```
1/720*(5*tan(1/2*d*x + 1/2*c)^9 - 18*tan(1/2*d*x + 1/2*c)^5 + 45*tan(1/2*d
*x + 1/2*c))/(a^5*d)
```

Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.32

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 - 18\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 45\right)}{720\,a^5\,d}$$

input
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^5),x)
```

output
```
(tan(c/2 + (d*x)/2)*(5*tan(c/2 + (d*x)/2)^8 - 18*tan(c/2 + (d*x)/2)^4 + 45
))/(720*a^5*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.32

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8-18\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+45\right)}{720a^5d}$$

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^5,x)
```

output
```
(tan((c + d*x)/2)*(5*tan((c + d*x)/2)**8 - 18*tan((c + d*x)/2)**4 + 45))/(
720*a**5*d)
```

3.85 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 143

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{5\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}+\frac{\tan(c+dx)}{21a^2d(a+a\sec(c+dx))^3}+\frac{2\tan(c+dx)}{63ad\left(a^2+a^2\sec(c+dx)\right)^2}+\frac{2\tan(c+dx)}{63d\left(a^5+a^5\sec(c+dx)\right)}$$

output
```
-1/9*tan(d*x+c)/d/(a+a*sec(d*x+c))^5+5/63*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^
4+1/21*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3+2/63*tan(d*x+c)/a/d/(a^2+a^2*se
c(d*x+c))^2+2/63*tan(d*x+c)/d/(a^5+a^5*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 4.05 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.87

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(315\sin\left(\frac{dx}{2}\right)-315\sin\left(c+\frac{dx}{2}\right)+273\sin\left(c+\frac{3dx}{2}\right)-147\sin\left(2c+\frac{3dx}{2}\right)+117\text{s}\right.}{16128a^5d}$$

input
```
Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^5,x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^9*(315*Sin[(d*x)/2] - 315*Sin[c + (d*x)/2] + 27
3*Sin[c + (3*d*x)/2] - 147*Sin[2*c + (3*d*x)/2] + 117*Sin[2*c + (5*d*x)/2]
 - 63*Sin[3*c + (5*d*x)/2] + 45*Sin[3*c + (7*d*x)/2] + 5*Sin[4*c + (9*d*x)
/2]))/(16128*a^5*d)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {3042, 4284, 3042, 4283, 3042, 4283, 3042, 4283, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a\sec(c+dx)+a)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$

$$\Big\downarrow \text{4284}$$

$$\frac{5\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^4}dx}{9a}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{4283}$$

$$\frac{5\left(\frac{3\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^3}dx}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 4283

$$\frac{5\left(\frac{3\left(\frac{2\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^2}dx}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 3042

$$\frac{5\left(\frac{3\left(\frac{2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 4283

$$\frac{5\left(\frac{3\left(\frac{2\left(\frac{\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 3042

$$\frac{5\left(\frac{3\left(\frac{2\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{ 4281}$$

$$\frac{5\left(\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}+\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{2\left(\frac{\tan(c+dx)}{3ad(a\sec(c+dx)+a)}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}\right)}{7a}\right)}{9a}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

input
```
Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^5,x]
```

output
```
-1/9*Tan[c + d*x]/(d*(a + a*Sec[c + d*x])^5) + (5*(Tan[c + d*x]/(7*d*(a +
a*Sec[c + d*x])^4) + (3*(Tan[c + d*x]/(5*d*(a + a*Sec[c + d*x])^3) + (2*(T
an[c + d*x]/(3*d*(a + a*Sec[c + d*x])^2) + Tan[c + d*x]/(3*a*d*(a + a*Sec[
c + d*x]))))/(5*a)))/(7*a)))/(9*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

rule 4284
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Simp[m/(b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.40

method	result	size
parallelrisch	$-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-\frac{18\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{7}+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-9\right)}{144d\,a^{5}}$	57
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$	58
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$	58
risch	$\frac{2i\left(63\,{\rm e}^{7i(dx+c)}+147\,{\rm e}^{6i(dx+c)}+315\,{\rm e}^{5i(dx+c)}+315\,{\rm e}^{4i(dx+c)}+273\,{\rm e}^{3i(dx+c)}+117\,{\rm e}^{2i(dx+c)}+45\,{\rm e}^{i(dx+c)}+5\right)}{63d\,a^{5}\left({\rm e}^{i(dx+c)}+1\right)^{9}}$	102
norman	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{48ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{24ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{56ad}+\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{1008ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{144ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)a^{4}}$	133

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
-1/144*tan(1/2*d*x+1/2*c)*(tan(1/2*d*x+1/2*c)^8-18/7*tan(1/2*d*x+1/2*c)^6+
6*tan(1/2*d*x+1/2*c)^2-9)/d/a^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.86

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx$$
$$= \frac{\left(5\cos\left(dx+c\right)^4+25\cos\left(dx+c\right)^3+21\cos\left(dx+c\right)^2+10\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)}{63\left(a^5d\cos\left(dx+c\right)^5+5\,a^5d\cos\left(dx+c\right)^4+10\,a^5d\cos\left(dx+c\right)^3+10\,a^5d\cos\left(dx+c\right)^2+5\,a^5d\cos\left(dx+\right.\right.}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/63*(5*cos(d*x + c)^4 + 25*cos(d*x + c)^3 + 21*cos(d*x + c)^2 + 10*cos(d*
x + c) + 2)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 +
10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) +
 a^5*d)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx$$
$$= \frac{\int \frac{\sec^2(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

input
```
integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**5,x)
```

output
```
Integral(sec(c + d*x)**2/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
 d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.61

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\frac{63\sin(dx+c)}{\cos(dx+c)+1} - \frac{42\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{18\sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{7\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{1008\,a^5d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/1008*(63*sin(d*x + c)/(cos(d*x + c) + 1) - 42*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 18*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 7*sin(d*x + c)^9/(cos
(d*x + c) + 1)^9)/(a^5*d)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.41

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 18\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 42\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 63\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{1008\,a^5d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/1008*(7*tan(1/2*d*x + 1/2*c)^9 - 18*tan(1/2*d*x + 1/2*c)^7 + 42*tan(1/2
*d*x + 1/2*c)^3 - 63*tan(1/2*d*x + 1/2*c))/(a^5*d)
```

Mupad [B] (verification not implemented)

Time = 9.39 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.41

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-18\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+42\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-63\right)}{1008\,a^5\,d}$$

input
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^5),x)
```

output
```
-(tan(c/2 + (d*x)/2)*(42*tan(c/2 + (d*x)/2)^2 - 18*tan(c/2 + (d*x)/2)^6 +
7*tan(c/2 + (d*x)/2)^8 - 63))/(1008*a^5*d)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.41

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+18\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6-42\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+63\right)}{1008a^5d}$$

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^5,x)
```

output
```
(tan((c + d*x)/2)*( - 7*tan((c + d*x)/2)**8 + 18*tan((c + d*x)/2)**6 - 42*
tan((c + d*x)/2)**2 + 63))/(1008*a**5*d)
```

3.86 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 143

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{4\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{4\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3} + \frac{8\tan(c+dx)}{315ad\,(a^2+a^2\sec(c+dx))^2} + \frac{8\tan(c+dx)}{315d\,(a^5+a^5\sec(c+dx))}$$

output

```
1/9*tan(d*x+c)/d/(a+a*sec(d*x+c))^5+4/63*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^4
+4/105*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3+8/315*tan(d*x+c)/a/d/(a^2+a^2*s
ec(d*x+c))^2+8/315*tan(d*x+c)/d/(a^5+a^5*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 3.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^5}\,dx$$
$$= \frac{\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(5418\sin\left(\frac{dx}{2}\right)-5040\sin\left(c+\frac{dx}{2}\right)+3612\sin\left(c+\frac{3dx}{2}\right)-3360\sin\left(2c+\frac{3dx}{2}\right)+\right.}{80640}$$

input
```
Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^5,x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^9*(5418*Sin[(d*x)/2] - 5040*Sin[c + (d*x)/2] +
3612*Sin[c + (3*d*x)/2] - 3360*Sin[2*c + (3*d*x)/2] + 1728*Sin[2*c + (5*d*
x)/2] - 1260*Sin[3*c + (5*d*x)/2] + 432*Sin[3*c + (7*d*x)/2] - 315*Sin[4*c
 + (7*d*x)/2] + 83*Sin[4*c + (9*d*x)/2]))/(80640*a^5*d)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 4283, 3042, 4283, 3042, 4283, 3042, 4283, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a\sec(c+dx)+a)^5}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$
$$\Big\downarrow \; 4283$$
$$\frac{4\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^4}dx}{9a}+\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$
$$\Big\downarrow \; 3042$$

$$\frac{4\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a}+\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 4283

$$\frac{4\left(\frac{3\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^3}dx}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}+\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 3042

$$\frac{4\left(\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}+\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 4283

$$\frac{4\left(\frac{3\left(\frac{2\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^2}dx}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}+\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 3042

$$\frac{4\left(\frac{3\left(\frac{2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}+\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 4283

$$\frac{4\left(\frac{3\left(\frac{2\left(\frac{\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}+\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 3042

$$\frac{4\left(\frac{3\left(\frac{2\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}+\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 4281

$$\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}+\frac{4\left(\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}+\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{2\left(\frac{\tan(c+dx)}{3ad(a\sec(c+dx)+a)}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}\right)}{7a}\right)}{9a}$$

input
```
Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^5,x]
```

output

```
Tan[c + d*x]/(9*d*(a + a*Sec[c + d*x])^5) + (4*(Tan[c + d*x]/(7*d*(a + a*S
ec[c + d*x])^4) + (3*(Tan[c + d*x]/(5*d*(a + a*Sec[c + d*x])^3) + (2*(Tan[
c + d*x]/(3*d*(a + a*Sec[c + d*x])^2) + Tan[c + d*x]/(3*a*d*(a + a*Sec[c +
 d*x]))))/(5*a)))/(7*a)))/(9*a)
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4283

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.50

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$
parallelrisch	$\frac{35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}-180\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+378\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-420\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+315\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{5040d\,a^{5}}$
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{12ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{40ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{28ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{144ad}}{a^{4}}$
risch	$\frac{2i\left(315\,\mathrm{e}^{8i(dx+c)}+1260\,\mathrm{e}^{7i(dx+c)}+3360\,\mathrm{e}^{6i(dx+c)}+5040\,\mathrm{e}^{5i(dx+c)}+5418\,\mathrm{e}^{4i(dx+c)}+3612\,\mathrm{e}^{3i(dx+c)}+1728\,\mathrm{e}^{2i(dx+c)}+43\right.}{315d\,a^{5}\left(\mathrm{e}^{i(dx+c)}+1\right)^{9}}$

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9-4/7*tan(1/2*d*x+1/2*c)^7+6/5*tan(1/2*
d*x+1/2*c)^5-4/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.86

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\left(83\cos\left(dx+c\right)^4+100\cos\left(dx+c\right)^3+84\cos\left(dx+c\right)^2+40\cos\left(dx+c\right)+8\right)\sin\left(dx+c\right)}{315\left(a^5d\cos\left(dx+c\right)^5+5\,a^5d\cos\left(dx+c\right)^4+10\,a^5d\cos\left(dx+c\right)^3+10\,a^5d\cos\left(dx+c\right)^2+5\,a^5d\cos\left(dx\right.\right.}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/315*(83*cos(d*x + c)^4 + 100*cos(d*x + c)^3 + 84*cos(d*x + c)^2 + 40*cos
(d*x + c) + 8)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4
 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c
) + a^5*d)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\int \frac{\sec(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))**5,x)
```

output
```
Integral(sec(c + d*x)/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c + d*
x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.75

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1} - \frac{420\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{378\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{180\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{35\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{5040\,a^5d}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/5040*(315*sin(d*x + c)/(cos(d*x + c) + 1) - 420*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 180*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.50

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 180\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 378\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 420\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 315\,\tan}{5040\,a^5d}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

output
```
1/5040*(35*tan(1/2*d*x + 1/2*c)^9 - 180*tan(1/2*d*x + 1/2*c)^7 + 378*tan(1
/2*d*x + 1/2*c)^5 - 420*tan(1/2*d*x + 1/2*c)^3 + 315*tan(1/2*d*x + 1/2*c))
/(a^5*d)
```

Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.89

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(315\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-420\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+378\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\right.}{5040\,a^5\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}$$

input
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^5),x)
```

output
```
(sin(c/2 + (d*x)/2)*(315*cos(c/2 + (d*x)/2)^8 + 35*sin(c/2 + (d*x)/2)^8 -
180*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^6 + 378*cos(c/2 + (d*x)/2)^4*s
in(c/2 + (d*x)/2)^4 - 420*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2))/(504
0*a^5*d*cos(c/2 + (d*x)/2)^9)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.50

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8-180\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+378\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4-420\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+315\right)}{5040a^5d}$$

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^5,x)
```

output
```
(tan((c + d*x)/2)*(35*tan((c + d*x)/2)**8 - 180*tan((c + d*x)/2)**6 + 378*
tan((c + d*x)/2)**4 - 420*tan((c + d*x)/2)**2 + 315))/(5040*a**5*d)
```

3.87 $\int \frac{1}{(a+a\sec(c+dx))^5}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 144

$$\int \frac{1}{(a+a\sec(c+dx))^5}\,dx = \frac{x}{a^5} - \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{13\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{34\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3} - \frac{173\tan(c+dx)}{315a^3d(a+a\sec(c+dx))^2} - \frac{488\tan(c+dx)}{315d\left(a^5+a^5\sec(c+dx)\right)}$$

output
```
x/a^5-1/9*tan(d*x+c)/d/(a+a*sec(d*x+c))^5-13/63*tan(d*x+c)/a/d/(a+a*sec(d*
x+c))^4-34/105*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3-173/315*tan(d*x+c)/a^3/
d/(a+a*sec(d*x+c))^2-488/315*tan(d*x+c)/d/(a^5+a^5*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 2.95 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.94

$$\int \frac{1}{(a+a\sec(c+dx))^5}\,dx$$
$$= \frac{\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(39690dx\cos\left(\frac{dx}{2}\right)+39690dx\cos\left(c+\frac{dx}{2}\right)+26460dx\cos\left(c+\frac{3dx}{2}\right)+26460dx\,\mathrm{c}\ldots\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(-5),x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^9*(39690*d*x*Cos[(d*x)/2] + 39690*d*x*Cos[c + (
d*x)/2] + 26460*d*x*Cos[c + (3*d*x)/2] + 26460*d*x*Cos[2*c + (3*d*x)/2] +
11340*d*x*Cos[2*c + (5*d*x)/2] + 11340*d*x*Cos[3*c + (5*d*x)/2] + 2835*d*x
*Cos[3*c + (7*d*x)/2] + 2835*d*x*Cos[4*c + (7*d*x)/2] + 315*d*x*Cos[4*c +
(9*d*x)/2] + 315*d*x*Cos[5*c + (9*d*x)/2] - 116676*Sin[(d*x)/2] + 100800*S
in[c + (d*x)/2] - 88284*Sin[c + (3*d*x)/2] + 56700*Sin[2*c + (3*d*x)/2] -
43236*Sin[2*c + (5*d*x)/2] + 18900*Sin[3*c + (5*d*x)/2] - 12384*Sin[3*c +
(7*d*x)/2] + 3150*Sin[4*c + (7*d*x)/2] - 1726*Sin[4*c + (9*d*x)/2]))/(1612
80*a^5*d)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.19, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {3042, 4264, 25, 3042, 4410, 27, 3042, 4410, 25, 3042, 4410, 25, 3042, 4407, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$

$$\Big\downarrow \text{4264}$$

$$-\frac{\int -\frac{9a-4a\sec(c+dx)}{(\sec(c+dx)a+a)^4}dx}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{9a-4a\sec(c+dx)}{(\sec(c+dx)a+a)^4}dx}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{9a-4a\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\big\downarrow$ 4410

$$\frac{-\frac{\int -\frac{3\left(21a^2-13a^2\sec(c+dx)\right)}{(\sec(c+dx)a+a)^3}dx}{7a^2}-\frac{13a\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\big\downarrow$ 27

$$\frac{\frac{3\int \frac{21a^2-13a^2\sec(c+dx)}{(\sec(c+dx)a+a)^3}dx}{7a^2}-\frac{13a\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\big\downarrow$ 3042

$$\frac{\frac{3\int \frac{21a^2-13a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{13a\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\big\downarrow$ 4410

$$\frac{\frac{3\left(-\frac{\int -\frac{105a^3-68a^3\sec(c+dx)}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{34a^2\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{13a\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\big\downarrow$ 25

$$\frac{\frac{3\left(\frac{\int \frac{105a^3-68a^3\sec(c+dx)}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{34a^2\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{13a\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\big\downarrow$ 3042

$$\frac{\frac{3\left(\frac{\int \frac{105a^3-68a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{34a^2\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{13a\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\big\downarrow$ 4410

$$\frac{\frac{3\left(\frac{-\frac{\int -\frac{315a^4-173a^4\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{173a^3\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{34a^2\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{13a\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3\left(\frac{\frac{\int \frac{315a^4-173a^4\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{173a^3\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{34a^2\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{13a\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3\left(\frac{\frac{\int \frac{315a^4-173a^4\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{173a^3\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{34a^2\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{13a\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 4407$$

$$\frac{\frac{3\left(\frac{\frac{315a^3x-488a^4\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{173a^3\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{34a^2\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{13a\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3\left(\frac{\frac{315a^3x-488a^4\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{173a^3\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{34a^2\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{13a\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \ 4281$$

$$\frac{\frac{3\left(\frac{\frac{315a^3x-\frac{488a^4\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{173a^3\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{34a^2\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{13a\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

input
```
Int[(a + a*Sec[c + d*x])^(-5),x]
```

output
```
-1/9*Tan[c + d*x]/(d*(a + a*Sec[c + d*x])^5) + ((-13*a*Tan[c + d*x])/(7*d*
(a + a*Sec[c + d*x])^4) + (3*((-34*a^2*Tan[c + d*x])/(5*d*(a + a*Sec[c + d
*x])^3) + ((-173*a^3*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + (315*a^3
*x - (488*a^4*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3*a^2))/(5*a^2)))/(
7*a^2))/(9*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4264
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Simp[1/(a^2*(2*n + 1))
   Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Int
egerQ[2*n]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4410
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e +
f*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&
EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.53

method	result
parallelrisch	$\frac{-35 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{9}+270 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{7}-1008 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{5}+2730 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}+5040 d x-9765 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{5040 d \,a^{5}}$
derivativedivides	$\frac{-\frac{\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{9}}{9}+\frac{6 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{7}}{7}-\frac{16 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{26 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{3}-31 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)+32 \arctan \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{16 d \,a^{5}}$
default	$\frac{-\frac{\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{9}}{9}+\frac{6 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{7}}{7}-\frac{16 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{26 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{3}-31 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)+32 \arctan \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{16 d \,a^{5}}$
norman	$\frac{\frac{x}{a}-\frac{31 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{16 a d}+\frac{13 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{24 a d}-\frac{\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}{5 a d}+\frac{3 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{7}}{56 a d}-\frac{\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{9}}{144 a d}}{a^{4}}$
risch	$\frac{x}{a^{5}}-\frac{2 i \left(1575 \,{\mathrm e}^{8 i \left(d x +c \right)}+9450 \,{\mathrm e}^{7 i \left(d x +c \right)}+28350 \,{\mathrm e}^{6 i \left(d x +c \right)}+50400 \,{\mathrm e}^{5 i \left(d x +c \right)}+58338 \,{\mathrm e}^{4 i \left(d x +c \right)}+44142 \,{\mathrm e}^{3 i \left(d x +c \right)}+2161\right.}{315 d \,a^{5} \left({\mathrm e}^{i \left(d x +c \right)}+1\right)^{9}}$

input
```
int(1/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/5040*(-35*tan(1/2*d*x+1/2*c)^9+270*tan(1/2*d*x+1/2*c)^7-1008*tan(1/2*d*x
+1/2*c)^5+2730*tan(1/2*d*x+1/2*c)^3+5040*d*x-9765*tan(1/2*d*x+1/2*c))/d/a^
5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.31

$$\int \frac{1}{(a+a\sec(c+dx))^5}\,dx = \frac{315\,dx\cos(dx+c)^5+1575\,dx\cos(dx+c)^4+3150\,dx\cos(dx+c)^3+3150\,dx\cos(dx+c)^2+1575\,dx\cos}{315\,(a^5d\cos(dx+c)^5+5\,a^5d\cos(dx+c)^4+10}$$

input
```
integrate(1/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/315*(315*d*x*cos(d*x + c)^5 + 1575*d*x*cos(d*x + c)^4 + 3150*d*x*cos(d*x
 + c)^3 + 3150*d*x*cos(d*x + c)^2 + 1575*d*x*cos(d*x + c) + 315*d*x - (863
*cos(d*x + c)^4 + 2740*cos(d*x + c)^3 + 3549*cos(d*x + c)^2 + 2125*cos(d*x
 + c) + 488)*sin(d*x + c))/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4
+ 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c)
 + a^5*d)
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(c+dx))^5}\,dx = \frac{\int \frac{1}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

input
```
integrate(1/(a+a*sec(d*x+c))**5,x)
```

output
```
Integral(1/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c + d*x)**3 + 10*
sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.92

$$\int \frac{1}{(a+a\sec(c+dx))^5}\,dx = -\frac{\frac{\frac{9765 \sin(dx+c)}{\cos(dx+c)+1} - \frac{2730 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{1008 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{270 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{35 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5} - \frac{10080 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^5}}{5040\,d}$$

input
```
integrate(1/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/5040*((9765*sin(d*x + c)/(cos(d*x + c) + 1) - 2730*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 1008*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 270*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^5 -
 10080*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^5)/d
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.69

$$\int \frac{1}{(a+a\sec(c+dx))^5}\,dx = \frac{\frac{5040\,(dx+c)}{a^5} - \frac{35\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 270\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 1008\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 2730\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 9765\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{45}}}{5040\,d}$$

input
```
integrate(1/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

output
```
1/5040*(5040*(d*x + c)/a^5 - (35*a^40*tan(1/2*d*x + 1/2*c)^9 - 270*a^40*ta
n(1/2*d*x + 1/2*c)^7 + 1008*a^40*tan(1/2*d*x + 1/2*c)^5 - 2730*a^40*tan(1/
2*d*x + 1/2*c)^3 + 9765*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

Mupad [B] (verification not implemented)

Time = 9.31 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a+a\sec(c+dx))^5}\,dx = \frac{x}{a^5} - \frac{\frac{863\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{315} - \frac{356\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{315} + \frac{169\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{420} - \frac{41\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{504}}{a^5\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}$$

input
```
int(1/(a + a/cos(c + d*x))^5,x)
```

output
```
x/a^5 - (sin(c/2 + (d*x)/2)/144 - (41*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)
/2))/504 + (169*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/420 - (356*cos(c/
2 + (d*x)/2)^6*sin(c/2 + (d*x)/2))/315 + (863*cos(c/2 + (d*x)/2)^8*sin(c/2
 + (d*x)/2))/315)/(a^5*d*cos(c/2 + (d*x)/2)^9)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.53

$$\int \frac{1}{(a+a\sec(c+dx))^5}\,dx = \frac{-35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9 + 270\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7 - 1008\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5 + 2730\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 - 9765\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{5040a^5d}$$

input
```
int(1/(a+a*sec(d*x+c))^5,x)
```

output
```
( - 35*tan((c + d*x)/2)**9 + 270*tan((c + d*x)/2)**7 - 1008*tan((c + d*x)/
2)**5 + 2730*tan((c + d*x)/2)**3 - 9765*tan((c + d*x)/2) + 5040*d*x)/(5040
*a**5*d)
```

3.88 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^5} dx$

Optimal result

Integrand size = 19, antiderivative size = 159

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^5} dx = -\frac{5x}{a^5} + \frac{496\sin(c+dx)}{63a^5d} - \frac{\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sin(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{29\sin(c+dx)}{63a^2d(a+a\sec(c+dx))^3} - \frac{67\sin(c+dx)}{63a^3d(a+a\sec(c+dx))^2} - \frac{5\sin(c+dx)}{d\left(a^5+a^5\sec(c+dx)\right)}$$

output

```
-5*x/a^5+496/63*sin(d*x+c)/a^5/d-1/9*sin(d*x+c)/d/(a+a*sec(d*x+c))^5-5/21*
sin(d*x+c)/a/d/(a+a*sec(d*x+c))^4-29/63*sin(d*x+c)/a^2/d/(a+a*sec(d*x+c))^
3-67/63*sin(d*x+c)/a^3/d/(a+a*sec(d*x+c))^2-5*sin(d*x+c)/d/(a^5+a^5*sec(d*
x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 319 vs. $2(159) = 318$.

Time = 4.38 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.01

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(79380dx\cos\left(\frac{dx}{2}\right)+79380dx\cos\left(c+\frac{dx}{2}\right)+52920dx\cos\left(c+\frac{3dx}{2}\right)+52920dx\right.}{}$$

input
```
Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^5,x]
```

output
```
-1/64512*(Sec[c/2]*Sec[(c + d*x)/2]^9*(79380*d*x*Cos[(d*x)/2] + 79380*d*x*
Cos[c + (d*x)/2] + 52920*d*x*Cos[c + (3*d*x)/2] + 52920*d*x*Cos[2*c + (3*d
*x)/2] + 22680*d*x*Cos[2*c + (5*d*x)/2] + 22680*d*x*Cos[3*c + (5*d*x)/2] +
 5670*d*x*Cos[3*c + (7*d*x)/2] + 5670*d*x*Cos[4*c + (7*d*x)/2] + 630*d*x*C
os[4*c + (9*d*x)/2] + 630*d*x*Cos[5*c + (9*d*x)/2] - 175014*Sin[(d*x)/2] +
 143010*Sin[c + (d*x)/2] - 138726*Sin[c + (3*d*x)/2] + 73290*Sin[2*c + (3*
d*x)/2] - 70389*Sin[2*c + (5*d*x)/2] + 20475*Sin[3*c + (5*d*x)/2] - 21141*
Sin[3*c + (7*d*x)/2] + 1575*Sin[4*c + (7*d*x)/2] - 3091*Sin[4*c + (9*d*x)/
2] - 567*Sin[5*c + (9*d*x)/2] - 63*Sin[5*c + (11*d*x)/2] - 63*Sin[6*c + (1
1*d*x)/2]))/(a^5*d)
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.20, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.895$, Rules used = {3042, 4304, 27, 3042, 4508, 3042, 4508, 27, 3042, 4508, 3042, 4508, 3042, 4274, 24, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a\sec(c+dx)+a)^5}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{5\cos(c+dx)(2a-a\sec(c+dx))}{(\sec(c+dx)a+a)^4}dx}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\int \frac{\cos(c+dx)(2a-a\sec(c+dx))}{(\sec(c+dx)a+a)^4}dx}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \frac{2a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{4508}$$

$$\frac{5\left(\frac{\int \frac{\cos(c+dx)\left(17a^2-12a^2\sec(c+dx)\right)}{(\sec(c+dx)a+a)^3}dx}{7a^2}-\frac{3a\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{\int \frac{17a^2-12a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{3a\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{4508}$$

$$\frac{5\left(\frac{\frac{\int \frac{3\cos(c+dx)\left(38a^3-29a^3\sec(c+dx)\right)}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{29a^2\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{3a\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\left(\frac{\frac{3\int \frac{\cos(c+dx)\left(38a^3-29a^3\sec(c+dx)\right)}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{29a^2\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{3a\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{\frac{3\int \frac{38a^3-29a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{29a^2\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{3a\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{4508}$$

$$\frac{5\left(\frac{\frac{3\left(\frac{\int \frac{\cos(c+dx)\left(181a^4-134a^4\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{67a^3\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{29a^2\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{3a\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{\frac{3\left(\frac{\int \frac{181a^4-134a^4\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{67a^3\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{29a^2\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{3a\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{4508}$$

$$\frac{5\left(\frac{\frac{3\left(\frac{\frac{\int \cos(c+dx)\left(496a^5-315a^5\sec(c+dx)\right)dx}{a^2}-\frac{315a^4\sin(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{67a^3\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{29a^2\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{3a\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{\frac{3\left(\frac{\frac{\int \frac{496a^5-315a^5\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{315a^4\sin(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{67a^3\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{29a^2\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{3a\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{4274}$$

$$\frac{5\left(\frac{\frac{3\left(\frac{\frac{496a^5\int \cos(c+dx)dx-315a^5\int 1dx}{a^2}-\frac{315a^4\sin(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{67a^3\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{29a^2\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{3a\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{24}$$

$$\frac{5\left(\frac{\frac{3\left(\frac{\frac{496a^5\int\cos(c+dx)dx-315a^5x}{a^2}-\frac{315a^4\sin(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{67a^3\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{29a^2\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{3a\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 3042

$$\frac{5\left(\frac{\frac{3\left(\frac{\frac{496a^5\int\sin\left(c+dx+\frac{\pi}{2}\right)dx-315a^5x}{a^2}-\frac{315a^4\sin(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{67a^3\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{29a^2\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{3a\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 3117

$$\frac{5\left(\frac{\frac{3\left(\frac{\frac{\frac{496a^5\sin(c+dx)}{d}-315a^5x}{a^2}-\frac{315a^4\sin(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{67a^3\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{29a^2\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{3a\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

input
```
Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^5,x]
```

output
```
-1/9*Sin[c + d*x]/(d*(a + a*Sec[c + d*x])^5) + (5*((-3*a*Sin[c + d*x])/(7*
d*(a + a*Sec[c + d*x])^4) + ((-29*a^2*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*
x])^3) + (3*((-67*a^3*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + ((-315*
a^4*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])) + (-315*a^5*x + (496*a^5*Sin[c
+ d*x])/d)/a^2)/(3*a^2)))/(5*a^2))/(7*a^2)))/(9*a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.57

method	result
parallelrisch	$-\frac{80640dx-(42676+63\cos(5dx+5c)+1892\cos(4dx+4c)+11675\cos(3dx+3c)+36632\cos(2dx+2c)+69350\cos(dx+c)}{16128d\,a^5}$
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{9}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5-24\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+129\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-160\arctan\left(\text{ta}\right.}{16d\,a^5}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{9}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5-24\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+129\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-160\arctan\left(\text{ta}\right.}{16d\,a^5}$
norman	$\frac{-\frac{5x}{a}+\frac{161\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}+\frac{105\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{16ad}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8ad}+\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{56ad}-\frac{65\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{1008ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{144ad}-\frac{5x\tan(}{}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)a^4}$
risch	$-\frac{5x}{a^5}-\frac{i\mathrm{e}^{i(dx+c)}}{2d\,a^5}+\frac{i\mathrm{e}^{-i(dx+c)}}{2d\,a^5}+\frac{2i(945\,\mathrm{e}^{8i(dx+c)}+6300\,\mathrm{e}^{7i(dx+c)}+19740\,\mathrm{e}^{6i(dx+c)}+36414\,\mathrm{e}^{5i(dx+c)}+43092\,\mathrm{e}^{4i}}{63d\,a^5\left(\mathrm{e}^{i(dx+c)}+1\right.}$

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
-1/16128*(80640*d*x-(42676+63*cos(5*d*x+5*c)+1892*cos(4*d*x+4*c)+11675*cos
(3*d*x+3*c)+36632*cos(2*d*x+2*c)+69350*cos(d*x+c))*tan(1/2*d*x+1/2*c)*sec(
1/2*d*x+1/2*c)^8)/d/a^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.25

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{315\,dx\cos\left(dx+c\right)^5+1575\,dx\cos\left(dx+c\right)^4+3150\,dx\cos\left(dx+c\right)^3+3150\,dx\cos\left(dx+c\right)^2+1575\,dx}{63\left(a^5d\cos\left(dx+c\right)^5+5\,a^5d\cos\left(dx\right.\right.}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

output
```
-1/63*(315*d*x*cos(d*x + c)^5 + 1575*d*x*cos(d*x + c)^4 + 3150*d*x*cos(d*x
 + c)^3 + 3150*d*x*cos(d*x + c)^2 + 1575*d*x*cos(d*x + c) + 315*d*x - (63*
cos(d*x + c)^5 + 946*cos(d*x + c)^4 + 2840*cos(d*x + c)^3 + 3633*cos(d*x +
 c)^2 + 2165*cos(d*x + c) + 496)*sin(d*x + c))/(a^5*d*cos(d*x + c)^5 + 5*a
^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 +
5*a^5*d*cos(d*x + c) + a^5*d)
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\int \frac{\cos\left(c+dx\right)}{\sec^5\left(c+dx\right)+5\sec^4\left(c+dx\right)+10\sec^3\left(c+dx\right)+10\sec^2\left(c+dx\right)+5\sec\left(c+dx\right)+1}\,dx}{a^5}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))**5,x)
```

output
```
Integral(cos(c + d*x)/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c + d*
x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.12

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^5}\,dx$$
$$= \frac{\frac{2016\,\sin(dx+c)}{\left(a^5+\frac{a^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} + \frac{\frac{8127\,\sin(dx+c)}{\cos(dx+c)+1} - \frac{1512\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{378\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{72\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{7\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5} - \frac{10080\,\arctan}{a}}{1008\,d}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/1008*(2016*sin(d*x + c)/((a^5 + a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1)) + (8127*sin(d*x + c)/(cos(d*x + c) + 1) - 1512*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 -
72*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 7*sin(d*x + c)^9/(cos(d*x + c) +
1)^9)/a^5 - 10080*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^5)/d
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.81

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^5}\,dx =$$
$$-\frac{\frac{5040\,(dx+c)}{a^5} - \frac{2016\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^5} - \frac{7\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 72\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 378\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 1512\,a^{40}\tan\left(\frac{1}{2}\,dx\right.}{a^{45}}}{1008\,d}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/1008*(5040*(d*x + c)/a^5 - 2016*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/
2*c)^2 + 1)*a^5) - (7*a^40*tan(1/2*d*x + 1/2*c)^9 - 72*a^40*tan(1/2*d*x +
1/2*c)^7 + 378*a^40*tan(1/2*d*x + 1/2*c)^5 - 1512*a^40*tan(1/2*d*x + 1/2*c
)^3 + 8127*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

Mupad [B] (verification not implemented)

Time = 9.50 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-100\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+636\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-2512\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{1008\,a}$$

input
```
int(cos(c + d*x)/(a + a/cos(c + d*x))^5,x)
```

output
```
(7*sin(c/2 + (d*x)/2) - 100*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 636*
cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) - 2512*cos(c/2 + (d*x)/2)^6*sin(c/
2 + (d*x)/2) + 10096*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 2016*cos(c/
2 + (d*x)/2)^10*sin(c/2 + (d*x)/2) - 5040*cos(c/2 + (d*x)/2)^9*(c + d*x))/
(1008*a^5*d*cos(c/2 + (d*x)/2)^9)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.75

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}-65\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9+306\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7-1134\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+6615\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-50}{1008a^5d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)}$$

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^5,x)
```

output
```
(7*tan((c + d*x)/2)**11 - 65*tan((c + d*x)/2)**9 + 306*tan((c + d*x)/2)**7
 - 1134*tan((c + d*x)/2)**5 + 6615*tan((c + d*x)/2)**3 - 5040*tan((c + d*x
)/2)**2*d*x + 10143*tan((c + d*x)/2) - 5040*d*x)/(1008*a**5*d*(tan((c + d*
x)/2)**2 + 1))
```

3.89 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 215

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx = {} & \frac{31x}{2a^5}-\frac{7664\sin(c+dx)}{315a^5d}+\frac{31\cos(c+dx)\sin(c+dx)}{2a^5d}\\ & -\frac{\cos(c+dx)\sin(c+dx)}{9d(a+a\sec(c+dx))^5}-\frac{17\cos(c+dx)\sin(c+dx)}{63ad(a+a\sec(c+dx))^4}\\ & -\frac{28\cos(c+dx)\sin(c+dx)}{45a^2d(a+a\sec(c+dx))^3}-\frac{577\cos(c+dx)\sin(c+dx)}{315a^3d(a+a\sec(c+dx))^2}\\ & -\frac{3832\cos(c+dx)\sin(c+dx)}{315d\,(a^5+a^5\sec(c+dx))}\end{aligned}$$

output
```
31/2*x/a^5-7664/315*sin(d*x+c)/a^5/d+31/2*cos(d*x+c)*sin(d*x+c)/a^5/d-1/9*
cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^5-17/63*cos(d*x+c)*sin(d*x+c)/a/d
/(a+a*sec(d*x+c))^4-28/45*cos(d*x+c)*sin(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3-5
77/315*cos(d*x+c)*sin(d*x+c)/a^3/d/(a+a*sec(d*x+c))^2-3832/315*cos(d*x+c)*
sin(d*x+c)/d/(a^5+a^5*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 6.54 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.60

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx$$
$$= \frac{\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(4921560dx\cos\left(\frac{dx}{2}\right)+4921560dx\cos\left(c+\frac{dx}{2}\right)+3281040dx\cos\left(c+\frac{3dx}{2}\right)+328\right.}{}$$

input
```
Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^5,x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^9*(4921560*d*x*Cos[(d*x)/2] + 4921560*d*x*Cos[c
 + (d*x)/2] + 3281040*d*x*Cos[c + (3*d*x)/2] + 3281040*d*x*Cos[2*c + (3*d*
x)/2] + 1406160*d*x*Cos[2*c + (5*d*x)/2] + 1406160*d*x*Cos[3*c + (5*d*x)/2
] + 351540*d*x*Cos[3*c + (7*d*x)/2] + 351540*d*x*Cos[4*c + (7*d*x)/2] + 39
060*d*x*Cos[4*c + (9*d*x)/2] + 39060*d*x*Cos[5*c + (9*d*x)/2] - 9163224*Si
n[(d*x)/2] + 7194600*Sin[c + (d*x)/2] - 7472241*Sin[c + (3*d*x)/2] + 34329
75*Sin[2*c + (3*d*x)/2] - 3871989*Sin[2*c + (5*d*x)/2] + 801675*Sin[3*c +
(5*d*x)/2] - 1186056*Sin[3*c + (7*d*x)/2] - 17640*Sin[4*c + (7*d*x)/2] - 1
75184*Sin[4*c + (9*d*x)/2] - 45360*Sin[5*c + (9*d*x)/2] - 3465*Sin[5*c + (
11*d*x)/2] - 3465*Sin[6*c + (11*d*x)/2] + 315*Sin[6*c + (13*d*x)/2] + 315*
Sin[7*c + (13*d*x)/2]))/(1290240*a^5*d)
```

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.12, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {3042, 4304, 25, 3042, 4508, 3042, 4508, 3042, 4508, 27, 3042, 4508, 3042, 4274, 3042, 3115, 24, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a\sec(c+dx)+a)^5}\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$

$\Big\downarrow$ 4304

$$-\frac{\int -\frac{\cos^2(c+dx)(11a-6a\sec(c+dx))}{(\sec(c+dx)a+a)^4}dx}{9a^2}-\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{\cos^2(c+dx)(11a-6a\sec(c+dx))}{(\sec(c+dx)a+a)^4}dx}{9a^2}-\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{11a-6a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a^2}-\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 4508

$$\frac{\frac{\int \frac{\cos^2(c+dx)\left(111a^2-85a^2\sec(c+dx)\right)}{(\sec(c+dx)a+a)^3}dx}{7a^2}-\frac{17a\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{111a^2-85a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{17a\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 4508

$$\frac{\frac{\frac{\int \frac{\cos^2(c+dx)\left(947a^3-784a^3\sec(c+dx)\right)}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{196a^2\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{17a\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{947a^3-784a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{196a^2\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{17a\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 4508$$

$$\frac{\frac{\frac{\frac{\int \frac{3\cos^2(c+dx)\left(2101a^4-1731a^4\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{577a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{196a^2\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{17a\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\frac{\int \frac{\cos^2(c+dx)\left(2101a^4-1731a^4\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{a^2}-\frac{577a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{196a^2\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{17a\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\frac{\int \frac{2101a^4-1731a^4\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{a^2}-\frac{577a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{196a^2\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{17a\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 4508$$

$$\frac{\frac{\frac{\frac{\frac{\int \cos^2(c+dx)\left(9765a^5-7664a^5\sec(c+dx)\right)dx}{a^2}-\frac{3832a^4\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}-\frac{577a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{196a^2\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{17a\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\frac{\frac{\int \frac{9765a^5-7664a^5\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx}{a^2}-\frac{3832a^4\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}-\frac{577a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{196a^2\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{17a\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 4274

$$\frac{\frac{\frac{\frac{\frac{9765a^5\int \cos^2(c+dx)dx-7664a^5\int \cos(c+dx)dx}{a^2}-\frac{3832a^4\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}-\frac{577a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{196a^2\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{17a\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\frac{\frac{9765a^5\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-7664a^5\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{3832a^4\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}-\frac{577a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{196a^2\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{17a\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 3115

$$\frac{\frac{\frac{\frac{\frac{9765a^5\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-7664a^5\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{3832a^4\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}-\frac{577a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{196a^2\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{17a\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 24

$$\frac{\frac{\frac{\frac{\frac{9765a^5\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-7664a^5\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{3832a^4\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}-\frac{577a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{196a^2\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{17a\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3117}$$

$$\frac{\frac{\frac{\frac{\frac{9765a^5\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{7664a^5\sin(c+dx)}{d}}{a^2}-\frac{3832a^4\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}-\frac{577a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{196a^2\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{17a\sin(c}{7d(a\,\mathrm{se}}}{9a^2}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

input
```
Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^5,x]
```

output
```
-1/9*(Cos[c + d*x]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^5) + ((-17*a*Cos[
c + d*x]*Sin[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + ((-196*a^2*Cos[c + d
*x]*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((-577*a^3*Cos[c + d*x]*S
in[c + d*x])/(d*(a + a*Sec[c + d*x])^2) + ((-3832*a^4*Cos[c + d*x]*Sin[c +
 d*x])/(d*(a + a*Sec[c + d*x])) + ((-7664*a^5*Sin[c + d*x])/d + 9765*a^5*(
x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/a^2)/a^2)/(5*a^2))/(7*a^2))/(9*a
^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.46

method	result
parallelrisch	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\cos(6dx+6c)-\frac{854012\cos(dx+c)}{63}-\frac{2250427\cos(2dx+2c)}{315}-\frac{143054\cos(3dx+3c)}{63}-\frac{113422\cos(4dx+4c)}{315}-10\cos(5d\right.}{1024d\,a^5}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{9}+\frac{10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}-\frac{48\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+50\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-351\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-176\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-144\tan\left(\frac{d}{2}\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}}{16d\,a^5}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{9}+\frac{10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}-\frac{48\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+50\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-351\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-176\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-144\tan\left(\frac{d}{2}\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}}{16d\,a^5}$
norman	$\frac{\frac{31x}{2a}-\frac{495\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}-\frac{207\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4ad}-\frac{1303\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{80ad}+\frac{141\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{70ad}-\frac{2159\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{5040ad}+\frac{19\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{252ad}-}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2a^4}$
risch	$\frac{31x}{2a^5}-\frac{ie^{2i(dx+c)}}{8d\,a^5}+\frac{5ie^{i(dx+c)}}{2d\,a^5}-\frac{5ie^{-i(dx+c)}}{2d\,a^5}+\frac{ie^{-2i(dx+c)}}{8d\,a^5}-\frac{2i\left(11025\,e^{8i(dx+c)}+77175\,e^{7i(dx+c)}+247695\,e\right.}{}$

input
```
int(cos(d*x+c)^2/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/1024*(tan(1/2*d*x+1/2*c)*(cos(6*d*x+6*c)-854012/63*cos(d*x+c)-2250427/31
5*cos(2*d*x+2*c)-143054/63*cos(3*d*x+3*c)-113422/315*cos(4*d*x+4*c)-10*cos
(5*d*x+5*c)-2627186/315)*sec(1/2*d*x+1/2*c)^8+15872*d*x)/d/a^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.96

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{9765\,dx\cos(dx+c)^5+48825\,dx\cos(dx+c)^4+97650\,dx\cos(dx+c)^3+97650\,dx\cos(dx+c)^2+4882}{630\left(a^5d\cos(dx+\right.}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/630*(9765*d*x*cos(d*x + c)^5 + 48825*d*x*cos(d*x + c)^4 + 97650*d*x*cos(
d*x + c)^3 + 97650*d*x*cos(d*x + c)^2 + 48825*d*x*cos(d*x + c) + 9765*d*x
+ (315*cos(d*x + c)^6 - 1575*cos(d*x + c)^5 - 28828*cos(d*x + c)^4 - 87440
*cos(d*x + c)^3 - 112119*cos(d*x + c)^2 - 66875*cos(d*x + c) - 15328)*sin(
d*x + c))/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*
x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\int \frac{\cos^2(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

input
```
integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**5,x)
```

output
```
Integral(cos(c + d*x)**2/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
 d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.04

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\frac{5040\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{11\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^5+\frac{2a^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{110565\sin(dx+c)}{\cos(dx+c)+1}-\frac{15750\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3024\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{450\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-156240}{5040\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/5040*(5040*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 11*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3)/(a^5 + 2*a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^5*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) + (110565*sin(d*x + c)/(cos(d*x + c) +
1) - 15750*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3024*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5 - 450*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c
)^9/(cos(d*x + c) + 1)^9)/a^5 - 156240*arctan(sin(d*x + c)/(cos(d*x + c) +
 1))/a^5)/d
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.67

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\frac{78120\,(dx+c)}{a^5} - \frac{5040\left(11\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a^5} - \frac{35\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-450\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+3024\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{a^{45}}}{5040\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

output
```
1/5040*(78120*(d*x + c)/a^5 - 5040*(11*tan(1/2*d*x + 1/2*c)^3 + 9*tan(1/2*
d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^5) - (35*a^40*tan(1/2*d*x
+ 1/2*c)^9 - 450*a^40*tan(1/2*d*x + 1/2*c)^7 + 3024*a^40*tan(1/2*d*x + 1/2
*c)^5 - 15750*a^40*tan(1/2*d*x + 1/2*c)^3 + 110565*a^40*tan(1/2*d*x + 1/2*
c))/a^45)/d
```

Mupad [B] (verification not implemented)

Time = 9.46 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.84

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{35\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-590\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+4584\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-23288\cos\left(\frac{c}{2}+\right.}{}$$

input
```
int(cos(c + d*x)^2/(a + a/cos(c + d*x))^5,x)
```

output
```
-(35*sin(c/2 + (d*x)/2) - 590*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 45
84*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) - 23288*cos(c/2 + (d*x)/2)^6*si
n(c/2 + (d*x)/2) + 129824*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 55440*
cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2) - 10080*cos(c/2 + (d*x)/2)^12*sin
(c/2 + (d*x)/2) - 78120*cos(c/2 + (d*x)/2)^9*(c + d*x))/(5040*a^5*d*cos(c/
2 + (d*x)/2)^9)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.74

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx$$
$$= \frac{-35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}+380\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}-2159\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}+10152\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}-82089\tan\left(\frac{dx}{2}+\right.}{5040a^5d\left(\tan\left(\frac{dx}{2}\right.\right.}$$

input
```
int(cos(d*x+c)^2/(a+a*sec(d*x+c))^5,x)
```

output
```
( - 35*tan((c + d*x)/2)**13 + 380*tan((c + d*x)/2)**11 - 2159*tan((c + d*x
)/2)**9 + 10152*tan((c + d*x)/2)**7 - 82089*tan((c + d*x)/2)**5 + 78120*ta
n((c + d*x)/2)**4*d*x - 260820*tan((c + d*x)/2)**3 + 156240*tan((c + d*x)/
2)**2*d*x - 155925*tan((c + d*x)/2) + 78120*d*x)/(5040*a**5*d*(tan((c + d*
x)/2)**4 + 2*tan((c + d*x)/2)**2 + 1))
```

3.90 $\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 122

$$\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{4a\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2a\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} - \frac{8\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{35d} + \frac{12(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{35ad}$$

output
```
4/5*a*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/7*a*sec(d*x+c)^3*tan(d*x+c)/d/
(a+a*sec(d*x+c))^(1/2)-8/35*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d+12/35*(a+a
*sec(d*x+c))^(3/2)*tan(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.48

$$\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2a(16+8\sec(c+dx)+6\sec^2(c+dx)+5\sec^3(c+dx))\tan(c+dx)}{35d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*a*(16 + 8*Sec[c + d*x] + 6*Sec[c + d*x]^2 + 5*Sec[c + d*x]^3)*Tan[c + d
*x])/(35*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4290, 3042, 4287, 27, 3042, 4489, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4290}$$

$$\frac{6}{7}\int \sec^3(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 4287$$

$$\frac{6}{7}\left(\frac{2\int \frac{1}{2}\sec(c+dx)(3a-2a\sec(c+dx))\sqrt{\sec(c+dx)a+a}dx}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 27$$

$$\frac{6}{7}\left(\frac{\int \sec(c+dx)(3a-2a\sec(c+dx))\sqrt{\sec(c+dx)a+a}dx}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{6}{7}\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(3a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a}{5ad}\right.$$
$$\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 4489$$

$$\frac{6}{7}\left(\frac{\frac{7}{3}a\int \sec(c+dx)\sqrt{\sec(c+dx)a+a}dx-\frac{4a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{6}{7}\left(\frac{\frac{7}{3}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{4a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)}{5ad}\right.$$
$$\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 4279$$

$$\frac{6}{7}\left(\frac{\frac{14a^2\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}-\frac{4a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Sec[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*a*Sec[c + d*x]^3*Tan[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]]) + (6*((2*
(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*a*d) + ((14*a^2*Tan[c + d*x])/
(3*d*Sqrt[a + a*Sec[c + d*x]]) - (4*a*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x
])/(3*d))/(5*a)))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4287
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 -
b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{2\left(16\cos(dx+c)^3+8\cos(dx+c)^2+6\cos(dx+c)+5\right)\sqrt{a(1+\sec(dx+c))}\,\tan(dx+c)\sec(dx+c)^2}{35d(\cos(dx+c)+1)}$	72

input
```
int(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/35/d*(16*cos(d*x+c)^3+8*cos(d*x+c)^2+6*cos(d*x+c)+5)*(a*(1+sec(d*x+c)))^
(1/2)/(cos(d*x+c)+1)*tan(d*x+c)*sec(d*x+c)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.67

$$\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$=\frac{2\left(16\cos(dx+c)^3+8\cos(dx+c)^2+6\cos(dx+c)+5\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{35\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)}$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/35*(16*cos(d*x + c)^3 + 8*cos(d*x + c)^2 + 6*cos(d*x + c) + 5)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x
+ c)^3)
```

Sympy [F]

$$\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sec^4\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**4*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**4, x)
```

Maxima [F]

$$\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
16/35*(35*(d*cos(2*d*x + 2*c)^2 + d*sin(2*d*x + 2*c)^2 + 2*d*cos(2*d*x + 2
*c) + d)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(3/4)*sqrt(a)*integrate((((cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8
*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos
(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*s
in(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)
*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)
^2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*
c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x
+ 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x
 + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*
d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(5/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*
cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) +
4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c)
- 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c
) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) - (cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x +
 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*...
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.98

$$\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx =$$

$$-\frac{2\sqrt{2}\left(35\,a^4-\left(35\,a^4+\left(9\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-49\,a^4\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right)\mathrm{sgn}(\cos}{35\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^3\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}d}$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-2/35*sqrt(2)*(35*a^4 - (35*a^4 + (9*a^4*tan(1/2*d*x + 1/2*c)^2 - 49*a^4)*
tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*sgn(cos(d*x + c))*tan(1/2*
d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a)*d)
```

Mupad [B] (verification not implemented)

Time = 13.92 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.71

$$\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = -\frac{e^{c\,1i+d\,x\,1i}\,\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,32i}{35\,d\,\left(e^{c\,1i+d\,x\,1i}+1\right)} - \frac{\left(\frac{16i}{7\,d}+\frac{e^{c\,1i+d\,x\,1i}\,16i}{7\,d}\right)\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}}{\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)^3} + \frac{\left(\frac{16i}{5\,d}+\frac{e^{c\,1i+d\,x\,1i}\,128i}{35\,d}\right)\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}}{\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)^2} - \frac{e^{c\,1i+d\,x\,1i}\,\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,16i}{35\,d\,\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)}$$

input
```
int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^4,x)
```

output
```
((16i/(5*d) + (exp(c*1i + d*x*1i)*128i)/(35*d))*(a + a/(exp(- c*1i - d*x*1
i)/2 + exp(c*1i + d*x*1i)/2))^(1/2))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i +
 d*x*2i) + 1)^2) - ((16i/(7*d) + (exp(c*1i + d*x*1i)*16i)/(7*d))*(a + a/(e
xp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2))/((exp(c*1i + d*x*1i)
 + 1)*(exp(c*2i + d*x*2i) + 1)^3) - (exp(c*1i + d*x*1i)*(a + a/(exp(- c*1i
 - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*32i)/(35*d*(exp(c*1i + d*x*1i)
 + 1)) - (exp(c*1i + d*x*1i)*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d
*x*1i)/2))^(1/2)*16i)/(35*d*(exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) +
 1))
```

Reduce [F]

$$\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \sqrt{a}\left(\int \sqrt{\sec(dx+c)+1}\,\sec(dx+c)^4\,dx\right)$$

input
```
int(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sec(c + d*x) + 1)*sec(c + d*x)**4,x)
```

3.91 $\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{14a\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} - \frac{4\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15d} + \frac{2(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5ad}$$

output
```
14/15*a*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)-4/15*(a+a*sec(d*x+c))^(1/2)*ta
n(d*x+c)/d+2/5*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.56

$$\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2a(8+4\sec(c+dx)+3\sec^2(c+dx))\tan(c+dx)}{15d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*a*(8 + 4*Sec[c + d*x] + 3*Sec[c + d*x]^2)*Tan[c + d*x])/(15*d*Sqrt[a*(1
 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 4287, 27, 3042, 4489, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4287}$$

$$\frac{2\int \frac{1}{2}\sec(c+dx)(3a-2a\sec(c+dx))\sqrt{\sec(c+dx)a+a}dx}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sec(c+dx)(3a-2a\sec(c+dx))\sqrt{\sec(c+dx)a+a}dx}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(3a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}$$

$$\Big\downarrow \text{4489}$$

$$\frac{\frac{7}{3}a\int \sec(c+dx)\sqrt{\sec(c+dx)a+a}dx-\frac{4a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{7}{3}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{4a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}$$

$$\Big\downarrow \text{4279}$$

$$\frac{\frac{14a^2\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}-\frac{4a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}$$

input
```
Int[Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*a*d) + ((14*a^2*Tan[c + d*x
])/(3*d*Sqrt[a + a*Sec[c + d*x]]) - (4*a*Sqrt[a + a*Sec[c + d*x]]*Tan[c +
d*x])/(3*d))/(5*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4287
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 -
b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{(16\sin(dx+c)+8\tan(dx+c)+6\sec(dx+c)\tan(dx+c))\sqrt{a(1+\sec(dx+c))}}{d(15\cos(dx+c)+15)}$	60

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(16*sin(d*x+c)+8*tan(d*x+c)+6*sec(d*x+c)*tan(d*x+c))/(15*cos(d*x+c)+15
)*(a*(1+sec(d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.84

$$\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\left(8\cos(dx+c)^2+4\cos(dx+c)+3\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{15\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/15*(8*cos(d*x + c)^2 + 4*cos(d*x + c) + 3)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sec^3\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**3, x)
```

Maxima [F]

$$\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
8/15*(15*(d*cos(2*d*x + 2*c)^2 + d*sin(2*d*x + 2*c)^2 + 2*d*cos(2*d*x + 2*
c) + d)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*sqrt(a)*integrate((((cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 3*cos(6*d*
x + 6*c)*cos(2*d*x + 2*c) + 3*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*
x + 2*c)^2 + sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 3*sin(6*d*x + 6*c)*sin(2*
d*x + 2*c) + 3*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(
8*d*x + 8*c) + 3*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 3*cos(2*d*x + 2*c)*si
n(4*d*x + 4*c) - cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 3*cos(6*d*x + 6*c)*si
n(2*d*x + 2*c) - 3*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(3/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 3*cos(2*d*x + 2*c
)*sin(6*d*x + 6*c) + 3*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(8*d*x + 8*c
)*sin(2*d*x + 2*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 3*cos(4*d*x + 4
*c)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - (cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 3*cos(6*d*x + 6*c)*cos(2*d*x + 2*c
) + 3*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(8*d*x +
 8*c)*sin(2*d*x + 2*c) + 3*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 3*sin(4*d*x
 + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x ...
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.17

$$\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\sqrt{2}\left(15a^3+\left(7a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-10a^3\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{15\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}d}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
2/15*sqrt(2)*(15*a^3 + (7*a^3*tan(1/2*d*x + 1/2*c)^2 - 10*a^3)*tan(1/2*d*x
 + 1/2*c)^2)*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*
c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

Mupad [B] (verification not implemented)

Time = 13.34 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.34

$$\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{8\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(e^{c\,2i+d\,x\,2i}\,5i-e^{c\,3i+d\,x\,3i}\,5i-e^{c\,5i+d\,x\,5i}\,2i+2i\right)}{15\,d\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)^2}$$

input
```
int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^3,x)
```

output
```
(8*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*2i
 + d*x*2i)*5i - exp(c*3i + d*x*3i)*5i - exp(c*5i + d*x*5i)*2i + 2i))/(15*d
*(exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2)
```

Reduce [F]

$$\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \sqrt{a}\left(\int \sqrt{\sec(dx+c)+1}\,\sec(dx+c)^3\,dx\right)$$

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sec(c + d*x) + 1)*sec(c + d*x)**3,x)
```

3.92 $\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 56

$$\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2a\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d}$$

output
```
2/3*a*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*(a+a*sec(d*x+c))^(1/2)*tan(d
*x+c)/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.64

$$\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2a(2+\sec(c+dx))\tan(c+dx)}{3d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*a*(2 + Sec[c + d*x])*Tan[c + d*x])/(3*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4285, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4285}$$

$$\frac{1}{3}\int \sec(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{4279}$$

$$\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}+\frac{2a\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Sec[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*a*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*Sqrt[a + a*Sec[c +
d*x]]*Tan[c + d*x])/(3*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4285
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[a*(m/(b*(m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x]
/; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{(4\sin(dx+c)+2\tan(dx+c))\sqrt{a(1+\sec(dx+c))}}{d(3\cos(dx+c)+3)}$	46

input
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(4*sin(d*x+c)+2*tan(d*x+c))/(3*cos(d*x+c)+3)*(a*(1+sec(d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07

$$\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx=\frac{2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(2\cos(dx+c)+1)\sin(dx+c)}{3\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(2*cos(d*x + c) + 1)*sin(d*x +
 c)/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F]

$$\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx=\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sec^2\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**2, x)
```

Maxima [F]

$$\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx=\int \sqrt{a\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
4/3*(3*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(3/4)*sqrt(a)*d*integrate((((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d
*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d
*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(
3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(6
*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2
*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*s
in(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*s
in(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (c
os(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + c
os(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)
*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(3/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)))/(((2*(2*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + cos(6*
d*x + 6*c)^2 + 4*cos(4*d*x + 4*c)^2 + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c)
+ cos(2*d*x + 2*c)^2 + 2*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x
 + 6*c) + sin(6*d*x + 6*c)^2 + 4*sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*s
in(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c),...
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.46

$$\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\sqrt{2}\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^2\right)\mathrm{sgn}(\cos\left(dx+c\right))\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{3\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}d}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
2/3*sqrt(2)*(a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2)*sgn(cos(d*x + c))*tan(1/2
*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a)*d)
```

Mupad [B] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.93

$$\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$=\frac{4\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}\,(3\,\sin(c+d\,x)+4\,\sin(2\,c+2\,d\,x)+3\,\sin(3\,c+3\,d\,x)+\sin(4\,c+4\,d\,x))}{3\,d\,\left(12\,\cos(c+d\,x)+8\,\cos(2\,c+2\,d\,x)+4\,\cos(3\,c+3\,d\,x)+\cos(4\,c+4\,d\,x)+7\right)}$$

input
```
int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^2,x)
```

output
```
(4*((a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2)*(3*sin(c + d*x) + 4*sin(2*c
 + 2*d*x) + 3*sin(3*c + 3*d*x) + sin(4*c + 4*d*x)))/(3*d*(12*cos(c + d*x)
+ 8*cos(2*c + 2*d*x) + 4*cos(3*c + 3*d*x) + cos(4*c + 4*d*x) + 7))
```

Reduce [F]

$$\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx=\sqrt{a}\left(\int \sqrt{\sec(dx+c)+1}\,\sec(dx+c)^2\,dx\right)$$

input
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sec(c + d*x) + 1)*sec(c + d*x)**2,x)
```

3.93 $\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 26

$$\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

output
```
2*a*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

input
```
Integrate[Sec[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$
$$\Big\downarrow \text{4279}$$
$$\frac{2a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Sec[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*a*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

method	result	size
default	$-\frac{2\sqrt{a(1+\sec(dx+c))}\,(\cot(dx+c)-\csc(dx+c))}{d}$	33

input
```
int(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(a*(1+sec(d*x+c)))^(1/2)*(cot(d*x+c)-csc(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{d\cos\left(dx+c\right)+d}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d
)
```

Sympy [F]

$$\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sec\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}\sec\left(dx+c\right)\,dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)*sec(d*x + c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(24) = 48.

Time = 0.34 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.38

$$\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = -\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}a\mathrm{sgn}(\cos\left(dx+c\right))\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)d}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a*sgn(cos(d*x + c))*tan(1/2
*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*d)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\,\sin\left(c+d\,x\right)\,\sqrt{\frac{a\,\left(\cos(c+d\,x)+1\right)}{\cos(c+d\,x)}}}{d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

input
```
int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x),x)
```

output
```
(2*sin(c + d*x)*((a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2))/(d*(cos(c + d
*x) + 1))
```

Reduce [F]

$$\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx=\sqrt{a}\left(\int \sqrt{\sec(dx+c)+1}\,\sec(dx+c)\,dx\right)$$

input
```
int(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sec(c + d*x) + 1)*sec(c + d*x),x)
```

3.94 $\int \sqrt{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 37

$$\int \sqrt{a+a\sec(c+dx)}\,dx=\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

output
```
2*a^(1/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \sqrt{a+a\sec(c+dx)}\,dx =\frac{\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}}{d}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]]*Sec[(c + d*x)
/2]*Sqrt[a*(1 + Sec[c + d*x])])/d
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4261}$$

$$-\frac{2a\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(31) = 62$.

Time = 0.88 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.38

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{-a(-1-\sec(dx+c))}\,\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\right)}{d}$	88

input
```
int((a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*2^(1/2)*(-a*(-1-sec(d*x+c)))^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*arctanh(2^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc
(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.59

$$\int \sqrt{a+a\sec(c+dx)}\,dx = \left[\frac{\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{d}, -\frac{2\,\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{d}\right]$$

input
```
integrate((a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c)
 + 1))/d, -2*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*
x + c)/(sqrt(a)*sin(d*x + c)))/d]
```

Sympy [F]

$$\int \sqrt{a + a \sec(c + dx)}\, dx = \int \sqrt{a \sec (c + dx) + a}\, dx$$

input
```
integrate((a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*sec(c + d*x) + a), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(31) = 62$.

Time = 0.16 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.95

$$\int \sqrt{a + a \sec(c + dx)}\, dx$$
$$= \frac{\sqrt{a} \arctan \left(\left(\cos (2\, dx + 2\, c)^2 + \sin (2\, dx + 2\, c)^2 + 2\, \cos (2\, dx + 2\, c) + 1 \right)^{\frac{1}{4}} \sin \left(\frac{1}{2} \arctan (\sin (2\, dx + 2\, c)\right.\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + co
s(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(31) = 62$.

Time = 0.31 (sec) , antiderivative size = 130, normalized size of antiderivative = 3.51

$$\int \sqrt{a+a\sec(c+dx)}\,dx = -\frac{\sqrt{-a}a\log\left(\left|\frac{2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a}{2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a}\right|\right)\mathrm{sgn}(\cos(dx+c))}{d|a|}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*d*x
+ 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a
))*sgn(cos(d*x + c))/(d*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+a\sec(c+dx)}\,dx=\sqrt{a}\left(\int \sqrt{\sec\left(dx+c\right)+1}dx\right)$$

input
```
int((a+a*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sec(c + d*x) + 1),x)
```

3.95 $\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx=\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

output
```
a^(1/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+a*sin(d*x+c)/d
/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.27

$$\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$=\frac{\left(\operatorname{arctanh}\left(\sqrt{1-\sec(c+dx)}\right)+\cos(c+dx)\sqrt{1-\sec(c+dx)}\right)\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
((ArcTanh[Sqrt[1 - Sec[c + d*x]]] + Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]])*S
qrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sqrt[1 - Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 4292, 3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4292}$$

$$\frac{1}{2}\int \sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4261}$$

$$\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{a\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*S
in[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(54) = 108.

Time = 2.57 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.11

method	result
default	$\frac{\left((-\cos(dx+c)-1)\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(\cot(dx+c)-\csc(dx+c))}{\sqrt{\csc(dx+c)^{2}-2\cot(dx+c)\csc(dx+c)+\cot(dx+c)^{2}-1}}\right)+2\sin(dx+c)\cos(dx+c)\right)\sqrt{a}}{2d(\cos(dx+c)+1)}$

input
```
int(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*((-cos(d*x+c)-1)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan
h(2^(1/2)*(cot(d*x+c)-csc(d*x+c))/(csc(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+co
t(d*x+c)^2-1)^(1/2))+2*sin(d*x+c)*cos(d*x+c))*(a*(1+sec(d*x+c)))^(1/2)/(co
s(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 242, normalized size of antiderivative = 3.90

$$\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \left[\frac{\sqrt{-a}(\cos(dx+c)+1)\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\sqrt{\frac{a\cos(dx+c}{\cos(dx+}}}{2\,(d\cos(dx+c)+d)}\right.$$

$$\left.-\frac{\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{d\cos(dx+c)+d}\right]$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x
 + c) - a)/(cos(d*x + c) + 1)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*cos(d*x + c)*sin(d*x + c))/(d*cos(d*x + c) + d), -(sqrt(a)*(cos(d*x + c)
+ 1)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*
sin(d*x + c))) - sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(
d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F]

$$\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\cos\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*cos(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 791 vs. $2(54) = 108$.

Time = 0.21 (sec) , antiderivative size = 791, normalized size of antiderivative = 12.76

$$\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x +
c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(54) = 108$.

Time = 0.26 (sec) , antiderivative size = 282, normalized size of antiderivative = 4.55

$$\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-4\sqrt{2}|a|-6\,a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2+4\sqrt{2}|a|-6\,a\right|}\right)}{|a|}+\frac{8\left(3\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\cdots\right.\right.}{\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\cdots}\right.}\right.}{4\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(
sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4
*sqrt(2)*abs(a) - 6*a))/abs(a) + 8*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a - sqrt(-a)*a^2)/((sqrt(-a)*
tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))*
sgn(cos(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx=\int \cos(c+d\,x)\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

input
```
int(cos(c + d*x)*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)*(a + a/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx=\sqrt{a}\left(\int \sqrt{\sec(dx+c)+1}\,\cos(dx+c)\,dx\right)$$

input
```
int(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sec(c + d*x) + 1)*cos(c + d*x),x)
```

3.96 $\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 102

$$\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{3\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{3a\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}$$

output
```
3/4*a^(1/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+3/4*a*sin(
d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x
+c))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.46

$$\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},1-\sec(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

input
```
Integrate[Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*
x])]*Tan[(c + d*x)/2])/d
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4292, 3042, 4292, 3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4292}$$

$$\frac{3}{4}\int \cos(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4292}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4261}$$

$$\frac{3}{4}\left(\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{a\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{216}$$

$$\frac{3}{4}\left(\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(a*Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (3*((Sqrt[a
]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sin[c +
d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 2.55 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.33

method	result
default	$\frac{\left(\sin(dx+c)\cos(dx+c)(3+2\cos(dx+c))-3(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(\cot(dx+c)-\csc(dx+c))}{\sqrt{\csc(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\cot(dx+c)^2}}\right.\right.}{4d(\cos(dx+c)+1)}$

input
```
int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*(sin(d*x+c)*cos(d*x+c)*(3+2*cos(d*x+c))-3*(cos(d*x+c)+1)*(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*arctanh(2^(1/2)*(cot(d*x+c)-csc(d*x+c))/(csc(d*x+c
)^2-2*cot(d*x+c)*csc(d*x+c)+cot(d*x+c)^2-1)^(1/2)))*(a*(1+sec(d*x+c)))^(1/
2)/(cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.65

$$\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \left[\frac{3\sqrt{-a}(\cos(dx+c)+1)\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(2\cos(d\right.}{8\left(d\cos(dx+c)+d\right)}\right.$$
$$-\frac{3\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(2\cos(dx+c)^2+3\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c}{\cos(dx+}}}{4\left(d\cos(dx+c)+d\right)}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(3*sqrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d
*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*cos(d*x + c)^2 + 3*cos(d*x + c))*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d),
 -1/4*(3*sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*cos(d*x + c)^2 + 3*cos(
d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c) + d)]
```

Sympy [F]

$$\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\cos^2\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*cos(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1059 vs. $2(86) = 172$.

Time = 0.22 (sec) , antiderivative size = 1059, normalized size of antiderivative = 10.38

$$\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
2*c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))),...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(86) = 172$.

Time = 0.34 (sec) , antiderivative size = 378, normalized size of antiderivative = 3.71

$$\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = -\frac{\sqrt{2}\left(\frac{3\sqrt{2}\sqrt{-a}a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}-\frac{8\left(5\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\cdots\right.\right.\right.}{\left(\left(\cdots\right.\right.}\right)}{\cdots}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*(3*sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(
2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2
+ 4*sqrt(2)*abs(a) - 6*a))/abs(a) - 8*(5*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(-a)*a + 19*(sqrt(-a)*tan(1/2*d
*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a^2 - 17*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(
-a)*a^3 + sqrt(-a)*a^4)/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)*sgn(cos(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \cos\left(c+d\,x\right)^2\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^2*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^2*(a + a/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \sqrt{a}\left(\int \sqrt{\sec\left(dx+c\right)+1}\,\cos\left(dx+c\right)^2 dx\right)$$

input
```
int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sec(c + d*x) + 1)*cos(c + d*x)**2,x)
```

3.97 $\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 138

$$\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{5\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{5a\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{5a\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}$$

output
```
5/8*a^(1/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+5/8*a*sin(
d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+5/12*a*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*
x+c))^(1/2)+1/3*a*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.34

$$\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},4,\frac{3}{2},1-\sec(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

input
```
Integrate[Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Hypergeometric2F1[1/2, 4, 3/2, 1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*
x])]*Tan[(c + d*x)/2])/d
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4292, 3042, 4292, 3042, 4292, 3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \; 4292$$

$$\frac{5}{6}\int \cos^2(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{5}{6}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$\downarrow$ 4292

$$\frac{5}{6}\left(\frac{3}{4}\int \cos(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$\downarrow$ 3042

$$\frac{5}{6}\left(\frac{3}{4}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$\downarrow$ 4292

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$\downarrow$ 3042

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$\downarrow$ 4261

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{a\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$\downarrow$ 216

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(a*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (5*((a*Co
s[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (3*((Sqrt[a]*Arc
Tan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sin[c + d*x])
/(d*Sqrt[a + a*Sec[c + d*x]])))/4))/6
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.07

method	result
default	$\frac{\left(\sin(dx+c)\cos(dx+c)\left(8\cos(dx+c)^2+10\cos(dx+c)+15\right)+(-15\cos(dx+c)-15)\operatorname{arctanh}\left(\frac{\sqrt{2}\,(\cot(dx+c)-\csc(dx+c))}{\sqrt{\csc(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\cot(}}\right.\right.}{24d(\cos(dx+c)+1)}$

input
```
int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/d*(sin(d*x+c)*cos(d*x+c)*(8*cos(d*x+c)^2+10*cos(d*x+c)+15)+(-15*cos(d
*x+c)-15)*arctanh(2^(1/2)*(cot(d*x+c)-csc(d*x+c))/(csc(d*x+c)^2-2*cot(d*x+
c)*csc(d*x+c)+cot(d*x+c)^2-1)^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(
a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.10

$$\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \left[\frac{15\sqrt{-a}(\cos(dx+c)+1)\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\,(8\cos(}{48\,(d\cos(dx+c)+d)}\right.$$

$$-\frac{15\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(8\cos(dx+c)^3+10\cos(dx+c)^2+15\text{ co}\right.}{24\,(d\cos(dx+c)+d)}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(15*sqrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos
(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*cos(d*x + c)^3 + 10*cos(d*x + c)
^2 + 15*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
)/(d*cos(d*x + c) + d), -1/24*(15*sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (
8*cos(d*x + c)^3 + 10*cos(d*x + c)^2 + 15*cos(d*x + c))*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1921 vs. $2(118) = 236$.

Time = 0.35 (sec) , antiderivative size = 1921, normalized size of antiderivative = 13.92

$$\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/96*(4*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin(3/2*a
rctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 1)^(1/4)*((sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 5*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))) + 3*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) - 4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) +
15*sqrt(a)*(arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(118) = 236$.

Time = 0.37 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.44

$$\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = -\frac{\sqrt{2}\left(\frac{15\sqrt{2}\sqrt{-a}a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}+\frac{8\left(63\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx \cdots\right.\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/96*sqrt(2)*(15*sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2
*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs
(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2
 + 4*sqrt(2)*abs(a) - 6*a))/abs(a) + 8*(63*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*sqrt(-a)*a - 369*(sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*sqrt(-a)*a^2 + 16
38*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6
*sqrt(-a)*a^3 - 1074*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a))^4*sqrt(-a)*a^4 + 171*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a^5 - 13*sqrt(-a)*a^6)/((sqrt
(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a
^2)^3)*sgn(cos(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \cos(c+d\,x)^3\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

input
```
int(cos(c + d*x)^3*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^3*(a + a/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \sqrt{a}\left(\int \sqrt{\sec(dx+c)+1}\,\cos(dx+c)^3\,dx\right)$$

input
```
int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sec(c + d*x) + 1)*cos(c + d*x)**3,x)
```

3.98 $\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 174

$$\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{35\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d} + \frac{35a\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{35a\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{7a\cos^2(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}$$

output
```
35/64*a^(1/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+35/64*a*
sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+35/96*a*cos(d*x+c)*sin(d*x+c)/d/(a+a*s
ec(d*x+c))^(1/2)+7/24*a*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1
/4*a*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.27

$$\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$=\frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},5,\frac{3}{2},1-\sec(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

input
```
Integrate[Cos[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Hypergeometric2F1[1/2, 5, 3/2, 1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*
x])]*Tan[(c + d*x)/2])/d
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3042, 4292, 3042, 4292, 3042, 4292, 3042, 4292, 3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{4292}$$

$$\frac{7}{8}\int \cos^3(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{8}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4292}$$

$$\frac{7}{8}\left(\frac{5}{6}\int \cos^2(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{8}\left(\frac{5}{6}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4292}$$

$$\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \cos(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4292}$$

$$\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos}{3d\sqrt{a\sec(c+}\right.$$
$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+\ldots}{3d\sqrt{a\,\mathrm{se}\ldots}}\right.$$
$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 4261

$$\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{a\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin\ldots}{3d\ldots}\right.$$
$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 216

$$\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\,\mathrm{c}\ldots}{3d\sqrt{a\sec(c\ldots}}\right.$$
$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Cos[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (7*((a*Co
s[c + d*x]^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (5*((a*Cos[c +
 d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (3*((Sqrt[a]*ArcTan[(
Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sin[c + d*x])/(d*S
qrt[a + a*Sec[c + d*x]])))/4))/6))/8
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.90

method	result
default	$\frac{\left(\sin(dx+c)\cos(dx+c)\left(48\cos(dx+c)^3+56\cos(dx+c)^2+70\cos(dx+c)+105\right)-105(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \operatorname{arctanh}\left(\frac{}{\sqrt{\csc(}}\right.\right.}{192d(\cos(dx+c)+1)}$

input
```
int(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/192/d*(sin(d*x+c)*cos(d*x+c)*(48*cos(d*x+c)^3+56*cos(d*x+c)^2+70*cos(d*x
+c)+105)-105*(cos(d*x+c)+1)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(2^(
1/2)*(cot(d*x+c)-csc(d*x+c))/(csc(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+cot(d*x
+c)^2-1)^(1/2)))*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.78

$$\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \left[\frac{105\sqrt{-a}(\cos(dx+c)+1)\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(48\cos\right.}{384\left(d\cos(dx+\right.}\right.$$
$$-\frac{105\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(48\cos(dx+c)^4+56\cos(dx+c)^3+70\right.}{192\left(d\cos(dx+c)+d\right)}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/384*(105*sqrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*c
os(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(48*cos(d*x + c)^4 + 56*cos(d*x +
 c)^3 + 70*cos(d*x + c)^2 + 105*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/192*(105*sqrt(a)*(cos(d
*x + c) + 1)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(
sqrt(a)*sin(d*x + c))) - (48*cos(d*x + c)^4 + 56*cos(d*x + c)^3 + 70*cos(d
*x + c)^2 + 105*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(
d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F]

$$\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\cos^4\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*cos(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6638 vs. $2(150) = 300$.

Time = 0.46 (sec) , antiderivative size = 6638, normalized size of antiderivative = 38.15

$$\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/768*(2*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((36*(sin(4*d*x + 4*c)^3 + (cos
(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*sin(4*d*x + 4*c))*cos(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 9*cos(4*d*x + 4*c)^2*sin(4*d*x
+ 4*c) + 9*sin(4*d*x + 4*c)^3 + 36*(sin(4*d*x + 4*c)^3 + (cos(4*d*x + 4*c)
^2 + 2*cos(4*d*x + 4*c) + 1)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c)))^2 + 9*(2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c)))*sin(4*d*x + 4*c) - 2*(cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c))*cos(3/4*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))) + 36*(sin(4*d*x + 4*c)^3 + (cos(4*d*x
+ 4*c)^2 - cos(4*d*x + 4*c))*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))) - (32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 -
 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
)))^2 + 32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) +
 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*cos(4*d*x +
 4*c)^2 + 2*(16*cos(4*d*x + 4*c)^2 + 16*sin(4*d*x + 4*c)^2 - 7*cos(4*d*x +
 4*c) - 9)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 8*sin(4*
d*x + 4*c)^2 - 2*(64*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*
sin(4*d*x + 4*c) + 7*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c)...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(150) = 300$.

Time = 0.35 (sec) , antiderivative size = 571, normalized size of antiderivative = 3.28

$$\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/768*sqrt(2)*(105*sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1
/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/a
bs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))
^2 + 4*sqrt(2)*abs(a) - 6*a))/abs(a) - 8*(279*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*sqrt(-a)*a + 285*(sqrt(-a)*ta
n(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*sqrt(-a)*a^2
- 4605*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
))^10*sqrt(-a)*a^3 + 37281*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^8*sqrt(-a)*a^4 - 35643*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(-a)*a^5 + 9175*(sqrt(-a)*
tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a^6
 - 1311*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
a))^2*sqrt(-a)*a^7 + 43*sqrt(-a)*a^8)/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^4)*sgn(cos(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \cos(c+d\,x)^4\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

input
```
int(cos(c + d*x)^4*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^4*(a + a/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \sqrt{a}\left(\int \sqrt{\sec(dx+c)+1}\,\cos(dx+c)^4\,dx\right)$$

input
```
int(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sec(c + d*x) + 1)*cos(c + d*x)**4,x)
```

3.99 $\int \sec^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 162

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{68a^2\tan(c+dx)}{45d\sqrt{a+a\sec(c+dx)}} + \frac{34a^2\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sec^4(c+dx)\tan(c+dx)}{9d\sqrt{a+a\sec(c+dx)}} - \frac{136a\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{315d} + \frac{68(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{105d}$$

output
```
68/45*a^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+34/63*a^2*sec(d*x+c)^3*tan(d
*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/9*a^2*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(
d*x+c))^(1/2)-136/315*a*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d+68/105*(a+a*se
c(d*x+c))^(3/2)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.43

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2a^2(272+136\sec(c+dx)+102\sec^2(c+dx)+85\sec^3(c+dx)+35\sec^4(c+dx))\tan(c+dx)}{315d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(2*a^2*(272 + 136*Sec[c + d*x] + 102*Sec[c + d*x]^2 + 85*Sec[c + d*x]^3 +
35*Sec[c + d*x]^4)*Tan[c + d*x])/(315*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4301, 27, 3042, 4290, 3042, 4287, 27, 3042, 4489, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \ 4301$$

$$\frac{2}{9}a\int \frac{17}{2}\sec^4(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{2a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \ 27$$

$$\frac{17}{9}a\int \sec^4(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{2a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{17}{9}a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4290}$$

$$\frac{17}{9}a\left(\frac{6}{7}\int \sec^3(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{17}{9}a\left(\frac{6}{7}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4287}$$

$$\frac{17}{9}a\left(\frac{6}{7}\left(\frac{2\int \frac{1}{2}\sec(c+dx)(3a-2a\sec(c+dx))\sqrt{\sec(c+dx)a+a}dx}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{17}{9}a\left(\frac{6}{7}\left(\frac{\int \sec(c+dx)(3a-2a\sec(c+dx))\sqrt{\sec(c+dx)a+a}dx}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a}{}\right.$$
$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{17}{9}a\left(\frac{6}{7}\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(3a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{5a}+\frac{2\tan(c+dx)(a\sec(c+d}{5ad}\right.\right.$$
$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4489}$$

$$\frac{17}{9}a\left(\frac{6}{7}\left(\frac{\frac{7}{3}a\int \sec(c+dx)\sqrt{\sec(c+dx)a+a}dx-\frac{4a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}\right.\right.$$
$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 3042

$$\frac{17}{9}a\left(\frac{6}{7}\left(\frac{\frac{7}{3}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{4a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)(a\sec(c-}{5ad}\right.\right.$$
$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 4279

$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}+$$
$$\frac{17}{9}a\left(\frac{6}{7}\left(\frac{\frac{14a^2\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}-\frac{4a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\tan(c+dx)}{7d\sqrt{a\sec(c-}\right.$$

input
```
Int[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(2*a^2*Sec[c + d*x]^4*Tan[c + d*x])/(9*d*Sqrt[a + a*Sec[c + d*x]]) + (17*a
*((2*a*Sec[c + d*x]^3*Tan[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]]) + (6*((
2*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*a*d) + ((14*a^2*Tan[c + d*x]
)/(3*d*Sqrt[a + a*Sec[c + d*x]]) - (4*a*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d
*x])/(3*d))/(5*a)))/7))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4287
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 -
b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.51

method	result	size
default	$\frac{2a\left(272\cos(dx+c)^4+136\cos(dx+c)^3+102\cos(dx+c)^2+85\cos(dx+c)+35\right)\sqrt{a(1+\sec(dx+c))}\,\tan(dx+c)\sec(dx+c)^3}{315d(\cos(dx+c)+1)}$	83

input
```
int(sec(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/315/d*a*(272*cos(d*x+c)^4+136*cos(d*x+c)^3+102*cos(d*x+c)^2+85*cos(d*x+c
)+35)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*tan(d*x+c)*sec(d*x+c)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.60

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2\left(272\,a\cos\left(dx+c\right)^4+136\,a\cos\left(dx+c\right)^3+102\,a\cos\left(dx+c\right)^2+85\,a\cos\left(dx+c\right)+35\,a\right)}{315\left(d\cos\left(dx+c\right)^5+d\cos\left(dx+c\right)^4\right)}$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/315*(272*a*cos(d*x + c)^4 + 136*a*cos(d*x + c)^3 + 102*a*cos(d*x + c)^2
+ 85*a*cos(d*x + c) + 35*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*
x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F]

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a(\sec(c+dx)+1))^{\frac{3}{2}}\sec^4(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**4*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)*sec(c + d*x)**4, x)
```

Maxima [F]

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
16/315*(315*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*((a*d*cos(2*d*x + 2*c)^4 + a*d*sin(2*d*x + 2*c)^4 + 4*a*d*cos(2
*d*x + 2*c)^3 + 6*a*d*cos(2*d*x + 2*c)^2 + 4*a*d*cos(2*d*x + 2*c) + 2*(a*d
*cos(2*d*x + 2*c)^2 + 2*a*d*cos(2*d*x + 2*c) + a*d)*sin(2*d*x + 2*c)^2 + a
*d)*integrate((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*(((cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*c
os(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)
*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*
c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x +
2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(7/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(10*d*x
 + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*
d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(
2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*si
n(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(7/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2
*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x +
 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x
 + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(...
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.11

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{4\left(315\sqrt{2}a^6\mathrm{sgn}(\cos(dx+c)) - \left(525\sqrt{2}a^6\mathrm{sgn}(\cos(dx+c)) - \left(819\sqrt{2}a^6\mathrm{sgn}(\cos(dx+c))\right.\right.\right.}{3}$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
4/315*(315*sqrt(2)*a^6*sgn(cos(d*x + c)) - (525*sqrt(2)*a^6*sgn(cos(d*x +
c)) - (819*sqrt(2)*a^6*sgn(cos(d*x + c)) + 47*(2*sqrt(2)*a^6*sgn(cos(d*x +
 c))*tan(1/2*d*x + 1/2*c)^2 - 9*sqrt(2)*a^6*sgn(cos(d*x + c)))*tan(1/2*d*x
 + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x +
 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a)*d)
```

Mupad [B] (verification not implemented)

Time = 15.28 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.65

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{\left(\frac{a\,32\mathrm{i}}{9\,d}-\frac{a\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,32\mathrm{i}}{9\,d}\right)\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^4} - \frac{\left(\frac{a\,80\mathrm{i}}{7\,d}-\frac{a\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,176\mathrm{i}}{63\,d}\right)\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^3} + \frac{\left(\frac{a\,48\mathrm{i}}{5\,d}+\frac{a\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,352\mathrm{i}}{105\,d}\right)\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^2} - \frac{a\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}\,544\mathrm{i}}{315\,d\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)} - \frac{a\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}\,272\mathrm{i}}{315\,d\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}$$

input
```
int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^4,x)
```

output
```
(((a*32i)/(9*d) - (a*exp(c*1i + d*x*1i)*32i)/(9*d))*(a + a/(exp(- c*1i - d
*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2))/((exp(c*1i + d*x*1i) + 1)*(exp(c*
2i + d*x*2i) + 1)^4) - (((a*80i)/(7*d) - (a*exp(c*1i + d*x*1i)*176i)/(63*d
))*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2))/((exp(c*
1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^3) + (((a*48i)/(5*d) + (a*exp(c
*1i + d*x*1i)*352i)/(105*d))*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d
*x*1i)/2))^(1/2))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2) -
(a*exp(c*1i + d*x*1i)*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/
2))^(1/2)*544i)/(315*d*(exp(c*1i + d*x*1i) + 1)) - (a*exp(c*1i + d*x*1i)*(
a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*272i)/(315*d*
(exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1))
```

Reduce [F]

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx=\sqrt{a}\,a\left(\int \sqrt{\sec(dx+c)+1}\,\sec(dx+c)^5\,dx+\int \sqrt{\sec(dx+c)+1}\,\sec(dx+c)^4\,dx\right)$$

input
```
int(sec(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sec(c + d*x) + 1)*sec(c + d*x)**5,x) + int(sqrt(sec(c
+ d*x) + 1)*sec(c + d*x)**4,x))
```

3.100 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 116

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{152a^2\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{38a\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{105d} - \frac{4(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{35d} + \frac{2(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{7ad}$$

output
```
152/105*a^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+38/105*a*(a+a*sec(d*x+c))^
(1/2)*tan(d*x+c)/d-4/35*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d+2/7*(a+a*sec(d
*x+c))^(5/2)*tan(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2a^2(104+52\sec(c+dx)+39\sec^2(c+dx)+15\sec^3(c+dx))\tan(c+dx)}{105d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(2*a^2*(104 + 52*Sec[c + d*x] + 39*Sec[c + d*x]^2 + 15*Sec[c + d*x]^3)*Tan
[c + d*x])/(105*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4287, 27, 3042, 4489, 3042, 4280, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4287}$$

$$\frac{2\int \frac{1}{2}\sec(c+dx)(5a-2a\sec(c+dx))(\sec(c+dx)a+a)^{3/2}dx}{7a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sec(c+dx)(5a-2a\sec(c+dx))(\sec(c+dx)a+a)^{3/2}dx}{7a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(5a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx}{7a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7ad}$$

$$\Big\downarrow \text{4489}$$

$$\frac{\frac{19}{5}a\int \sec(c+dx)(\sec(c+dx)a+a)^{3/2}dx-\frac{4a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}}{7a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7ad}$$

$\downarrow$ 3042

$$\frac{\frac{19}{5}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx-\frac{4a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}}{7a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7ad}$$

$\downarrow$ 4280

$$\frac{\frac{19}{5}a\left(\frac{4}{3}a\int \sec(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)-\frac{4a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}}{7a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7ad}$$

$\downarrow$ 3042

$$\frac{\frac{19}{5}a\left(\frac{4}{3}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)-\frac{4a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}}{7a}$$
$$\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7ad}$$

$\downarrow$ 4279

$$\frac{\frac{19}{5}a\left(\frac{8a^2\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)-\frac{4a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}}{7a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7ad}$$

input
```
Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(2*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*a*d) + ((-4*a*(a + a*Sec[c
+ d*x])^(3/2)*Tan[c + d*x])/(5*d) + (19*a*((8*a^2*Tan[c + d*x])/(3*d*Sqrt[
a + a*Sec[c + d*x]]) + (2*a*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*d)))
/5)/(7*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4280
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Simp[a*((2*m - 1)/m)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
 x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Intege
rQ[2*m]
```

rule 4287
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 -
b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{2a\left(104\cos(dx+c)^{3}+52\cos(dx+c)^{2}+39\cos(dx+c)+15\right)\sqrt{a(1+\sec(dx+c))}\,\tan(dx+c)\sec(dx+c)^{2}}{105d(\cos(dx+c)+1)}$	73

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/105/d*a*(104*cos(d*x+c)^3+52*cos(d*x+c)^2+39*cos(d*x+c)+15)*(a*(1+sec(d*
x+c)))^(1/2)/(cos(d*x+c)+1)*tan(d*x+c)*sec(d*x+c)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.75

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2\left(104\,a\cos\left(dx+c\right)^{3}+52\,a\cos\left(dx+c\right)^{2}+39\,a\cos\left(dx+c\right)+15\,a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+\right.}{105\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/105*(104*a*cos(d*x + c)^3 + 52*a*cos(d*x + c)^2 + 39*a*cos(d*x + c) + 15
*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4
 + d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a(\sec(c+dx)+1))^{\frac{3}{2}}\sec^3(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)*sec(c + d*x)**3, x)
```

Maxima [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
8/105*(105*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(3/4)*(3*(a*d*cos(2*d*x + 2*c)^2 + a*d*sin(2*d*x + 2*c)^2 + 2*a*d*cos(
2*d*x + 2*c) + a*d)*integrate((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 3*cos(
6*d*x + 6*c)*cos(2*d*x + 2*c) + 3*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(
2*d*x + 2*c)^2 + sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 3*sin(6*d*x + 6*c)*si
n(2*d*x + 2*c) + 3*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)
*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*
sin(8*d*x + 8*c) + 3*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 3*cos(2*d*x + 2*c
)*sin(4*d*x + 4*c) - cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 3*cos(6*d*x + 6*c
)*sin(2*d*x + 2*c) - 3*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(5/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 3*cos(2*d*x +
 2*c)*sin(6*d*x + 6*c) + 3*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(8*d*x +
 8*c)*sin(2*d*x + 2*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 3*cos(4*d*x
 + 4*c)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - (cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 3*cos(6*d*x + 6*c)*cos(2*d*x +
 2*c) + 3*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(8*d
*x + 8*c)*sin(2*d*x + 2*c) + 3*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 3*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(5/2*arctan2(sin(...
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.30

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = -\frac{4\left(105\sqrt{2}a^5\mathrm{sgn}(\cos(dx+c)) - \left(140\sqrt{2}a^5\mathrm{sgn}(\cos(dx+c)) + 19\left(2\sqrt{2}a^5\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\right.\right.\right.\right.}{105\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^3\sqrt{-a\,t}}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-4/105*(105*sqrt(2)*a^5*sgn(cos(d*x + c)) - (140*sqrt(2)*a^5*sgn(cos(d*x +
 c)) + 19*(2*sqrt(2)*a^5*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 - 7*sqrt
(2)*a^5*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)
*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a)*d)
```

Mupad [B] (verification not implemented)

Time = 13.28 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.98

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = -\frac{\left(\frac{a\,16\mathrm{i}}{7\,d}+\frac{a\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,16\mathrm{i}}{7\,d}\right)\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^3} + \frac{\left(\frac{a\,8\mathrm{i}}{3\,d}-\frac{a\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,104\mathrm{i}}{105\,d}\right)\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} + \frac{\left(\frac{a\,8\mathrm{i}}{5\,d}+\frac{a\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,184\mathrm{i}}{35\,d}\right)\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^2} - \frac{a\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}\,208\mathrm{i}}{105\,d\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)}$$

input
```
int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^3,x)
```

output
```
(((a*8i)/(3*d) - (a*exp(c*1i + d*x*1i)*104i)/(105*d))*(a + a/(exp(- c*1i -
 d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2))/((exp(c*1i + d*x*1i) + 1)*(exp(
c*2i + d*x*2i) + 1)) - (((a*16i)/(7*d) + (a*exp(c*1i + d*x*1i)*16i)/(7*d))
*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2))/((exp(c*1i
 + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^3) + (((a*8i)/(5*d) + (a*exp(c*1i
 + d*x*1i)*184i)/(35*d))*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1
i)/2))^(1/2))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2) - (a*e
xp(c*1i + d*x*1i)*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^
(1/2)*208i)/(105*d*(exp(c*1i + d*x*1i) + 1))
```

Reduce [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \sqrt{a}\,a\left(\int \sqrt{\sec(dx+c)+1}\,\sec(dx+c)^4\,dx + \int \sqrt{\sec(dx+c)+1}\,\sec(dx+c)^3\,dx\right)$$

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sec(c + d*x) + 1)*sec(c + d*x)**4,x) + int(sqrt(sec(c
+ d*x) + 1)*sec(c + d*x)**3,x))
```

3.101 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{8a^2\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2a\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{5d} + \frac{2(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5d}$$

output
```
8/5*a^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/5*a*(a+a*sec(d*x+c))^(1/2)*t
an(d*x+c)/d+2/5*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.56

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2a^2(6+3\sec(c+dx)+\sec^2(c+dx))\tan(c+dx)}{5d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(2*a^2*(6 + 3*Sec[c + d*x] + Sec[c + d*x]^2)*Tan[c + d*x])/(5*d*Sqrt[a*(1
+ Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4285, 3042, 4280, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{ 4285}$$

$$\frac{3}{5}\int \sec(c+dx)(\sec(c+dx)a+a)^{3/2}dx+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{ 4280}$$

$$\frac{3}{5}\left(\frac{4}{3}a\int \sec(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3}{5}\left(\frac{4}{3}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{ 4279}$$

$$\frac{3}{5}\left(\frac{8a^2\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

input
```
Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(2*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d) + (3*((8*a^2*Tan[c + d*x
])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Sqrt[a + a*Sec[c + d*x]]*Tan[c +
d*x])/(3*d)))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4280
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Simp[a*((2*m - 1)/m)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
 x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Intege
rQ[2*m]
```

rule 4285
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[a*(m/(b*(m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x]
/; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.71

method	result	size
default	$\frac{(12\sin(dx+c)+6\tan(dx+c)+2\sec(dx+c)\tan(dx+c))a\sqrt{a(1+\sec(dx+c))}}{d(5\cos(dx+c)+5)}$	61

input
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(12*sin(d*x+c)+6*tan(d*x+c)+2*sec(d*x+c)*tan(d*x+c))/(5*cos(d*x+c)+5)*
a*(a*(1+sec(d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2\left(6\,a\cos\left(dx+c\right)^2+3\,a\cos\left(dx+c\right)+a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{5\left(d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2\right)}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/5*(6*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx=\int (a(\sec{(c+dx)}+1))^{\frac{3}{2}}\sec^2{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)*sec(c + d*x)**2, x)
```

Maxima [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx=\int (a\sec{(dx+c)}+a)^{\frac{3}{2}}\sec{(dx+c)}^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
4/5*(5*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*((a*d*cos(2*d*x + 2*c)^2 + a*d*sin(2*d*x + 2*c)^2 + 2*a*d*cos(2*d*x
+ 2*c) + a*d)*integrate((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*(((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x
+ 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x
+ 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(5/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(6*d*
x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*
x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(5/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(
4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(
2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(
6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(
2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))
/((cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 4*(cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)^...
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.41

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{4\left(5\sqrt{2}a^4\mathrm{sgn}(\cos(dx+c))+\left(2\sqrt{2}a^4\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-5\sqrt{2}a^4\mathrm{sgn}(\cos(\right.\right.}{5\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
4/5*(5*sqrt(2)*a^4*sgn(cos(d*x + c)) + (2*sqrt(2)*a^4*sgn(cos(d*x + c))*ta
n(1/2*d*x + 1/2*c)^2 - 5*sqrt(2)*a^4*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*
c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a)*d)
```

Mupad [B] (verification not implemented)

Time = 12.99 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.35

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{4\,a\,\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,5\mathrm{i}-\mathrm{e}^{c\,3\mathrm{i}+d\,x\,3\mathrm{i}}\,5\mathrm{i}-\mathrm{e}^{c\,5\mathrm{i}+d\,x\,5\mathrm{i}}\,3\mathrm{i}+3\mathrm{i}\right)}{5\,d\,\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}$$

input
```
int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^2,x)
```

output
```
(4*a*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*
2i + d*x*2i)*5i - exp(c*3i + d*x*3i)*5i - exp(c*5i + d*x*5i)*3i + 3i))/(5*
d*(exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2)
```

Reduce [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \sqrt{a}\,a\left(\int \sqrt{\sec(dx+c)+1}\,\sec(dx+c)^3\,dx + \int \sqrt{\sec(dx+c)+1}\,\sec(dx+c)^2\,dx\right)$$

input
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sec(c + d*x) + 1)*sec(c + d*x)**3,x) + int(sqrt(sec(c
+ d*x) + 1)*sec(c + d*x)**2,x))
```

3.102 $\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{8a^2\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2a\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d}$$

output
```
8/3*a^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a*(a+a*sec(d*x+c))^(1/2)*t
an(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.64

$$\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2a^2(5+\sec(c+dx))\tan(c+dx)}{3d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(2*a^2*(5 + Sec[c + d*x])*Tan[c + d*x])/(3*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4280, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4280}$$

$$\frac{4}{3}a\int \sec(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{4279}$$

$$\frac{8a^2\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

input
```
Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(8*a^2*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Sqrt[a + a*Sec[
c + d*x]]*Tan[c + d*x])/(3*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4280
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Simp[a*((2*m - 1)/m)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
 x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Intege
rQ[2*m]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{(10\sin(dx+c)+2\tan(dx+c))a\sqrt{a(1+\sec(dx+c))}}{d(3\cos(dx+c)+3)}$	47

input
```
int(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(10*sin(d*x+c)+2*tan(d*x+c))/(3*cos(d*x+c)+3)*a*(a*(1+sec(d*x+c)))^(1/
2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2\,(5\,a\cos(dx+c)+a)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{3\,\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/3*(5*a*cos(d*x + c) + a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
 + c)/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a(\sec(c+dx)+1))^{\frac{3}{2}}\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.58

$$\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{4\left(2\sqrt{2}a^3\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3\sqrt{2}a^3\mathrm{sgn}(\cos(dx+c))\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{3\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}d}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
4/3*(2*sqrt(2)*a^3*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 - 3*sqrt(2)*a^
3*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

Mupad [B] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.88

$$\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2\,a\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}\,(12\,\sin(c+d\,x)+14\,\sin(2\,c+2\,d\,x)+12\,\sin(3\,c+3\,d\,x)+5\,\sin(4\,c+}{3\,d\,(12\,\cos(c+d\,x)+8\,\cos(2\,c+2\,d\,x)+4\,\cos(3\,c+3\,d\,x)+\cos(4\,c+4\,d\,x)+7)}$$

input
```
int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x),x)
```

output
```
(2*a*((a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2)*(12*sin(c + d*x) + 14*sin
(2*c + 2*d*x) + 12*sin(3*c + 3*d*x) + 5*sin(4*c + 4*d*x)))/(3*d*(12*cos(c
+ d*x) + 8*cos(2*c + 2*d*x) + 4*cos(3*c + 3*d*x) + cos(4*c + 4*d*x) + 7))
```

Reduce [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\,dx=\sqrt{a}\,a\left(\int \sqrt{\sec(dx+c)+1}\,\sec(dx+c)^2\,dx+\int \sqrt{\sec(dx+c)+1}\,\sec(dx+c)\,dx\right)$$

input
```
int(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sec(c + d*x) + 1)*sec(c + d*x)**2,x) + int(sqrt(sec(c
+ d*x) + 1)*sec(c + d*x),x))
```

3.103 $\int (a+a\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 66

$$\int (a+a\sec(c+dx))^{3/2}\,dx=\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

output
```
2*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2*a^2*tan(d*
x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

$$\int (a+a\sec(c+dx))^{3/2}\,dx=\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2\sin\right.}{d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*ArcSin[Sqrt[2]*Sin
[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*Sin[(c + d*x)/2]))/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4262, 27, 3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4262}$$

$$2a\int \frac{1}{2}\sqrt{\sec(c+dx)a+a}dx+\frac{2a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$a\int \sqrt{\sec(c+dx)a+a}dx+\frac{2a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4261}$$

$$\frac{2a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{2a^2\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a^2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4262
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[a/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Inte
gerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(58) = 116.

Time = 2.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.88

method	result
default	$-\frac{a\sqrt{a(1+\sec(dx+c))}\left(\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}\,(\cos(dx+c)+1)\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,(\cot(dx+c)-\csc(dx+c))}{\sqrt{\csc(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\cot(dx+c)^2-1}}\right)-2\sin(dx$ … $}{d(\cos(dx+c)+1)}$

input
```
int((a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*a*(a*(1+sec(d*x+c)))^(1/2)*(2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)+1)*arctanh(2^(1/2)*(cot(d*x+c)-csc(d*x+c))/(csc(d*x+c)^2-2
*cot(d*x+c)*csc(d*x+c)+cot(d*x+c)^2-1)^(1/2))-2*sin(d*x+c))/(cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.56

$$\int (a+a\sec(c+dx))^{3/2}\,dx = \left[\frac{(a\cos(dx+c)+a)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right) - \cdots}{d\cos(dx+c)+d}\right.$$

$$\left.-\frac{2\left((a\cos(dx+c)+a)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)\right)}{d\cos(dx+c)+d}\right]$$

input
```
integrate((a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[((a*cos(d*x + c) + a)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x +
 c) - a)/(cos(d*x + c) + 1)) + 2*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sin(d*x + c))/(d*cos(d*x + c) + d), -2*((a*cos(d*x + c) + a)*sqrt(a)*arct
an(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x +
 c))) - a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c) + d)]
```

Sympy [F]

$$\int (a+a\sec(c+dx))^{3/2}\,dx=\int (a\sec{(c+dx)}+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a*sec(c + d*x) + a)**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 997 vs. $2(58) = 116$.

Time $= 0.21$ (sec) , antiderivative size $= 997$, normalized size of antiderivative $= 15.11$

$$\int (a+a\sec(c+dx))^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/2*((a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c),...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(58) = 116$.

Time = 0.39 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.95

$$\int (a + a\sec(c + dx))^{3/2}\, dx =$$

$$-\frac{\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\,a^2\operatorname{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a} + \frac{\sqrt{-a}a^2\log\left(\left|\frac{2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-4\sqrt{2}|a|-6\,a}{2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2+4\sqrt{2}|a|-6\,a}\right.\right.}{|a|}}{d}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-(2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^2*sgn(cos(d*x + c))*tan(
1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a) + sqrt(-a)*a^2*log(abs(2*(
sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4
*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))*sgn(cos(d*x + c))/ab
s(a))/d
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^{3/2}\,dx=\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

input
```
int((a + a/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int (a+a\sec(c+dx))^{3/2}\,dx=\sqrt{a}\,a\left(\int \sqrt{\sec\left(dx+c\right)+1}dx\right.$$
$$\left.+\int \sqrt{\sec\left(dx+c\right)+1}\,\sec\left(dx+c\right)dx\right)$$

input
```
int((a+a*sec(d*x+c))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sec(c + d*x) + 1),x) + int(sqrt(sec(c + d*x) + 1)*sec(
c + d*x),x))
```

3.104 $\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{3a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{a^2\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

output
```
3*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+a^2*sin(d*x+
c)/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.26

$$\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a\left(3\text{arctanh}\left(\sqrt{1-\sec(c+dx)}\right)+\cos(c+dx)\sqrt{1-\sec(c+dx)}\right)\sqrt{a(1+\sec(c+dx))}\tan}{d\sqrt{1-\sec(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(a*(3*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]
])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sqrt[1 - Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.45, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 4301, 27, 3042, 4292, 3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4301}$$

$$-2a\int -\frac{3}{2}\cos(c+dx)\sqrt{\sec(c+dx)a+a}dx-\frac{2a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$3a\int \cos(c+dx)\sqrt{\sec(c+dx)a+a}dx-\frac{2a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$3a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{2a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4292}$$

$$3a\left(\frac{1}{2}\int \sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)-\frac{2a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$3a\left(\frac{1}{2}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)-\frac{2a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4261}$$

$$3a\left(\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{a\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)-\frac{2a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{216}$$

$$3a\left(\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)-\frac{2a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*a^2*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + 3*a*((Sqrt[a]*ArcTan[
(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sin[c + d*x])/(d*
Sqrt[a + a*Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(57) = 114$.

Time = 2.98 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.03

method	result
default	$\frac{a\left((3\cos(dx+c)+3)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{\sqrt{\csc(dx+c)^{2}-2\cot(dx+c)\csc(dx+c)+\cot(dx+c)^{2}-1}}\right)\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}+2\sin(dx+c)\cos(dx+c)\right)\sqrt{a}}{2d(\cos(dx+c)+1)}$

input
```
int(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*a*((3*cos(d*x+c)+3)*2^(1/2)*arctanh(2^(1/2)/(csc(d*x+c)^2-2*cot(d*x+
c)*csc(d*x+c)+cot(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c)))*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)+2*sin(d*x+c)*cos(d*x+c))*(a*(1+sec(d*x+c)))^(1/2)
/(cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 248, normalized size of antiderivative = 3.82

$$\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left[\frac{2\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3\,(a\cos(dx+c)+a)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{\ldots}}{\ldots}\right.}{2\,(d\cos(dx+c)+d)}\right.$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c
) + 3*(a*cos(d*x + c) + a)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d
*x + c) - a)/(cos(d*x + c) + 1)))/(d*cos(d*x + c) + d), (a*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*(a*cos(d*x + c) + a
)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqr
t(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)]
```

Sympy [F]

$$\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a(\sec(c+dx)+1))^{\frac{3}{2}}\cos(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)*cos(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 803 vs. $2(57) = 114$.

Time = 0.24 (sec) , antiderivative size = 803, normalized size of antiderivative = 12.35

$$\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/4*(2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
 + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(57) = 114$.

Time = 0.39 (sec) , antiderivative size = 278, normalized size of antiderivative = 4.28

$$\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\,dx =$$

$$-\frac{\sqrt{2}\sqrt{-a}a^3\left(\frac{3\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-4\sqrt{2}|a|-6\,a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2+4\sqrt{2}|a|-6\,a\right|}\right)}{a|a|}+\frac{8\left(3\left(\sqrt{-a}\tan\left(\frac{1}{2}\right.\right.\right.}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}\right.\right.}\right.}{4\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*sqrt(-a)*a^3*(3*sqrt(2)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2
*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs
(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2
 + 4*sqrt(2)*abs(a) - 6*a))/(a*abs(a)) + 8*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)/((((sqrt(-a)*tan(1/2*d*x +
 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x
 + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)*a))*sgn(cos(d*
x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int \cos\left(c+d\,x\right)\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

input
```
int(cos(c + d*x)*(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)*(a + a/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \sqrt{a}\,a\left(\int \sqrt{\sec(dx+c)+1}\,\cos(dx+c)\sec(dx+c)\,dx + \int \sqrt{\sec(dx+c)+1}\,\cos(dx+c)\,dx\right)$$

input
```
int(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sec(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x),x) + int(s
qrt(sec(c + d*x) + 1)*cos(c + d*x),x))
```

3.105 $\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 106

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{7a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{7a^2\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}$$

output
```
7/4*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+7/4*a^2*si
n(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a^2*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec
(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.02

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a\cos(c+dx)\sqrt{a(1+\sec(c+dx))}\left(\sqrt{1-\sec(c+dx)}(7\sin(c+dx)+\sin(2(c+dx)))+7\mathrm{ar}\right.}{4d(1+\cos(c+dx))\sqrt{1-\sec(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(a*Cos[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[1 - Sec[c + d*x]]*(7*Sin[
c + d*x] + Sin[2*(c + d*x)]) + 7*ArcTanh[Sqrt[1 - Sec[c + d*x]]]*Tan[c + d
*x]))/(4*d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4300, 27, 3042, 4292, 3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4300}$$

$$\frac{1}{2}a\int \frac{7}{2}\cos(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{7}{4}a\int \cos(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4292}$$

$$\frac{7}{4}a\left(\frac{1}{2}\int \sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}a\left(\frac{1}{2}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4261}$$

$$\frac{7}{4}a\left(\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{a\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{216}$$

$$\frac{a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}+\frac{7}{4}a\left(\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

input
```
Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (7*a*((Sq
rt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sin[
c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 3.23 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.29

method	result
default	$\frac{a\left(\sin(dx+c)\cos(dx+c)(2\cos(dx+c)+7)+7(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{\sqrt{\csc(dx+c)^{2}-2\cot(dx+c)\csc(dx+c)+\cot(dx+c)}}\right.\right.}{4d(\cos(dx+c)+1)}$

input
```
int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*a*(sin(d*x+c)*cos(d*x+c)*(2*cos(d*x+c)+7)+7*(cos(d*x+c)+1)*(-cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*arctanh(2^(1/2)/(csc(d*x+c)^2-2*cot(d*x+c)*csc(d
*x+c)+cot(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c))))*(a*(1+sec(d*x+c)))^
(1/2)/(cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.62

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left[\frac{7\,(a\cos(dx+c)+a)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{8\,(d\cos(dx+c)+d)}\right.$$

$$-\frac{7\,(a\cos(dx+c)+a)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(2\,a\cos(dx+c)^2+7\,a\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{4\,(d\cos(dx+c)+d)}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(7*(a*cos(d*x + c) + a)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos
(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*a*cos(d*x + c)^2 + 7*a*cos(d*x +
 c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)
 + d), -1/4*(7*(a*cos(d*x + c) + a)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*a*cos(d*x + c)^
2 + 7*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
)/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{%%{[469762048,0]:[1,0,-2]%%},[14]%%%},0]:[1,0,%%%{
1,[1]%%%}
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int \cos(c+d\,x)^2\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

input
```
int(cos(c + d*x)^2*(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^2*(a + a/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \sqrt{a}\,a\left(\int \sqrt{\sec(dx+c)+1}\,\cos(dx+c)^2\sec(dx+c)\,dx + \int \sqrt{\sec(dx+c)+1}\,\cos(dx+c)^2\,dx\right)$$

input
```
int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sec(c + d*x) + 1)*cos(c + d*x)**2*sec(c + d*x),x) + in
t(sqrt(sec(c + d*x) + 1)*cos(c + d*x)**2,x))
```

3.106 $\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 144

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{11a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{11a^2\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{11a^2\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}$$

output
```
11/8*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+11/8*a^2*
sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+11/12*a^2*cos(d*x+c)*sin(d*x+c)/d/(a+a
*sec(d*x+c))^(1/2)+1/3*a^2*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2
)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.83

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a\cos(c+dx)\sqrt{a(1+\sec(c+dx))}\left(\sqrt{1-\sec(c+dx)}(35\sin(c+dx)+11\sin(2(c+dx))+2\sin(3(c+dx)))+33\operatorname{arctanh}\left(\sqrt{1-\sec(c+dx)}\right)\tan(c+dx)\right)}{24d(1+\cos(c+dx))\sqrt{1-\sec(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(a*Cos[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[1 - Sec[c + d*x]]*(35*Sin
[c + d*x] + 11*Sin[2*(c + d*x)] + 2*Sin[3*(c + d*x)]) + 33*ArcTanh[Sqrt[1
- Sec[c + d*x]]]*Tan[c + d*x]))/(24*d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c +
d*x]])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4300, 27, 3042, 4292, 3042, 4292, 3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{ 4300}$$

$$\frac{1}{3}a\int \frac{11}{2}\cos^2(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{11}{6}a\int \cos^2(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11}{6}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4292}$$

$$\frac{11}{6}a\left(\frac{3}{4}\int \cos(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11}{6}a\left(\frac{3}{4}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4292}$$

$$\frac{11}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int \sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4261}$$

$$\frac{11}{6}a\left(\frac{3}{4}\left(\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{a\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$\downarrow$ 216

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{11}{6}a\left(\frac{3}{4}\left(\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)$$

input
```
Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(a^2*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (11*a*(
(a*Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (3*((Sqrt[a
]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sin[c +
d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 3.24 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.03

method	result
default	$\frac{a\left(\sin(dx+c)\cos(dx+c)\left(8\cos(dx+c)^2+22\cos(dx+c)+33\right)+(-33\cos(dx+c)-33)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \operatorname{arctanh}\left(\frac{\sqrt{2}\,(\cot(d}{\sqrt{\csc(dx+c)^2-2\cot(dx}}\right.\right.}{24d(\cos(dx+c)+1)}$

input
```
int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/d*a*(sin(d*x+c)*cos(d*x+c)*(8*cos(d*x+c)^2+22*cos(d*x+c)+33)+(-33*cos
(d*x+c)-33)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(2^(1/2)*(cot(d*x+c)
-csc(d*x+c))/(csc(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+cot(d*x+c)^2-1)^(1/2))
*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.08

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left[\frac{33\,(a\cos(dx+c)+a)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right.}{48\,(d\cos(d}\right.$$

$$-\frac{33\,(a\cos(dx+c)+a)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(8\,a\cos(dx+c)^3+22\,a\cos(dx+c)^2+33\,a\right.}{24\,(d\cos(dx+c)+d)}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/48*(33*(a*cos(d*x + c) + a)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*c
os(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*a*cos(d*x + c)^3 + 22*a*cos(d*
x + c)^2 + 33*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(
d*x + c))/(d*cos(d*x + c) + d), -1/24*(33*(a*cos(d*x + c) + a)*sqrt(a)*arc
tan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x
+ c))) - (8*a*cos(d*x + c)^3 + 22*a*cos(d*x + c)^2 + 33*a*cos(d*x + c))*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(124) = 248$.

Time = 0.66 (sec) , antiderivative size = 535, normalized size of antiderivative = 3.72

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-1/48*(33*sqrt(-a)*a*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))*sgn(cos(d*x + c)) - 33*sq
rt(-a)*a*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))*sgn(cos(d*x + c)) + 4*(33*sqrt(2)*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*sqrt
(-a)*a^2*sgn(cos(d*x + c)) - 303*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 23
94*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
 + a))^6*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 1806*sqrt(2)*(sqrt(-a)*tan(1/2*d
*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a^5*sgn(cos(
d*x + c)) + 309*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^2*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 19*sqrt(2)*sqrt(-a)
*a^7*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int \cos\left(c+d\,x\right)^3\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

input
```
int(cos(c + d*x)^3*(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^3*(a + a/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \sqrt{a}\,a\left(\int \sqrt{\sec\left(dx+c\right)+1}\,\cos\left(dx+c\right)^3\sec\left(dx+c\right)dx + \int \sqrt{\sec\left(dx+c\right)+1}\,\cos\left(dx+c\right)^3dx\right)$$

input
```
int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sec(c + d*x) + 1)*cos(c + d*x)**3*sec(c + d*x),x) + in
t(sqrt(sec(c + d*x) + 1)*cos(c + d*x)**3,x))
```

3.107 $\int \sec^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 203

$$\begin{aligned}\int \sec^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx &= \frac{284a^3\tan(c+dx)}{99d\sqrt{a+a\sec(c+dx)}} \\ &+ \frac{710a^3\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}} + \frac{46a^3\sec^4(c+dx)\tan(c+dx)}{99d\sqrt{a+a\sec(c+dx)}} \\ &- \frac{568a^2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{693d} \\ &+ \frac{2a^2\sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{11d} \\ &+ \frac{284a(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{231d}\end{aligned}$$

output
```
284/99*a^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+710/693*a^3*sec(d*x+c)^3*ta
n(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+46/99*a^3*sec(d*x+c)^4*tan(d*x+c)/d/(a+a
*sec(d*x+c))^(1/2)-568/693*a^2*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d+2/11*a^
2*sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d+284/231*a*(a+a*sec(d*x+
c))^(3/2)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.39

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a^3(1136+568\sec(c+dx)+426\sec^2(c+dx)+355\sec^3(c+dx)+224\sec^4(c+dx)+63\sec^5(c+dx))\tan(c+dx)}{693d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(2*a^3*(1136 + 568*Sec[c + d*x] + 426*Sec[c + d*x]^2 + 355*Sec[c + d*x]^3
+ 224*Sec[c + d*x]^4 + 63*Sec[c + d*x]^5)*Tan[c + d*x])/(693*d*Sqrt[a*(1 +
 Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4301, 27, 3042, 4504, 3042, 4290, 3042, 4287, 27, 3042, 4489, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \; 4301$$

$$\frac{2}{11}a\int \frac{1}{2}\sec^4(c+dx)\sqrt{\sec(c+dx)a+a}(23\sec(c+dx)a+19a)dx+\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{11}a\int \sec^4(c+dx)\sqrt{\sec(c+dx)a+a}(23\sec(c+dx)a+19a)dx+\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{11}a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(23\csc\left(c+dx+\frac{\pi}{2}\right)a+19a\right)dx+\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}$$

$$\Big\downarrow \; 4504$$

$$\frac{1}{11}a\left(\frac{355}{9}a\int \sec^4(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{46a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{11}a\left(\frac{355}{9}a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{46a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}$$

$$\Big\downarrow \; 4290$$

$$\frac{1}{11}a\left(\frac{355}{9}a\left(\frac{6}{7}\int \sec^3(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}\right)+\frac{46a^2\tan(c+dx)\sec^4(c+}{9d\sqrt{a\sec(c+dx)+}}\right.\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{11}a\left(\frac{355}{9}a\left(\frac{6}{7}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}\right)+\frac{46a^2\tan(c}{9d\sqrt{a\,\mathrm{se}}}\right.\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}$$

$$\Big\downarrow \; 4287$$

$$\frac{1}{11}a\left(\frac{355}{9}a\left(\frac{6}{7}\left(\frac{2\int\frac{1}{2}\sec(c+dx)(3a-2a\sec(c+dx))\sqrt{\sec(c+dx)a+a}dx}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)}{5ad}\right.\right.\right.$$
$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}$$

$\Big\downarrow$ 27

$$\frac{1}{11}a\left(\frac{355}{9}a\left(\frac{6}{7}\left(\frac{\int\sec(c+dx)(3a-2a\sec(c+dx))\sqrt{\sec(c+dx)a+a}dx}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}\right.\right.\right.$$
$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}$$

$\Big\downarrow$ 3042

$$\frac{1}{11}a\left(\frac{355}{9}a\left(\frac{6}{7}\left(\frac{\int\csc\left(c+dx+\frac{\pi}{2}\right)\left(3a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{5a}+\frac{2\tan(c+dx)(a\sec}{5a}\right.\right.\right.$$
$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}$$

$\Big\downarrow$ 4489

$$\frac{1}{11}a\left(\frac{355}{9}a\left(\frac{6}{7}\left(\frac{\frac{7}{3}a\int\sec(c+dx)\sqrt{\sec(c+dx)a+a}dx-\frac{4a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx}{5ad}\right.\right.\right.$$
$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}$$

$\Big\downarrow$ 3042

$$\frac{1}{11}a\left(\frac{355}{9}a\left(\frac{6}{7}\left(\frac{\frac{7}{3}a\int\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{4a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)(}{}\right.\right.\right.$$
$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}$$

$\Big\downarrow$ 4279

$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}+$$
$$\frac{1}{11}a\left(\frac{46a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}+\frac{355}{9}a\left(\frac{6}{7}\left(\frac{\frac{14a^2\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}-\frac{4a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)(a}{}\right.\right.\right.$$

input
```
Int[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(2*a^2*Sec[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(11*d) + (a*(
(46*a^2*Sec[c + d*x]^4*Tan[c + d*x])/(9*d*Sqrt[a + a*Sec[c + d*x]]) + (355
*a*((2*a*Sec[c + d*x]^3*Tan[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]]) + (6*
((2*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*a*d) + ((14*a^2*Tan[c + d*
x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) - (4*a*Sqrt[a + a*Sec[c + d*x]]*Tan[c +
 d*x])/(3*d))/(5*a)))/7))/9))/11
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4287
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 -
b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

rule 4504
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Simp[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1))   Int[Sqrt[a + b*Csc[e + f*
x]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&
!LtQ[n, 0]
```

Maple [A] (verified)

Time = 47.62 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.47

method	result
default	$\frac{2a^2\left(1136\cos(dx+c)^5+568\cos(dx+c)^4+426\cos(dx+c)^3+355\cos(dx+c)^2+224\cos(dx+c)+63\right)\sqrt{a(1+\sec(dx+c))}\ \tan(dx+c)\sec(}{693d(\cos(dx+c)+1)}$

input
```
int(sec(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/693/d*a^2*(1136*cos(d*x+c)^5+568*cos(d*x+c)^4+426*cos(d*x+c)^3+355*cos(d
*x+c)^2+224*cos(d*x+c)+63)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*tan(d*x
+c)*sec(d*x+c)^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.60

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2\left(1136\,a^2\cos\left(dx+c\right)^5+568\,a^2\cos\left(dx+c\right)^4+426\,a^2\cos\left(dx+c\right)^3+355\,a^2\cos\left(dx+c\right)^2+\right.}{693\left(d\cos\left(dx+c\right)^6+d\cos\left(dx+c\right)^5\right)}$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/693*(1136*a^2*cos(d*x + c)^5 + 568*a^2*cos(d*x + c)^4 + 426*a^2*cos(d*x
+ c)^3 + 355*a^2*cos(d*x + c)^2 + 224*a^2*cos(d*x + c) + 63*a^2)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^6 + d*cos(d*x
+ c)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**4*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.03

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx =$$
$$-\frac{8\left(693\sqrt{2}a^8\mathrm{sgn}(\cos(dx+c)) - \left(1617\sqrt{2}a^8\mathrm{sgn}(\cos(dx+c)) - \left(3003\sqrt{2}a^8\mathrm{sgn}(\cos(dx+c)) - 25\left(99\right.\right.\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-8/693*(693*sqrt(2)*a^8*sgn(cos(d*x + c)) - (1617*sqrt(2)*a^8*sgn(cos(d*x
+ c)) - (3003*sqrt(2)*a^8*sgn(cos(d*x + c)) - 25*(99*sqrt(2)*a^8*sgn(cos(d
*x + c)) + 4*(2*sqrt(2)*a^8*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 - 11*
sqrt(2)*a^8*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c
)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/
((a*tan(1/2*d*x + 1/2*c)^2 - a)^5*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

Mupad [B] (verification not implemented)

Time = 20.16 (sec) , antiderivative size = 542, normalized size of antiderivative = 2.67

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^4,x)
```

output
```
((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*64i)/
(11*d) + (a^2*exp(c*1i + d*x*1i)*64i)/(11*d)))/((exp(c*1i + d*x*1i) + 1)*(
exp(c*2i + d*x*2i) + 1)^5) + ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i +
d*x*1i)/2))^(1/2)*((a^2*16i)/d + (a^2*exp(c*1i + d*x*1i)*640i)/(231*d)))/(
(exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2) - ((a + a/(exp(- c*1i
 - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*64i)/(9*d) + (a^2*exp(c*
1i + d*x*1i)*2176i)/(99*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i)
 + 1)^4) - ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*
((a^2*80i)/(7*d) - (a^2*exp(c*1i + d*x*1i)*12688i)/(693*d)))/((exp(c*1i +
d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^3) - (a^2*exp(c*1i + d*x*1i)*(a + a/
(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*2272i)/(693*d*(exp(
c*1i + d*x*1i) + 1)) - (a^2*exp(c*1i + d*x*1i)*(a + a/(exp(- c*1i - d*x*1i
)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*1136i)/(693*d*(exp(c*1i + d*x*1i) + 1)*
(exp(c*2i + d*x*2i) + 1))
```

Reduce [F]

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \sqrt{a}\,a^2\left(\int \sqrt{\sec(dx+c)+1}\,\sec(dx+c)^6\,dx + 2\left(\int \sqrt{\sec(dx+c)+1}\,\sec(dx+c)^5\,dx\right) + \int \sqrt{\sec(dx+c)+1}\,\sec(dx+c)^4\,dx\right)$$

input
```
int(sec(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sec(c + d*x) + 1)*sec(c + d*x)**6,x) + 2*int(sqrt(s
ec(c + d*x) + 1)*sec(c + d*x)**5,x) + int(sqrt(sec(c + d*x) + 1)*sec(c + d
*x)**4,x))
```

3.108 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 146

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx=\frac{832a^3\tan(c+dx)}{315d\sqrt{a+a\sec(c+dx)}}+\frac{208a^2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{315d}+\frac{26a(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{105d}-\frac{4(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{63d}+\frac{2(a+a\sec(c+dx))^{7/2}\tan(c+dx)}{9ad}$$

output
```
832/315*a^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+208/315*a^2*(a+a*sec(d*x+c
))^(1/2)*tan(d*x+c)/d+26/105*a*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d-4/63*(a
+a*sec(d*x+c))^(5/2)*tan(d*x+c)/d+2/9*(a+a*sec(d*x+c))^(7/2)*tan(d*x+c)/a/
d
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.48

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx=\frac{2a^3(584+292\sec(c+dx)+219\sec^2(c+dx)+130\sec^3(c+dx)+35\sec^4(c+dx))\tan(c+}{315d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(2*a^3*(584 + 292*Sec[c + d*x] + 219*Sec[c + d*x]^2 + 130*Sec[c + d*x]^3 +
 35*Sec[c + d*x]^4)*Tan[c + d*x])/(315*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4287, 27, 3042, 4489, 3042, 4280, 3042, 4280, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4287}$$

$$\frac{2\int \frac{1}{2}\sec(c+dx)(7a-2a\sec(c+dx))(\sec(c+dx)a+a)^{5/2}dx}{9a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{7/2}}{9ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sec(c+dx)(7a-2a\sec(c+dx))(\sec(c+dx)a+a)^{5/2}dx}{9a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{7/2}}{9ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(7a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}dx}{9a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{7/2}}{9ad}$$

$$\Big\downarrow \text{4489}$$

$$\frac{\frac{39}{7}a\int \sec(c+dx)(\sec(c+dx)a+a)^{5/2}dx-\frac{4a\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d}}{9a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{7/2}}{9ad}$$

$\big\downarrow$ 3042

$$\frac{\frac{39}{7}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}dx-\frac{4a\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d}}{9a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{7/2}}{9ad}$$

$\big\downarrow$ 4280

$$\frac{\frac{39}{7}a\left(\frac{8}{5}a\int \sec(c+dx)(\sec(c+dx)a+a)^{3/2}dx+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}\right)-\frac{4a\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d}}{9a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{7/2}}{9ad}$$

$\big\downarrow$ 3042

$$\frac{\frac{39}{7}a\left(\frac{8}{5}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}\right)-\frac{4a\tan(c+dx)(a\sec(c+dx)-}{7d}}{9a}$$
$$\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{7/2}}{9ad}$$

$\big\downarrow$ 4280

$$\frac{\frac{39}{7}a\left(\frac{8}{5}a\left(\frac{4}{3}a\int \sec(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}\right)-4a}{9a}$$
$$\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{7/2}}{9ad}$$

$\big\downarrow$ 3042

$$\frac{\frac{39}{7}a\left(\frac{8}{5}a\left(\frac{4}{3}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)+\frac{2a\tan(c+dx)(a\sec(c+dx)+}{5d}}{9a}$$
$$\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{7/2}}{9ad}$$

$\big\downarrow$ 4279

$$\frac{\frac{39}{7}a\left(\frac{8}{5}a\left(\frac{8a^2\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}\right)-\frac{4a\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d}}{9a}$$

$$\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{7/2}}{9ad}$$

input
```
Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(2*(a + a*Sec[c + d*x])^(7/2)*Tan[c + d*x])/(9*a*d) + ((-4*a*(a + a*Sec[c
+ d*x])^(5/2)*Tan[c + d*x])/(7*d) + (39*a*((2*a*(a + a*Sec[c + d*x])^(3/2)
*Tan[c + d*x])/(5*d) + (8*a*((8*a^2*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c +
d*x]]) + (2*a*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*d)))/5))/7)/(9*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4280
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Simp[a*((2*m - 1)/m)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
 x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Intege
rQ[2*m]
```

rule 4287
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 -
b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 17.39 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{2a^{2}\left(584\cos\left(dx+c\right)^{4}+292\cos\left(dx+c\right)^{3}+219\cos\left(dx+c\right)^{2}+130\cos\left(dx+c\right)+35\right)\sqrt{a\left(1+\sec\left(dx+c\right)\right)}\,\tan\left(dx+c\right)\sec\left(dx+c\right)^{3}}{315d\left(\cos\left(dx+c\right)+1\right)}$	85

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/315/d*a^2*(584*cos(d*x+c)^4+292*cos(d*x+c)^3+219*cos(d*x+c)^2+130*cos(d*
x+c)+35)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*tan(d*x+c)*sec(d*x+c)^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.74

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2\left(584\,a^2\cos\left(dx+c\right)^4+292\,a^2\cos\left(dx+c\right)^3+219\,a^2\cos\left(dx+c\right)^2+130\,a^2\cos\left(dx+c\right)+3\right.}{315\left(d\cos\left(dx+c\right)^5+d\cos\left(dx+c\right)^4\right)}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/315*(584*a^2*cos(d*x + c)^4 + 292*a^2*cos(d*x + c)^3 + 219*a^2*cos(d*x +
 c)^2 + 130*a^2*cos(d*x + c) + 35*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
8/315*(315*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*(5*(a^2*d*cos(2*d*x + 2*c)^4 + a^2*d*sin(2*d*x + 2*c)^4 + 4*a^2*
d*cos(2*d*x + 2*c)^3 + 6*a^2*d*cos(2*d*x + 2*c)^2 + 4*a^2*d*cos(2*d*x + 2*
c) + a^2*d + 2*(a^2*d*cos(2*d*x + 2*c)^2 + 2*a^2*d*cos(2*d*x + 2*c) + a^2*
d)*sin(2*d*x + 2*c)^2)*integrate((((cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 3*
cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 3*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) +
cos(2*d*x + 2*c)^2 + sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 3*sin(6*d*x + 6*c
)*sin(2*d*x + 2*c) + 3*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c
)^2)*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2
*c)*sin(8*d*x + 8*c) + 3*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 3*cos(2*d*x +
 2*c)*sin(4*d*x + 4*c) - cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 3*cos(6*d*x +
 6*c)*sin(2*d*x + 2*c) - 3*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(7/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 3*cos(2*d
*x + 2*c)*sin(6*d*x + 6*c) + 3*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(8*d
*x + 8*c)*sin(2*d*x + 2*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 3*cos(4
*d*x + 4*c)*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - (cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 3*cos(6*d*x + 6*c)*cos(2*d
*x + 2*c) + 3*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin
(8*d*x + 8*c)*sin(2*d*x + 2*c) + 3*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + ...
```

Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.23

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{8\left(315\sqrt{2}a^7\mathrm{sgn}(\cos(dx+c)) - \left(630\sqrt{2}a^7\mathrm{sgn}(\cos(dx+c)) - 13\left(63\sqrt{2}a^7\mathrm{sgn}(\cos(dx+c)\right.\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
8/315*(315*sqrt(2)*a^7*sgn(cos(d*x + c)) - (630*sqrt(2)*a^7*sgn(cos(d*x +
c)) - 13*(63*sqrt(2)*a^7*sgn(cos(d*x + c)) + 4*(2*sqrt(2)*a^7*sgn(cos(d*x
+ c))*tan(1/2*d*x + 1/2*c)^2 - 9*sqrt(2)*a^7*sgn(cos(d*x + c)))*tan(1/2*d*
x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x
+ 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a)*d)
```

Mupad [B] (verification not implemented)

Time = 16.73 (sec) , antiderivative size = 456, normalized size of antiderivative = 3.12

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^2\,32i}{9\,d}-\frac{a^2\,e^{c\,1i+d\,x\,1i}\,32i}{9\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\,\left(e^{c\,2i+d\,x\,2i}+1\right)^4} - \frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^2\,96i}{7\,d}-\frac{a^2\,e^{c\,1i+d\,x\,1i}\,32i}{63\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\,\left(e^{c\,2i+d\,x\,2i}+1\right)^3} + \frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^2\,8i}{3\,d}-\frac{a^2\,e^{c\,1i+d\,x\,1i}\,584i}{315\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\,\left(e^{c\,2i+d\,x\,2i}+1\right)} + \frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^2\,56i}{5\,d}+\frac{a^2\,e^{c\,1i+d\,x\,1i}\,904i}{105\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\,\left(e^{c\,2i+d\,x\,2i}+1\right)^2} - \frac{a^2\,e^{c\,1i+d\,x\,1i}\,\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,1168i}{315\,d\,\left(e^{c\,1i+d\,x\,1i}+1\right)}$$

input
```
int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^3,x)
```

output
```
((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*32i)/
(9*d) - (a^2*exp(c*1i + d*x*1i)*32i)/(9*d)))/((exp(c*1i + d*x*1i) + 1)*(ex
p(c*2i + d*x*2i) + 1)^4) - ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*
x*1i)/2))^(1/2)*((a^2*96i)/(7*d) - (a^2*exp(c*1i + d*x*1i)*32i)/(63*d)))/(
(exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^3) + ((a + a/(exp(- c*1i
 - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*8i)/(3*d) - (a^2*exp(c*1
i + d*x*1i)*584i)/(315*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i)
+ 1)) + ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a
^2*56i)/(5*d) + (a^2*exp(c*1i + d*x*1i)*904i)/(105*d)))/((exp(c*1i + d*x*1
i) + 1)*(exp(c*2i + d*x*2i) + 1)^2) - (a^2*exp(c*1i + d*x*1i)*(a + a/(exp(
- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*1168i)/(315*d*(exp(c*1i
+ d*x*1i) + 1))
```

Reduce [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \sqrt{a}\,a^2\left(\int \sqrt{\sec(dx+c)+1}\,\sec(dx+c)^5\,dx + 2\left(\int \sqrt{\sec(dx+c)+1}\,\sec(dx+c)^4\,dx\right) + \int \sqrt{\sec(dx+c)+1}\,\sec(dx+c)^3\,dx\right)$$

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sec(c + d*x) + 1)*sec(c + d*x)**5,x) + 2*int(sqrt(s
ec(c + d*x) + 1)*sec(c + d*x)**4,x) + int(sqrt(sec(c + d*x) + 1)*sec(c + d
*x)**3,x))
```

3.109 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 116

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{64a^3\tan(c+dx)}{21d\sqrt{a+a\sec(c+dx)}} + \frac{16a^2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{21d} + \frac{2a(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{7d} + \frac{2(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{7d}$$

output
```
64/21*a^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+16/21*a^2*(a+a*sec(d*x+c))^(
1/2)*tan(d*x+c)/d+2/7*a*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d+2/7*(a+a*sec(d
*x+c))^(5/2)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a^3(46+23\sec(c+dx)+12\sec^2(c+dx)+3\sec^3(c+dx))\tan(c+dx)}{21d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(2*a^3*(46 + 23*Sec[c + d*x] + 12*Sec[c + d*x]^2 + 3*Sec[c + d*x]^3)*Tan[c
 + d*x])/(21*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4285, 3042, 4280, 3042, 4280, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4285}$$

$$\frac{5}{7}\int \sec(c+dx)(\sec(c+dx)a+a)^{5/2}dx+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}dx+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \text{4280}$$

$$\frac{5}{7}\left(\frac{8}{5}a\int \sec(c+dx)(\sec(c+dx)a+a)^{3/2}dx+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}\right)+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}\left(\frac{8}{5}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}\right)+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d}$$

$\Big\downarrow$ 4280

$$\frac{5}{7}\left(\frac{8}{5}a\left(\frac{4}{3}a\int \sec(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)+\frac{2a\tan(c+dx)(a\sec(c}{5d}\right.$$
$$\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d}$$

$\Big\downarrow$ 3042

$$\frac{5}{7}\left(\frac{8}{5}a\left(\frac{4}{3}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)+\frac{2a\tan(c-}{}\right.$$
$$\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d}$$

$\Big\downarrow$ 4279

$$\frac{5}{7}\left(\frac{8}{5}a\left(\frac{8a^2\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}\right)-$$
$$\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d}$$

input
```
Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(2*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*d) + (5*((2*a*(a + a*Sec[c
+ d*x])^(3/2)*Tan[c + d*x])/(5*d) + (8*a*((8*a^2*Tan[c + d*x])/(3*d*Sqrt[a
 + a*Sec[c + d*x]]) + (2*a*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*d)))/
5))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4280
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Simp[a*((2*m - 1)/m)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
 x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Intege
rQ[2*m]
```

rule 4285
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[a*(m/(b*(m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x]
/; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 6.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{2a^2\left(46\cos(dx+c)^3+23\cos(dx+c)^2+12\cos(dx+c)+3\right)\sqrt{a(1+\sec(dx+c))}\,\tan(dx+c)\sec(dx+c)^2}{21d(\cos(dx+c)+1)}$	75

input
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/d*a^2*(46*cos(d*x+c)^3+23*cos(d*x+c)^2+12*cos(d*x+c)+3)*(a*(1+sec(d*x
+c)))^(1/2)/(cos(d*x+c)+1)*tan(d*x+c)*sec(d*x+c)^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.82

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2\,\left(46\,a^2\cos(dx+c)^3+23\,a^2\cos(dx+c)^2+12\,a^2\cos(dx+c)+3\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sin(dx}{21\,\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/21*(46*a^2*cos(d*x + c)^3 + 23*a^2*cos(d*x + c)^2 + 12*a^2*cos(d*x + c)
+ 3*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x +
 c)^4 + d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a(\sec(c+dx)+1))^{\frac{5}{2}}\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(5/2)*sec(c + d*x)**2, x)
```

Maxima [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
4/21*(21*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(3/4)*((a^2*d*cos(2*d*x + 2*c)^2 + a^2*d*sin(2*d*x + 2*c)^2 + 2*a^2*d*co
s(2*d*x + 2*c) + a^2*d)*integrate((((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2
*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)
*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)
^2)*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*
c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*
c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(7/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x
+ 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x
+ 4*c)*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - (cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x +
2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(7/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)))/(((cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 4*(
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*
x + 4*c)^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2...
```

Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.30

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx =$$

$$-\frac{8\left(21\sqrt{2}a^6\mathrm{sgn}(\cos(dx+c))-\left(35\sqrt{2}a^6\mathrm{sgn}(\cos(dx+c))+4\left(2\sqrt{2}a^6\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right.\right.\right.}{21\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^3\sqrt{-a\tan}}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-8/21*(21*sqrt(2)*a^6*sgn(cos(d*x + c)) - (35*sqrt(2)*a^6*sgn(cos(d*x + c)
) + 4*(2*sqrt(2)*a^6*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 - 7*sqrt(2)*
a^6*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan
(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a)*d)
```

Mupad [B] (verification not implemented)

Time = 13.22 (sec) , antiderivative size = 349, normalized size of antiderivative = 3.01

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^2\,20i}{3\,d}-\frac{a^2\,e^{c\,1i+d\,x\,1i}\,4i}{21\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)} - \frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^2\,16i}{7\,d}+\frac{a^2\,e^{c\,1i+d\,x\,1i}\,16i}{7\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)^3} - \frac{a^2\,e^{c\,1i+d\,x\,1i}\,\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,92i}{21\,d\,\left(e^{c\,1i+d\,x\,1i}+1\right)} + \frac{a^2\,e^{c\,1i+d\,x\,1i}\,\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,48i}{7\,d\,\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)^2}$$

input
```
int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^2,x)
```

output
```
((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*20i)/
(3*d) - (a^2*exp(c*1i + d*x*1i)*4i)/(21*d)))/((exp(c*1i + d*x*1i) + 1)*(ex
p(c*2i + d*x*2i) + 1)) - ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*
1i)/2))^(1/2)*((a^2*16i)/(7*d) + (a^2*exp(c*1i + d*x*1i)*16i)/(7*d)))/((ex
p(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^3) - (a^2*exp(c*1i + d*x*1i
)*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*92i)/(21*d
*(exp(c*1i + d*x*1i) + 1)) + (a^2*exp(c*1i + d*x*1i)*(a + a/(exp(- c*1i -
d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*48i)/(7*d*(exp(c*1i + d*x*1i) + 1
)*(exp(c*2i + d*x*2i) + 1)^2)
```

Reduce [F]

$$
\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \sqrt{a}\,a^2\left(\int \sqrt{\sec(dx+c)+1}\,\sec(dx+c)^4\,dx + 2\left(\int \sqrt{\sec(dx+c)+1}\,\sec(dx+c)^3\,dx\right) + \int \sqrt{\sec(dx+c)+1}\,\sec(dx+c)^2\,dx\right)
$$

input
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sec(c + d*x) + 1)*sec(c + d*x)**4,x) + 2*int(sqrt(s
ec(c + d*x) + 1)*sec(c + d*x)**3,x) + int(sqrt(sec(c + d*x) + 1)*sec(c + d
*x)**2,x))
```

3.110 $\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{64a^3\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{16a^2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15d} + \frac{2a(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5d}$$

output
```
64/15*a^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+16/15*a^2*(a+a*sec(d*x+c))^(
1/2)*tan(d*x+c)/d+2/5*a*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.56

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a^3(43+14\sec(c+dx)+3\sec^2(c+dx))\tan(c+dx)}{15d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(2*a^3*(43 + 14*Sec[c + d*x] + 3*Sec[c + d*x]^2)*Tan[c + d*x])/(15*d*Sqrt[
a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4280, 3042, 4280, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\sec(c+dx)+a)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4280}$$

$$\frac{8}{5}a\int \sec(c+dx)(\sec(c+dx)a+a)^{3/2}dx+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{4280}$$

$$\frac{8}{5}a\left(\frac{4}{3}a\int \sec(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}a\left(\frac{4}{3}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow 4279$$

$$\frac{8}{5}a\left(\frac{8a^2\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

input
```
Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(2*a*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d) + (8*a*((8*a^2*Tan[c +
 d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Sqrt[a + a*Sec[c + d*x]]*Tan[
c + d*x])/(3*d)))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4280
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Simp[a*((2*m - 1)/m)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
 x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Intege
rQ[2*m]
```

Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.71

method	result	size
default	$\frac{(86\sin(dx+c)+28\tan(dx+c)+6\sec(dx+c)\tan(dx+c))a^2\sqrt{a(1+\sec(dx+c))}}{d(15\cos(dx+c)+15)}$	63

input
```
int(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(86*sin(d*x+c)+28*tan(d*x+c)+6*sec(d*x+c)*tan(d*x+c))/(15*cos(d*x+c)+1
5)*a^2*(a*(1+sec(d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2\left(43\,a^2\cos\left(dx+c\right)^2+14\,a^2\cos\left(dx+c\right)+3\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2\right)}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/15*(43*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 3*a^2)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2
)
```

Sympy [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\,dx=\int (a(\sec{(c+dx)}+1))^{\frac{5}{2}}\sec{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(5/2)*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\,dx=\int (a\sec{(dx+c)}+a)^{\frac{5}{2}}\sec{(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.37

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\,dx=\frac{8\left(15\sqrt{2}a^5\mathrm{sgn}(\cos{(dx+c)})+4\left(2\sqrt{2}a^5\mathrm{sgn}(\cos{(dx+c)})\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}^2-5\sqrt{2}a^5\mathrm{sgn}(\mathrm{co}\right.\right.}{15\left(a\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}^2-a\right)^2\sqrt{-a\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
8/15*(15*sqrt(2)*a^5*sgn(cos(d*x + c)) + 4*(2*sqrt(2)*a^5*sgn(cos(d*x + c)
)*tan(1/2*d*x + 1/2*c)^2 - 5*sqrt(2)*a^5*sgn(cos(d*x + c)))*tan(1/2*d*x +
1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a)*d)
```

Mupad [B] (verification not implemented)

Time = 13.15 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.64

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\,dx =$$
$$-\frac{2\,a^2\,\sqrt{a+\frac{a}{\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}}\,\left({\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}\,15{}\mathrm{i}-{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,70{}\mathrm{i}+{\mathrm{e}}^{c\,3{}\mathrm{i}+d\,x\,3{}\mathrm{i}}\,70{}\mathrm{i}-{\mathrm{e}}^{c\,4{}\mathrm{i}+d\,x\,4{}\mathrm{i}}\,15{}\mathrm{i}+{\mathrm{e}}^{c\,5{}\mathrm{i}+d\,x\,5{}\mathrm{i}}\,43{}\mathrm{i}-}{15\,d\,\left({\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}+1\right)\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^2}$$

input
```
int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x),x)
```

output
```
-(2*a^2*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp
(c*1i + d*x*1i)*15i - exp(c*2i + d*x*2i)*70i + exp(c*3i + d*x*3i)*70i - ex
p(c*4i + d*x*4i)*15i + exp(c*5i + d*x*5i)*43i - 43i))/(15*d*(exp(c*1i + d*
x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2)
```

Reduce [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \sqrt{a}\,a^2\left(\left(\int \sqrt{\sec(dx+c)+1}\,\sec(dx+c)^3\,dx\right)+2\left(\int \sqrt{\sec(dx+c)+1}\,\sec(dx+c)^2\,dx\right)+\int \sqrt{\sec(dx+c)+1}\,\sec(dx+c)\,dx\right)$$

input
```
int(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sec(c + d*x) + 1)*sec(c + d*x)**3,x) + 2*int(sqrt(s
ec(c + d*x) + 1)*sec(c + d*x)**2,x) + int(sqrt(sec(c + d*x) + 1)*sec(c + d
*x),x))
```

3.111 $\int (a+a\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 98

$$\int (a+a\sec(c+dx))^{5/2}\,dx=\frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{14a^3\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d}$$

output
```
2*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+14/3*a^3*tan
(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a^2*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)
/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.93 (sec) , antiderivative size = 360, normalized size of antiderivative = 3.67

$$\int (a+a\sec(c+dx))^{5/2}\,dx=\frac{\csc^3\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)(a(1+\sec(c+dx)))^{5/2}\sqrt{\frac{1}{1-2\sin^2\left(\frac{1}{2}(c+dx)\right)}}\sqrt{1-2\sin^2\left(\frac{1}{2}(c\right.}}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(Csc[(c + d*x)/2]^3*Sec[(c + d*x)/2]^5*(a*(1 + Sec[c + d*x]))^(5/2)*Sqrt[(
1 - 2*Sin[(c + d*x)/2]^2)^(-1)]*Sqrt[1 - 2*Sin[(c + d*x)/2]^2]*(256*Cos[(c
 + d*x)/2]^4*HypergeometricPFQ[{3/2, 2, 7/2}, {1, 9/2}, 2*Sin[(c + d*x)/2]
^2]*Sin[(c + d*x)/2]^6 + 512*Hypergeometric2F1[3/2, 7/2, 9/2, 2*Sin[(c + d
*x)/2]^2]*Sin[(c + d*x)/2]^6*(2 - 3*Sin[(c + d*x)/2]^2 + Sin[(c + d*x)/2]^
4) + (21*Sqrt[2]*ArcSin[Sqrt[2]*Sqrt[Sin[(c + d*x)/2]^2]]*(15 - 10*Sin[(c
+ d*x)/2]^2 + 3*Sin[(c + d*x)/2]^4))/Sqrt[Sin[(c + d*x)/2]^2] - 14*Sqrt[1
- 2*Sin[(c + d*x)/2]^2]*(45 + 30*Sin[(c + d*x)/2]^2 - 31*Sin[(c + d*x)/2]^
4 + 12*Sin[(c + d*x)/2]^6)))/(672*d*Sec[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3042, 4262, 27, 3042, 4403, 3042, 4261, 216, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4262}$$

$$\frac{2}{3}a\int \frac{1}{2}\sqrt{\sec(c+dx)a+a}(7\sec(c+dx)a+3a)dx+\frac{2a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}a\int \sqrt{\sec(c+dx)a+a}(7\sec(c+dx)a+3a)dx+\frac{2a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(7\csc\left(c+dx+\frac{\pi}{2}\right)a+3a\right)dx+\frac{2a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$\big\downarrow$ 4403

$$\frac{1}{3}a\left(3a\int\sqrt{\sec(c+dx)a+a}dx+7a\int\sec(c+dx)\sqrt{\sec(c+dx)a+a}dx\right)+\frac{2a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$\big\downarrow$ 3042

$$\frac{1}{3}a\left(3a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+7a\int\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx\right)+\frac{2a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$\big\downarrow$ 4261

$$\frac{1}{3}a\left(7a\int\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{6a^2\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{2a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$\big\downarrow$ 216

$$\frac{1}{3}a\left(7a\int\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{6a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}\right)+\frac{2a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$\big\downarrow$ 4279

$$\frac{2a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}+\frac{1}{3}a\left(\frac{6a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{14a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

input

```
Int[(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(2*a^2*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*d) + (a*((6*a^(3/2)*ArcTa
n[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (14*a^2*Tan[c + d*
x])/(d*Sqrt[a + a*Sec[c + d*x]])))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4262
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[a/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Inte
gerQ[2*n]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4403
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Simp[c   Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Si
mp[d   Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.38

method	result
default	$\frac{a^2\sqrt{a(1+\sec(dx+c))}\left(16\sin(dx+c)+2\tan(dx+c)-3\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}(\cos(dx+c)+1)\operatorname{arctanh}\left(\frac{\sqrt{2}(\cot(dx+c)-\csc(d}{\sqrt{\csc(dx+c)^2-2\cot(dx+c)\csc(dx-}}\right.\right.}{3d(\cos(dx+c)+1)}$

input
```
int((a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(16*sin(d*x+c)+2*tan(d*x
+c)-3*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1)*arctanh(
2^(1/2)*(cot(d*x+c)-csc(d*x+c))/(csc(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+cot(
d*x+c)^2-1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 310, normalized size of antiderivative = 3.16

$$\int (a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{3\left(a^2\cos(dx+c)^2+a^2\cos(dx+c)\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+}{\cos(dx+c)+1}\right.}{3\left(d\cos(dx+c)^2+d\cos(d\right.}\right.$$

$$-\frac{2\left(3\left(a^2\cos(dx+c)^2+a^2\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(8a^2\cos(dx+c)+a^2\right)\sqrt{\frac{a}{\ }}\right.}{3\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqrt(-a)*log((2*a*cos(d*x
+ c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*s
in(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*a^2*cos(d*x +
 c) + a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*
x + c)^2 + d*cos(d*x + c)), -2/3*(3*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c)
)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqr
t(a)*sin(d*x + c))) - (8*a^2*cos(d*x + c) + a^2)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F]

$$\int (a + a\sec(c + dx))^{5/2} \, dx = \int (a\sec{(c + dx)} + a)^{\frac{5}{2}} \, dx$$

input
```
integrate((a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral((a*sec(c + d*x) + a)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1395 vs. $2(84) = 168$.

Time = 0.23 (sec) , antiderivative size = 1395, normalized size of antiderivative = 14.23

$$\int (a + a\sec(c + dx))^{5/2} \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/6*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*((12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
 2*c) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3
*a^2*cos(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(
a) + 3*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x
 + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(84) = 168$.

Time = 0.48 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.30

$$\int (a+a\sec(c+dx))^{5/2}\,dx =$$

$$-\frac{\frac{3\sqrt{-a}a^3\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}(\cos(dx+c))}{|a|}-\frac{2\left(7\sqrt{2}a^4\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\cdots\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{\cdots}}}{3d}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-1/3*(3*sqrt(-a)*a^3*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan
(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs
(a) - 6*a))*sgn(cos(d*x + c))/abs(a) - 2*(7*sqrt(2)*a^4*sgn(cos(d*x + c))*
tan(1/2*d*x + 1/2*c)^2 - 9*sqrt(2)*a^4*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/
2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))
/d
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^{5/2}\,dx=\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}\,dx$$

input
```
int((a + a/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int (a+a\sec(c+dx))^{5/2}\,dx=\sqrt{a}\,a^2\left(\int\sqrt{\sec\left(dx+c\right)+1}dx+\int\sqrt{\sec\left(dx+c\right)+1}\,\sec\left(dx+c\right)^2dx+2\left(\int\sqrt{\sec\left(dx+c\right)+1}\,\sec\left(dx+c\right)dx\right)\right)$$

input
```
int((a+a*sec(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sec(c + d*x) + 1),x) + int(sqrt(sec(c + d*x) + 1)*s
ec(c + d*x)**2,x) + 2*int(sqrt(sec(c + d*x) + 1)*sec(c + d*x),x))
```

3.112 $\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 94

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{5a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{a^3\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
5*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d-a^3*sin(d*x+
c)/d/(a+a*sec(d*x+c))^(1/2)+2*a^2*(a+a*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.87

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^3\left(5\text{arctanh}\left(\sqrt{1-\sec(c+dx)}\right)+(2+\cos(c+dx))\sqrt{1-\sec(c+dx)}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^3*(5*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + (2 + Cos[c + d*x])*Sqrt[1 - Sec[
c + d*x]])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x
])])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4301, 27, 3042, 4503, 3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\sec(c+dx)+a)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4301}$$

$$2a\int -\frac{1}{2}\cos(c+dx)(a-3a\sec(c+dx))\sqrt{\sec(c+dx)a+a}dx+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}-a\int \cos(c+dx)(a-3a\sec(c+dx))\sqrt{\sec(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}-a\int \frac{\left(a-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4503}$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}-a\left(\frac{a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{5}{2}a\int \sqrt{\sec(c+dx)a+a}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}-a\left(\frac{a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{5}{2}a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx\right)$$

$$\Big\downarrow \text{4261}$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}-a\left(\frac{5a^2\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}-a\left(\frac{a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{5a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}\right)$$

input
```
Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(2*a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/d - a*((-5*a^(3/2)*ArcTan[(S
qrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a^2*Sin[c + d*x])/(d*
Sqrt[a + a*Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4503
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Simp
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n)   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[
e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a
*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Maple [A] (verified)

Time = 10.62 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.46

method	result
default	$-\frac{a^{2}\left(\left(-2\cos\left(dx+c\right)-4\right)\sin\left(dx+c\right)+\left(5\cos\left(dx+c\right)+5\right)\sqrt{2}\,\sqrt{-\frac{2\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,\left(\cot\left(dx+c\right)-\csc\left(dx+c\right)\right)}{\sqrt{\csc\left(dx+c\right)^{2}-2\cot\left(dx+c\right)\csc\left(dx+c\right)+\cot\left(dx+c\right)^{2}}}\right.\right.}{2d\left(\cos\left(dx+c\right)+1\right)}$

input
```
int(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*a^2*((-2*cos(d*x+c)-4)*sin(d*x+c)+(5*cos(d*x+c)+5)*2^(1/2)*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(2^(1/2)*(cot(d*x+c)-csc(d*x+c))/(csc(
d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+cot(d*x+c)^2-1)^(1/2)))*(a*(1+sec(d*x+c))
)^(1/2)/(cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.94

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{5\left(a^2\cos\left(dx+c\right)+a^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right.}{2\left(d\cos\left(dx+c\right)+d\right)}\right.$$

$$-\frac{5\left(a^2\cos\left(dx+c\right)+a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(a^2\cos\left(dx+c\right)+2\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx\right.}{d\cos\left(dx+c\right)+d}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/2*(5*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt
(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a
*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(a^2*cos(d*x + c) + 2*a^2)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -(
5*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (a^2*cos(d*x + c) + 2*a^2)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)
]
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1383 vs. $2(84) = 168$.

Time = 0.24 (sec) , antiderivative size = 1383, normalized size of antiderivative = 14.71

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/4*(18*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*((4*a^2*sin(3*d*x + 3*c) + 5*a^2*sin(2*d*x + 2*c) + 4*a^2*sin(d*x + c))*c
os(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (a^2*cos(2*d*x +
 2*c)^2*sin(d*x + c) + a^2*sin(2*d*x + 2*c)^2*sin(d*x + c) + 2*a^2*cos(2*d
*x + 2*c)*sin(d*x + c) + a^2*sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) - (4*a^2*cos(3*d*x + 3*c) + 5*a^2*cos(2*d*x + 2*
c) + 4*a^2*cos(d*x + c) + 5*a^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)) - ((a^2*cos(d*x + c) - a^2)*cos(2*d*x + 2*c)^2 + a^2*cos(d
*x + c) + (a^2*cos(d*x + c) - a^2)*sin(2*d*x + 2*c)^2 - a^2 + 2*(a^2*cos(d
*x + c) - a^2)*cos(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)))*sqrt(a) + 5*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*
c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(84) = 168$.

Time = 0.67 (sec) , antiderivative size = 368, normalized size of antiderivative = 3.91

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\,dx =$$

$$-\frac{\frac{4\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}a^3\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}+5\sqrt{-a}a^2\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\right.}\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-1/2*(4*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^3*sgn(cos(d*x + c))*
tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a) + 5*sqrt(-a)*a^2*log(a
bs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2
 - a*(2*sqrt(2) + 3)))*sgn(cos(d*x + c)) - 5*sqrt(-a)*a^2*log(abs((sqrt(-a
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqr
t(2) - 3)))*sgn(cos(d*x + c)) + 4*(3*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c
) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a^3*sgn(cos(d*x + c))
- sqrt(2)*sqrt(-a)*a^4*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c
) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \cos(c+d\,x)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)*(a + a/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \sqrt{a}\,a^2\left(\int \sqrt{\sec(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^2\,dx + 2\left(\int \sqrt{\sec(dx+c)+1}\,\cos(dx+c)\sec(dx+c)\,dx\right) + \int \sqrt{\sec(dx+c)+1}\,\cos(dx+c)\,dx\right)$$

input
```
int(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sec(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x)**2,x) +
 2*int(sqrt(sec(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x),x) + int(sqrt(sec(
c + d*x) + 1)*cos(c + d*x),x))
```

3.113 $\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 106

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{19a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{9a^3\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{2d}$$

output
```
19/4*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+9/4*a^3*s
in(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a^2*cos(d*x+c)*(a+a*sec(d*x+c))^(1/
2)*sin(d*x+c)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.42

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = -\frac{a^2\cos(c+dx)\sqrt{a(1+\sec(c+dx))}\left(\sqrt{1-\sec(c+dx)}(\sin(c+dx)+3\sin(2(c+dx)))-7\operatorname{arctanh}\left(\sqrt{1\ldots}\right.\right.}{4d(1+\cos(c+\ldots}$$

input
```
Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
-1/4*(a^2*Cos[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[1 - Sec[c + d*x]]*
(Sin[c + d*x] + 3*Sin[2*(c + d*x)]) - 7*ArcTanh[Sqrt[1 - Sec[c + d*x]]]*Ta
n[c + d*x] - 32*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 -
Sec[c + d*x]]*Tan[c + d*x]))/(d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4300, 27, 3042, 4503, 3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4300}$$

$$\frac{1}{2}a\int \frac{1}{2}\cos(c+dx)\sqrt{\sec(c+dx)a+a}(5\sec(c+dx)a+9a)dx+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}a\int \cos(c+dx)\sqrt{\sec(c+dx)a+a}(5\sec(c+dx)a+9a)dx+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(5\csc\left(c+dx+\frac{\pi}{2}\right)a+9a\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{4503}$$

$$\frac{1}{4}a\left(\frac{19}{2}a\int \sqrt{\sec(c+dx)a+a}dx+\frac{9a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}a\left(\frac{19}{2}a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{9a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{4261}$$

$$\frac{1}{4}a\left(\frac{9a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{19a^2\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{216}$$

$$\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}+\frac{1}{4}a\left(\frac{19a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{9a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

input
```
Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^2*Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (a*((19*a
^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (9*a^2
*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

rule 4503
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Simp
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n)   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[
e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a
*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Maple [A] (verified)

Time = 33.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.31

method	result
default	$\frac{a^{2}\left(\sin(dx+c)\cos(dx+c)(2\cos(dx+c)+11)+19(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{\sqrt{\csc(dx+c)^{2}-2\cot(dx+c)\csc(dx+c)+\cot(dx}}\right.\right.}{4d(\cos(dx+c)+1)}$

input
```
int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*a^2*(sin(d*x+c)*cos(d*x+c)*(2*cos(d*x+c)+11)+19*(cos(d*x+c)+1)*(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(2^(1/2)/(csc(d*x+c)^2-2*cot(d*x+c)*c
sc(d*x+c)+cot(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c))))*(a*(1+sec(d*x+c
)))^(1/2)/(cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.77

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{19\left(a^2\cos(dx+c)+a^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-}{\cos(dx+c)+1}\right.}{8\left(d\cos(dx+c)+\right.}\right.$$

$$-\frac{19\left(a^2\cos(dx+c)+a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(2\,a^2\cos(dx+c)^2+11\,a^2\cos(dx+c)\right)\sqrt{}}{4\left(d\cos(dx+c)+d\right)}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/8*(19*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqr
t(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) +
a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*a^2*cos(d*x + c)^2 + 11*a^2
*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c) + d), -1/4*(19*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*
a^2*cos(d*x + c)^2 + 11*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c + dx)(a + a\sec(c + dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \cos^2(c + dx)(a + a\sec(c + dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(90) = 180$.

Time = 0.52 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.43

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx =$$

$$-\frac{\sqrt{2}\sqrt{-a}a^5\left(\frac{19\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{a^2|a|}+\frac{8\left(19\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx\right.}\right.\right.}{\left(\left(\sqrt{-a}\tan\right.\right.}\right.}{}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*sqrt(-a)*a^5*(19*sqrt(2)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1
/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/a
bs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))
^2 + 4*sqrt(2)*abs(a) - 6*a))/(a^2*abs(a)) + 8*(19*(sqrt(-a)*tan(1/2*d*x +
 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6 - 171*(sqrt(-a)*tan(1/2*d
*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a + 89*(sqrt(-a)*tan(
1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^2 - 9*a^3)/(((
sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6
*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a
 + a^2)^2*a^2))*sgn(cos(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \cos\left(c+d\,x\right)^2\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

input
```
int(cos(c + d*x)^2*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^2*(a + a/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx=\sqrt{a}\,a^2\left(\int \sqrt{\sec (dx+c)+1}\,\cos (dx+c)^2\sec (dx+c)^2\,dx+2\left(\int \sqrt{\sec (dx+c)+1}\,\cos (dx+c)^2\sec (dx+c)\,dx\right)+\int \sqrt{\sec (dx+c)+1}\,\cos (dx+c)^2\,dx\right)$$

input
```
int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sec(c + d*x) + 1)*cos(c + d*x)**2*sec(c + d*x)**2,x
) + 2*int(sqrt(sec(c + d*x) + 1)*cos(c + d*x)**2*sec(c + d*x),x) + int(sqr
t(sec(c + d*x) + 1)*cos(c + d*x)**2,x))
```

3.114 $\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 144

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{25a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{25a^3\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{13a^3\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d}$$

output
```
25/8*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+25/8*a^3*
sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+13/12*a^3*cos(d*x+c)*sin(d*x+c)/d/(a+a
*sec(d*x+c))^(1/2)+1/3*a^2*cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2)*sin(d*x+c)/
d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.54 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.05

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^2\left(165\text{arctanh}\left(\sqrt{1-\sec(c+dx)}\right)+(31+159\cos(c+dx)+31\cos(2(c+dx))-2\cos(3(c$$

input
```
Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^2*(165*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + (31 + 159*Cos[c + d*x] + 31*Co
s[2*(c + d*x)] - 2*Cos[3*(c + d*x)])*Sqrt[1 - Sec[c + d*x]] + 192*Hypergeo
metric2F1[1/2, 4, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])*Sqrt[a*(1
 + Sec[c + d*x])]*Sin[c + d*x])/(72*d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c +
d*x]])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4300, 27, 3042, 4503, 3042, 4292, 3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{4300}$$

$$\frac{1}{3}a\int \frac{1}{2}\cos^2(c+dx)\sqrt{\sec(c+dx)a+a}(9\sec(c+dx)a+13a)dx+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}a\int \cos^2(c+dx)\sqrt{\sec(c+dx)a+a}(9\sec(c+dx)a+13a)dx+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(9\csc\left(c+dx+\frac{\pi}{2}\right)a+13a\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{4503}$$

$$\frac{1}{6}a\left(\frac{75}{4}a\int \cos(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{13a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}a\left(\frac{75}{4}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{13a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{4292}$$

$$\frac{1}{6}a\left(\frac{75}{4}a\left(\frac{1}{2}\int \sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{13a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}a\left(\frac{75}{4}a\left(\frac{1}{2}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{13a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{4261}$$

$$\frac{1}{6}a\left(\frac{75}{4}a\left(\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{a\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{13a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{6}a\left(\frac{13a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}+\frac{75}{4}a\left(\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

input
```
Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^2*Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (a*((13
*a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (75*a*((S
qrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sin
[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

rule 4503
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Simp
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n)   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[
e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a
*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Maple [A] (verified)

Time = 125.66 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.04

method	result
default	$\frac{a^2\left(\sin(dx+c)\cos(dx+c)\left(8\cos(dx+c)^2+34\cos(dx+c)+75\right)+(75\cos(dx+c)+75)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \text{arctanh}\left(\frac{\sqrt{2}\,(-\cot(d}{\sqrt{\csc(dx+c)^2-2\cot(dx-}}\right.\right.}{24d(\cos(dx+c)+1)}$

input
```
int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/d*a^2*(sin(d*x+c)*cos(d*x+c)*(8*cos(d*x+c)^2+34*cos(d*x+c)+75)+(75*co
s(d*x+c)+75)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(2^(1/2)/(csc(d*x+c
)^2-2*cot(d*x+c)*csc(d*x+c)+cot(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c))
))*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.22

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{75\left(a^2\cos(dx+c)+a^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-}{\cos(dx+c)+1}\right.}{48\,(d\cos}\right.$$

$$-\frac{75\left(a^2\cos(dx+c)+a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(8\,a^2\cos(dx+c)^3+34\,a^2\cos(dx+c)^2+\right.}{24\,(d\cos(dx+c)+d)}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(75*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) +
 a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*a^2*cos(d*x + c)^3 + 34*a^
2*cos(d*x + c)^2 + 75*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/24*(75*(a^2*cos(d*x + c) + a^
2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sq
rt(a)*sin(d*x + c))) - (8*a^2*cos(d*x + c)^3 + 34*a^2*cos(d*x + c)^2 + 75*
a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d
*cos(d*x + c) + d)]
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(124) = 248$.

Time = 0.74 (sec) , antiderivative size = 539, normalized size of antiderivative = 3.74

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-1/48*(75*sqrt(-a)*a^2*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))*sgn(cos(d*x + c)) - 75*
sqrt(-a)*a^2*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))*sgn(cos(d*x + c)) + 4*(75*sqrt(2)
*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*
sqrt(-a)*a^3*sgn(cos(d*x + c)) - 1125*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*sqrt(-a)*a^4*sgn(cos(d*x + c))
 + 6174*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a))^6*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 4314*sqrt(2)*(sqrt(-a)*tan(
1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a^6*sgn
(cos(d*x + c)) + 807*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a^7*sgn(cos(d*x + c)) - 49*sqrt(2)*sqr
t(-a)*a^8*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \cos\left(c+d\,x\right)^3\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)^3*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^3*(a + a/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \sqrt{a}\,a^2\left(\int \sqrt{\sec(dx+c)+1}\,\cos(dx+c)^3\sec(dx+c)^2\,dx + 2\left(\int \sqrt{\sec(dx+c)+1}\,\cos(dx+c)^3\sec(dx+c)\,dx\right) + \int \sqrt{\sec(dx+c)+1}\,\cos(dx+c)^3\,dx\right)$$

input
```
int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sec(c + d*x) + 1)*cos(c + d*x)**3*sec(c + d*x)**2,x
) + 2*int(sqrt(sec(c + d*x) + 1)*cos(c + d*x)**3*sec(c + d*x),x) + int(sqr
t(sec(c + d*x) + 1)*cos(c + d*x)**3,x))
```

3.115 $\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 182

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{163a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d} + \frac{163a^3\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{17a^3\cos^2(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d}$$

output
```
163/64*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+163/64*
a^3*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+163/96*a^3*cos(d*x+c)*sin(d*x+c)/d
/(a+a*sec(d*x+c))^(1/2)+17/24*a^3*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*sec(d*x+c
))^(1/2)+1/4*a^2*cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.61 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.51

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = -\frac{2a^2\cos(c+dx)\sqrt{a(1+\sec(c+dx))}\left(\cos^2(c+dx)(7+16\cos(c+dx))\sin(c+dx)-163\,\text{Hypergeometri}\right.}{35d(1+\cos(c+dx))}$$

input
```
Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*a^2*Cos[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(Cos[c + d*x]^2*(7 + 16*Co
s[c + d*x])*Sin[c + d*x] - 163*Hypergeometric2F1[1/2, 5, 3/2, 1 - Sec[c +
d*x]]*Tan[c + d*x]))/(35*d*(1 + Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4300, 27, 3042, 4503, 3042, 4292, 3042, 4292, 3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{4300}$$

$$\frac{1}{4}a\int \frac{1}{2}\cos^3(c+dx)\sqrt{\sec(c+dx)a+a}(13\sec(c+dx)a+17a)dx+\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{8}a\int \cos^3(c+dx)\sqrt{\sec(c+dx)a+a}(13\sec(c+dx)a+17a)dx+\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{8}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(13\csc\left(c+dx+\frac{\pi}{2}\right)a+17a\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \; 4503$$

$$\frac{1}{8}a\left(\frac{163}{6}a\int \cos^2(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{17a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{8}a\left(\frac{163}{6}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{17a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \; 4292$$

$$\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\int \cos(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{17a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{17a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \; 4292$$

$$\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{17a^2\sin(c-}{3d\sqrt{a\,\mathrm{se}}}\right.$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{17a^2}{3}\right.$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \text{4261}$$

$$\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\left(\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{a\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\right.$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{8}a\left(\frac{17a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{163}{6}a\left(\frac{3}{4}\left(\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c}{2d\sqrt{}}\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

input
```
Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^2*Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (a*((17
*a^2*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (163*a*
((a*Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (3*((Sqrt[
a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sin[c +
 d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4))/6))/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

rule 4503
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Simp
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n)   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[
e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a
*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.88

$$\frac{a^2\left(\sin(dx+c)\cos(dx+c)\left(48\cos(dx+c)^3+184\cos(dx+c)^2+326\cos(dx+c)+489\right)+(489\cos(dx\right.}{192d\,(\cos(dx}$$

input
```
int(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x)
```

output
```
1/192/d*a^2*(sin(d*x+c)*cos(d*x+c)*(48*cos(d*x+c)^3+184*cos(d*x+c)^2+326*c
os(d*x+c)+489)+(489*cos(d*x+c)+489)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arc
tanh(2^(1/2)/(csc(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+cot(d*x+c)^2-1)^(1/2)*(
-cot(d*x+c)+csc(d*x+c))))*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.90

$$\int\cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx=\left[\frac{489\left(a^2\cos(dx+c)+a^2\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)}{\cos(dx+c)+1}\right.}{}\right.$$

$$-\frac{489\left(a^2\cos(dx+c)+a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(48a^2\cos(dx+c)^4+184a^2\cos(dx+c)^3\right.}{192\,(d\cos(dx+c)+d)}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/384*(489*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*
sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)
 + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(48*a^2*cos(d*x + c)^4 + 18
4*a^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 489*a^2*cos(d*x + c))*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1
/192*(489*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (48*a^2*cos(d*x + c
)^4 + 184*a^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 489*a^2*cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)
+ d)]
```

Sympy [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{%%{[-2309237210123256509497344,0]:[1,0,-2]%%},[35]
%%%},0]:[
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \cos\left(c+d\,x\right)^4\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,dx$$

input
```
int(cos(c + d*x)^4*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^4*(a + a/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \sqrt{a}\,a^2\left(\int \sqrt{\sec(dx+c)+1}\,\cos(dx+c)^4\sec(dx+c)^2\,dx + 2\left(\int \sqrt{\sec(dx+c)+1}\,\cos(dx+c)^4\sec(dx+c)\,dx\right) + \int \sqrt{\sec(dx+c)+1}\,\cos(dx+c)^4\,dx\right)$$

input
```
int(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sec(c + d*x) + 1)*cos(c + d*x)**4*sec(c + d*x)**2,x
) + 2*int(sqrt(sec(c + d*x) + 1)*cos(c + d*x)**4*sec(c + d*x),x) + int(sqr
t(sec(c + d*x) + 1)*cos(c + d*x)**4,x))
```

3.116 $\int \sec(c+dx)\sqrt{a-a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 27

$$\int \sec(c+dx)\sqrt{a-a\sec(c+dx)}\,dx = -\frac{2a\tan(c+dx)}{d\sqrt{a-a\sec(c+dx)}}$$

output
```
-2*a*tan(d*x+c)/d/(a-a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \sec(c+dx)\sqrt{a-a\sec(c+dx)}\,dx = \frac{2\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{a-a\sec(c+dx)}}{d}$$

input
```
Integrate[Sec[c + d*x]*Sqrt[a - a*Sec[c + d*x]],x]
```

output
```
(2*Cot[(c + d*x)/2]*Sqrt[a - a*Sec[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)\sqrt{a-a\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4279}$$

$$-\frac{2a\tan(c+dx)}{d\sqrt{a-a\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]*Sqrt[a - a*Sec[c + d*x]],x]
```

output
```
(-2*a*Tan[c + d*x])/(d*Sqrt[a - a*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

method	result	size
default	$-\frac{\sqrt{2}\,\sqrt{-2a(-1+\sec(dx+c))}\,\sin(dx+c)}{d(\cos(dx+c)-1)}$	38

input
```
int(sec(d*x+c)*(a-a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*2^(1/2)*(-2*a*(-1+sec(d*x+c)))^(1/2)*sin(d*x+c)/(cos(d*x+c)-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int \sec(c+dx)\sqrt{a-a\sec(c+dx)}\,dx = \frac{2\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}(\cos(dx+c)+1)}{d\sin(dx+c)}$$

input
```
integrate(sec(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*(cos(d*x + c) + 1)/(d*sin(d*x +
c))
```

Sympy [F]

$$\int \sec(c+dx)\sqrt{a-a\sec(c+dx)}\,dx = \int \sqrt{-a(\sec(c+dx)-1)}\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a-a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(-a*(sec(c + d*x) - 1))*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)\sqrt{a-a\sec(c+dx)}\,dx = \int \sqrt{-a\sec(dx+c)+a}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a*sec(d*x + c) + a)*sec(d*x + c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(25) = 50.

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.11

$$\int \sec(c+dx)\sqrt{a-a\sec(c+dx)}\,dx = -\frac{2\sqrt{2}a\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\mathrm{sgn}(\cos(dx+c))}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}d}$$

input
```
integrate(sec(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(2)*a*sgn(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*sgn(cos(d*
x + c))/(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*d)
```

Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \sec(c+dx)\sqrt{a-a\sec(c+dx)}\,dx = \frac{\sin(c+dx)\sqrt{a-\frac{a}{\cos(c+dx)}}}{d\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2}$$

input
```
int((a - a/cos(c + d*x))^(1/2)/cos(c + d*x),x)
```

output
```
(sin(c + d*x)*(a - a/cos(c + d*x))^(1/2))/(d*sin(c/2 + (d*x)/2)^2)
```

Reduce [F]

$$\int \sec(c+dx)\sqrt{a-a\sec(c+dx)}\,dx=\sqrt{a}\left(\int \sqrt{-\sec(dx+c)+1}\,\sec(dx+c)\,dx\right)$$

input
```
int(sec(d*x+c)*(a-a*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt( - sec(c + d*x) + 1)*sec(c + d*x),x)
```

3.117 $\int \sqrt{a-a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \sqrt{a-a\sec(c+dx)}\,dx=\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}$$

output
```
2*a^(1/2)*arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.24

$$\int \sqrt{a-a\sec(c+dx)}\,dx = -\frac{i\sqrt{1+e^{2i(c+dx)}}\left(\operatorname{arcsinh}\left(e^{i(c+dx)}\right)+\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{a-a\sec(c+dx)}}{d\left(-1+e^{i(c+dx)}\right)}$$

input
```
Integrate[Sqrt[a - a*Sec[c + d*x]],x]
```

output
```
((-I)*Sqrt[1 + E^((2*I)*(c + d*x))]*(ArcSinh[E^(I*(c + d*x))] + ArcTanh[Sq
rt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a - a*Sec[c + d*x]])/(d*(-1 + E^(I*(c +
 d*x))))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-a\sec(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{4261}$$
$$\frac{2a\int \frac{1}{\frac{a^2\tan^2(c+dx)}{a-a\sec(c+dx)}+a}d\frac{a\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}}{d}$$
$$\Big\downarrow \text{216}$$
$$\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}$$

input
```
Int[Sqrt[a - a*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/d
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(32) = 64$.

Time = 1.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.13

method	result	size
default	$-\frac{\sqrt{-2a(-1+\sec(dx+c))}\,\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}}{2}\right)}{d(\cos(dx+c)-1)}$	81

input
```
int((a-a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(-2*a*(-1+sec(d*x+c)))^(1/2)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))/(cos(d*x+
c)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(32) = 64$.

Time = 0.11 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.79

$$\int \sqrt{a - a\sec(c + dx)}\, dx = \left[\frac{\sqrt{-a}\log\left(-\frac{4\left(2\cos(dx+c)^3+3\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}+\left(8a\cos(dx+c)^2+8a\cos(dx+c)+a\right)\sin(dx+c)}{\sin(dx+c)}\right)}{2d}, -\frac{\sqrt{a}\arctan\left(\frac{2\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}}{(2a\cos(dx+c)+a)\sin(dx+c)}\right)}{d}\right]$$

input
```
integrate((a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(-a)*log(-(4*(2*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x + c))
*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) + (8*a*cos(d*x + c)^2 +
8*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))/d, -sqrt(a)*arctan(2*(co
s(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c
))/((2*a*cos(d*x + c) + a)*sin(d*x + c)))/d]
```

Sympy [F]

$$\int \sqrt{a - a\sec(c + dx)}\, dx = \int \sqrt{-a\sec(c + dx) + a}\, dx$$

input
```
integrate((a-a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(-a*sec(c + d*x) + a), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(32) = 64$.

Time = 0.24 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.84

$$\int \sqrt{a - a\sec(c + dx)}\, dx$$
$$= \frac{\sqrt{a}\arctan\left(\left(\cos\left(2\,dx + 2\,c\right)^{2} + \sin\left(2\,dx + 2\,c\right)^{2} + 2\,\cos\left(2\,dx + 2\,c\right) + 1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan\left(\sin\left(2\,dx + 2\,c\right)\right.\right.\right.}{}$$

input
```
integrate((a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + co
s(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(32) = 64$.

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \sqrt{a - a\sec(c + dx)}\, dx =$$
$$-\frac{2\,\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{2} - a}}{2\,\sqrt{a}}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{3} + \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)\mathrm{sgn}(\cos\left(dx + c\right))}{d}$$

input
```
integrate((a-a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))*
sgn(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*sgn(cos(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a-a\sec(c+dx)}\,dx=\int \sqrt{a-\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((a - a/cos(c + d*x))^(1/2),x)
```

output
```
int((a - a/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a-a\sec(c+dx)}\,dx=\sqrt{a}\left(\int \sqrt{-\sec\left(dx+c\right)+1}dx\right)$$

input
```
int((a-a*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt( - sec(c + d*x) + 1),x)
```

3.118 $\int \cos(c+dx)\sqrt{a-a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 65

$$\int \cos(c+dx)\sqrt{a-a\sec(c+dx)}\,dx = -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}}$$

output
```
-a^(1/2)*arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))/d+a*sin(d*x+c)/
d/(a-a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.18

$$\int \cos(c+dx)\sqrt{a-a\sec(c+dx)}\,dx = -\frac{\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{a-a\sec(c+dx)}\left(-\operatorname{arctanh}\left(\sqrt{1+\sec(c+dx)}\right)+\cos(c+dx)\sqrt{1+\sec(c+dx)}\right)}{d\sqrt{1+\sec(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]*Sqrt[a - a*Sec[c + d*x]],x]
```

output
```
-((Cot[(c + d*x)/2]*Sqrt[a - a*Sec[c + d*x]]*(-ArcTanh[Sqrt[1 + Sec[c + d*
x]]] + Cos[c + d*x]*Sqrt[1 + Sec[c + d*x]]))/(d*Sqrt[1 + Sec[c + d*x]]))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3042, 4292, 3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)\sqrt{a-a\sec(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4292$$

$$\frac{a\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}}-\frac{1}{2}\int \sqrt{a-a\sec(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\frac{a\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}}-\frac{1}{2}\int \sqrt{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4261$$

$$\frac{a\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}}-\frac{a\int \frac{1}{\frac{a^2\tan^2(c+dx)}{a-a\sec(c+dx)}+a}d\frac{a\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}}{d}$$

$$\Big\downarrow \; 216$$

$$\frac{a\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}$$

input
```
Int[Cos[c + d*x]*Sqrt[a - a*Sec[c + d*x]],x]
```

output
```
-((Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/d) + (
a*Sin[c + d*x])/(d*Sqrt[a - a*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.46

method	result	s
default	$-\frac{\sqrt{2}\,(\cos(dx+c)+1)\left(\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}}{2}\right)+2\cos(dx+c)\right)\sqrt{-2a(-1+\sec(dx+c))}\,\csc(dx+c)}{4d}$	9

input
```
int(cos(d*x+c)*(a-a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d*2^(1/2)*(cos(d*x+c)+1)*((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2
)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+2*cos(d*x+c))*(
-2*a*(-1+sec(d*x+c)))^(1/2)*csc(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(57) = 114$.

Time = 0.11 (sec) , antiderivative size = 294, normalized size of antiderivative = 4.52

$$\int \cos(c+dx)\sqrt{a-a\sec(c+dx)}\,dx$$
$$= \left[\frac{\sqrt{-a}\log\left(\frac{4\left(2\cos(dx+c)^3+3\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}-\left(8a\cos(dx+c)^2+8a\cos(dx+c)+a\right)\sin(dx+c)}{\sin(dx+c)}\right)\sin(a}{4d\sin(dx+c)}\right.$$

input
```
integrate(cos(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(-a)*log((4*(2*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x + c))
*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) - (8*a*cos(d*x + c)^2 +
8*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) - 4*(cos(d*
x + c)^2 + cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/(d*sin(d
*x + c)), 1/2*(sqrt(a)*arctan(2*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sq
rt((a*cos(d*x + c) - a)/cos(d*x + c))/((2*a*cos(d*x + c) + a)*sin(d*x + c)
))*sin(d*x + c) - 2*(cos(d*x + c)^2 + cos(d*x + c))*sqrt((a*cos(d*x + c) -
 a)/cos(d*x + c)))/(d*sin(d*x + c))]
```

Sympy [F]

$$\int \cos(c+dx)\sqrt{a-a\sec(c+dx)}\,dx = \int \sqrt{-a\left(\sec\left(c+dx\right)-1\right)}\cos\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)*(a-a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(-a*(sec(c + d*x) - 1))*cos(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 791 vs. 2(57) = 114.

Time = 0.29 (sec) , antiderivative size = 791, normalized size of antiderivative = 12.17

$$\int \cos(c+dx)\sqrt{a-a\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x +
 c) - (cos(d*x + c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) + arctan2((cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)) + 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(57) = 114$.

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.06

$$\int \cos(c+dx)\sqrt{a-a\sec(c+dx)}\,dx$$
$$= \frac{\sqrt{2}\left(\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}}{2\,\sqrt{a}}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-\frac{2\,\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{}\right.}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(sqrt(2)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c
)^2 - a)/sqrt(a))*sgn(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c)) - 2*s
qrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*a*sgn(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d
*x + 1/2*c))/(a*tan(1/2*d*x + 1/2*c)^2 + a))*sgn(cos(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt{a-a\sec(c+dx)}\,dx = \int \cos\left(c+d\,x\right)\,\sqrt{a-\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)*(a - a/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)*(a - a/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cos(c+dx)\sqrt{a-a\sec(c+dx)}\,dx = \sqrt{a}\left(\int \sqrt{-\sec\left(dx+c\right)+1}\,\cos\left(dx+c\right)dx\right)$$

input
```
int(cos(d*x+c)*(a-a*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt( - sec(c + d*x) + 1)*cos(c + d*x),x)
```

3.119 $\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 140

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{28\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} - \frac{2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15ad}$$

output
```
-2^(1/2)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(
1/2)/d+28/15*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/5*sec(d*x+c)^2*tan(d*x+
c)/d/(a+a*sec(d*x+c))^(1/2)-2/15*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.76

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\left(-15\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)+2\sqrt{1-\sec(c+dx)}(13-\sec(c+dx)+3\sec^2(c+dx))\right)\tan(c+dx)}{15d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
((-15*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + 2*Sqrt[1 - Sec[c +
 d*x]]*(13 - Sec[c + d*x] + 3*Sec[c + d*x]^2))*Tan[c + d*x])/(15*d*Sqrt[1
- Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4309, 3042, 4498, 27, 3042, 4489, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4309}$$

$$\frac{\int \frac{\sec^2(c+dx)(4a-a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{5a}+\frac{2\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(4a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}+\frac{2\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \textbf{4498}$$

$$\frac{\frac{2\int -\frac{\sec(c+dx)\left(a^2-14a^2\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)a+a}}dx}{3a}-\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{-\frac{\int \frac{\sec(c+dx)\left(a^2-14a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{3a}-\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a^2-14a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}-\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \textbf{4489}$$

$$\frac{-\frac{15a^2\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx-\frac{28a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{-\frac{15a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{28a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \textbf{4282}$$

$$\frac{-\frac{-\frac{30a^2\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{28a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \textbf{216}$$

$$-\frac{\frac{\frac{15\sqrt{2}a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{28a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Sec[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Sec[c + d*x]^2*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]]) + ((-2*Sqrt
[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*d) - ((15*Sqrt[2]*a^(3/2)*ArcTan[(Sq
rt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d - (28*a^2*Tan[c
 + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(3*a))/(5*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4309
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(
f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[d^2/(b*(2*n - 3))   Int[(
d*Csc[e + f*x])^(n - 2)*((2*b*(n - 2) - a*Csc[e + f*x])/Sqrt[a + b*Csc[e +
f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n,
2] && IntegerQ[2*n]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

rule 4498
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int
[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)
*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a
*B, 0] &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left(26\sin(dx+c)-2\tan(dx+c)+6\sec(dx+c)\tan(dx+c)-15\sqrt{2}\,(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\ln\left(\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)-}}\right.\right.}{15da(\cos(dx+c)+1)}$

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15/d/a*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(26*sin(d*x+c)-2*tan(d*x+
c)+6*sec(d*x+c)*tan(d*x+c)-15*2^(1/2)*(cos(d*x+c)+1)*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*ln((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cot(d*x+c)+csc(d*x+c
)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.48

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \left[\frac{15\sqrt{2}\left(a\cos(dx+c)^3+a\cos(dx+c)^2\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos\right.}{\cos(dx+c)^2+2\cos(dx+c)+1}}{30\left(ad\cos(dx+c)^3+ad\cos(dx\right.}\right.$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/30*(15*sqrt(2)*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sqrt(-1/a)*log((2*
sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*si
n(d*x + c) + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) + 4*(13*cos(d*x + c)^2 - cos(d*x + c) + 3)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x
+ c)^2), 1/15*(2*(13*cos(d*x + c)^2 - cos(d*x + c) + 3)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sin(d*x + c) + 15*sqrt(2)*(a*cos(d*x + c)^3 + a*cos(
d*x + c)^2)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
 + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x +
 c)^2)]
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sec^4(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**4/sqrt(a*(sec(c + d*x) + 1)), x)
```

Maxima [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^4}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^4/sqrt(a*sec(d*x + c) + a), x)
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.07

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\left(\frac{15\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}}+\frac{2\left(\left(17a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-20a^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+15a^2\right)\tan\left(\frac{1}{2}dx+\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}\right.}{15\,d\operatorname{sgn}(\cos(dx+c))}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/15*sqrt(2)*(15*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a)))/sqrt(-a) + 2*((17*a^2*tan(1/2*d*x + 1/2*c)^2 - 20*a^
2)*tan(1/2*d*x + 1/2*c)^2 + 15*a^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x +
 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(d*sgn(cos(d*x + c)
))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^4\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sec(dx+c)^4}{\sec(dx+c)+1}dx\right)}{a}$$

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sec(c + d*x)**4)/(sec(c + d*x) + 1),x
))/a
```

3.120 $\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 104

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{4\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3ad}$$

output
```
2^(1/2)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(1
/2)/d-4/3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*(a+a*sec(d*x+c))^(1/2)*t
an(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\left(-\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)+\frac{2}{3}(1-\sec(c+dx))^{3/2}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
-(((-(Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]) + (2*(1 - Sec[c + d
*x])^(3/2))/3)*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c +
 d*x])]))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4287, 27, 3042, 4489, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4287}$$

$$\frac{2\int \frac{\sec(c+dx)(a-2a\sec(c+dx))}{2\sqrt{\sec(c+dx)a+a}}dx}{3a}+\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sec(c+dx)(a-2a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{3a}+\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}+\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3ad}$$

$$\Big\downarrow \text{4489}$$

$$\frac{3a\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx-\frac{4a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}+\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{4a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}+\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3ad}$$

$$\Big\downarrow \text{4282}$$

$$\frac{-\frac{6a\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{4a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}+\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3ad}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{3\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{4a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}+\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3ad}$$

input
```
Int[Sec[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*a*d) + ((3*Sqrt[2]*Sqrt[a]*Ar
cTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d - (4*a*
Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(3*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4287
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 -
b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.12

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left(-2\sin(dx+c)+2\tan(dx+c)+3\sqrt{2}\,(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\ln\left(\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}-\cot(dx+c)+\csc(dx+c\right.\right.}{3da(\cos(dx+c)+1)}$

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d/a*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(-2*sin(d*x+c)+2*tan(d*x+c
)+3*2^(1/2)*(cos(d*x+c)+1)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln((-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)-cot(d*x+c)+csc(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.04

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \left[\frac{3\sqrt{2}\left(a\cos(dx+c)^2+a\cos(dx+c)\right)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-2\cos\cdots}{\cos(dx+c)^2+2\cos(dx+c)+1}\cdots\right.}{6\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}\cdots\right.$$

$$-\frac{2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-1)\sin(dx+c)+\frac{3\sqrt{2}\left(a\cos(dx+c)^2+a\cos(dx+c)\right)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\cdots\right.}{\sqrt{a}}\cdots}{3\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(2)*(a*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(-1/a)*log(-(2*sqr
t(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d
*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(cos(d*x + c)^2 + 2*cos(d
*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) -
1)*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c)), -1/3*(2*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 1)*sin(d*x + c) + 3*sqrt(2
)*(a*cos(d*x + c)^2 + a*cos(d*x + c))*arctan(sqrt(2)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(
d*x + c)^2 + a*d*cos(d*x + c))]
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sec^3(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**3/sqrt(a*(sec(c + d*x) + 1)), x)
```

Maxima [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^3}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/sqrt(a*sec(d*x + c) + a), x)
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\left(\frac{4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}+\frac{3\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{\sqrt{-a}}\right)}{3\,d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/3*sqrt(2)*(4*a*tan(1/2*d*x + 1/2*c)^3/((a*tan(1/2*d*x + 1/2*c)^2 - a)*s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)) + 3*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/
2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/sqrt(-a))/(d*sgn(cos(d*x + c)
))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^3\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sec(dx+c)^3}{\sec(dx+c)+1}dx\right)}{a}$$

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sec(c + d*x)**3)/(sec(c + d*x) + 1),x
))/a
```

3.121 $\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 73

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx=-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}+\frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

output
```
-2^(1/2)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(
1/2)/d+2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\left(\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)-2\sqrt{1-\sec(c+dx)}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
-(((Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] - 2*Sqrt[1 - Sec[c + d
*x]])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4285, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4285}$$

$$\frac{2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4282}$$

$$\frac{2\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

input
```
Int[Sec[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
-((Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]]
)])/(Sqrt[a]*d)) + (2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4285
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[a*(m/(b*(m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x]
/; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.30

method	result	size
default	$\frac{\left(-\ln\left(\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}-\cot(dx+c)+\csc(dx+c)\right)\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}-2\cot(dx+c)+2\csc(dx+c)\right)\sqrt{a(1+\sec(dx+c))}}{da}$	95

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-ln((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cot(d*x+c)+csc(d*x+c))*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)-2*cot(d*x+c)+2*csc(d*x+c))/a*(a*(1+sec(d*x
+c)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.59

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \left[\frac{\sqrt{2}(a\cos(dx+c)+a)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)-1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4\sqrt{\cdot}}{2\,(ad\cos(dx+c)+ad)}\right.$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) + 3*cos(d*
x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) +
a*d), (sqrt(2)*(a*cos(d*x + c) + a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a) + 2*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sec^2(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**2/sqrt(a*(sec(c + d*x) + 1)), x)
```

Maxima [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^2}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/sqrt(a*sec(d*x + c) + a), x)
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.48

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\left(\frac{\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{\sqrt{-a}} - \frac{2\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}\right)}{d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*(log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a)))/sqrt(-a) - 2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x
 + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a))/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)^2\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sec(dx+c)^2}{\sec(dx+c)+1}dx\right)}{a}$$

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sec(c + d*x)**2)/(sec(c + d*x) + 1),x
))/a
```

3.122 $\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
2^(1/2)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(1
/2)/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.39

$$\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Tan[c + d*x])/(d*Sqrt[1 -
 Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4282}$$

$$-\frac{2\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

input
```
Int[Sec[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]
)/(Sqrt[a]*d)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(37) = 74$.

Time = 0.87 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.65

method	result	size
default	$\frac{\sqrt{a(1+\sec(dx+c))}\,\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}\,\ln\left(\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}-\cot(dx+c)+\csc(dx+c)\right)}{da}$	76

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(a*(1+sec(d*x+c)))^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln((-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cot(d*x+c)+csc(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 158, normalized size of antiderivative = 3.43

$$\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \left[\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2d}, -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}d}\right]$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x
+ c) + 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/d, -sqrt(2)*arctan(sqrt(2
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x +
c)))/(sqrt(a)*d)]
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x)
```

Maxima [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/sqrt(a*sec(d*x + c) + a), x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28

$$\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right|\right)}{\sqrt{-a}d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a)))/(sqrt(-a)*d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sec(dx+c)}{\sec(dx+c)+1}dx\right)}{a}$$

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sec(c + d*x))/(sec(c + d*x) + 1),x))/
a
```

3.123 $\int \frac{1}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 85

$$\int \frac{1}{\sqrt{a+a\sec(c+dx)}}\,dx=\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(1/2)/d-2^(1/2)*arct
an(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(1/2)/d
```

Mathematica [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.53

$$\int \frac{1}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)-\sqrt{2}\arctan\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}}\right)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx}}{d\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[1/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(-2*(ArcSin[Tan[(c + d*x)/2]] - Sqrt[2]*ArcTan[Tan[(c + d*x)/2]/Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])]])*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[S
ec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(d*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[a*(1
 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4263, 3042, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4263}$$

$$\frac{\int \sqrt{\sec(c+dx)a+a}dx}{a}-\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{a}-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4261}$$

$$-\frac{2\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{216}$$

$$\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4282}$$

$$\frac{2\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

input
```
Int[1/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) -
(Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]
)/(Sqrt[a]*d)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4263
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[1/a   I
nt[Sqrt[a + b*Csc[c + d*x]], x], x] - Simp[b/a   Int[Csc[c + d*x]/Sqrt[a +
b*Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.54

method	result
default	$-\frac{\sqrt{-a(-1-\sec(dx+c))}\,\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}\,\left(\ln\left(\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}-\cot(dx+c)+\csc(dx+c)\right)-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+}{\sqrt{(1-\cos(dx+c))^2\csc(dx+c}}\right.\right.}{da}$

input
```
int(1/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d/a*(-a*(-1-sec(d*x+c)))^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(ln
((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cot(d*x+c)+csc(d*x+c))-2^(1/2)*arcta
nh(2^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c)
)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.46

$$\int \frac{1}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \left[\frac{\sqrt{2}a\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)-1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-2\sqrt{-a}\log\left(\frac{2a\cos(dx+c)}{}\right.}{2ad}\right.$$

input
```
integrate(1/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(2)*a*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) + 3*cos(d*x + c)^2 + 2*cos(d*
x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 2*sqrt(-a)*log((2*a*c
os(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
 + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)))/(a*d), (sqrt
(2)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
 + c)/(sqrt(a)*sin(d*x + c))) - 2*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/(a*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{a\sec\left(c+dx\right)+a}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a*sec(c + d*x) + a), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 698, normalized size of antiderivative = 8.21

$$\int \frac{1}{\sqrt{a+a\sec(c+dx)}}\,dx=\text{Too large to display}$$

input
```
integrate(1/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-(sqrt(2)*sqrt(a)*arctan2(((abs(2*e^(I*d*x + I*c) + 2)^4 + 16*cos(d*x + c)
^4 + 16*sin(d*x + c)^4 + 8*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x +
c) + 1)*abs(2*e^(I*d*x + I*c) + 2)^2 - 64*cos(d*x + c)^3 + 32*(cos(d*x + c
)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 96*cos(d*x + c)^2 - 64*cos(d*x
+ c) + 16)^(1/4)*sin(1/2*arctan2(8*(cos(d*x + c) - 1)*sin(d*x + c)/abs(2*e
^(I*d*x + I*c) + 2)^2, (abs(2*e^(I*d*x + I*c) + 2)^2 + 4*cos(d*x + c)^2 -
4*sin(d*x + c)^2 - 8*cos(d*x + c) + 4)/abs(2*e^(I*d*x + I*c) + 2)^2)) + 2*
sin(d*x + c))/abs(2*e^(I*d*x + I*c) + 2), ((abs(2*e^(I*d*x + I*c) + 2)^4 +
 16*cos(d*x + c)^4 + 16*sin(d*x + c)^4 + 8*(cos(d*x + c)^2 - sin(d*x + c)^
2 - 2*cos(d*x + c) + 1)*abs(2*e^(I*d*x + I*c) + 2)^2 - 64*cos(d*x + c)^3 +
 32*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 96*cos(d*x + c)
^2 - 64*cos(d*x + c) + 16)^(1/4)*cos(1/2*arctan2(8*(cos(d*x + c) - 1)*sin(
d*x + c)/abs(2*e^(I*d*x + I*c) + 2)^2, (abs(2*e^(I*d*x + I*c) + 2)^2 + 4*c
os(d*x + c)^2 - 4*sin(d*x + c)^2 - 8*cos(d*x + c) + 4)/abs(2*e^(I*d*x + I*
c) + 2)^2)) + 2*cos(d*x + c) - 2)/abs(2*e^(I*d*x + I*c) + 2)) - sqrt(a)*ar
ctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x +
c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/
4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos(d*x + c)
))/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{a+a\sec(c+dx)}}\,dx=\frac{\frac{\sqrt{2}\arctan\left(\frac{\sqrt{-a+\frac{a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\arctan\left(\frac{\sqrt{2}\sqrt{-a+\frac{a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}}{2\sqrt{a}}\right)}{\sqrt{a}}}{d}$$

input
```
integrate(1/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
(sqrt(2)*arctan(sqrt(-a + a/tan(1/2*d*x + 1/2*c)^2)/sqrt(a))/sqrt(a) - 2*a
rctan(1/2*sqrt(2)*sqrt(-a + a/tan(1/2*d*x + 1/2*c)^2)/sqrt(a))/sqrt(a))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(1/(a + a/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+a\sec(c+dx)}}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}}{\sec(dx+c)+1}dx\right)}{a}$$

input
```
int(1/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int(sqrt(sec(c + d*x) + 1)/(sec(c + d*x) + 1),x))/a
```

3.124 $\int \frac{\cos(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \frac{\cos(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

output
```
-arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(1/2)/d+2^(1/2)*arcta
n(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(1/2)/d+sin(d*x
+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.97

$$\int \frac{\cos(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\left(\operatorname{arctanh}\left(\sqrt{1-\sec(c+dx)}\right)-\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)-\cos(c+dx)\sqrt{1-\sec(c+dx)}\right)\tan(c+}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Cos[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
-(((ArcTanh[Sqrt[1 - Sec[c + d*x]]] - Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x
]]/Sqrt[2]] - Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]])*Tan[c + d*x])/(d*Sqrt[1
 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4310, 3042, 4392, 3042, 4375, 383, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4310}$$

$$\frac{\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{a-a\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}$$

$$\Big\downarrow \text{4392}$$

$$\frac{1}{2}a\int \frac{\tan^2(c+dx)}{(\sec(c+dx)a+a)^{3/2}}dx+\frac{\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^2}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx+\frac{\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 4375$$

$$\frac{\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{a\int \frac{\tan^2(c+dx)}{(\sec(c+dx)a+a)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \; 383$$

$$\frac{\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{a\left(\frac{2\int \frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{a}-\frac{\int \frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{a}\right)}{d}$$

$$\Big\downarrow \; 216$$

$$\frac{\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{a\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}}\right)}{d}$$

input
```
Int[Cos[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
-((a*(ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]/a^(3/2) - (S
qrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/
a^(3/2)))/d) + Sin[c + d*x]/(d*Sqrt[a + a*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 383
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[(-a)*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(a + b*x^2), x], x]
 + Simp[c*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; Free
Q[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4310
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a +
b*Csc[e + f*x]])), x] + Simp[1/(2*b*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*((a
 + b*(2*n + 1)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(91) = 182$.

Time = 2.48 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.84

method	result
default	$-\frac{\left((\cos(dx+c)+1)\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}\ \operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{\sqrt{\csc(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\cot(dx+c)^2-1}}\right)+(-2\cos(dx+c)-2)\ln\left(\sqrt{-\frac{}{c}}\right.\right.}{2da(\cos(dx+c)+1)}$

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d/a*((cos(d*x+c)+1)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arct
anh(2^(1/2)/(csc(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+cot(d*x+c)^2-1)^(1/2)*(-
cot(d*x+c)+csc(d*x+c)))+(-2*cos(d*x+c)-2)*ln((-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)-cot(d*x+c)+csc(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-2*sin(
d*x+c)*cos(d*x+c))*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 417, normalized size of antiderivative = 3.86

$$\int \frac{\cos(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \left[\frac{\sqrt{2}(a\cos(dx+c)+a)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-\sqrt{\cdot}}{}\right.$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) - 3*cos(d
*x + c)^2 - 2*cos(d*x + c) + 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - s
qrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) -
 a)/(cos(d*x + c) + 1)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*
x + c)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d), (sqrt(a)*(cos(d*x + c) + 1)
*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(
d*x + c))) + sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
+ c) - sqrt(2)*(a*cos(d*x + c) + a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*
x + c) + a*d)]
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cos{(c+dx)}}{\sqrt{a\left(\sec{(c+dx)}+1\right)}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x)
```

Maxima [F]

$$\int \frac{\cos(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cos{(dx+c)}}{\sqrt{a\sec{(dx+c)}+a}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/sqrt(a*sec(d*x + c) + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(91) = 182$.

Time = 0.40 (sec) , antiderivative size = 326, normalized size of antiderivative = 3.02

$$\int \frac{\cos(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{-a}\log\left(\left|\frac{2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a}{2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a}\right|\right)}{|a|} - \frac{2\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right.\right.}{\sqrt{-a}}\right.}{4d\,\mathrm{sgn}\left(\cos\left(dx\right.\right.}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(sqrt(2)*sqrt(-a)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqr
t(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sq
rt(2)*abs(a) - 6*a))/abs(a) - 2*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/sqrt(-a) - 8*(3*(sqrt(-a)*tan(1/2*d*x +
 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a) - sqrt(-a)*a)/((
sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6
*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a
 + a^2))/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(cos(c + d*x)/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)/(a + a/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\cos(dx+c)}{\sec(dx+c)+1}dx\right)}{a}$$

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*cos(c + d*x))/(sec(c + d*x) + 1),x))/
a
```

3.125 $\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{7\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{a}d} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}$$

output
```
7/4*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(1/2)/d-2^(1/2)*ar
ctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(1/2)/d-1/4*
sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d
*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.80

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\left(7\text{arctanh}\left(\sqrt{1-\sec(c+dx)}\right) - 4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right) + \cos(c+dx)(-1+2\cos(c+dx))\sqrt{1-s}\right.}{4d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Cos[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
((7*ArcTanh[Sqrt[1 - Sec[c + d*x]]] - 4*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d
*x]]/Sqrt[2]] + Cos[c + d*x]*(-1 + 2*Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]])
*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {3042, 4310, 3042, 4510, 27, 3042, 4408, 3042, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4310}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{\cos(c+dx)(a-3a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{4a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{a-3a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}$$

$$\Big\downarrow \text{4510}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{\int -\frac{7a^2-a^2\sec(c+dx)}{2\sqrt{\sec(c+dx)a+a}}dx}{a}+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{7a^2-a^2\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{2a}}{4a}$$

$\Big\downarrow$ 3042

$$\frac{\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{7a^2-a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}}{4a}$$

$\Big\downarrow$ 4408

$$\frac{\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{7a\int\sqrt{\sec(c+dx)a+a}dx-8a^2\int\frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{2a}}{4a}$$

$\Big\downarrow$ 3042

$$\frac{\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{7a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-8a^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}}{4a}$$

$\Big\downarrow$ 4261

$$\frac{\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{-8a^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{14a^2\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{2a}}{4a}$$

$\Big\downarrow$ 216

$$\frac{\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{14a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-8a^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}}{4a}$$

$\Big\downarrow$ 4282

$$\frac{\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{16a^2\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{14a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{2a}}{4a}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{14a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{8\sqrt{2}a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{2a}}{4a}$$

input
```
Int[Cos[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) - (-1/2*((14*a^
(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (8*Sqrt
[2]*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]
])])/d)/a + (a*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(4*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4310
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a +
b*Csc[e + f*x]])), x] + Simp[1/(2*b*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*((a
 + b*(2*n + 1)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

rule 4408
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c/a   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[(b*c - a*d)/a   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.41

method	result
default	$-\frac{\left(\sin(dx+c)\cos(dx+c)(1-2\cos(dx+c))+7(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \operatorname{arctanh}\left(\frac{\sqrt{2}\,(\cot(dx+c)-\csc(dx+c))}{\sqrt{\csc(dx+c)^{2}-2\cot(dx+c)\csc(dx+c)+\cot(dx+}}\right.\right.}{4da(\cos(dx+c)+}$

input
```
int(cos(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/a*(sin(d*x+c)*cos(d*x+c)*(1-2*cos(d*x+c))+7*(cos(d*x+c)+1)*(-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(2^(1/2)*(cot(d*x+c)-csc(d*x+c))/(csc(d*
x+c)^2-2*cot(d*x+c)*csc(d*x+c)+cot(d*x+c)^2-1)^(1/2))+4*2^(1/2)*(cos(d*x+c
)+1)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln((-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)-cot(d*x+c)+csc(d*x+c)))*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 446, normalized size of antiderivative = 3.03

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \left[\frac{4\sqrt{2}(a\cos(dx+c)+a)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)-1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-7\sqrt{}\right.$$

$$-\frac{7\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(2\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+}{\cos(dx+c)}}}{4\left(ad\cos(dx+c)+ad\right)}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(4*sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) + 3*cos(
d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) -
7*sqrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c
) - a)/(cos(d*x + c) + 1)) + 2*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d), -1/
4*(7*sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*cos(d*x + c)^2 - cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 4*sqrt(2)*(a*co
s(d*x + c) + a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos
(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cos^2{(c+dx)}}{\sqrt{a\,(\sec{(c+dx)}+1)}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)**2/sqrt(a*(sec(c + d*x) + 1)), x)
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cos{(dx+c)}^2}{\sqrt{a\sec{(dx+c)}+a}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^2/sqrt(a*sec(d*x + c) + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(122) = 244$.

Time = 0.42 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.88

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx =$$

$$-\frac{\sqrt{2}\left(\frac{7\sqrt{2}\sqrt{-a}\log\left(\left|\frac{2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-4\sqrt{2}|a|-6\,a}{2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2+4\sqrt{2}|a|-6\,a}\right|\right)}{|a|}-\frac{8\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\right.}\right.\right.}{\sqrt{-a}}\right.}{}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*(7*sqrt(2)*sqrt(-a)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*
(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 +
4*sqrt(2)*abs(a) - 6*a))/abs(a) - 8*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/sqrt(-a) - 8*(17*(sqrt(-a)*tan(1/2*
d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(-a) - 57*(sqrt(
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)
*a + 19*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
a))^2*sqrt(-a)*a^2 - 3*sqrt(-a)*a^3)/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int(cos(c + d*x)^2/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^2/(a + a/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\cos(dx+c)^2}{\sec(dx+c)+1}dx\right)}{a}$$

input
```
int(cos(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*cos(c + d*x)**2)/(sec(c + d*x) + 1),x
))/a
```

3.126 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 183

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{15\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{31\tan(c+dx)}{5ad\sqrt{a+a\sec(c+dx)}} + \frac{9\sec^2(c+dx)\tan(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}} - \frac{13\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{10a^2d}$$

output
```
-15/4*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2
)/a^(3/2)/d-1/2*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)+31/5*tan(
d*x+c)/a/d/(a+a*sec(d*x+c))^(1/2)+9/10*sec(d*x+c)^2*tan(d*x+c)/a/d/(a+a*se
c(d*x+c))^(1/2)-13/10*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/a^2/d
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.68

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\left(-75\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)(1+\sec(c+dx))+2\sqrt{1-\sec(c+dx)}(49\right.}{20d\sqrt{1-\sec(c+dx)}(a(1+}$$

input
```
Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
((-75*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*(1 + Sec[c + d*x]) +
 2*Sqrt[1 - Sec[c + d*x]]*(49 + 36*Sec[c + d*x] - 4*Sec[c + d*x]^2 + 4*Sec
[c + d*x]^3))*Tan[c + d*x])/(20*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d
*x]))^(3/2))
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4303, 27, 3042, 4509, 27, 3042, 4498, 27, 3042, 4489, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4303}$$

$$-\frac{\int \frac{3\sec^3(c+dx)(2a-3a\sec(c+dx))}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3\int \frac{\sec^3(c+dx)(2a-3a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(2a-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 4509

$$-\frac{3\left(\frac{2\int -\frac{\sec^2(c+dx)\left(12a^2-13a^2\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)a+a}}dx}{5a}-\frac{6a\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 27

$$-\frac{3\left(-\frac{\int \frac{\sec^2(c+dx)\left(12a^2-13a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{5a}-\frac{6a\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{3\left(-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(12a^2-13a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}-\frac{6a\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 4498

$$-\frac{3\left(-\frac{\frac{2\int -\frac{\sec(c+dx)\left(13a^3-62a^3\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)a+a}}dx}{3a}-\frac{26a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}-\frac{6a\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 27

$$-\frac{3\left(-\frac{-\frac{\int \frac{\sec(c+dx)\left(13a^3-62a^3\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{3a}-\frac{26a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}-\frac{6a\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(-\frac{-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(13a^3-62a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}-\frac{26a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}-\frac{6a\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4489}$$

$$-\frac{3\left(-\frac{-\frac{75a^3\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx-\frac{124a^3\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{26a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}-\frac{6a\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(-\frac{-\frac{75a^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{124a^3\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{26a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}-\frac{6a\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4282}$$

$$-\frac{3\left(-\frac{-\frac{-\frac{150a^3\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{124a^3\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{26a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}-\frac{6a\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 216

$$-\frac{3\left(-\frac{-\frac{\frac{75\sqrt{2}a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{124a^3\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{26a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}-\frac{6a\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

input
```
Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
-1/2*(Sec[c + d*x]^3*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^(3/2)) - (3*((-
6*a*Sec[c + d*x]^2*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]]) - ((-26*a*
Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*d) - ((75*Sqrt[2]*a^(5/2)*ArcTan
[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d - (124*a^3*
Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(3*a))/(5*a)))/(4*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

rule 4498
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int
[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)
*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a
*B, 0] &&  !LtQ[m, -1]
```

rule 4509
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Simp[d/(b*(m + n))   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
 1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[n, 1]
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.90

method	result
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\left(\left(-98\cos(dx+c)^3-72\cos(dx+c)^2+8\cos(dx+c)-8\right)\tan(dx+c)\sec(dx+c)+\left(75\cos(dx+c)^2+150\cos(dx+c)\right.}{20d\,a^2\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)}$

input
```
int(sec(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20/d/a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)^2+2*cos(d*x+c)+1)*((-98*c
os(d*x+c)^3-72*cos(d*x+c)^2+8*cos(d*x+c)-8)*tan(d*x+c)*sec(d*x+c)+(75*cos(
d*x+c)^2+150*cos(d*x+c)+75)*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cot(d*x+c)+csc(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.26

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[-\frac{75\sqrt{2}\left(\cos(dx+c)^4+2\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{}}{}\right.}{}\right.$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/40*(75*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*sq
rt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(
d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(49*cos(d*x + c)^3 + 36*cos(d*x + c)
^2 - 4*cos(d*x + c) + 4)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
 c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2
), 1/20*(75*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*s
qrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
/(sqrt(a)*sin(d*x + c))) + 2*(49*cos(d*x + c)^3 + 36*cos(d*x + c)^2 - 4*co
s(d*x + c) + 4)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2
*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)]
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^5(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**5/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^5}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^5/(a*sec(d*x + c) + a)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.15

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx =$$
$$-\frac{\frac{\left(\left(\left(\frac{5\sqrt{2}a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{\text{sgn}(\cos(dx+c))}-\frac{127\sqrt{2}a}{\text{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{175\sqrt{2}a}{\text{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{85\sqrt{2}a}{\text{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}-\frac{75\sqrt{2}\log}{}}{20\,d}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-1/20*((((5*sqrt(2)*a*tan(1/2*d*x + 1/2*c)^2/sgn(cos(d*x + c)) - 127*sqrt(
2)*a/sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2 + 175*sqrt(2)*a/sgn(cos(d*x
 + c)))*tan(1/2*d*x + 1/2*c)^2 - 85*sqrt(2)*a/sgn(cos(d*x + c)))*tan(1/2*d
*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a)) - 75*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^5\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sec(dx+c)^5}{\sec(dx+c)^2+2\sec(dx+c)+1}dx\right)}{a^2}$$

input
```
int(sec(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sec(c + d*x)**5)/(sec(c + d*x)**2 + 2
*sec(c + d*x) + 1),x))/a**2
```

3.127 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 145

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{11\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{13\tan(c+dx)}{3ad\sqrt{a+a\sec(c+dx)}} + \frac{7\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{6a^2d}$$

output
```
11/4*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)
/a^(3/2)/d-1/2*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)-13/3*tan(d
*x+c)/a/d/(a+a*sec(d*x+c))^(1/2)+7/6*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/a^2
/d
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.79

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\left(33\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)(1+\sec(c+dx))+2\sqrt{1-\sec(c+dx)}(-19\right.}{12d\sqrt{1-\sec(c+dx)}(a(1+\sec(c+}$$

input
```
Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
((33*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*(1 + Sec[c + d*x]) +
2*Sqrt[1 - Sec[c + d*x]]*(-19 - 12*Sec[c + d*x] + 4*Sec[c + d*x]^2))*Tan[c
 + d*x])/(12*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4303, 27, 3042, 4498, 27, 3042, 4489, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{\sec^2(c+dx)(4a-7a\sec(c+dx))}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sec^2(c+dx)(4a-7a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(4a-7a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4498}$$

$$-\frac{\frac{2\int -\frac{\sec(c+dx)\left(7a^2-26a^2\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)a+a}}dx}{3a}-\frac{14\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{4a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{\sec(c+dx)\left(7a^2-26a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{3a}-\frac{14\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{4a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(7a^2-26a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}-\frac{14\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{4a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4489}$$

$$-\frac{-\frac{33a^2\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx-\frac{52a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{14\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{4a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{33a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{52a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{14\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{4a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4282}$$

$$-\frac{-\frac{-\frac{66a^2\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{52a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{14\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{4a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{-\frac{\frac{33\sqrt{2}a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{52a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{14\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{4a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

input
```
Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
-1/2*(Sec[c + d*x]^2*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^(3/2)) - ((-14*
Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*d) - ((33*Sqrt[2]*a^(3/2)*ArcTan
[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d - (52*a^2*T
an[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(3*a))/(4*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

rule 4498
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int
[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)
*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a
*B, 0] &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.03

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left(\left(-38\cos(dx+c)^2-24\cos(dx+c)+8\right)\tan(dx+c)+\left(33\cos(dx+c)^2+66\cos(dx+c)+33\right)\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ln(\right.}{12d\,a^2\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)}$

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/d/a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)^2+2*cos(d*x+c)+1)*((-38*co
s(d*x+c)^2-24*cos(d*x+c)+8)*tan(d*x+c)+(33*cos(d*x+c)^2+66*cos(d*x+c)+33)*
2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln((-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)-cot(d*x+c)+csc(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 387, normalized size of antiderivative = 2.67

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \Bigg[-\frac{33\sqrt{2}\left(\cos(dx+c)^3+2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\cdots}{\cdots}\right)\cdots}{24(\cdots}$$

$$-\frac{33\sqrt{2}\left(\cos(dx+c)^3+2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+2\left(19\cos(\cdots\right.}{12\left(a^2d\cos(dx+c)^3+2a^2d\cos(dx+c)^2+a^2d\cos(d\cdots\right.}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/24*(33*sqrt(2)*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt
(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d
*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x
 + c)^2 + 2*cos(d*x + c) + 1)) + 4*(19*cos(d*x + c)^2 + 12*cos(d*x + c) -
4)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x +
c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c)), -1/12*(33*sqrt(2)*(co
s(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*arctan(sqrt(2)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
+ 2*(19*cos(d*x + c)^2 + 12*cos(d*x + c) - 4)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 +
 a^2*d*cos(d*x + c))]
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^4(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^4}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.23

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\frac{\left(\left(\frac{3\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{\mathrm{sgn}(\cos(dx+c))}-\frac{46\sqrt{2}}{\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{27\sqrt{2}}{\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}-\frac{33\sqrt{2}\,l}{}}{12\,d}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/12*(((3*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/sgn(cos(d*x + c)) - 46*sqrt(2)/sg
n(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2 + 27*sqrt(2)/sgn(cos(d*x + c)))*ta
n(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a)) - 33*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^4\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sec(dx+c)^4}{\sec(dx+c)^2+2\sec(dx+c)+1}dx\right)}{a^2}$$

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sec(c + d*x)**4)/(sec(c + d*x)**2 + 2
*sec(c + d*x) + 1),x))/a**2
```

3.128 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{7\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}}$$

output
```
-7/4*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)
/a^(3/2)/d+1/2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)+2*tan(d*x+c)/a/d/(a+a*s
ec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\left(-7\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)(1+\sec(c+dx)) + 2\sqrt{1-\sec(c+dx)}(5+ \cdots}{4d\sqrt{1-\sec(c+dx)}(a(1+\sec(c+dx)))^{3/2}}$$

input
```
Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
((-7*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*(1 + Sec[c + d*x]) +
2*Sqrt[1 - Sec[c + d*x]]*(5 + 4*Sec[c + d*x]))*Tan[c + d*x])/(4*d*Sqrt[1 -
 Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4286, 27, 3042, 4489, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \ 4286$$

$$\frac{\int -\frac{\sec(c+dx)(3a-4a\sec(c+dx))}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{\int \frac{\sec(c+dx)(3a-4a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3a-4a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}$$

$$\Big\downarrow \ 4489$$

$$\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{7a\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx-\frac{8a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{7a\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{8a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}$$

$\Big\downarrow$ 4282

$$\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{-\frac{14a\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{8a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}$$

$\Big\downarrow$ 216

$$\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{\frac{7\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{8a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}$$

input
```
Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
Tan[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2)) - ((7*Sqrt[2]*Sqrt[a]*ArcTan
[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d - (8*a*Tan[
c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(4*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4286
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Simp[1/(a^2*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[
a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.14

method	result
default	$\frac{\left(-\frac{(1-\cos(dx+c))^3\csc(dx+c)^3}{4}-\frac{7\ln\left(\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}-\cot(dx+c)+\csc(dx+c)\right)\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}}{4}+\frac{9\csc(dx+c)}{4}-\frac{9\cot(dx+c)}{4}\right)\sqrt{-a(-1-s}}{d\,a^2}$

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4*(1-cos(d*x+c))^3*csc(d*x+c)^3-7/4*ln((-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)-cot(d*x+c)+csc(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+9/4*
csc(d*x+c)-9/4*cot(d*x+c))/a^2*(-a*(-1-sec(d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 336, normalized size of antiderivative = 3.20

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[-\frac{7\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+c}{\cos(dx+c)}}}{\cdots}\right.}{8\left(a^2d\cos(d\right.\cdots}\right.$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log(-(2*sq
rt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*
x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(5*cos(d*x +
c) + 4)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d
), 1/4*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqr
t(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x
 + c))) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(5*cos(d*x + c) + 4)*s
in(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^3}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.50

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}-\frac{9\,\sqrt{2}}{a\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}+\frac{7\,\sqrt{2}\log\left(\left|-\sqrt{-a}\right.\right.}{}}{4\,d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/4*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(sqrt(2)*tan(1/2*d*x + 1/2*c)^2/(
a*sgn(cos(d*x + c))) - 9*sqrt(2)/(a*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*
c)/(a*tan(1/2*d*x + 1/2*c)^2 - a) + 7*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*
x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(cos(d*x
 + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{{\cos\left(c+d\,x\right)}^3\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sec(dx+c)^3}{\sec(dx+c)^2+2\sec(dx+c)+1}dx\right)}{a^2}$$

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sec(c + d*x)**3)/(sec(c + d*x)**2 + 2
*sec(c + d*x) + 1),x))/a**2
```

3.129 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{3\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

output
```
3/4*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/
a^(3/2)/d-1/2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.22

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\left(-2\sqrt{1-\sec(c+dx)}+3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)(1+\sec(c+dx))\right)\tan}{4d\sqrt{1-\sec(c+dx)}(a(1+\sec(c+dx)))^{3/2}}$$

input
```
Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
((-2*Sqrt[1 - Sec[c + d*x]] + 3*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqr
t[2]]*(1 + Sec[c + d*x]))*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*(1
+ Sec[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4284, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4284}$$

$$\frac{3\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{4a}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4282}$$

$$-\frac{3\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2ad}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{3\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

input
```
Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(3*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*S
qrt[2]*a^(3/2)*d) - Tan[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4284
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Simp[m/(b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.49

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left((2\cos(dx+c)-2)\cot(dx+c)+3(\cos(dx+c)+1)\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}\ \ln\left(\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}-\cot(dx+c)+\csc(dx+c)\right)\right)}{4d\,a^2(\cos(dx+c)+1)}$

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*((2*cos(d*x+c)-2)*cot(d*
x+c)+3*(cos(d*x+c)+1)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln((-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)-cot(d*x+c)+csc(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(62) = 124$.

Time = 0.10 (sec) , antiderivative size = 329, normalized size of antiderivative = 4.27

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[-\frac{3\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos\ldots}{c\ldots}\right)\ldots}{8\left(a^2d\cos(dx+c\ldots\right.}\right.$$

$$-\frac{3\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx\ldots}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*sqr
t(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
 + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos
(d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*s
in(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(
3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
+ 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c))/(a^
2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.40

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\frac{3\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}a\operatorname{sgn}(\cos(dx+c))}+\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2\operatorname{sgn}(\cos(dx+c))}}{4d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-1/4*(3*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(cos(d*x + c))) + sqrt(2)*sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a^2*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^2\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sec(dx+c)^2}{\sec(dx+c)^2+2\sec(dx+c)+1}dx\right)}{a^2}$$

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sec(c + d*x)**2)/(sec(c + d*x)**2 + 2
*sec(c + d*x) + 1),x))/a**2
```

3.130 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx=\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

output
```
1/4*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/
a^(3/2)/d+1/2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.21

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx=\frac{\left(2\sqrt{1-\sec(c+dx)}+\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)(1+\sec(c+dx))\right)\tan(c-}{4d\sqrt{1-\sec(c+dx)}(a(1+\sec(c+dx)))^{3/2}}$$

input
```
Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
((2*Sqrt[1 - Sec[c + d*x]] + Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2
]]*(1 + Sec[c + d*x]))*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + S
ec[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 4283, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4283}$$

$$\frac{\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{4a}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4282}$$

$$\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2ad}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

input
```
Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]/(2*Sqrt[
2]*a^(3/2)*d) + Tan[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.48

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left((\cos(dx+c)+1)\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}\,\ln\left(\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}-\cot(dx+c)+\csc(dx+c)\right)+(-2\cos(dx+c)+2)\cot(dx+c)\right)}{4d\,a^2(\cos(dx+c)+1)}$

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*((cos(d*x+c)+1)*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*ln((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cot(d
*x+c)+csc(d*x+c))+(-2*cos(d*x+c)+2)*cot(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(62) = 124$.

Time = 0.11 (sec) , antiderivative size = 327, normalized size of antiderivative = 4.25

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[-\frac{\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos\cdots}{\cos\cdots}\right)\cdots}{8\left(a^2d\cos(dx+c)\cdots\right)}\right.$$

$$-\frac{\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+\cdots}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*sqrt(
2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
 c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d
*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin
(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(sq
rt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c))/(a^2*d*
cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.40

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\frac{\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{\sqrt{-a}a\mathrm{sgn}(\cos(dx+c))} - \frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2\mathrm{sgn}(\cos(dx+c))}}{4\,d}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-1/4*(sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(cos(d*x + c))) - sqrt(2)*sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a^2*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos(c+d\,x)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sec(dx+c)}{\sec(dx+c)^2+2\sec(dx+c)+1}dx\right)}{a^2}$$

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sec(c + d*x))/(sec(c + d*x)**2 + 2*se
c(c + d*x) + 1),x))/a**2
```

3.131 $\int \frac{1}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 114

$$\int \frac{1}{(a+a\sec(c+dx))^{3/2}}\,dx=\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d}-\frac{5\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

output
```
2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d-5/4*arctan(1
/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d-1/
2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.68 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.59

$$\int \frac{1}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sec^{\frac{3}{2}}(c+dx)\left(5\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{1+\sec(c+dx)}+\sqrt{2}\left(-4\arctan\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{1}{1+\sec(c+dx)}}}\right)\right.\right.}{d\sec^2\left(\frac{1}{2}(c+dx)\right)^{3/2}(a(1+\sec(c+dx)))^{3/2}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(-3/2),x]
```

output
```
-((Sec[c + d*x]^(3/2)*(5*ArcSin[Tan[(c + d*x)/2]]*Sqrt[(1 + Sec[c + d*x])^
(-1)]*Sqrt[1 + Sec[c + d*x]] + Sqrt[2]*(-4*ArcTan[Tan[(c + d*x)/2]/Sqrt[(1
 + Sec[c + d*x])^(-1)]]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x
]] + (Sqrt[(1 + Cos[c + d*x])^(-1)]*Tan[(c + d*x)/2])/Sqrt[Sec[c + d*x]]))
)/(d*(Sec[(c + d*x)/2]^2)^(3/2)*(a*(1 + Sec[c + d*x]))^(3/2)))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4264, 27, 3042, 4408, 3042, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4264}$$

$$-\frac{\int -\frac{4a-a\sec(c+dx)}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{4a-a\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{4a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 4408}$$

$$\frac{4\int \sqrt{\sec(c+dx)a+a}dx-5a\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\downarrow$ 3042

$$\frac{4\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-5a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\downarrow$ 4261

$$\frac{-\frac{8a\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-5a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\downarrow$ 216

$$\frac{\frac{8\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-5a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\downarrow$ 4282

$$\frac{\frac{10a\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{8\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\downarrow$ 216

$$\frac{\frac{8\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{5\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

input
```
Int[(a + a*Sec[c + d*x])^(-3/2),x]
```

output
```
((8*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (
5*Sqrt[2]*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
+ d*x]])])/d)/(4*a^2) - Tan[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4264
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Simp[1/(a^2*(2*n + 1))
   Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Int
egerQ[2*n]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4408
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c/a   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[(b*c - a*d)/a   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [A] (warning: unable to verify)

Time = 1.37 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.48

method	result
default	$\frac{\sqrt{-a(-1-\sec(dx+c))}\,\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}\left(4\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\right)+\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}\,(-\cot(dx+c)+\csc(dx+\right.}{4d\,a^2}$

input
```
int(1/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^2*(-a*(-1-sec(d*x+c)))^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(4*2^(1/2)*arctanh(2^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d
*x+c)+csc(d*x+c)))+(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d
*x+c))-5*ln((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cot(d*x+c)+csc(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(93) = 186.

Time = 0.14 (sec) , antiderivative size = 491, normalized size of antiderivative = 4.31

$$\int \frac{1}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[-\frac{5\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{}\right.}{}\right.$$

input
```
integrate(1/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log(-(2*sq
rt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*
x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) + 8*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2
*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos
(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 4*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c))/(a^2*d*cos(d
*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/4*(5*sqrt(2)*(cos(d*x + c)^2
+ 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 8*(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos
(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*cos(d*x + c)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c
) + a^2*d)]
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(a\sec{(c+dx)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a*sec(c + d*x) + a)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(a\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(-3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.41

$$\int \frac{1}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{4\,a^2d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(1/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a^2
*d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(1/(a + a/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^2+2\sec(dx+c)+1}dx\right)}{a^2}$$

input
```
int(1/(a+a*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*int(sqrt(sec(c + d*x) + 1)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1)
,x))/a**2
```

3.132 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 144

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{3\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{9\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{3\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}}$$

output
```
-3*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d+9/4*arctan(
1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d-1
/2*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)+3/2*sin(d*x+c)/a/d/(a+a*sec(d*x+c))
^(1/2)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.90

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\left(2(3+2\cos(c+dx))\sqrt{1-\sec(c+dx)} - 12\text{arctanh}\left(\sqrt{1-\sec(c+dx)}\right)(\cdots\right.}{4d\sqrt{1-\sec(c+dx)}(\cdots}$$

input
```
Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
((2*(3 + 2*Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] - 12*ArcTanh[Sqrt[1 - Sec[
c + d*x]]]*(1 + Sec[c + d*x]) + 9*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/S
qrt[2]]*(1 + Sec[c + d*x]))*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*(
1 + Sec[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 4304, 27, 3042, 4510, 25, 3042, 4408, 3042, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{3\cos(c+dx)(2a-a\sec(c+dx))}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{\cos(c+dx)(2a-a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{2a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4510}$$

$$\frac{3\left(\frac{\int -\frac{2a^2-a^2\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{a}+\frac{2a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{3\left(\frac{2a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{2a^2-a^2\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3\left(\frac{2a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{2a^2-a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 4408}$$

$$\frac{3\left(\frac{2a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{2a\int\sqrt{\sec(c+dx)a+a}dx-3a^2\int\frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3\left(\frac{2a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{2a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-3a^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 4261}$$

$$\frac{3\left(\frac{2a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{-3a^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{4a^2\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{3\left(\frac{2a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{4a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-3a^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4282}$$

$$\frac{3\left(\frac{2a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{6a^2\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{4a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{3\left(\frac{2a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{4a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{3\sqrt{2}a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

input
```
Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
-1/2*Sin[c + d*x]/(d*(a + a*Sec[c + d*x])^(3/2)) + (3*(-(((4*a^(3/2)*ArcTa
n[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (3*Sqrt[2]*a^(3/2)
*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d)/a)
+ (2*a*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/(4*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4408
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c/a   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[(b*c - a*d)/a   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(119) = 238$.

Time = 3.13 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.67

method	result
default	$\frac{\left(\sin(dx+c)\cos(dx+c)(4\cos(dx+c)+6)+\left(9\cos(dx+c)^{2}+18\cos(dx+c)+9\right)\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}\,\ln\left(\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}-\cot(dx+c)+\csc(\right.\right.}{4d\,a^{2}\left(\cos(\right.}$

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^2*(sin(d*x+c)*cos(d*x+c)*(4*cos(d*x+c)+6)+(9*cos(d*x+c)^2+18*cos(d
*x+c)+9)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln((-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)-cot(d*x+c)+csc(d*x+c))+(6*cos(d*x+c)^2+12*cos(d*x+c)+6)*2^(1/2
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(2^(1/2)*(cot(d*x+c)-csc(d*x
+c))/(csc(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+cot(d*x+c)^2-1)^(1/2)))*(a*(1+s
ec(d*x+c)))^(1/2)/(cos(d*x+c)^2+2*cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 518, normalized size of antiderivative = 3.60

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[-\frac{9\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\mathrm{c}\cdots}{\mathrm{c}\cdots}\right.}{\cdots}\right.$$

$$-\frac{9\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-12\left(\cos(dx+c)^2+2\,\mathrm{co}\cdots\right.}{4\left(a^2d\cos(dx+c)^2\cdots\right.}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(9*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*sqr
t(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
 + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos
(d*x + c) + 1)) + 12*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2
*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos
(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 4*(2*co
s(d*x + c)^2 + 3*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin
(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(9*
sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) -
12*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*(2*cos(d*x
+ c)^2 + 3*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
 c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(dx+c\right)}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(119) = 238$.

Time = 0.51 (sec) , antiderivative size = 387, normalized size of antiderivative = 2.69

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{16\sqrt{2}\left(3\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}-a\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{4}-6\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}\right.\right.}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/8*(16*sqrt(2)*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^2 - a)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a))^2*a + a^2)*sqrt(-a)*sgn(cos(d*x + c))) - 9*sqrt(2)*l
og((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2
)/(sqrt(-a)*a*sgn(cos(d*x + c))) + 2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a)*tan(1/2*d*x + 1/2*c)/(a^2*sgn(cos(d*x + c))) + 12*log(abs(-2*(sqrt(
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt
(2)*abs(a) + 6*a)/abs(-2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) + 6*a))/(sqrt(-a)*abs(a)*sgn(cos
(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)/(a + a/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\cos(dx+c)}{\sec(dx+c)^2+2\sec(dx+c)+1}dx\right)}{a^2}$$

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*cos(c + d*x))/(sec(c + d*x)**2 + 2*se
c(c + d*x) + 1),x))/a**2
```

3.133 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 185

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{19\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a^{3/2}d} - \frac{13\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{7\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{ad\sqrt{a+a\sec(c+dx)}}$$

output
```
19/4*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d-13/4*arct
an(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/
d-1/2*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)-7/4*sin(d*x+c)/a/d/(a
+a*sec(d*x+c))^(1/2)+cos(d*x+c)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.18 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.96

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{-\sin(2(c+dx)) + \frac{\left(\frac{13}{4}\left(7\text{arctanh}\left(\sqrt{1-\sec(c+dx)}\right)-4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)+\cos(\cdots}{\cdots}}{4a\cdots}$$

input
```
Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(-Sin[2*(c + d*x)] + (((13*(7*ArcTanh[Sqrt[1 - Sec[c + d*x]]] - 4*Sqrt[2]*
ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + Cos[c + d*x]*(-1 + 2*Cos[c + d*x
])*Sqrt[1 - Sec[c + d*x]]))/4 - 10*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[
c + d*x]]*Sqrt[1 - Sec[c + d*x]])*(1 + Sec[c + d*x])*Tan[c + d*x])/Sqrt[1
- Sec[c + d*x]])/(4*d*(a*(1 + Sec[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.08, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.696, Rules used = {3042, 4304, 27, 3042, 4510, 27, 3042, 4510, 27, 3042, 4408, 3042, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a\sec(c+dx)+a)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{\cos^2(c+dx)(8a-5a\sec(c+dx))}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cos^2(c+dx)(8a-5a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{8a-5a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4510}$$

$$\frac{\frac{\int -\frac{2\cos(c+dx)\left(7a^2-6a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{2a}+\frac{4a\sin(c+dx)\cos(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{4a\sin(c+dx)\cos(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{\cos(c+dx)\left(7a^2-6a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{a}}{4a^2}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{4a\sin(c+dx)\cos(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{7a^2-6a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}}{4a^2}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4510}$$

$$\frac{\frac{4a\sin(c+dx)\cos(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{\int -\frac{19a^3-7a^3\sec(c+dx)}{2\sqrt{\sec(c+dx)a+a}}dx}{a}+\frac{7a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{a}}{4a^2}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{4a\sin(c+dx)\cos(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{7a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{19a^3-7a^3\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{2a}}{a}}{4a^2}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{4a\sin(c+dx)\cos(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{7a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{19a^3-7a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}}{a}}{4a^2}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\downarrow$ 4408

$$\frac{\frac{4a\sin(c+dx)\cos(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{7a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{19a^2\int\sqrt{\sec(c+dx)a+a}dx-26a^3\int\frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{2a}}{a}}{4a^2}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{4a\sin(c+dx)\cos(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{7a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{19a^2\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-26a^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}}{a}}{4a^2}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\downarrow$ 4261

$$\frac{\frac{4a\sin(c+dx)\cos(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{7a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{-26a^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{38a^3\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{2a}}{a}}{4a^2}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\downarrow$ 216

$$\frac{\frac{4a\sin(c+dx)\cos(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{7a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{38a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-26a^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}}{a}}{4a^2}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\downarrow$ 4282

$$\frac{\frac{4a\sin(c+dx)\cos(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{7a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{52a^3\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{38a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{2a}}{a}}{4a^2}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{4a\sin(c+dx)\cos(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{7a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{38a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{26\sqrt{2}a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{2a}}{a}}{4a^2}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

input

```
Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

output

```
-1/2*(Cos[c + d*x]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^(3/2)) + ((4*a*Co
s[c + d*x]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) - (-1/2*((38*a^(5/2)
*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (26*Sqrt[2]*
a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])
/d)/a + (7*a^2*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/a)/(4*a^2)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4408
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c/a   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[(b*c - a*d)/a   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Maple [A] (warning: unable to verify)

Time = 3.60 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.35

method	result
default	$-\frac{\left(\sin(dx+c)\cos(dx+c)\left(-2\cos(dx+c)^2+3\cos(dx+c)+7\right)+\left(19\cos(dx+c)^2+38\cos(dx+c)+19\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \operatorname{arctanh}\left(\frac{}{\sqrt{\csc}}\right.}{}$

input
```
int(cos(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/a^2*(sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+c)^2+3*cos(d*x+c)+7)+(19*cos
(d*x+c)^2+38*cos(d*x+c)+19)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(2^(
1/2)*(cot(d*x+c)-csc(d*x+c))/(csc(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+cot(d*x
+c)^2-1)^(1/2))+(13*cos(d*x+c)^2+26*cos(d*x+c)+13)*2^(1/2)*(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*ln((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cot(d*x+c)+csc
(d*x+c)))*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)^2+2*cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.90

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[-\frac{13\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)}{\cos(dx+c)}}}{}\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(13*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log(-(2*s
qrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d
*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*c
os(d*x + c) + 1)) + 19*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log(
(2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*c
os(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(2*
cos(d*x + c)^3 - 3*cos(d*x + c)^2 - 7*cos(d*x + c))*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x +
c) + a^2*d), 1/4*(13*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)
*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt
(a)*sin(d*x + c))) - 19*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arct
an(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x +
 c))) + (2*cos(d*x + c)^3 - 3*cos(d*x + c)^2 - 7*cos(d*x + c))*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d
*cos(d*x + c) + a^2*d)]
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos^2{(c+dx)}}{(a(\sec{(c+dx)}+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)**2/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos{(dx+c)}^2}{(a\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^2/(a*sec(d*x + c) + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(156) = 312$.

Time = 0.56 (sec) , antiderivative size = 473, normalized size of antiderivative = 2.56

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/8*(13*sqrt(2)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a))^2)/(sqrt(-a)*a*sgn(cos(d*x + c))) - 2*sqrt(2)*sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a^2*sgn(cos(d*x + c))) + 19*
log(abs(8*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a))^2 - 16*sqrt(2)*abs(a) - 24*a)/abs(8*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
 sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 16*sqrt(2)*abs(a) - 24*a))/(sqrt
(-a)*abs(a)*sgn(cos(d*x + c))) - 4*sqrt(2)*(29*(sqrt(-a)*tan(1/2*d*x + 1/2
*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6 - 133*(sqrt(-a)*tan(1/2*d*x +
 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a + 55*(sqrt(-a)*tan(1/2*
d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^2 - 7*a^3)/(((sqrt
(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a
^2)^2*sqrt(-a)*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^2/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^2/(a + a/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\cos(dx+c)^2}{\sec(dx+c)^2+2\sec(dx+c)+1}dx\right)}{a^2}$$

input
```
int(cos(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*cos(c + d*x)**2)/(sec(c + d*x)**2 + 2
*sec(c + d*x) + 1),x))/a**2
```

3.134 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 183

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{163\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sec^2(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{197\tan(c+dx)}{24a^2d\sqrt{a+a\sec(c+dx)}} + \frac{95\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{48a^3d}$$

output
```
163/32*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/
2)/a^(5/2)/d-1/4*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-17/16*se
c(d*x+c)^2*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)-197/24*tan(d*x+c)/a^2/d/(
a+a*sec(d*x+c))^(1/2)+95/48*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/a^3/d
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.74

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\left(978\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)+\sqrt{1-\sec(c\right.}{48d\sqrt{1-\sec(c+d}}$$

input
```
Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
((978*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^4*S
ec[c + d*x]^2 + Sqrt[1 - Sec[c + d*x]]*(-299 - 503*Sec[c + d*x] - 160*Sec[
c + d*x]^2 + 32*Sec[c + d*x]^3))*Tan[c + d*x])/(48*d*Sqrt[1 - Sec[c + d*x]
]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {3042, 4303, 27, 3042, 4507, 27, 3042, 4498, 27, 3042, 4489, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{\sec^3(c+dx)(6a-11a\sec(c+dx))}{2(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sec^3(c+dx)(6a-11a\sec(c+dx))}{(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(6a-11a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 4507

$$-\frac{\frac{\int \frac{\sec^2(c+dx)\left(68a^2-95a^2\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}+\frac{17a\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\int \frac{\sec^2(c+dx)\left(68a^2-95a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}+\frac{17a\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(68a^2-95a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{17a\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 4498

$$-\frac{\frac{\frac{2\int -\frac{\sec(c+dx)\left(95a^3-394a^3\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)a+a}}dx}{3a}-\frac{190a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{4a^2}+\frac{17a\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 27

$$-\frac{\frac{-\frac{\int \frac{\sec(c+dx)\left(95a^3-394a^3\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{3a}-\frac{190a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{4a^2}+\frac{17a\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(95a^3-394a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}-\frac{190a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{4a^2}+\frac{17a\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 4489

$$-\frac{\frac{-\frac{489a^3\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx-\frac{788a^3\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{190a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{4a^2}+\frac{17a\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{-\frac{489a^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{788a^3\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{190a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{4a^2}+\frac{17a\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 4282

$$-\frac{\frac{-\frac{-\frac{978a^3\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{788a^3\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{190a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{4a^2}+\frac{17a\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 216

$$-\frac{\frac{-\frac{\frac{489\sqrt{2}a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{788a^3\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{190a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{4a^2}+\frac{17a\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

input
```
Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
-1/4*(Sec[c + d*x]^3*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^(5/2)) - ((17*a
*Sec[c + d*x]^2*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + ((-190*a*
Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*d) - ((489*Sqrt[2]*a^(5/2)*ArcTa
n[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d - (788*a^3
*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(3*a))/(4*a^2))/(8*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

rule 4498
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int
[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)
*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a
*B, 0] &&  !LtQ[m, -1]
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.98

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left(\left(-598\cos(dx+c)^3-1006\cos(dx+c)^2-320\cos(dx+c)+64\right)\tan(dx+c)+\left(489\cos(dx+c)^3+1467\cos(dx+c)^2+\right.\right.}{96d\,a^3\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+\right.}$

input
```
int(sec(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/96/d/a^3*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)^3+3*cos(d*x+c)^2+3*cos(d*x
+c)+1)*((-598*cos(d*x+c)^3-1006*cos(d*x+c)^2-320*cos(d*x+c)+64)*tan(d*x+c)
+(489*cos(d*x+c)^3+1467*cos(d*x+c)^2+1467*cos(d*x+c)+489)*2^(1/2)*(-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*ln((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cot(d*x
+c)+csc(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 455, normalized size of antiderivative = 2.49

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[-\frac{489\sqrt{2}\left(\cos(dx+c)^4+3\cos(dx+c)^3+3\cos(dx+c)^2+\cos(dx+c)\right)\ldots}{\ldots}\right.$$

$$-\frac{489\sqrt{2}\left(\cos(dx+c)^4+3\cos(dx+c)^3+3\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(d\ldots}{\sqrt{a}\sin(dx+c)}\ldots\right.}{96\left(a^3d\cos(dx+c)^4+3a^3d\cos(dx+c)^3+\ldots\right.}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[-1/192*(489*sqrt(2)*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2
 + cos(d*x + c))*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d
*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(299*cos(d*x + c)^
3 + 503*cos(d*x + c)^2 + 160*cos(d*x + c) - 32)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3
 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c)), -1/96*(489*sqrt(2)*(cos(d
*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*ar
ctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)
*sin(d*x + c))) + 2*(299*cos(d*x + c)^3 + 503*cos(d*x + c)^2 + 160*cos(d*x
 + c) - 32)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*c
os(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*co
s(d*x + c))]
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^5{(c+dx)}}{(a(\sec{(c+dx)}+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**5/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec{(dx+c)}^5}{(a\sec{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^5/(a*sec(d*x + c) + a)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.20

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\frac{\left(\left(3\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a\mathrm{sgn}(\cos(dx+c))}+\frac{23\sqrt{2}}{a\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{668\sqrt{2}}{a\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+ \frac{}{as}}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}}{96\,a}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/96*(((3*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/(a*sgn(cos(d*x + c))) + 23*sqr
t(2)/(a*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 - 668*sqrt(2)/(a*sgn(co
s(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 + 465*sqrt(2)/(a*sgn(cos(d*x + c))))*
tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a)) - 489*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^5\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sec(dx+c)^5}{\sec(dx+c)^3+3\sec(dx+c)^2+3\sec(dx+c)+1}dx\right)}{a^3}$$

input
```
int(sec(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sec(c + d*x)**5)/(sec(c + d*x)**3 + 3
*sec(c + d*x)**2 + 3*sec(c + d*x) + 1),x))/a**3
```

3.135 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 145

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{75\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{13\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{9\tan(c+dx)}{4a^2d\sqrt{a+a\sec(c+dx)}}$$

output
```
-75/32*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/
2)/a^(5/2)/d-1/4*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)+13/16*ta
n(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)+9/4*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^
(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.86

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\left(-150\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx) + \sqrt{1-\sec(\cdots}{16d\sqrt{1-\sec(c+dx)}(a(1+\text{s}\cdots}$$

input
```
Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
((-150*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^4*
Sec[c + d*x]^2 + Sqrt[1 - Sec[c + d*x]]*(49 + 85*Sec[c + d*x] + 32*Sec[c +
 d*x]^2))*Tan[c + d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x])
)^(5/2))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4303, 27, 3042, 4496, 27, 3042, 4489, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{\sec^2(c+dx)(4a-9a\sec(c+dx))}{2(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sec^2(c+dx)(4a-9a\sec(c+dx))}{(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(4a-9a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{4496}$$

$$-\frac{-\frac{\int -\frac{3\sec(c+dx)\left(13a^2-12a^2\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{13a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{3\int \frac{\sec(c+dx)\left(13a^2-12a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{13a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(13a^2-12a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{13a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 4489$$

$$-\frac{\frac{3\left(25a^2\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx-\frac{24a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{13a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{3\left(25a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{24a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{13a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 4282$$

$$-\frac{\frac{3\left(-\frac{50a^2\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{24a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{13a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 216$$

$$-\frac{\frac{3\left(\frac{25\sqrt{2}a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{24a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{13a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

input
```
Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
-1/4*(Sec[c + d*x]^2*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^(5/2)) - ((-13*
a*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + (3*((25*Sqrt[2]*a^(3/2)
*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d - (2
4*a^2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/(4*a^2))/(8*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

rule 4496
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(b^2*(2*m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b
*B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Ne
Q[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.98

method	result
default	$\frac{\left(-\frac{(1-\cos(dx+c))^5 \csc(dx+c)^5}{16}-\frac{17(1-\cos(dx+c))^3 \csc(dx+c)^3}{32}-\frac{75 \ln\left(\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}-\cot(dx+c)+\csc(dx+c)\right)\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}}{32}+\frac{83 \csc(d}{32}\right.}{d\,a^3}$

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/16*(1-cos(d*x+c))^5*csc(d*x+c)^5-17/32*(1-cos(d*x+c))^3*csc(d*x+c)
^3-75/32*ln((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cot(d*x+c)+csc(d*x+c))*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+83/32*csc(d*x+c)-83/32*cot(d*x+c))/a^3*
(-a*(-1-sec(d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 404, normalized size of antiderivative = 2.79

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[-\frac{75\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\log\left(-\right.}{64\,(}\right.$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[-1/64*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(49*cos(d*x + c)^2 + 85*cos(d*x
+ c) + 32)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*co
s(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/3
2*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqr
t(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(
sqrt(a)*sin(d*x + c))) + 2*(49*cos(d*x + c)^2 + 85*cos(d*x + c) + 32)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 +
3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^4(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^4}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.30

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\,\Big(\Big(\frac{2\,\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^2\mathrm{sgn}(\cos(dx+c))}+\frac{17\,\sqrt{2}}{a^2\mathrm{sgn}(\cos(dx+c))}\Big)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\frac{83\,\sqrt{2}}{a^2\mathrm{sgn}(\cos(dx+c)}}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}}{32\,d}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*((2*sqrt(2)*tan(1/2*d*x + 1/2*c)
^2/(a^2*sgn(cos(d*x + c))) + 17*sqrt(2)/(a^2*sgn(cos(d*x + c))))*tan(1/2*d
*x + 1/2*c)^2 - 83*sqrt(2)/(a^2*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)/(
a*tan(1/2*d*x + 1/2*c)^2 - a) + 75*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x +
 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(cos(d*x
+ c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^4\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sec(dx+c)^4}{\sec(dx+c)^3+3\sec(dx+c)^2+3\sec(dx+c)+1}dx\right)}{a^3}$$

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sec(c + d*x)**4)/(sec(c + d*x)**3 + 3
*sec(c + d*x)**2 + 3*sec(c + d*x) + 1),x))/a**3
```

3.136 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 107

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{19\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{13\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

output
```
19/32*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2
)/a^(5/2)/d+1/4*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-13/16*tan(d*x+c)/a/d/(
a+a*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.08

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\left(76\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx) - 2\sqrt{1-\sec(c}{32d\sqrt{1-\sec(c+dx)}(a(1+\sec(c+dx)}$$

input
```
Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
((76*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^4*Se
c[c + d*x]^2 - 2*Sqrt[1 - Sec[c + d*x]]*(9 + 13*Sec[c + d*x]))*Tan[c + d*x
])/(32*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4286, 27, 3042, 4488, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4286}$$

$$\frac{\int -\frac{\sec(c+dx)(5a-8a\sec(c+dx))}{2(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-\frac{\int \frac{\sec(c+dx)(5a-8a\sec(c+dx))}{(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(5a-8a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}$$

$$\Big\downarrow \text{4488}$$

$$\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-\frac{\frac{13a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{19}{4}\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{8a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-\frac{\frac{13a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{19}{4}\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{8a^2}$$

$$\Big\downarrow \text{4282}$$

$$\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-\frac{\frac{19\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2d}+\frac{13a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-\frac{\frac{13a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{19\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\sqrt{a}d}}{8a^2}$$

input
```
Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
Tan[c + d*x]/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((-19*ArcTan[(Sqrt[a]*Tan[
c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*Sqrt[a]*d) + (13
*a*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)))/(8*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4286
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Simp[1/(a^2*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[
a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4488
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
 /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.51

method	result
default	$\frac{\sqrt{-a\left(-1-\sec\left(dx+c\right)\right)}\,\sqrt{-\frac{2\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\,\left(2\left(1-\cos\left(dx+c\right)\right)^{3}\sqrt{-\frac{2\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\,\csc\left(dx+c\right)^{3}+11\sqrt{-\frac{2\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\,\left(-\cot\left(dx+c\right)+\csc\left(dx\right.\right.\right.}{32d\,a^{3}}$

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/d/a^3*(-a*(-1-sec(d*x+c)))^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(2*(1-cos(d*x+c))^3*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*csc(d*x+c)^3+11*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c))+19*ln((-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)-cot(d*x+c)+csc(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(88) = 176$.

Time = 0.11 (sec) , antiderivative size = 403, normalized size of antiderivative = 3.77

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[-\frac{19\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2}{-}\right.}{64\left(a^3d\right.}\right.$$

$$-\frac{19\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+2\left(9\right.}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx\right.}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[-1/64*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(9*cos(d*x + c)^2 + 13*cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x +
 c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(19*
sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*a
rctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a
)*sin(d*x + c))) + 2*(9*cos(d*x + c)^2 + 13*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(
d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx=\int \frac{\sec\left(dx+c\right)^3}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.30

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx=-\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\,\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3\mathrm{sgn}(\cos(dx+c))}+\frac{11\,\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{19\,\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\right.\right.\right.}{\sqrt{-a}a^2\mathrm{sg}}}{32\,d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)
^2/(a^3*sgn(cos(d*x + c))) + 11*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d
*x + 1/2*c) + 19*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^3\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sec(dx+c)^3}{\sec(dx+c)^3+3\sec(dx+c)^2+3\sec(dx+c)+1}dx\right)}{a^3}$$

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sec(c + d*x)**3)/(sec(c + d*x)**3 + 3
*sec(c + d*x)**2 + 3*sec(c + d*x) + 1),x))/a**3
```

3.137 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 107

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{5\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{5\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

output
```
5/32*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)
/a^(5/2)/d-1/4*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)+5/16*tan(d*x+c)/a/d/(a+
a*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.07

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\left(10\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)+\sqrt{1-\sec(c+\cdots}\right.}{16d\sqrt{1-\sec(c+dx)}(a(1+\sec(c+dx)))\cdots}$$

input
```
Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
((10*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^4*Se
c[c + d*x]^2 + Sqrt[1 - Sec[c + d*x]]*(1 + 5*Sec[c + d*x]))*Tan[c + d*x])/
(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4284, 3042, 4283, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4284}$$

$$\frac{5\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^{3/2}}dx}{8a}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{4283}$$

$$\frac{5\left(\frac{\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{4a}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{ 4282}$$

$$\frac{5\left(\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2ad}\right)}{8a}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{5\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

input
```
Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
-1/4*Tan[c + d*x]/(d*(a + a*Sec[c + d*x])^(5/2)) + (5*(ArcTan[(Sqrt[a]*Tan
[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) + Tan
[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2))))/(8*a)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

rule 4284
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Simp[m/(b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(88) = 176$.

Time = 1.42 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.79

method	result
default	$\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\,\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\,\left(-2\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^{\frac{3}{2}}(-\cot(dx+c)+\csc(dx\right.$

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/d/a^3*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*((1-cos(d*x+c))^
2*csc(d*x+c)^2-1)^(1/2)*(-2*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(3/2)*(-cot(
d*x+c)+csc(d*x+c))-5*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+
csc(d*x+c))+5*ln(csc(d*x+c)-cot(d*x+c)+((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(
1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 399, normalized size of antiderivative = 3.73

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[-\frac{5\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{}}{}\right.}{64\left(a^3d\cos\right.}\right.$$

$$-\frac{5\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-2\left(\cos\right.}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+\right.}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[-1/64*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(co
s(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(cos(d*x + c)^2 + 5*cos(d*x + c))*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^
3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(5*sqrt(
2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan
(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin
(d*x + c))) - 2*(cos(d*x + c)^2 + 5*cos(d*x + c))*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)
^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^2}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.30

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3\mathrm{sgn}(\cos(dx+c))}+\frac{3\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\right.}{32\,d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^
2/(a^3*sgn(cos(d*x + c))) + 3*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d*x
 + 1/2*c) - 5*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^2\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sec(dx+c)^2}{\sec(dx+c)^3+3\sec(dx+c)^2+3\sec(dx+c)+1}dx\right)}{a^3}$$

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sec(c + d*x)**2)/(sec(c + d*x)**3 + 3
*sec(c + d*x)**2 + 3*sec(c + d*x) + 1),x))/a**3
```

3.138 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{3\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{3\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

output
```
3/32*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)
/a^(5/2)/d+1/4*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)+3/16*tan(d*x+c)/a/d/(a+
a*sec(d*x+c))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.49

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)\tan(c+dx)}{4a^2d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(Hypergeometric2F1[1/2, 3, 3/2, (1 - Sec[c + d*x])/2]*Tan[c + d*x])/(4*a^2
*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4283, 3042, 4283, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4283}$$

$$\frac{3\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^{3/2}}dx}{8a}+\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a}+\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{4283}$$

$$\frac{3\left(\frac{\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{4a}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}+\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}+\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 4282$$

$$\frac{3\left(\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2ad}\right)}{8a}+\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 216$$

$$\frac{3\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}+\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

input
```
Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
Tan[c + d*x]/(4*d*(a + a*Sec[c + d*x])^(5/2)) + (3*(ArcTan[(Sqrt[a]*Tan[c
+ d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) + Tan[c
+ d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2))))/(8*a)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(88) = 176$.

Time = 1.47 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.79

method	result
default	$\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\left(2\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^{\frac{3}{2}}(-\cot(dx+c)+\csc(dx+c$ ... $}{3}$

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/d/a^3*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*((1-cos(d*x+c))^
2*csc(d*x+c)^2-1)^(1/2)*(2*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(3/2)*(-cot(d
*x+c)+csc(d*x+c))-3*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+c
sc(d*x+c))+3*ln(csc(d*x+c)-cot(d*x+c)+((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1
/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(88) = 176$.

Time = 0.11 (sec) , antiderivative size = 403, normalized size of antiderivative = 3.77

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \Bigg[-\frac{3\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{\ldots}}{\ldots}\right.}{64\left(a^3d\cos\ldots\right.}$$

$$-\frac{3\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-2\left(7\cos\ldots\right.}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+\ldots\right.}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[-1/64*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(co
s(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(7*cos(d*x + c)^2 + 3*cos(d*x + c)
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c
)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(3*sqr
t(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arct
an(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*s
in(d*x + c))) - 2*(7*cos(d*x + c)^2 + 3*cos(d*x + c))*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x
+ c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.30

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\,\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3\mathrm{sgn}(\cos(dx+c))}-\frac{5\,\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{3\,\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\right.\right.\right.}{\sqrt{-a}a^2\mathrm{sg}}}{32\,d}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)
^2/(a^3*sgn(cos(d*x + c))) - 5*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d*
x + 1/2*c) + 3*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos(c+d\,x)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sec(dx+c)}{\sec(dx+c)^3+3\sec(dx+c)^2+3\sec(dx+c)+1}dx\right)}{a^3}$$

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sec(c + d*x))/(sec(c + d*x)**3 + 3*se
c(c + d*x)**2 + 3*sec(c + d*x) + 1),x))/a**3
```

3.139 $\int \frac{1}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 144

$$\int \frac{1}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{43\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{11\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

output
```
2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d-43/32*arctan
(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d-
1/4*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-11/16*tan(d*x+c)/a/d/(a+a*sec(d*x+
c))^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 4.16 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.47

$$\int \frac{1}{(a+a\sec(c+dx))^{5/2}}\,dx =$$
$$-\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(-64\arctan\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{1}{1+\sec(c+dx)}}}\right)\sqrt{\frac{\cos(c+dx)}{(1+\cos(c+dx))^2}}\sqrt{1+\sec(c+dx)}+43\arcsin\left(\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(-5/2),x]
```

output
```
-1/4*(Cos[(c + d*x)/2]^6*Sec[c + d*x]^(5/2)*(-64*ArcTan[Tan[(c + d*x)/2]/S
qrt[(1 + Sec[c + d*x])^(-1)]]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])^2]*Sqrt
[1 + Sec[c + d*x]] + 43*ArcSin[Tan[(c + d*x)/2]]*Sqrt[Sec[(c + d*x)/2]^2]*
Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x]] + (30 - 19*Sec[(c + d
*x)/2]^2 + 2*Sec[(c + d*x)/2]^4)*Sqrt[Sec[c + d*x]]*Tan[(c + d*x)/2]))/(d*
(a*(1 + Sec[c + d*x]))^(5/2))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {3042, 4264, 27, 3042, 4410, 27, 3042, 4408, 3042, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)^{5/2}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$
$$\Big\downarrow \text{ 4264}$$

$$-\frac{\int -\frac{8a-3a\sec(c+dx)}{2(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{8a-3a\sec(c+dx)}{(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{8a-3a\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 4410

$$\frac{-\frac{\int -\frac{32a^2-11a^2\sec(c+dx)}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{11a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{32a^2-11a^2\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{11a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{32a^2-11a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{11a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 4408

$$\frac{\frac{32a\int \sqrt{\sec(c+dx)a+a}dx-43a^2\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{11a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{32a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-43a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{11a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 4261

$$\frac{\frac{-43a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{64a^2\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{4a^2}-\frac{11a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{64a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-43a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{11a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{4282}$$

$$\frac{\frac{\frac{86a^2\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{64a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}-\frac{11a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{64a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{43\sqrt{2}a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}-\frac{11a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

input
```
Int[(a + a*Sec[c + d*x])^(-5/2),x]
```

output
```
-1/4*Tan[c + d*x]/(d*(a + a*Sec[c + d*x])^(5/2)) + (((64*a^(3/2)*ArcTan[(S
qrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (43*Sqrt[2]*a^(3/2)*Ar
cTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d)/(4*a^2
) - (11*a*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)))/(8*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4264
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Simp[1/(a^2*(2*n + 1))
   Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Int
egerQ[2*n]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4408
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c/a   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[(b*c - a*d)/a   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 4410
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e +
f*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&
EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Maple [A] (warning: unable to verify)

Time = 1.62 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.47

method	result
default	$\frac{\sqrt{-a(-1-\sec(dx+c))}\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}\left(-2(1-\cos(dx+c))^3\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}\csc(dx+c)^3+32\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(d}{\sqrt{(1-\cos(dx+c))^2\csc(dx}}\right.\right.}{32d\,a^3}$

input
```
int(1/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/d/a^3*(-a*(-1-sec(d*x+c)))^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(-2*(1-cos(d*x+c))^3*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*csc(d*x+c)^3+32
*2^(1/2)*arctanh(2^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x
+c)+csc(d*x+c)))+13*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(
d*x+c))-43*ln((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cot(d*x+c)+csc(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(119) = 238.

Time = 0.27 (sec) , antiderivative size = 585, normalized size of antiderivative = 4.06

$$\int \frac{1}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[-1/64*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 64*(cos(d*x + c)^3 + 3*cos(d*x + c
)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*
x + c) - a)/(cos(d*x + c) + 1)) + 4*(15*cos(d*x + c)^2 + 11*cos(d*x + c))*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^
3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(43*sqrt(
2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan
(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin
(d*x + c))) - 64*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(
a)*sin(d*x + c))) - 2*(15*cos(d*x + c)^2 + 11*cos(d*x + c))*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*co
s(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{(a\sec(c+dx)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral((a*sec(c + d*x) + a)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+a\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(-5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.54

$$\int \frac{1}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3\mathrm{sgn}(\cos(dx+c))}-\frac{13\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\right.}{32\,d}$$

input
```
integrate(1/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/32*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2
/(a^3*sgn(cos(d*x + c))) - 13*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d*x
 + 1/2*c)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(1/(a + a/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^3+3\sec(dx+c)^2+3\sec(dx+c)+1}dx\right)}{a^3}$$

input
```
int(1/(a+a*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*int(sqrt(sec(c + d*x) + 1)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 +
 3*sec(c + d*x) + 1),x))/a**3
```

3.140 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 174

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{5\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{115\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{15\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{35\sin(c+dx)}{16a^2d\sqrt{a+a\sec(c+dx)}}$$

output

```
-5*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d+115/32*arct
an(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/
d-1/4*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-15/16*sin(d*x+c)/a/d/(a+a*sec(d*
x+c))^(3/2)+35/16*sin(d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.97

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{460\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\cos^5\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right) -$$

input
```
Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(460*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^5*Se
c[c + d*x]^3*Sin[(c + d*x)/2] - 80*ArcTanh[Sqrt[1 - Sec[c + d*x]]]*(1 + Se
c[c + d*x])^2*Tan[c + d*x] + Sqrt[1 - Sec[c + d*x]]*(16*Sin[c + d*x] + 5*(
11 + 7*Sec[c + d*x])*Tan[c + d*x]))/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + S
ec[c + d*x]))^(5/2))
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.09, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {3042, 4304, 27, 3042, 4508, 27, 3042, 4510, 25, 3042, 4408, 3042, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4304}$$

$$-\frac{\int -\frac{5\cos(c+dx)(2a-a\sec(c+dx))}{2(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{5\int \frac{\cos(c+dx)(2a-a\sec(c+dx))}{(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{5\int \frac{2a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 4508

$$\frac{5\left(\frac{\int \frac{\cos(c+dx)\left(14a^2-9a^2\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{3a\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 27

$$\frac{5\left(\frac{\int \frac{\cos(c+dx)\left(14a^2-9a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{3a\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{5\left(\frac{\int \frac{14a^2-9a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{3a\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 4510

$$\frac{5\left(\frac{\frac{\int -\frac{16a^3-7a^3\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{a}+\frac{14a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}-\frac{3a\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 25

$$\frac{5\left(\frac{\frac{14a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{16a^3-7a^3\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{a}}{4a^2}-\frac{3a\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{5\left(\frac{\frac{14a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{16a^3-7a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}}{4a^2}-\frac{3a\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 4408

$$\frac{5\left(\frac{\frac{14a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{16a^2\int\sqrt{\sec(c+dx)a+a}dx-23a^3\int\frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{a}}{4a^2}-\frac{3a\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{5\left(\frac{\frac{14a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{16a^2\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-23a^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}}{4a^2}-\frac{3a\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 4261

$$\frac{5\left(\frac{\frac{14a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{-23a^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{32a^3\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{a}}{4a^2}-\frac{3a\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 216

$$\frac{5\left(\frac{\frac{14a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{32a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-23a^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}}{4a^2}-\frac{3a\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 4282

$$\frac{5\left(\frac{\frac{14a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{46a^3\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{32a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}}{4a^2}-\frac{3a\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 216

$$\frac{5\left(\frac{\frac{14a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{32a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{23\sqrt{2}a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}}{4a^2}-\frac{3a\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

input
```
Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
-1/4*Sin[c + d*x]/(d*(a + a*Sec[c + d*x])^(5/2)) + (5*((-3*a*Sin[c + d*x])
/(2*d*(a + a*Sec[c + d*x])^(3/2)) + (-(((32*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c
+ d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (23*Sqrt[2]*a^(5/2)*ArcTan[(Sqrt[a]
*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d)/a) + (14*a^2*Sin[c
+ d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(4*a^2)))/(8*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4408
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c/a   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[(b*c - a*d)/a   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
 - a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Maple [A] (warning: unable to verify)

Time = 3.46 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.61

method	result
default	$\frac{\left(\sin(dx+c)\cos(dx+c)\left(32\cos(dx+c)^2+110\cos(dx+c)+70\right)+\left(80\cos(dx+c)^3+240\cos(dx+c)^2+240\cos(dx+c)+80\right)\sqrt{2}\,\sqrt{-\frac{2\cos(}{\cos(dx}}}{}$

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/d/a^3*(sin(d*x+c)*cos(d*x+c)*(32*cos(d*x+c)^2+110*cos(d*x+c)+70)+(80*
cos(d*x+c)^3+240*cos(d*x+c)^2+240*cos(d*x+c)+80)*2^(1/2)*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*arctanh(2^(1/2)*(cot(d*x+c)-csc(d*x+c))/(csc(d*x+c)^2-
2*cot(d*x+c)*csc(d*x+c)+cot(d*x+c)^2-1)^(1/2))+(115*cos(d*x+c)^3+345*cos(d
*x+c)^2+345*cos(d*x+c)+115)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln((-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)-cot(d*x+c)+csc(d*x+c)))*(a*(1+sec(d*x+c)))^
(1/2)/(cos(d*x+c)^3+3*cos(d*x+c)^2+3*cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 606, normalized size of antiderivative = 3.48

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[-1/64*(115*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) +
1)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 160*(cos(d*x + c)^3 + 3*cos(d*x +
c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d
*x + c) - a)/(cos(d*x + c) + 1)) - 4*(16*cos(d*x + c)^3 + 55*cos(d*x + c)^
2 + 35*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))
/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a
^3*d), -1/32*(115*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x +
 c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c)/(sqrt(a)*sin(d*x + c))) - 160*(cos(d*x + c)^3 + 3*cos(d*x + c)^
2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*(16*cos(d*x + c)^3 + 55*cos(
d*x + c)^2 + 35*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(
d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x
 + c) + a^3*d)]
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral(cos(c + d*x)/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(145) = 290$.

Time = 0.64 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.44

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx =$$

$$-\frac{2\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3\mathrm{sgn}(\cos(dx+c))}-\frac{21\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{\cdots}\right.\right.}}{}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-1/64*(2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*
c)^2/(a^3*sgn(cos(d*x + c))) - 21*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2
*d*x + 1/2*c) - 128*sqrt(2)*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2 - a)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)*sqrt(-a)*a*sgn(cos(d*x + c)))
 + 115*sqrt(2)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^2)/(sqrt(-a)*a^2*sgn(cos(d*x + c))) - 160*log(abs(-2*(sqrt(
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt
(2)*abs(a) + 6*a)/abs(-2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) + 6*a))/(sqrt(-a)*a*abs(a)*sgn(c
os(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)/(a + a/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\cos(dx+c)}{\sec(dx+c)^3+3\sec(dx+c)^2+3\sec(dx+c)+1}dx\right)}{a^3}$$

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*cos(c + d*x))/(sec(c + d*x)**3 + 3*se
c(c + d*x)**2 + 3*sec(c + d*x) + 1),x))/a**3
```

3.141 $\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 48

$$\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
-2^(1/2)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a-a*sec(d*x+c))^(1/2))/a^(
1/2)/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.29

$$\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\sec(c+dx)}}{\sqrt{2}}\right)\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}\sqrt{a-a\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]/Sqrt[a - a*Sec[c + d*x]],x]
```

output
```
-((Sqrt[2]*ArcTanh[Sqrt[1 + Sec[c + d*x]]/Sqrt[2]]*Tan[c + d*x])/(d*Sqrt[1
 + Sec[c + d*x]]*Sqrt[a - a*Sec[c + d*x]]))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4282}$$

$$-\frac{2\int \frac{1}{\frac{a^2\tan^2(c+dx)}{a-a\sec(c+dx)}+2a}d\frac{a\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}}{d}$$

$$\Big\downarrow \text{ 216}$$

$$-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

input
```
Int[Sec[c + d*x]/Sqrt[a - a*Sec[c + d*x]],x]
```

output
```
-((Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]]
)])/(Sqrt[a]*d))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{\sec(dx+c)}{\sqrt{a-a\sec(dx+c)}}dx$$

input
```
int(sec(d*x+c)/(a-a*sec(d*x+c))^(1/2),x)
```

output
```
int(sec(d*x+c)/(a-a*sec(d*x+c))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 161, normalized size of antiderivative = 3.35

$$\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx = \left[\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)}{2d}, \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx}{\cos(d}}}{\sqrt{a}s}\right.}{\sqrt{a}d}\right.$$

input
```
integrate(sec(d*x+c)/(a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*sqrt(-1/a)*log(-(2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x + c))*sq
rt((a*cos(d*x + c) - a)/cos(d*x + c))*sqrt(-1/a) - (3*cos(d*x + c) + 1)*si
n(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))/d, sqrt(2)*arctan(sqrt(2)*s
qrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))
)/(sqrt(a)*d)]
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx = \int \frac{\sec(c+dx)}{\sqrt{-a(\sec(c+dx)-1)}}\,dx$$

input
```
integrate(sec(d*x+c)/(a-a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)/sqrt(-a*(sec(c + d*x) - 1)), x)
```

Maxima [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)}{\sqrt{-a\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)/(a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/sqrt(-a*sec(d*x + c) + a), x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.40

$$\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}}{\sqrt{a}}\right)}{\sqrt{a}d\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)/(a-a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(sqrt(a)*d*sgn(
tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)\,\sqrt{a-\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a - a/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)*(a - a/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx = -\frac{\sqrt{a}\left(\int \frac{\sqrt{-\sec(dx+c)+1}\,\sec(dx+c)}{\sec(dx+c)-1}dx\right)}{a}$$

input
```
int(sec(d*x+c)/(a-a*sec(d*x+c))^(1/2),x)
```

output
```
( - sqrt(a)*int((sqrt( - sec(c + d*x) + 1)*sec(c + d*x))/(sec(c + d*x) - 1
),x))/a
```

3.142 $\int \frac{1}{\sqrt{a-a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 87

$$\int \frac{1}{\sqrt{a-a\sec(c+dx)}}\,dx=\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
2*arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))/a^(1/2)/d-2^(1/2)*arct
an(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a-a*sec(d*x+c))^(1/2))/a^(1/2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.46

$$\int \frac{1}{\sqrt{a-a\sec(c+dx)}}\,dx = -\frac{i\left(-1+e^{i(c+dx)}\right)\left(\operatorname{arcsinh}\left(e^{i(c+dx)}\right)-\sqrt{2}\operatorname{arctanh}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)}{d\sqrt{1+e^{2i(c+dx)}}\sqrt{a-a\sec(c+dx)}}$$

input
```
Integrate[1/Sqrt[a - a*Sec[c + d*x]],x]
```

output
```
((-I)*(-1 + E^(I*(c + d*x)))*(ArcSinh[E^(I*(c + d*x))] - Sqrt[2]*ArcTanh[(
1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + ArcTanh[Sq
rt[1 + E^((2*I)*(c + d*x))]]))/(d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a - a
*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 4263, 3042, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-a\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4263}$$

$$\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}dx+\frac{\int \sqrt{a-a\sec(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\int \sqrt{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}$$

$$\Big\downarrow \text{4261}$$

$$\frac{2\int \frac{1}{\frac{a^2\tan^2(c+dx)}{a-a\sec(c+dx)}+a}d\frac{a\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}}{d}+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{216}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

$$\Big\downarrow \text{4282}$$

$$\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}-\frac{2\int \frac{1}{\frac{a^2\tan^2(c+dx)}{a-a\sec(c+dx)}+2a}d\frac{a\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

input
```
Int[1/Sqrt[a - a*Sec[c + d*x]],x]
```

output
```
(2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(Sqrt[a]*d) -
(Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])]
)/(Sqrt[a]*d)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4263
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[1/a   I
nt[Sqrt[a + b*Csc[c + d*x]], x], x] - Simp[b/a   Int[Csc[c + d*x]/Sqrt[a +
b*Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{1}{\sqrt{a - a\sec(dx + c)}}dx$$

input
```
int(1/(a-a*sec(d*x+c))^(1/2),x)
```

output
```
int(1/(a-a*sec(d*x+c))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 301, normalized size of antiderivative = 3.46

$$\int \frac{1}{\sqrt{a - a\sec(c + dx)}}\,dx = \left[\frac{\sqrt{2}a\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right) - 2\sqrt{-a}\log\left(\frac{2\left(\cos(dx\right.}{}\right.}{2\,ad}\right.$$

input
```
integrate(1/(a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(2)*a*sqrt(-1/a)*log(-(2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x + c))
*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*sqrt(-1/a) - (3*cos(d*x + c) + 1)
*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c))) - 2*sqrt(-a)*log((2*(cos
(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c
)) - (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c)))/(a*d), (sqrt(2)*s
qrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)
/(sqrt(a)*sin(d*x + c))) - 2*sqrt(a)*arctan(sqrt((a*cos(d*x + c) - a)/cos(
d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/(a*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a - a\sec(c + dx)}}\,dx = \int \frac{1}{\sqrt{-a\sec{(c + dx)} + a}}\,dx$$

input
```
integrate(1/(a-a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(-a*sec(c + d*x) + a), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 698, normalized size of antiderivative = 8.02

$$\int \frac{1}{\sqrt{a - a\sec(c + dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-(sqrt(2)*sqrt(a)*arctan2(((abs(2*e^(I*d*x + I*c) - 2)^4 + 16*cos(d*x + c)
^4 + 16*sin(d*x + c)^4 + 8*(cos(d*x + c)^2 - sin(d*x + c)^2 + 2*cos(d*x +
c) + 1)*abs(2*e^(I*d*x + I*c) - 2)^2 + 64*cos(d*x + c)^3 + 32*(cos(d*x + c
)^2 + 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 96*cos(d*x + c)^2 + 64*cos(d*x
+ c) + 16)^(1/4)*sin(1/2*arctan2(8*(cos(d*x + c) + 1)*sin(d*x + c)/abs(2*e
^(I*d*x + I*c) - 2)^2, (abs(2*e^(I*d*x + I*c) - 2)^2 + 4*cos(d*x + c)^2 -
4*sin(d*x + c)^2 + 8*cos(d*x + c) + 4)/abs(2*e^(I*d*x + I*c) - 2)^2)) + 2*
sin(d*x + c))/abs(2*e^(I*d*x + I*c) - 2), ((abs(2*e^(I*d*x + I*c) - 2)^4 +
 16*cos(d*x + c)^4 + 16*sin(d*x + c)^4 + 8*(cos(d*x + c)^2 - sin(d*x + c)^
2 + 2*cos(d*x + c) + 1)*abs(2*e^(I*d*x + I*c) - 2)^2 + 64*cos(d*x + c)^3 +
 32*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 96*cos(d*x + c)
^2 + 64*cos(d*x + c) + 16)^(1/4)*cos(1/2*arctan2(8*(cos(d*x + c) + 1)*sin(
d*x + c)/abs(2*e^(I*d*x + I*c) - 2)^2, (abs(2*e^(I*d*x + I*c) - 2)^2 + 4*c
os(d*x + c)^2 - 4*sin(d*x + c)^2 + 8*cos(d*x + c) + 4)/abs(2*e^(I*d*x + I*
c) - 2)^2)) + 2*cos(d*x + c) + 2)/abs(2*e^(I*d*x + I*c) - 2)) - sqrt(a)*ar
ctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x +
c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/
4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos(d*x + c)
))/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{a-a\sec(c+dx)}}\,dx = \frac{\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{2\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{2\sqrt{a}}\right)}{\sqrt{a}}}{d}$$

input
```
integrate(1/(a-a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
(sqrt(2)*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/sqrt(a) - 2*ar
ctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/sqrt(a))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a - a\sec(c + dx)}}\, dx = \int \frac{1}{\sqrt{a - \frac{a}{\cos(c+d\,x)}}}\, dx$$

input
```
int(1/(a - a/cos(c + d*x))^(1/2),x)
```

output
```
int(1/(a - a/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a - a\sec(c + dx)}}\, dx = -\frac{\sqrt{a}\left(\int \frac{\sqrt{-\sec(dx+c)+1}}{\sec(dx+c)-1}dx\right)}{a}$$

input
```
int(1/(a-a*sec(d*x+c))^(1/2),x)
```

output
```
( - sqrt(a)*int(sqrt( - sec(c + d*x) + 1)/(sec(c + d*x) - 1),x))/a
```

3.143 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 383

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = -\frac{9(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40d} + \frac{57(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{80d(1+\sec(c+dx))} + \frac{3(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{8ad} - \frac{19\, 3^{3/4}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)(a+a\sec(c+dx))^{2/3}\left(\sqrt[3]{2}-\cdots\right.}{80\sqrt[3]{2}d(1-\sec(c+dx))(1+\sec(c+dx))\sqrt{-\frac{\sqrt[3]{1+\sec(c\cdots}}{\left(\sqrt[3]{2}-(1\cdots\right.}}}$$

output
```
-9/40*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/d+57/80*(a+a*sec(d*x+c))^(2/3)*tan
(d*x+c)/d/(1+sec(d*x+c))+3/8*(a+a*sec(d*x+c))^(5/3)*tan(d*x+c)/a/d-19/160*
3^(3/4)*InverseJacobiAM(arccos((2^(1/3)-(1-3^(1/2))*(1+sec(d*x+c))^(1/3))/
(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))*(a+a*
sec(d*x+c))^(2/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(
d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1
/3))^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(1+sec(d*x+c))/(-(1+sec(
d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d
*x+c))^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.27

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{(a(1+\sec(c+dx)))^{2/3}\left(38\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)+3\sqrt[6]{1+\right.}{40d(1+\sec(c+dx))^{7/6}}$$

input
```
Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(2/3),x]
```

output
```
((a*(1 + Sec[c + d*x]))^(2/3)*(38*2^(1/6)*Hypergeometric2F1[-1/6, 1/2, 3/2
, (1 - Sec[c + d*x])/2] + 3*(1 + Sec[c + d*x])^(1/6)*(2 + 7*Sec[c + d*x] +
 5*Sec[c + d*x]^2))*Tan[c + d*x])/(40*d*(1 + Sec[c + d*x])^(7/6))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4287, 27, 3042, 4489, 3042, 4315, 3042, 4314, 60, 73, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\sec(c+dx)+a)^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{2/3}dx$$

$$\Big\downarrow \text{4287}$$

$$\frac{3\int \frac{1}{3}\sec(c+dx)(5a-3a\sec(c+dx))(\sec(c+dx)a+a)^{2/3}dx}{8a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8ad}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \sec(c+dx)(5a-3a\sec(c+dx))(\sec(c+dx)a+a)^{2/3}dx}{8a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8ad}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(5a-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{2/3}dx}{8a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8ad}$$

$$\Big\downarrow \ 4489$$

$$\frac{\frac{19}{5}a\int \sec(c+dx)(\sec(c+dx)a+a)^{2/3}dx-\frac{9a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8ad}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{19}{5}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{2/3}dx-\frac{9a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8ad}$$

$$\Big\downarrow \ 4315$$

$$\frac{\frac{19a(a\sec(c+dx)+a)^{2/3}\int \sec(c+dx)(\sec(c+dx)+1)^{2/3}dx}{5(\sec(c+dx)+1)^{2/3}}-\frac{9a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8ad}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{19a(a\sec(c+dx)+a)^{2/3}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{2/3}dx}{5(\sec(c+dx)+1)^{2/3}}-\frac{9a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8ad}$$

$$\Big\downarrow \ 4314$$

$$\frac{-\frac{19a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int \frac{\sqrt[6]{\sec(c+dx)+1}}{\sqrt{1-\sec(c+dx)}}d\sec(c+dx)}{5d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}-\frac{9a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8ad}$$

$\Big\downarrow$ 60

$$-\frac{\frac{19a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}d\sec(c+dx)-\frac{3}{2}\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\right)}{5d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}-\frac{9a\tan(c+}{}}{8a}$$

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8ad}$$

$\Big\downarrow$ 73

$$-\frac{\frac{19a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(3\int\frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}-\frac{3}{2}\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\right)}{5d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}-\frac{9a\tan(c+dx)(}{}}{8a}$$

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8ad}$$

$\Big\downarrow$ 766

$$-\frac{19a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{3^{3/4}\sqrt[6]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+}\right.}}}{2\sqrt[3]{2}\sqrt{1-\sec(c+dx)}\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\right.}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(\right.}}}}\right.}{5d\sqrt{1-\sec(c+dx)}(\sec(c+dx}$$

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8ad}$$

input
```
Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(2/3),x]
```

output
```
(3*(a + a*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(8*a*d) + ((-9*a*(a + a*Sec[c
+ d*x])^(2/3)*Tan[c + d*x])/(5*d) - (19*a*(a + a*Sec[c + d*x])^(2/3)*((-3*
Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])^(1/6))/2 + (3^(3/4)*EllipticF[Ar
cCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sq
rt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x])^(1/
6)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c
 + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + S
ec[c + d*x])^(1/3))^2])/(2*2^(1/3)*Sqrt[1 - Sec[c + d*x]]*Sqrt[-(((1 + Sec
[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqr
t[3])*(1 + Sec[c + d*x])^(1/3))^2)]))*Tan[c + d*x])/(5*d*Sqrt[1 - Sec[c +
d*x]]*(1 + Sec[c + d*x])^(7/6)))/(8*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4287
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 -
b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Maple [F]

$$\int \sec (dx + c)^3 (a + a \sec (dx + c))^{\frac{2}{3}} dx$$

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(2/3),x)
```

output
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int \sec^3(c + dx)(a + a\sec(c + dx))^{2/3} dx = \int (a \sec (dx + c) + a)^{\frac{2}{3}} \sec (dx + c)^3 \, dx$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

Sympy [F]

$$\int \sec^3(c + dx)(a + a\sec(c + dx))^{2/3} dx = \int (a(\sec (c + dx) + 1))^{\frac{2}{3}} \sec^3 (c + dx) dx$$

input
```
integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(2/3),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(2/3)*sec(c + d*x)**3, x)
```

Maxima [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

Giac [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{2/3}}{\cos(c+d\,x)^3}\,dx$$

input
```
int((a + a/cos(c + d*x))^(2/3)/cos(c + d*x)^3,x)
```

output
```
int((a + a/cos(c + d*x))^(2/3)/cos(c + d*x)^3, x)
```

Reduce [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = a^{\frac{2}{3}}\left(\int (\sec(dx+c)+1)^{\frac{2}{3}}\sec(dx+c)^3\,dx\right)$$

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(2/3),x)
```

output
```
a**(2/3)*int((sec(c + d*x) + 1)**(2/3)*sec(c + d*x)**3,x)
```

3.144 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 353

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d(1+\sec(c+dx))} - \frac{3^{3/4}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)(a+a\sec(c+dx))^{2/3}\left(\sqrt[3]{2}-\sqrt[3]{1\ldots}\right.}{5\sqrt[3]{2}d(1-\sec(c+dx))(1+\sec(c+dx))\sqrt{-\frac{\sqrt[3]{1+\sec(c+\ldots}}{\left(\sqrt[3]{2}-\left(1+\ldots\right.\right.}}}$$

output
```
3/5*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/d+3/5*(a+a*sec(d*x+c))^(2/3)*tan(d*x
+c)/d/(1+sec(d*x+c))-1/10*3^(3/4)*InverseJacobiAM(arccos((2^(1/3)-(1-3^(1/
2))*(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))),1/4*
6^(1/2)+1/4*2^(1/2))*(a+a*sec(d*x+c))^(2/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))
*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+
3^(1/2))*(1+sec(d*x+c))^(1/3))^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c)
)/(1+sec(d*x+c))/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^
(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.24

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{(a(1+\sec(c+dx)))^{2/3}\left(4\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)+3(1+\mathrm{sec}\right.}{5d(1+\sec(c+dx))^{7/6}}$$

input
```
Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(2/3),x]
```

output
```
((a*(1 + Sec[c + d*x]))^(2/3)*(4*2^(1/6)*Hypergeometric2F1[-1/6, 1/2, 3/2,
 (1 - Sec[c + d*x])/2] + 3*(1 + Sec[c + d*x])^(7/6))*Tan[c + d*x])/(5*d*(1
 + Sec[c + d*x])^(7/6))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4285, 3042, 4315, 3042, 4314, 60, 73, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\sec(c+dx)+a)^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{2/3}dx$$

$$\Big\downarrow \text{4285}$$

$$\frac{2}{5}\int \sec(c+dx)(\sec(c+dx)a+a)^{2/3}dx+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{2/3}dx+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}$$

$\Big\downarrow$ 4315

$$\frac{2(a\sec(c+dx)+a)^{2/3}\int \sec(c+dx)(\sec(c+dx)+1)^{2/3}dx}{5(\sec(c+dx)+1)^{2/3}}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}$$

$\Big\downarrow$ 3042

$$\frac{2(a\sec(c+dx)+a)^{2/3}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{2/3}dx}{5(\sec(c+dx)+1)^{2/3}}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}$$

$\Big\downarrow$ 4314

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}-\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int \frac{\sqrt[6]{\sec(c+dx)+1}}{\sqrt{1-\sec(c+dx)}}d\sec(c+dx)}{5d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$\Big\downarrow$ 60

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}-\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}d\sec(c+dx)-\frac{3}{2}\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+\right.}{5d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$\Big\downarrow$ 73

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}-\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(3\int \frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}-\frac{3}{2}\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1\right.}{5d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$\Big\downarrow$ 766

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}-\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{3^{3/4}\sqrt[6]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+}}}}{2\sqrt[3]{2}\sqrt{1-\sec(c+dx)}\sqrt{-\frac{\sqrt[3]{\sec(c+dx}}{\left(\sqrt[3]{2}-(1\right.}}}\right.}{5d\sqrt{1-\sec(c+dx}}$$

input
```
Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(2/3),x]
```

output
```
(3*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*d) - (2*(a + a*Sec[c + d*x]
)^(2/3)*((-3*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])^(1/6))/2 + (3^(3/4)
*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1
/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[
c + d*x])^(1/6)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/
3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sq
rt[3])*(1 + Sec[c + d*x])^(1/3))^2])/(2*2^(1/3)*Sqrt[1 - Sec[c + d*x]]*Sqr
t[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/
3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]))*Tan[c + d*x])/(5*d*Sqrt
[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])^(7/6))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4285
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[a*(m/(b*(m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x]
/; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \sec(dx+c)^2 (a+a\sec(dx+c))^{\frac{2}{3}}\,dx$$

input
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(2/3),x)
```

output
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a(\sec(c+dx)+1))^{\frac{2}{3}}\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(2/3),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(2/3)*sec(c + d*x)**2, x)
```

Maxima [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

Giac [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{2/3}}{\cos(c+d\,x)^2}\,dx$$

input
```
int((a + a/cos(c + d*x))^(2/3)/cos(c + d*x)^2,x)
```

output
```
int((a + a/cos(c + d*x))^(2/3)/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = a^{\frac{2}{3}}\left(\int (\sec(dx+c)+1)^{\frac{2}{3}}\sec(dx+c)^2\,dx\right)$$

input
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(2/3),x)
```

output
```
a**(2/3)*int((sec(c + d*x) + 1)**(2/3)*sec(c + d*x)**2,x)
```

3.145 $\int \sec(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 326

$$\int \sec(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{2d(1+\sec(c+dx))} - \frac{3^{3/4}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right),\frac{1}{4}(2+\sqrt{3})\right)(a+a\sec(c+dx))^{2/3}\left(\sqrt[3]{2}-\sqrt[3]{1\ldots}\right.}{2\sqrt[3]{2}d(1-\sec(c+dx))(1+\sec(c+dx))\sqrt{-\frac{\sqrt[3]{1+\sec(c+\ldots}}{\left(\sqrt[3]{2}-(1+\ldots\right.}}}$$

output
```
3/2*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/d/(1+sec(d*x+c))-1/4*3^(3/4)*Inverse
JacobiAM(arccos((2^(1/3)-(1-3^(1/2))*(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+3^(
1/2))*(1+sec(d*x+c))^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))*(a+a*sec(d*x+c))^(2/
3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(
1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))^2)^(1/2)*t
an(d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(1+sec(d*x+c))/(-(1+sec(d*x+c))^(1/3)*(
2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))^2
)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.20

$$\int \sec(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{2\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)(a(1+\sec(c+dx)))^{2/3}\tan(c+dx)}{d(1+\sec(c+dx))^{7/6}}$$

input
```
Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(2/3),x]
```

output
```
(2*2^(1/6)*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Sec[c + d*x])/2]*(a*(1 +
 Sec[c + d*x]))^(2/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(7/6))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4315, 3042, 4314, 60, 73, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\sec(c+dx)+a)^{2/3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{2/3}dx$$

$$\Big\downarrow \; 4315$$

$$\frac{(a\sec(c+dx)+a)^{2/3}\int \sec(c+dx)(\sec(c+dx)+1)^{2/3}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \; 3042$$

$$\frac{(a\sec(c+dx)+a)^{2/3}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{2/3}dx}{(\sec(c+dx)+1)^{2/3}}$$

$\Big\downarrow$ 4314

$$-\frac{\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int \frac{\sqrt[6]{\sec(c+dx)+1}}{\sqrt{1-\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$\Big\downarrow$ 60

$$-\frac{\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}d\sec(c+dx)-\frac{3}{2}\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+\right.}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$\Big\downarrow$ 73

$$-\frac{\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(3\int \frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}-\frac{3}{2}\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1\right.}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$\Big\downarrow$ 766

$$-\frac{\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{3^{3/4}\sqrt[6]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c-}}}}{2\sqrt[3]{2}\sqrt{1-\sec(c+dx)}\sqrt{-\frac{\sqrt[3]{\sec(c+dx}}{\left(\sqrt[3]{2}-(\right.}}}\right.}{d\sqrt{1-\sec(c+dx}}$$

input
```
Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(2/3),x]
```

output
```
-(((a + a*Sec[c + d*x])^(2/3)*((-3*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x
])^(1/6))/2 + (3^(3/4)*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[
c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 +
 Sqrt[3])/4]*(1 + Sec[c + d*x])^(1/6)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))
*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/
3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]])/(2*2^(1/3)*Sqrt
[1 - Sec[c + d*x]]*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c
+ d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]))*T
an[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])^(7/6)))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \sec (dx + c) (a + a \sec (dx + c))^{\frac{2}{3}} dx$$

input
```
int(sec(d*x+c)*(a+a*sec(d*x+c))^(2/3),x)
```

output
```
int(sec(d*x+c)*(a+a*sec(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int \sec(c + dx)(a + a\sec(c + dx))^{2/3} dx = \int (a \sec (dx + c) + a)^{\frac{2}{3}} \sec (dx + c) \, dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

Sympy [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a(\sec(c+dx)+1))^{\frac{2}{3}}\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(2/3),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(2/3)*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

Giac [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{2/3}}{\cos\left(c+d\,x\right)}\,dx$$

input
```
int((a + a/cos(c + d*x))^(2/3)/cos(c + d*x),x)
```

output
```
int((a + a/cos(c + d*x))^(2/3)/cos(c + d*x), x)
```

Reduce [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = a^{\frac{2}{3}}\left(\int \left(\sec\left(dx+c\right)+1\right)^{\frac{2}{3}}\sec\left(dx+c\right)dx\right)$$

input
```
int(sec(d*x+c)*(a+a*sec(d*x+c))^(2/3),x)
```

output
```
a**(2/3)*int((sec(c + d*x) + 1)**(2/3)*sec(c + d*x),x)
```

3.146 $\int (a + a\sec(c + dx))^{2/3} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 77

$$\int (a + a\sec(c + dx))^{2/3} \, dx = \frac{3\sqrt{2}\,\text{AppellF1}\left(\frac{7}{6}, \frac{1}{2}, 1, \frac{13}{6}, \frac{1}{2}(1 + \sec(c + dx)), 1 + \sec(c + dx)\right) (a + a\sec(c + dx))^{2/3} \tan(c +}{7d\sqrt{1 - \sec(c + dx)}}$$

output
```
3/7*2^(1/2)*AppellF1(7/6,1,1/2,13/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*(a+a*
sec(d*x+c))^(2/3)*tan(d*x+c)/d/(1-sec(d*x+c))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 691 vs. $2(77) = 154$.

Time = 3.53 (sec) , antiderivative size = 691, normalized size of antiderivative = 8.97

$$\int (a + a\sec(c + dx))^{2/3} \, dx = \text{Too large to display}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(2/3),x]
```

output
```
(45*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(a
*(1 + Sec[c + d*x]))^(5/3)*Sin[c + d*x]*(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[
(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Ta
n[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[
(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))/(a*d*(40*(3*App
ellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*Appel
lF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])^2*Sec[c +
d*x]*Sin[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + 6*AppellF1[1/2, 2/3, 1, 3/2,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[c + d*x]^2*Sin[(c + d*x)/2]^2
*(15*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(
-7 + 16*Cos[c + d*x] - 3*Cos[2*(c + d*x)]) + 10*AppellF1[3/2, 5/3, 1, 5/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(7 - 16*Cos[c + d*x] + 3*Cos[2*(
c + d*x)]) - 24*(9*AppellF1[5/2, 2/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c
+ d*x)/2]^2] - 6*AppellF1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2] + 5*AppellF1[5/2, 8/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2])*Cos[c + d*x]*Tan[(c + d*x)/2]^2) + 135*AppellF1[1/2, 2/3, 1, 3/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]^2*(3 + 3*Cos[c + d*x] + 2*Tan[c
 + d*x]^2)))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4266, 3042, 4265, 149, 25, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{2/3}dx$$

$$\Big\downarrow \text{4266}$$

$$\frac{(a\sec(c+dx)+a)^{2/3}\int(\sec(c+dx)+1)^{2/3}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a\sec(c+dx)+a)^{2/3}\int\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{2/3}dx}{(\sec(c+dx)+1)^{2/3}}$$

$\big\downarrow$ 4265

$$-\frac{\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int\frac{\cos(c+dx)\sqrt[6]{\sec(c+dx)+1}}{\sqrt{1-\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$\big\downarrow$ 149

$$-\frac{6\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int\frac{\cos(c+dx)(\sec(c+dx)+1)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$\big\downarrow$ 25

$$\frac{6\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int-\frac{\cos(c+dx)(\sec(c+dx)+1)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$\big\downarrow$ 1012

$$\frac{3\sqrt{2}\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\operatorname{AppellF1}\left(\frac{7}{6},1,\frac{1}{2},\frac{13}{6},\sec(c+dx)+1,\frac{1}{2}(\sec(c+dx)+1)\right)}{7d\sqrt{1-\sec(c+dx)}}$$

input
```
Int[(a + a*Sec[c + d*x])^(2/3),x]
```

output
```
(3*Sqrt[2]*AppellF1[7/6, 1, 1/2, 13/6, 1 + Sec[c + d*x], (1 + Sec[c + d*x]
)/2]*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(7*d*Sqrt[1 - Sec[c + d*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 149
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1
) - 1)*(c - a*(d/b) + d*(x^k/b))^n*(e - a*(f/b) + f*(x^k/b))^p, x], x, (a +
 b*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && FractionQ[m] &&
IntegerQ[2*n] && IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4265
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[a^n*(Cot
[c + d*x]/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]))   Subst[Int[(1
 + b*(x/a))^(n - 1/2)/(x*Sqrt[1 - b*(x/a)]), x], x, Csc[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0
]
```

rule 4266
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Csc[c + d*x])^FracPart[n]/(1 + (b/a)*Csc[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Maple [F]

$$\int (a + a\sec(dx + c))^{\frac{2}{3}} \, dx$$

input
```
int((a+a*sec(d*x+c))^(2/3),x)
```

output
```
int((a+a*sec(d*x+c))^(2/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{2/3} \, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int (a + a\sec(c + dx))^{2/3} \, dx = \int (a \sec(c + dx) + a)^{\frac{2}{3}} \, dx$$

input
```
integrate((a+a*sec(d*x+c))**(2/3),x)
```

output
```
Integral((a*sec(c + d*x) + a)**(2/3), x)
```

Maxima [F]

$$\int (a + a\sec(c + dx))^{2/3} \, dx = \int (a \sec(dx + c) + a)^{\frac{2}{3}} \, dx$$

input
```
integrate((a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(2/3), x)
```

Giac [F]

$$\int (a+a\sec(c+dx))^{2/3}\,dx=\int (a\sec(dx+c)+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^{2/3}\,dx=\int \left(a+\frac{a}{\cos(c+d\,x)}\right)^{2/3}\,dx$$

input
```
int((a + a/cos(c + d*x))^(2/3),x)
```

output
```
int((a + a/cos(c + d*x))^(2/3), x)
```

Reduce [F]

$$\int (a+a\sec(c+dx))^{2/3}\,dx=a^{\frac{2}{3}}\left(\int (\sec(dx+c)+1)^{\frac{2}{3}}\,dx\right)$$

input
```
int((a+a*sec(d*x+c))^(2/3),x)
```

output
```
a**(2/3)*int((sec(c + d*x) + 1)**(2/3),x)
```

3.147 $\int \cos(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \cos(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = -\frac{3\sqrt{2}\operatorname{AppellF1}\left(\frac{7}{6},\frac{1}{2},2,\frac{13}{6},\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{7d\sqrt{1-\sec(c+dx)}}$$

output
```
-3/7*2^(1/2)*AppellF1(7/6,2,1/2,13/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*(a+a
*sec(d*x+c))^(2/3)*tan(d*x+c)/d/(1-sec(d*x+c))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2700 vs. 2(77) = 154.

Time = 14.50 (sec) , antiderivative size = 2700, normalized size of antiderivative = 35.06

$$\int \cos(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(2/3),x]
```

output
```
(((1 + Cos[c + d*x])*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(2/3)*(Sin
[c + d*x] - Tan[(c + d*x)/2]))/(d*(1 + Sec[c + d*x])^(2/3)) - (2^(2/3)*(Co
s[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(2/3)*((Sec[(c
 + d*x)/2]^2*(1 + Sec[c + d*x])^(2/3))/6 + (Cos[c + d*x]*Sec[(c + d*x)/2]^
2*(1 + Sec[c + d*x])^(2/3))/3)*Tan[(c + d*x)/2]*(AppellF1[3/2, 2/3, 1, 5/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^
2)^(2/3)*Tan[(c + d*x)/2]^2 + (81*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(-9*AppellF1[1/2, 2/3, 1,
3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 2/3, 2,
 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/(9*d*(
1 + Sec[c + d*x])^(2/3)*(-1/9*(Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*Sec[
c + d*x])^(2/3)*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (8
1*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[
(c + d*x)/2]^2)/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c
+ d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/2^(1/3) - (2^(2/3)*(Cos[(c + d*x)/2]^2
*Sec[c + d*x])^(2/3)*Tan[(c + d*x)/2]*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(...
```

Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4315, 3042, 4314, 149, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\sec(c+dx)+a)^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{2/3}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{(a\sec(c+dx)+a)^{2/3}\int\cos(c+dx)(\sec(c+dx)+1)^{2/3}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a\sec(c+dx)+a)^{2/3}\int\frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{2/3}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \text{4314}$$

$$-\frac{\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int\frac{\cos^2(c+dx)\sqrt[6]{\sec(c+dx)+1}}{\sqrt{1-\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \text{149}$$

$$-\frac{6\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int\frac{\cos^2(c+dx)(\sec(c+dx)+1)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{3\sqrt{2}\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\operatorname{AppellF1}\left(\frac{7}{6},2,\frac{1}{2},\frac{13}{6},\sec(c+dx)+1,\frac{1}{2}(\sec(c+dx)+1)\right)}{7d\sqrt{1-\sec(c+dx)}}$$

input
```
Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(2/3),x]
```

output
```
(-3*Sqrt[2]*AppellF1[7/6, 2, 1/2, 13/6, 1 + Sec[c + d*x], (1 + Sec[c + d*x
])/2]*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(7*d*Sqrt[1 - Sec[c + d*x]]
)
```

Defintions of rubi rules used

rule 149
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1
) - 1)*(c - a*(d/b) + d*(x^k/b))^n*(e - a*(f/b) + f*(x^k/b))^p, x], x, (a +
 b*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && FractionQ[m] &&
IntegerQ[2*n] && IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \cos (dx+c) (a+a\sec (dx+c))^{\frac{2}{3}} dx$$

input
```
int(cos(d*x+c)*(a+a*sec(d*x+c))^(2/3),x)
```

output
```
int(cos(d*x+c)*(a+a*sec(d*x+c))^(2/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \cos(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a(\sec(c+dx)+1))^{\frac{2}{3}}\cos(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(2/3),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(2/3)*cos(c + d*x), x)
```

Maxima [F]

$$\int \cos(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{2}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(2/3)*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{2}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(2/3)*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int \cos(c+d\,x)\left(a+\frac{a}{\cos(c+d\,x)}\right)^{2/3}\,dx$$

input
```
int(cos(c + d*x)*(a + a/cos(c + d*x))^(2/3),x)
```

output
```
int(cos(c + d*x)*(a + a/cos(c + d*x))^(2/3), x)
```

Reduce [F]

$$\int \cos(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = a^{\frac{2}{3}}\left(\int (\sec(dx+c)+1)^{\frac{2}{3}}\cos(dx+c)\,dx\right)$$

input
```
int(cos(d*x+c)*(a+a*sec(d*x+c))^(2/3),x)
```

output
```
a**(2/3)*int((sec(c + d*x) + 1)**(2/3)*cos(c + d*x),x)
```

3.148 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 413

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \frac{147a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{440d}$$
$$+\frac{1029a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{880d(1+\sec(c+dx))}$$
$$-\frac{9(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{88d}+\frac{3(a+a\sec(c+dx))^{8/3}\tan(c+dx)}{11ad}$$
$$-\frac{343\ 3^{3/4}a\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)(a+a\sec(c+dx))^{2/3}\left(\sqrt[3]{2}\ \cdots\right.}{880\sqrt[3]{2}d(1-\sec(c+dx))(1+\sec(c+dx))\sqrt{-\frac{\sqrt[3]{1+\sec(c\ \cdots}}{\left(\sqrt[3]{2}-\cdots\right.}}}$$

output
```
147/440*a*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/d+1029/880*a*(a+a*sec(d*x+c))^
(2/3)*tan(d*x+c)/d/(1+sec(d*x+c))-9/88*(a+a*sec(d*x+c))^(5/3)*tan(d*x+c)/d
+3/11*(a+a*sec(d*x+c))^(8/3)*tan(d*x+c)/a/d-343/1760*3^(3/4)*a*InverseJaco
biAM(arccos((2^(1/3)-(1-3^(1/2))*(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+3^(1/2)
)*(1+sec(d*x+c))^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))*(a+a*sec(d*x+c))^(2/3)*(
2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+se
c(d*x+c))^(2/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))^2)^(1/2)*tan(d
*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(1+sec(d*x+c))/(-(1+sec(d*x+c))^(1/3)*(2^(1
/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))^2)^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.23

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \frac{a(a(1+\sec(c+dx)))^{2/3}\left(196\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)+3(1+\cdots\right.}{88d(1+\sec(c+dx))^{7/6}}$$

input
```
Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
(a*(a*(1 + Sec[c + d*x]))^(2/3)*(196*2^(1/6)*Hypergeometric2F1[-7/6, 1/2,
3/2, (1 - Sec[c + d*x])/2] + 3*(1 + Sec[c + d*x])^(13/6)*(5 + 8*Sec[c + d*
x]))*Tan[c + d*x])/(88*d*(1 + Sec[c + d*x])^(7/6))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4287, 27, 3042, 4489, 3042, 4315, 3042, 4314, 60, 60, 73, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\sec(c+dx)+a)^{5/3}\,dx$$

$\Big\downarrow$ 3042

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/3}dx$$

$\Big\downarrow$ 4287

$$\frac{3\int \frac{1}{3}\sec(c+dx)(8a-3a\sec(c+dx))(\sec(c+dx)a+a)^{5/3}dx}{11a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{8/3}}{11ad}$$

$\Big\downarrow$ 27

$$\frac{\int \sec(c+dx)(8a-3a\sec(c+dx))(\sec(c+dx)a+a)^{5/3}dx}{11a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{8/3}}{11ad}$$

$\Big\downarrow$ 3042

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(8a-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/3}dx}{11a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{8/3}}{11ad}$$

$\Big\downarrow$ 4489

$$\frac{\frac{49}{8}a\int \sec(c+dx)(\sec(c+dx)a+a)^{5/3}dx-\frac{9a\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}}{11a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{8/3}}{11ad}$$

$\Big\downarrow$ 3042

$$\frac{\frac{49}{8}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/3}dx-\frac{9a\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}}{11a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{8/3}}{11ad}$$

$\Big\downarrow$ 4315

$$\frac{\frac{49a^2(a\sec(c+dx)+a)^{2/3}\int \sec(c+dx)(\sec(c+dx)+1)^{5/3}dx}{8(\sec(c+dx)+1)^{2/3}}-\frac{9a\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}}{11a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{8/3}}{11ad}$$

$\Big\downarrow$ 3042

$$\frac{\frac{49a^2(a\sec(c+dx)+a)^{2/3}\int\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{5/3}dx}{8(\sec(c+dx)+1)^{2/3}}-\frac{9a\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}}{11a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{8/3}}{11ad}$$

$\downarrow$ 4314

$$\frac{-\frac{49a^2\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int\frac{(\sec(c+dx)+1)^{7/6}}{\sqrt{1-\sec(c+dx)}}d\sec(c+dx)}{8d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}-\frac{9a\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}}{11a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{8/3}}{11ad}$$

$\downarrow$ 60

$$\frac{-\frac{49a^2\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{7}{5}\int\frac{\sqrt[6]{\sec(c+dx)+1}}{\sqrt{1-\sec(c+dx)}}d\sec(c+dx)-\frac{3}{5}\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}\right)}{8d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}-\frac{9a\tan(c+dx)(a\sec(}{8d}}{11a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{8/3}}{11ad}$$

$\downarrow$ 60

$$\frac{-\frac{49a^2\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{7}{5}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}d\sec(c+dx)-\frac{3}{2}\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\right)-\frac{3}{5}\sqrt{1-se}\right.}{8d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}}{11a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{8/3}}{11ad}$$

$\downarrow$ 73

$$\frac{-\frac{49a^2\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{7}{5}\left(3\int\frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}-\frac{3}{2}\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\right)-\frac{3}{5}\sqrt{1-\sec(c\right.}{8d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}}{11a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{8/3}}{11ad}$$

$\downarrow$ 766

$$-\frac{49a^2\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{7}{5}\left(\frac{3^{3/4}\sqrt[6]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+d}}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx}\right.}}}{2\sqrt[3]{2}\sqrt{1-\sec(c+dx)}\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\right.}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{}\right.}}}\right.\right.}{8d\sqrt{1}}$$

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{8/3}}{11ad}$$

input

```
Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/3),x]
```

output

```
(3*(a + a*Sec[c + d*x])^(8/3)*Tan[c + d*x])/(11*a*d) + ((-9*a*(a + a*Sec[c
 + d*x])^(5/3)*Tan[c + d*x])/(8*d) - (49*a^2*(a + a*Sec[c + d*x])^(2/3)*((
-3*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])^(7/6))/5 + (7*((-3*Sqrt[1 - S
ec[c + d*x]]*(1 + Sec[c + d*x])^(1/6))/2 + (3^(3/4)*EllipticF[ArcCos[(2^(1
/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1
+ Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x])^(1/6)*(2^(1/3
) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(
1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x
])^(1/3))^2])/(2*2^(1/3)*Sqrt[1 - Sec[c + d*x]]*Sqrt[-(((1 + Sec[c + d*x])
^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 +
 Sec[c + d*x])^(1/3))^2)])))/5)*Tan[c + d*x])/(8*d*Sqrt[1 - Sec[c + d*x]]*
(1 + Sec[c + d*x])^(7/6)))/(11*a)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4287
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 -
b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Maple [F]

$$\int \sec (dx+c)^3 (a+a\sec (dx+c))^{\frac{5}{3}} dx$$

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/3),x)
```

Fricas [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \int (a\sec (dx+c)+a)^{\frac{5}{3}} \sec (dx+c)^3 \, dx$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c)^4 + a*sec(d*x + c)^3)*(a*sec(d*x + c) + a)^(2/3),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(5/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^3, x)
```

Giac [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/3}}{\cos\left(c+d\,x\right)^3}\,dx$$

input
```
int((a + a/cos(c + d*x))^(5/3)/cos(c + d*x)^3,x)
```

output
```
int((a + a/cos(c + d*x))^(5/3)/cos(c + d*x)^3, x)
```

Reduce [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = a^{\frac{5}{3}}\left(\int (\sec(dx+c)+1)^{\frac{2}{3}}\sec(dx+c)^4\,dx + \int (\sec(dx+c)+1)^{\frac{2}{3}}\sec(dx+c)^3\,dx\right)$$

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/3),x)
```

output
```
a**(2/3)*a*(int((sec(c + d*x) + 1)**(2/3)*sec(c + d*x)**4,x) + int((sec(c
+ d*x) + 1)**(2/3)*sec(c + d*x)**3,x))
```

3.149 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 383

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/3}\,dx=\frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{8d}$$
$$+\frac{21a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{16d(1+\sec(c+dx))}+\frac{3(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{8d}$$
$$-\frac{7\,3^{3/4}a\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)(a+a\sec(c+dx))^{2/3}\left(\sqrt[3]{2}-\right.}{16\sqrt[3]{2}d(1-\sec(c+dx))(1+\sec(c+dx))\sqrt{-\frac{\sqrt[3]{1+\sec(c-}}{\left(\sqrt[3]{2}-(1\right.}}}$$

output
```
3/8*a*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/d+21/16*a*(a+a*sec(d*x+c))^(2/3)*t
an(d*x+c)/d/(1+sec(d*x+c))+3/8*(a+a*sec(d*x+c))^(5/3)*tan(d*x+c)/d-7/32*3^
(3/4)*a*InverseJacobiAM(arccos((2^(1/3)-(1-3^(1/2))*(1+sec(d*x+c))^(1/3))/
(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))*(a+a*
sec(d*x+c))^(2/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(
d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1
/3))^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(1+sec(d*x+c))/(-(1+sec(
d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d
*x+c))^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.31 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.28

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \frac{a(a(1+\sec(c+dx)))^{2/3}\left(5\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)+3\cos^4\left(\right.\right.}{2d(1+\sec(c+dx))^{7/6}}$$

input
```
Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
(a*(a*(1 + Sec[c + d*x]))^(2/3)*(5*2^(1/6)*Hypergeometric2F1[-7/6, 1/2, 3/
2, (1 - Sec[c + d*x])/2] + 3*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(1 + Sec[c
+ d*x])^(1/6))*Tan[c + d*x])/(2*d*(1 + Sec[c + d*x])^(7/6))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4285, 3042, 4315, 3042, 4314, 60, 60, 73, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\sec(c+dx)+a)^{5/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/3}dx$$

$$\Big\downarrow \text{4285}$$

$$\frac{5}{8}\int \sec(c+dx)(\sec(c+dx)a+a)^{5/3}dx+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{8}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/3}dx+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}$$

$\downarrow$ 4315

$$\frac{5a(a\sec(c+dx)+a)^{2/3}\int \sec(c+dx)(\sec(c+dx)+1)^{5/3}dx}{8(\sec(c+dx)+1)^{2/3}}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}$$

$\downarrow$ 3042

$$\frac{5a(a\sec(c+dx)+a)^{2/3}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{5/3}dx}{8(\sec(c+dx)+1)^{2/3}}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}$$

$\downarrow$ 4314

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}-\frac{5a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int \frac{(\sec(c+dx)+1)^{7/6}}{\sqrt{1-\sec(c+dx)}}d\sec(c+dx)}{8d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$\downarrow$ 60

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}-\frac{5a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{7}{5}\int \frac{\sqrt[6]{\sec(c+dx)+1}}{\sqrt{1-\sec(c+dx)}}d\sec(c+dx)-\frac{3}{5}\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1\right.}{8d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$\downarrow$ 60

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}-\frac{5a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{7}{5}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}d\sec(c+dx)-\frac{3}{2}\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec\right.\right.}{8d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$\downarrow$ 73

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}-\frac{5a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{7}{5}\left(3\int \frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}-\frac{3}{2}\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)\right.\right.}{8d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$\downarrow$ 766

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}-$$
$$\frac{5a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{7}{5}\left(\frac{3^{3/4}\sqrt[6]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{s}}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{se}\right.}}}{2\sqrt[3]{2}\sqrt{1-\sec(c+dx)}\sqrt{-\frac{\sqrt[3]{\sec(c}}{\left(\sqrt[3]{\right.}}}}\right.\right.}{}$$

input
```
Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
(3*(a + a*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(8*d) - (5*a*(a + a*Sec[c + d*
x])^(2/3)*((-3*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])^(7/6))/5 + (7*((-
3*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])^(1/6))/2 + (3^(3/4)*EllipticF[
ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 +
Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x])^(
1/6)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec
[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 +
 Sec[c + d*x])^(1/3))^2])/(2*2^(1/3)*Sqrt[1 - Sec[c + d*x]]*Sqrt[-(((1 + S
ec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + S
qrt[3])*(1 + Sec[c + d*x])^(1/3))^2])]))/5)*Tan[c + d*x])/(8*d*Sqrt[1 - Se
c[c + d*x]]*(1 + Sec[c + d*x])^(7/6))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4285
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[a*(m/(b*(m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x]
/; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \sec (dx+c)^2 (a+a\sec (dx+c))^{\frac{5}{3}}\, dx$$

input
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/3),x)
```

Fricas [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/3}\, dx = \int (a\sec (dx+c)+a)^{\frac{5}{3}} \sec (dx+c)^2\, dx$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c)^3 + a*sec(d*x + c)^2)*(a*sec(d*x + c) + a)^(2/3),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/3}\, dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(5/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^2, x)
```

Giac [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/3}}{\cos(c+d\,x)^2}\,dx$$

input
```
int((a + a/cos(c + d*x))^(5/3)/cos(c + d*x)^2,x)
```

output
```
int((a + a/cos(c + d*x))^(5/3)/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = a^{\frac{5}{3}}\left(\int (\sec(dx+c)+1)^{\frac{2}{3}}\sec(dx+c)^3\,dx + \int (\sec(dx+c)+1)^{\frac{2}{3}}\sec(dx+c)^2\,dx\right)$$

input
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/3),x)
```

output
```
a**(2/3)*a*(int((sec(c + d*x) + 1)**(2/3)*sec(c + d*x)**3,x) + int((sec(c
+ d*x) + 1)**(2/3)*sec(c + d*x)**2,x))
```

3.150 $\int \sec(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 356

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{21a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{10d(1+\sec(c+dx))} - \frac{7\,3^{3/4}a\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right),\frac{1}{4}(2+\sqrt{3})\right)(a+a\sec(c+dx))^{2/3}\left(\sqrt[3]{2}-\cdots\right.}{10\sqrt[3]{2}d(1-\sec(c+dx))(1+\sec(c+dx))\sqrt{-\frac{\sqrt[3]{1+\sec(c-\cdots}}{\left(\sqrt[3]{2}-(1\cdots\right.}}}$$

output
```
3/5*a*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/d+21/10*a*(a+a*sec(d*x+c))^(2/3)*t
an(d*x+c)/d/(1+sec(d*x+c))-7/20*3^(3/4)*a*InverseJacobiAM(arccos((2^(1/3)-
(1-3^(1/2))*(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3
))),1/4*6^(1/2)+1/4*2^(1/2))*(a+a*sec(d*x+c))^(2/3)*(2^(1/3)-(1+sec(d*x+c)
)^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(
1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-se
c(d*x+c))/(1+sec(d*x+c))/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1
/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.19

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \frac{4\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)(a(1+\sec(c+dx)))^{5/3}\tan(c+dx)}{d(1+\sec(c+dx))^{13/6}}$$

input
```
Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
(4*2^(1/6)*Hypergeometric2F1[-7/6, 1/2, 3/2, (1 - Sec[c + d*x])/2]*(a*(1 +
 Sec[c + d*x]))^(5/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(13/6))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4315, 3042, 4314, 60, 60, 73, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\sec(c+dx)+a)^{5/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/3}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{a(a\sec(c+dx)+a)^{2/3}\int \sec(c+dx)(\sec(c+dx)+1)^{5/3}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(a\sec(c+dx)+a)^{2/3}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{5/3}dx}{(\sec(c+dx)+1)^{2/3}}$$

$\downarrow$ 4314

$$-\frac{a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int \frac{(\sec(c+dx)+1)^{7/6}}{\sqrt{1-\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$\downarrow$ 60

$$-\frac{a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{7}{5}\int \frac{\sqrt[6]{\sec(c+dx)+1}}{\sqrt{1-\sec(c+dx)}}d\sec(c+dx)-\frac{3}{5}\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1\right.}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$\downarrow$ 60

$$-\frac{a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{7}{5}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}d\sec(c+dx)-\frac{3}{2}\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec\right.\right.}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$\downarrow$ 73

$$-\frac{a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{7}{5}\left(3\int \frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}-\frac{3}{2}\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx\right.\right.}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$\downarrow$ 766

$$-\frac{a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{7}{5}\left(\frac{3^{3/4}\sqrt[6]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\ldots}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\mathrm{s}\ldots}\right.}}}{2\sqrt[3]{2}\sqrt{1-\sec(c+dx)}\sqrt{-\frac{\sqrt[3]{\sec(c\ldots}}{(\ldots}}}\right.\right.}{}$$

input
```
Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
-((a*(a + a*Sec[c + d*x])^(2/3)*((-3*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d
*x])^(7/6))/5 + (7*((-3*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])^(1/6))/2
 + (3^(3/4)*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(
1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4
]*(1 + Sec[c + d*x])^(1/6)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2
/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3
) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2])/(2*2^(1/3)*Sqrt[1 - Sec[c
+ d*x]]*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/
3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])))/5)*Tan[c +
d*x])/(d*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])^(7/6)))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \sec(dx+c)\,(a+a\sec(dx+c))^{\frac{5}{3}}\,dx$$

input
```
int(sec(d*x+c)*(a+a*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)*(a+a*sec(d*x+c))^(5/3),x)
```

Fricas [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c)^2 + a*sec(d*x + c))*(a*sec(d*x + c) + a)^(2/3), x
)
```

Sympy [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/3}\,dx=\int (a(\sec{(c+dx)}+1))^{\frac{5}{3}}\sec{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(5/3),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(5/3)*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/3}\,dx=\int (a\sec{(dx+c)}+a)^{\frac{5}{3}}\sec{(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c), x)
```

Giac [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/3}\,dx=\int (a\sec{(dx+c)}+a)^{\frac{5}{3}}\sec{(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/3}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/3}}{\cos\left(c+d\,x\right)}\,dx$$

input
```
int((a + a/cos(c + d*x))^(5/3)/cos(c + d*x),x)
```

output
```
int((a + a/cos(c + d*x))^(5/3)/cos(c + d*x), x)
```

Reduce [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/3}\,dx=a^{\frac{5}{3}}\left(\int \left(\sec\left(dx+c\right)+1\right)^{\frac{2}{3}}\sec\left(dx+c\right)^{2}dx+\int \left(\sec\left(dx+c\right)+1\right)^{\frac{2}{3}}\sec\left(dx+c\right)dx\right)$$

input
```
int(sec(d*x+c)*(a+a*sec(d*x+c))^(5/3),x)
```

output
```
a**(2/3)*a*(int((sec(c + d*x) + 1)**(2/3)*sec(c + d*x)**2,x) + int((sec(c
+ d*x) + 1)**(2/3)*sec(c + d*x),x))
```

3.151 $\int (a+a\sec(c+dx))^{5/3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 78

$$\int (a+a\sec(c+dx))^{5/3}\,dx = \frac{4\sqrt[6]{2}a\,\text{AppellF1}\left(\frac{1}{2},-\frac{7}{6},1,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),1-\sec(c+dx)\right)(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{d(1+\sec(c+dx))^{7/6}}$$

output
```
4*2^(1/6)*a*AppellF1(1/2,1,-7/6,3/2,1-sec(d*x+c),1/2-1/2*sec(d*x+c))*(a+a*
sec(d*x+c))^(2/3)*tan(d*x+c)/d/(1+sec(d*x+c))^(7/6)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2694 vs. 2(78) = 156.

Time = 14.39 (sec) , antiderivative size = 2694, normalized size of antiderivative = 34.54

$$\int (a+a\sec(c+dx))^{5/3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
(3*((1 + Cos[c + d*x])*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(5/3)*Ta
n[(c + d*x)/2])/(2*d*(1 + Sec[c + d*x])^(5/3)) + ((Cos[(c + d*x)/2]^2*Sec[
c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(5/3)*((3*Sec[(c + d*x)/2]^2*(1 + S
ec[c + d*x])^(2/3))/4 + (Cos[c + d*x]*Sec[(c + d*x)/2]^2*(1 + Sec[c + d*x]
)^(2/3))/2)*Tan[(c + d*x)/2]*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d
*x)/2]^2 + (135*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d
*x)/2]^2]*Cos[(c + d*x)/2]^2)/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/(3*2^(1/3)*d*(1 + Sec[c
+ d*x])^(5/3)*((Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)
*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos
[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (135*AppellF1[1/2
, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2
)/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] +
 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
 + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*
Tan[(c + d*x)/2]^2)))/(6*2^(1/3)) + ((Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/
3)*Tan[(c + d*x)/2]*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Ta...
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4266, 3042, 4265, 149, 25, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^{5/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/3}dx$$

$$\Big\downarrow \text{4266}$$

$$\frac{a(a\sec(c+dx)+a)^{2/3}\int(\sec(c+dx)+1)^{5/3}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(a\sec(c+dx)+a)^{2/3}\int\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{5/3}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \text{4265}$$

$$-\frac{a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int\frac{\cos(c+dx)(\sec(c+dx)+1)^{7/6}}{\sqrt{1-\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \text{149}$$

$$-\frac{6a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int\frac{\cos(c+dx)(\sec(c+dx)+1)^{2}}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \text{25}$$

$$\frac{6a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int-\frac{\cos(c+dx)(\sec(c+dx)+1)^{2}}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{3\sqrt{2}a\tan(c+dx)(\sec(c+dx)+1)(a\sec(c+dx)+a)^{2/3}\operatorname{AppellF1}\left(\frac{13}{6},1,\frac{1}{2},\frac{19}{6},\sec(c+dx)+1,\frac{1}{2}(\sec(c+dx)+\right.}{13d\sqrt{1-\sec(c+dx)}}$$

input
```
Int[(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
(3*Sqrt[2]*a*AppellF1[13/6, 1, 1/2, 19/6, 1 + Sec[c + d*x], (1 + Sec[c + d
*x])/2]*(1 + Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(13*d*
Sqrt[1 - Sec[c + d*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 149
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1
) - 1)*(c - a*(d/b) + d*(x^k/b))^n*(e - a*(f/b) + f*(x^k/b))^p, x], x, (a +
 b*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && FractionQ[m] &&
IntegerQ[2*n] && IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4265
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[a^n*(Cot
[c + d*x]/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]))   Subst[Int[(1
 + b*(x/a))^(n - 1/2)/(x*Sqrt[1 - b*(x/a)]), x], x, Csc[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0
]
```

rule 4266
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Csc[c + d*x])^FracPart[n]/(1 + (b/a)*Csc[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Maple [F]

$$\int (a + a\sec(dx + c))^{\frac{5}{3}} dx$$

input
```
int((a+a*sec(d*x+c))^(5/3),x)
```

output
```
int((a+a*sec(d*x+c))^(5/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{5/3} dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int (a + a\sec(c + dx))^{5/3} dx = \int (a\sec(c + dx) + a)^{\frac{5}{3}} dx$$

input
```
integrate((a+a*sec(d*x+c))**(5/3),x)
```

output
```
Integral((a*sec(c + d*x) + a)**(5/3), x)
```

Maxima [F]

$$\int (a + a\sec(c + dx))^{5/3} \, dx = \int (a\sec(dx + c) + a)^{\frac{5}{3}} \, dx$$

input
```
integrate((a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(5/3), x)
```

Giac [F]

$$\int (a + a\sec(c + dx))^{5/3} \, dx = \int (a\sec(dx + c) + a)^{\frac{5}{3}} \, dx$$

input
```
integrate((a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{5/3} \, dx = \int \left(a + \frac{a}{\cos(c + d\,x)}\right)^{5/3} \, dx$$

input
```
int((a + a/cos(c + d*x))^(5/3),x)
```

output
```
int((a + a/cos(c + d*x))^(5/3), x)
```

Reduce [F]

$$\int (a + a\sec(c + dx))^{5/3}\,dx = a^{\frac{5}{3}}\left(\int (\sec(dx + c) + 1)^{\frac{2}{3}}\,dx + \int (\sec(dx + c) + 1)^{\frac{2}{3}}\sec(dx + c)\,dx\right)$$

input

```
int((a+a*sec(d*x+c))^(5/3),x)
```

output

```
a**(2/3)*a*(int((sec(c + d*x) + 1)**(2/3),x) + int((sec(c + d*x) + 1)**(2/
3)*sec(c + d*x),x))
```

3.152 $\int \cos(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 78

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \frac{4\sqrt[6]{2}a\,\text{AppellF1}\left(\frac{1}{2},-\frac{7}{6},2,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),1-\sec(c+dx)\right)(a+a\sec(c+dx))^{2/3}\tan(}{d(1+\sec(c+dx))^{7/6}}$$

output
```
4*2^(1/6)*a*AppellF1(1/2,2,-7/6,3/2,1-sec(d*x+c),1/2-1/2*sec(d*x+c))*(a+a*
sec(d*x+c))^(2/3)*tan(d*x+c)/d/(1+sec(d*x+c))^(7/6)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2700 vs. $2(78) = 156$.

Time = 14.52 (sec) , antiderivative size = 2700, normalized size of antiderivative = 34.62

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
(((1 + Cos[c + d*x])*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(5/3)*(Sin
[c + d*x] - Tan[(c + d*x)/2]))/(d*(1 + Sec[c + d*x])^(5/3)) - (2^(2/3)*(Co
s[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(5/3)*((2*Sec[
(c + d*x)/2]^2*(1 + Sec[c + d*x])^(2/3))/3 + (5*Cos[c + d*x]*Sec[(c + d*x)
/2]^2*(1 + Sec[c + d*x])^(2/3))/6)*Tan[(c + d*x)/2]*(AppellF1[3/2, 2/3, 1,
 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)
/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (243*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(-9*AppellF1[1/2, 2/3
, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 2/
3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3,
 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/(
9*d*(1 + Sec[c + d*x])^(5/3)*(-1/9*(Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2
*Sec[c + d*x])^(2/3)*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2
 + (243*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
]*Cos[(c + d*x)/2]^2)/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/2^(1/3) - (2^(2/3)*(Cos[(c + d*x
)/2]^2*Sec[c + d*x])^(2/3)*Tan[(c + d*x)/2]*(AppellF1[3/2, 2/3, 1, 5/2,...
```

Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4315, 3042, 4314, 149, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\sec(c+dx)+a)^{5/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/3}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{a(a\sec(c+dx)+a)^{2/3}\int\cos(c+dx)(\sec(c+dx)+1)^{5/3}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a(a\sec(c+dx)+a)^{2/3}\int\frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{5/3}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \text{ 4314}$$

$$-\frac{a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int\frac{\cos^2(c+dx)(\sec(c+dx)+1)^{7/6}}{\sqrt{1-\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \text{ 149}$$

$$-\frac{6a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int\frac{\cos^2(c+dx)(\sec(c+dx)+1)^2}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \text{ 1012}$$

$$-\frac{3\sqrt{2}a\tan(c+dx)(\sec(c+dx)+1)(a\sec(c+dx)+a)^{2/3}\operatorname{AppellF1}\left(\frac{13}{6},2,\frac{1}{2},\frac{19}{6},\sec(c+dx)+1,\frac{1}{2}(\sec(c+dx)\right.}{13d\sqrt{1-\sec(c+dx)}}$$

input
```
Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
(-3*Sqrt[2]*a*AppellF1[13/6, 2, 1/2, 19/6, 1 + Sec[c + d*x], (1 + Sec[c +
d*x])/2]*(1 + Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(13*d
*Sqrt[1 - Sec[c + d*x]])
```

Defintions of rubi rules used

rule 149
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1
) - 1)*(c - a*(d/b) + d*(x^k/b))^n*(e - a*(f/b) + f*(x^k/b))^p, x], x, (a +
 b*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && FractionQ[m] &&
IntegerQ[2*n] && IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \cos(dx+c)\,(a+a\sec(dx+c))^{\frac{5}{3}}\,dx$$

input
```
int(cos(d*x+c)*(a+a*sec(d*x+c))^(5/3),x)
```

output
```
int(cos(d*x+c)*(a+a*sec(d*x+c))^(5/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(5/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(5/3)*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(5/3)*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \int \cos(c+d\,x)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/3}\,dx$$

input
```
int(cos(c + d*x)*(a + a/cos(c + d*x))^(5/3),x)
```

output
```
int(cos(c + d*x)*(a + a/cos(c + d*x))^(5/3), x)
```

Reduce [F]

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = a^{\frac{5}{3}}\left(\int (\sec(dx+c)+1)^{\frac{2}{3}}\cos(dx+c)\sec(dx+c)\,dx + \int (\sec(dx+c)+1)^{\frac{2}{3}}\cos(dx+c)\,dx\right)$$

input
```
int(cos(d*x+c)*(a+a*sec(d*x+c))^(5/3),x)
```

output
```
a**(2/3)*a*(int((sec(c + d*x) + 1)**(2/3)*cos(c + d*x)*sec(c + d*x),x) + i
nt((sec(c + d*x) + 1)**(2/3)*cos(c + d*x),x))
```

3.153 $\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 371

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{99\tan(c+dx)}{80d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{8d\sqrt[3]{a+a\sec(c+dx)}} - \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40ad} + \frac{37\,3^{3/4}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)\left(\sqrt[3]{2}-\sqrt[3]{1+\sec(c+dx)}\right)\cdots}{80\sqrt[3]{2}d(1-\sec(c+dx))\sqrt[3]{a+a\sec(c+dx)}\sqrt{-\frac{\sqrt[3]{1+\sec(c+dx)}\left(\cdots\right.}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}\right.}}}$$

output

```
99/80*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/3)+3/8*sec(d*x+c)^2*tan(d*x+c)/d/(a
+a*sec(d*x+c))^(1/3)-3/40*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/a/d+37/160*3^(
3/4)*InverseJacobiAM(arccos((2^(1/3)-(1-3^(1/2))*(1+sec(d*x+c))^(1/3))/(2^
(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))*(2^(1/3)
-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c
))^(2/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))^2)^(1/2)*tan(d*x+c)*2
^(2/3)/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(1/3)/(-(1+sec(d*x+c))^(1/3)*(2^(
1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))^2)^(
1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.26 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.42

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$$
$$= \frac{\left(-4\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)+16\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\right.}{8d\sqrt[6]{1+\sec(}}$$

input
```
Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(1/3),x]
```

output
```
((-4*2^(1/6)*Hypergeometric2F1[-7/6, 1/2, 3/2, (1 - Sec[c + d*x])/2] + 16*
2^(1/6)*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Sec[c + d*x])/2] - 7*2^(1/6
)*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sec[c + d*x])/2] + 3*Sec[c + d*x]^
2*(1 + Sec[c + d*x])^(1/6))*Tan[c + d*x])/(8*d*(1 + Sec[c + d*x])^(1/6)*(a
*(1 + Sec[c + d*x]))^(1/3))
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4311, 27, 3042, 4498, 27, 3042, 4489, 3042, 4315, 3042, 4314, 73, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a\sec(c+dx)+a}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\sqrt[3]{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$
$$\Big\downarrow \text{4311}$$

$$\frac{3\int \frac{\sec^2(c+dx)(6a-a\sec(c+dx))}{3\sqrt[3]{\sec(c+dx)a+a}}dx}{8a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sec^2(c+dx)(6a-a\sec(c+dx))}{\sqrt[3]{\sec(c+dx)a+a}}dx}{8a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(6a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{8a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4498}$$

$$\frac{\frac{3\int -\frac{\sec(c+dx)\left(2a^2-33a^2\sec(c+dx)\right)}{3\sqrt[3]{\sec(c+dx)a+a}}dx}{5a}-\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{\sec(c+dx)\left(2a^2-33a^2\sec(c+dx)\right)}{\sqrt[3]{\sec(c+dx)a+a}}dx}{5a}-\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2a^2-33a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}-\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4489}$$

$$\frac{-\frac{\frac{37}{2}a^2\int \frac{\sec(c+dx)}{\sqrt[3]{\sec(c+dx)a+a}}dx-\frac{99a^2\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}}{5a}-\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\frac{37}{2}a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{99a^2\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}}{5a}-\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}$$

$\downarrow$ 4315

$$\frac{-\frac{\frac{37a^2\sqrt[3]{\sec(c+dx)+1}\int \frac{\sec(c+dx)}{\sqrt[3]{\sec(c+dx)+1}}dx}{2\sqrt[3]{a\sec(c+dx)+a}}-\frac{99a^2\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}}{5a}-\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}$$

$\downarrow$ 3042

$$\frac{-\frac{\frac{37a^2\sqrt[3]{\sec(c+dx)+1}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx}{2\sqrt[3]{a\sec(c+dx)+a}}-\frac{99a^2\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}}{5a}-\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}$$

$\downarrow$ 4314

$$\frac{-\frac{-\frac{37a^2\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}d\sec(c+dx)}{2d\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a\sec(c+dx)+a}}-\frac{99a^2\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}}{5a}-\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}$$

$\downarrow$ 73

$$\frac{-\frac{-\frac{111a^2\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a\sec(c+dx)+a}}-\frac{99a^2\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}}{5a}-\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}$$

$\downarrow$ 766

$$-\frac{-\frac{37\ 3^{3/4}a^2\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-(1-\sqrt{3})\sqrt[3]{\cdots}}{\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\cdots}}\right.\right.}{2\sqrt[3]{2}d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}}}{5a}}{8a}$$

$$\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}$$

input
```
Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(1/3),x]
```

output
```
(3*Sec[c + d*x]^2*Tan[c + d*x])/(8*d*(a + a*Sec[c + d*x])^(1/3)) + ((-3*(a
 + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*d) - ((-99*a^2*Tan[c + d*x])/(2*
d*(a + a*Sec[c + d*x])^(1/3)) - (37*3^(3/4)*a^2*EllipticF[ArcCos[(2^(1/3)
- (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Se
c[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))
*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/
3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2
*2^(1/3)*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Sqrt[-(((1 + Sec[
c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt
[3])*(1 + Sec[c + d*x])^(1/3))^2)]))/(5*a))/(8*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4311
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(m + n - 1))), x] + Simp[d^2/(b*(m + n - 1))   In
t[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*m*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && GtQ[n
, 2] && NeQ[m + n - 1, 0] && IntegerQ[n]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

rule 4498
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int
[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)
*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a
*B, 0] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{\sec\left(dx+c\right)^{4}}{\left(a+a\sec\left(dx+c\right)\right)^{\frac{1}{3}}}dx$$

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^{4}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(1/3), x)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\sec^4(c+dx)}{\sqrt[3]{a(\sec(c+dx)+1)}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**(1/3),x)
```

output
```
Integral(sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(1/3), x)
```

Maxima [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^4}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(1/3), x)
```

Giac [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^4}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^4\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{1/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(1/3)),x)
```

output
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(1/3)), x)
```

Reduce [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx=\frac{\int \frac{\sec(dx+c)^4}{(\sec(dx+c)+1)^{\frac{1}{3}}}dx}{a^{\frac{1}{3}}}$$

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(c + d*x)**4/(sec(c + d*x) + 1)**(1/3),x)/a**(1/3)
```

3.154 $\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 336

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = -\frac{9\tan(c+dx)}{10d\sqrt[3]{a+a\sec(c+dx)}}+\frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5ad}$$
$$-\frac{7\,3^{3/4}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)\left(\sqrt[3]{2}-\sqrt[3]{1+\sec(c+dx)}\right)\sqrt{\cdots}}{10\sqrt[3]{2}d(1-\sec(c+dx))\sqrt[3]{a+a\sec(c+dx)}\sqrt{-\frac{\sqrt[3]{1+\sec(c+dx)}\left(\sqrt[3]{\cdots}\right.}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}\right.}}}$$

output
```
-9/10*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/3)+3/5*(a+a*sec(d*x+c))^(2/3)*tan(d
*x+c)/a/d-7/20*3^(3/4)*InverseJacobiAM(arccos((2^(1/3)-(1-3^(1/2))*(1+sec(
d*x+c))^(1/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))),1/4*6^(1/2)+1/4
*2^(1/2))*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^
(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))^2)^
(1/2)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(1/3)/(-(1+sec(
d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d
*x+c))^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.28

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{\left(7\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)+3\sqrt[6]{1+\sec(c+dx)}(-1+2\sec(c+dx))\right)\tan(}{10d\sqrt[6]{1+\sec(c+dx)}\sqrt[3]{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(1/3),x]
```

output
```
((7*2^(1/6)*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sec[c + d*x])/2] + 3*(1
+ Sec[c + d*x])^(1/6)*(-1 + 2*Sec[c + d*x]))*Tan[c + d*x])/(10*d*(1 + Sec[
c + d*x])^(1/6)*(a*(1 + Sec[c + d*x]))^(1/3))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4287, 27, 3042, 4489, 3042, 4315, 3042, 4314, 73, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\sqrt[3]{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4287}$$

$$\frac{3\int \frac{\sec(c+dx)(2a-3a\sec(c+dx))}{3\sqrt[3]{\sec(c+dx)a+a}}dx}{5a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sec(c+dx)(2a-3a\sec(c+dx))}{\sqrt[3]{\sec(c+dx)a+a}}dx}{5a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2a-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5ad}$$

$$\Big\downarrow \text{4489}$$

$$\frac{\frac{7}{2}a\int \frac{\sec(c+dx)}{\sqrt[3]{\sec(c+dx)a+a}}dx-\frac{9a\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}}{5a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{7}{2}a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{9a\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}}{5a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5ad}$$

$$\Big\downarrow \text{4315}$$

$$\frac{\frac{7a\sqrt[3]{\sec(c+dx)+1}\int \frac{\sec(c+dx)}{\sqrt[3]{\sec(c+dx)+1}}dx}{2\sqrt[3]{a\sec(c+dx)+a}}-\frac{9a\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}}{5a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{7a\sqrt[3]{\sec(c+dx)+1}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx}{2\sqrt[3]{a\sec(c+dx)+a}}-\frac{9a\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}}{5a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5ad}$$

$$\Big\downarrow \text{4314}$$

$$\frac{-\frac{7a\tan(c+dx)\int\frac{1}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}d\sec(c+dx)}{2d\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a\sec(c+dx)+a}}-\frac{9a\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}}{5a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5ad}$$

$$\Big\downarrow \text{73}$$

$$\frac{-\frac{21a\tan(c+dx)\int\frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a\sec(c+dx)+a}}-\frac{9a\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}}{5a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5ad}$$

$$\Big\downarrow \text{766}$$

$$\frac{-\frac{7\,3^{3/4}a\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\ldots}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\ldots}}\right.\right.}{2\sqrt[3]{2}d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+\ldots}}}{5a}$$

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5ad}$$

input
```
Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(1/3),x]
```

output
```
(3*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*a*d) + ((-9*a*Tan[c + d*x])
/(2*d*(a + a*Sec[c + d*x])^(1/3)) - (7*3^(3/4)*a*EllipticF[ArcCos[(2^(1/3)
 - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + S
ec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)
)*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2
/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(
2*2^(1/3)*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Sqrt[-(((1 + Sec
[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqr
t[3])*(1 + Sec[c + d*x])^(1/3))^2)]))/(5*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4287
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 -
b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Maple [F]

$$\int \frac{\sec \left(dx+c\right)^{3}}{\left(a+a\sec \left(dx+c\right)\right)^{\frac{1}{3}}}dx$$

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\sec \left(dx+c\right)^{3}}{\left(a\sec \left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(1/3), x)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\sec^3(c+dx)}{\sqrt[3]{a(\sec(c+dx)+1)}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**(1/3),x)
```

output
```
Integral(sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(1/3), x)
```

Maxima [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^3}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(1/3), x)
```

Giac [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^3}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^3\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{1/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(1/3)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(1/3)), x)
```

Reduce [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx=\frac{\int \frac{\sec(dx+c)^3}{(\sec(dx+c)+1)^{\frac{1}{3}}}dx}{a^{\frac{1}{3}}}$$

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(c + d*x)**3/(sec(c + d*x) + 1)**(1/3),x)/a**(1/3)
```

3.155 $\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 306

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{3\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3^{3/4}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-(1-\sqrt{3})\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{1+\sec(c+dx)}}\right),\frac{1}{4}(2+\sqrt{3})\right)\left(\sqrt[3]{2}-\sqrt[3]{1+\sec(c+dx)}\right)\sqrt{\cdots}}{2\sqrt[3]{2}d(1-\sec(c+dx))\sqrt[3]{a+a\sec(c+dx)}\sqrt{-\frac{\sqrt[3]{1+\sec(c+dx)}\left(\sqrt[3]{2}\cdots\right.}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{1\cdots}\right.}}}$$

output

```
3/2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/3)+1/4*3^(3/4)*InverseJacobiAM(arccos
((2^(1/3)-(1-3^(1/2))*(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*
x+c))^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^
(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+3^(1/
2))*(1+sec(d*x+c))^(1/3))^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(a+
a*sec(d*x+c))^(1/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/
(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.28

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{\left(-\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)+3\sqrt[6]{1+\sec(c+dx)}\right)\tan(c+dx)}{2d\sqrt[6]{1+\sec(c+dx)}\sqrt[3]{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(1/3),x]
```

output
```
((-(2^(1/6)*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sec[c + d*x])/2]) + 3*(1
 + Sec[c + d*x])^(1/6))*Tan[c + d*x])/(2*d*(1 + Sec[c + d*x])^(1/6)*(a*(1
+ Sec[c + d*x]))^(1/3))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4285, 3042, 4315, 3042, 4314, 73, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a\sec(c+dx)+a}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt[3]{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{ 4285}$$

$$\frac{3\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}-\frac{1}{2}\int \frac{\sec(c+dx)}{\sqrt[3]{\sec(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}-\frac{1}{2}\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{3\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}-\frac{\sqrt[3]{\sec(c+dx)+1}\int\frac{\sec(c+dx)}{\sqrt[3]{\sec(c+dx)+1}}dx}{2\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}-\frac{\sqrt[3]{\sec(c+dx)+1}\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx}{2\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4314}$$

$$\frac{\tan(c+dx)\int\frac{1}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}d\sec(c+dx)}{2d\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a\sec(c+dx)+a}}+\frac{3\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{73}$$

$$\frac{3\tan(c+dx)\int\frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a\sec(c+dx)+a}}+\frac{3\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{766}$$

$$\frac{3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}\ldots}{\sqrt[3]{2}\ldots}\right.\right.}{2\sqrt[3]{2}d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(\ldots}$$
$$\frac{3\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}$$

input

```
Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(1/3),x]
```

output
```
(3*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(1/3)) + (3^(3/4)*EllipticF[Arc
Cos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqr
t[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1 + Sec[c +
 d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[
c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan
[c + d*x])/(2*2^(1/3)*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Sqrt
[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3
) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4285
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[a*(m/(b*(m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x]
/; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{\sec(dx+c)^2}{(a+a\sec(dx+c))^{\frac{1}{3}}}dx$$

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(1/3), x)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\sec^2(c+dx)}{\sqrt[3]{a(\sec(c+dx)+1)}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**(1/3),x)
```

output
```
Integral(sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(1/3), x)
```

Maxima [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(1/3), x)
```

Giac [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)^2\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(1/3)),x)
```

output
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(1/3)), x)
```

Reduce [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{\int \frac{\sec(dx+c)^2}{(\sec(dx+c)+1)^{\frac{1}{3}}}dx}{a^{\frac{1}{3}}}$$

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(c + d*x)**2/(sec(c + d*x) + 1)**(1/3),x)/a**(1/3)
```

3.156 $\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 276

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = -\frac{3^{3/4}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)\left(\sqrt[3]{2}-\sqrt[3]{1+\sec(c+dx)}\right)\sqrt{\cdots}}{\sqrt[3]{2}d(1-\sec(c+dx))\sqrt[3]{a+a\sec(c+dx)}\sqrt{-\frac{\sqrt[3]{1+\sec(c+dx)}\left(\sqrt[3]{2}\cdots\right.}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1\cdots}\right.}}}$$

output
```
-1/2*3^(3/4)*InverseJacobiAM(arccos((2^(1/3)-(1-3^(1/2))*(1+sec(d*x+c))^(1
/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))*
(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+s
ec(d*x+c))^(2/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))^2)^(1/2)*tan(
d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(1/3)/(-(1+sec(d*x+c))^(1
/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/
3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.24

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)\tan(c+dx)}{d\sqrt[6]{1+\sec(c+dx)}\sqrt[3]{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^(1/3),x]
```

output
```
(2^(1/6)*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sec[c + d*x])/2]*Tan[c + d*
x])/(d*(1 + Sec[c + d*x])^(1/6)*(a*(1 + Sec[c + d*x]))^(1/3))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4315, 3042, 4314, 73, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{\sqrt[3]{\sec(c+dx)+1}\int \frac{\sec(c+dx)}{\sqrt[3]{\sec(c+dx)+1}}dx}{\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{\sec(c+dx)+1}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx}{\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4314}$$

$$-\frac{\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{6\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{766}$$

$$-\frac{3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\text{EllipticF}\left(\arccos\left(\right.\right.}{\sqrt[3]{2}d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec}}$$

input
```
Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^(1/3),x]
```

output
```
-((3^(3/4)*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1
/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]
*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c +
 d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec
[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2^(1/3)*d*(1 - Sec[c + d*x])*(a + a*Se
c[c + d*x])^(1/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c +
 d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]))
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{\sec(dx+c)}{(a+a\sec(dx+c))^{\frac{1}{3}}}dx$$

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)/(a*sec(d*x + c) + a)^(1/3), x)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(c+dx)}{\sqrt[3]{a(\sec(c+dx)+1)}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))**(1/3),x)
```

output
```
Integral(sec(c + d*x)/(a*(sec(c + d*x) + 1))**(1/3), x)
```

Maxima [F]

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(1/3), x)
```

Giac [F]

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(1/3)),x)
```

output
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(1/3)), x)
```

Reduce [F]

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{\int \frac{\sec(dx+c)}{(\sec(dx+c)+1)^{\frac{1}{3}}}dx}{a^{\frac{1}{3}}}$$

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(c + d*x)/(sec(c + d*x) + 1)**(1/3),x)/a**(1/3)
```

3.157 $\int \frac{1}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 75

$$\int \frac{1}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{3\sqrt{2}\,\text{AppellF1}\left(\frac{1}{6},\frac{1}{2},1,\frac{7}{6},\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{a+a\sec(c+dx)}}$$

output
```
3*2^(1/2)*AppellF1(1/6,1,1/2,7/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*tan(d*x+
c)/d/(1-sec(d*x+c))^(1/2)/(a+a*sec(d*x+c))^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 718 vs. 2(75) = 150.

Time = 3.09 (sec) , antiderivative size = 718, normalized size of antiderivative = 9.57

$$\int \frac{1}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(-1/3),x]
```

output
```
(45*AppellF1[1/2, -1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*C
os[c + d*x]*(1 + Sec[c + d*x])^2*Tan[(c + d*x)/2]*(9*AppellF1[1/2, -1/3, 1
, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*(3*AppellF1[3/2, -1/3,
 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 2/3, 1,
5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))/(d*(a*
(1 + Sec[c + d*x]))^(1/3)*(40*(3*AppellF1[3/2, -1/3, 2, 5/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2])^2*Sec[c + d*x]*Sin[(c + d*x)/2]^2*Tan[(c + d*x)/2
]^2 + 6*AppellF1[1/2, -1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*Sec[c + d*x]^2*Sin[(c + d*x)/2]^2*(-15*AppellF1[3/2, -1/3, 2, 5/2, Tan[
(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 - 10*Cos[c + d*x] + 3*Cos[2*(c + d
*x)]) - 5*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2]*(1 - 10*Cos[c + d*x] + 3*Cos[2*(c + d*x)]) - 24*(9*AppellF1[5/2, -1/3,
 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 3*AppellF1[5/2, 2/3, 2
, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[5/2, 5/3, 1, 7/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Tan[(c + d*x)/2]
^2) + 135*AppellF1[1/2, -1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]^2*(3 + 3*Sec[c + d*x] - 3*Sin[c + d*x]*Tan[c + d*x] - Tan[c + d*x]^2)
))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4266, 3042, 4265, 149, 25, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt[3]{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4266}$$

$$\frac{\sqrt[3]{\sec(c+dx)+1}\int \frac{1}{\sqrt[3]{\sec(c+dx)+1}}dx}{\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\sqrt[3]{\sec(c+dx)+1}\int \frac{1}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx}{\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \textbf{4265}$$

$$-\frac{\tan(c+dx)\int \frac{\cos(c+dx)}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \textbf{149}$$

$$-\frac{6\tan(c+dx)\int \frac{\cos(c+dx)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{6\tan(c+dx)\int -\frac{\cos(c+dx)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \textbf{936}$$

$$\frac{3\sqrt{2}\tan(c+dx)\operatorname{AppellF1}\left(\frac{1}{6},1,\frac{1}{2},\frac{7}{6},\sec(c+dx)+1,\frac{1}{2}(\sec(c+dx)+1)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{a\sec(c+dx)+a}}$$

input
```
Int[(a + a*Sec[c + d*x])^(-1/3),x]
```

output
```
(3*Sqrt[2]*AppellF1[1/6, 1, 1/2, 7/6, 1 + Sec[c + d*x], (1 + Sec[c + d*x])
/2]*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(1/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 149
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1
) - 1)*(c - a*(d/b) + d*(x^k/b))^n*(e - a*(f/b) + f*(x^k/b))^p, x], x, (a +
 b*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && FractionQ[m] &&
IntegerQ[2*n] && IntegerQ[p]
```

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4265
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[a^n*(Cot
[c + d*x]/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]))   Subst[Int[(1
 + b*(x/a))^(n - 1/2)/(x*Sqrt[1 - b*(x/a)]), x], x, Csc[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0
]
```

rule 4266
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Csc[c + d*x])^FracPart[n]/(1 + (b/a)*Csc[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{(a + a\sec(dx + c))^{\frac{1}{3}}} dx$$

input
```
int(1/(a+a*sec(d*x+c))^(1/3),x)
```

output
```
int(1/(a+a*sec(d*x+c))^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a + a\sec(c + dx)}} dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a + a\sec(c + dx)}} dx = \int \frac{1}{\sqrt[3]{a\sec(c + dx) + a}} dx$$

input
```
integrate(1/(a+a*sec(d*x+c))**(1/3),x)
```

output
```
Integral((a*sec(c + d*x) + a)**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+a\sec(c+dx)}}\,dx=\int \frac{1}{(a\sec{(dx+c)}+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a+a\sec(c+dx)}}\,dx=\int \frac{1}{(a\sec{(dx+c)}+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(-1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+a\sec(c+dx)}}\,dx=\int \frac{1}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{1/3}}\,dx$$

input
```
int(1/(a + a/cos(c + d*x))^(1/3),x)
```

output
```
int(1/(a + a/cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{a + a\sec(c + dx)}}\,dx = \frac{\int \frac{1}{(\sec(dx+c)+1)^{\frac{1}{3}}}dx}{a^{\frac{1}{3}}}$$

input
```
int(1/(a+a*sec(d*x+c))^(1/3),x)
```

output
```
int(1/(sec(c + d*x) + 1)**(1/3),x)/a**(1/3)
```

3.158 $\int \frac{\cos(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 75

$$\int \frac{\cos(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = -\frac{3\sqrt{2}\,\text{AppellF1}\left(\frac{1}{6},\frac{1}{2},2,\frac{7}{6},\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{a+a\sec(c+dx)}}$$

output
```
-3*2^(1/2)*AppellF1(1/6,2,1/2,7/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*tan(d*x
+c)/d/(1-sec(d*x+c))^(1/2)/(a+a*sec(d*x+c))^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 240 vs. 2(75) = 150.

Time = 1.27 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.20

$$\int \frac{\cos(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{(a(1+\sec(c+dx)))^{2/3}\left(\frac{20\,\text{AppellF1}\left(\frac{3}{2},\frac{2}{3},1,\frac{5}{2},\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}\right.\right.}{6\left(3\,\text{AppellF1}\left(\frac{5}{2},\frac{2}{3},2,\frac{7}{2},\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-2\,\text{AppellF1}\left(\frac{5}{2},\frac{5}{3},1,\frac{7}{2},\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(\right.\right.\right.}\right.}{}$$

input
```
Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^(1/3),x]
```

output
```
((a*(1 + Sec[c + d*x]))^(2/3)*((20*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]*Sin[(c + d*x)/2]^3)/(6*(3*Ap
pellF1[5/2, 2/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*Appe
llF1[5/2, 5/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos
[c + d*x]) + 45*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d
*x)/2]^2]*(1 + Cos[c + d*x])) + Sin[c + d*x] - Tan[(c + d*x)/2]))/(a*d)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4315, 3042, 4314, 149, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{\sqrt[3]{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt[3]{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{\sqrt[3]{\sec(c+dx)+1}\int \frac{\cos(c+dx)}{\sqrt[3]{\sec(c+dx)+1}}dx}{\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{\sec(c+dx)+1}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx}{\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4314}$$

$$-\frac{\tan(c+dx)\int \frac{\cos^2(c+dx)}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{149}$$

$$-\frac{6\tan(c+dx)\int \frac{\cos^2(c+dx)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{936}$$

$$-\frac{3\sqrt{2}\tan(c+dx)\,\text{AppellF1}\left(\frac{1}{6},2,\frac{1}{2},\frac{7}{6},\sec(c+dx)+1,\frac{1}{2}(\sec(c+dx)+1)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{a\sec(c+dx)+a}}$$

input
```
Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^(1/3),x]
```

output
```
(-3*Sqrt[2]*AppellF1[1/6, 2, 1/2, 7/6, 1 + Sec[c + d*x], (1 + Sec[c + d*x]
)/2]*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(1/3))
```

Defintions of rubi rules used

rule 149
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1
) - 1)*(c - a*(d/b) + d*(x^k/b))^n*(e - a*(f/b) + f*(x^k/b))^p, x], x, (a +
 b*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && FractionQ[m] &&
IntegerQ[2*n] && IntegerQ[p]
```

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{\cos(dx+c)}{(a+a\sec(dx+c))^{\frac{1}{3}}}dx$$

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^(1/3),x)
```

output
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\cos(c+dx)}{\sqrt[3]{a\left(\sec(c+dx)+1\right)}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))**(1/3),x)
```

output
```
Integral(cos(c + d*x)/(a*(sec(c + d*x) + 1))**(1/3), x)
```

Maxima [F]

$$\int \frac{\cos(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\cos(dx+c)}{\left(a\sec(dx+c)+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(1/3), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\cos(dx+c)}{\left(a\sec(dx+c)+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx=\int \frac{\cos\left(c+d\,x\right)}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{1/3}}\,dx$$

input
```
int(cos(c + d*x)/(a + a/cos(c + d*x))^(1/3),x)
```

output
```
int(cos(c + d*x)/(a + a/cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx=\frac{\int \frac{\cos(dx+c)}{(\sec(dx+c)+1)^{\frac{1}{3}}}dx}{a^{\frac{1}{3}}}$$

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^(1/3),x)
```

output
```
int(cos(c + d*x)/(sec(c + d*x) + 1)**(1/3),x)/a**(1/3)
```

3.159 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 766

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = -\frac{33\tan(c+dx)}{28d(a+a\sec(c+dx))^{5/3}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/3}} + \frac{135\tan(c+dx)}{14ad(a+a\sec(c+dx))^{2/3}} + \frac{375\left(1+\sqrt{3}\right)\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{28a^2d(1+\sec(c+dx))^{2/3}\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}\right)} - \frac{375\sqrt[4]{3}E\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)\sqrt[3]{a+a\sec(c+dx)}\left(\sqrt[3]{2}-\sqrt[3]{1+\sec(c\ldots}\right.}{14\,2^{2/3}a^2d(1-\sec(c+dx))(1+\sec(c+dx))^{2/3}\sqrt{-\frac{\sqrt[3]{1+\sec(c+\ldots}}{\left(\sqrt[3]{2}-\left(1+\ldots\right.}}} - \frac{125\,3^{3/4}\left(1-\sqrt{3}\right)\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)\sqrt[3]{a+a\sec(c+dx)\ldots}}{28\,2^{2/3}a^2d(1-\sec(c+dx))(1+\sec(c+dx))^{2/3}\sqrt{-\frac{\sqrt[3]{1+\ldots}}{\ldots}}}$$

output
```
-33/28*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/3)+3/4*sec(d*x+c)^2*tan(d*x+c)/d/(
a+a*sec(d*x+c))^(5/3)+135/14*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^(2/3)+375/28*
(1+3^(1/2))*(a+a*sec(d*x+c))^(1/3)*tan(d*x+c)/a^2/d/(1+sec(d*x+c))^(2/3)/(
2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))-375/28*3^(1/4)*EllipticE((1-(2^(
1/3)-(1-3^(1/2))*(1+sec(d*x+c))^(1/3))^2/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c
))^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a+a*sec(d*x+c))^(1/3)*(2^(1/3
)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+
c))^(2/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))^2)^(1/2)*tan(d*x+c)*
2^(1/3)/a^2/d/(1-sec(d*x+c))/(1+sec(d*x+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(
2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))^2
)^(1/2)-125/56*3^(3/4)*(1-3^(1/2))*InverseJacobiAM(arccos((2^(1/3)-(1-3^(1
/2))*(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))),1/4
*6^(1/2)+1/4*2^(1/2))*(a+a*sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3)
)*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1
+3^(1/2))*(1+sec(d*x+c))^(1/3))^2)^(1/2)*tan(d*x+c)*2^(1/3)/a^2/d/(1-sec(d
*x+c))/(1+sec(d*x+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))
^(1/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.38 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.14

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{\left(-2502^{5/6}\cos^2\left(\frac{1}{2}(c+dx)\right)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)\right)\right.}{28d(a(1}$$

input
```
Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
((-250*2^(5/6)*Cos[(c + d*x)/2]^2*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Se
c[c + d*x])/2]*Sec[c + d*x]*(1 + Sec[c + d*x])^(1/6) + 3*(79 + 90*Sec[c +
d*x] + 7*Sec[c + d*x]^2))*Tan[c + d*x])/(28*d*(a*(1 + Sec[c + d*x]))^(5/3)
)
```

Rubi [A] (warning: unable to verify)

Time = 1.42 (sec) , antiderivative size = 781, normalized size of antiderivative = 1.02, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4311, 27, 3042, 4496, 27, 3042, 4488, 3042, 4315, 3042, 4314, 73, 837, 25, 27, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a\sec(c+dx)+a)^{5/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/3}}dx$$

$$\Big\downarrow \text{4311}$$

$$\frac{3\int \frac{\sec^2(c+dx)(6a-5a\sec(c+dx))}{3(\sec(c+dx)a+a)^{5/3}}dx}{4a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sec^2(c+dx)(6a-5a\sec(c+dx))}{(\sec(c+dx)a+a)^{5/3}}dx}{4a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(6a-5a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/3}}dx}{4a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{4496}$$

$$\frac{-\frac{3\int -\frac{5\sec(c+dx)\left(11a^2-7a^2\sec(c+dx)\right)}{3(\sec(c+dx)a+a)^{2/3}}dx}{7a^2}-\frac{33a\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}}{4a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{5\int \frac{\sec(c+dx)\left(11a^2-7a^2\sec(c+dx)\right)}{(\sec(c+dx)a+a)^{2/3}}dx}{7a^2}-\frac{33a\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}}{4a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(11a^2-7a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{2/3}}dx}{7a^2}-\frac{33a\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}}{4a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{4488}$$

$$\frac{\frac{5\left(\frac{54a^2\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}-25a\int \sec(c+dx)\sqrt[3]{\sec(c+dx)a+a}dx\right)}{7a^2}-\frac{33a\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}}{4a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5\left(\frac{54a^2\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}-25a\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx\right)}{7a^2}-\frac{33a\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}}{4a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{4315}$$

$$\frac{\frac{5\left(\frac{54a^2\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}-\frac{25a\sqrt[3]{a\sec(c+dx)+a}\int \sec(c+dx)\sqrt[3]{\sec(c+dx)+1}dx}{\sqrt[3]{\sec(c+dx)+1}}\right)}{7a^2}-\frac{33a\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}}{4a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5\left(\frac{54a^2\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}-\frac{25a\sqrt[3]{a\sec(c+dx)+a}\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)+1}dx}{\sqrt[3]{\sec(c+dx)+1}}\right)}{7a^2}-\frac{33a\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}}{4a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{4314}$$

$$\frac{5\left(\frac{25a\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\int\frac{1}{\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}+\frac{54a^2\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}\right)}{7a^2}-\frac{33a\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}}{4a} -$$

$$\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{5\left(\frac{150a\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\int\frac{(\sec(c+dx)+1)^{2/3}}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}+\frac{54a^2\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}\right)}{7a^2}-\frac{33a\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}}{4a}+$$

$$\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{837}$$

$$\frac{5\left(\frac{150a\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\left(-\frac{(1-\sqrt{3})\int\frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{\sqrt[3]{2}}-\frac{1}{2}\int-\frac{2(\sec(c+dx)+1)^{2/3}+2^{2/3}(1-\sqrt{3})}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c\cdots}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}\cdots}{7a^2}\cdots}{4a}$$

$$\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{25}$$

$$\frac{5\left(\frac{150a\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\left(\frac{1}{2}\int\frac{2^{2/3}\left(\sqrt[3]{2}(\sec(c+dx)+1)^{2/3}-\sqrt{3}+1\right)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}-\frac{(1-\sqrt{3})\int\frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c\cdots}{\sqrt[3]{2}}\cdots}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}\cdots}{7a^2}\cdots}{4a}$$

$$\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\left(\frac{150a\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\left(\frac{\int\frac{\sqrt[3]{2}(\sec(c+dx)+1)^{2/3}-\sqrt{3}+1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{\sqrt[3]{2}}-\frac{(1-\sqrt{3})\int\frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)}}{\sqrt[3]{2}}\right.}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}\right.}{7a^2}$$

$$4a$$

$$\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$\downarrow$ 766

$$5\left(\frac{150a\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\left(\frac{\int\frac{\sqrt[3]{2}(\sec(c+dx)+1)^{2/3}-\sqrt{3}+1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{\sqrt[3]{2}}-\frac{(1-\sqrt{3})\sqrt[6]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{s\ldots}\right.}{\ldots}\right.}{d\sqrt{1-\sec(c+dx)}\ldots}\right.$$

$$\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$\downarrow$ 2420

$$\sqrt[4]{3}\sqrt[6]{\sec(c+dx)+1}\left(\sqrt[3]{\cdots}\right.$$

$$\frac{\left(1+\sqrt{3}\right)\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}}{2^{2/3}\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)}-\frac{\cdots}{\cdots}$$

$$150a\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\left(\frac{\cdots}{\cdots}\right.$$

$$5\left(\frac{54a^2\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}+\frac{\cdots}{\cdots}\right.$$

$$\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

input
```
Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
(3*Sec[c + d*x]^2*Tan[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/3)) + ((-33*a
*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^(5/3)) + (5*((54*a^2*Tan[c + d*x]
)/(d*(a + a*Sec[c + d*x])^(2/3)) + (150*a*(a + a*Sec[c + d*x])^(1/3)*(-1/2
*((1 - Sqrt[3])*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x
])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3
])/4]*(1 + Sec[c + d*x])^(1/6)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(
2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^
(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2])/(2^(2/3)*3^(1/4)*Sqrt[
1 - Sec[c + d*x]]*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c +
 d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) + (
((1 + Sqrt[3])*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])^(1/6))/(2^(2/3)*(
2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) - (3^(1/4)*EllipticE[Ar
cCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sq
rt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x])^(1/
6)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c
 + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + S
ec[c + d*x])^(1/3))^2])/(2^(1/3)*Sqrt[1 - Sec[c + d*x]]*Sqrt[-(((1 + Sec[c
 + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[
3])*(1 + Sec[c + d*x])^(1/3))^2)]))/2^(1/3))*Tan[c + d*x])/(d*Sqrt[1 - Sec
[c + d*x]]*(1 + Sec[c + d*x])^(5/6))))/(7*a^2))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4311
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(m + n - 1))), x] + Simp[d^2/(b*(m + n - 1))   In
t[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*m*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && GtQ[n
, 2] && NeQ[m + n - 1, 0] && IntegerQ[n]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

rule 4488
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
 /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

rule 4496
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(b^2*(2*m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b
*B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Ne
Q[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [F]

$$\int \frac{\sec(dx+c)^4}{(a+a\sec(dx+c))^{\frac{5}{3}}}\,dx$$

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/3),x)
```

Fricas [F]

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)^4}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^4/(a^2*sec(d*x + c)^2 + 2
*a^2*sec(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec^4(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**(5/3),x)
```

output
```
Integral(sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(5/3), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx=\int \frac{\sec\left(dx+c\right)^4}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^4\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(5/3)),x)
```

output
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(5/3)), x)
```

Reduce [F]

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx=\frac{\int \frac{\sec(dx+c)^4}{(\sec(dx+c)+1)^{\frac{2}{3}}\sec(dx+c)+(\sec(dx+c)+1)^{\frac{2}{3}}}dx}{a^{\frac{5}{3}}}$$

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(c + d*x)**4/((sec(c + d*x) + 1)**(2/3)*sec(c + d*x) + (sec(c + d*x
) + 1)**(2/3)),x)/(a**(2/3)*a)
```

3.160 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 731

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{36\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}}$$

$$-\frac{57\left(1+\sqrt{3}\right)\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{7a^2d(1+\sec(c+dx))^{2/3}\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}\right)}$$

$$+\frac{57\sqrt[3]{2}\sqrt[4]{3}E\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)\sqrt[3]{a+a\sec(c+dx)}\left(\sqrt[3]{2}-\sqrt[3]{1+\sec\ldots}\right.}{7a^2d(1-\sec(c+dx))(1+\sec(c+dx))^{2/3}\sqrt{-\frac{\sqrt[3]{1+\sec(c+dx)}\ldots}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\ldots\right.}}}$$

$$+\frac{19\ 3^{3/4}\left(1-\sqrt{3}\right)\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)\sqrt[3]{a+a\sec(c+dx)}\left(\ldots\right.}{7\ 2^{2/3}a^2d(1-\sec(c+dx))(1+\sec(c+dx))^{2/3}\sqrt{-\frac{\sqrt[3]{1+\mathrm{s}\ldots}}{\left(\ldots\right.}}}$$

output
```
3/7*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/3)-36/7*tan(d*x+c)/a/d/(a+a*sec(d*x+c
))^(2/3)-57/7*(1+3^(1/2))*(a+a*sec(d*x+c))^(1/3)*tan(d*x+c)/a^2/d/(1+sec(d
*x+c))^(2/3)/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))+57/7*3^(1/4)*Ellip
ticE((1-(2^(1/3)-(1-3^(1/2))*(1+sec(d*x+c))^(1/3))^2/(2^(1/3)-(1+3^(1/2))*
(1+sec(d*x+c))^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a+a*sec(d*x+c))^(
1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)
+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))^2)^(1/2)
*tan(d*x+c)*2^(1/3)/a^2/d/(1-sec(d*x+c))/(1+sec(d*x+c))^(2/3)/(-(1+sec(d*x
+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+
c))^(1/3))^2)^(1/2)+19/14*3^(3/4)*(1-3^(1/2))*InverseJacobiAM(arccos((2^(1
/3)-(1-3^(1/2))*(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^
(1/3))),1/4*6^(1/2)+1/4*2^(1/2))*(a+a*sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*
x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/
(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))^2)^(1/2)*tan(d*x+c)*2^(1/3)/a^2
/d/(1-sec(d*x+c))/(1+sec(d*x+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+
sec(d*x+c))^(1/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.19 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.13

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{\left(-33-36\sec(c+dx)+38\,2^{5/6}\cos^2\left(\frac{1}{2}(c+dx)\right)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2}\right.\right.}{7d(a(1+\sec(c+$$

input
```
Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
((-33 - 36*Sec[c + d*x] + 38*2^(5/6)*Cos[(c + d*x)/2]^2*Hypergeometric2F1[
1/6, 1/2, 3/2, (1 - Sec[c + d*x])/2]*Sec[c + d*x]*(1 + Sec[c + d*x])^(1/6)
)*Tan[c + d*x])/(7*d*(a*(1 + Sec[c + d*x]))^(5/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.99 (sec) , antiderivative size = 734, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4286, 27, 3042, 4488, 3042, 4315, 3042, 4314, 73, 837, 25, 27, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a\sec(c+dx)+a)^{5/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/3}}dx$$

$$\Big\downarrow \text{4286}$$

$$\frac{3\int -\frac{\sec(c+dx)(5a-7a\sec(c+dx))}{3(\sec(c+dx)a+a)^{2/3}}dx}{7a^2}+\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}-\frac{\int \frac{\sec(c+dx)(5a-7a\sec(c+dx))}{(\sec(c+dx)a+a)^{2/3}}dx}{7a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(5a-7a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{2/3}}dx}{7a^2}$$

$$\Big\downarrow \text{4488}$$

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}-\frac{\frac{36a\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}-19\int \sec(c+dx)\sqrt[3]{\sec(c+dx)a+a}dx}{7a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}-\frac{\frac{36a\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}-19\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{7a^2}$$

$$\Big\downarrow \text{4315}$$

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}-\frac{\frac{36a\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}-\frac{19\sqrt[3]{a\sec(c+dx)+a}\int\sec(c+dx)\sqrt[3]{\sec(c+dx)+1}dx}{\sqrt[3]{\sec(c+dx)+1}}}{7a^2}$$

$\Big\downarrow$ 3042

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}-\frac{\frac{36a\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}-\frac{19\sqrt[3]{a\sec(c+dx)+a}\int\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)+1}dx}{\sqrt[3]{\sec(c+dx)+1}}}{7a^2}$$

$\Big\downarrow$ 4314

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}-\frac{\frac{19\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\int\frac{1}{\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}+\frac{36a\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}}{7a^2}$$

$\Big\downarrow$ 73

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}-\frac{\frac{114\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\int\frac{(\sec(c+dx)+1)^{2/3}}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}+\frac{36a\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}}{7a^2}$$

$\Big\downarrow$ 837

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}-\frac{\frac{114\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\left(-\frac{\left(1-\sqrt{3}\right)\int\frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{\sqrt[3]{2}}-\frac{1}{2}\int-\frac{2(\sec(c+dx)+1)^{2/3}+2^{2/3}\left(1-\sqrt{3}\right)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+\cdots\right.}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}}{7a^2}$$

$\Big\downarrow$ 25

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}-\frac{\frac{114\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\left(\frac{1}{2}\int\frac{2^{2/3}\left(\sqrt[3]{2}(\sec(c+dx)+1)^{2/3}-\sqrt{3}+1\right)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}-\frac{\left(1-\sqrt{3}\right)\int\frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c\cdots}{\sqrt[3]{2}}\right.}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}}{7a^2}$$

$\downarrow$ 27

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}} - \frac{\frac{114\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\left(\frac{\int \frac{\sqrt[3]{2}(\sec(c+dx)+1)^{2/3}-\sqrt{3}+1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{\sqrt[3]{2}} - \frac{\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+}}{\sqrt[3]{2}}\right.}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}}{7a^2}$$

$\downarrow$ 766

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}} - \frac{114\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\left(\frac{\int \frac{\sqrt[3]{2}(\sec(c+dx)+1)^{2/3}-\sqrt{3}+1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{\sqrt[3]{2}} - \frac{\left(1-\sqrt{3}\right)\sqrt[6]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\mathrm{se}\right.}{}\right.}{d\sqrt{1-\sec(c+dx)}(\mathrm{s}}$$

$\downarrow$ 2420

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}} - \frac{114\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}}{2^{2/3}\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)} - \frac{\sqrt[4]{3}\sqrt[6]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)}\right.}{\sqrt[3]{2}\surd}\right.}{}$$

input
```
Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
(3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^(5/3)) - ((36*a*Tan[c + d*x])/(
d*(a + a*Sec[c + d*x])^(2/3)) + (114*(a + a*Sec[c + d*x])^(1/3)*(-1/2*((1
- Sqrt[3])*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1
/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]
*(1 + Sec[c + d*x])^(1/6)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/
3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3)
 - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2])/(2^(2/3)*3^(1/4)*Sqrt[1 - S
ec[c + d*x]]*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x]
)^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) + (((1 +
 Sqrt[3])*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])^(1/6))/(2^(2/3)*(2^(1/
3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) - (3^(1/4)*EllipticE[ArcCos[
(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3]
)*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x])^(1/6)*(2
^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*
x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c
+ d*x])^(1/3))^2])/(2^(1/3)*Sqrt[1 - Sec[c + d*x]]*Sqrt[-(((1 + Sec[c + d*
x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(
1 + Sec[c + d*x])^(1/3))^2)]))/2^(1/3))*Tan[c + d*x])/(d*Sqrt[1 - Sec[c +
d*x]]*(1 + Sec[c + d*x])^(5/6)))/(7*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4286
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Simp[1/(a^2*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[
a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

rule 4488
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
 /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Maple [F]

$$\int \frac{\sec (dx+c)^3}{(a+a\sec (dx+c))^{\frac{5}{3}}}dx$$

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/3),x)
```

Fricas [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec (dx+c)^3}{(a\sec (dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^3/(a^2*sec(d*x + c)^2 + 2
*a^2*sec(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**(5/3),x)
```

output
```
Integral(sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(5/3), x)
```

Maxima [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)^3}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(5/3), x)
```

Giac [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)^3}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{1}{\cos(c+d\,x)^3\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(5/3)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(5/3)), x)
```

Reduce [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{\int \frac{\sec(dx+c)^3}{(\sec(dx+c)+1)^{\frac{2}{3}}\sec(dx+c)+(\sec(dx+c)+1)^{\frac{2}{3}}}dx}{a^{\frac{5}{3}}}$$

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(c + d*x)**3/((sec(c + d*x) + 1)**(2/3)*sec(c + d*x) + (sec(c + d*x
) + 1)**(2/3)),x)/(a**(2/3)*a)
```

3.161 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 731

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = -\frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} + \frac{15\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}}$$

$$+\frac{15\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}\right)}$$

$$-\frac{15\sqrt[3]{2}\sqrt[4]{3}E\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)\sqrt[3]{1+\sec(c+dx)}\left(\sqrt[3]{2}-\sqrt[3]{1+\sec(c\ldots}{7ad(1-\sec(c+dx))(a+a\sec(c+dx))^{2/3}\sqrt{-\frac{\sqrt[3]{1+\sec(c+dx)}\ldots}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\ldots}}}$$

$$-\frac{5\ 3^{3/4}\left(1-\sqrt{3}\right)\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)\sqrt[3]{1+\sec(c+dx)}\left(\sqrt[3]{\ldots}{7\ 2^{2/3}ad(1-\sec(c+dx))(a+a\sec(c+dx))^{2/3}\sqrt{-\frac{\sqrt[3]{1+\ldots}}{\left(\sqrt[3]{\ldots}}}}$$

output
```
-3/7*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/3)+15/7*tan(d*x+c)/a/d/(a+a*sec(d*x+
c))^(2/3)+15/7*(1+3^(1/2))*(1+sec(d*x+c))^(1/3)*tan(d*x+c)/a/d/(a+a*sec(d*
x+c))^(2/3)/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))-15/7*2^(1/3)*3^(1/4
)*EllipticE((1-(2^(1/3)-(1-3^(1/2))*(1+sec(d*x+c))^(1/3))^2/(2^(1/3)-(1+3^
(1/2))*(1+sec(d*x+c))^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1+sec(d*x+
c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^
(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))^2)^
(1/2)*tan(d*x+c)/a/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(2/3)/(-(1+sec(d*x+c)
)^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))
^(1/3))^2)^(1/2)-5/14*3^(3/4)*(1-3^(1/2))*InverseJacobiAM(arccos((2^(1/3)-
(1-3^(1/2))*(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3
))),1/4*6^(1/2)+1/4*2^(1/2))*(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^
(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/
3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))^2)^(1/2)*tan(d*x+c)*2^(1/3)/a/d/(1-se
c(d*x+c))/(a+a*sec(d*x+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*
x+c))^(1/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.12

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{\left(-3+5\ 2^{5/6}\cos^2\left(\frac{1}{2}(c+dx)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{3}{2},\frac{1}{2}(1-\sec(c+d\right.\right.}{7d(a(1+\sec(c+dx)))^{5/3}}$$

input
```
Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
((-3 + 5*2^(5/6)*Cos[(c + d*x)/2]^2*Hypergeometric2F1[1/2, 7/6, 3/2, (1 -
Sec[c + d*x])/2]*Sec[c + d*x]*(1 + Sec[c + d*x])^(1/6))*Tan[c + d*x])/(7*d
*(a*(1 + Sec[c + d*x]))^(5/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.83 (sec) , antiderivative size = 736, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {3042, 4284, 3042, 4315, 3042, 4314, 61, 73, 837, 25, 27, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a\sec(c+dx)+a)^{5/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/3}}dx$$

$$\Big\downarrow \text{4284}$$

$$\frac{5\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^{2/3}}dx}{7a}-\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{2/3}}dx}{7a}-\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{4315}$$

$$\frac{5(\sec(c+dx)+1)^{2/3}\int \frac{\sec(c+dx)}{(\sec(c+dx)+1)^{2/3}}dx}{7a(a\sec(c+dx)+a)^{2/3}}-\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5(\sec(c+dx)+1)^{2/3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{2/3}}dx}{7a(a\sec(c+dx)+a)^{2/3}}-\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{4314}$$

$$-\frac{5\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\int \frac{1}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}d\sec(c+dx)}{7ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}-\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}$$

$\Big\downarrow$ 61

$$-\frac{5\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(-\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}}d\sec(c+dx)-\frac{3\sqrt{1-\sec(c+dx)}}{\sqrt[6]{\sec(c+dx)+1}}\right)}{7ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}-\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}$$

$\Big\downarrow$ 73

$$-\frac{5\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(-6\int \frac{(\sec(c+dx)+1)^{2/3}}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}-\frac{3\sqrt{1-\sec(c+dx)}}{\sqrt[6]{\sec(c+dx)+1}}\right)}{7ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}-\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}$$

$\Big\downarrow$ 837

$$-\frac{5\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(-6\left(-\frac{\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{\sqrt[3]{2}}-\frac{1}{2}\int -\frac{2(\sec(c+dx)+1)^{2/3}+2^{2/3}\left(1-\right.}{\sqrt{1-\sec(c+dx)}}\right.\right.}{7ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}$$

$\Big\downarrow$ 25

$$-\frac{5\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(-6\left(\frac{1}{2}\int \frac{2^{2/3}\left(\sqrt[3]{2}(\sec(c+dx)+1)^{2/3}-\sqrt{3}+1\right)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}-\frac{\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{1-\sec(}}}{}\right.\right.}{7ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}$$

$\Big\downarrow$ 27

$$-\frac{5\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(-6\left(\frac{\int \frac{\sqrt[3]{2}(\sec(c+dx)+1)^{2/3}-\sqrt{3}+1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{\sqrt[3]{2}}-\frac{\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec}}{\sqrt[3]{2}}\right.\right.}{7ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \ 766$$

$$-\frac{5\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(-6\left(\frac{\int \frac{\sqrt[3]{2}(\sec(c+dx)+1)^{2/3}-\sqrt{3}+1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{\sqrt[3]{2}}-\frac{\left(1-\sqrt{3}\right)\sqrt[6]{\sec(c+dx)+1}\left(\right.}{}\right.\right.}{7d}$$

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \ 2420$$

$$-\frac{5\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(-6\left(\frac{\frac{\left(1+\sqrt{3}\right)\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}}{2^{2/3}\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)}-\frac{\sqrt[4]{3}\sqrt[6]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c\right.}{}}{}\right.\right.}{}$$

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}$$

input

```
Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
(-3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^(5/3)) - (5*(1 + Sec[c + d*x])
^(1/6)*((-3*Sqrt[1 - Sec[c + d*x]])/(1 + Sec[c + d*x])^(1/6) - 6*(-1/2*((1
 - Sqrt[3])*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(
1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4
]*(1 + Sec[c + d*x])^(1/6)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2
/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3
) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2])/(2^(2/3)*3^(1/4)*Sqrt[1 -
Sec[c + d*x]]*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x
])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) + (((1
+ Sqrt[3])*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])^(1/6))/(2^(2/3)*(2^(1
/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) - (3^(1/4)*EllipticE[ArcCos
[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3
])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x])^(1/6)*(
2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d
*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c
 + d*x])^(1/3))^2])/(2^(1/3)*Sqrt[1 - Sec[c + d*x]]*Sqrt[-(((1 + Sec[c + d
*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*
(1 + Sec[c + d*x])^(1/3))^2)]))/2^(1/3)))*Tan[c + d*x])/(7*a*d*Sqrt[1 - Se
c[c + d*x]]*(a + a*Sec[c + d*x])^(2/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4284
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Simp[m/(b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{\sec \left(dx+c\right)^{2}}{\left(a+a\sec \left(dx+c\right)\right)^{\frac{5}{3}}}dx$$

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/3),x)
```

Fricas [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx=\int \frac{\sec \left(dx+c\right)^{2}}{\left(a\sec \left(dx+c\right)+a\right)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^2/(a^2*sec(d*x + c)^2 + 2
*a^2*sec(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec^2{(c+dx)}}{(a(\sec{(c+dx)}+1))^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**(5/3),x)
```

output
```
Integral(sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(5/3), x)
```

Maxima [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec{(dx+c)}^2}{(a\sec{(dx+c)}+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(5/3), x)
```

Giac [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec{(dx+c)}^2}{(a\sec{(dx+c)}+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{1}{\cos(c+d\,x)^2\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(5/3)),x)
```

output
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(5/3)), x)
```

Reduce [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{\int \frac{\sec(dx+c)^2}{(\sec(dx+c)+1)^{\frac{2}{3}}\sec(dx+c)+(\sec(dx+c)+1)^{\frac{2}{3}}}dx}{a^{\frac{5}{3}}}$$

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(c + d*x)**2/((sec(c + d*x) + 1)**(2/3)*sec(c + d*x) + (sec(c + d*x
) + 1)**(2/3)),x)/(a**(2/3)*a)
```

3.162 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 744

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{6\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}}$$
$$+ \frac{3\tan(c+dx)}{7ad(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}}$$
$$+ \frac{6(1+\sqrt{3})\sqrt[3]{1+\sec(c+dx)}\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{1+\sec(c+dx)}\right)}$$
$$- \frac{6\sqrt[3]{2}\sqrt[4]{3}E\left(\arccos\left(\frac{\sqrt[3]{2}-(1-\sqrt{3})\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{1+\sec(c+dx)}}\right)|\frac{1}{4}(2+\sqrt{3})\right)\sqrt[3]{1+\sec(c+dx)}\left(\sqrt[3]{2}-\sqrt[3]{1+\sec(c\right.}{7ad(1-\sec(c+dx))(a+a\sec(c+dx))^{2/3}\sqrt{-\frac{\sqrt[3]{1+\sec(c+dx)}}{\left(\sqrt[3]{2}-(1+\sqrt{3})\right.}}}$$
$$- \frac{\sqrt[3]{2}3^{3/4}(1-\sqrt{3})\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-(1-\sqrt{3})\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{1+\sec(c+dx)}}\right),\frac{1}{4}(2+\sqrt{3})\right)\sqrt[3]{1+\sec(c+dx)}\left(\right.}{7ad(1-\sec(c+dx))(a+a\sec(c+dx))^{2/3}\sqrt{-\frac{\sqrt[3]{1+\sec}}{\left(\sqrt[3]{2}\right.}}}$$

output
```
6/7*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^(2/3)+3/7*tan(d*x+c)/a/d/(1+sec(d*x+c)
)/(a+a*sec(d*x+c))^(2/3)+6/7*(1+3^(1/2))*(1+sec(d*x+c))^(1/3)*tan(d*x+c)/a
/d/(a+a*sec(d*x+c))^(2/3)/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))-6/7*2
^(1/3)*3^(1/4)*EllipticE((1-(2^(1/3)-(1-3^(1/2))*(1+sec(d*x+c))^(1/3))^2/(
2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2)
)*(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1
+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c
))^(1/3))^2)^(1/2)*tan(d*x+c)/a/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(2/3)/(-
(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+3^(1/2))*(
1+sec(d*x+c))^(1/3))^2)^(1/2)-1/7*3^(3/4)*(1-3^(1/2))*InverseJacobiAM(arcc
os((2^(1/3)-(1-3^(1/2))*(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(
d*x+c))^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))*(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+
sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(
2/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))^2)^(1/2)*tan(d*x+c)*2^(1/
3)/a/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/
3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+3^(1/2))*(1+sec(d*x+c))^(1/3))^2)^(1/
2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.09

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)(1+\sec(c+dx))^{7/6}\tan(c}{2\sqrt[6]{2}d(a(1+\sec(c+dx)))^{5/3}}$$

input
```
Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
(Hypergeometric2F1[1/2, 13/6, 3/2, (1 - Sec[c + d*x])/2]*(1 + Sec[c + d*x]
)^(7/6)*Tan[c + d*x])/(2*2^(1/6)*d*(a*(1 + Sec[c + d*x]))^(5/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 741, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4315, 3042, 4314, 61, 61, 73, 837, 25, 27, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a\sec(c+dx)+a)^{5/3}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/3}}dx$$

$$\Big\downarrow \text{ 4315}$$

$$\frac{(\sec(c+dx)+1)^{2/3}\int \frac{\sec(c+dx)}{(\sec(c+dx)+1)^{5/3}}dx}{a(a\sec(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(\sec(c+dx)+1)^{2/3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{5/3}}dx}{a(a\sec(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \text{ 4314}$$

$$-\frac{\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\int \frac{1}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{13/6}}d\sec(c+dx)}{ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \text{ 61}$$

$$-\frac{\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(\frac{2}{7}\int \frac{1}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}d\sec(c+dx)-\frac{3\sqrt{1-\sec(c+dx)}}{7(\sec(c+dx)+1)^{7/6}}\right)}{ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \text{ 61}$$

$$-\frac{\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(\frac{2}{7}\left(-\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}}d\sec(c+dx)-\frac{3\sqrt{1-\sec(c+dx)}}{\sqrt[6]{\sec(c+dx)+1}}\right)-\frac{3}{7(}\cdots\right.}{ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

$\downarrow$ 73

$$-\frac{\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(\frac{2}{7}\left(-6\int\frac{(\sec(c+dx)+1)^{2/3}}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}-\frac{3\sqrt{1-\sec(c+dx)}}{\sqrt[6]{\sec(c+dx)+1}}\right)-\frac{3\sqrt{1-\sec(c}{7(\sec(c+dx)-}\right.}{ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

$\downarrow$ 837

$$-\frac{\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(\frac{2}{7}\left(-6\left(-\frac{\left(1-\sqrt{3}\right)\int\frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{\sqrt[3]{2}}-\frac{1}{2}\int-\frac{2(\sec(c+dx)+1)^{2/3}+2^{2/3}\left(\right.}{\sqrt{1-\sec(c+dx)}}\right.\right.\right.}{ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/}}$$

$\downarrow$ 25

$$-\frac{\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(\frac{2}{7}\left(-6\left(\frac{1}{2}\int\frac{2^{2/3}\left(\sqrt[3]{2}(\sec(c+dx)+1)^{2/3}-\sqrt{3}+1\right)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}-\frac{\left(1-\sqrt{3}\right)\int\frac{}{\sqrt{1-\mathrm{se}}}}{}\right.\right.\right.}{ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2}}$$

$\downarrow$ 27

$$-\frac{\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(\frac{2}{7}\left(-6\left(\frac{\int\frac{\sqrt[3]{2}(\sec(c+dx)+1)^{2/3}-\sqrt{3}+1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{\sqrt[3]{2}}-\frac{\left(1-\sqrt{3}\right)\int\frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\mathrm{s}}}{\sqrt[3]{2}}\right.\right.\right.}{ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

$\downarrow$ 766

$$-\frac{\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(\frac{2}{7}\left(-6\left(\frac{\int\frac{\sqrt[3]{2}(\sec(c+dx)+1)^{2/3}-\sqrt{3}+1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{\sqrt[3]{2}}-\frac{\left(1-\sqrt{3}\right)\sqrt[6]{\sec(c+dx)+}}{}\right.\right.\right.}{}$$

$\downarrow$ 2420

$$-\frac{\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(\frac{2}{7}\left(-6\left(\frac{\frac{(1+\sqrt{3})\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}}{2^{2/3}\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)}-\frac{\sqrt[4]{3}\sqrt[6]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec\cdots}\right.}{\cdots}}{\cdots}\right.\right.\right.}{\cdots}$$

input
```
Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
-(((1 + Sec[c + d*x])^(1/6)*((-3*Sqrt[1 - Sec[c + d*x]])/(7*(1 + Sec[c + d
*x])^(7/6)) + (2*((-3*Sqrt[1 - Sec[c + d*x]])/(1 + Sec[c + d*x])^(1/6) - 6
*(-1/2*((1 - Sqrt[3])*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c
 + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 +
Sqrt[3])/4]*(1 + Sec[c + d*x])^(1/6)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*
Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3
))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2])/(2^(2/3)*3^(1/4)
*Sqrt[1 - Sec[c + d*x]]*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + S
ec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)
]) + (((1 + Sqrt[3])*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])^(1/6))/(2^(
2/3)*(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) - (3^(1/4)*Ellipt
icE[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (
1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x
])^(1/6)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 +
 Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*
(1 + Sec[c + d*x])^(1/3))^2])/(2^(1/3)*Sqrt[1 - Sec[c + d*x]]*Sqrt[-(((1 +
 Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 +
 Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2])))/2^(1/3))))/7)*Tan[c + d*x])/(a*d
*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(2/3)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{\sec(dx+c)}{(a+a\sec(dx+c))^{\frac{5}{3}}}dx$$

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^(5/3),x)
```

Fricas [F]

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)/(a^2*sec(d*x + c)^2 + 2*a
^2*sec(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))**(5/3),x)
```

output
```
Integral(sec(c + d*x)/(a*(sec(c + d*x) + 1))**(5/3), x)
```

Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(5/3), x)
```

Giac [F]

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{1}{\cos(c+dx)\left(a+\frac{a}{\cos(c+dx)}\right)^{5/3}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(5/3)),x)
```

output
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(5/3)), x)
```

Reduce [F]

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{\int \frac{\sec(dx+c)}{(\sec(dx+c)+1)^{\frac{2}{3}}\sec(dx+c)+(\sec(dx+c)+1)^{\frac{2}{3}}}dx}{a^{\frac{5}{3}}}$$

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(c + d*x)/((sec(c + d*x) + 1)**(2/3)*sec(c + d*x) + (sec(c + d*x) +
 1)**(2/3)),x)/(a**(2/3)*a)
```

3.163 $\int \frac{1}{(a+a\sec(c+dx))^{5/3}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 82

$$\int \frac{1}{(a+a\sec(c+dx))^{5/3}}\,dx=\frac{\text{AppellF1}\left(\frac{1}{2},\frac{13}{6},1,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),1-\sec(c+dx)\right)\sqrt[6]{1+\sec(c+dx)}}{2\sqrt[6]{2}ad(a+a\sec(c+dx))^{2/3}}$$

output
```
1/4*AppellF1(1/2,1,13/6,3/2,1-sec(d*x+c),1/2-1/2*sec(d*x+c))*(1+sec(d*x+c)
)^(1/6)*tan(d*x+c)*2^(5/6)/a/d/(a+a*sec(d*x+c))^(2/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3007 vs. 2(82) = 164.

Time = 14.84 (sec) , antiderivative size = 3007, normalized size of antiderivative = 36.67

$$\int \frac{1}{(a+a\sec(c+dx))^{5/3}}\,dx=\text{Result too large to show}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(-5/3),x]
```

output
```
(((1 + Cos[c + d*x])*Sec[c + d*x])^(1/3)*(1 + Sec[c + d*x])^(5/3)*((27*Sin
[c + d*x])/7 - (30*Tan[(c + d*x)/2])/7 + (3*Sec[(c + d*x)/2]^2*Tan[(c + d*
x)/2])/14))/(d*(a*(1 + Sec[c + d*x]))^(5/3)) + (2^(1/3)*(1 + Sec[c + d*x])
^(5/3)*((16*(1 + Sec[c + d*x])^(1/3))/7 - (27*Cos[c + d*x]*(1 + Sec[c + d*
x])^(1/3))/7)*Tan[(c + d*x)/2]*((-3*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d
*x)/2]^2)^(2/3) + Cos[(c + d*x)/2]^2*(-27 - (5*AppellF1[1/2, 1/3, 1, 3/2,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/((-1 + Tan[(c + d*x)/2]^2)*(Appe
llF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*A
ppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + Appel
lF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d
*x)/2]^2)/9)))))/(7*d*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(a*(1 + Sec[
c + d*x]))^(5/3)*((Sec[(c + d*x)/2]^2*((-3*AppellF1[3/2, 1/3, 1, 5/2, Tan[
(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec
[(c + d*x)/2]^2)^(2/3) + Cos[(c + d*x)/2]^2*(-27 - (5*AppellF1[1/2, 1/3, 1
, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/((-1 + Tan[(c + d*x)/2]^2
)*(AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (
2*(-3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
+ AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan
[(c + d*x)/2]^2)/9)))))/(7*2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2...
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4266, 3042, 4265, 149, 25, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)^{5/3}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/3}}dx$$

$$\Big\downarrow \text{4266}$$

$$\frac{(\sec(c+dx)+1)^{2/3}\int \frac{1}{(\sec(c+dx)+1)^{5/3}}dx}{a(a\sec(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(\sec(c+dx)+1)^{2/3}\int \frac{1}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{5/3}}dx}{a(a\sec(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \text{4265}$$

$$-\frac{\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\int \frac{\cos(c+dx)}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{13/6}}d\sec(c+dx)}{ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \text{149}$$

$$-\frac{6\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\int \frac{\cos^9(c+dx)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \text{25}$$

$$\frac{6\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\int -\frac{\cos^9(c+dx)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{3\sqrt{2}\sin(c+dx)\cos^6(c+dx)\sqrt[6]{\sec(c+dx)+1}\,\text{AppellF1}\left(-\frac{7}{6},1,\frac{1}{2},-\frac{1}{6},\sec(c+dx)+1,\frac{1}{2}(\sec(c+dx)+1)\right)}{7ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

input
```
Int[(a + a*Sec[c + d*x])^(-5/3),x]
```

output
```
(-3*Sqrt[2]*AppellF1[-7/6, 1, 1/2, -1/6, 1 + Sec[c + d*x], (1 + Sec[c + d*
x])/2]*Cos[c + d*x]^6*(1 + Sec[c + d*x])^(1/6)*Sin[c + d*x])/(7*a*d*Sqrt[1
 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(2/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 149
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1
) - 1)*(c - a*(d/b) + d*(x^k/b))^n*(e - a*(f/b) + f*(x^k/b))^p, x], x, (a +
 b*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && FractionQ[m] &&
IntegerQ[2*n] && IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4265
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[a^n*(Cot
[c + d*x]/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]))   Subst[Int[(1
 + b*(x/a))^(n - 1/2)/(x*Sqrt[1 - b*(x/a)]), x], x, Csc[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0
]
```

rule 4266
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Csc[c + d*x])^FracPart[n]/(1 + (b/a)*Csc[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{(a + a\sec(dx + c))^{\frac{5}{3}}} dx$$

input
```
int(1/(a+a*sec(d*x+c))^(5/3),x)
```

output
```
int(1/(a+a*sec(d*x+c))^(5/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a + a\sec(c + dx))^{5/3}} dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a + a\sec(c + dx))^{5/3}} dx = \int \frac{1}{(a\sec(c + dx) + a)^{\frac{5}{3}}} dx$$

input
```
integrate(1/(a+a*sec(d*x+c))**(5/3),x)
```

output
```
Integral((a*sec(c + d*x) + a)**(-5/3), x)
```

Maxima [F]

$$\int \frac{1}{(a+a\sec(c+dx))^{5/3}}\,dx=\int \frac{1}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(-5/3), x)
```

Giac [F]

$$\int \frac{1}{(a+a\sec(c+dx))^{5/3}}\,dx=\int \frac{1}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(-5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))^{5/3}}\,dx=\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

input
```
int(1/(a + a/cos(c + d*x))^(5/3),x)
```

output
```
int(1/(a + a/cos(c + d*x))^(5/3), x)
```

Reduce [F]

$$\int \frac{1}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{\int \frac{1}{(\sec(dx+c)+1)^{\frac{2}{3}}\sec(dx+c)+(\sec(dx+c)+1)^{\frac{2}{3}}}dx}{a^{\frac{5}{3}}}$$

input
```
int(1/(a+a*sec(d*x+c))^(5/3),x)
```

output
```
int(1/((sec(c + d*x) + 1)**(2/3)*sec(c + d*x) + (sec(c + d*x) + 1)**(2/3))
,x)/(a**(2/3)*a)
```

3.164 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 82

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{\operatorname{AppellF1}\left(\frac{1}{2},\frac{13}{6},2,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),1-\sec(c+dx)\right)\sqrt[6]{1+\sec(c+dx}}{2\sqrt[6]{2}ad(a+a\sec(c+dx))^{2/3}}$$

output
```
1/4*AppellF1(1/2,2,13/6,3/2,1-sec(d*x+c),1/2-1/2*sec(d*x+c))*(1+sec(d*x+c)
)^(1/6)*tan(d*x+c)*2^(5/6)/a/d/(a+a*sec(d*x+c))^(2/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3011 vs. 2(82) = 164.

Time = 14.82 (sec) , antiderivative size = 3011, normalized size of antiderivative = 36.72

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
(((1 + Cos[c + d*x])*Sec[c + d*x])^(1/3)*(1 + Sec[c + d*x])^(5/3)*((-48*Si
n[c + d*x])/7 + (51*Tan[(c + d*x)/2])/7 - (3*Sec[(c + d*x)/2]^2*Tan[(c + d
*x)/2])/14))/(d*(a*(1 + Sec[c + d*x]))^(5/3)) - (5*2^(1/3)*(1 + Sec[c + d*
x])^(5/3)*((-30*(1 + Sec[c + d*x])^(1/3))/7 + (55*Cos[c + d*x]*(1 + Sec[c
+ d*x])^(1/3))/7)*Tan[(c + d*x)/2]*((-11*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(
c + d*x)/2]^2)^(2/3) + 9*Cos[(c + d*x)/2]^2*(-11 - AppellF1[1/2, 1/3, 1, 3
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]/((-1 + Tan[(c + d*x)/2]^2)*(A
ppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-
3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + Ap
pellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c
+ d*x)/2]^2)/9)))))/(63*d*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(a*(1 +
Sec[c + d*x]))^(5/3)*((-5*Sec[(c + d*x)/2]^2*((-11*AppellF1[3/2, 1/3, 1, 5
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c +
d*x]*Sec[(c + d*x)/2]^2)^(2/3) + 9*Cos[(c + d*x)/2]^2*(-11 - AppellF1[1/2,
 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]/((-1 + Tan[(c + d*x
)/2]^2)*(AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2] + (2*(-3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2] + AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
])*Tan[(c + d*x)/2]^2)/9)))))/(63*2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d...
```

Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4315, 3042, 4314, 149, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a\sec(c+dx)+a)^{5/3}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/3}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{(\sec(c+dx)+1)^{2/3}\int \frac{\cos(c+dx)}{(\sec(c+dx)+1)^{5/3}}dx}{a(a\sec(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{(\sec(c+dx)+1)^{2/3}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{5/3}}dx}{a(a\sec(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \textbf{4314}$$

$$-\frac{\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\int \frac{\cos^2(c+dx)}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{13/6}}d\sec(c+dx)}{ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \textbf{149}$$

$$-\frac{6\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\int \frac{\cos^{10}(c+dx)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \textbf{1012}$$

$$\frac{3\sqrt{2}\sin(c+dx)\cos^6(c+dx)\sqrt[6]{\sec(c+dx)+1}\,\text{AppellF1}\left(-\frac{7}{6},2,\frac{1}{2},-\frac{1}{6},\sec(c+dx)+1,\frac{1}{2}(\sec(c+dx)+1)\right)}{7ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

input
```
Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
(3*Sqrt[2]*AppellF1[-7/6, 2, 1/2, -1/6, 1 + Sec[c + d*x], (1 + Sec[c + d*x
])/2]*Cos[c + d*x]^6*(1 + Sec[c + d*x])^(1/6)*Sin[c + d*x])/(7*a*d*Sqrt[1
- Sec[c + d*x]]*(a + a*Sec[c + d*x])^(2/3))
```

Defintions of rubi rules used

rule 149
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1
) - 1)*(c - a*(d/b) + d*(x^k/b))^n*(e - a*(f/b) + f*(x^k/b))^p, x], x, (a +
 b*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && FractionQ[m] &&
IntegerQ[2*n] && IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{\cos(dx+c)}{(a+a\sec(dx+c))^{\frac{5}{3}}}dx$$

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^(5/3),x)
```

output
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^(5/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\cos(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))**(5/3),x)
```

output
```
Integral(cos(c + d*x)/(a*(sec(c + d*x) + 1))**(5/3), x)
```

Maxima [F]

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\cos(dx+c)}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(5/3), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\cos(dx+c)}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\cos(c+d\,x)}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

input
```
int(cos(c + d*x)/(a + a/cos(c + d*x))^(5/3),x)
```

output
```
int(cos(c + d*x)/(a + a/cos(c + d*x))^(5/3), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{\int \frac{\cos(dx+c)}{(\sec(dx+c)+1)^{\frac{2}{3}}\sec(dx+c)+(\sec(dx+c)+1)^{\frac{2}{3}}}dx}{a^{\frac{5}{3}}}$$

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^(5/3),x)
```

output
```
int(cos(c + d*x)/((sec(c + d*x) + 1)**(2/3)*sec(c + d*x) + (sec(c + d*x) +
 1)**(2/3)),x)/(a**(2/3)*a)
```

3.165 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 151

$$\begin{aligned}\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx &= -\frac{6a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &\quad+\frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\ &\quad+\frac{6a\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} \\ &\quad+\frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}\end{aligned}$$

output

```
-6/5*a*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(
1/2)/d+2/3*a*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d
*x+c)^(1/2)/d+6/5*a*sec(d*x+c)^(1/2)*sin(d*x+c)/d+2/3*a*sec(d*x+c)^(3/2)*s
in(d*x+c)/d+2/5*a*sec(d*x+c)^(5/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.76

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx$$
$$=\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))\left(-9\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+5\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c\right.\right.}{15d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x]),x]
```

output
```
(a*Sec[(c + d*x)/2]^2*(1 + Sec[c + d*x])*(-9*Sqrt[Cos[c + d*x]]*EllipticE[
(c + d*x)/2, 2] + 5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 9*Sin[c
 + d*x] + 5*Tan[c + d*x] + 3*Sec[c + d*x]*Tan[c + d*x]))/(15*d*Sqrt[Sec[c
+ d*x]])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4274, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \sec^{\frac{5}{2}}(c+dx)dx+a\int \sec^{\frac{7}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}dx$$

$\downarrow$ 4255

$$a\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+a\left(\frac{3}{5}\int \sec^{\frac{3}{2}}(c+dx)dx+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$\downarrow$ 3042

$$a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+a\left(\frac{3}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$\downarrow$ 4255

$$a\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)+a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$\downarrow$ 3042

$$a\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)+a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$\downarrow$ 4258

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+a\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+d}{5d}$$

$\downarrow$ 3042

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+$$
$$a\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}}{5d}\right.$$

$$\Big\downarrow \textbf{3119}$$

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+$$
$$a\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)$$

$$\Big\downarrow \textbf{3120}$$

$$a\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+$$
$$a\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)$$

input
```
Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x]),x]
```

output
```
a*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)) + a*((2*Sec[c + d*x]^(5/2)
*Sin[c + d*x])/(5*d) + (3*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/5)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(130) = 260$.

Time = 3.99 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.54

method	result
default	$-\frac{a\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{10\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{1}{2}\right)^{3}}-\frac{12\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{5\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}}\right.}{}$
parts	$-\frac{2a\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\sqrt{\frac{1}{2}-\frac{\cos(}{}}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right.}$

input
```
int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/10*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^3-12/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*co
s(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+28/15*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6/5*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2)/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.25

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx$$

$$= \frac{-5i\sqrt{2}a\cos(dx+c)^2 \text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}a\cos(dx+c)^2 \text{w}}{}$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/15*(-5*I*sqrt(2)*a*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) + 5*I*sqrt(2)*a*cos(d*x + c)^2*weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*a*cos(d*x + c)^2*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
+ 9*I*sqrt(2)*a*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*a*cos(d*x + c)^2 + 5*a*cos(d
*x + c) + 3*a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \int (a\sec(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \int (a\sec(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx=\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,dx$$

input
```
int((a + a/cos(c + d*x))*(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))*(1/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx=a\left(\int \sqrt{\sec\left(dx+c\right)}\,\sec\left(dx+c\right)^{3}dx+\int \sqrt{\sec\left(dx+c\right)}\,\sec\left(dx+c\right)^{2}dx\right)$$

input
```
int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x)
```

output
```
a*(int(sqrt(sec(c + d*x))*sec(c + d*x)**3,x) + int(sqrt(sec(c + d*x))*sec(
c + d*x)**2,x))
```

3.166 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx &= -\frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}\\&\quad+\frac{2a\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}\\&\quad+\frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}\end{aligned}$$

output

```
-2*a*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/
2)/d+2/3*a*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x
+c)^(1/2)/d+2*a*sec(d*x+c)^(1/2)*sin(d*x+c)/d+2/3*a*sec(d*x+c)^(3/2)*sin(d
*x+c)/d
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.67

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{a\sec^{\frac{3}{2}}(c+dx)\left(-6\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\cos^{\frac{3}{2}}(c+dx)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2\sin(c+d\right.}{3d}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x]),x]
```

output
```
(a*Sec[c + d*x]^(3/2)*(-6*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 2
*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 2*Sin[c + d*x] + 3*Sin[2*(
c + d*x)]))/(3*d)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$
$$\Big\downarrow \text{4274}$$
$$a\int \sec^{\frac{3}{2}}(c+dx)dx+a\int \sec^{\frac{5}{2}}(c+dx)dx$$
$$\Big\downarrow \text{3042}$$
$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

input
```
Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x]),x]
```

output
```
a*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d) + a*((2*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[
c + d*x])/(3*d))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(110) = 220.

Time = 2.05 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.99

method	result
default	$-\frac{2a\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(12\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\text{EllipticF}\left(\text{co}\right.\right.}{}$
parts	$-\frac{2a\left(-2\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$

input
```
int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/3*a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(12*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/
2*c)^2-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-8*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.36

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx$$
$$=\frac{-i\sqrt{2}a\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}a\cos(dx+c)\,\text{weiers}}{}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) + I*sqrt(2)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*a*cos(d*x + c)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(
2)*a*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))) + 2*(3*a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos
(d*x + c)))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \sec^{\frac{3}{2}}(c+dx)\,dx + \int \sec^{\frac{5}{2}}(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(sec(c + d*x)**(3/2), x) + Integral(sec(c + d*x)**(5/2), x))
```

Maxima [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \int (a\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \int (a\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

input
```
int((a + a/cos(c + d*x))*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))*(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^2\,dx + \int \sqrt{\sec(dx+c)}\,\sec(dx+c)\,dx\right)$$

input
```
int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x)
```

output
```
a*(int(sqrt(sec(c + d*x))*sec(c + d*x)**2,x) + int(sqrt(sec(c + d*x))*sec(
c + d*x),x))
```

3.167 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\begin{aligned}\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))\,dx &= -\frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}\\ &\quad+\frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d}\\ &\quad+\frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{d}\end{aligned}$$

output

```
-2*a*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/
2)/d+2*a*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c
)^(1/2)/d+2*a*sec(d*x+c)^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.70

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))\,dx$$
$$=\frac{2a\sqrt{\sec(c+dx)}\left(-\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(c+dx\right.}{d}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

output
```
(2*a*Sqrt[Sec[c + d*x]]*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) +
 Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[c + d*x]))/d
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a\sec(c+dx)+a)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$
$$\Big\downarrow \text{4274}$$
$$a\int \sec^{\frac{3}{2}}(c+dx)dx+a\int \sqrt{\sec(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx$$

$$\Big\downarrow \text{4255}$$

$$a\int \sqrt{\csc \left(c+dx+\frac{\pi }{2}\right)}dx+a\left(\frac{2 \sin (c+dx) \sqrt{\sec (c+dx)}}{d}-\int \frac{1}{\sqrt{\sec (c+dx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a\int \sqrt{\csc \left(c+dx+\frac{\pi }{2}\right)}dx+a\left(\frac{2 \sin (c+dx) \sqrt{\sec (c+dx)}}{d}-\int \frac{1}{\sqrt{\csc \left(c+dx+\frac{\pi }{2}\right)}}dx\right)$$

$$\Big\downarrow \text{4258}$$

$$a \sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} \int \frac{1}{\sqrt{\cos (c+dx)}}dx+ a\left(\frac{2 \sin (c+dx) \sqrt{\sec (c+dx)}}{d}-\sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} \int \sqrt{\cos (c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a \sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} \int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi }{2}\right)}}dx+ a\left(\frac{2 \sin (c+dx) \sqrt{\sec (c+dx)}}{d}-\sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} \int \sqrt{\sin \left(c+dx+\frac{\pi }{2}\right)}dx\right)$$

$$\Big\downarrow \text{3119}$$

$$a \sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} \int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi }{2}\right)}}dx+ a\left(\frac{2 \sin (c+dx) \sqrt{\sec (c+dx)}}{d}-\frac{2 \sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} E\left(\left.\frac{1}{2} (c+dx)\right|2\right)}{d}\right)$$

$$\Big\downarrow \text{3120}$$

$$\frac{2 a \sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} \operatorname{EllipticF}\left(\frac{1}{2} (c+dx),2\right)}{d}+ a\left(\frac{2 \sin (c+dx) \sqrt{\sec (c+dx)}}{d}-\frac{2 \sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} E\left(\left.\frac{1}{2} (c+dx)\right|2\right)}{d}\right)$$

input

```
Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

output
```
(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
a*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.53

method	result
default	$\frac{2a\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\mathrm{EllipticE}\left(\cos\right.\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$
parts	$-\frac{2a\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}-\frac{2a\left(-2\sqrt{-2\sin\right.}{}$

input
```
int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*a*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-(
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.28

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))\,dx = \frac{-i\sqrt{2}a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}a\mathrm{weierstrassPInverse}(-4,0,\cos(}{}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) +
I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - I*
sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) + I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*a*sin(d*x + c)/sqrt(cos(d*x +
c)))/d
```

Sympy [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))\,dx = a\left(\int \sqrt{\sec(c+dx)}\,dx + \int \sec^{\frac{3}{2}}(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(sqrt(sec(c + d*x)), x) + Integral(sec(c + d*x)**(3/2), x))
```

Maxima [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))\,dx = \int (a\sec(dx+c)+a)\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))\,dx = \int (a\sec(dx+c)+a)\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

input
```
int((a + a/cos(c + d*x))*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))*(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))\,dx = a\left(\int \sqrt{\sec(dx+c)}dx + \int \sqrt{\sec(dx+c)}\sec(dx+c)\,dx\right)$$

input
```
int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x)
```

output
```
a*(int(sqrt(sec(c + d*x)),x) + int(sqrt(sec(c + d*x))*sec(c + d*x),x))
```

3.168 $\int \frac{a+a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 75

$$\int \frac{a+a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = \frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d}$$

output
```
2*a*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2
)/d+2*a*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)
^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.65

$$\int \frac{a+a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = \frac{2a\sqrt{\cos(c+dx)}\left(E\left(\frac{1}{2}(c+dx)\middle|2\right)+\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right)\sqrt{\sec(c+dx)}}{d}$$

input
```
Integrate[(a + a*Sec[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*a*Sqrt[Cos[c + d*x]]*(EllipticE[(c + d*x)/2, 2] + EllipticF[(c + d*x)/2
, 2])*Sqrt[Sec[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sec(c+dx)+a}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \frac{1}{\sqrt{\sec(c+dx)}}dx+a\int \sqrt{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4258}$$

$$a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3119}$$

$$a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

input
```
Int[(a + a*Sec[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(70) = 140$.

Time = 1.34 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.00

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\left(\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-\operatorname{EllipticE}\left(\cos\left(\frac{d}{2}\cdots\right.\right.\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$
parts	$\frac{2a\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\operatorname{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}-\frac{2a\sqrt{\left(2\cos\left(\frac{dx}{2}+\cdots\right.\right.}}{\cdots}$
risch	$-\frac{ia\sqrt{2}}{d\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{i\left(\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(dx+c)}-i\right)}\,\sqrt{ie^{i(dx+c)}}\,\operatorname{EllipticF}\left(\sqrt{-i\left(e^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)}{\sqrt{e^{3i(dx+c)}+e^{i(dx+c)}}}-\frac{2\left(e^{2i(dx+c)}+1\right)}{\sqrt{e^{i(dx+c)}\left(e^{2i(dx+c)}+\cdots\right.}}\cdots\right.}{\cdots}$

input
```
int((a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.43

$$\int \frac{a + a\sec(c + dx)}{\sqrt{\sec(c + dx)}}\,dx$$
$$= \frac{-i\sqrt{2}a\mathrm{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)) + i\sqrt{2}a\mathrm{weierstrassPInverse}(-4, 0, \cos(}{}$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) +
I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + I*
sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) - I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F]

$$\int \frac{a + a\sec(c + dx)}{\sqrt{\sec(c + dx)}}\,dx = a\left(\int \frac{1}{\sqrt{\sec(c + dx)}}\,dx + \int \sqrt{\sec(c + dx)}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

output
```
a*(Integral(1/sqrt(sec(c + d*x)), x) + Integral(sqrt(sec(c + d*x)), x))
```

Maxima [F]

$$\int \frac{a + a\sec(c + dx)}{\sqrt{\sec(c + dx)}}\,dx = \int \frac{a\sec(dx + c) + a}{\sqrt{\sec(dx + c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{a + a\sec(c + dx)}{\sqrt{\sec(c + dx)}}\,dx = \int \frac{a\sec(dx + c) + a}{\sqrt{\sec(dx + c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + a\sec(c + dx)}{\sqrt{\sec(c + dx)}}\,dx = \int \frac{a + \frac{a}{\cos(c+d\,x)}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{a + a\sec(c + dx)}{\sqrt{\sec(c + dx)}}\,dx = a\left(\int \frac{\sqrt{\sec(dx + c)}}{\sec(dx + c)}dx + \int \sqrt{\sec(dx + c)}dx\right)$$

input
```
int((a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x)
```

output
```
a*(int(sqrt(sec(c + d*x))/sec(c + d*x),x) + int(sqrt(sec(c + d*x)),x))
```

3.169 $\int \frac{a+a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
2*a*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2
)/d+2/3*a*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+
c)^(1/2)/d+2/3*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{a\sqrt{\sec(c+dx)}\left(6\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+d\right.}{3d}$$

input
```
Integrate[(a + a*Sec[c + d*x])/Sec[c + d*x]^(3/2),x]
```

output
```
(a*Sqrt[Sec[c + d*x]]*(6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)]))/(3*d)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sec(c+dx)+a}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx + a\int \frac{1}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\csc \left(c+dx+\frac{\pi }{2}\right)^{3/2}}dx+a\int \frac{1}{\sqrt{\csc \left(c+dx+\frac{\pi }{2}\right)}}dx$$

$$\Big\downarrow \textbf{4256}$$

$$a\int \frac{1}{\sqrt{\csc \left(c+dx+\frac{\pi }{2}\right)}}dx+a\left(\frac{1}{3}\int \sqrt{\sec (c+dx)}dx+\frac{2 \sin (c+dx)}{3 d \sqrt{\sec (c+dx)}}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$a\int \frac{1}{\sqrt{\csc \left(c+dx+\frac{\pi }{2}\right)}}dx+a\left(\frac{1}{3}\int \sqrt{\csc \left(c+dx+\frac{\pi }{2}\right)}dx+\frac{2 \sin (c+dx)}{3 d \sqrt{\sec (c+dx)}}\right)$$

$$\Big\downarrow \textbf{4258}$$

$$a \sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} \int \sqrt{\cos (c+dx)}dx+a\left(\frac{1}{3} \sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} \int \frac{1}{\sqrt{\cos (c+dx)}}dx+\frac{2 \sin (c+dx)}{3 d \sqrt{\sec (c+dx)}}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$a\left(\frac{1}{3} \sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} \int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi }{2}\right)}}dx+\frac{2 \sin (c+dx)}{3 d \sqrt{\sec (c+dx)}}\right)+a \sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} \int \sqrt{\sin \left(c+dx+\frac{\pi }{2}\right)}dx$$

$$\Big\downarrow \textbf{3119}$$

$$a\left(\frac{1}{3} \sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} \int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi }{2}\right)}}dx+\frac{2 \sin (c+dx)}{3 d \sqrt{\sec (c+dx)}}\right)+\frac{2 a \sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} E\left(\frac{1}{2} (c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \textbf{3120}$$

$$\frac{2 a \sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} E\left(\frac{1}{2} (c+dx)\middle|2\right)}{d}+a\left(\frac{2 \sin (c+dx)}{3 d \sqrt{\sec (c+dx)}}+\frac{2 \sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} \text{EllipticF}\left(\frac{1}{2} (c+dx),2\right)}{3 d}\right)$$

input
```
Int[(a + a*Sec[c + d*x])/Sec[c + d*x]^(3/2),x]
```

output
```
(2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
a*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(90) = 180.

Time = 3.58 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.23

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, a\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}\right.}}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sin\left(\frac{dx}{2}+\right.}$
parts	$-\frac{2a\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}\right.}}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\, d}$

input
```
int((a+a*sec(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.24

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$=\frac{2\,a\sqrt{\cos(dx+c)}\sin(dx+c)-i\sqrt{2}a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}a\mathrm{w}}{}$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(2*a*sqrt(cos(d*x + c))*sin(d*x + c) - I*sqrt(2)*a*weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*a*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)
*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))))/d
```

Sympy [F]

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = a\left(\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{1}{\sqrt{\sec(c+dx)}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

output
```
a*(Integral(sec(c + d*x)**(-3/2), x) + Integral(1/sqrt(sec(c + d*x)), x))
```

Maxima [F]

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{a\sec(dx+c)+a}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{a\sec(dx+c)+a}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{a+\frac{a}{\cos(c+d\,x)}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = a\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx + \int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)$$

input
```
int((a+a*sec(d*x+c))/sec(d*x+c)^(3/2),x)
```

output
```
a*(int(sqrt(sec(c + d*x))/sec(c + d*x)**2,x) + int(sqrt(sec(c + d*x))/sec(
c + d*x),x))
```

3.170 $\int \frac{a+a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 127

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{6a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
6/5*a*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1
/2)/d+2/3*a*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*
x+c)^(1/2)/d+2/5*a*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/3*a*sin(d*x+c)/d/sec(d*
x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.73

$$\int \frac{a + a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{a\sqrt{\sec(c+dx)}\left(36\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right) + 20\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) + 3\sin(c+\right.}{30d}$$

input
```
Integrate[(a + a*Sec[c + d*x])/Sec[c + d*x]^(5/2),x]
```

output
```
(a*Sqrt[Sec[c + d*x]]*(36*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2
0*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 3*Sin[c + d*x] + 10*Sin[2
*(c + d*x)] + 3*Sin[3*(c + d*x)]))/(30*d)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sec(c+dx)+a}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx + a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+a\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+a\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+a\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

input
```
Int[(a + a*Sec[c + d*x])/Sec[c + d*x]^(5/2),x]
```

output
```
a*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*
d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))) + a*((2*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3
*d*Sqrt[Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [A] (verified)

Time = 6.12 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.72

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}-28\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \mathrm{EllipticF}}{15\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\right.}$
parts	$-\frac{2a\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\right.\right.}{5\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\right.}}$

input
```
int((a+a*sec(d*x+c))/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(24*cos(1/
2*d*x+1/2*c)^7-28*cos(1/2*d*x+1/2*c)^5+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))+4*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.14

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$=\frac{-5i\sqrt{2}a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}a\mathrm{weierstrassPInverse}(-4,0,\mathrm{co}}{}$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/15*(-5*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
 c)) + 5*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c)) + 9*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c))) - 9*I*sqrt(2)*a*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*a*cos(d*x + c)
^2 + 5*a*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{a + a\sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)}\,dx = a\left(\int \frac{1}{\sec^{\frac{5}{2}}(c + dx)}\,dx + \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

output
```
a*(Integral(sec(c + d*x)**(-5/2), x) + Integral(sec(c + d*x)**(-3/2), x))
```

Maxima [F]

$$\int \frac{a + a\sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)}\,dx = \int \frac{a\sec(dx + c) + a}{\sec(dx + c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{a\sec(dx+c)+a}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{a+\frac{a}{\cos(c+d\,x)}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx=a\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3}dx+\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx\right)$$

input
```
int((a+a*sec(d*x+c))/sec(d*x+c)^(5/2),x)
```

output
```
a*(int(sqrt(sec(c + d*x))/sec(c + d*x)**3,x) + int(sqrt(sec(c + d*x))/sec(
c + d*x)**2,x))
```

3.171 $\int \frac{a+a\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 151

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{6a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{10a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} + \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{10a\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}$$

output
```
6/5*a*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1
/2)/d+10/21*a*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(
d*x+c)^(1/2)/d+2/7*a*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/5*a*sin(d*x+c)/d/sec(
d*x+c)^(3/2)+10/21*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.68

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{a\sqrt{\sec(c+dx)}\left(504\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+200\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+42\sin(\right.}{420d}$$

input
```
Integrate[(a + a*Sec[c + d*x])/Sec[c + d*x]^(7/2),x]
```

output
```
(a*Sqrt[Sec[c + d*x]]*(504*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] +
200*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 42*Sin[c + d*x] + 130*S
in[2*(c + d*x)] + 42*Sin[3*(c + d*x)] + 15*Sin[4*(c + d*x)]))/(420*d)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4274, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sec(c+dx)+a}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}dx+a\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{5}{7}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+a\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{7}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+ a\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+ a\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+ a\left(\frac{5}{7}\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+ a\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+a\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+a\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{5}{7}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)\right)+a\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

input
```
Int[(a + a*Sec[c + d*x])/Sec[c + d*x]^(7/2),x]
```

output
```
a*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*
d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))) + a*((2*Sin[c + d*x])/(7*d
*Sec[c + d*x]^(5/2)) + (5*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/7
)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(130) = 260$.

Time = 9.61 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.79

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a\left(240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-528\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+448\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos(\right.}{105\sqrt{-2\,\mathrm{s}}}$
parts	$-\frac{2a\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(48\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}-120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+128\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-72\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+5\sqrt{\frac{1}{2}-\frac{\cos}{}}\right.}{21\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}\right.}}$

input
```
int((a+a*sec(d*x+c))/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-528*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1
/2*c)+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-122*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2+25*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*
c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.03

$$\int \frac{a + a\sec(c + dx)}{\sec^{\frac{7}{2}}(c + dx)}\, dx$$
$$= \frac{-25i\sqrt{2}a\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)) + 25i\sqrt{2}a\text{weierstrassPInverse}(-4, 0,$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
1/105*(-25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c)) + 25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)) + 63*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*a*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(15*a*cos(d*
x + c)^3 + 21*a*cos(d*x + c)^2 + 25*a*cos(d*x + c))*sin(d*x + c)/sqrt(cos(
d*x + c)))/d
```

Sympy [F]

$$\int \frac{a + a\sec(c + dx)}{\sec^{\frac{7}{2}}(c + dx)}\,dx = a\left(\int \frac{1}{\sec^{\frac{7}{2}}(c + dx)}\,dx + \int \frac{1}{\sec^{\frac{5}{2}}(c + dx)}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)**(7/2),x)
```

output
```
a*(Integral(sec(c + d*x)**(-7/2), x) + Integral(sec(c + d*x)**(-5/2), x))
```

Maxima [F]

$$\int \frac{a + a\sec(c + dx)}{\sec^{\frac{7}{2}}(c + dx)}\,dx = \int \frac{a\sec(dx + c) + a}{\sec(dx + c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \frac{a + a\sec(c + dx)}{\sec^{\frac{7}{2}}(c + dx)}\,dx = \int \frac{a\sec(dx + c) + a}{\sec(dx + c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{a+\frac{a}{\cos(c+d\,x)}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(7/2),x)
```

output
```
int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx=a\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^4}dx+\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3}dx\right)$$

input
```
int((a+a*sec(d*x+c))/sec(d*x+c)^(7/2),x)
```

output
```
a*(int(sqrt(sec(c + d*x))/sec(c + d*x)**4,x) + int(sqrt(sec(c + d*x))/sec(
c + d*x)**3,x))
```

3.172 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 187

$$\begin{aligned}\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx &= -\frac{12a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &+ \frac{8a^2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{7d} \\ &+ \frac{12a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{8a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{7d} \\ &+ \frac{4a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}\end{aligned}$$

output
```
-12/5*a^2*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c
)^(1/2)/d+8/7*a^2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*
sec(d*x+c)^(1/2)/d+12/5*a^2*sec(d*x+c)^(1/2)*sin(d*x+c)/d+8/7*a^2*sec(d*x+
c)^(3/2)*sin(d*x+c)/d+4/5*a^2*sec(d*x+c)^(5/2)*sin(d*x+c)/d+2/7*a^2*sec(d*
x+c)^(7/2)*sin(d*x+c)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.96 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.53

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$$

$$= \frac{a^2\sec^4\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^2\left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos^2(c+dx)\left(21\left(1+e^{2i(c+dx)}\right)+21\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c}}\right.}{}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
 + d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^2*(21*(1 + E^((2*I)*(c +
d*x))) + 21*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 10*E^(I*(c + d*x))*(-1 + E^((
2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E
^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (42*Cos[d*x]*
Csc[c] + (15 + 14*Cos[c + d*x] + 10*Cos[2*(c + d*x)])*Sec[c + d*x]^2*Tan[c
 + d*x])/Sec[c + d*x]^(3/2)))/(70*d)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4275, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119, 4534, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$2a^2\int \sec^{\frac{7}{2}}(c+dx)dx+\int \sec^{\frac{5}{2}}(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}dx+\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx$$

$$\Big\downarrow \text{4255}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{3}{5}\int \sec^{\frac{3}{2}}(c+dx)dx+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{3}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{4255}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{4258}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+$$
$$2a^2\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2\sin(c+dx)\sec}{5d}\right.$$

$$\Big\downarrow \text{3119}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+$$
$$2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right|2}{d}\right.\right.$$

$$\Big\downarrow \text{4534}$$

$$\frac{12}{7}a^2\int\sec^{\frac{5}{2}}(c+dx)dx+\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+$$
$$2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right|2}{d}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{7}a^2\int\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+$$
$$2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right|2}{d}\right.\right.$$

$$\Big\downarrow \text{4255}$$

$$\frac{12}{7}a^2\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+$$
$$2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right|2}{d}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{7}a^2\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+$$
$$\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+$$
$$2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right|2}{d}\right.\right.$$

$$\Big\downarrow \text{4258}$$

$$\frac{12}{7}a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right.}{d}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{7}a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right.}{d}\right.\right.$$

$$\Big\downarrow \text{3120}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{12}{7}a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right.}{d}\right.\right.$$

input
```
Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
(2*a^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d) + (12*a^2*((2*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]
^(3/2)*Sin[c + d*x])/(3*d)))/7 + 2*a^2*((2*Sec[c + d*x]^(5/2)*Sin[c + d*x]
)/(5*d) + (3*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/5)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. 2(162) = 324.

Time = 7.72 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.35

method	result
default	$-\frac{a^{2}\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{28\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{1}{2}\right)^{4}}-\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\right.}}{7\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{1}{2}\right)^{2}}\right.}{}$
parts	Expression too large to display

input

```
int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output

```
-a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/28*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^4-4/7*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+124/35*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/5*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(
1/2*d*x+1/2*c)^2-1/2)^3-24/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2
*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-12/5*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.15

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx =$$

$$-\frac{2\left(10i\sqrt{2}a^{2}\cos\left(dx+c\right)^{3}\operatorname{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))-10i\sqrt{2}a^{2}\cos\left(dx\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/35*(10*I*sqrt(2)*a^2*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) - 10*I*sqrt(2)*a^2*cos(d*x + c)^3*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*I*sqrt(2)*a^2*cos(d*x + c)^3
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c))) - 21*I*sqrt(2)*a^2*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (42*a^2*cos(d*x + c)^
3 + 20*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 5*a^2)*sin(d*x + c)/sqrt
(cos(d*x + c)))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \int (a\sec(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx=\int (a\sec(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx=\int \left(a+\frac{a}{\cos(c+d\,x)}\right)^2\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

input
```
int((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx=a^2\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^4\,dx+2\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^3\,dx\right)+\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^2\,dx\right)$$

input
```
int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x)
```

output
```
a**2*(int(sqrt(sec(c + d*x))*sec(c + d*x)**4,x) + 2*int(sqrt(sec(c + d*x))
*sec(c + d*x)**3,x) + int(sqrt(sec(c + d*x))*sec(c + d*x)**2,x))
```

3.173 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 161

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{16a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{16a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{4a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}$$

output
```
-16/5*a^2*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c
)^(1/2)/d+4/3*a^2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*
sec(d*x+c)^(1/2)/d+16/5*a^2*sec(d*x+c)^(1/2)*sin(d*x+c)/d+4/3*a^2*sec(d*x+
c)^(3/2)*sin(d*x+c)/d+2/5*a^2*sec(d*x+c)^(5/2)*sin(d*x+c)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.63 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.67

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{a^2\sec^4\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^2\left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos^2(c+dx)\left(12\left(1+e^{2i(c+dx)}\right)+12\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c}}\right.}{}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
 + d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^2*(12*(1 + E^((2*I)*(c +
d*x))) + 12*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2
*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^
((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (24*Cos[d*x]*C
sc[c] + (10 + 3*Sec[c + d*x])*Tan[c + d*x])/Sec[c + d*x]^(3/2)))/(30*d)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.99, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4275, 3042, 4255, 3042, 4258, 3042, 3120, 4534, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$2a^2\int \sec^{\frac{5}{2}}(c+dx)dx+\int \sec^{\frac{3}{2}}(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx$$

$$\Big\downarrow \text{4255}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{4258}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{3120}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \text{4534}$$

$$\frac{8}{5}a^2\int \sec^{\frac{3}{2}}(c+dx)dx+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \text{4255}$$

$$\frac{8}{5}a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \text{4258}$$

$$\frac{8}{5}a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \; 3119$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+\frac{8}{5}a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

input
```
Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
(2*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + (8*a^2*((-2*Sqrt[Cos[c + d
*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/d))/5 + 2*a^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d
))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(140) = 280$.

Time = 3.54 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.40

method	result
default	$-\frac{a^2\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{10\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^3}-\frac{32\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{5\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}}\right.}{}$
parts	Expression too large to display

input
```
int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/10*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^3-32/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*
cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+68/15*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-16/5*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2/3*cos(1/2*d*x+1/2*c)*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2)/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.25

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx =$$

$$-\frac{2\left(5i\sqrt{2}a^2\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))-5i\sqrt{2}a^2\cos(dx+\right.}{}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/15*(5*I*sqrt(2)*a^2*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) - 5*I*sqrt(2)*a^2*cos(d*x + c)^2*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 12*I*sqrt(2)*a^2*cos(d*x + c)^2*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c))) - 12*I*sqrt(2)*a^2*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (24*a^2*cos(d*x + c)^2
+ 10*a^2*cos(d*x + c) + 3*a^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x
 + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \int (a\sec(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \int (a\sec(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx=\int \left(a+\frac{a}{\cos(c+d\,x)}\right)^2\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx=a^2\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^3\,dx+2\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^2\,dx\right)+\int \sqrt{\sec(dx+c)}\,\sec(dx+c)\,dx\right)$$

input
```
int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x)
```

output
```
a**2*(int(sqrt(sec(c + d*x))*sec(c + d*x)**3,x) + 2*int(sqrt(sec(c + d*x))
*sec(c + d*x)**2,x) + int(sqrt(sec(c + d*x))*sec(c + d*x),x))
```

3.174 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 131

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\,dx = -\frac{4a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{8a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{4a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}$$

output
```
-4*a^2*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(
1/2)/d+8/3*a^2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec
(d*x+c)^(1/2)/d+4*a^2*sec(d*x+c)^(1/2)*sin(d*x+c)/d+2/3*a^2*sec(d*x+c)^(3/
2)*sin(d*x+c)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.40 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.02

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\,dx = \frac{1}{3}a^2\sec^4\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^2\left(-\frac{i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos^2(c+dx)\left(3\left(1+e^{2i(c+dx)}\right)+3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hyperg}\ldots\right.}{\ldots} + \frac{6\cos(dx)\csc(c)+\tan(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)}\right)$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(((-I)*Sqrt[2]*Sqrt[E^(I*(c +
 d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^2*(3*(1 + E^((2*I)*(c + d*x
))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1
[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 2*E^(I*(c + d*x))*(-1 + E^((2*I)*
c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*
I)*(c + d*x))]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (6*Cos[d*x]*Csc[
c] + Tan[c + d*x])/(2*d*Sec[c + d*x]^(3/2))))/3
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4275, 3042, 4255, 3042, 4258, 3042, 3119, 4534, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \; 4275$$

$$2a^2\int \sec^{\frac{3}{2}}(c+dx)dx+\int \sqrt{\sec(c+dx)}\left(\sec^2(c+dx)a^2+a^2\right)dx$$

$$\Big\downarrow \; 3042$$

$$2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx$$

$$\Big\downarrow \; 4255$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \; 4258$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \; 3119$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)$$

$$\Big\downarrow \text{4534}$$

$$\frac{4}{3}a^2\int\sqrt{\sec(c+dx)}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}a^2\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{4258}$$

$$\frac{4}{3}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{3120}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

input
```
Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

output
```
(8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + 2*a^2*((-2*Sqrt[Cos[
c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c
+ d*x]]*Sin[c + d*x])/d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(118) = 236.

Time = 2.24 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.83

method	result
default	$-\frac{4a^2\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(12\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\ \text{EllipticF}(c}{}$
parts	$-\frac{2a^2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}-\frac{2a^2\left(-2\sqrt{\frac{1}{2}-}\right.}{}$

input
```
int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-4/3*a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(
1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(12*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+
1/2*c)^2-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-7*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.37

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\,dx =$$

$$-\frac{2\left(2i\sqrt{2}a^2\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-2i\sqrt{2}a^2\cos(dx+c\right.}{}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/3*(2*I*sqrt(2)*a^2*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) - 2*I*sqrt(2)*a^2*cos(d*x + c)*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a^2*cos(d*x + c)*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) -
 3*I*sqrt(2)*a^2*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c))) - (6*a^2*cos(d*x + c) + a^2)*sin(d*x
 + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\,dx = a^2\left(\int \sqrt{\sec{(c+dx)}}\,dx + \int 2\sec^{\frac{3}{2}}{(c+dx)}\,dx + \int \sec^{\frac{5}{2}}{(c+dx)}\,dx\right)$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(sqrt(sec(c + d*x)), x) + Integral(2*sec(c + d*x)**(3/2), x)
 + Integral(sec(c + d*x)**(5/2), x))
```

Maxima [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\,dx = \int (a\sec{(dx+c)}+a)^2\sqrt{\sec{(dx+c)}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\,dx = \int (a\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)^2\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

input
```
int((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\,dx = a^2\left(\int \sqrt{\sec(dx+c)}dx + \int \sqrt{\sec(dx+c)}\sec(dx+c)^2\,dx + 2\left(\int \sqrt{\sec(dx+c)}\sec(dx+c)\,dx\right)\right)$$

input
```
int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x)
```

output
```
a**2*(int(sqrt(sec(c + d*x)),x) + int(sqrt(sec(c + d*x))*sec(c + d*x)**2,x
) + 2*int(sqrt(sec(c + d*x))*sec(c + d*x),x))
```

3.175 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 64

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \frac{4a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
4*a^2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(
1/2)/d+2*a^2*sec(d*x+c)^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \frac{2a^2\sqrt{\sec(c+dx)}\left(2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(c+dx)\right)}{d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*a^2*Sqrt[Sec[c + d*x]]*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
+ Sin[c + d*x]))/d
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4275, 3042, 4258, 3042, 3120, 4531}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^2}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4275}$$

$$\int \frac{\sec^2(c+dx)a^2+a^2}{\sqrt{\sec(c+dx)}}dx+2a^2\int \sqrt{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4258}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3120}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

$$\Big\downarrow \; 4531$$

$$\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

output
```
(4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4531
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] /;
 FreeQ[{b, e, f, A, C, m}, x] && EqQ[C*m + A*(m + 1), 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(59) = 118.

Time = 2.26 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.89

method	result
default	$-\frac{4a^2\left(-\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$
parts	$\frac{2a^2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}-\frac{2a^2\left(-2\sqrt{-2\sin\right.}{}$

input
```
int((a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4*a^2*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.20

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx =$$
$$-\frac{2\left(i\sqrt{2}a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-i\sqrt{2}a^2\mathrm{weierstrassPInverse}(-4,0\right.}{d}$$

input
```
integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-2*(I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)
) - I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
) - a^2*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a + a\sec(c + dx))^2}{\sqrt{\sec(c + dx)}}\,dx = a^2\left(\int \frac{1}{\sqrt{\sec(c + dx)}}\,dx + \int 2\sqrt{\sec(c + dx)}\,dx + \int \sec^{\frac{3}{2}}(c + dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

output
```
a**2*(Integral(1/sqrt(sec(c + d*x)), x) + Integral(2*sqrt(sec(c + d*x)), x
) + Integral(sec(c + d*x)**(3/2), x))
```

Maxima [F]

$$\int \frac{(a + a\sec(c + dx))^2}{\sqrt{\sec(c + dx)}}\,dx = \int \frac{(a\sec(dx + c) + a)^2}{\sqrt{\sec(dx + c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = a^2\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx + 2\left(\int \sqrt{\sec(dx+c)}dx\right) + \int \sqrt{\sec(dx+c)}\,\sec(dx+c)\,dx\right)$$

input
```
int((a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x)
```

output
```
a**2*(int(sqrt(sec(c + d*x))/sec(c + d*x),x) + 2*int(sqrt(sec(c + d*x)),x)
 + int(sqrt(sec(c + d*x))*sec(c + d*x),x))
```

3.176 $\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)} dx$

Optimal result

Integrand size = 23, antiderivative size = 107

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)} dx = \frac{4a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{8a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
4*a^2*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1
/2)/d+8/3*a^2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(
d*x+c)^(1/2)/d+2/3*a^2*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.36 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.46

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{a^2\left(\cos\left(\frac{c}{2}\right)-i\sin\left(\frac{c}{2}\right)\right)\left(-i\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(12-\frac{24\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+8\sqrt{1+e^{2i(c+dx)}}\right.}{3d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

output
```
(a^2*(Cos[c/2] - I*Sin[c/2])*((-I)*Cos[c/2] + Sin[c/2])*(12 - (24*Hypergeo
metric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*
x))] + 8*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E
^((2*I)*(c + d*x))]*Sec[c + d*x] + (2*I)*Sin[c + d*x]))/(3*d*Sqrt[Sec[c +
d*x]])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4275, 3042, 4258, 3042, 3119, 4533, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4275}$$

$$\int \frac{\sec^2(c+dx)a^2+a^2}{\sec^{\frac{3}{2}}(c+dx)}dx+2a^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4258}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3119}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{4533}$$

$$\frac{4}{3}a^2\int \sqrt{\sec(c+dx)}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{4}{3}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$\Big\downarrow$ 3120

$$\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

output
```
(4*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(3*d) + (2*a^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(96) = 192$.

Time = 2.16 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.13

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\, a^2\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{d}{}\right.}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\right.}$
parts	$-\frac{2a^2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\right.}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\, d}$

input
```
int((a+a*sec(d*x+c))^2/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(2*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.25

$$\int \frac{(a + a\sec(c + dx))^2}{\sec^{\frac{3}{2}}(c + dx)}\,dx$$
$$= \frac{2\left(a^2\sqrt{\cos(dx + c)}\sin(dx + c) - 2i\sqrt{2}a^2\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)) + 2i\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(a^2*sqrt(cos(d*x + c))*sin(d*x + c) - 2*I*sqrt(2)*a^2*weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 2*I*sqrt(2)*a^2*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a^2*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3
*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c))))/d
```

Sympy [F]

$$\int \frac{(a + a\sec(c + dx))^2}{\sec^{\frac{3}{2}}(c + dx)}\,dx = a^2\left(\int \frac{1}{\sec^{\frac{3}{2}}(c + dx)}\,dx + \int \frac{2}{\sqrt{\sec(c + dx)}}\,dx + \int \sqrt{\sec(c + dx)}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

output
```
a**2*(Integral(sec(c + d*x)**(-3/2), x) + Integral(2/sqrt(sec(c + d*x)), x
) + Integral(sqrt(sec(c + d*x)), x))
```

Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = a^2\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx + 2\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right) + \int \sqrt{\sec(dx+c)}dx\right)$$

input
```
int((a+a*sec(d*x+c))^2/sec(d*x+c)^(3/2),x)
```

output
```
a**2*(int(sqrt(sec(c + d*x))/sec(c + d*x)**2,x) + 2*int(sqrt(sec(c + d*x))
/sec(c + d*x),x) + int(sqrt(sec(c + d*x)),x))
```

3.177 $\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{16a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output

```
16/5*a^2*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)
^(1/2)/d+4/3*a^2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*s
ec(d*x+c)^(1/2)/d+2/5*a^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)+4/3*a^2*sin(d*x+c)
/d/sec(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.65 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{a^2\left(-96i+\frac{192i\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-40i\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-e^{2i(c+d}\right.\right.}{30d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(5/2),x]
```

output
```
(a^2*(-96*I + ((192*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*
x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (40*I)*Sqrt[1 + E^((2*I)*(c + d*x))]
*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 40*
Sin[c + d*x] + 6*Sin[2*(c + d*x)]))/(30*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4275, 3042, 4256, 3042, 4258, 3042, 3120, 4533, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4275}$$

$$2a^2\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\int \frac{\sec^2(c+dx)a^2+a^2}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2a^2\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2a^2\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{4258}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+ \\ 2a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+ \\ 2a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3120}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+ \\ 2a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \text{4533}$$

$$\frac{8}{5}a^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+ \\ 2a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}a^2\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+$$
$$2a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$\Big\downarrow$ 4258

$$\frac{8}{5}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+$$
$$2a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$\Big\downarrow$ 3042

$$\frac{8}{5}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+$$
$$2a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$\Big\downarrow$ 3119

$$\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{16a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+$$
$$2a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

input
```
Int[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(5/2),x]
```

output
```
(16*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (2*a^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + 2*a^2*((2*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c +
 d*x])/(3*d*Sqrt[Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(118) = 236.

Time = 6.30 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.85

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a^{2}\left(-12\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+32\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-13\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\right.\right.}{15\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin}}$
parts	$-\frac{2a^{2}\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\right.\right.}{5\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\right.}}$

input
```
int((a+a*sec(d*x+c))^2/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*sin
(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+32*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)-13*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.16

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx =$$
$$-\frac{2\left(5i\sqrt{2}a^{2}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}a^{2}\text{weierstrassPInverse}(-\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-2/15*(5*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c)) - 5*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c)) - 12*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c))) + 12*I*sqrt(2)*a^2*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*a^2*co
s(d*x + c)^2 + 10*a^2*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx = a^2\left(\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \int \frac{2}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{1}{\sqrt{\sec(c+dx)}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2/sec(d*x+c)**(5/2),x)
```

output
```
a**2*(Integral(sec(c + d*x)**(-5/2), x) + Integral(2/sec(c + d*x)**(3/2),
x) + Integral(1/sqrt(sec(c + d*x)), x))
```

Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(a\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx=a^2\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3}dx+2\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx\right)+\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)$$

input
```
int((a+a*sec(d*x+c))^2/sec(d*x+c)^(5/2),x)
```

output
```
a**2*(int(sqrt(sec(c + d*x))/sec(c + d*x)**3,x) + 2*int(sqrt(sec(c + d*x))
/sec(c + d*x)**2,x) + int(sqrt(sec(c + d*x))/sec(c + d*x),x))
```

3.178 $\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 161

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{12a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{8a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{7d} + \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{8a^2\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}$$

output
```
12/5*a^2*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)
^(1/2)/d+8/7*a^2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*s
ec(d*x+c)^(1/2)/d+2/7*a^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)+4/5*a^2*sin(d*x+c)
/d/sec(d*x+c)^(3/2)+8/7*a^2*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.90 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.93

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{a^2\left(\frac{672i\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+2\left(-168i-80i\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-e^{2i\ldots}\right.\right.}{140d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(7/2),x]
```

output
```
(a^2*(((672*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sq
rt[1 + E^((2*I)*(c + d*x))] + 2*(-168*I - (80*I)*Sqrt[1 + E^((2*I)*(c + d*
x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] +
 85*Sin[c + d*x] + 28*Sin[2*(c + d*x)] + 5*Sin[3*(c + d*x)])))/(140*d*Sqrt
[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4275, 3042, 4256, 3042, 4258, 3042, 3119, 4533, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4275}$$

$$2a^2\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx+\int \frac{\sec^2(c+dx)a^2+a^2}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$\Big\downarrow$ 3042

$$2a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$\Big\downarrow$ 4256

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2a^2\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$\Big\downarrow$ 3042

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2a^2\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$\Big\downarrow$ 4258

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2a^2\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$\Big\downarrow$ 3042

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2a^2\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$\Big\downarrow$ 3119

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$\Big\downarrow$ 4533

$$\frac{12}{7}a^2\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$\big\downarrow$ 3042

$$\frac{12}{7}a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$\big\downarrow$ 4256

$$\frac{12}{7}a^2\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$\big\downarrow$ 3042

$$\frac{12}{7}a^2\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$\big\downarrow$ 4258

$$\frac{12}{7}a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$\big\downarrow$ 3042

$$\frac{12}{7}a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$\big\downarrow$ 3120

$$\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{12}{7}a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

input
```
Int[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(7/2),x]
```

output
```
(2*a^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + 2*a^2*((6*Sqrt[Cos[c + d*x
]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*Sin[c + d*x])/
(5*d*Sec[c + d*x]^(3/2))) + (12*a^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c +
d*x]])))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Maple [A] (verified)

Time = 10.06 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.69

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a^{2}\left(40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-116\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+126\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\right.\right.}{35\sqrt{-2\sin}}$
parts	$-\frac{2a^{2}\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(48\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}-120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+128\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-72\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+5\sqrt{\frac{1}{2}-\right.}{21\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}\right.}}$

input
```
int((a+a*sec(d*x+c))^2/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-4/35*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(40*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-116*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1
/2*c)+126*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-39*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^2+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-21*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx =$$

$$-\frac{2\left(10i\sqrt{2}a^2\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-10i\sqrt{2}a^2\text{weierstrassPInverse}(\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-2/35*(10*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) - 10*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c)) - 21*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*I*sqrt(2)*a^2*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (5*a^2*
cos(d*x + c)^3 + 14*a^2*cos(d*x + c)^2 + 20*a^2*cos(d*x + c))*sin(d*x + c)
/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a + a\sec(c + dx))^2}{\sec^{\frac{7}{2}}(c + dx)}\,dx = a^2\left(\int \frac{1}{\sec^{\frac{7}{2}}(c + dx)}\,dx + \int \frac{2}{\sec^{\frac{5}{2}}(c + dx)}\,dx + \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2/sec(d*x+c)**(7/2),x)
```

output
```
a**2*(Integral(sec(c + d*x)**(-7/2), x) + Integral(2/sec(c + d*x)**(5/2),
x) + Integral(sec(c + d*x)**(-3/2), x))
```

Maxima [F]

$$\int \frac{(a + a\sec(c + dx))^2}{\sec^{\frac{7}{2}}(c + dx)}\,dx = \int \frac{(a\sec(dx + c) + a)^2}{\sec(dx + c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(a + a\sec(c + dx))^2}{\sec^{\frac{7}{2}}(c + dx)}\,dx = \int \frac{(a\sec(dx + c) + a)^2}{\sec(dx + c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(7/2),x)
```

output
```
int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx=a^2\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^4}dx+2\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3}dx\right)+\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx\right)$$

input
```
int((a+a*sec(d*x+c))^2/sec(d*x+c)^(7/2),x)
```

output
```
a**2*(int(sqrt(sec(c + d*x))/sec(c + d*x)**4,x) + 2*int(sqrt(sec(c + d*x))
/sec(c + d*x)**3,x) + int(sqrt(sec(c + d*x))/sec(c + d*x)**2,x))
```

3.179 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 187

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx &= -\frac{28a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &+ \frac{52a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\ &+ \frac{28a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{52a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} \\ &+ \frac{6a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a^3\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}\end{aligned}$$

output
```
-28/5*a^3*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c
)^(1/2)/d+52/21*a^3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)
)*sec(d*x+c)^(1/2)/d+28/5*a^3*sec(d*x+c)^(1/2)*sin(d*x+c)/d+52/21*a^3*sec(
d*x+c)^(3/2)*sin(d*x+c)/d+6/5*a^3*sec(d*x+c)^(5/2)*sin(d*x+c)/d+2/7*a^3*se
c(d*x+c)^(7/2)*sin(d*x+c)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.88 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.53

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{a^3\sec^6\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^3\left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos^3(c+dx)\left(147\left(1+e^{2i(c+dx)}\right)+147\left(-1+e^{2ic}\right)\sqrt{1+e^{2}}\right.}{}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3,x]
```

output
```
(a^3*Sec[(c + d*x)/2]^6*(1 + Sec[c + d*x])^3*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
 + d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^3*(147*(1 + E^((2*I)*(c +
 d*x))) + 147*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 65*E^(I*(c + d*x))*(-1 + E^
((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4,
-E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (294*Cos[d*
x]*Csc[c] + (80 + 63*Cos[c + d*x] + 65*Cos[2*(c + d*x)])*Sec[c + d*x]^2*Ta
n[c + d*x])/Sec[c + d*x]^(5/2)))/(420*d)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^3\sec^{\frac{9}{2}}(c+dx)+3a^3\sec^{\frac{7}{2}}(c+dx)+3a^3\sec^{\frac{5}{2}}(c+dx)+a^3\sec^{\frac{3}{2}}(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^3\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{6a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{52a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{28a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{52a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}-\frac{28a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

input
```
Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3,x]
```

output
```
(-28*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(5*d) + (52*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(21*d) + (28*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (52*a^3*S
ec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (6*a^3*Sec[c + d*x]^(5/2)*Sin[c +
 d*x])/(5*d) + (2*a^3*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. 2(162) = 324.

Time = 6.08 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.35

method	result
default	$-\frac{a^3\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{28\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^4}-\frac{26\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin}}{21\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^2}\right.}{}$
parts	Expression too large to display

input
```
int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/28*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^4-26/21*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+848/105*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/10
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
cos(1/2*d*x+1/2*c)^2-1/2)^3-56/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(
-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-28/5*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.15

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{2\left(65i\sqrt{2}a^3\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-65i\sqrt{2}a^3\cos(dx\right.}{}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-2/105*(65*I*sqrt(2)*a^3*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c)) - 65*I*sqrt(2)*a^3*cos(d*x + c)^3*weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 147*I*sqrt(2)*a^3*cos(d*x + c)
^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c))) - 147*I*sqrt(2)*a^3*cos(d*x + c)^3*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (294*a^3*cos(d*x +
 c)^3 + 130*a^3*cos(d*x + c)^2 + 63*a^3*cos(d*x + c) + 15*a^3)*sin(d*x + c
)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \int (a\sec(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx=\int (a\sec(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx=\int \left(a+\frac{a}{\cos(c+d\,x)}\right)^3\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx=a^3\Bigg(\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^4\,dx\right)\\+3\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^3\,dx\right)\\+3\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^2\,dx\right)\\+\int \sqrt{\sec(dx+c)}\,\sec(dx+c)\,dx\Bigg)\end{aligned}$$

input
```
int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x)
```

output

```
a**3*(int(sqrt(sec(c + d*x))*sec(c + d*x)**4,x) + 3*int(sqrt(sec(c + d*x))
*sec(c + d*x)**3,x) + 3*int(sqrt(sec(c + d*x))*sec(c + d*x)**2,x) + int(sq
rt(sec(c + d*x))*sec(c + d*x),x))
```

3.180 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 157

$$\begin{aligned}\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\,dx &= -\frac{36a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}\\ &\quad+\frac{4a^3\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d}\\ &\quad+\frac{36a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}\\ &\quad+\frac{2a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d}+\frac{2a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}\end{aligned}$$

output
```
-36/5*a^3*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c
)^(1/2)/d+4*a^3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*se
c(d*x+c)^(1/2)/d+36/5*a^3*sec(d*x+c)^(1/2)*sin(d*x+c)/d+2*a^3*sec(d*x+c)^(
3/2)*sin(d*x+c)/d+2/5*a^3*sec(d*x+c)^(5/2)*sin(d*x+c)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.62 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.70

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\,dx$$
$$= \frac{a^3\sec^6\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^3\left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos^3(c+dx)\left(9\left(1+e^{2i(c+dx)}\right)+9\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+d}}\right.}{}\right.}{}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3,x]
```

output
```
(a^3*Sec[(c + d*x)/2]^6*(1 + Sec[c + d*x])^3*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
 + d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^3*(9*(1 + E^((2*I)*(c + d
*x))) + 9*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2
F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I
)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((
2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (18*Cos[d*x]*Csc
[c] + (5 + Sec[c + d*x])*Tan[c + d*x])/Sec[c + d*x]^(5/2)))/(20*d)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^3 \sec^{\frac{7}{2}}(c+dx) + 3a^3 \sec^{\frac{5}{2}}(c+dx) + 3a^3 \sec^{\frac{3}{2}}(c+dx) + a^3 \sqrt{\sec(c+dx)}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^3 \sin(c+dx) \sec^{\frac{5}{2}}(c+dx)}{5d} + \frac{2a^3 \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{d} + \frac{36a^3 \sin(c+dx) \sqrt{\sec(c+dx)}}{5d} + \frac{4a^3 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \,\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d} - \frac{36a^3 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} E\left(\frac{1}{2}(c+dx)\middle| 2\right)}{5d}$$

input
```
Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3,x]
```

output
```
(-36*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(5*d) + (4*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (36*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a^3*Sec[c +
d*x]^(3/2)*Sin[c + d*x])/d + (2*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(140) = 280$.

Time = 3.79 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.46

method	result
default	$-\frac{a^{3}\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(\frac{56\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{5\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2}}{10(\cos}\right.}{}$
parts	Expression too large to display

input
```
int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(56/5*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-1/10*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-72/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2
*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-36/5*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2)/sin(1/2
*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.27

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{2\left(5i\sqrt{2}a^{3}\cos(dx+c)^{2}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}a^{3}\cos(dx+\right.}{}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-2/5*(5*I*sqrt(2)*a^3*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) - 5*I*sqrt(2)*a^3*cos(d*x + c)^2*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*I*sqrt(2)*a^3*cos(d*x + c)^2*wei
erstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c))) - 9*I*sqrt(2)*a^3*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (18*a^3*cos(d*x + c)^2 + 5
*a^3*cos(d*x + c) + a^3)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^
2)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\,dx = \int (a\sec(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\,dx = \int (a\sec(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\,dx = \int \left(a+\frac{a}{\cos(c+dx)}\right)^3\sqrt{\frac{1}{\cos(c+dx)}}\,dx$$

input
```
int((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\,dx = a^3\left(\int \sqrt{\sec(dx+c)}dx + \int \sqrt{\sec(dx+c)}\sec(dx+c)^3\,dx + 3\left(\int \sqrt{\sec(dx+c)}\sec(dx+c)^2\,dx\right) + 3\left(\int \sqrt{\sec(dx+c)}\sec(dx+c)\,dx\right)\right)$$

input
```
int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x)
```

output
```
a**3*(int(sqrt(sec(c + d*x)),x) + int(sqrt(sec(c + d*x))*sec(c + d*x)**3,x
) + 3*int(sqrt(sec(c + d*x))*sec(c + d*x)**2,x) + 3*int(sqrt(sec(c + d*x))
*sec(c + d*x),x))
```

3.181 $\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 131

$$\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx = -\frac{4a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{20a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{6a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}$$

output
```
-4*a^3*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(
1/2)/d+20/3*a^3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*se
c(d*x+c)^(1/2)/d+6*a^3*sec(d*x+c)^(1/2)*sin(d*x+c)/d+2/3*a^3*sec(d*x+c)^(3
/2)*sin(d*x+c)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.31 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.43

$$\int \frac{(a + a\sec(c + dx))^3}{\sqrt{\sec(c + dx)}}\,dx = \frac{a^3 e^{-2i(c+dx)}\sec^{\frac{3}{2}}(c + dx)\left(-6 - 6\cos(2(c + dx)) + 6e^{-2i(c+dx)}\left(1 + e^{2i(c+dx)}\right)^{3/2}\text{Hypergeometric2F1}\left(-\frac{1}{4}\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```

output
```
(a^3*Sec[c + d*x]^(3/2)*(-6 - 6*Cos[2*(c + d*x)] + (6*(1 + E^((2*I)*(c + d
*x)))^(3/2)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/E^((2
*I)*(c + d*x)) + 20*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]*Hypergeomet
ric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))] + (2*I)*Sin[c + d*x] + (9*I)*S
in[2*(c + d*x)])*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)]))/(3*d*E^((2*I)
*(c + d*x)))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c + dx) + a)^3}{\sqrt{\sec(c + dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c + dx + \frac{\pi}{2}\right) + a\right)^3}{\sqrt{\csc\left(c + dx + \frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^3 \sec^{\frac{5}{2}}(c+dx)+3a^3 \sec^{\frac{3}{2}}(c+dx)+3a^3 \sqrt{\sec(c+dx)}+\frac{a^3}{\sqrt{\sec(c+dx)}}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2a^3 \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{6a^3 \sin(c+dx) \sqrt{\sec(c+dx)}}{d}+\frac{20a^3 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{4a^3 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```

output
```
(-4*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
 + (20*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(3*d) + (6*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a^3*Sec[c + d*x]^
(3/2)*Sin[c + d*x])/(3*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(118) = 236.

Time = 3.23 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.83

method	result
default	$-\frac{4a^3\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(18\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\text{EllipticF}\left(\right.\right.}{}$
parts	$\frac{2a^3\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}-\frac{2a^3\left(-2\sqrt{\frac{1}{2}-\frac{\text{co}}{}}\right.}{}$

input
```
int((a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(
1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(18*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x
+1/2*c)^2-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-10*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^2+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.37

$$\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{2\left(5i\sqrt{2}a^3\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}a^3\cos(dx+\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(5*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) - 5*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a^3*cos(d*x + c)*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) -
 3*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c))) - (9*a^3*cos(d*x + c) + a^3)*sin(d*x
 + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx = a^3\left(\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \int 3\sqrt{\sec(c+dx)}\,dx + \int 3\sec^{\frac{3}{2}}(c+dx)\,dx + \int \sec^{\frac{5}{2}}(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

output
```
a**3*(Integral(1/sqrt(sec(c + d*x)), x) + Integral(3*sqrt(sec(c + d*x)), x
) + Integral(3*sec(c + d*x)**(3/2), x) + Integral(sec(c + d*x)**(5/2), x))
```

Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a\sec(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a + a\sec(c + dx))^3}{\sqrt{\sec(c + dx)}}\,dx = \int \frac{(a\sec(dx + c) + a)^3}{\sqrt{\sec(dx + c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(c + dx))^3}{\sqrt{\sec(c + dx)}}\,dx = \int \frac{\left(a + \frac{a}{\cos(c+d\,x)}\right)^3}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(a + a\sec(c + dx))^3}{\sqrt{\sec(c + dx)}}\,dx = a^3\left(\int \frac{\sqrt{\sec(dx + c)}}{\sec(dx + c)}dx + 3\left(\int \sqrt{\sec(dx + c)}dx\right) + \int \sqrt{\sec(dx + c)}\,\sec(dx + c)^2\,dx + 3\left(\int \sqrt{\sec(dx + c)}\,\sec(dx + c)\,dx\right)\right)$$

input
```
int((a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x)
```

output

```
a**3*(int(sqrt(sec(c + d*x))/sec(c + d*x),x) + 3*int(sqrt(sec(c + d*x)),x)
 + int(sqrt(sec(c + d*x))*sec(c + d*x)**2,x) + 3*int(sqrt(sec(c + d*x))*se
c(c + d*x),x))
```

3.182 $\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 131

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{4a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{20a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
4*a^3*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1
/2)/d+20/3*a^3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec
(d*x+c)^(1/2)/d+2/3*a^3*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2*a^3*sec(d*x+c)^(1/
2)*sin(d*x+c)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.14 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.29

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{a^3\left(\cos\left(\frac{c}{2}\right)-i\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+i\sin\left(\frac{c}{2}\right)\right)\left(\frac{24i\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+2\left(-6i-10i\sqrt{1+e^{2i(\cdots}}\right.\right.}{3d\sqrt{\sec(c+dx\cdots}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

output
```
(a^3*(Cos[c/2] - I*Sin[c/2])*(Cos[c/2] + I*Sin[c/2])*(((24*I)*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))]
 + 2*(-6*I - (10*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1
/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + Sin[c + d*x] + 3*Tan[c + d*x
])))/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^3 \sec^{\frac{3}{2}}(c+dx) + \frac{a^3}{\sec^{\frac{3}{2}}(c+dx)} + 3a^3\sqrt{\sec(c+dx)} + \frac{3a^3}{\sqrt{\sec(c+dx)}}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2a^3 \sin(c+dx)\sqrt{\sec(c+dx)}}{d} + \frac{2a^3 \sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{20a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{4a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

output
```
(4*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (20*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(3*d) + (2*a^3*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a^3*Sqrt[Sec[c
 + d*x]]*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [A] (verified)

Time = 3.45 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.31

method	result
default	$-\frac{4a^3\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{\right.}\right.}{3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$
parts	$-\frac{2a^3\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\right.}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$

input
```
int((a+a*sec(d*x+c))^3/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*a^3*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-4*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.13

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx =$$
$$-\frac{2\left(5i\sqrt{2}a^3\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}a^3\mathrm{weierstrassPInverse}(-4\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c)) - 5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)) - 3*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*a^3*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (a^3*cos(d*x
 + c) + 3*a^3)*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = a^3\left(\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{3}{\sqrt{\sec(c+dx)}}\,dx + \int 3\sqrt{\sec(c+dx)}\,dx + \int \sec^{\frac{3}{2}}(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

output
```
a**3*(Integral(sec(c + d*x)**(-3/2), x) + Integral(3/sqrt(sec(c + d*x)), x
) + Integral(3*sqrt(sec(c + d*x)), x) + Integral(sec(c + d*x)**(3/2), x))
```

Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = a^3\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx + 3\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right) + 3\left(\int \sqrt{\sec(dx+c)}dx\right) + \int \sqrt{\sec(dx+c)}\,\sec(dx+c)\,dx\right)$$

input
```
int((a+a*sec(d*x+c))^3/sec(d*x+c)^(3/2),x)
```

output
```
a**3*(int(sqrt(sec(c + d*x))/sec(c + d*x)**2,x) + 3*int(sqrt(sec(c + d*x))
/sec(c + d*x),x) + 3*int(sqrt(sec(c + d*x)),x) + int(sqrt(sec(c + d*x))*se
c(c + d*x),x))
```

3.183 $\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 131

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{36a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}}$$

output
```
36/5*a^3*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)
^(1/2)/d+4*a^3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec
(d*x+c)^(1/2)/d+2/5*a^3*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2*a^3*sin(d*x+c)/d/s
ec(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.89 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.31

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{a^3\left(\cos\left(\frac{c}{2}\right)-i\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+i\sin\left(\frac{c}{2}\right)\right)\left(\frac{144i\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+2\left(-36i-20i\sqrt{1+e\right.\right.}{10d\sqrt{\sec(c+}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(5/2),x]
```

output
```
(a^3*(Cos[c/2] - I*Sin[c/2])*(Cos[c/2] + I*Sin[c/2])*(((144*I)*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))
] + 2*(-36*I - (20*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4,
 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 10*Sin[c + d*x] + Sin[2*(c
 + d*x)])))/(10*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4278}$$

$$\int \left(\frac{3a^3}{\sec^{\frac{3}{2}}(c+dx)}+\frac{a^3}{\sec^{\frac{5}{2}}(c+dx)}+a^3\sqrt{\sec(c+dx)}+\frac{3a^3}{\sqrt{\sec(c+dx)}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}}+\frac{4a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{36a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

input
```
Int[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(5/2),x]
```

output
```
(36*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (4*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (2*a^3*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a^3*Sin[c + d*
x])/(d*Sqrt[Sec[c + d*x]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(118) = 236.

Time = 4.02 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.91

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,a^3\left(-4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+14\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\right.\right.}{5\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\right.}}$
parts	$-\frac{2a^3\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\right.\right.}{5\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\right.}}$

input
```
int((a+a*sec(d*x+c))^3/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-4/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-4*sin(1
/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)-6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.19

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx =$$
$$-\frac{2\left(5i\sqrt{2}a^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}a^3\text{weierstrassPInverse}(-\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-2/5*(5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c)) - 5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)) - 9*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c))) + 9*I*sqrt(2)*a^3*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (a^3*cos(d*x
 + c)^2 + 5*a^3*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a + a\sec(c + dx))^3}{\sec^{\frac{5}{2}}(c + dx)}\,dx = a^3\left(\int \frac{1}{\sec^{\frac{5}{2}}(c + dx)}\,dx + \int \frac{3}{\sec^{\frac{3}{2}}(c + dx)}\,dx + \int \frac{3}{\sqrt{\sec(c + dx)}}\,dx + \int \sqrt{\sec(c + dx)}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**3/sec(d*x+c)**(5/2),x)
```

output
```
a**3*(Integral(sec(c + d*x)**(-5/2), x) + Integral(3/sec(c + d*x)**(3/2),
x) + Integral(3/sqrt(sec(c + d*x)), x) + Integral(sqrt(sec(c + d*x)), x))
```

Maxima [F]

$$\int \frac{(a + a\sec(c + dx))^3}{\sec^{\frac{5}{2}}(c + dx)}\,dx = \int \frac{(a\sec(dx + c) + a)^3}{\sec(dx + c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx = a^3\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3}dx+3\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx\right)\right.$$
$$\left.+3\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)+\int \sqrt{\sec(dx+c)}dx\right)$$

input
```
int((a+a*sec(d*x+c))^3/sec(d*x+c)^(5/2),x)
```

output
```
a**3*(int(sqrt(sec(c + d*x))/sec(c + d*x)**3,x) + 3*int(sqrt(sec(c + d*x))
/sec(c + d*x)**2,x) + 3*int(sqrt(sec(c + d*x))/sec(c + d*x),x) + int(sqrt(
sec(c + d*x)),x))
```

3.184 $\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 161

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{28a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{52a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} + \frac{2a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{52a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}$$

output

```
28/5*a^3*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)
^(1/2)/d+52/21*a^3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))
*sec(d*x+c)^(1/2)/d+2/7*a^3*sin(d*x+c)/d/sec(d*x+c)^(5/2)+6/5*a^3*sin(d*x+
c)/d/sec(d*x+c)^(3/2)+52/21*a^3*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.91

$$\int \frac{(a + a\sec(c + dx))^3}{\sec^{\frac{7}{2}}(c + dx)}\,dx$$
$$= \frac{a^3\left(-2352i + \frac{4704i\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 1040i\sqrt{1 + e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-e\right.\right.}{420d\sqrt{\sec(c + dx)}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(7/2),x]
```

output
```
(a^3*(-2352*I + ((4704*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c +
 d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (1040*I)*Sqrt[1 + E^((2*I)*(c + d
*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x]
+ 1070*Sin[c + d*x] + 252*Sin[2*(c + d*x)] + 30*Sin[3*(c + d*x)]))/(420*d*
Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c + dx) + a)^3}{\sec^{\frac{7}{2}}(c + dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a\csc\left(c + dx + \frac{\pi}{2}\right) + a\right)^3}{\csc\left(c + dx + \frac{\pi}{2}\right)^{7/2}}dx$$
$$\Big\downarrow \text{4278}$$

$$\int \left(\frac{3a^3}{\sec^{\frac{3}{2}}(c+dx)}+\frac{3a^3}{\sec^{\frac{5}{2}}(c+dx)}+\frac{a^3}{\sec^{\frac{7}{2}}(c+dx)}+\frac{a^3}{\sqrt{\sec(c+dx)}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{52a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{52a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{28a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

input

```
Int[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(7/2),x]
```

output

```
(28*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (52*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(21*d) + (2*a^3*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (6*a^3*Sin[
c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (52*a^3*Sin[c + d*x])/(21*d*Sqrt[Sec[
c + d*x]])
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [A] (verified)

Time = 9.76 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.69

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, a^{3}\left(120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-432\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+602\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\cdots\right.}{105\sqrt{-2\cdots}}$
parts	$-\frac{2a^{3}\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(48\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}-120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+128\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-72\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+5\sqrt{\frac{1}{2}-\cdots}\right.}{21\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\cdots\right.}}$

input
```
int((a+a*sec(d*x+c))^3/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(120*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-432*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)+602*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-208*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^2+65*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1
/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx =$$
$$-\frac{2\left(65i\sqrt{2}a^{3}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-65i\sqrt{2}a^{3}\mathrm{weierstrassPInverse}(\cdots\right.}{\cdots}$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-2/105*(65*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c)) - 65*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) - 147*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 147*I*sqrt(2)*a^3*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (15*
a^3*cos(d*x + c)^3 + 63*a^3*cos(d*x + c)^2 + 130*a^3*cos(d*x + c))*sin(d*x
 + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx = a^3\left(\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + \int \frac{3}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \int \frac{3}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{1}{\sqrt{\sec(c+dx)}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**3/sec(d*x+c)**(7/2),x)
```

output
```
a**3*(Integral(sec(c + d*x)**(-7/2), x) + Integral(3/sec(c + d*x)**(5/2),
x) + Integral(3/sec(c + d*x)**(3/2), x) + Integral(1/sqrt(sec(c + d*x)), x
))
```

Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(7/2),x)
```

output
```
int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx = a^3\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^4}dx+3\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3}dx\right)+3\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx\right)+\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)$$

input
```
int((a+a*sec(d*x+c))^3/sec(d*x+c)^(7/2),x)
```

output
```
a**3*(int(sqrt(sec(c + d*x))/sec(c + d*x)**4,x) + 3*int(sqrt(sec(c + d*x))
/sec(c + d*x)**3,x) + 3*int(sqrt(sec(c + d*x))/sec(c + d*x)**2,x) + int(sq
rt(sec(c + d*x))/sec(c + d*x),x))
```

3.185 $\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 187

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{68a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{44a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} + \frac{2a^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{68a^3\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{44a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}$$

output
```
68/15*a^3*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c
)^(1/2)/d+44/21*a^3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)
)*sec(d*x+c)^(1/2)/d+2/9*a^3*sin(d*x+c)/d/sec(d*x+c)^(7/2)+6/7*a^3*sin(d*x
+c)/d/sec(d*x+c)^(5/2)+68/45*a^3*sin(d*x+c)/d/sec(d*x+c)^(3/2)+44/21*a^3*s
in(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.69 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.83

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \frac{a^3\left(-11424i+\frac{22848i\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-5280i\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\right.\right.}{2520d\sqrt{}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(9/2),x]
```

output
```
(a^3*(-11424*I + ((22848*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c
 + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (5280*I)*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x
] + 5820*Sin[c + d*x] + 2044*Sin[2*(c + d*x)] + 540*Sin[3*(c + d*x)] + 70*
Sin[4*(c + d*x)]))/(2520*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$
$$\Big\downarrow \text{4278}$$

$$\int \left(\frac{a^3}{\sec^{\frac{3}{2}}(c+dx)} + \frac{3a^3}{\sec^{\frac{5}{2}}(c+dx)} + \frac{3a^3}{\sec^{\frac{7}{2}}(c+dx)} + \frac{a^3}{\sec^{\frac{9}{2}}(c+dx)} \right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{68a^3\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{44a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{44a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} + \frac{68a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}$$

input
```
Int[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(9/2),x]
```

output
```
(68*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
15*d) + (44*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(21*d) + (2*a^3*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (6*a^3*Sin
[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (68*a^3*Sin[c + d*x])/(45*d*Sec[c +
d*x]^(3/2)) + (44*a^3*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [A] (verified)

Time = 15.69 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.39

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a^{3}\left(560\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}-600\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}+212\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+66\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-430\cos\right.}{315\sqrt{-2\sin}}$
parts	Expression too large to display

input
```
int((a+a*sec(d*x+c))^3/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(560*co
s(1/2*d*x+1/2*c)^11-600*cos(1/2*d*x+1/2*c)^9+212*cos(1/2*d*x+1/2*c)^7+66*c
os(1/2*d*x+1/2*c)^5-430*cos(1/2*d*x+1/2*c)^3+165*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-357*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))+192*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.98

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx =$$
$$-\frac{2\left(165i\sqrt{2}a^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-165i\sqrt{2}a^3\text{weierstrassPInvers}\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
-2/315*(165*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) - 165*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*
sin(d*x + c)) - 357*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 357*I*sqrt(2)*a^3*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3
5*a^3*cos(d*x + c)^4 + 135*a^3*cos(d*x + c)^3 + 238*a^3*cos(d*x + c)^2 + 3
30*a^3*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(c + dx))^3}{\sec^{\frac{9}{2}}(c + dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**3/sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a + a\sec(c + dx))^3}{\sec^{\frac{9}{2}}(c + dx)}\,dx = \int \frac{(a\sec(dx + c) + a)^3}{\sec(dx + c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(9/2), x)
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(9/2),x)
```

output
```
int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(9/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx=a^3\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^5}dx+3\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^4}dx\right)+3\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3}dx\right)+\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx\right)$$

input
```
int((a+a*sec(d*x+c))^3/sec(d*x+c)^(9/2),x)
```

output

```
a**3*(int(sqrt(sec(c + d*x))/sec(c + d*x)**5,x) + 3*int(sqrt(sec(c + d*x))
/sec(c + d*x)**4,x) + 3*int(sqrt(sec(c + d*x))/sec(c + d*x)**3,x) + int(sq
rt(sec(c + d*x))/sec(c + d*x)**2,x))
```

3.186 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal result

Integrand size = 23, antiderivative size = 213

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^4\,dx &= -\frac{152a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d}\\&\quad+\frac{32a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{7d}\\&\quad+\frac{152a^4\sqrt{\sec(c+dx)}\sin(c+dx)}{15d}\\&\quad+\frac{32a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{122a^4\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{45d}\\&\quad+\frac{8a^4\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{2a^4\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d}\end{aligned}$$

output
```
-152/15*a^4*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x
+c)^(1/2)/d+32/7*a^4*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2
))*sec(d*x+c)^(1/2)/d+152/15*a^4*sec(d*x+c)^(1/2)*sin(d*x+c)/d+32/7*a^4*se
c(d*x+c)^(3/2)*sin(d*x+c)/d+122/45*a^4*sec(d*x+c)^(5/2)*sin(d*x+c)/d+8/7*a
^4*sec(d*x+c)^(7/2)*sin(d*x+c)/d+2/9*a^4*sec(d*x+c)^(9/2)*sin(d*x+c)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.01 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.36

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^4\,dx$$

$$= \frac{a^4\sec^8\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^4\left(-\frac{12i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos^4(c+dx)\left(133\left(1+e^{2i(c+dx)}\right)+133\left(-1+e^{2ic}\right)\sqrt{1+e\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^4,x]
```

output
```
(a^4*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(((-12*I)*Sqrt[2]*Sqrt[E^(I*(
c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^4*(133*(1 + E^((2*I)*(c
+ d*x))) + 133*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeome
tric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 60*E^(I*(c + d*x))*(-1 + E
^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4,
 -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (1596*Cos[
d*x]*Csc[c] + (720 + 427*Sec[c + d*x] + 180*Sec[c + d*x]^2 + 35*Sec[c + d*
x]^3)*Tan[c + d*x])/Sec[c + d*x]^(7/2)))/(2520*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^4 \sec^{\frac{11}{2}}(c+dx)+4a^4 \sec^{\frac{9}{2}}(c+dx)+6a^4 \sec^{\frac{7}{2}}(c+dx)+4a^4 \sec^{\frac{5}{2}}(c+dx)+a^4 \sec^{\frac{3}{2}}(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^4 \sin(c+dx) \sec^{\frac{9}{2}}(c+dx)}{9d}+\frac{8a^4 \sin(c+dx) \sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{122a^4 \sin(c+dx) \sec^{\frac{5}{2}}(c+dx)}{45d}+\frac{32a^4 \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{7d}+\frac{152a^4 \sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{7d}-\frac{152a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}$$

input
```
Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^4,x]
```

output
```
(-152*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(15*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(7*d) + (152*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (32*a^
4*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(7*d) + (122*a^4*Sec[c + d*x]^(5/2)*Sin
[c + d*x])/(45*d) + (8*a^4*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d) + (2*a^4
*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(184) = 368$.

Time = 8.24 (sec) , antiderivative size = 492, normalized size of antiderivative = 2.31

method	result
default	$-\frac{a^4\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{72\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^5}-\frac{61\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin}}{90\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^3}\right.}{}$
parts	Expression too large to display

input
```
int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-a^4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/72*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^5-61/90*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-304/15*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2
*c)^2)^(1/2)+1544/105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-152/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2)))-1/7*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-16/7*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.07

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^4\,dx =$$

$$-\frac{2\left(360i\sqrt{2}a^4\cos(dx+c)^4\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-360i\sqrt{2}a^4\cos(\right.}{}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
-2/315*(360*I*sqrt(2)*a^4*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) - 360*I*sqrt(2)*a^4*cos(d*x + c)^4*weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 798*I*sqrt(2)*a^4*cos(d*x +
c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c))) - 798*I*sqrt(2)*a^4*cos(d*x + c)^4*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (1596*a^4*cos(d*
x + c)^4 + 720*a^4*cos(d*x + c)^3 + 427*a^4*cos(d*x + c)^2 + 180*a^4*cos(d
*x + c) + 35*a^4)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**4,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^4\,dx = \int (a\sec(dx+c)+a)^4\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^4*sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)^4\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

input
```
int((a + a/cos(c + d*x))^4*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^4*(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^4\,dx = a^4\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^5\,dx + 4\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^4\,dx\right) + 6\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^3\,dx\right) + 4\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^2\,dx\right) + \int \sqrt{\sec(dx+c)}\,\sec(dx+c)\,dx\right)$$

input

```
int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^4,x)
```

output

```
a**4*(int(sqrt(sec(c + d*x))*sec(c + d*x)**5,x) + 4*int(sqrt(sec(c + d*x))
*sec(c + d*x)**4,x) + 6*int(sqrt(sec(c + d*x))*sec(c + d*x)**3,x) + 4*int(
sqrt(sec(c + d*x))*sec(c + d*x)**2,x) + int(sqrt(sec(c + d*x))*sec(c + d*x
),x))
```

3.187 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^4\,dx$

Optimal result

Integrand size = 23, antiderivative size = 187

$$\begin{aligned}\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^4\,dx &= -\frac{64a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}\\ &\quad+\frac{136a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d}\\ &\quad+\frac{64a^4\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{94a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d}\\ &\quad+\frac{8a^4\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2a^4\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}\end{aligned}$$

output
```
-64/5*a^4*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c
)^(1/2)/d+136/21*a^4*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2
))*sec(d*x+c)^(1/2)/d+64/5*a^4*sec(d*x+c)^(1/2)*sin(d*x+c)/d+94/21*a^4*sec
(d*x+c)^(3/2)*sin(d*x+c)/d+8/5*a^4*sec(d*x+c)^(5/2)*sin(d*x+c)/d+2/7*a^4*s
ec(d*x+c)^(7/2)*sin(d*x+c)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.17 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.49

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^4\,dx$$
$$= \frac{a^4\sec^8\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^4\left(-\frac{4i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos^4(c+dx)\left(168\left(1+e^{2i(c+dx)}\right)+168\left(-1+e^{2ic}\right)\sqrt{1+e^2}\right.}{}\right.}{}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^4,x]
```

output
```
(a^4*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(((-4*I)*Sqrt[2]*Sqrt[E^(I*(c
 + d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^4*(168*(1 + E^((2*I)*(c +
 d*x))) + 168*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 85*E^(I*(c + d*x))*(-1 + E^
((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4,
-E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (672*Cos[d*
x]*Csc[c] + (235 + 84*Sec[c + d*x] + 15*Sec[c + d*x]^2)*Tan[c + d*x])/Sec[
c + d*x]^(7/2)))/(840*d)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^4\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^4 \sec^{\frac{9}{2}}(c+dx)+4a^4 \sec^{\frac{7}{2}}(c+dx)+6a^4 \sec^{\frac{5}{2}}(c+dx)+4a^4 \sec^{\frac{3}{2}}(c+dx)+a^4 \sqrt{\sec(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^4 \sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{8a^4 \sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{94a^4 \sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{64a^4 \sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{136a^4 \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}-\frac{64a^4 \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

input
```
Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^4,x]
```

output
```
(-64*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(5*d) + (136*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(21*d) + (64*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (94*a^4*
Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (8*a^4*Sec[c + d*x]^(5/2)*Sin[c
+ d*x])/(5*d) + (2*a^4*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(162) = 324$.

Time = 6.10 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.35

method	result
default	$-\frac{a^4\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\frac{2024\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{105\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-}}{28(}\right.}{}$
parts	Expression too large to display

input
```
int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-a^4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2024/105*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-1/28*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-47/21*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2-2/5*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(c
os(1/2*d*x+1/2*c)^2-1/2)^3-128/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(
-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-64/5*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.15

$$\int\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^4\,dx =$$

$$-\frac{2\left(170i\sqrt{2}a^4\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-170i\sqrt{2}a^4\cos(\right.}{}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
-2/105*(170*I*sqrt(2)*a^4*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) - 170*I*sqrt(2)*a^4*cos(d*x + c)^3*weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 336*I*sqrt(2)*a^4*cos(d*x +
c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c))) - 336*I*sqrt(2)*a^4*cos(d*x + c)^3*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (672*a^4*cos(d*x
 + c)^3 + 235*a^4*cos(d*x + c)^2 + 84*a^4*cos(d*x + c) + 15*a^4)*sin(d*x +
 c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^4\,dx = \int (a\sec(dx+c)+a)^4\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^4*sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^4\,dx = \int (a\sec(dx+c)+a)^4\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^4*sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^4\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)^4\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

input
```
int((a + a/cos(c + d*x))^4*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^4*(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\begin{aligned}\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^4\,dx = a^4\Bigg(\int \sqrt{\sec(dx+c)}dx \\ +\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^4\,dx \\ +4\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^3\,dx\right) \\ +6\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^2\,dx\right) \\ +4\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)\,dx\right)\Bigg)\end{aligned}$$

input
```
int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^4,x)
```

output
```
a**4*(int(sqrt(sec(c + d*x)),x) + int(sqrt(sec(c + d*x))*sec(c + d*x)**4,x
) + 4*int(sqrt(sec(c + d*x))*sec(c + d*x)**3,x) + 6*int(sqrt(sec(c + d*x))
*sec(c + d*x)**2,x) + 4*int(sqrt(sec(c + d*x))*sec(c + d*x),x))
```

3.188 $\int \frac{(a+a\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 161

$$\int \frac{(a+a\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx = -\frac{56a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{32a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{66a^4\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{8a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a^4\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}$$

output
```
-56/5*a^4*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c
)^(1/2)/d+32/3*a^4*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))
*sec(d*x+c)^(1/2)/d+66/5*a^4*sec(d*x+c)^(1/2)*sin(d*x+c)/d+8/3*a^4*sec(d*x
+c)^(3/2)*sin(d*x+c)/d+2/5*a^4*sec(d*x+c)^(5/2)*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.49 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.78

$$\int \frac{(a+a\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{a^4\sec^8\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^4\left(-\frac{8i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos^4(c+dx)\left(21\left(1+e^{2i(c+dx)}\right)+21\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c}}\right.}{}\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^4/Sqrt[Sec[c + d*x]],x]
```

output
```
(a^4*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(((-8*I)*Sqrt[2]*Sqrt[E^(I*(c
 + d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^4*(21*(1 + E^((2*I)*(c +
d*x))) + 21*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 20*E^(I*(c + d*x))*(-1 + E^((
2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E
^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (-3*(-61 + 5*
Cos[2*c])*Cos[d*x]*Csc[c] + 30*Cos[c]*Sin[d*x] + 2*(20 + 3*Sec[c + d*x])*T
an[c + d*x])/Sec[c + d*x]^(7/2)))/(240*d)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^4}{\sqrt{\sec(c+dx)}}\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a \csc \left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\sqrt{\csc \left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^4 \sec^{\frac{7}{2}}(c+dx)+4a^4 \sec^{\frac{5}{2}}(c+dx)+6a^4 \sec^{\frac{3}{2}}(c+dx)+4a^4\sqrt{\sec(c+dx)}+\frac{a^4}{\sqrt{\sec(c+dx)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^4 \sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{8a^4 \sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{66a^4 \sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{56a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

input
```
Int[(a + a*Sec[c + d*x])^4/Sqrt[Sec[c + d*x]],x]
```

output
```
(-56*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(5*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(3*d) + (66*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (8*a^4*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a^4*Sec[c + d*x]^(5/2)*Sin[c + d*
x])/(5*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(140) = 280.

Time = 4.95 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.40

method	result
default	$-\frac{a^4\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\frac{328\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{15\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}-\frac{56\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}}{}\right.}{}$
parts	Expression too large to display

input

```
int((a+a*sec(d*x+c))^4/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-a^4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(328/15*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-56/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/10*cos(1/2*d*x+1/2
*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*
c)^2-1/2)^3-132/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*
x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-4/3*cos(1/2*d*x+1/2*c)*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.25

$$\int \frac{(a+a\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx = -\frac{2\left(40i\sqrt{2}a^4\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-40i\sqrt{2}a^4\cos(dx\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-2/15*(40*I*sqrt(2)*a^4*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) - 40*I*sqrt(2)*a^4*cos(d*x + c)^2*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 42*I*sqrt(2)*a^4*cos(d*x + c)^2
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c))) - 42*I*sqrt(2)*a^4*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (99*a^4*cos(d*x + c)^
2 + 20*a^4*cos(d*x + c) + 3*a^4)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d
*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a\sec{(dx+c)}+a)^4}{\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(a\sec(dx+c)+a)^4}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^4}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx=a^4\Bigg(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx+4\left(\int \sqrt{\sec(dx+c)}dx\right)\\+\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^3\,dx\\+4\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^2\,dx\right)\\+6\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)\,dx\right)\Bigg)\end{aligned}$$

input
```
int((a+a*sec(d*x+c))^4/sec(d*x+c)^(1/2),x)
```

output
```
a**4*(int(sqrt(sec(c + d*x))/sec(c + d*x),x) + 4*int(sqrt(sec(c + d*x)),x)
 + int(sqrt(sec(c + d*x))*sec(c + d*x)**3,x) + 4*int(sqrt(sec(c + d*x))*se
c(c + d*x)**2,x) + 6*int(sqrt(sec(c + d*x))*sec(c + d*x),x))
```

3.189 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 118

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{40a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{8a^4\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}$$

output

```
40/3*a^4*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c
)^(1/2)/d+2/3*a^4*sin(d*x+c)/d/sec(d*x+c)^(1/2)+8*a^4*sec(d*x+c)^(1/2)*sin
(d*x+c)/d+2/3*a^4*sec(d*x+c)^(3/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 2.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.59

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{a^4\sec^{\frac{3}{2}}(c+dx)\left(80\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+5\sin(c+dx)+24\sin(2(c+dx))+\sin(3(c-\right.}{6d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(3/2),x]
```

output
```
(a^4*Sec[c + d*x]^(3/2)*(80*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] +
 5*Sin[c + d*x] + 24*Sin[2*(c + d*x)] + Sin[3*(c + d*x)]))/(6*d)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$
$$\Big\downarrow \; 4278$$
$$\int \left(a^4\sec^{\frac{5}{2}}(c+dx)+4a^4\sec^{\frac{3}{2}}(c+dx)+\frac{a^4}{\sec^{\frac{3}{2}}(c+dx)}+6a^4\sqrt{\sec(c+dx)}+\frac{4a^4}{\sqrt{\sec(c+dx)}}\right)dx$$
$$\Big\downarrow \; 2009$$

$$\frac{2a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{8a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{40a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}$$

input
```
Int[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(3/2),x]
```

output
```
(40*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (2*a^4*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (8*a^4*Sqrt[Sec[c +
 d*x]]*Sin[c + d*x])/d + (2*a^4*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(103) = 206$.

Time = 4.16 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.47

method	result
default	$\frac{8a^4\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-14\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2s\cdots}\right.}{3\left(4\sin\cdots\right.}$
parts	Expression too large to display

input
```
int((a+a*sec(d*x+c))^4/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
8/3*a^4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1
/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+7*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^2-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2\left(10i\sqrt{2}a^4\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-10i\sqrt{2}a^4\cos(dx\right.}{3\,d\cos}$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(10*I*sqrt(2)*a^4*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) - 10*I*sqrt(2)*a^4*cos(d*x + c)*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) - (a^4*cos(d*x + c)^2 + 12*a^4*cos(d*x
 + c) + a^4)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{(a + a\sec(c + dx))^4}{\sec^{\frac{3}{2}}(c + dx)}\,dx = a^4\left(\int \frac{1}{\sec^{\frac{3}{2}}(c + dx)}\,dx + \int \frac{4}{\sqrt{\sec(c + dx)}}\,dx + \int 6\sqrt{\sec(c + dx)}\,dx + \int 4\sec^{\frac{3}{2}}(c + dx)\,dx + \int \sec^{\frac{5}{2}}(c + dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(3/2),x)
```

output
```
a**4*(Integral(sec(c + d*x)**(-3/2), x) + Integral(4/sqrt(sec(c + d*x)), x
) + Integral(6*sqrt(sec(c + d*x)), x) + Integral(4*sec(c + d*x)**(3/2), x)
 + Integral(sec(c + d*x)**(5/2), x))
```

Maxima [F]

$$\int \frac{(a + a\sec(c + dx))^4}{\sec^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{(a\sec(dx + c) + a)^4}{\sec(dx + c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx=a^4\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx+4\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)+6\left(\int \sqrt{\sec(dx+c)}dx\right)+\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^2\,dx+4\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)\,dx\right)\right)$$

input
```
int((a+a*sec(d*x+c))^4/sec(d*x+c)^(3/2),x)
```

output
```
a**4*(int(sqrt(sec(c + d*x))/sec(c + d*x)**2,x) + 4*int(sqrt(sec(c + d*x))
/sec(c + d*x),x) + 6*int(sqrt(sec(c + d*x)),x) + int(sqrt(sec(c + d*x))*se
c(c + d*x)**2,x) + 4*int(sqrt(sec(c + d*x))*sec(c + d*x),x))
```

3.190 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 159

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{56a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{32a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2a^4\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
56/5*a^4*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)
^(1/2)/d+32/3*a^4*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*
sec(d*x+c)^(1/2)/d+2/5*a^4*sin(d*x+c)/d/sec(d*x+c)^(3/2)+8/3*a^4*sin(d*x+c
)/d/sec(d*x+c)^(1/2)+2*a^4*sec(d*x+c)^(1/2)*sin(d*x+c)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.22 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.16

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{a^4\left(\cos\left(\frac{c}{2}\right)-i\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+i\sin\left(\frac{c}{2}\right)\right)\left(-336i+\frac{672i\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-320i\sqrt{1+e^{2}}\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(5/2),x]
```

output
```
(a^4*(Cos[c/2] - I*Sin[c/2])*(Cos[c/2] + I*Sin[c/2])*(-336*I + ((672*I)*Hy
pergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(
c + d*x))] - (320*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4,
1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 80*Sin[c + d*x] + 3*Sec[c +
 d*x]*Sin[3*(c + d*x)] + 63*Tan[c + d*x]))/(30*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$
$$\Big\downarrow \; 4278$$

$$\int \left(a^4 \sec^{\frac{3}{2}}(c+dx) + \frac{4a^4}{\sec^{\frac{3}{2}}(c+dx)} + \frac{a^4}{\sec^{\frac{5}{2}}(c+dx)} + 4a^4\sqrt{\sec(c+dx)} + \frac{6a^4}{\sqrt{\sec(c+dx)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{d} + \frac{8a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{56a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

input
```
Int[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(5/2),x]
```

output
```
(56*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(3*d) + (2*a^4*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (8*a^4*Sin[c
 + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [A] (verified)

Time = 5.97 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.22

method	result
default	$\frac{8a^4\left(6\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-26\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+19\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-20\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}\right.}{15\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}}$
parts	Expression too large to display

input
```
int((a+a*sec(d*x+c))^4/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
8/15*a^4*(6*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-26*sin(1/2*d*x+1/2*c)^
4*cos(1/2*d*x+1/2*c)+19*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-20*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.02

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx =$$
$$-\frac{2\left(40i\sqrt{2}a^4\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-40i\sqrt{2}a^4\text{weierstrassPInverse}(\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-2/15*(40*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) - 40*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c)) - 42*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c))) + 42*I*sqrt(2)*a^4*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*a^4*
cos(d*x + c)^2 + 20*a^4*cos(d*x + c) + 15*a^4)*sin(d*x + c)/sqrt(cos(d*x +
 c)))/d
```

Sympy [F]

$$\int \frac{(a + a\sec(c + dx))^4}{\sec^{\frac{5}{2}}(c + dx)}\,dx = a^4\left(\int \frac{1}{\sec^{\frac{5}{2}}(c + dx)}\,dx + \int \frac{4}{\sec^{\frac{3}{2}}(c + dx)}\,dx + \int \frac{6}{\sqrt{\sec(c + dx)}}\,dx + \int 4\sqrt{\sec(c + dx)}\,dx + \int \sec^{\frac{3}{2}}(c + dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(5/2),x)
```

output
```
a**4*(Integral(sec(c + d*x)**(-5/2), x) + Integral(4/sec(c + d*x)**(3/2),
x) + Integral(6/sqrt(sec(c + d*x)), x) + Integral(4*sqrt(sec(c + d*x)), x)
 + Integral(sec(c + d*x)**(3/2), x))
```

Maxima [F]

$$\int \frac{(a + a\sec(c + dx))^4}{\sec^{\frac{5}{2}}(c + dx)}\,dx = \int \frac{(a\sec(dx + c) + a)^4}{\sec(dx + c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a + a\sec(c + dx))^4}{\sec^{\frac{5}{2}}(c + dx)}\, dx = \int \frac{(a\sec(dx + c) + a)^4}{\sec(dx + c)^{\frac{5}{2}}}\, dx$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(c + dx))^4}{\sec^{\frac{5}{2}}(c + dx)}\, dx = \int \frac{\left(a + \frac{a}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\, dx$$

input
```
int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{(a + a\sec(c + dx))^4}{\sec^{\frac{5}{2}}(c + dx)}\, dx = a^4 \Bigg(\int \frac{\sqrt{\sec(dx + c)}}{\sec(dx + c)^3} dx + 4 \left(\int \frac{\sqrt{\sec(dx + c)}}{\sec(dx + c)^2} dx \right) \\ + 6 \left(\int \frac{\sqrt{\sec(dx + c)}}{\sec(dx + c)} dx \right) + 4 \left(\int \sqrt{\sec(dx + c)} dx \right) \\ + \int \sqrt{\sec(dx + c)}\, \sec(dx + c)\, dx \Bigg)\end{aligned}$$

input
```
int((a+a*sec(d*x+c))^4/sec(d*x+c)^(5/2),x)
```

output
```
a**4*(int(sqrt(sec(c + d*x))/sec(c + d*x)**3,x) + 4*int(sqrt(sec(c + d*x))
/sec(c + d*x)**2,x) + 6*int(sqrt(sec(c + d*x))/sec(c + d*x),x) + 4*int(sqr
t(sec(c + d*x)),x) + int(sqrt(sec(c + d*x))*sec(c + d*x),x))
```

3.191 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 161

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{64a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{136a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} + \frac{2a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{94a^4\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}$$

output

```
64/5*a^4*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)
^(1/2)/d+136/21*a^4*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)
)*sec(d*x+c)^(1/2)/d+2/7*a^4*sin(d*x+c)/d/sec(d*x+c)^(5/2)+8/5*a^4*sin(d*x
+c)/d/sec(d*x+c)^(3/2)+94/21*a^4*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.14 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.12

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{a^4\left(\cos\left(\frac{c}{2}\right)-i\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+i\sin\left(\frac{c}{2}\right)\right)\left(-5376i+\frac{10752i\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-2720i\sqrt{1}\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(7/2),x]
```

output
```
(a^4*(Cos[c/2] - I*Sin[c/2])*(Cos[c/2] + I*Sin[c/2])*(-5376*I + ((10752*I)
*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I
)*(c + d*x))] - (2720*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1
/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 1910*Sin[c + d*x] + 336
*Sin[2*(c + d*x)] + 30*Sin[3*(c + d*x)]))/(420*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$
$$\Big\downarrow \text{4278}$$

$$\int \left(\frac{6a^4}{\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^4}{\sec^{\frac{5}{2}}(c+dx)}+\frac{a^4}{\sec^{\frac{7}{2}}(c+dx)}+a^4\sqrt{\sec(c+dx)}+\frac{4a^4}{\sqrt{\sec(c+dx)}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{8a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{94a^4\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{136a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{64a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

input
```
Int[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(7/2),x]
```

output
```
(64*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (136*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(21*d) + (2*a^4*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (8*a^4*Sin
[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (94*a^4*Sin[c + d*x])/(21*d*Sqrt[Sec
[c + d*x]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [A] (verified)

Time = 7.00 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.69

method	result
default	$-\frac{8\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, a^{4}\left(60\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-258\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+448\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\right.\right.}{105\sqrt{-2}}$
parts	Expression too large to display

input
```
int((a+a*sec(d*x+c))^4/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-8/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(60*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-258*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+
1/2*c)+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-167*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^2+85*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-168*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx =$$

$$-\frac{2\left(170i\sqrt{2}a^{4}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-170i\sqrt{2}a^{4}\text{weierstrassPInvers}\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-2/105*(170*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) - 170*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*
sin(d*x + c)) - 336*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 336*I*sqrt(2)*a^4*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (1
5*a^4*cos(d*x + c)^3 + 84*a^4*cos(d*x + c)^2 + 235*a^4*cos(d*x + c))*sin(d
*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx = a^4\left(\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + \int \frac{4}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \int \frac{6}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{4}{\sqrt{\sec(c+dx)}}\,dx + \int \sqrt{\sec(c+dx)}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(7/2),x)
```

output
```
a**4*(Integral(sec(c + d*x)**(-7/2), x) + Integral(4/sec(c + d*x)**(5/2),
x) + Integral(6/sec(c + d*x)**(3/2), x) + Integral(4/sqrt(sec(c + d*x)), x
) + Integral(sqrt(sec(c + d*x)), x))
```

Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(7/2),x)
```

output
```
int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx = a^4\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^4}dx + 4\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3}dx\right) + 6\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx\right) + 4\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right) + \int \sqrt{\sec(dx+c)}dx\right)$$

input
```
int((a+a*sec(d*x+c))^4/sec(d*x+c)^(7/2),x)
```

output
```
a**4*(int(sqrt(sec(c + d*x))/sec(c + d*x)**4,x) + 4*int(sqrt(sec(c + d*x))
/sec(c + d*x)**3,x) + 6*int(sqrt(sec(c + d*x))/sec(c + d*x)**2,x) + 4*int(
sqrt(sec(c + d*x))/sec(c + d*x),x) + int(sqrt(sec(c + d*x)),x))
```

3.192 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 187

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{152a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{32a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{7d} + \frac{2a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{122a^4\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{32a^4\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}$$

output
```
152/15*a^4*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+
c)^(1/2)/d+32/7*a^4*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)
)*sec(d*x+c)^(1/2)/d+2/9*a^4*sin(d*x+c)/d/sec(d*x+c)^(7/2)+8/7*a^4*sin(d*x
+c)/d/sec(d*x+c)^(5/2)+122/45*a^4*sin(d*x+c)/d/sec(d*x+c)^(3/2)+32/7*a^4*s
in(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.36 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.83

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{a^4\left(-25536i+\frac{51072i\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-11520i\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4}\right.\right.}{2520d\sqrt{}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(9/2),x]
```

output
```
(a^4*(-25536*I + ((51072*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c
 + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (11520*I)*Sqrt[1 + E^((2*I)*(c
+ d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*
x] + 12240*Sin[c + d*x] + 3556*Sin[2*(c + d*x)] + 720*Sin[3*(c + d*x)] + 7
0*Sin[4*(c + d*x)]))/(2520*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \ 4278$$

$$\int \left(\frac{4a^4}{\sec^{\frac{3}{2}}(c+dx)}+\frac{6a^4}{\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^4}{\sec^{\frac{7}{2}}(c+dx)}+\frac{a^4}{\sec^{\frac{9}{2}}(c+dx)}+\frac{a^4}{\sqrt{\sec(c+dx)}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{122a^4\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{32a^4\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{7d}+\frac{152a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}$$

input
```
Int[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(9/2),x]
```

output
```
(152*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(15*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(7*d) + (2*a^4*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (8*a^4*Sin
[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (122*a^4*Sin[c + d*x])/(45*d*Sec[c +
 d*x]^(3/2)) + (32*a^4*Sin[c + d*x])/(7*d*Sqrt[Sec[c + d*x]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [A] (verified)

Time = 15.52 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.39

method	result
default	$-\frac{8\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a^{4}\left(280\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}-120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}+34\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+72\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-485\cos\left(\right.\right.}{315\sqrt{-2\sin\left(\right.}}$
parts	Expression too large to display

input
```
int((a+a*sec(d*x+c))^4/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-8/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(280*co
s(1/2*d*x+1/2*c)^11-120*cos(1/2*d*x+1/2*c)^9+34*cos(1/2*d*x+1/2*c)^7+72*co
s(1/2*d*x+1/2*c)^5-485*cos(1/2*d*x+1/2*c)^3+180*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-399*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+219*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.98

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx =$$

$$-\frac{2\left(360i\sqrt{2}a^4\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-360i\sqrt{2}a^4\text{weierstrassPInvers}\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
-2/315*(360*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) - 360*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*
sin(d*x + c)) - 798*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 798*I*sqrt(2)*a^4*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3
5*a^4*cos(d*x + c)^4 + 180*a^4*cos(d*x + c)^3 + 427*a^4*cos(d*x + c)^2 + 7
20*a^4*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(9/2), x)
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(9/2),x)
```

output
```
int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(9/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx=a^4\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^5}dx+4\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^4}dx\right)+6\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3}dx\right)+4\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx\right)+\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)$$

input
```
int((a+a*sec(d*x+c))^4/sec(d*x+c)^(9/2),x)
```

output
```
a**4*(int(sqrt(sec(c + d*x))/sec(c + d*x)**5,x) + 4*int(sqrt(sec(c + d*x))
/sec(c + d*x)**4,x) + 6*int(sqrt(sec(c + d*x))/sec(c + d*x)**3,x) + 4*int(
sqrt(sec(c + d*x))/sec(c + d*x)**2,x) + int(sqrt(sec(c + d*x))/sec(c + d*x
),x))
```

3.193 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 213

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{128a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{904a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{231d} + \frac{2a^4\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{150a^4\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)} + \frac{128a^4\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{904a^4\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}$$

output
```
128/15*a^4*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+
c)^(1/2)/d+904/231*a^4*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1
/2))*sec(d*x+c)^(1/2)/d+2/11*a^4*sin(d*x+c)/d/sec(d*x+c)^(9/2)+8/9*a^4*sin
(d*x+c)/d/sec(d*x+c)^(7/2)+150/77*a^4*sin(d*x+c)/d/sec(d*x+c)^(5/2)+128/45
*a^4*sin(d*x+c)/d/sec(d*x+c)^(3/2)+904/231*a^4*sin(d*x+c)/d/sec(d*x+c)^(1/
2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.98 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.44

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx =$$
$$-\frac{ia^4e^{-6i(c+dx)}\left(-315-3080e^{i(c+dx)}-14760e^{2i(c+dx)}-48664e^{3i(c+dx)}-137055e^{4i(c+dx)}+427504e^{5i(c+d}\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(11/2),x]
```

output
```
((-1/1774080*I)*a^4*(-315 - 3080*E^(I*(c + d*x)) - 14760*E^((2*I)*(c + d*x
)) - 48664*E^((3*I)*(c + d*x)) - 137055*E^((4*I)*(c + d*x)) + 427504*E^((5
*I)*(c + d*x)) + 518672*E^((7*I)*(c + d*x)) + 137055*E^((8*I)*(c + d*x)) +
 48664*E^((9*I)*(c + d*x)) + 14760*E^((10*I)*(c + d*x)) + 3080*E^((11*I)*(
c + d*x)) + 315*E^((12*I)*(c + d*x)) - 946176*E^((5*I)*(c + d*x))*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x
))] + 433920*E^((6*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeomet
ric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[(c + d*x)/2]^8*(1 + Sec[c
 + d*x])^4)/(d*E^((6*I)*(c + d*x))*Sec[c + d*x]^(7/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^4}{\sec^{\frac{11}{2}}(c+dx)}dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a \csc \left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\csc \left(c+dx+\frac{\pi}{2}\right)^{11/2}}dx$$
$$\Big\downarrow \text{4278}$$
$$\int \left(\frac{a^4}{\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^4}{\sec^{\frac{5}{2}}(c+dx)}+\frac{6a^4}{\sec^{\frac{7}{2}}(c+dx)}+\frac{4a^4}{\sec^{\frac{9}{2}}(c+dx)}+\frac{a^4}{\sec^{\frac{11}{2}}(c+dx)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{128a^4 \sin(c+dx)}{45d \sec^{\frac{3}{2}}(c+dx)}+\frac{150a^4 \sin(c+dx)}{77d \sec^{\frac{5}{2}}(c+dx)}+\frac{8a^4 \sin(c+dx)}{9d \sec^{\frac{7}{2}}(c+dx)}+\frac{2a^4 \sin(c+dx)}{11d \sec^{\frac{9}{2}}(c+dx)}+\frac{904a^4 \sin(c+dx)}{231d \sqrt{\sec(c+dx)}}+\frac{904a^4 \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d}+\frac{128a^4 \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}$$

input
```
Int[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(11/2),x]
```

output
```
(128*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(15*d) + (904*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(231*d) + (2*a^4*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9/2)) + (8*a^4
*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (150*a^4*Sin[c + d*x])/(77*d*Sec
[c + d*x]^(5/2)) + (128*a^4*Sin[c + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (904
*a^4*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Maple [A] (verified)

Time = 22.15 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.28

method	result
default	$-\frac{8\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,a^4\left(5040\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}-5320\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}+1740\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^9+326\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^7+6\right.}{}$
parts	Expression too large to display

input
```
int((a+a*sec(d*x+c))^4/sec(d*x+c)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-8/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(5040*
cos(1/2*d*x+1/2*c)^13-5320*cos(1/2*d*x+1/2*c)^11+1740*cos(1/2*d*x+1/2*c)^9
+326*cos(1/2*d*x+1/2*c)^7+678*cos(1/2*d*x+1/2*c)^5-4465*cos(1/2*d*x+1/2*c)
^3+1695*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3696*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2001*co
s(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.92

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx =$$

$$-\frac{2\left(3390i\sqrt{2}a^4\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-3390i\sqrt{2}a^4\text{weierstrassPInv}\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="fricas")
```

output
```
-2/3465*(3390*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) - 3390*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c)) - 7392*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 7392*I*sqrt(2)*a^4*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))
 - (315*a^4*cos(d*x + c)^5 + 1540*a^4*cos(d*x + c)^4 + 3375*a^4*cos(d*x +
c)^3 + 4928*a^4*cos(d*x + c)^2 + 6780*a^4*cos(d*x + c))*sin(d*x + c)/sqrt(
cos(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(11/2), x)
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{11/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(11/2),x)
```

output
```
int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(11/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx = a^4\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^6}dx+4\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^5}dx\right)+6\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^4}dx\right)+4\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3}dx\right)+\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx\right)$$

input
```
int((a+a*sec(d*x+c))^4/sec(d*x+c)^(11/2),x)
```

output

```
a**4*(int(sqrt(sec(c + d*x))/sec(c + d*x)**6,x) + 4*int(sqrt(sec(c + d*x))
/sec(c + d*x)**5,x) + 6*int(sqrt(sec(c + d*x))/sec(c + d*x)**4,x) + 4*int(
sqrt(sec(c + d*x))/sec(c + d*x)**3,x) + int(sqrt(sec(c + d*x))/sec(c + d*x
)**2,x))
```

3.194 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 164

$$\begin{aligned}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx &= \frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} \\ &\quad+\frac{5\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3ad} \\ &\quad-\frac{3\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} \\ &\quad+\frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad}-\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}\end{aligned}$$

output
```
3*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)/
a/d+5/3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)
^(1/2)/a/d-3*sec(d*x+c)^(1/2)*sin(d*x+c)/a/d+5/3*sec(d*x+c)^(3/2)*sin(d*x+
c)/a/d-sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.06 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.77

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(9\left(1+e^{2i(c+dx)}\right)+9\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4}\right.\right.}{-1+e}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x]),x]
```

output
```
(Cos[(c + d*x)/2]^2*Sec[c + d*x]*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 +
 E^((2*I)*(c + d*x)))]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 + E^((2*I)*c))
*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)
*(c + d*x))] - 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c
+ d*x))*(-1 + E^((2*I)*c))) - Sqrt[Sec[c + d*x]]*(18*Cos[d*x]*Csc[c] + Sec
[c + d*x]*(-5*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + Tan[(c + d*x)/2]))))
/(3*a*d*(1 + Sec[c + d*x]))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4305, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a\sec(c+dx)+a}\,dx$$
$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \; 4305$$

$$-\frac{\int \frac{1}{2}\sec^{\frac{3}{2}}(c+dx)(3a-5a\sec(c+dx))dx}{a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \sec^{\frac{3}{2}}(c+dx)(3a-5a\sec(c+dx))dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3a-5a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \; 4274$$

$$-\frac{3a\int \sec^{\frac{3}{2}}(c+dx)dx-5a\int \sec^{\frac{5}{2}}(c+dx)dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx-5a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \; 4255$$

$$-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)-5a\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-5a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \; 4258$$

$$-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)-5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{2a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$\downarrow$ 3042

$$-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)-5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{2a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$\downarrow$ 3119

$$-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)-5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx+}}\right.}{2a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$\downarrow$ 3120

$$-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)-5a\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{3a}\right.}{2a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

input
```
Int[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x]),x]
```

output
```
-((Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))) - (3*a*((-2*
Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/d) - 5*a*((2*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d
*x])/(3*d)))/(2*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4305
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Simp[d^2/(a*b)   Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
 - 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ
[a^2 - b^2, 0] && GtQ[n, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(149) = 298.

Time = 3.76 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.52

method	result
default	$\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(10\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\mathrm{EllipticF}\left(\cos\left(\right.\right.\right.}{}$

input
```
int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a/cos(1/2*d*
x+1/2*c)/sin(1/2*d*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)
^2+1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(10*cos(1/2*d*x
+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-18*cos(1/2*d*x+1/2*
c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-36*sin(1/2*d*x+1/2*c)^6-
5*cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)+9*cos(1/2*d*x+1/2*c)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*
d*x+1/2*c)^2)^(1/2)+44*sin(1/2*d*x+1/2*c)^4-11*sin(1/2*d*x+1/2*c)^2)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.51

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx =$$

$$-\frac{5\left(i\sqrt{2}\cos\left(dx+c\right)^{2}+i\sqrt{2}\cos\left(dx+c\right)\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+5}{}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/6*(5*(I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassPIn
verse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^2
 - I*sqrt(2)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c)) + 9*(-I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(2)*cos(d*x + c))*weiers
trassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
) + 9*(I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*c
os(d*x + c)^2 + 4*cos(d*x + c) - 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*
cos(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)/(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{7}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \int \frac{\sec(dx+c)^{\frac{7}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^3}{\sec(dx+c)+1}dx}{a}$$

input
```
int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x)
```

output
```
int((sqrt(sec(c + d*x))*sec(c + d*x)**3)/(sec(c + d*x) + 1),x)/a
```

3.195 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 136

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{3\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}$$

output
```
-3*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)
/a/d-cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1
/2)/a/d+3*sec(d*x+c)^(1/2)*sin(d*x+c)/a/d-sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a
+a*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.06 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.93

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$=\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3\left(1+e^{2i(c+dx)}\right)+3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\cdots\right.\right.}{d(-1\cdots}\right.}{a(1+\cdots}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x]),x]
```

output
```
(Cos[(c + d*x)/2]^2*Sec[c + d*x]*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1
+ E^((2*I)*(c + d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c)
)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I
)*(c + d*x))] - E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c +
d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*E^(I*(c
 + d*x))*(-1 + E^((2*I)*c))) + (Sqrt[Sec[c + d*x]]*(6*Cos[d*x]*Csc[c] - 2*
Tan[(c + d*x)/2]))/d))/(a*(1 + Sec[c + d*x]))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {3042, 4305, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a\sec(c+dx)+a}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4305}$$

$$-\frac{\int \frac{1}{2}\sqrt{\sec(c+dx)}(a-3a\sec(c+dx))dx}{a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \sqrt{\sec(c+dx)}(a-3a\sec(c+dx))dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4274}$$

$$-\frac{a\int \sqrt{\sec(c+dx)}dx-3a\int \sec^{\frac{3}{2}}(c+dx)dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-3a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4255}$$

$$-\frac{a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4258}$$

$$-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c\right.}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow 3042$$

$$-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin}\right.}{2a^2}$$
$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow 3119$$

$$-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)}{2a^2}$$
$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow 3120$$

$$-\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)}{2a^2}-$$
$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

input
```
Int[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x]),x]
```

output
```
-((Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))) - ((2*a*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - 3*a*((-2*
Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/d))/(2*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4305
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Simp[d^2/(a*b)   Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
 - 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ
[a^2 - b^2, 0] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.86

method	result
default	$-\frac{-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\left(\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-3\,\mathrm{EllipticE}\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\right.}}$

input
```
int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-(-cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+6*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-5*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/
a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.44

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\left(i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+\left(-i\sqrt{2}\cos\left(dx+c\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + (-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(I*sqrt(2)*cos(d*x +
 c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c))) - 3*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))
 + 2*(3*cos(d*x + c) + 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x +
c) + a*d)
```

Sympy [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**(5/2)/(sec(c + d*x) + 1), x)/a
```

Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \int \frac{\sec(dx+c)^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx=\frac{\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^{2}}{\sec(dx+c)+1}dx}{a}$$

input
```
int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x)
```

output
```
int((sqrt(sec(c + d*x))*sec(c + d*x)**2)/(sec(c + d*x) + 1),x)/a
```

3.196 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 110

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))}$$

output
```
cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)/a/
d+cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)
/a/d-sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.28 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.83

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{2ie^{-i(c+dx)}\cos^2\left(\frac{1}{2}(c+dx)\right)\left(1+e^{2i(c+dx)}-\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4}\right.\right.}{ad\left(1+e^{i(c+dx)}\right.}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x]),x]
```

output
```
((-2*I)*Cos[(c + d*x)/2]^2*(1 + E^((2*I)*(c + d*x)) - (1 + E^(I*(c + d*x))
)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I
)*(c + d*x))] + E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c
 + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c +
d*x]^(3/2))/(a*d*E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*(1 + Sec[c + d*x]))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4305, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4305}$$

$$-\frac{\int -\frac{\sec(c+dx)a+a}{2\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sec(c+dx)a+a}{\sqrt{\sec(c+dx)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4274}$$

$$\frac{a\int \frac{1}{\sqrt{\sec(c+dx)}}dx+a\int \sqrt{\sec(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

input
```
Int[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x]),x]
```

output
```
((2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/
(2*a^2) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4305
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Simp[d^2/(a*b)   Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
 - 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ
[a^2 - b^2, 0] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.82

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-\text{Elliptic}\right.\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}$

input
```
int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*
c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin
(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.67

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{\left(-i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\left(i\sqrt{2}\cos\left(dx+c\right)\right.}{}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c
) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c))) + (-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2
*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**(3/2)/(sec(c + d*x) + 1), x)/a
```

Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)}{\sec(dx+c)+1}dx}{a}$$

input

```
int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x)
```

output

```
int((sqrt(sec(c + d*x))*sec(c + d*x))/(sec(c + d*x) + 1),x)/a
```

3.197 $\int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 110

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx = -\frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))}$$

output
```
-cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)/a
/d+cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2
)/a/d+sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.02 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx = -\frac{2ie^{-i(c+dx)}\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-1-e^{2i(c+dx)}+\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2}\right.\right.}{ad\left(1+e^{i(c+dx}\right.}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

output
```
((-2*I)*Cos[(c + d*x)/2]^2*(-1 - E^((2*I)*(c + d*x)) + (1 + E^(I*(c + d*x)
))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*
I)*(c + d*x))] + E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(
c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c +
 d*x]^(3/2))/(a*d*E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*(1 + Sec[c + d*x])
)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4307, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4307}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int \frac{a-a\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int \frac{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}$$

$$\Big\downarrow \; 4274$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{a\int \frac{1}{\sqrt{\sec(c+dx)}}dx-a\int \sqrt{\sec(c+dx)}dx}{2a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}$$

$$\Big\downarrow \; 4258$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{2a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}$$

$$\Big\downarrow \; 3119$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}$$

$$\Big\downarrow \; 3120$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{2a^2}$$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

output
```
-1/2*((2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/d - (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/d)/a^2 + (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(a*f*
(a + b*Csc[e + f*x]))), x] + Simp[d*((n - 1)/(a*b))   Int[(d*Csc[e + f*x])^
(n - 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ
[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.80

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+\text{Elliptic}\right.\right.}{a\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}}$

input
```
int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*
c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin
(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.67

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx$$
$$= \frac{\left(-i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\left(i\sqrt{2}\cos\left(dx+c\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + (-I*sqrt(2)*cos(d*x +
c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c))) + (I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2
*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sqrt{\sec(c+dx)}}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(sqrt(sec(c + d*x))/(sec(c + d*x) + 1), x)/a
```

Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}}{a\sec\left(dx+c\right)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)+1}dx}{a}$$

input
```
int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x)
```

output
```
int(sqrt(sec(c + d*x))/(sec(c + d*x) + 1),x)/a
```

3.198 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 112

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx = \frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))}$$

output
```
3*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)/
a/d-cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/
2)/a/d-sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.66 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.83

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx$$
$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3(1+e^{2i(c+dx)})+3(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)\right.}{d(-1+e^{2ic})}\right.}{}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

output
```
(Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
+ d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))]
+ E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyperge
ometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*E^(I*(c + d*x))*(-1 +
 E^((2*I)*c))) - ((Cos[(c - d*x)/2] + 2*Cos[(3*c + d*x)/2] + 2*Cos[(c + 3*
d*x)/2] + Cos[(5*c + 3*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]*Sqrt[Se
c[c + d*x]])/(2*d))*Sec[c + d*x])/(a*(1 + Sec[c + d*x]))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4306, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$
$$\Big\downarrow \; 4306$$

$$-\frac{\int -\frac{3a-a\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3a-a\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4274}$$

$$\frac{3a\int \frac{1}{\sqrt{\sec(c+dx)}}dx-a\int \sqrt{\sec(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\downarrow \text{3120}$$

$$\frac{\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

output
```
((6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d -
 (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/
(2*a^2) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.78

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\left(\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+3\,\mathrm{EllipticE}\right.\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-}}$

input
```
int(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/a*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1
/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+
2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.66

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx$$
$$= \frac{\left(i\sqrt{2}\cos{(dx+c)}+i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin{(dx+c)})+\left(-i\sqrt{2}\cos{(dx+c)}\right.}{}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + (-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(-I*sqrt(2)*cos(d*x
+ c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c))) - 3*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))
 - 2*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx}{a}$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(1/(sec(c + d*x)**(3/2) + sqrt(sec(c + d*x))), x)/a
```

Maxima [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2+\sec(dx+c)}dx}{a}$$

input
```
int(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x)
```

output
```
int(sqrt(sec(c + d*x))/(sec(c + d*x)**2 + sec(c + d*x)),x)/a
```

3.199 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 140

$$\begin{aligned}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx &= -\frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}\\ &\quad+\frac{5\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3ad}\\ &\quad+\frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\end{aligned}$$

output
```
-3*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)
/a/d+5/3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c
)^(1/2)/a/d+5/3*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)-sin(d*x+c)/d/sec(d*x+c)^(1
/2)/(a+a*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.44 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.27

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$$
$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(9\left(1+e^{2i(c+dx)}\right)+9\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}(\cdots\right.}{-1\cdots}\right.}{\cdots}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
```

output
```
(Cos[(c + d*x)/2]^2*Sec[c + d*x]*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1
+ E^((2*I)*(c + d*x)))]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 + E^((2*I)*c)
)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I
)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c
+ d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c
 + d*x))*(-1 + E^((2*I)*c))) + 2*Sqrt[Sec[c + d*x]]*(3*(2 + Cos[2*c])*Cos[
d*x]*Csc[c] + Cos[2*d*x]*Sin[2*c] - 3*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/
2] - 6*Cos[c]*Sin[d*x] + Cos[2*c]*Sin[2*d*x] - 3*Tan[c/2])))/(3*a*d*(1 + S
ec[c + d*x]))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4306, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$\Big\downarrow$ 4306

$$-\frac{\int -\frac{5a-3a\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{5a-3a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{5a-3a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 4274

$$\frac{5a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx-3a\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 3042

$$\frac{5a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx-3a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 4256

$$\frac{5a\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-3a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 3042

$$\frac{5a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-3a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 4258

$$\frac{5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx}}{2a^2}$$
$$\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 3042

$$\frac{5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c\right.}}{2a^2}$$
$$\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 3119

$$\frac{5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a^2}-$$
$$\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 3120

$$\frac{5a\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)-\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a^2}-$$
$$\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

input
```
Int[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
```

output
```
-(Sin[c + d*x]/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]))) + ((-6*a*Sqrt[
Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + 5*a*((2*Sq
rt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*
Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/(2*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.54

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\left(5\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+9\,\mathrm{Elli}\right.\right.}{3a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\mathrm{c}}}$

input
```
int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*
x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(5*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2)))-8*sin(1/2*d*x+1/2*c)^6+18*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2
)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx =$$

$$-\frac{5\left(i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+5\left(-i\sqrt{2}\cos\left(\right.\right.}{}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/6*(5*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, co
s(d*x + c) + I*sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*(I*sqrt(2)*cos(
d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) + I*sin(d*x + c))) + 9*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c))) - 2*(2*cos(d*x + c)^2 + 5*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x +
c)))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \frac{\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)+\sec^{\frac{3}{2}}(c+dx)}\,dx}{a}$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(1/(sec(c + d*x)**(5/2) + sec(c + d*x)**(3/2)), x)/a
```

Maxima [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3+\sec(dx+c)^2}dx}{a}$$

input
```
int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x)
```

output
```
int(sqrt(sec(c + d*x))/(sec(c + d*x)**3 + sec(c + d*x)**2),x)/a
```

3.200 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 168

$$\begin{aligned}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx &= \frac{21\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} \\ &\quad - \frac{5\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3ad} + \frac{7\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} \\ &\quad - \frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\end{aligned}$$

output
```
21/5*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/
2)/a/d-5/3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x
+c)^(1/2)/a/d+7/5*sin(d*x+c)/a/d/sec(d*x+c)^(3/2)-5/3*sin(d*x+c)/a/d/sec(d
*x+c)^(1/2)-sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.51 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.07

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$$
$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\frac{8i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(63\left(1+e^{2i(c+dx)}\right)+63\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\right.\right.}{-1+}\right.}{}$$

input
```
Integrate[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```

output
```
(Cos[(c + d*x)/2]^2*Sec[c + d*x]*(((8*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 +
 E^((2*I)*(c + d*x)))]*(63*(1 + E^((2*I)*(c + d*x))) + 63*(-1 + E^((2*I)*c
))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*
I)*(c + d*x))] + 25*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(
c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*
(c + d*x))*(-1 + E^((2*I)*c))) - Sqrt[Sec[c + d*x]]*(18*(17 + 11*Cos[2*c])
*Cos[d*x]*Csc[c] + 4*(10*Cos[2*d*x]*Sin[2*c] - 3*Cos[3*d*x]*Sin[3*c] - 30*
Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] - 99*Cos[c]*Sin[d*x] + 10*Cos[2*c]*
Sin[2*d*x] - 3*Cos[3*c]*Sin[3*d*x] - 30*Tan[c/2]))))/(60*a*d*(1 + Sec[c +
d*x]))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4306, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{4306}$$

$$-\frac{\int -\frac{7a-5a\sec(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)}dx}{a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{7a-5a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{7a-5a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4274}$$

$$\frac{7a\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx-5a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx-5a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4256}$$

$$\frac{7a\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-5a\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)}{2a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7a\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-5a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)}{2a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{7a\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}\right.}{2a^2}$$

$$\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7a\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\right.}{2a^2}$$

$$\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{7a\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{5d}\right)-5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{}{3}\right.}{2a^2}$$

$$\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{7a\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{5d}\right)-5a\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\right.}{3d}\right.}{2a^2}$$

$$\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

input
```
Int[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```

output
```
-(Sin[c + d*x]/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x]))) + (7*a*((6*Sqr
t[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*S
in[c + d*x])/(5*d*Sec[c + d*x]^(3/2))) - 5*a*((2*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt
[Sec[c + d*x]])))/(2*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

Maple [A] (verified)

Time = 3.39 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.36

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(25\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+63\right.\right.}{15a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\right.}$

input
```
int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/15/a*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*
d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
25*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))+48*sin(1/2*d*x+1/2*c)^8-56*sin(1/2*d*x+1/2*c)^6-30*sin(1/2*d*x+1/
2*c)^4+23*sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.29

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx =$$

$$-\frac{25\left(-i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+25\left(i\sqrt{2}\cos\right.}{}$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/30*(25*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c)) + 25*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 63*(-I*sqrt(2)
*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) + 63*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2
))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) - 2*(6*cos(d*x + c)^3 - 4*cos(d*x + c)^2 - 25*cos(d*x + c))*sin
(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \frac{\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)+\sec^{\frac{5}{2}}(c+dx)}\,dx}{a}$$

input
```
integrate(1/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(1/(sec(c + d*x)**(7/2) + sec(c + d*x)**(5/2)), x)/a
```

Maxima [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^4+\sec(dx+c)^3}dx}{a}$$

input
```
int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x)
```

output
```
int(sqrt(sec(c + d*x))/(sec(c + d*x)**4 + sec(c + d*x)**3),x)/a
```

3.201 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 202

$$\begin{aligned}\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \frac{7\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}\\
&+\frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d}\\
&-\frac{7\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d}+\frac{10\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d}\\
&-\frac{7\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}\end{aligned}$$

output
```
7*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)/
a^2/d+10/3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x
+c)^(1/2)/a^2/d-7*sec(d*x+c)^(1/2)*sin(d*x+c)/a^2/d+10/3*sec(d*x+c)^(3/2)*
sin(d*x+c)/a^2/d-7/3*sec(d*x+c)^(5/2)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*
sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.67 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.42

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{e^{-\frac{1}{2}i(4c+3dx)}(-1+e^{ic})\cos\left(\frac{1}{2}(c+dx)\right)\csc\left(\frac{c}{2}\right)\left(-10-37e^{i(c+dx)}-65e^{2i(c+dx)}-82e^{3i(c+dx)}-68e^{4i(c+d}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/12*((-1 + E^(I*c))*Cos[(c + d*x)/2]*Csc[c/2]*(-10 - 37*E^(I*(c + d*x))
- 65*E^((2*I)*(c + d*x)) - 82*E^((3*I)*(c + d*x)) - 68*E^((4*I)*(c + d*x))
 - 53*E^((5*I)*(c + d*x)) - 21*E^((6*I)*(c + d*x)) + (10*I)*(1 + E^(I*(c +
 d*x)))^3*(1 + E^((2*I)*(c + d*x)))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2] + 7*E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))^3*(1 + E^((2*I)*(c + d*x)
))^(3/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c + d
*x]^(5/2))/(a^2*d*E^((I/2)*(4*c + 3*d*x))*(1 + E^((2*I)*(c + d*x)))*(1 + S
ec[c + d*x])^2)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4303, 27, 3042, 4507, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{ 4303}$$

$$-\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)(5a-9a\sec(c+dx))}{2(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)(5a-9a\sec(c+dx))}{\sec(c+dx)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(5a-9a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 4507}$$

$$-\frac{\frac{\int 3\sec^{\frac{3}{2}}(c+dx)\left(7a^2-10a^2\sec(c+dx)\right)dx}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{3\int \sec^{\frac{3}{2}}(c+dx)\left(7a^2-10a^2\sec(c+dx)\right)dx}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{3\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(7a^2-10a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 4274}$$

$$-\frac{\frac{3\left(7a^2\int \sec^{\frac{3}{2}}(c+dx)dx-10a^2\int \sec^{\frac{5}{2}}(c+dx)dx\right)}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{3\left(7a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx-10a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx\right)}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4255}$$

$$-\frac{\frac{3\left(7a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)-10a^2\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)\right)}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{3\left(7a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-10a^2\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)\right)}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+d}{d(\sec(c+dx)+1)}}{6a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4258}$$

$$-\frac{\frac{3\left(7a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)-10a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\,s}{3}\right.\right.}{a^2}}{6a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{3\left(7a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)-10a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\,\mathrm{si}}{}\right.\right.}{a^2}}{6a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{\frac{3\left(7a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)-10a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\,\mathrm{se}}{3d}\right.\right.}{a^2}}{6a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{\frac{3\left(7a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)-10a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx)\right.}{3d}\right.}{a^2}}{6a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/3*(Sec[c + d*x]^(7/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^2) - ((14*S
ec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(1 + Sec[c + d*x])) + (3*(7*a^2*((-2*Sq
rt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/d) - 10*a^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[c +
d*x])/(3*d))))/a^2)/(6*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(181) = 362.

Time = 8.14 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.04

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\frac{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{6\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{22\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}}{3}\right.}{}$

input
```
int(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(1/3*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3+6*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)-22
/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+14*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2/3*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1
/2*c)^2-1/2)^2+16*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*
x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.62

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx =$$

$$-\frac{10\left(i\sqrt{2}\cos\left(dx+c\right)^3+2i\sqrt{2}\cos\left(dx+c\right)^2+i\sqrt{2}\cos\left(dx+c\right)\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+\right.}{}$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/6*(10*(I*sqrt(2)*cos(d*x + c)^3 + 2*I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2
)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
+ 10*(-I*sqrt(2)*cos(d*x + c)^3 - 2*I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(2)*c
os(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2
1*(-I*sqrt(2)*cos(d*x + c)^3 - 2*I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(2)*cos(
d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) + 21*(I*sqrt(2)*cos(d*x + c)^3 + 2*I*sqrt(2)*cos(d*x + c
)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(21*cos(d*x + c)^3 + 32*cos(d*x
+ c)^2 + 8*cos(d*x + c) - 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d
*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(9/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sec(dx+c)^{\frac{9}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(9/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int((1/cos(c + d*x))^(9/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(9/2)/(a + a/cos(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^4}{\sec(dx+c)^2+2\sec(dx+c)+1}dx}{a^2}$$

input
```
int(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x)
```

output
```
int((sqrt(sec(c + d*x))*sec(c + d*x)**4)/(sec(c + d*x)**2 + 2*sec(c + d*x)
 + 1),x)/a**2
```

3.202 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 176

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} - \frac{5\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d} + \frac{4\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} - \frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

output
```
-4*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)
/a^2/d-5/3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x
+c)^(1/2)/a^2/d+4*sec(d*x+c)^(1/2)*sin(d*x+c)/a^2/d-5/3*sec(d*x+c)^(3/2)*s
in(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(
d*x+c))^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.05 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.43

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(-4ie^{-i(c+dx)}\left(1+e^{i(c+dx)}\right)^3\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/6*(Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(((-4*I)*(1 + E^(I*(c + d*x)))^3
*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*
(c + d*x))])/E^(I*(c + d*x)) + 40*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*(29 +
50*Cos[c + d*x] + 17*Cos[2*(c + d*x)] + (12*I)*Sin[c + d*x] + (7*I)*Sin[2*
(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(a^2*d*E^(I*d*x)
*(1 + Sec[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4303, 27, 3042, 4507, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)(3a-7a\sec(c+dx))}{2(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)(3a-7a\sec(c+dx))}{\sec(c+dx)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3a-7a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{\frac{\int \sqrt{\sec(c+dx)}\left(5a^2-12a^2\sec(c+dx)\right)dx}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5a^2-12a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4274}$$

$$-\frac{\frac{5a^2\int \sqrt{\sec(c+dx)}dx-12a^2\int \sec^{\frac{3}{2}}(c+dx)dx}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{5a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-12a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4255}$$

$$-\frac{\frac{5a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{5a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 4258

$$-\frac{\frac{5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3119

$$-\frac{\frac{5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3120

$$-\frac{\frac{\frac{10a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/3*(Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^2) - ((10*S
ec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(1 + Sec[c + d*x])) + ((10*a^2*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - 12*a^2*((-2*S
qrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/d))/a^2)/(6*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(159) = 318$.

Time = 7.48 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.30

method	result
default	$-\frac{2\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(5\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-12\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{}$

input
```
int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)*cos(1/2*d*x+1/2*c)-48*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*sin(1/2*d*x+1/2*c)^6+86*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*sin(1/2*d*x+1/2*c)^4-37*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.58

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{5\left(-i\sqrt{2}\cos\left(dx+c\right)^2-2i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx\right.}{}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/6*(5*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))
*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(I*sqrt(2)*
cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 12*(I*sqrt(2)*cos(d*x + c)^2 + 2*
I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 12*(-I*sqrt(2)*cos(d*x + c)^
2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(12*cos(d*x + c)^2 +
 19*cos(d*x + c) + 6)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)
^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{7}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^3}{\sec(dx+c)^2+2\sec(dx+c)+1}dx}{a^2}$$

input
```
int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x)
```

output
```
int((sqrt(sec(c + d*x))*sec(c + d*x)**3)/(sec(c + d*x)**2 + 2*sec(c + d*x)
 + 1),x)/a**2
```

3.203 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 149

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2} \, dx = \frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

output
```
cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)/a^
2/d+2/3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)
^(1/2)/a^2/d-sec(d*x+c)^(1/2)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*sec(d*x+
c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.98 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.62

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(-ie^{-i(c+dx)}\left(1+e^{i(c+dx)}\right)^3\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4}\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(((-I)*(1 + E^(I*(c + d*x)))^3*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*
x))])/E^(I*(c + d*x)) + 16*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*(5 + 14*Cos[c
 + d*x] + 5*Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)])*(Cos[(c + 3*d*x)/2] +
I*Sin[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Sec[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4303, 27, 3042, 4507, 25, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}(a-5a\sec(c+dx))}{2(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 27

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}(a-5a\sec(c+dx))}{\sec(c+dx)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a-5a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 4507

$$-\frac{\frac{\int -\frac{2\sec(c+dx)a^2+3a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 25

$$-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\int \frac{2\sec(c+dx)a^2+3a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\int \frac{2\csc\left(c+dx+\frac{\pi}{2}\right)a^2+3a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 4274

$$-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx+2a^2\int \sqrt{\sec(c+dx)}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 4258

$$-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/3*(Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^2) - (-(((6
*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)
/a^2) + (6*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(1 + Sec[c + d*x])))/(6*a^2
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

Maple [A] (verified)

Time = 6.04 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.72

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^6-4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\cos\right.}{6a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin}}$

input
```
int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*
x+1/2*c)^6-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))-16*cos(1/2*d*x+1/2*c)^4+3*cos(1/2*d*x
+1/2*c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.86

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx =$$

$$-\frac{2\left(i\sqrt{2}\cos\left(dx+c\right)^2+2i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+\right.}{}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/6*(2*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 2*(-I*sqrt(2)*
cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(-I*sqrt(2)*cos(d*x + c)^2 - 2*
I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(I*sqrt(2)*cos(d*x + c)^2
+ 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*cos(d*x + c)^2 + 4*
cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a
^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**(5/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a*
*2
```

Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx=\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx=\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^2}{\sec(dx+c)^2+2\sec(dx+c)+1}dx}{a^2}$$

input
```
int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x)
```

output
```
int((sqrt(sec(c + d*x))*sec(c + d*x)**2)/(sec(c + d*x)**2 + 2*sec(c + d*x)
 + 1),x)/a**2
```

3.204 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d} + \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

output
```
1/3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/
2)/a^2/d+1/3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.27

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(4\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\sin\left(\frac{1}{2}(c+dx\right.\right.}{3a^2d(1+\sec(c+dx))^2}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(4*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] - Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/
(3*a^2*d*(1 + Sec[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4302, 27, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4302}$$

$$\frac{\int \frac{1}{2}\sqrt{\sec(c+dx)}dx}{3a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt{\sec(c+dx)}dx}{6a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{6a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{6a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{6a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d
) + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4302
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] - Simp[d/(a*b*(2*m + 1))   Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*(a*(n - 1) - b*(m + n)*C
sc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && L
tQ[m, -1] && LtQ[1, n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(68) = 136$.

Time = 2.16 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.44

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+2\cos\right.}{6a^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$

input
```
int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+2*cos(1/2*d*x+1/2*c)^4-3*cos(1/2*d*x
+1/2*c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.95

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\left(-i\sqrt{2}\cos(dx+c)^2-2i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+\ldots}{6\left(a^{2}\ldots\right.}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*((-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*cos(d
*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(cos(d*x + c))*sin(d*x + c))/(a^
2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**(3/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a*
*2
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)}{\sec(dx+c)^2+2\sec(dx+c)+1}dx}{a^2}$$

input
```
int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x)
```

output
```
int((sqrt(sec(c + d*x))*sec(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) +
1),x)/a**2
```

3.205 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 149

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = -\frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d} + \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

output
```
-cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)/a
^2/d+2/3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c
)^(1/2)/a^2/d+sec(d*x+c)^(1/2)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*sec(d*x
+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.12 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.60

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(16\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(\cos\left(\frac{1}{2}(c+\right.\right.\right.}{}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(16*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I
*(-7 - 10*Cos[c + d*x] - 7*Cos[2*(c + d*x)] + ((1 + E^(I*(c + d*x)))^3*Sqr
t[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c +
 d*x))])/E^(I*(c + d*x)) + I*Sin[2*(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*Si
n[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Sec[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4304, 27, 3042, 4507, 25, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a\sec(c+dx)+a)^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$
$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{\sqrt{\sec(c+dx)}(5a-a\sec(c+dx))}{2(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{\sec(c+dx)}(5a-a\sec(c+dx))}{\sec(c+dx)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4507}$$

$$\frac{\frac{\int -\frac{3a^2-2a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\int \frac{3a^2-2a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\int \frac{3a^2-2a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4274}$$

$$\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx-2a^2\int \sqrt{\sec(c+dx)}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-2a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\frac{6a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\frac{6a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/3*(Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^2) + (-(((6
*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d -
(4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)
/a^2) + (6*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(1 + Sec[c + d*x])))/(6*a^2
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.72

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)c\right.}{6a^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+}}$

input
```
int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1
/2*d*x+1/2*c)^6+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c
)^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-20*cos(1/2*d*x+1/2*c)^4+9*cos(1/
2*d*x+1/2*c)^2-1)/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.86

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx =$$

$$-\frac{2\left(i\sqrt{2}\cos\left(dx+c\right)^2+2i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)) \cdots}{\cdots}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/6*(2*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 2*(-I*sqrt(2)*
cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(I*sqrt(2)*cos(d*x + c)^2 + 2*I
*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(-I*sqrt(2)*cos(d*x + c)^2
- 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(3*cos(d*x + c)^2 + 2*
cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a
^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{\sec(c+dx)}}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(sqrt(sec(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**
2
```

Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2+2\sec(dx+c)+1}dx}{a^2}$$

input
```
int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x)
```

output
```
int(sqrt(sec(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1),x)/a**2
```

3.206 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 152

$$\begin{aligned}\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx &= \frac{4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}\\ &\quad-\frac{5\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d}\\ &\quad-\frac{5\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2}\end{aligned}$$

output

```
4*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)/
a^2/d-5/3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+
c)^(1/2)/a^2/d-5/3*sec(d*x+c)^(1/2)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*se
c(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.77 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.71

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-3-16e^{i(c+dx)}-23e^{2i(c+dx)}-25e^{3i(c+dx)}-20e^{4i(c+dx)}-9e^{5i(c+dx)}-5ie^{i(c-}\right.}{}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

output
```
((-1/3*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(-3 - 16
*E^(I*(c + d*x)) - 23*E^((2*I)*(c + d*x)) - 25*E^((3*I)*(c + d*x)) - 20*E^
((4*I)*(c + d*x)) - 9*E^((5*I)*(c + d*x)) - (5*I)*E^(I*(c + d*x))*(1 + E^(
I*(c + d*x)))^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 4*E^((2*I)*
(c + d*x))*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeom
etric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/(a^2*d*E^(I*(c + d*x))*(1
+ E^(I*(c + d*x)))^3)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4304, 27, 3042, 4508, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{7a-3a\sec(c+dx)}{2\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{7a-3a\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{7a-3a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4508}$$

$$\frac{\frac{\int \frac{12a^2-5a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{12a^2-5a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4274}$$

$$\frac{\frac{12a^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx-5a^2\int \sqrt{\sec(c+dx)}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{12a^2\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-5a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{12a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{12a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{\frac{24a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{\frac{24a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{10a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

output
```
-1/3*(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^2) + (((24*
a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (
10*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)
/a^2 - (10*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(1 + Sec[c + d*x])))/(6*a^2
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 2.59 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.69

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\mathrm{co}\right.}{6a^{2}\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\mathrm{c}}$

input
```
int(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(1/
2*d*x+1/2*c)^6+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+24*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*
c)^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*cos(1/2*d*x+1/2*c)^4+15*cos(
1/2*d*x+1/2*c)^2-1)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.82

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx =$$

$$-\frac{5\left(-i\sqrt{2}\cos\left(dx+c\right)^{2}-2i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx\right.}{}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/6*(5*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))
*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(I*sqrt(2)*
cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 12*(-I*sqrt(2)*cos(d*x + c)^2 - 2
*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 12*(I*sqrt(2)*cos(d*x + c)^
2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(6*cos(d*x + c)^2 +
5*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2
*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)+2\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx}{a^2}$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(1/(sec(c + d*x)**(5/2) + 2*sec(c + d*x)**(3/2) + sqrt(sec(c + d*x
))), x)/a**2
```

Maxima [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3+2\sec(dx+c)^2+\sec(dx+c)}dx}{a^2}$$

input
```
int(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x)
```

output
```
int(sqrt(sec(c + d*x))/(sec(c + d*x)**3 + 2*sec(c + d*x)**2 + sec(c + d*x)
),x)/a**2
```

3.207 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 178

$$\begin{aligned}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx &= -\frac{7\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} \\ &\quad+\frac{10\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d}+\frac{10\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} \\ &\quad-\frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(1+\sec(c+dx))}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\end{aligned}$$

output
```
-7*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)
/a^2/d+10/3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*
x+c)^(1/2)/a^2/d+10/3*sin(d*x+c)/a^2/d/sec(d*x+c)^(1/2)-7/3*sin(d*x+c)/a^2
/d/sec(d*x+c)^(1/2)/(1+sec(d*x+c))-1/3*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*
sec(d*x+c))^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.37 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(-84i\cos\left(\frac{1}{2}(c+dx)\right)-63i\cos\left(\frac{3}{2}(c+dx)\right)-2\right.}{}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(Cos[d*x] + I*Sin[d*x])*((-84*I)*Cos[
(c + d*x)/2] - (63*I)*Cos[(3*(c + d*x))/2] - (21*I)*Cos[(5*(c + d*x))/2] +
 80*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + ((7*
I)*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1
[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((I/2)*(c + d*x)) + 3*Sin[(c + d*
x)/2] + 10*Sin[(3*(c + d*x))/2] + 12*Sin[(5*(c + d*x))/2] + Sin[(7*(c + d*
x))/2]))/(6*a^2*d*E^(I*d*x)*(1 + Sec[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4304, 27, 3042, 4508, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{9a-5a\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{9a-5a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{9a-5a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4508}$$

$$\frac{\frac{\int \frac{3\left(10a^2-7a^2\sec(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\int \frac{10a^2-7a^2\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3\int \frac{10a^2-7a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4274}$$

$$\frac{\frac{3\left(10a^2\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx-7a^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3\left(10a^2\int\frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx-7a^2\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 4256

$$\frac{\frac{3\left(10a^2\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-7a^2\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3\left(10a^2\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-7a^2\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 4258

$$\frac{\frac{3\left(10a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-7a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3\left(10a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-7a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3119

$$\frac{\frac{3\left(10a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-\frac{14a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)}{a^2}-\frac{14\sin(c+dx}{d\sqrt{\sec(c+dx)}(\sec(c}}{6a^2}$$
$$\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3120

$$\frac{\frac{3\left(10a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)-\frac{14a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)}}{6a^2}$$
$$\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

input
```
Int[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
-1/3*Sin[c + d*x]/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2) + ((-14*Si
n[c + d*x])/(d*Sqrt[Sec[c + d*x]]*(1 + Sec[c + d*x])) + (3*((-14*a^2*Sqrt[
Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + 10*a^2*((2
*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) +
(2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]))))/a^2)/(6*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.52

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(16\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+20\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \mathrm{EllipticF}(}{6a^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}\sqrt{-2\sin(}}$

input
```
int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*cos(1
/2*d*x+1/2*c)^8+12*cos(1/2*d*x+1/2*c)^6+20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(
1/2*d*x+1/2*c)^3+42*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48*cos
(1/2*d*x+1/2*c)^4+21*cos(1/2*d*x+1/2*c)^2-1)/cos(1/2*d*x+1/2*c)^3/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.61

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx =$$

$$-\frac{10\left(i\sqrt{2}\cos\left(dx+c\right)^{2}+2i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx\right.}{}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/6*(10*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))
*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 10*(-I*sqrt(2
)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*(I*sqrt(2)*cos(d*x + c)^2 +
2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*(-I*sqrt(2)*cos(d*x + c
)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(2*cos(d*x + c)^3
+ 13*cos(d*x + c)^2 + 10*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a
^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)+2\sec^{\frac{5}{2}}(c+dx)+\sec^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(1/(sec(c + d*x)**(7/2) + 2*sec(c + d*x)**(5/2) + sec(c + d*x)**(3
/2)), x)/a**2
```

Maxima [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(c+dx)}\right)^2\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^4+2\sec(dx+c)^3+\sec(dx+c)^2}dx}{a^2}$$

input
```
int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x)
```

output
```
int(sqrt(sec(c + d*x))/(sec(c + d*x)**4 + 2*sec(c + d*x)**3 + sec(c + d*x)
**2),x)/a**2
```

3.208 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 200

$$\begin{aligned}
&\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2} dx \\
&= \frac{56\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5a^2d} \\
&\quad - \frac{5\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a^2d} \\
&\quad + \frac{56\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{a^2d\sqrt{\sec(c+dx)}} \\
&\quad - \frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}
\end{aligned}$$

output
```
56/5*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/
2)/a^2/d-5*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x
+c)^(1/2)/a^2/d+56/15*sin(d*x+c)/a^2/d/sec(d*x+c)^(3/2)-5*sin(d*x+c)/a^2/d
/sec(d*x+c)^(1/2)-3*sin(d*x+c)/a^2/d/sec(d*x+c)^(3/2)/(1+sec(d*x+c))-1/3*s
in(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.48 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.36

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(1344i\cos\left(\frac{1}{2}(c+dx)\right)+1008i\cos\left(\frac{3}{2}(c+dx)\right)-\right.}{}$$

input
```
Integrate[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(Cos[d*x] + I*Sin[d*x])*((1344*I)*Cos
[(c + d*x)/2] + (1008*I)*Cos[(3*(c + d*x))/2] + (336*I)*Cos[(5*(c + d*x))/
2] - 1200*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
- ((112*I)*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeom
etric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((I/2)*(c + d*x)) - 34*Si
n[(c + d*x)/2] - 148*Sin[(3*(c + d*x))/2] - 168*Sin[(5*(c + d*x))/2] - 11*
Sin[(7*(c + d*x))/2] + 3*Sin[(9*(c + d*x))/2]))/(60*a^2*d*E^(I*d*x)*(1 + S
ec[c + d*x])^2)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4304, 27, 3042, 4508, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$\Big\downarrow$ 4304

$$-\frac{\int -\frac{11a-7a\sec(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{11a-7a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{11a-7a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 4508

$$\frac{\frac{\int \frac{56a^2-45a^2\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx}{a^2}-\frac{18\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{56a^2-45a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx}{a^2}-\frac{18\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 4274

$$\frac{\frac{56a^2\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx-45a^2\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{18\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{56a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx-45a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{a^2}-\frac{18\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 4256$$

$$\frac{\frac{56a^2\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-45a^2\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)}{a^2}-\frac{18\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{56a^2\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-45a^2\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)}{a^2}-\frac{18\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 4258$$

$$\frac{\frac{56a^2\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-45a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)}{a^2}-\frac{}{d}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{56a^2\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-45a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right.}{a^2}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{56a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)-45a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)}{a^2}-\frac{}{d\sec}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{56a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)-45a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)}{a^2}-\frac{1}{d\sec^{\frac{3}{2}}(c}}{6a^2}$$

$$\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

input
```
Int[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
-1/3*Sin[c + d*x]/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2) + ((-18*Si
n[c + d*x])/(d*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x])) + (56*a^2*((6*Sqrt[C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*Sin[
c + d*x])/(5*d*Sec[c + d*x]^(3/2))) - 45*a^2*((2*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt
[Sec[c + d*x]])))/a^2)/(6*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 4.49 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.42

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(96\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}-352\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^8+120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^6-150\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos(}\right.}{30a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right.}$

input
```
int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/30/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*cos(
1/2*d*x+1/2*c)^10-352*cos(1/2*d*x+1/2*c)^8+120*cos(1/2*d*x+1/2*c)^6-150*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-336*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))+266*cos(1/2*d*x+1/2*c)^4-135*cos(1/2*d*x+1/2*c)^2+
5)/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx =$$

$$-\frac{75\left(-i\sqrt{2}\cos(dx+c)^2-2i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(}{}$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/30*(75*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2
))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 75*(I*sqrt(
2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 168*(-I*sqrt(2)*cos(d*x + c)^2
 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 168*(I*sqrt(2)*cos(d*x
+ c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(6*cos(d*x + c)
^4 - 8*cos(d*x + c)^3 - 94*cos(d*x + c)^2 - 75*cos(d*x + c))*sin(d*x + c)/
sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{1}{\sec^{\frac{9}{2}}(c+dx)+2\sec^{\frac{7}{2}}(c+dx)+\sec^{\frac{5}{2}}(c+dx)}\,dx}{a^2}$$

input
```
integrate(1/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(1/(sec(c + d*x)**(9/2) + 2*sec(c + d*x)**(7/2) + sec(c + d*x)**(5
/2)), x)/a**2
```

Maxima [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^5+2\sec(dx+c)^4+\sec(dx+c)^3}dx}{a^2}$$

input
```
int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x)
```

output
```
int(sqrt(sec(c + d*x))/(sec(c + d*x)**5 + 2*sec(c + d*x)**4 + sec(c + d*x)
**3),x)/a**2
```

3.209 $\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 247

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{119\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{11\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{2a^3d} - \frac{119\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} + \frac{11\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a^3d} - \frac{\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} - \frac{119\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{30d\left(a^3+a^3\sec(c+dx)\right)}$$

output
```
119/10*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(
1/2)/a^3/d+11/2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*se
c(d*x+c)^(1/2)/a^3/d-119/10*sec(d*x+c)^(1/2)*sin(d*x+c)/a^3/d+11/2*sec(d*x
+c)^(3/2)*sin(d*x+c)/a^3/d-1/5*sec(d*x+c)^(9/2)*sin(d*x+c)/d/(a+a*sec(d*x+
c))^3-2/3*sec(d*x+c)^(7/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-119/30*sec(d*
x+c)^(5/2)*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.51 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.53

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{e^{-idx}\csc\left(\frac{c}{2}\right)\left(-119\sqrt{2}e^{2idx}(-1+e^{2ic})\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos^6\left(\frac{1}{2}(c+dx)\right)\text{Hypergeometric2F1}\left(\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(11/2)/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Csc[c/2]*(-119*Sqrt[2]*E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*
x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[(c + d*x)
/2]^6*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]*Sec[c/2]*Sec[
c + d*x]^3 + ((-1 + E^(I*c))*Cos[(c + d*x)/2]*(165 + 944*E^(I*(c + d*x)) +
 2476*E^((2*I)*(c + d*x)) + 4148*E^((3*I)*(c + d*x)) + 5134*E^((4*I)*(c +
d*x)) + 4664*E^((5*I)*(c + d*x)) + 3340*E^((6*I)*(c + d*x)) + 1620*E^((7*I
)*(c + d*x)) + 357*E^((8*I)*(c + d*x)) - (165*I)*(1 + E^(I*(c + d*x)))^5*(
1 + E^((2*I)*(c + d*x)))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])*Sec
[c + d*x]^(7/2))/(16*E^(((3*I)/2)*(2*c + d*x))*(1 + E^((2*I)*(c + d*x)))))
)/(15*a^3*d*E^(I*d*x)*(1 + Sec[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.06, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4303, 27, 3042, 4507, 3042, 4507, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{11/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{\sec^{\frac{7}{2}}(c+dx)(7a-13a\sec(c+dx))}{2(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sec^{\frac{7}{2}}(c+dx)(7a-13a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(7a-13a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(50a^2-69a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}+\frac{20a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(50a^2-69a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{20a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{\frac{\frac{\int \frac{3}{2}\sec^{\frac{3}{2}}(c+dx)\left(119a^3-165a^3\sec(c+dx)\right)dx}{a^2}+\frac{119a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{20a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\frac{3\int \sec^{\frac{3}{2}}(c+dx)\left(119a^3-165a^3\sec(c+dx)\right)dx}{2a^2}+\frac{119a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{20a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{3\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(119a^3-165a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}+\frac{119a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{20a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4274

$$-\frac{\frac{\frac{3\left(119a^3\int \sec^{\frac{3}{2}}(c+dx)dx-165a^3\int \sec^{\frac{5}{2}}(c+dx)dx\right)}{2a^2}+\frac{119a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{20a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{3\left(119a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx-165a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx\right)}{2a^2}+\frac{119a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{20a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4255

$$-\frac{\frac{\frac{3\left(119a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)-165a^3\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)\right)}{2a^2}+\frac{119a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{20a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{3\left(119a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-165a^3\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)\right)}{2a^2}+\frac{119a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4258

$$-\frac{\frac{3\left(119a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)-165a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx}{3d}\right.}{2a^2}}{3a^2}}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{3\left(119a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)-165a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\text{sec}}{3d}\right.}{2a^2}}{3a^2}}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3119

$$-\frac{\frac{3\left(119a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)-165a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(}{3d}\right.}{2a^2}}{3a^2}}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3120

$$-\frac{\frac{\frac{119a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{3\left(119a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)-165a^3\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+}}{}\right.}{2a^2}}{3a^2}}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[Sec[c + d*x]^(11/2)/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/5*(Sec[c + d*x]^(9/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^3) - ((20*a
*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + ((119*a^2
*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])) + (3*(119*a^3*(
(-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d) - 165*a^3*((2*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*S
in[c + d*x])/(3*d))))/(2*a^2))/(3*a^2))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(218) = 436$.

Time = 21.63 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.83

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(\frac{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}+\frac{32\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{15\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}+\frac{118\sqrt{-2\sin\left(\frac{dx}{2}\right.}}{5\cos}\right.}{}$

input
```
int(sec(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^3*(1/5*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5+32
/15*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)^3+118/5*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)-128/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+238/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-4/
3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(cos(1/2*d*x+1/2*c)^2-1/2)^2+48*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-
(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.64

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$

$$-\frac{165\left(i\sqrt{2}\cos(dx+c)^4+3i\sqrt{2}\cos(dx+c)^3+3i\sqrt{2}\cos(dx+c)^2+i\sqrt{2}\cos(dx+c)\right)\text{weierstrassPI}}{}$$

input
```
integrate(sec(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/60*(165*(I*sqrt(2)*cos(d*x + c)^4 + 3*I*sqrt(2)*cos(d*x + c)^3 + 3*I*sq
rt(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c)) + 165*(-I*sqrt(2)*cos(d*x + c)^4 - 3*I*sqrt
(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(2)*cos(d*x + c))*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 357*(-I*sqrt(2
)*cos(d*x + c)^4 - 3*I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2
 - I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) + 357*(I*sqrt(2)*cos(d*x + c)^4 + 3*I*s
qrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(d*x + c
))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) + 2*(357*cos(d*x + c)^4 + 906*cos(d*x + c)^3 + 695*cos(d*x + c)
^2 + 120*cos(d*x + c) - 20)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*
x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x
 + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(11/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{11}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(11/2)/(a*sec(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{11/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int((1/cos(c + d*x))^(11/2)/(a + a/cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(11/2)/(a + a/cos(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^5}{\sec(dx+c)^3+3\sec(dx+c)^2+3\sec(dx+c)+1}dx}{a^3}$$

input
```
int(sec(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x)
```

output
```
int((sqrt(sec(c + d*x))*sec(c + d*x)**5)/(sec(c + d*x)**3 + 3*sec(c + d*x)
**2 + 3*sec(c + d*x) + 1),x)/a**3
```

3.210 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 221

$$\begin{aligned}\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = &-\frac{49\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}\\&-\frac{13\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{6a^3d}\\&+\frac{49\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d}-\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}\\&-\frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}-\frac{13\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d\left(a^3+a^3\sec(c+dx)\right)}\end{aligned}$$

output
```
-49/10*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(
1/2)/a^3/d-13/6*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*se
c(d*x+c)^(1/2)/a^3/d+49/10*sec(d*x+c)^(1/2)*sin(d*x+c)/a^3/d-1/5*sec(d*x+c
)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-8/15*sec(d*x+c)^(5/2)*sin(d*x+c)/a
/d/(a+a*sec(d*x+c))^2-13/6*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a^3+a^3*sec(d*x+
c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.72 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.68

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(147\left(1+e^{2i(c+dx)}\right)+147\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\right.\right.}{-1+e^{2ic}}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^3,x]
```

output
```
(2*Cos[(c + d*x)/2]^6*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*
(c + d*x)))]*(147*(1 + E^((2*I)*(c + d*x))) + 147*(-1 + E^((2*I)*c))*Sqrt[
1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c +
d*x))] - 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x)
)]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x
))*(-1 + E^((2*I)*c))) + ((1284*Cos[(c - d*x)/2] + 921*Cos[(3*c + d*x)/2]
+ 1243*Cos[(c + 3*d*x)/2] + 374*Cos[(5*c + 3*d*x)/2] + 670*Cos[(3*c + 5*d*
x)/2] + 65*Cos[(7*c + 5*d*x)/2] + 147*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c
/2]*Sec[(c + d*x)/2]^5*Sqrt[Sec[c + d*x]])/32)*Sec[c + d*x]^3)/(15*a^3*d*(
1 + Sec[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.06, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4303, 27, 3042, 4507, 3042, 4507, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{ 4303}$$

$$-\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)(5a-11a\sec(c+dx))}{2(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)(5a-11a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(5a-11a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 4507}$$

$$-\frac{\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(24a^2-41a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(24a^2-41a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 4507}$$

$$-\frac{\frac{\frac{\int \frac{1}{2}\sqrt{\sec(c+dx)}\left(65a^3-147a^3\sec(c+dx)\right)dx}{a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{\frac{\int \sqrt{\sec(c+dx)}\left(65a^3-147a^3\sec(c+dx)\right)dx}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(65a^3-147a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4274

$$-\frac{\frac{\frac{65a^3\int \sqrt{\sec(c+dx)}dx-147a^3\int \sec^{\frac{3}{2}}(c+dx)dx}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{65a^3\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-147a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4255

$$-\frac{\frac{\frac{65a^3\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{65a^3\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-$$
$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 4258

$$-\frac{\frac{\frac{65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}\ -}{10a^2}$$
$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(}{d(a\sec(c+dx)+a}}{3a^2}}{10a^2}$$
$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3119

$$-\frac{\frac{\frac{65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}$$
$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3120

$$-\frac{\frac{\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{\frac{130a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{d}\right.}{2a^2}}{3a^2}}{10a^2}$$
$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/5*(Sec[c + d*x]^(7/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^3) - ((16*a
*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + ((65*a^2*
Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])) + ((130*a^3*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - 147*a^3*(
(-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/(2*a^2))/(3*a^2))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(196) = 392$.

Time = 21.61 (sec) , antiderivative size = 555, normalized size of antiderivative = 2.51

method	result
default	$-\frac{-2\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(65\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-147\,\mathrm{EllipticE}\left(\cos\left(\right.\right.\right.}{}$

input
```
int(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/60*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^4+4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(65
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(65*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)+588*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8-1634*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+1488*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-439*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(
1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.60

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$

$$-\frac{65\left(-i\sqrt{2}\cos(dx+c)^3-3i\sqrt{2}\cos(dx+c)^2-3i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/60*(65*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sq
rt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) + 65*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^
2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) + 147*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*co
s(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 147*(-I*sqr
t(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c
) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c))) - 2*(147*cos(d*x + c)^3 + 376*cos(d*x + c)^2 + 295*
cos(d*x + c) + 60)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3
+ 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**(9/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{9}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(9/2)/(a*sec(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int((1/cos(c + d*x))^(9/2)/(a + a/cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(9/2)/(a + a/cos(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^4}{\sec(dx+c)^3+3\sec(dx+c)^2+3\sec(dx+c)+1}dx}{a^3}$$

input
```
int(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x)
```

output
```
int((sqrt(sec(c + d*x))*sec(c + d*x)**4)/(sec(c + d*x)**3 + 3*sec(c + d*x)
**2 + 3*sec(c + d*x) + 1),x)/a**3
```

3.211 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 195

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{9\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{2a^3d} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{9\sqrt{\sec(c+dx)}\sin(c+dx)}{10d\,(a^3+a^3\sec(c+dx))}$$

output
```
9/10*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/
2)/a^3/d+1/2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d
*x+c)^(1/2)/a^3/d-1/5*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-2/5
*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-9/10*sec(d*x+c)^(1/2)*
sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.34 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.41

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\left(-3ie^{-2i(c+dx)}\left(1+e^{i(c+dx)}\right)^5\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4}\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^(7/2)*(((-3*I)*(1 + E^(I*(c + d*x)))^5*Sqrt
[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c +
d*x))])/E^((2*I)*(c + d*x)) + 160*Cos[(c + d*x)/2]^5*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + (2*I)*(3
4 + 69*Cos[c + d*x] + 34*Cos[2*(c + d*x)] + 7*Cos[3*(c + d*x)] + (2*I)*Sin
[c + d*x] + (6*I)*Sin[2*(c + d*x)] + (2*I)*Sin[3*(c + d*x)]))*(Cos[(c + 3*
d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(40*a^3*d*E^(I*d*x)*(1 + Sec[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4303, 27, 3042, 4507, 3042, 4507, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{3\sec^{\frac{3}{2}}(c+dx)(a-3a\sec(c+dx))}{2(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\int \frac{\sec^{\frac{3}{2}}(c+dx)(a-3a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{3\left(\frac{\int \frac{\sqrt{\sec(c+dx)}\left(2a^2-7a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(2a^2-7a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{3\left(\frac{\frac{\int -\frac{5\sec(c+dx)a^3+9a^3}{2\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int\frac{5\sec(c+dx)a^3+9a^3}{\sqrt{\sec(c+dx)}}dx}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int\frac{5\csc\left(c+dx+\frac{\pi}{2}\right)a^3+9a^3}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4274

$$-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{9a^3\int\frac{1}{\sqrt{\sec(c+dx)}}dx+5a^3\int\sqrt{\sec(c+dx)}dx}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{9a^3\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5a^3\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4258

$$-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+9a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+da}{3d(a\sec(c+dx)+a)^2}\right.}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3042

$$-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+9a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec ...}{3d(a\sec(c+dx ...}\right)}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3119

$$-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{18a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c ...}{3d(a\sec(c+dx)+ ...}\right)}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3120

$$-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{10a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{18a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}} ...}{3d(a\sec(c+dx) ...}\right)}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/5*(Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^3) - (3*((4
*a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + (-1/2*(
(18*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
 + (10*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/d)/a^2 + (9*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])
))/(3*a^2)))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

Maple [A] (verified)

Time = 19.38 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.37

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(36\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\operatorname{co}\right.}{20a^{3}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}\sqrt{-2\sin\left(\frac{dx}{2}+\right.}}$

input
```
int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*d
*x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^
5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-46*cos(1/2*d*x+1/2*c)^6+8*cos(1/2*
d*x+1/2*c)^4+cos(1/2*d*x+1/2*c)^2+1)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.81

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$

$$-\frac{5\left(i\sqrt{2}\cos(dx+c)^3+3i\sqrt{2}\cos(dx+c)^2+3i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos}{}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/20*(5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt
(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2
- 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) + 9*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d
*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*(I*sqrt(2)*c
os(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*
sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*(9*cos(d*x + c)^3 + 22*cos(d*x + c)^2 + 15*cos(d*x +
c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d
*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\sec(dx+c)^{\frac{7}{2}}}{(a\sec(dx+c)+a)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^3}{\sec(dx+c)^3+3\sec(dx+c)^2+3\sec(dx+c)+1}dx}{a^3}$$

input
```
int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x)
```

output
```
int((sqrt(sec(c + d*x))*sec(c + d*x)**3)/(sec(c + d*x)**3 + 3*sec(c + d*x)
**2 + 3*sec(c + d*x) + 1),x)/a**3
```

3.212 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 195

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{6a^3d} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d\left(a^3+a^3\sec(c+dx)\right)}$$

output
```
1/10*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/
2)/a^3/d+1/6*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d
*x+c)^(1/2)/a^3/d-1/5*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-4/1
5*sec(d*x+c)^(1/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2+1/6*sec(d*x+c)^(1/2)*
sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.50 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.90

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3\left(1+e^{2i(c+dx)}\right)+3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+\cdots}\right.\right.}{-1+e^{2ic}}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^3,x]
```

output
```
(2*Cos[(c + d*x)/2]^6*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(
c + d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E
^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))
] - 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyp
ergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1
 + E^((2*I)*c))) - ((36*Cos[(c - d*x)/2] + 9*Cos[(3*c + d*x)/2] + 7*Cos[(c
 + 3*d*x)/2] + 26*Cos[(5*c + 3*d*x)/2] + 10*Cos[(3*c + 5*d*x)/2] + 5*Cos[(
7*c + 5*d*x)/2] + 3*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/
2]^5*Sqrt[Sec[c + d*x]])/32)*Sec[c + d*x]^3)/(15*a^3*d*(1 + Sec[c + d*x])^
3)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 4303, 27, 3042, 4507, 25, 3042, 4508, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \; 4303$$

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}(a-7a\sec(c+dx))}{2(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}(a-7a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a-7a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 4507$$

$$-\frac{\frac{\int -\frac{9\sec(c+dx)a^2+4a^2}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}+\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\int \frac{9\sec(c+dx)a^2+4a^2}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\int \frac{9\csc\left(c+dx+\frac{\pi}{2}\right)a^2+4a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 4508$$

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int \frac{5\sec(c+dx)a^3+3a^3}{2\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 27

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int \frac{5\sec(c+dx)a^3+3a^3}{\sqrt{\sec(c+dx)}}dx}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3042

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int \frac{5\csc\left(c+dx+\frac{\pi}{2}\right)a^3+3a^3}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 4274

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{3a^3\int \frac{1}{\sqrt{\sec(c+dx)}}dx+5a^3\int \sqrt{\sec(c+dx)}dx}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3042

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{3a^3\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5a^3\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 4258

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3042

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c}{d(a\sec(c+dx)+a}}{3a^2}}{10a^2}\;\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3119

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3120

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{\frac{10a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input

```
Int[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^3,x]
```

output

```
-1/5*(Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^3) - ((8*a*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) - (((6*a^3*S
qrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (10*a^
3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/(2*a
^2) + (5*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3
*a^2))/(10*a^2)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Maple [A] (verified)

Time = 21.55 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.38

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\mathrm{co}\right.}{60a^{3}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}\sqrt{-2\sin\left(\frac{dx}{2}+\right.}}$

input
```
int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d
*x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-22*cos(1/2*d*x+1/2*c)^6+6*cos(1/2*d
*x+1/2*c)^4+7*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.81

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{5\left(i\sqrt{2}\cos\left(dx+c\right)^3+3i\sqrt{2}\cos\left(dx+c\right)^2+3i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos\ldots}{}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/60*(5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt
(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2
- 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) + 3*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d
*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(I*sqrt(2)*c
os(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*
sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*(3*cos(d*x + c)^3 + 4*cos(d*x + c)^2 - 5*cos(d*x + c)
)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x
 + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**(5/2)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(
c + d*x) + 1), x)/a**3
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^2}{\sec(dx+c)^3+3\sec(dx+c)^2+3\sec(dx+c)+1}dx}{a^3}$$

input
```
int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x)
```

output
```
int((sqrt(sec(c + d*x))*sec(c + d*x)**2)/(sec(c + d*x)**3 + 3*sec(c + d*x)
**2 + 3*sec(c + d*x) + 1),x)/a**3
```

3.213 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 195

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{6a^3d} + \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d\,(a^3+a^3\sec(c+dx))}$$

output
```
-1/10*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1
/2)/a^3/d+1/6*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(
d*x+c)^(1/2)/a^3/d+1/5*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-1/
15*sec(d*x+c)^(1/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2+1/6*sec(d*x+c)^(1/2)
*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.50 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.90

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3\left(1+e^{2i(c+dx)}\right)+3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(\cdots}\right.\right.}{-1+e^{2ic}}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^3,x]
```

output
```
(2*Cos[(c + d*x)/2]^6*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*
(c + d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 +
E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x)
)] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hy
pergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-
1 + E^((2*I)*c))) + ((36*Cos[(c - d*x)/2] + 9*Cos[(3*c + d*x)/2] + 17*Cos[
(c + 3*d*x)/2] + 16*Cos[(5*c + 3*d*x)/2] + 20*Cos[(3*c + 5*d*x)/2] - 5*Cos
[(7*c + 5*d*x)/2] + 3*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x
)/2]^5*Sqrt[Sec[c + d*x]])/32)*Sec[c + d*x]^3)/(15*a^3*d*(1 + Sec[c + d*x]
)^3)
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4302, 27, 3042, 4507, 3042, 4508, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a\sec(c+dx)+a)^3}dx$$

$\Big\downarrow$ 3042

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$\Big\downarrow$ 4302

$$\frac{\int \frac{\sqrt{\sec(c+dx)}(3\sec(c+dx)a+a)}{2(\sec(c+dx)a+a)^2}dx}{5a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{\sqrt{\sec(c+dx)}(3\sec(c+dx)a+a)}{(\sec(c+dx)a+a)^2}dx}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(3\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4507

$$\frac{\frac{\int \frac{6\sec(c+dx)a^2+a^2}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{6\csc\left(c+dx+\frac{\pi}{2}\right)a^2+a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4508

$$\frac{\frac{\frac{\int -\frac{3a^3-5a^3\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int \frac{3a^3-5a^3\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int \frac{3a^3-5a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 4274$$

$$\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{3a^3\int \frac{1}{\sqrt{\sec(c+dx)}}dx-5a^3\int \sqrt{\sec(c+dx)}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{3a^3\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-5a^3\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 4258$$

$$\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3120

$$\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{10a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((-2*a*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + (-1/2*((6*a^
3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (10
*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a
^2 + (5*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3*
a^2))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4302
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] - Simp[d/(a*b*(2*m + 1))   Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*(a*(n - 1) - b*(m + n)*C
sc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && L
tQ[m, -1] && LtQ[1, n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 3.44 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.38

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{60a^{3}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}\sqrt{-2\sin\left(\right.}}$

input
```
int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*
d*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^
5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*cos(1/2*d*x+1/2*c)^6-24*cos(1/2*
d*x+1/2*c)^4+17*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*
d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.81

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$

$$-\frac{5\left(i\sqrt{2}\cos\left(dx+c\right)^{3}+3i\sqrt{2}\cos\left(dx+c\right)^{2}+3i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos}{}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/60*(5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt
(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2
- 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) + 3*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*
x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(-I*sqrt(2)*c
os(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*
sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) - 2*(3*cos(d*x + c)^3 + 14*cos(d*x + c)^2 + 5*cos(d*x + c
))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*
x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx=\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**(3/2)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(
c + d*x) + 1), x)/a**3
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{(a\sec(dx+c)+a)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)}{\sec(dx+c)^3+3\sec(dx+c)^2+3\sec(dx+c)+1}dx}{a^3}$$

input
```
int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x)
```

output
```
int((sqrt(sec(c + d*x))*sec(c + d*x))/(sec(c + d*x)**3 + 3*sec(c + d*x)**2
 + 3*sec(c + d*x) + 1),x)/a**3
```

3.214 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 195

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = -\frac{9\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{2a^3d} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{2d\left(a^3+a^3\sec(c+dx)\right)}$$

output
```
-9/10*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1
/2)/a^3/d+1/2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(
d*x+c)^(1/2)/a^3/d-1/5*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3+2/
5*sec(d*x+c)^(1/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2+1/2*sec(d*x+c)^(1/2)*
sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.63 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = \frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\left(160\cos^5\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(\cos\left(\frac{1}{2}(c\right.\right.\right.}{}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^(7/2)*(160*Cos[(c + d*x)/2]^5*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) +
I*(-68 - 128*Cos[c + d*x] - 68*Cos[2*(c + d*x)] - 24*Cos[3*(c + d*x)] + (3
*(1 + E^(I*(c + d*x)))^5*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1
/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((2*I)*(c + d*x)) + (6*I)*Sin[c + d
*x] + (8*I)*Sin[2*(c + d*x)] + (6*I)*Sin[3*(c + d*x)]))*(Cos[(c + 3*d*x)/2
] + I*Sin[(c + 3*d*x)/2]))/(40*a^3*d*E^(I*d*x)*(1 + Sec[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 4304, 27, 3042, 4507, 25, 3042, 4508, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{3\sqrt{\sec(c+dx)}(3a-a\sec(c+dx))}{2(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{\sqrt{\sec(c+dx)}(3a-a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(3a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4507}$$

$$\frac{3\left(\frac{\int -\frac{2a^2-3a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}+\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\int \frac{2a^2-3a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\int \frac{2a^2-3a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4508}$$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int \frac{9a^3-5a^3\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int\frac{9a^3-5a^3\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int\frac{9a^3-5a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4274}$$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{9a^3\int\frac{1}{\sqrt{\sec(c+dx)}}dx-5a^3\int\sqrt{\sec(c+dx)}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{9a^3\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-5a^3\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4258}$$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{9a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{9a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec}}{d(a\sec(c+dx)}}{3a^2}\right.}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3119$$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\frac{18a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+}}{d(a\sec(c+dx)+a)}}{3a^2}\right.}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3120$$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\frac{18a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{10a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(}}{d(a\sec(c+dx)+}}{3a^2}\right.}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/5*(Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^3) + (3*((4
*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) - (((18*a
^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (1
0*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/
(2*a^2) - (5*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
)/(3*a^2)))/(10*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 3.22 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.38

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(36\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right.}{20a^{3}\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin}}$

input
```
int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/20/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(
1/2*d*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/
2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-66*cos(1/2*d*x+1/2*c)^6+38*co
s(1/2*d*x+1/2*c)^4-9*cos(1/2*d*x+1/2*c)^2+1)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.81

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx =$$

$$-\frac{5\left(i\sqrt{2}\cos\left(dx+c\right)^3+3i\sqrt{2}\cos\left(dx+c\right)^2+3i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/20*(5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt
(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2
- 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) + 9*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*
x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*(-I*sqrt(2)*c
os(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*
sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) - 2*(9*cos(d*x + c)^3 + 12*cos(d*x + c)^2 + 5*cos(d*x + c
))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*
x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sqrt{\sec{(c+dx)}}}{\sec^3{(c+dx)}+3\sec^2{(c+dx)}+3\sec{(c+dx)}+1}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(sqrt(sec(c + d*x))/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
 + d*x) + 1), x)/a**3
```

Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^3, x)
```

Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3+3\sec(dx+c)^2+3\sec(dx+c)+1}dx}{a^3}$$

input
```
int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x)
```

output
```
int(sqrt(sec(c + d*x))/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*
x) + 1),x)/a**3
```

3.215 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 195

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx = \frac{49\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{13\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{6a^3d} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{13\sqrt{\sec(c+dx)}\sin(c+dx)}{6d\,(a^3+a^3\sec(c+dx))}$$

output
```
49/10*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1
/2)/a^3/d-13/6*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec
(d*x+c)^(1/2)/a^3/d-1/5*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-8
/15*sec(d*x+c)^(1/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-13/6*sec(d*x+c)^(1/
2)*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.78 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.98

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(147\left(1+e^{2i(c+dx)}\right)+147\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2}\right.\right.}{-1+e^{2ic}}\right.}{}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3),x]
```

output
```
(2*Cos[(c + d*x)/2]^6*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(
c + d*x)))]*(147*(1 + E^((2*I)*(c + d*x))) + 147*(-1 + E^((2*I)*c))*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d
*x))] + 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))
]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x)
)*(-1 + E^((2*I)*c))) - ((1134*Cos[(c - d*x)/2] + 1071*Cos[(3*c + d*x)/2]
+ 923*Cos[(c + 3*d*x)/2] + 694*Cos[(5*c + 3*d*x)/2] + 470*Cos[(3*c + 5*d*x
)/2] + 265*Cos[(7*c + 5*d*x)/2] + 117*Cos[(5*c + 7*d*x)/2] + 30*Cos[(9*c +
 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5*Sqrt[Sec[c + d*x]])/32)*S
ec[c + d*x]^3)/(15*a^3*d*(1 + Sec[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4304, 27, 3042, 4508, 3042, 4508, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{11a-5a\sec(c+dx)}{2\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{11a-5a\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{11a-5a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4508}$$

$$\frac{\frac{\int \frac{41a^2-24a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{41a^2-24a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4508}$$

$$\frac{\frac{\frac{\int \frac{147a^3-65a^3\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{\int \frac{147a^3-65a^3\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\int \frac{147a^3-65a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 4274

$$\frac{\frac{\frac{147a^3\int \frac{1}{\sqrt{\sec(c+dx)}}dx-65a^3\int \sqrt{\sec(c+dx)}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{147a^3\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-65a^3\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 4258

$$\frac{\frac{\frac{147a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{147a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3119

$$\frac{\frac{\frac{\frac{294a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+}}{3d(a\sec(c+dx)+a)^2}}{10a^2}$$
$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3120

$$\frac{\frac{\frac{\frac{294a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{130a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(}}{3d(a\sec(c+dx)+}}{10a^2}$$
$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3),x]
```

output
```
-1/5*(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^3) + ((-16*
a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + (((294*a
^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (1
30*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)
/(2*a^2) - (65*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]
)))/(3*a^2))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.38

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(348\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^8+130\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right.}{60a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^5\sqrt{-2\sin(}}$

input
```
int(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/60/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(348*cos(
1/2*d*x+1/2*c)^8+130*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+294*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+
1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-578*cos(1/2*d*x+1/2*c)^6+26
4*cos(1/2*d*x+1/2*c)^4-37*cos(1/2*d*x+1/2*c)^2+3)/cos(1/2*d*x+1/2*c)^5/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.81

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx =$$

$$-\frac{65\left(-i\sqrt{2}\cos\left(dx+c\right)^3-3i\sqrt{2}\cos\left(dx+c\right)^2-3i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/60*(65*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sq
rt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) + 65*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^
2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) + 147*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*c
os(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 147*(I*sqr
t(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c
) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c))) + 2*(87*cos(d*x + c)^3 + 146*cos(d*x + c)^2 + 65*co
s(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3
*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)+3\sec^{\frac{5}{2}}(c+dx)+3\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx}{a^3}$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(1/(sec(c + d*x)**(7/2) + 3*sec(c + d*x)**(5/2) + 3*sec(c + d*x)**
(3/2) + sqrt(sec(c + d*x))), x)/a**3
```

Maxima [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(c+dx)}\right)^3\sqrt{\frac{1}{\cos(c+dx)}}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^4+3\sec(dx+c)^3+3\sec(dx+c)^2+\sec(dx+c)}dx}{a^3}$$

input
```
int(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x)
```

output
```
int(sqrt(sec(c + d*x))/(sec(c + d*x)**4 + 3*sec(c + d*x)**3 + 3*sec(c + d*
x)**2 + sec(c + d*x)),x)/a**3
```

3.216 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 221

$$\begin{aligned}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx &= -\frac{119\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} \\ &\quad+\frac{11\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{2a^3d} \\ &\quad+\frac{11\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} \\ &\quad-\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}-\frac{119\sin(c+dx)}{30d\sqrt{\sec(c+dx)}\left(a^3+a^3\sec(c+dx)\right)}\end{aligned}$$

output

```
-119/10*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^
(1/2)/a^3/d+11/2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*s
ec(d*x+c)^(1/2)/a^3/d+11/2*sin(d*x+c)/a^3/d/sec(d*x+c)^(1/2)-1/5*sin(d*x+c
)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3-2/3*sin(d*x+c)/a/d/sec(d*x+c)^(1/2
)/(a+a*sec(d*x+c))^2-119/30*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a^3+a^3*sec(d*x
+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.97 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.29

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(-5355i\cos\left(\frac{1}{2}(c+dx)\right)-3927i\cos\left(\frac{3}{2}(c+dx)\right)\right.}{}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^(7/2)*(Cos[d*x] + I*Sin[d*x])*((-5355*I)*Co
s[(c + d*x)/2] - (3927*I)*Cos[(3*(c + d*x))/2] - (1785*I)*Cos[(5*(c + d*x)
)/2] - (357*I)*Cos[(7*(c + d*x))/2] + 5280*Cos[(c + d*x)/2]^5*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2] + ((119*I)*(1 + E^(I*(c + d*x)))^5*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*
x))])/E^(((3*I)/2)*(c + d*x)) + 193*Sin[(c + d*x)/2] + 579*Sin[(3*(c + d*x
))/2] + 555*Sin[(5*(c + d*x))/2] + 227*Sin[(7*(c + d*x))/2] + 10*Sin[(9*(c
 + d*x))/2]))/(120*a^3*d*E^(I*d*x)*(1 + Sec[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.08, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4304, 27, 3042, 4508, 3042, 4508, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{13a-7a\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{13a-7a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{13a-7a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4508}$$

$$\frac{\frac{\int \frac{69a^2-50a^2\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{69a^2-50a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4508}$$

$$\frac{\frac{\frac{\int \frac{3\left(165a^3-119a^3\sec(c+dx)\right)}{2\sec^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3\int \frac{165a^3-119a^3\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{3\int \frac{165a^3-119a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4274

$$\frac{\frac{\frac{3\left(165a^3\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx-119a^3\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{3\left(165a^3\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx-119a^3\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4256

$$\frac{\frac{\frac{3\left(165a^3\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-119a^3\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{3\left(165a^3\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-119a^3\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow 4258$$

$$\frac{\frac{\frac{3\left(165a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-119a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-}{10a^2}$$

$$\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{\frac{3\left(165a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-119a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx}}{3a^2}}{10a^2}$$

$$\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow 3119$$

$$\frac{\frac{\frac{3\left(165a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-\frac{238a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}}{10a^2}$$

$$\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow 3120$$

$$\frac{\frac{\frac{3\left(165a^3\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)-\frac{238a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a}}{3a^2}}{10a^2}$$

$$\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

input
```
Int[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),x]
```

output
```
-1/5*Sin[c + d*x]/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3) + ((-20*a*
Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2) + ((-119*a^2
*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])) + (3*((-238*a^3
*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + 165*
a^3*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]))))/(2*a^2))/(3*a^2))/(10*
a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
 + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 4.63 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.28

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(160\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+468\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+330\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \text{Ellipt}\right.}{60a^{3}\sqrt{-2\sin\left(\right.}}$

input
```
int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/60/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*cos
(1/2*d*x+1/2*c)^10+468*cos(1/2*d*x+1/2*c)^8+330*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*cos(1/2*d*x+1/2*c)^5+714*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-
1058*cos(1/2*d*x+1/2*c)^6+474*cos(1/2*d*x+1/2*c)^4-47*cos(1/2*d*x+1/2*c)^2
+3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.64

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx =$$

$$-\frac{165\left(i\sqrt{2}\cos\left(dx+c\right)^3+3i\sqrt{2}\cos\left(dx+c\right)^2+3i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,c}{}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/60*(165*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sq
rt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) + 165*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c
)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) + 357*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*
cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 357*(-I*s
qrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x +
 c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c))) - 2*(20*cos(d*x + c)^4 + 237*cos(d*x + c)^3 + 376
*cos(d*x + c)^2 + 165*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*
d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{1}{\sec^{\frac{9}{2}}(c+dx)+3\sec^{\frac{7}{2}}(c+dx)+3\sec^{\frac{5}{2}}(c+dx)+\sec^{\frac{3}{2}}(c+dx)}\,dx}{a^3}$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(1/(sec(c + d*x)**(9/2) + 3*sec(c + d*x)**(7/2) + 3*sec(c + d*x)**
(5/2) + sec(c + d*x)**(3/2)), x)/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^5+3\sec(dx+c)^4+3\sec(dx+c)^3+\sec(dx+c)^2}dx}{a^3}$$

input
```
int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x)
```

output
```
int(sqrt(sec(c + d*x))/(sec(c + d*x)**5 + 3*sec(c + d*x)**4 + 3*sec(c + d*
x)**3 + sec(c + d*x)**2),x)/a**3
```

3.217 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 247

$$\begin{aligned}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx &= \frac{231\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} \\ &\quad - \frac{21\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{2a^3d} + \frac{77\sin(c+dx)}{10a^3d\sec^{\frac{3}{2}}(c+dx)} \\ &\quad - \frac{21\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} \\ &\quad - \frac{4\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} - \frac{63\sin(c+dx)}{10d\sec^{\frac{3}{2}}(c+dx)\left(a^3+a^3\sec(c+dx)\right)}\end{aligned}$$

output
```
231/10*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(
1/2)/a^3/d-21/2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*se
c(d*x+c)^(1/2)/a^3/d+77/10*sin(d*x+c)/a^3/d/sec(d*x+c)^(3/2)-21/2*sin(d*x+
c)/a^3/d/sec(d*x+c)^(1/2)-1/5*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c
))^3-4/5*sin(d*x+c)/a/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2-63/10*sin(d*x+
c)/d/sec(d*x+c)^(3/2)/(a^3+a^3*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.31 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = -\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(-3465i\cos\left(\frac{1}{2}(c+dx)\right)-2541i\cos\left(\frac{3}{2}(c+dx\right.\right.}{}$$

input
```
Integrate[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),x]
```

output
```
-1/40*(Cos[(c + d*x)/2]*Sec[c + d*x]^(7/2)*(Cos[d*x] + I*Sin[d*x])*((-3465
*I)*Cos[(c + d*x)/2] - (2541*I)*Cos[(3*(c + d*x))/2] - (1155*I)*Cos[(5*(c
+ d*x))/2] - (231*I)*Cos[(7*(c + d*x))/2] + 3360*Cos[(c + d*x)/2]^5*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2] + ((77*I)*(1 + E^(I*(c + d*x)))^5*S
qrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c
 + d*x))])/E^(((3*I)/2)*(c + d*x)) + 125*Sin[(c + d*x)/2] + 359*Sin[(3*(c
+ d*x))/2] + 350*Sin[(5*(c + d*x))/2] + 138*Sin[(7*(c + d*x))/2] + 5*Sin[(
9*(c + d*x))/2] - Sin[(11*(c + d*x))/2]))/(a^3*d*E^(I*d*x)*(1 + Sec[c + d*
x])^3)
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.08, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4304, 27, 3042, 4508, 27, 3042, 4508, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$\Big\downarrow$ 4304

$$-\frac{\int -\frac{3(5a-3a\sec(c+dx))}{2\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 27

$$\frac{3\int \frac{5a-3a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{3\int \frac{5a-3a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4508

$$\frac{3\left(\frac{\int \frac{7\left(5a^2-4a^2\sec(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 27

$$\frac{3\left(\frac{7\int \frac{5a^2-4a^2\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{3\left(\frac{7\int \frac{5a^2-4a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4508}$$

$$\frac{3\left(\frac{7\left(\frac{\int \frac{5\left(11a^3-9a^3\sec(c+dx)\right)}{2\sec^{\frac{5}{2}}(c+dx)}dx}{a^2}-\frac{9a^2\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(\frac{7\left(\frac{5\int \frac{11a^3-9a^3\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx}{2a^2}-\frac{9a^2\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{7\left(\frac{5\int \frac{11a^3-9a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx}{2a^2}-\frac{9a^2\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4274}$$

$$\frac{3\left(\frac{7\left(\frac{5\left(11a^3\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx-9a^3\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx\right)}{2a^2}-\frac{9a^2\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{7\left(\frac{5\left(11a^3\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx-9a^3\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx\right)}{2a^2}-\frac{9a^2\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4256}$$

$$\frac{3\left(\frac{7\left(\frac{5\left(11a^3\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-9a^3\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right)}{2a^2}-\frac{9a^2\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{7\left(\frac{5\left(11a^3\left(\frac{3}{5}\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-9a^3\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right)}{2a^2}-\frac{9a^2\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)}{3a^2}-\frac{}{3d\sec^{\frac{3}{2}}}\right)}{10a^2}$$

$$\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$\downarrow$ 4258

$$\frac{3\left(\frac{7\left(\frac{5\left(11a^3\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-9a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right)}{2a^2}-\frac{}{d\sec^{\frac{3}{2}}}\right)}{3a^2}\right)}{10a^2}$$

$$\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3042

$$\frac{3\left(\frac{7\left(\frac{5\left(11a^3\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-9a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right)}{2a^2}\right)}{3a^2}\right)}{10a^2}$$

$$\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3119

$$3\left(\frac{7\left(\frac{5\left(11a^3\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)-9a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right)}{2a^2}-\frac{}{d\,s}\right.}{3a^2}\right.}{10a^2}$$

$$\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3120

$$3\left(\frac{7\left(\frac{5\left(11a^3\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)-9a^3\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)\right)}{2a^2}-\frac{}{d\,s}\right.}{3a^2}\right.}{10a^2}$$

$$\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

input
```
Int[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),x]
```

output
```
-1/5*Sin[c + d*x]/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3) + (3*((-8*
a*Sin[c + d*x])/(3*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2) + (7*((-9*
a^2*Sin[c + d*x])/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])) + (5*(11*a^3
*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d
) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))) - 9*a^3*((2*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])
/(3*d*Sqrt[Sec[c + d*x]]))))/(2*a^2)))/(3*a^2)))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Maple [A] (verified)

Time = 5.05 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.20

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(64\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}-288\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}-76\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-210\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\right.}\right.}{20a^{3}\,c}$

input
```
int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/20/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(64*cos(
1/2*d*x+1/2*c)^12-288*cos(1/2*d*x+1/2*c)^10-76*cos(1/2*d*x+1/2*c)^8-210*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-462*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))+530*cos(1/2*d*x+1/2*c)^6-248*cos(1/2*d*x+1/2*c)^4+
19*cos(1/2*d*x+1/2*c)^2-1)/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.51

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx =$$

$$-\frac{105\left(-i\sqrt{2}\cos(dx+c)^3-3i\sqrt{2}\cos(dx+c)^2-3i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0}{}$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/20*(105*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*s
qrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + 105*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c
)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) + 231*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)
*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 231*(I*s
qrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x +
 c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c))) - 2*(4*cos(d*x + c)^5 - 8*cos(d*x + c)^4 - 147*co
s(d*x + c)^3 - 238*cos(d*x + c)^2 - 105*cos(d*x + c))*sin(d*x + c)/sqrt(co
s(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(
d*x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{1}{\sec^{\frac{11}{2}}(c+dx)+3\sec^{\frac{9}{2}}(c+dx)+3\sec^{\frac{7}{2}}(c+dx)+\sec^{\frac{5}{2}}(c+dx)}\,dx}{a^3}$$

input
```
integrate(1/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(1/(sec(c + d*x)**(11/2) + 3*sec(c + d*x)**(9/2) + 3*sec(c + d*x)*
*(7/2) + sec(c + d*x)**(5/2)), x)/a**3
```

Maxima [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^3*sec(d*x + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^3*sec(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^6+3\sec(dx+c)^5+3\sec(dx+c)^4+\sec(dx+c)^3}dx}{a^3}$$

input
```
int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x)
```

output
```
int(sqrt(sec(c + d*x))/(sec(c + d*x)**6 + 3*sec(c + d*x)**5 + 3*sec(c + d*
x)**4 + sec(c + d*x)**3),x)/a**3
```

3.218 $\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{3\sqrt{a}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{3a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}$$

output
```
3/4*a^(1/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+3/4*a*sec
(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a*sec(d*x+c)^(5/2)*s
in(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \frac{a\left(3\arcsin\left(\sqrt{1-\sec(c+dx)}\right)+2\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+3\sqrt{-((-1+\sec(c+dx))\sec(c+ \right.}{4d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(a*(3*ArcSin[Sqrt[1 - Sec[c + d*x]]] + 2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*
x]^(3/2) + 3*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Tan[c + d*x])/(4*d
*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 4290, 3042, 4290, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4290}$$

$$\frac{3}{4}\int \sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4290}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4288}$$

$$\frac{3}{4}\left(\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{222}$$

$$\frac{3}{4}\left(\frac{\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(a*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (3*((
Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*S
ec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.53

method	result
default	$\frac{\sec(dx+c)^{\frac{5}{2}}\sqrt{a(1+\sec(dx+c))}\left(3\cos(dx+c)^3\arctan\left(\frac{\cot(dx+c)-\csc(dx+c)+1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+3\cos(dx+c)^3\arctan\left(\frac{\cot(dx+c)-\csc(dx+c)-1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\right.}{8d(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*sec(d*x+c)^(5/2)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*
x+c)+1))^(1/2)*(3*cos(d*x+c)^3*arctan(1/2/(-1/(cos(d*x+c)+1))^(1/2)*(cot(d
*x+c)-csc(d*x+c)+1))+3*cos(d*x+c)^3*arctan(1/2*(cot(d*x+c)-csc(d*x+c)-1)/(
-1/(cos(d*x+c)+1))^(1/2))+sin(d*x+c)*(2+3*cos(d*x+c))*2^(1/2)*(-2/(cos(d*x
+c)+1))^(1/2)*cos(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 353, normalized size of antiderivative = 3.04

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \left[\frac{3\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+}{\sqrt{\cos(dx+c)}}}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{16\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}\right.$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7
*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(
d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
(3*cos(d*x + c) + 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 +
d*cos(d*x + c)), 1/8*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*arctan(1/
2*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))/(a*sqrt(cos(d*x + c))*sin(d*x + c))) + 2*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*(3*cos(d*x + c) + 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d
*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1264 vs. $2(96) = 192$.

Time = 0.24 (sec) , antiderivative size = 1264, normalized size of antiderivative = 10.90

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/16*(12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*
arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqr
t(2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(s
qrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(
d*x + c), cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*
d*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d
*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^
2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x
 + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt
(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4
*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)
^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*
d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(
sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 2) - 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*
d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4
*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*l...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(96) = 192$.

Time = 0.60 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.89

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \frac{\sqrt{2}\left(\frac{3\sqrt{2}a^{\frac{3}{2}}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|} - \frac{8\left(5\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^6 a^{\cdots}\right.}{\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\cdots\right.\right.}\right.}{\cdots}$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/16*sqrt(2)*(3*sqrt(2)*a^(3/2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt
(2)*abs(a) - 6*a))/abs(a) - 8*(5*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))^6*a^(3/2) + 19*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(5/2) - 17*(sqrt(a)*tan(1/2*d*x +
1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(7/2) + a^(9/2))/((sqrt(a
)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a
)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)
*sgn(cos(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx=\int \sqrt{a+\frac{a}{\cos(c+d\,x)}}\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}dx$$

input
```
int((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx=\sqrt{a}\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}\,\sec(dx+c)^2\,dx\right)$$

input
```
int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1)*sec(c + d*x)**2,x)
```

3.219 $\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 72

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx=\frac{\sqrt{a}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

output
```
a^(1/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+a*sec(d*x+c)^
(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx=\frac{a\left(\arcsin\left(\sqrt{1-\sec(c+dx)}\right)+\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(a*(ArcSin[Sqrt[1 - Sec[c + d*x]]] + Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*
x])])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4290, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4290}$$

$$\frac{1}{2}\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4288}$$

$$\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{222}$$

$$\frac{\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*
Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(62) = 124$.

Time = 2.24 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.29

method	result
default	$\frac{\sec(dx+c)^{\frac{3}{2}}\sqrt{a(1+\sec(dx+c))}\left(\sin(dx+c)\sqrt{2}\sqrt{-\frac{2}{\cos(dx+c)+1}}\cos(dx+c)+\cos(dx+c)^{2}\arctan\left(\frac{\cot(dx+c)-\csc(dx+c)-1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+\cos(dx}{2d(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*sec(d*x+c)^(3/2)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*
x+c)+1))^(1/2)*(sin(d*x+c)*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+co
s(d*x+c)^2*arctan(1/2*(cot(d*x+c)-csc(d*x+c)-1)/(-1/(cos(d*x+c)+1))^(1/2))
+cos(d*x+c)^2*arctan(1/2/(-1/(cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+
1)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(62) = 124$.

Time = 0.10 (sec) , antiderivative size = 299, normalized size of antiderivative = 4.15

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \left[\frac{\sqrt{a}(\cos(dx+c)+1)\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\sqrt{\cdots}}{4\left(d\cos(dx+c)+d\right)}\right.$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a)*(cos(d*x + c) + 1)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^
2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos
(d*x + c)^2)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqr
t(cos(d*x + c)))/(d*cos(d*x + c) + d), 1/2*(sqrt(-a)*(cos(d*x + c) + 1)*ar
ctan(1/2*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))/(a*sqrt(cos(d*x + c))*sin(d*x + c))) + 2*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)
+ d)]
```

Sympy [F]

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx=\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sec^{\frac{3}{2}}\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(62) = 124$.

Time = 0.22 (sec) , antiderivative size = 662, normalized size of antiderivative = 9.19

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx=\text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2
*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2
*c) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*l
og(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2)
+ (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2
*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
 + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos
(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2
*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
- 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)
*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c)))
+ 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(62) = 124$.

Time = 0.55 (sec) , antiderivative size = 256, normalized size of antiderivative = 3.56

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \frac{\sqrt{2}\left(\frac{\sqrt{2}a^{\frac{3}{2}}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|} + \frac{8\left(3\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\right.}\right.\right.}{\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4-6\left(\sqrt{a}\right.}\right.}{4d}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(sqrt(2)*a^(3/2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a
)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)
*abs(a) - 6*a))/abs(a) + 8*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))^2*a^(3/2) - a^(5/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c
) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c
) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))*sgn(cos(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx=\int\sqrt{a+\frac{a}{\cos(c+d\,x)}}\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx=\sqrt{a}\left(\int\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}\,\sec(dx+c)\,dx\right)$$

input
```
int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1)*sec(c + d*x),x)
```

3.220 $\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 37

$$\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx=\frac{2\sqrt{a}\,\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

output
```
2*a^(1/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.46

$$\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = -\frac{2\arcsin\left(\sqrt{\sec(c+dx)}\right)\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(-2*ArcSin[Sqrt[Sec[c + d*x]]]*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2]
)/(d*Sqrt[1 - Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \ 4288$$

$$-\frac{2\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \ 222$$

$$\frac{2\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

input
```
Int[Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(31) = 62$.

Time = 2.22 (sec) , antiderivative size = 122, normalized size of antiderivative = 3.30

method	result	size
default	$\frac{\left(\arctan\left(\frac{\cot(dx+c)-\csc(dx+c)-1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+\arctan\left(\frac{\cot(dx+c)-\csc(dx+c)+1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\right)\sqrt{\sec(dx+c)}\,\sqrt{a(1+\sec(dx+c))}\,\cos(dx+c)}{d(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$	122

input
```
int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(arctan(1/2*(cot(d*x+c)-csc(d*x+c)-1)/(-1/(cos(d*x+c)+1))^(1/2))+arcta
n(1/2/(-1/(cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)))*sec(d*x+c)^(1/
2)*(a*(1+sec(d*x+c)))^(1/2)*cos(d*x+c)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^
(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(31) = 62$.

Time = 0.09 (sec) , antiderivative size = 186, normalized size of antiderivative = 5.03

$$\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \left[\frac{\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{2\,d}, \frac{\sqrt{-a}\arctan\left(\frac{(\cos(dx+c)}{}\right.}{}\right.$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^
2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*
x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))/d, sqr
t(-a)*arctan(1/2*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))/(a*sqrt(cos(d*x + c))*sin(d*x + c)))/d]
```

Sympy [F]

$$\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sqrt{\sec\left(c+dx\right)}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*sqrt(sec(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(31) = 62.

Time = 0.22 (sec) , antiderivative size = 241, normalized size of antiderivative = 6.51

$$\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \frac{\sqrt{a}\left(\log\left(2\,\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sqrt{2}\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right)\right.}{}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(a)*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(31) = 62$.

Time = 0.44 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.24

$$\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = \frac{a^{\frac{3}{2}}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\operatorname{sgn}(\cos(dx+c))}{d|a|}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
a^(3/2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c
) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))*sgn(c
os(d*x + c))/(d*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a+\frac{a}{\cos(c+d\,x)}}\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = \sqrt{a}\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}dx\right)$$

input
```
int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1),x)
```

3.221 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 36

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx=\frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

output
```
2*a*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx=\frac{2\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{\sec(c+dx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{4291}$$
$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

method	result	size
default	$-\frac{2\sqrt{a(1+\sec(dx+c))}\,(\cot(dx+c)-\csc(dx+c))}{d\sqrt{\sec(dx+c)}}$	41
risch	$-\frac{i\sqrt{2}\,\sqrt{\frac{a\left(e^{i(dx+c)}+1\right)^{2}}{e^{2i(dx+c)}+1}}\,\left(e^{i(dx+c)}-1\right)}{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\left(e^{i(dx+c)}+1\right)d}$	89

input
```
int((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(a*(1+sec(d*x+c)))^(1/2)/sec(d*x+c)^(1/2)*(cot(d*x+c)-csc(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \frac{2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{d\cos(dx+c)+d}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/
(d*cos(d*x + c) + d)
```

Sympy [F]

$$\int \frac{\sqrt{a + a\sec(c + dx)}}{\sqrt{\sec(c + dx)}}\,dx = \int \frac{\sqrt{a\left(\sec\left(c + dx\right) + 1\right)}}{\sqrt{\sec\left(c + dx\right)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))/sqrt(sec(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{a + a\sec(c + dx)}}{\sqrt{\sec(c + dx)}}\,dx = \frac{2\sqrt{2}\sqrt{a}\sin\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{d}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(2)*sqrt(a)*sin(1/2*d*x + 1/2*c)/d
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{a + a\sec(c + dx)}}{\sqrt{\sec(c + dx)}}\,dx = \frac{2\sqrt{2}\sqrt{a}\operatorname{sgn}(\cos\left(dx + c\right))\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{d}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(2)*sqrt(a)*sgn(cos(d*x + c))*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x
+ 1/2*c)/d
```

Mupad [B] (verification not implemented)

Time = 10.60 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \frac{\sin(2c+2dx)\,\sqrt{\frac{1}{\cos(c+dx)}}\,\sqrt{\frac{a\,(\cos(c+dx)+1)}{\cos(c+dx)}}}{d\,(\cos(c+dx)+1)}$$

input
```
int((a + a/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/2),x)
```

output
```
(sin(2*c + 2*d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d
*x))^(1/2))/(d*(cos(c + d*x) + 1))
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)}dx\right)$$

input
```
int((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

output
```
sqrt(a)*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/sec(c + d*x),x)
```

3.222 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{4a\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}$$

output
```
2/3*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+4/3*a*sec(d*x+c
)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2(2+\cos(c+dx))\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{3d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

output
```
(2*(2 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(3*d*Sq
rt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 4292, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(c+dx)+a}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4292}$$

$$\frac{2}{3}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{ 4291}$$

$$\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

output
```
(2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (4*
a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{(2\sin(dx+c)+4\tan(dx+c))\sqrt{a(1+\sec(dx+c))}}{d(3\cos(dx+c)+3)\sec(dx+c)^{\frac{3}{2}}}$	54

input
```
int((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*sin(d*x+c)+4*tan(d*x+c))/(3*cos(d*x+c)+3)*(a*(1+sec(d*x+c)))^(1/2)/
sec(d*x+c)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(cos(d*x + c)^2 + 2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))/sec(c + d*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{2}\left(3\,\cos\left(\frac{2}{3}\,\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right)\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-3\,\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\sin\left(\frac{2}{3}\,\mathrm{arct}\right.\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(2)*(3*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)*sin(3/2*d*x + 3/2*c) - 3*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(3/2*d*x + 3/2*c) + 3*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)/d
```

Giac [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{a\sec(dx+c)+a}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.54 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\cos(c+d\,x)\,\left(4\,\sin(c+d\,x)+\sin(2\,c+2\,d\,x)\right)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}}{3\,d\,(\cos(c+d\,x)+1)}$$

input
```
int((a + a/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(3/2),x)
```

output
```
(cos(c + d*x)*(4*sin(c + d*x) + sin(2*c + 2*d*x))*(1/cos(c + d*x))^(1/2)*(
(a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2))/(3*d*(cos(c + d*x) + 1))
```

Reduce [F]

$$\int \frac{\sqrt{a + a\sec(c + dx)}}{\sec^{\frac{3}{2}}(c + dx)}\,dx = \sqrt{a}\left(\int \frac{\sqrt{\sec(dx + c)}\,\sqrt{\sec(dx + c) + 1}}{\sec(dx + c)^2}dx\right)$$

input
```
int((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x)
```

output
```
sqrt(a)*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/sec(c + d*x)**2,x)
```

3.223 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{8a\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{16a\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}$$

output
```
2/5*a*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+8/15*a*sin(d*x+
c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+16/15*a*sec(d*x+c)^(1/2)*sin(
d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{(19+8\cos(c+dx)+3\cos(2(c+dx)))\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{15d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(5/2),x]
```

output
```
((19 + 8*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan
[(c + d*x)/2])/(15*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 4292, 3042, 4292, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(c+dx)+a}}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4292}$$

$$\frac{4}{5}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{4}{5}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4292}$$

$$\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4291}$$

$$\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{4}{5}\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(5/2),x]
```

output
```
(2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (4*
((2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (4
*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{(6\sin(dx+c)+8\tan(dx+c)+16\sec(dx+c)\tan(dx+c))\sqrt{a(1+\sec(dx+c))}}{d(15\cos(dx+c)+15)\sec(dx+c)^{\frac{5}{2}}}$	68

input
```
int((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(6*sin(d*x+c)+8*tan(d*x+c)+16*sec(d*x+c)*tan(d*x+c))/(15*cos(d*x+c)+15
)*(a*(1+sec(d*x+c)))^(1/2)/sec(d*x+c)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\left(3\cos\left(dx+c\right)^{3}+4\cos\left(dx+c\right)^{2}+8\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
2/15*(3*cos(d*x + c)^3 + 4*cos(d*x + c)^2 + 8*cos(d*x + c))*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x
+ c)))
```

Sympy [F]

$$\int \frac{\sqrt{a + a\sec(c + dx)}}{\sec^{\frac{5}{2}}(c + dx)}\, dx = \int \frac{\sqrt{a\left(\sec\left(c + dx\right) + 1\right)}}{\sec^{\frac{5}{2}}\left(c + dx\right)}\, dx$$

input
```
integrate((a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(5/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))/sec(c + d*x)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(97) = 194.

Time = 0.21 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.77

$$\int \frac{\sqrt{a + a\sec(c + dx)}}{\sec^{\frac{5}{2}}(c + dx)}\, dx = \frac{\sqrt{2}\left(30\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx + \frac{5}{2}c\right), \cos\left(\frac{5}{2}dx + \frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx + \frac{5}{2}c\right) + 5\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx + \right.\right.\right.\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
1/60*sqrt(2)*(30*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))*sin(5/2*d*x + 5/2*c) + 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 30*cos(5/2*d*x + 5/2*c)*sin(4/5*arc
tan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*cos(5/2*d*x + 5/2*c)
*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 6*sin(5/2*
d*x + 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
))) + 30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sqr
t(a)/d
```

Giac [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\sqrt{a\sec(dx+c)+a}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.69 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\cos(c+d\,x)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}\,(35\,\sin(c+d\,x)+8\,\sin(2\,c+2\,d\,x)+3\,\sin(3\,c+3\,d\,x))}{30\,d\,(\cos(c+d\,x)+1)}$$

input
```
int((a + a/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(5/2),x)
```

output
```
(cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*x))
^(1/2)*(35*sin(c + d*x) + 8*sin(2*c + 2*d*x) + 3*sin(3*c + 3*d*x)))/(30*d*
(cos(c + d*x) + 1))
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^{3}}dx\right)$$

input
```
int((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x)
```

output
```
sqrt(a)*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/sec(c + d*x)**3,x)
```

3.224 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 153

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{12a\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{16a\sin(c+dx)}{35d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{32a\sqrt{\sec(c+dx)}\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}}$$

output
```
2/7*a*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+12/35*a*sin(d*x
+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+16/35*a*sin(d*x+c)/d/sec(d*x
+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+32/35*a*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+
a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{(76+47\cos(c+dx)+12\cos(2(c+dx))+5\cos(3(c+dx)))\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{70d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(7/2),x]
```

output
```
((76 + 47*Cos[c + d*x] + 12*Cos[2*(c + d*x)] + 5*Cos[3*(c + d*x)])*Sqrt[a*
(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(70*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4292, 3042, 4292, 3042, 4292, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(c+dx)+a}}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4292}$$

$$\frac{6}{7}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4292}$$

$$\frac{6}{7}\left(\frac{4}{5}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\left(\frac{4}{5}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4292}$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+c}}\right.$$
$$\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+c}}\right.$$
$$\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4291}$$

$$\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+$$
$$\frac{6}{7}\left(\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{4}{5}\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right.\right.$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(7/2),x]
```

output
```
(2*a*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (6*
((2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (4
*((2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (
4*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])))/5))/
7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.47

method	result	size
default	$\frac{2\left(5\cos(dx+c)^{3}+6\cos(dx+c)^{2}+8\cos(dx+c)+16\right)\sqrt{a(1+\sec(dx+c))}\,\tan(dx+c)}{35d(\cos(dx+c)+1)\sec(dx+c)^{\frac{3}{2}}}$	72

input
```
int((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/35/d*(5*cos(d*x+c)^3+6*cos(d*x+c)^2+8*cos(d*x+c)+16)*(a*(1+sec(d*x+c)))^
(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^(3/2)*tan(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\left(5\cos\left(dx+c\right)^{4}+6\cos\left(dx+c\right)^{3}+8\cos\left(dx+c\right)^{2}+16\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{35\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
2/35*(5*cos(d*x + c)^4 + 6*cos(d*x + c)^3 + 8*cos(d*x + c)^2 + 16*cos(d*x
+ c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c
) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(129) = 258.

Time = 0.21 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.92

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{2}\left(105\cos\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+35\cos\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}dx\right.\right.\right.\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
1/280*sqrt(2)*(105*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2
*c)))*sin(7/2*d*x + 7/2*c) + 35*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(
7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 7*cos(2/7*arctan2(sin(7/2*d*x +
7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 105*cos(7/2*d*x + 7/
2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 35*cos
(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*
c))) - 7*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/
2*d*x + 7/2*c))) + 10*sin(7/2*d*x + 7/2*c) + 7*sin(5/7*arctan2(sin(7/2*d*x
 + 7/2*c), cos(7/2*d*x + 7/2*c))) + 35*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c))) + 105*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(
7/2*d*x + 7/2*c))))*sqrt(a)/d
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\sqrt{2}\left(35\,a^4+\left(35\,a^4+\left(9\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+49\,a^4\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right)\mathrm{sgn}(\cos\left(dx\right.}{35\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\right)^{\frac{7}{2}}d}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
2/35*sqrt(2)*(35*a^4 + (35*a^4 + (9*a^4*tan(1/2*d*x + 1/2*c)^2 + 49*a^4)*t
an(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*sgn(cos(d*x + c))*tan(1/2*d
*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 + a)^(7/2)*d)
```

Mupad [B] (verification not implemented)

Time = 11.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{\cos(c+d\,x)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}\,(140\,\sin(c+d\,x)+42\,\sin(2\,c+2\,d\,x)+12\,\sin(3\,c+3\,d\,x)+5}{140\,d\,(\cos(c+d\,x)+1)}$$

input
```
int((a + a/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(7/2),x)
```

output
```
(cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*x))
^(1/2)*(140*sin(c + d*x) + 42*sin(2*c + 2*d*x) + 12*sin(3*c + 3*d*x) + 5*s
in(4*c + 4*d*x)))/(140*d*(cos(c + d*x) + 1))
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^4}dx\right)$$

input
```
int((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x)
```

output
```
sqrt(a)*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/sec(c + d*x)**4,x)
```

3.225 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 160

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx=\frac{11a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d}+\frac{11a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{11a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}$$

output
```
11/8*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+11/8*a^2
*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+11/12*a^2*sec(d*x+c)
^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/3*a^2*sec(d*x+c)^(7/2)*sin(d*
x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.85

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a^2\left(33\arcsin\left(\sqrt{1-\sec(c+dx)}\right)+22\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+8\sqrt{1-\sec(c+dx)}\right.}{24d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\mathrm{se}}}$$

input
```
Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(a^2*(33*ArcSin[Sqrt[1 - Sec[c + d*x]]] + 22*Sqrt[1 - Sec[c + d*x]]*Sec[c
+ d*x]^(3/2) + 8*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2) + 33*Sqrt[-((-1
 + Sec[c + d*x])*Sec[c + d*x])])*Tan[c + d*x])/(24*d*Sqrt[1 - Sec[c + d*x]
]*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4301, 27, 3042, 4290, 3042, 4290, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{ 4301}$$

$$\frac{1}{3}a\int \frac{11}{2}\sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{11}{6}a\int \sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$\big\downarrow$ 3042

$$\frac{11}{6}a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$\big\downarrow$ 4290

$$\frac{11}{6}a\left(\frac{3}{4}\int \sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$\big\downarrow$ 3042

$$\frac{11}{6}a\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$\big\downarrow$ 4290

$$\frac{11}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$\big\downarrow$ 3042

$$\frac{11}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\text{se}}{2d\sqrt{a\sec(c+}}\right.$$
$$\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$\big\downarrow$ 4288

$$\frac{11}{6}a\left(\frac{3}{4}\left(\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 222$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{11}{6}a\left(\frac{3}{4}\left(\frac{\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)$$

input
```
Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(a^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (11
*a*((a*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (
3*((Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 3.05 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.14

method	result
default	$\frac{a\left(33\cos(dx+c)^3\arctan\left(\frac{\cot(dx+c)-\csc(dx+c)-1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+33\cos(dx+c)^3\arctan\left(\frac{\cot(dx+c)-\csc(dx+c)+1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+\sin(dx+c)\left(33\cos(dx+c)^2-\right.\right.}{48d(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/d*a*(33*cos(d*x+c)^3*arctan(1/2*(cot(d*x+c)-csc(d*x+c)-1)/(-1/(cos(d*
x+c)+1))^(1/2))+33*cos(d*x+c)^3*arctan(1/2/(-1/(cos(d*x+c)+1))^(1/2)*(cot(
d*x+c)-csc(d*x+c)+1))+sin(d*x+c)*(33*cos(d*x+c)^2+22*cos(d*x+c)+8)*2^(1/2)
*(-2/(cos(d*x+c)+1))^(1/2))*(a*(1+sec(d*x+c)))^(1/2)*sec(d*x+c)^(5/2)/(cos
(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 397, normalized size of antiderivative = 2.48

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left[\frac{33\left(a\cos\left(dx+c\right)^3+a\cos\left(dx+c\right)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\cdots}{\sqrt{\cos(dx\cdots}}\cdots}{\cos(dx+c)^3+\cos(dx+c)^2}\cdots\right.}{96\left(d\cos\left(dx+c\right)^3+d\cos\cdots\right.}\cdots\right.$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/96*(33*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c
)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a
)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(33*a*cos(d*x + c)^2 + 22*a*cos(d
*x + c) + 8*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(c
os(d*x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), 1/48*(33*(a*cos(d*x +
 c)^3 + a*cos(d*x + c)^2)*sqrt(-a)*arctan(1/2*(cos(d*x + c)^2 - 2*cos(d*x
+ c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))/(a*sqrt(cos(d*x + c
))*sin(d*x + c))) + 2*(33*a*cos(d*x + c)^2 + 22*a*cos(d*x + c) + 8*a)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*co
s(d*x + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2361 vs. $2(134) = 268$.

Time = 0.32 (sec) , antiderivative size = 2361, normalized size of antiderivative = 14.76

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/96*(132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*
sqrt(2)*a*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) +
3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c)
+ 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c
) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) - 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*
c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) - 132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x +
4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*
cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*
d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x
 + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*
c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(134) = 268$.

Time = 1.98 (sec) , antiderivative size = 472, normalized size of antiderivative = 2.95

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/48*(33*a^(3/2)*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))*sgn(cos(d*x + c)) - 33*a^(3/2)*
log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)
)^2 + a*(2*sqrt(2) - 3)))*sgn(cos(d*x + c)) + 4*(33*sqrt(2)*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*a^(5/2)*sgn(cos(d
*x + c)) - 303*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
+ 1/2*c)^2 + a))^8*a^(7/2)*sgn(cos(d*x + c)) + 2394*sqrt(2)*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*a^(9/2)*sgn(cos(d*
x + c)) - 1806*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
+ 1/2*c)^2 + a))^4*a^(11/2)*sgn(cos(d*x + c)) + 309*sqrt(2)*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(13/2)*sgn(cos(d
*x + c)) - 19*sqrt(2)*a^(15/2)*sgn(cos(d*x + c)))/((sqrt(a)*tan(1/2*d*x +
1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x +
1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

input
```
int((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx=\sqrt{a}\,a\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}\,\sec(dx+c)^{3}\,dx+\int \sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}\,\sec(dx+c)^{2}\,dx\right)$$

input
```
int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1)*sec(c + d*x)**3,x
) + int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1)*sec(c + d*x)**2,x))
```

3.226 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 120

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx=\frac{7a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}+\frac{7a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}$$

output
```
7/4*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+7/4*a^2*s
ec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a^2*sec(d*x+c)^(5/
2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx=\frac{a^2\left(7\arcsin\left(\sqrt{1-\sec(c+dx)}\right)+2\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+7\sqrt{-((-1+\sec(c+\ldots}{4d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(a^2*(7*ArcSin[Sqrt[1 - Sec[c + d*x]]] + 2*Sqrt[1 - Sec[c + d*x]]*Sec[c +
d*x]^(3/2) + 7*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Tan[c + d*x])/(4
*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4301, 27, 3042, 4290, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{1}{2}a\int \frac{7}{2}\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{7}{4}a\int \sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4290}$$

$$\frac{7}{4}a\left(\frac{1}{2}\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}a\left(\frac{1}{2}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4288}$$

$$\frac{7}{4}a\left(\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{222}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}+\frac{7}{4}a\left(\frac{\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

input
```
Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (7*
a*((Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.43

method	result
default	$\frac{a\left(7\cos(dx+c)^2\arctan\left(\frac{\cot(dx+c)-\csc(dx+c)+1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+7\cos(dx+c)^2\arctan\left(\frac{\cot(dx+c)-\csc(dx+c)-1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+\sin(dx+c)(7\cos(dx+c)+2)\sqrt{}\right.}{8d(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*a*(7*cos(d*x+c)^2*arctan(1/2/(-1/(cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-c
sc(d*x+c)+1))+7*cos(d*x+c)^2*arctan(1/2*(cot(d*x+c)-csc(d*x+c)-1)/(-1/(cos
(d*x+c)+1))^(1/2))+sin(d*x+c)*(7*cos(d*x+c)+2)*2^(1/2)*(-2/(cos(d*x+c)+1))
^(1/2))*(a*(1+sec(d*x+c)))^(1/2)*sec(d*x+c)^(3/2)/(cos(d*x+c)+1)/(-1/(cos(
d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 367, normalized size of antiderivative = 3.06

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left[\frac{7\left(a\cos\left(dx+c\right)^2+a\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\cdots}}{\sqrt{\cos(dx+c\cdots}}}{\cos(dx+c)^3+\cos(dx+c)^2}\cdots\right.}{16\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+\cdots\right.\right.}\right.$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(7*(a*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3
 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(
cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(7*a*cos(d*x + c) + 2*a)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x +
 c)^2 + d*cos(d*x + c)), 1/8*(7*(a*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(-
a)*arctan(1/2*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))/(a*sqrt(cos(d*x + c))*sin(d*x + c))) + 2*(7*a*cos(d
*x + c) + 2*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(c
os(d*x + c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2244 vs. $2(100) = 200$.

Time = 0.34 (sec) , antiderivative size = 2244, normalized size of antiderivative = 18.70

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/16*(56*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 24*s
qrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(
4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*sqrt(2)*a*si
n(3/2*d*x + 3/2*c) + 28*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 4*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*s
in(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*
sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*(3*sqrt(2)*a*sin(3/2
*d*x + 3/2*c) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) - 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*si
n(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos...
```

Giac [F(-2)]

Exception generated.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{%%{[
%%%{%%{[-469762048,0]:[1,0,-2]%%},[14]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,1
]%%%} / %%%{
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

input
```
int((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \sqrt{a}\,a\left(\int \sqrt{\sec\left(dx+c\right)}\,\sqrt{\sec\left(dx+c\right)+1}\,\sec\left(dx+c\right)^{2}dx + \int \sqrt{\sec\left(dx+c\right)}\,\sqrt{\sec\left(dx+c\right)+1}\,\sec\left(dx+c\right)dx\right)$$

input
```
int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1)*sec(c + d*x)**2,x
) + int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1)*sec(c + d*x),x))
```

3.227 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 75

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx = \frac{3a^{3/2}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

output
```
3*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+a^2*sec(d*x
+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx = \frac{a^2\left(-3\arcsin\left(\sqrt{\sec(c+dx)}\right)+\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(a^2*(-3*ArcSin[Sqrt[Sec[c + d*x]]] + Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d
*x])])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 4301, 27, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4301}$$

$$a\int \frac{3}{2}\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{2}a\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4288}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{3a\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{222}$$

$$\frac{3a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(3*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(65) = 130.

Time = 2.89 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.11

method	result
default	$\frac{a\sqrt{\sec(dx+c)}\,\sqrt{a(1+\sec(dx+c))}\left(\sin(dx+c)\sqrt{2}\,\sqrt{-\frac{2}{\cos(dx+c)+1}}-3\cos(dx+c)\arctan\left(\frac{-\cot(dx+c)+\csc(dx+c)+1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-3\cos(dx+c)\,a\right.}{2d(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*a*sec(d*x+c)^(1/2)*(a*(1+sec(d*x+c)))^(1/2)*(sin(d*x+c)*2^(1/2)*(-2/
(cos(d*x+c)+1))^(1/2)-3*cos(d*x+c)*arctan(1/2/(-1/(cos(d*x+c)+1))^(1/2)*(-
cot(d*x+c)+csc(d*x+c)+1))-3*cos(d*x+c)*arctan(1/2*(-cot(d*x+c)+csc(d*x+c)-
1)/(-1/(cos(d*x+c)+1))^(1/2)))/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(65) = 130.

Time = 0.12 (sec) , antiderivative size = 307, normalized size of antiderivative = 4.09

$$\int\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx=\left[\frac{3\,(a\cos(dx+c)+a)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx}{\sqrt{\cos(dx+c)}}}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{4\,(d\cos(dx+c)+d)}\right.$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(3*(a*cos(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x +
 c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 +
 cos(d*x + c)^2)) + 4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d), 1/2*(3*(a*cos(d*x + c) + a)*s
qrt(-a)*arctan(1/2*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(-a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))/(a*sqrt(cos(d*x + c))*sin(d*x + c))) + 2*a*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*c
os(d*x + c) + d)]
```

Sympy [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx = \int (a(\sec(c+dx)+1))^{\frac{3}{2}}\sqrt{\sec(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)*sqrt(sec(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1143 vs. $2(65) = 130$.

Time = 0.22 (sec) , antiderivative size = 1143, normalized size of antiderivative = 15.24

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/4*(3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 + a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*
c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a
*sin(3/2*d*x + 3/2*c) - 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 2*(2*sqrt(2)*a*
sin(3/2*d*x + 3/2*c) - 2*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*
c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(65) = 130$.

Time = 1.35 (sec) , antiderivative size = 258, normalized size of antiderivative = 3.44

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx = \frac{\sqrt{2}a^{\frac{7}{2}}\left(\frac{3\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-4\sqrt{2}|a|-6\,a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2+4\sqrt{2}|a|-6\,a\right|}\right)}{a|a|}+\frac{8\left(3\left(\sqrt{a}\tan\ldots\right.\right.}{\left(\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\ldots\right.}\right.\right.}\right.}{4\,d}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*a^(7/2)*(3*sqrt(2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt
(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(
2)*abs(a) - 6*a))/(a*abs(a)) + 8*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a
*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)*a))*sgn(cos(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx=\int \left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx=\sqrt{a}\,a\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}\,\sec(dx+c)\,dx+\int \sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}dx\right)$$

input
```
int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1)*sec(c + d*x),x) +
 int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1),x))
```

3.228 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 76

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx=\frac{2a^{3/2}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

output
```
2*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2*a^2*sec(d
*x+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx=\frac{2a^2\left(\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\sin(c+dx)+\arcsin\left(\sqrt{1-\sec(c\right.\right.}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*a^2*(Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sin[c + d*x] + ArcSin[Sq
rt[1 - Sec[c + d*x]]]*Tan[c + d*x]))/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 +
 Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4300, 27, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{3/2}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4300}$$

$$2a\int \frac{1}{2}\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$a\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4288}$$

$$\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{2a\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{222}$$

$$\frac{2a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[(a + a*Sec[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(66) = 132.

Time = 2.90 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.93

method	result
default	$\frac{\left(-\frac{\sqrt{2}\,\arctan\left(\frac{\cot(dx+c)-\csc(dx+c)-1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{2}{\cos(dx+c)+1}}}{2}-\frac{\sqrt{2}\,\arctan\left(\frac{\cot(dx+c)-\csc(dx+c)+1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{2}{\cos(dx+c)+1}}}{2}-2\cot(dx+c)+2\csc(\right.}{d\sqrt{\sec(dx+c)}}$

input
```
int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*2^(1/2)*arctan(1/2*(cot(d*x+c)-csc(d*x+c)-1)/(-1/(cos(d*x+c)+1))
^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)-1/2*2^(1/2)*arctan(1/2/(-1/(cos(d*x+c)+1
))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1))*(-2/(cos(d*x+c)+1))^(1/2)-2*cot(d*x+c)
+2*csc(d*x+c))*a*(a*(1+sec(d*x+c)))^(1/2)/sec(d*x+c)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(66) = 132.

Time = 0.10 (sec) , antiderivative size = 304, normalized size of antiderivative = 4.00

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \left[\frac{4\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+(a\cos(dx+c)+a)\sqrt{a}\log\left(\right.}{2\,(d\cos(dx+}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c) + (a*cos(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x
 + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3
 + cos(d*x + c)^2)))/(d*cos(d*x + c) + d), (2*a*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + (a*cos(d*x + c) + a)*sqrt(
-a)*arctan(1/2*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))/(a*sqrt(cos(d*x + c))*sin(d*x + c))))/(d*cos(d*x +
 c) + d)]
```

Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a(\sec(c+dx)+1))^{\frac{3}{2}}}{\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)/sqrt(sec(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(66) = 132$.

Time = 0.22 (sec) , antiderivative size = 274, normalized size of antiderivative = 3.61

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{\sqrt{2}\left(\sqrt{2}a\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}\right.\right.\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*(sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) - sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(
2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8*a*sin(1/2*d*x + 1/2*c
))*sqrt(a)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(66) = 132$.

Time = 1.22 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.14

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{\frac{2\sqrt{2}a^2\operatorname{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}} + \frac{a^{\frac{5}{2}}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-4\sqrt{2}\ldots\right.}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2+4\sqrt{2}\ldots\right.}\ldots\right.}{|a|}\ldots}{d}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
(2*sqrt(2)*a^2*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x +
 1/2*c)^2 + a) + a^(5/2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*
tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a)*ta
n(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs
(a) - 6*a))*sgn(cos(d*x + c))/abs(a))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \sqrt{a}\,a\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)}dx + \int \sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}dx\right)$$

input
```
int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)
```

output
```
sqrt(a)*a*(int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/sec(c + d*x),x)
 + int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1),x))
```

3.229 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 79

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{8a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2a\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
8/3*a^2*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a*(a+a*se
c(d*x+c))^(1/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.63

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a(5+\cos(c+dx))\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{3d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

output
```
(2*a*(5 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(3*d*
Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 4296, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4296}$$

$$\frac{4}{3}a\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4291}$$

$$\frac{8a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

input
```
Int[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

output
```
(8*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (
2*a*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4296
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)
*((d*Csc[e + f*x])^n/(f*m)), x] + Simp[b*((2*m - 1)/(d*m))   Int[(a + b*Csc
[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f
, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{(2\sin(dx+c)+10\tan(dx+c))a\sqrt{a(1+\sec(dx+c))}}{d(3\cos(dx+c)+3)\sec(dx+c)^{\frac{3}{2}}}$	55

input
```
int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*sin(d*x+c)+10*tan(d*x+c))/(3*cos(d*x+c)+3)*a*(a*(1+sec(d*x+c)))^(1/
2)/sec(d*x+c)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.87

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\left(a\cos\left(dx+c\right)^{2}+5\,a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(a*cos(d*x + c)^2 + 5*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(a(\sec\left(c+dx\right)+1)\right)^{\frac{3}{2}}}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)/sec(c + d*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.48

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\left(\sqrt{2}a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+9\,\sqrt{2}a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{3\,d}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
1/3*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*sq
rt(a)/d
```

Giac [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{4\left(2\sqrt{2}a^3\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3\sqrt{2}a^3\mathrm{sgn}(\cos(dx+c))\right)\mathrm{ta}}{3\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}d}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
4/3*(2*sqrt(2)*a^3*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 + 3*sqrt(2)*a^
3*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 + a)^
(3/2)*d)
```

Mupad [B] (verification not implemented)

Time = 10.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{a\,\cos(c+d\,x)\,(10\,\sin(c+d\,x)+\sin(2\,c+2\,d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(c+d\,x)+}{\cos(c+d\,x)}}}{3\,d\,(\cos(c+d\,x)+1)}$$

input
```
int((a + a/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(3/2),x)
```

output
```
(a*cos(c + d*x)*(10*sin(c + d*x) + sin(2*c + 2*d*x))*(1/cos(c + d*x))^(1/2
)*((a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2))/(3*d*(cos(c + d*x) + 1))
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \sqrt{a}\,a\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^2}dx + \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)}dx\right)$$

input
```
int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x)
```

output
```
sqrt(a)*a*(int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/sec(c + d*x)**2
,x) + int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/sec(c + d*x),x))
```

3.230 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\frac{8a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}}+\frac{2a\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{5d\sqrt{\sec(c+dx)}}+\frac{2(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

output
```
8/5*a^2*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/5*a*(a+a*se
c(d*x+c))^(1/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/5*(a+a*sec(d*x+c))^(3/2)*s
in(d*x+c)/d/sec(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\frac{a(13+6\cos(c+dx)+\cos(2(c+dx)))\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{5d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/2),x]
```

output
```
(a*(13 + 6*Cos[c + d*x] + Cos[2*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan
[(c + d*x)/2])/(5*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4299, 3042, 4296, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4299}$$

$$\frac{3}{5}\int \frac{(\sec(c+dx)a+a)^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4296}$$

$$\frac{3}{5}\left(\frac{4}{3}a\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}\left(\frac{4}{3}a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 4291

$$\frac{3}{5}\left(\frac{8a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/2),x]
```

output
```
(2*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (3*
((8*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) +
(2*a*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4296
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)
*((d*Csc[e + f*x])^n/(f*m)), x] + Simp[b*((2*m - 1)/(d*m))   Int[(a + b*Csc
[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f
, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

rule 4299
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(m + 1))), x] + Simp[a*(m/(b*d*(m + 1)))   Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{(2\sin(dx+c)+6\tan(dx+c)+12\sec(dx+c)\tan(dx+c))a\sqrt{a(1+\sec(dx+c))}}{d(5\cos(dx+c)+5)\sec(dx+c)^{\frac{5}{2}}}$	69

input
```
int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*sin(d*x+c)+6*tan(d*x+c)+12*sec(d*x+c)*tan(d*x+c))/(5*cos(d*x+c)+5)*
a*(a*(1+sec(d*x+c)))^(1/2)/sec(d*x+c)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\left(a\cos(dx+c)^3+3\,a\cos(dx+c)^2+6\,a\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(d}{5\left(d\cos(dx+c)+d\right)\sqrt{\cos(dx+c)}}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
2/5*(a*cos(d*x + c)^3 + 3*a*cos(d*x + c)^2 + 6*a*cos(d*x + c))*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d
*x + c)))
```

Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(a(\sec{(c+dx)}+1))^{\frac{3}{2}}}{\sec^{\frac{5}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(5/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)/sec(c + d*x)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(98) = 196$.

Time = 0.20 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.81

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\frac{\sqrt{2}\left(20\,a\cos\left(\frac{4}{5}\,\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
1/20*sqrt(2)*(20*a*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2
*c)))*sin(5/2*d*x + 5/2*c) + 5*a*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos
(5/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 20*a*cos(5/2*d*x + 5/2*c)*sin(4
/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*a*cos(5/2*d*x
+ 5/2*c)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 2*
a*sin(5/2*d*x + 5/2*c) + 5*a*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
*d*x + 5/2*c))) + 20*a*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x +
 5/2*c))))*sqrt(a)/d
```

Giac [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.87

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{4\left(5\sqrt{2}a^4\text{sgn}(\cos(dx+c)) + \left(2\sqrt{2}a^4\text{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + \right.\right.}{5\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right.}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
4/5*(5*sqrt(2)*a^4*sgn(cos(d*x + c)) + (2*sqrt(2)*a^4*sgn(cos(d*x + c))*ta
n(1/2*d*x + 1/2*c)^2 + 5*sqrt(2)*a^4*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*
c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 + a)^(5/2)*d)
```

Mupad [B] (verification not implemented)

Time = 10.38 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{a\,\cos(c+d\,x)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,(25\,\sin(c+d\,x)+6\,\sin(2\,c+2\,d\,x)+\sin(3\,c+}{10\,d\,(\cos(c+d\,x)+1)}$$

input
```
int((a + a/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(5/2),x)
```

output
```
(a*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*(25*sin(c + d*x) + 6*sin(2*c + 2*d*
x) + sin(3*c + 3*d*x))*((a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2))/(10*d*
(cos(c + d*x) + 1))
```

Reduce [F]

$$\int \frac{(a + a\sec(c + dx))^{3/2}}{\sec^{\frac{5}{2}}(c + dx)}\,dx = \sqrt{a}\,a\left(\int \frac{\sqrt{\sec\left(dx + c\right)}\,\sqrt{\sec\left(dx + c\right) + 1}}{\sec\left(dx + c\right)^{3}}dx + \int \frac{\sqrt{\sec\left(dx + c\right)}\,\sqrt{\sec\left(dx + c\right) + 1}}{\sec\left(dx + c\right)^{2}}dx\right)$$

input
```
int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x)
```

output
```
sqrt(a)*a*(int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/sec(c + d*x)**3
,x) + int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/sec(c + d*x)**2,x))
```

3.231 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 161

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{26a^2\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{104a^2\sin(c+dx)}{105d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{208a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}}$$

output
```
2/7*a^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+26/35*a^2*sin
(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+104/105*a^2*sin(d*x+c)/d
/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+208/105*a^2*sec(d*x+c)^(1/2)*sin(
d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.45

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{a(494+253\cos(c+dx)+78\cos(2(c+dx))+15\cos(3(c+dx)))\sqrt{a(1+\mathrm{se}}}{210d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/2),x]
```

output
```
(a*(494 + 253*Cos[c + d*x] + 78*Cos[2*(c + d*x)] + 15*Cos[3*(c + d*x)])*Sq
rt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(210*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 4300, 27, 3042, 4292, 3042, 4292, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4300}$$

$$\frac{2}{7}a\int \frac{13\sqrt{\sec(c+dx)a+a}}{2\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{13}{7}a\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{13}{7}a\int \frac{\sqrt{\csc \left(c+dx+\frac{\pi}{2}\right) a+a}}{\csc \left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4292}$$

$$\frac{13}{7}a\left(\frac{4}{5}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{13}{7}a\left(\frac{4}{5}\int \frac{\sqrt{\csc \left(c+dx+\frac{\pi}{2}\right) a+a}}{\csc \left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4292}$$

$$\frac{13}{7}a\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+}}\right.$$
$$\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{13}{7}a\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\csc \left(c+dx+\frac{\pi}{2}\right) a+a}}{\sqrt{\csc \left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c}}\right.$$
$$\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4291}$$

$$\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+$$
$$\frac{13}{7}a\left(\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{4}{5}\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+}}\right.\right.$$

input
```
Int[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/2),x]
```

output
```
(2*a^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (
13*a*((2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
 + (4*((2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]
) + (4*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])))
/5))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.45

method	result	size
default	$\frac{2a\left(15\cos(dx+c)^3+39\cos(dx+c)^2+52\cos(dx+c)+104\right)\sqrt{a(1+\sec(dx+c))}\,\tan(dx+c)}{105d(\cos(dx+c)+1)\sec(dx+c)^{\frac{3}{2}}}$	73

input
```
int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/105/d*a*(15*cos(d*x+c)^3+39*cos(d*x+c)^2+52*cos(d*x+c)+104)*(a*(1+sec(d*
x+c)))^(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^(3/2)*tan(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.57

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\left(15\,a\cos\left(dx+c\right)^4+39\,a\cos\left(dx+c\right)^3+52\,a\cos\left(dx+c\right)^2+104\,a\cos\left(dx\right.\right.}{105\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
2/105*(15*a*cos(d*x + c)^4 + 39*a*cos(d*x + c)^3 + 52*a*cos(d*x + c)^2 + 1
04*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((
d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(137) = 274.

Time = 0.20 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.88

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
1/840*sqrt(2)*(735*a*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7
/2*c)))*sin(7/2*d*x + 7/2*c) + 175*a*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c),
 cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 63*a*cos(2/7*arctan2(sin(7/
2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 735*a*cos(7/
2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))
) - 175*a*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7
/2*d*x + 7/2*c))) - 63*a*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x
+ 7/2*c), cos(7/2*d*x + 7/2*c))) + 30*a*sin(7/2*d*x + 7/2*c) + 63*a*sin(5/
7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*a*sin(3/7*arc
tan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 735*a*sin(1/7*arctan2(
sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(a)/d
```

Giac [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.81

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{4\left(105\sqrt{2}a^5\text{sgn}(\cos(dx+c)) + \left(140\sqrt{2}a^5\text{sgn}(\cos(dx+c)) + 19\left(2\sqrt{2}a^5\text{s}\right.\right.\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
4/105*(105*sqrt(2)*a^5*sgn(cos(d*x + c)) + (140*sqrt(2)*a^5*sgn(cos(d*x +
c)) + 19*(2*sqrt(2)*a^5*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 + 7*sqrt(
2)*a^5*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*
tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 + a)^(7/2)*d)
```

Mupad [B] (verification not implemented)

Time = 10.83 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.58

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{a\,\cos\left(c+d\,x\right)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}\,\left(910\,\sin\left(c+d\,x\right)+238\,\sin\left(2\,c+\right.\right.}{420\,d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

input
```
int((a + a/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(7/2),x)
```

output
```
(a*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*x
))^(1/2)*(910*sin(c + d*x) + 238*sin(2*c + 2*d*x) + 78*sin(3*c + 3*d*x) +
15*sin(4*c + 4*d*x)))/(420*d*(cos(c + d*x) + 1))
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \sqrt{a}\,a\left(\int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{\sec\left(dx+c\right)+1}}{\sec\left(dx+c\right)^{4}}dx + \int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{\sec\left(dx+c\right)+1}}{\sec\left(dx+c\right)^{3}}dx\right)$$

input
```
int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x)
```

output
```
sqrt(a)*a*(int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/sec(c + d*x)**4
,x) + int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/sec(c + d*x)**3,x))
```

3.232 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 201

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{34a^2\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{68a^2\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{272a^2\sin(c+dx)}{315d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{544a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}}$$

output
```
2/9*a^2*sin(d*x+c)/d/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2)+34/63*a^2*sin
(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+68/105*a^2*sin(d*x+c)/d/
sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+272/315*a^2*sin(d*x+c)/d/sec(d*x+c
)^(1/2)/(a+a*sec(d*x+c))^(1/2)+544/315*a^2*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(
a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.40

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a^2(35+85\sec(c+dx)+102\sec^2(c+dx)+136\sec^3(c+dx)+272\sec^4(c}{315d\sec^{\frac{7}{2}}(c+dx)\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(9/2),x]
```

output
```
(2*a^2*(35 + 85*Sec[c + d*x] + 102*Sec[c + d*x]^2 + 136*Sec[c + d*x]^3 + 2
72*Sec[c + d*x]^4)*Sin[c + d*x])/(315*d*Sec[c + d*x]^(7/2)*Sqrt[a*(1 + Sec
[c + d*x])])
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4300, 27, 3042, 4292, 3042, 4292, 3042, 4292, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{4300}$$

$$\frac{2}{9}a\int \frac{17\sqrt{\sec(c+dx)a+a}}{2\sec^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{17}{9}a\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{17}{9}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4292}$$

$$\frac{17}{9}a\left(\frac{6}{7}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{17}{9}a\left(\frac{6}{7}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4292}$$

$$\frac{17}{9}a\left(\frac{6}{7}\left(\frac{4}{5}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+}}\right.$$
$$\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{17}{9}a\left(\frac{6}{7}\left(\frac{4}{5}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+}}\right.$$
$$\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4292}$$

$$\frac{17}{9}a\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+a}}\right.\right.$$
$$\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{17}{9}a\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right.\right.$$
$$\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4291}$$

$$\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+$$
$$\frac{17}{9}a\left(\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{6}{7}\left(\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{4}{5}\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}\right.\right.\right.$$

input
```
Int[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(9/2),x]
```

output
```
(2*a^2*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (
17*a*((2*a*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]])
 + (6*((2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]
) + (4*((2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]
]) + (4*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]))
)/5))/7))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.41

method	result	size
default	$\frac{2a\left(35\cos(dx+c)^4+85\cos(dx+c)^3+102\cos(dx+c)^2+136\cos(dx+c)+272\right)\sqrt{a(1+\sec(dx+c))}\,\tan(dx+c)}{315d(\cos(dx+c)+1)\sec(dx+c)^{\frac{3}{2}}}$	83

input
```
int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/315/d*a*(35*cos(d*x+c)^4+85*cos(d*x+c)^3+102*cos(d*x+c)^2+136*cos(d*x+c)
+272)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^(3/2)*tan(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.51

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2\left(35\,a\cos\left(dx+c\right)^5+85\,a\cos\left(dx+c\right)^4+102\,a\cos\left(dx+c\right)^3+136\,a\cos\left(a\right.\right.}{315\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos}}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
2/315*(35*a*cos(d*x + c)^5 + 85*a*cos(d*x + c)^4 + 102*a*cos(d*x + c)^3 +
136*a*cos(d*x + c)^2 + 272*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(c + dx))^{3/2}}{\sec^{\frac{9}{2}}(c + dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(171) = 342$.

Time = 0.21 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.97

$$\int \frac{(a + a\sec(c + dx))^{3/2}}{\sec^{\frac{9}{2}}(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
1/5040*sqrt(2)*(3780*a*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x +
 9/2*c)))*sin(9/2*d*x + 9/2*c) + 1050*a*cos(2/3*arctan2(sin(9/2*d*x + 9/2*
c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 378*a*cos(4/9*arctan2(si
n(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 135*a*co
s(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9
/2*c) - 3780*a*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*c),
cos(9/2*d*x + 9/2*c))) - 1050*a*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(sin(9
/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 378*a*cos(9/2*d*x + 9/2*c)*sin(4
/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 135*a*cos(9/2*d*
x + 9/2*c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) +
70*a*sin(9/2*d*x + 9/2*c) + 135*a*sin(7/9*arctan2(sin(9/2*d*x + 9/2*c), co
s(9/2*d*x + 9/2*c))) + 378*a*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2
*d*x + 9/2*c))) + 1050*a*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
 + 9/2*c))) + 3780*a*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9
/2*c))))*sqrt(a)/d
```

Giac [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.79

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{4\left(315\sqrt{2}a^6\mathrm{sgn}(\cos(dx+c)) + \left(525\sqrt{2}a^6\mathrm{sgn}(\cos(dx+c)) + \left(819\sqrt{2}a^6\mathrm{s}\right.\right.\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
4/315*(315*sqrt(2)*a^6*sgn(cos(d*x + c)) + (525*sqrt(2)*a^6*sgn(cos(d*x +
c)) + (819*sqrt(2)*a^6*sgn(cos(d*x + c)) + 47*(2*sqrt(2)*a^6*sgn(cos(d*x +
 c))*tan(1/2*d*x + 1/2*c)^2 + 9*sqrt(2)*a^6*sgn(cos(d*x + c)))*tan(1/2*d*x
 + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x +
 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 + a)^(9/2)*d)
```

Mupad [B] (verification not implemented)

Time = 11.51 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.52

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{a\,\cos\left(c+d\,x\right)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}\,\left(4830\,\sin\left(c+d\,x\right)+1428\,\sin\left(2\,c\right.\right.}{2520\,d\,\left(\cos\left(\right.\right.}$$

input
```
int((a + a/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(9/2),x)
```

output
```
(a*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*x
))^(1/2)*(4830*sin(c + d*x) + 1428*sin(2*c + 2*d*x) + 513*sin(3*c + 3*d*x)
 + 170*sin(4*c + 4*d*x) + 35*sin(5*c + 5*d*x)))/(2520*d*(cos(c + d*x) + 1)
)
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \sqrt{a}\,a\left(\int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{\sec\left(dx+c\right)+1}}{\sec\left(dx+c\right)^{5}}dx + \int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{\sec\left(dx+c\right)+1}}{\sec\left(dx+c\right)^{4}}dx\right)$$

input
```
int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x)
```

output
```
sqrt(a)*a*(int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/sec(c + d*x)**5
,x) + int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/sec(c + d*x)**4,x))
```

3.233 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 200

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{163a^{5/2}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d} + \frac{163a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{17a^3\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d}$$

output

```
163/64*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+163/64
*a^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+163/96*a^3*sec(d
*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+17/24*a^3*sec(d*x+c)^(7/2)
*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/4*a^2*sec(d*x+c)^(7/2)*(a+a*sec(d*x
+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.81

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^3\left(489\arcsin\left(\sqrt{1-\sec(c+dx)}\right)+326\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+184\sqrt{1-\sec(c+dx)}\ldots\right)}{192d\sqrt{1-s\ldots}}$$

input
```
Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^3*(489*ArcSin[Sqrt[1 - Sec[c + d*x]]] + 326*Sqrt[1 - Sec[c + d*x]]*Sec[
c + d*x]^(3/2) + 184*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2) + 48*Sqrt[1
 - Sec[c + d*x]]*Sec[c + d*x]^(7/2) + 489*Sqrt[-((-1 + Sec[c + d*x])*Sec[c
 + d*x])])*Tan[c + d*x])/(192*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c +
 d*x])])
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4301, 27, 3042, 4504, 3042, 4290, 3042, 4290, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{1}{4}a\int \frac{1}{2}\sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}(17\sec(c+dx)a+13a)dx+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{8}a\int \sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}(17\sec(c+dx)a+13a)dx+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{8}a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(17\csc\left(c+dx+\frac{\pi}{2}\right)a+13a\right)dx+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \; 4504$$

$$\frac{1}{8}a\left(\frac{163}{6}a\int \sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{17a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{8}a\left(\frac{163}{6}a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{17a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \; 4290$$

$$\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\int \sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{17a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+a}{3d\sqrt{a\sec(c+dx)+a}}\right.$$
$$\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{17a^2\sin(c+}{3d\sqrt{a\,\mathrm{se}}}\right.$$
$$\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \; 4290$$

$$\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+\right.\right.$$
$$\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$\big\downarrow$ 3042

$$\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+}{2d\sqrt{a\,s}}\right.\right.$$
$$\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$\big\downarrow$ 4288

$$\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\left(\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx}{2d\sqrt{a\sec(c+dx)+a}}\right.\right.$$
$$\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$\big\downarrow$ 222

$$\frac{1}{8}a\left(\frac{17a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{163}{6}a\left(\frac{3}{4}\left(\frac{\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a}{}\right.\right.$$
$$\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

input
```
Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^2*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (a*
((17*a^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) +
 (163*a*((a*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]
) + (3*((Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])
/d + (a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4)
)/6))/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4504
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Simp[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1))   Int[Sqrt[a + b*Csc[e + f*
x]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&
!LtQ[n, 0]
```

Maple [A] (verified)

Time = 17.62 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.97

method	result
default	$\frac{a^2 \sqrt{a(1+\sec(dx+c))}\, \sec(dx+c)^{\frac{5}{2}} \left(489 \cos(dx+c)^3 \arctan\left(\frac{\cot(dx+c)-\csc(dx+c)+1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+489 \cos(dx+c)^3 \arctan\left(\frac{\cot(dx+c)-\csc(dx+}{2\sqrt{-\frac{1}{\cos(dx+c)+}}}\right.\right.}{384 d(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)-}}}$

input
```
int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/384/d*a^2*(a*(1+sec(d*x+c)))^(1/2)*sec(d*x+c)^(5/2)/(cos(d*x+c)+1)/(-1/(
cos(d*x+c)+1))^(1/2)*(489*cos(d*x+c)^3*arctan(1/2/(-1/(cos(d*x+c)+1))^(1/2
)*(cot(d*x+c)-csc(d*x+c)+1))+489*cos(d*x+c)^3*arctan(1/2*(cot(d*x+c)-csc(d
*x+c)-1)/(-1/(cos(d*x+c)+1))^(1/2))+(489*cos(d*x+c)^3+326*cos(d*x+c)^2+184
*cos(d*x+c)+48)*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.22

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{489\left(a^2\cos(dx+c)^4+a^2\cos(dx+c)^3\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c\right.}{\sqrt{c}}}{\cos(dx+c)^3+\cos(dx+c)}\right.}{768\left(d\cos(dx\right.}\right.$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/768*(489*(a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d
*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c))
 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(489*a^2*cos(d*x + c)^3 + 3
26*a^2*cos(d*x + c)^2 + 184*a^2*cos(d*x + c) + 48*a^2)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^4 +
d*cos(d*x + c)^3), 1/384*(489*(a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*sq
rt(-a)*arctan(1/2*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))/(a*sqrt(cos(d*x + c))*sin(d*x + c))) + 2*(489*a
^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 184*a^2*cos(d*x + c) + 48*a^2
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))
/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3860 vs. $2(168) = 336$.

Time = 0.46 (sec) , antiderivative size = 3860, normalized size of antiderivative = 19.30

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-1/768*(1956*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c
) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(1
5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 652*(sqrt(2)*a^2*sin(8*
d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*
c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(13/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 6204*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(
6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x +
2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2060*(sqrt(2
)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*si
n(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 2060*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(
2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*s
in(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 62
04*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt
(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(5/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 652*(sqrt(2)*a^2*sin(8*d*x + 8*c)
+ 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt
(2)*a^2*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - 1956*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c)
 + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos...
```

Giac [F(-2)]

Exception generated.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{%%{[
%%%{%%{[2309237210123256509497344,0]:[1,0,-2]%%},[35]%%%},0]:[1,0,%%%{-1,[
1]%%%}]%%},[
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}dx$$

input
```
int((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \sqrt{a}\,a^2\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}\,\sec(dx+c)^4\,dx + 2\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}\,\sec(dx+c)^3\,dx\right) + \int \sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}\,\sec(dx+c)^2\,dx\right)$$

input
```
int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1)*sec(c + d*x)**
4,x) + 2*int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1)*sec(c + d*x)**3,x)
+ int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1)*sec(c + d*x)**2,x))
```

3.234 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 160

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{25a^{5/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{25a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{13a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d}$$

output
```
25/8*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+25/8*a^3
*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+13/12*a^3*sec(d*x+c)
^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/3*a^2*sec(d*x+c)^(5/2)*(a+a*s
ec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.85

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^3\left(75\arcsin\left(\sqrt{1-\sec(c+dx)}\right)+34\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+8\sqrt{1-\sec(c+dx)}\right.}{24d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec}}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^3*(75*ArcSin[Sqrt[1 - Sec[c + d*x]]] + 34*Sqrt[1 - Sec[c + d*x]]*Sec[c
+ d*x]^(3/2) + 8*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2) + 75*Sqrt[-((-1
 + Sec[c + d*x])*Sec[c + d*x])])*Tan[c + d*x])/(24*d*Sqrt[1 - Sec[c + d*x]
]*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4301, 27, 3042, 4504, 3042, 4290, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{1}{3}a\int \frac{1}{2}\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}(13\sec(c+dx)a+9a)dx+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}a\int \sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}(13\sec(c+dx)a+9a)dx+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(13\csc\left(c+dx+\frac{\pi}{2}\right)a+9a\right)dx+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{4504}$$

$$\frac{1}{6}a\left(\frac{75}{4}a\int \sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{13a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}a\left(\frac{75}{4}a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{13a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{4290}$$

$$\frac{1}{6}a\left(\frac{75}{4}a\left(\frac{1}{2}\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{13a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+d}{2d\sqrt{a\sec(c+dx)+a}}\right.$$
$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}a\left(\frac{75}{4}a\left(\frac{1}{2}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{13a^2\sin(c+d}{2d\sqrt{a\sec(}}\right.$$
$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{4288}$$

$$\frac{1}{6}a\left(\frac{75}{4}a\left(\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{13a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right.$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow 222$$

$$\frac{1}{6}a\left(\frac{13a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}+\frac{75}{4}a\left(\frac{\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)\right)+$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

input
```
Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^2*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (a*
((13*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) +
 (75*a*((Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])
/d + (a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4)
)/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4504
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Simp[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1))   Int[Sqrt[a + b*Csc[e + f*
x]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&
!LtQ[n, 0]
```

Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.15

method	result
default	$\frac{a^2\sqrt{a(1+\sec(dx+c))}\,\sec(dx+c)^{\frac{3}{2}}\left(-75\cos(dx+c)^2\arctan\left(\frac{-\cot(dx+c)+\csc(dx+c)+1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-75\cos(dx+c)^2\arctan\left(\frac{-\cot(dx+c)+\csc(dx+c)}{2\sqrt{-\frac{1}{\cos(dx+c)}}}\right.\right.}{48d(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/d*a^2*(a*(1+sec(d*x+c)))^(1/2)*sec(d*x+c)^(3/2)/(cos(d*x+c)+1)/(-1/(c
os(d*x+c)+1))^(1/2)*(-75*cos(d*x+c)^2*arctan(1/2/(-1/(cos(d*x+c)+1))^(1/2)
*(-cot(d*x+c)+csc(d*x+c)+1))-75*cos(d*x+c)^2*arctan(1/2*(-cot(d*x+c)+csc(d
*x+c)-1)/(-1/(cos(d*x+c)+1))^(1/2))+(75*cos(d*x+c)^2+34*cos(d*x+c)+8)*2^(1
/2)*(-2/(cos(d*x+c)+1))^(1/2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.61

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{75\left(a^2\cos(dx+c)^3+a^2\cos(dx+c)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\cdots}{\sqrt{\cos\cdots}}}{\cos(dx+c)^3+\cos(dx+c)^2\cdots}\right.}{96\left(d\cos(dx+c)^3+d\cdots\right.}\right.$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/96*(75*(a^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x
 + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) +
 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(75*a^2*cos(d*x + c)^2 + 34*a
^2*cos(d*x + c) + 8*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
 c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), 1/48*(75*(a
^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2)*sqrt(-a)*arctan(1/2*(cos(d*x + c)^
2 - 2*cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))/(a*sq
rt(cos(d*x + c))*sin(d*x + c))) + 2*(75*a^2*cos(d*x + c)^2 + 34*a^2*cos(d*
x + c) + 8*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(
cos(d*x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3469 vs. 2(134) = 268.

Time = 0.38 (sec) , antiderivative size = 3469, normalized size of antiderivative = 21.68

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/96*(300*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))*sin(6*d*x + 6*c) - 28*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 28*sqrt(
2)*a^2*sin(3/2*d*x + 3/2*c) - 28*(sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - sqrt(
2)*a^2*sin(3/2*d*x + 3/2*c))*cos(6*d*x + 6*c) - 300*(sqrt(2)*a^2*sin(6*d*x
 + 6*c) + 3*sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))))*cos(11/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) - 12*(7*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x +
3/2*c) - 114*sqrt(2)*a^2*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 114*sqrt(2)*a^2*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + 75*sqrt(2)*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) - 456*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*sin(4/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 456*(sqrt(2)*a^2*sin(6*d*x + 6*c) +
3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
)*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*(7*sqr
t(2)*a^2*sin(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 75*sq
rt(2)*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*co
s(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 75*(a^2*co...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(134) = 268$.

Time = 2.11 (sec) , antiderivative size = 472, normalized size of antiderivative = 2.95

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/48*(75*a^(5/2)*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))*sgn(cos(d*x + c)) - 75*a^(5/2)*
log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)
)^2 + a*(2*sqrt(2) - 3)))*sgn(cos(d*x + c)) + 4*(75*sqrt(2)*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*a^(7/2)*sgn(cos(d
*x + c)) - 1125*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a))^8*a^(9/2)*sgn(cos(d*x + c)) + 6174*sqrt(2)*(sqrt(a)*tan(
1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*a^(11/2)*sgn(cos(
d*x + c)) - 4314*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))^4*a^(13/2)*sgn(cos(d*x + c)) + 807*sqrt(2)*(sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(15/2)*sgn(cos
(d*x + c)) - 49*sqrt(2)*a^(17/2)*sgn(cos(d*x + c)))/((sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx=\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

input
```
int((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx=\sqrt{a}\,a^{2}\left(\int \sqrt{\sec\left(dx+c\right)}\,\sqrt{\sec\left(dx+c\right)+1}\,\sec\left(dx+c\right)^{3}dx+2\left(\int \sqrt{\sec\left(dx+c\right)}\,\sqrt{\sec\left(dx+c\right)+1}\,\sec\left(dx+c\right)^{2}dx\right)+\int \sqrt{\sec\left(dx+c\right)}\,\sqrt{\sec\left(dx+c\right)+1}\,\sec\left(dx+c\right)dx\right)$$

input
```
int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1)*sec(c + d*x)**
3,x) + 2*int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1)*sec(c + d*x)**2,x)
+ int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1)*sec(c + d*x),x))
```

3.235 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 120

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx = \frac{19a^{5/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{9a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{2d}$$

output
```
19/4*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+9/4*a^3*
sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a^2*sec(d*x+c)^(3
/2)*(a+a*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.88

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^3\left(-19\arcsin\left(\sqrt{\sec(c+dx)}\right)+2\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+11\sqrt{-((-1+\sec(c+\right.}{4d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^3*(-19*ArcSin[Sqrt[Sec[c + d*x]]] + 2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*
x]^(3/2) + 11*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Tan[c + d*x])/(4*
d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4301, 27, 3042, 4504, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{1}{2}a\int \frac{1}{2}\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}(9\sec(c+dx)a+5a)dx+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}a\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}(9\sec(c+dx)a+5a)dx+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(9\csc\left(c+dx+\frac{\pi}{2}\right)a+5a\right)dx+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{4504}$$

$$\frac{1}{4}a\left(\frac{19}{2}a\int\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{9a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}a\left(\frac{19}{2}a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{9a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

$\Big\downarrow$ 4288

$$\frac{1}{4}a\left(\frac{9a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{19a\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

$\Big\downarrow$ 222

$$\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}+\frac{1}{4}a\left(\frac{19a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{9a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

input
```
Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^2*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (a*
((19*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
 (9*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4504
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Simp[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1))   Int[Sqrt[a + b*Csc[e + f*
x]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&
!LtQ[n, 0]
```

Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.42

method	result
default	$\frac{a^2\sqrt{a(1+\sec(dx+c))}\,\sqrt{\sec(dx+c)}\left(19\cos(dx+c)\arctan\left(\frac{\cot(dx+c)-\csc(dx+c)+1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+19\cos(dx+c)\arctan\left(\frac{\cot(dx+c)-\csc(dx+c)-1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right.\right.}{8d(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*a^2*(a*(1+sec(d*x+c)))^(1/2)*sec(d*x+c)^(1/2)/(cos(d*x+c)+1)/(-1/(co
s(d*x+c)+1))^(1/2)*(19*cos(d*x+c)*arctan(1/2/(-1/(cos(d*x+c)+1))^(1/2)*(co
t(d*x+c)-csc(d*x+c)+1))+19*cos(d*x+c)*arctan(1/2*(cot(d*x+c)-csc(d*x+c)-1)
/(-1/(cos(d*x+c)+1))^(1/2))+2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(11*sin(d*x+
c)+2*tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 383, normalized size of antiderivative = 3.19

$$\int\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx=\left[\frac{19\left(a^2\cos(dx+c)^2+a^2\cos(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{\ldots}}{\sqrt{\cos(\ldots}}}{\cos(dx+c)^3+\cos(dx+c)^2\ldots}\right.}{16\left(d\cos(dx+c)^2+d\cos(dx\ldots\right.}\right.$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/16*(19*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqrt(a)*log((a*cos(d*x +
 c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8
*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(11*a^2*cos(d*x + c) + 2*a^2)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d
*cos(d*x + c)^2 + d*cos(d*x + c)), 1/8*(19*(a^2*cos(d*x + c)^2 + a^2*cos(d
*x + c))*sqrt(-a)*arctan(1/2*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(-a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))/(a*sqrt(cos(d*x + c))*sin(d*x + c)))
 + 2*(11*a^2*cos(d*x + c) + 2*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2826 vs. $2(100) = 200$.

Time = 2.92 (sec) , antiderivative size = 2826, normalized size of antiderivative = 23.55

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-1/16*(88*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) - 56*sqrt(2)*a
^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 28*sqrt(2)*a^2*sin(3/2*d*x + 3/
2*c) + 44*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*(a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sq
rt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x +
4*c)^2 - 76*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^
2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(
1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c
) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 - 19*a^2*log(2*cos(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(100) = 200$.

Time = 2.05 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.82

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx = \frac{\sqrt{2}a^{\frac{11}{2}}\left(\frac{19\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{a^2|a|}+\frac{8\left(19\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\cdots\right.}\right.\right.}{\left(\left(\sqrt{a}\cdots\right.\right.}\right.}{\cdots}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/16*sqrt(2)*a^(11/2)*(19*sqrt(2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(s
qrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sq
rt(2)*abs(a) - 6*a))/(a^2*abs(a)) + 8*(19*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6 - 171*(sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a + 89*(sqrt(a)*tan(1/2*d*x + 1/2*
c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^2 - 9*a^3)/(((sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2*a^2))*sgn
(cos(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx = \sqrt{a}\,a^2\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}\,\sec(dx+c)^2\,dx + 2\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}\,\sec(dx+c)\,dx\right) + \int \sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}dx\right)$$

input
```
int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1)*sec(c + d*x)**
2,x) + 2*int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1)*sec(c + d*x),x) + i
nt(sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1),x))
```

3.236 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 112

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{5a^{5/2}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
5*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+a^3*sec(d*x
+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+a^2*sec(d*x+c)^(1/2)*(a+a*se
c(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.81

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{a^3\left(5\arcsin\left(\sqrt{1-\sec(c+dx)}\right) + (1+2\cos(c+dx))\sqrt{(-1+\cos(c+dx)}\right.}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

output
```
(a^3*(5*ArcSin[Sqrt[1 - Sec[c + d*x]]] + (1 + 2*Cos[c + d*x])*Sqrt[(-1 + C
os[c + d*x])*Sec[c + d*x]^2])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt
[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4301, 27, 3042, 4503, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{5/2}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4301}$$

$$a\int \frac{\sqrt{\sec(c+dx)a+a}(5\sec(c+dx)a+a)}{2\sqrt{\sec(c+dx)}}dx+\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}a\int \frac{\sqrt{\sec(c+dx)a+a}(5\sec(c+dx)a+a)}{\sqrt{\sec(c+dx)}}dx+\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(5\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{d}$$

$$\Big\downarrow \text{4503}$$

$$\frac{1}{2}a\left(5a\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\left(5a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{d}$$

$$\Big\downarrow \text{4288}$$

$$\frac{1}{2}a\left(\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{10a\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{d}$$

$$\Big\downarrow \text{222}$$

$$\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{d}+\frac{1}{2}a\left(\frac{10a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}\right)$$

input
```
Int[(a + a*Sec[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

output
```
(a^2*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/d + (a*((10
*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2*
a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4503
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Simp
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n)   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[
e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a
*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.57

method	result
default	$-\frac{a^{2}\sqrt{a(1+\sec(dx+c))}\left(-8\sin(dx+c)-4\tan(dx+c)+5\sqrt{2}\,(\cos(dx+c)+1)\sqrt{-\frac{2}{\cos(dx+c)+1}}\,\arctan\left(\frac{\cot(dx+c)-\csc(dx+c)+1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+5\sqrt{}\right.}{4d(\cos(dx+c)+1)\sqrt{\sec(dx+c)}}$

input
```
int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d*a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^(1/2)*(-8*si
n(d*x+c)-4*tan(d*x+c)+5*2^(1/2)*(cos(d*x+c)+1)*(-2/(cos(d*x+c)+1))^(1/2)*a
rctan(1/2/(-1/(cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1))+5*2^(1/2)*(
cos(d*x+c)+1)*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/2*(cot(d*x+c)-csc(d*x+c)-
1)/(-1/(cos(d*x+c)+1))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 343, normalized size of antiderivative = 3.06

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \left[\frac{5\left(a^{2}\cos(dx+c)+a^{2}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-\frac{4\left(\cos(dx+c)^{2}-2\,\cos(dx+c)\right.}{\sqrt{\cos}}}{\cos(dx+c)^{3}+\cos(dx+c)}\right.}{4\left(d\cos(dx+\right.}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(5*(a^2*cos(d*x + c) + a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d
*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)
^3 + cos(d*x + c)^2)) + 4*(2*a^2*cos(d*x + c) + a^2)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d),
1/2*(5*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*arctan(1/2*(cos(d*x + c)^2 - 2*co
s(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))/(a*sqrt(cos(d
*x + c))*sin(d*x + c))) + 2*(2*a^2*cos(d*x + c) + a^2)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(c + dx))^{5/2}}{\sqrt{\sec(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11494 vs. $2(98) = 196$.

Time = 0.32 (sec) , antiderivative size = 11494, normalized size of antiderivative = 102.62

$$\int \frac{(a + a\sec(c + dx))^{5/2}}{\sqrt{\sec(c + dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(8*a^2*cos(1/2*d*x + 1/2*c)^4*sin(1/2*d*x + 1/2*c) + 16*a^2*cos(1/2*d*
x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^3 + 8*a^2*sin(1/2*d*x + 1/2*c)^5 + 5*(sq
rt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a
^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(1/2*d*x + 1/2*c)^4 + 10*(sq
rt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a
^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(1/2*d*x + 1/2*c)^2*sin(1/2*
d*x + 1/2*c)^2 + 5*(sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(98) = 196$.

Time = 1.97 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.79

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{\frac{4\sqrt{2}a^3\operatorname{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}} + 5a^{\frac{5}{2}}\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\right.}\right.\right.\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*(4*sqrt(2)*a^3*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a) + 5*a^(5/2)*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))*sgn(cos(d*x + c))
 - 5*a^(5/2)*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))*sgn(cos(d*x + c)) + 4*(3*sqrt(2)*(s
qrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(7/2
)*sgn(cos(d*x + c)) - sqrt(2)*a^(9/2)*sgn(cos(d*x + c)))/((sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx=\sqrt{a}\,a^2\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)}dx+\int \sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}\,\sec(dx+c)\,dx+2\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}dx\right)\right)$$

input
```
int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)
```

output
```
sqrt(a)*a**2*(int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/sec(c + d*x)
,x) + int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1)*sec(c + d*x),x) + 2*in
t(sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1),x))
```

3.237 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 118

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a^{5/2}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{14a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
2*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+14/3*a^3*se
c(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a^2*(a+a*sec(d*x+c)
)^(1/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.87

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a^3\left(3\arcsin\left(\sqrt{1-\sec(c+dx)}\right)\sec^{\frac{3}{2}}(c+dx)+\sqrt{1-\sec(c+dx)}(1+8\operatorname{se}\right.}{3d\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\sqrt{a(1+\sec(c+}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

output
```
(2*a^3*(3*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]^(3/2) + Sqrt[1 - Sec
[c + d*x]]*(1 + 8*Sec[c + d*x]))*Sin[c + d*x])/(3*d*Sqrt[-((-1 + Sec[c + d
*x])*Sec[c + d*x])]*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 4300, 27, 3042, 4503, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \textbf{4300}$$

$$\frac{2}{3}a\int \frac{\sqrt{\sec(c+dx)a+a}(3\sec(c+dx)a+7a)}{2\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{3}a\int \frac{\sqrt{\sec(c+dx)a+a}(3\sec(c+dx)a+7a)}{\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{1}{3}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(3\csc\left(c+dx+\frac{\pi}{2}\right)a+7a\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \textbf{4503}$$

$$\frac{1}{3}a\left(3a\int\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{14a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}a\left(3a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{14a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4288}$$

$$\frac{1}{3}a\left(\frac{14a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{6a\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{222}$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}+\frac{1}{3}a\left(\frac{6a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{14a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}\right)$$

input
```
Int[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

output
```
(2*a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (
a*((6*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d
+ (14*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/
3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

rule 4503
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Simp
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n)   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[
e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a
*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Maple [A] (warning: unable to verify)

Time = 3.01 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.51

method	result
default	$-\frac{a^2\sqrt{a(1+\sec(dx+c))}\left(-4\sin(dx+c)-32\tan(dx+c)+\sqrt{2}\arctan\left(\frac{\cot(dx+c)-\csc(dx+c)-1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{2}{\cos(dx+c)+1}}\,(3+3\sec(dx+c))+\right.}{6d(\cos(dx+c)+1)\sec(dx+c)^{\frac{3}{2}}}$

input
```
int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/d*a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^(3/2)*(-4*si
n(d*x+c)-32*tan(d*x+c)+2^(1/2)*arctan(1/2*(cot(d*x+c)-csc(d*x+c)-1)/(-1/(c
os(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*(3+3*sec(d*x+c))+2^(1/2)*ar
ctan(1/2/(-1/(cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1))*(-2/(cos(d*x
+c)+1))^(1/2)*(3+3*sec(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 361, normalized size of antiderivative = 3.06

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left[\frac{3\left(a^2\cos(dx+c)+a^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right.}{\sqrt{\mathrm{co}}}}{\cos(dx+c)^3+\cos(dx+c)}\right.}{6\left(d\cos\left(d\right.\right.}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(a^2*cos(d*x + c) + a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d
*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)
^3 + cos(d*x + c)^2)) + 4*(a^2*cos(d*x + c)^2 + 8*a^2*cos(d*x + c))*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(
d*x + c) + d), 1/3*(3*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*arctan(1/2*(cos(d*
x + c)^2 - 2*cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)/(a*sqrt(cos(d*x + c))*sin(d*x + c))) + 2*(a^2*cos(d*x + c)^2 + 8*a^2*cos
(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d
*x + c)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(100) = 200$.

Time = 0.24 (sec) , antiderivative size = 593, normalized size of antiderivative = 5.03

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
1/12*sqrt(2)*(30*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))*sin(3/2*d*x + 3/2*c) - 30*a^2*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*log(2*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2
*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 3*sqrt(2)*a^
2*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2
)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqr
t(2)*a^2*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2
*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*
sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2)
+ 4*a^2*sin(3/2*d*x + 3/2*c) + 30*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2...
```

Giac [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.64

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\frac{3\,a^{\frac{7}{2}}\log\left(\left|\frac{2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-4\,\sqrt{2}|a|-6\,a}{2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2+4\,\sqrt{2}|a|-6\,a}\right|\right)\mathrm{sgn}(\cos(dx+c))}{|a|}+\frac{2\left(7\,\sqrt{2}a^4\mathrm{s}\cdots\right.}{\cdots}}{3\,d}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
1/3*(3*a^(7/2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a)
)*sgn(cos(d*x + c))/abs(a) + 2*(7*sqrt(2)*a^4*sgn(cos(d*x + c))*tan(1/2*d*
x + 1/2*c)^2 + 9*sqrt(2)*a^4*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)/(a*ta
n(1/2*d*x + 1/2*c)^2 + a)^(3/2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\sqrt{a}\,a^2\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^2}dx+2\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)}dx\right)+\int \sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}dx\right)$$

input
```
int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x)
```

output
```
sqrt(a)*a**2*(int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/sec(c + d*x)
**2,x) + 2*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/sec(c + d*x),x)
 + int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1),x))
```

3.238 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 119

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{64a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{16a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2a(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

output
```
64/15*a^3*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+16/15*a^2*(
a+a*sec(d*x+c))^(1/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/5*a*(a+a*sec(d*x+c))
^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.54

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{a^2(89+28\cos(c+dx)+3\cos(2(c+dx)))\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+\right.}{15d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/2),x]
```

output
```
(a^2*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])
]*Tan[(c + d*x)/2])/(15*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 4296, 3042, 4296, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4296}$$

$$\frac{8}{5}a\int \frac{(\sec(c+dx)a+a)^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}a\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4296}$$

$$\frac{8}{5}a\left(\frac{4}{3}a\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}a\left(\frac{4}{3}a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4291}$$

$$\frac{8}{5}a\left(\frac{8a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/2),x]
```

output
```
(2*a*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (
8*a*((8*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]
) + (2*a*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))
/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4296
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)
*((d*Csc[e + f*x])^n/(f*m)), x] + Simp[b*((2*m - 1)/(d*m))   Int[(a + b*Csc
[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f
, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{(6\sin(dx+c)+28\tan(dx+c)+86\sec(dx+c)\tan(dx+c))a^2\sqrt{a(1+\sec(dx+c))}}{d(15\cos(dx+c)+15)\sec(dx+c)^{\frac{5}{2}}}$	71

input
```
int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(6*sin(d*x+c)+28*tan(d*x+c)+86*sec(d*x+c)*tan(d*x+c))/(15*cos(d*x+c)+1
5)*a^2*(a*(1+sec(d*x+c)))^(1/2)/sec(d*x+c)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.73

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\left(3\,a^2\cos\left(dx+c\right)^3+14\,a^2\cos\left(dx+c\right)^2+43\,a^2\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+c}{\cos(dx+c)}}}{15\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
2/15*(3*a^2*cos(d*x + c)^3 + 14*a^2*cos(d*x + c)^2 + 43*a^2*cos(d*x + c))*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)
*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.50

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\left(3\sqrt{2}a^2\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+25\sqrt{2}a^2\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+150\sqrt{2}a^2\sin\left(\frac{1}{2}\,dx+\right.\right.}{30\,d}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
1/30*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x + 3/
2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Giac [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.86

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{8\left(15\sqrt{2}a^5\text{sgn}(\cos(dx+c))+4\left(2\sqrt{2}a^5\text{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right.\right.}{15\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
8/15*(15*sqrt(2)*a^5*sgn(cos(d*x + c)) + 4*(2*sqrt(2)*a^5*sgn(cos(d*x + c)
)*tan(1/2*d*x + 1/2*c)^2 + 5*sqrt(2)*a^5*sgn(cos(d*x + c)))*tan(1/2*d*x +
1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 + a)^(5/2)*d)
```

Mupad [B] (verification not implemented)

Time = 10.60 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{a^2\cos(c+d\,x)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}\,(175\,\sin(c+d\,x)+28\,\sin(2\,c+$$
$$30\,d\,(\cos(c+d\,x)+1)$$

input
```
int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(5/2),x)
```

output
```
(a^2*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d
*x))^(1/2)*(175*sin(c + d*x) + 28*sin(2*c + 2*d*x) + 3*sin(3*c + 3*d*x)))/
(30*d*(cos(c + d*x) + 1))
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \sqrt{a}\,a^2\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^3}dx + 2\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^2}dx\right) + \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)}dx\right)$$

input
```
int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x)
```

output
```
sqrt(a)*a**2*(int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/sec(c + d*x)
**3,x) + 2*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/sec(c + d*x)**2
,x) + int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/sec(c + d*x),x))
```

3.239 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 156

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{64a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{21d\sqrt{a+a\sec(c+dx)}} + \frac{16a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2a(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{7d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

output
```
64/21*a^3*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+16/21*a^2*(
a+a*sec(d*x+c))^(1/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/7*a*(a+a*sec(d*x+c))
^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/7*(a+a*sec(d*x+c))^(5/2)*sin(d*x+c)
/d/sec(d*x+c)^(5/2)
```

Mathematica [A] (verified)

Time = 6.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.60

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(315\sin\left(\frac{1}{2}(c+dx)\right)+77\sin\left(\frac{3}{2}(c+\right.\right.}{84d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/2),x]
```

output
```
(a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(315*Sin[(c + d*x)/2] + 7
7*Sin[(3*(c + d*x))/2] + 3*(7*Sin[(5*(c + d*x))/2] + Sin[(7*(c + d*x))/2])
))/(84*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 4299, 3042, 4296, 3042, 4296, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4299}$$

$$\frac{5}{7}\int \frac{(\sec(c+dx)a+a)^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4296}$$

$$\frac{5}{7}\left(\frac{8}{5}a\int \frac{(\sec(c+dx)a+a)^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}\left(\frac{8}{5}a\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4296}$$

$$\frac{5}{7}\left(\frac{8}{5}a\left(\frac{4}{3}a\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3}}{5d\sec^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}\left(\frac{8}{5}a\left(\frac{4}{3}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin(c+dx)(a\sec(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4291}$$

$$\frac{5}{7}\left(\frac{8}{5}a\left(\frac{8a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)}{5d\sec^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

input

```
Int[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/2),x]
```

output
```
(2*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (5*
((2*a*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) +
(8*a*((8*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]
]) + (2*a*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]))
)/5))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4296
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)
*((d*Csc[e + f*x])^n/(f*m)), x] + Simp[b*((2*m - 1)/(d*m))   Int[(a + b*Csc
[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f
, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

rule 4299
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(m + 1))), x] + Simp[a*(m/(b*d*(m + 1)))   Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{2a^{2}\left(3\cos(dx+c)^{3}+12\cos(dx+c)^{2}+23\cos(dx+c)+46\right)\sqrt{a(1+\sec(dx+c))}\,\tan(dx+c)}{21d(\cos(dx+c)+1)\sec(dx+c)^{\frac{3}{2}}}$	75

input
```
int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/d*a^2*(3*cos(d*x+c)^3+12*cos(d*x+c)^2+23*cos(d*x+c)+46)*(a*(1+sec(d*x
+c)))^(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^(3/2)*tan(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.64

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\left(3\,a^{2}\cos\left(dx+c\right)^{4}+12\,a^{2}\cos\left(dx+c\right)^{3}+23\,a^{2}\cos\left(dx+c\right)^{2}+46\,a^{2}\cos\left(c\right.\right.}{21\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
2/21*(3*a^2*cos(d*x + c)^4 + 12*a^2*cos(d*x + c)^3 + 23*a^2*cos(d*x + c)^2
 + 46*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(132) = 264.

Time = 0.21 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.07

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
1/168*sqrt(2)*(315*a^2*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
 7/2*c)))*sin(7/2*d*x + 7/2*c) + 77*a^2*cos(4/7*arctan2(sin(7/2*d*x + 7/2*
c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*a^2*cos(2/7*arctan2(s
in(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 315*a^2
*cos(7/2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
7/2*c))) - 77*a^2*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c))) - 21*a^2*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(si
n(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 6*a^2*sin(7/2*d*x + 7/2*c) +
21*a^2*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 77*a
^2*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 315*a^2*
sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(a)/d
```

Giac [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.83

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{8\left(21\sqrt{2}a^6\text{sgn}(\cos(dx+c)) + \left(35\sqrt{2}a^6\text{sgn}(\cos(dx+c)) + 4\left(2\sqrt{2}a^6\text{sgn}(\right.\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
8/21*(21*sqrt(2)*a^6*sgn(cos(d*x + c)) + (35*sqrt(2)*a^6*sgn(cos(d*x + c))
 + 4*(2*sqrt(2)*a^6*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 + 7*sqrt(2)*a
^6*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(
1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 + a)^(7/2)*d)
```

Mupad [B] (verification not implemented)

Time = 11.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.62

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\frac{a^2\cos(c+d\,x)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}\,(392\sin(c+d\,x)+98\sin(2\,c+}{84\,d\,(\cos(c+d\,x)+1)}$$

input
```
int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(7/2),x)
```

output
```
(a^2*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d
*x))^(1/2)*(392*sin(c + d*x) + 98*sin(2*c + 2*d*x) + 24*sin(3*c + 3*d*x) +
 3*sin(4*c + 4*d*x)))/(84*d*(cos(c + d*x) + 1))
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\sqrt{a}\,a^2\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^4}dx+2\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^3}dx\right)+\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^2}dx\right)$$

input
```
int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x)
```

output
```
sqrt(a)*a**2*(int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/sec(c + d*x)
**4,x) + 2*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/sec(c + d*x)**3
,x) + int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/sec(c + d*x)**2,x))
```

3.240 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 201

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx &= \frac{38a^3\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\\ &+\frac{146a^3\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{584a^3\sin(c+dx)}{315d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\\ &+\frac{1168a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}\end{aligned}$$

output
```
38/63*a^3*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+146/105*a^3
*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+584/315*a^3*sin(d*x+
c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+1168/315*a^3*sec(d*x+c)^(1/2)
*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/9*a^2*(a+a*sec(d*x+c))^(1/2)*sin(d*
x+c)/d/sec(d*x+c)^(7/2)
```

Mathematica [A] (verified)

Time = 6.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.52

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx=\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(8190\sin\left(\frac{1}{2}(c+dx)\right)+2100\sin\left(\frac{3}{2}(\right.\right.}{2520d\sqrt{\mathrm{se}}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(9/2),x]
```

output
```
(a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(8190*Sin[(c + d*x)/2] +
2100*Sin[(3*(c + d*x))/2] + 756*Sin[(5*(c + d*x))/2] + 225*Sin[(7*(c + d*x
))/2] + 35*Sin[(9*(c + d*x))/2]))/(2520*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 4300, 27, 3042, 4503, 3042, 4292, 3042, 4292, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{4300}$$

$$\frac{2}{9}a\int \frac{\sqrt{\sec(c+dx)a+a}(15\sec(c+dx)a+19a)}{2\sec^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}a\int \frac{\sqrt{\sec(c+dx)a+a}(15\sec(c+dx)a+19a)}{\sec^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(15\csc\left(c+dx+\frac{\pi}{2}\right)a+19a\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4503}$$

$$\frac{1}{9}a\left(\frac{219}{7}a\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{38a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}a\left(\frac{219}{7}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{38a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4292}$$

$$\frac{1}{9}a\left(\frac{219}{7}a\left(\frac{4}{5}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{38a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}a\left(\frac{219}{7}a\left(\frac{4}{5}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{38a^2\sin(c+d}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4292}$$

$$\frac{1}{9}a\left(\frac{219}{7}a\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c-}\right.\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}a\left(\frac{219}{7}a\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c-}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a}}\right.\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4291}$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{9d\sec^{\frac{7}{2}}(c+dx)}+$$
$$\frac{1}{9}a\left(\frac{38a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{219}{7}a\left(\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{4}{5}\left(\frac{4a\sin(c+dx)\sqrt{\sec(}}{3d\sqrt{a\sec(c+dx}}\right.\right.\right.$$

input
```
Int[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(9/2),x]
```

output
```
(2*a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (
a*((38*a^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]])
 + (219*a*((2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d
*x]]) + (4*((2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c +
d*x]]) + (4*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x
]])))/5))/7))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

rule 4503
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Simp
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n)   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[
e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a
*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.42

method	result	size
default	$\frac{2a^{2}\left(35\cos(dx+c)^{4}+130\cos(dx+c)^{3}+219\cos(dx+c)^{2}+292\cos(dx+c)+584\right)\sqrt{a(1+\sec(dx+c))}\,\tan(dx+c)}{315d(\cos(dx+c)+1)\sec(dx+c)^{\frac{3}{2}}}$	85

input
```
int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/315/d*a^2*(35*cos(d*x+c)^4+130*cos(d*x+c)^3+219*cos(d*x+c)^2+292*cos(d*x
+c)+584)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^(3/2)*tan(d*x+
c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.56

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2\left(35\,a^2\cos\left(dx+c\right)^5+130\,a^2\cos\left(dx+c\right)^4+219\,a^2\cos\left(dx+c\right)^3+292\,a^2 \cdots\right.}{315\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cdots}}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
2/315*(35*a^2*cos(d*x + c)^5 + 130*a^2*cos(d*x + c)^4 + 219*a^2*cos(d*x +
c)^3 + 292*a^2*cos(d*x + c)^2 + 584*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(171) = 342$.

Time = 0.21 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.10

$$\int \frac{(a + a\sec(c + dx))^{5/2}}{\sec^{\frac{9}{2}}(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
1/5040*sqrt(2)*(8190*a^2*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
 + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 2100*a^2*cos(2/3*arctan2(sin(9/2*d*x +
9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 756*a^2*cos(4/9*arct
an2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 22
5*a^2*cos(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2
*d*x + 9/2*c) - 8190*a^2*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x
+ 9/2*c), cos(9/2*d*x + 9/2*c))) - 2100*a^2*cos(9/2*d*x + 9/2*c)*sin(2/3*a
rctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 756*a^2*cos(9/2*d*x
+ 9/2*c)*sin(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 22
5*a^2*cos(9/2*d*x + 9/2*c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d
*x + 9/2*c))) + 70*a^2*sin(9/2*d*x + 9/2*c) + 225*a^2*sin(7/9*arctan2(sin(
9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 756*a^2*sin(5/9*arctan2(sin(9/2
*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 2100*a^2*sin(1/3*arctan2(sin(9/2*d
*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 8190*a^2*sin(1/9*arctan2(sin(9/2*d*x
 + 9/2*c), cos(9/2*d*x + 9/2*c))))*sqrt(a)/d
```

Giac [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.79

$$\int \frac{(a + a\sec(c + dx))^{5/2}}{\sec^{\frac{9}{2}}(c + dx)}\,dx = \frac{8\left(315\sqrt{2}a^7\mathrm{sgn}(\cos(dx + c)) + \left(630\sqrt{2}a^7\mathrm{sgn}(\cos(dx + c)) + 13\left(63\sqrt{2}a^7\right.\right.\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
8/315*(315*sqrt(2)*a^7*sgn(cos(d*x + c)) + (630*sqrt(2)*a^7*sgn(cos(d*x +
c)) + 13*(63*sqrt(2)*a^7*sgn(cos(d*x + c)) + 4*(2*sqrt(2)*a^7*sgn(cos(d*x
+ c))*tan(1/2*d*x + 1/2*c)^2 + 9*sqrt(2)*a^7*sgn(cos(d*x + c)))*tan(1/2*d*
x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x
+ 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 + a)^(9/2)*d)
```

Mupad [B] (verification not implemented)

Time = 11.74 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.53

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx=\frac{a^2\,\cos\left(c+d\,x\right)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}\,\left(10290\,\sin\left(c+d\,x\right)+2856\,\sin\left(\right.\right.}{2520\,d\,\left(\cos\right.}$$

input
```
int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(9/2),x)
```

output
```
(a^2*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d
*x))^(1/2)*(10290*sin(c + d*x) + 2856*sin(2*c + 2*d*x) + 981*sin(3*c + 3*d
*x) + 260*sin(4*c + 4*d*x) + 35*sin(5*c + 5*d*x)))/(2520*d*(cos(c + d*x) +
 1))
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx=\sqrt{a}\,a^2\left(\int\frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^5}dx+2\left(\int\frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^4}dx\right)+\int\frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^3}dx\right)$$

input
```
int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x)
```

output
```
sqrt(a)*a**2*(int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/sec(c + d*x)
**5,x) + 2*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/sec(c + d*x)**4
,x) + int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/sec(c + d*x)**3,x))
```

3.241 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 241

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{46a^3\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{710a^3\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{284a^3\sin(c+dx)}{231d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1136a^3\sin(c+dx)}{693d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2272a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{693d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)}$$

output
```
46/99*a^3*sin(d*x+c)/d/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2)+710/693*a^3
*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+284/231*a^3*sin(d*x+
c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+1136/693*a^3*sin(d*x+c)/d/sec
(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2272/693*a^3*sec(d*x+c)^(1/2)*sin(d*x
+c)/d/(a+a*sec(d*x+c))^(1/2)+2/11*a^2*(a+a*sec(d*x+c))^(1/2)*sin(d*x+c)/d/
sec(d*x+c)^(9/2)
```

Mathematica [A] (verified)

Time = 6.38 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.49

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(31878\sin\left(\frac{1}{2}(c+dx)\right)+8778\sin\left(\frac{3}{2}\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(11/2),x]
```

output
```
(a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(31878*Sin[(c + d*x)/2] +
 8778*Sin[(3*(c + d*x))/2] + 3465*Sin[(5*(c + d*x))/2] + 1287*Sin[(7*(c +
d*x))/2] + 385*Sin[(9*(c + d*x))/2] + 63*Sin[(11*(c + d*x))/2]))/(11088*d*
Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4300, 27, 3042, 4503, 3042, 4292, 3042, 4292, 3042, 4292, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{11/2}}dx$$

$$\Big\downarrow \ 4300$$

$$\frac{2}{11}a\int \frac{\sqrt{\sec(c+dx)a+a}(19\sec(c+dx)a+23a)}{2\sec^{\frac{9}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{11}a\int \frac{\sqrt{\sec(c+dx)a+a}(19\sec(c+dx)a+23a)}{\sec^{\frac{9}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(19\csc\left(c+dx+\frac{\pi}{2}\right)a+23a\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{4503}$$

$$\frac{1}{11}a\left(\frac{355}{9}a\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{7}{2}}(c+dx)}dx+\frac{46a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}a\left(\frac{355}{9}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{46a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{4292}$$

$$\frac{1}{11}a\left(\frac{355}{9}a\left(\frac{6}{7}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{46a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+d}}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}a\left(\frac{355}{9}a\left(\frac{6}{7}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{46a^2\sin(c+}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec}}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{4292}$$

$$\frac{1}{11}a\left(\frac{355}{9}a\left(\frac{6}{7}\left(\frac{4}{5}\int\frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right.\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{11}a\left(\frac{355}{9}a\left(\frac{6}{7}\left(\frac{4}{5}\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a}}\right.\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 4292

$$\frac{1}{11}a\left(\frac{355}{9}a\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec}}\right.\right.\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{11}a\left(\frac{355}{9}a\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\,\mathrm{si}}{5d\sec^{\frac{3}{2}}(c+dx}\right.\right.\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 4291

$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{11d\sec^{\frac{9}{2}}(c+dx)}+$$
$$\frac{1}{11}a\left(\frac{46a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{355}{9}a\left(\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{6}{7}\left(\frac{2a\sin(c}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a}}\right.\right.\right.$$

input

```
Int[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(11/2),x]
```

output
```
(2*a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9/2)) +
(a*((46*a^2*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]
) + (355*a*((2*a*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c +
d*x]]) + (6*((2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c +
 d*x]]) + (4*((2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c
+ d*x]]) + (4*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d
*x]])))/5))/7))/9))/11
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

rule 4503
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Simp
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n)   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[
e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a
*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.39

method	result	s
default	$\frac{2a^2\left(63\cos(dx+c)^5+224\cos(dx+c)^4+355\cos(dx+c)^3+426\cos(dx+c)^2+568\cos(dx+c)+1136\right)\sqrt{a(1+\sec(dx+c))}\,\tan(dx+c)}{693d(\cos(dx+c)+1)\sec(dx+c)^{\frac{3}{2}}}$	9

input
```
int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
2/693/d*a^2*(63*cos(d*x+c)^5+224*cos(d*x+c)^4+355*cos(d*x+c)^3+426*cos(d*x
+c)^2+568*cos(d*x+c)+1136)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/sec(d*x
+c)^(3/2)*tan(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.52

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{2\left(63\,a^2\cos(dx+c)^6+224\,a^2\cos(dx+c)^5+355\,a^2\cos(dx+c)^4+426\,a^2\right.}{693\,(d\cos(dx}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x, algorithm="fricas")
```

output
```
2/693*(63*a^2*cos(d*x + c)^6 + 224*a^2*cos(d*x + c)^5 + 355*a^2*cos(d*x +
c)^4 + 426*a^2*cos(d*x + c)^3 + 568*a^2*cos(d*x + c)^2 + 1136*a^2*cos(d*x
+ c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c
) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(11/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. 2(205) = 410.

Time = 0.23 (sec) , antiderivative size = 521, normalized size of antiderivative = 2.16

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x, algorithm="maxima")
```

output
```
1/22176*sqrt(2)*(31878*a^2*cos(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(1
1/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 8778*a^2*cos(8/11*arctan2(sin
(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 346
5*a^2*cos(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*si
n(11/2*d*x + 11/2*c) + 1287*a^2*cos(4/11*arctan2(sin(11/2*d*x + 11/2*c), c
os(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 385*a^2*cos(2/11*arctan2(
sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) -
31878*a^2*cos(11/2*d*x + 11/2*c)*sin(10/11*arctan2(sin(11/2*d*x + 11/2*c),
 cos(11/2*d*x + 11/2*c))) - 8778*a^2*cos(11/2*d*x + 11/2*c)*sin(8/11*arcta
n2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 3465*a^2*cos(11/2*d*
x + 11/2*c)*sin(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c
))) - 1287*a^2*cos(11/2*d*x + 11/2*c)*sin(4/11*arctan2(sin(11/2*d*x + 11/2
*c), cos(11/2*d*x + 11/2*c))) - 385*a^2*cos(11/2*d*x + 11/2*c)*sin(2/11*ar
ctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 126*a^2*sin(11/2*
d*x + 11/2*c) + 385*a^2*sin(9/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*
d*x + 11/2*c))) + 1287*a^2*sin(7/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11
/2*d*x + 11/2*c))) + 3465*a^2*sin(5/11*arctan2(sin(11/2*d*x + 11/2*c), cos
(11/2*d*x + 11/2*c))) + 8778*a^2*sin(3/11*arctan2(sin(11/2*d*x + 11/2*c),
cos(11/2*d*x + 11/2*c))) + 31878*a^2*sin(1/11*arctan2(sin(11/2*d*x + 11/2*
c), cos(11/2*d*x + 11/2*c))))*sqrt(a)/d
```

Giac [A] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.78

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{8\left(693\sqrt{2}a^8\text{sgn}(\cos(dx+c))+\left(1617\sqrt{2}a^8\text{sgn}(\cos(dx+c))+\left(3003\sqrt{2}a\right.\right.\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x, algorithm="giac")
```

output
```
8/693*(693*sqrt(2)*a^8*sgn(cos(d*x + c)) + (1617*sqrt(2)*a^8*sgn(cos(d*x +
 c)) + (3003*sqrt(2)*a^8*sgn(cos(d*x + c)) + 25*(99*sqrt(2)*a^8*sgn(cos(d*
x + c)) + 4*(2*sqrt(2)*a^8*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 + 11*s
qrt(2)*a^8*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)
^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/(
(a*tan(1/2*d*x + 1/2*c)^2 + a)^(11/2)*d)
```

Mupad [B] (verification not implemented)

Time = 15.54 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.48

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{\sqrt{a-\frac{a}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}}\left(2\sin\left(\frac{11\,c}{4}+\frac{11\,d\,x}{4}\right)^2+\sin\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)\,1\mathrm{i}-1\right)\left(\frac{23\,a^2}{\cdots}\right.}{\cdots}$$

input
```
int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(11/2),x)
```

output
```
((a - a/(2*sin(c/2 + (d*x)/2)^2 - 1))^(1/2)*(sin((11*c)/2 + (11*d*x)/2)*1i
 + 2*sin((11*c)/4 + (11*d*x)/4)^2 - 1)*((23*a^2*sin(c/2 + (d*x)/2)*(sin((1
1*c)/2 + (11*d*x)/2)*1i - 2*sin((11*c)/4 + (11*d*x)/4)^2 + 1))/(4*d) + (19
*a^2*sin((3*c)/2 + (3*d*x)/2)*(sin((11*c)/2 + (11*d*x)/2)*1i - 2*sin((11*c
)/4 + (11*d*x)/4)^2 + 1))/(12*d) + (5*a^2*sin((5*c)/2 + (5*d*x)/2)*(sin((1
1*c)/2 + (11*d*x)/2)*1i - 2*sin((11*c)/4 + (11*d*x)/4)^2 + 1))/(8*d) + (13
*a^2*sin((7*c)/2 + (7*d*x)/2)*(sin((11*c)/2 + (11*d*x)/2)*1i - 2*sin((11*c
)/4 + (11*d*x)/4)^2 + 1))/(56*d) + (5*a^2*sin((9*c)/2 + (9*d*x)/2)*(sin((1
1*c)/2 + (11*d*x)/2)*1i - 2*sin((11*c)/4 + (11*d*x)/4)^2 + 1))/(72*d) + (a
^2*sin((11*c)/2 + (11*d*x)/2)*(sin((11*c)/2 + (11*d*x)/2)*1i - 2*sin((11*c
)/4 + (11*d*x)/4)^2 + 1))/(88*d)))/(2*(-1/(2*sin(c/2 + (d*x)/2)^2 - 1))^(1
/2)*(2*sin(c/4 + (d*x)/4)^2 - 1))
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \sqrt{a}\,a^2\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^6}dx + 2\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^5}dx\right) + \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^4}dx\right)$$

input
```
int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x)
```

output
```
sqrt(a)*a**2*(int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/sec(c + d*x)
**6,x) + 2*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/sec(c + d*x)**5
,x) + int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/sec(c + d*x)**4,x))
```

3.242 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt[4]{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 38

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt[4]{\sec(c+dx)}}\,dx = \frac{4a^2\sec^{\frac{3}{4}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

output
```
4*a^2*sec(d*x+c)^(3/4)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt[4]{\sec(c+dx)}}\,dx = \frac{2\sec^2\left(\frac{1}{2}(c+dx)\right)(a(1+\sec(c+dx)))^{3/2}\tan\left(\frac{1}{2}(c+dx)\right)}{d\sec^{\frac{5}{4}}(c+dx)}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/4),x]
```

output
```
(2*Sec[(c + d*x)/2]^2*(a*(1 + Sec[c + d*x]))^(3/2)*Tan[(c + d*x)/2])/(d*Se
c[c + d*x]^(5/4))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3042, 4301, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{3/2}}{\sqrt[4]{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\sqrt[4]{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4301}$$

$$4a\int 0dx+\frac{4a^2\sin(c+dx)\sec^{\frac{3}{4}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{24}$$

$$\frac{4a^2\sin(c+dx)\sec^{\frac{3}{4}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/4),x]
```

output
```
(4*a^2*Sec[c + d*x]^(3/4)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Maple [F]

$$\int \frac{(a + a \sec(dx + c))^{\frac{3}{2}}}{\sec(dx + c)^{\frac{1}{4}}} dx$$

input
```
int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/4),x)
```

output
```
int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/4),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \frac{(a + a\sec(c + dx))^{3/2}}{\sqrt[4]{\sec(c + dx)}} dx = \frac{4 a \sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}} \cos(dx + c)^{\frac{1}{4}} \sin(dx + c)}{d\cos(dx + c) + d}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/4),x, algorithm="fricas")
```

output
```
4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)^(1/4)*sin(d*x + c
)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt[4]{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/4),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(34) = 68.

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.18

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt[4]{\sec(c+dx)}}\,dx = \frac{4\left(\frac{\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1} - \frac{\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{4}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{4}}\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^{\frac{1}{4}}}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/4),x, algorithm="maxima")
```

output
```
4*(sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*a^(3/2)*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(
5/4)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/4)*(sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 1)^(1/4))
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt[4]{\sec(c+dx)}}\,dx=\int \frac{(a\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{1}{4}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/4),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(1/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt[4]{\sec(c+dx)}}\,dx=\frac{2\,a\,\sin(2\,c+2\,d\,x)\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/4}\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}}{d\,(\cos(c+d\,x)+1)}$$

input
```
int((a + a/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/4),x)
```

output
```
(2*a*sin(2*c + 2*d*x)*(1/cos(c + d*x))^(3/4)*((a*(cos(c + d*x) + 1))/cos(c
 + d*x))^(1/2))/(d*(cos(c + d*x) + 1))
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt[4]{\sec(c+dx)}}\,dx=\sqrt{a}\,a\left(\int \frac{\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^{\frac{1}{4}}}dx+\int \sqrt{\sec(dx+c)+1}\,\sec(dx+c)^{\frac{3}{4}}\,dx\right)$$

input
```
int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/4),x)
```

output
```
sqrt(a)*a*(int(sqrt(sec(c + d*x) + 1)/sec(c + d*x)**(1/4),x) + int((sqrt(s
ec(c + d*x) + 1)*sec(c + d*x))/sec(c + d*x)**(1/4),x))
```

3.243 $\int \sqrt{\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 37

$$\int \sqrt{\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx=\frac{2\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

output
```
2*a^(1/2)*arcsinh(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.46

$$\int \sqrt{\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx = -\frac{2\arcsin\left(\sqrt{\sec(e+fx)}\right)\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}}$$

input
```
Integrate[Sqrt[Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(-2*ArcSin[Sqrt[Sec[e + f*x]]]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2]
)/(f*Sqrt[1 - Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(e+fx)}\sqrt{a\sec(e+fx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4288}$$

$$-\frac{2\int \frac{1}{\sqrt{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{222}$$

$$\frac{2\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}$$

input
```
Int[Sqrt[Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(31) = 62$.

Time = 2.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 3.30

method	result	size
default	$\frac{\left(\arctan\left(\frac{\cot(fx+e)-\csc(fx+e)-1}{2\sqrt{-\frac{1}{\cos(fx+e)+1}}}\right)+\arctan\left(\frac{\cot(fx+e)-\csc(fx+e)+1}{2\sqrt{-\frac{1}{\cos(fx+e)+1}}}\right)\right)\sqrt{\sec(fx+e)}\,\sqrt{a(1+\sec(fx+e))}\,\cos(fx+e)}{f(\cos(fx+e)+1)\sqrt{-\frac{1}{\cos(fx+e)+1}}}$	122

input
```
int(sec(f*x+e)^(1/2)*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(arctan(1/2/(-1/(cos(f*x+e)+1))^(1/2)*(cot(f*x+e)-csc(f*x+e)-1))+arcta
n(1/2/(-1/(cos(f*x+e)+1))^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)))*sec(f*x+e)^(1/
2)*(a*(1+sec(f*x+e)))^(1/2)*cos(f*x+e)/(cos(f*x+e)+1)/(-1/(cos(f*x+e)+1))^
(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(31) = 62$.

Time = 0.10 (sec) , antiderivative size = 186, normalized size of antiderivative = 5.03

$$\int\sqrt{\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx$$

$$= \left[\frac{\sqrt{a}\log\left(\frac{a\cos(fx+e)^3-7\,a\cos(fx+e)^2-\frac{4\left(\cos(fx+e)^2-2\,\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)}{\sqrt{\cos(fx+e)}}+8\,a}{\cos(fx+e)^3+\cos(fx+e)^2}\right)}{2\,f}, \frac{\sqrt{-a}\arctan\left(\frac{(\cos(fx+\cdots\right.}{}\right.$$

input
```
integrate(sec(f*x+e)^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(a)*log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^
2 - 2*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*
x + e)/sqrt(cos(f*x + e)) + 8*a)/(cos(f*x + e)^3 + cos(f*x + e)^2))/f, sqr
t(-a)*arctan(1/2*(cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))/(a*sqrt(cos(f*x + e))*sin(f*x + e)))/f]
```

Sympy [F]

$$\int \sqrt{\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx = \int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sqrt{\sec\left(e+fx\right)}\,dx$$

input
```
integrate(sec(f*x+e)**(1/2)*(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*sqrt(sec(e + f*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(31) = 62$.

Time = 0.20 (sec) , antiderivative size = 241, normalized size of antiderivative = 6.51

$$\int \sqrt{\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx$$
$$= \frac{\sqrt{a}\left(\log\left(2\,\cos\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,\sin\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,\sqrt{2}\cos\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,\sqrt{2}\sin\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\right)\right.}{}$$

input
```
integrate(sec(f*x+e)^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(a)*(log(2*cos(1/2*f*x + 1/2*e)^2 + 2*sin(1/2*f*x + 1/2*e)^2 + 2*s
qrt(2)*cos(1/2*f*x + 1/2*e) + 2*sqrt(2)*sin(1/2*f*x + 1/2*e) + 2) - log(2*
cos(1/2*f*x + 1/2*e)^2 + 2*sin(1/2*f*x + 1/2*e)^2 + 2*sqrt(2)*cos(1/2*f*x
+ 1/2*e) - 2*sqrt(2)*sin(1/2*f*x + 1/2*e) + 2) + log(2*cos(1/2*f*x + 1/2*e
)^2 + 2*sin(1/2*f*x + 1/2*e)^2 - 2*sqrt(2)*cos(1/2*f*x + 1/2*e) + 2*sqrt(2
)*sin(1/2*f*x + 1/2*e) + 2) - log(2*cos(1/2*f*x + 1/2*e)^2 + 2*sin(1/2*f*x
 + 1/2*e)^2 - 2*sqrt(2)*cos(1/2*f*x + 1/2*e) - 2*sqrt(2)*sin(1/2*f*x + 1/2
*e) + 2))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(31) = 62$.

Time = 0.46 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.24

$$\int \sqrt{\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx = \frac{a^{\frac{3}{2}}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\operatorname{sgn}(\cos(fx+e))}{f|a|}$$

input
```
integrate(sec(f*x+e)^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
a^(3/2)*log(abs(2*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2
*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a)*tan(1/2*f*x + 1/2*e
) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))*sgn(c
os(f*x + e))/(f*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx=\int \sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\sqrt{\frac{1}{\cos\left(e+f\,x\right)}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(1/cos(e + f*x))^(1/2),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(1/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx=\sqrt{a}\left(\int \sqrt{\sec\left(fx+e\right)}\,\sqrt{\sec\left(fx+e\right)+1}dx\right)$$

input
```
int(sec(f*x+e)^(1/2)*(a+a*sec(f*x+e))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sec(e + f*x))*sqrt(sec(e + f*x) + 1),x)
```

3.244 $\int \sqrt{-\sec(e+fx)}\sqrt{a-a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 38

$$\int \sqrt{-\sec(e+fx)}\sqrt{a-a\sec(e+fx)}\,dx = \frac{2\sqrt{a}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a-a\sec(e+fx)}}\right)}{f}$$

output
```
2*a^(1/2)*arcsinh(a^(1/2)*tan(f*x+e)/(a-a*sec(f*x+e))^(1/2))/f
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45

$$\int \sqrt{-\sec(e+fx)}\sqrt{a-a\sec(e+fx)}\,dx = \frac{2\arcsin\left(\sqrt{-\sec(e+fx)}\right)\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{a-a\sec(e+fx)}}{f\sqrt{1+\sec(e+fx)}}$$

input
```
Integrate[Sqrt[-Sec[e + f*x]]*Sqrt[a - a*Sec[e + f*x]],x]
```

output
```
(2*ArcSin[Sqrt[-Sec[e + f*x]]]*Cot[(e + f*x)/2]*Sqrt[a - a*Sec[e + f*x]])/
(f*Sqrt[1 + Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-\sec(e+fx)}\sqrt{a-a\sec(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{-\csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{a-a\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4288$$

$$\frac{2\int \frac{1}{\sqrt{\frac{a\tan^2(e+fx)}{a-a\sec(e+fx)}+1}}d\frac{a\tan(e+fx)}{\sqrt{a-a\sec(e+fx)}}}{f}$$

$$\Big\downarrow \; 222$$

$$\frac{2\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a-a\sec(e+fx)}}\right)}{f}$$

input
```
Int[Sqrt[-Sec[e + f*x]]*Sqrt[a - a*Sec[e + f*x]],x]
```

output
```
(2*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[e + f*x])/Sqrt[a - a*Sec[e + f*x]]])/f
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(32) = 64$.

Time = 2.43 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.92

method	result
default	$\frac{\left(\operatorname{arctanh}\left(\frac{-\cot(fx+e)+\csc(fx+e)+1}{2\sqrt{\frac{1}{\cos(fx+e)+1}}}\right)-\operatorname{arctanh}\left(\frac{-\cot(fx+e)+\csc(fx+e)-1}{2\sqrt{\frac{1}{\cos(fx+e)+1}}}\right)\right)\sqrt{-\sec(fx+e)}\,\sqrt{-a(-1+\sec(fx+e))}\,\cot(fx+e)}{f\sqrt{\frac{1}{\cos(fx+e)+1}}}$

input
```
int((-sec(f*x+e))^(1/2)*(a-a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(arctanh(1/2*(-cot(f*x+e)+csc(f*x+e)+1)/(1/(cos(f*x+e)+1))^(1/2))-arct
anh(1/2*(-cot(f*x+e)+csc(f*x+e)-1)/(1/(cos(f*x+e)+1))^(1/2)))*(-sec(f*x+e)
)^(1/2)*(-a*(-1+sec(f*x+e)))^(1/2)/(1/(cos(f*x+e)+1))^(1/2)*cot(f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(32) = 64$.

Time = 0.10 (sec) , antiderivative size = 206, normalized size of antiderivative = 5.42

$$\int \sqrt{-\sec(e+fx)}\sqrt{a-a\sec(e+fx)}\,dx$$
$$= \left[\frac{\sqrt{a}\log\left(\frac{4\left(\cos(fx+e)^3+3\cos(fx+e)^2+2\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)-a}{\cos(fx+e)}}\sqrt{-\frac{1}{\cos(fx+e)}}+\left(a\cos(fx+e)^2+8a\cos(fx+e)+8a\right)\sin(fx+e}{\cos(fx+e)^2\sin(fx+e)}\right.}{2f}, -\frac{\sqrt{-a}\arctan\left(\frac{\left(\cos(fx+e)^2+2\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)-a}{\cos(fx+e)}}\sqrt{-\frac{1}{\cos(fx+e)}}}{2a\sin(fx+e)}\right)}{f}\right]$$

input
```
integrate((-sec(f*x+e))^(1/2)*(a-a*sec(f*x+e))^(1/2),x, algorithm="fricas"
)
```

output
```
[1/2*sqrt(a)*log((4*(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + 2*cos(f*x + e))*s
qrt(a)*sqrt((a*cos(f*x + e) - a)/cos(f*x + e))*sqrt(-1/cos(f*x + e)) + (a*
cos(f*x + e)^2 + 8*a*cos(f*x + e) + 8*a)*sin(f*x + e))/(cos(f*x + e)^2*sin
(f*x + e)))/f, -sqrt(-a)*arctan(1/2*(cos(f*x + e)^2 + 2*cos(f*x + e))*sqrt
(-a)*sqrt((a*cos(f*x + e) - a)/cos(f*x + e))*sqrt(-1/cos(f*x + e))/(a*sin(
f*x + e)))/f]
```

Sympy [F]

$$\int \sqrt{-\sec(e+fx)}\sqrt{a-a\sec(e+fx)}\,dx$$
$$= \int \sqrt{-\sec{(e+fx)}}\sqrt{-a\,(\sec{(e+fx)}-1)}\,dx$$

input
```
integrate((-sec(f*x+e))**(1/2)*(a-a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(-sec(e + f*x))*sqrt(-a*(sec(e + f*x) - 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(32) = 64$.

Time = 0.23 (sec) , antiderivative size = 353, normalized size of antiderivative = 9.29

$$\int \sqrt{-\sec(e+fx)}\sqrt{a-a\sec(e+fx)}\,dx =$$

$$-\frac{\sqrt{a}\left(\log\left(2\cos\left(\frac{1}{2}\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)\right)\right)^2+2\sin\left(\frac{1}{2}\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)\right)\right)\right.\right.}{}$$

input
```
integrate((-sec(f*x+e))^(1/2)*(a-a*sec(f*x+e))^(1/2),x, algorithm="maxima"
)
```

output
```
-1/2*sqrt(a)*(log(2*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sin
(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sqrt(2)*cos(1/2*arctan2(si
n(f*x + e), cos(f*x + e))) + 2*sqrt(2)*sin(1/2*arctan2(sin(f*x + e), cos(f
*x + e))) + 2) + log(2*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*
sin(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sqrt(2)*cos(1/2*arctan2
(sin(f*x + e), cos(f*x + e))) - 2*sqrt(2)*sin(1/2*arctan2(sin(f*x + e), co
s(f*x + e))) + 2) - log(2*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 +
 2*sin(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 - 2*sqrt(2)*cos(1/2*arct
an2(sin(f*x + e), cos(f*x + e))) + 2*sqrt(2)*sin(1/2*arctan2(sin(f*x + e),
 cos(f*x + e))) + 2) - log(2*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^
2 + 2*sin(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 - 2*sqrt(2)*cos(1/2*a
rctan2(sin(f*x + e), cos(f*x + e))) - 2*sqrt(2)*sin(1/2*arctan2(sin(f*x +
e), cos(f*x + e))) + 2))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(32) = 64$.

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.39

$$\int \sqrt{-\sec(e+fx)}\sqrt{a-a\sec(e+fx)}\,dx = \frac{\sqrt{2}\left(\frac{\sqrt{2}a\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}}{2\sqrt{-a}}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\sqrt{-a}} - \frac{\sqrt{2}a\arctan\left(\frac{\sqrt{2}\sqrt{a}}{2\sqrt{-a}}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\sqrt{-a}}\right)}{f}$$

input
```
integrate((-sec(f*x+e))^(1/2)*(a-a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*(sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a)/s
qrt(-a))*sgn(tan(1/2*f*x + 1/2*e))/sqrt(-a) - sqrt(2)*a*arctan(1/2*sqrt(2)
*sqrt(a)/sqrt(-a))*sgn(tan(1/2*f*x + 1/2*e))/sqrt(-a))/f
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{-\sec(e+fx)}\sqrt{a-a\sec(e+fx)}\,dx = \int \sqrt{a-\frac{a}{\cos(e+f\,x)}}\,\sqrt{-\frac{1}{\cos(e+f\,x)}}\,dx$$

input
```
int((a - a/cos(e + f*x))^(1/2)*(-1/cos(e + f*x))^(1/2),x)
```

output
```
int((a - a/cos(e + f*x))^(1/2)*(-1/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{-\sec(e+fx)}\sqrt{a-a\sec(e+fx)}\,dx$$
$$=\sqrt{a}\left(\int \sqrt{\sec(fx+e)}\,\sqrt{-\sec(fx+e)+1}dx\right)i$$

input
```
int((-sec(f*x+e))^(1/2)*(a-a*sec(f*x+e))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sec(e + f*x))*sqrt( - sec(e + f*x) + 1),x)*i
```

3.245 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 128

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

output
```
-arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(1/2)/d+2^(1/2)*arct
anh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2)
)/a^(1/2)/d+sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.98

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\left(\arcsin\left(\sqrt{1-\sec(c+dx)}\right)+2\arcsin\left(\sqrt{\sec(c+dx)}\right)-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)+\sqrt{-((-1+\sec(}\right.}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
((ArcSin[Sqrt[1 - Sec[c + d*x]]] + 2*ArcSin[Sqrt[Sec[c + d*x]]] - Sqrt[2]*
ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] + Sqrt[-((-1 +
 Sec[c + d*x])*Sec[c + d*x])])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqr
t[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 4309, 3042, 4511, 3042, 4288, 222, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a\sec(c+dx)+a}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4309}$$

$$\frac{\int \frac{\sqrt{\sec(c+dx)}(a-a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{2a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 4511$$

$$\frac{2a\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx-\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx}{2a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{2a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{2a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 4288$$

$$\frac{2a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{2\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{2a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 222$$

$$\frac{2a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{2\sqrt{a}\,\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{2a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 4295$$

$$\frac{-\frac{4a\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{2\sqrt{a}\,\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{2a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{2\sqrt{2}\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{2\sqrt{a}\,\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{2a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Sec[c + d*x]^(5/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
((-2*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
 (2*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqr
t[2]*Sqrt[a + a*Sec[c + d*x]])])/d)/(2*a) + (Sec[c + d*x]^(3/2)*Sin[c + d*
x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4309
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(
f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[d^2/(b*(2*n - 3))   Int[(
d*Csc[e + f*x])^(n - 2)*((2*b*(n - 2) - a*Csc[e + f*x])/Sqrt[a + b*Csc[e +
f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n,
2] && IntegerQ[2*n]
```

rule 4511
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b -
a*B)/b   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Simp[B/b
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b
, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(107) = 214$.

Time = 2.28 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.73

method	result
default	$-\frac{\sec(dx+c)^{\frac{5}{2}}\sqrt{a(1+\sec(dx+c))}\left(2\cos(dx+c)^{3}\sqrt{2}\arctan\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-\sqrt{2}\sin(dx+c)\sqrt{-\frac{2}{\cos(dx+c)+1}}\cos(dx}{2da(\cos(dx+c)+1)\sqrt{-\frac{}{\cos(dx}}}$

input
```
int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d/a*sec(d*x+c)^(5/2)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/(-1/(cos
(d*x+c)+1))^(1/2)*(2*cos(d*x+c)^3*2^(1/2)*arctan(1/2*2^(1/2)/(-1/(cos(d*x+
c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)))-2^(1/2)*sin(d*x+c)*(-2/(cos(d*x+c)+
1))^(1/2)*cos(d*x+c)^2-cos(d*x+c)^3*arctan(1/2/(-1/(cos(d*x+c)+1))^(1/2)*(
-cot(d*x+c)+csc(d*x+c)+1))-cos(d*x+c)^3*arctan(1/2*(-cot(d*x+c)+csc(d*x+c)
-1)/(-1/(cos(d*x+c)+1))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 478, normalized size of antiderivative = 3.73

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \left[\frac{\sqrt{a}(\cos(dx+c)+1)\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2+\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+ \frac{2\sqrt{2}}{}}{4\,(ad\cos(dx+c}\right.$$

$$-\frac{2\sqrt{2}(a\cos(dx+c)+a)\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)+\sqrt{-a}(\cos(dx+c)+1)\,\mathrm{arctan}}{2\,(ad\cos(dx+c)+ad)}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a)*(cos(d*x + c) + 1)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^
2 + 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos
(d*x + c)^2)) + 2*sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 - 2*sq
rt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
 c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/s
qrt(a) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d
*x + c)))/(a*d*cos(d*x + c) + a*d), -1/2*(2*sqrt(2)*(a*cos(d*x + c) + a)*s
qrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a
)*sqrt(cos(d*x + c))/sin(d*x + c)) + sqrt(-a)*(cos(d*x + c) + 1)*arctan(1/
2*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))/(a*sqrt(cos(d*x + c))*sin(d*x + c))) - 2*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d
)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 876 vs. $2(107) = 214$.

Time = 0.24 (sec) , antiderivative size = 876, normalized size of antiderivative = 6.84

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2
*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2
*c) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*l
og(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2)
- (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2
*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
 + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos
(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2
*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
- 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 2*(sqrt(2)
*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2
*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(107) = 214$.

Time = 0.79 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.34

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx =$$

$$-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{a}\log\left(\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\right)}{|a|}+\frac{2\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)\right.}{\sqrt{a}}\right.}{4d\,\mathrm{sgn}\left(\cos\left(dx+c\right.\right.}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(sqrt(2)*sqrt(a)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2
)*abs(a) - 6*a))/abs(a) + 2*log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan
(1/2*d*x + 1/2*c)^2 + a))^2)/sqrt(a) - 8*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(a) - a^(3/2))/((sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))/(d*sgn(c
os(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}\,\sec(dx+c)^2}{\sec(dx+c)+1}dx\right)}{a}$$

input
```
int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1)*sec(c + d*x)**2)/(
sec(c + d*x) + 1),x))/a
```

3.246 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx=\frac{2\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
2*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(1/2)/d-2^(1/2)*arc
tanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2
))/a^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx =\frac{\left(-2\arcsin\left(\sqrt{\sec(c+dx)}\right)+\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
((-2*ArcSin[Sqrt[Sec[c + d*x]]] + Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x
]])/Sqrt[1 - Sec[c + d*x]]])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[
a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4308, 3042, 4288, 222, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4308}$$

$$\frac{\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx}{a}-\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{a}-\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4288}$$

$$-\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{2\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{ad}$$

$$\Big\downarrow \text{222}$$

$$\frac{2\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4295}$$

$$\frac{2\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{2\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

input
```
Int[Sec[c + d*x]^(3/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) -
 (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Sec[c + d*x]])])/(Sqrt[a]*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4308
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d/b   Int[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[
e + f*x]], x], x] - Simp[a*(d/b)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(78) = 156$.

Time = 2.27 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.79

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\cos(dx+c)^2\sec(dx+c)^{\frac{3}{2}}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-\arctan\left(\frac{-\cot(dx+c)+\csc(dx+c)+1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-\text{arct}\right.}{da(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(a*(1+sec(d*x+c)))^(1/2)*cos(d*x+c)^2*sec(d*x+c)^(3/2)*(2^(1/2)*arct
an(1/2*2^(1/2)/(-1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)))-arctan(
1/2/(-1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1))-arctan(1/2*(-cot
(d*x+c)+csc(d*x+c)-1)/(-1/(cos(d*x+c)+1))^(1/2)))/(cos(d*x+c)+1)/(-1/(cos(
d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 348, normalized size of antiderivative = 3.66

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \left[\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2 \cdots}{\cdots}\right.}{2ad} \right.$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(2)*sqrt(a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x +
 c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + sqrt(a)*log((a*cos(d*x +
 c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8
*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(a*d), (sqrt(2)*a*sqrt(-1/a)*arcta
n(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x
+ c))/sin(d*x + c)) + sqrt(-a)*arctan(1/2*(cos(d*x + c)^2 - 2*cos(d*x + c)
)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))/(a*sqrt(cos(d*x + c))*s
in(d*x + c))))/(a*d)]
```

Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a\left(\sec(c+dx)+1\right)}}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**(3/2)/sqrt(a*(sec(c + d*x) + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(78) = 156$.

Time = 0.24 (sec) , antiderivative size = 476, normalized size of antiderivative = 5.01

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2
*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c))) + 1) - sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x +
c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sq
rt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arc
tan2(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c
), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2
*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c))) + 2) - log(2*cos(1/2*arctan2(sin(d*x
+ c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
- 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d
*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*si
n(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2))/(sqrt(a)*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}\,\sec(dx+c)}{\sec(dx+c)+1}dx\right)}{a}$$

input
```
int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1)*sec(c + d*x))/(sec
(c + d*x) + 1),x))/a
```

3.247 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 56

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
2^(1/2)*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d
*x+c))^(1/2))/a^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
-((Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan
[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \; 4295$$

$$-\frac{2\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

input
```
Int[Sqrt[Sec[c + d*x]]/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
 + a*Sec[c + d*x]])])/(Sqrt[a]*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(45) = 90$.

Time = 1.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.71

method	result	size
default	$\frac{\sqrt{a(1+\sec(dx+c))}\,\sqrt{\sec(dx+c)}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(\cot(dx+c)-\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\cos(dx+c)}{da(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$	96

input
```
int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(a*(1+sec(d*x+c)))^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)*arctan(1/2*2^(1/2)
*(cot(d*x+c)-csc(d*x+c))/(-1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)/(cos(d*x+c)
+1)/(-1/(cos(d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.86

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \left[\frac{\sqrt{2}\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2\sqrt{a}d}, -\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)}{d}\right]$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/
(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/(sqrt(a)*d), -sqrt(2)*sqrt(-1/a)*ar
ctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d
*x + c))/sin(d*x + c))/d]
```

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/sqrt(a*(sec(c + d*x) + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\right.}{2\,\sqrt{a}d}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))/(sqrt(a)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(45) = 90$.

Time = 0.42 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\left(\frac{\log\left(\left|\frac{1}{\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right)}{\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{\log\left(\left|\frac{1}{\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right|\right)}{\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}\right)}{4\,\sqrt{a}d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(log(abs(1/sin(1/2*d*x + 1/2*c) + sin(1/2*d*x + 1/2*c) + 2))/s
gn(cos(1/2*d*x + 1/2*c)) - log(abs(1/sin(1/2*d*x + 1/2*c) + sin(1/2*d*x +
1/2*c) - 2))/sgn(cos(1/2*d*x + 1/2*c)))/(sqrt(a)*d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)+1}dx\right)}{a}$$

input
```
int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/(sec(c + d*x) + 1
),x))/a
```

3.248 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 93

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

output
```
-2^(1/2)*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(
d*x+c))^(1/2))/a^(1/2)/d+2*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^
(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right) + \frac{2\sqrt{1-\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
((Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] + (2
*Sqrt[1 - Sec[c + d*x]])/Sqrt[Sec[c + d*x]])*Tan[c + d*x])/(d*Sqrt[1 - Sec
[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4299, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4299}$$

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4295}$$

$$\frac{2\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
-((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt
[a + a*Sec[c + d*x]])]])/(Sqrt[a]*d)) + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(d*Sqrt[a + a*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4299
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(m + 1))), x] + Simp[a*(m/(b*d*(m + 1)))   Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{\left(\arctan\left(\frac{\sqrt{2}\,(\cot(dx+c)-\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{2}{\cos(dx+c)+1}}-2\cot(dx+c)+2\csc(dx+c)\right)\sqrt{a(1+\sec(dx+c))}}{da\sqrt{\sec(dx+c)}}$	95

input
```
int(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(arctan(1/2*2^(1/2)*(cot(d*x+c)-csc(d*x+c))/(-1/(cos(d*x+c)+1))^(1/2))
*(-2/(cos(d*x+c)+1))^(1/2)-2*cot(d*x+c)+2*csc(d*x+c))/a*(a*(1+sec(d*x+c)))
^(1/2)/sec(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.02

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \left[\frac{\frac{\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}+4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+}}{2\,(ad\cos(dx+c)+ad)}\right.$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a)
- 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) + 4*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a
*d*cos(d*x + c) + a*d), (sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*arctan(sq
rt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c)
)/sin(d*x + c)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sqrt{\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(sec(c + d*x) + 1))*sqrt(sec(c + d*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\right.}{2\,\sqrt{a}d}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))/
(sqrt(a)*d)
```

Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\left(\frac{\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{\sqrt{a}}+\frac{2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}\right)}{d\text{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*(log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a)))/sqrt(a) + 2*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/2*c)^2
 + a))/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^2+\sec(dx+c)}dx\right)}{a}$$

input
```
int(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/(sec(c + d*x)**2
+ sec(c + d*x)),x))/a
```

3.249 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}$$

output
```
2^(1/2)*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d
*x+c))^(1/2))/a^(1/2)/d+2/3*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^(1/2)-2/3*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.17 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\left(2(-1+\cos(c+dx))\sqrt{1-\sec(c+dx)}-3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sqrt{\sec(c+dx)}\right)\tan(c+dx)}{3d\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
((2*(-1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] - 3*Sqrt[2]*ArcTan[(Sqrt[2]
*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]])*Tan[c + d
*x])/(3*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sqrt[a*(1 + Sec[c + d*
x])])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4310, 3042, 4501, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4310}$$

$$\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{a-2a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}$$

$$\Big\downarrow \ 3042$$

$$\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{a-2a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}$$

$$\Big\downarrow \ 4501$$

$$\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-3a\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{3a}$$

$$\Big\downarrow \ 3042$$

$$\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-3a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}$$

$$\Big\downarrow \ 4295$$

$$\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{6a\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}}{3a}$$

$$\Big\downarrow \ 219$$

$$\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{3\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{3a}$$

input
```
Int[1/(Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
(2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((-3*
Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[a + a*Sec[c + d*x]])])/d + (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*
Sqrt[a + a*Sec[c + d*x]]))/(3*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4310
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a +
b*Csc[e + f*x]])), x] + Simp[1/(2*b*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*((a
 + b*(2*n + 1)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.89

method	result	siz
default	$-\frac{\sqrt{a\left(1+\sec\left(dx+c\right)\right)}\left(2\tan\left(dx+c\right)-2\sin\left(dx+c\right)+\arctan\left(\frac{\sqrt{2}\left(\cot\left(dx+c\right)-\csc\left(dx+c\right)\right)}{2\sqrt{-\frac{1}{\cos\left(dx+c\right)+1}}}\right)\sqrt{-\frac{2}{\cos\left(dx+c\right)+1}}\left(3+3\sec\left(dx+c\right)\right)\right)}{3da\left(\cos\left(dx+c\right)+1\right)\sec\left(dx+c\right)^{\frac{3}{2}}}$	11(

input
```
int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/d/a*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^(3/2)*(2*tan(d
*x+c)-2*sin(d*x+c)+arctan(1/2*2^(1/2)*(cot(d*x+c)-csc(d*x+c))/(-1/(cos(d*x
+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*(3+3*sec(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.43

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \left[\frac{\frac{3\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}+\frac{4\left(\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{\frac{a\cos(}{\cos(}}}{\sqrt{\cos(dx+c)}}}{6\left(ad\cos\left(dx+c\right)+ad\right)}\right.$$

$$-\frac{3\sqrt{2}(a\cos\left(dx+c\right)+a)\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)-\frac{2\left(\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+}{\cos(dx+}}}{\sqrt{\cos(dx+c)}}}{3\left(ad\cos\left(dx+c\right)+ad\right)}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a
) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) + 4
*(cos(d*x + c)^2 - cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
in(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d), -1/3*(3*sqrt(2)*
(a*cos(d*x + c) + a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) - 2*(cos(d*x + c)
^2 - cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sq
rt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**(3/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(108) = 216$.

Time = 0.21 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.15

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{3\sqrt{2}\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3\sqrt{2}\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\sin\left(\frac{2}{3}\right.}{}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/6*(3*sqrt(2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*log(cos(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))/(sqrt(a)*d)
```

Giac [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\left(\frac{4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a\right)^{\frac{3}{2}}}+\frac{3\,\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right|\right)}{\sqrt{a}}\right)}{3\,d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/3*sqrt(2)*(4*a*tan(1/2*d*x + 1/2*c)^3/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3
/2) + 3*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a)))/sqrt(a))/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^3+\sec(dx+c)^2}dx\right)}{a}$$

input
```
int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/(sec(c + d*x)**3
+ sec(c + d*x)**2),x))/a
```

3.250 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 169

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{26\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}$$

output
```
-2^(1/2)*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(
d*x+c))^(1/2))/a^(1/2)/d+2/5*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)
)^(1/2)-2/15*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+26/15*se
c(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{(29-2\cos(c+dx)+3\cos(2(c+dx)))\sqrt{\sec(c+dx)}\sin(c+dx)+\frac{15\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\tan(c+dx)}{\sqrt{1-\sec(c+dx)}}}{15d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[1/(Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
((29 - 2*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sqrt[Sec[c + d*x]]*Sin[c + d*x
] + (15*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]
]*Tan[c + d*x])/Sqrt[1 - Sec[c + d*x]])/(15*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4310, 3042, 4510, 27, 3042, 4501, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4310}$$

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{a-4a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{5a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{a-4a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}$$

$$\Big\downarrow \text{4510}$$

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2\int -\frac{13a^2-2a^2\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}}{5a}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{13a^2-2a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}}{5a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{13a^2-2a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{5a}$$

$$\Big\downarrow \text{4501}$$

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-15a^2\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{3a}}{5a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-15a^2\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{5a}$$

$$\Big\downarrow \text{4295}$$

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{30a^2\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{26a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}}{3a}}{5a}$$

$\Big\downarrow$ 219

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{15\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{3a}}{5a}$$

input
```
Int[1/(Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
(2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) - ((2*a
*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((-15*S
qrt[2]*a^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[a + a*Sec[c + d*x]])])/d + (26*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
d*Sqrt[a + a*Sec[c + d*x]]))/(3*a))/(5*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4310
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a +
b*Csc[e + f*x]])), x] + Simp[1/(2*b*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*((a
 + b*(2*n + 1)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.82

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left(6\sin(dx+c)-2\tan(dx+c)+26\sec(dx+c)\tan(dx+c)+\sqrt{-\frac{2}{\cos(dx+c)+1}}\,\arctan\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\right.}{15da(\cos(dx+c)+1)\sec(dx+c)^{\frac{5}{2}}}$

input
```
int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15/d/a*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^(5/2)*(6*sin(d
*x+c)-2*tan(d*x+c)+26*sec(d*x+c)*tan(d*x+c)+(-2/(cos(d*x+c)+1))^(1/2)*arct
an(1/2*2^(1/2)/(-1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)))*(-15*se
c(d*x+c)-15*sec(d*x+c)^2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.02

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \left[\frac{\frac{15\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}+\frac{4\left(3\cos(dx+c)^3-\cos(dx+c)^2+13\,c\right.}{\sqrt{\cos}}}{30\,(ad\cos(dx+c)+ad)}\right.$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/30*(15*sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt
(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) +
 4*(3*cos(d*x + c)^3 - cos(d*x + c)^2 + 13*cos(d*x + c))*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)
+ a*d), 1/15*(15*sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*arctan(sqrt(2)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*
x + c)) + 2*(3*cos(d*x + c)^3 - cos(d*x + c)^2 + 13*cos(d*x + c))*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(
d*x + c) + a*d)]
```

Sympy [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sec^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**(5/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(140) = 280$.

Time = 0.22 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/60*sqrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))*sin(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c)*sin(4/5*arc
tan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*cos(5/2*d*x + 5/2*c)
*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 30*log(cos
(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arct
an2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin
(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 1) + 30*log(cos(1/5*arctan2(si
n(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x
 + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*
c), cos(5/2*d*x + 5/2*c))) + 1) + 6*sin(5/2*d*x + 5/2*c) - 5*sin(3/5*arcta
n2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 60*sin(1/5*arctan2(sin(5
/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))/(sqrt(a)*d)
```

Giac [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\left(\frac{15\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{a}}+\frac{2\left(\left(17a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+20a^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+15a^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{5}{2}}}\right)}{15\,d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/15*sqrt(2)*(15*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a)))/sqrt(a) + 2*((17*a^2*tan(1/2*d*x + 1/2*c)^2 + 20*a^2)*
tan(1/2*d*x + 1/2*c)^2 + 15*a^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2
*c)^2 + a)^(5/2))/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^{4}+\sec(dx+c)^{3}}dx\right)}{a}$$

input
```
int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/(sec(c + d*x)**4
+ sec(c + d*x)**3),x))/a
```

3.251 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 174

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{3\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{9\text{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}}$$

output
```
-3*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d+9/4*arctan
h(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*
2^(1/2)/a^(3/2)/d-1/2*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)
+3/2*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.45

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{6\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)+4\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+$$

input
```
Integrate[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(6*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] + 4*Sqrt[1 - Sec
[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] - 9*Sqrt[2]*ArcTan[(Sqrt[2]*Sqr
t[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - 9*Sqrt[2]*ArcTan[(
Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[c + d
*x] + 6*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c + d*x] + 1
8*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c + d*x])/(4*d*Sqrt[1
- Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3042, 4303, 27, 3042, 4509, 25, 3042, 4511, 3042, 4288, 222, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{3\sec^{\frac{3}{2}}(c+dx)(a-2a\sec(c+dx))}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\int \frac{\sec^{\frac{3}{2}}(c+dx)(a-2a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4509}$$

$$-\frac{3\left(\frac{\int -\frac{\sqrt{\sec(c+dx)}\left(a^2-2a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3\left(-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-2a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2-2a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4511}$$

$$-\frac{3\left(-\frac{3a^2\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx-2a\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(-\frac{3a^2\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-2a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4288}$$

$$-\frac{3\left(-\frac{3a^2\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{4a\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{222}$$

$$-\frac{3\left(-\frac{3a^2\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{4a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4295}$$

$$-\frac{3\left(-\frac{-\frac{6a^2\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{4a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{3\left(-\frac{\frac{3\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{4a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

input
```
Int[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
-1/2*(Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^(3/2)) - (3
*(-(((-4*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])
/d + (3*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d)/a) - (2*a*Sec[c + d*x]^(3/2)*Sin[c
 + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/(4*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4509
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Simp[d/(b*(m + n))   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
 1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[n, 1]
```

rule 4511
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b -
a*B)/b   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Simp[B/b
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b
, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 3.00 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.55

method	result
default	$\frac{\cos(dx+c)^3\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}\,(\cot(dx+c)-\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\left(9\cos(dx+c)^2+9\cos(dx+c)\right)+\arctan\left(\frac{\cot(dx+c)-\csc(dx+c)+1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\left(-6\cos(dx\right.\right.}{4d\,a^2\left(\right.}$

input
```
int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^2*cos(d*x+c)^3*(2^(1/2)*arctan(1/2*2^(1/2)*(cot(d*x+c)-csc(d*x+c))
/(-1/(cos(d*x+c)+1))^(1/2))*(9*cos(d*x+c)^2+9*cos(d*x+c))+arctan(1/2/(-1/(
cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1))*(-6*cos(d*x+c)^2-6*cos(d*x
+c))+arctan(1/2*(cot(d*x+c)-csc(d*x+c)-1)/(-1/(cos(d*x+c)+1))^(1/2))*(-6*c
os(d*x+c)^2-6*cos(d*x+c))+sin(d*x+c)*(2+3*cos(d*x+c))*2^(1/2)*(-2/(cos(d*x
+c)+1))^(1/2))*(a*(1+sec(d*x+c)))^(1/2)*sec(d*x+c)^(7/2)/(cos(d*x+c)^2+2*c
os(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 576, normalized size of antiderivative = 3.31

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(9*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(
d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1)) + 6*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(
(a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2*cos(d*x + c
))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d
*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*(3*cos(d*x + c) + 2)*sin(d*x + c)/sqrt(cos(d*x + c)))
/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(9*sqrt(2)*(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) +
6*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(1/2*(cos(d*x + c)^
2 - 2*cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))/(a*sq
rt(cos(d*x + c))*sin(d*x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*(3*cos(d*x + c) + 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c
)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4934 vs. $2(143) = 286$.

Time $= 0.43$ (sec) , antiderivative size $= 4934$, normalized size of antiderivative $= 28.36$

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(12*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 8*(s
in(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - sin(3/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*
(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 12*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 3*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2
)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x
 + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*
d*x + 4*c) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + 2*
sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))...
```

Giac [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{7}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}\,\sec(dx+c)^{3}}{\sec(dx+c)^{2}+2\sec(dx+c)+1}dx\right)}{a^{2}}$$

input
```
int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1)*sec(c + d*x)**3)/(
sec(c + d*x)**2 + 2*sec(c + d*x) + 1),x))/a**2
```

3.252 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 134

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{5\arctan\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

output
```
2*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d-5/4*arctanh
(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2
^(1/2)/a^(3/2)/d-1/2*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.64

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{-2\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)+5\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)}{}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] + 5*Sqrt[2]*Arc
Tan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] + 5*
Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c
+ d*x]*Tan[c + d*x] - 2*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])*
Tan[c + d*x] - 10*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c + d*
x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4303, 27, 3042, 4511, 3042, 4288, 222, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}(a-4a\sec(c+dx))}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}(a-4a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a-4a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4511}$$

$$-\frac{5a\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx-4\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-4\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4288}$$

$$-\frac{5a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{8\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{222}$$

$$-\frac{5a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{8\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4295}$$

$$-\frac{-\frac{10a\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{8\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{5\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{8\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

input
```
Int[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
-1/4*((-8*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]
)/d + (5*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d)/a^2 - (Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4511
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b -
a*B)/b   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Simp[B/b
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b
, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(109) = 218$.

Time = 2.97 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.66

method	result
default	$-\frac{\cos(dx+c)^3 \sec(dx+c)^{\frac{5}{2}} \sqrt{a(1+\sec(dx+c))} \left(-5\sqrt{2}\,(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+(4\cos(dx+c)+4)\operatorname{arct}\right.}{4d\,a^2(\cos(dx+c)+1)^2\sqrt{}}$

input
```
int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/a^2*cos(d*x+c)^3*sec(d*x+c)^(5/2)*(a*(1+sec(d*x+c)))^(1/2)*(-5*2^(1
/2)*(cos(d*x+c)+1)*arctan(1/2*2^(1/2)/(-1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+
c)+csc(d*x+c)))+(4*cos(d*x+c)+4)*arctan(1/2*(-cot(d*x+c)+csc(d*x+c)-1)/(-1
/(cos(d*x+c)+1))^(1/2))+(4*cos(d*x+c)+4)*arctan(1/2/(-1/(cos(d*x+c)+1))^(1
/2)*(-cot(d*x+c)+csc(d*x+c)+1))+sin(d*x+c)*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/
2))/(cos(d*x+c)+1)^2/(-1/(cos(d*x+c)+1))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(109) = 218$.

Time = 0.12 (sec) , antiderivative size = 556, normalized size of antiderivative = 4.15

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(
d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1)) + 4*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(
(a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c
))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d
*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^
2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/4*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(
d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 4*(cos(d*x + c)^2 + 2*
cos(d*x + c) + 1)*sqrt(-a)*arctan(1/2*(cos(d*x + c)^2 - 2*cos(d*x + c))*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))/(a*sqrt(cos(d*x + c))*sin(d
*x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2122 vs. $2(109) = 218$.

Time = 0.27 (sec) , antiderivative size = 2122, normalized size of antiderivative = 15.84

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/4*(4*(sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)
*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) +
 sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)
*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt
(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)
*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) +
 sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)
*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt
(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 2*(sqrt...
```

Giac [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.48

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\frac{5\sqrt{2}\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2\right)}{a^{\frac{3}{2}}} - \frac{2\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\tan\left(\frac{1}{2}\,dx-\right.}{a^2}}{}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/8*(5*sqrt(2)*log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^2)/a^(3/2) - 2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*tan
(1/2*d*x + 1/2*c)/a^2 + 8*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/a^(3/2) - 8*log(abs((s
qrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2
*sqrt(2) - 3)))/a^(3/2))/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}\,\sec(dx+c)^2}{\sec(dx+c)^2+2\sec(dx+c)+1}dx\right)}{a^2}$$

input
```
int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1)*sec(c + d*x)**2)/(
sec(c + d*x)**2 + 2*sec(c + d*x) + 1),x))/a**2
```

3.253 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

output
```
1/4*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d*x+c
))^(1/2))*2^(1/2)/a^(3/2)/d+1/2*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x
+c))^(3/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 220 vs. 2(97) = 194.

Time = 0.33 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.27

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx=\frac{2\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\tan}{}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] - Sqrt[2]*ArcTan
[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - Sqrt[
2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x
]*Tan[c + d*x] + 2*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c
 + d*x] + 2*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c + d*x])/(4
*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4297, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4297}$$

$$\frac{\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{4a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 4295}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2ad}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

input
```
Int[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[
c + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d
*(a + a*Sec[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4297
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[d*((m + 1)/(b*(2*m + 1)))
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ
[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m,
-2^(-1)] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.34

method	result
default	$\frac{\left((\cos(dx+c)+1)\arctan\left(\frac{\sqrt{2}\,(\cot(dx+c)-\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+\sqrt{-\frac{2}{\cos(dx+c)+1}}\,\sin(dx+c)\right)\sqrt{2}\,\sqrt{a(1+\sec(dx+c))}\,\sec(dx+c)^{\frac{3}{2}}\cos(dx+c}{4d\,a^{2}(\cos(dx+c)+1)^{2}\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^2*((cos(d*x+c)+1)*arctan(1/2*2^(1/2)*(cot(d*x+c)-csc(d*x+c))/(-1/(
cos(d*x+c)+1))^(1/2))+(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*2^(1/2)*(a*(1+
sec(d*x+c)))^(1/2)*sec(d*x+c)^(3/2)*cos(d*x+c)^2/(cos(d*x+c)+1)^2/(-1/(cos
(d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 338, normalized size of antiderivative = 3.48

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{\sqrt{2}\left(\cos(dx+c)^{2}+2\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx}{\cos(d}}}{\cos(d}\right.}{8\left(a^{2}d\cos(dx+c)^{2}\right.}\right.$$

$$-\frac{\sqrt{2}\left(\cos(dx+c)^{2}+2\cos(dx+c)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)-2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{c}}{4\left(a^{2}d\cos(dx+c)^{2}+2a^{2}d\cos(dx+c)+a^{2}d\right)}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d*
x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*c
os(d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
 + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
, -1/4*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt
(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a
*sin(d*x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**(3/2)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15721 vs. $2(78) = 156$.

Time = 0.71 (sec) , antiderivative size = 15721, normalized size of antiderivative = 162.07

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/4*(32*(cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c)*sin(3/2*
d*x + 3/2*c) + cos(d*x + c)*sin(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*si
n(d*x + c))*cos(3*d*x + 3*c)^2 + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)
 + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*
d*x + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*
sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*cos(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 96*(cos(3/2*d*x + 3/2*c
)*sin(3*d*x + 3*c) + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x
+ c) + 1)*sin(3/2*d*x + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3
*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x +
c))*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 32*(c
os(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*
c) + cos(d*x + c)*sin(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*sin(d*x + c)
)*sin(3*d*x + 3*c)^2 + 32*(6*cos(d*x + c) + 1)*cos(2*d*x + 2*c)*sin(3/2*d*
x + 3/2*c) + 96*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 96*sin(2*d*x + 2
*c)^2*sin(3/2*d*x + 3/2*c) + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 3
*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*d*x
+ 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(
3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*sin(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 96*(cos(3/2*d*x + 3/2*c)...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}\,\sec(dx+c)}{\sec(dx+c)^2+2\sec(dx+c)+1}dx\right)}{a^2}$$

input
```
int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1)*sec(c + d*x))/(sec
(c + d*x)**2 + 2*sec(c + d*x) + 1),x))/a**2
```

3.254 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx=\frac{3\text{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

output
```
3/4*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d*x+c
))^(1/2))*2^(1/2)/a^(3/2)/d-1/2*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x
+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx=\frac{-2\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)-3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)}{4d\sqrt{1-\sec(c+dx)}(a(1+\sec(c+dx)))^{3/2}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] - 3*Sqrt[2]*Arc
Tan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x]
)*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4298, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4298}$$

$$\frac{3\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{4a}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 4295}$$

$$-\frac{3\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2ad}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{3\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(3*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/
(2*d*(a + a*Sec[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4298
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[m/(a*(2*m + 1))   Int[(a + b*Csc[e +
 f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.34

method	result
default	$-\frac{\left((3\cos(dx+c)+3)\arctan\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+\sqrt{-\frac{2}{\cos(dx+c)+1}}\,\sin(dx+c)\right)\sqrt{2}\,\sqrt{\sec(dx+c)}\,\sqrt{a(1+\sec(dx+c))}\,\cos}{4d\,a^2(\cos(dx+c)+1)^2\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/a^2*((3*cos(d*x+c)+3)*arctan(1/2*2^(1/2)/(-1/(cos(d*x+c)+1))^(1/2)*
(-cot(d*x+c)+csc(d*x+c)))+(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*2^(1/2)*se
c(d*x+c)^(1/2)*(a*(1+sec(d*x+c)))^(1/2)*cos(d*x+c)/(cos(d*x+c)+1)^2/(-1/(c
os(d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 340, normalized size of antiderivative = 3.51

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{3\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(}{\cos}}}{\cos}\right.}{8\left(a^2d\cos(dx+c)\right.}\right.$$

$$-\frac{3\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)+2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{}}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(
d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*
d), -1/4*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(
sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c)
)/(a*sin(d*x + c))) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*
d)]
```

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec\left(c+dx\right)}}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1031 vs. $2(78) = 156$.

Time = 0.21 (sec) , antiderivative size = 1031, normalized size of antiderivative = 10.63

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/4*(3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*
sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos
(d*x + c)^2 + 3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
+ 1))*sin(d*x + c)^2 + 2*(6*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/
2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 3*log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1
/2*c) + 1) - 2*sin(3/2*d*x + 3/2*c) + 2*sin(1/2*d*x + 1/2*c))*cos(2*d*x +
2*c) + 4*(3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 2*sin(1/2*d*x + 1/2*c))*cos(d*x + c) + 4
*(3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^2+2\sec(dx+c)+1}dx\right)}{a^2}$$

input
```
int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/(sec(c + d*x)**2
+ 2*sec(c + d*x) + 1),x))/a**2
```

3.255 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 137

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{7\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{5\sqrt{\sec(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}}$$

output
```
-7/4*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d*x+
c))^(1/2))*2^(1/2)/a^(3/2)/d-1/2*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*
x+c))^(3/2)+5/2*sec(d*x+c)^(1/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2\left(5\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+4\sqrt{-((-1+\sec(c+dx}{4d\sqrt{1-}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)),x]
```

output
```
(2*(5*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + 4*Sqrt[-((-1 + Sec[c + d
*x])*Sec[c + d*x])])*Sin[c + d*x] + 7*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c +
 d*x]])/Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c + d*x])/(4*d*Sqrt
[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4304, 27, 3042, 4501, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{5a-2a\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5a-2a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{5a-2a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4501}$$

$$\frac{\frac{10a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-7a\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{10a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-7a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4295}$$

$$\frac{\frac{14a\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{10a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{10a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{7\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)),x]
```

output
```
-1/2*(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^(3/2)) + ((
-7*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt
[2]*Sqrt[a + a*Sec[c + d*x]])])/d + (10*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(d*Sqrt[a + a*Sec[c + d*x]]))/(4*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.88

method	result
default	$\frac{\left(\frac{(1-\cos(dx+c))^3\csc(dx+c)^3}{4}+\frac{7\arctan\left(\frac{\sqrt{2}\,(\cot(dx+c)-\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{2}{\cos(dx+c)+1}}}{4}+\frac{9\csc(dx+c)}{4}-\frac{9\cot(dx+c)}{4}\right)\sqrt{-a(-1-\sec(dx+c}}{d\,a^2\sqrt{\sec(dx+c)}}$

input
```
int(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*(1-cos(d*x+c))^3*csc(d*x+c)^3+7/4*arctan(1/2*2^(1/2)*(cot(d*x+c)-
csc(d*x+c))/(-1/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+9/4*csc(d
*x+c)-9/4*cot(d*x+c))/a^2*(-a*(-1-sec(d*x+c)))^(1/2)/sec(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.76

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{7\sqrt{2}\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)}{}\right.}{8\,(a}\right.$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(
d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1)) + 4*(4*cos(d*x + c)^2 + 5*cos(d*x + c))*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x
+ c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/4*(7*sqrt(2)*(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(4*cos(d*x + c)
^2 + 5*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/
sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(1/((a*(sec(c + d*x) + 1))**(3/2)*sqrt(sec(c + d*x))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7176 vs. $2(112) = 224$.

Time = 0.27 (sec) , antiderivative size = 7176, normalized size of antiderivative = 52.38

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(4*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2
*c)^4 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^4 + 4*(7*log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c
) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^4 + 70*(log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^2 + 7*(log(cos(1/2*d*x
+ 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1
))*sin(1/2*d*x + 1/2*c)^4 - 8*sin(1/2*d*x + 1/2*c)^5 + 28*(7*(log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))
*cos(3/2*d*x + 3/2*c)^3 + 4*(21*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 +...
```

Giac [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\frac{\left(\frac{\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a}+\frac{9\sqrt{2}}{a}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}+\frac{7\sqrt{2}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{\right.}\right.}{a^{\frac{3}{2}}}}{4\,d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/4*((sqrt(2)*tan(1/2*d*x + 1/2*c)^2/a + 9*sqrt(2)/a)*tan(1/2*d*x + 1/2*c)
/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + 7*sqrt(2)*log(abs(-sqrt(a)*tan(1/2*d
*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(3/2))/(d*sgn(cos(d*x
 + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^3+2\sec(dx+c)^2+\sec(dx+c)}dx\right)}{a^2}$$

input
```
int(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/(sec(c + d*x)**3
+ 2*sec(c + d*x)**2 + sec(c + d*x)),x))/a**2
```

3.256 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 177

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \frac{11\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{7\sin(c+dx)}{6ad\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{19\sqrt{\sec(c+dx)}\sin(c+dx)}{6ad\sqrt{a+a\sec(c+dx)}}$$

output
```
11/4*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d*x+
c))^(1/2))*2^(1/2)/a^(3/2)/d-1/2*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*
x+c))^(3/2)+7/6*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)-19/
6*sec(d*x+c)^(1/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \frac{-33\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sin}{6d\sqrt{-((-1+\sec(c+}}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

output
```
(-33*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*C
os[(c + d*x)/2]^2*Sec[c + d*x]^(5/2)*Sin[c + d*x] + Sqrt[1 - Sec[c + d*x]]
*(4*Sin[c + d*x] - (12 + 19*Sec[c + d*x])*Tan[c + d*x]))/(6*d*Sqrt[-((-1 +
 Sec[c + d*x])*Sec[c + d*x])]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4304, 27, 3042, 4510, 27, 3042, 4501, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{7a-4a\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{7a-4a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{7a-4a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 4510}$$

$$\frac{\frac{2\int -\frac{19a^2-14a^2\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}+\frac{14a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{14a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{19a^2-14a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{14a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{19a^2-14a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 4501}$$

$$\frac{\frac{14a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{38a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-33a^2\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{14a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{38a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-33a^2\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4295}$$

$$\frac{\frac{14a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{66a^2\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{38a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}}{3a}}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{14a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{38a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{33\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{3a}}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}$$

input
```
Int[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

output
```
-1/2*Sin[c + d*x]/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)) + ((14
*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((-33
*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2
]*Sqrt[a + a*Sec[c + d*x]])])/d + (38*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(d*Sqrt[a + a*Sec[c + d*x]]))/(3*a))/(4*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.82

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left(\left(8\cos(dx+c)^2-24\cos(dx+c)-38\right)\tan(dx+c)+\arctan\left(\frac{\sqrt{2}\,(\cot(dx+c)-\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{2}{\cos(dx+c)+1}}\,(-33\cos\right.}{12d\,a^2\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sec(dx+c)^{\frac{3}{2}}}$

input
```
int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/d/a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)^2+2*cos(d*x+c)+1)/sec(d*x+
c)^(3/2)*((8*cos(d*x+c)^2-24*cos(d*x+c)-38)*tan(d*x+c)+arctan(1/2*2^(1/2)*
(cot(d*x+c)-csc(d*x+c))/(-1/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/
2)*(-33*cos(d*x+c)-66-33*sec(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.25

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{33\sqrt{2}\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c}{\cdots}\right.}{\cdots}\right.$$

$$-\frac{33\sqrt{2}\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)-\frac{2\left(4\cos(dx+c)^3-12\cdots\right.}{\cdots}}{12\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/24*(33*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*co
s(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 +
 2*cos(d*x + c) + 1)) + 4*(4*cos(d*x + c)^3 - 12*cos(d*x + c)^2 - 19*cos(d
*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
 + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/12*(33*s
qrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(
-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x
+ c))) - 2*(4*cos(d*x + c)^3 - 12*cos(d*x + c)^2 - 19*cos(d*x + c))*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*
cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(a(\sec(c+dx)+1))^{\frac{3}{2}}\sec^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(1/((a*(sec(c + d*x) + 1))**(3/2)*sec(c + d*x)**(3/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33960 vs. 2(146) = 292.

Time = 0.60 (sec) , antiderivative size = 33960, normalized size of antiderivative = 191.86

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/12*(4*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(
3/2*d*x + 3/2*c) - 9*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 64*(cos(3*d*x + 3*c)^2*sin(3/2*d
*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 9*(cos(3*d*x + 3*c
)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))))*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^4 + 4*sin(3/2*d*x + 3/2*c)^5 + 4*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c)
 + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 9*(cos(3*d*x + 3*c)^2 + sin(3
*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
))*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 64*(co
s(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3
/2*c) - 9*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(5/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c)))^4 + 4*(2*cos(3*d*x + 3*c)^2*cos(3/2*d*x + 3/2*c
)*sin(3/2*d*x + 3/2*c) + 2*cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)^2*sin(3/2
*d*x + 3/2*c) + 2*cos(3*d*x + 3*c)*cos(3/2*d*x + 3/2*c)*sin(3/2*d*x + 3/2*
c) + 8*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3
/2*d*x + 3/2*c) - 9*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arctan2(sin(3...
```

Giac [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\frac{\left(\left(3\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+46\sqrt{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+27\sqrt{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\right)^{\frac{3}{2}}}+\frac{33\sqrt{2}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{a^{\frac{3}{2}}}}{12\,d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-1/12*(((3*sqrt(2)*tan(1/2*d*x + 1/2*c)^2 + 46*sqrt(2))*tan(1/2*d*x + 1/2*
c)^2 + 27*sqrt(2))*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/
2) + 33*sqrt(2)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a)))/a^(3/2))/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^4+2\sec(dx+c)^3+\sec(dx+c)^2}dx\right)}{a^2}$$

input
```
int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/(sec(c + d*x)**4
+ 2*sec(c + d*x)**3 + sec(c + d*x)**2),x))/a**2
```

3.257 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 217

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{15\text{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{9\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{13\sin(c+dx)}{10ad\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{49\sqrt{\sec(c+dx)}\sin(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}}$$

output
```
-15/4*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d*x
+c))^(1/2))*2^(1/2)/a^(3/2)/d-1/2*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d
*x+c))^(3/2)+9/10*sin(d*x+c)/a/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)-1
3/10*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+49/10*sec(d*x+
c)^(1/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.87 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \frac{75\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c}{10d\sqrt{-(}}$$

input
```
Integrate[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

output
```
(75*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Co
s[(c + d*x)/2]^2*Sec[c + d*x]^(5/2)*Sin[c + d*x] + (47 + 39*Cos[c + d*x] -
 2*Cos[2*(c + d*x)] + Cos[3*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x
]*Tan[c + d*x])/(10*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*(a*(1 + Se
c[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4304, 27, 3042, 4510, 27, 3042, 4510, 27, 3042, 4501, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{3(3a-2a\sec(c+dx))}{2\sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{3a-2a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3\int \frac{3a-2a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 4510}$$

$$\frac{3\left(\frac{2\int -\frac{13a^2-12a^2\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{5a}+\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{13a^2-12a^2\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{5a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{13a^2-12a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 4510}$$

$$\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2\int -\frac{49a^3-26a^3\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}+\frac{26a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}}{5a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{49a^3-26a^3\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}}{5a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{49a^3-26a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{5a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 4501}$$

$$\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{98a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-75a^3\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{3a}}{5a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{98a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-75a^3\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{5a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 4295}$$

$$\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{150a^3\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{98a^3\sin(c+}{d\sqrt{a\,s}}}{3a}}{5a}\right)}{4a^2}$$

$$\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 219

$$\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{98a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{75\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{3a}}{5a}\right)}{4a^2}$$

$$\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

input

```
Int[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

output

```
-1/2*Sin[c + d*x]/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)) + (3*(
(6*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) - ((2
6*a^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((
-75*Sqrt[2]*a^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqr
t[2]*Sqrt[a + a*Sec[c + d*x]])])/d + (98*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(3*a))/(5*a)))/(4*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.75

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left(\left(8\cos(dx+c)^3-8\cos(dx+c)^2+72\cos(dx+c)+98\right)\tan(dx+c)\sec(dx+c)+\arctan\left(\frac{\sqrt{2}\,(\cot(dx+c)-\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\right.}{20d\,a^2\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sec(dx+c)^{\frac{5}{2}}}$

input
```
int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/20/d/a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)^2+2*cos(d*x+c)+1)/sec(d*x+
c)^(5/2)*((8*cos(d*x+c)^3-8*cos(d*x+c)^2+72*cos(d*x+c)+98)*tan(d*x+c)*sec(
d*x+c)+arctan(1/2*2^(1/2)*(cot(d*x+c)-csc(d*x+c))/(-1/(cos(d*x+c)+1))^(1/2
))*(-2/(cos(d*x+c)+1))^(1/2)*(75+150*sec(d*x+c)+75*sec(d*x+c)^2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.93

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{75\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c}{}\right.}{}\right.$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/40*(75*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*co
s(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 +
 2*cos(d*x + c) + 1)) + 4*(4*cos(d*x + c)^4 - 4*cos(d*x + c)^3 + 36*cos(d*
x + c)^2 + 49*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*
x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) +
a^2*d), 1/20*(75*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*ar
ctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
 + c))/(a*sin(d*x + c))) + 2*(4*cos(d*x + c)^4 - 4*cos(d*x + c)^3 + 36*cos
(d*x + c)^2 + 49*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin
(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c)
 + a^2*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument V
alue
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^5+2\sec(dx+c)^4+\sec(dx+c)^3}dx\right)}{a^2}$$

input
```
int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/(sec(c + d*x)**5
+ 2*sec(c + d*x)**4 + sec(c + d*x)**3),x))/a**2
```

3.258 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 214

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{5\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{115\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{15\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{35\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16a^2d\sqrt{a+a\sec(c+dx)}}$$

output

```
-5*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d+115/32*arc
tanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2
))*2^(1/2)/a^(5/2)/d-1/4*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5
/2)-15/16*sec(d*x+c)^(5/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)+35/16*sec
(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.59

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{70\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)+110\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)}{}$$

input
```
Integrate[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(70*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] + 110*Sqrt[1 -
Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] + 32*Sqrt[1 - Sec[c + d*x]]*
Sec[c + d*x]^(7/2)*Sin[c + d*x] - 115*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c +
 d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - 230*Sqrt[2]*ArcTan[(Sqrt[2]
*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[c + d*x] - 1
15*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec
[c + d*x]^2*Tan[c + d*x] + 70*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c +
d*x])^2*Tan[c + d*x] + 230*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])^2
*Tan[c + d*x])/(32*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4303, 27, 3042, 4507, 27, 3042, 4509, 25, 3042, 4511, 3042, 4288, 222, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{5\sec^{\frac{5}{2}}(c+dx)(a-2a\sec(c+dx))}{2(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{5\int \frac{\sec^{\frac{5}{2}}(c+dx)(a-2a\sec(c+dx))}{(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{5\left(\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(9a^2-14a^2\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{5\left(\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(9a^2-14a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(9a^2-14a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{4509}$$

$$-\frac{5\left(\frac{\frac{\int -\frac{\sqrt{\sec(c+dx)}\left(7a^3-16a^3\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{5\left(\frac{-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(7a^3-16a^3\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5\left(\frac{-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(7a^3-16a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 4511$$

$$-\frac{5\left(\frac{-\frac{23a^3\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx-16a^2\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5\left(\frac{-\frac{23a^3\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-16a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 4288$$

$$-\frac{5\left(\frac{-\frac{23a^3\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{32a^2\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 222$$

$$-\frac{5\left(\frac{-\frac{23a^3\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{32a^{5/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-$$

$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 4295$$

$$-\frac{5\left(\frac{-\frac{-\frac{46a^3\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{32a^{5/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}+\frac{3a\sin\ldots}{2d(\ldots}\right)}{8a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 219$$

$$-\frac{5\left(\frac{-\frac{\frac{23\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{32a^{5/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c\cdots}{2d(a\sec(c+dx)+a)}\right.}{8a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

input
```
Int[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
-1/4*(Sec[c + d*x]^(7/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^(5/2)) - (5
*((3*a*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) +
 (-(((-32*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]
)/d + (23*Sqrt[2]*a^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d)/a) - (14*a^2*Sec[c + d*x]^(3/2)*
Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(4*a^2)))/(8*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4509
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Simp[d/(b*(m + n))   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
 1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[n, 1]
```

rule 4511
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b -
a*B)/b   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Simp[B/b
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b
, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 3.08 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.50

method	result
default	$\frac{\cos(dx+c)^4\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\left(-115\cos(dx+c)^3-230\cos(dx+c)^2-115\cos(dx+c)\right)+\arctan\left(\frac{-\cot(dx+c)}{2\sqrt{-\frac{}{\cos}}}\right.}{}$

input
```
int(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/d/a^3*cos(d*x+c)^4*(2^(1/2)*arctan(1/2*2^(1/2)/(-1/(cos(d*x+c)+1))^(1
/2)*(-cot(d*x+c)+csc(d*x+c)))*(-115*cos(d*x+c)^3-230*cos(d*x+c)^2-115*cos(
d*x+c))+arctan(1/2/(-1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1))*(
80*cos(d*x+c)^3+160*cos(d*x+c)^2+80*cos(d*x+c))+arctan(1/2*(-cot(d*x+c)+cs
c(d*x+c)-1)/(-1/(cos(d*x+c)+1))^(1/2))*(80*cos(d*x+c)^3+160*cos(d*x+c)^2+8
0*cos(d*x+c))+sin(d*x+c)*(35*cos(d*x+c)^2+55*cos(d*x+c)+16)*2^(1/2)*(-2/(c
os(d*x+c)+1))^(1/2))*sec(d*x+c)^(9/2)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)
^3+3*cos(d*x+c)^2+3*cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 664, normalized size of antiderivative = 3.10

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(115*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*
a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 80*(cos(d*x + c)^3 + 3*cos(d*x
 + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x
+ c)^2 + 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3
+ cos(d*x + c)^2)) + 4*(35*cos(d*x + c)^2 + 55*cos(d*x + c) + 16)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*co
s(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/
32*(115*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sq
rt(cos(d*x + c))/(a*sin(d*x + c))) + 80*(cos(d*x + c)^3 + 3*cos(d*x + c)^2
 + 3*cos(d*x + c) + 1)*sqrt(-a)*arctan(1/2*(cos(d*x + c)^2 - 2*cos(d*x + c
))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))/(a*sqrt(cos(d*x + c))*
sin(d*x + c))) - 2*(35*cos(d*x + c)^2 + 55*cos(d*x + c) + 16)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*
x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(9/2)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9048 vs. $2(177) = 354$.

Time = 2.29 (sec) , antiderivative size = 9048, normalized size of antiderivative = 42.28

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-1/32*(140*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 4
*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*sin(3/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - 16*(75*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 24*sin(7/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 24*sin(5/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 75*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) - 35*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*
cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 300*(sin(6*d*x + 6*
c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 8*sin(3/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 96*(sin
(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 8*sin(3/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 32*(24*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 75*sin(3
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 35*sin(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - 96*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*
c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/4*ar...
```

Giac [F]

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{9}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(9/2)/(a*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(9/2)/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(9/2)/(a + a/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}\,\sec(dx+c)^4}{\sec(dx+c)^3+3\sec(dx+c)^2+3\sec(dx+c)+1}dx\right)}{a^3}$$

input
```
int(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1)*sec(c + d*x)**4)/(
sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1),x))/a**3
```

3.259 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 174

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{2\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{43\text{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{11\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

output
```
2*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d-43/32*arcta
nh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))
*2^(1/2)/a^(5/2)/d-1/4*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2
)-11/16*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.77

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{-22\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)-30\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(}{}$$

input
```
Integrate[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(-22*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] - 30*Sqrt[1 -
Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] + 43*Sqrt[2]*ArcTan[(Sqrt[2]
*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] + 86*Sqrt[2]*Arc
Tan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[
c + d*x] + 43*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c +
 d*x]]]*Sec[c + d*x]^2*Tan[c + d*x] - 22*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1
 + Sec[c + d*x])^2*Tan[c + d*x] - 86*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c
 + d*x])^2*Tan[c + d*x])/(32*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]
))^(5/2))
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4303, 27, 3042, 4507, 27, 3042, 4511, 3042, 4288, 222, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)(3a-8a\sec(c+dx))}{2(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\big\downarrow$ 27

$$-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)(3a-8a\sec(c+dx))}{(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\big\downarrow$ 3042

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3a-8a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\big\downarrow$ 4507

$$-\frac{\frac{\int \frac{\sqrt{\sec(c+dx)}\left(11a^2-32a^2\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\big\downarrow$ 27

$$-\frac{\frac{\int \frac{\sqrt{\sec(c+dx)}\left(11a^2-32a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\big\downarrow$ 3042

$$-\frac{\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(11a^2-32a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\big\downarrow$ 4511

$$-\frac{\frac{43a^2\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx-32a\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\big\downarrow$ 3042

$$-\frac{\frac{43a^2\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-32a\int \sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{\csc(c+dx+\frac{\pi}{2})a+a}dx}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{4288}$$

$$-\frac{\frac{43a^2\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{64a\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{222}$$

$$-\frac{\frac{43a^2\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{64a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{4295}$$

$$-\frac{\frac{-\frac{86a^2\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{64a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{\frac{43\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{64a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

input
```
Int[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
-1/4*(Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^(5/2)) - ((
(-64*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
 (43*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sq
rt[2]*Sqrt[a + a*Sec[c + d*x]])])/d)/(4*a^2) + (11*a*Sec[c + d*x]^(3/2)*Si
n[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)))/(8*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4511
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b -
a*B)/b   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Simp[B/b
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b
, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Maple [A] (warning: unable to verify)

Time = 3.00 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.61

method	result
default	$-\frac{\sec(dx+c)^{\frac{7}{2}}\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^4\sqrt{-a(-1-\sec(dx+c))}\left(2\sqrt{-\frac{2}{\cos(dx+c)+1}}\,(1-\cos(dx+c))^3\csc(dx+c)^3-16\sqrt{2}\,a\right.}{64d\,a^3\left((\right.}$

input
```
int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/64/d/a^3*sec(d*x+c)^(7/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^4*(-a*(-1-s
ec(d*x+c)))^(1/2)*(2*(-2/(cos(d*x+c)+1))^(1/2)*(1-cos(d*x+c))^3*csc(d*x+c)
^3-16*2^(1/2)*arctan(1/2*(cot(d*x+c)-csc(d*x+c)-1)/(-1/(cos(d*x+c)+1))^(1/
2))-16*2^(1/2)*arctan(1/2/(-1/(cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)
+1))+13*(-2/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c))+43*arctan(1/2*2
^(1/2)*(cot(d*x+c)-csc(d*x+c))/(-1/(cos(d*x+c)+1))^(1/2)))/((1-cos(d*x+c))
^2*csc(d*x+c)^2+1)^3*2^(1/2)/(-1/(cos(d*x+c)+1))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(143) = 286$.

Time = 0.12 (sec) , antiderivative size = 662, normalized size of antiderivative = 3.80

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 32*(cos(d*x + c)^3 + 3*cos(d*x
+ c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x +
 c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 +
 cos(d*x + c)^2)) - 4*(11*cos(d*x + c)^2 + 15*cos(d*x + c))*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x
+ c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(43*
sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*
arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))/(a*sin(d*x + c))) + 32*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos
(d*x + c) + 1)*sqrt(-a)*arctan(1/2*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(
-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))/(a*sqrt(cos(d*x + c))*sin(d*x
+ c))) - 2*(11*cos(d*x + c)^2 + 15*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*
a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4988 vs. $2(143) = 286$.

Time = 0.44 (sec) , antiderivative size = 4988, normalized size of antiderivative = 28.67

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx=\text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/32*(44*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 16*(
19*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 19*sin(3/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 11*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 76*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - 76*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - 44*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c))*cos(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*cos(4*d*x + 4*c)^2
 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c)
 + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos
(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +...
```

Giac [A] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.26

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx =$$

$$-\frac{2\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3}+\frac{13\sqrt{2}}{a^3}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{43\sqrt{2}\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan(\cdots}\right.\right.}{a^{\frac{5}{2}}}\cdots}{\cdots}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-1/64*(2*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c
)^2/a^3 + 13*sqrt(2)/a^3)*tan(1/2*d*x + 1/2*c) - 43*sqrt(2)*log((sqrt(a)*t
an(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a^(5/2) - 64*
log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)
)^2 - a*(2*sqrt(2) + 3)))/a^(5/2) + 64*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*
c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/a^(5/2))/
(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}\,\sec(dx+c)^3}{\sec(dx+c)^3+3\sec(dx+c)^2+3\sec(dx+c)+1}dx\right)}{a^3}$$

input
```
int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1)*sec(c + d*x)**3)/(
sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1),x))/a**3
```

3.260 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 137

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{3\text{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

output
```
3/32*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d*x+
c))^(1/2))*2^(1/2)/a^(5/2)/d+1/4*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*
x+c))^(5/2)+3/16*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 308 vs. 2(137) = 274.

Time = 0.47 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.25

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{6\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx) + 14\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+\cdots}{}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(6*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] + 14*Sqrt[1 - Se
c[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] - 3*Sqrt[2]*ArcTan[(Sqrt[2]*Sq
rt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - 6*Sqrt[2]*ArcTan[
(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[c +
d*x] - 3*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]
]]*Sec[c + d*x]^2*Tan[c + d*x] + 6*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec
[c + d*x])^2*Tan[c + d*x] + 6*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x]
)^2*Tan[c + d*x])/(32*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2
))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4297, 3042, 4297, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4297}$$

$$\frac{3\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(\sec(c+dx)a+a)^{3/2}}dx}{8a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \ 4297$$

$$\frac{3\left(\frac{\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{4a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{3\left(\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \ 4295$$

$$\frac{3\left(\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2ad}\right)}{8a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \ 219$$

$$\frac{3\left(\frac{\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

input
```
Int[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(Sec[c + d*x]^(5/2)*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) + (3*(A
rcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
 + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*
(a + a*Sec[c + d*x])^(3/2))))/(8*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4297
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[d*((m + 1)/(b*(2*m + 1)))
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ
[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m,
-2^(-1)] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.26

method	result
default	$\frac{\left(\left(3\cos(dx+c)^2+6\cos(dx+c)+3\right)\arctan\left(\frac{\sqrt{2}\,(\cot(dx+c)-\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+(3\cos(dx+c)+7)\sin(dx+c)\sqrt{-\frac{2}{\cos(dx+c)+1}}\right)\cos(dx+c)^3}{32d\,a^3\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/d/a^3*((3*cos(d*x+c)^2+6*cos(d*x+c)+3)*arctan(1/2*2^(1/2)*(cot(d*x+c)
-csc(d*x+c))/(-1/(cos(d*x+c)+1))^(1/2))+(3*cos(d*x+c)+7)*sin(d*x+c)*(-2/(c
os(d*x+c)+1))^(1/2))*cos(d*x+c)^3*2^(1/2)*(a*(1+sec(d*x+c)))^(1/2)*sec(d*x
+c)^(5/2)/(cos(d*x+c)^3+3*cos(d*x+c)^2+3*cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))
^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 426, normalized size of antiderivative = 3.11

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{3\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(\cdots}{\cdots}\right.}{64\left(a^3d\cos(dx+c)\cdots\right.}\right.$$

$$-\frac{3\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)-\cdots}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+\cdots\right.}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)
/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(3*cos(d*x + c)^2 + 7*cos(d*x
+ c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x +
c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c)
 + a^3*d), -1/32*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x
 + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*(3*cos(d*x + c)^2 + 7*c
os(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos
(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d
*x + c) + a^3*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84332 vs. 2(112) = 224.

Time = 14.41 (sec) , antiderivative size = 84332, normalized size of antiderivative = 615.56

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/32*(512*((2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(
5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 2*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c
) + (2*cos(2*d*x + 2*c) + cos(d*x + c))*sin(5/2*d*x + 5/2*c) + cos(4*d*x +
 4*c)*sin(5/2*d*x + 5/2*c) + 2*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(
5*d*x + 5*c)^2 + 2560*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x +
 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*s
in(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x
 + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(
5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x
+ 3*c)*sin(5/2*d*x + 5/2*c))*cos(8/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
*d*x + 5/2*c)))^2 + 10240*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d
*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*
c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2
*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*
sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*
d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos
(5/2*d*x + 5/2*c)))^2 + 10240*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5
/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x +
5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*c
os(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x ...
```

Giac [A] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.82

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\,\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3}+\frac{5\,\sqrt{2}}{a^3}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{3\,\sqrt{2}1}{}}{32\,d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/32*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2
/a^3 + 5*sqrt(2)/a^3)*tan(1/2*d*x + 1/2*c) - 3*sqrt(2)*log(abs(-sqrt(a)*ta
n(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(5/2))/(d*sgn(
cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}\,\sec(dx+c)^2}{\sec(dx+c)^3+3\sec(dx+c)^2+3\sec(dx+c)+1}dx\right)}{a^3}$$

input
```
int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1)*sec(c + d*x)**2)/(
sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1),x))/a**3
```

3.261 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 137

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{5\text{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

output
```
5/32*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d*x+
c))^(1/2))*2^(1/2)/a^(5/2)/d-1/4*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*
x+c))^(5/2)+5/16*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.26 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.49

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx) + \frac{5(1+\sec(c+dx))\left(2\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)-(1+\sec(c+dx))\left(2\arcsin\left(\sqrt{1-\sec(c+dx)}\right) \cdots\right.\right.}{\sqrt{1-\sec \cdots}}}{32d(a(1+\sec(c+dx)))^{5/2}}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
-1/32*(8*Sec[c + d*x]^(5/2)*Sin[c + d*x] + (5*(1 + Sec[c + d*x])*(2*Sqrt[1
 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] - (1 + Sec[c + d*x])*(2*A
rcSin[Sqrt[1 - Sec[c + d*x]]] + 2*ArcSin[Sqrt[Sec[c + d*x]]] - Sqrt[2]*Arc
Tan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] + 2*Sqrt[-((-1 +
Sec[c + d*x])*Sec[c + d*x])])*Tan[c + d*x]))/Sqrt[1 - Sec[c + d*x]])/(d*(a
*(1 + Sec[c + d*x]))^(5/2))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4298, 3042, 4297, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4298}$$

$$\frac{5\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(\sec(c+dx)a+a)^{3/2}}dx}{8a}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{4297}$$

$$\frac{5\left(\frac{\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{4a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{4295}$$

$$\frac{5\left(\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2ad}\right)}{8a}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{5\left(\frac{\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

input
```
Int[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
-1/4*(Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^(5/2)) + (5
*(ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2
*d*(a + a*Sec[c + d*x])^(3/2))))/(8*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4297
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[d*((m + 1)/(b*(2*m + 1)))
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ
[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m,
-2^(-1)] && IntegerQ[2*m]
```

rule 4298
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[m/(a*(2*m + 1))   Int[(a + b*Csc[e +
 f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.26

method	result
default	$-\frac{\left(\left(5\cos(dx+c)^{2}+10\cos(dx+c)+5\right)\arctan\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+(-5\cos(dx+c)-1)\sin(dx+c)\sqrt{-\frac{2}{\cos(dx+c)+1}}\right)\cos(}{32d\,a^{3}\left(\cos(dx+c)^{3}+3\cos(dx+c)^{2}+3\cos(dx+c)+1\right)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/d/a^3*((5*cos(d*x+c)^2+10*cos(d*x+c)+5)*arctan(1/2*2^(1/2)/(-1/(cos(
d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)))+(-5*cos(d*x+c)-1)*sin(d*x+c)*(-
2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*2^(1/2)*(a*(1+sec(d*x+c)))^(1/2)*sec
(d*x+c)^(3/2)/(cos(d*x+c)^3+3*cos(d*x+c)^2+3*cos(d*x+c)+1)/(-1/(cos(d*x+c)
+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 422, normalized size of antiderivative = 3.08

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{5\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(\cdots}{\cdots}\right.}{64\left(a^3d\cos(dx+c)^3\cdots\right.}\right.$$

$$-\frac{5\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)-\cdots}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+\cdots\right.}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)
/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(5*cos(d*x + c)^2 + cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)
))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) +
 a^3*d), -1/32*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x +
 c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*(5*cos(d*x + c)^2 + cos(d
*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
 + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
 c) + a^3*d)]
```

Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**(3/2)/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2875 vs. $2(112) = 224$.

Time = 0.40 (sec) , antiderivative size = 2875, normalized size of antiderivative = 20.99

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/32*(4*(3*sin(3/2*d*x + 3/2*c) + 5*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) - 3*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 40*(2
*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))
*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(2*sin(
3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(
5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(3*sin(3/2*d
*x + 3/2*c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*(3*s
in(3/2*d*x + 3/2*c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))))*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 5*(16*cos(3*d*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c)))^2 + 12*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}\,\sec(dx+c)}{\sec(dx+c)^3+3\sec(dx+c)^2+3\sec(dx+c)+1}dx\right)}{a^3}$$

input
```
int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1)*sec(c + d*x))/(sec
(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1),x))/a**3
```

3.262 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 137

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{19\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{9\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

output
```
19/32*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d*x
+c))^(1/2))*2^(1/2)/a^(5/2)/d-1/4*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d
*x+c))^(5/2)-9/16*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{-76\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^5\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)}{16d\sqrt{1-\sec(c+dx)}(a(1+}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(-76*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*C
os[(c + d*x)/2]^5*Sec[c + d*x]^3*Sin[(c + d*x)/2] - (9 + 13*Cos[c + d*x])*
Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(16*d*Sqrt[1 - Sec
[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 4304, 27, 3042, 4500, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{\sqrt{\sec(c+dx)}(7a-2a\sec(c+dx))}{2(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{\sec(c+dx)}(7a-2a\sec(c+dx))}{(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(7a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{4500}$$

$$\frac{\frac{19}{4}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx-\frac{9a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{19}{4}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{9a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \textbf{4295}$$

$$\frac{-\frac{19\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2d}-\frac{9a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{\frac{19\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\sqrt{a}d}-\frac{9a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

input

```
Int[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^(5/2),x]
```

output

```
-1/4*(Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^(5/2)) + ((
19*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])])/(2*Sqrt[2]*Sqrt[a]*d) - (9*a*Sec[c + d*x]^(3/2)*Sin[c + d*
x])/(2*d*(a + a*Sec[c + d*x])^(3/2)))/(8*a^2)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4500
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] + Simp[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1))   Int[(a + b*Csc[e +
 f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}
, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[
m, -1]
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.24

method	result
default	$-\frac{\left(\left(-19\cos(dx+c)^{2}-38\cos(dx+c)-19\right)\arctan\left(\frac{\sqrt{2}\,(\cot(dx+c)-\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+(13\cos(dx+c)+9)\sin(dx+c)\sqrt{-\frac{2}{\cos(dx+c)+1}}\right)\text{co}}{32d\,a^{3}\left(\cos(dx+c)^{3}+3\cos(dx+c)^{2}+3\cos(dx+c)+1\right)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/d/a^3*((-19*cos(d*x+c)^2-38*cos(d*x+c)-19)*arctan(1/2*2^(1/2)*(cot(d
*x+c)-csc(d*x+c))/(-1/(cos(d*x+c)+1))^(1/2))+(13*cos(d*x+c)+9)*sin(d*x+c)*
(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*2^(1/2)*sec(d*x+c)^(1/2)*(a*(1+sec(d
*x+c)))^(1/2)/(cos(d*x+c)^3+3*cos(d*x+c)^2+3*cos(d*x+c)+1)/(-1/(cos(d*x+c)
+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 426, normalized size of antiderivative = 3.11

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{19\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos\cdots}{\cdots}\right.}{64\left(a^3d\cos(dx+c\cdots\right.}\right.$$

$$-\frac{19\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)\cdots}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+\cdots\right.}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(13*cos(d*x + c)^2 + 9*cos(d*
x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
+ c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
c) + a^3*d), -1/32*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(
d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(13*cos(d*x + c)^2 +
 9*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt
(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*c
os(d*x + c) + a^3*d)]
```

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec{(c+dx)}}}{(a\,(\sec{(c+dx)}+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3049 vs. $2(112) = 224$.

Time = 0.51 (sec) , antiderivative size = 3049, normalized size of antiderivative = 22.26

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/32*(19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
2*sin(1/2*d*x + 1/2*c) + 1))*cos(4*d*x + 4*c)^2 + 304*(log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*
cos(3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 304*(log
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
+ 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
 + 1/2*c) + 1))*cos(d*x + c)^2 + 19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(4*d*x + 4*c)^2
 + 304*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*
sin(1/2*d*x + 1/2*c) + 1))*sin(3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*si
n(2*d*x + 2*c)^2 + 304*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^3+3\sec(dx+c)^2+3\sec(dx+c)+1}dx\right)}{a^3}$$

input
```
int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/(sec(c + d*x)**3
+ 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1),x))/a**3
```

3.263 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 177

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{75\text{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{13\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{49\sqrt{\sec(c+dx)}\sin(c+dx)}{16a^2d\sqrt{a+a\sec(c+dx)}}$$

output
```
-75/32*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d*
x+c))^(1/2))*2^(1/2)/a^(5/2)/d-1/4*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(
d*x+c))^(5/2)-13/16*sec(d*x+c)^(1/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)
+49/16*sec(d*x+c)^(1/2)*sin(d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.91 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = \frac{300\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^5\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sin}{}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)),x]
```

output
```
(300*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*C
os[(c + d*x)/2]^5*Sec[c + d*x]^3*Sin[(c + d*x)/2] + (85*Sqrt[1 - Sec[c + d
*x]]*Sec[c + d*x]^(3/2) + 49*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2) + 3
2*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Sin[c + d*x])/(16*d*Sqrt[1 -
Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4304, 27, 3042, 4508, 27, 3042, 4501, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4304}$$

$$-\frac{\int -\frac{9a-4a\sec(c+dx)}{2\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{9a-4a\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{9a-4a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 4508$$

$$\frac{\frac{\int \frac{49a^2-26a^2\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{13a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{49a^2-26a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{13a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{49a^2-26a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{13a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 4501$$

$$\frac{\frac{\frac{98a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-75a^2\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{13a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{98a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-75a^2\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{13a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 4295$$

$$\frac{\frac{\frac{150a^2\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{98a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}-\frac{13a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\frac{98a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{75\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}-\frac{13a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)),x]
```

output
```
-1/4*(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^(5/2)) + ((
-13*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) +
((-75*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(S
qrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d + (98*a^2*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(4*a^2))/(8*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [A] (warning: unable to verify)

Time = 1.86 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.81

method	result
default	$\frac{\left(-\frac{(1-\cos(dx+c))^5\csc(dx+c)^5}{16}+\frac{17(1-\cos(dx+c))^3\csc(dx+c)^3}{32}+\frac{75\arctan\left(\frac{\sqrt{2}\,(\cot(dx+c)-\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{2}{\cos(dx+c)+1}}}{32}+\frac{83\csc(dx+c)}{32}-\cdots\right.}{d\,a^3\sqrt{\sec(dx+c)}}$

input
```
int(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/16*(1-cos(d*x+c))^5*csc(d*x+c)^5+17/32*(1-cos(d*x+c))^3*csc(d*x+c)
^3+75/32*arctan(1/2*2^(1/2)*(cot(d*x+c)-csc(d*x+c))/(-1/(cos(d*x+c)+1))^(1
/2))*(-2/(cos(d*x+c)+1))^(1/2)+83/32*csc(d*x+c)-83/32*cot(d*x+c))/a^3*(-a*
(-1-sec(d*x+c)))^(1/2)/sec(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 446, normalized size of antiderivative = 2.52

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{75\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\cdots}{\cdots}\right.$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(32*cos(d*x + c)^3 + 85*cos(d
*x + c)^2 + 49*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d
*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2
 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos
(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*
(32*cos(d*x + c)^3 + 85*cos(d*x + c)^2 + 49*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x +
c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}\sqrt{\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral(1/((a*(sec(c + d*x) + 1))**(5/2)*sqrt(sec(c + d*x))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258456 vs. 2(146) = 292.

Time = 2.65 (sec) , antiderivative size = 258456, normalized size of antiderivative = 1460.20

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-1/32*(576*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2
*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x
 + 5/2*c)^6 + 14400*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*co
s(3/2*d*x + 3/2*c)^6 + 187500*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(
1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^6 +
 576*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2
*c)^6 + 5184*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1
/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*sin(3/2*d
*x + 3/2*c)^6 + 262500*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
 + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^4*sin(1/2*
d*x + 1/2*c)^2 + 77700*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*...
```

Giac [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\frac{\left(\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^2}-\frac{17\sqrt{2}}{a^2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\frac{83\sqrt{2}}{a^2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}-\frac{75\sqrt{2}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{a^{\frac{5}{2}}}}{32\,d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-1/32*(((2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/a^2 - 17*sqrt(2)/a^2)*tan(1/2*d*
x + 1/2*c)^2 - 83*sqrt(2)/a^2)*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1
/2*c)^2 + a) - 75*sqrt(2)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a)))/a^(5/2))/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^4+3\sec(dx+c)^3+3\sec(dx+c)^2+\sec(dx+c)}dx\right)}{a^3}$$

input
```
int(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/(sec(c + d*x)**4
+ 3*sec(c + d*x)**3 + 3*sec(c + d*x)**2 + sec(c + d*x)),x))/a**3
```

3.264 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 217

$$\begin{aligned}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx &= \frac{163\text{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} \\ &\quad - \frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}} \\ &\quad - \frac{17\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}} \\ &\quad + \frac{95\sin(c+dx)}{48a^2d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{299\sqrt{\sec(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

output
```
163/32*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d*
x+c))^(1/2))*2^(1/2)/a^(5/2)/d-1/4*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(
d*x+c))^(5/2)-17/16*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2)
+95/48*sin(d*x+c)/a^2/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)-299/48*sec
(d*x+c)^(1/2)*sin(d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.57 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx =$$
$$-\frac{\sec(c+dx)\left(978\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)+\sqrt{1-\sec(c+dx)}}{48d\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}(a(1}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

output
```
-1/48*(Sec[c + d*x]*(978*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[
1 - Sec[c + d*x]]]*Cos[(c + d*x)/2]^4*Sec[c + d*x]^(5/2)*Sin[c + d*x] + Sq
rt[1 - Sec[c + d*x]]*(487 + 379*Sec[c + d*x] + 16*Cos[2*(c + d*x)]*(-1 + 5
*Sec[c + d*x]))*Tan[c + d*x]))/(d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])
]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4304, 27, 3042, 4508, 27, 3042, 4510, 27, 3042, 4501, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{11a-6a\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{11a-6a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{11a-6a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 4508$$

$$\frac{\frac{\int \frac{95a^2-68a^2\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{95a^2-68a^2\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{95a^2-68a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 4510$$

$$\frac{\frac{\frac{2\int -\frac{299a^3-190a^3\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}+\frac{190a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}}{4a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{299a^3-190a^3\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{299a^3-190a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 4501

$$\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{598a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-489a^3\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{598a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-489a^3\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 4295

$$\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{978a^3\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{598a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}}{3a}}{4a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{598a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{489\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{3a}}{4a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/}}}{8a^2}$$

$$\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}$$

input
```
Int[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

output
```
-1/4*Sin[c + d*x]/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)) + ((-1
7*a*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)) + ((
190*a^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) -
((-489*Sqrt[2]*a^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d + (598*a^3*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(3*a))/(4*a^2))/(8*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.81

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left(\left(64\cos(dx+c)^3-320\cos(dx+c)^2-1006\cos(dx+c)-598\right)\tan(dx+c)+\arctan\left(\frac{\sqrt{2}\,(\cot(dx+c)-\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-}\right.}{96d\,a^3\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sec(dx}$

input
```
int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/96/d/a^3*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)^3+3*cos(d*x+c)^2+3*cos(d*x
+c)+1)/sec(d*x+c)^(3/2)*((64*cos(d*x+c)^3-320*cos(d*x+c)^2-1006*cos(d*x+c)
-598)*tan(d*x+c)+arctan(1/2*2^(1/2)*(cot(d*x+c)-csc(d*x+c))/(-1/(cos(d*x+c
)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*(-489*cos(d*x+c)^2-1467*cos(d*x+c)-
1467-489*sec(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.15

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{489\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)}{}\right.$$

$$-\frac{489\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)}{96\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\right.}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/192*(489*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) +
1)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3
*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(32*cos(d*x + c)^4 - 160*co
s(d*x + c)^3 - 503*cos(d*x + c)^2 - 299*cos(d*x + c))*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3
 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/96*(489*sqrt
(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arct
an(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))/(a*sin(d*x + c))) - 2*(32*cos(d*x + c)^4 - 160*cos(d*x + c)^3 - 503*c
os(d*x + c)^2 - 299*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x +
 c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148823 vs. 2(180) = 360.

Time = 2.60 (sec) , antiderivative size = 148823, normalized size of antiderivative = 685.82

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/96*(32*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin
(3/2*d*x + 3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(11/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 41472*(cos(3*d*x + 3*c)^2*sin
(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 15*(cos(3*d*
x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^4 + 8192*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*
c)^2*sin(3/2*d*x + 3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 288*sin(3/2*d*x + 3/2*
c)^5 + 32*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*si
n(3/2*d*x + 3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(11/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 41472*(cos(3*d*x + 3*c)^2*si
n(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 15*(cos(3*d
*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))))*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))^4 + 8192*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3
*c)^2*sin(3/2*d*x + 3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^...
```

Giac [A] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\frac{\left(\left(3\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a}-\frac{23\sqrt{2}}{a}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{668\sqrt{2}}{a}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{465}{a}}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}}}{96\,d\,\mathrm{sgn}\,(\mathrm{co}}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/96*(((3*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/a - 23*sqrt(2)/a)*tan(1/2*d*x
+ 1/2*c)^2 - 668*sqrt(2)/a)*tan(1/2*d*x + 1/2*c)^2 - 465*sqrt(2)/a)*tan(1/
2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) - 489*sqrt(2)*log(abs(
-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(5/
2))/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^5+3\sec(dx+c)^4+3\sec(dx+c)^3+\sec(dx+c)^2}dx\right)}{a^3}$$

input
```
int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/(sec(c + d*x)**5
+ 3*sec(c + d*x)**4 + 3*sec(c + d*x)**3 + sec(c + d*x)**2),x))/a**3
```

3.265 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 126

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\text{arcsinh}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d} + \frac{7\text{arcsinh}\left(\frac{\tan(c+dx)}{\sqrt{1+\sec(c+dx)}}\right)}{4d} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{1+\sec(c+dx)}} + \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1+\sec(c+dx)}}$$

output
```
-2^(1/2)*arcsinh(tan(d*x+c)/(1+sec(d*x+c)))/d+7/4*arcsinh(tan(d*x+c)/(1+se
c(d*x+c))^(1/2))/d-1/4*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(1+sec(d*x+c))^(1/2)+
1/2*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(1+sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.11

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \frac{\cot(c+dx)\left(\arcsin\left(\sqrt{1-\sec(c+dx)}\right)+8\arcsin\left(\sqrt{\sec(c+dx)}\right)-4\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)-2\sqrt{\ldots}\right.}{4d}$$

input
```
Integrate[Sec[c + d*x]^(7/2)/Sqrt[1 + Sec[c + d*x]],x]
```

output
```
(Cot[c + d*x]*(ArcSin[Sqrt[1 - Sec[c + d*x]]] + 8*ArcSin[Sqrt[Sec[c + d*x]
]] - 4*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]
 - 2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + Sqrt[-((-1 + Sec[c + d*x]
)*Sec[c + d*x])])*Sqrt[-Tan[c + d*x]^2])/(4*d)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4309, 3042, 4509, 27, 3042, 4511, 3042, 4288, 222, 4294, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{\sec(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4309}$$

$$\frac{1}{4}\int \frac{(3-\sec(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{\sec(c+dx)+1}}dx+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{\left(3-\csc\left(c+dx+\frac{\pi}{2}\right)\right)\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{4509}$$

$$\frac{1}{4}\left(\int -\frac{(1-7\sec(c+dx))\sqrt{\sec(c+dx)}}{2\sqrt{\sec(c+dx)+1}}dx-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}\right)+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{\sec(c+dx)+1}}$$

$$\downarrow 27$$

$$\frac{1}{4}\left(-\frac{1}{2}\int \frac{(1-7\sec(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)+1}}dx-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}\right)+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{\sec(c+dx)+1}}$$

$$\downarrow 3042$$

$$\frac{1}{4}\left(-\frac{1}{2}\int \frac{\left(1-7\csc\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}\right)+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{\sec(c+dx)+1}}$$

$$\downarrow 4511$$

$$\frac{1}{4}\left(\frac{1}{2}\left(7\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}dx-8\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)+1}}dx\right)-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}\right)+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{\sec(c+dx)+1}}$$

$$\downarrow 3042$$

$$\frac{1}{4}\left(\frac{1}{2}\left(7\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}dx-8\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx\right)-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}\right)+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{\sec(c+dx)+1}}$$

$$\downarrow 4288$$

$$\frac{1}{4}\left(\frac{1}{2}\left(-8\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx-\frac{14\int \frac{1}{\sqrt{\frac{\tan^2(c+dx)}{\sec(c+dx)+1}+1}}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}\right)-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}\right)+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{\sec(c+dx)+1}}$$

$$\downarrow 222$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{14\text{arcsinh}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}-8\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx\right)-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}\right)+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{ 4294}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{8\sqrt{2}\int\frac{1}{\sqrt{\frac{\tan^2(c+dx)}{(\sec(c+dx)+1)^2}+1}}d\left(-\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}+\frac{14\text{arcsinh}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}\right)-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}\right)+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{14\text{arcsinh}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}-\frac{8\sqrt{2}\text{arcsinh}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}\right)-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}\right)+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{\sec(c+dx)+1}}$$

input
```
Int[Sec[c + d*x]^(7/2)/Sqrt[1 + Sec[c + d*x]],x]
```

output
```
(Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[1 + Sec[c + d*x]]) + (((-8*Sqr
t[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d + (14*ArcSinh[Tan[c + d*x
]/Sqrt[1 + Sec[c + d*x]]])/d)/2 - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqr
t[1 + Sec[c + d*x]]))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4294
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[(-Sqrt[2])*(Sqrt[a]/(b*f))   Subst[Int[1/Sqrt[1
+ x^2], x], x, b*(Cot[e + f*x]/(a + b*Csc[e + f*x]))], x] /; FreeQ[{a, b, d
, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

rule 4309
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(
f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[d^2/(b*(2*n - 3))   Int[(
d*Csc[e + f*x])^(n - 2)*((2*b*(n - 2) - a*Csc[e + f*x])/Sqrt[a + b*Csc[e +
f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n,
2] && IntegerQ[2*n]
```

rule 4509
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Simp[d/(b*(m + n))   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
 1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[n, 1]
```

rule 4511
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b -
a*B)/b   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Simp[B/b
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b
, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(108) = 216$.

Time = 2.15 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.79

method	result
default	$\frac{\sqrt{1+\sec(dx+c)}\,\sec(dx+c)^{\frac{7}{2}}\left(8\sqrt{2}\,\cos(dx+c)^4\arctan\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-7\cos(dx+c)^4\arctan\left(\frac{-\cot(dx+c)+\csc(dx+c}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right.\right.}{8d(\cos(dx+c)+1)\sqrt{-}}$

input
```
int(sec(d*x+c)^(7/2)/(1+sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*(1+sec(d*x+c))^(1/2)*sec(d*x+c)^(7/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)
+1))^(1/2)*(8*2^(1/2)*cos(d*x+c)^4*arctan(1/2*2^(1/2)/(-1/(cos(d*x+c)+1))^
(1/2)*(-cot(d*x+c)+csc(d*x+c)))-7*cos(d*x+c)^4*arctan(1/2*(-cot(d*x+c)+csc
(d*x+c)-1)/(-1/(cos(d*x+c)+1))^(1/2))-7*cos(d*x+c)^4*arctan(1/2/(-1/(cos(d
*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1))+sin(d*x+c)*(-cos(d*x+c)+2)*2^(
1/2)*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(108) = 216$.

Time = 0.10 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.67

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \frac{8\left(\sqrt{2}\cos(dx+c)^2+\sqrt{2}\cos(dx+c)\right)\log\left(-\frac{2\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\cos(dx+c)^2-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-}{}$$

input
```
integrate(sec(d*x+c)^(7/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/16*(8*(sqrt(2)*cos(d*x + c)^2 + sqrt(2)*cos(d*x + c))*log(-(2*sqrt(2)*sq
rt((cos(d*x + c) + 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + cos(
d*x + c)^2 - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) -
7*(cos(d*x + c)^2 + cos(d*x + c))*log(-(cos(d*x + c)^2 + 2*sqrt((cos(d*x +
 c) + 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - cos(d*x + c) - 2)
/(cos(d*x + c) + 1)) + 7*(cos(d*x + c)^2 + cos(d*x + c))*log(-(cos(d*x + c
)^2 - 2*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
 c) - cos(d*x + c) - 2)/(cos(d*x + c) + 1)) - 4*sqrt((cos(d*x + c) + 1)/co
s(d*x + c))*(cos(d*x + c) - 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x
 + c)^2 + d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)/(1+sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1643 vs. $2(108) = 216$.

Time = 0.26 (sec) , antiderivative size = 1643, normalized size of antiderivative = 13.04

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(7/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/16*(4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*ar
ctan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt
(2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(s
qrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(
d*x + c), cos(d*x + c))) - 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d
*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 7*(2*(2*cos(2*d*
x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2
 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x
+ 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(
2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*
d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^
2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d
*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*si
n(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(s
in(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(
d*x + c))) + 2) + 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d
*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*lo...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(108) = 216.

Time = 0.98 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.35

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx$$
$$= \frac{\sqrt{2}\left(7\sqrt{2}\log\left(\left|\frac{2\left(\sqrt{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^2-4\sqrt{2}-6}{2\left(\sqrt{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^2+4\sqrt{2}-6}\right|\right)-\frac{8\left(17\left(\sqrt{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^6-57\left(\sqrt{\ldots}\right.}{\left(\left(\sqrt{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}-\tan\ldots\right.\right.}}{}$$

input
```
integrate(sec(d*x+c)^(7/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/16*sqrt(2)*(7*sqrt(2)*log(abs(2*(sqrt(tan(1/2*d*x + 1/2*c)^2 + 1) - tan(
1/2*d*x + 1/2*c))^2 - 4*sqrt(2) - 6)/abs(2*(sqrt(tan(1/2*d*x + 1/2*c)^2 +
1) - tan(1/2*d*x + 1/2*c))^2 + 4*sqrt(2) - 6)) - 8*(17*(sqrt(tan(1/2*d*x +
 1/2*c)^2 + 1) - tan(1/2*d*x + 1/2*c))^6 - 57*(sqrt(tan(1/2*d*x + 1/2*c)^2
 + 1) - tan(1/2*d*x + 1/2*c))^4 + 19*(sqrt(tan(1/2*d*x + 1/2*c)^2 + 1) - t
an(1/2*d*x + 1/2*c))^2 - 3)/((sqrt(tan(1/2*d*x + 1/2*c)^2 + 1) - tan(1/2*d
*x + 1/2*c))^4 - 6*(sqrt(tan(1/2*d*x + 1/2*c)^2 + 1) - tan(1/2*d*x + 1/2*c
))^2 + 1)^2 + 16*log(sqrt(tan(1/2*d*x + 1/2*c)^2 + 1) - tan(1/2*d*x + 1/2*
c)))/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}+1}}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)/(1/cos(c + d*x) + 1)^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(7/2)/(1/cos(c + d*x) + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}\,\sec(dx+c)^3}{\sec(dx+c)+1}dx$$

input
```
int(sec(d*x+c)^(7/2)/(1+sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1)*sec(c + d*x)**3)/(sec(c + d
*x) + 1),x)
```

3.266 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \frac{\sqrt{2}\text{arcsinh}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d} - \frac{\text{arcsinh}\left(\frac{\tan(c+dx)}{\sqrt{1+\sec(c+dx)}}\right)}{d} + \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{1+\sec(c+dx)}}$$

output

```
2^(1/2)*arcsinh(tan(d*x+c)/(1+sec(d*x+c)))/d-arcsinh(tan(d*x+c)/(1+sec(d*x
+c))^(1/2))/d+sec(d*x+c)^(3/2)*sin(d*x+c)/d/(1+sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.31

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \frac{\left(\arcsin\left(\sqrt{1-\sec(c+dx)}\right)+2\arcsin\left(\sqrt{\sec(c+dx)}\right)-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)+\sqrt{-((-1+\sec(}{d\sqrt{-\tan^2(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/Sqrt[1 + Sec[c + d*x]],x]
```

output
```
((ArcSin[Sqrt[1 - Sec[c + d*x]]] + 2*ArcSin[Sqrt[Sec[c + d*x]]] - Sqrt[2]*
ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] + Sqrt[-((-1 +
 Sec[c + d*x])*Sec[c + d*x])])*Tan[c + d*x])/(d*Sqrt[-Tan[c + d*x]^2])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3042, 4309, 3042, 4511, 3042, 4288, 222, 4294, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{\sec(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4309}$$

$$\frac{1}{2}\int \frac{(1-\sec(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)+1}}dx+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\left(1-\csc\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{4511}$$

$$\frac{1}{2}\left(2\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)+1}}dx-\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}dx\right)+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(2\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx-\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}dx\right)+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{4288}$$

$$\frac{1}{2}\left(2\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx+\frac{2\int \frac{1}{\sqrt{\frac{\tan^2(c+dx)}{\sec(c+dx)+1}+1}}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}\right)+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\left(2\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx-\frac{2\text{arcsinh}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}\right)+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{4294}$$

$$\frac{1}{2}\left(-\frac{2\sqrt{2}\int \frac{1}{\sqrt{\frac{\tan^2(c+dx)}{(\sec(c+dx)+1)^2}+1}}d\left(-\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}-\frac{2\text{arcsinh}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}\right)+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\left(\frac{2\sqrt{2}\text{arcsinh}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}-\frac{2\text{arcsinh}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}\right)+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}$$

input

```
Int[Sec[c + d*x]^(5/2)/Sqrt[1 + Sec[c + d*x]],x]
```

output
```
((2*Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d - (2*ArcSinh[Tan[c
 + d*x]/Sqrt[1 + Sec[c + d*x]]])/d)/2 + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/
(d*Sqrt[1 + Sec[c + d*x]])
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4294
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[(-Sqrt[2])*(Sqrt[a]/(b*f))   Subst[Int[1/Sqrt[1
+ x^2], x], x, b*(Cot[e + f*x]/(a + b*Csc[e + f*x]))], x] /; FreeQ[{a, b, d
, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

rule 4309
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(
f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[d^2/(b*(2*n - 3))   Int[(
d*Csc[e + f*x])^(n - 2)*((2*b*(n - 2) - a*Csc[e + f*x])/Sqrt[a + b*Csc[e +
f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n,
2] && IntegerQ[2*n]
```

rule 4511
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b -
a*B)/b   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Simp[B/b
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b
, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(77) = 154$.

Time = 2.10 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.51

method	result
default	$\frac{\sqrt{1+\sec(dx+c)}\,\sec(dx+c)^{\frac{5}{2}}\left(-2\cos(dx+c)^3\sqrt{2}\arctan\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+\sqrt{2}\sin(dx+c)\sqrt{-\frac{2}{\cos(dx+c)+1}}\cos(dx+c)}{2d(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)}}}$

input
```
int(sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(1+sec(d*x+c))^(1/2)*sec(d*x+c)^(5/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)
+1))^(1/2)*(-2*cos(d*x+c)^3*2^(1/2)*arctan(1/2*2^(1/2)/(-1/(cos(d*x+c)+1))
^(1/2)*(-cot(d*x+c)+csc(d*x+c)))+2^(1/2)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1
/2)*cos(d*x+c)^2+cos(d*x+c)^3*arctan(1/2/(-1/(cos(d*x+c)+1))^(1/2)*(-cot(d
*x+c)+csc(d*x+c)+1))+cos(d*x+c)^3*arctan(1/2*(-cot(d*x+c)+csc(d*x+c)-1)/(-
1/(cos(d*x+c)+1))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(77) = 154$.

Time = 0.10 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.49

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \frac{2\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\log\left(\frac{2\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-\cos(dx+c)^2+2\cos(dx+c)+3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+(\cos(dx+c)+}{}$$

input
```
integrate(sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/4*(2*(sqrt(2)*cos(d*x + c) + sqrt(2))*log((2*sqrt(2)*sqrt((cos(d*x + c)
+ 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - cos(d*x + c)^2 + 2*co
s(d*x + c) + 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + (cos(d*x + c) + 1
)*log(-(cos(d*x + c)^2 + 2*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sqrt(cos(
d*x + c))*sin(d*x + c) - cos(d*x + c) - 2)/(cos(d*x + c) + 1)) - (cos(d*x
+ c) + 1)*log(-(cos(d*x + c)^2 - 2*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c) - cos(d*x + c) - 2)/(cos(d*x + c) + 1)) + 4
*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d
*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)/(1+sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 873 vs. $2(77) = 154$.

Time = 0.23 (sec) , antiderivative size = 873, normalized size of antiderivative = 10.27

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2
*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2
*c) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*l
og(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2)
- (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2
*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
 + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos
(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2
*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
- 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 2*(sqrt(2)
*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2
*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(77) = 154$.

Time = 0.89 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.74

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\left(\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)^2-4\sqrt{2}-6\right|}{\left|2\left(\sqrt{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)^2+4\sqrt{2}-6\right|}\right)-\frac{8\left(3\left(\sqrt{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}-\tan\left(\frac{1}{2}\right.\right.\right.}{\left(\sqrt{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)^4-6\left(\sqrt{\tan\left(\right.\right.}\right.}}{4\,d\mathrm{sgn}\left(\cos\left(dx+c\right)\right.}$$

input
```
integrate(sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(sqrt(2)*log(abs(2*(sqrt(tan(1/2*d*x + 1/2*c)^2 + 1) - tan(1/
2*d*x + 1/2*c))^2 - 4*sqrt(2) - 6)/abs(2*(sqrt(tan(1/2*d*x + 1/2*c)^2 + 1)
 - tan(1/2*d*x + 1/2*c))^2 + 4*sqrt(2) - 6)) - 8*(3*(sqrt(tan(1/2*d*x + 1/
2*c)^2 + 1) - tan(1/2*d*x + 1/2*c))^2 - 1)/((sqrt(tan(1/2*d*x + 1/2*c)^2 +
 1) - tan(1/2*d*x + 1/2*c))^4 - 6*(sqrt(tan(1/2*d*x + 1/2*c)^2 + 1) - tan(
1/2*d*x + 1/2*c))^2 + 1) + 4*log(sqrt(tan(1/2*d*x + 1/2*c)^2 + 1) - tan(1/
2*d*x + 1/2*c)))/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}+1}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(1/cos(c + d*x) + 1)^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(1/cos(c + d*x) + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}\,\sec(dx+c)^{2}}{\sec(dx+c)+1}dx$$

input
```
int(sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1)*sec(c + d*x)**2)/(sec(c + d
*x) + 1),x)
```

3.267 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx=-\frac{\sqrt{2}\text{arcsinh}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}+\frac{2\text{arcsinh}\left(\frac{\tan(c+dx)}{\sqrt{1+\sec(c+dx)}}\right)}{d}$$

output
```
-2^(1/2)*arcsinh(tan(d*x+c)/(1+sec(d*x+c)))/d+2*arcsinh(tan(d*x+c)/(1+sec(
d*x+c))^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.41

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx =\frac{\left(2\arcsin\left(\sqrt{\sec(c+dx)}\right)-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)\cot(c+dx)\sqrt{-\tan^2(c+dx)}}{d}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/Sqrt[1 + Sec[c + d*x]],x]
```

output
```
((2*ArcSin[Sqrt[Sec[c + d*x]]] - Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]
])/Sqrt[1 - Sec[c + d*x]]])*Cot[c + d*x]*Sqrt[-Tan[c + d*x]^2])/d
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 4308, 3042, 4288, 222, 4294, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{\sec(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4308}$$

$$\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}dx-\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}dx-\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4288}$$

$$-\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx-\frac{2\int \frac{1}{\sqrt{\frac{\tan^2(c+dx)}{\sec(c+dx)+1}+1}}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}$$

$$\Big\downarrow \text{222}$$

$$\frac{2\text{arcsinh}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}-\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\downarrow \textbf{4294}$$

$$\frac{\sqrt{2}\int \frac{1}{\sqrt{\frac{\tan^2(c+dx)}{(\sec(c+dx)+1)^2}+1}}d\left(-\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}+\frac{2\text{arcsinh}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}$$

$$\downarrow \textbf{222}$$

$$\frac{2\text{arcsinh}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}-\frac{\sqrt{2}\text{arcsinh}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

input
```
Int[Sec[c + d*x]^(3/2)/Sqrt[1 + Sec[c + d*x]],x]
```

output
```
-((Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d) + (2*ArcSinh[Tan[c
 + d*x]/Sqrt[1 + Sec[c + d*x]]])/d
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4294
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[(-Sqrt[2])*(Sqrt[a]/(b*f))   Subst[Int[1/Sqrt[1
+ x^2], x], x, b*(Cot[e + f*x]/(a + b*Csc[e + f*x]))], x] /; FreeQ[{a, b, d
, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

rule 4308
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d/b   Int[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[
e + f*x]], x], x] - Simp[a*(d/b)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(50) = 100$.

Time = 2.05 (sec) , antiderivative size = 166, normalized size of antiderivative = 3.07

method	result
default	$-\frac{\cos(dx+c)^2\sqrt{1+\sec(dx+c)}\,\sec(dx+c)^{\frac{3}{2}}\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(\cot(dx+c)-\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-\arctan\left(\frac{\cot(dx+c)-\csc(dx+c)-1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-\arctan\left(\right.\right.}{d(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*cos(d*x+c)^2*(1+sec(d*x+c))^(1/2)*sec(d*x+c)^(3/2)*(2^(1/2)*arctan(1/
2*2^(1/2)*(cot(d*x+c)-csc(d*x+c))/(-1/(cos(d*x+c)+1))^(1/2))-arctan(1/2*(c
ot(d*x+c)-csc(d*x+c)-1)/(-1/(cos(d*x+c)+1))^(1/2))-arctan(1/2/(-1/(cos(d*x
+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)))/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1
))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(50) = 100$.

Time = 0.09 (sec) , antiderivative size = 223, normalized size of antiderivative = 4.13

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \frac{\sqrt{2}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\cos(dx+c)^2-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-\log\left(-\frac{\cos(dx+c)^2+2\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(d}{\cos(dx+c)}\right.}{2d}$$

input
```
integrate(sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*log(-(2*sqrt(2)*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c) + cos(d*x + c)^2 - 2*cos(d*x + c) - 3)/(cos(d*x +
 c)^2 + 2*cos(d*x + c) + 1)) - log(-(cos(d*x + c)^2 + 2*sqrt((cos(d*x + c)
 + 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - cos(d*x + c) - 2)/(c
os(d*x + c) + 1)) + log(-(cos(d*x + c)^2 - 2*sqrt((cos(d*x + c) + 1)/cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - cos(d*x + c) - 2)/(cos(d*x + c)
 + 1)))/d
```

Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{\sec(c+dx)+1}}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(1+sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**(3/2)/sqrt(sec(c + d*x) + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(50) = 100$.

Time = 0.24 (sec) , antiderivative size = 473, normalized size of antiderivative = 8.76

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2
*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c))) + 1) - sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x +
c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sq
rt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arc
tan2(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c
), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2
*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c))) + 2) - log(2*cos(1/2*arctan2(sin(d*x
+ c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
- 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d
*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*si
n(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}+1}}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(1/cos(c + d*x) + 1)^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(1/cos(c + d*x) + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{\sec\left(dx+c\right)+1}\,\sec\left(dx+c\right)}{\sec\left(dx+c\right)+1}dx$$

input
```
int(sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1)*sec(c + d*x))/(sec(c + d*x)
 + 1),x)
```

3.268 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 27

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx = \frac{\sqrt{2}\operatorname{arcsinh}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}$$

output
```
2^(1/2)*arcsinh(tan(d*x+c)/(1+sec(d*x+c)))/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx = \frac{2\coth^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{1}{1+\cos(c+dx)}}}{d}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/Sqrt[1 + Sec[c + d*x]],x]
```

output
```
(2*ArcCoth[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]*Sqrt[(1 + Cos[c + d*x])^(-1)
])/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 4294, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4294}$$

$$-\frac{\sqrt{2}\int \frac{1}{\sqrt{\frac{\tan^2(c+dx)}{(\sec(c+dx)+1)^2}+1}}d\left(-\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

$$\Big\downarrow \text{222}$$

$$\frac{\sqrt{2}\text{arcsinh}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

input
```
Int[Sqrt[Sec[c + d*x]]/Sqrt[1 + Sec[c + d*x]],x]
```

output
```
(Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4294
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[(-Sqrt[2])*(Sqrt[a]/(b*f))   Subst[Int[1/Sqrt[1
+ x^2], x], x, b*(Cot[e + f*x]/(a + b*Csc[e + f*x]))], x] /; FreeQ[{a, b, d
, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(25) = 50$.

Time = 1.00 (sec) , antiderivative size = 92, normalized size of antiderivative = 3.41

method	result	size
default	$-\frac{\sqrt{2}\,\sqrt{1+\sec(dx+c)}\,\sqrt{\sec(dx+c)}\,\arctan\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\cos(dx+c)}{d(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$	92

input
```
int(sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*2^(1/2)*(1+sec(d*x+c))^(1/2)*sec(d*x+c)^(1/2)*arctan(1/2*2^(1/2)/(-1/
(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)))*cos(d*x+c)/(cos(d*x+c)+1)/
(-1/(cos(d*x+c)+1))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(25) = 50$.

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.26

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx = \frac{\sqrt{2}\log\left(\frac{2\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-\cos(dx+c)^2+2\cos(dx+c)+3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2d}$$

input
```
integrate(sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*log((2*sqrt(2)*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sqrt(cos(
d*x + c))*sin(d*x + c) - cos(d*x + c)^2 + 2*cos(d*x + c) + 3)/(cos(d*x + c
)^2 + 2*cos(d*x + c) + 1))/d
```

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)+1}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(1+sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/sqrt(sec(c + d*x) + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(25) = 50$.

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.22

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx = \frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\right.}{2\,d}$$

input
```
integrate(sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(25) = 50$.

Time = 0.44 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.44

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx = \frac{\sqrt{2}\left(\frac{\log\left(\left|\frac{1}{\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right)}{\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{\log\left(\left|\frac{1}{\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right|\right)}{\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}\right)}{4\,d\operatorname{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(log(abs(1/sin(1/2*d*x + 1/2*c) + sin(1/2*d*x + 1/2*c) + 2))/s
gn(cos(1/2*d*x + 1/2*c)) - log(abs(1/sin(1/2*d*x + 1/2*c) + sin(1/2*d*x +
1/2*c) - 2))/sgn(cos(1/2*d*x + 1/2*c)))/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{\frac{1}{\cos(c+d\,x)}+1}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(1/cos(c + d*x) + 1)^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(1/cos(c + d*x) + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)+1}dx$$

input
```
int(sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/(sec(c + d*x) + 1),x)
```

3.269 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 62

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\operatorname{arcsinh}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d} + \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{1+\sec(c+dx)}}$$

output
```
-2^(1/2)*arcsinh(tan(d*x+c)/(1+sec(d*x+c)))/d+2*sec(d*x+c)^(1/2)*sin(d*x+c
)/d/(1+sec(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.45

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\,dx = \frac{2\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\sin(c+dx)+\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\tan(c+dx)}{d\sqrt{-\tan^2(c+dx)}}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]),x]
```

output
```
(2*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sin[c + d*x] + Sqrt[2]*ArcTan
[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x])/(d*Sqr
t[-Tan[c + d*x]^2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4299, 3042, 4294, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4299}$$

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}-\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}-\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4294}$$

$$\frac{\sqrt{2}\int \frac{1}{\sqrt{\frac{\tan^2(c+dx)}{(\sec(c+dx)+1)^2}+1}}d\left(-\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{222}$$

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}-\frac{\sqrt{2}\text{arcsinh}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]),x]
```

output
```
-((Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d) + (2*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(d*Sqrt[1 + Sec[c + d*x]])
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4294
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[(-Sqrt[2])*(Sqrt[a]/(b*f))   Subst[Int[1/Sqrt[1
+ x^2], x], x, b*(Cot[e + f*x]/(a + b*Csc[e + f*x]))], x] /; FreeQ[{a, b, d
, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

rule 4299
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(m + 1))), x] + Simp[a*(m/(b*d*(m + 1)))   Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.47

method	result	size
default	$\frac{\left(-\arctan\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{2}{\cos(dx+c)+1}}-2\cot(dx+c)+2\csc(dx+c)\right)\sqrt{1+\sec(dx+c)}}{d\sqrt{\sec(dx+c)}}$	91

input
```
int(1/sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-arctan(1/2*2^(1/2)/(-1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)
))*(-2/(cos(d*x+c)+1))^(1/2)-2*cot(d*x+c)+2*csc(d*x+c))*(1+sec(d*x+c))^(1/
2)/sec(d*x+c)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(56) = 112$.

Time = 0.09 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.32

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\,dx = \frac{\left(\sqrt{2}\cos\left(dx+c\right)+\sqrt{2}\right)\log\left(-\frac{2\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\cos(dx+c)^2-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}}{2\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/2*((sqrt(2)*cos(d*x + c) + sqrt(2))*log(-(2*sqrt(2)*sqrt((cos(d*x + c) +
 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + cos(d*x + c)^2 - 2*cos
(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt((cos(d*x +
c) + 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d
)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\sec\left(c+dx\right)+1}\sqrt{\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(1+sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(sec(c + d*x) + 1)*sqrt(sec(c + d*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.63

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\right.}{2\,d}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))/
d
```

Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\,dx = \frac{\sqrt{2}\left(\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}+\log\left(\sqrt{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)}{d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*(2*tan(1/2*d*x + 1/2*c)/sqrt(tan(1/2*d*x + 1/2*c)^2 + 1) + log(sqr
t(tan(1/2*d*x + 1/2*c)^2 + 1) - tan(1/2*d*x + 1/2*c)))/(d*sgn(cos(d*x + c)
))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}+1}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((1/cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((1/cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^2+\sec(dx+c)}dx$$

input
```
int(1/sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/(sec(c + d*x)**2 + sec(c +
 d*x)),x)
```

3.270 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 98

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx = \frac{\sqrt{2}\operatorname{arcsinh}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d} + \frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}} - \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{1+\sec(c+dx)}}$$

output
```
2^(1/2)*arcsinh(tan(d*x+c)/(1+sec(d*x+c)))/d+2/3*sin(d*x+c)/d/sec(d*x+c)^(
1/2)/(1+sec(d*x+c))^(1/2)-2/3*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(1+sec(d*x+c))
^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.19 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx = \frac{\left(2(-1+\cos(c+dx))\sqrt{1-\sec(c+dx)}-3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sqrt{\sec(c+dx)}\right)\tan(c+dx)}{3d\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\sqrt{1+\sec(c+dx)}}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*Sqrt[1 + Sec[c + d*x]]),x]
```

output
```
((2*(-1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] - 3*Sqrt[2]*ArcTan[(Sqrt[2]
*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]])*Tan[c + d
*x])/(3*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sqrt[1 + Sec[c + d*x]]
)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4310, 3042, 4501, 3042, 4294, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4310}$$

$$\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{1}{3}\int \frac{1-2\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{1}{3}\int\frac{1-2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4501}$$

$$\frac{1}{3}\left(3\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)+1}}dx-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}\right)+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(3\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}\right)+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{4294}$$

$$\frac{1}{3}\left(-\frac{3\sqrt{2}\int\frac{1}{\sqrt{\frac{\tan^2(c+dx)}{(\sec(c+dx)+1)^2}+1}}d\left(-\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}\right)+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{3}\left(\frac{3\sqrt{2}\text{arcsinh}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}\right)+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

input
```
Int[1/(Sec[c + d*x]^(3/2)*Sqrt[1 + Sec[c + d*x]]),x]
```

output
```
(2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]) + ((3*Sqr
t[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d - (2*Sqrt[Sec[c + d*x]]*S
in[c + d*x])/(d*Sqrt[1 + Sec[c + d*x]]))/3
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4294
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[(-Sqrt[2])*(Sqrt[a]/(b*f))   Subst[Int[1/Sqrt[1
+ x^2], x], x, b*(Cot[e + f*x]/(a + b*Csc[e + f*x]))], x] /; FreeQ[{a, b, d
, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

rule 4310
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a +
b*Csc[e + f*x]])), x] + Simp[1/(2*b*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*((a
 + b*(2*n + 1)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.13

method	result	size
default	$\frac{\sqrt{1+\sec(dx+c)}\left(-2\tan(dx+c)+2\sin(dx+c)+\sqrt{-\frac{2}{\cos(dx+c)+1}}\arctan\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)(3+3\sec(dx+c))\right)}{3d(\cos(dx+c)+1)\sec(dx+c)^{\frac{3}{2}}}$	111

input
```
int(1/sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*(1+sec(d*x+c))^(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^(3/2)*(-2*tan(d*x+c)+
2*sin(d*x+c)+(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)/(-1/(cos(d*x+c)+
1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)))*(3+3*sec(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.66

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx = \frac{3\left(\sqrt{2}\cos\left(dx+c\right)+\sqrt{2}\right)\log\left(\frac{2\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-\cos(dx+c)^2+2\cos(dx+c)+3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\frac{4\left(\cos(dx+c)^2-\cos\right.}{}}{6\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/6*(3*(sqrt(2)*cos(d*x + c) + sqrt(2))*log((2*sqrt(2)*sqrt((cos(d*x + c)
+ 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - cos(d*x + c)^2 + 2*co
s(d*x + c) + 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(cos(d*x + c)^2
 - cos(d*x + c))*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sin(d*x + c)/sqrt(c
os(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\sec\left(c+dx\right)+1}\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(1+sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(sec(c + d*x) + 1)*sec(c + d*x)**(3/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(84) = 168$.

Time = 0.20 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.85

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx = -\frac{3\sqrt{2}\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3\sqrt{2}\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\sin\left(\frac{2}{3}\right.}{}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/6*(3*sqrt(2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*log(cos(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))/d
```

Giac [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\left(\frac{4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^{\frac{3}{2}}}+3\log\left(\sqrt{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\right)}{3\,d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/3*sqrt(2)*(4*tan(1/2*d*x + 1/2*c)^3/(tan(1/2*d*x + 1/2*c)^2 + 1)^(3/2)
+ 3*log(sqrt(tan(1/2*d*x + 1/2*c)^2 + 1) - tan(1/2*d*x + 1/2*c)))/(d*sgn(c
os(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}+1}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((1/cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((1/cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^3+\sec(dx+c)^2}dx$$

input
```
int(1/sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/(sec(c + d*x)**3 + sec(c +
 d*x)**2),x)
```

3.271 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 134

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\,\text{arcsinh}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d} + \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}} - \frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}} + \frac{26\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{1+\sec(c+dx)}}$$

output
```
-2^(1/2)*arcsinh(tan(d*x+c)/(1+sec(d*x+c)))/d+2/5*sin(d*x+c)/d/sec(d*x+c)^
(3/2)/(1+sec(d*x+c))^(1/2)-2/15*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(1+sec(d*x+c
))^(1/2)+26/15*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(1+sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx = \frac{\left(15\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sec^{\frac{5}{2}}(c+dx)+2\sqrt{1-\sec(c+dx)}(3-\sec(c+dx)+13\sec^2(c+dx))\right)\sin(c+dx)}{15d\sec^{\frac{3}{2}}(c+dx)\sqrt{-\tan^2(c+dx)}}$$

input
```
Integrate[1/(Sec[c + d*x]^(5/2)*Sqrt[1 + Sec[c + d*x]]),x]
```

output
```
((15*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*S
ec[c + d*x]^(5/2) + 2*Sqrt[1 - Sec[c + d*x]]*(3 - Sec[c + d*x] + 13*Sec[c
+ d*x]^2))*Sin[c + d*x])/(15*d*Sec[c + d*x]^(3/2)*Sqrt[-Tan[c + d*x]^2])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4310, 3042, 4510, 27, 3042, 4501, 3042, 4294, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4310}$$

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}-\frac{1}{5}\int \frac{1-4\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}-\frac{1}{5}\int\frac{1-4\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4510}$$

$$\frac{1}{5}\left(-\frac{2}{3}\int-\frac{13-2\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}dx-\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}\right)+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(\frac{1}{3}\int\frac{13-2\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}dx-\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}\right)+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\int\frac{13-2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx-\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}\right)+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{4501}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{26\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}-15\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)+1}}dx\right)-\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}\right)+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{26\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}-15\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx\right)-\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}\right)+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{4294}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{15\sqrt{2}\int \frac{1}{\sqrt{\frac{\tan^2(c+dx)}{(\sec(c+dx)+1)^2}+1}}d\left(-\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}+\frac{26\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}\right)-\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+d}\right.$$

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{26\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}-\frac{15\sqrt{2}\text{arcsinh}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}\right)-\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}\right)+$$

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}$$

input
```
Int[1/(Sec[c + d*x]^(5/2)*Sqrt[1 + Sec[c + d*x]]),x]
```

output
```
(2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*Sqrt[1 + Sec[c + d*x]]) + ((-2*Si
n[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]) + ((-15*Sqrt[2
]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d + (26*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(d*Sqrt[1 + Sec[c + d*x]]))/3)/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4294
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[(-Sqrt[2])*(Sqrt[a]/(b*f))   Subst[Int[1/Sqrt[1
+ x^2], x], x, b*(Cot[e + f*x]/(a + b*Csc[e + f*x]))], x] /; FreeQ[{a, b, d
, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

rule 4310
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a +
b*Csc[e + f*x]])), x] + Simp[1/(2*b*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*((a
 + b*(2*n + 1)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00

method	result
default	$-\frac{\sqrt{1+\sec(dx+c)}\left(-6\sin(dx+c)+2\tan(dx+c)-26\sec(dx+c)\tan(dx+c)+\arctan\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{2}{\cos(dx+c)+1}}}{15d(\cos(dx+c)+1)\sec(dx+c)^{\frac{5}{2}}}$

input
```
int(1/sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/d*(1+sec(d*x+c))^(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^(5/2)*(-6*sin(d*x+c
)+2*tan(d*x+c)-26*sec(d*x+c)*tan(d*x+c)+arctan(1/2*2^(1/2)/(-1/(cos(d*x+c)
+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)*(15*sec(d*x
+c)+15*sec(d*x+c)^2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx = \frac{15\left(\sqrt{2}\cos\left(dx+c\right)+\sqrt{2}\right)\log\left(-\frac{2\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\cos(dx+c)^2-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\frac{4\left(3\cos(dx+c)\right.}{}}{30\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/30*(15*(sqrt(2)*cos(d*x + c) + sqrt(2))*log(-(2*sqrt(2)*sqrt((cos(d*x +
c) + 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + cos(d*x + c)^2 - 2
*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(3*cos(d*x +
 c)^3 - cos(d*x + c)^2 + 13*cos(d*x + c))*sqrt((cos(d*x + c) + 1)/cos(d*x
+ c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\sec\left(c+dx\right)+1}\sec^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(5/2)/(1+sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(sec(c + d*x) + 1)*sec(c + d*x)**(5/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(114) = 228.

Time = 0.22 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.64

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/60*sqrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))*sin(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c)*sin(4/5*arc
tan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*cos(5/2*d*x + 5/2*c)
*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 30*log(cos
(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arct
an2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin
(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 1) + 30*log(cos(1/5*arctan2(si
n(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x
 + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*
c), cos(5/2*d*x + 5/2*c))) + 1) + 6*sin(5/2*d*x + 5/2*c) - 5*sin(3/5*arcta
n2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 60*sin(1/5*arctan2(sin(5
/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))/d
```

Giac [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.77

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx = \frac{\sqrt{2}\left(\frac{2\left(\left(17\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+20\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+15\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^{\frac{5}{2}}}+15\log\left(\sqrt{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}-\tan\left(\frac{1}{2}dx+\right.\right.\right.}{15\,d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/15*sqrt(2)*(2*((17*tan(1/2*d*x + 1/2*c)^2 + 20)*tan(1/2*d*x + 1/2*c)^2 +
 15)*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1)^(5/2) + 15*log(sqrt
(tan(1/2*d*x + 1/2*c)^2 + 1) - tan(1/2*d*x + 1/2*c)))/(d*sgn(cos(d*x + c))
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}+1}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/((1/cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/((1/cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx=\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^4+\sec(dx+c)^3}dx$$

input
```
int(1/sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x) + 1))/(sec(c + d*x)**4 + sec(c +
 d*x)**3),x)
```

3.272 $\int (e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 325

$$\int (e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}\,dx = \frac{6ae\sqrt[3]{e\sec(c+dx)}\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}}$$
$$+\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}a^2e\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right),-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right.}{5d(a-a\sec(c+dx))\sqrt{a+a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec}\right.}}}$$

output
```
6/5*a*e*(e*sec(d*x+c))^(1/3)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+4/5*3^(3/
4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^2*e*EllipticF(((1-3^(1/2))*e^(1/3)-(e*sec(d
*x+c))^(1/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3)),I*3^(1/2)+2*I)*(e
^(1/3)-(e*sec(d*x+c))^(1/3))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e*sec
(d*x+c))^(2/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3))^2)^(1/2)*tan(d*
x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*sec(d*
x+c))^(1/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.22

$$\int (e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{3}{2},1-\sec(c+dx)\right)(e\sec(c+dx))^{4/3}\sqrt{a(1}{d\sec^{\frac{4}{3}}(c+dx)}$$

input
```
Integrate[(e*Sec[c + d*x])^(4/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Hypergeometric2F1[-1/3, 1/2, 3/2, 1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(4
/3)*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sec[c + d*x]^(4/3))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4293, 60, 73, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(c+dx)+a}(e\sec(c+dx))^{4/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}dx$$

$$\Big\downarrow \text{4293}$$

$$-\frac{a^2e\tan(c+dx)\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{60}$$

$$-\frac{a^2 e\tan(c+dx)\left(\frac{2}{5}e\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a-a\sec(c+dx)}}d\sec(c+dx)-\frac{6\sqrt{a-a\sec(c+dx)}\sqrt[3]{e\sec(c+dx)}}{5a}\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{a^2 e\tan(c+dx)\left(\frac{6}{5}\int \frac{1}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}-\frac{6\sqrt{a-a\sec(c+dx)}\sqrt[3]{e\sec(c+dx)}}{5a}\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{759}$$

$$-\frac{a^2 e\tan(c+dx)\left(-\frac{4\ 3^{3/4}\sqrt{2+\sqrt{3}}\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\text{EllipticF}\left(\arcsin\left(\frac{(1\ldots}{(1\ldots}\right.\right.}{5\sqrt{a-a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}\right.}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[(e*Sec[c + d*x])^(4/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
-((a^2*e*((-6*(e*Sec[c + d*x])^(1/3)*Sqrt[a - a*Sec[c + d*x]])/(5*a) - (4*
3^(3/4)*Sqrt[2 + Sqrt[3]]*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec
[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 -
4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*S
ec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*S
ec[c + d*x])^(1/3))^2])/(5*Sqrt[a - a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3)
 - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3
))^2]))*Tan[c + d*x])/(d*Sqrt[a - a*Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]
))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int (e\sec(dx+c))^{\frac{4}{3}}\sqrt{a+a\sec(dx+c)}dx$$

input
```
int((e*sec(d*x+c))^(4/3)*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
int((e*sec(d*x+c))^(4/3)*(a+a*sec(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int (e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}(e\sec(dx+c))^{\frac{4}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(4/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(1/3)*e*sec(d*x + c), x
)
```

Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(4/3)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}(e\sec(dx+c))^{\frac{4}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(4/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(4/3), x)
```

Giac [F]

$$\int (e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}(e\sec(dx+c))^{\frac{4}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(4/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a+\frac{a}{\cos(c+d\,x)}}\left(\frac{e}{\cos(c+d\,x)}\right)^{4/3}\,dx$$

input
```
int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(4/3),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(4/3), x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}\,dx = e^{\frac{4}{3}}\sqrt{a}\left(\int \sec(dx+c)^{\frac{4}{3}}\sqrt{\sec(dx+c)+1}dx\right)$$

input
```
int((e*sec(d*x+c))^(4/3)*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
e**(1/3)*sqrt(a)*int(sec(c + d*x)**(1/3)*sqrt(sec(c + d*x) + 1)*sec(c + d*
x),x)*e
```

3.273 $\int \sqrt[3]{e \sec(c+dx)} \sqrt{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 280

$$\int \sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\, 3^{3/4}\sqrt{2+\sqrt{3}}a^2 \operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right),-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right.}{d(a-a\sec(c+dx))\sqrt{a+a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec}\right.}}}$$

output
```
2*3^(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^2*EllipticF(((1-3^(1/2))*e^(1/3)-(e*
sec(d*x+c))^(1/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3)),I*3^(1/2)+2*
I)*(e^(1/3)-(e*sec(d*x+c))^(1/3))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(
e*sec(d*x+c))^(2/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3))^2)^(1/2)*t
an(d*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*s
ec(d*x+c))^(1/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.25

$$\int \sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx$$
$$=\frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},1-\sec(c+dx)\right)\sqrt[3]{e\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d\sqrt[3]{\sec(c+dx)}}$$

input
```
Integrate[(e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Hypergeometric2F1[1/2, 2/3, 3/2, 1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(1/
3)*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sec[c + d*x]^(1/3))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3042, 4293, 73, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(c+dx)+a}\sqrt[3]{e\sec(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}\sqrt[3]{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \; 4293$$
$$-\frac{a^2e\tan(c+dx)\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a-a\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$
$$\Big\downarrow \; 73$$

$$-\frac{3a^2\tan(c+dx)\int\frac{1}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{759}$$

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\text{EllipticF}\left(\text{arcsi}\right.}{d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+d}\right.}}}$$

input
```
Int[(e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3) -
 (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))]
, -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/
3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3)
 - (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(d*(a - a*Sec[c + d*x])*Sqrt[a
 + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 +
 Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int (e\sec(dx+c))^{\frac{1}{3}}\sqrt{a+a\sec(dx+c)}dx$$

input
```
int((e*sec(d*x+c))^(1/3)*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
int((e*sec(d*x+c))^(1/3)*(a+a*sec(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx=\int \sqrt{a\sec(dx+c)+a}(e\sec(dx+c))^{\frac{1}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(1/3), x)
```

Sympy [F]

$$\int \sqrt[3]{e \sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sqrt[3]{e\sec\left(c+dx\right)}\,dx$$

input
```
integrate((e*sec(d*x+c))**(1/3)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*(e*sec(c + d*x))**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{e \sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}\left(e\sec\left(dx+c\right)\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{e \sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}\left(e\sec\left(dx+c\right)\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx=\int \sqrt{a+\frac{a}{\cos(c+d\,x)}}\left(\frac{e}{\cos(c+d\,x)}\right)^{1/3}dx$$

input
```
int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(1/3),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx=e^{\frac{1}{3}}\sqrt{a}\left(\int \sec(dx+c)^{\frac{1}{3}}\sqrt{\sec(dx+c)+1}dx\right)$$

input
```
int((e*sec(d*x+c))^(1/3)*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
e**(1/3)*sqrt(a)*int(sec(c + d*x)**(1/3)*sqrt(sec(c + d*x) + 1),x)
```

3.274 $\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 326

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{2/3}}\,dx = \frac{3a\tan(c+dx)}{2d(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}a^2\,\text{EllipticF}\left(\arcsin\left(\frac{(1-\sqrt{3})\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{(1+\sqrt{3})\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right),-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\ldots}{2de(a-a\sec(c+dx))\sqrt{a+a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(\ldots}{\left((1+\sqrt{3})\sqrt[3]{e}-\sqrt[3]{e\sec\ldots}}}}$$

output
```
3/2*a*tan(d*x+c)/d/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2)+1/2*3^(3/4)
*(1/2*6^(1/2)+1/2*2^(1/2))*a^2*EllipticF(((1-3^(1/2))*e^(1/3)-(e*sec(d*x+c
))^(1/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3)),I*3^(1/2)+2*I)*(e^(1/
3)-(e*sec(d*x+c))^(1/3))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e*sec(d*x
+c))^(2/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3))^2)^(1/2)*tan(d*x+c)
/d/e/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*sec(d*x+
c))^(1/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.22

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{2/3}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{3}{2},1-\sec(c+dx)\right)\sec^{\frac{2}{3}}(c+dx)\sqrt{a(1+\sec(c+dx))}}{d(e\sec(c+dx))^{2/3}}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(2/3),x]
```

output
```
(2*Hypergeometric2F1[1/2, 5/3, 3/2, 1 - Sec[c + d*x]]*Sec[c + d*x]^(2/3)*S
qrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*(e*Sec[c + d*x])^(2/3))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3042, 4293, 61, 73, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(c+dx)+a}}{(e\sec(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{4293}$$

$$-\frac{a^2e\tan(c+dx)\int \frac{1}{(e\sec(c+dx))^{5/3}\sqrt{a-a\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{a^2 e\tan(c+dx)\left(\frac{\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a-a\sec(c+dx)}}d\sec(c+dx)}{4e}-\frac{3\sqrt{a-a\sec(c+dx)}}{2ae(e\sec(c+dx))^{2/3}}\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{a^2 e\tan(c+dx)\left(\frac{3\int \frac{1}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}}{4e^2}-\frac{3\sqrt{a-a\sec(c+dx)}}{2ae(e\sec(c+dx))^{2/3}}\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{759}$$

$$-\frac{a^2 e\tan(c+dx)\left(-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\text{EllipticF}\left(\arcsin\left(\frac{(1-}{(1+}\right.\right.}{2e^2\sqrt{a-a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}\right.}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(2/3),x]
```

output
```
-((a^2*e*((-3*Sqrt[a - a*Sec[c + d*x]])/(2*a*e*(e*Sec[c + d*x])^(2/3)) - (
3^(3/4)*Sqrt[2 + Sqrt[3]]*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec
[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 -
4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*S
ec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*S
ec[c + d*x])^(1/3))^2])/(2*e^2*Sqrt[a - a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(
1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^
(1/3))^2]))*Tan[c + d*x])/(d*Sqrt[a - a*Sec[c + d*x]]*Sqrt[a + a*Sec[c + d
*x]]))
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{\sqrt{a+a\sec(dx+c)}}{(e\sec(dx+c))^{\frac{2}{3}}}dx$$

input
```
int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(2/3),x)
```

output
```
int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{2/3}}\,dx=\int \frac{\sqrt{a\sec(dx+c)+a}}{(e\sec(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(2/3),x, algorithm="fricas
")
```

output
```
integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(1/3)/(e*sec(d*x + c)),
 x)
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{2/3}}\,dx=\int \frac{\sqrt{a(\sec(c+dx)+1)}}{(e\sec(c+dx))^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(1/2)/(e*sec(d*x+c))**(2/3),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))/(e*sec(c + d*x))**(2/3), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{2/3}}\,dx = \int \frac{\sqrt{a\sec\left(dx+c\right)+a}}{\left(e\sec\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(2/3),x, algorithm="maxima
")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)/(e*sec(d*x + c))^(2/3), x)
```

Giac [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{2/3}}\,dx = \int \frac{\sqrt{a\sec\left(dx+c\right)+a}}{\left(e\sec\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)/(e*sec(d*x + c))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{2/3}}\,dx = \int \frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{{\left(\frac{e}{\cos(c+d\,x)}\right)}^{2/3}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(1/2)/(e/cos(c + d*x))^(2/3),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2)/(e/cos(c + d*x))^(2/3), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{2/3}}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^{\frac{2}{3}}}dx\right)}{e^{\frac{2}{3}}}$$

input
```
int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(2/3),x)
```

output
```
(sqrt(a)*int(sqrt(sec(c + d*x) + 1)/sec(c + d*x)**(2/3),x))/e**(2/3)
```

3.275 $\int (e\sec(c+dx))^{8/3}\sqrt{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 716

$$\int (e\sec(c+dx))^{8/3}\sqrt{a+a\sec(c+dx)}\,dx = \frac{60ae^2(e\sec(c+dx))^{2/3}\tan(c+dx)}{91d\sqrt{a+a\sec(c+dx)}}$$

$$+\frac{6ae(e\sec(c+dx))^{5/3}\tan(c+dx)}{13d\sqrt{a+a\sec(c+dx)}}$$

$$-\frac{240ae^3\tan(c+dx)}{91d\sqrt{a+a\sec(c+dx)}\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}$$

$$+\frac{120\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^2e^{7/3}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)|-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{}}{91d(a-a\sec(c+dx))\sqrt{a+a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec}\right.}}}$$

$$-\frac{80\sqrt{2}3^{3/4}a^2e^{7/3}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right),-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{91d(a-a\sec(c+dx))\sqrt{a+a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec}\right.}}}$$

output
```
60/91*a*e^2*(e*sec(d*x+c))^(2/3)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+6/13*
a*e*(e*sec(d*x+c))^(5/3)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)-240/91*a*e^3*
tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1
/3))+120/91*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^2*e^(7/3)*EllipticE(((1-3^
(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(
1/3)),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x+c))^(1/3))*((e^(2/3)+e^(1/3)*(e*s
ec(d*x+c))^(1/3)+(e*sec(d*x+c))^(2/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))
^(1/3))^2)^(1/2)*tan(d*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(
1/3)*(e^(1/3)-(e*sec(d*x+c))^(1/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1
/3))^2)^(1/2)-80/91*2^(1/2)*3^(3/4)*a^2*e^(7/3)*EllipticF(((1-3^(1/2))*e^(
1/3)-(e*sec(d*x+c))^(1/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3)),I*3^
(1/2)+2*I)*(e^(1/3)-(e*sec(d*x+c))^(1/3))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))
^(1/3)+(e*sec(d*x+c))^(2/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3))^2)
^(1/2)*tan(d*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1
/3)-(e*sec(d*x+c))^(1/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3))^2)^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.10

$$\int (e\sec(c+dx))^{8/3}\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{1}{2},\frac{3}{2},1-\sec(c+dx)\right)(e\sec(c+dx))^{8/3}\sqrt{a(1}{d\sec^{\frac{8}{3}}(c+dx)}$$

input
```
Integrate[(e*Sec[c + d*x])^(8/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Hypergeometric2F1[-5/3, 1/2, 3/2, 1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(8
/3)*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sec[c + d*x]^(8/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.67 (sec) , antiderivative size = 708, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {3042, 4293, 60, 60, 73, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a \sec(c+dx)+a}(e \sec(c+dx))^{8/3} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a \csc\left(c+dx+\frac{\pi}{2}\right)+a}\left(e \csc\left(c+dx+\frac{\pi}{2}\right)\right)^{8/3} dx$$

$$\Big\downarrow \text{4293}$$

$$-\frac{a^2 e \tan(c+dx) \int \frac{(e \sec(c+dx))^{5/3}}{\sqrt{a-a \sec(c+dx)}}d\sec(c+dx)}{d\sqrt{a-a \sec(c+dx)}\sqrt{a \sec(c+dx)+a}}$$

$$\Big\downarrow \text{60}$$

$$-\frac{a^2 e \tan(c+dx) \left(\frac{10}{13}e \int \frac{(e \sec(c+dx))^{2/3}}{\sqrt{a-a \sec(c+dx)}}d\sec(c+dx)-\frac{6\sqrt{a-a \sec(c+dx)}(e \sec(c+dx))^{5/3}}{13a}\right)}{d\sqrt{a-a \sec(c+dx)}\sqrt{a \sec(c+dx)+a}}$$

$$\Big\downarrow \text{60}$$

$$-\frac{a^2 e \tan(c+dx) \left(\frac{10}{13}e\left(\frac{4}{7}e \int \frac{1}{\sqrt[3]{e \sec(c+dx)}\sqrt{a-a \sec(c+dx)}}d\sec(c+dx)-\frac{6\sqrt{a-a \sec(c+dx)}(e \sec(c+dx))^{2/3}}{7a}\right)-\frac{6\sqrt{a-}\cdots}{\cdots}\right.}{d\sqrt{a-a \sec(c+dx)}\sqrt{a \sec(c+dx)+a}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{a^2 e \tan(c+dx) \left(\frac{10}{13}e\left(\frac{12}{7}\int \frac{\sqrt[3]{e \sec(c+dx)}}{\sqrt{a-a \sec(c+dx)}}d\sqrt[3]{e \sec(c+dx)}-\frac{6\sqrt{a-a \sec(c+dx)}(e \sec(c+dx))^{2/3}}{7a}\right)-\frac{6\sqrt{a-a \sec(c+dx)}\cdots}{13\cdots}\right.}{d\sqrt{a-a \sec(c+dx)}\sqrt{a \sec(c+dx)+a}}$$

$$\Big\downarrow \text{832}$$

$$-\frac{a^2 e\tan(c+dx)\left(\frac{10}{13}e\left(\frac{12}{7}\left(\left(1-\sqrt{3}\right)\sqrt[3]{e}\int\frac{1}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}-\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\right.\right.\right.}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)}}$$

$\downarrow$ 759

$$-\frac{a^2 e\tan(c+dx)\left(\frac{10}{13}e\left(\frac{12}{7}\left(-\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{e}\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec}}{\left(\left(1+\sqrt{3}\right)\right.}}}{}\right.\right.\right.}{}$$

$\downarrow$ 2416

$$-\frac{a^2 e\tan(c+dx)\left(\frac{10}{13}e\left(\frac{12}{7}\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}{\sqrt[4]{3}\sqrt{a-a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(\right.}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(\right.}}}}\right.\right.\right.}{}$$

input
```
Int[(e*Sec[c + d*x])^(8/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
-((a^2*e*((-6*(e*Sec[c + d*x])^(5/3)*Sqrt[a - a*Sec[c + d*x]])/(13*a) + (1
0*e*((-6*(e*Sec[c + d*x])^(2/3)*Sqrt[a - a*Sec[c + d*x]])/(7*a) + (12*((2*
e*Sqrt[a - a*Sec[c + d*x]])/(a*((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(
1/3))) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*e^(1/3)*EllipticE[ArcSin[((1 - Sqrt[3]
)*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*
x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2
/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[
3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])/(Sqrt[a - a*Sec[c + d*x]]*Sqrt[(
e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Se
c[c + d*x])^(1/3))^2]) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*e^(1/3)*Ellipt
icF[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])
*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c +
 d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c +
d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])/(3^(1/4)
*Sqrt[a - a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3))
)/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])))/7))/13)*Tan[c + d
*x])/(d*Sqrt[a - a*Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int (e\sec(dx+c))^{\frac{8}{3}}\sqrt{a+a\sec(dx+c)}dx$$

input
```
int((e*sec(d*x+c))^(8/3)*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
int((e*sec(d*x+c))^(8/3)*(a+a*sec(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int (e\sec(c+dx))^{8/3}\sqrt{a+a\sec(c+dx)}\,dx=\int \sqrt{a\sec(dx+c)+a}(e\sec(dx+c))^{\frac{8}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(8/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3)*e^2*sec(d*x + c)^
2, x)
```

Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{8/3}\sqrt{a+a\sec(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(8/3)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (e\sec(c+dx))^{8/3}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}(e\sec(dx+c))^{\frac{8}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(8/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(8/3), x)
```

Giac [F]

$$\int (e\sec(c+dx))^{8/3}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}(e\sec(dx+c))^{\frac{8}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(8/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(8/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{8/3}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a+\frac{a}{\cos(c+d\,x)}}\left(\frac{e}{\cos(c+d\,x)}\right)^{8/3}\,dx$$

input
```
int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(8/3),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(8/3), x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{8/3}\sqrt{a+a\sec(c+dx)}\,dx = e^{\frac{8}{3}}\sqrt{a}\left(\int \sec(dx+c)^{\frac{8}{3}}\sqrt{\sec(dx+c)+1}dx\right)$$

input
```
int((e*sec(d*x+c))^(8/3)*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
e**(2/3)*sqrt(a)*int(sec(c + d*x)**(2/3)*sqrt(sec(c + d*x) + 1)*sec(c + d*
x)**2,x)*e**2
```

3.276 $\int (e\sec(c+dx))^{5/3}\sqrt{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 673

$$\int (e\sec(c+dx))^{5/3}\sqrt{a+a\sec(c+dx)}\,dx=\frac{6ae(e\sec(c+dx))^{2/3}\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}}$$

$$-\frac{24ae^2\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}$$

$$+\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^2e^{4/3}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)\middle|-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\cdots}}{7d(a-a\sec(c+dx))\sqrt{a+a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c\cdots}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(\cdots}\right.}}}$$

$$-\frac{8\sqrt{2}3^{3/4}a^2e^{4/3}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right),-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\cdots}}{7d(a-a\sec(c+dx))\sqrt{a+a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(\cdots}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec\cdots}\right.}}}$$

output
```
6/7*a*e*(e*sec(d*x+c))^(2/3)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)-24/7*a*e^
2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^
(1/3))+12/7*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^2*e^(4/3)*EllipticE(((1-3^
(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(
1/3)),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x+c))^(1/3))*((e^(2/3)+e^(1/3)*(e*s
ec(d*x+c))^(1/3)+(e*sec(d*x+c))^(2/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))
^(1/3))^2)^(1/2)*tan(d*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(
1/3)*(e^(1/3)-(e*sec(d*x+c))^(1/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1
/3))^2)^(1/2)-8/7*2^(1/2)*3^(3/4)*a^2*e^(4/3)*EllipticF(((1-3^(1/2))*e^(1/
3)-(e*sec(d*x+c))^(1/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3)),I*3^(1
/2)+2*I)*(e^(1/3)-(e*sec(d*x+c))^(1/3))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(
1/3)+(e*sec(d*x+c))^(2/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3))^2)^(
1/2)*tan(d*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3
)-(e*sec(d*x+c))^(1/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3))^2)^(1/2
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.11

$$\int (e\sec(c+dx))^{5/3}\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{3}{2},1-\sec(c+dx)\right)(e\sec(c+dx))^{5/3}\sqrt{a(1}{d\sec^{\frac{5}{3}}(c+dx)}$$

input
```
Integrate[(e*Sec[c + d*x])^(5/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Hypergeometric2F1[-2/3, 1/2, 3/2, 1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(5
/3)*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sec[c + d*x]^(5/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.61 (sec) , antiderivative size = 668, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3042, 4293, 60, 73, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(c+dx)+a}(e\sec(c+dx))^{5/3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx$$

$$\Big\downarrow \text{4293}$$

$$-\frac{a^2e\tan(c+dx)\int\frac{(e\sec(c+dx))^{2/3}}{\sqrt{a-a\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{60}$$

$$-\frac{a^2e\tan(c+dx)\left(\frac{4}{7}e\int\frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a-a\sec(c+dx)}}d\sec(c+dx)-\frac{6\sqrt{a-a\sec(c+dx)}(e\sec(c+dx))^{2/3}}{7a}\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{a^2e\tan(c+dx)\left(\frac{12}{7}\int\frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}-\frac{6\sqrt{a-a\sec(c+dx)}(e\sec(c+dx))^{2/3}}{7a}\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{832}$$

$$-\frac{a^2e\tan(c+dx)\left(\frac{12}{7}\left(\left(1-\sqrt{3}\right)\sqrt[3]{e}\int\frac{1}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}-\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}\right.\right.}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{759}$$

$$-\frac{a^2 e\tan(c+dx)\left(\frac{12}{7}\left(-\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{e}\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+a}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{}\right.}}}{}\right.\right.}{d\sqrt{a-a\,s}}$$

$\Big\downarrow$ 2416

$$-\frac{a^2 e\tan(c+dx)\left(\frac{12}{7}\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,\text{Elliptic}}{\sqrt[4]{3}\sqrt{a-a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+d}\right.}}}\right.\right.}{}$$

input
```
Int[(e*Sec[c + d*x])^(5/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
-((a^2*e*((-6*(e*Sec[c + d*x])^(2/3)*Sqrt[a - a*Sec[c + d*x]])/(7*a) + (12
*((2*e*Sqrt[a - a*Sec[c + d*x]])/(a*((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*
x])^(1/3))) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*e^(1/3)*EllipticE[ArcSin[((1 - Sq
rt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c
 + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[
(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 +
Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])/(Sqrt[a - a*Sec[c + d*x]]*S
qrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) -
(e*Sec[c + d*x])^(1/3))^2]) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*e^(1/3)*E
llipticF[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqr
t[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Se
c[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec
[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])/(3^
(1/4)*Sqrt[a - a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(
1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])))/7)*Tan[c + d
*x])/(d*Sqrt[a - a*Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int (e \sec (dx + c))^{\frac{5}{3}} \sqrt{a + a \sec (dx + c)}dx$$

input
```
int((e*sec(d*x+c))^(5/3)*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
int((e*sec(d*x+c))^(5/3)*(a+a*sec(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int (e \sec(c + dx))^{5/3} \sqrt{a + a \sec(c + dx)}\, dx = \int \sqrt{a \sec (dx + c) + a}(e \sec (dx + c))^{\frac{5}{3}}\, dx$$

input
```
integrate((e*sec(d*x+c))^(5/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3)*e*sec(d*x + c), x
)
```

Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{5/3}\sqrt{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(5/3)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (e\sec(c+dx))^{5/3}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}(e\sec(dx+c))^{\frac{5}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(5/3), x)
```

Giac [F]

$$\int (e\sec(c+dx))^{5/3}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}(e\sec(dx+c))^{\frac{5}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{5/3}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a+\frac{a}{\cos(c+d\,x)}}\left(\frac{e}{\cos(c+d\,x)}\right)^{5/3} dx$$

input
```
int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(5/3),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(5/3), x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{5/3}\sqrt{a+a\sec(c+dx)}\,dx = e^{\frac{5}{3}}\sqrt{a}\left(\int \sec(dx+c)^{\frac{5}{3}}\sqrt{\sec(dx+c)+1}dx\right)$$

input
```
int((e*sec(d*x+c))^(5/3)*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
e**(2/3)*sqrt(a)*int(sec(c + d*x)**(2/3)*sqrt(sec(c + d*x) + 1)*sec(c + d*
x),x)*e
```

3.277 $\int (e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 624

$$\int (e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}\,dx = -\frac{6ae\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}$$

$$+\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^2\sqrt[3]{e}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)|-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{e^{2/}\cdots}{\cdots}}}{d(a-a\sec(c+dx))\sqrt{a+a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+\cdots}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c\cdots}\right)\cdots}}}$$

$$-\frac{2\sqrt{2}3^{3/4}a^2\sqrt[3]{e}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right),-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\cdots}}{d(a-a\sec(c+dx))\sqrt{a+a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c\cdots}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(\cdots}\right)\cdots}}}$$

output
```
-6*a*e*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)/((1+3^(1/2))*e^(1/3)-(e*sec(d*x
+c))^(1/3))+3*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^2*e^(1/3)*EllipticE(((1-
3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))
^(1/3)),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x+c))^(1/3))*((e^(2/3)+e^(1/3)*(e
*sec(d*x+c))^(1/3)+(e*sec(d*x+c))^(2/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c
))^(1/3))^2)^(1/2)*tan(d*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e
^(1/3)*(e^(1/3)-(e*sec(d*x+c))^(1/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^
(1/3))^2)^(1/2)-2*2^(1/2)*3^(3/4)*a^2*e^(1/3)*EllipticF(((1-3^(1/2))*e^(1/
3)-(e*sec(d*x+c))^(1/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3)),I*3^(1
/2)+2*I)*(e^(1/3)-(e*sec(d*x+c))^(1/3))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(
1/3)+(e*sec(d*x+c))^(2/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3))^2)^(
1/2)*tan(d*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3
)-(e*sec(d*x+c))^(1/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3))^2)^(1/2
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.11

$$\int (e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},1-\sec(c+dx)\right)(e\sec(c+dx))^{2/3}\sqrt{a(1+}}{d\sec^{\frac{2}{3}}(c+dx)}$$

input
```
Integrate[(e*Sec[c + d*x])^(2/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Hypergeometric2F1[1/3, 1/2, 3/2, 1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(2/
3)*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sec[c + d*x]^(2/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 628, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3042, 4293, 73, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(c+dx)+a}(e\sec(c+dx))^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx$$

$$\Big\downarrow \text{4293}$$

$$-\frac{a^2e\tan(c+dx)\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a-a\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{3a^2\tan(c+dx)\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{832}$$

$$-\frac{3a^2\tan(c+dx)\left(\left(1-\sqrt{3}\right)\sqrt[3]{e}\int \frac{1}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}-\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+a}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{759}$$

$$-\frac{3a^2\tan(c+dx)\left(-\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{e}\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\,\mathrm{se}}\right.}}}{}\right.}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(}$$

$\downarrow$ 2416

$$-\frac{3a^2\tan(c+dx)\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\text{EllipticF}\left(\text{ar}\right.}{\sqrt[4]{3}\sqrt{a-a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}\right.}{}$$

input
```
Int[(e*Sec[c + d*x])^(2/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(-3*a^2*((2*e*Sqrt[a - a*Sec[c + d*x]])/(a*((1 + Sqrt[3])*e^(1/3) - (e*Sec
[c + d*x])^(1/3))) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*e^(1/3)*EllipticE[ArcSin[(
(1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (
e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3)
)*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))
/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])/(Sqrt[a - a*Sec[c +
d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(
1/3) - (e*Sec[c + d*x])^(1/3))^2]) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*e^
(1/3)*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((
1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3)
- (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) +
 (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^
2])/(3^(1/4)*Sqrt[a - a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c +
d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]))*Tan[c
+ d*x])/(d*Sqrt[a - a*Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int (e\sec(dx+c))^{\frac{2}{3}}\sqrt{a+a\sec(dx+c)}dx$$

input
```
int((e*sec(d*x+c))^(2/3)*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
int((e*sec(d*x+c))^(2/3)*(a+a*sec(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int (e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}(e\sec(dx+c))^{\frac{2}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(2/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a(\sec(c+dx)+1)}(e\sec(c+dx))^{\frac{2}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))**(2/3)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*(e*sec(c + d*x))**(2/3), x)
```

Maxima [F]

$$\int (e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}(e\sec(dx+c))^{\frac{2}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(2/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3), x)
```

Giac [F]

$$\int (e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}(e\sec(dx+c))^{\frac{2}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(2/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a+\frac{a}{\cos(c+d\,x)}}\left(\frac{e}{\cos(c+d\,x)}\right)^{2/3}\,dx$$

input
```
int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(2/3),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(2/3), x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}\,dx = e^{\frac{2}{3}}\sqrt{a}\left(\int \sec(dx+c)^{\frac{2}{3}}\sqrt{\sec(dx+c)+1}dx\right)$$

input
```
int((e*sec(d*x+c))^(2/3)*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
e**(2/3)*sqrt(a)*int(sec(c + d*x)**(2/3)*sqrt(sec(c + d*x) + 1),x)
```

3.278 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt[3]{e\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 662

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt[3]{e\sec(c+dx)}}\,dx = \frac{3a\tan(c+dx)}{d\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$
$$+\frac{3a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}$$
$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^2E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)\middle|-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{e^{2/\cdots}}{\cdots}}}{2de^{2/3}(a-a\sec(c+dx))\sqrt{a+a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec\cdots}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec\cdots}\right.}}}$$
$$+\frac{\sqrt{2}3^{3/4}a^2\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right),-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{e^{\cdots}}{\cdots}}}{de^{2/3}(a-a\sec(c+dx))\sqrt{a+a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec\cdots}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\se\cdots}\right.}}}$$

output
```
3*a*tan(d*x+c)/d/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2)+3*a*tan(d*x+c
)/d/(a+a*sec(d*x+c))^(1/2)/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3))-3/2*
3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^2*EllipticE(((1-3^(1/2))*e^(1/3)-(e*se
c(d*x+c))^(1/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3)),I*3^(1/2)+2*I)
*(e^(1/3)-(e*sec(d*x+c))^(1/3))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e*
sec(d*x+c))^(2/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3))^2)^(1/2)*tan
(d*x+c)/d/e^(2/3)/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3
)-(e*sec(d*x+c))^(1/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3))^2)^(1/2
)+2^(1/2)*3^(3/4)*a^2*EllipticF(((1-3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3))
/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3)),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec
(d*x+c))^(1/3))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e*sec(d*x+c))^(2/3
))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3))^2)^(1/2)*tan(d*x+c)/d/e^(2/3
)/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*sec(d*x+c))
^(1/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.11

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt[3]{e\sec(c+dx)}}\,dx$$
$$=\frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{3}{2},1-\sec(c+dx)\right)\sqrt[3]{\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d\sqrt[3]{e\sec(c+dx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(1/3),x]
```

output
```
(2*Hypergeometric2F1[1/2, 4/3, 3/2, 1 - Sec[c + d*x]]*Sec[c + d*x]^(1/3)*S
qrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*(e*Sec[c + d*x])^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3042, 4293, 61, 73, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a \sec(c+dx)+a}}{\sqrt[3]{e \sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a \csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\sqrt[3]{e \csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4293}$$

$$-\frac{a^2 e \tan(c+dx)\int \frac{1}{(e \sec(c+dx))^{4/3}\sqrt{a-a \sec(c+dx)}}d\sec(c+dx)}{d\sqrt{a-a \sec(c+dx)}\sqrt{a \sec(c+dx)+a}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{a^2 e \tan(c+dx)\left(-\frac{\int \frac{1}{\sqrt[3]{e \sec(c+dx)}\sqrt{a-a \sec(c+dx)}}d\sec(c+dx)}{2e}-\frac{3\sqrt{a-a \sec(c+dx)}}{ae\sqrt[3]{e \sec(c+dx)}}\right)}{d\sqrt{a-a \sec(c+dx)}\sqrt{a \sec(c+dx)+a}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{a^2 e \tan(c+dx)\left(-\frac{3\int \frac{\sqrt[3]{e \sec(c+dx)}}{\sqrt{a-a \sec(c+dx)}}d\sqrt[3]{e \sec(c+dx)}}{2e^2}-\frac{3\sqrt{a-a \sec(c+dx)}}{ae\sqrt[3]{e \sec(c+dx)}}\right)}{d\sqrt{a-a \sec(c+dx)}\sqrt{a \sec(c+dx)+a}}$$

$$\Big\downarrow \text{832}$$

$$-\frac{a^2 e \tan(c+dx)\left(-\frac{3\left(\left(1-\sqrt{3}\right)\sqrt[3]{e}\int \frac{1}{\sqrt{a-a \sec(c+dx)}}d\sqrt[3]{e \sec(c+dx)}-\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e \sec(c+dx)}}{\sqrt{a-a \sec(c+dx)}}d\sqrt[3]{e \sec(c+dx)}\right)}{2e^2}\right.}{d\sqrt{a-a \sec(c+dx)}\sqrt{a \sec(c+dx)+a}}$$

$\Big\downarrow$ 759

$$-\frac{a^2 e\tan(c+dx)\left(-\frac{3\left(-\int\frac{(1-\sqrt{3})\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}-\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}\sqrt[3]{e}\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e s}{((1+\sqrt{3})\sqrt[3]{e}-\sqrt[3]{e\sec}}}}{}\right)}{2e^2}\right)}{d\sqrt{a-a\sec(c+dx)}}$$

$\Big\downarrow$ 2416

$$-\frac{a^2 e\tan(c+dx)\left(-\frac{3\left(-\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left((1+\sqrt{3})\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\text{EllipticF}\left(\text{arc}\right.}{\sqrt[4]{3}\sqrt{a-a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left((1+\sqrt{3})\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}\right)}{}\right)}{}$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(1/3),x]
```

output
```
-((a^2*e*((-3*Sqrt[a - a*Sec[c + d*x]])/(a*e*(e*Sec[c + d*x])^(1/3)) - (3*
((2*e*Sqrt[a - a*Sec[c + d*x]])/(a*((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x
])^(1/3))) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*e^(1/3)*EllipticE[ArcSin[((1 - Sqr
t[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c
+ d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(
e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + S
qrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])/(Sqrt[a - a*Sec[c + d*x]]*Sq
rt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (
e*Sec[c + d*x])^(1/3))^2]) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*e^(1/3)*El
lipticF[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt
[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec
[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[
c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])/(3^(
1/4)*Sqrt[a - a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1
/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])))/(2*e^2))*Tan[
c + d*x])/(d*Sqrt[a - a*Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{\sqrt{a + a \sec (dx + c)}}{(e \sec (dx + c))^{\frac{1}{3}}} dx$$

input
```
int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/3),x)
```

output
```
int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{\sqrt{a + a \sec(c + dx)}}{\sqrt[3]{e \sec(c + dx)}}\, dx = \int \frac{\sqrt{a \sec (dx + c) + a}}{(e \sec (dx + c))^{\frac{1}{3}}}\, dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/3),x, algorithm="fricas
")
```

output
```
integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3)/(e*sec(d*x + c)),
 x)
```

Sympy [F]

$$\int \frac{\sqrt{a + a \sec(c + dx)}}{\sqrt[3]{e \sec(c + dx)}}\, dx = \int \frac{\sqrt{a (\sec (c + dx) + 1)}}{\sqrt[3]{e \sec (c + dx)}}\, dx$$

input
```
integrate((a+a*sec(d*x+c))**(1/2)/(e*sec(d*x+c))**(1/3),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))/(e*sec(c + d*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt[3]{e\sec(c+dx)}}\,dx=\int \frac{\sqrt{a\sec\left(dx+c\right)+a}}{\left(e\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/3),x, algorithm="maxima
")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)/(e*sec(d*x + c))^(1/3), x)
```

Giac [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt[3]{e\sec(c+dx)}}\,dx=\int \frac{\sqrt{a\sec\left(dx+c\right)+a}}{\left(e\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)/(e*sec(d*x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt[3]{e\sec(c+dx)}}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{{\left(\frac{e}{\cos(c+d\,x)}\right)}^{1/3}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(1/2)/(e/cos(c + d*x))^(1/3),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2)/(e/cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt[3]{e\sec(c+dx)}}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^{\frac{1}{3}}}dx\right)}{e^{\frac{1}{3}}}$$

input
```
int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/3),x)
```

output
```
(sqrt(a)*int(sqrt(sec(c + d*x) + 1)/sec(c + d*x)**(1/3),x))/e**(1/3)
```

3.279 $\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 715

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{4/3}}\,dx = \frac{3a\tan(c+dx)}{4d(e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}}$$

$$+\frac{15a\tan(c+dx)}{8de\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$+\frac{15a\tan(c+dx)}{8de\sqrt{a+a\sec(c+dx)}\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}$$

$$-\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^2E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)|-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{e^{2/3}\cdots}{\cdots}}}{16de^{5/3}(a-a\sec(c+dx))\sqrt{a+a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec\cdots}}}}$$

$$+\frac{5\ 3^{3/4}a^2\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right),-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{e^{2/3}+\cdots}{(\cdots}}}{4\sqrt{2}de^{5/3}(a-a\sec(c+dx))\sqrt{a+a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec\cdots}}}}$$

output
```
3/4*a*tan(d*x+c)/d/(e*sec(d*x+c))^(4/3)/(a+a*sec(d*x+c))^(1/2)+15/8*a*tan(
d*x+c)/d/e/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2)+15/8*a*tan(d*x+c)/d
/e/(a+a*sec(d*x+c))^(1/2)/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3))-15/16
*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^2*EllipticE(((1-3^(1/2))*e^(1/3)-(e*s
ec(d*x+c))^(1/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3)),I*3^(1/2)+2*I
)*(e^(1/3)-(e*sec(d*x+c))^(1/3))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e
*sec(d*x+c))^(2/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3))^2)^(1/2)*ta
n(d*x+c)/d/e^(5/3)/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/
3)-(e*sec(d*x+c))^(1/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3))^2)^(1/
2)+5/8*3^(3/4)*a^2*EllipticF(((1-3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3))/((
1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3)),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*
x+c))^(1/3))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e*sec(d*x+c))^(2/3))/
((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3))^2)^(1/2)*tan(d*x+c)*2^(1/2)/d/e
^(5/3)/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*sec(d*
x+c))^(1/3))/((1+3^(1/2))*e^(1/3)-(e*sec(d*x+c))^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.10

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{4/3}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{3},\frac{3}{2},1-\sec(c+dx)\right)\sec^{\frac{4}{3}}(c+dx)\sqrt{a(1+\sec(c+dx))}}{d(e\sec(c+dx))^{4/3}}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(4/3),x]
```

output
```
(2*Hypergeometric2F1[1/2, 7/3, 3/2, 1 - Sec[c + d*x]]*Sec[c + d*x]^(4/3)*S
qrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*(e*Sec[c + d*x])^(4/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 717, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {3042, 4293, 61, 61, 73, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(c+dx)+a}}{(e\sec(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$

$$\Big\downarrow \text{4293}$$

$$-\frac{a^2e\tan(c+dx)\int \frac{1}{(e\sec(c+dx))^{7/3}\sqrt{a-a\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{a^2e\tan(c+dx)\left(\frac{5\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a-a\sec(c+dx)}}d\sec(c+dx)}{8e}-\frac{3\sqrt{a-a\sec(c+dx)}}{4ae(e\sec(c+dx))^{4/3}}\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{a^2e\tan(c+dx)\left(\frac{5\left(-\frac{\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a-a\sec(c+dx)}}d\sec(c+dx)}{2e}-\frac{3\sqrt{a-a\sec(c+dx)}}{ae\sqrt[3]{e\sec(c+dx)}}\right)}{8e}-\frac{3\sqrt{a-a\sec(c+dx)}}{4ae(e\sec(c+dx))^{4/3}}\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{a^2 e\tan(c+dx)\left(\frac{5\left(-\frac{3\int\frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}}{2e^2}-\frac{3\sqrt{a-a\sec(c+dx)}}{ae\sqrt[3]{e\sec(c+dx)}}\right)}{8e}-\frac{3\sqrt{a-a\sec(c+dx)}}{4ae(e\sec(c+dx))^{4/3}}\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 832

$$-\frac{a^2 e\tan(c+dx)\left(\frac{5\left(-\frac{3\left(\left(1-\sqrt{3}\right)\sqrt[3]{e}\int\frac{1}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}-\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}\right)}{2e^2}\right.}{8e}\right.}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 759

$$-\frac{a^2 e\tan(c+dx)\left(\frac{5\left(-\frac{3\left(-\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{e}\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\ldots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec\ldots}\right.}}}{\ldots}\right.}{2\ldots}\right.}{\ldots}\right.}{d\sqrt{a-a\sec\ldots}}$$

$\downarrow$ 2416

$$-\frac{a^2 e\tan(c+dx)\left(\frac{5\left(-\frac{3\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,\mathrm{EllipticF}(\ldots)}{\sqrt[4]{3}\sqrt{a-a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\ldots}}}\ldots\right)}{\ldots}\ldots\right)}{\ldots}\ldots\right)}{\ldots}$$

input

```
Int[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(4/3),x]
```

output
```
-((a^2*e*((-3*Sqrt[a - a*Sec[c + d*x]])/(4*a*e*(e*Sec[c + d*x])^(4/3)) + (
5*((-3*Sqrt[a - a*Sec[c + d*x]])/(a*e*(e*Sec[c + d*x])^(1/3)) - (3*((2*e*S
qrt[a - a*Sec[c + d*x]])/(a*((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3
))) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*e^(1/3)*EllipticE[ArcSin[((1 - Sqrt[3])*e
^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])
^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3)
 + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])
*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])/(Sqrt[a - a*Sec[c + d*x]]*Sqrt[(e^(
1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c
 + d*x])^(1/3))^2]) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*e^(1/3)*EllipticF
[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^
(1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*
x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x
])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])/(3^(1/4)*Sq
rt[a - a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/(
(1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])))/(2*e^2)))/(8*e))*Tan
[c + d*x])/(d*Sqrt[a - a*Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{\sqrt{a+a\sec(dx+c)}}{(e\sec(dx+c))^{\frac{4}{3}}}dx$$

input
```
int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(4/3),x)
```

output
```
int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{4/3}}\,dx = \int \frac{\sqrt{a\sec(dx+c)+a}}{(e\sec(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(4/3),x, algorithm="fricas
")
```

output
```
integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3)/(e^2*sec(d*x + c)
^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{4/3}}\,dx = \int \frac{\sqrt{a(\sec(c+dx)+1)}}{(e\sec(c+dx))^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(1/2)/(e*sec(d*x+c))**(4/3),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))/(e*sec(c + d*x))**(4/3), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{4/3}}\,dx=\int \frac{\sqrt{a\sec\left(dx+c\right)+a}}{\left(e\sec\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(4/3),x, algorithm="maxima
")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)/(e*sec(d*x + c))^(4/3), x)
```

Giac [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{4/3}}\,dx=\int \frac{\sqrt{a\sec\left(dx+c\right)+a}}{\left(e\sec\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)/(e*sec(d*x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{4/3}}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{{\left(\frac{e}{\cos(c+d\,x)}\right)}^{4/3}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(1/2)/(e/cos(c + d*x))^(4/3),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2)/(e/cos(c + d*x))^(4/3), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{4/3}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^{\frac{4}{3}}}dx\right)}{e^{\frac{4}{3}}}$$

input
```
int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(4/3),x)
```

output
```
(sqrt(a)*int(sqrt(sec(c + d*x) + 1)/(sec(c + d*x)**(1/3)*sec(c + d*x)),x))
/(e**(1/3)*e)
```

3.280 $\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 78

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{3\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\sec(c+dx),-\sec(c+dx)\right)(e\sec(c+dx))^{2/3}\tan(c+dx)}{2d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

output
```
-3/2*AppellF1(2/3,1,1/2,5/3,-sec(d*x+c),sec(d*x+c))*(e*sec(d*x+c))^(2/3)*t
an(d*x+c)/d/(1-sec(d*x+c))^(1/2)/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 760 vs. 2(78) = 156.

Time = 4.57 (sec) , antiderivative size = 760, normalized size of antiderivative = 9.74

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+a\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[(e*Sec[c + d*x])^(2/3)/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(90*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*
Cos[(c + d*x)/2]*Cos[c + d*x]^2*(e*Sec[c + d*x])^(2/3)*Sqrt[a*(1 + Sec[c +
 d*x])]*Sin[(c + d*x)/2]*(9*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2] + (-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2
]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))/(a*d*(270*AppellF1[1/2, 1/6,
 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]^2*Cos[(c + d*x)/2]^4*(
1 + 2*Cos[c + d*x]) + 10*(-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2])^2*Cos[c + d*x]*Sin[(c + d*x)/2]^2*Tan[(c + d*x)/2]
^2 - 3*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*(10*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*Cos[(c + d*x)/2]^2*(2 - 9*Cos[c + d*x] + Cos[2*(c + d*x)]) - 5*AppellF1
[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x
)/2]^2*(2 - 9*Cos[c + d*x] + Cos[2*(c + d*x)]) + 6*(16*AppellF1[5/2, 1/6,
7/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 4*AppellF1[5/2, 7/6,
4/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 7*AppellF1[5/2, 13/6,
 1/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Sin[(c +
 d*x)/2]^2)*Tan[(c + d*x)/2]^2))
```

Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3042, 4315, 3042, 4314, 148, 27, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{\sqrt{\sec(c+dx)+1}\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{\sec(c+dx)+1}}dx}{\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\sec(c+dx)+1}\int \frac{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx}{\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{ 4314}$$

$$-\frac{e\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt[3]{e\sec(c+dx)}(\sec(c+dx)+1)}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{ 148}$$

$$-\frac{3\tan(c+dx)\int \frac{e\sqrt[3]{e\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)e+e)}d\sqrt[3]{e\sec(c+dx)}}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3e\tan(c+dx)\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)e+e)}d\sqrt[3]{e\sec(c+dx)}}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{ 1012}$$

$$-\frac{3\tan(c+dx)\,\text{AppellF1}\left(\frac{2}{3},1,\frac{1}{2},\frac{5}{3},-\sec(c+dx),\sec(c+dx)\right)(e\sec(c+dx))^{2/3}}{2d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[(e*Sec[c + d*x])^(2/3)/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(-3*AppellF1[2/3, 1, 1/2, 5/3, -Sec[c + d*x], Sec[c + d*x]]*(e*Sec[c + d*x
])^(2/3)*Tan[c + d*x])/(2*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]
])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{(e\sec(dx+c))^{\frac{2}{3}}}{\sqrt{a+a\sec(dx+c)}}dx$$

input
```
int((e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
int((e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+a\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\sec(c+dx))^{\frac{2}{3}}}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

input
```
integrate((e*sec(d*x+c))**(2/3)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral((e*sec(c + d*x))**(2/3)/sqrt(a*(sec(c + d*x) + 1)), x)
```

Maxima [F]

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{2}{3}}}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima
")
```

output
```
integrate((e*sec(d*x + c))^(2/3)/sqrt(a*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{2}{3}}}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(2/3)/sqrt(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{2/3}}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int((e/cos(c + d*x))^(2/3)/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int((e/cos(c + d*x))^(2/3)/(a + a/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{e^{\frac{2}{3}}\sqrt{a}\left(\int \frac{\sec(dx+c)^{\frac{2}{3}}\sqrt{\sec(dx+c)+1}}{\sec(dx+c)+1}dx\right)}{a}$$

input
```
int((e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
(e**(2/3)*sqrt(a)*int((sec(c + d*x)**(2/3)*sqrt(sec(c + d*x) + 1))/(sec(c
+ d*x) + 1),x))/a
```

3.281 $\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 76

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{3\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},\sec(c+dx),-\sec(c+dx)\right)\sqrt[3]{e\sec(c+dx)}\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

output
```
-3*AppellF1(1/3,1,1/2,4/3,-sec(d*x+c),sec(d*x+c))*(e*sec(d*x+c))^(1/3)*tan
(d*x+c)/d/(1-sec(d*x+c))^(1/2)/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 761 vs. 2(76) = 152.

Time = 6.79 (sec) , antiderivative size = 761, normalized size of antiderivative = 10.01

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[(e*Sec[c + d*x])^(1/3)/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(90*AppellF1[1/2, -1/6, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
*Cos[(c + d*x)/2]*(e*Sec[c + d*x])^(1/3)*Sin[(c + d*x)/2]*(9*AppellF1[1/2,
 -1/6, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - (4*AppellF1[3/
2, -1/6, 5/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2
, 5/6, 2/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2
]^2))/(d*((1 + Cos[c + d*x])^(-1))^(2/3)*(Sec[c + d*x]/(1 + Sec[c + d*x]))
^(1/3)*Sqrt[a*(1 + Sec[c + d*x])]*(270*AppellF1[1/2, -1/6, 2/3, 3/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]^2*Cos[(c + d*x)/2]^4*(-1 + 4*Cos[c + d
*x])*Sec[c + d*x] + 10*(4*AppellF1[3/2, -1/6, 5/3, 5/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 5/6, 2/3, 5/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2])^2*Sin[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 - 3*AppellF1
[1/2, -1/6, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sin[(c + d*
x)/2]^2*(20*AppellF1[3/2, -1/6, 5/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2]*(-9 + (1 + 2*Cos[2*(c + d*x)])*Sec[c + d*x]) + 5*AppellF1[3/2, 5/
6, 2/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(-9 + (1 + 2*Cos[2*(
c + d*x)])*Sec[c + d*x]) + 6*(40*AppellF1[5/2, -1/6, 8/3, 7/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2] + 8*AppellF1[5/2, 5/6, 5/3, 7/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2] - 5*AppellF1[5/2, 11/6, 2/3, 7/2, Tan[(c + d
*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))
```

Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3042, 4315, 3042, 4314, 148, 27, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{e \sec(c+dx)}}{\sqrt{a \sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{e \csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a \csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{\sqrt{\sec(c+dx)+1}\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{\sec(c+dx)+1}}dx}{\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 3042

$$\frac{\sqrt{\sec(c+dx)+1}\int \frac{\sqrt[3]{e\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx}{\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 4314

$$-\frac{e\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}(e\sec(c+dx))^{2/3}(\sec(c+dx)+1)}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 148

$$-\frac{3\tan(c+dx)\int \frac{e}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)e+e)}d\sqrt[3]{e\sec(c+dx)}}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 27

$$-\frac{3e\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)e+e)}d\sqrt[3]{e\sec(c+dx)}}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 936

$$-\frac{3\tan(c+dx)\operatorname{AppellF1}\left(\frac{1}{3},1,\frac{1}{2},\frac{4}{3},-\sec(c+dx),\sec(c+dx)\right)\sqrt[3]{e\sec(c+dx)}}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[(e*Sec[c + d*x])^(1/3)/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(-3*AppellF1[1/3, 1, 1/2, 4/3, -Sec[c + d*x], Sec[c + d*x]]*(e*Sec[c + d*x
])^(1/3)*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{(e \sec (dx+c))^{\frac{1}{3}}}{\sqrt{a+a \sec (dx+c)}}dx$$

input
```
int((e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
int((e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{e \sec(c+dx)}}{\sqrt{a+a \sec(c+dx)}}\, dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{e \sec(c+dx)}}{\sqrt{a+a \sec(c+dx)}}\, dx = \int \frac{\sqrt[3]{e \sec (c+dx)}}{\sqrt{a (\sec (c+dx)+1)}}\, dx$$

input
```
integrate((e*sec(d*x+c))**(1/3)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral((e*sec(c + d*x))**(1/3)/sqrt(a*(sec(c + d*x) + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{e \sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{1}{3}}}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima
")
```

output
```
integrate((e*sec(d*x + c))^(1/3)/sqrt(a*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{e \sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{1}{3}}}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(1/3)/sqrt(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{e \sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{1/3}}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int((e/cos(c + d*x))^(1/3)/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int((e/cos(c + d*x))^(1/3)/(a + a/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{e^{\frac{1}{3}}\sqrt{a}\left(\int \frac{\sec(dx+c)^{\frac{1}{3}}\sqrt{\sec(dx+c)+1}}{\sec(dx+c)+1}dx\right)}{a}$$

input
```
int((e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
(e**(1/3)*sqrt(a)*int((sec(c + d*x)**(1/3)*sqrt(sec(c + d*x) + 1))/(sec(c
+ d*x) + 1),x))/a
```

3.282 $\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 76

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \frac{3\,\text{AppellF1}\left(-\frac{1}{3},\frac{1}{2},1,\frac{2}{3},\sec(c+dx),-\sec(c+dx)\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

output
```
3*AppellF1(-1/3,1,1/2,2/3,-sec(d*x+c),sec(d*x+c))*tan(d*x+c)/d/(1-sec(d*x+
c))^(1/2)/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3346 vs. 2(76) = 152.

Time = 17.01 (sec) , antiderivative size = 3346, normalized size of antiderivative = 44.03

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
-(((Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/6)*Tan[(c + d*x)/2]*(-1 + Tan[(c +
 d*x)/2]^2)*((2*AppellF1[3/2, 1/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/6) +
(3*(1 + (3*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2])/((-1 + Tan[(c + d*x)/2]^2)*(9*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(Sec[(c + d*x)/2
]^2)^(1/3)))/(d*(e*Sec[c + d*x])^(1/3)*Sqrt[a*(1 + Sec[c + d*x])]*(-(Sec[(
c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/6)*Tan[(c + d*x)/2]^2*(
(2*AppellF1[3/2, 1/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*T
an[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/6) + (3*(1 + (3*Ap
pellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/((-1
+ Tan[(c + d*x)/2]^2)*(9*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2] + (-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(Sec[(c + d*x)/2]^2)^(1/3)))
- (Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/6)*(-1 + Tan[(c
 + d*x)/2]^2)*((2*AppellF1[3/2, 1/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/...
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3042, 4315, 3042, 4314, 148, 27, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sec(c+dx)+a}\sqrt[3]{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}\sqrt[3]{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{\sqrt{\sec(c+dx)+1}\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{\sec(c+dx)+1}}dx}{\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(c+dx)+1}\int \frac{1}{\sqrt[3]{e\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx}{\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4314}$$

$$-\frac{e\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}(e\sec(c+dx))^{4/3}(\sec(c+dx)+1)}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{148}$$

$$-\frac{3\tan(c+dx)\int \frac{e\cos^2(c+dx)}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)e+e)}d\sqrt[3]{e\sec(c+dx)}}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3e\tan(c+dx)\int \frac{\cos^2(c+dx)}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)e+e)}d\sqrt[3]{e\sec(c+dx)}}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{3\sin(c+dx)\operatorname{AppellF1}\left(-\frac{1}{3},1,\frac{1}{2},\frac{2}{3},-\sec(c+dx),\sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[1/((e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
(3*AppellF1[-1/3, 1, 1/2, 2/3, -Sec[c + d*x], Sec[c + d*x]]*Sin[c + d*x])/
(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{1}{3}}\sqrt{a+a\sec\left(dx+c\right)}}dx$$

input
```
int(1/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
int(1/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sqrt[3]{e\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))**(1/3)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(sec(c + d*x) + 1))*(e*sec(c + d*x))**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\sec\left(dx+c\right)+a}\left(e\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\sec\left(dx+c\right)+a}\left(e\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,{\left(\frac{e}{\cos(c+d\,x)}\right)}^{1/3}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(1/3)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(1/3)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^{\frac{4}{3}}+\sec(dx+c)^{\frac{1}{3}}}dx\right)}{e^{\frac{1}{3}}a}$$

input
```
int(1/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int(sqrt(sec(c + d*x) + 1)/(sec(c + d*x)**(1/3)*sec(c + d*x) + se
c(c + d*x)**(1/3)),x))/(e**(1/3)*a)
```

3.283 $\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 78

$$\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}\,dx = \frac{3\,\text{AppellF1}\left(-\frac{2}{3},\frac{1}{2},1,\frac{1}{3},\sec(c+dx),-\sec(c+dx)\right)\tan(c+}{2d\sqrt{1-\sec(c+dx)}(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}$$

output
```
3/2*AppellF1(-2/3,1,1/2,1/3,-sec(d*x+c),sec(d*x+c))*tan(d*x+c)/d/(1-sec(d*
x+c))^(1/2)/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 585 vs. 2(78) = 156.

Time = 4.78 (sec) , antiderivative size = 585, normalized size of antiderivative = 7.50

$$\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(2/3)*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
(Sec[c + d*x]^(7/6)*((-3*Cos[(c + d*x)/2]*Sec[c + d*x]^(5/6)*(Sin[(c + d*x
)/2] - Sin[(3*(c + d*x))/2]))/2 + (5*Sqrt[(1 + Cos[c + d*x])^(-1)]*(-1 + 3
*Cos[c + d*x])*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(5/6)*Sin[(c + d*x)/2]*(-
3*Cos[c + d*x]^(5/6)*Hypergeometric2F1[1/2, 5/6, 3/2, 2*Sin[(c + d*x)/2]^2
]*(Sec[(c + d*x)/2]^2)^(1/3) + 2*AppellF1[3/2, 5/6, 2/3, 5/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/6)*Tan[(
c + d*x)/2]^2))/(-120*AppellF1[3/2, 5/6, 2/3, 5/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2]*((1 + Cos[c + d*x])^(-1))^(2/3)*(Cos[c + d*x]/(1 + Cos[c
 + d*x]))^(5/6)*Sin[(c + d*x)/2]*Tan[(c + d*x)/2] + 32*AppellF1[5/2, 5/6,
5/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*((1 + Cos[c + d*x])^(-1
))^(2/3)*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(5/6)*Sin[(c + d*x)/2]*Tan[(c +
 d*x)/2]^3 + 5*Sqrt[2]*Cos[(c + d*x)/2]*(3 - 4*Sqrt[2]*AppellF1[5/2, 11/6,
 2/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*((1 + Cos[c + d*x])^(-
1))^(2/3)*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(5/6)*Tan[(c + d*x)/2]^4))))/(
d*(e*Sec[c + d*x])^(2/3)*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3042, 4315, 3042, 4314, 148, 27, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sec(c+dx)+a}(e\sec(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{\sqrt{\sec(c+dx)+1}\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{\sec(c+dx)+1}}dx}{\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(c+dx)+1}\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx}{\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{ 4314}$$

$$-\frac{e\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}(e\sec(c+dx))^{5/3}(\sec(c+dx)+1)}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{ 148}$$

$$-\frac{3\tan(c+dx)\int \frac{e\cos^3(c+dx)}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)e+e)}d\sqrt[3]{e\sec(c+dx)}}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3e\tan(c+dx)\int \frac{\cos^3(c+dx)}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)e+e)}d\sqrt[3]{e\sec(c+dx)}}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{3\sin(c+dx)\cos(c+dx)\operatorname{AppellF1}\left(-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},-\sec(c+dx),\sec(c+dx)\right)}{2d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[1/((e*Sec[c + d*x])^(2/3)*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
(3*AppellF1[-2/3, 1, 1/2, 1/3, -Sec[c + d*x], Sec[c + d*x]]*Cos[c + d*x]*S
in[c + d*x])/(2*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{2}{3}}\sqrt{a+a\sec\left(dx+c\right)}}dx$$

input
```
int(1/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
int(1/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(e\sec\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))**(2/3)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(sec(c + d*x) + 1))*(e*sec(c + d*x))**(2/3)), x)
```

Maxima [F]

$$\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\sec\left(dx+c\right)+a}\left(e\sec\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3)), x)
```

Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\sec\left(dx+c\right)+a}\left(e\sec\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,\left(\frac{e}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(2/3)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(2/3)), x)
```

Reduce [F]

$$\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sec(dx+c)^{\frac{1}{3}}\sqrt{\sec(dx+c)+1}}{\sec(dx+c)^2+\sec(dx+c)}dx\right)}{e^{\frac{2}{3}}a}$$

input
```
int(1/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
(e**(1/3)*sqrt(a)*int((sec(c + d*x)**(1/3)*sqrt(sec(c + d*x) + 1))/(sec(c
+ d*x)**2 + sec(c + d*x)),x))/(a*e)
```

3.284 $\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 78

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\sec(c+dx)}\,dx = \frac{2^{5/6}\operatorname{AppellF1}\left(\frac{1}{2},-\frac{1}{3},\frac{1}{6},\frac{3}{2},1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{d(1+\sec(c+dx))^{5/6}}$$

output
```
2^(5/6)*AppellF1(1/2,-1/3,1/6,3/2,1-sec(d*x+c),1/2-1/2*sec(d*x+c))*(a+a*se
c(d*x+c))^(1/3)*tan(d*x+c)/d/(1+sec(d*x+c))^(5/6)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1982 vs. 2(78) = 156.

Time = 20.92 (sec) , antiderivative size = 1982, normalized size of antiderivative = 25.41

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[c + d*x]^(4/3)*(a + a*Sec[c + d*x])^(1/3),x]
```

output
```
(3*Sec[c + d*x]^(1/3)*((1 + Cos[c + d*x])*Sec[c + d*x])^(1/3)*(a*(1 + Sec[
c + d*x]))^(1/3)*Sin[c + d*x])/(2*d*(1 + Sec[c + d*x])^(1/3)) + (3*(a*(1 +
 Sec[c + d*x]))^(1/3)*(-((1 + Sec[c + d*x])^(1/3)/Sec[c + d*x]^(2/3)) + (S
ec[c + d*x]^(1/3)*(1 + Sec[c + d*x])^(1/3))/2)*Tan[(c + d*x)/2]*(-1 + (3*A
ppellF1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(9*
AppellF1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2
*(2*AppellF1[3/2, -1/3, 5/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
 + HypergeometricPFQ[{2/3, 3/4}, {7/4}, Tan[(c + d*x)/2]^4])*Tan[(c + d*x)
/2]^2)))/(2^(2/3)*d*(Sec[(c + d*x)/2]^2)^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c +
 d*x])^(1/3)*(1 + Sec[c + d*x])^(1/3)*((3*(Sec[(c + d*x)/2]^2)^(1/3)*(-1 +
 (3*AppellF1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
)/(9*AppellF1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
] - 2*(2*AppellF1[3/2, -1/3, 5/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2] + HypergeometricPFQ[{2/3, 3/4}, {7/4}, Tan[(c + d*x)/2]^4])*Tan[(c +
 d*x)/2]^2)))/(2*2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/3)) - (2^(1/
3)*Tan[(c + d*x)/2]^2*(-1 + (3*AppellF1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2])/(9*AppellF1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2] - 2*(2*AppellF1[3/2, -1/3, 5/3, 5/2, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2] + HypergeometricPFQ[{2/3, 3/4}, {7/4}, Ta
n[(c + d*x)/2]^4])*Tan[(c + d*x)/2]^2)))/((Sec[(c + d*x)/2]^2)^(2/3)*(C...
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4315, 3042, 4312, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}\sqrt[3]{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{\sqrt[3]{a\sec(c+dx)+a}\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{\sec(c+dx)+1}dx}{\sqrt[3]{\sec(c+dx)+1}}$$

$\Big\downarrow$ 3042

$$\frac{\sqrt[3]{a\sec(c+dx)+a}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)+1}dx}{\sqrt[3]{\sec(c+dx)+1}}$$

$\Big\downarrow$ 4312

$$\frac{\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}}d(1-\sec(c+dx))}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}$$

$\Big\downarrow$ 150

$$\frac{2^{5/6}\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{3},\frac{1}{6},\frac{3}{2},1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{5/6}}$$

input
```
Int[Sec[c + d*x]^(4/3)*(a + a*Sec[c + d*x])^(1/3),x]
```

output
```
(2^(5/6)*AppellF1[1/2, -1/3, 1/6, 3/2, 1 - Sec[c + d*x], (1 - Sec[c + d*x]
)/2]*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(5/6))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4312
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a - x)^(n - 1)
*((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \sec(dx+c)^{\frac{4}{3}} (a+a\sec(dx+c))^{\frac{1}{3}} \, dx$$

input
```
int(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\sec(c+dx)}\, dx = \int (a\sec(dx+c)+a)^{\frac{1}{3}} \sec(dx+c)^{\frac{4}{3}} \, dx$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^(4/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(4/3)*(a+a*sec(d*x+c))**(1/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\sec(c+dx)}\,dx = \int (a\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)^{\frac{4}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^(4/3), x)
```

Giac [F]

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\sec(c+dx)}\,dx = \int (a\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)^{\frac{4}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\sec(c+dx)}\,dx=\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{1/3}\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{4/3}\,dx$$

input
```
int((a + a/cos(c + d*x))^(1/3)*(1/cos(c + d*x))^(4/3),x)
```

output
```
int((a + a/cos(c + d*x))^(1/3)*(1/cos(c + d*x))^(4/3), x)
```

Reduce [F]

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\sec(c+dx)}\,dx=a^{\frac{1}{3}}\left(\int \sec\left(dx+c\right)^{\frac{4}{3}}\left(\sec\left(dx+c\right)+1\right)^{\frac{1}{3}}dx\right)$$

input
```
int(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(1/3),x)
```

output
```
a**(1/3)*int(sec(c + d*x)**(1/3)*(sec(c + d*x) + 1)**(1/3)*sec(c + d*x),x)
```

3.285 $\int \sec^{\frac{4}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 79

$$\int \sec^{\frac{4}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{2\sqrt[6]{2}\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{3},-\frac{1}{6},\frac{3}{2},1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)(a+a\sec(c+dx))^{2/3}\tan(}{d(1+\sec(c+dx))^{7/6}}$$

output
```
2*2^(1/6)*AppellF1(1/2,-1/3,-1/6,3/2,1-sec(d*x+c),1/2-1/2*sec(d*x+c))*(a+a
*sec(d*x+c))^(2/3)*tan(d*x+c)/d/(1+sec(d*x+c))^(7/6)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 22.38 (sec) , antiderivative size = 1566, normalized size of antiderivative = 19.82

$$\int \sec^{\frac{4}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[c + d*x]^(4/3)*(a + a*Sec[c + d*x])^(2/3),x]
```

output
```
((a*(1 + Sec[c + d*x]))^(2/3)*(3*Sec[c + d*x]^(1/3)*Tan[(c + d*x)/2] + (2^
(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/
2])/(1 + Sec[c + d*x])^(2/3) - ((3 - 3*I)*Sec[(c + d*x)/2]^2*Sec[c + d*x]^
(1/3)*(AppellF1[-2/3, -1/3, -1/3, 1/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (
-1 + I)/(-1 + Tan[(c + d*x)/2])]/(((-I + Tan[(c + d*x)/2])/(-1 + Tan[(c +
d*x)/2]))^(1/3)*((I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(1/3)) -
AppellF1[-2/3, -1/3, -1/3, 1/3, (1 - I)/(1 + Tan[(c + d*x)/2]), (1 + I)/(1
 + Tan[(c + d*x)/2])]/(((-I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1
/3)*((I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3))))/((2*((-I)*App
ellF1[1/3, -1/3, 2/3, 4/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/(-1
+ Tan[(c + d*x)/2])] + AppellF1[1/3, 2/3, -1/3, 4/3, (-1 - I)/(-1 + Tan[(c
 + d*x)/2]), (-1 + I)/(-1 + Tan[(c + d*x)/2])]))/((-1 + Sin[c + d*x])*((-I
 + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(1/3)*((I + Tan[(c + d*x)/2]
)/(-1 + Tan[(c + d*x)/2]))^(1/3)) + (I*AppellF1[-2/3, -1/3, -1/3, 1/3, (-1
 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c + d*x)/2])]*Sec[(c +
d*x)/2]^2*((-I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(2/3))/((-I +
Tan[(c + d*x)/2])^2*((I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(1/3)
) - (AppellF1[-2/3, -1/3, -1/3, 1/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1
 + I)/(-1 + Tan[(c + d*x)/2])]*Sec[(c + d*x)/2]^2*((I + Tan[(c + d*x)/2])/
(-1 + Tan[(c + d*x)/2]))^(2/3))/(((-I + Tan[(c + d*x)/2])/(-1 + Tan[(c ...
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4315, 3042, 4312, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{4}{3}}(c+dx)(a\sec(c+dx)+a)^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{2/3}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{(a\sec(c+dx)+a)^{2/3}\int \sec^{\frac{4}{3}}(c+dx)(\sec(c+dx)+1)^{2/3}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a\sec(c+dx)+a)^{2/3}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{2/3}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \text{4312}$$

$$\frac{\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int \frac{\sqrt[3]{\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}}{\sqrt{1-\sec(c+dx)}}d(1-\sec(c+dx))}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \text{150}$$

$$\frac{2\sqrt[6]{2}\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},-\frac{1}{3},-\frac{1}{6},\frac{3}{2},1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{7/6}}$$

input
```
Int[Sec[c + d*x]^(4/3)*(a + a*Sec[c + d*x])^(2/3),x]
```

output
```
(2*2^(1/6)*AppellF1[1/2, -1/3, -1/6, 3/2, 1 - Sec[c + d*x], (1 - Sec[c + d
*x])/2]*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(7/
6))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4312
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a - x)^(n - 1)
*((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \sec (dx+c)^{\frac{4}{3}} (a+a\sec (dx+c))^{\frac{2}{3}} dx$$

input
```
int(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(2/3),x)
```

output
```
int(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(2/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \sec^{\frac{4}{3}}(c+dx)(a+a\sec(c+dx))^{2/3} dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{4}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**(4/3)*(a+a*sec(d*x+c))**(2/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^{\frac{4}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx=\int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^{\frac{4}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^(4/3), x)
```

Giac [F]

$$\int \sec^{\frac{4}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx=\int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^{\frac{4}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{4}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx=\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{2/3}\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{4/3}dx$$

input
```
int((a + a/cos(c + d*x))^(2/3)*(1/cos(c + d*x))^(4/3),x)
```

output
```
int((a + a/cos(c + d*x))^(2/3)*(1/cos(c + d*x))^(4/3), x)
```

Reduce [F]

$$\int \sec^{\frac{4}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx=a^{\frac{2}{3}}\left(\int \sec\left(dx+c\right)^{\frac{4}{3}}\left(\sec\left(dx+c\right)+1\right)^{\frac{2}{3}}dx\right)$$

input
```
int(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(2/3),x)
```

output
```
a**(2/3)*int(sec(c + d*x)**(1/3)*(sec(c + d*x) + 1)**(2/3)*sec(c + d*x),x)
```

3.286 $\int \sec^{\frac{5}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 327

$$\int \sec^{\frac{5}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = -\frac{3a\sec^{\frac{5}{3}}(c+dx)\sin(c+dx)}{2d\sqrt[3]{a(1+\sec(c+dx))}}$$
$$+\frac{9\sec^{\frac{2}{3}}(c+dx)(a(1+\sec(c+dx)))^{2/3}\sin(c+dx)}{4d}$$
$$-\frac{9(a(1+\sec(c+dx)))^{2/3}\tan(c+dx)}{4d\sqrt[3]{\frac{1}{1+\cos(c+dx)}}(1+\sec(c+dx))^{7/3}}$$
$$+\frac{\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{5}{4},\tan^4\left(\frac{1}{2}(c+dx)\right)\right)\sqrt[3]{\cos(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)}(a(1+\sec(c+dx)))^{2/3}\text{ta}}{8d\sqrt[3]{\frac{1}{1+\cos(c+dx)}}(1+\sec(c+dx))^{4/3}}$$
$$-\frac{5\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{7}{4},\tan^4\left(\frac{1}{2}(c+dx)\right)\right)\sqrt[3]{\cos(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)}(a(1+\sec(c+dx)))^{2/3}}{8d\sqrt[3]{\frac{1}{1+\cos(c+dx)}}(1+\sec(c+dx))^{10/3}}$$

output
```
-3/2*a*sec(d*x+c)^(5/3)*sin(d*x+c)/d/(a*(1+sec(d*x+c)))^(1/3)+9/4*sec(d*x+
c)^(2/3)*(a*(1+sec(d*x+c)))^(2/3)*sin(d*x+c)/d-9/4*(a*(1+sec(d*x+c)))^(2/3
)*tan(d*x+c)/d/(1/(1+cos(d*x+c)))^(1/3)/(1+sec(d*x+c))^(7/3)+1/8*hypergeom
([1/4, 1/3],[5/4],tan(1/2*d*x+1/2*c)^4)*(cos(d*x+c)*sec(1/2*d*x+1/2*c)^4)^
(1/3)*(a*(1+sec(d*x+c)))^(2/3)*tan(d*x+c)/d/(1/(1+cos(d*x+c)))^(1/3)/(1+se
c(d*x+c))^(4/3)-5/8*hypergeom([1/3, 3/4],[7/4],tan(1/2*d*x+1/2*c)^4)*(cos(
d*x+c)*sec(1/2*d*x+1/2*c)^4)^(1/3)*(a*(1+sec(d*x+c)))^(2/3)*tan(d*x+c)^3/d
/(1/(1+cos(d*x+c)))^(1/3)/(1+sec(d*x+c))^(10/3)
```

Mathematica [A] (warning: unable to verify)

Time = 6.85 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.84

$$\int \sec^{\frac{5}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{(a(1+\sec(c+dx)))^{2/3}\left(-3\sec^3\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt[3]{1+\sec(c+dx)}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(5/3)*(a + a*Sec[c + d*x])^(2/3),x]
```

output
```
((a*(1 + Sec[c + d*x]))^(2/3)*(-3*Sec[(c + d*x)/2]^3*Sec[c + d*x]*(1 + Sec
[c + d*x])^(1/3)*(Sin[(c + d*x)/2] - 2*Sin[(3*(c + d*x))/2]) + 2^(1/3)*Hyp
ergeometric2F1[1/4, 1/3, 5/4, Tan[(c + d*x)/2]^4]*(Cos[c + d*x]*Sec[(c + d
*x)/2]^4)^(1/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/3)*Tan[(c + d*x)/2] -
 5*2^(1/3)*Hypergeometric2F1[1/3, 3/4, 7/4, Tan[(c + d*x)/2]^4]*(Cos[c + d
*x]*Sec[(c + d*x)/2]^4)^(1/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/3)*Tan[
(c + d*x)/2]^3))/(8*d*((1 + Cos[c + d*x])^(-1))^(1/3)*(1 + Sec[c + d*x])^(
2/3))
```

Rubi [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.40 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.24, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4315, 3042, 4312, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{3}}(c+dx)(a\sec(c+dx)+a)^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/3}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{2/3}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{(a\sec(c+dx)+a)^{2/3}\int \sec^{\frac{5}{3}}(c+dx)(\sec(c+dx)+1)^{2/3}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a\sec(c+dx)+a)^{2/3}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/3}\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{2/3}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \text{4312}$$

$$\frac{\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int \frac{\sec^{\frac{2}{3}}(c+dx)\sqrt[6]{\sec(c+dx)+1}}{\sqrt{1-\sec(c+dx)}}d(1-\sec(c+dx))}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \text{150}$$

$$\frac{2\sqrt[6]{2}\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},-\frac{2}{3},-\frac{1}{6},\frac{3}{2},1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{7/6}}$$

input
```
Int[Sec[c + d*x]^(5/3)*(a + a*Sec[c + d*x])^(2/3),x]
```

output
```
(2*2^(1/6)*AppellF1[1/2, -2/3, -1/6, 3/2, 1 - Sec[c + d*x], (1 - Sec[c + d
*x])/2]*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(7/
6))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4312
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a - x)^(n - 1)
*((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \sec (dx + c)^{\frac{5}{3}} (a + a \sec (dx + c))^{\frac{2}{3}} dx$$

input
```
int(sec(d*x+c)^(5/3)*(a+a*sec(d*x+c))^(2/3),x)
```

output
```
int(sec(d*x+c)^(5/3)*(a+a*sec(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int \sec^{\frac{5}{3}}(c + dx)(a + a\sec(c + dx))^{2/3} dx = \int (a \sec (dx + c) + a)^{\frac{2}{3}} \sec (dx + c)^{\frac{5}{3}} dx$$

input
```
integrate(sec(d*x+c)^(5/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^(5/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{3}}(c + dx)(a + a\sec(c + dx))^{2/3} dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/3)*(a+a*sec(d*x+c))**(2/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^{\frac{5}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^{\frac{5}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^(5/3), x)
```

Giac [F]

$$\int \sec^{\frac{5}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^{\frac{5}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)^{2/3}\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}dx$$

input
```
int((a + a/cos(c + d*x))^(2/3)*(1/cos(c + d*x))^(5/3),x)
```

output
```
int((a + a/cos(c + d*x))^(2/3)*(1/cos(c + d*x))^(5/3), x)
```

Reduce [F]

$$\int \sec^{\frac{5}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = a^{\frac{2}{3}}\left(\int \sec\left(dx+c\right)^{\frac{5}{3}}\left(\sec\left(dx+c\right)+1\right)^{\frac{2}{3}}dx\right)$$

input
```
int(sec(d*x+c)^(5/3)*(a+a*sec(d*x+c))^(2/3),x)
```

output
```
a**(2/3)*int(sec(c + d*x)**(2/3)*(sec(c + d*x) + 1)**(2/3)*sec(c + d*x),x)
```

3.287 $\int \frac{(a+a\sec(c+dx))^{4/3}}{\sqrt[3]{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 80

$$\int \frac{(a+a\sec(c+dx))^{4/3}}{\sqrt[3]{\sec(c+dx)}}\,dx = \frac{2\,2^{5/6} a \operatorname{AppellF1}\left(\frac{1}{2},\frac{4}{3},-\frac{5}{6},\frac{3}{2},1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)\sqrt[3]{a+a\,\mathrm{s}}}{d(1+\sec(c+dx))^{5/6}}$$

output
```
2*2^(5/6)*a*AppellF1(1/2,4/3,-5/6,3/2,1-sec(d*x+c),1/2-1/2*sec(d*x+c))*(a+
a*sec(d*x+c))^(1/3)*tan(d*x+c)/d/(1+sec(d*x+c))^(5/6)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 34.83 (sec) , antiderivative size = 2325, normalized size of antiderivative = 29.06

$$\int \frac{(a+a\sec(c+dx))^{4/3}}{\sqrt[3]{\sec(c+dx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(4/3)/Sec[c + d*x]^(1/3),x]
```

output
```
(-3*(a*(1 + Sec[c + d*x]))^(4/3)*((1 + Sec[c + d*x])^(1/3)/Sec[c + d*x]^(1
/3) + Sec[c + d*x]^(2/3)*(1 + Sec[c + d*x])^(1/3))*(-8*Tan[(c + d*x)/2] +
(AppellF1[-4/3, -2/3, -2/3, -1/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 +
I)/(-1 + Tan[(c + d*x)/2])]*Sec[(c + d*x)/2]^2)/(((-I + Tan[(c + d*x)/2])/
(-1 + Tan[(c + d*x)/2]))^(2/3)*((I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)
/2]))^(2/3)) - AppellF1[-4/3, -2/3, -2/3, -1/3, (1 - I)/(1 + Tan[(c + d*x)
/2]), (1 + I)/(1 + Tan[(c + d*x)/2])]*((-I + Tan[(c + d*x)/2])/(1 + Tan[(c
 + d*x)/2]))^(1/3)*((I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*(
1 + Tan[(c + d*x)/2])^2))/(4*2^(2/3)*d*(Sec[(c + d*x)/2]^2)^(1/3)*(1 + Sec
[c + d*x])^(4/3)*((Tan[(c + d*x)/2]*(-8*Tan[(c + d*x)/2] + (AppellF1[-4/3,
 -2/3, -2/3, -1/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c
 + d*x)/2])]*Sec[(c + d*x)/2]^2)/(((-I + Tan[(c + d*x)/2])/(-1 + Tan[(c +
d*x)/2]))^(2/3)*((I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(2/3)) -
AppellF1[-4/3, -2/3, -2/3, -1/3, (1 - I)/(1 + Tan[(c + d*x)/2]), (1 + I)/(
1 + Tan[(c + d*x)/2])]*((-I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1
/3)*((I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*(1 + Tan[(c + d*
x)/2])^2))/(4*2^(2/3)*(Sec[(c + d*x)/2]^2)^(1/3)) - (3*(-4*Sec[(c + d*x)/2
]^2 + (Sec[(c + d*x)/2]^2*(((-4/3 + (4*I)/3)*AppellF1[-1/3, -2/3, 1/3, 2/3
, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c + d*x)/2])]*Sec[
(c + d*x)/2]^2)/(-1 + Tan[(c + d*x)/2])^2 - ((4/3 + (4*I)/3)*AppellF1[-...
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4315, 3042, 4312, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{4/3}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{4/3}}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{a\sqrt[3]{a\sec(c+dx)+a}\int \frac{(\sec(c+dx)+1)^{4/3}}{\sqrt[3]{\sec(c+dx)}}dx}{\sqrt[3]{\sec(c+dx)+1}}$$

$\Big\downarrow$ 3042

$$\frac{a\sqrt[3]{a\sec(c+dx)+a}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{4/3}}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt[3]{\sec(c+dx)+1}}$$

$\Big\downarrow$ 4312

$$\frac{a\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\int \frac{(\sec(c+dx)+1)^{5/6}}{\sqrt{1-\sec(c+dx)}\sec^{\frac{4}{3}}(c+dx)}d(1-\sec(c+dx))}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}$$

$\Big\downarrow$ 150

$$\frac{2\ 2^{5/6}a\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\operatorname{AppellF1}\left(\frac{1}{2},\frac{4}{3},-\frac{5}{6},\frac{3}{2},1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{5/6}}$$

input
```
Int[(a + a*Sec[c + d*x])^(4/3)/Sec[c + d*x]^(1/3),x]
```

output
```
(2*2^(5/6)*a*AppellF1[1/2, 4/3, -5/6, 3/2, 1 - Sec[c + d*x], (1 - Sec[c +
d*x])/2]*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(5
/6))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4312
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a - x)^(n - 1)
*((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{(a+a\sec(dx+c))^{\frac{4}{3}}}{\sec(dx+c)^{\frac{1}{3}}}dx$$

input
```
int((a+a*sec(d*x+c))^(4/3)/sec(d*x+c)^(1/3),x)
```

output
```
int((a+a*sec(d*x+c))^(4/3)/sec(d*x+c)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{4/3}}{\sqrt[3]{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))^(4/3)/sec(d*x+c)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{4/3}}{\sqrt[3]{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(4/3)/sec(d*x+c)**(1/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^{4/3}}{\sqrt[3]{\sec(c+dx)}}\,dx = \int \frac{(a\sec(dx+c)+a)^{\frac{4}{3}}}{\sec(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(4/3)/sec(d*x+c)^(1/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(4/3)/sec(d*x + c)^(1/3), x)
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^{4/3}}{\sqrt[3]{\sec(c+dx)}}\,dx = \int \frac{(a\sec(dx+c)+a)^{\frac{4}{3}}}{\sec(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(4/3)/sec(d*x+c)^(1/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(4/3)/sec(d*x + c)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{4/3}}{\sqrt[3]{\sec(c+dx)}}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{4/3}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(4/3)/(1/cos(c + d*x))^(1/3),x)
```

output
```
int((a + a/cos(c + d*x))^(4/3)/(1/cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^{4/3}}{\sqrt[3]{\sec(c+dx)}}\,dx=a^{\frac{4}{3}}\left(\int \frac{(\sec(dx+c)+1)^{\frac{1}{3}}}{\sec(dx+c)^{\frac{1}{3}}}dx+\int (\sec(dx+c)+1)^{\frac{1}{3}}\sec(dx+c)^{\frac{2}{3}}\,dx\right)$$

input
```
int((a+a*sec(d*x+c))^(4/3)/sec(d*x+c)^(1/3),x)
```

output
```
a**(1/3)*a*(int((sec(c + d*x) + 1)**(1/3)/sec(c + d*x)**(1/3),x) + int(((s
ec(c + d*x) + 1)**(1/3)*sec(c + d*x))/sec(c + d*x)**(1/3),x))
```

3.288 $\int \sec^n(e+fx)(a+a\sec(e+fx))^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 304

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^4\,dx=\frac{a^4(30+21n+4n^2)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)(2+n)(3+n)}+\frac{\sec^{1+n}(e+fx)\left(a^2+a^2\sec(e+fx)\right)^2\sin(e+fx)}{f(3+n)}+\frac{2(4+n)\sec^{1+n}(e+fx)\left(a^4+a^4\sec(e+fx)\right)\sin(e+fx)}{f(2+n)(3+n)}-\frac{a^4(3+24n+8n^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sec^{-1+n}(e+fx)\sin(e+fx)}{f(1-n)(1+n)(3+n)\sqrt{\sin^2(e+fx)}}+\frac{4a^4(3+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)\sec^n(e+fx)\sin(e+fx)}{fn(2+n)\sqrt{\sin^2(e+fx)}}$$

output
```
a^4*(4*n^2+21*n+30)*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(1+n)/(2+n)/(3+n)+sec(f*
x+e)^(1+n)*(a^2+a^2*sec(f*x+e))^2*sin(f*x+e)/f/(3+n)+2*(4+n)*sec(f*x+e)^(1
+n)*(a^4+a^4*sec(f*x+e))*sin(f*x+e)/f/(2+n)/(3+n)-a^4*(8*n^2+24*n+3)*hyper
geom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-1+n)*sin(f*x+
e)/f/(1-n)/(1+n)/(3+n)/(sin(f*x+e)^2)^(1/2)+4*a^4*(3+2*n)*hypergeom([1/2,
-1/2*n],[1-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^n*sin(f*x+e)/f/n/(2+n)/(sin(f*
+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 3.56 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.68

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^4\,dx$$
$$= \frac{a^4\csc(e+fx)\sec^{-1+n}(e+fx)\left(n(40+34n+7n^2+4(3+4n+n^2)\sec(e+fx)+(2+3n+n^2)\sec^2(e\right.}{}$$

input
```
Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^4,x]
```

output
```
(a^4*Csc[e + f*x]*Sec[e + f*x]^(-1 + n)*(n*(40 + 34*n + 7*n^2 + 4*(3 + 4*n
 + n^2)*Sec[e + f*x] + (2 + 3*n + n^2)*Sec[e + f*x]^2)*Tan[e + f*x]^2 + (6
 + 51*n + 40*n^2 + 8*n^3)*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sec[e + f
*x]^2]*Sqrt[-Tan[e + f*x]^2] + 4*n*(9 + 9*n + 2*n^2)*Hypergeometric2F1[1/2
, (1 + n)/2, (3 + n)/2, Sec[e + f*x]^2]*Sec[e + f*x]*Sqrt[-Tan[e + f*x]^2]
))/(f*n*(1 + n)*(2 + n)*(3 + n))
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4301, 3042, 4506, 3042, 4485, 3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^4\sec^n(e+fx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^4\csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \text{ 4301}$$

$$\frac{a\int \sec^n(e+fx)(\sec(e+fx)a+a)^2(a(2n+3)+2a(n+4)\sec(e+fx))dx}{n+3}+\frac{\sin(e+fx)\left(a^2\sec(e+fx)+a^2\right)^2\sec^{n+1}(e+fx)}{f(n+3)}$$

$\big\downarrow$ 3042

$$\frac{a\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2\left(a(2n+3)+2a(n+4)\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx}{n+3}+\frac{\sin(e+fx)\left(a^2\sec(e+fx)+a^2\right)^2\sec^{n+1}(e+fx)}{f(n+3)}$$

$\big\downarrow$ 4506

$$\frac{a\left(\frac{\int \sec^n(e+fx)(\sec(e+fx)a+a)\left(\left(4n^2+15n+6\right)a^2+\left(4n^2+21n+30\right)\sec(e+fx)a^2\right)dx}{n+2}+\frac{2(n+4)\sin(e+fx)\left(a^3\sec(e+fx)+a^3\right)\sec^{n+1}(e+fx)}{f(n+2)}\right.}{n+3}+\frac{\sin(e+fx)\left(a^2\sec(e+fx)+a^2\right)^2\sec^{n+1}(e+fx)}{f(n+3)}$$

$\big\downarrow$ 3042

$$\frac{a\left(\frac{\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)\left(\left(4n^2+15n+6\right)a^2+\left(4n^2+21n+30\right)\csc\left(e+fx+\frac{\pi}{2}\right)a^2\right)dx}{n+2}+\frac{2(n+4)\sin(e+fx)\left(a^3\sec(e+fx)+a^3\right)\mathrm{sec}\right.}{f(n+2)}}{n+3}+\frac{\sin(e+fx)\left(a^2\sec(e+fx)+a^2\right)^2\sec^{n+1}(e+fx)}{f(n+3)}$$

$\big\downarrow$ 4485

$$\frac{a\left(\frac{\frac{\int \sec^n(e+fx)\left((n+2)\left(8n^2+24n+3\right)a^3+4(n+1)(n+3)(2n+3)\sec(e+fx)a^3\right)dx}{n+1}+\frac{a^3\left(4n^2+21n+30\right)\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)}}{n+2}+\frac{2(n+4)\sin(e+fx)\left(a^3\mathrm{se}\right.}{f}\right.}{n+3}+\frac{\sin(e+fx)\left(a^2\sec(e+fx)+a^2\right)^2\sec^{n+1}(e+fx)}{f(n+3)}$$

$\big\downarrow$ 3042

$$\frac{a\left(\frac{\frac{\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\left((n+2)\left(8n^2+24n+3\right)a^3+4(n+1)(n+3)(2n+3)\csc\left(e+fx+\frac{\pi}{2}\right)a^3\right)dx}{n+1}+\frac{a^3\left(4n^2+21n+30\right)\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)}}{n+2}+2(n+4)\sin(e+\right.}{n+3}+\frac{\sin(e+fx)\left(a^2\sec(e+fx)+a^2\right)^2\sec^{n+1}(e+fx)}{f(n+3)}$$

$\downarrow$ 4274

$$a\left(\frac{\frac{\frac{a^3(n+2)\left(8n^2+24n+3\right)\int \sec^n(e+fx)dx+4a^3(n+1)(n+3)(2n+3)\int \sec^{n+1}(e+fx)dx}{n+1}+\frac{a^3\left(4n^2+21n+30\right)\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)}}{n+2}+\frac{2(n+4)\sin(e+fx)}{}}{n+3}\right.$$

$$\frac{\sin(e+fx)\left(a^2\sec(e+fx)+a^2\right)^2\sec^{n+1}(e+fx)}{f(n+3)}$$

$\downarrow$ 3042

$$a\left(\frac{\frac{\frac{a^3(n+2)\left(8n^2+24n+3\right)\int \csc\left(e+fx+\frac{\pi}{2}\right)^n dx+4a^3(n+1)(n+3)(2n+3)\int \csc\left(e+fx+\frac{\pi}{2}\right)^{n+1}dx}{n+1}+\frac{a^3\left(4n^2+21n+30\right)\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)}}{n+2}+\frac{2(n+4)\text{si}}{}}{n+3}\right.$$

$$\frac{\sin(e+fx)\left(a^2\sec(e+fx)+a^2\right)^2\sec^{n+1}(e+fx)}{f(n+3)}$$

$\downarrow$ 4259

$$a\left(\frac{\frac{\frac{a^3(n+2)\left(8n^2+24n+3\right)\cos^n(e+fx)\sec^n(e+fx)\int \cos^{-n}(e+fx)dx+4a^3(n+1)(n+3)(2n+3)\cos^n(e+fx)\sec^n(e+fx)\int \cos^{-n-1}(e+fx)dx}{n+1}+\frac{a^3\left(4n^2+21n+3\right.}{}}{n+2}}{n+3}\right.$$

$$\frac{\sin(e+fx)\left(a^2\sec(e+fx)+a^2\right)^2\sec^{n+1}(e+fx)}{f(n+3)}$$

$\downarrow$ 3042

$$a\left(\frac{\frac{\frac{a^3(n+2)\left(8n^2+24n+3\right)\cos^n(e+fx)\sec^n(e+fx)\int \sin\left(e+fx+\frac{\pi}{2}\right)^{-n}dx+4a^3(n+1)(n+3)(2n+3)\cos^n(e+fx)\sec^n(e+fx)\int \sin\left(e+fx+\frac{\pi}{2}\right)^{-n-1}dx}{n+1}+\frac{a^3\left(4n^2\right.}{}}{n+2}}{n+3}\right.$$

$$\frac{\sin(e+fx)\left(a^2\sec(e+fx)+a^2\right)^2\sec^{n+1}(e+fx)}{f(n+3)}$$

$\downarrow$ 3122

$$a\left(\frac{\frac{\frac{\frac{4a^3(n+1)(n+3)(2n+3)\sin(e+fx)\sec^n(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}-\frac{a^3(n+2)\left(8n^2+24n+3\right)\sin(e+fx)\sec^{n-1}(e+fx)\text{Hypergeom}}{f(1-n)\sqrt{\sin^2(e+fx)}}}{n+1}}{n+2}}{n+3}\right.$$

$$\frac{\sin(e+fx)\left(a^2\sec(e+fx)+a^2\right)^2\sec^{n+1}(e+fx)}{f(n+3)}$$

input
```
Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^4,x]
```

output
```
(Sec[e + f*x]^(1 + n)*(a^2 + a^2*Sec[e + f*x])^2*Sin[e + f*x])/(f*(3 + n))
 + (a*((2*(4 + n)*Sec[e + f*x]^(1 + n)*(a^3 + a^3*Sec[e + f*x])*Sin[e + f*
x])/(f*(2 + n)) + ((a^3*(30 + 21*n + 4*n^2)*Sec[e + f*x]^(1 + n)*Sin[e + f
*x])/(f*(1 + n)) + (-((a^3*(2 + n)*(3 + 24*n + 8*n^2)*Hypergeometric2F1[1/
2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^(-1 + n)*Sin[e + f*x
])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2])) + (4*a^3*(1 + n)*(3 + n)*(3 + 2*n)*Hy
pergeometric2F1[1/2, -1/2*n, (2 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^n*Sin
[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2]))/(1 + n))/(2 + n)))/(3 + n)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

rule 4506
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Simp[1/(d*(m + n))   Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x]
)^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
 a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Maple [F]

$$\int \sec (fx+e)^n (a+a\sec (fx+e))^4\,dx$$

input
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^4,x)
```

output
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^4,x)
```

Fricas [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^4\,dx=\int (a\sec(fx+e)+a)^4\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^4,x, algorithm="fricas")
```

output
```
integral((a^4*sec(f*x + e)^4 + 4*a^4*sec(f*x + e)^3 + 6*a^4*sec(f*x + e)^2
 + 4*a^4*sec(f*x + e) + a^4)*sec(f*x + e)^n, x)
```

Sympy [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^4\,dx=a^4\left(\int 4\sec(e+fx)\sec^n(e+fx)\,dx+\int 6\sec^2(e+fx)\sec^n(e+fx)\,dx+\int 4\sec^3(e+fx)\sec^n(e+fx)\,dx+\int \sec^4(e+fx)\sec^n(e+fx)\,dx+\int \sec^n(e+fx)\,dx\right)$$

input
```
integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**4,x)
```

output
```
a**4*(Integral(4*sec(e + f*x)*sec(e + f*x)**n, x) + Integral(6*sec(e + f*x
)**2*sec(e + f*x)**n, x) + Integral(4*sec(e + f*x)**3*sec(e + f*x)**n, x)
+ Integral(sec(e + f*x)**4*sec(e + f*x)**n, x) + Integral(sec(e + f*x)**n,
 x))
```

Maxima [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^4\,dx=\int (a\sec(fx+e)+a)^4\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^4,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^4*sec(f*x + e)^n, x)
```

Giac [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^4\,dx=\int (a\sec(fx+e)+a)^4\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^4,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^4*sec(f*x + e)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^4\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^4\left(\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a + a/cos(e + f*x))^4*(1/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^4*(1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^4\,dx = a^4\left(\int \sec(fx+e)^n\,dx + \int \sec(fx+e)^n\sec(fx+e)^4\,dx + 4\left(\int \sec(fx+e)^n\sec(fx+e)^3\,dx\right) + 6\left(\int \sec(fx+e)^n\sec(fx+e)^2\,dx\right) + 4\left(\int \sec(fx+e)^n\sec(fx+e)\,dx\right)\right)$$

input
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^4,x)
```

output
```
a**4*(int(sec(e + f*x)**n,x) + int(sec(e + f*x)**n*sec(e + f*x)**4,x) + 4*
int(sec(e + f*x)**n*sec(e + f*x)**3,x) + 6*int(sec(e + f*x)**n*sec(e + f*x
)**2,x) + 4*int(sec(e + f*x)**n*sec(e + f*x),x))
```

3.289 $\int \sec^n(e+fx)(a+a\sec(e+fx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 230

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^3\,dx = \frac{a^3(5+2n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)(2+n)} + \frac{\sec^{1+n}(e+fx)\left(a^3+a^3\sec(e+fx)\right)\sin(e+fx)}{f(2+n)} - \frac{a^3(1+4n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sec^{-1+n}(e+fx)\sin(e+fx)}{f\left(1-n^2\right)\sqrt{\sin^2(e+fx)}} + \frac{a^3(7+4n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)\sec^n(e+fx)\sin(e+fx)}{fn(2+n)\sqrt{\sin^2(e+fx)}}$$

output
```
a^3*(5+2*n)*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(1+n)/(2+n)+sec(f*x+e)^(1+n)*(a^
3+a^3*sec(f*x+e))*sin(f*x+e)/f/(2+n)-a^3*(1+4*n)*hypergeom([1/2, 1/2-1/2*n
],[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-1+n)*sin(f*x+e)/f/(-n^2+1)/(sin(f
*x+e)^2)^(1/2)+a^3*(7+4*n)*hypergeom([1/2, -1/2*n],[1-1/2*n],cos(f*x+e)^2)
*sec(f*x+e)^n*sin(f*x+e)/f/n/(2+n)/(sin(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.72

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^3\,dx$$
$$= \frac{a^3\csc(e+fx)\sec^{-1+n}(e+fx)\left(n(3(2+n)+(1+n)\sec(e+fx))\tan^2(e+fx)+(2+9n+4n^2)\,\text{Hype}\right.}{}$$

input
```
Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^3,x]
```

output
```
(a^3*Csc[e + f*x]*Sec[e + f*x]^(-1 + n)*(n*(3*(2 + n) + (1 + n)*Sec[e + f*
x])*Tan[e + f*x]^2 + (2 + 9*n + 4*n^2)*Hypergeometric2F1[1/2, n/2, (2 + n)
/2, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2] + n*(7 + 4*n)*Hypergeometric2F1[
1/2, (1 + n)/2, (3 + n)/2, Sec[e + f*x]^2]*Sec[e + f*x]*Sqrt[-Tan[e + f*x]
^2]))/(f*n*(1 + n)*(2 + n))
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4301, 3042, 4485, 3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^3\sec^n(e+fx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \; 4301$$

$$\frac{a\int \sec^n(e+fx)(\sec(e+fx)a+a)(2a(n+1)+a(2n+5)\sec(e+fx))dx}{n+2}+\frac{\sin(e+fx)\left(a^3\sec(e+fx)+a^3\right)\sec^{n+1}(e+fx)}{f(n+2)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)\left(2a(n+1)+a(2n+5)\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx}{n+2}+\frac{\sin(e+fx)\left(a^3\sec(e+fx)+a^3\right)\sec^{n+1}(e+fx)}{f(n+2)}$$

$$\Big\downarrow \; 4485$$

$$\frac{a\left(\frac{\int \sec^n(e+fx)\left((n+2)(4n+1)a^2+(n+1)(4n+7)\sec(e+fx)a^2\right)dx}{n+1}+\frac{a^2(2n+5)\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)}\right)}{n+2}+\frac{\sin(e+fx)\left(a^3\sec(e+fx)+a^3\right)\sec^{n+1}(e+fx)}{f(n+2)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\left(\frac{\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\left((n+2)(4n+1)a^2+(n+1)(4n+7)\csc\left(e+fx+\frac{\pi}{2}\right)a^2\right)dx}{n+1}+\frac{a^2(2n+5)\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)}\right)}{n+2}+\frac{\sin(e+fx)\left(a^3\sec(e+fx)+a^3\right)\sec^{n+1}(e+fx)}{f(n+2)}$$

$$\Big\downarrow \; 4274$$

$$\frac{a\left(\frac{a^2(n+1)(4n+7)\int \sec^{n+1}(e+fx)dx+a^2(n+2)(4n+1)\int \sec^n(e+fx)dx}{n+1}+\frac{a^2(2n+5)\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)}\right)}{n+2}+\frac{\sin(e+fx)\left(a^3\sec(e+fx)+a^3\right)\sec^{n+1}(e+fx)}{f(n+2)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\left(\frac{a^2(n+2)(4n+1)\int \csc\left(e+fx+\frac{\pi}{2}\right)^n dx+a^2(n+1)(4n+7)\int \csc\left(e+fx+\frac{\pi}{2}\right)^{n+1}dx}{n+1}+\frac{a^2(2n+5)\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)}\right)}{n+2}+\frac{\sin(e+fx)\left(a^3\sec(e+fx)+a^3\right)\sec^{n+1}(e+fx)}{f(n+2)}$$

$$\Big\downarrow \; 4259$$

$$\frac{a\left(\frac{a^2(n+1)(4n+7)\cos^n(e+fx)\sec^n(e+fx)\int \cos^{-n-1}(e+fx)dx+a^2(n+2)(4n+1)\cos^n(e+fx)\sec^n(e+fx)\int \cos^{-n}(e+fx)dx}{n+1}+\frac{a^2(2n+5)\sin\cdots}{\cdots}\right)}{n+2}+\frac{\sin(e+fx)\left(a^3\sec(e+fx)+a^3\right)\sec^{n+1}(e+fx)}{f(n+2)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\left(\frac{a^2(n+1)(4n+7)\cos^n(e+fx)\sec^n(e+fx)\int\sin(e+fx+\frac{\pi}{2})^{-n-1}dx+a^2(n+2)(4n+1)\cos^n(e+fx)\sec^n(e+fx)\int\sin(e+fx+\frac{\pi}{2})^{-n}dx}{n+1}+\frac{a^2(2\ldots}{\ldots}\right.}{n+2}$$

$$\frac{\sin(e+fx)\left(a^3\sec(e+fx)+a^3\right)\sec^{n+1}(e+fx)}{f(n+2)}$$

$$\Big\downarrow \ 3122$$

$$\frac{\sin(e+fx)\left(a^3\sec(e+fx)+a^3\right)\sec^{n+1}(e+fx)}{f(n+2)}+$$

$$\frac{a\left(\frac{\frac{a^2(n+1)(4n+7)\sin(e+fx)\sec^n(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}-\frac{a^2(n+2)(4n+1)\sin(e+fx)\sec^{n-1}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-\ldots}{\ldots}\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}}{n+1}\right.}{n+2}$$

input
```
Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^3,x]
```

output
```
(Sec[e + f*x]^(1 + n)*(a^3 + a^3*Sec[e + f*x])*Sin[e + f*x])/(f*(2 + n)) +
 (a*((a^2*(5 + 2*n)*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + n)) + (-((a
^2*(2 + n)*(1 + 4*n)*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e +
f*x]^2]*Sec[e + f*x]^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2
])) + (a^2*(1 + n)*(7 + 4*n)*Hypergeometric2F1[1/2, -1/2*n, (2 - n)/2, Cos
[e + f*x]^2]*Sec[e + f*x]^n*Sin[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2]))/(1 +
 n)))/(2 + n)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

Maple [F]

$$\int \sec(fx+e)^n (a+a\sec(fx+e))^3 \, dx$$

input
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^3,x)
```

output
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^3,x)
```

Fricas [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^3\,dx=\int (a\sec(fx+e)+a)^3\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
integral((a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) +
 a^3)*sec(f*x + e)^n, x)
```

Sympy [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^3\,dx=a^3\left(\int 3\sec(e+fx)\sec^n(e+fx)\,dx+\int 3\sec^2(e+fx)\sec^n(e+fx)\,dx+\int \sec^3(e+fx)\sec^n(e+fx)\,dx+\int \sec^n(e+fx)\,dx\right)$$

input
```
integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**3,x)
```

output
```
a**3*(Integral(3*sec(e + f*x)*sec(e + f*x)**n, x) + Integral(3*sec(e + f*x
)**2*sec(e + f*x)**n, x) + Integral(sec(e + f*x)**3*sec(e + f*x)**n, x) +
Integral(sec(e + f*x)**n, x))
```

Maxima [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^3\,dx=\int (a\sec(fx+e)+a)^3\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^3*sec(f*x + e)^n, x)
```

Giac [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^3\,dx=\int (a\sec(fx+e)+a)^3\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^3*sec(f*x + e)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^3\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^3\left(\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a + a/cos(e + f*x))^3*(1/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^3*(1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^3\,dx = a^3\left(\int \sec(fx+e)^n\,dx + \int \sec(fx+e)^n\sec(fx+e)^3\,dx + 3\left(\int \sec(fx+e)^n\sec(fx+e)^2\,dx\right) + 3\left(\int \sec(fx+e)^n\sec(fx+e)\,dx\right)\right)$$

input
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^3,x)
```

output
```
a**3*(int(sec(e + f*x)**n,x) + int(sec(e + f*x)**n*sec(e + f*x)**3,x) + 3*
int(sec(e + f*x)**n*sec(e + f*x)**2,x) + 3*int(sec(e + f*x)**n*sec(e + f*x
),x))
```

3.290 $\int \sec^n(e+fx)(a+a\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 172

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^2\,dx = \frac{a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)} - \frac{a^2(1+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sec^{-1+n}(e+fx)\sin(e+fx)}{f(1-n^2)\sqrt{\sin^2(e+fx)}} + \frac{2a^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)\sec^n(e+fx)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}}$$

output
```
a^2*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(1+n)-a^2*(1+2*n)*hypergeom([1/2, 1/2-1/
2*n],[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-1+n)*sin(f*x+e)/f/(-n^2+1)/(si
n(f*x+e)^2)^(1/2)+2*a^2*hypergeom([1/2, -1/2*n],[1-1/2*n],cos(f*x+e)^2)*se
c(f*x+e)^n*sin(f*x+e)/f/n/(sin(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.78

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^2\,dx$$
$$=\frac{a^2\csc(e+fx)\sec^{-1+n}(e+fx)\left((1+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\sec^2(e+fx)\right)\sqrt{-\tan^2(e+fx}\right.}{fn(1}$$

input
```
Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^2,x]
```

output
```
(a^2*Csc[e + f*x]*Sec[e + f*x]^(-1 + n)*((1 + 2*n)*Hypergeometric2F1[1/2,
n/2, (2 + n)/2, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2] + n*(Tan[e + f*x]^2
+ 2*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sec[e + f*x]^2]*Sec[e + f
*x]*Sqrt[-Tan[e + f*x]^2])))/(f*n*(1 + n))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {3042, 4275, 3042, 4259, 3042, 3122, 4534, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^2\sec^n(e+fx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \; 4275$$

$$2a^2\int \sec^{n+1}(e+fx)dx+\int \sec^n(e+fx)\left(\sec^2(e+fx)a^2+a^2\right)dx$$

$$\Big\downarrow \; 3042$$

$$2a^2\int \csc\left(e+fx+\frac{\pi}{2}\right)^{n+1}dx+\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx$$

$$\Big\downarrow \; 4259$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\cos^n(e+fx)\sec^n(e+fx)\int \cos^{-n-1}(e+fx)dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\cos^n(e+fx)\sec^n(e+fx)\int \sin\left(e+fx+\frac{\pi}{2}\right)^{-n-1}dx$$

$$\Big\downarrow \; 3122$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+\frac{2a^2\sin(e+fx)\sec^n(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}$$

$$\Big\downarrow \; 4534$$

$$\frac{a^2(2n+1)\int \sec^n(e+fx)dx}{n+1}+\frac{2a^2\sin(e+fx)\sec^n(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}+\frac{a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a^2(2n+1)\int \csc\left(e+fx+\frac{\pi}{2}\right)^n dx}{n+1}+\frac{2a^2\sin(e+fx)\sec^n(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}+\frac{a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)}$$

$$\Big\downarrow \; 4259$$

$$\frac{a^2(2n+1)\cos^n(e+fx)\sec^n(e+fx)\int \cos^{-n}(e+fx)dx}{n+1}+\frac{2a^2\sin(e+fx)\sec^n(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}+\frac{a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2(2n+1)\cos^n(e+fx)\sec^n(e+fx)\int \sin\left(e+fx+\frac{\pi}{2}\right)^{-n}dx}{n+1}+\frac{2a^2\sin(e+fx)\sec^n(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}+\frac{a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{a^2(2n+1)\sin(e+fx)\sec^{n-1}(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)(n+1)\sqrt{\sin^2(e+fx)}}+\frac{2a^2\sin(e+fx)\sec^n(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}+\frac{a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)}$$

input
```
Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^2,x]
```

output
```
(a^2*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + n)) - (a^2*(1 + 2*n)*Hyper
geometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^(-1 +
 n)*Sin[e + f*x])/(f*(1 - n)*(1 + n)*Sqrt[Sin[e + f*x]^2]) + (2*a^2*Hyperg
eometric2F1[1/2, -1/2*n, (2 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^n*Sin[e +
 f*x])/(f*n*Sqrt[Sin[e + f*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Maple [F]

$$\int \sec(fx+e)^n (a+a\sec(fx+e))^2\,dx$$

input
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^2,x)
```

output
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^2,x)
```

Fricas [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^2\,dx=\int (a\sec(fx+e)+a)^2\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sec(f*x + e)^n, x
)
```

Sympy [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^2\,dx=a^2\left(\int 2\sec(e+fx)\sec^n(e+fx)\,dx+\int \sec^2(e+fx)\sec^n(e+fx)\,dx+\int \sec^n(e+fx)\,dx\right)$$

input
```
integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**2,x)
```

output
```
a**2*(Integral(2*sec(e + f*x)*sec(e + f*x)**n, x) + Integral(sec(e + f*x)*
*2*sec(e + f*x)**n, x) + Integral(sec(e + f*x)**n, x))
```

Maxima [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^2\,dx=\int (a\sec(fx+e)+a)^2\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^2*sec(f*x + e)^n, x)
```

Giac [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^2\,dx = \int (a\sec(fx+e)+a)^2\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^2*sec(f*x + e)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^2\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)^2\left(\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a + a/cos(e + f*x))^2*(1/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^2*(1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^2\,dx = a^2\left(\int \sec(fx+e)^n\,dx + \int \sec(fx+e)^n\sec(fx+e)^2\,dx + 2\left(\int \sec(fx+e)^n\sec(fx+e)\,dx\right)\right)$$

input
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^2,x)
```

output
```
a**2*(int(sec(e + f*x)**n,x) + int(sec(e + f*x)**n*sec(e + f*x)**2,x) + 2*
int(sec(e + f*x)**n*sec(e + f*x),x))
```

3.291 $\int \sec^n(e+fx)(a+a\sec(e+fx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 132

$$\int \sec^n(e+fx)(a+a\sec(e+fx))\,dx = -\frac{a\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sec^{-1+n}(e+fx)\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}} + \frac{a\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)\sec^n(e+fx)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}}$$

output
```
-a*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-1+n)*
sin(f*x+e)/f/(1-n)/(sin(f*x+e)^2)^(1/2)+a*hypergeom([1/2, -1/2*n],[1-1/2*n
],cos(f*x+e)^2)*sec(f*x+e)^n*sin(f*x+e)/f/n/(sin(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.80

$$\int \sec^n(e+fx)(a+a\sec(e+fx))\,dx = \frac{a\csc(e+fx)\sec^{-1+n}(e+fx)\left((1+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\sec^2(e+fx)\right)+n\,\text{Hypergeometr}\right.}{fn(1+n)}$$

input
```
Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x]),x]
```

output
```
(a*Csc[e + f*x]*Sec[e + f*x]^(-1 + n)*((1 + n)*Hypergeometric2F1[1/2, n/2,
 (2 + n)/2, Sec[e + f*x]^2] + n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/
2, Sec[e + f*x]^2]*Sec[e + f*x])*Sqrt[-Tan[e + f*x]^2])/(f*n*(1 + n))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)\sec^n(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \sec^{n+1}(e+fx)dx+a\int \sec^n(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(e+fx+\frac{\pi}{2}\right)^n dx+a\int \csc\left(e+fx+\frac{\pi}{2}\right)^{n+1} dx$$

$$\Big\downarrow \text{4259}$$

$$a\cos^n(e+fx)\sec^n(e+fx)\int \cos^{-n-1}(e+fx)dx+a\cos^n(e+fx)\sec^n(e+fx)\int \cos^{-n}(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\cos^n(e+fx)\sec^n(e+fx)\int \sin\left(e+fx+\frac{\pi}{2}\right)^{-n-1} dx+a\cos^n(e+fx)\sec^n(e+fx)\int \sin\left(e+fx+\frac{\pi}{2}\right)^{-n} dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{a\sin(e+fx)\sec^n(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}-\frac{a\sin(e+fx)\sec^{n-1}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

input
```
Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x]),x]
```

output
```
-((a*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e +
f*x]^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2])) + (a*Hyperge
ometric2F1[1/2, -1/2*n, (2 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^n*Sin[e +
f*x])/(f*n*Sqrt[Sin[e + f*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [F]

$$\int \sec (fx+e)^n (a+a\sec (fx+e))\,dx$$

input
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e)),x)
```

output
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e)),x)
```

Fricas [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))\,dx = \int (a\sec (fx+e)+a)\sec (fx+e)^n \,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

Sympy [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))\,dx = a\left(\int \sec (e+fx)\sec^n (e+fx)\,dx + \int \sec^n (e+fx)\,dx\right)$$

input
```
integrate(sec(f*x+e)**n*(a+a*sec(f*x+e)),x)
```

output
```
a*(Integral(sec(e + f*x)*sec(e + f*x)**n, x) + Integral(sec(e + f*x)**n, x
))
```

Maxima [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))\,dx = \int (a\sec(fx+e)+a)\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

Giac [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))\,dx = \int (a\sec(fx+e)+a)\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(a+a\sec(e+fx))\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)\left(\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a + a/cos(e + f*x))*(1/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))*(1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))\,dx = a\left(\int \sec(fx+e)^n\,dx + \int \sec(fx+e)^n \sec(fx+e)\,dx\right)$$

input
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e)),x)
```

output
```
a*(int(sec(e + f*x)**n,x) + int(sec(e + f*x)**n*sec(e + f*x),x))
```

3.292 $\int \frac{\sec^n(e+fx)}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 174

$$\int \frac{\sec^n(e+fx)}{a+a\sec(e+fx)}\,dx = \frac{\sec^n(e+fx)\sin(e+fx)}{f(a+a\sec(e+fx))} + \frac{(1-n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-n}{2},\frac{4-n}{2},\cos^2(e+fx)\right)\sec^{-2+n}(e+fx)\sin(e+fx)}{af(2-n)\sqrt{\sin^2(e+fx)}} - \frac{\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sec^{-1+n}(e+fx)\sin(e+fx)}{af\sqrt{\sin^2(e+fx)}}$$

output
```
sec(f*x+e)^n*sin(f*x+e)/f/(a+a*sec(f*x+e))+(1-n)*hypergeom([1/2, 1-1/2*n],
[2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-2+n)*sin(f*x+e)/a/f/(2-n)/(sin(f*x+e)
^2)^(1/2)-hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^
(-1+n)*sin(f*x+e)/a/f/(sin(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.80

$$\int \frac{\sec^n(e+fx)}{a+a\sec(e+fx)}\,dx = \frac{\cot\left(\frac{1}{2}(e+fx)\right)\sec^n(e+fx)\left(n-n\cos(e+fx)+n\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+}{2}\right.\right.}{afn(1}$$

input
```
Integrate[Sec[e + f*x]^n/(a + a*Sec[e + f*x]),x]
```

output
```
(Cot[(e + f*x)/2]*Sec[e + f*x]^n*(n - n*Cos[e + f*x] + n*Cos[e + f*x]*Hype
rgeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*
x]^2] - (-1 + n)*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sec[e + f*x]^2]*Sq
rt[-Tan[e + f*x]^2]))/(a*f*n*(1 + Sec[e + f*x]))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4307, 3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^n(e+fx)}{a\sec(e+fx)+a}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^n}{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \ 4307$$

$$\frac{\sin(e+fx)\sec^n(e+fx)}{f(a\sec(e+fx)+a)}-\frac{(1-n)\int \sec^{n-1}(e+fx)(a-a\sec(e+fx))dx}{a^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sin(e+fx)\sec^n(e+fx)}{f(a\sec(e+fx)+a)}-\frac{(1-n)\int \csc\left(e+fx+\frac{\pi}{2}\right)^{n-1}\left(a-a\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx}{a^2}$$

$\Big\downarrow$ 4274

$$\frac{\sin(e+fx)\sec^n(e+fx)}{f(a\sec(e+fx)+a)}-\frac{(1-n)\left(a\int \sec^{n-1}(e+fx)dx-a\int \sec^n(e+fx)dx\right)}{a^2}$$

$\Big\downarrow$ 3042

$$\frac{\sin(e+fx)\sec^n(e+fx)}{f(a\sec(e+fx)+a)}-\frac{(1-n)\left(a\int \csc\left(e+fx+\frac{\pi}{2}\right)^{n-1}dx-a\int \csc\left(e+fx+\frac{\pi}{2}\right)^{n}dx\right)}{a^2}$$

$\Big\downarrow$ 4259

$$\frac{\sin(e+fx)\sec^n(e+fx)}{f(a\sec(e+fx)+a)}-\frac{(1-n)\left(a\cos^n(e+fx)\sec^n(e+fx)\int \cos^{1-n}(e+fx)dx-a\cos^n(e+fx)\sec^n(e+fx)\int \cos^{-n}(e+fx)dx\right)}{a^2}$$

$\Big\downarrow$ 3042

$$\frac{\sin(e+fx)\sec^n(e+fx)}{f(a\sec(e+fx)+a)}-\frac{(1-n)\left(a\cos^n(e+fx)\sec^n(e+fx)\int \sin\left(e+fx+\frac{\pi}{2}\right)^{1-n}dx-a\cos^n(e+fx)\sec^n(e+fx)\int \sin\left(e+fx+\frac{\pi}{2}\right)\right.}{a^2}$$

$\Big\downarrow$ 3122

$$\frac{\sin(e+fx)\sec^n(e+fx)}{f(a\sec(e+fx)+a)}-\frac{(1-n)\left(\frac{a\sin(e+fx)\sec^{n-1}(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}-\frac{a\sin(e+fx)\sec^{n-2}(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-}{2}\right.}{f(2-n)\sqrt{\sin^2(e+fx)}}\right.}{a^2}$$

input
```
Int[Sec[e + f*x]^n/(a + a*Sec[e + f*x]),x]
```

output
```
(Sec[e + f*x]^n*Sin[e + f*x])/(f*(a + a*Sec[e + f*x])) - ((1 - n)*(-((a*Hy
pergeometric2F1[1/2, (2 - n)/2, (4 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^(-
2 + n)*Sin[e + f*x])/(f*(2 - n)*Sqrt[Sin[e + f*x]^2])) + (a*Hypergeometric
2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^(-1 + n)*Sin[e
 + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2])))/a^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(a*f*
(a + b*Csc[e + f*x]))), x] + Simp[d*((n - 1)/(a*b))   Int[(d*Csc[e + f*x])^
(n - 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ
[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{\sec (fx+e)^n}{a+a\sec (fx+e)}dx$$

input
```
int(sec(f*x+e)^n/(a+a*sec(f*x+e)),x)
```

output
```
int(sec(f*x+e)^n/(a+a*sec(f*x+e)),x)
```

Fricas [F]

$$\int \frac{\sec^n(e+fx)}{a+a\sec(e+fx)}\,dx=\int \frac{\sec(fx+e)^n}{a\sec(fx+e)+a}\,dx$$

input
```
integrate(sec(f*x+e)^n/(a+a*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral(sec(f*x + e)^n/(a*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{\sec^n(e+fx)}{a+a\sec(e+fx)}\,dx=\frac{\int \frac{\sec^n(e+fx)}{\sec(e+fx)+1}\,dx}{a}$$

input
```
integrate(sec(f*x+e)**n/(a+a*sec(f*x+e)),x)
```

output
```
Integral(sec(e + f*x)**n/(sec(e + f*x) + 1), x)/a
```

Maxima [F]

$$\int \frac{\sec^n(e+fx)}{a+a\sec(e+fx)}\,dx=\int \frac{\sec(fx+e)^n}{a\sec(fx+e)+a}\,dx$$

input
```
integrate(sec(f*x+e)^n/(a+a*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^n/(a*sec(f*x + e) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^n(e+fx)}{a+a\sec(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(f*x+e)^n/(a+a*sec(f*x+e)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[0,1,0]%%%} / %%%{2,[0,0,1]%%%} Error: Bad Argument Val
ue
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^n(e+fx)}{a+a\sec(e+fx)}\,dx = \int \frac{\left(\frac{1}{\cos(e+f\,x)}\right)^n}{a+\frac{a}{\cos(e+f\,x)}}\,dx$$

input
```
int((1/cos(e + f*x))^n/(a + a/cos(e + f*x)),x)
```

output
```
int((1/cos(e + f*x))^n/(a + a/cos(e + f*x)), x)
```

Reduce [F]

$$\int \frac{\sec^n(e+fx)}{a+a\sec(e+fx)}\,dx = \frac{\int \frac{\sec(fx+e)^n}{\sec(fx+e)+1}dx}{a}$$

input
```
int(sec(f*x+e)^n/(a+a*sec(f*x+e)),x)
```

output
```
int(sec(e + f*x)**n/(sec(e + f*x) + 1),x)/a
```

3.293 $\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 217

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^2}\,dx = -\frac{2(2-n)\sec^{1+n}(e+fx)\sin(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{\sec^{1+n}(e+fx)\sin(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{(3-2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sec^{-1+n}(e+fx)\sin(e+fx)}{3a^2f\sqrt{\sin^2(e+fx)}} + \frac{2(2-n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)\sec^n(e+fx)\sin(e+fx)}{3a^2f\sqrt{\sin^2(e+fx)}}$$

output
```
-2/3*(2-n)*sec(f*x+e)^(1+n)*sin(f*x+e)/a^2/f/(1+sec(f*x+e))-1/3*sec(f*x+e)
^(1+n)*sin(f*x+e)/f/(a+a*sec(f*x+e))^2-1/3*(3-2*n)*hypergeom([1/2, 1/2-1/2
*n],[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-1+n)*sin(f*x+e)/a^2/f/(sin(f*x+
e)^2)^(1/2)+2/3*(2-n)*hypergeom([1/2, -1/2*n],[1-1/2*n],cos(f*x+e)^2)*sec(
f*x+e)^n*sin(f*x+e)/a^2/f/(sin(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.88

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{\csc(e+fx)\sec^n(e+fx)\left(-n(1+n)\sin(e+fx)\tan(e+fx)+(1+\sec(e+fx))\left(2(-2+n)n(1+n)\right.\right.}{}$$

input
```
Integrate[Sec[e + f*x]^n/(a + a*Sec[e + f*x])^2,x]
```

output
```
(Csc[e + f*x]*Sec[e + f*x]^n*(-(n*(1 + n)*Sin[e + f*x]*Tan[e + f*x]) + (1
+ Sec[e + f*x])*(2*(-2 + n)*n*(1 + n)*Sin[e + f*x]*Tan[e + f*x] + ((-1 + n
)*(1 + n)*(-3 + 2*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/2, S
ec[e + f*x]^2] - 2*(-2 + n)*n^2*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/
2, Sec[e + f*x]^2])*(1 + Sec[e + f*x])*Sqrt[-Tan[e + f*x]^2])))/(3*a^2*f*n
*(1 + n)*(1 + Sec[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4304, 25, 3042, 4508, 3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^n(e+fx)}{(a\sec(e+fx)+a)^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^n}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$
$$\Big\downarrow \text{4304}$$
$$-\frac{\int -\frac{\sec^n(e+fx)(a(3-n)-a(1-n)\sec(e+fx))}{\sec(e+fx)a+a}dx}{3a^2}-\frac{\sin(e+fx)\sec^{n+1}(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\sec^n(e+fx)(a(3-n)-a(1-n)\sec(e+fx))}{\sec(e+fx)a+a}dx}{3a^2}-\frac{\sin(e+fx)\sec^{n+1}(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^n\left(a(3-n)-a(1-n)\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{\sin(e+fx)\sec^{n+1}(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \ 4508$$

$$\frac{\frac{\int \sec^n(e+fx)\left((3-2n)(1-n)a^2+2(2-n)n\sec(e+fx)a^2\right)dx}{a^2}-\frac{2(2-n)\sin(e+fx)\sec^{n+1}(e+fx)}{f(\sec(e+fx)+1)}}{3a^2}-\frac{\sin(e+fx)\sec^{n+1}(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\left((3-2n)(1-n)a^2+2(2-n)n\csc\left(e+fx+\frac{\pi}{2}\right)a^2\right)dx}{a^2}-\frac{2(2-n)\sin(e+fx)\sec^{n+1}(e+fx)}{f(\sec(e+fx)+1)}}{3a^2}-\frac{\sin(e+fx)\sec^{n+1}(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \ 4274$$

$$\frac{\frac{2a^2(2-n)n\int \sec^{n+1}(e+fx)dx+a^2(3-2n)(1-n)\int \sec^n(e+fx)dx}{a^2}-\frac{2(2-n)\sin(e+fx)\sec^{n+1}(e+fx)}{f(\sec(e+fx)+1)}}{3a^2}-\frac{\sin(e+fx)\sec^{n+1}(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{a^2(3-2n)(1-n)\int \csc\left(e+fx+\frac{\pi}{2}\right)^n dx+2a^2(2-n)n\int \csc\left(e+fx+\frac{\pi}{2}\right)^{n+1}dx}{a^2}-\frac{2(2-n)\sin(e+fx)\sec^{n+1}(e+fx)}{f(\sec(e+fx)+1)}}{3a^2}-\frac{\sin(e+fx)\sec^{n+1}(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \ 4259$$

$$\frac{\frac{2a^2(2-n)n\cos^n(e+fx)\sec^n(e+fx)\int \cos^{-n-1}(e+fx)dx+a^2(3-2n)(1-n)\cos^n(e+fx)\sec^n(e+fx)\int \cos^{-n}(e+fx)dx}{a^2}-\frac{2(2-n)\sin(e+fx)\sec}{f(\sec(e+fx)}}{3a^2}$$

$$\frac{\sin(e+fx)\sec^{n+1}(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{2a^2(2-n)n\cos^n(e+fx)\sec^n(e+fx)\int\sin\left(e+fx+\frac{\pi}{2}\right)^{-n-1}dx+a^2(3-2n)(1-n)\cos^n(e+fx)\sec^n(e+fx)\int\sin\left(e+fx+\frac{\pi}{2}\right)^{-n}dx}{a^2}-\frac{2(2-n)\sin(e-}{f(\sec}}{3a^2}$$
$$\frac{\sin(e+fx)\sec^{n+1}(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

$\downarrow$ 3122

$$\frac{\frac{\frac{2a^2(2-n)\sin(e+fx)\sec^n(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{f\sqrt{\sin^2(e+fx)}}-\frac{a^2(3-2n)\sin(e+fx)\sec^{n-1}(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+\right.}{f\sqrt{\sin^2(e+fx)}}}{a^2}}{3a^2}$$
$$\frac{\sin(e+fx)\sec^{n+1}(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

input
```
Int[Sec[e + f*x]^n/(a + a*Sec[e + f*x])^2,x]
```

output
```
-1/3*(Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(a + a*Sec[e + f*x])^2) + ((-2
*(2 - n)*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + Sec[e + f*x])) + (-((a
^2*(3 - 2*n)*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*
Sec[e + f*x]^(-1 + n)*Sin[e + f*x])/(f*Sqrt[Sin[e + f*x]^2])) + (2*a^2*(2
- n)*Hypergeometric2F1[1/2, -1/2*n, (2 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x
]^n*Sin[e + f*x])/(f*Sqrt[Sin[e + f*x]^2]))/a^2)/(3*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [F]

$$\int \frac{\sec (fx+e)^n}{(a+a\sec (fx+e))^2}dx$$

input
```
int(sec(f*x+e)^n/(a+a*sec(f*x+e))^2,x)
```

output
```
int(sec(f*x+e)^n/(a+a*sec(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^2}\,dx = \int \frac{\sec\left(fx+e\right)^n}{\left(a\sec\left(fx+e\right)+a\right)^2}\,dx$$

input
```
integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(sec(f*x + e)^n/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2), x
)
```

Sympy [F]

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^2}\,dx = \frac{\int \frac{\sec^n(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx}{a^2}$$

input
```
integrate(sec(f*x+e)**n/(a+a*sec(f*x+e))**2,x)
```

output
```
Integral(sec(e + f*x)**n/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)/a**2
```

Maxima [F]

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^2}\,dx = \int \frac{\sec\left(fx+e\right)^n}{\left(a\sec\left(fx+e\right)+a\right)^2}\,dx$$

input
```
integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^n/(a*sec(f*x + e) + a)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,2,0]%%%}+%%%{-3,[0,1,0,0]%%%} / %%%{4,[0,0,0,2]%%%}
 Error: B
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^2}\,dx = \int \frac{\left(\frac{1}{\cos(e+f\,x)}\right)^n}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int((1/cos(e + f*x))^n/(a + a/cos(e + f*x))^2,x)
```

output
```
int((1/cos(e + f*x))^n/(a + a/cos(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^2}\,dx = \frac{\int \frac{\sec(fx+e)^n}{\sec(fx+e)^2+2\sec(fx+e)+1}dx}{a^2}$$

input
```
int(sec(f*x+e)^n/(a+a*sec(f*x+e))^2,x)
```

output
```
int(sec(e + f*x)**n/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),x)/a**2
```

3.294 $\int \sec^n(e+fx)(1+\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 162

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{5/2}\,dx=\frac{2(7+4n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)(3+2n)\sqrt{1+\sec(e+fx)}}+\frac{2\sec^{1+n}(e+fx)\sqrt{1+\sec(e+fx)}\sin(e+fx)}{f(3+2n)}+\frac{2(3+24n+16n^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)\tan(e+fx)}{f(1+2n)(3+2n)\sqrt{1+\sec(e+fx)}}$$

output
```
2*(7+4*n)*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(1+2*n)/(3+2*n)/(1+sec(f*x+e))^(1/
2)+2*sec(f*x+e)^(1+n)*(1+sec(f*x+e))^(1/2)*sin(f*x+e)/f/(3+2*n)+2*(16*n^2+
24*n+3)*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*tan(f*x+e)/f/(1+2*n)/(3+2
*n)/(1+sec(f*x+e))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 23.49 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.46

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{5/2}\,dx =$$

$$-\frac{i2^{-\frac{5}{2}+n}e^{-\frac{1}{2}i(3+2n)(e+fx)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{\frac{3}{2}+n}\left(\frac{10e^{i(2+n)(e+fx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-1-n),\frac{4+n}{2},-e^{2i(e+fx)}\right)}{2+n}+\frac{5e^{i(4+n)(e+fx)}}{}\right.}{}$$

input
```
Integrate[Sec[e + f*x]^n*(1 + Sec[e + f*x])^(5/2),x]
```

output
```
((-I)*2^(-5/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(3/2 + n)*(
(10*E^(I*(2 + n)*(e + f*x))*Hypergeometric2F1[1, (-1 - n)/2, (4 + n)/2, -E
^((2*I)*(e + f*x))])/(2 + n) + (5*E^(I*(4 + n)*(e + f*x))*Hypergeometric2F
1[1, (1 - n)/2, (6 + n)/2, -E^((2*I)*(e + f*x))])/(4 + n) + (E^(I*n*(e + f
*x))*Hypergeometric2F1[1, -3/2 - n/2, 1 + n/2, -E^((2*I)*(e + f*x))])/n +
(5*E^(I*(1 + n)*(e + f*x))*Hypergeometric2F1[1, -1 - n/2, (3 + n)/2, -E^((
2*I)*(e + f*x))])/(1 + n) + (E^(I*(5 + n)*(e + f*x))*Hypergeometric2F1[1,
1 - n/2, (7 + n)/2, -E^((2*I)*(e + f*x))])/(5 + n) + (10*E^(I*(3 + n)*(e +
 f*x))*Hypergeometric2F1[1, -1/2*n, (5 + n)/2, -E^((2*I)*(e + f*x))])/(3 +
 n))*Sec[(e + f*x)/2]^5*(1 + Sec[e + f*x])^(5/2))/(E^((I/2)*(3 + 2*n)*(e +
 f*x))*f*Sec[e + f*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4301, 27, 3042, 4504, 3042, 4293, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sec(e+fx)+1)^{5/2}\sec^n(e+fx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(\csc \left(e+fx+\frac{\pi}{2}\right)+1\right)^{5/2} \csc \left(e+fx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{2 \int \frac{1}{2} \sec^n(e+fx) \sqrt{\sec(e+fx)+1}(4n+(4n+7) \sec(e+fx)+3)dx}{2n+3}+\frac{2 \sin(e+fx) \sqrt{\sec(e+fx)+1} \sec^{n+1}(e+fx)}{f(2n+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sec^n(e+fx) \sqrt{\sec(e+fx)+1}(4n+(4n+7) \sec(e+fx)+3)dx}{2n+3}+\frac{2 \sin(e+fx) \sqrt{\sec(e+fx)+1} \sec^{n+1}(e+fx)}{f(2n+3)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc \left(e+fx+\frac{\pi}{2}\right)^n \sqrt{\csc \left(e+fx+\frac{\pi}{2}\right)+1}\left(4n+(4n+7) \csc \left(e+fx+\frac{\pi}{2}\right)+3\right) dx}{2n+3}+\frac{2 \sin(e+fx) \sqrt{\sec(e+fx)+1} \sec^{n+1}(e+fx)}{f(2n+3)}$$

$$\Big\downarrow \text{4504}$$

$$\frac{\frac{\left(16n^2+24n+3\right) \int \sec^n(e+fx) \sqrt{\sec(e+fx)+1}dx}{2n+1}+\frac{2(4n+7) \sin(e+fx) \sec^{n+1}(e+fx)}{f(2n+1) \sqrt{\sec(e+fx)+1}}}{2n+3}+\frac{2 \sin(e+fx) \sqrt{\sec(e+fx)+1} \sec^{n+1}(e+fx)}{f(2n+3)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\left(16n^2+24n+3\right) \int \csc\left(e+fx+\frac{\pi}{2}\right)^n \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)+1}dx}{2n+1}+\frac{2(4n+7) \sin(e+fx) \sec^{n+1}(e+fx)}{f(2n+1) \sqrt{\sec(e+fx)+1}}}{2n+3}+\frac{2 \sin(e+fx) \sqrt{\sec(e+fx)+1} \sec^{n+1}(e+fx)}{f(2n+3)}$$

$$\Big\downarrow \text{4293}$$

$$\frac{\frac{2(4n+7) \sin(e+fx) \sec^{n+1}(e+fx)}{f(2n+1) \sqrt{\sec(e+fx)+1}}-\frac{\left(16n^2+24n+3\right) \tan(e+fx) \int \frac{\sec^{n-1}(e+fx)}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f(2n+1) \sqrt{1-\sec(e+fx)} \sqrt{\sec(e+fx)+1}}}{2n+3}+\frac{2 \sin(e+fx) \sqrt{\sec(e+fx)+1} \sec^{n+1}(e+fx)}{f(2n+3)}$$

$$\Big\downarrow \text{75}$$

$$\frac{\frac{2(16n^2+24n+3)\tan(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f(2n+1)\sqrt{\sec(e+fx)+1}}+\frac{2(4n+7)\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{\sec(e+fx)+1}}}{2n+3}+\frac{2\sin(e+fx)\sqrt{\sec(e+fx)+1}\sec^{n+1}(e+fx)}{f(2n+3)}$$

input
```
Int[Sec[e + f*x]^n*(1 + Sec[e + f*x])^(5/2),x]
```

output
```
(2*Sec[e + f*x]^(1 + n)*Sqrt[1 + Sec[e + f*x]]*Sin[e + f*x])/(f*(3 + 2*n))
 + ((2*(7 + 4*n)*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[1 +
Sec[e + f*x]]) + (2*(3 + 24*n + 16*n^2)*Hypergeometric2F1[1/2, 1 - n, 3/2,
 1 - Sec[e + f*x]]*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[1 + Sec[e + f*x]]))/(3
+ 2*n)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4504
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Simp[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1))   Int[Sqrt[a + b*Csc[e + f*
x]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&
!LtQ[n, 0]
```

Maple [F]

$$\int \sec (fx+e)^{n} (1+\sec (fx+e))^{\frac{5}{2}} dx$$

input
```
int(sec(f*x+e)^n*(1+sec(f*x+e))^(5/2),x)
```

output
```
int(sec(f*x+e)^n*(1+sec(f*x+e))^(5/2),x)
```

Fricas [F]

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{5/2}\,dx=\int \sec (fx+e)^{n} (\sec (fx+e)+1)^{\frac{5}{2}} dx$$

input
```
integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
integral((sec(f*x + e)^2 + 2*sec(f*x + e) + 1)*sec(f*x + e)^n*sqrt(sec(f*x
 + e) + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**n*(1+sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{5/2}\,dx = \int \sec\left(fx+e\right)^n\left(\sec\left(fx+e\right)+1\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^n*(sec(f*x + e) + 1)^(5/2), x)
```

Giac [F]

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{5/2}\,dx = \int \sec\left(fx+e\right)^n\left(\sec\left(fx+e\right)+1\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^n*(sec(f*x + e) + 1)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{5/2}\,dx = \int \left(\frac{1}{\cos(e+f\,x)}+1\right)^{5/2}\left(\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((1/cos(e + f*x) + 1)^(5/2)*(1/cos(e + f*x))^n,x)
```

output
```
int((1/cos(e + f*x) + 1)^(5/2)*(1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\begin{aligned}\int \sec^n(e+fx)(1 \\ &+\sec(e+fx))^{5/2}\,dx = \int \sec(fx+e)^n\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2\,dx \\ &+2\left(\int \sec(fx+e)^n\sqrt{\sec(fx+e)+1}\,\sec(fx+e)\,dx\right) \\ &+\int \sec(fx+e)^n\sqrt{\sec(fx+e)+1}dx\end{aligned}$$

input
```
int(sec(f*x+e)^n*(1+sec(f*x+e))^(5/2),x)
```

output
```
int(sec(e + f*x)**n*sqrt(sec(e + f*x) + 1)*sec(e + f*x)**2,x) + 2*int(sec(
e + f*x)**n*sqrt(sec(e + f*x) + 1)*sec(e + f*x),x) + int(sec(e + f*x)**n*s
qrt(sec(e + f*x) + 1),x)
```

3.295 $\int \sec^n(e+fx)(1+\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{3/2}\,dx=\frac{2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}}+\frac{2(1+4n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}}$$

output
```
2*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(1+2*n)/(1+sec(f*x+e))^(1/2)+2*(1+4*n)*hyp
ergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*tan(f*x+e)/f/(1+2*n)/(1+sec(f*x+e))^
(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{3/2}\,dx=\frac{\left(-1+(1+4n)\cos^{\frac{1}{2}+n}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{2}+n,\frac{3}{2},2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\sec^n(}{fn}$$

input
```
Integrate[Sec[e + f*x]^n*(1 + Sec[e + f*x])^(3/2),x]
```

output
```
((-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n, 3/
2, 2*Sin[(e + f*x)/2]^2])*Sec[e + f*x]^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f
*x)/2])/(f*n)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4301, 27, 2011, 3042, 4293, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sec(e+fx)+1)^{3/2}\sec^n(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^{3/2}\csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{2\int \frac{\sec^n(e+fx)(4n+(4n+1)\sec(e+fx)+1)}{2\sqrt{\sec(e+fx)+1}}dx}{2n+1}+\frac{2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sec^n(e+fx)(4n+(4n+1)\sec(e+fx)+1)}{\sqrt{\sec(e+fx)+1}}dx}{2n+1}+\frac{2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{2011}$$

$$\frac{(4n+1)\int \sec^n(e+fx)\sqrt{\sec(e+fx)+1}dx}{2n+1}+\frac{2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(4n+1)\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)+1}dx}{2n+1}+\frac{2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{4293}$$

$$\frac{2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{\sec(e+fx)+1}}-\frac{(4n+1)\tan(e+fx)\int\frac{\sec^{n-1}(e+fx)}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f(2n+1)\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{ 75}$$

$$\frac{2(4n+1)\tan(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f(2n+1)\sqrt{\sec(e+fx)+1}}+\frac{2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

input
```
Int[Sec[e + f*x]^n*(1 + Sec[e + f*x])^(3/2),x]
```

output
```
(2*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[1 + Sec[e + f*x]])
 + (2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e
 + f*x])/(f*(1 + 2*n)*Sqrt[1 + Sec[e + f*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Maple [F]

$$\int \sec(fx+e)^n (1+\sec(fx+e))^{\frac{3}{2}}\, dx$$

input
```
int(sec(f*x+e)^n*(1+sec(f*x+e))^(3/2),x)
```

output
```
int(sec(f*x+e)^n*(1+sec(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{3/2}\, dx = \int \sec(fx+e)^n (\sec(fx+e)+1)^{\frac{3}{2}}\, dx$$

input
```
integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sec(f*x + e)^n*(sec(f*x + e) + 1)^(3/2), x)
```

Sympy [F]

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{3/2}\,dx = \int (\sec(e+fx)+1)^{\frac{3}{2}}\sec^n(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**n*(1+sec(f*x+e))**(3/2),x)
```

output
```
Integral((sec(e + f*x) + 1)**(3/2)*sec(e + f*x)**n, x)
```

Maxima [F]

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{3/2}\,dx = \int \sec(fx+e)^n(\sec(fx+e)+1)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^n*(sec(f*x + e) + 1)^(3/2), x)
```

Giac [F]

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{3/2}\,dx = \int \sec(fx+e)^n(\sec(fx+e)+1)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^n*(sec(f*x + e) + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{3/2}\,dx=\int \left(\frac{1}{\cos\left(e+f\,x\right)}+1\right)^{3/2}\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^n\,dx$$

input
```
int((1/cos(e + f*x) + 1)^(3/2)*(1/cos(e + f*x))^n,x)
```

output
```
int((1/cos(e + f*x) + 1)^(3/2)*(1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{3/2}\,dx=\int \sec\left(fx+e\right)^n\sqrt{\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)dx+\int \sec\left(fx+e\right)^n\sqrt{\sec\left(fx+e\right)+1}dx$$

input
```
int(sec(f*x+e)^n*(1+sec(f*x+e))^(3/2),x)
```

output
```
int(sec(e + f*x)**n*sqrt(sec(e + f*x) + 1)*sec(e + f*x),x) + int(sec(e + f
*x)**n*sqrt(sec(e + f*x) + 1),x)
```

3.296 $\int \sec^n(e+fx)\sqrt{1+\sec(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 45

$$\int \sec^n(e+fx)\sqrt{1+\sec(e+fx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)\tan(e+fx)}{f\sqrt{1+\sec(e+fx)}}$$

output
```
2*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*tan(f*x+e)/f/(1+sec(f*x+e))^(1/
2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \sec^n(e+fx)\sqrt{1+\sec(e+fx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)\tan(e+fx)}{f\sqrt{1+\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]^n*Sqrt[1 + Sec[e + f*x]],x]
```

output
```
(2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(f*S
qrt[1 + Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4293, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(e+fx)+1}\sec^n(e+fx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)+1}\csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$
$$\Big\downarrow \text{4293}$$
$$-\frac{\tan(e+fx)\int \frac{\sec^{n-1}(e+fx)}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$
$$\Big\downarrow \text{75}$$
$$\frac{2\tan(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f\sqrt{\sec(e+fx)+1}}$$

input
```
Int[Sec[e + f*x]^n*Sqrt[1 + Sec[e + f*x]],x]
```

output
```
(2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(f*S
qrt[1 + Sec[e + f*x]])
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sec (fx+e)^{n} \sqrt{1+\sec (fx+e)}dx$$

input
```
int(sec(f*x+e)^n*(1+sec(f*x+e))^(1/2),x)
```

output
```
int(sec(f*x+e)^n*(1+sec(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \sec^n(e+fx)\sqrt{1+\sec(e+fx)}\,dx = \int \sec (fx+e)^{n} \sqrt{\sec (fx+e)+1}\,dx$$

input
```
integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(f*x + e)^n*sqrt(sec(f*x + e) + 1), x)
```

Sympy [F]

$$\int \sec^n(e+fx)\sqrt{1+\sec(e+fx)}\,dx = \int \sqrt{\sec(e+fx)+1}\sec^n(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**n*(1+sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**n, x)
```

Maxima [F]

$$\int \sec^n(e+fx)\sqrt{1+\sec(e+fx)}\,dx = \int \sec(fx+e)^n\sqrt{\sec(fx+e)+1}\,dx$$

input
```
integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^n*sqrt(sec(f*x + e) + 1), x)
```

Giac [F]

$$\int \sec^n(e+fx)\sqrt{1+\sec(e+fx)}\,dx = \int \sec(fx+e)^n\sqrt{\sec(fx+e)+1}\,dx$$

input
```
integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^n*sqrt(sec(f*x + e) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)\sqrt{1+\sec(e+fx)}\,dx=\int \sqrt{\frac{1}{\cos(e+f\,x)}+1}\,\left(\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((1/cos(e + f*x) + 1)^(1/2)*(1/cos(e + f*x))^n,x)
```

output
```
int((1/cos(e + f*x) + 1)^(1/2)*(1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int \sec^n(e+fx)\sqrt{1+\sec(e+fx)}\,dx=\int \sec(fx+e)^n\,\sqrt{\sec(fx+e)+1}dx$$

input
```
int(sec(f*x+e)^n*(1+sec(f*x+e))^(1/2),x)
```

output
```
int(sec(e + f*x)**n*sqrt(sec(e + f*x) + 1),x)
```

3.297 $\int \frac{\sec^n(e+fx)}{\sqrt{1+\sec(e+fx)}} dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sec^n(e+fx)}{\sqrt{1+\sec(e+fx)}} dx = \frac{\text{AppellF1}\left(\frac{1}{2},1-n,1,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e+fx)}{f\sqrt{1+\sec(e+fx)}}$$

output
```
AppellF1(1/2,1-n,1,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*tan(f*x+e)/f/(1+se
c(f*x+e))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2938 vs. 2(59) = 118.

Time = 14.44 (sec) , antiderivative size = 2938, normalized size of antiderivative = 49.80

$$\int \frac{\sec^n(e+fx)}{\sqrt{1+\sec(e+fx)}} dx = \text{Result too large to show}$$

input
```
Integrate[Sec[e + f*x]^n/Sqrt[1 + Sec[e + f*x]],x]
```

output
```
(3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(-1/2 + (-1 + 2*n)/2)*(C
os[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(e + f*x)/2])/(f*(3*AppellF1[1/2, -1
/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)
*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*AppellF1[1/2, -1/2 + n, 1 - n,
 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)
/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]])
/(Sqrt[2]*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n
, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) - (3
*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqr
t[1 + Sec[e + f*x]]*Sin[e + f*x]*Tan[(e + f*x)/2])/(3*AppellF1[1/2, -1/2 +
 n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*App
ellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
+ (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 +...
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4312, 148, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^n(e+fx)}{\sqrt{\sec(e+fx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^n}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4312}$$

$$\frac{\tan(e+fx)\int \frac{\sec^{n-1}(e+fx)}{\sqrt{1-\sec(e+fx)}(\sec(e+fx)+1)}d(1-\sec(e+fx))}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{ 148}$$

$$\frac{2\tan(e+fx)\int \frac{\sec^{n-1}(e+fx)}{\sec(e+fx)+1}d\sqrt{1-\sec(e+fx)}}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{ 333}$$

$$\frac{\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1-n,1,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f\sqrt{\sec(e+fx)+1}}$$

input
```
Int[Sec[e + f*x]^n/Sqrt[1 + Sec[e + f*x]],x]
```

output
```
(AppellF1[1/2, 1 - n, 1, 3/2, 1 - Sec[e + f*x], (1 - Sec[e + f*x])/2]*Tan[
e + f*x])/(f*Sqrt[1 + Sec[e + f*x]])
```

Defintions of rubi rules used

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4312
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a - x)^(n - 1)
*((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

Maple [F]

$$\int \frac{\sec (fx+e)^n}{\sqrt{1+\sec (fx+e)}}dx$$

input
```
int(sec(f*x+e)^n/(1+sec(f*x+e))^(1/2),x)
```

output
```
int(sec(f*x+e)^n/(1+sec(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{\sec^n(e+fx)}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{\sec (fx+e)^n}{\sqrt{\sec (fx+e)+1}}\,dx$$

input
```
integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(f*x + e)^n/sqrt(sec(f*x + e) + 1), x)
```

Sympy [F]

$$\int \frac{\sec^n(e+fx)}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{\sec^n{(e+fx)}}{\sqrt{\sec{(e+fx)}+1}}\,dx$$

input
```
integrate(sec(f*x+e)**n/(1+sec(f*x+e))**(1/2),x)
```

output
```
Integral(sec(e + f*x)**n/sqrt(sec(e + f*x) + 1), x)
```

Maxima [F]

$$\int \frac{\sec^n(e+fx)}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{\sec{(fx+e)}^n}{\sqrt{\sec{(fx+e)}+1}}\,dx$$

input
```
integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^n/sqrt(sec(f*x + e) + 1), x)
```

Giac [F]

$$\int \frac{\sec^n(e+fx)}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{\sec{(fx+e)}^n}{\sqrt{\sec{(fx+e)}+1}}\,dx$$

input
```
integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^n/sqrt(sec(f*x + e) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^n(e+fx)}{\sqrt{1+\sec(e+fx)}}\,dx=\int \frac{\left(\frac{1}{\cos(e+f\,x)}\right)^n}{\sqrt{\frac{1}{\cos(e+f\,x)}+1}}\,dx$$

input
```
int((1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(1/2),x)
```

output
```
int((1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sec^n(e+fx)}{\sqrt{1+\sec(e+fx)}}\,dx=\int \frac{\sec(fx+e)^n\sqrt{\sec(fx+e)+1}}{\sec(fx+e)+1}dx$$

input
```
int(sec(f*x+e)^n/(1+sec(f*x+e))^(1/2),x)
```

output
```
int((sec(e + f*x)**n*sqrt(sec(e + f*x) + 1))/(sec(e + f*x) + 1),x)
```

3.298 $\int \frac{\sec^n(e+fx)}{(1+\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{\sec^n(e+fx)}{(1+\sec(e+fx))^{3/2}}\,dx = \frac{\operatorname{AppellF1}\left(\frac{1}{2},1-n,2,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e+fx)}{2f\sqrt{1+\sec(e+fx)}}$$

output
```
1/2*AppellF1(1/2,1-n,2,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*tan(f*x+e)/f/(
1+sec(f*x+e))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2990 vs. 2(62) = 124.

Time = 14.71 (sec) , antiderivative size = 2990, normalized size of antiderivative = 48.23

$$\int \frac{\sec^n(e+fx)}{(1+\sec(e+fx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[e + f*x]^n/(1 + Sec[e + f*x])^(3/2),x]
```

output
```
(6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(1/2 + (-3 + 2*n)/2)*(Cos[(e + f
*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2
)^2)/(f*(1 + Sec[e + f*x])^(3/2)*(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 +
n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*Appel
lF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*T
an[(e + f*x)/2]^2)*((12*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[
(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1 + Tan[(e + f
*x)/2]^2))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 -
 n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (
3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e +
 f*x])^(3/2 + n)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1
 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1
[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3
 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e...
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4312, 148, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^n(e+fx)}{(\sec(e+fx)+1)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^n}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{4312}$$

$$\frac{\tan(e+fx)\int \frac{\sec^{n-1}(e+fx)}{\sqrt{1-\sec(e+fx)}(\sec(e+fx)+1)^2}d(1-\sec(e+fx))}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{148}$$

$$\frac{2\tan(e+fx)\int \frac{\sec^{n-1}(e+fx)}{(\sec(e+fx)+1)^2}d\sqrt{1-\sec(e+fx)}}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{333}$$

$$\frac{\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1-n,2,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{2f\sqrt{\sec(e+fx)+1}}$$

input
```
Int[Sec[e + f*x]^n/(1 + Sec[e + f*x])^(3/2),x]
```

output
```
(AppellF1[1/2, 1 - n, 2, 3/2, 1 - Sec[e + f*x], (1 - Sec[e + f*x])/2]*Tan[
e + f*x])/(2*f*Sqrt[1 + Sec[e + f*x]])
```

Defintions of rubi rules used

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4312
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a - x)^(n - 1)
*((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

Maple [F]

$$\int \frac{\sec (fx+e)^{n}}{(1+\sec (fx+e))^{\frac{3}{2}}}dx$$

input
```
int(sec(f*x+e)^n/(1+sec(f*x+e))^(3/2),x)
```

output
```
int(sec(f*x+e)^n/(1+sec(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \frac{\sec^n(e+fx)}{(1+\sec(e+fx))^{3/2}}\,dx=\int \frac{\sec (fx+e)^{n}}{(\sec (fx+e)+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sec(f*x + e)^n*sqrt(sec(f*x + e) + 1)/(sec(f*x + e)^2 + 2*sec(f*x
 + e) + 1), x)
```

Sympy [F]

$$\int \frac{\sec^n(e+fx)}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{\sec^n(e+fx)}{(\sec(e+fx)+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)**n/(1+sec(f*x+e))**(3/2),x)
```

output
```
Integral(sec(e + f*x)**n/(sec(e + f*x) + 1)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec^n(e+fx)}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)^n}{(\sec(fx+e)+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^n/(sec(f*x + e) + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{\sec^n(e+fx)}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)^n}{(\sec(fx+e)+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^n/(sec(f*x + e) + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^n(e+fx)}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(\frac{1}{\cos(e+f\,x)}\right)^n}{\left(\frac{1}{\cos(e+f\,x)}+1\right)^{3/2}}\,dx$$

input
```
int((1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(3/2),x)
```

output
```
int((1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sec^n(e+fx)}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)^n\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^2+2\sec(fx+e)+1}dx$$

input
```
int(sec(f*x+e)^n/(1+sec(f*x+e))^(3/2),x)
```

output
```
int((sec(e + f*x)**n*sqrt(sec(e + f*x) + 1))/(sec(e + f*x)**2 + 2*sec(e +
f*x) + 1),x)
```

3.299 $\int (-\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 120

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \frac{2(1+4n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)(-\sec(e+fx))^n\sec^{1-n}(e+fx}{f(1+2n)\sqrt{1+\sec(e+fx)}} + \frac{2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}}$$

output
```
2*(1+4*n)*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*(-sec(f*x+e))^n*sec(f*x
+e)^(1-n)*sin(f*x+e)/f/(1+2*n)/(1+sec(f*x+e))^(1/2)+2*(-sec(f*x+e))^n*tan(
f*x+e)/f/(1+2*n)/(1+sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \frac{\left(-1+(1+4n)\cos^{\frac{1}{2}+n}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{2}+n,\frac{3}{2},2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\right)(-\text{se}}{fn}$$

input
```
Integrate[(-Sec[e + f*x])^n*(1 + Sec[e + f*x])^(3/2),x]
```

output
```
((-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n, 3/
2, 2*Sin[(e + f*x)/2]^2])*(-Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e
+ f*x)/2])/(f*n)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4301, 27, 2011, 3042, 4293, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sec(e+fx)+1)^{3/2}(-\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^{3/2}\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{2\int \frac{(-\sec(e+fx))^n(4n+(4n+1)\sec(e+fx)+1)}{2\sqrt{\sec(e+fx)+1}}dx}{2n+1}+\frac{2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(-\sec(e+fx))^n(4n+(4n+1)\sec(e+fx)+1)}{\sqrt{\sec(e+fx)+1}}dx}{2n+1}+\frac{2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{2011}$$

$$\frac{(4n+1)\int (-\sec(e+fx))^n\sqrt{\sec(e+fx)+1}dx}{2n+1}+\frac{2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(4n+1)\int \left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)+1}dx}{2n+1}+\frac{2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{4293}$$

$$\frac{(4n+1)\tan(e+fx)\int\frac{(-\sec(e+fx))^{n-1}}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f(2n+1)\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}+\frac{2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{74}$$

$$\frac{2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}-\frac{(4n+1)\tan(e+fx)(-\sec(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},n,n+1,\sec(e+fx)\right)}{fn(2n+1)\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

input
```
Int[(-Sec[e + f*x])^n*(1 + Sec[e + f*x])^(3/2),x]
```

output
```
(2*(-Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[1 + Sec[e + f*x]]) -
((1 + 4*n)*Hypergeometric2F1[1/2, n, 1 + n, Sec[e + f*x]]*(-Sec[e + f*x])^
n*Tan[e + f*x])/(f*n*(1 + 2*n)*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x
]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Maple [F]

$$\int (-\sec (fx+e))^n (1+\sec (fx+e))^{\frac{3}{2}} dx$$

input
```
int((-sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x)
```

output
```
int((-sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^{3/2} dx = \int (-\sec (fx+e))^n (\sec (fx+e)+1)^{\frac{3}{2}} dx$$

input
```
integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral((-sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

Sympy [F]

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \int (-\sec(e+fx))^n\,(\sec(e+fx)+1)^{\frac{3}{2}}\,dx$$

input
```
integrate((-sec(f*x+e))**n*(1+sec(f*x+e))**(3/2),x)
```

output
```
Integral((-sec(e + f*x))**n*(sec(e + f*x) + 1)**(3/2), x)
```

Maxima [F]

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \int (-\sec(fx+e))^n\,(\sec(fx+e)+1)^{\frac{3}{2}}\,dx$$

input
```
integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

Giac [F]

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \int (-\sec(fx+e))^n\,(\sec(fx+e)+1)^{\frac{3}{2}}\,dx$$

input
```
integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((-sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \int \left(\frac{1}{\cos(e+f\,x)}+1\right)^{3/2}\left(-\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((1/cos(e + f*x) + 1)^(3/2)*(-1/cos(e + f*x))^n,x)
```

output
```
int((1/cos(e + f*x) + 1)^(3/2)*(-1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = (-1)^n\left(\int \sec(fx+e)^n\sqrt{\sec(fx+e)+1}\,\sec(fx+e)\,dx + \int \sec(fx+e)^n\sqrt{\sec(fx+e)+1}dx\right)$$

input
```
int((-sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x)
```

output
```
( - 1)**n*(int(sec(e + f*x)**n*sqrt(sec(e + f*x) + 1)*sec(e + f*x),x) + in
t(sec(e + f*x)**n*sqrt(sec(e + f*x) + 1),x))
```

3.300 $\int (-\sec(e+fx))^n \sqrt{1+\sec(e+fx)}\, dx$

Optimal result

Integrand size = 23, antiderivative size = 67

$$\int (-\sec(e+fx))^n \sqrt{1+\sec(e+fx)}\, dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)(-\sec(e+fx))^n \sec^{1-n}(e+fx)\sin(e+fx)}{f\sqrt{1+\sec(e+fx)}}$$

output
```
2*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*(-sec(f*x+e))^n*sec(f*x+e)^(1-n
)*sin(f*x+e)/f/(1+sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int (-\sec(e+fx))^n \sqrt{1+\sec(e+fx)}\, dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)(-\sec(e+fx))^n \sec^{1-n}(e+fx)\sin(e+fx)}{f\sqrt{1+\sec(e+fx)}}$$

input
```
Integrate[(-Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]],x]
```

output
```
(2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*(-Sec[e + f*x])^n*
Sec[e + f*x]^(1 - n)*Sin[e + f*x])/(f*Sqrt[1 + Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4293, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(e+fx)+1}(-\sec(e+fx))^n\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)+1}\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$
$$\Big\downarrow \text{ 4293}$$
$$\frac{\tan(e+fx)\int \frac{(-\sec(e+fx))^{n-1}}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$
$$\Big\downarrow \text{ 74}$$
$$-\frac{\tan(e+fx)(-\sec(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},n,n+1,\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

input
```
Int[(-Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]],x]
```

output
```
-((Hypergeometric2F1[1/2, n, 1 + n, Sec[e + f*x]]*(-Sec[e + f*x])^n*Tan[e
+ f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int (-\sec (fx+e))^{n} \sqrt{1+\sec (fx+e)}dx$$

input
```
int((-sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x)
```

output
```
int((-sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int (-\sec(e+fx))^n\sqrt{1+\sec(e+fx)}\,dx = \int (-\sec (fx+e))^{n} \sqrt{\sec (fx+e)+1}\,dx$$

input
```
integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral((-sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

Sympy [F]

$$\int (-\sec(e+fx))^n\sqrt{1+\sec(e+fx)}\,dx = \int (-\sec{(e+fx)})^n\,\sqrt{\sec{(e+fx)}+1}\,dx$$

input
```
integrate((-sec(f*x+e))**n*(1+sec(f*x+e))**(1/2),x)
```

output
```
Integral((-sec(e + f*x))**n*sqrt(sec(e + f*x) + 1), x)
```

Maxima [F]

$$\int (-\sec(e+fx))^n\sqrt{1+\sec(e+fx)}\,dx = \int (-\sec{(fx+e)})^n\,\sqrt{\sec{(fx+e)}+1}\,dx$$

input
```
integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((-sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

Giac [F]

$$\int (-\sec(e+fx))^n\sqrt{1+\sec(e+fx)}\,dx = \int (-\sec{(fx+e)})^n\,\sqrt{\sec{(fx+e)}+1}\,dx$$

input
```
integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((-sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (-\sec(e+fx))^n \sqrt{1+\sec(e+fx)}\,dx = \int \sqrt{\frac{1}{\cos(e+f\,x)}+1}\,\left(-\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((1/cos(e + f*x) + 1)^(1/2)*(-1/cos(e + f*x))^n,x)
```

output
```
int((1/cos(e + f*x) + 1)^(1/2)*(-1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (-\sec(e+fx))^n \sqrt{1+\sec(e+fx)}\,dx = (-1)^n \left(\int \sec(fx+e)^n \sqrt{\sec(fx+e)+1}dx\right)$$

input
```
int((-sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x)
```

output
```
( - 1)**n*int(sec(e + f*x)**n*sqrt(sec(e + f*x) + 1),x)
```

3.301 $\int \frac{(-\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = -\frac{\operatorname{AppellF1}\left(\frac{1}{2},1-n,1,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)(-\sec(e+fx))^{-1+n}\sec^{2-n}(e+fx)\sin}{f\sqrt{1+\sec(e+fx)}}$$

output
```
-AppellF1(1/2,1-n,1,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*(-sec(f*x+e))^(-1
+n)*sec(f*x+e)^(2-n)*sin(f*x+e)/f/(1+sec(f*x+e))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2951 vs. 2(84) = 168.

Time = 6.13 (sec) , antiderivative size = 2951, normalized size of antiderivative = 35.13

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(-Sec[e + f*x])^n/Sqrt[1 + Sec[e + f*x]],x]
```

output
```
(3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(-Sec[e + f*x])^n*Sec[e + f*x]^(-1/2
- n + (-1 + 2*n)/2)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(e + f*x)/2])/
(f*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*AppellF1
[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e
 + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*S
qrt[1 + Sec[e + f*x]])/(Sqrt[2]*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n
, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*Appell
F1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan
[(e + f*x)/2]^2)) - (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]
^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Sin[e + f*x]*Tan[(e + f*x)/2])/(
3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*Sqrt[2]*...
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4313, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{\sec(e+fx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4313}$$

$$\frac{\tan(e+fx)\int \frac{(-\sec(e+fx))^{n-1}}{\sqrt{1-\sec(e+fx)}(\sec(e+fx)+1)}d(\sec(e+fx)+1)}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \; 153$$

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n \operatorname{AppellF1}\left(n,\frac{1}{2},1,n+1,\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

input
```
Int[(-Sec[e + f*x])^n/Sqrt[1 + Sec[e + f*x]],x]
```

output
```
-((AppellF1[n, 1/2, 1, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(-Sec[e + f*x])
^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] && !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4313
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-((-a)*(d/b))^n)*(Cot[e + f*x]/(a^(n - 1)*f*S
qrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[x^(m - 1/2)*
((a - x)^(n - 1)/Sqrt[2*a - x]), x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a
, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
&&  !IntegerQ[n] && LtQ[a*(d/b), 0]
```

Maple [F]

$$\int \frac{(-\sec(fx+e))^n}{\sqrt{1+\sec(fx+e)}}dx$$

input
```
int((-sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x)
```

output
```
int((-sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{(-\sec(fx+e))^n}{\sqrt{\sec(fx+e)+1}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral((-sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

Sympy [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{(-\sec(e+fx))^n}{\sqrt{\sec(e+fx)+1}}\,dx$$

input
```
integrate((-sec(f*x+e))**n/(1+sec(f*x+e))**(1/2),x)
```

output
```
Integral((-sec(e + f*x))**n/sqrt(sec(e + f*x) + 1), x)
```

Maxima [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{(-\sec(fx+e))^n}{\sqrt{\sec(fx+e)+1}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((-sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

Giac [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{(-\sec(fx+e))^n}{\sqrt{\sec(fx+e)+1}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((-sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{\left(-\frac{1}{\cos(e+f\,x)}\right)^n}{\sqrt{\frac{1}{\cos(e+f\,x)}+1}}\,dx$$

input
```
int((-1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(1/2),x)
```

output
```
int((-1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = (-1)^n \left(\int \frac{\sec(fx+e)^n\sqrt{\sec(fx+e)+1}}{\sec(fx+e)+1}dx\right)$$

input
```
int((-sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x)
```

output
```
( - 1)**n*int((sec(e + f*x)**n*sqrt(sec(e + f*x) + 1))/(sec(e + f*x) + 1),
x)
```

3.302 $\int \frac{(-\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{(-\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = -\frac{\operatorname{AppellF1}\left(\frac{1}{2},1-n,2,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)(-\sec(e+fx))^{-1+n}\sec^{2-n}(e+fx)\sin(e+}{2f\sqrt{1+\sec(e+fx)}}$$

output
```
-1/2*AppellF1(1/2,1-n,2,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*(-sec(f*x+e))
^(-1+n)*sec(f*x+e)^(2-n)*sin(f*x+e)/f/(1+sec(f*x+e))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3003 vs. 2(86) = 172.

Time = 6.13 (sec) , antiderivative size = 3003, normalized size of antiderivative = 34.92

$$\int \frac{(-\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(-Sec[e + f*x])^n/(1 + Sec[e + f*x])^(3/2),x]
```

output
```
(6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*(Sec[(e + f*x)/2]^2)^n*(-Sec[e + f*x])^n*Sec[e + f*x]^(1/2 - n + (-3
 + 2*n)/2)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-
1 + Tan[(e + f*x)/2]^2)^2)/(f*(1 + Sec[e + f*x])^(3/2)*(3*AppellF1[1/2, -3
/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)
*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((12*AppellF1[1/2, -3/2 + n, 1 -
n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*
x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)
/2]^2*(-1 + Tan[(e + f*x)/2]^2))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 +
n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*Appel
lF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*T
an[(e + f*x)/2]^2) + (3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[
(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*App
ellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
+ (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(...
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4313, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-\sec(e+fx))^n}{(\sec(e+fx)+1)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{4313}$$

$$\frac{\tan(e+fx)\int \frac{(-\sec(e+fx))^{n-1}}{\sqrt{1-\sec(e+fx)}(\sec(e+fx)+1)^2}d(\sec(e+fx)+1)}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{153}$$

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n \operatorname{AppellF1}\left(n,\frac{1}{2},2,n+1,\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

input
```
Int[(-Sec[e + f*x])^n/(1 + Sec[e + f*x])^(3/2),x]
```

output
```
-((AppellF1[n, 1/2, 2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(-Sec[e + f*x])
^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] && !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4313
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-((-a)*(d/b))^n)*(Cot[e + f*x]/(a^(n - 1)*f*S
qrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[x^(m - 1/2)*
((a - x)^(n - 1)/Sqrt[2*a - x]), x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a
, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
&&  !IntegerQ[n] && LtQ[a*(d/b), 0]
```

Maple [F]

$$\int \frac{(-\sec (fx+e))^n}{(1+\sec (fx+e))^{\frac{3}{2}}}dx$$

input
```
int((-sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x)
```

output
```
int((-sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \frac{(-\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{(-\sec (fx+e))^n}{(\sec (fx+e)+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral((-sec(f*x + e))^n*sqrt(sec(f*x + e) + 1)/(sec(f*x + e)^2 + 2*sec(
f*x + e) + 1), x)
```

Sympy [F]

$$\int \frac{(-\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{(-\sec (e+fx))^n}{(\sec (e+fx)+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-sec(f*x+e))**n/(1+sec(f*x+e))**(3/2),x)
```

output
```
Integral((-sec(e + f*x))**n/(sec(e + f*x) + 1)**(3/2), x)
```

Maxima [F]

$$\int \frac{(-\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{(-\sec(fx+e))^n}{(\sec(fx+e)+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-sec(f*x + e))^n/(sec(f*x + e) + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{(-\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{(-\sec(fx+e))^n}{(\sec(fx+e)+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((-sec(f*x + e))^n/(sec(f*x + e) + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(-\frac{1}{\cos(e+f\,x)}\right)^n}{\left(\frac{1}{\cos(e+f\,x)}+1\right)^{3/2}}\,dx$$

input
```
int((-1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(3/2),x)
```

output
```
int((-1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(3/2), x)
```

Reduce [F]

$$\int \frac{(-\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = (-1)^n \left(\int \frac{\sec(fx+e)^n \sqrt{\sec(fx+e)+1}}{\sec(fx+e)^2+2\sec(fx+e)+1}dx\right)$$

input
```
int((-sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x)
```

output
```
( - 1)**n*int((sec(e + f*x)**n*sqrt(sec(e + f*x) + 1))/(sec(e + f*x)**2 +
2*sec(e + f*x) + 1),x)
```

3.303 $\int (d\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 120

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \frac{2(1+4n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)\sec^{1-n}(e+fx)(d\sec(e+fx))^n}{f(1+2n)\sqrt{1+\sec(e+fx)}} + \frac{2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}}$$

output
```
2*(1+4*n)*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*sec(f*x+e)^(1-n)*(d*sec
(f*x+e))^n*sin(f*x+e)/f/(1+2*n)/(1+sec(f*x+e))^(1/2)+2*(d*sec(f*x+e))^n*ta
n(f*x+e)/f/(1+2*n)/(1+sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \frac{\left(-1+(1+4n)\cos^{\frac{1}{2}+n}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{2}+n,\frac{3}{2},2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\right)(d\sec}{fn}$$

input
```
Integrate[(d*Sec[e + f*x])^n*(1 + Sec[e + f*x])^(3/2),x]
```

output
```
((-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n, 3/
2, 2*Sin[(e + f*x)/2]^2])*(d*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e
 + f*x)/2])/(f*n)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 4301, 27, 2011, 3042, 4293, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sec(e+fx)+1)^{3/2}(d\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^{3/2}\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{2\int \frac{(d\sec(e+fx))^n(4n+(4n+1)\sec(e+fx)+1)}{2\sqrt{\sec(e+fx)+1}}dx}{2n+1}+\frac{2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(d\sec(e+fx))^n(4n+(4n+1)\sec(e+fx)+1)}{\sqrt{\sec(e+fx)+1}}dx}{2n+1}+\frac{2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{2011}$$

$$\frac{(4n+1)\int (d\sec(e+fx))^n\sqrt{\sec(e+fx)+1}dx}{2n+1}+\frac{2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(4n+1)\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)+1}dx}{2n+1}+\frac{2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \; 4293$$

$$\frac{2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}-\frac{d(4n+1)\tan(e+fx)\int\frac{(d\sec(e+fx))^{n-1}}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f(2n+1)\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \; 74$$

$$\frac{2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}-\frac{(4n+1)\tan(e+fx)(d\sec(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},n,n+1,\sec(e+fx)\right)}{fn(2n+1)\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

input
```
Int[(d*Sec[e + f*x])^n*(1 + Sec[e + f*x])^(3/2),x]
```

output
```
(2*(d*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[1 + Sec[e + f*x]]) -
 ((1 + 4*n)*Hypergeometric2F1[1/2, n, 1 + n, Sec[e + f*x]]*(d*Sec[e + f*x]
)^n*Tan[e + f*x])/(f*n*(1 + 2*n)*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f
*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Maple [F]

$$\int (d\sec(fx+e))^{n} (1+\sec(fx+e))^{\frac{3}{2}} dx$$

input
```
int((d*sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x)
```

output
```
int((d*sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \int (d\sec(fx+e))^{n} (\sec(fx+e)+1)^{\frac{3}{2}} dx$$

input
```
integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

Sympy [F]

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \int (d\sec{(e+fx)})^n\,(\sec{(e+fx)}+1)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))**n*(1+sec(f*x+e))**(3/2),x)
```

output
```
Integral((d*sec(e + f*x))**n*(sec(e + f*x) + 1)**(3/2), x)
```

Maxima [F]

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \int (d\sec{(fx+e)})^n\,(\sec{(fx+e)}+1)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

Giac [F]

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \int (d\sec{(fx+e)})^n\,(\sec{(fx+e)}+1)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \int \left(\frac{1}{\cos\left(e+f\,x\right)}+1\right)^{3/2}\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^n\,dx$$

input
```
int((1/cos(e + f*x) + 1)^(3/2)*(d/cos(e + f*x))^n,x)
```

output
```
int((1/cos(e + f*x) + 1)^(3/2)*(d/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = d^n\left(\int \sec\left(fx+e\right)^n\sqrt{\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)dx + \int \sec\left(fx+e\right)^n\sqrt{\sec\left(fx+e\right)+1}dx\right)$$

input
```
int((d*sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x)
```

output
```
d**n*(int(sec(e + f*x)**n*sqrt(sec(e + f*x) + 1)*sec(e + f*x),x) + int(sec
(e + f*x)**n*sqrt(sec(e + f*x) + 1),x))
```

3.304 $\int (d\sec(e+fx))^n\sqrt{1+\sec(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 67

$$\int (d\sec(e+fx))^n\sqrt{1+\sec(e+fx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\sin(e+fx)}{f\sqrt{1+\sec(e+fx)}}$$

output
```
2*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*sec(f*x+e)^(1-n)*(d*sec(f*x+e))
^n*sin(f*x+e)/f/(1+sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int (d\sec(e+fx))^n\sqrt{1+\sec(e+fx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\sin(e+fx)}{f\sqrt{1+\sec(e+fx)}}$$

input
```
Integrate[(d*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]],x]
```

output
```
(2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Sec[e + f*x]^(1 -
n)*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*Sqrt[1 + Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4293, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(e+fx)+1}(d\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)+1}\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4293}$$

$$-\frac{d\tan(e+fx)\int \frac{(d\sec(e+fx))^{n-1}}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{74}$$

$$-\frac{\tan(e+fx)(d\sec(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},n,n+1,\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

input
```
Int[(d*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]],x]
```

output
```
-((Hypergeometric2F1[1/2, n, 1 + n, Sec[e + f*x]]*(d*Sec[e + f*x])^n*Tan[e
 + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int (d\sec (fx+e))^n \sqrt{1+\sec (fx+e)}dx$$

input
```
int((d*sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x)
```

output
```
int((d*sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int (d\sec(e+fx))^n\sqrt{1+\sec(e+fx)}\,dx = \int (d\sec (fx+e))^n \sqrt{\sec (fx+e)+1}\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

Sympy [F]

$$\int (d\sec(e+fx))^n\sqrt{1+\sec(e+fx)}\,dx=\int (d\sec{(e+fx)})^n\,\sqrt{\sec{(e+fx)}+1}\,dx$$

input
```
integrate((d*sec(f*x+e))**n*(1+sec(f*x+e))**(1/2),x)
```

output
```
Integral((d*sec(e + f*x))**n*sqrt(sec(e + f*x) + 1), x)
```

Maxima [F]

$$\int (d\sec(e+fx))^n\sqrt{1+\sec(e+fx)}\,dx=\int (d\sec{(fx+e)})^n\,\sqrt{\sec{(fx+e)}+1}\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

Giac [F]

$$\int (d\sec(e+fx))^n\sqrt{1+\sec(e+fx)}\,dx=\int (d\sec{(fx+e)})^n\,\sqrt{\sec{(fx+e)}+1}\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^n\sqrt{1+\sec(e+fx)}\,dx=\int \sqrt{\frac{1}{\cos(e+f\,x)}+1}\,\left(\frac{d}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((1/cos(e + f*x) + 1)^(1/2)*(d/cos(e + f*x))^n,x)
```

output
```
int((1/cos(e + f*x) + 1)^(1/2)*(d/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (d\sec(e+fx))^n\sqrt{1+\sec(e+fx)}\,dx=d^n\left(\int \sec(fx+e)^n\sqrt{\sec(fx+e)+1}dx\right)$$

input
```
int((d*sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x)
```

output
```
d**n*int(sec(e + f*x)**n*sqrt(sec(e + f*x) + 1),x)
```

3.305 $\int \frac{(d\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 81

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = \frac{\text{AppellF1}\left(\frac{1}{2},1-n,1,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\sin(e+fx}{f\sqrt{1+\sec(e+fx)}}$$

output
```
AppellF1(1/2,1-n,1,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*sec(f*x+e)^(1-n)*(
d*sec(f*x+e))^n*sin(f*x+e)/f/(1+sec(f*x+e))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1021 vs. 2(81) = 162.

Time = 5.73 (sec) , antiderivative size = 1021, normalized size of antiderivative = 12.60

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[(d*Sec[e + f*x])^n/Sqrt[1 + Sec[e + f*x]],x]
```

output
```
(30*d*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*(d*Sec[e + f*x])^(-1 + n)*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]
)/(f*(30*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] + 60*n*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Sin[(e + f*x)/2]^2 + 60*n*AppellF1[1/2, -1/2 + n, 1
- n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[e + f*x]*Sin[(e + f
*x)/2]^2 + 5*(2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*(1 + Cos[e + f*x])*Sec[(e + f*
x)/2]^2*Tan[(e + f*x)/2]^2 - (9*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x])*Sec[(e + f*x)/2]^2*T
an[(e + f*x)/2]^2*(10*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 5*(-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1
- n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (4*(2 - 3*n + n^2)*Ap
pellF1[5/2, -1/2 + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + (-1 + 2*n)*(4*(-1 + n)*AppellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2] + (1 + 2*n)*AppellF1[5/2, 3/2 + n, 1 - n, 7/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*Tan[(e + f*x)/2]^2))/(3*Appell
F1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (
2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan...
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4314, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{\sec(e+fx)+1}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{ 4314}$$

$$-\frac{d\tan(e+fx)\int \frac{(d\sec(e+fx))^{n-1}}{\sqrt{1-\sec(e+fx)}(\sec(e+fx)+1)}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{150}$$

$$-\frac{\tan(e+fx)\operatorname{AppellF1}\left(n,\frac{1}{2},1,n+1,\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

input
```
Int[(d*Sec[e + f*x])^n/Sqrt[1 + Sec[e + f*x]],x]
```

output
```
-((AppellF1[n, 1/2, 1, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e + f*x]
)^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Maple [F]

$$\int \frac{(d\sec(fx+e))^n}{\sqrt{1+\sec(fx+e)}}dx$$

input
```
int((d*sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x)
```

output
```
int((d*sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^n}{\sqrt{\sec(fx+e)+1}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

Sympy [F]

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{(d\sec(e+fx))^n}{\sqrt{\sec(e+fx)+1}}\,dx$$

input
```
integrate((d*sec(f*x+e))**n/(1+sec(f*x+e))**(1/2),x)
```

output
```
Integral((d*sec(e + f*x))**n/sqrt(sec(e + f*x) + 1), x)
```

Maxima [F]

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^n}{\sqrt{\sec(fx+e)+1}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^n}{\sqrt{\sec(fx+e)+1}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^n}{\sqrt{\frac{1}{\cos(e+f\,x)}+1}}\,dx$$

input
```
int((d/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(1/2),x)
```

output
```
int((d/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = d^n\left(\int \frac{\sec\left(fx+e\right)^n\sqrt{\sec\left(fx+e\right)+1}}{\sec\left(fx+e\right)+1}dx\right)$$

input
```
int((d*sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x)
```

output
```
d**n*int((sec(e + f*x)**n*sqrt(sec(e + f*x) + 1))/(sec(e + f*x) + 1),x)
```

3.306 $\int \frac{(d\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \frac{(d\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = \frac{\text{AppellF1}\left(\frac{1}{2},1-n,2,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\sec^{1-n}(e+fx)}{2f\sqrt{1+\sec(e+fx)}}$$

output
```
1/2*AppellF1(1/2,1-n,2,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*sec(f*x+e)^(1-
n)*(d*sec(f*x+e))^n*sin(f*x+e)/f/(1+sec(f*x+e))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3003 vs. 2(84) = 168.

Time = 6.14 (sec) , antiderivative size = 3003, normalized size of antiderivative = 35.75

$$\int \frac{(d\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Sec[e + f*x])^n/(1 + Sec[e + f*x])^(3/2),x]
```

output
```
(6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(1/2 - n + (-3 + 2*n)/2)*(d*Sec[
e + f*x])^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(
-1 + Tan[(e + f*x)/2]^2)^2)/(f*(1 + Sec[e + f*x])^(3/2)*(3*AppellF1[1/2, -
3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n
)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((12*AppellF1[1/2, -3/2 + n, 1 -
 n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f
*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x
)/2]^2*(-1 + Tan[(e + f*x)/2]^2))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 +
 n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*Appe
llF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*
Tan[(e + f*x)/2]^2) + (3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos
[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*Ap
pellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[...
```

Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4314, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^n}{(\sec(e+fx)+1)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{4314}$$

$$-\frac{d\tan(e+fx)\int \frac{(d\sec(e+fx))^{n-1}}{\sqrt{1-\sec(e+fx)}(\sec(e+fx)+1)^2}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{150}$$

$$-\frac{\tan(e+fx)\,\text{AppellF1}\left(n,\frac{1}{2},2,n+1,\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

input
```
Int[(d*Sec[e + f*x])^n/(1 + Sec[e + f*x])^(3/2),x]
```

output
```
-((AppellF1[n, 1/2, 2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e + f*x]
)^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Maple [F]

$$\int \frac{(d\sec(fx+e))^n}{(1+\sec(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((d*sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x)
```

output
```
int((d*sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \frac{(d\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(fx+e))^n}{(\sec(fx+e)+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^n*sqrt(sec(f*x + e) + 1)/(sec(f*x + e)^2 + 2*sec
(f*x + e) + 1), x)
```

Sympy [F]

$$\int \frac{(d\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(e+fx))^n}{(\sec(e+fx)+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))**n/(1+sec(f*x+e))**(3/2),x)
```

output
```
Integral((d*sec(e + f*x))**n/(sec(e + f*x) + 1)**(3/2), x)
```

Maxima [F]

$$\int \frac{(d\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{(d\sec{(fx+e)})^n}{(\sec{(fx+e)}+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^n/(sec(f*x + e) + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{(d\sec{(fx+e)})^n}{(\sec{(fx+e)}+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^n/(sec(f*x + e) + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^n}{\left(\frac{1}{\cos(e+f\,x)}+1\right)^{3/2}}\,dx$$

input
```
int((d/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(3/2),x)
```

output
```
int((d/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = d^n\left(\int \frac{\sec(fx+e)^n\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^2+2\sec(fx+e)+1}dx\right)$$

input
```
int((d*sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x)
```

output
```
d**n*int((sec(e + f*x)**n*sqrt(sec(e + f*x) + 1))/(sec(e + f*x)**2 + 2*sec
(e + f*x) + 1),x)
```

3.307 $\int \sec^n(e+fx)(a+a\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 177

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{5/2}\,dx = \frac{2a^3(7+4n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)(3+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2a^2\sec^{1+n}(e+fx)\sqrt{a+a\sec(e+fx)}\sin(e+fx)}{f(3+2n)} + \frac{2a^3(3+24n+16n^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)\tan(e+fx)}{f(1+2n)(3+2n)\sqrt{a+a\sec(e+fx)}}$$

output

```
2*a^3*(7+4*n)*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(1+2*n)/(3+2*n)/(a+a*sec(f*x+e
))^(1/2)+2*a^2*sec(f*x+e)^(1+n)*(a+a*sec(f*x+e))^(1/2)*sin(f*x+e)/f/(3+2*n
)+2*a^3*(16*n^2+24*n+3)*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*tan(f*x+e
)/f/(1+2*n)/(3+2*n)/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.20 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.26

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{5/2}\,dx =$$
$$-\frac{i2^{-\frac{5}{2}+n}e^{-\frac{1}{2}i(3+2n)(e+fx)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{\frac{3}{2}+n}\left(\frac{10e^{i(2+n)(e+fx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-1-n),\frac{4+n}{2},-e^{2i(e+fx)}\right)}{2+n}+\frac{5e^{i(4+n)(e+fx)}}{}\right.}{}$$

input
```
Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
((-I)*2^(-5/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(3/2 + n)*(
(10*E^(I*(2 + n)*(e + f*x))*Hypergeometric2F1[1, (-1 - n)/2, (4 + n)/2, -E
^((2*I)*(e + f*x))])/(2 + n) + (5*E^(I*(4 + n)*(e + f*x))*Hypergeometric2F
1[1, (1 - n)/2, (6 + n)/2, -E^((2*I)*(e + f*x))])/(4 + n) + (E^(I*n*(e + f
*x))*Hypergeometric2F1[1, -3/2 - n/2, 1 + n/2, -E^((2*I)*(e + f*x))])/n +
(5*E^(I*(1 + n)*(e + f*x))*Hypergeometric2F1[1, -1 - n/2, (3 + n)/2, -E^((
2*I)*(e + f*x))])/(1 + n) + (E^(I*(5 + n)*(e + f*x))*Hypergeometric2F1[1,
1 - n/2, (7 + n)/2, -E^((2*I)*(e + f*x))])/(5 + n) + (10*E^(I*(3 + n)*(e +
 f*x))*Hypergeometric2F1[1, -1/2*n, (5 + n)/2, -E^((2*I)*(e + f*x))])/(3 +
 n))*Sec[(e + f*x)/2]^5*(a*(1 + Sec[e + f*x]))^(5/2))/(E^((I/2)*(3 + 2*n)*
(e + f*x))*f*Sec[e + f*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4301, 27, 3042, 4504, 3042, 4293, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{5/2}\sec^n(e+fx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{2a\int \frac{1}{2}\sec^n(e+fx)\sqrt{\sec(e+fx)a+a}(a(4n+3)+a(4n+7)\sec(e+fx))dx}{2n+3}+\frac{2a^2\sin(e+fx)\sqrt{a\sec(e+fx)+a}\sec^{n+1}(e+fx)}{f(2n+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \sec^n(e+fx)\sqrt{\sec(e+fx)a+a}(a(4n+3)+a(4n+7)\sec(e+fx))dx}{2n+3}+\frac{2a^2\sin(e+fx)\sqrt{a\sec(e+fx)+a}\sec^{n+1}(e+fx)}{f(2n+3)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\left(a(4n+3)+a(4n+7)\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx}{2n+3}+\frac{2a^2\sin(e+fx)\sqrt{a\sec(e+fx)+a}\sec^{n+1}(e+fx)}{f(2n+3)}$$

$$\Big\downarrow \text{4504}$$

$$\frac{a\left(\frac{a(16n^2+24n+3)\int \sec^n(e+fx)\sqrt{\sec(e+fx)a+a}dx}{2n+1}+\frac{2a^2(4n+7)\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}\right)}{2n+3}+\frac{2a^2\sin(e+fx)\sqrt{a\sec(e+fx)+a}\sec^{n+1}(e+fx)}{f(2n+3)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\left(\frac{a(16n^2+24n+3)\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{2n+1}+\frac{2a^2(4n+7)\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}\right)}{2n+3}+\frac{2a^2\sin(e+fx)\sqrt{a\sec(e+fx)+a}\sec^{n+1}(e+fx)}{f(2n+3)}$$

$$\Big\downarrow \text{4293}$$

$$\frac{a\left(\frac{2a^2(4n+7)\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}-\frac{a^3(16n^2+24n+3)\tan(e+fx)\int \frac{\sec^{n-1}(e+fx)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}\right)}{2n+3}+\frac{2a^2\sin(e+fx)\sqrt{a\sec(e+fx)+a}\sec^{n+1}(e+fx)}{f(2n+3)}$$

$$\Big\downarrow \text{75}$$

$$\frac{a\left(\frac{2a^2(16n^2+24n+3)\tan(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2(4n+7)\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}\right)}{2n+3}+\frac{2a^2\sin(e+fx)\sqrt{a\sec(e+fx)+a}\sec^{n+1}(e+fx)}{f(2n+3)}$$

input
```
Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
(2*a^2*Sec[e + f*x]^(1 + n)*Sqrt[a + a*Sec[e + f*x]]*Sin[e + f*x])/(f*(3 +
 2*n)) + (a*((2*a^2*(7 + 4*n)*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2
*n)*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*(3 + 24*n + 16*n^2)*Hypergeometric2
F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a +
a*Sec[e + f*x]])))/(3 + 2*n)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4504
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Simp[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1))   Int[Sqrt[a + b*Csc[e + f*
x]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&
!LtQ[n, 0]
```

Maple [F]

$$\int \sec (fx+e)^{n} (a+a\sec (fx+e))^{\frac{5}{2}}\, dx$$

input
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(5/2),x)
```

output
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(5/2),x)
```

Fricas [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{5/2}\, dx=\int (a\sec (fx+e)+a)^{\frac{5}{2}} \sec (fx+e)^{n}\, dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x +
e) + a)*sec(f*x + e)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{5/2}\,dx = \int (a\sec(fx+e)+a)^{\frac{5}{2}}\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^(5/2)*sec(f*x + e)^n, x)
```

Giac [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{5/2}\,dx = \int (a\sec(fx+e)+a)^{\frac{5}{2}}\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^(5/2)*sec(f*x + e)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{5/2}\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)*(1/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)*(1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{5/2}\,dx = \sqrt{a}\,a^2\left(\int \sec(fx+e)^n\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2\,dx + 2\left(\int \sec(fx+e)^n\sqrt{\sec(fx+e)+1}\,\sec(fx+e)\,dx\right) + \int \sec(fx+e)^n\sqrt{\sec(fx+e)+1}dx\right)$$

input
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(5/2),x)
```

output
```
sqrt(a)*a**2*(int(sec(e + f*x)**n*sqrt(sec(e + f*x) + 1)*sec(e + f*x)**2,x
) + 2*int(sec(e + f*x)**n*sqrt(sec(e + f*x) + 1)*sec(e + f*x),x) + int(sec
(e + f*x)**n*sqrt(sec(e + f*x) + 1),x))
```

3.308 $\int \sec^n(e+fx)(a+a\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{3/2}\,dx = \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(1+4n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

output
```
2*a^2*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*a^2*(
1+4*n)*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*tan(f*x+e)/f/(1+2*n)/(a+a*
sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{3/2}\,dx = \frac{a\left(-1+(1+4n)\cos^{\frac{1}{2}+n}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{2}+n,\frac{3}{2},2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\sec^n}{fn}$$

input
```
Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(a*(-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n,
3/2, 2*Sin[(e + f*x)/2]^2])*Sec[e + f*x]^n*Sqrt[a*(1 + Sec[e + f*x])]*Tan[
(e + f*x)/2])/(f*n)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4301, 27, 2011, 3042, 4293, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{3/2}\sec^n(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{2a\int \frac{\sec^n(e+fx)(a(4n+1)+a\sec(e+fx)(4n+1))}{2\sqrt{\sec(e+fx)a+a}}dx}{2n+1}+\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{\sec^n(e+fx)(a(4n+1)+a\sec(e+fx)(4n+1))}{\sqrt{\sec(e+fx)a+a}}dx}{2n+1}+\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{2011}$$

$$\frac{a(4n+1)\int \sec^n(e+fx)\sqrt{\sec(e+fx)a+a}dx}{2n+1}+\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(4n+1)\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{2n+1}+\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{4293}$$

$$\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}-\frac{a^3(4n+1)\tan(e+fx)\int\frac{\sec^{n-1}(e+fx)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{75}$$

$$\frac{2a^2(4n+1)\tan(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(2*a^2*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e +
f*x]]) + (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f
*x]]*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Maple [F]

$$\int \sec \left(fx+e\right)^{n} \left(a+a\sec \left(fx+e\right)\right)^{\frac{3}{2}}dx$$

input
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(3/2),x)
```

output
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{3/2}\,dx=\int \left(a\sec \left(fx+e\right)+a\right)^{\frac{3}{2}}\sec \left(fx+e\right)^{n}\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^(3/2)*sec(f*x + e)^n, x)
```

Sympy [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{3/2}\,dx=\int (a(\sec(e+fx)+1))^{\frac{3}{2}}\sec^n(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(3/2)*sec(e + f*x)**n, x)
```

Maxima [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{3/2}\,dx=\int (a\sec(fx+e)+a)^{\frac{3}{2}}\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^(3/2)*sec(f*x + e)^n, x)
```

Giac [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{3/2}\,dx=\int (a\sec(fx+e)+a)^{\frac{3}{2}}\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^(3/2)*sec(f*x + e)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{3/2}\,dx=\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{3/2}\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^n\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)*(1/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)*(1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{3/2}\,dx=\sqrt{a}\,a\left(\int \sec\left(fx+e\right)^n\sqrt{\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)dx+\int \sec\left(fx+e\right)^n\sqrt{\sec\left(fx+e\right)+1}dx\right)$$

input
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sec(e + f*x)**n*sqrt(sec(e + f*x) + 1)*sec(e + f*x),x) + in
t(sec(e + f*x)**n*sqrt(sec(e + f*x) + 1),x))
```

3.309 $\int \sec^n(e+fx)\sqrt{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 48

$$\int \sec^n(e+fx)\sqrt{a+a\sec(e+fx)}\,dx = \frac{2a\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

output
```
2*a*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))
^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \sec^n(e+fx)\sqrt{a+a\sec(e+fx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{f}$$

input
```
Integrate[Sec[e + f*x]^n*Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Sqrt[a*(1 + Sec[e
+ f*x])]*Tan[(e + f*x)/2])/f
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 4293, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}\sec^n(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \text{4293}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\sec^{n-1}(e+fx)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{75}$$

$$\frac{2a\tan(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sec[e + f*x]^n*Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(f
*Sqrt[a + a*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sec(fx+e)^n \sqrt{a+a\sec(fx+e)}dx$$

input
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(1/2),x)
```

output
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \sec^n(e+fx)\sqrt{a+a\sec(e+fx)}\,dx = \int \sqrt{a\sec(fx+e)+a}\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

Sympy [F]

$$\int \sec^n(e+fx)\sqrt{a+a\sec(e+fx)}\,dx=\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sec^n\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)**n, x)
```

Maxima [F]

$$\int \sec^n(e+fx)\sqrt{a+a\sec(e+fx)}\,dx=\int \sqrt{a\sec\left(fx+e\right)+a}\sec\left(fx+e\right)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

Giac [F]

$$\int \sec^n(e+fx)\sqrt{a+a\sec(e+fx)}\,dx=\int \sqrt{a\sec\left(fx+e\right)+a}\sec\left(fx+e\right)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)\sqrt{a+a\sec(e+fx)}\,dx=\int \sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^n\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(1/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int \sec^n(e+fx)\sqrt{a+a\sec(e+fx)}\,dx=\sqrt{a}\left(\int \sec\left(fx+e\right)^{n}\sqrt{\sec\left(fx+e\right)+1}dx\right)$$

input
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(1/2),x)
```

output
```
sqrt(a)*int(sec(e + f*x)**n*sqrt(sec(e + f*x) + 1),x)
```

3.310 $\int \frac{\sec^n(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 61

$$\int \frac{\sec^n(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{\operatorname{AppellF1}\left(\frac{1}{2},1-n,1,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

output
```
AppellF1(1/2,1-n,1,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*tan(f*x+e)/f/(a+a*
sec(f*x+e))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1023 vs. 2(61) = 122.

Time = 5.95 (sec) , antiderivative size = 1023, normalized size of antiderivative = 16.77

$$\int \frac{\sec^n(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[e + f*x]^n/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(30*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*Sec[e + f*x]^(-1 + n)*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/
(a*f*(30*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] + 60*n*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Sin[(e + f*x)/2]^2 + 60*n*AppellF1[1/2, -1/2 + n, 1
- n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[e + f*x]*Sin[(e + f
*x)/2]^2 + 5*(2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*(1 + Cos[e + f*x])*Sec[(e + f*
x)/2]^2*Tan[(e + f*x)/2]^2 - (9*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x])*Sec[(e + f*x)/2]^2*T
an[(e + f*x)/2]^2*(10*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 5*(-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1
- n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (4*(2 - 3*n + n^2)*Ap
pellF1[5/2, -1/2 + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + (-1 + 2*n)*(4*(-1 + n)*AppellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2] + (1 + 2*n)*AppellF1[5/2, 3/2 + n, 1 - n, 7/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*Tan[(e + f*x)/2]^2))/(3*Appell
F1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (
2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan...
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4315, 3042, 4312, 148, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^n(e+fx)}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^n}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{\sqrt{\sec(e+fx)+1}\int \frac{\sec^n(e+fx)}{\sqrt{\sec(e+fx)+1}}dx}{\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\sec(e+fx)+1}\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^n}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)+1}}dx}{\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 4312}$$

$$\frac{\tan(e+fx)\int \frac{\sec^{n-1}(e+fx)}{\sqrt{1-\sec(e+fx)}(\sec(e+fx)+1)}d(1-\sec(e+fx))}{f\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 148}$$

$$\frac{2\tan(e+fx)\int \frac{\sec^{n-1}(e+fx)}{\sec(e+fx)+1}d\sqrt{1-\sec(e+fx)}}{f\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 333}$$

$$\frac{\tan(e+fx)\,\text{AppellF1}\left(\frac{1}{2},1-n,1,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sec[e + f*x]^n/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(AppellF1[1/2, 1 - n, 1, 3/2, 1 - Sec[e + f*x], (1 - Sec[e + f*x])/2]*Tan[
e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4312
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a - x)^(n - 1)
*((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{\sec (fx+e)^n}{\sqrt{a+a\sec (fx+e)}}dx$$

input
```
int(sec(f*x+e)^n/(a+a*sec(f*x+e))^(1/2),x)
```

output
```
int(sec(f*x+e)^n/(a+a*sec(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{\sec^n(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\sec (fx+e)^n}{\sqrt{a\sec (fx+e)+a}}\,dx$$

input
```
integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(f*x + e)^n/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{\sec^n(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\sec^n (e+fx)}{\sqrt{a(\sec (e+fx)+1)}}\,dx$$

input
```
integrate(sec(f*x+e)**n/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sec(e + f*x)**n/sqrt(a*(sec(e + f*x) + 1)), x)
```

Maxima [F]

$$\int \frac{\sec^n(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\sec(fx+e)^n}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^n/sqrt(a*sec(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{\sec^n(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\sec(fx+e)^n}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^n/sqrt(a*sec(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^n(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\left(\frac{1}{\cos(e+fx)}\right)^n}{\sqrt{a+\frac{a}{\cos(e+fx)}}}\,dx$$

input
```
int((1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sec^n(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sec(fx+e)^n\sqrt{\sec(fx+e)+1}}{\sec(fx+e)+1}dx\right)}{a}$$

input
```
int(sec(f*x+e)^n/(a+a*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(a)*int((sec(e + f*x)**n*sqrt(sec(e + f*x) + 1))/(sec(e + f*x) + 1),x
))/a
```

3.311 $\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 67

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx=\frac{\operatorname{AppellF1}\left(\frac{1}{2},1-n,2,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e+fx)}{2af\sqrt{a+a\sec(e+fx)}}$$

output
```
1/2*AppellF1(1/2,1-n,2,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*tan(f*x+e)/a/f
/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2992 vs. 2(67) = 134.

Time = 6.17 (sec) , antiderivative size = 2992, normalized size of antiderivative = 44.66

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx=\text{Result too large to show}$$

input
```
Integrate[Sec[e + f*x]^n/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(1/2 + (-3 + 2*n)/2)*(Cos[(e + f
*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2
)^2)/(f*(a*(1 + Sec[e + f*x]))^(3/2)*(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/
2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*A
ppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
])*Tan[(e + f*x)/2]^2)*((12*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(
Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1 + Tan[(e
 + f*x)/2]^2))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n,
 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)
 + (3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec
[e + f*x])^(3/2 + n)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 +
n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*Appe
llF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Ta...
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4315, 3042, 4312, 148, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^n(e+fx)}{(a\sec(e+fx)+a)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^n}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{\sqrt{\sec(e+fx)+1}\int \frac{\sec^n(e+fx)}{(\sec(e+fx)+1)^{3/2}}dx}{a\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(e+fx)+1}\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^n}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^{3/2}}dx}{a\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{4312}$$

$$\frac{\tan(e+fx)\int \frac{\sec^{n-1}(e+fx)}{\sqrt{1-\sec(e+fx)}(\sec(e+fx)+1)^2}d(1-\sec(e+fx))}{af\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{148}$$

$$\frac{2\tan(e+fx)\int \frac{\sec^{n-1}(e+fx)}{(\sec(e+fx)+1)^2}d\sqrt{1-\sec(e+fx)}}{af\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{333}$$

$$\frac{\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1-n,2,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{2af\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sec[e + f*x]^n/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(AppellF1[1/2, 1 - n, 2, 3/2, 1 - Sec[e + f*x], (1 - Sec[e + f*x])/2]*Tan[
e + f*x])/(2*a*f*Sqrt[a + a*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4312
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a - x)^(n - 1)
*((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{\sec(fx+e)^n}{(a+a\sec(fx+e))^{\frac{3}{2}}}dx$$

input
```
int(sec(f*x+e)^n/(a+a*sec(f*x+e))^(3/2),x)
```

output
```
int(sec(f*x+e)^n/(a+a*sec(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{\sec\left(fx+e\right)^n}{\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)^n/(a^2*sec(f*x + e)^2 + 2*a
^2*sec(f*x + e) + a^2), x)
```

Sympy [F]

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{\sec^n\left(e+fx\right)}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)**n/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Integral(sec(e + f*x)**n/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{\sec\left(fx+e\right)^n}{\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^n/(a*sec(f*x + e) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)^n}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^n/(a*sec(f*x + e) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(\frac{1}{\cos(e+f\,x)}\right)^n}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sec(fx+e)^n\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^2+2\sec(fx+e)+1}dx\right)}{a^2}$$

input
```
int(sec(f*x+e)^n/(a+a*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(a)*int((sec(e + f*x)**n*sqrt(sec(e + f*x) + 1))/(sec(e + f*x)**2 + 2
*sec(e + f*x) + 1),x))/a**2
```

3.312 $\int (-\sec(e+fx))^n (a+a\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 130

$$\int (-\sec(e+fx))^n (a+a\sec(e+fx))^{3/2}\,dx = \frac{2a^2(1+4n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)(-\sec(e+fx))^n \sec^{1-n}(e+ \ldots}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

output
```
2*a^2*(1+4*n)*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*(-sec(f*x+e))^n*sec
(f*x+e)^(1-n)*sin(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*a^2*(-sec(f*x+
e))^n*tan(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.68

$$\int (-\sec(e+fx))^n (a+a\sec(e+fx))^{3/2}\,dx = \frac{a\left(-1+(1+4n)\cos^{\frac{1}{2}+n}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{2}+n,\frac{3}{2},2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\right)(-\ldots}{fn}$$

input
```
Integrate[(-Sec[e + f*x])^n*(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(a*(-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n,
3/2, 2*Sin[(e + f*x)/2]^2])*(-Sec[e + f*x])^n*Sqrt[a*(1 + Sec[e + f*x])]*T
an[(e + f*x)/2])/(f*n)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4301, 27, 2011, 3042, 4293, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{3/2}(-\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{2a\int \frac{(-\sec(e+fx))^n(a(4n+1)+a\sec(e+fx)(4n+1))}{2\sqrt{\sec(e+fx)a+a}}dx}{2n+1}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{(-\sec(e+fx))^n(a(4n+1)+a\sec(e+fx)(4n+1))}{\sqrt{\sec(e+fx)a+a}}dx}{2n+1}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{2011}$$

$$\frac{a(4n+1)\int(-\sec(e+fx))^n\sqrt{\sec(e+fx)a+a}dx}{2n+1}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(4n+1)\int\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{2n+1}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 4293

$$\frac{a^3(4n+1)\tan(e+fx)\int\frac{(-\sec(e+fx))^{n-1}}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 77

$$\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}-\frac{a^3(4n+1)\sin(e+fx)(-\sec(e+fx))^n\sec^{1-n}(e+fx)\int\frac{\sec^{n-1}(e+fx)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 75

$$\frac{2a^2(4n+1)\sin(e+fx)(-\sec(e+fx))^n\sec^{1-n}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(-Sec[e + f*x])^n*(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*(-Se
c[e + f*x])^n*Sec[e + f*x]^(1 - n)*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*S
ec[e + f*x]]) + (2*a^2*(-Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a
 + a*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Maple [F]

$$\int (-\sec (fx+e))^n (a+a\sec (fx+e))^{\frac{3}{2}} dx$$

input
```
int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x)
```

output
```
int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int (-\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx=\int (a\sec (fx+e)+a)^{\frac{3}{2}}(-\sec (fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^(3/2)*(-sec(f*x + e))^n, x)
```

Sympy [F]

$$\int (-\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx=\int (-\sec (e+fx))^n (a(\sec (e+fx)+1))^{\frac{3}{2}}\,dx$$

input
```
integrate((-sec(f*x+e))**n*(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Integral((-sec(e + f*x))**n*(a*(sec(e + f*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int (-\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx = \int (a\sec(fx+e)+a)^{\frac{3}{2}}(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^(3/2)*(-sec(f*x + e))^n, x)
```

Giac [F]

$$\int (-\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx = \int (a\sec(fx+e)+a)^{\frac{3}{2}}(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^(3/2)*(-sec(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (-\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(-\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)*(-1/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)*(-1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int(-\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx=\sqrt{a}\,(-1)^n\,a\left(\int\sec\left(fx+e\right)^n\sqrt{\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)dx+\int\sec\left(fx+e\right)^n\sqrt{\sec\left(fx+e\right)+1}dx\right)$$

input
```
int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x)
```

output
```
sqrt(a)*( - 1)**n*a*(int(sec(e + f*x)**n*sqrt(sec(e + f*x) + 1)*sec(e + f*
x),x) + int(sec(e + f*x)**n*sqrt(sec(e + f*x) + 1),x))
```

3.313 $\int (-\sec(e+fx))^n \sqrt{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 70

$$\int (-\sec(e+fx))^n \sqrt{a+a\sec(e+fx)}\,dx = \frac{2a\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)(-\sec(e+fx))^n \sec^{1-n}(e+fx)\sin(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

output
```
2*a*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*(-sec(f*x+e))^n*sec(f*x+e)^(1
-n)*sin(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int (-\sec(e+fx))^n \sqrt{a+a\sec(e+fx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)(-\sec(e+fx))^n \sec^{-n}(e+fx)\sqrt{a(1+\sec(e+fx)}}{f}$$

input
```
Integrate[(-Sec[e + f*x])^n*Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*(-Sec[e + f*x])^n*
Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sec[e + f*x]^n)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 4293, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}(-\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4293}$$

$$\frac{a^2\tan(e+fx)\int \frac{(-\sec(e+fx))^{n-1}}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{77}$$

$$-\frac{a^2\sin(e+fx)(-\sec(e+fx))^n\sec^{1-n}(e+fx)\int \frac{\sec^{n-1}(e+fx)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{75}$$

$$\frac{2a\sin(e+fx)(-\sec(e+fx))^n\sec^{1-n}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(-Sec[e + f*x])^n*Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*(-Sec[e + f*x])^
n*Sec[e + f*x]^(1 - n)*Sin[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int (-\sec (fx+e))^n \sqrt{a+a\sec (fx+e)}dx$$

input
```
int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x)
```

output
```
int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int (-\sec(e+fx))^n\sqrt{a+a\sec(e+fx)}\,dx = \int \sqrt{a\sec(fx+e)+a}(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*(-sec(f*x + e))^n, x)
```

Sympy [F]

$$\int (-\sec(e+fx))^n\sqrt{a+a\sec(e+fx)}\,dx = \int (-\sec(e+fx))^n\sqrt{a(\sec(e+fx)+1)}\,dx$$

input
```
integrate((-sec(f*x+e))**n*(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral((-sec(e + f*x))**n*sqrt(a*(sec(e + f*x) + 1)), x)
```

Maxima [F]

$$\int (-\sec(e+fx))^n\sqrt{a+a\sec(e+fx)}\,dx = \int \sqrt{a\sec(fx+e)+a}(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)*(-sec(f*x + e))^n, x)
```

Giac [F]

$$\int (-\sec(e+fx))^n \sqrt{a+a\sec(e+fx)}\,dx = \int \sqrt{a\sec(fx+e)+a}(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)*(-sec(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (-\sec(e+fx))^n \sqrt{a+a\sec(e+fx)}\,dx = \int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(-\frac{1}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(-1/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(-1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (-\sec(e+fx))^n \sqrt{a+a\sec(e+fx)}\,dx = \sqrt{a}\,(-1)^n\left(\int \sec(fx+e)^n\sqrt{\sec(fx+e)+1}dx\right)$$

input
```
int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x)
```

output
```
sqrt(a)*( - 1)**n*int(sec(e + f*x)**n*sqrt(sec(e + f*x) + 1),x)
```

3.314 $\int \frac{(-\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 86

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{\operatorname{AppellF1}\left(\frac{1}{2},1-n,1,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)(-\sec(e+fx))^{-1+n}\sec^{2-n}(e+fx)\sin}{f\sqrt{a+a\sec(e+fx)}}$$

output
```
-AppellF1(1/2,1-n,1,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*(-sec(f*x+e))^(-1
+n)*sec(f*x+e)^(2-n)*sin(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1025 vs. 2(86) = 172.

Time = 6.17 (sec) , antiderivative size = 1025, normalized size of antiderivative = 11.92

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[(-Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(-30*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*(-Sec[e + f*x])^(-1 + n)*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/
2])/(a*f*(30*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + 60*n*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2]*Sin[(e + f*x)/2]^2 + 60*n*AppellF1[1/2, -1/2 + n
, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[e + f*x]*Sin[(e
 + f*x)/2]^2 + 5*(2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*(1 + Cos[e + f*x])*Sec[(e
+ f*x)/2]^2*Tan[(e + f*x)/2]^2 - (9*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x])*Sec[(e + f*x)/2]
^2*Tan[(e + f*x)/2]^2*(10*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 5*(-1 + 2*n)*AppellF1[3/2, 1/2 + n
, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (4*(2 - 3*n + n^2
)*AppellF1[5/2, -1/2 + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] + (-1 + 2*n)*(4*(-1 + n)*AppellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 + 2*n)*AppellF1[5/2, 3/2 + n, 1 - n,
7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*Tan[(e + f*x)/2]^2))/(3*Ap
pellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, ...
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4315, 3042, 4313, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{\sqrt{\sec(e+fx)+1}\int \frac{(-\sec(e+fx))^n}{\sqrt{\sec(e+fx)+1}}dx}{\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(e+fx)+1}\int \frac{\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)+1}}dx}{\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{4313}$$

$$\frac{\tan(e+fx)\int \frac{(-\sec(e+fx))^{n-1}}{\sqrt{1-\sec(e+fx)}(\sec(e+fx)+1)}d(\sec(e+fx)+1)}{f\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{153}$$

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n \operatorname{AppellF1}\left(n,\frac{1}{2},1,n+1,\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(-Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
-((AppellF1[n, 1/2, 1, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(-Sec[e + f*x])
^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] && !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4313
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-((-a)*(d/b))^n)*(Cot[e + f*x]/(a^(n - 1)*f*S
qrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[x^(m - 1/2)*
((a - x)^(n - 1)/Sqrt[2*a - x]), x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a
, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
&&  !IntegerQ[n] && LtQ[a*(d/b), 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{(-\sec(fx+e))^n}{\sqrt{a+a\sec(fx+e)}}dx$$

input
```
int((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

output
```
int((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(-\sec(fx+e))^n}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral((-sec(f*x + e))^n/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(-\sec{(e+fx)})^n}{\sqrt{a\left(\sec{(e+fx)}+1\right)}}\,dx$$

input
```
integrate((-sec(f*x+e))**n/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral((-sec(e + f*x))**n/sqrt(a*(sec(e + f*x) + 1)), x)
```

Maxima [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(-\sec{(fx+e)})^n}{\sqrt{a\sec{(fx+e)}+a}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((-sec(f*x + e))^n/sqrt(a*sec(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(-\sec{(fx+e)})^n}{\sqrt{a\sec{(fx+e)}+a}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((-sec(f*x + e))^n/sqrt(a*sec(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\left(-\frac{1}{\cos(e+f\,x)}\right)^n}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((-1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((-1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{\sqrt{a}\,(-1)^n\left(\int \frac{\sec(fx+e)^n\sqrt{\sec(fx+e)+1}}{\sec(fx+e)+1}dx\right)}{a}$$

input
```
int((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(a)*( - 1)**n*int((sec(e + f*x)**n*sqrt(sec(e + f*x) + 1))/(sec(e + f
*x) + 1),x))/a
```

3.315 $\int \frac{(-\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 91

$$\int \frac{(-\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{\operatorname{AppellF1}\left(\frac{1}{2},1-n,2,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)(-\sec(e+fx))^{-1+n}\sec^{2-n}(e+fx)\sin(e+}{2af\sqrt{a+a\sec(e+fx)}}$$

output
```
-1/2*AppellF1(1/2,1-n,2,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*(-sec(f*x+e))
^(-1+n)*sec(f*x+e)^(2-n)*sin(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2030 vs. 2(91) = 182.

Time = 6.05 (sec) , antiderivative size = 2030, normalized size of antiderivative = 22.31

$$\int \frac{(-\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(-Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(2*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*Cos[(e + f*x)/2]*(-Sec[e + f*x])^n*Sin[(e + f*x)/2]*(-1 + Tan[(e + f
*x)/2]^2)*(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 -
n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))/(f*
(a*(1 + Sec[e + f*x]))^(3/2)*(4*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2*(3*AppellF1[1/2, -3/
2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*
AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + AppellF1[1/2, -3/2 + n, 1 - n, 3
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(-1 + Tan[(e + f*x)/2]^2)*(3*
AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + 2*n*AppellF1[1/
2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[(e +
 f*x)/2]^2*(-1 + Tan[(e + f*x)/2]^2)*(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, ...
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4315, 3042, 4313, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-\sec(e+fx))^n}{(a\sec(e+fx)+a)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{\sqrt{\sec(e+fx)+1}\int \frac{(-\sec(e+fx))^n}{(\sec(e+fx)+1)^{3/2}}dx}{a\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\sec(e+fx)+1}\int \frac{\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^{3/2}}dx}{a\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 4313}$$

$$\frac{\tan(e+fx)\int \frac{(-\sec(e+fx))^{n-1}}{\sqrt{1-\sec(e+fx)}(\sec(e+fx)+1)^2}d(\sec(e+fx)+1)}{af\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 153}$$

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n \operatorname{AppellF1}\left(n,\frac{1}{2},2,n+1,\sec(e+fx),-\sec(e+fx)\right)}{afn\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(-Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
-((AppellF1[n, 1/2, 2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(-Sec[e + f*x])
^n*Tan[e + f*x])/(a*f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4313

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-((-a)*(d/b))^n)*(Cot[e + f*x]/(a^(n - 1)*f*S
qrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[x^(m - 1/2)*
((a - x)^(n - 1)/Sqrt[2*a - x]), x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a
, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
&&  !IntegerQ[n] && LtQ[a*(d/b), 0]
```

rule 4315

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{(-\sec (fx+e))^{n}}{(a+a\sec (fx+e))^{\frac{3}{2}}}dx$$

input

```
int((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

output

```
int((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \frac{(-\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{(-\sec (fx+e))^{n}}{(a\sec (fx+e)+a)^{\frac{3}{2}}}\,dx$$

input

```
integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output

```
integral(sqrt(a*sec(f*x + e) + a)*(-sec(f*x + e))^n/(a^2*sec(f*x + e)^2 +
2*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F]

$$\int \frac{(-\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{(-\sec(e+fx))^n}{(a(\sec(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((-sec(f*x+e))**n/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Integral((-sec(e + f*x))**n/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(-\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{(-\sec(fx+e))^n}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-sec(f*x + e))^n/(a*sec(f*x + e) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(-\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{(-\sec(fx+e))^n}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((-sec(f*x + e))^n/(a*sec(f*x + e) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(-\frac{1}{\cos(e+f\,x)}\right)^n}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((-1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((-1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(-\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{a}\,(-1)^n \left(\int \frac{\sec(fx+e)^n\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^2+2\sec(fx+e)+1}dx\right)}{a^2}$$

input
```
int((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(a)*( - 1)**n*int((sec(e + f*x)**n*sqrt(sec(e + f*x) + 1))/(sec(e + f
*x)**2 + 2*sec(e + f*x) + 1),x))/a**2
```

3.316 $\int (d\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 130

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx = \frac{2a^2(1+4n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)\sec^{1-n}(e+fx)(d\sec(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

output
```
2*a^2*(1+4*n)*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*sec(f*x+e)^(1-n)*(d
*sec(f*x+e))^n*sin(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*a^2*(d*sec(f*
x+e))^n*tan(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.68

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx = \frac{a\left(-1+(1+4n)\cos^{\frac{1}{2}+n}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{2}+n,\frac{3}{2},2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\right)(d\,\text{se}}{fn}$$

input
```
Integrate[(d*Sec[e + f*x])^n*(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(a*(-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n,
3/2, 2*Sin[(e + f*x)/2]^2])*(d*Sec[e + f*x])^n*Sqrt[a*(1 + Sec[e + f*x])]*
Tan[(e + f*x)/2])/(f*n)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4301, 27, 2011, 3042, 4293, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{3/2}(d\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{ 4301}$$

$$\frac{2a\int \frac{(d\sec(e+fx))^n(a(4n+1)+a\sec(e+fx)(4n+1))}{2\sqrt{\sec(e+fx)a+a}}dx}{2n+1}+\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a\int \frac{(d\sec(e+fx))^n(a(4n+1)+a\sec(e+fx)(4n+1))}{\sqrt{\sec(e+fx)a+a}}dx}{2n+1}+\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 2011}$$

$$\frac{a(4n+1)\int (d\sec(e+fx))^n\sqrt{\sec(e+fx)a+a}dx}{2n+1}+\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a(4n+1)\int \left(d\csc \left(e+fx+\frac{\pi}{2}\right)\right)^n \sqrt{\csc \left(e+fx+\frac{\pi}{2}\right) a+a}dx}{2n+1}+\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 4293

$$\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}-\frac{a^3d(4n+1)\tan(e+fx)\int \frac{(d\sec(e+fx))^{n-1}}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 77

$$\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}-\frac{a^3(4n+1)\sin(e+fx)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\int \frac{\sec^{n-1}(e+fx)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 75

$$\frac{2a^2(4n+1)\sin(e+fx)\sec^{1-n}(e+fx)(d\sec(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(d*Sec[e + f*x])^n*(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Sec[
e + f*x]^(1 - n)*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*
Sec[e + f*x]]) + (2*a^2*(d*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt
[a + a*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Maple [F]

$$\int (d\sec(fx+e))^n (a+a\sec(fx+e))^{\frac{3}{2}} dx$$

input
```
int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x)
```

output
```
int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx = \int (a\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

Sympy [F]

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx = \int (a(\sec(e+fx)+1))^{\frac{3}{2}}(d\sec(e+fx))^n\,dx$$

input
```
integrate((d*sec(f*x+e))**n*(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(3/2)*(d*sec(e + f*x))**n, x)
```

Maxima [F]

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx=\int (a\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

Giac [F]

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx=\int (a\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(\frac{d}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)*(d/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)*(d/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx = d^n\sqrt{a}\,a\left(\int \sec(fx+e)^n\sqrt{\sec(fx+e)+1}\,\sec(fx+e)\,dx+\int \sec(fx+e)^n\sqrt{\sec(fx+e)+1}dx\right)$$

input
```
int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x)
```

output
```
d**n*sqrt(a)*a*(int(sec(e + f*x)**n*sqrt(sec(e + f*x) + 1)*sec(e + f*x),x)
 + int(sec(e + f*x)**n*sqrt(sec(e + f*x) + 1),x))
```

3.317 $\int (d\sec(e+fx))^n\sqrt{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 70

$$\int (d\sec(e+fx))^n\sqrt{a+a\sec(e+fx)}\,dx = \frac{2a\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\sin(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

output
```
2*a*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*sec(f*x+e)^(1-n)*(d*sec(f*x+e
))^n*sin(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int (d\sec(e+fx))^n\sqrt{a+a\sec(e+fx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)\sec^{-n}(e+fx)(d\sec(e+fx))^n\sqrt{a(1+\sec(e+fx))}}{f}$$

input
```
Integrate[(d*Sec[e + f*x])^n*Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*(d*Sec[e + f*x])^n
*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sec[e + f*x]^n)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 4293, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}(d\sec(e+fx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \; 4293$$

$$-\frac{a^2 d\tan(e+fx)\int \frac{(d\sec(e+fx))^{n-1}}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 77$$

$$-\frac{a^2\sin(e+fx)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\int \frac{\sec^{n-1}(e+fx)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 75$$

$$\frac{2a\sin(e+fx)\sec^{1-n}(e+fx)(d\sec(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(d*Sec[e + f*x])^n*Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Sec[e + f*x]^(1
- n)*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int (d\sec(fx+e))^n \sqrt{a+a\sec(fx+e)}dx$$

input
```
int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x)
```

output
```
int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int (d\sec(e+fx))^n\sqrt{a+a\sec(e+fx)}\,dx=\int \sqrt{a\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)\right)^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Sympy [F]

$$\int (d\sec(e+fx))^n\sqrt{a+a\sec(e+fx)}\,dx=\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(d\sec\left(e+fx\right)\right)^n\,dx$$

input
```
integrate((d*sec(f*x+e))**n*(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*(d*sec(e + f*x))**n, x)
```

Maxima [F]

$$\int (d\sec(e+fx))^n\sqrt{a+a\sec(e+fx)}\,dx=\int \sqrt{a\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)\right)^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Giac [F]

$$\int (d\sec(e+fx))^n\sqrt{a+a\sec(e+fx)}\,dx=\int \sqrt{a\sec(fx+e)+a}(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^n\sqrt{a+a\sec(e+fx)}\,dx=\int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(\frac{d}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(d/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(d/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (d\sec(e+fx))^n\sqrt{a+a\sec(e+fx)}\,dx=d^n\sqrt{a}\left(\int \sec(fx+e)^n\sqrt{\sec(fx+e)+1}dx\right)$$

input
```
int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x)
```

output
```
d**n*sqrt(a)*int(sec(e + f*x)**n*sqrt(sec(e + f*x) + 1),x)
```

3.318 $\int \frac{(d\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 83

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{\text{AppellF1}\left(\frac{1}{2},1-n,1,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\sin(e+fx}{f\sqrt{a+a\sec(e+fx)}}$$

output
```
AppellF1(1/2,1-n,1,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*sec(f*x+e)^(1-n)*(
d*sec(f*x+e))^n*sin(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1026 vs. 2(83) = 166.

Time = 5.83 (sec) , antiderivative size = 1026, normalized size of antiderivative = 12.36

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[(d*Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(30*d*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*(d*Sec[e + f*x])^(-1 + n)*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x
)/2])/(a*f*(30*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2] + 60*n*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*Sin[(e + f*x)/2]^2 + 60*n*AppellF1[1/2, -1/2 +
 n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[e + f*x]*Sin[
(e + f*x)/2]^2 + 5*(2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 -
n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*(1 + Cos[e + f*x])*Sec[(
e + f*x)/2]^2*Tan[(e + f*x)/2]^2 - (9*AppellF1[1/2, -1/2 + n, 1 - n, 3/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x])*Sec[(e + f*x)/
2]^2*Tan[(e + f*x)/2]^2*(10*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 5*(-1 + 2*n)*AppellF1[3/2, 1/2 +
 n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (4*(2 - 3*n + n
^2)*AppellF1[5/2, -1/2 + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2] + (-1 + 2*n)*(4*(-1 + n)*AppellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 + 2*n)*AppellF1[5/2, 3/2 + n, 1 - n
, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*Tan[(e + f*x)/2]^2))/(3*
AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2...
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4315, 3042, 4314, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \ 4315$$

$$\frac{\sqrt{\sec(e+fx)+1}\int \frac{(d\sec(e+fx))^n}{\sqrt{\sec(e+fx)+1}}dx}{\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\sec(e+fx)+1}\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)+1}}dx}{\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 4314}$$

$$-\frac{d\tan(e+fx)\int \frac{(d\sec(e+fx))^{n-1}}{\sqrt{1-\sec(e+fx)}(\sec(e+fx)+1)}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 150}$$

$$-\frac{\tan(e+fx)\operatorname{AppellF1}\left(n,\frac{1}{2},1,n+1,\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{fn\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(d*Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
-((AppellF1[n, 1/2, 1, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e + f*x]
)^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{(d \sec (fx+e))^n}{\sqrt{a+a \sec (fx+e)}}dx$$

input
```
int((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

output
```
int((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{(d \sec(e+fx))^n}{\sqrt{a+a \sec(e+fx)}}\,dx = \int \frac{(d \sec (fx+e))^n}{\sqrt{a \sec (fx+e)+a}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^n/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(d\sec(e+fx))^n}{\sqrt{a(\sec(e+fx)+1)}}\,dx$$

input
```
integrate((d*sec(f*x+e))**n/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral((d*sec(e + f*x))**n/sqrt(a*(sec(e + f*x) + 1)), x)
```

Maxima [F]

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^n}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^n/sqrt(a*sec(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^n}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^n/sqrt(a*sec(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^n}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((d/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((d/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{d^n\sqrt{a}\left(\int \frac{\sec(fx+e)^n\sqrt{\sec(fx+e)+1}}{\sec(fx+e)+1}dx\right)}{a}$$

input
```
int((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

output
```
(d**n*sqrt(a)*int((sec(e + f*x)**n*sqrt(sec(e + f*x) + 1))/(sec(e + f*x) +
 1),x))/a
```

3.319 $\int \frac{(d\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 89

$$\int \frac{(d\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\operatorname{AppellF1}\left(\frac{1}{2},1-n,2,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\sec^{1-n}(e+f}{2af\sqrt{a+a\sec(e+fx)}}$$

output
```
1/2*AppellF1(1/2,1-n,2,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*sec(f*x+e)^(1-
n)*(d*sec(f*x+e))^n*sin(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2030 vs. $2(89) = 178$.

Time = 6.04 (sec) , antiderivative size = 2030, normalized size of antiderivative = 22.81

$$\int \frac{(d\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(2*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*Cos[(e + f*x)/2]*(d*Sec[e + f*x])^n*Sin[(e + f*x)/2]*(-1 + Tan[(e +
f*x)/2]^2)*(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 -
 n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))/(f
*(a*(1 + Sec[e + f*x]))^(3/2)*(4*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2*(3*AppellF1[1/2, -3
/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)
*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + AppellF1[1/2, -3/2 + n, 1 - n,
3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(-1 + Tan[(e + f*x)/2]^2)*(3
*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + 2*n*AppellF1[1
/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[(e
+ f*x)/2]^2*(-1 + Tan[(e + f*x)/2]^2)*(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2,...
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4315, 3042, 4314, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^n}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{\sqrt{\sec(e+fx)+1}\int \frac{(d\sec(e+fx))^n}{(\sec(e+fx)+1)^{3/2}}dx}{a\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\sec(e+fx)+1}\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^{3/2}}dx}{a\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 4314}$$

$$-\frac{d\tan(e+fx)\int \frac{(d\sec(e+fx))^{n-1}}{\sqrt{1-\sec(e+fx)}(\sec(e+fx)+1)^2}d\sec(e+fx)}{af\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 150}$$

$$-\frac{\tan(e+fx)\operatorname{AppellF1}\left(n,\frac{1}{2},2,n+1,\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{afn\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(d*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
-((AppellF1[n, 1/2, 2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e + f*x]
)^n*Tan[e + f*x])/(a*f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{(d\sec (fx+e))^n}{(a+a\sec (fx+e))^{\frac{3}{2}}}dx$$

input
```
int((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

output
```
int((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \frac{(d\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{(d\sec (fx+e))^n}{(a\sec (fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e))^n/(a^2*sec(f*x + e)^2 +
 2*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F]

$$\int \frac{(d\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{(d\sec{(e+fx)})^n}{(a(\sec{(e+fx)}+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))**n/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Integral((d*sec(e + f*x))**n/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(d\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{(d\sec{(fx+e)})^n}{(a\sec{(fx+e)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^n/(a*sec(f*x + e) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{(d\sec{(fx+e)})^n}{(a\sec{(fx+e)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^n/(a*sec(f*x + e) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^n}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((d/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((d/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(d\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx=\frac{d^n\sqrt{a}\left(\int \frac{\sec(fx+e)^n\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^2+2\sec(fx+e)+1}dx\right)}{a^2}$$

input
```
int((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

output
```
(d**n*sqrt(a)*int((sec(e + f*x)**n*sqrt(sec(e + f*x) + 1))/(sec(e + f*x)**
2 + 2*sec(e + f*x) + 1),x))/a**2
```

3.320 $\int (-\sec(e+fx))^n(a-a\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 178

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^{5/2}\,dx = \frac{2a^3(3+24n+16n^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1+\sec(e+fx)\right)\tan(e+fx)}{f(1+2n)(3+2n)\sqrt{a-a\sec(e+fx)}} + \frac{2a^3(7+4n)(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)(3+2n)\sqrt{a-a\sec(e+fx)}} + \frac{2a^2(-\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\tan(e+fx)}{f(3+2n)}$$

output
```
2*a^3*(16*n^2+24*n+3)*hypergeom([1/2, 1-n],[3/2],1+sec(f*x+e))*tan(f*x+e)/
f/(1+2*n)/(3+2*n)/(a-a*sec(f*x+e))^(1/2)+2*a^3*(7+4*n)*(-sec(f*x+e))^n*tan
(f*x+e)/f/(1+2*n)/(3+2*n)/(a-a*sec(f*x+e))^(1/2)+2*a^2*(-sec(f*x+e))^n*(a-
a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(3+2*n)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.74 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.35

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^{5/2}\,dx =$$
$$-\frac{2^{-\frac{5}{2}+n}e^{-\frac{1}{2}i(e+fx)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{\frac{1}{2}+n}\left(1+e^{2i(e+fx)}\right)^{\frac{1}{2}+n}\csc^5\left(\frac{1}{2}(e+fx)\right)\left(-\frac{\text{Hypergeometric2F1}\left(\frac{n}{2},\frac{5}{2}+n,\frac{2+n}{2},-e^{2i(e+fx}\right.}{n}\right.}{}$$

input
```
Integrate[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(5/2),x]
```

output
```
-((2^(-5/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(1/2 + n)*(1 +
 E^((2*I)*(e + f*x)))^(1/2 + n)*Csc[(e + f*x)/2]^5*(-(Hypergeometric2F1[n/
2, 5/2 + n, (2 + n)/2, -E^((2*I)*(e + f*x))]/n) + (5*E^(I*(e + f*x))*Hyper
geometric2F1[(1 + n)/2, 5/2 + n, (3 + n)/2, -E^((2*I)*(e + f*x))])/(1 + n)
 - (10*E^((2*I)*(e + f*x))*Hypergeometric2F1[(2 + n)/2, 5/2 + n, (4 + n)/2
, -E^((2*I)*(e + f*x))])/(2 + n) + (10*E^((3*I)*(e + f*x))*Hypergeometric2
F1[5/2 + n, (3 + n)/2, (5 + n)/2, -E^((2*I)*(e + f*x))])/(3 + n) - (5*E^((
4*I)*(e + f*x))*Hypergeometric2F1[5/2 + n, (4 + n)/2, (6 + n)/2, -E^((2*I)
*(e + f*x))])/(4 + n) + (E^((5*I)*(e + f*x))*Hypergeometric2F1[5/2 + n, (5
 + n)/2, (7 + n)/2, -E^((2*I)*(e + f*x))])/(5 + n))*(-Sec[e + f*x])^n*Sec[
e + f*x]^(-5/2 - n)*(a - a*Sec[e + f*x])^(5/2))/(E^((I/2)*(e + f*x))*f))
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 4301, 27, 3042, 4504, 3042, 4293, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-a\sec(e+fx))^{5/2}(-\sec(e+fx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a-a\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{ 4301}$$

$$\frac{2a^2\tan(e+fx)\sqrt{a-a\sec(e+fx)}(-\sec(e+fx))^n}{f(2n+3)}-\frac{2a\int -\frac{1}{2}(-\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}(a(4n+3)-a(4n+7)\sec(e+fx))dx}{2n+3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a\int(-\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}(a(4n+3)-a(4n+7)\sec(e+fx))dx}{2n+3}+\frac{2a^2\tan(e+fx)\sqrt{a-a\sec(e+fx)}(-\sec(e+fx))^n}{f(2n+3)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a\int\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\sqrt{a-a\csc\left(e+fx+\frac{\pi}{2}\right)}\left(a(4n+3)-a(4n+7)\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx}{2n+3}+\frac{2a^2\tan(e+fx)\sqrt{a-a\sec(e+fx)}(-\sec(e+fx))^n}{f(2n+3)}$$

$$\Big\downarrow \text{ 4504}$$

$$\frac{a\left(\frac{a(16n^2+24n+3)\int(-\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}dx}{2n+1}+\frac{2a^2(4n+7)\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}\right)}{2n+3}+\frac{2a^2\tan(e+fx)\sqrt{a-a\sec(e+fx)}(-\sec(e+fx))^n}{f(2n+3)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a\left(\frac{a(16n^2+24n+3)\int\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\sqrt{a-a\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{2n+1}+\frac{2a^2(4n+7)\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}\right)}{2n+3}+\frac{2a^2\tan(e+fx)\sqrt{a-a\sec(e+fx)}(-\sec(e+fx))^n}{f(2n+3)}$$

$$\Big\downarrow \text{ 4293}$$

$$\frac{a\left(\frac{a^3(16n^2+24n+3)\tan(e+fx)\int\frac{(-\sec(e+fx))^{n-1}}{\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^2(4n+7)\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}\right)}{2n+3}+\frac{2a^2\tan(e+fx)\sqrt{a-a\sec(e+fx)}(-\sec(e+fx))^n}{f(2n+3)}$$

$$\Big\downarrow 75$$

$$\frac{a\left(\frac{2a^2(16n^2+24n+3)\tan(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},\sec(e+fx)+1\right)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2(4n+7)\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}\right)}{2n+3}+\frac{2a^2\tan(e+fx)\sqrt{a-a\sec(e+fx)}(-\sec(e+fx))^n}{f(2n+3)}$$

input
```
Int[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(5/2),x]
```

output
```
(2*a^2*(-Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]]*Tan[e + f*x])/(f*(3 + 2*
n)) + (a*((2*a^2*(3 + 24*n + 16*n^2)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1
+ Sec[e + f*x]]*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a - a*Sec[e + f*x]]) + (2*
a^2*(7 + 4*n)*(-Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a - a*Sec[
e + f*x]])))/(3 + 2*n)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4504
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Simp[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1))   Int[Sqrt[a + b*Csc[e + f*
x]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&
!LtQ[n, 0]
```

Maple [F]

$$\int (-\sec (fx+e))^{n} (a-a\sec (fx+e))^{\frac{5}{2}} dx$$

input
```
int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(5/2),x)
```

output
```
int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(5/2),x)
```

Fricas [F]

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^{5/2}\,dx=\int (-a\sec (fx+e)+a)^{\frac{5}{2}}(-\sec (fx+e))^{n}\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
integral((a^2*sec(f*x + e)^2 - 2*a^2*sec(f*x + e) + a^2)*sqrt(-a*sec(f*x +
 e) + a)*(-sec(f*x + e))^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((-sec(f*x+e))**n*(a-a*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (-\sec(e+fx))^n(a - a\sec(e+fx))^{5/2}\,dx = \int (-a\sec(fx+e)+a)^{\frac{5}{2}}(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((-a*sec(f*x + e) + a)^(5/2)*(-sec(f*x + e))^n, x)
```

Giac [F]

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^{5/2}\,dx = \int (-a\sec(fx+e)+a)^{\frac{5}{2}}(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate((-a*sec(f*x + e) + a)^(5/2)*(-sec(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^{5/2}\,dx = \int \left(a-\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(-\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a - a/cos(e + f*x))^(5/2)*(-1/cos(e + f*x))^n,x)
```

output
```
int((a - a/cos(e + f*x))^(5/2)*(-1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^{5/2}\,dx = \sqrt{a}\,(-1)^n a^2\left(\left(\int \sec(fx+e)^n\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2\,dx\right.\right.$$
$$\left.-2\left(\int \sec(fx+e)^n\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)\right.$$
$$\left.+\int \sec(fx+e)^n\sqrt{-\sec(fx+e)+1}dx\right)$$

input
```
int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(5/2),x)
```

output
```
sqrt(a)*( - 1)**n*a**2*(int(sec(e + f*x)**n*sqrt( - sec(e + f*x) + 1)*sec(
e + f*x)**2,x) - 2*int(sec(e + f*x)**n*sqrt( - sec(e + f*x) + 1)*sec(e + f
*x),x) + int(sec(e + f*x)**n*sqrt( - sec(e + f*x) + 1),x))
```

3.321 $\int (-\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 108

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx = \frac{2a^2(1+4n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1+\sec(e+fx)\right)\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{2a^2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}}$$

output
```
2*a^2*(1+4*n)*hypergeom([1/2, 1-n],[3/2],1+sec(f*x+e))*tan(f*x+e)/f/(1+2*n
)/(a-a*sec(f*x+e))^(1/2)+2*a^2*(-sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(a-a*s
ec(f*x+e))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.12 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.93

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx = \frac{2^{-\frac{3}{2}+n}e^{-\frac{1}{2}i(e+fx)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{\frac{1}{2}+n}\left(1+e^{2i(e+fx)}\right)^{\frac{1}{2}+n}\csc^3\left(\frac{1}{2}(e+fx)\right)\left(-\frac{\text{Hypergeometric2F1}\left(\frac{n}{2},\frac{3}{2}+\cdots\right.}{n}\right.}{\cdots}$$

input
```
Integrate[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(3/2),x]
```

output
```
(2^(-3/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(1/2 + n)*(1 + E
^((2*I)*(e + f*x)))^(1/2 + n)*Csc[(e + f*x)/2]^3*(-(Hypergeometric2F1[n/2,
 3/2 + n, (2 + n)/2, -E^((2*I)*(e + f*x))]/n) + (3*E^(I*(e + f*x))*Hyperge
ometric2F1[(1 + n)/2, 3/2 + n, (3 + n)/2, -E^((2*I)*(e + f*x))])/(1 + n) -
 (3*E^((2*I)*(e + f*x))*Hypergeometric2F1[3/2 + n, (2 + n)/2, (4 + n)/2, -
E^((2*I)*(e + f*x))])/(2 + n) + (E^((3*I)*(e + f*x))*Hypergeometric2F1[3/2
 + n, (3 + n)/2, (5 + n)/2, -E^((2*I)*(e + f*x))])/(3 + n))*(-Sec[e + f*x]
)^n*Sec[e + f*x]^(-3/2 - n)*(a - a*Sec[e + f*x])^(3/2))/(E^((I/2)*(e + f*x
))*f)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3042, 4301, 27, 2011, 3042, 4293, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-a\sec(e+fx))^{3/2}(-\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}-\frac{2a\int -\frac{(-\sec(e+fx))^n(a(4n+1)-a(4n+1)\sec(e+fx))}{2\sqrt{a-a\sec(e+fx)}}dx}{2n+1}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{(-\sec(e+fx))^n(a(4n+1)-a(4n+1)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}}dx}{2n+1}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{2011}$$

$$\frac{a(4n+1)\int(-\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}dx}{2n+1}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(4n+1)\int\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\sqrt{a-a\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{2n+1}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{4293}$$

$$\frac{a^3(4n+1)\tan(e+fx)\int\frac{(-\sec(e+fx))^{n-1}}{\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{75}$$

$$\frac{2a^2(4n+1)\tan(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},\sec(e+fx)+1\right)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

input
```
Int[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(3/2),x]
```

output
```
(2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]*Tan[
e + f*x])/(f*(1 + 2*n)*Sqrt[a - a*Sec[e + f*x]]) + (2*a^2*(-Sec[e + f*x])^
n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a - a*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Maple [F]

$$\int (-\sec (fx+e))^n (a-a\sec (fx+e))^{\frac{3}{2}} dx$$

input
```
int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x)
```

output
```
int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int(-\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx=\int(-a\sec(fx+e)+a)^{\frac{3}{2}}(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(-(a*sec(f*x + e) - a)*sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n
, x)
```

Sympy [F]

$$\int(-\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx=\int(-\sec(e+fx))^n(-a(\sec(e+fx)-1))^{\frac{3}{2}}\,dx$$

input
```
integrate((-sec(f*x+e))**n*(a-a*sec(f*x+e))**(3/2),x)
```

output
```
Integral((-sec(e + f*x))**n*(-a*(sec(e + f*x) - 1))**(3/2), x)
```

Maxima [F]

$$\int(-\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx=\int(-a\sec(fx+e)+a)^{\frac{3}{2}}(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-a*sec(f*x + e) + a)^(3/2)*(-sec(f*x + e))^n, x)
```

Giac [F]

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx = \int (-a\sec(fx+e)+a)^{\frac{3}{2}}(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((-a*sec(f*x + e) + a)^(3/2)*(-sec(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx = \int \left(a-\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(-\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a - a/cos(e + f*x))^(3/2)*(-1/cos(e + f*x))^n,x)
```

output
```
int((a - a/cos(e + f*x))^(3/2)*(-1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx = \sqrt{a}\,(-1)^n\,a\left(-\left(\int \sec(fx+e)^n\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)+\int \sec(fx+e)^n\sqrt{-\sec(fx+e)+1}dx\right)$$

input
```
int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x)
```

output
```
sqrt(a)*( - 1)**n*a*( - int(sec(e + f*x)**n*sqrt( - sec(e + f*x) + 1)*sec(
e + f*x),x) + int(sec(e + f*x)**n*sqrt( - sec(e + f*x) + 1),x))
```

3.322 $\int (-\sec(e+fx))^n \sqrt{a-a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 47

$$\int (-\sec(e+fx))^n \sqrt{a-a\sec(e+fx)}\,dx = \frac{2a\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1+\sec(e+fx)\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}}$$

output
```
2*a*hypergeom([1/2, 1-n],[3/2],1+sec(f*x+e))*tan(f*x+e)/f/(a-a*sec(f*x+e))
^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int (-\sec(e+fx))^n \sqrt{a-a\sec(e+fx)}\,dx = -\frac{2\cot\left(\frac{1}{2}(e+fx)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1+\sec(e+fx)\right)\sqrt{a-a\sec(e+fx)}}{f}$$

input
```
Integrate[(-Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]],x]
```

output
```
(-2*Cot[(e + f*x)/2]*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]*
Sqrt[a - a*Sec[e + f*x]])/f
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 4293, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-a\sec(e+fx)}(-\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a-a\csc\left(e+fx+\frac{\pi}{2}\right)}\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4293}$$

$$\frac{a^2\tan(e+fx)\int \frac{(-\sec(e+fx))^{n-1}}{\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{75}$$

$$\frac{2a\tan(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},\sec(e+fx)+1\right)}{f\sqrt{a-a\sec(e+fx)}}$$

input
```
Int[(-Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]],x]
```

output
```
(2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]*Tan[e + f*x])/(f
*Sqrt[a - a*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int (-\sec (fx+e))^n \sqrt{a-a\sec (fx+e)}dx$$

input
```
int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x)
```

output
```
int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int (-\sec(e+fx))^n \sqrt{a-a\sec(e+fx)}\,dx = \int \sqrt{-a\sec (fx+e)+a}(-\sec (fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n, x)
```

Sympy [F]

$$\int (-\sec(e+fx))^n \sqrt{a-a\sec(e+fx)}\,dx = \int (-\sec{(e+fx)})^n \sqrt{-a(\sec{(e+fx)}-1)}\,dx$$

input
```
integrate((-sec(f*x+e))**n*(a-a*sec(f*x+e))**(1/2),x)
```

output
```
Integral((-sec(e + f*x))**n*sqrt(-a*(sec(e + f*x) - 1)), x)
```

Maxima [F]

$$\int (-\sec(e+fx))^n \sqrt{a-a\sec(e+fx)}\,dx = \int \sqrt{-a\sec{(fx+e)}+a}(-\sec{(fx+e)})^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n, x)
```

Giac [F]

$$\int (-\sec(e+fx))^n \sqrt{a-a\sec(e+fx)}\,dx = \int \sqrt{-a\sec{(fx+e)}+a}(-\sec{(fx+e)})^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (-\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\,dx=\int \sqrt{a-\frac{a}{\cos\left(e+f\,x\right)}}\,\left(-\frac{1}{\cos\left(e+f\,x\right)}\right)^n\,dx$$

input
```
int((a - a/cos(e + f*x))^(1/2)*(-1/cos(e + f*x))^n,x)
```

output
```
int((a - a/cos(e + f*x))^(1/2)*(-1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (-\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\,dx=\sqrt{a}\,(-1)^n\left(\int \sec\left(fx+e\right)^n\sqrt{-\sec\left(fx+e\right)+1}dx\right)$$

input
```
int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x)
```

output
```
sqrt(a)*( - 1)**n*int(sec(e + f*x)**n*sqrt( - sec(e + f*x) + 1),x)
```

3.323 $\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 58

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}\,dx = \frac{\operatorname{AppellF1}\left(\frac{1}{2},1-n,1,\frac{3}{2},1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}}$$

output
```
AppellF1(1/2,1-n,1,3/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*tan(f*x+e)/f/(a-a*
sec(f*x+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}\,dx = \int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}\,dx$$

input
```
Integrate[(-Sec[e + f*x])^n/Sqrt[a - a*Sec[e + f*x]],x]
```

output
```
Integrate[(-Sec[e + f*x])^n/Sqrt[a - a*Sec[e + f*x]], x]
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 4315, 3042, 4312, 148, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\sqrt{a-a\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{\sqrt{1-\sec(e+fx)}\int \frac{(-\sec(e+fx))^n}{\sqrt{1-\sec(e+fx)}}dx}{\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{1-\sec(e+fx)}\int \frac{\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\sqrt{1-\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{4312}$$

$$\frac{\tan(e+fx)\int \frac{(-\sec(e+fx))^{n-1}}{(1-\sec(e+fx))\sqrt{\sec(e+fx)+1}}d(\sec(e+fx)+1)}{f\sqrt{\sec(e+fx)+1}\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{148}$$

$$\frac{2\tan(e+fx)\int \frac{(-\sec(e+fx))^{n-1}}{1-\sec(e+fx)}d\sqrt{\sec(e+fx)+1}}{f\sqrt{\sec(e+fx)+1}\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{333}$$

$$\frac{\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1-n,1,\frac{3}{2},\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f\sqrt{a-a\sec(e+fx)}}$$

input
```
Int[(-Sec[e + f*x])^n/Sqrt[a - a*Sec[e + f*x]],x]
```

output
```
(AppellF1[1/2, 1 - n, 1, 3/2, 1 + Sec[e + f*x], (1 + Sec[e + f*x])/2]*Tan[
e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4312
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a - x)^(n - 1)
*((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{(-\sec(fx+e))^n}{\sqrt{a-a\sec(fx+e)}}dx$$

input
```
int((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(1/2),x)
```

output
```
int((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}\,dx = \int \frac{(-\sec(fx+e))^n}{\sqrt{-a\sec(fx+e)+a}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n/(a*sec(f*x + e) - a)
, x)
```

Sympy [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}\,dx = \int \frac{(-\sec(e+fx))^n}{\sqrt{-a(\sec(e+fx)-1)}}\,dx$$

input
```
integrate((-sec(f*x+e))**n/(a-a*sec(f*x+e))**(1/2),x)
```

output
```
Integral((-sec(e + f*x))**n/sqrt(-a*(sec(e + f*x) - 1)), x)
```

Maxima [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}\,dx = \int \frac{(-\sec(fx+e))^n}{\sqrt{-a\sec(fx+e)+a}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((-sec(f*x + e))^n/sqrt(-a*sec(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}\,dx = \int \frac{(-\sec(fx+e))^n}{\sqrt{-a\sec(fx+e)+a}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((-sec(f*x + e))^n/sqrt(-a*sec(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}\,dx = \int \frac{\left(-\frac{1}{\cos(e+f\,x)}\right)^n}{\sqrt{a-\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((-1/cos(e + f*x))^n/(a - a/cos(e + f*x))^(1/2),x)
```

output
```
int((-1/cos(e + f*x))^n/(a - a/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}\,dx = -\frac{\sqrt{a}\,(-1)^n \left(\int \frac{\sec(fx+e)^n\sqrt{-\sec(fx+e)+1}}{\sec(fx+e)-1}dx\right)}{a}$$

input
```
int((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(1/2),x)
```

output
```
( - sqrt(a)*( - 1)**n*int((sec(e + f*x)**n*sqrt( - sec(e + f*x) + 1))/(sec
(e + f*x) - 1),x))/a
```

3.324 $\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 64

$$\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx = \frac{\text{AppellF1}\left(\frac{1}{2},1-n,2,\frac{3}{2},1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)\tan(e+fx)}{2af\sqrt{a-a\sec(e+fx)}}$$

output
```
1/2*AppellF1(1/2,1-n,2,3/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*tan(f*x+e)/a/f
/(a-a*sec(f*x+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx = \int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx$$

input
```
Integrate[(-Sec[e + f*x])^n/(a - a*Sec[e + f*x])^(3/2),x]
```

output
```
Integrate[(-Sec[e + f*x])^n/(a - a*Sec[e + f*x])^(3/2), x]
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 4315, 3042, 4312, 148, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\left(a-a\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{\sqrt{1-\sec(e+fx)}\int \frac{(-\sec(e+fx))^n}{(1-\sec(e+fx))^{3/2}}dx}{a\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{1-\sec(e+fx)}\int \frac{\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\left(1-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{a\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{4312}$$

$$\frac{\tan(e+fx)\int \frac{(-\sec(e+fx))^{n-1}}{(1-\sec(e+fx))^2\sqrt{\sec(e+fx)+1}}d(\sec(e+fx)+1)}{af\sqrt{\sec(e+fx)+1}\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{148}$$

$$\frac{2\tan(e+fx)\int \frac{(-\sec(e+fx))^{n-1}}{(1-\sec(e+fx))^2}d\sqrt{\sec(e+fx)+1}}{af\sqrt{\sec(e+fx)+1}\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{333}$$

$$\frac{\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1-n,2,\frac{3}{2},\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{2af\sqrt{a-a\sec(e+fx)}}$$

input
```
Int[(-Sec[e + f*x])^n/(a - a*Sec[e + f*x])^(3/2),x]
```

output
```
(AppellF1[1/2, 1 - n, 2, 3/2, 1 + Sec[e + f*x], (1 + Sec[e + f*x])/2]*Tan[
e + f*x])/(2*a*f*Sqrt[a - a*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4312
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a - x)^(n - 1)
*((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{(-\sec(fx+e))^n}{(a-a\sec(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(3/2),x)
```

output
```
int((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx = \int \frac{(-\sec(fx+e))^n}{(-a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n/(a^2*sec(f*x + e)^2 -
 2*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F]

$$\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx = \int \frac{(-\sec(e+fx))^n}{(-a(\sec(e+fx)-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((-sec(f*x+e))**n/(a-a*sec(f*x+e))**(3/2),x)
```

output
```
Integral((-sec(e + f*x))**n/(-a*(sec(e + f*x) - 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx = \int \frac{(-\sec\left(fx+e\right))^n}{(-a\sec\left(fx+e\right)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-sec(f*x + e))^n/(-a*sec(f*x + e) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx = \int \frac{(-\sec\left(fx+e\right))^n}{(-a\sec\left(fx+e\right)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((-sec(f*x + e))^n/(-a*sec(f*x + e) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(-\frac{1}{\cos(e+f\,x)}\right)^n}{\left(a-\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((-1/cos(e + f*x))^n/(a - a/cos(e + f*x))^(3/2),x)
```

output
```
int((-1/cos(e + f*x))^n/(a - a/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{a}\,(-1)^n \left(\int \frac{\sec(fx+e)^n\sqrt{-\sec(fx+e)+1}}{\sec(fx+e)^2-2\sec(fx+e)+1}dx\right)}{a^2}$$

input
```
int((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(a)*( - 1)**n*int((sec(e + f*x)**n*sqrt( - sec(e + f*x) + 1))/(sec(e
+ f*x)**2 - 2*sec(e + f*x) + 1),x))/a**2
```

3.325 $\int \sec^n(e+fx)(a-a\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 130

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^{3/2}\,dx=\frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2(1+4n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1+\sec(e+fx)\right)(-\sec(e+fx))^{-n}\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}}$$

output

```
2*a^2*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(1+2*n)/(a-a*sec(f*x+e))^(1/2)+2*a^2*(
1+4*n)*hypergeom([1/2, 1-n],[3/2],1+sec(f*x+e))*sec(f*x+e)^(1+n)*sin(f*x+e
)/f/(1+2*n)/((-sec(f*x+e))^n)/(a-a*sec(f*x+e))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.78 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.32

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^{3/2}\,dx=\frac{2^{-\frac{3}{2}+n}e^{-\frac{1}{2}i(e+fx)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{\frac{1}{2}+n}\left(1+e^{2i(e+fx)}\right)^{\frac{1}{2}+n}\csc^3\left(\frac{1}{2}(e+fx)\right)\left(-\frac{\text{Hypergeometric2F1}\left(\frac{n}{2},\frac{3}{2}+\cdots\right.}{n}\right.}{\cdots}$$

input
```
Integrate[Sec[e + f*x]^n*(a - a*Sec[e + f*x])^(3/2),x]
```

output
```
(2^(-3/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(1/2 + n)*(1 + E
^((2*I)*(e + f*x)))^(1/2 + n)*Csc[(e + f*x)/2]^3*(-(Hypergeometric2F1[n/2,
 3/2 + n, (2 + n)/2, -E^((2*I)*(e + f*x))]/n) + (3*E^(I*(e + f*x))*Hyperge
ometric2F1[(1 + n)/2, 3/2 + n, (3 + n)/2, -E^((2*I)*(e + f*x))])/(1 + n) -
 (3*E^((2*I)*(e + f*x))*Hypergeometric2F1[3/2 + n, (2 + n)/2, (4 + n)/2, -
E^((2*I)*(e + f*x))])/(2 + n) + (E^((3*I)*(e + f*x))*Hypergeometric2F1[3/2
 + n, (3 + n)/2, (5 + n)/2, -E^((2*I)*(e + f*x))])/(3 + n))*(a - a*Sec[e +
 f*x])^(3/2))/(E^((I/2)*(e + f*x))*f*Sec[e + f*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4301, 27, 2011, 3042, 4293, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-a\sec(e+fx))^{3/2}\sec^n(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}\csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}-\frac{2a\int -\frac{\sec^n(e+fx)(a(4n+1)-a(4n+1)\sec(e+fx))}{2\sqrt{a-a\sec(e+fx)}}dx}{2n+1}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{\sec^n(e+fx)(a(4n+1)-a(4n+1)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}}dx}{2n+1}+\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{2011}$$

$$\frac{a(4n+1)\int \sec^n(e+fx)\sqrt{a-a\sec(e+fx)}dx}{2n+1}+\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

$\Big\downarrow$ 3042

$$\frac{a(4n+1)\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\sqrt{a-a\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{2n+1}+\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

$\Big\downarrow$ 4293

$$\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}-\frac{a^3(4n+1)\tan(e+fx)\int \frac{\sec^{n-1}(e+fx)}{\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 77

$$\frac{a^3(4n+1)\sin(e+fx)\sec^{n+1}(e+fx)(-\sec(e+fx))^{-n}\int \frac{(-\sec(e+fx))^{n-1}}{\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

$\Big\downarrow$ 75

$$\frac{2a^2(4n+1)\sin(e+fx)\sec^{n+1}(e+fx)(-\sec(e+fx))^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},\sec(e+fx)+1\right)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

input
```
Int[Sec[e + f*x]^n*(a - a*Sec[e + f*x])^(3/2),x]
```

output
```
(2*a^2*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[a - a*Sec[e +
f*x]]) + (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f
*x]]*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*(-Sec[e + f*x])^n*Sqr
t[a - a*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Maple [F]

$$\int \sec(fx+e)^n (a - a\sec(fx+e))^{\frac{3}{2}} dx$$

input
```
int(sec(f*x+e)^n*(a-a*sec(f*x+e))^(3/2),x)
```

output
```
int(sec(f*x+e)^n*(a-a*sec(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^{3/2} dx = \int (-a\sec(fx+e)+a)^{\frac{3}{2}} \sec(fx+e)^n \, dx$$

input
```
integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(-(a*sec(f*x + e) - a)*sqrt(-a*sec(f*x + e) + a)*sec(f*x + e)^n, x
)
```

Sympy [F]

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^{3/2} dx = \int (-a(\sec(e+fx)-1))^{\frac{3}{2}} \sec^n(e+fx) \, dx$$

input
```
integrate(sec(f*x+e)**n*(a-a*sec(f*x+e))**(3/2),x)
```

output
```
Integral((-a*(sec(e + f*x) - 1))**(3/2)*sec(e + f*x)**n, x)
```

Maxima [F]

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^{3/2}\,dx=\int (-a\sec(fx+e)+a)^{\frac{3}{2}}\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-a*sec(f*x + e) + a)^(3/2)*sec(f*x + e)^n, x)
```

Giac [F]

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^{3/2}\,dx=\int (-a\sec(fx+e)+a)^{\frac{3}{2}}\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((-a*sec(f*x + e) + a)^(3/2)*sec(f*x + e)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^{3/2}\,dx=\int \left(a-\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a - a/cos(e + f*x))^(3/2)*(1/cos(e + f*x))^n,x)
```

output
```
int((a - a/cos(e + f*x))^(3/2)*(1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^{3/2}\,dx = \sqrt{a}\,a\left(-\left(\int \sec(fx+e)^n\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)+\int \sec(fx+e)^n\sqrt{-\sec(fx+e)+1}dx\right)$$

input
```
int(sec(f*x+e)^n*(a-a*sec(f*x+e))^(3/2),x)
```

output
```
sqrt(a)*a*( - int(sec(e + f*x)**n*sqrt( - sec(e + f*x) + 1)*sec(e + f*x),x
) + int(sec(e + f*x)**n*sqrt( - sec(e + f*x) + 1),x))
```

3.326 $\int \sec^n(e+fx)\sqrt{a-a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 69

$$\int \sec^n(e+fx)\sqrt{a-a\sec(e+fx)}\,dx = \frac{2a\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1+\sec(e+fx)\right)(-\sec(e+fx))^{-n}\sec^{1+n}(e+fx)\sin(e+fx)}{f\sqrt{a-a\sec(e+fx)}}$$

output
```
2*a*hypergeom([1/2, 1-n],[3/2],1+sec(f*x+e))*sec(f*x+e)^(1+n)*sin(f*x+e)/f
/((-sec(f*x+e))^n)/(a-a*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \sec^n(e+fx)\sqrt{a-a\sec(e+fx)}\,dx = \frac{\cot\left(\frac{1}{2}(e+fx)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},n,1+n,-\sec(e+fx)\right)\sec^n(e+fx)\sqrt{a-a\sec(e+fx)}}{fn\sqrt{1+\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]^n*Sqrt[a - a*Sec[e + f*x]],x]
```

output
```
(Cot[(e + f*x)/2]*Hypergeometric2F1[1/2, n, 1 + n, -Sec[e + f*x]]*Sec[e +
f*x]^n*Sqrt[a - a*Sec[e + f*x]])/(f*n*Sqrt[1 + Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4293, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-a\sec(e+fx)}\sec^n(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a-a\csc\left(e+fx+\frac{\pi}{2}\right)}\csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \text{4293}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\sec^{n-1}(e+fx)}{\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{77}$$

$$\frac{a^2\sin(e+fx)(-\sec(e+fx))^{-n}\sec^{n+1}(e+fx)\int \frac{(-\sec(e+fx))^{n-1}}{\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{75}$$

$$\frac{2a\sin(e+fx)(-\sec(e+fx))^{-n}\sec^{n+1}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},\sec(e+fx)+1\right)}{f\sqrt{a-a\sec(e+fx)}}$$

input
```
Int[Sec[e + f*x]^n*Sqrt[a - a*Sec[e + f*x]],x]
```

output
```
(2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]*Sec[e + f*x]^(1
+ n)*Sin[e + f*x])/(f*(-Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sec(fx+e)^n \sqrt{a-a\sec(fx+e)}dx$$

input
```
int(sec(f*x+e)^n*(a-a*sec(f*x+e))^(1/2),x)
```

output
```
int(sec(f*x+e)^n*(a-a*sec(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \sec^n(e+fx)\sqrt{a-a\sec(e+fx)}\,dx=\int \sqrt{-a\sec\left(fx+e\right)+a}\sec\left(fx+e\right)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

Sympy [F]

$$\int \sec^n(e+fx)\sqrt{a-a\sec(e+fx)}\,dx=\int \sqrt{-a\left(\sec\left(e+fx\right)-1\right)}\sec^n\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)**n*(a-a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(-a*(sec(e + f*x) - 1))*sec(e + f*x)**n, x)
```

Maxima [F]

$$\int \sec^n(e+fx)\sqrt{a-a\sec(e+fx)}\,dx=\int \sqrt{-a\sec\left(fx+e\right)+a}\sec\left(fx+e\right)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

Giac [F]

$$\int \sec^n(e+fx)\sqrt{a-a\sec(e+fx)}\,dx=\int \sqrt{-a\sec\left(fx+e\right)+a}\sec\left(fx+e\right)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)\sqrt{a-a\sec(e+fx)}\,dx=\int \sqrt{a-\frac{a}{\cos\left(e+f\,x\right)}}\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^n\,dx$$

input
```
int((a - a/cos(e + f*x))^(1/2)*(1/cos(e + f*x))^n,x)
```

output
```
int((a - a/cos(e + f*x))^(1/2)*(1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int \sec^n(e+fx)\sqrt{a-a\sec(e+fx)}\,dx=\sqrt{a}\left(\int \sec\left(fx+e\right)^n\sqrt{-\sec\left(fx+e\right)+1}dx\right)$$

input
```
int(sec(f*x+e)^n*(a-a*sec(f*x+e))^(1/2),x)
```

output
```
sqrt(a)*int(sec(e + f*x)**n*sqrt( - sec(e + f*x) + 1),x)
```

3.327 $\int (d\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 130

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx=\frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}}$$
$$+\frac{2a^2(1+4n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1+\sec(e+fx)\right)(-\sec(e+fx))^{-n}(d\sec(e+fx))^n\tan(e+\ldots}{f(1+2n)\sqrt{a-a\sec(e+fx)}}$$

output
```
2*a^2*(d*sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(a-a*sec(f*x+e))^(1/2)+2*a^2*(
1+4*n)*hypergeom([1/2, 1-n],[3/2],1+sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+e
)/f/(1+2*n)/((-sec(f*x+e))^n)/(a-a*sec(f*x+e))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.43

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx=\frac{2^{-\frac{3}{2}+n}e^{-\frac{1}{2}i(e+fx)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{\frac{1}{2}+n}\left(1+e^{2i(e+fx)}\right)^{\frac{1}{2}+n}\csc^3\left(\frac{1}{2}(e+fx)\right)\left(-\frac{\text{Hypergeometric2F1}\left(\frac{n}{2},\frac{3}{2}+\ldots\right)}{n}\ldots\right.}{\ldots}$$

input
```
Integrate[(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(3/2),x]
```

output
```
(2^(-3/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(1/2 + n)*(1 + E
^((2*I)*(e + f*x)))^(1/2 + n)*Csc[(e + f*x)/2]^3*(-(Hypergeometric2F1[n/2,
 3/2 + n, (2 + n)/2, -E^((2*I)*(e + f*x))]/n) + (3*E^(I*(e + f*x))*Hyperge
ometric2F1[(1 + n)/2, 3/2 + n, (3 + n)/2, -E^((2*I)*(e + f*x))])/(1 + n) -
 (3*E^((2*I)*(e + f*x))*Hypergeometric2F1[3/2 + n, (2 + n)/2, (4 + n)/2, -
E^((2*I)*(e + f*x))])/(2 + n) + (E^((3*I)*(e + f*x))*Hypergeometric2F1[3/2
 + n, (3 + n)/2, (5 + n)/2, -E^((2*I)*(e + f*x))])/(3 + n))*Sec[e + f*x]^(
-3/2 - n)*(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(3/2))/(E^((I/2)*(e + f*
x))*f)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 4301, 27, 2011, 3042, 4293, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-a\sec(e+fx))^{3/2}(d\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}-\frac{2a\int -\frac{(d\sec(e+fx))^n(a(4n+1)-a(4n+1)\sec(e+fx))}{2\sqrt{a-a\sec(e+fx)}}dx}{2n+1}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{(d\sec(e+fx))^n(a(4n+1)-a(4n+1)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}}dx}{2n+1}+\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{2011}$$

$$\frac{a(4n+1)\int (d\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}dx}{2n+1}+\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(4n+1)\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\sqrt{a-a\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{2n+1}+\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{4293}$$

$$\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}-\frac{a^3d(4n+1)\tan(e+fx)\int \frac{(d\sec(e+fx))^{n-1}}{\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{77}$$

$$\frac{a^3(4n+1)\tan(e+fx)(-\sec(e+fx))^{-n}(d\sec(e+fx))^n\int \frac{(-\sec(e+fx))^{n-1}}{\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{75}$$

$$\frac{2a^2(4n+1)\tan(e+fx)(-\sec(e+fx))^{-n}(d\sec(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},\sec(e+fx)+1\right)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(3/2),x]
```

output
```
(2*a^2*(d*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a - a*Sec[e + f*
x]]) + (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x
]]*(d*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*(-Sec[e + f*x])^n*Sqrt[a
- a*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Maple [F]

$$\int (d\sec(fx+e))^n (a-a\sec(fx+e))^{\frac{3}{2}} dx$$

input
```
int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x)
```

output
```
int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx = \int (-a\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(-(a*sec(f*x + e) - a)*sqrt(-a*sec(f*x + e) + a)*(d*sec(f*x + e))^
n, x)
```

Sympy [F]

$$\int (d\sec(e+fx))^n(a - a\sec(e+fx))^{3/2}\,dx = \int (d\sec(e+fx))^n\,(-a(\sec(e+fx)-1))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))**n*(a-a*sec(f*x+e))**(3/2),x)
```

output
```
Integral((d*sec(e + f*x))**n*(-a*(sec(e + f*x) - 1))**(3/2), x)
```

Maxima [F]

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx=\int (-a\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

Giac [F]

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx=\int (-a\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((-a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx=\int \left(a-\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(\frac{d}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a - a/cos(e + f*x))^(3/2)*(d/cos(e + f*x))^n,x)
```

output
```
int((a - a/cos(e + f*x))^(3/2)*(d/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx = d^n\sqrt{a}\,a\left(-\left(\int \sec(fx+e)^n\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)+\int \sec(fx+e)^n\sqrt{-\sec(fx+e)+1}dx\right)$$

input
```
int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x)
```

output
```
d**n*sqrt(a)*a*( - int(sec(e + f*x)**n*sqrt( - sec(e + f*x) + 1)*sec(e + f
*x),x) + int(sec(e + f*x)**n*sqrt( - sec(e + f*x) + 1),x))
```

3.328 $\int (d\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 69

$$\int (d\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\,dx = \frac{2a\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1+\sec(e+fx)\right)(-\sec(e+fx))^{-n}(d\sec(e+fx))^n\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}}$$

output
```
2*a*hypergeom([1/2, 1-n],[3/2],1+sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+e)/f
/((-sec(f*x+e))^n)/(a-a*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int (d\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\,dx = \frac{\cot\left(\frac{1}{2}(e+fx)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},n,1+n,-\sec(e+fx)\right)(d\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}}{fn\sqrt{1+\sec(e+fx)}}$$

input
```
Integrate[(d*Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]],x]
```

output
```
(Cot[(e + f*x)/2]*Hypergeometric2F1[1/2, n, 1 + n, -Sec[e + f*x]]*(d*Sec[e
 + f*x])^n*Sqrt[a - a*Sec[e + f*x]])/(f*n*Sqrt[1 + Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 4293, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-a\sec(e+fx)}(d\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a-a\csc\left(e+fx+\frac{\pi}{2}\right)}\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4293}$$

$$-\frac{a^2d\tan(e+fx)\int \frac{(d\sec(e+fx))^{n-1}}{\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{77}$$

$$\frac{a^2\tan(e+fx)(-\sec(e+fx))^{-n}(d\sec(e+fx))^n\int \frac{(-\sec(e+fx))^{n-1}}{\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{75}$$

$$\frac{2a\tan(e+fx)(-\sec(e+fx))^{-n}(d\sec(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},\sec(e+fx)+1\right)}{f\sqrt{a-a\sec(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]],x]
```

output
```
(2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]*(d*Sec[e + f*x])
^n*Tan[e + f*x])/(f*(-Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int (d\sec(fx+e))^n \sqrt{a-a\sec(fx+e)}dx$$

input
```
int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x)
```

output
```
int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int (d\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\,dx = \int \sqrt{-a\sec\left(fx+e\right)+a}(d\sec\left(fx+e\right))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Sympy [F]

$$\int (d\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\,dx$$
$$= \int (d\sec\left(e+fx\right))^n\sqrt{-a\left(\sec\left(e+fx\right)-1\right)}\,dx$$

input
```
integrate((d*sec(f*x+e))**n*(a-a*sec(f*x+e))**(1/2),x)
```

output
```
Integral((d*sec(e + f*x))**n*sqrt(-a*(sec(e + f*x) - 1)), x)
```

Maxima [F]

$$\int (d\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\,dx = \int \sqrt{-a\sec\left(fx+e\right)+a}(d\sec\left(fx+e\right))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Giac [F]

$$\int (d\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\,dx=\int \sqrt{-a\sec(fx+e)+a}(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-a*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\,dx=\int \sqrt{a-\frac{a}{\cos(e+f\,x)}}\left(\frac{d}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a - a/cos(e + f*x))^(1/2)*(d/cos(e + f*x))^n,x)
```

output
```
int((a - a/cos(e + f*x))^(1/2)*(d/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (d\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\,dx=d^n\sqrt{a}\left(\int \sec(fx+e)^n\sqrt{-\sec(fx+e)+1}dx\right)$$

input
```
int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x)
```

output
```
d**n*sqrt(a)*int(sec(e + f*x)**n*sqrt( - sec(e + f*x) + 1),x)
```

3.329 $\int \sec^n(e+fx)(1+\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \sec^n(e+fx)(1+\sec(e+fx))^m\,dx = \frac{2^{\frac{1}{2}+m}\operatorname{AppellF1}\left(\frac{1}{2},1-n,\frac{1}{2}-m,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e+fx)}{f\sqrt{1+\sec(e+fx)}}$$

output
```
2^(1/2+m)*AppellF1(1/2,1-n,1/2-m,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*tan(
f*x+e)/f/(1+sec(f*x+e))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 821 vs. 2(72) = 144.

Time = 13.45 (sec) , antiderivative size = 821, normalized size of antiderivative = 11.40

$$\int \sec^n(e+fx)(1+\sec(e+fx))^m\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[e + f*x]^n*(1 + Sec[e + f*x])^m,x]
```

output
```
(30*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*Cos[(e + f*x)/2]*Sec[e + f*x]^n*(1 + Sec[e + f*x])^m*Sin[(e + f*x)/2])
/(f*(15*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2] + 30*(-1 + n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Sin[(e + f*x)/2]^2 + 10*((-1 + n)*AppellF1[3/2, m +
n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1
[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan
[(e + f*x)/2]^2 - (18*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2*(5*(-1 + n)*AppellF1[3/2, m + n,
2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 5*(m + n)*AppellF1[
3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*(
(2 - 3*n + n^2)*AppellF1[5/2, m + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + (m + n)*(2*(-1 + n)*AppellF1[5/2, 1 + m + n, 2 - n, 7/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 + m + n)*AppellF1[5/2, 2 +
m + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*Tan[(e + f*x
)/2]^2))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + 15*(m + n)*
AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^...
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 4312, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sec(e+fx)+1)^m \sec^n(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m \csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \text{4312}$$

$$\frac{\tan(e+fx)\int \frac{\sec^{n-1}(e+fx)(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}}d(1-\sec(e+fx))}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{ 150}$$

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)\,\text{AppellF1}\left(\frac{1}{2},1-n,\frac{1}{2}-m,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f\sqrt{\sec(e+fx)+1}}$$

input
```
Int[Sec[e + f*x]^n*(1 + Sec[e + f*x])^m,x]
```

output
```
(2^(1/2 + m)*AppellF1[1/2, 1 - n, 1/2 - m, 3/2, 1 - Sec[e + f*x], (1 - Sec
[e + f*x])/2]*Tan[e + f*x])/(f*Sqrt[1 + Sec[e + f*x]])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4312
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a - x)^(n - 1)
*((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

Maple [F]

$$\int \sec (fx+e)^n (1+\sec (fx+e))^m \, dx$$

input
```
int(sec(f*x+e)^n*(1+sec(f*x+e))^m,x)
```

output
```
int(sec(f*x+e)^n*(1+sec(f*x+e))^m,x)
```

Fricas [F]

$$\int \sec^n(e+fx)(1+\sec(e+fx))^m \, dx = \int (\sec (fx+e)+1)^m \sec (fx+e)^n \, dx$$

input
```
integrate(sec(f*x+e)^n*(1+sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

Sympy [F]

$$\int \sec^n(e+fx)(1+\sec(e+fx))^m \, dx = \int (\sec (e+fx)+1)^m \sec^n (e+fx) \, dx$$

input
```
integrate(sec(f*x+e)**n*(1+sec(f*x+e))**m,x)
```

output
```
Integral((sec(e + f*x) + 1)**m*sec(e + f*x)**n, x)
```

Maxima [F]

$$\int \sec^n(e+fx)(1+\sec(e+fx))^m\,dx = \int (\sec(fx+e)+1)^m \sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(1+sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

Giac [F]

$$\int \sec^n(e+fx)(1+\sec(e+fx))^m\,dx = \int (\sec(fx+e)+1)^m \sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(1+sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(1+\sec(e+fx))^m\,dx = \int \left(\frac{1}{\cos(e+f\,x)}+1\right)^m \left(\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((1/cos(e + f*x) + 1)^m*(1/cos(e + f*x))^n,x)
```

output
```
int((1/cos(e + f*x) + 1)^m*(1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int \sec^n(e+fx)(1+\sec(e+fx))^m\,dx = \int \sec\left(fx+e\right)^n\left(\sec\left(fx+e\right)+1\right)^m dx$$

input
```
int(sec(f*x+e)^n*(1+sec(f*x+e))^m,x)
```

output
```
int(sec(e + f*x)**n*(sec(e + f*x) + 1)**m,x)
```

3.330 $\int (1-\sec(e+fx))^m \sec^n(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 93

$$\int (1-\sec(e+fx))^m \sec^n(e+fx)\,dx = -\frac{2^{\frac{1}{2}+m}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2}-m,1-n,\frac{3}{2},\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(-\sec(e+fx))^{1-n}\sec^n(e+$$
$$f\sqrt{1-\sec(e+fx)}$$

output
```
-2^(1/2+m)*AppellF1(1/2,1-n,1/2-m,3/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(-s
ec(f*x+e))^(1-n)*sec(f*x+e)^n*sin(f*x+e)/f/(1-sec(f*x+e))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 255 vs. 2(93) = 186.

Time = 2.02 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.74

$$\int (1-\sec(e+fx))^m \sec^n(e+fx)\,dx$$
$$= \frac{(3+2m)\operatorname{Appell}}{f(1+2m)\left((3+2m)\operatorname{AppellF1}\left(\frac{1}{2}+m,m+n,1-n,\frac{3}{2}+m,\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+2\right.}$$

input
```
Integrate[(1 - Sec[e + f*x])^m*Sec[e + f*x]^n,x]
```

output
```
((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*(1 - Sec[e + f*x])^m*Sec[e + f*x]^n*Sin[e + f*x])/(f*(1
 + 2*m)*((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2 + m, m + n, 2 - n, 5
/2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2 +
m, 1 + m + n, 1 - n, 5/2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Ta
n[(e + f*x)/2]^2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4313, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-\sec(e+fx))^m \sec^n(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(1-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m \csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \text{4313}$$

$$\frac{\tan(e+fx)\int \frac{(1-\sec(e+fx))^{m-\frac{1}{2}}\sec^{n-1}(e+fx)}{\sqrt{\sec(e+fx)+1}}d(1-\sec(e+fx))}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{150}$$

$$\frac{\sqrt{2}\tan(e+fx)(1-\sec(e+fx))^m \text{AppellF1}\left(m+\frac{1}{2},1-n,\frac{1}{2},m+\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f(2m+1)\sqrt{\sec(e+fx)+1}}$$

input
```
Int[(1 - Sec[e + f*x])^m*Sec[e + f*x]^n,x]
```

output
```
(Sqrt[2]*AppellF1[1/2 + m, 1 - n, 1/2, 3/2 + m, 1 - Sec[e + f*x], (1 - Sec
[e + f*x])/2]*(1 - Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 + Sec
[e + f*x]])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4313
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-((-a)*(d/b))^n)*(Cot[e + f*x]/(a^(n - 1)*f*S
qrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[x^(m - 1/2)*
((a - x)^(n - 1)/Sqrt[2*a - x]), x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a
, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
&&  !IntegerQ[n] && LtQ[a*(d/b), 0]
```

Maple [F]

$$\int (1-\sec (fx+e))^m \sec (fx+e)^n \, dx$$

input
```
int((1-sec(f*x+e))^m*sec(f*x+e)^n,x)
```

output
```
int((1-sec(f*x+e))^m*sec(f*x+e)^n,x)
```

Fricas [F]

$$\int (1-\sec(e+fx))^m \sec^n(e+fx)\,dx = \int (-\sec(fx+e)+1)^m \sec(fx+e)^n\,dx$$

input
```
integrate((1-sec(f*x+e))^m*sec(f*x+e)^n,x, algorithm="fricas")
```

output
```
integral((-sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

Sympy [F]

$$\int (1-\sec(e+fx))^m \sec^n(e+fx)\,dx = \int (1-\sec(e+fx))^m \sec^n(e+fx)\,dx$$

input
```
integrate((1-sec(f*x+e))**m*sec(f*x+e)**n,x)
```

output
```
Integral((1 - sec(e + f*x))**m*sec(e + f*x)**n, x)
```

Maxima [F]

$$\int (1-\sec(e+fx))^m \sec^n(e+fx)\,dx = \int (-\sec(fx+e)+1)^m \sec(fx+e)^n\,dx$$

input
```
integrate((1-sec(f*x+e))^m*sec(f*x+e)^n,x, algorithm="maxima")
```

output
```
integrate((-sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

Giac [F]

$$\int (1-\sec(e+fx))^m \sec^n(e+fx)\,dx = \int (-\sec(fx+e)+1)^m \sec(fx+e)^n\,dx$$

input
```
integrate((1-sec(f*x+e))^m*sec(f*x+e)^n,x, algorithm="giac")
```

output
```
integrate((-sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (1-\sec(e+fx))^m \sec^n(e+fx)\,dx = \int \left(1-\frac{1}{\cos(e+f\,x)}\right)^m \left(\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((1 - 1/cos(e + f*x))^m*(1/cos(e + f*x))^n,x)
```

output
```
int((1 - 1/cos(e + f*x))^m*(1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (1-\sec(e+fx))^m \sec^n(e+fx)\,dx = \int \sec(fx+e)^n (-\sec(fx+e)+1)^m\,dx$$

input
```
int((1-sec(f*x+e))^m*sec(f*x+e)^n,x)
```

output
```
int(sec(e + f*x)**n*( - sec(e + f*x) + 1)**m,x)
```

3.331 $\int \sec^n(e+fx)(a+a\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 21, antiderivative size = 88

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^m\,dx = \frac{2^{\frac{1}{2}+m}\operatorname{AppellF1}\left(\frac{1}{2},1-n,\frac{1}{2}-m,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\,\mathrm{s}}{f}$$

output
```
2^(1/2+m)*AppellF1(1/2,1-n,1/2-m,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*(1+s
ec(f*x+e))^(-1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 823 vs. 2(88) = 176.

Time = 6.02 (sec) , antiderivative size = 823, normalized size of antiderivative = 9.35

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^m\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^m,x]
```

output
```
(30*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*Cos[(e + f*x)/2]*Sec[e + f*x]^n*(a*(1 + Sec[e + f*x]))^m*Sin[(e + f*x)
/2])/(f*(15*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] + 30*(-1 + n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2]*Sin[(e + f*x)/2]^2 + 10*((-1 + n)*AppellF1[3/2,
m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*Appe
llF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])
*Tan[(e + f*x)/2]^2 - (18*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2*(5*(-1 + n)*AppellF1[3/2, m +
 n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 5*(m + n)*Appel
lF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 2*((2 - 3*n + n^2)*AppellF1[5/2, m + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] + (m + n)*(2*(-1 + n)*AppellF1[5/2, 1 + m + n, 2 - n,
7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 + m + n)*AppellF1[5/2,
2 + m + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*Tan[(e +
 f*x)/2]^2))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + 15*(m +
 n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)...
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4315, 3042, 4312, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^n(e+fx)(a\sec(e+fx)+a)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m dx$$

$$\Big\downarrow \text{4315}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int \sec^n(e+fx)(\sec(e+fx)+1)^m dx$$

$$\Big\downarrow \text{3042}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m dx$$

$$\Big\downarrow \text{4312}$$

$$\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\int \frac{\sec^{n-1}(e+fx)(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}}d(1-\sec(e+fx))}{f\sqrt{1-\sec(e+fx)}}$$

$$\Big\downarrow \text{150}$$

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m \operatorname{AppellF1}\left(\frac{1}{2},1-n,\frac{1}{2}-m,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(\right.}{f}$$

input
```
Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^m,x]
```

output
```
(2^(1/2 + m)*AppellF1[1/2, 1 - n, 1/2 - m, 3/2, 1 - Sec[e + f*x], (1 - Sec
[e + f*x])/2]*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e +
 f*x])/f
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4312
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a - x)^(n - 1)
*((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \sec(fx+e)^n (a+a\sec(fx+e))^m \, dx$$

input
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^m,x)
```

output
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^m \, dx = \int (a\sec(fx+e)+a)^m \sec(fx+e)^n \, dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

Sympy [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^m\,dx = \int (a(\sec(e+fx)+1))^m \sec^n(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**m,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)**n, x)
```

Maxima [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^m\,dx = \int (a\sec(fx+e)+a)^m \sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

Giac [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^m\,dx = \int (a\sec(fx+e)+a)^m \sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^m\,dx=\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^m\,\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^n\,dx$$

input
```
int((a + a/cos(e + f*x))^m*(1/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^m*(1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^m\,dx=\int \sec\left(fx+e\right)^n\left(\sec\left(fx+e\right)a+a\right)^m dx$$

input
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^m,x)
```

output
```
int(sec(e + f*x)**n*(sec(e + f*x)*a + a)**m,x)
```

3.332 $\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 22, antiderivative size = 110

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx = -\frac{2^{\frac{1}{2}+m}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2}-m,1-n,\frac{3}{2},\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{-\frac{1}{2}-m}(-\text{se}}{f}$$

output
```
-2^(1/2+m)*AppellF1(1/2,1-n,1/2-m,3/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(1-
sec(f*x+e))^(-1/2-m)*(-sec(f*x+e))^(1-n)*sec(f*x+e)^n*(a-a*sec(f*x+e))^m*s
in(f*x+e)/f
```

Mathematica [F]

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx = \int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx$$

input
```
Integrate[Sec[e + f*x]^n*(a - a*Sec[e + f*x])^m,x]
```

output
```
Integrate[Sec[e + f*x]^n*(a - a*Sec[e + f*x])^m, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3042, 4315, 3042, 4313, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\left(a-a\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{4315}$$

$$(1-\sec(e+fx))^{-m}(a-a\sec(e+fx))^m\int(1-\sec(e+fx))^m\sec^n(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$(1-\sec(e+fx))^{-m}(a-a\sec(e+fx))^m\int\left(1-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m\csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \text{4313}$$

$$\frac{\tan(e+fx)(1-\sec(e+fx))^{-m-\frac{1}{2}}(a-a\sec(e+fx))^m\int\frac{(1-\sec(e+fx))^{m-\frac{1}{2}}\sec^{n-1}(e+fx)}{\sqrt{\sec(e+fx)+1}}d(1-\sec(e+fx))}{f\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{150}$$

$$\frac{\sqrt{2}\tan(e+fx)(a-a\sec(e+fx))^m\operatorname{AppellF1}\left(m+\frac{1}{2},1-n,\frac{1}{2},m+\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f(2m+1)\sqrt{\sec(e+fx)+1}}$$

input
```
Int[Sec[e + f*x]^n*(a - a*Sec[e + f*x])^m,x]
```

output
```
(Sqrt[2]*AppellF1[1/2 + m, 1 - n, 1/2, 3/2 + m, 1 - Sec[e + f*x], (1 - Sec
[e + f*x])/2]*(a - a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 + S
ec[e + f*x]])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4313
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-((-a)*(d/b))^n)*(Cot[e + f*x]/(a^(n - 1)*f*S
qrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[x^(m - 1/2)*
((a - x)^(n - 1)/Sqrt[2*a - x]), x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a
, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
&&  !IntegerQ[n] && LtQ[a*(d/b), 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \sec(fx+e)^n (a-a\sec(fx+e))^m\,dx$$

input
```
int(sec(f*x+e)^n*(a-a*sec(f*x+e))^m,x)
```

output
```
int(sec(f*x+e)^n*(a-a*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx=\int (-a\sec(fx+e)+a)^m\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((-a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

Sympy [F]

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx=\int (-a(\sec(e+fx)-1))^m\sec^n(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**n*(a-a*sec(f*x+e))**m,x)
```

output
```
Integral((-a*(sec(e + f*x) - 1))**m*sec(e + f*x)**n, x)
```

Maxima [F]

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx=\int (-a\sec(fx+e)+a)^m\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((-a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

Giac [F]

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx=\int(-a\sec(fx+e)+a)^m\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((-a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx=\int\left(a-\frac{a}{\cos(e+f\,x)}\right)^m\left(\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a - a/cos(e + f*x))^m*(1/cos(e + f*x))^n,x)
```

output
```
int((a - a/cos(e + f*x))^m*(1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx=\int\sec(fx+e)^n(-\sec(fx+e)\,a+a)^m\,dx$$

input
```
int(sec(f*x+e)^n*(a-a*sec(f*x+e))^m,x)
```

output
```
int(sec(e + f*x)**n*( - sec(e + f*x)*a + a)**m,x)
```

3.333 $\int (-\sec(e+fx))^n(1+\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^m\,dx = -\frac{2^{\frac{1}{2}+m}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2}-m,1-n,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx)),1-\sec(e+fx)\right)(-\sec(e+fx))^{-1+n}\sec^{2-n}(e}{f\sqrt{1+\sec(e+fx)}}$$

output
```
-2^(1/2+m)*AppellF1(1/2,1-n,1/2-m,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*(-s
ec(f*x+e))^(-1+n)*sec(f*x+e)^(2-n)*sin(f*x+e)/f/(1+sec(f*x+e))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 823 vs. 2(97) = 194.

Time = 1.91 (sec) , antiderivative size = 823, normalized size of antiderivative = 8.48

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^m\,dx = \text{Too large to display}$$

input
```
Integrate[(-Sec[e + f*x])^n*(1 + Sec[e + f*x])^m,x]
```

output
```
(30*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*Cos[(e + f*x)/2]*(-Sec[e + f*x])^n*(1 + Sec[e + f*x])^m*Sin[(e + f*x)/
2])/(f*(15*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2] + 30*(-1 + n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2]*Sin[(e + f*x)/2]^2 + 10*((-1 + n)*AppellF1[3/2, m
 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*Appel
lF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*
Tan[(e + f*x)/2]^2 - (18*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2*(5*(-1 + n)*AppellF1[3/2, m +
n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 5*(m + n)*Appell
F1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
2*((2 - 3*n + n^2)*AppellF1[5/2, m + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2] + (m + n)*(2*(-1 + n)*AppellF1[5/2, 1 + m + n, 2 - n, 7
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 + m + n)*AppellF1[5/2, 2
 + m + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*Tan[(e +
f*x)/2]^2))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + 15*(m +
n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/...
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4313, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sec(e+fx)+1)^m(-\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4313}$$

$$\frac{\tan(e+fx)\int\frac{(-\sec(e+fx))^{n-1}(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}}d(\sec(e+fx)+1)}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{150}$$

$$\frac{\sqrt{2}\tan(e+fx)(\sec(e+fx)+1)^m \operatorname{AppellF1}\left(m+\frac{1}{2},1-n,\frac{1}{2},m+\frac{3}{2},\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f(2m+1)\sqrt{1-\sec(e+fx)}}$$

input
```
Int[(-Sec[e + f*x])^n*(1 + Sec[e + f*x])^m,x]
```

output
```
(Sqrt[2]*AppellF1[1/2 + m, 1 - n, 1/2, 3/2 + m, 1 + Sec[e + f*x], (1 + Sec
[e + f*x])/2]*(1 + Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 - Sec
[e + f*x]])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4313
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-((-a)*(d/b))^n)*(Cot[e + f*x]/(a^(n - 1)*f*S
qrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[x^(m - 1/2)*
((a - x)^(n - 1)/Sqrt[2*a - x]), x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a
, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
&&  !IntegerQ[n] && LtQ[a*(d/b), 0]
```

Maple [F]

$$\int (-\sec(fx+e))^n (1+\sec(fx+e))^m\, dx$$

input
```
int((-sec(f*x+e))^n*(1+sec(f*x+e))^m,x)
```

output
```
int((-sec(f*x+e))^n*(1+sec(f*x+e))^m,x)
```

Fricas [F]

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^m\, dx = \int (-\sec(fx+e))^n (\sec(fx+e)+1)^m\, dx$$

input
```
integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((-sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

Sympy [F]

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^m\, dx = \int (-\sec(e+fx))^n (\sec(e+fx)+1)^m\, dx$$

input
```
integrate((-sec(f*x+e))**n*(1+sec(f*x+e))**m,x)
```

output
```
Integral((-sec(e + f*x))**n*(sec(e + f*x) + 1)**m, x)
```

Maxima [F]

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^m\,dx = \int (-\sec(fx+e))^n(\sec(fx+e)+1)^m\,dx$$

input
```
integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((-sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

Giac [F]

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^m\,dx = \int (-\sec(fx+e))^n(\sec(fx+e)+1)^m\,dx$$

input
```
integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((-sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^m\,dx = \int \left(\frac{1}{\cos(e+f\,x)}+1\right)^m \left(-\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((1/cos(e + f*x) + 1)^m*(-1/cos(e + f*x))^n,x)
```

output
```
int((1/cos(e + f*x) + 1)^m*(-1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^m\,dx = (-1)^n\left(\int \sec(fx+e)^n(\sec(fx+e)+1)^m\,dx\right)$$

input
```
int((-sec(f*x+e))^n*(1+sec(f*x+e))^m,x)
```

output
```
( - 1)**n*int(sec(e + f*x)**n*(sec(e + f*x) + 1)**m,x)
```

3.334 $\int (1-\sec(e+fx))^m(-\sec(e+fx))^n\,dx$

Optimal result

Integrand size = 23, antiderivative size = 70

$$\int (1-\sec(e+fx))^m(-\sec(e+fx))^n\,dx = \frac{2^{\frac{1}{2}+m}\operatorname{AppellF1}\left(\frac{1}{2},1-n,\frac{1}{2}-m,\frac{3}{2},1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)\tan(e+fx)}{f\sqrt{1-\sec(e+fx)}}$$

output
```
2^(1/2+m)*AppellF1(1/2,1-n,1/2-m,3/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*tan(
f*x+e)/f/(1-sec(f*x+e))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 257 vs. 2(70) = 140.

Time = 0.39 (sec) , antiderivative size = 257, normalized size of antiderivative = 3.67

$$\int (1-\sec(e+fx))^m(-\sec(e+fx))^n\,dx = \frac{(3+2m)\operatorname{AppellF}}{f(1+2m)\left((3+2m)\operatorname{AppellF1}\left(\frac{1}{2}+m,m+n,1-n,\frac{3}{2}+m,\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+2\right.}$$

input
```
Integrate[(1 - Sec[e + f*x])^m*(-Sec[e + f*x])^n,x]
```

output
```
((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*(1 - Sec[e + f*x])^m*(-Sec[e + f*x])^n*Sin[e + f*x])/(f
*(1 + 2*m)*((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2 + m, m + n, 2 - n
, 5/2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2
 + m, 1 + m + n, 1 - n, 5/2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])
*Tan[(e + f*x)/2]^2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4312, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-\sec(e+fx))^m(-\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(1-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4312}$$

$$\frac{\tan(e+fx)\int \frac{(1-\sec(e+fx))^{m-\frac{1}{2}}(-\sec(e+fx))^{n-1}}{\sqrt{\sec(e+fx)+1}}d(\sec(e+fx)+1)}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{150}$$

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)\,\text{AppellF1}\left(\frac{1}{2},1-n,\frac{1}{2}-m,\frac{3}{2},\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f\sqrt{1-\sec(e+fx)}}$$

input
```
Int[(1 - Sec[e + f*x])^m*(-Sec[e + f*x])^n,x]
```

output
```
(2^(1/2 + m)*AppellF1[1/2, 1 - n, 1/2 - m, 3/2, 1 + Sec[e + f*x], (1 + Sec
[e + f*x])/2]*Tan[e + f*x])/(f*Sqrt[1 - Sec[e + f*x]])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4312
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a - x)^(n - 1)
*((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

Maple [F]

$$\int (1-\sec(fx+e))^m (-\sec(fx+e))^n \, dx$$

input
```
int((1-sec(f*x+e))^m*(-sec(f*x+e))^n,x)
```

output
```
int((1-sec(f*x+e))^m*(-sec(f*x+e))^n,x)
```

Fricas [F]

$$\int (1-\sec(e+fx))^m(-\sec(e+fx))^n \, dx = \int (-\sec(fx+e))^n (-\sec(fx+e)+1)^m \, dx$$

input
```
integrate((1-sec(f*x+e))^m*(-sec(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((-sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)
```

Sympy [F]

$$\int (1-\sec(e+fx))^m(-\sec(e+fx))^n\,dx = \int (-\sec{(e+fx)})^n\,(1-\sec{(e+fx)})^m\,dx$$

input
```
integrate((1-sec(f*x+e))**m*(-sec(f*x+e))**n,x)
```

output
```
Integral((-sec(e + f*x))**n*(1 - sec(e + f*x))**m, x)
```

Maxima [F]

$$\int (1-\sec(e+fx))^m(-\sec(e+fx))^n\,dx = \int (-\sec{(fx+e)})^n\,(-\sec{(fx+e)}+1)^m\,dx$$

input
```
integrate((1-sec(f*x+e))^m*(-sec(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((-sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)
```

Giac [F]

$$\int (1-\sec(e+fx))^m(-\sec(e+fx))^n\,dx = \int (-\sec{(fx+e)})^n\,(-\sec{(fx+e)}+1)^m\,dx$$

input
```
integrate((1-sec(f*x+e))^m*(-sec(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((-sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (1-\sec(e+fx))^m(-\sec(e+fx))^n\,dx$$
$$=\int \left(1-\frac{1}{\cos(e+f\,x)}\right)^m\left(-\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((1 - 1/cos(e + f*x))^m*(-1/cos(e + f*x))^n,x)
```

output
```
int((1 - 1/cos(e + f*x))^m*(-1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (1-\sec(e+fx))^m(-\sec(e+fx))^n\,dx = (-1)^n\left(\int \sec(fx+e)^n\,(-\sec(fx+e)+1)^m\,dx\right)$$

input
```
int((1-sec(f*x+e))^m*(-sec(f*x+e))^n,x)
```

output
```
( - 1)**n*int(sec(e + f*x)**n*( - sec(e + f*x) + 1)**m,x)
```

3.335 $\int (-\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 113

$$\int (-\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx =$$
$$-\frac{2^{\frac{1}{2}+m}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2}-m,1-n,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx)),1-\sec(e+fx)\right)(-\sec(e+fx))^{-1+n}\sec^{2-n}(}{f}$$

output
```
-2^(1/2+m)*AppellF1(1/2,1-n,1/2-m,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*(-s
ec(f*x+e))^(-1+n)*sec(f*x+e)^(2-n)*(1+sec(f*x+e))^(-1/2-m)*(a+a*sec(f*x+e)
)^m*sin(f*x+e)/f
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 825 vs. 2(113) = 226.

Time = 5.25 (sec) , antiderivative size = 825, normalized size of antiderivative = 7.30

$$\int (-\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx = \text{Too large to display}$$

input
```
Integrate[(-Sec[e + f*x])^n*(a + a*Sec[e + f*x])^m,x]
```

output
```
(30*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*Cos[(e + f*x)/2]*(-Sec[e + f*x])^n*(a*(1 + Sec[e + f*x]))^m*Sin[(e + f
*x)/2])/(f*(15*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2] + 30*(-1 + n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[(e + f*x)/2]^2 + 10*((-1 + n)*AppellF1[3/
2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*A
ppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2])*Tan[(e + f*x)/2]^2 - (18*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2*(5*(-1 + n)*AppellF1[3/2,
m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 5*(m + n)*Ap
pellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
] + 2*((2 - 3*n + n^2)*AppellF1[5/2, m + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] + (m + n)*(2*(-1 + n)*AppellF1[5/2, 1 + m + n, 2 -
n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 + m + n)*AppellF1[5/
2, 2 + m + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*Tan[(
e + f*x)/2]^2))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + 15*(
m + n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f...
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4315, 3042, 4313, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (-\sec(e+fx))^n(a\sec(e+fx)+a)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m dx$$

$$\Big\downarrow \text{4315}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int (-\sec(e+fx))^n(\sec(e+fx)+1)^m dx$$

$$\Big\downarrow \text{3042}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m dx$$

$$\Big\downarrow \text{4313}$$

$$\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\int\frac{(-\sec(e+fx))^{n-1}(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}}d(\sec(e+fx)+1)}{f\sqrt{1-\sec(e+fx)}}$$

$$\Big\downarrow \text{150}$$

$$\frac{\sqrt{2}\tan(e+fx)(a\sec(e+fx)+a)^m\operatorname{AppellF1}\left(m+\frac{1}{2},1-n,\frac{1}{2},m+\frac{3}{2},\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f(2m+1)\sqrt{1-\sec(e+fx)}}$$

input
```
Int[(-Sec[e + f*x])^n*(a + a*Sec[e + f*x])^m,x]
```

output
```
(Sqrt[2]*AppellF1[1/2 + m, 1 - n, 1/2, 3/2 + m, 1 + Sec[e + f*x], (1 + Sec
[e + f*x])/2]*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 - S
ec[e + f*x]])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4313
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-((-a)*(d/b))^n)*(Cot[e + f*x]/(a^(n - 1)*f*S
qrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[x^(m - 1/2)*
((a - x)^(n - 1)/Sqrt[2*a - x]), x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a
, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
&&  !IntegerQ[n] && LtQ[a*(d/b), 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int (-\sec (fx+e))^{n} (a+a\sec (fx+e))^{m} dx$$

input
```
int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x)
```

output
```
int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int (-\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx = \int (a\sec (fx+e)+a)^{m}(-\sec (fx+e))^{n}\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

Sympy [F]

$$\int (-\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx = \int (-\sec(e+fx))^n\,(a(\sec(e+fx)+1))^m\,dx$$

input
```
integrate((-sec(f*x+e))**n*(a+a*sec(f*x+e))**m,x)
```

output
```
Integral((-sec(e + f*x))**n*(a*(sec(e + f*x) + 1))**m, x)
```

Maxima [F]

$$\int (-\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx = \int (a\sec(fx+e)+a)^m(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

Giac [F]

$$\int (-\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx = \int (a\sec(fx+e)+a)^m(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (-\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)^m\left(-\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a + a/cos(e + f*x))^m*(-1/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^m*(-1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (-\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx = (-1)^n\left(\int \sec(fx+e)^n(\sec(fx+e)a+a)^m\,dx\right)$$

input
```
int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x)
```

output
```
( - 1)**n*int(sec(e + f*x)**n*(sec(e + f*x)*a + a)**m,x)
```

3.336 $\int (-\sec(e+fx))^n (a-a\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int (-\sec(e+fx))^n (a-a\sec(e+fx))^m\,dx = \frac{2^{\frac{1}{2}+m}\operatorname{AppellF1}\left(\frac{1}{2},1-n,\frac{1}{2}-m,\frac{3}{2},1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)(1-\sec(e+fx))^{-\frac{1}{2}-m}(a-a\,\mathrm{s}}{f}$$

output
```
2^(1/2+m)*AppellF1(1/2,1-n,1/2-m,3/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(1-s
ec(f*x+e))^(-1/2-m)*(a-a*sec(f*x+e))^m*tan(f*x+e)/f
```

Mathematica [F]

$$\int (-\sec(e+fx))^n (a-a\sec(e+fx))^m\,dx = \int (-\sec(e+fx))^n (a-a\sec(e+fx))^m\,dx$$

input
```
Integrate[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m,x]
```

output
```
Integrate[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m, x]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3042, 4315, 3042, 4312, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\left(a-a\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \; 4315$$

$$(1-\sec(e+fx))^{-m}(a-a\sec(e+fx))^m\int (1-\sec(e+fx))^m(-\sec(e+fx))^n dx$$

$$\Big\downarrow \; 3042$$

$$(1-\sec(e+fx))^{-m}(a-a\sec(e+fx))^m\int \left(1-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \; 4312$$

$$\frac{\tan(e+fx)(1-\sec(e+fx))^{-m-\frac{1}{2}}(a-a\sec(e+fx))^m\int \frac{(1-\sec(e+fx))^{m-\frac{1}{2}}(-\sec(e+fx))^{n-1}}{\sqrt{\sec(e+fx)+1}}d(\sec(e+fx)+1)}{f\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \; 150$$

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{-m-\frac{1}{2}}(a-a\sec(e+fx))^m\operatorname{AppellF1}\left(\frac{1}{2},1-n,\frac{1}{2}-m,\frac{3}{2},\sec(e+fx)+1,\frac{1}{2}(\mathrm{s}\right.}{f}$$

input
```
Int[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m,x]
```

output
```
(2^(1/2 + m)*AppellF1[1/2, 1 - n, 1/2 - m, 3/2, 1 + Sec[e + f*x], (1 + Sec
[e + f*x])/2]*(1 - Sec[e + f*x])^(-1/2 - m)*(a - a*Sec[e + f*x])^m*Tan[e +
 f*x])/f
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4312
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a - x)^(n - 1)
*((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int (-\sec(fx+e))^n (a-a\sec(fx+e))^m\,dx$$

input
```
int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x)
```

output
```
int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx = \int (-a\sec(fx+e)+a)^m(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((-a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

Sympy [F]

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx = \int (-\sec(e+fx))^n\,(-a(\sec(e+fx) - 1))^m\,dx$$

input
```
integrate((-sec(f*x+e))**n*(a-a*sec(f*x+e))**m,x)
```

output
```
Integral((-sec(e + f*x))**n*(-a*(sec(e + f*x) - 1))**m, x)
```

Maxima [F]

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx$$
$$=\int (-a\sec(fx+e)+a)^m(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((-a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

Giac [F]

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx$$
$$=\int (-a\sec(fx+e)+a)^m(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((-a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx$$
$$=\int \left(a-\frac{a}{\cos(e+f\,x)}\right)^m\left(-\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a - a/cos(e + f*x))^m*(-1/cos(e + f*x))^n,x)
```

output
```
int((a - a/cos(e + f*x))^m*(-1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (-\sec(e+fx))^n (a-a\sec(e+fx))^m \, dx = (-1)^n \left(\int \sec(fx+e)^n (-\sec(fx+e)a + a)^m \, dx \right)$$

input
```
int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x)
```

output
```
( - 1)**n*int(sec(e + f*x)**n*( - sec(e + f*x)*a + a)**m,x)
```

3.337 $\int (d\sec(e+fx))^n(1+\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^m\,dx$$
$$= \frac{2^{\frac{1}{2}+m}d\,\text{AppellF1}\left(\frac{1}{2},1-n,\frac{1}{2}-m,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\sec^{2-n}(e+fx)(d\sec(e+fx)}{f\sqrt{1+\sec(e+fx)}}$$

output
```
2^(1/2+m)*d*AppellF1(1/2,1-n,1/2-m,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*se
c(f*x+e)^(2-n)*(d*sec(f*x+e))^(-1+n)*sin(f*x+e)/f/(1+sec(f*x+e))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 823 vs. 2(97) = 194.

Time = 5.05 (sec) , antiderivative size = 823, normalized size of antiderivative = 8.48

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^m\,dx = \text{Too large to display}$$

input
```
Integrate[(d*Sec[e + f*x])^n*(1 + Sec[e + f*x])^m,x]
```

output
```
(30*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*Cos[(e + f*x)/2]*(d*Sec[e + f*x])^n*(1 + Sec[e + f*x])^m*Sin[(e + f*x)
/2])/(f*(15*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] + 30*(-1 + n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2]*Sin[(e + f*x)/2]^2 + 10*((-1 + n)*AppellF1[3/2,
m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*Appe
llF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])
*Tan[(e + f*x)/2]^2 - (18*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2*(5*(-1 + n)*AppellF1[3/2, m +
 n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 5*(m + n)*Appel
lF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 2*((2 - 3*n + n^2)*AppellF1[5/2, m + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] + (m + n)*(2*(-1 + n)*AppellF1[5/2, 1 + m + n, 2 - n,
7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 + m + n)*AppellF1[5/2,
2 + m + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*Tan[(e +
 f*x)/2]^2))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + 15*(m +
 n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)...
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{3042, 4314, 150\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sec(e+fx)+1)^m(d\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4314}$$

$$-\frac{d\tan(e+fx)\int \frac{(d\sec(e+fx))^{n-1}(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{ 150}$$

$$-\frac{\tan(e+fx)(d\sec(e+fx))^n \operatorname{AppellF1}\left(n,\frac{1}{2},\frac{1}{2}-m,n+1,\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

input
```
Int[(d*Sec[e + f*x])^n*(1 + Sec[e + f*x])^m,x]
```

output
```
-((AppellF1[n, 1/2, 1/2 - m, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e
+ f*x])^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]
))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Maple [F]

$$\int (d\sec(fx+e))^n (1+\sec(fx+e))^m \, dx$$

input
```
int((d*sec(f*x+e))^n*(1+sec(f*x+e))^m,x)
```

output
```
int((d*sec(f*x+e))^n*(1+sec(f*x+e))^m,x)
```

Fricas [F]

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^m \, dx = \int (d\sec(fx+e))^n (\sec(fx+e)+1)^m \, dx$$

input
```
integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

Sympy [F]

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^m \, dx = \int (d\sec(e+fx))^n (\sec(e+fx)+1)^m \, dx$$

input
```
integrate((d*sec(f*x+e))**n*(1+sec(f*x+e))**m,x)
```

output
```
Integral((d*sec(e + f*x))**n*(sec(e + f*x) + 1)**m, x)
```

Maxima [F]

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^m\,dx = \int (d\sec(fx+e))^n(\sec(fx+e)+1)^m\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

Giac [F]

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^m\,dx = \int (d\sec(fx+e))^n(\sec(fx+e)+1)^m\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^m\,dx = \int \left(\frac{1}{\cos(e+f\,x)}+1\right)^m \left(\frac{d}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((1/cos(e + f*x) + 1)^m*(d/cos(e + f*x))^n,x)
```

output
```
int((1/cos(e + f*x) + 1)^m*(d/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^m\,dx = d^n\left(\int \sec\left(fx+e\right)^n\left(\sec\left(fx+e\right)+1\right)^m dx\right)$$

input
```
int((d*sec(f*x+e))^n*(1+sec(f*x+e))^m,x)
```

output
```
d**n*int(sec(e + f*x)**n*(sec(e + f*x) + 1)**m,x)
```

3.338 $\int (1-\sec(e+fx))^m (d\sec(e+fx))^n\,dx$

Optimal result

Integrand size = 23, antiderivative size = 98

$$\int (1-\sec(e+fx))^m (d\sec(e+fx))^n\,dx = -\frac{2^{\frac{1}{2}+m} d\,\text{AppellF1}\left(\frac{1}{2},1-n,\frac{1}{2}-m,\frac{3}{2},1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)(-\sec(e+fx))^{1-n}(d\sec(e}{f\sqrt{1-\sec(e+fx)}}$$

output
```
-2^(1/2+m)*d*AppellF1(1/2,1-n,1/2-m,3/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(
-sec(f*x+e))^(1-n)*(d*sec(f*x+e))^(-1+n)*tan(f*x+e)/f/(1-sec(f*x+e))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 257 vs. 2(98) = 196.

Time = 0.36 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.62

$$\int (1-\sec(e+fx))^m (d\sec(e+fx))^n\,dx$$

$$= \frac{(3+2m)\,\text{AppellF}}{f(1+2m)\left((3+2m)\,\text{AppellF1}\left(\frac{1}{2}+m,m+n,1-n,\frac{3}{2}+m,\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+2\right.}$$

input
```
Integrate[(1 - Sec[e + f*x])^m*(d*Sec[e + f*x])^n,x]
```

output
```
((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*(1 - Sec[e + f*x])^m*(d*Sec[e + f*x])^n*Sin[e + f*x])/(
f*(1 + 2*m)*((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2 + m, m + n, 2 -
n, 5/2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/
2 + m, 1 + m + n, 1 - n, 5/2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
)*Tan[(e + f*x)/2]^2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4314, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-\sec(e+fx))^m (d\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(1-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4314}$$

$$-\frac{d\tan(e+fx)\int \frac{(1-\sec(e+fx))^{m-\frac{1}{2}}(d\sec(e+fx))^{n-1}}{\sqrt{\sec(e+fx)+1}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{150}$$

$$-\frac{\tan(e+fx)(d\sec(e+fx))^n \operatorname{AppellF1}\left(n,\frac{1}{2}-m,\frac{1}{2},n+1,\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

input
```
Int[(1 - Sec[e + f*x])^m*(d*Sec[e + f*x])^n,x]
```

output
```
-((AppellF1[n, 1/2 - m, 1/2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e
+ f*x])^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]
))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

Maple [F]

$$\int (1-\sec (fx+e))^m (d \sec (fx+e))^n \, dx$$

input
```
int((1-sec(f*x+e))^m*(d*sec(f*x+e))^n,x)
```

output
```
int((1-sec(f*x+e))^m*(d*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int (1-\sec(e+fx))^m (d\sec(e+fx))^n\,dx = \int (d\sec(fx+e))^n (-\sec(fx+e)+1)^m\,dx$$

input
```
integrate((1-sec(f*x+e))^m*(d*sec(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)
```

Sympy [F]

$$\int (1-\sec(e+fx))^m (d\sec(e+fx))^n\,dx = \int (d\sec(e+fx))^n (1-\sec(e+fx))^m\,dx$$

input
```
integrate((1-sec(f*x+e))**m*(d*sec(f*x+e))**n,x)
```

output
```
Integral((d*sec(e + f*x))**n*(1 - sec(e + f*x))**m, x)
```

Maxima [F]

$$\int (1-\sec(e+fx))^m (d\sec(e+fx))^n\,dx = \int (d\sec(fx+e))^n (-\sec(fx+e)+1)^m\,dx$$

input
```
integrate((1-sec(f*x+e))^m*(d*sec(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)
```

Giac [F]

$$\int (1-\sec(e+fx))^m (d\sec(e+fx))^n \, dx = \int (d\sec(fx+e))^n (-\sec(fx+e)+1)^m \, dx$$

input
```
integrate((1-sec(f*x+e))^m*(d*sec(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (1-\sec(e+fx))^m (d\sec(e+fx))^n \, dx = \int \left(1-\frac{1}{\cos(e+f\,x)}\right)^m \left(\frac{d}{\cos(e+f\,x)}\right)^n \, dx$$

input
```
int((1 - 1/cos(e + f*x))^m*(d/cos(e + f*x))^n,x)
```

output
```
int((1 - 1/cos(e + f*x))^m*(d/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (1-\sec(e+fx))^m (d\sec(e+fx))^n \, dx = d^n \left(\int \sec(fx+e)^n (-\sec(fx+e)+1)^m \, dx\right)$$

input
```
int((1-sec(f*x+e))^m*(d*sec(f*x+e))^n,x)
```

output
```
d**n*int(sec(e + f*x)**n*( - sec(e + f*x) + 1)**m,x)
```

3.339 $\int (d\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 113

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx$$
$$= \frac{2^{\frac{1}{2}+m}d\,\text{AppellF1}\left(\frac{1}{2},1-n,\frac{1}{2}-m,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\sec^{2-n}(e+fx)(d\sec(e+fx)}{f}$$

output
```
2^(1/2+m)*d*AppellF1(1/2,1-n,1/2-m,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*se
c(f*x+e)^(2-n)*(d*sec(f*x+e))^(-1+n)*(1+sec(f*x+e))^(-1/2-m)*(a+a*sec(f*x+
e))^m*sin(f*x+e)/f
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 825 vs. 2(113) = 226.

Time = 5.13 (sec) , antiderivative size = 825, normalized size of antiderivative = 7.30

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx = \text{Too large to display}$$

input
```
Integrate[(d*Sec[e + f*x])^n*(a + a*Sec[e + f*x])^m,x]
```

output
```
(30*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*Cos[(e + f*x)/2]*(d*Sec[e + f*x])^n*(a*(1 + Sec[e + f*x]))^m*Sin[(e +
f*x)/2])/(f*(15*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + 30*(-1 + n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[(e + f*x)/2]^2 + 10*((-1 + n)*AppellF1[3
/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*
AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2])*Tan[(e + f*x)/2]^2 - (18*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2*(5*(-1 + n)*AppellF1[3/2,
 m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 5*(m + n)*A
ppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2] + 2*((2 - 3*n + n^2)*AppellF1[5/2, m + n, 3 - n, 7/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2] + (m + n)*(2*(-1 + n)*AppellF1[5/2, 1 + m + n, 2 -
 n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 + m + n)*AppellF1[5
/2, 2 + m + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*Tan[
(e + f*x)/2]^2))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n
, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + 15*
(m + n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + ...
```

Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4315, 3042, 4314, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^m(d\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4315}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int (d\sec(e+fx))^n(\sec(e+fx)+1)^m dx$$

$$\Big\downarrow \text{3042}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m dx$$

$$\Big\downarrow \text{4314}$$

$$-\frac{d\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\int\frac{(d\sec(e+fx))^{n-1}(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}}$$

$$\Big\downarrow \text{150}$$

$$-\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m(d\sec(e+fx))^n\operatorname{AppellF1}\left(n,\frac{1}{2},\frac{1}{2}-m,n+1,\sec(e+\right.}{fn\sqrt{1-\sec(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^n*(a + a*Sec[e + f*x])^m,x]
```

output
```
-((AppellF1[n, 1/2, 1/2 - m, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e
+ f*x])^n*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x
])/(f*n*Sqrt[1 - Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int (d\sec (fx+e))^n (a+a\sec (fx+e))^m \, dx$$

input
```
int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x)
```

output
```
int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^m \, dx = \int (a\sec (fx+e)+a)^m(d\sec (fx+e))^n \, dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

Sympy [F]

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx = \int (a(\sec(e+fx)+1))^m\,(d\sec(e+fx))^n\,dx$$

input
```
integrate((d*sec(f*x+e))**n*(a+a*sec(f*x+e))**m,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m*(d*sec(e + f*x))**n, x)
```

Maxima [F]

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx = \int (a\sec(fx+e)+a)^m(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

Giac [F]

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx = \int (a\sec(fx+e)+a)^m(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx=\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^m\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^n\,dx$$

input
```
int((a + a/cos(e + f*x))^m*(d/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^m*(d/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx=d^n\left(\int \sec\left(fx+e\right)^n\left(\sec\left(fx+e\right)a+a\right)^m dx\right)$$

input
```
int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x)
```

output
```
d**n*int(sec(e + f*x)**n*(sec(e + f*x)*a + a)**m,x)
```

3.340 $\int (d\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 24, antiderivative size = 115

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx = -\frac{2^{\frac{1}{2}+m}d\,\text{AppellF1}\left(\frac{1}{2},1-n,\frac{1}{2}-m,\frac{3}{2},1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)(1-\sec(e+fx))^{-\frac{1}{2}-m}(-\text{s}}{f}$$

output
```
-2^(1/2+m)*d*AppellF1(1/2,1-n,1/2-m,3/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(
1-sec(f*x+e))^(-1/2-m)*(-sec(f*x+e))^(1-n)*(d*sec(f*x+e))^(-1+n)*(a-a*sec(
f*x+e))^m*tan(f*x+e)/f
```

Mathematica [F]

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx = \int (d\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx$$

input
```
Integrate[(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m,x]
```

output
```
Integrate[(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3042, 4315, 3042, 4314, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-a\sec(e+fx))^m(d\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4315}$$

$$(1-\sec(e+fx))^{-m}(a-a\sec(e+fx))^m\int (1-\sec(e+fx))^m(d\sec(e+fx))^n dx$$

$$\Big\downarrow \text{3042}$$

$$(1-\sec(e+fx))^{-m}(a-a\sec(e+fx))^m\int \left(1-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4314}$$

$$-\frac{d\tan(e+fx)(1-\sec(e+fx))^{-m-\frac{1}{2}}(a-a\sec(e+fx))^m\int \frac{(1-\sec(e+fx))^{m-\frac{1}{2}}(d\sec(e+fx))^{n-1}}{\sqrt{\sec(e+fx)+1}}d\sec(e+fx)}{f\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{150}$$

$$-\frac{\tan(e+fx)(1-\sec(e+fx))^{-m-\frac{1}{2}}(a-a\sec(e+fx))^m(d\sec(e+fx))^n \operatorname{AppellF1}\left(n,\frac{1}{2}-m,\frac{1}{2},n+1,\sec(e+\right.}{fn\sqrt{\sec(e+fx)+1}}$$

input
```
Int[(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m,x]
```

output
```
-((AppellF1[n, 1/2 - m, 1/2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(1 - Sec[
e + f*x])^(-1/2 - m)*(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m*Tan[e + f*x
])/(f*n*Sqrt[1 + Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int (d\sec (fx+e))^n (a-a\sec (fx+e))^m dx$$

input
```
int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x)
```

output
```
int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx = \int (-a\sec(fx+e)+a)^m(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((-a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

Sympy [F]

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx = \int (d\sec(e+fx))^n\,(-a(\sec(e+fx)-1))^m\,dx$$

input
```
integrate((d*sec(f*x+e))**n*(a-a*sec(f*x+e))**m,x)
```

output
```
Integral((d*sec(e + f*x))**n*(-a*(sec(e + f*x) - 1))**m, x)
```

Maxima [F]

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx = \int (-a\sec(fx+e)+a)^m(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((-a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

Giac [F]

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx = \int (-a\sec(fx+e)+a)^m(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((-a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx = \int \left(a-\frac{a}{\cos(e+fx)}\right)^m\left(\frac{d}{\cos(e+fx)}\right)^n\,dx$$

input
```
int((a - a/cos(e + f*x))^m*(d/cos(e + f*x))^n,x)
```

output
```
int((a - a/cos(e + f*x))^m*(d/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx = d^n\left(\int \sec(fx+e)^n(-\sec(fx+e)a+a)^m\,dx\right)$$

input
```
int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x)
```

output
```
d**n*int(sec(e + f*x)**n*( - sec(e + f*x)*a + a)**m,x)
```

3.341 $\int \sec^4(e+fx)(a+a\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 21, antiderivative size = 211

$$\int \sec^4(e+fx)(a+a\sec(e+fx))^m\,dx = \frac{(4+m)(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)(2+m)(3+m)} + \frac{\sec^2(e+fx)(a+a\sec(e+fx))^m\tan(e+fx)}{f(3+m)} + \frac{2^{\frac{1}{2}+m}m(5+3m+m^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(}{f(1+m)(2+m)(3+m)} + \frac{m(a+a\sec(e+fx))^{1+m}\tan(e+fx)}{af\,(6+5m+m^2)}$$

output
```
(4+m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(1+m)/(2+m)/(3+m)+sec(f*x+e)^2*(a+a*
sec(f*x+e))^m*tan(f*x+e)/f/(3+m)+2^(1/2+m)*m*(m^2+3*m+5)*hypergeom([1/2, 1
/2-m],[3/2],1/2-1/2*sec(f*x+e))*(1+sec(f*x+e))^(-1/2-m)*(a+a*sec(f*x+e))^m
*tan(f*x+e)/f/(1+m)/(2+m)/(3+m)+m*(a+a*sec(f*x+e))^(1+m)*tan(f*x+e)/a/f/(m
^2+5*m+6)
```

Mathematica [A] (warning: unable to verify)

Time = 0.91 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.73

$$\int \sec^4(e+fx)(a+a\sec(e+fx))^m\,dx$$
$$= \frac{(1+\sec(e+fx))^{-\frac{1}{2}-m}(a(1+\sec(e+fx)))^m\left(2^{\frac{3}{2}+m}m(5+3m+m^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1}{2}-m,\right.\right.}{}$$

input
```
Integrate[Sec[e + f*x]^4*(a + a*Sec[e + f*x])^m,x]
```

output
```
((1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))^m*(2^(3/2 + m)*m*(5
+ 3*m + m^2)*Hypergeometric2F1[1/2, -1/2 - m, 3/2, (1 - Sec[e + f*x])/2] +
 (1 + Sec[e + f*x])^(1/2 + m)*(4 + m + m^2 + m*(1 + 2*m)*Sec[e + f*x] + (2
 + 5*m + 2*m^2)*Sec[e + f*x]^2))*Tan[e + f*x])/(f*(2 + m)*(3 + m)*(1 + 2*m
))
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4311, 3042, 4498, 3042, 4489, 3042, 4315, 3042, 4314, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(e+fx)(a\sec(e+fx)+a)^m\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^4\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m dx$$
$$\Big\downarrow \text{4311}$$
$$\frac{\int \sec^2(e+fx)(\sec(e+fx)a+a)^m(m\sec(e+fx)a+2a)dx}{a(m+3)}+\frac{\tan(e+fx)\sec^2(e+fx)(a\sec(e+fx)+a)^m}{f(m+3)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \csc\left(e+fx+\frac{\pi}{2}\right)^2\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^m\left(m\csc\left(e+fx+\frac{\pi}{2}\right)a+2a\right)dx}{a(m+3)}+\frac{\tan(e+fx)\sec^2(e+fx)(a\sec(e+fx)+a)^m}{f(m+3)}$$

$$\Big\downarrow \; 4498$$

$$\frac{\frac{\int \sec(e+fx)(\sec(e+fx)a+a)^m\left(m(m+1)a^2+(m+4)\sec(e+fx)a^2\right)dx}{a(m+2)}+\frac{m\tan(e+fx)(a\sec(e+fx)+a)^{m+1}}{f(m+2)}}{a(m+3)}+\frac{\tan(e+fx)\sec^2(e+fx)(a\sec(e+fx)+a)^m}{f(m+3)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^m\left(m(m+1)a^2+(m+4)\csc\left(e+fx+\frac{\pi}{2}\right)a^2\right)dx}{a(m+2)}+\frac{m\tan(e+fx)(a\sec(e+fx)+a)^{m+1}}{f(m+2)}}{a(m+3)}+\frac{\tan(e+fx)\sec^2(e+fx)(a\sec(e+fx)+a)^m}{f(m+3)}$$

$$\Big\downarrow \; 4489$$

$$\frac{\frac{\frac{a^2m\left(m^2+3m+5\right)\int \sec(e+fx)(\sec(e+fx)a+a)^m dx}{m+1}+\frac{a^2(m+4)\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}}{a(m+2)}+\frac{m\tan(e+fx)(a\sec(e+fx)+a)^{m+1}}{f(m+2)}}{a(m+3)}+\frac{\tan(e+fx)\sec^2(e+fx)(a\sec(e+fx)+a)^m}{f(m+3)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{a^2m\left(m^2+3m+5\right)\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^m dx}{m+1}+\frac{a^2(m+4)\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}}{a(m+2)}+\frac{m\tan(e+fx)(a\sec(e+fx)+a)^{m+1}}{f(m+2)}}{a(m+3)}+\frac{\tan(e+fx)\sec^2(e+fx)(a\sec(e+fx)+a)^m}{f(m+3)}$$

$$\Big\downarrow \; 4315$$

$$\frac{\frac{\frac{a^2m\left(m^2+3m+5\right)(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int \sec(e+fx)(\sec(e+fx)+1)^m dx}{m+1}+\frac{a^2(m+4)\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}}{a(m+2)}+\frac{m\tan(e+fx)(a\sec(e+}{f(m+2)}}{a(m+3)}$$

$$\frac{\tan(e+fx)\sec^2(e+fx)(a\sec(e+fx)+a)^m}{f(m+3)}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{a^2m(m^2+3m+5)(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m dx}{m+1}+\frac{a^2(m+4)\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}}{a(m+2)}+\frac{m\tan(e+fx)(}{}}{a(m+3)}$$

$$\frac{\tan(e+fx)\sec^2(e+fx)(a\sec(e+fx)+a)^m}{f(m+3)}$$

$\Big\downarrow$ 4314

$$\frac{\frac{\frac{a^2(m+4)\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}-\frac{a^2m(m^2+3m+5)\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\int\frac{(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f(m+1)\sqrt{1-\sec(e+fx)}}}{a(m+2)}+\frac{m\tan(}{}}{a(m+3)}$$

$$\frac{\tan(e+fx)\sec^2(e+fx)(a\sec(e+fx)+a)^m}{f(m+3)}$$

$\Big\downarrow$ 79

$$\frac{\frac{\frac{a^2 2^{m+\frac{1}{2}}m(m^2+3m+5)\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx))\right)}{f(m+1)}+\frac{a^2(m+4)\tan(e+fx)(a\sec(e+}{f(m+1)}}{a(m+2)}}{a(m+3)}$$

$$\frac{\tan(e+fx)\sec^2(e+fx)(a\sec(e+fx)+a)^m}{f(m+3)}$$

input
```
Int[Sec[e + f*x]^4*(a + a*Sec[e + f*x])^m,x]
```

output
```
(Sec[e + f*x]^2*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(3 + m)) + ((m*(a
+ a*Sec[e + f*x])^(1 + m)*Tan[e + f*x])/(f*(2 + m)) + ((a^2*(4 + m)*(a + a
*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m)) + (2^(1/2 + m)*a^2*m*(5 + 3*m +
 m^2)*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f*x])/2]*(1 + Sec[
e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m)))/(a*
(2 + m)))/(a*(3 + m))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4311
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(m + n - 1))), x] + Simp[d^2/(b*(m + n - 1))   In
t[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*m*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && GtQ[n
, 2] && NeQ[m + n - 1, 0] && IntegerQ[n]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

rule 4498
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int
[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)
*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a
*B, 0] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \sec(fx+e)^4 (a+a\sec(fx+e))^m \, dx$$

input
```
int(sec(f*x+e)^4*(a+a*sec(f*x+e))^m,x)
```

output
```
int(sec(f*x+e)^4*(a+a*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int \sec^4(e+fx)(a+a\sec(e+fx))^m \, dx = \int (a\sec(fx+e)+a)^m \sec(fx+e)^4 \, dx$$

input
```
integrate(sec(f*x+e)^4*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^m*sec(f*x + e)^4, x)
```

Sympy [F]

$$\int \sec^4(e+fx)(a+a\sec(e+fx))^m\,dx=\int (a(\sec{(e+fx)}+1))^m\sec^4{(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)**4*(a+a*sec(f*x+e))**m,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)**4, x)
```

Maxima [F]

$$\int \sec^4(e+fx)(a+a\sec(e+fx))^m\,dx=\int (a\sec{(fx+e)}+a)^m\sec{(fx+e)}^4\,dx$$

input
```
integrate(sec(f*x+e)^4*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^4, x)
```

Giac [F]

$$\int \sec^4(e+fx)(a+a\sec(e+fx))^m\,dx=\int (a\sec{(fx+e)}+a)^m\sec{(fx+e)}^4\,dx$$

input
```
integrate(sec(f*x+e)^4*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^4(e+fx)(a+a\sec(e+fx))^m\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\cos\left(e+f\,x\right)^4}\,dx$$

input
```
int((a + a/cos(e + f*x))^m/cos(e + f*x)^4,x)
```

output
```
int((a + a/cos(e + f*x))^m/cos(e + f*x)^4, x)
```

Reduce [F]

$$\int \sec^4(e+fx)(a+a\sec(e+fx))^m\,dx=\int \left(\sec\left(fx+e\right)a+a\right)^m\sec\left(fx+e\right)^4dx$$

input
```
int(sec(f*x+e)^4*(a+a*sec(f*x+e))^m,x)
```

output
```
int((sec(e + f*x)*a + a)**m*sec(e + f*x)**4,x)
```

3.342 $\int \sec^3(e+fx)(a+a\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \sec^3(e+fx)(a+a\sec(e+fx))^m\,dx = -\frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f(2+3m+m^2)} + \frac{2^{\frac{1}{2}+m}(1+m+m^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+}{f(1+m)(2+m)} + \frac{(a+a\sec(e+fx))^{1+m}\tan(e+fx)}{af(2+m)}$$

output
```
-(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(m^2+3*m+2)+2^(1/2+m)*(m^2+m+1)*hypergeom
([1/2, 1/2-m],[3/2],1/2-1/2*sec(f*x+e))*(1+sec(f*x+e))^(-1/2-m)*(a+a*sec(f
*x+e))^m*tan(f*x+e)/f/(1+m)/(2+m)+(a+a*sec(f*x+e))^(1+m)*tan(f*x+e)/a/f/(2
+m)
```

Mathematica [F(-1)]

Timed out.

$$\int \sec^3(e+fx)(a+a\sec(e+fx))^m\,dx=\$\text{Aborted}$$

input
```
Integrate[Sec[e + f*x]^3*(a + a*Sec[e + f*x])^m,x]
```

output
```
$Aborted
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4287, 3042, 4489, 3042, 4315, 3042, 4314, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(e+fx)(a\sec(e+fx)+a)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^3\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m dx$$

$$\Big\downarrow \text{4287}$$

$$\frac{\int \sec(e+fx)(a(m+1)-a\sec(e+fx))(\sec(e+fx)a+a)^m dx}{a(m+2)}+\frac{\tan(e+fx)(a\sec(e+fx)+a)^{m+1}}{af(m+2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a(m+1)-a\csc\left(e+fx+\frac{\pi}{2}\right)\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^m dx}{a(m+2)}+\frac{\tan(e+fx)(a\sec(e+fx)+a)^{m+1}}{af(m+2)}$$

$$\Big\downarrow \text{4489}$$

$$\frac{\frac{a(m^2+m+1)\int \sec(e+fx)(\sec(e+fx)a+a)^m dx}{m+1}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}}{a(m+2)}+\frac{\tan(e+fx)(a\sec(e+fx)+a)^{m+1}}{af(m+2)}$$

$\downarrow$ 3042

$$\frac{\frac{a(m^2+m+1)\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^m dx}{m+1}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}}{a(m+2)}+\frac{\tan(e+fx)(a\sec(e+fx)+a)^{m+1}}{af(m+2)}$$

$\downarrow$ 4315

$$\frac{\frac{a(m^2+m+1)(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int \sec(e+fx)(\sec(e+fx)+1)^m dx}{m+1}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}}{a(m+2)}+\frac{\tan(e+fx)(a\sec(e+fx)+a)^{m+1}}{af(m+2)}$$

$\downarrow$ 3042

$$\frac{\frac{a(m^2+m+1)(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m dx}{m+1}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}}{a(m+2)}+\frac{\tan(e+fx)(a\sec(e+fx)+a)^{m+1}}{af(m+2)}$$

$\downarrow$ 4314

$$\frac{-\frac{a(m^2+m+1)\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\int \frac{(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f(m+1)\sqrt{1-\sec(e+fx)}}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}}{a(m+2)}+\frac{\tan(e+fx)(a\sec(e+fx)+a)^{m+1}}{af(m+2)}$$

$\downarrow$ 79

$$\frac{\frac{a2^{m+\frac{1}{2}}(m^2+m+1)\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx))\right)}{f(m+1)}-\frac{a\tan(e+fx)(a\,\text{se}}{f(m}}{a(m+2)}$$
$$\frac{\tan(e+fx)(a\sec(e+fx)+a)^{m+1}}{af(m+2)}$$

input
```
Int[Sec[e + f*x]^3*(a + a*Sec[e + f*x])^m,x]
```

output
```
((a + a*Sec[e + f*x])^(1 + m)*Tan[e + f*x])/(a*f*(2 + m)) + (-((a*(a + a*S
ec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m))) + (2^(1/2 + m)*a*(1 + m + m^2)*H
ypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f*x])/2]*(1 + Sec[e + f*x
])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m)))/(a*(2 + m)
)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4287
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 -
b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Maple [F]

$$\int \sec\left(fx+e\right)^{3}\left(a+a\sec\left(fx+e\right)\right)^{m}dx$$

input
```
int(sec(f*x+e)^3*(a+a*sec(f*x+e))^m,x)
```

output
```
int(sec(f*x+e)^3*(a+a*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int \sec^3(e+fx)(a+a\sec(e+fx))^m\,dx=\int \left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{3}\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

Sympy [F]

$$\int \sec^3(e+fx)(a+a\sec(e+fx))^m\,dx = \int (a(\sec(e+fx)+1))^m \sec^3(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**3*(a+a*sec(f*x+e))**m,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)**3, x)
```

Maxima [F]

$$\int \sec^3(e+fx)(a+a\sec(e+fx))^m\,dx = \int (a\sec(fx+e)+a)^m \sec(fx+e)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

Giac [F]

$$\int \sec^3(e+fx)(a+a\sec(e+fx))^m\,dx = \int (a\sec(fx+e)+a)^m \sec(fx+e)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(e+fx)(a+a\sec(e+fx))^m\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\cos\left(e+f\,x\right)^3}\,dx$$

input
```
int((a + a/cos(e + f*x))^m/cos(e + f*x)^3,x)
```

output
```
int((a + a/cos(e + f*x))^m/cos(e + f*x)^3, x)
```

Reduce [F]

$$\int \sec^3(e+fx)(a+a\sec(e+fx))^m\,dx = \int \left(\sec\left(fx+e\right)a+a\right)^m\sec\left(fx+e\right)^3dx$$

input
```
int(sec(f*x+e)^3*(a+a*sec(f*x+e))^m,x)
```

output
```
int((sec(e + f*x)*a + a)**m*sec(e + f*x)**3,x)
```

3.343 $\int \sec^2(e+fx)(a+a\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^m\,dx=\frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)}+\frac{2^{\frac{1}{2}+m}m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+f}{f(1+m)}$$

output
```
(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(1+m)+2^(1/2+m)*m*hypergeom([1/2, 1/2-m],[
3/2],1/2-1/2*sec(f*x+e))*(1+sec(f*x+e))^(-1/2-m)*(a+a*sec(f*x+e))^m*tan(f*
x+e)/f/(1+m)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.89

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^m\,dx=\frac{(1+\sec(e+fx))^{-\frac{1}{2}-m}(a(1+\sec(e+fx)))^m\left(2^{\frac{3}{2}+m}m\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sec(e-\right.\right.}{f+2fm}$$

input
```
Integrate[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^m,x]
```

output
```
((1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))^m*(2^(3/2 + m)*m*Hyp
ergeometric2F1[1/2, -1/2 - m, 3/2, (1 - Sec[e + f*x])/2] + (1 + Sec[e + f*
x])^(1/2 + m))*Tan[e + f*x])/(f + 2*f*m)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4285, 3042, 4315, 3042, 4314, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)(a\sec(e+fx)+a)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^2\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m dx$$

$$\Big\downarrow \text{4285}$$

$$\frac{m\int \sec(e+fx)(\sec(e+fx)a+a)^m dx}{m+1}+\frac{\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{m\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^m dx}{m+1}+\frac{\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}$$

$$\Big\downarrow \text{4315}$$

$$\frac{m(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int \sec(e+fx)(\sec(e+fx)+1)^m dx}{m+1}+\frac{\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{m(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m dx}{m+1}+\frac{\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}$$

$$\Big\downarrow \text{4314}$$

$$\frac{\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}-\frac{m\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\int\frac{(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f(m+1)\sqrt{1-\sec(e+fx)}}$$

$$\Big\downarrow \text{79}$$

$$\frac{2^{m+\frac{1}{2}}m\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+\right.}{f(m+1)}$$
$$\frac{\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}$$

input
```
Int[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^m,x]
```

output
```
((a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m)) + (2^(1/2 + m)*m*Hyperge
ometric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f*x])/2]*(1 + Sec[e + f*x])^(-1
/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4285
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[a*(m/(b*(m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x]
/; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \sec(fx+e)^2 (a+a\sec(fx+e))^m\, dx$$

input
```
int(sec(f*x+e)^2*(a+a*sec(f*x+e))^m,x)
```

output
```
int(sec(f*x+e)^2*(a+a*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^m\, dx = \int (a\sec(fx+e)+a)^m \sec(fx+e)^2\, dx$$

input
```
integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

Sympy [F]

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^m\,dx=\int (a(\sec(e+fx)+1))^m\sec^2(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**2*(a+a*sec(f*x+e))**m,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)**2, x)
```

Maxima [F]

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^m\,dx=\int (a\sec(fx+e)+a)^m\sec(fx+e)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

Giac [F]

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^m\,dx=\int (a\sec(fx+e)+a)^m\sec(fx+e)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^m\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\cos\left(e+f\,x\right)^2}\,dx$$

input
```
int((a + a/cos(e + f*x))^m/cos(e + f*x)^2,x)
```

output
```
int((a + a/cos(e + f*x))^m/cos(e + f*x)^2, x)
```

Reduce [F]

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^m\,dx=\int \left(\sec\left(fx+e\right)a+a\right)^m\sec\left(fx+e\right)^2dx$$

input
```
int(sec(f*x+e)^2*(a+a*sec(f*x+e))^m,x)
```

output
```
int((sec(e + f*x)*a + a)**m*sec(e + f*x)**2,x)
```

3.344 $\int \sec(e+fx)(a+a\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\,dx = \frac{2^{\frac{1}{2}+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^{m}}{f}$$

output
```
2^(1/2+m)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sec(f*x+e))*(1+sec(f*x+e))^
(-1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\,dx = \frac{2^{\frac{1}{2}+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a(1+\sec(e+fx)))}{f}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m,x]
```

output
```
(2^(1/2 + m)*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f*x])/2]*(1
 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))^m*Tan[e + f*x])/f
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 4315, 3042, 4314, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m dx$$

$$\Big\downarrow \text{4315}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int \sec(e+fx)(\sec(e+fx)+1)^m dx$$

$$\Big\downarrow \text{3042}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m dx$$

$$\Big\downarrow \text{4314}$$

$$-\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\int \frac{(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}}$$

$$\Big\downarrow \text{79}$$

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+f}{f}$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m,x]
```

output
```
(2^(1/2 + m)*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f*x])/2]*(1
 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/f
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \sec(fx+e)\,(a+a\sec(fx+e))^m\,dx$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m,x)
```

output
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\,dx=\int (a\sec(fx+e)+a)^m\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\,dx=\int (a(\sec(e+fx)+1))^m\sec(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x), x)
```

Maxima [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\,dx=\int (a\sec(fx+e)+a)^m\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\,dx=\int (a\sec(fx+e)+a)^m\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\cos(e+f\,x)}\,dx$$

input
```
int((a + a/cos(e + f*x))^m/cos(e + f*x),x)
```

output
```
int((a + a/cos(e + f*x))^m/cos(e + f*x), x)
```

Reduce [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\,dx=\int (\sec(fx+e)\,a+a)^m\sec(fx+e)\,dx$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m,x)
```

output
```
int((sec(e + f*x)*a + a)**m*sec(e + f*x),x)
```

3.345 $\int (a + a\sec(e + fx))^m \, dx$

Optimal result

Integrand size = 12, antiderivative size = 84

$$\int (a + a\sec(e + fx))^m \, dx = \frac{2^{\frac{1}{2}+m} \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{2} - m, 1, \frac{3}{2}, \frac{1}{2}(1 - \sec(e + fx)), 1 - \sec(e + fx)\right) (1 + \sec(e + fx))^{-\frac{1}{2}-m} (a + a\sec(e$$

output
```
2^(1/2+m)*AppellF1(1/2,1,1/2-m,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*(1+sec
(f*x+e))^(-1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 711 vs. 2(84) = 168.

Time = 4.31 (sec) , antiderivative size = 711, normalized size of antiderivative = 8.46

$$\int (a + a\sec(e + fx))^m \, dx = \text{Too large to display}$$

input
```
Integrate[(a + a*Sec[e + f*x])^m,x]
```

output
```
(30*AppellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[
(e + f*x)/2]^2*Cos[e + f*x]*(a*(1 + Sec[e + f*x]))^m*Sin[e + f*x]*(3*Appel
lF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*(AppellF1
[3/2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - m*AppellF1[3/2
, 1 + m, 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2
]^2))/(f*(45*AppellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]^2*Cos[(e + f*x)/2]^2*(1 + 2*m - 2*m*Cos[e + f*x] + Cos[2*(e + f*x)])
+ 6*AppellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[
(e + f*x)/2]^2*(-5*AppellF1[3/2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2]*(1 + 2*m - 2*(2 + m)*Cos[e + f*x] + Cos[2*(e + f*x)]) + 5*m*App
ellF1[3/2, 1 + m, 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + 2*
m - 2*(2 + m)*Cos[e + f*x] + Cos[2*(e + f*x)]) - 48*(2*AppellF1[5/2, m, 3,
 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*m*AppellF1[5/2, 1 + m,
2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*(1 + m)*AppellF1[5/2,
 2 + m, 1, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Cot[e + f*x]*Csc
[e + f*x]*Sin[(e + f*x)/2]^4) + 40*(AppellF1[3/2, m, 2, 5/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2] - m*AppellF1[3/2, 1 + m, 1, 5/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2])^2*Cos[e + f*x]*Sin[(e + f*x)/2]^2*Tan[(e + f*
x)/2]^2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 4266, 3042, 4265, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^m\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m\,dx$$

$$\Big\downarrow \ 4266$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int (\sec(e+fx)+1)^m dx$$

$$\Big\downarrow \text{3042}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m dx$$

$$\Big\downarrow \text{4265}$$

$$-\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\int\frac{\cos(e+fx)(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}}$$

$$\Big\downarrow \text{153}$$

$$\frac{\sqrt{2}\tan(e+fx)(a\sec(e+fx)+a)^m\operatorname{AppellF1}\left(m+\frac{1}{2},\frac{1}{2},1,m+\frac{3}{2},\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)}{f(2m+1)\sqrt{1-\sec(e+fx)}}$$

input
```
Int[(a + a*Sec[e + f*x])^m,x]
```

output
```
(Sqrt[2]*AppellF1[1/2 + m, 1/2, 1, 3/2 + m, (1 + Sec[e + f*x])/2, 1 + Sec[
e + f*x]]*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 - Sec[e
 + f*x]])
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4265
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[a^n*(Cot
[c + d*x]/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]))   Subst[Int[(1
 + b*(x/a))^(n - 1/2)/(x*Sqrt[1 - b*(x/a)]), x], x, Csc[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0
]
```

rule 4266
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Csc[c + d*x])^FracPart[n]/(1 + (b/a)*Csc[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Maple [F]

$$\int (a+a\sec (fx+e))^m\,dx$$

input
```
int((a+a*sec(f*x+e))^m,x)
```

output
```
int((a+a*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int (a+a\sec(e+fx))^m\,dx=\int (a\sec (fx+e)+a)^m\,dx$$

input
```
integrate((a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^m, x)
```

Sympy [F]

$$\int (a + a \sec(e + fx))^m \, dx = \int (a \sec(e + fx) + a)^m \, dx$$

input
```
integrate((a+a*sec(f*x+e))**m,x)
```

output
```
Integral((a*sec(e + f*x) + a)**m, x)
```

Maxima [F]

$$\int (a + a \sec(e + fx))^m \, dx = \int (a \sec(fx + e) + a)^m \, dx$$

input
```
integrate((a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^m, x)
```

Giac [F]

$$\int (a + a \sec(e + fx))^m \, dx = \int (a \sec(fx + e) + a)^m \, dx$$

input
```
integrate((a+a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(e + fx))^m \, dx = \int \left(a + \frac{a}{\cos(e + f x)}\right)^m dx$$

input
```
int((a + a/cos(e + f*x))^m,x)
```

output
```
int((a + a/cos(e + f*x))^m, x)
```

Reduce [F]

$$\int (a + a\sec(e + fx))^m \, dx = \int (\sec(fx + e) a + a)^m \, dx$$

input
```
int((a+a*sec(f*x+e))^m,x)
```

output
```
int((sec(e + f*x)*a + a)**m,x)
```

3.346 $\int \cos(e+fx)(a+a\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \cos(e+fx)(a+a\sec(e+fx))^m\,dx = \frac{2^{\frac{1}{2}+m}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2}-m,2,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx)),1-\sec(e+fx)\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e$$

output
```
2^(1/2+m)*AppellF1(1/2,2,1/2-m,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*(1+sec
(f*x+e))^(-1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3781 vs. 2(84) = 168.

Time = 14.99 (sec) , antiderivative size = 3781, normalized size of antiderivative = 45.01

$$\int \cos(e+fx)(a+a\sec(e+fx))^m\,dx = \text{Result too large to show}$$

input
```
Integrate[Cos[e + f*x]*(a + a*Sec[e + f*x])^m,x]
```

output
```
(2^(1 + m)*Cos[(e + f*x)/2]^3*Cos[e + f*x]*(Cos[(e + f*x)/2]^2*Sec[e + f*x
])^m*(a*(1 + Sec[e + f*x]))^m*Sin[(e + f*x)/2]*((-3*AppellF1[1/2, m, 1, 3/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2)/(3*AppellF
1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*(AppellF1[3
/2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - m*AppellF1[3/2,
1 + m, 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^
2) + (2*AppellF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])
/(AppellF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(
-2*AppellF1[3/2, m, 3, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*A
ppellF1[3/2, 1 + m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[
(e + f*x)/2]^2)/3)))/(f*(2^m*Cos[(e + f*x)/2]^4*(Cos[(e + f*x)/2]^2*Sec[e
+ f*x])^m*((-3*AppellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*Sec[(e + f*x)/2]^2)/(3*AppellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] - 2*(AppellF1[3/2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2] - m*AppellF1[3/2, 1 + m, 1, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (2*AppellF1[1/2, m, 2, 3/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])/(AppellF1[1/2, m, 2, 3/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-2*AppellF1[3/2, m, 3, 5/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/2, 1 + m, 2, 5/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3)) - 3*2^m*Cos[(e...
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 4315, 3042, 4314, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(e+fx)(a\sec(e+fx)+a)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m}{\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4315}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int \cos(e+fx)(\sec(e+fx)+1)^m dx$$

$$\Big\downarrow \ 3042$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m}{\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 4314$$

$$-\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\int \frac{\cos^2(e+fx)(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}}$$

$$\Big\downarrow \ 153$$

$$-\frac{\sqrt{2}\tan(e+fx)(a\sec(e+fx)+a)^m \operatorname{AppellF1}\left(m+\frac{1}{2},\frac{1}{2},2,m+\frac{3}{2},\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)}{f(2m+1)\sqrt{1-\sec(e+fx)}}$$

input
```
Int[Cos[e + f*x]*(a + a*Sec[e + f*x])^m,x]
```

output
```
-((Sqrt[2]*AppellF1[1/2 + m, 1/2, 2, 3/2 + m, (1 + Sec[e + f*x])/2, 1 + Se
c[e + f*x]]*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 - Sec
[e + f*x]]))
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \cos \left(fx+e\right) \left(a+a\sec \left(fx+e\right)\right)^{m} dx$$

input
```
int(cos(f*x+e)*(a+a*sec(f*x+e))^m,x)
```

output
```
int(cos(f*x+e)*(a+a*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int \cos(e+fx)(a+a\sec(e+fx))^m\,dx=\int \left(a\sec \left(fx+e\right)+a\right)^{m}\cos \left(fx+e\right)\,dx$$

input
```
integrate(cos(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

Sympy [F]

$$\int \cos(e+fx)(a+a\sec(e+fx))^m\,dx = \int (a(\sec(e+fx)+1))^m \cos(e+fx)\,dx$$

input
```
integrate(cos(f*x+e)*(a+a*sec(f*x+e))**m,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m*cos(e + f*x), x)
```

Maxima [F]

$$\int \cos(e+fx)(a+a\sec(e+fx))^m\,dx = \int (a\sec(fx+e)+a)^m \cos(fx+e)\,dx$$

input
```
integrate(cos(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

Giac [F]

$$\int \cos(e+fx)(a+a\sec(e+fx))^m\,dx = \int (a\sec(fx+e)+a)^m \cos(fx+e)\,dx$$

input
```
integrate(cos(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(e+fx)(a+a\sec(e+fx))^m\,dx=\int \cos\left(e+f\,x\right)\,\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^m\,dx$$

input
```
int(cos(e + f*x)*(a + a/cos(e + f*x))^m,x)
```

output
```
int(cos(e + f*x)*(a + a/cos(e + f*x))^m, x)
```

Reduce [F]

$$\int \cos(e+fx)(a+a\sec(e+fx))^m\,dx=\int \left(\sec\left(fx+e\right)a+a\right)^m\cos\left(fx+e\right)dx$$

input
```
int(cos(f*x+e)*(a+a*sec(f*x+e))^m,x)
```

output
```
int((sec(e + f*x)*a + a)**m*cos(e + f*x),x)
```

3.347 $\int (d\sec(e+fx))^{3/2}(a+a\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 25, antiderivative size = 98

$$\int (d\sec(e+fx))^{3/2}(a+a\sec(e+fx))^m\,dx = -\frac{2\,\mathrm{AppellF1}\left(\frac{3}{2},\frac{1}{2},\frac{1}{2}-m,\frac{5}{2},\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^{3/2}(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\,\mathrm{s}}{3f\sqrt{1-\sec(e+fx)}}$$

output
```
-2/3*AppellF1(3/2,1/2-m,1/2,5/2,-sec(f*x+e),sec(f*x+e))*(d*sec(f*x+e))^(3/
2)*(1+sec(f*x+e))^(-1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(1-sec(f*x+e))^
(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2529 vs. 2(98) = 196.

Time = 15.59 (sec) , antiderivative size = 2529, normalized size of antiderivative = 25.81

$$\int (d\sec(e+fx))^{3/2}(a+a\sec(e+fx))^m\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Sec[e + f*x])^(3/2)*(a + a*Sec[e + f*x])^m,x]
```

output
```
(-3*2^(1 + m)*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2]*Sqrt[Sec[e + f*x]]*(d*Sec[e + f*x])^(3/2)*(Cos[(e + f*x)/2]
^2*Sec[e + f*x])^m*(a*(1 + Sec[e + f*x]))^m*Tan[(e + f*x)/2])/(f*(-1 + Tan
[(e + f*x)/2]^2)*(3*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] + (AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2] + (3 + 2*m)*AppellF1[3/2, 5/2 + m, -1/2, 5/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^(1 + m)*Ap
pellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*S
ec[(e + f*x)/2]^2*Sqrt[Sec[e + f*x]]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^m*T
an[(e + f*x)/2]^2)/((-1 + Tan[(e + f*x)/2]^2)^2*(3*AppellF1[1/2, 3/2 + m,
-1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (AppellF1[3/2, 3/2 +
 m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + 2*m)*AppellF
1[3/2, 5/2 + m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(
e + f*x)/2]^2)) - (3*2^m*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Sqrt[Sec[e + f*x]]*(Cos[(e +
f*x)/2]^2*Sec[e + f*x])^m)/((-1 + Tan[(e + f*x)/2]^2)*(3*AppellF1[1/2, 3/2
 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (AppellF1[3/2,
 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + 2*m)*A
ppellF1[3/2, 5/2 + m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])
*Tan[(e + f*x)/2]^2)) - (3*2^m*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e...
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4315, 3042, 4314, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^{3/2}(a\sec(e+fx)+a)^m\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m dx$$

$$\Big\downarrow \text{ 4315}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int (d\sec(e+fx))^{3/2}(\sec(e+fx)+1)^m dx$$

$$\Big\downarrow \text{3042}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m dx$$

$$\Big\downarrow \text{4314}$$

$$-\frac{d\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\int \frac{\sqrt{d\sec(e+fx)}(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}}$$

$$\Big\downarrow \text{150}$$

$$-\frac{2\tan(e+fx)(d\sec(e+fx))^{3/2}(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m \operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{2},\frac{1}{2}-m,\frac{5}{2},\sec(e+\right.}{3f\sqrt{1-\sec(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^(3/2)*(a + a*Sec[e + f*x])^m,x]
```

output
```
(-2*AppellF1[3/2, 1/2, 1/2 - m, 5/2, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e
 + f*x])^(3/2)*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e
+ f*x])/(3*f*Sqrt[1 - Sec[e + f*x]])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int (d\sec(fx+e))^{\frac{3}{2}} (a+a\sec(fx+e))^m \, dx$$

input
```
int((d*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^m,x)
```

output
```
int((d*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int (d\sec(e+fx))^{3/2}(a+a\sec(e+fx))^m \, dx = \int (d\sec(fx+e))^{\frac{3}{2}} (a\sec(fx+e)+a)^m \, dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m*d*sec(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(a+a\sec(e+fx))^m\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(3/2)*(a+a*sec(f*x+e))**m,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\sec(e+fx))^{3/2}(a+a\sec(e+fx))^m\,dx = \int (d\sec(fx+e))^{\frac{3}{2}}\,(a\sec(fx+e)+a)^m\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(3/2)*(a*sec(f*x + e) + a)^m, x)
```

Giac [F]

$$\int (d\sec(e+fx))^{3/2}(a+a\sec(e+fx))^m\,dx = \int (d\sec(fx+e))^{\frac{3}{2}}\,(a\sec(fx+e)+a)^m\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(3/2)*(a*sec(f*x + e) + a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(a+a\sec(e+fx))^m\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)^m\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}dx$$

input
```
int((a + a/cos(e + f*x))^m*(d/cos(e + f*x))^(3/2),x)
```

output
```
int((a + a/cos(e + f*x))^m*(d/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int (d\sec(e+fx))^{3/2}(a+a\sec(e+fx))^m\,dx = \sqrt{d}\left(\int \sqrt{\sec(fx+e)}\,(\sec(fx+e)\,a+a)^m\sec(fx+e)\,dx\right)d$$

input
```
int((d*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^m,x)
```

output
```
sqrt(d)*int(sqrt(sec(e + f*x))*(sec(e + f*x)*a + a)**m*sec(e + f*x),x)*d
```

3.348 $\int \sqrt{d\sec(e+fx)}(a+a\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 25, antiderivative size = 96

$$\int \sqrt{d\sec(e+fx)}(a+a\sec(e+fx))^m\,dx = -\frac{2\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\sec(e+fx),-\sec(e+fx)\right)\sqrt{d\sec(e+fx)}(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\,\text{s}}{f\sqrt{1-\sec(e+fx)}}$$

output
```
-2*AppellF1(1/2,1/2-m,1/2,3/2,-sec(f*x+e),sec(f*x+e))*(d*sec(f*x+e))^(1/2)
*(1+sec(f*x+e))^(-1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(1-sec(f*x+e))^(1
/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 782 vs. 2(96) = 192.

Time = 8.91 (sec) , antiderivative size = 782, normalized size of antiderivative = 8.15

$$\int \sqrt{d\sec(e+fx)}(a+a\sec(e+fx))^m\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[d*Sec[e + f*x]]*(a + a*Sec[e + f*x])^m,x]
```

output
```
(120*AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*Cos[(e + f*x)/2]*Sqrt[d*Sec[e + f*x]]*(a*(1 + Sec[e + f*x]))^m*Sin[(e
 + f*x)/2]*(3*AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2] + (-AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2] + (1 + 2*m)*AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))/(f*(45*AppellF1[1/2, 1
/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]^2*(3 + 4*m - 4*
m*Cos[e + f*x] + Cos[2*(e + f*x)])*Sec[e + f*x] + 20*(AppellF1[3/2, 1/2 +
m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - (1 + 2*m)*AppellF1
[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])^2*Tan[(
e + f*x)/2]^4 - 3*AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2*(5*AppellF1[3/2, 1/2 + m, 3/2, 5/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(3 + 4*m - 4*(2 + m)*Cos[e + f*x]
+ Cos[2*(e + f*x)])*Sec[e + f*x] - 5*(1 + 2*m)*AppellF1[3/2, 3/2 + m, 1/2,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(3 + 4*m - 4*(2 + m)*Cos[e
+ f*x] + Cos[2*(e + f*x)])*Sec[e + f*x] + 12*(3*AppellF1[5/2, 1/2 + m, 5/2
, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 + 2*m)*(-2*AppellF1[5
/2, 3/2 + m, 3/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + 2*m
)*AppellF1[5/2, 5/2 + m, 1/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
]))*Tan[(e + f*x)/2]^2)))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4315, 3042, 4314, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d\sec(e+fx)}(a\sec(e+fx)+a)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m dx$$

$$\Big\downarrow \text{4315}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int \sqrt{d\sec(e+fx)}(\sec(e+fx)+1)^m dx$$

$$\Big\downarrow \text{3042}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int\sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m dx$$

$$\Big\downarrow \text{4314}$$

$$-\frac{d\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\int\frac{(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}\sqrt{d\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}}$$

$$\Big\downarrow \text{150}$$

$$-\frac{2\tan(e+fx)\sqrt{d\sec(e+fx)}(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\sec(e+fx\right.}{f\sqrt{1-\sec(e+fx)}}$$

input
```
Int[Sqrt[d*Sec[e + f*x]]*(a + a*Sec[e + f*x])^m,x]
```

output
```
(-2*AppellF1[1/2, 1/2, 1/2 - m, 3/2, Sec[e + f*x], -Sec[e + f*x]]*Sqrt[d*S
ec[e + f*x]]*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e +
f*x])/(f*Sqrt[1 - Sec[e + f*x]])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \sqrt{d \sec (fx+e)}\,(a+a \sec (fx+e))^m\,dx$$

input
```
int((d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^m,x)
```

output
```
int((d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int \sqrt{d \sec(e+fx)}(a+a \sec(e+fx))^m\,dx = \int \sqrt{d \sec (fx+e)}(a \sec (fx+e)+a)^m\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m, x)
```

Sympy [F]

$$\int \sqrt{d\sec(e+fx)}(a+a\sec(e+fx))^m\,dx = \int (a(\sec(e+fx)+1))^m\sqrt{d\sec(e+fx)}\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/2)*(a+a*sec(f*x+e))**m,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m*sqrt(d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \sqrt{d\sec(e+fx)}(a+a\sec(e+fx))^m\,dx = \int \sqrt{d\sec(fx+e)}(a\sec(fx+e)+a)^m\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m, x)
```

Giac [F]

$$\int \sqrt{d\sec(e+fx)}(a+a\sec(e+fx))^m\,dx = \int \sqrt{d\sec(fx+e)}(a\sec(fx+e)+a)^m\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d \sec(e+fx)}(a+a\sec(e+fx))^m\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)^m\sqrt{\frac{d}{\cos(e+f\,x)}}\,dx$$

input
```
int((a + a/cos(e + f*x))^m*(d/cos(e + f*x))^(1/2),x)
```

output
```
int((a + a/cos(e + f*x))^m*(d/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d \sec(e+fx)}(a+a\sec(e+fx))^m\,dx = \sqrt{d}\left(\int \sqrt{\sec(fx+e)}\,(\sec(fx+e)\,a + a)^m\,dx\right)$$

input
```
int((d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^m,x)
```

output
```
sqrt(d)*int(sqrt(sec(e + f*x))*(sec(e + f*x)*a + a)**m,x)
```

3.349 $\int \frac{(a+a\sec(e+fx))^m}{\sqrt{d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 96

$$\int \frac{(a+a\sec(e+fx))^m}{\sqrt{d\sec(e+fx)}}\,dx = \frac{2\,\mathrm{AppellF1}\left(-\frac{1}{2},\frac{1}{2},\frac{1}{2}-m,\frac{1}{2},\sec(e+fx),-\sec(e+fx)\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\,\mathrm{ta}}{f\sqrt{1-\sec(e+fx)}\sqrt{d\sec(e+fx)}}$$

output
```
2*AppellF1(-1/2,1/2-m,1/2,1/2,-sec(f*x+e),sec(f*x+e))*(1+sec(f*x+e))^(-1/2
-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(1-sec(f*x+e))^(1/2)/(d*sec(f*x+e))^(1
/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.59 (sec) , antiderivative size = 820, normalized size of antiderivative = 8.54

$$\int \frac{(a+a\sec(e+fx))^m}{\sqrt{d\sec(e+fx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + a*Sec[e + f*x])^m/Sqrt[d*Sec[e + f*x]],x]
```

output
```
(120*d*AppellF1[1/2, -1/2 + m, 3/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*Cos[(e + f*x)/2]*(a*(1 + Sec[e + f*x]))^m*Sin[(e + f*x)/2]*(Cos[e
+ f*x] + I*Sin[e + f*x])*(3*AppellF1[1/2, -1/2 + m, 3/2, 3/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] - (3*AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 3/2,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*(1 - I
*Tan[e + f*x]))/(f*(d*Sec[e + f*x])^(3/2)*(45*AppellF1[1/2, -1/2 + m, 3/2,
 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]^2*(1 + 4*m - 4*m*Cos[e + f*
x] + 3*Cos[2*(e + f*x)])*Sec[e + f*x] + 20*(3*AppellF1[3/2, -1/2 + m, 5/2,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1
/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])^2*Tan[(e + f*x
)/2]^4 - 3*AppellF1[1/2, -1/2 + m, 3/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*Tan[(e + f*x)/2]^2*(15*AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + 4*m - 4*(2 + m)*Cos[e + f*x] + 3*
Cos[2*(e + f*x)])*Sec[e + f*x] - 5*(-1 + 2*m)*AppellF1[3/2, 1/2 + m, 3/2,
5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + 4*m - 4*(2 + m)*Cos[e +
 f*x] + 3*Cos[2*(e + f*x)])*Sec[e + f*x] + 12*(15*AppellF1[5/2, -1/2 + m,
7/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*m)*(-6*Appell
F1[5/2, 1/2 + m, 5/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 +
 2*m)*AppellF1[5/2, 3/2 + m, 3/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*...
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4315, 3042, 4314, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^m}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m}{\sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4315}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int\frac{(\sec(e+fx)+1)^m}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int\frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m}{\sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4314}$$

$$-\frac{d\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\int\frac{(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}(d\sec(e+fx))^{3/2}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}}$$

$$\Big\downarrow \text{150}$$

$$\frac{2\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\operatorname{AppellF1}\left(-\frac{1}{2},\frac{1}{2},\frac{1}{2}-m,\frac{1}{2},\sec(e+fx),-\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{d\sec(e+fx)}}$$

input
```
Int[(a + a*Sec[e + f*x])^m/Sqrt[d*Sec[e + f*x]],x]
```

output
```
(2*AppellF1[-1/2, 1/2, 1/2 - m, 1/2, Sec[e + f*x], -Sec[e + f*x]]*(1 + Sec
[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*Sqrt[1 - Sec
[e + f*x]]*Sqrt[d*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{(a + a\sec(fx + e))^m}{\sqrt{d\sec(fx + e)}}dx$$

input
```
int((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(1/2),x)
```

output
```
int((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{(a + a\sec(e + fx))^m}{\sqrt{d\sec(e + fx)}}\,dx = \int \frac{(a\sec(fx + e) + a)^m}{\sqrt{d\sec(fx + e)}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m/(d*sec(f*x + e)), x)
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^m}{\sqrt{d\sec(e+fx)}}\,dx=\int \frac{(a(\sec{(e+fx)}+1))^m}{\sqrt{d\sec{(e+fx)}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**m/(d*sec(f*x+e))**(1/2),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m/sqrt(d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(a+a\sec(e+fx))^m}{\sqrt{d\sec(e+fx)}}\,dx=\int \frac{(a\sec{(fx+e)}+a)^m}{\sqrt{d\sec{(fx+e)}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^m/sqrt(d*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{(a+a\sec(e+fx))^m}{\sqrt{d\sec(e+fx)}}\,dx=\int \frac{(a\sec{(fx+e)}+a)^m}{\sqrt{d\sec{(fx+e)}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m/sqrt(d*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^m}{\sqrt{d\sec(e+fx)}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\sqrt{\frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int((a + a/cos(e + f*x))^m/(d/cos(e + f*x))^(1/2),x)
```

output
```
int((a + a/cos(e + f*x))^m/(d/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(e+fx))^m}{\sqrt{d\sec(e+fx)}}\,dx=\frac{\sqrt{d}\left(\int \frac{\sqrt{\sec(fx+e)}\,(\sec(fx+e)a+a)^m}{\sec(fx+e)}dx\right)}{d}$$

input
```
int((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(sec(e + f*x))*(sec(e + f*x)*a + a)**m)/sec(e + f*x),x))
/d
```

3.350 $\int \frac{(a+a\sec(e+fx))^m}{(d\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 98

$$\int \frac{(a+a\sec(e+fx))^m}{(d\sec(e+fx))^{3/2}}\,dx = \frac{2\,\text{AppellF1}\left(-\frac{3}{2},\frac{1}{2},\frac{1}{2}-m,-\frac{1}{2},\sec(e+fx),-\sec(e+fx)\right)(1+\sec(e+fx)}{3f\sqrt{1-\sec(e+fx)}(d\sec(e+fx))^{3/}}$$

output
```
2/3*AppellF1(-3/2,1/2-m,1/2,-1/2,-sec(f*x+e),sec(f*x+e))*(1+sec(f*x+e))^(-
1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(1-sec(f*x+e))^(1/2)/(d*sec(f*x+e))
^(3/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.26 (sec) , antiderivative size = 3349, normalized size of antiderivative = 34.17

$$\int \frac{(a+a\sec(e+fx))^m}{(d\sec(e+fx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + a*Sec[e + f*x])^m/(d*Sec[e + f*x])^(3/2),x]
```

output
```
(2^(1 + m)*Sec[e + f*x]^(3/2)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(1/2 + m)*
(a*(1 + Sec[e + f*x]))^m*((Cos[2*(e + f*x)]^3*Sqrt[Sec[e + f*x]]*(1 + Sec[
e + f*x])^m)/4 + Cos[2*(e + f*x)]^2*Sqrt[Sec[e + f*x]]*((1 + Sec[e + f*x])
^m/2 + (I/4)*(1 + Sec[e + f*x])^m*Sin[2*(e + f*x)]) + Cos[2*(e + f*x)]*Sqr
t[Sec[e + f*x]]*((1 + Sec[e + f*x])^m/4 + ((1 + Sec[e + f*x])^m*Sin[2*(e +
 f*x)]^2)/4) + Sqrt[Sec[e + f*x]]*((-1/4*I)*(1 + Sec[e + f*x])^m*Sin[2*(e
+ f*x)] + ((1 + Sec[e + f*x])^m*Sin[2*(e + f*x)]^2)/2 + (I/4)*(1 + Sec[e +
 f*x])^m*Sin[2*(e + f*x)]^3))*Tan[(e + f*x)/2]*(-(AppellF1[3/2, -1/2 + m,
5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e +
f*x)/2]^2)^(1/2 + m)*Tan[(e + f*x)/2]^2) - (9*AppellF1[1/2, -1/2 + m, 5/2,
 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x])/((Sec[(e + f*
x)/2]^2)^(3/2)*(-3*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] + (5*AppellF1[3/2, -1/2 + m, 7/2, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 5/2, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))))/(3*f*(d*Sec[
e + f*x])^(3/2)*(1 + Sec[e + f*x])^m*((2^m*Sec[(e + f*x)/2]^2*(Cos[(e + f*
x)/2]^2*Sec[e + f*x])^(1/2 + m)*(-(AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(1/2
 + m)*Tan[(e + f*x)/2]^2) - (9*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x])/((Sec[(e + f*x)/2]^2)^(3/...
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4315, 3042, 4314, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^m}{(d\sec(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m}{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4315}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int\frac{(\sec(e+fx)+1)^m}{(d\sec(e+fx))^{3/2}}dx$$

$\Big\downarrow$ 3042

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int\frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m}{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$\Big\downarrow$ 4314

$$-\frac{d\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\int\frac{(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}(d\sec(e+fx))^{5/2}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}}$$

$\Big\downarrow$ 150

$$\frac{2\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\operatorname{AppellF1}\left(-\frac{3}{2},\frac{1}{2},\frac{1}{2}-m,-\frac{1}{2},\sec(e+fx),-\sec(e+f\right.}{3f\sqrt{1-\sec(e+fx)}(d\sec(e+fx))^{3/2}}$$

input
```
Int[(a + a*Sec[e + f*x])^m/(d*Sec[e + f*x])^(3/2),x]
```

output
```
(2*AppellF1[-3/2, 1/2, 1/2 - m, -1/2, Sec[e + f*x], -Sec[e + f*x]]*(1 + Se
c[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(3*f*Sqrt[1 -
Sec[e + f*x]]*(d*Sec[e + f*x])^(3/2))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{(a+a\sec(fx+e))^{m}}{(d\sec(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(3/2),x)
```

output
```
int((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^m}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^{m}}{(d\sec(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m/(d^2*sec(f*x + e)^2),
 x)
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^m}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(a(\sec(e+fx)+1))^m}{(d\sec(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**m/(d*sec(f*x+e))**(3/2),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m/(d*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+a\sec(e+fx))^m}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^m}{(d\sec(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^m/(d*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{(a+a\sec(e+fx))^m}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^m}{(d\sec(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m/(d*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^m}{(d\sec(e+fx))^{3/2}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^m/(d/cos(e + f*x))^(3/2),x)
```

output
```
int((a + a/cos(e + f*x))^m/(d/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(e+fx))^m}{(d\sec(e+fx))^{3/2}}\,dx=\frac{\sqrt{d}\left(\int \frac{\sqrt{\sec(fx+e)}\,(\sec(fx+e)a+a)^m}{\sec(fx+e)^2}dx\right)}{d^2}$$

input
```
int((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(sec(e + f*x))*(sec(e + f*x)*a + a)**m)/sec(e + f*x)**2,
x))/d**2
```

3.351 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{10a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} + \frac{10a\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}$$

output
```
6/5*a*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+10/21*a*InverseJacobiAM(1/2*
d*x+1/2*c,2^(1/2))/d+10/21*a*cos(d*x+c)^(1/2)*sin(d*x+c)/d+2/5*a*cos(d*x+c
)^(3/2)*sin(d*x+c)/d+2/7*a*cos(d*x+c)^(5/2)*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.77 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.17

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{63(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}}-100\cos(c\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x]),x]
```

output
```
(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((63*(3*Cos[c - d*x - ArcTan[Tan[
c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 100*
Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeometr
icPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot
[c]]]*Sin[c] + Cos[c + d*x]*(-252*Cot[c] + 115*Sin[c + d*x] + 42*Sin[2*(c
+ d*x)] + 15*Sin[3*(c + d*x)]) - 126*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*Hype
rgeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c
]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(420*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4713, 3042, 3227, 3042, 3115, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{7}{2}}(c+dx)(a\sec(c+dx)+a)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{4713}$$

$$\int \cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \cos^{\frac{5}{2}}(c+dx)dx+a\int \cos^{\frac{7}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}dx$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{3}{5}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+a\left(\frac{5}{7}\int \cos^{\frac{3}{2}}(c+dx)dx+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+a\left(\frac{5}{7}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)+a\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)+ a\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)+ a\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{5}{7}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)+ a\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

input
```
Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x]),x]
```

output
```
a*((6*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*Cos[c + d*x]^(3/2)*Sin[c + d*x
])/(5*d)) + a*((2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (5*((2*Elliptic
F[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/7)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(98) = 196$.

Time = 6.08 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.43

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, a\left(240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-528\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+448\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos(}{105\sqrt{-2\,\mathrm{s}}}$

input
```
int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-528*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1
/2*c)+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-122*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2+25*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*
c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.33

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{2\left(15\,a\cos\left(dx+c\right)^2+21\,a\cos\left(dx+c\right)+25\,a\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-25i\,\sqrt{2}a\text{weierstrassPInvers}}{}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/105*(2*(15*a*cos(d*x + c)^2 + 21*a*cos(d*x + c) + 25*a)*sqrt(cos(d*x + c
))*sin(d*x + c) - 25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + 25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c)) + 63*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*a*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \int (a\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \int (a\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\,dx$$
$$= -\frac{2\,a\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$-\frac{2\,a\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x)),x)
```

output
```
- (2*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c +
 d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*a*cos(c + d*x)^(9/2)*sin(c + d
*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/
2))
```

Reduce [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)dx + \int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^{3}dx\right)$$

input
```
int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c)),x)
```

output
```
a*(int(sqrt(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x),x) + int(sqrt(cos(c
 + d*x))*cos(c + d*x)**3,x))
```

3.352 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}$$

output
```
6/5*a*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*InverseJacobiAM(1/2*d*
x+1/2*c,2^(1/2))/d+2/3*a*cos(d*x+c)^(1/2)*sin(d*x+c)/d+2/5*a*cos(d*x+c)^(3
/2)*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.65 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.67

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{9(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}} - 20\cos(c+\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x]),x]
```

output
```
(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((9*(3*Cos[c - d*x - ArcTan[Tan[c
]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 20*Co
s[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeometric
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c
]]]*Sin[c] + 2*Cos[c + d*x]*(-18*Cot[c] + 10*Sin[c + d*x] + 3*Sin[2*(c + d
*x)]) - 18*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}
, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan
[Tan[c]]]^2]))/(60*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4713, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{4713}$$

$$\int \cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \cos^{\frac{3}{2}}(c+dx)dx+a\int \cos^{\frac{5}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{3}{5}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+a\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+a\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+a\left(\frac{2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

input
```
Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x]),x]
```

output
```
a*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(3*d)) + a*((6*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*Cos[c + d*x]^(3/2)
*Sin[c + d*x])/(5*d))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(78) = 156$.

Time = 3.91 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.52

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, a\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}-28\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \mathrm{EllipticF}}{15\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\ \sin\left(\frac{dx}{2}+\right.}$

input
```
int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(24*cos(1/
2*d*x+1/2*c)^7-28*cos(1/2*d*x+1/2*c)^5+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))+4*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.57

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{2\left(3\,a\cos\left(dx+c\right)+5\,a\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-5i\,\sqrt{2}a\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/15*(2*(3*a*cos(d*x + c) + 5*a)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*I*sqr
t(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqr
t(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*I*sqr
t(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c))) - 9*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \int (a\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \int (a\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \frac{2\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,a\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3\,d} - \frac{2\,a\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+d\,x)^2\right)}{7\,d\,\sqrt{\sin(c+d\,x)^2}}$$

input
```
int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x)),x)
```

output
```
(2*a*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*a*cos(c + d*x)^(1/2)*sin(c +
d*x))/(3*d) - (2*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 1
1/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)\,dx + \int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)$$

input
```
int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x)
```

output
```
a*(int(sqrt(cos(c + d*x))*cos(c + d*x)**2*sec(c + d*x),x) + int(sqrt(cos(c
 + d*x))*cos(c + d*x)**2,x))
```

3.353 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
2*a*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*InverseJacobiAM(1/2*d*x+
1/2*c,2^(1/2))/d+2/3*a*cos(d*x+c)^(1/2)*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.18 (sec) , antiderivative size = 222, normalized size of antiderivative = 3.64

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{3(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}} - 4\cos(c+d\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x]),x]
```

output
```
(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((3*(3*Cos[c - d*x - ArcTan[Tan[c
]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 4*Cos
[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricP
FQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]
]]*Sin[c] - 4*Cos[c + d*x]*(3*Cot[c] - Sin[c + d*x]) - 6*Cos[c]*Csc[d*x +
ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Ta
n[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(12*d*Sqrt[Co
s[c + d*x]])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4713, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{4713}$$

$$\int \sqrt{\cos(c+dx)}(a\cos(c+dx)+a)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \cos^{\frac{3}{2}}(c+dx)dx+a\int \sqrt{\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sqrt{\sin \left(c+dx+\frac{\pi }{2}\right)}dx+a\int \sin \left(c+dx+\frac{\pi }{2}\right)^{3/2}dx$$

$$\Big\downarrow \text{3115}$$

$$a\int \sqrt{\sin \left(c+dx+\frac{\pi }{2}\right)}dx+a\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos (c+dx)}}dx+\frac{2 \sin (c+dx) \sqrt{\cos (c+dx)}}{3 d}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\int \sqrt{\sin \left(c+dx+\frac{\pi }{2}\right)}dx+a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi }{2}\right)}}dx+\frac{2 \sin (c+dx) \sqrt{\cos (c+dx)}}{3 d}\right)$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi }{2}\right)}}dx+\frac{2 \sin (c+dx) \sqrt{\cos (c+dx)}}{3 d}\right)+\frac{2 a E\left(\frac{1}{2} (c+dx)\right|2)}{d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2 a E\left(\frac{1}{2} (c+dx)\right|2)}{d}+a\left(\frac{2 \text{EllipticF}\left(\frac{1}{2} (c+dx),2\right)}{3 d}+\frac{2 \sin (c+dx) \sqrt{\cos (c+dx)}}{3 d}\right)$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x]),x]
```

output
```
(2*a*EllipticE[(c + d*x)/2, 2])/d + a*((2*EllipticF[(c + d*x)/2, 2])/(3*d)
 + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(58) = 116$.

Time = 2.30 (sec) , antiderivative size = 225, normalized size of antiderivative = 3.69

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}\right.}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sin\left(\frac{dx}{2}+\right.}$

input
```
int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.05

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{2\,a\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right) - i\,\sqrt{2}a\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)) + i\,\sqrt{2}a\text{w}}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/3*(2*a*sqrt(cos(d*x + c))*sin(d*x + c) - I*sqrt(2)*a*weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*a*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)
*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \int (a\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \int (a\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 9.87 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \frac{2\,a\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d} + \frac{2\,a\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}$$

input
```
int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x)),x)
```

output
```
(2*a*ellipticE(c/2 + (d*x)/2, 2))/d + (2*a*ellipticF(c/2 + (d*x)/2, 2))/(3
*d) + (2*a*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d)
```

Reduce [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)\sec\left(dx+c\right)dx + \int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)dx\right)$$

input
```
int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x)
```

output
```
a*(int(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x),x) + int(sqrt(cos(c +
d*x))*cos(c + d*x),x))
```

3.354 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 35

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))\,dx=\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

output
```
2*a*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a*InverseJacobiAM(1/2*d*x+1/
2*c,2^(1/2))/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.34 (sec) , antiderivative size = 155, normalized size of antiderivative = 4.43

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))\,dx$$
$$=\frac{a\sqrt{\cos(c+dx)}(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-2\sqrt{\cos^2(dx-\arctan(\cot(c)))}\sqrt{\csc^2(c)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\right.\right.}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

output
```
(a*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(-2*Sqrt[Cos[d
*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}
, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + Tan[d*x
+ ArcTan[Tan[c]]] - (HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcT
an[Tan[c]]]^2]*Tan[d*x + ArcTan[Tan[c]]])/Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2
]))/(2*d)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4713, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a\sec(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{4713}$$

$$\int \frac{a\cos(c+dx)+a}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \frac{1}{\sqrt{\cos(c+dx)}}dx+a\int \sqrt{\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3119}$$

$$a\int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

output
```
(2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(38) = 76.

Time = 0.70 (sec) , antiderivative size = 150, normalized size of antiderivative = 4.29

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, a\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\left(\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-\text{EllipticE}\left(\cos\left(\frac{dx}{2}\right.\right.\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\, d}$
risch	$-\frac{ia\sqrt{2}\,\sqrt{\left(e^{2i(dx+c)}+1\right)e^{-i(dx+c)}}}{d}-\frac{i\left(\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(dx+c)}-i\right)}\,\sqrt{ie^{i(dx+c)}}\,\text{EllipticF}\left(\sqrt{-i\left(e^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)}{\sqrt{e^{3i(dx+c)}+e^{i(dx+c)}}}-\frac{2}{\sqrt{e^{i(c}}}\right.}{}$

input
```
int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.06

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))\,dx$$
$$=\frac{-i\sqrt{2}a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}a\text{weierstrassPInverse}(-4,0,\cos(}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) +
I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + I*
sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) - I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F]

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))\,dx = a\left(\int \sqrt{\cos(c+dx)}\sec(c+dx)\,dx + \int \sqrt{\cos(c+dx)}\,dx\right)$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(sqrt(cos(c + d*x))*sec(c + d*x), x) + Integral(sqrt(cos(c + d*
x)), x))
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))\,dx = \int (a\sec(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))\,dx = \int (a\sec(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))\,dx = \frac{2a\left(\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)+\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)\right)}{d}$$

input
```
int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x)),x)
```

output
```
(2*a*(ellipticE(c/2 + (d*x)/2, 2) + ellipticF(c/2 + (d*x)/2, 2)))/d
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))\,dx = a\left(\int \sqrt{\cos(dx+c)}dx + \int \sqrt{\cos(dx+c)}\sec(dx+c)\,dx\right)$$

input
```
int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x)
```

output
```
a*(int(sqrt(cos(c + d*x)),x) + int(sqrt(cos(c + d*x))*sec(c + d*x),x))
```

3.355 $\int \frac{a+a\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{a+a\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = -\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d} + \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
-2*a*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a*InverseJacobiAM(1/2*d*x+1
/2*c,2^(1/2))/d+2*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.29 (sec) , antiderivative size = 209, normalized size of antiderivative = 3.67

$$\int \frac{a+a\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(4\cos(dx)\csc(c) - \frac{(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\,\mathrm{s}}{\sqrt{\sec^2(c)}}\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

output
```
(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(4*Cos[d*x]*Csc[c] - ((3*Cos[c -
d*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt
[Sec[c]^2] - 4*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^
2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d
*x - ArcTan[Cot[c]]]*Sin[c] + 2*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*Hypergeom
etricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*
Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(4*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4713, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sec(c+dx)+a}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4713}$$

$$\int \frac{a\cos(c+dx)+a}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx+a\int \frac{1}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3116}$$

$$a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3119}$$

$$a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{3120}$$

$$\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

input

```
Int[(a + a*Sec[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

output

```
(2*a*EllipticF[(c + d*x)/2, 2])/d + a*((-2*EllipticE[(c + d*x)/2, 2])/d +
(2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(58) = 116$.

Time = 1.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.60

method	result
default	$\frac{2a\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\text{EllipticE}\left(\cos\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$

input
```
int((a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-(
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.74

$$\int \frac{a+a\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = \frac{-i\sqrt{2}a\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+i\sqrt{2}a\cos(dx+c)\,\mathrm{weiers}}{}$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c)) + I*sqrt(2)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) - I*sqrt(2)*a*cos(d*x + c)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*a*cos
(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))) + 2*a*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{a+a\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = a\left(\int \frac{\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx + \int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

output
```
a*(Integral(sec(c + d*x)/sqrt(cos(c + d*x)), x) + Integral(1/sqrt(cos(c +
d*x)), x))
```

Maxima [F]

$$\int \frac{a+a\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{a\sec(dx+c)+a}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{a+a\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{a\sec(dx+c)+a}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 10.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{a + a\sec(c + dx)}{\sqrt{\cos(c + dx)}} dx = \frac{2\,a\,\mathrm{F}\left(\frac{c}{2} + \frac{d\,x}{2}\middle|2\right)}{d} + \frac{2\,a\,\sin\left(c + d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4}, \frac{1}{2};\ \frac{3}{4};\ \cos\left(c + d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c + d\,x\right)}\,\sqrt{\sin\left(c + d\,x\right)^2}}$$

input
```
int((a + a/cos(c + d*x))/cos(c + d*x)^(1/2),x)
```

output
```
(2*a*ellipticF(c/2 + (d*x)/2, 2))/d + (2*a*sin(c + d*x)*hypergeom([-1/4, 1
/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{a + a\sec(c + dx)}{\sqrt{\cos(c + dx)}} dx = a\left(\int \frac{\sqrt{\cos\left(dx + c\right)}}{\cos\left(dx + c\right)}dx + \int \frac{\sqrt{\cos\left(dx + c\right)}\,\sec\left(dx + c\right)}{\cos\left(dx + c\right)}dx\right)$$

input
```
int((a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x)
```

output
```
a*(int(sqrt(cos(c + d*x))/cos(c + d*x),x) + int((sqrt(cos(c + d*x))*sec(c
+ d*x))/cos(c + d*x),x))
```

3.356 $\int \frac{a+a\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
-2*a*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*InverseJacobiAM(1/2*d*x
+1/2*c,2^(1/2))/d+2/3*a*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2*a*sin(d*x+c)/d/cos
(d*x+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.58 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.98

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(2(3\cos(c)+\cos(dx)-\cos(2c+dx)+3\cos(c+2dx))\csc(c)-4\,\text{co}\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])/Cos[c + d*x]^(3/2),x]
```

output
```
(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(2*(3*Cos[c] + Cos[d*x] - Cos[2*c
 + d*x] + 3*Cos[c + 2*d*x])*Csc[c] - 4*Cos[c + d*x]^2*Sqrt[Cos[d*x - ArcTa
n[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
- ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] - (3*Cos[c + d*x]*Se
c[c]*(-2*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^
2]*Sin[d*x + ArcTan[Tan[c]]] + (3*Cos[c - d*x - ArcTan[Tan[c]]] + Cos[c +
d*x + ArcTan[Tan[c]]])*Csc[c]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(Sqrt[Se
c[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])))/(12*d*Cos[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4713, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sec(c+dx)+a}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4713}$$

$$\int \frac{a\cos(c+dx)+a}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx + a\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx + a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx + \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right) + a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \int \sqrt{\cos(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx + \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right) + a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx + \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right) + a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right) + a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

input
```
Int[(a + a*Sec[c + d*x])/Cos[c + d*x]^(3/2),x]
```

output
```
a*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x
]^(3/2))) + a*((-2*EllipticE[(c + d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt
[Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(78) = 156$.

Time = 1.84 (sec) , antiderivative size = 368, normalized size of antiderivative = 4.43

method	result
default	$-\frac{2\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a\left(12\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\text{EllipticF}\left(\text{co}\right.\right.}{}$

input
```
int((a+a*sec(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(12*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/
2*c)^2-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-8*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.11

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{-i\sqrt{2}a\cos{(dx+c)}^2\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+i\sqrt{2}a\cos{(dx+c)}^2\,\text{weie}}{}$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*a*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + I*sqrt(2)*a*cos(d*x + c)^2*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*a*cos(d*x + c)^2*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I
*sqrt(2)*a*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{a + a\sec(c + dx)}{\cos^{\frac{3}{2}}(c + dx)}\, dx = a\left(\int \frac{\sec(c + dx)}{\cos^{\frac{3}{2}}(c + dx)}\, dx + \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)}\, dx\right)$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)**(3/2),x)
```

output
```
a*(Integral(sec(c + d*x)/cos(c + d*x)**(3/2), x) + Integral(cos(c + d*x)**
(-3/2), x))
```

Maxima [F]

$$\int \frac{a + a\sec(c + dx)}{\cos^{\frac{3}{2}}(c + dx)}\, dx = \int \frac{a\sec(dx + c) + a}{\cos(dx + c)^{\frac{3}{2}}}\, dx$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{a + a\sec(c + dx)}{\cos^{\frac{3}{2}}(c + dx)}\, dx = \int \frac{a\sec(dx + c) + a}{\cos(dx + c)^{\frac{3}{2}}}\, dx$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\,a\,\sin(c+d\,x)\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\;\frac{3}{4};\;\cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}} + \frac{2\,a\,\sin(c+d\,x)\;{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\;\frac{1}{4};\;\cos(c+d\,x)^2\right)}{3\,d\cos(c+d\,x)^{3/2}\,\sqrt{\sin(c+d\,x)^2}}$$

input
```
int((a + a/cos(c + d*x))/cos(c + d*x)^(3/2),x)
```

output
```
(2*a*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c +
d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*a*sin(c + d*x)*hypergeom([-3/4, 1/
2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = a\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx + \int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)}{\cos(dx+c)^2}dx\right)$$

input
```
int((a+a*sec(d*x+c))/cos(d*x+c)^(3/2),x)
```

output
```
a*(int(sqrt(cos(c + d*x))/cos(c + d*x)**2,x) + int((sqrt(cos(c + d*x))*sec
(c + d*x))/cos(c + d*x)**2,x))
```

3.357 $\int \frac{a+a\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = -\frac{6aE\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} + \frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6a\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output
```
-6/5*a*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*InverseJacobiAM(1/2*d
*x+1/2*c,2^(1/2))/d+2/5*a*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/3*a*sin(d*x+c)/d
/cos(d*x+c)^(3/2)+6/5*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.28 (sec) , antiderivative size = 477, normalized size of antiderivative = 4.30

$$\int \frac{a + a\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = a\left(\sqrt{\cos(c+dx)}(1+\cos(c+dx))\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3\csc(c)\sec(c)}{5d}+\frac{\sec(c)\sec^3(c+dx)\sin(dx)}{5d}+\frac{\sec(c)\sec^2(c+dx)(3\sin(c)+5\sin(dx))}{15d}+\frac{\sec(c)\sec(c+dx)(5\sin(c)+9\sin(dx))}{15d}\right)-\frac{(1+\cos(c+dx))\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\arctan(\cot(c)))\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(dx-\arctan(\cot(c)))}{3d}+\frac{3(1+\cos(c+dx))\csc(c)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sin(dx+\arctan(\text{ta}}{\sqrt{1-\cos(dx+\arctan(\tan(c)))}\sqrt{1+\cos(dx+\arctan(\tan(c)))}\sqrt{\cos(c)\cos(dx+\arctan}}\right.}{10d}\right.$$

input
```
Integrate[(a + a*Sec[c + d*x])/Cos[c + d*x]^(5/2),x]
```

output
```
a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((3*Csc[c]*S
ec[c])/(5*d) + (Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) + (Sec[c]*Sec[c + d*
x]^2*(3*Sin[c] + 5*Sin[d*x]))/(15*d) + (Sec[c]*Sec[c + d*x]*(5*Sin[c] + 9*
Sin[d*x]))/(15*d)) - ((1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/
2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - Arc
Tan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]
*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
3*d*Sqrt[1 + Cot[c]^2]) + (3*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^
2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Si
n[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[
1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/S
qrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
Tan[c]^2]]))/(10*d))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4713, 3042, 3227, 3042, 3116, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sec(c+dx)+a}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4713}$$

$$\int \frac{a\cos(c+dx)+a}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}dx+a\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$a\left(\frac{3}{5}\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)+a\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)+ a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3116}$$

$$a\left(\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)+ a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+ a\left(\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+ a\left(\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+a\left(\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)$$

input
```
Int[(a + a*Sec[c + d*x])/Cos[c + d*x]^(5/2),x]
```

output
```
a*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x
]^(3/2))) + a*((2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (3*((-2*Ellipti
cE[(c + d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])))/5)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(98) = 196$.

Time = 3.02 (sec) , antiderivative size = 384, normalized size of antiderivative = 3.46

method	result
default	$-\frac{4\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\, a\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{40\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^3}-\frac{3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{5\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}}$

input
```
int((a+a*sec(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1/40*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^3-3/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*c
os(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/10*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/12*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2)/si
n(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.69

$$\int \frac{a + a\sec(c + dx)}{\cos^{\frac{5}{2}}(c + dx)}\,dx$$
$$= \frac{-5i\sqrt{2}a\cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)) + 5i\sqrt{2}a\cos(dx + c)^3 \operatorname{w}}{}$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/15*(-5*I*sqrt(2)*a*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) + 5*I*sqrt(2)*a*cos(d*x + c)^3*weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*a*cos(d*x + c)^3*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
+ 9*I*sqrt(2)*a*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*a*cos(d*x + c)^2 + 5*a*cos(d
*x + c) + 3*a)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + a\sec(c + dx)}{\cos^{\frac{5}{2}}(c + dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a + a\sec(c + dx)}{\cos^{\frac{5}{2}}(c + dx)}\,dx = \int \frac{a\sec(dx + c) + a}{\cos(dx + c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{a + a\sec(c + dx)}{\cos^{\frac{5}{2}}(c + dx)}\,dx = \int \frac{a\sec(dx + c) + a}{\cos(dx + c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.41 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{a + a\sec(c + dx)}{\cos^{\frac{5}{2}}(c + dx)}\,dx = \frac{2\,a\,\sin(c + d\,x)\,{}_2\mathrm{F}_1\left(-\frac{3}{4}, \frac{1}{2};\ \frac{1}{4};\ \cos(c + d\,x)^2\right)}{3\,d\cos(c + d\,x)^{3/2}\,\sqrt{\sin(c + d\,x)^2}} + \frac{2\,a\,\sin(c + d\,x)\,{}_2\mathrm{F}_1\left(-\frac{5}{4}, \frac{1}{2};\ -\frac{1}{4};\ \cos(c + d\,x)^2\right)}{5\,d\cos(c + d\,x)^{5/2}\,\sqrt{\sin(c + d\,x)^2}}$$

input
```
int((a + a/cos(c + d*x))/cos(c + d*x)^(5/2),x)
```

output
```
(2*a*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c
+ d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*a*sin(c + d*x)*hypergeom([-5/4,
1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1/2
))
```

Reduce [F]

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = a\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx + \int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)}{\cos(dx+c)^3}dx\right)$$

input
```
int((a+a*sec(d*x+c))/cos(d*x+c)^(5/2),x)
```

output
```
a*(int(sqrt(cos(c + d*x))/cos(c + d*x)**3,x) + int((sqrt(cos(c + d*x))*sec
(c + d*x))/cos(c + d*x)**3,x))
```

3.358 $\int \frac{a+a\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 135

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = -\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{10a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{10a\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output
```
-6/5*a*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+10/21*a*InverseJacobiAM(1/2
*d*x+1/2*c,2^(1/2))/d+2/7*a*sin(d*x+c)/d/cos(d*x+c)^(7/2)+2/5*a*sin(d*x+c)
/d/cos(d*x+c)^(5/2)+10/21*a*sin(d*x+c)/d/cos(d*x+c)^(3/2)+6/5*a*sin(d*x+c)
/d/cos(d*x+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.08 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.18

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left((189\cos(c)+85\cos(dx)-85\cos(2c+dx)+231\cos(c+2dx)+2\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])/Cos[c + d*x]^(7/2),x]
```

output
```
(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((189*Cos[c] + 85*Cos[d*x] - 85*C
os[2*c + d*x] + 231*Cos[c + 2*d*x] + 21*Cos[3*c + 2*d*x] + 25*Cos[2*c + 3*
d*x] - 25*Cos[4*c + 3*d*x] + 63*Cos[3*c + 4*d*x])*Csc[c] - 200*Cos[c + d*x
]^4*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/
4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin
[c] - (126*Cos[c + d*x]^3*Sec[c]*(-2*HypergeometricPFQ[{-1/2, -1/4}, {3/4}
, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]] + (3*Cos[c - d*x
- ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sqrt[Sin[d*x + A
rcTan[Tan[c]]]^2]))/(Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])))/(
840*d*Cos[c + d*x]^(7/2))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4713, 3042, 3227, 3042, 3116, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sec(c+dx)+a}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{a \csc \left(c+dx+\frac{\pi}{2}\right)+a}{\sin \left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 4713}$$

$$\int \frac{a \cos(c+dx)+a}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a \sin \left(c+dx+\frac{\pi}{2}\right)+a}{\sin \left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{ 3227}$$

$$a\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)}dx+a\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$a\int \frac{1}{\sin \left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+a\int \frac{1}{\sin \left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 3116}$$

$$a\left(\frac{5}{7}\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}\right)+a\left(\frac{3}{5}\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$a\left(\frac{5}{7}\int \frac{1}{\sin \left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}\right)+a\left(\frac{3}{5}\int \frac{1}{\sin \left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{ 3116}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}\right)+a\left(\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}\right)+$$
$$a\left(\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}\right)+$$
$$a\left(\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{5}{7}\left(\frac{2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)\right)+$$
$$a\left(\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)$$

input
```
Int[(a + a*Sec[c + d*x])/Cos[c + d*x]^(7/2),x]
```

output
```
a*((2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (5*((2*EllipticF[(c + d*x)/
2, 2])/(3*d) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))))/7) + a*((2*Sin[
c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (3*((-2*EllipticE[(c + d*x)/2, 2])/d
+ (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])))/5)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(118) = 236$.

Time = 4.45 (sec) , antiderivative size = 437, normalized size of antiderivative = 3.24

method	result
default	$-\frac{4\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{112\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{1}{2}\right)^{4}}-\frac{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin(}{84\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{1}{2}\right)^{2}}$

input
```
int((a+a*sec(d*x+c))/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1/112*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(
1/2*d*x+1/2*c)^2-1/2)^4-5/84*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+44/105*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/40*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(c
os(1/2*d*x+1/2*c)^2-1/2)^3-3/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(
-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-3/10*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.47

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{-25i\sqrt{2}a\cos(dx+c)^4 \text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+25i\sqrt{2}a\cos(dx+c)^4}{}$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
1/105*(-25*I*sqrt(2)*a*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + 25*I*sqrt(2)*a*cos(d*x + c)^4*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*I*sqrt(2)*a*cos(d*x + c)^4*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c
))) + 63*I*sqrt(2)*a*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(63*a*cos(d*x + c)^3 + 25*
a*cos(d*x + c)^2 + 21*a*cos(d*x + c) + 15*a)*sqrt(cos(d*x + c))*sin(d*x +
c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + a\sec(c + dx)}{\cos^{\frac{7}{2}}(c + dx)}\, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a + a\sec(c + dx)}{\cos^{\frac{7}{2}}(c + dx)}\, dx = \int \frac{a\sec(dx + c) + a}{\cos(dx + c)^{\frac{7}{2}}}\, dx$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \frac{a + a\sec(c + dx)}{\cos^{\frac{7}{2}}(c + dx)}\, dx = \int \frac{a\sec(dx + c) + a}{\cos(dx + c)^{\frac{7}{2}}}\, dx$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.64

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{5\,d\cos\left(c+d\,x\right)^{5/2}\,\sqrt{\sin\left(c+d\,x\right)^2}} + \frac{2\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\ -\frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\cos\left(c+d\,x\right)^{7/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int((a + a/cos(c + d*x))/cos(c + d*x)^(7/2),x)
```

output
```
(2*a*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c
 + d*x)^(5/2)*(sin(c + d*x)^2)^(1/2)) + (2*a*sin(c + d*x)*hypergeom([-7/4,
 1/2], -3/4, cos(c + d*x)^2))/(7*d*cos(c + d*x)^(7/2)*(sin(c + d*x)^2)^(1/
2))
```

Reduce [F]

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = a\left(\int \frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^4}dx + \int \frac{\sqrt{\cos\left(dx+c\right)}\,\sec\left(dx+c\right)}{\cos\left(dx+c\right)^4}dx\right)$$

input
```
int((a+a*sec(d*x+c))/cos(d*x+c)^(7/2),x)
```

output
```
a*(int(sqrt(cos(c + d*x))/cos(c + d*x)**4,x) + int((sqrt(cos(c + d*x))*sec
(c + d*x))/cos(c + d*x)**4,x))
```

3.359 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{32a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{20a^2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} + \frac{20a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{32a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{4a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2a^2\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d}$$

output
```
32/15*a^2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+20/21*a^2*InverseJacobiA
M(1/2*d*x+1/2*c,2^(1/2))/d+20/21*a^2*cos(d*x+c)^(1/2)*sin(d*x+c)/d+32/45*a
^2*cos(d*x+c)^(3/2)*sin(d*x+c)/d+4/7*a^2*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9
*a^2*cos(d*x+c)^(7/2)*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.89 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.73

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{672(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}} - 1200\,\mathrm{co}\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*((672*(3*Cos[c - d*x - ArcTan
[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] -
 1200*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hyperg
eometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcT
an[Cot[c]]]*Sin[c] + Cos[c + d*x]*(-2688*Cot[c] + 1380*Sin[c + d*x] + 518*
Sin[2*(c + d*x)] + 180*Sin[3*(c + d*x)] + 35*Sin[4*(c + d*x)]) - 1344*Cos[
c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*
x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/
(5040*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.44, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.826$, Rules used = {3042, 4752, 3042, 4275, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120, 4533, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{9}{2}}(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^2}{\sec^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{4275}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a^2\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}dx+\int \frac{\sec^2(c+dx)a^2+a^2}{\sec^{\frac{9}{2}}(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx\right)$$

$$\Big\downarrow \text{4256}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2a^2\left(\frac{5}{7}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2a^2\left(\frac{5}{7}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)\right)$$

$$\Big\downarrow \text{4256}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2a^2\left(\frac{5}{7}\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2a^2\left(\frac{5}{7}\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right.\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2a^2\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c}}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2a^2\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{}{\sqrt{\sin(}}\right.\right.\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2a^2\left(\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{5}{7}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2}{}\right.\right.\right.$$

$\downarrow$ 4533

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{16}{9}a^2\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{5}{7}\left(\frac{2\sin(c}{3d\sqrt{\sec(}}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{16}{9}a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{5}{7}\left(\frac{2}{3d}\right.\right.\right.$$

$\downarrow$ 4256

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{16}{9}a^2\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+2a^2\left(\frac{2\sin}{7d\sec}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{16}{9}a^2\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+2a^2\left(\right.\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{16}{9}a^2\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2}{9d}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{16}{9}a^2\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)\right.$$

$$\Big\downarrow \text{3119}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{5}{7}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}}{}\right.\right.\right.$$

input
```
Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sin[c + d*x])/(9*d*Sec[c + d
*x]^(7/2)) + (16*a^2*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(5*d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))))/9 + 2*
a^2*((2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (5*((2*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3
*d*Sqrt[Sec[c + d*x]])))/7))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 9.87 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.77

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, a^{2}\left(560\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}-960\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}+608\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}-96\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-205\cos\right.}{315\sqrt{-2\sin\left(\right.}}$

input
```
int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(560*co
s(1/2*d*x+1/2*c)^11-960*cos(1/2*d*x+1/2*c)^9+608*cos(1/2*d*x+1/2*c)^7-96*c
os(1/2*d*x+1/2*c)^5-205*cos(1/2*d*x+1/2*c)^3+75*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-168*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+93*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.19

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx =$$

$$-\frac{2\left(75i\sqrt{2}a^2\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})-75i\sqrt{2}a^2\text{weierstrassPInverse}(\right.}{}$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/315*(75*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c)) - 75*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) - 168*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 168*I*sqrt(2)*a^2*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (35*
a^2*cos(d*x + c)^3 + 90*a^2*cos(d*x + c)^2 + 112*a^2*cos(d*x + c) + 150*a^
2)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \int (a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(9/2), x)
```

Giac [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \int (a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(9/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.45 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.93

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= -\frac{2\,a^2\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$-\frac{4\,a^2\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$-\frac{2\,a^2\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right)^2\right)}{11\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^2,x)
```

output
```
- (2*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c
 + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (4*a^2*cos(c + d*x)^(9/2)*sin(c
 + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)
^(1/2)) - (2*a^2*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 1
5/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= a^2\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^4\sec\left(dx+c\right)^2dx + 2\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^4\sec\left(dx+c\right)dx\right) + \int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^4dx\right)$$

input
```
int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2,x)
```

output
```
a**2*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4*sec(c + d*x)**2,x) + 2*int(sq
rt(cos(c + d*x))*cos(c + d*x)**4*sec(c + d*x),x) + int(sqrt(cos(c + d*x))*
cos(c + d*x)**4,x))
```

3.360 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{8a^2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{7d} + \frac{8a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{7d} + \frac{4a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}$$

output
```
12/5*a^2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+8/7*a^2*InverseJacobiAM(1
/2*d*x+1/2*c,2^(1/2))/d+8/7*a^2*cos(d*x+c)^(1/2)*sin(d*x+c)/d+4/5*a^2*cos(
d*x+c)^(3/2)*sin(d*x+c)/d+2/7*a^2*cos(d*x+c)^(5/2)*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.86 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.02

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{42(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}} - 80\cos(c\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*((42*(3*Cos[c - d*x - ArcTan[
Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] -
80*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeom
etricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[
Cot[c]]]*Sin[c] + Cos[c + d*x]*(-168*Cot[c] + 85*Sin[c + d*x] + 28*Sin[2*(
c + d*x)] + 5*Sin[3*(c + d*x)]) - 84*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*Hype
rgeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c
]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(280*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.52, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4752, 3042, 4275, 3042, 4256, 3042, 4258, 3042, 3119, 4533, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{7}{2}}(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^2}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4275}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a^2\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx+\int \frac{\sec^2(c+dx)a^2+a^2}{\sec^{\frac{7}{2}}(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx\right)$$

$$\Big\downarrow \text{4256}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2a^2\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2a^2\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx}{5d\sec^{\frac{3}{2}}(c+d}\right.\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2a^2\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+d}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2a^2\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+\right.}\right.\right.$$

$$\Big\downarrow \text{3119}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{5d}\right.\right.$$

$$\Big\downarrow \text{4533}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{12}{7}a^2\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+d}}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{12}{7}a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\text{co}}}{}\right.\right.$$

$$\Big\downarrow \text{4256}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{12}{7}a^2\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2a^2\left(\frac{2\sin}{5d\sec}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{12}{7}a^2\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2a^2\left(\right.\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{12}{7}a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a}{7d}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{12}{7}a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right.$$

$$\Big\downarrow \text{3120}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{12}{7}a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{Elliptic}}{3d}\right.\right.$$

input
```
Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sin[c + d*x])/(7*d*Sec[c + d
*x]^(5/2)) + 2*a^2*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(5*d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))) + (12*a^2
*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d
) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/7)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(108) = 216$.

Time = 7.23 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.25

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a^{2}\left(40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-116\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+126\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos(}{35\sqrt{-2\sin}}$

input
```
int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-4/35*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(40*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-116*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1
/2*c)+126*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-39*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^2+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-21*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.34

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{2\left(10i\sqrt{2}a^2\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-10i\sqrt{2}a^2\text{weierstrassPInverse}(\right.}{}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/35*(10*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) - 10*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c)) - 21*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*I*sqrt(2)*a^2*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (5*a^2*
cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 20*a^2)*sqrt(cos(d*x + c))*sin(d*x
+ c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx=\int (a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx=\int (a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.42 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.07

$$\begin{aligned}&\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx\\&=\frac{2\left(a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{3\,d}\\&\quad-\frac{4\,a^2\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}\\&\quad-\frac{2\,a^2\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}\end{aligned}$$

input
```
int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^2,x)
```

output
```
(2*(a^2*ellipticF(c/2 + (d*x)/2, 2) + a^2*cos(c + d*x)^(1/2)*sin(c + d*x))
)/(3*d) - (4*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/
4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*a^2*cos(c + d*x)^(9/
2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c +
 d*x)^2)^(1/2))
```

Reduce [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\sec(dx+c)^2dx + 2\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\sec(dx+c)\,dx\right) + \int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3dx\right)$$

input
```
int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2,x)
```

output
```
a**2*(int(sqrt(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x)**2,x) + 2*int(sq
rt(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x),x) + int(sqrt(cos(c + d*x))*
cos(c + d*x)**3,x))
```

3.361 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 95

$$\begin{aligned}\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx =& \frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} \\ &+ \frac{4a^2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} \\ &+ \frac{4a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} \\ &+ \frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}\end{aligned}$$

output
```
16/5*a^2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a^2*InverseJacobiAM(1
/2*d*x+1/2*c,2^(1/2))/d+4/3*a^2*cos(d*x+c)^(1/2)*sin(d*x+c)/d+2/5*a^2*cos(
d*x+c)^(3/2)*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.11 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.47

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{12(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}}-20\cos(c$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*((12*(3*Cos[c - d*x - ArcTan[
Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] -
20*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeom
etricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[
Cot[c]]]*Sin[c] + Cos[c + d*x]*(-48*Cot[c] + 20*Sin[c + d*x] + 3*Sin[2*(c
+ d*x)]) - 24*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1
/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + Arc
Tan[Tan[c]]]^2]))/(60*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.64, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4752, 3042, 4275, 3042, 4256, 3042, 4258, 3042, 3120, 4533, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \ 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^2}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \ 4275$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a^2\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\int \frac{\sec^2(c+dx)a^2+a^2}{\sec^{\frac{5}{2}}(c+dx)}dx\right)$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx\right)$$

$$\Big\downarrow \ 4256$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2a^2\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right)$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2a^2\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right)$$

$$\Big\downarrow \ 4258$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}\right.\right.$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+\right.}}\right.\right.$$

$$\Big\downarrow \ 3120$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.\right.$$

$\Big\downarrow$ 4533

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a^2\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos\right.\right.$$

$\Big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+2a^2\left(\frac{2\,\mathrm{si}}{3d\sqrt{}}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+2a^2\right.$$

$\Big\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{16a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+2a^2\left(\frac{2\sin(}{3d\sqrt{\mathrm{se}}}\right.\right.$$

input
```
Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((16*a^2*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*a^2*Sin[c + d*x])/(5*d*Se
c[c + d*x]^(3/2)) + 2*a^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(86) = 172.

Time = 5.72 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.63

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a^{2}\left(-12\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+32\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-13\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{c}{}\right.\right.}{15\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin}}$

input
```
int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*sin
(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+32*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)-13*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.57

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx =$$

$$-\frac{2\left(5i\sqrt{2}a^{2}\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-5i\sqrt{2}a^{2}\text{weierstrassPInverse}(-4\right.}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/15*(5*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c)) - 5*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c)) - 12*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c))) + 12*I*sqrt(2)*a^2*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*a^2*co
s(d*x + c) + 10*a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \int (a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \int (a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.09

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{d}+\frac{4a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{3d}+\frac{4a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} - \frac{2a^2\cos(c+dx)^{7/2}\sin(c+dx)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+dx)^2\right)}{7d\sqrt{\sin(c+dx)^2}}$$

input
```
int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^2,x)
```

output
```
(2*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a^2*ellipticF(c/2 + (d*x)/2, 2)
)/(3*d) + (4*a^2*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) - (2*a^2*cos(c + d
*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(
sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$
\begin{aligned}
&\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx \\
&= a^2\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^2\,dx\right. \\
&\quad + 2\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)\,dx\right) \\
&\quad \left. + \int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)
\end{aligned}
$$

input
```
int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x)
```

output
```
a**2*(int(sqrt(cos(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**2,x) + 2*int(sq
rt(cos(c + d*x))*cos(c + d*x)**2*sec(c + d*x),x) + int(sqrt(cos(c + d*x))*
cos(c + d*x)**2,x))
```

3.362 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 67

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{8a^2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
4*a^2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+8/3*a^2*InverseJacobiAM(1/2*
d*x+1/2*c,2^(1/2))/d+2/3*a^2*cos(d*x+c)^(1/2)*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.90 (sec) , antiderivative size = 224, normalized size of antiderivative = 3.34

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{3(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}} - 8\cos(c+\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*((3*(3*Cos[c - d*x - ArcTan[T
an[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 8
*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeomet
ricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Co
t[c]]]*Sin[c] + 2*Cos[c + d*x]*(-6*Cot[c] + Sin[c + d*x]) - 6*Cos[c]*Csc[d
*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcT
an[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(12*d*Sq
rt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.91, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4752, 3042, 4275, 3042, 4258, 3042, 3119, 4533, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^2}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4275}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\sec^2(c+dx)a^2+a^2}{\sec^{\frac{3}{2}}(c+dx)}dx+2a^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a^2\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx\right)$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}d\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx\right.}\right.$$

$$\Big\downarrow \text{3119}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{4533}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{3}a^2\int \sqrt{\sec(c+dx)}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(\right.}{d}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{3}a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{3}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2\sqrt{\cos}}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{3}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2}{}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{4a^2\sqrt{}}{}\right.$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((4*a^2*Sqrt[Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (8*a^2*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a^2*Sin[c + d*x])/(3*d*S
qrt[Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(64) = 128$.

Time = 0.79 (sec) , antiderivative size = 228, normalized size of antiderivative = 3.40

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a^{2}\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}\cdots\right.}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\cdots\right.}$

input
```
int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-4/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(2*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.00

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{2\left(a^2\sqrt{\cos(dx+c)}\sin(dx+c) - 2i\sqrt{2}a^2\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + 2i\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
2/3*(a^2*sqrt(cos(d*x + c))*sin(d*x + c) - 2*I*sqrt(2)*a^2*weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 2*I*sqrt(2)*a^2*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a^2*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3
*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \int (a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \int (a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.34 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{4a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{d} + \frac{8a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{3d} + \frac{2a^2\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3d}$$

input
```
int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^2,x)
```

output
```
(4*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (8*a^2*ellipticF(c/2 + (d*x)/2, 2)
)/(3*d) + (2*a^2*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d)
```

Reduce [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= a^2\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)^2\,dx + 2\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)\,dx\right) + \int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)$$

input
```
int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x)
```

output
```
a**2*(int(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x)**2,x) + 2*int(sqrt(
cos(c + d*x))*cos(c + d*x)*sec(c + d*x),x) + int(sqrt(cos(c + d*x))*cos(c
+ d*x),x))
```

3.363 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 44

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2\,dx = \frac{4a^2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d} + \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
4*a^2*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+2*a^2*sin(d*x+c)/d/cos(d*x+
c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.55

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2\,dx =$$

$$-\frac{2a^2\csc(c+dx)\left(-3\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},\cos^2(c+dx)\right)+\cos(c+dx)\left(6\,\text{Hypergeometric2F}\right.\right.}{3d\sqrt{\cos(c}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

output
```
(-2*a^2*Csc[c + d*x]*(-3*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2]
 + Cos[c + d*x]*(6*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2] + Cos[
c + d*x]*Hypergeometric2F1[1/2, 3/4, 7/4, Cos[c + d*x]^2]))*Sqrt[Sin[c + d
*x]^2])/(3*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.93, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4752, 3042, 4275, 3042, 4258, 3042, 3120, 4531}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^2}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4275}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\sec^2(c+dx)a^2+a^2}{\sqrt{\sec(c+dx)}}dx+2a^2\int \sqrt{\sec(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx\right.}}\right.$$

$$\Big\downarrow \text{3120}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+\right.}{d}\right.$$

$$\Big\downarrow \text{4531}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx\right.}{d}\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((4*a^2*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4531
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] /;
 FreeQ[{b, e, f, A, C, m}, x] && EqQ[C*m + A*(m + 1), 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(43) = 86$.

Time = 1.50 (sec) , antiderivative size = 185, normalized size of antiderivative = 4.20

method	result
default	$-\frac{4a^2\left(-\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$

input
```
int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-4*a^2*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.20

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2\,dx =$$

$$-\frac{2\left(i\sqrt{2}a^2\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-i\sqrt{2}a^2\cos(dx+c)\,\cdots\right.}{d\cos(dx+c\cdots}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2*(I*sqrt(2)*a^2*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) - I*sqrt(2)*a^2*cos(d*x + c)*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) - a^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(
d*x + c))
```

Sympy [F]

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\sqrt{\cos(c+dx)}\sec(c+dx)\,dx + \int \sqrt{\cos(c+dx)}\sec^2(c+dx)\,dx + \int \sqrt{\cos(c+dx)}\,dx\right)$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*sqrt(cos(c + d*x))*sec(c + d*x), x) + Integral(sqrt(cos(c
 + d*x))*sec(c + d*x)**2, x) + Integral(sqrt(cos(c + d*x)), x))
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2\,dx = \int (a\sec(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2\,dx = \int (a\sec(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 10.46 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.86

$$\begin{aligned}\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2\,dx &= \frac{2\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}\\ &\quad+\frac{2\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}\end{aligned}$$

input
```
int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^2,x)
```

output
```
(2*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a^2*ellipticF(c/2 + (d*x)/2, 2)
)/d + (2*a^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*
cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2\,dx = a^2\left(\int \sqrt{\cos(dx+c)}dx + \int \sqrt{\cos(dx+c)}\sec(dx+c)^2\,dx + 2\left(\int \sqrt{\cos(dx+c)}\sec(dx+c)\,dx\right)\right)$$

input
```
int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x)
```

output
```
a**2*(int(sqrt(cos(c + d*x)),x) + int(sqrt(cos(c + d*x))*sec(c + d*x)**2,x
) + 2*int(sqrt(cos(c + d*x))*sec(c + d*x),x))
```

3.364 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = -\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{8a^2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
-4*a^2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+8/3*a^2*InverseJacobiAM(1/2
*d*x+1/2*c,2^(1/2))/d+2/3*a^2*sin(d*x+c)/d/cos(d*x+c)^(3/2)+4*a^2*sin(d*x+
c)/d/cos(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.22

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{2a^2\csc(c+dx)\left(\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},\cos^2(c+dx)\right)+6\cos(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\right.\right.}{3d\cos^{\frac{3}{2}}(c+}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

output
```
(2*a^2*Csc[c + d*x]*(Hypergeometric2F1[-3/4, 1/2, 1/4, Cos[c + d*x]^2] + 6
*Cos[c + d*x]*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2] - 3*Cos[c
+ d*x]^2*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*
x]^2])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.67, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4752, 3042, 4275, 3042, 4255, 3042, 4258, 3042, 3119, 4534, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^2}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \ 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^2dx$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2dx$$

$$\Big\downarrow \ 4275$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a^2\int \sec^{\frac{3}{2}}(c+dx)dx+\int \sqrt{\sec(c+dx)}\left(\sec^2(c+dx)a^2+a^2\right)dx\right)$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right.\right.$$

$$\Big\downarrow \ 4255$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(}}{d}\right.\right.$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(}}{d}\right.\right.$$

$$\Big\downarrow \ 4258$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(}}{d}\right.\right.$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(}}{d}\right.\right.$$

$$\Big\downarrow \ 3119$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(}}{d}\right.\right.$$

$$\Big\downarrow \ 4534$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{3}a^2\int \sqrt{\sec(c+dx)}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c}}{d}\right.\right.$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{3}a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+2a^2\left(\frac{2\sin(c+dx)}{d}\right.\right.$$

$$\Big\downarrow \ 4258$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{3}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{3}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c-\right.$$

$\Big\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx\right.}{3d}\right.$$

input
```
Int[(a + a*Sec[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((8*a^2*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a^2*Sec[c + d*x]^(3/2)*Sin
[c + d*x])/(3*d) + 2*a^2*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(86) = 172$.

Time = 1.99 (sec) , antiderivative size = 371, normalized size of antiderivative = 4.08

method	result
default	$-\frac{4a^2\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(12\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\text{EllipticF}\left(c\right.\right.}{}$

input
```
int((a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(
1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(12*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+
1/2*c)^2-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-7*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.05

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = -\frac{2\left(2i\sqrt{2}a^2\cos{(dx+c)}^2\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})-2i\sqrt{2}a^2\cos{(dx+}\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(2*I*sqrt(2)*a^2*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) - 2*I*sqrt(2)*a^2*cos(d*x + c)^2*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a^2*cos(d*x + c)^2*wei
erstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c))) - 3*I*sqrt(2)*a^2*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (6*a^2*cos(d*x + c) + a^2)
*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = a^2\left(\int \frac{2\sec{(c+dx)}}{\sqrt{\cos{(c+dx)}}}\,dx + \int \frac{\sec^2{(c+dx)}}{\sqrt{\cos{(c+dx)}}}\,dx + \int \frac{1}{\sqrt{\cos{(c+dx)}}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2/cos(d*x+c)**(1/2),x)
```

output
```
a**2*(Integral(2*sec(c + d*x)/sqrt(cos(c + d*x)), x) + Integral(sec(c + d*
x)**2/sqrt(cos(c + d*x)), x) + Integral(1/sqrt(cos(c + d*x)), x))
```

Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(a\sec{(dx+c)}+a)^2}{\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(a\sec{(dx+c)}+a)^2}{\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 10.51 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.20

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = \frac{2\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{4\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}} + \frac{2\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int((a + a/cos(c + d*x))^2/cos(c + d*x)^(1/2),x)
```

output
```
(2*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (4*a^2*sin(c + d*x)*hypergeom([-1/
4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2
)) + (2*a^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d
*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = a^2\left(\int \frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)}dx + \int \frac{\sqrt{\cos\left(dx+c\right)}\,\sec\left(dx+c\right)^2}{\cos\left(dx+c\right)}dx + 2\left(\int \frac{\sqrt{\cos\left(dx+c\right)}\,\sec\left(dx+c\right)}{\cos\left(dx+c\right)}dx\right)\right)$$

input
```
int((a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x)
```

output
```
a**2*(int(sqrt(cos(c + d*x))/cos(c + d*x),x) + int((sqrt(cos(c + d*x))*sec
(c + d*x)**2)/cos(c + d*x),x) + 2*int((sqrt(cos(c + d*x))*sec(c + d*x))/co
s(c + d*x),x))
```

3.365 $\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{16a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output
```
-16/5*a^2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a^2*InverseJacobiAM(
1/2*d*x+1/2*c,2^(1/2))/d+2/5*a^2*sin(d*x+c)/d/cos(d*x+c)^(5/2)+4/3*a^2*sin
(d*x+c)/d/cos(d*x+c)^(3/2)+16/5*a^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.30 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.94

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a^2\csc(c+dx)\left(3\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},-\frac{1}{4},\cos^2(c+dx)\right)+5\cos(c+dx)\left(2\,\text{Hypergeometric2F1}\right.\right.}{15d\cos^{\frac{5}{2}}(}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

output
```
(2*a^2*Csc[c + d*x]*(3*Hypergeometric2F1[-5/4, 1/2, -1/4, Cos[c + d*x]^2]
+ 5*Cos[c + d*x]*(2*Hypergeometric2F1[-3/4, 1/2, 1/4, Cos[c + d*x]^2] + 3*
Cos[c + d*x]*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2]))*Sqrt[Sin[
c + d*x]^2])/(15*d*Cos[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.49, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4752, 3042, 4275, 3042, 4255, 3042, 4258, 3042, 3120, 4534, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^2}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a^2\int \sec^{\frac{5}{2}}(c+dx)dx+\int \sec^{\frac{3}{2}}(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a\right.\right.$$

$$\Big\downarrow \text{4255}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx\right.}\right.\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\right.\right.$$

$$\Big\downarrow \text{3120}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(}{3d}\right.\right.$$

$$\Big\downarrow \text{4534}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a^2\int \sec^{\frac{3}{2}}(c+dx)dx+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+}{3d}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2a^2\left(\frac{2\sin(c+dx)}{3}\right.\right.$$

$$\Big\downarrow \text{4255}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2a^2\sin(c+dx)\mathrm{se}}{5d}\right.$$

$\big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\right.}\right.\right.$$

$\big\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{}}{}\right.\right.$$

input
```
Int[(a + a*Sec[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sec[c + d*x]^(5/2)*Sin[c + d
*x])/(5*d) + (8*a^2*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/5 + 2*a^2*((2*
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (
2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(108) = 216$.

Time = 3.26 (sec) , antiderivative size = 386, normalized size of antiderivative = 3.19

method	result
default	$-\frac{8\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, a^{2}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{80\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{1}{2}\right)^{3}}-\frac{4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{5\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\right.}}$

input
```
int((a+a*sec(d*x+c))^2/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-1/80*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^3-4/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2
*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+17/30*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/5*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/12*cos(1/2*d*x+1/2*c)*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.67

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2\left(5i\sqrt{2}a^2\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) - 5i\sqrt{2}a^2\cos(dx+\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-2/15*(5*I*sqrt(2)*a^2*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) - 5*I*sqrt(2)*a^2*cos(d*x + c)^3*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 12*I*sqrt(2)*a^2*cos(d*x + c)^3*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c))) - 12*I*sqrt(2)*a^2*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (24*a^2*cos(d*x + c)^2
+ 10*a^2*cos(d*x + c) + 3*a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c)^3)
```

Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = a^2\left(\int \frac{2\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2/cos(d*x+c)**(3/2),x)
```

output
```
a**2*(Integral(2*sec(c + d*x)/cos(c + d*x)**(3/2), x) + Integral(sec(c + d
*x)**2/cos(c + d*x)**(3/2), x) + Integral(cos(c + d*x)**(-3/2), x))
```

Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.49 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.94

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{6\,a^2\,\sin(c+d\,x)\;{}_2F_1\left(-\frac{5}{4},\frac{1}{2};\;-\frac{1}{4};\;\cos(c+d\,x)^2\right)+20\,a^2\,\cos(c+d\,x)\,\sin(c+d\,x)\;{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\;\frac{1}{4};\;\cos(c\right.}{15\,d\cos(c+d\,x)^{5/2}\,\sqrt{1-\cos(c+}}$$

input
```
int((a + a/cos(c + d*x))^2/cos(c + d*x)^(3/2),x)
```

output
```
(6*a^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 20*a^2*
cos(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2) + 30
*a^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^
2))/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{(a + a\sec(c + dx))^2}{\cos^{\frac{3}{2}}(c + dx)}\, dx = a^2 \left(\int \frac{\sqrt{\cos(dx + c)}}{\cos(dx + c)^2} dx + \int \frac{\sqrt{\cos(dx + c)}\, \sec(dx + c)^2}{\cos(dx + c)^2} dx + 2 \left(\int \frac{\sqrt{\cos(dx + c)}\, \sec(dx + c)}{\cos(dx + c)^2} dx \right) \right)$$

input
```
int((a+a*sec(d*x+c))^2/cos(d*x+c)^(3/2),x)
```

output
```
a**2*(int(sqrt(cos(c + d*x))/cos(c + d*x)**2,x) + int((sqrt(cos(c + d*x))*
sec(c + d*x)**2)/cos(c + d*x)**2,x) + 2*int((sqrt(cos(c + d*x))*sec(c + d*
x))/cos(c + d*x)**2,x))
```

3.366 $\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = -\frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{7d}+\frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{8a^2\sin(c+dx)}{7d\cos^{\frac{3}{2}}(c+dx)}+\frac{12a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output
```
-12/5*a^2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+8/7*a^2*InverseJacobiAM(
1/2*d*x+1/2*c,2^(1/2))/d+2/7*a^2*sin(d*x+c)/d/cos(d*x+c)^(7/2)+4/5*a^2*sin
(d*x+c)/d/cos(d*x+c)^(5/2)+8/7*a^2*sin(d*x+c)/d/cos(d*x+c)^(3/2)+12/5*a^2*
sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.42 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.78

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2a^2\csc(c+dx)\left(15\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{1}{2},-\frac{3}{4},\cos^2(c+dx)\right)+7\cos(c+dx)\left(6\,\text{Hypergeometric2F}\right.\right.}{105d\cos}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

output
```
(2*a^2*Csc[c + d*x]*(15*Hypergeometric2F1[-7/4, 1/2, -3/4, Cos[c + d*x]^2]
 + 7*Cos[c + d*x]*(6*Hypergeometric2F1[-5/4, 1/2, -1/4, Cos[c + d*x]^2] +
5*Cos[c + d*x]*Hypergeometric2F1[-3/4, 1/2, 1/4, Cos[c + d*x]^2]))*Sqrt[Si
n[c + d*x]^2])/(105*d*Cos[c + d*x]^(7/2))
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.41, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.826$, Rules used = {3042, 4752, 3042, 4275, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119, 4534, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a^2\int \sec^{\frac{7}{2}}(c+dx)dx+\int \sec^{\frac{5}{2}}(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}dx+\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a\right.\right.$$

$$\Big\downarrow \text{4255}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{3}{5}\int \sec^{\frac{3}{2}}(c+dx)a\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{3}{5}\int \csc\left(c+dx+\right.\right.\right.$$

$$\Big\downarrow \text{4255}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{}}{d}\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{}}{d}\right.\right.\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{}}{d}\right.\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2\right)dx+2a^2\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{}}{d}\right.\right.\right.$$

$\Big\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2\right)dx+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(}{5d}\right.\right.$$

$\Big\downarrow$ 4534

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{12}{7}a^2\int \sec^{\frac{5}{2}}(c+dx)dx+\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c}{5d}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{12}{7}a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+2a^2\left(\frac{2\sin(c+dx}{}\right.\right.$$

$\Big\downarrow$ 4255

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{12}{7}a^2\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(}{7d}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{12}{7}a^2\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+\frac{2a^2\sin(c+dx}{7}\right.$$

$\Big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{12}{7}a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+d}{3d}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{12}{7}a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(}{3d}\right.\right.$$

$$\Big\downarrow \text{3120}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{12}{7}a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}}{}\right.\right.$$

input
```
Int[(a + a*Sec[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sec[c + d*x]^(7/2)*Sin[c + d
*x])/(7*d) + (12*a^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)))/7 + 2*
a^2*((2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + (3*((-2*Sqrt[Cos[c + d*x]
]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/d))/5))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(130) = 260$.

Time = 5.21 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.99

method	result
default	$-\frac{8\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\, a^2\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{224\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^4}-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin(}}{14\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^2}\right.}{}$

input
```
int((a+a*sec(d*x+c))^2/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-1/224*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(co
s(1/2*d*x+1/2*c)^2-1/2)^4-1/14*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+31/70*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/40
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
cos(1/2*d*x+1/2*c)^2-1/2)^3-3/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-
(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-3/10*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.46

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{2\left(10i\sqrt{2}a^2\cos{(dx+c)}^4\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})-10i\sqrt{2}a^2\cos{(dx}\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-2/35*(10*I*sqrt(2)*a^2*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) - 10*I*sqrt(2)*a^2*cos(d*x + c)^4*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*I*sqrt(2)*a^2*cos(d*x + c)^4
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c))) - 21*I*sqrt(2)*a^2*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (42*a^2*cos(d*x + c)^
3 + 20*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 5*a^2)*sqrt(cos(d*x + c)
)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**2/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 11.31 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.78

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{30\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\ -\frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)+84\,a^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \mathrm{co}\right.}{105\,d\cos\left(c+d\,x\right)^{7/2}\,\sqrt{1-\cos\left(c\right.}}$$

input
```
int((a + a/cos(c + d*x))^2/cos(c + d*x)^(5/2),x)
```

output
```
(30*a^2*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2) + 84*a^2
*cos(c + d*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) +
70*a^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x
)^2))/(105*d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = a^2\left(\int \frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^3}dx + \int \frac{\sqrt{\cos\left(dx+c\right)}\,\sec\left(dx+c\right)^2}{\cos\left(dx+c\right)^3}dx + 2\left(\int \frac{\sqrt{\cos\left(dx+c\right)}\,\sec\left(dx+c\right)}{\cos\left(dx+c\right)^3}dx\right)\right)$$

input
```
int((a+a*sec(d*x+c))^2/cos(d*x+c)^(5/2),x)
```

output
```
a**2*(int(sqrt(cos(c + d*x))/cos(c + d*x)**3,x) + int((sqrt(cos(c + d*x))*
sec(c + d*x)**2)/cos(c + d*x)**3,x) + 2*int((sqrt(cos(c + d*x))*sec(c + d*
x))/cos(c + d*x)**3,x))
```

3.367 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{68a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{44a^3\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} + \frac{44a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{68a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{6a^3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2a^3\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d}$$

output
```
68/15*a^3*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+44/21*a^3*InverseJacobiA
M(1/2*d*x+1/2*c,2^(1/2))/d+44/21*a^3*cos(d*x+c)^(1/2)*sin(d*x+c)/d+68/45*a
^3*cos(d*x+c)^(3/2)*sin(d*x+c)/d+6/7*a^3*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9
*a^3*cos(d*x+c)^(7/2)*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.57 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.73

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{1428(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}}-2640\,c\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^3,x]
```

output
```
(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*((1428*(3*Cos[c - d*x - ArcTa
n[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2]
- 2640*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hyper
geometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - Arc
Tan[Cot[c]]]*Sin[c] + Cos[c + d*x]*(-5712*Cot[c] + 2910*Sin[c + d*x] + 102
2*Sin[2*(c + d*x)] + 270*Sin[3*(c + d*x)] + 35*Sin[4*(c + d*x)]) - 2856*Co
s[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[
d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])
)/(10080*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.41, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4752, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{9}{2}}(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^3}{\sec^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{4278}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \left(\frac{a^3}{\sec^{\frac{3}{2}}(c+dx)}+\frac{3a^3}{\sec^{\frac{5}{2}}(c+dx)}+\frac{3a^3}{\sec^{\frac{7}{2}}(c+dx)}+\frac{a^3}{\sec^{\frac{9}{2}}(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{68a^3\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{44a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{44a^3}{\cdots}\right.$$

input
```
Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^3,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((68*a^3*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (44*a^3*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^3*Sin[c + d*x]
)/(9*d*Sec[c + d*x]^(7/2)) + (6*a^3*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
 + (68*a^3*Sin[c + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (44*a^3*Sin[c + d*x])
/(21*d*Sqrt[Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 19.18 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.77

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, a^{3}\left(560\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}-600\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}+212\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+66\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-430\cos\right.}{315\sqrt{-2\sin}}$

input
```
int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(560*co
s(1/2*d*x+1/2*c)^11-600*cos(1/2*d*x+1/2*c)^9+212*cos(1/2*d*x+1/2*c)^7+66*c
os(1/2*d*x+1/2*c)^5-430*cos(1/2*d*x+1/2*c)^3+165*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-357*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))+192*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.19

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{2\left(165i\sqrt{2}a^3\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin{(dx+c)})-165i\sqrt{2}a^3\text{weierstrassPInvers}\right.}{}$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-2/315*(165*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) - 165*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*
sin(d*x + c)) - 357*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 357*I*sqrt(2)*a^3*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3
5*a^3*cos(d*x + c)^3 + 135*a^3*cos(d*x + c)^2 + 238*a^3*cos(d*x + c) + 330
*a^3)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \int (a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2), x)
```

Giac [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \int (a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2), x)
```

Mupad [B] (verification not implemented)

Time = 11.00 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.40

$$\begin{aligned}&\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx\\&= \frac{2\left(a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a^3\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{3\,d}\\&\quad-\frac{2\left(\frac{33\,a^3\,\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)}{\sqrt{\sin(c+d\,x)^2}}-\frac{5\,a^3\,\cos(c+d\,x)^{11/2}\,\sin(c+d\,x)}{\sqrt{\sin(c+d\,x)^2}}\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right)^2\right)}{77\,d}\\&\quad-\frac{2\,a^3\,\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}\\&\quad-\frac{104\,a^3\,\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{19}{4};\ \cos\left(c+d\,x\right)^2\right)}{385\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}\end{aligned}$$

input
```
int(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^3,x)
```

output
```
(2*(a^3*ellipticF(c/2 + (d*x)/2, 2) + a^3*cos(c + d*x)^(1/2)*sin(c + d*x))
)/(3*d) - (2*((33*a^3*cos(c + d*x)^(7/2)*sin(c + d*x))/(sin(c + d*x)^2)^(1
/2) - (5*a^3*cos(c + d*x)^(11/2)*sin(c + d*x))/(sin(c + d*x)^2)^(1/2))*hyp
ergeom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(77*d) - (2*a^3*cos(c + d*x)^(9
/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin(c
+ d*x)^2)^(1/2)) - (104*a^3*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/
2, 11/4], 19/4, cos(c + d*x)^2))/(385*d*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= a^3\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^4\sec(dx+c)^3\,dx + 3\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^4\sec(dx+c)^2\,dx\right) + 3\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^4\sec(dx+c)\,dx\right) + \int \sqrt{\cos(dx+c)}\,\cos(dx+c)^4\,dx\right)$$

input
```
int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3,x)
```

output
```
a**3*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4*sec(c + d*x)**3,x) + 3*int(sq
rt(cos(c + d*x))*cos(c + d*x)**4*sec(c + d*x)**2,x) + 3*int(sqrt(cos(c + d
*x))*cos(c + d*x)**4*sec(c + d*x),x) + int(sqrt(cos(c + d*x))*cos(c + d*x)
**4,x))
```

3.368 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{52a^3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} + \frac{52a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{6a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a^3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}$$

output
```
28/5*a^3*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+52/21*a^3*InverseJacobiAM
(1/2*d*x+1/2*c,2^(1/2))/d+52/21*a^3*cos(d*x+c)^(1/2)*sin(d*x+c)/d+6/5*a^3*
cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*a^3*cos(d*x+c)^(5/2)*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.56 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.02

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{294(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}}-520\cos\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3,x]
```

output
```
(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*((294*(3*Cos[c - d*x - ArcTan
[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] -
 520*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hyperge
ometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTa
n[Cot[c]]]*Sin[c] + Cos[c + d*x]*(-1176*Cot[c] + 535*Sin[c + d*x] + 126*Si
n[2*(c + d*x)] + 15*Sin[3*(c + d*x)]) - 588*Cos[c]*Csc[d*x + ArcTan[Tan[c]
]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqr
t[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(1680*d*Sqrt[Cos[c + d*x]]
)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.50, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4752, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{7}{2}}(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^3}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4278}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \left(\frac{a^3}{\sqrt{\sec(c+dx)}}+\frac{3a^3}{\sec^{\frac{3}{2}}(c+dx)}+\frac{3a^3}{\sec^{\frac{5}{2}}(c+dx)}+\frac{a^3}{\sec^{\frac{7}{2}}(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{52a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{52a^3\sqrt{\cos(c+dx)}\sqrt{\sec}}{}\right.$$

input

```
Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3,x]
```

output

```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((28*a^3*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (52*a^3*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^3*Sin[c + d*x])
/(7*d*Sec[c + d*x]^(5/2)) + (6*a^3*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
+ (52*a^3*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(108) = 216$.

Time = 17.17 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.25

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, a^{3}\left(120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-432\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+602\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos}{105\sqrt{-2}}$

input
```
int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(120*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-432*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)+602*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-208*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^2+65*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1
/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.34

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{2\left(65i\sqrt{2}a^3\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))-65i\sqrt{2}a^3\text{weierstrassPInverse}(\right.}{}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-2/105*(65*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c)) - 65*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) - 147*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 147*I*sqrt(2)*a^3*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (15*
a^3*cos(d*x + c)^2 + 63*a^3*cos(d*x + c) + 130*a^3)*sqrt(cos(d*x + c))*sin
(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \int (a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \int (a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.79 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.18

$$\begin{aligned}\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx &= \frac{2\left(a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a^3\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{d}\\ &\quad-\frac{6\,a^3\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}\\ &\quad-\frac{2\,a^3\,{\cos\left(c+d\,x\right)}^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{9\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}\end{aligned}$$

input
```
int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^3,x)
```

output
```
(2*(a^3*ellipticE(c/2 + (d*x)/2, 2) + a^3*ellipticF(c/2 + (d*x)/2, 2) + a^
3*cos(c + d*x)^(1/2)*sin(c + d*x)))/d - (6*a^3*cos(c + d*x)^(7/2)*sin(c +
d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1
/2)) - (2*a^3*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4,
cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\begin{aligned}&\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx\\&=a^3\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\sec(dx+c)^3\,dx\right.\\&\quad+3\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\sec(dx+c)^2\,dx\right)\\&\quad+3\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\sec(dx+c)\,dx\right)\\&\quad\left.+\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)\end{aligned}$$

input
```
int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3,x)
```

output
```
a**3*(int(sqrt(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x)**3,x) + 3*int(sq
rt(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x)**2,x) + 3*int(sqrt(cos(c + d
*x))*cos(c + d*x)**3*sec(c + d*x),x) + int(sqrt(cos(c + d*x))*cos(c + d*x)
**3,x))
```

3.369 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d} + \frac{2a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{d} + \frac{2a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}$$

output
```
36/5*a^3*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4*a^3*InverseJacobiAM(1/2
*d*x+1/2*c,2^(1/2))/d+2*a^3*cos(d*x+c)^(1/2)*sin(d*x+c)/d+2/5*a^3*cos(d*x+
c)^(3/2)*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.16 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.56

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{9(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}}-20\cos(c\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3,x]
```

output
```
(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*((9*(3*Cos[c - d*x - ArcTan[T
an[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 2
0*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeome
tricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[C
ot[c]]]*Sin[c] + Cos[c + d*x]*(-36*Cot[c] + 10*Sin[c + d*x] + Sin[2*(c + d
*x)]) - 18*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}
, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan
[Tan[c]]]^2]))/(40*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.67, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4752, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^3}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \; 4278$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \left(\sqrt{\sec(c+dx)}a^3+\frac{3a^3}{\sqrt{\sec(c+dx)}}+\frac{3a^3}{\sec^{\frac{3}{2}}(c+dx)}+\frac{a^3}{\sec^{\frac{5}{2}}(c+dx)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}}+\frac{4a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c\right.}{d}\right.$$

input
```
Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((36*a^3*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (4*a^3*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a^3*Sin[c + d*x])/(5*d*
Sec[c + d*x]^(3/2)) + (2*a^3*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(86) = 172$.

Time = 14.00 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.75

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a^{3}\left(-4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+14\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right.\right.}{5\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\right.}}$

input
```
int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-4/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-4*sin(1
/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)-6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.63

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx =$$
$$-\frac{2\left(5i\sqrt{2}a^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}a^3\text{weierstrassPInverse}(-4\right.}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-2/5*(5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c)) - 5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)) - 9*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c))) + 9*I*sqrt(2)*a^3*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (a^3*cos(d*x
 + c) + 5*a^3)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx=\int (a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx=\int (a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{6\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a^3\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{d}-\frac{2\,a^3\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

input
```
int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^3,x)
```

output
```
(6*a^3*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a^3*ellipticF(c/2 + (d*x)/2, 2)
)/d + (2*a^3*cos(c + d*x)^(1/2)*sin(c + d*x))/d - (2*a^3*cos(c + d*x)^(7/2
)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c +
d*x)^2)^(1/2))
```

Reduce [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= a^3\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^3\,dx\right.\right.$$
$$\left.+3\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^2\,dx\right)\right.$$
$$+3\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)\,dx\right)$$
$$\left.+\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)$$

input
```
int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3,x)
```

output
```
a**3*(int(sqrt(cos(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**3,x) + 3*int(sq
rt(cos(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**2,x) + 3*int(sqrt(cos(c + d
*x))*cos(c + d*x)**2*sec(c + d*x),x) + int(sqrt(cos(c + d*x))*cos(c + d*x)
**2,x))
```

3.370 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{20a^3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
4*a^3*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+20/3*a^3*InverseJacobiAM(1/2
*d*x+1/2*c,2^(1/2))/d+2*a^3*sin(d*x+c)/d/cos(d*x+c)^(1/2)+2/3*a^3*cos(d*x+
c)^(1/2)*sin(d*x+c)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.37 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.60

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx =$$
$$-\frac{2a^3\csc(c+dx)\left(-3\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}+\cos(c+dx)\left(-1+\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3,x]
```

output
```
(-2*a^3*Csc[c + d*x]*(-3*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2]
*Sqrt[Sin[c + d*x]^2] + Cos[c + d*x]*(-1 + Cos[c + d*x]^2 + 10*Hypergeomet
ric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 3*Cos[c + d*x
]*Hypergeometric2F1[1/2, 3/4, 7/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))
/(3*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.67, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4752, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^3}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4278}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \left(\sec^{\frac{3}{2}}(c+dx)a^3+3\sqrt{\sec(c+dx)}a^3+\frac{3a^3}{\sqrt{\sec(c+dx)}}+\frac{a^3}{\sec^{\frac{3}{2}}(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{20a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{3d}\right.$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((4*a^3*Sqrt[Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (20*a^3*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a^3*Sin[c + d*x])/(3*d*
Sqrt[Sec[c + d*x]]) + (2*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.89

method	result
default	$-\frac{4a^3\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{}\right.}{3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$

input
```
int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-4/3*a^3*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-4*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.98

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{2\left(5i\sqrt{2}a^3\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}a^3\cos(dx+c\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-2/3*(5*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) - 5*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*a^3*cos(d*x + c)*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) +
 3*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c))) - (a^3*cos(d*x + c) + 3*a^3)*sqrt(co
s(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \int (a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \int (a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.17 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{6\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{20\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d} + \frac{2\,a^3\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3\,d} + \frac{2\,a^3\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}$$

input
```
int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^3,x)
```

output
```
(6*a^3*ellipticE(c/2 + (d*x)/2, 2))/d + (20*a^3*ellipticF(c/2 + (d*x)/2, 2
))/(3*d) + (2*a^3*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) + (2*a^3*sin(c +
d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(s
in(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= a^3\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)^3\,dx + 3\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)^2\,dx\right) + 3\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)\,dx\right) + \int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)$$

input
```
int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x)
```

output
```
a**3*(int(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x)**3,x) + 3*int(sqrt(
cos(c + d*x))*cos(c + d*x)*sec(c + d*x)**2,x) + 3*int(sqrt(cos(c + d*x))*c
os(c + d*x)*sec(c + d*x),x) + int(sqrt(cos(c + d*x))*cos(c + d*x),x))
```

3.371 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3\,dx = -\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{20a^3\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2a^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output

```
-4*a^3*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+20/3*a^3*InverseJacobiAM(1/
2*d*x+1/2*c,2^(1/2))/d+2/3*a^3*sin(d*x+c)/d/cos(d*x+c)^(3/2)+6*a^3*sin(d*x
+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.35 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.53

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3\,dx$$

$$= \frac{2a^3\csc(c+dx)\left(\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},\cos^2(c+dx)\right)+9\cos(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\right.\right.}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3,x]
```

output
```
(2*a^3*Csc[c + d*x]*(Hypergeometric2F1[-3/4, 1/2, 1/4, Cos[c + d*x]^2] + 9
*Cos[c + d*x]*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2] - Cos[c +
d*x]^2*(9*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2] + Cos[c + d*x]*
Hypergeometric2F1[1/2, 3/4, 7/4, Cos[c + d*x]^2]))*Sqrt[Sin[c + d*x]^2])/(
3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.67, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4752, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a\sec(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^3}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4278}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \left(\sec^{\frac{5}{2}}(c+dx)a^3+3\sec^{\frac{3}{2}}(c+dx)a^3+3\sqrt{\sec(c+dx)}a^3+\frac{a^3}{\sqrt{\sec(c+dx)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{6a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{20a^3\sqrt{\cos(c+dx)}}{}\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-4*a^3*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (20*a^3*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (6*a^3*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/d + (2*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(86) = 172.

Time = 2.62 (sec) , antiderivative size = 371, normalized size of antiderivative = 4.08

method	result
default	$-\frac{4a^3\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(18\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\ \mathrm{EllipticF}\left(\right.\right.}{}$

input
```
int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-4/3*a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(
1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(18*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x
+1/2*c)^2-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-10*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^2+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.05

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3\,dx =$$
$$-\frac{2\left(5i\sqrt{2}a^3\cos\left(dx+c\right)^2 \mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))-5i\sqrt{2}a^3\cos\left(dx+\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-2/3*(5*I*sqrt(2)*a^3*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) - 5*I*sqrt(2)*a^3*cos(d*x + c)^2*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a^3*cos(d*x + c)^2*wei
erstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c))) - 3*I*sqrt(2)*a^3*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (9*a^3*cos(d*x + c) + a^3)
*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3\,dx = a^3\left(\int 3\sqrt{\cos(c+dx)}\sec(c+dx)\,dx + \int 3\sqrt{\cos(c+dx)}\sec^2(c+dx)\,dx + \int \sqrt{\cos(c+dx)}\sec^3(c+dx)\,dx + \int \sqrt{\cos(c+dx)}\,dx\right)$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+a*sec(d*x+c))**3,x)
```

output
```
a**3*(Integral(3*sqrt(cos(c + d*x))*sec(c + d*x), x) + Integral(3*sqrt(cos
(c + d*x))*sec(c + d*x)**2, x) + Integral(sqrt(cos(c + d*x))*sec(c + d*x)*
*3, x) + Integral(sqrt(cos(c + d*x)), x))
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3\,dx = \int (a\sec(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3\,dx=\int (a\sec(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 10.72 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.38

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3\,dx = \frac{2\left(a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+3\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d} + \frac{6\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}} + \frac{2\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\,\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^3,x)
```

output
```
(2*(a^3*ellipticE(c/2 + (d*x)/2, 2) + 3*a^3*ellipticF(c/2 + (d*x)/2, 2)))/
d + (6*a^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*co
s(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*a^3*sin(c + d*x)*hypergeom([
-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)
^(1/2))
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3\,dx = a^3\left(\int \sqrt{\cos(dx+c)}dx + \int \sqrt{\cos(dx+c)}\,\sec(dx+c)^3\,dx + 3\left(\int \sqrt{\cos(dx+c)}\,\sec(dx+c)^2\,dx\right) + 3\left(\int \sqrt{\cos(dx+c)}\,\sec(dx+c)\,dx\right)\right)$$

input
```
int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x)
```

output
```
a**3*(int(sqrt(cos(c + d*x)),x) + int(sqrt(cos(c + d*x))*sec(c + d*x)**3,x
) + 3*int(sqrt(cos(c + d*x))*sec(c + d*x)**2,x) + 3*int(sqrt(cos(c + d*x))
*sec(c + d*x),x))
```

3.372 $\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = -\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{36a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output
```
-36/5*a^3*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4*a^3*InverseJacobiAM(1/
2*d*x+1/2*c,2^(1/2))/d+2/5*a^3*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2*a^3*sin(d*x
+c)/d/cos(d*x+c)^(3/2)+36/5*a^3*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.46 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.18

$$\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2a^3\csc(c+dx)\left(\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},-\frac{1}{4},\cos^2(c+dx)\right)+5\cos(c+dx)\left(\text{Hypergeometric2F1}\left(-\right.\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

output
```
(2*a^3*Csc[c + d*x]*(Hypergeometric2F1[-5/4, 1/2, -1/4, Cos[c + d*x]^2] +
5*Cos[c + d*x]*(Hypergeometric2F1[-3/4, 1/2, 1/4, Cos[c + d*x]^2] + Cos[c
+ d*x]*(3*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2] - Cos[c + d*x]
*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]))*Sqrt[Sin[c + d*x]^2])
/(5*d*Cos[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.52, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4752, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^3}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^3dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3dx$$

$$\Big\downarrow \text{4278}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \left(a^3\sec^{\frac{7}{2}}(c+dx)+3a^3\sec^{\frac{5}{2}}(c+dx)+3a^3\sec^{\frac{3}{2}}(c+dx)+a^3\sqrt{\sec(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d}+\frac{36a^3\sin(c+dx)\sqrt{\mathrm{s}}}{5d}\right.$$

input
```
Int[(a + a*Sec[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-36*a^3*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (4*a^3*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (36*a^3*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/(5*d) + (2*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/d + (2*a^3
*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(108) = 216$.

Time = 3.61 (sec) , antiderivative size = 386, normalized size of antiderivative = 3.30

method	result
default	$-\frac{16\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a^{3}\left(\frac{7\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{10\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-}}{160(c}\right.}{}$

input
```
int((a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(7/10*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-1/160*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-9/10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-9/20*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/16*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.71

$$\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx =$$
$$-\frac{2\left(5i\sqrt{2}a^3\cos(dx+c)^3\ \text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}a^3\cos(dx+\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-2/5*(5*I*sqrt(2)*a^3*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) - 5*I*sqrt(2)*a^3*cos(d*x + c)^3*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*I*sqrt(2)*a^3*cos(d*x + c)^3*wei
erstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c))) - 9*I*sqrt(2)*a^3*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (18*a^3*cos(d*x + c)^2 + 5
*a^3*cos(d*x + c) + a^3)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^
3)
```

Sympy [F]

$$\int \frac{(a + a\sec(c + dx))^3}{\sqrt{\cos(c + dx)}}\,dx = a^3 \left(\int \frac{3\sec(c + dx)}{\sqrt{\cos(c + dx)}}\,dx + \int \frac{3\sec^2(c + dx)}{\sqrt{\cos(c + dx)}}\,dx + \int \frac{\sec^3(c + dx)}{\sqrt{\cos(c + dx)}}\,dx + \int \frac{1}{\sqrt{\cos(c + dx)}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

output
```
a**3*(Integral(3*sec(c + d*x)/sqrt(cos(c + d*x)), x) + Integral(3*sec(c +
d*x)**2/sqrt(cos(c + d*x)), x) + Integral(sec(c + d*x)**3/sqrt(cos(c + d*x
)), x) + Integral(1/sqrt(cos(c + d*x)), x))
```

Maxima [F]

$$\int \frac{(a + a\sec(c + dx))^3}{\sqrt{\cos(c + dx)}}\,dx = \int \frac{(a\sec(dx + c) + a)^3}{\sqrt{\cos(dx + c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a + a\sec(c + dx))^3}{\sqrt{\cos(c + dx)}}\,dx = \int \frac{(a\sec(dx + c) + a)^3}{\sqrt{\cos(dx + c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 10.69 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.32

$$\int \frac{(a + a\sec(c + dx))^3}{\sqrt{\cos(c + dx)}}\,dx = \frac{2a^3\,\mathrm{F}\left(\frac{c}{2} + \frac{dx}{2}\middle|2\right)}{d} + \frac{6a^3\,\sin(c + dx)\;{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c + dx)^2\right)}{d\sqrt{\cos(c + dx)}\,\sqrt{\sin(c + dx)^2}} + \frac{2a^3\,\sin(c + dx)\;{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c + dx)^2\right)}{d\cos(c + dx)^{3/2}\,\sqrt{\sin(c + dx)^2}} + \frac{2a^3\,\sin(c + dx)\;{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos(c + dx)^2\right)}{5d\cos(c + dx)^{5/2}\,\sqrt{\sin(c + dx)^2}}$$

input
```
int((a + a/cos(c + d*x))^3/cos(c + d*x)^(1/2),x)
```

output
```
(2*a^3*ellipticF(c/2 + (d*x)/2, 2))/d + (6*a^3*sin(c + d*x)*hypergeom([-1/
4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2
)) + (2*a^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(d*c
os(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*a^3*sin(c + d*x)*hypergeom(
[-5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^
2)^(1/2))
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = a^3\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx + \int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)^3}{\cos(dx+c)}dx + 3\left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)^2}{\cos(dx+c)}dx\right) + 3\left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)}{\cos(dx+c)}dx\right)\right)$$

input
```
int((a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x)
```

output
```
a**3*(int(sqrt(cos(c + d*x))/cos(c + d*x),x) + int((sqrt(cos(c + d*x))*sec
(c + d*x)**3)/cos(c + d*x),x) + 3*int((sqrt(cos(c + d*x))*sec(c + d*x)**2)
/cos(c + d*x),x) + 3*int((sqrt(cos(c + d*x))*sec(c + d*x))/cos(c + d*x),x)
)
```

3.373 $\int \frac{(a+a\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{(a+a\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{52a^3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{52a^3\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{28a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output
```
-28/5*a^3*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+52/21*a^3*InverseJacobiA
M(1/2*d*x+1/2*c,2^(1/2))/d+2/7*a^3*sin(d*x+c)/d/cos(d*x+c)^(7/2)+6/5*a^3*s
in(d*x+c)/d/cos(d*x+c)^(5/2)+52/21*a^3*sin(d*x+c)/d/cos(d*x+c)^(3/2)+28/5*
a^3*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.65 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.95

$$\int \frac{(a+a\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2a^3\csc(c+dx)\left(5\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{1}{2},-\frac{3}{4},\cos^2(c+dx)\right)+7\cos(c+dx)\left(3\,\text{Hypergeometric2F1}\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

output
```
(2*a^3*Csc[c + d*x]*(5*Hypergeometric2F1[-7/4, 1/2, -3/4, Cos[c + d*x]^2]
+ 7*Cos[c + d*x]*(3*Hypergeometric2F1[-5/4, 1/2, -1/4, Cos[c + d*x]^2] + 5
*Cos[c + d*x]*(Hypergeometric2F1[-3/4, 1/2, 1/4, Cos[c + d*x]^2] + Cos[c +
 d*x]*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2])))*Sqrt[Sin[c + d*
x]^2])/(35*d*Cos[c + d*x]^(7/2))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.41, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4752, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$
$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^3dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3dx$$

$$\Big\downarrow \text{4278}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \left(a^3\sec^{\frac{9}{2}}(c+dx)+3a^3\sec^{\frac{7}{2}}(c+dx)+3a^3\sec^{\frac{5}{2}}(c+dx)+a^3\sec^{\frac{3}{2}}(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^3\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{6a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{52a^3\sin(c+dx)\sec}{21d}\right.$$

input
```
Int[(a + a*Sec[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-28*a^3*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (52*a^3*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (28*a^3*Sqrt[Sec[c
+ d*x]]*Sin[c + d*x])/(5*d) + (52*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21
*d) + (6*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + (2*a^3*Sec[c + d*x]^
(7/2)*Sin[c + d*x])/(7*d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(130) = 260$.

Time = 5.73 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.99

method	result
default	$-\frac{16\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a^{3}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{448\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{1}{2}\right)^{4}}-\frac{13\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\cdots}}{168\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{1}{2}\right)\cdots}\cdots}{\cdots}$

input
```
int((a+a*sec(d*x+c))^3/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-1/448*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(c
os(1/2*d*x+1/2*c)^2-1/2)^4-13/168*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+53/105*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
3/160*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-7/10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2
*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-7/20*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.46

$$\int \frac{(a + a\sec(c + dx))^3}{\cos^{\frac{3}{2}}(c + dx)}\,dx =$$

$$-\frac{2\left(65i\sqrt{2}a^3\cos(dx+c)^4\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)) - 65i\sqrt{2}a^3\cos(dx\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-2/105*(65*I*sqrt(2)*a^3*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c)) - 65*I*sqrt(2)*a^3*cos(d*x + c)^4*weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 147*I*sqrt(2)*a^3*cos(d*x + c)
^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c))) - 147*I*sqrt(2)*a^3*cos(d*x + c)^4*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (294*a^3*cos(d*x +
 c)^3 + 130*a^3*cos(d*x + c)^2 + 63*a^3*cos(d*x + c) + 15*a^3)*sqrt(cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = a^3\left(\int \frac{3\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{3\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{\sec^3(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**3/cos(d*x+c)**(3/2),x)
```

output
```
a**3*(Integral(3*sec(c + d*x)/cos(c + d*x)**(3/2), x) + Integral(3*sec(c +
 d*x)**2/cos(c + d*x)**(3/2), x) + Integral(sec(c + d*x)**3/cos(c + d*x)**
(3/2), x) + Integral(cos(c + d*x)**(-3/2), x))
```

Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^3}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^3}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.74 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99

$$\int \frac{(a+a\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\frac{2a^3\sin(c+dx)\,{}_2F_1\left(-\frac{7}{4},\frac{1}{2};\,-\frac{3}{4};\,\cos(c+dx)^2\right)}{7}+\frac{6a^3\cos(c+dx)\sin(c+dx)\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};\,-\frac{1}{4};\,\cos(c+dx)^2\right)}{5}+2a^3\cos(c+dx)^2\sin}{d\cos(c+dx)^{7/2}\sqrt{}}$$

input
```
int((a + a/cos(c + d*x))^3/cos(c + d*x)^(3/2),x)
```

output
```
((2*a^3*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2))/7 + (6*
a^3*cos(c + d*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2)
)/5 + 2*a^3*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c
+ d*x)^2) + 2*a^3*cos(c + d*x)^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4,
cos(c + d*x)^2))/(d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = a^3\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx + \int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)^3}{\cos(dx+c)^2}dx + 3\left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)^2}{\cos(dx+c)^2}dx\right) + 3\left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)}{\cos(dx+c)^2}dx\right)\right)$$

input
```
int((a+a*sec(d*x+c))^3/cos(d*x+c)^(3/2),x)
```

output
```
a**3*(int(sqrt(cos(c + d*x))/cos(c + d*x)**2,x) + int((sqrt(cos(c + d*x))*
sec(c + d*x)**3)/cos(c + d*x)**2,x) + 3*int((sqrt(cos(c + d*x))*sec(c + d*
x)**2)/cos(c + d*x)**2,x) + 3*int((sqrt(cos(c + d*x))*sec(c + d*x))/cos(c
+ d*x)**2,x))
```

3.374 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 128

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{21E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} - \frac{5\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} + \frac{7\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}$$

output
```
21/5*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d-5/3*InverseJacobiAM(1/2*d*x
+1/2*c,2^(1/2))/a/d-5/3*cos(d*x+c)^(1/2)*sin(d*x+c)/a/d+7/5*cos(d*x+c)^(3/
2)*sin(d*x+c)/a/d-cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.40 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.45

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{2i\sqrt{2}e^{-i(c+dx)}\left(63\left(1+e^{2i(c+dx)}\right)+63\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)+25e^{i(c+dx}\cdots}{d\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\cdots}\cdots}{\cdots}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x]),x]
```

output
```
(Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*(63*(1 + E^((2*I)*(c + d*x))) + 63*(-1
 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
 3/4, -E^((2*I)*(c + d*x))] + 25*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*
x))])*Sec[c + d*x])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*
I)*(c + d*x)))/E^(I*(c + d*x))]) + (-96*Cot[c] - 30*Csc[c] - 20*Cos[d*x]*S
in[c] + 6*Cos[2*d*x]*Sin[2*c] - 30*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2]
- 20*Cos[c]*Sin[d*x] + 6*Cos[2*c]*Sin[2*d*x])/(d*Sqrt[Cos[c + d*x]])))/(15
*a*(1 + Sec[c + d*x]))
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.51, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4752, 3042, 4306, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx$$

$$\Big\downarrow \; 4306$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{7a-5a\sec(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)}dx}{a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{7a-5a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{7a-5a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \; 4274$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7a\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx-5a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx-5a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right.$$

$$\Big\downarrow \; 4256$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7a\left(\frac{3}{5}\int\frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-5a\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)}{2a^2}-\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7a\left(\frac{3}{5}\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-5a\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c}{3d\sqrt{\sec(}}\right.}{2a^2}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7a\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-5a\left(\frac{1}{3}\sqrt{\cos(}\right.}{2a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7a\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-5a\left(\frac{1}{3}\right.}{2a^2}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7a\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{5d}\right)-5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(}\right.}{2a^2}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7a\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{5d}\right)-5a\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(}}{}\right.}{2a^2}\right.$$

input
```
Int[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x]),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-(Sin[c + d*x]/(d*Sec[c + d*x]^(3/2
)*(a + a*Sec[c + d*x]))) + (7*a*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2
))) - 5*a*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/(2*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 5.00 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.79

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(25\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+63\right.\right.}{15a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin(}$

input
```
int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*
x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(25
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2)))+48*sin(1/2*d*x+1/2*c)^8-56*sin(1/2*d*x+1/2*c)^6-30*sin(1/2*d*x+1/2*
c)^4+23*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.62

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{2\left(6\cos\left(dx+c\right)^{2}-4\cos\left(dx+c\right)-25\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-25\left(-i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\text{we}}{}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/30*(2*(6*cos(d*x + c)^2 - 4*cos(d*x + c) - 25)*sqrt(cos(d*x + c))*sin(d*
x + c) - 25*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) - 25*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2)
)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*(-I*sqrt(
2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*(I*sqrt(2)*cos(d*x + c) + I*sqrt
(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c))))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx=\int \frac{\cos\left(c+d\,x\right)^{5/2}}{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)^2}{\sec(dx+c)+1}dx}{a}$$

input
```
int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x)
```

output
```
int((sqrt(cos(c + d*x))*cos(c + d*x)**2)/(sec(c + d*x) + 1),x)/a
```

3.375 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{5\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad} + \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))}$$

output
```
-3*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+5/3*InverseJacobiAM(1/2*d*x+1
/2*c,2^(1/2))/a/d+5/3*cos(d*x+c)^(1/2)*sin(d*x+c)/a/d-cos(d*x+c)^(1/2)*sin
(d*x+c)/d/(a+a*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.32 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.92

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$$

$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\left(9\left(1+e^{2i(c+dx)}\right)+9\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)+5e^{i(c+dx)}\right.}{d\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right.}}}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x]),x]
```

output
```
(Cos[(c + d*x)/2]^2*(((-2*I)*Sqrt[2]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1
+ E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x)
)])*Sec[c + d*x])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)
*(c + d*x)))/E^(I*(c + d*x))]) + (12*Cot[c] + 6*Csc[c] + 4*Cos[d*x]*Sin[c]
 + 6*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] + 4*Cos[c]*Sin[d*x])/(d*Sqrt[C
os[c + d*x]])))/(3*a*(1 + Sec[c + d*x]))
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.65, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4752, 3042, 4306, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx$$

$$\Big\downarrow \text{4306}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{5a-3a\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{5a-3a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{5a-3a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \; 4274$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx-3a\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx-3a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}\right.$$

$$\Big\downarrow \; 4256$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5a\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-3a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(}{d\sqrt{\sec(c+dx)}}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-3a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{}{d\sqrt{\sec(c-}}\right.$$

$$\Big\downarrow \; 4258$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-3a\sqrt{\cos(c+dx}}{2a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-3a\sqrt{\cos(c-}}{2a^2}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-\frac{6a\sqrt{\cos(c+dx}}{}}{2a^2}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5a\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)-\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{d}}{2a^2}\right.$$

input
```
Int[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x]),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-(Sin[c + d*x]/(d*Sqrt[Sec[c + d*x]
]*(a + a*Sec[c + d*x]))) + ((-6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/d + 5*a*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x
]])))/(2*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(95) = 190$.

Time = 3.61 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.15

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\left(5\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+9\,\mathrm{Ellip}\right.\right.}{3a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,c}}$

input
```
int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+
1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(5*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))-8*sin(1/2*d*x+1/2*c)^6+18*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2)/
a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.98

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{2\,(2\,\cos(dx+c)+5)\sqrt{\cos(dx+c)}\sin(dx+c)-5\left(i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,}{}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(2*(2*cos(d*x + c) + 5)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*(I*sqrt(2)
*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c)) - 5*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*(I*sqrt(2)*cos(d*x + c) + I*sqrt(
2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c))) - 9*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*
x + c) + a*d)
```

Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)**(3/2)/(sec(c + d*x) + 1), x)/a
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(dx+c)^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \int \frac{\cos\left(c+d\,x\right)^{3/2}}{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)}{\sec(dx+c)+1}dx}{a}$$

input
```
int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x)
```

output
```
int((sqrt(cos(c + d*x))*cos(c + d*x))/(sec(c + d*x) + 1),x)/a
```

3.376 $\int \frac{\sqrt{\cos(c+dx)}}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \frac{\sqrt{\cos(c+dx)}}{a+a\sec(c+dx)}\,dx=\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}$$

output
```
3*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d-InverseJacobiAM(1/2*d*x+1/2*c,
2^(1/2))/a/d-sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.00 (sec) , antiderivative size = 270, normalized size of antiderivative = 3.75

$$\int \frac{\sqrt{\cos(c+dx)}}{a+a\sec(c+dx)}\,dx$$
$$=\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{2i\sqrt{2}e^{-i(c+dx)}\left(3\left(1+e^{2i(c+dx)}\right)+3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)+e^{i(c+dx)}\left(-\right.\right.}{d\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}}{a(1+\sec(c-}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

output
```
(Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 +
 E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3
/4, -E^((2*I)*(c + d*x))] + E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^
((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])
*Sec[c + d*x])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c
 + d*x)))/E^(I*(c + d*x))]) - (2*(2*Cot[c] + Csc[c] + Sec[c/2]*Sec[(c + d*
x)/2]*Sin[(d*x)/2]))/(d*Sqrt[Cos[c + d*x]])))/(a*(1 + Sec[c + d*x]))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.90, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {3042, 4752, 3042, 4306, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx$$

$$\Big\downarrow \text{4306}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{3a-a\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{3a-a\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{3a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}\right)$$

$\Big\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3a\int \frac{1}{\sqrt{\sec(c+dx)}}dx-a\int \sqrt{\sec(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}\right)$$

$\Big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int}{2a^2}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-a\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{2a^2}\right.$$

$\Big\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}d\right.}{2a^2}$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{2a^2}-\frac{\sin}{c}\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((6*a*Sqrt[Cos[c + d*x]]*EllipticE[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (2*a*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/(2*a^2) - (Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(d*(a + a*Sec[c + d*x])))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(73) = 146$.

Time = 2.13 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.76

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\left(\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+3\,\text{EllipticE}\right.\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-}}$

input
```
int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*si
n(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.58

$$\int \frac{\sqrt{\cos(c+dx)}}{a+a\sec(c+dx)}\,dx = \frac{\left(i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+\left(-i\sqrt{2}\cos\left(dx+c\right)\right.}{}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + (-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(-I*sqrt(2)*cos(d*x
+ c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c))) - 3*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))
 - 2*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sqrt{\cos(c+dx)}}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(sqrt(cos(c + d*x))/(sec(c + d*x) + 1), x)/a
```

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\sqrt{\cos(c+d\,x)}}{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sqrt{\cos(dx+c)}}{\sec(dx+c)+1}dx}{a}$$

input
```
int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))/(sec(c + d*x) + 1),x)/a
```

3.377 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}\,dx=-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}+\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}$$

output
```
-EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+InverseJacobiAM(1/2*d*x+1/2*c,2
^(1/2))/a/d+sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.36 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.74

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}\,dx = \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\left(1+e^{2i(c+dx)}+(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)+e^{i(c+dx)}(-1+\cdots\right)}{d(-1+e^{2ic})\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}\cdots}{a(1+\sec(c+dx\cdots}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

output
```
(Cos[(c + d*x)/2]^2*(((-2*I)*Sqrt[2]*(1 + E^((2*I)*(c + d*x)) + (-1 + E^((
2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -
E^((2*I)*(c + d*x))] + E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I
)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[
c + d*x])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*
x)))/E^(I*(c + d*x))]) + (2*(Csc[c] + Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/
2]))/(d*Sqrt[Cos[c + d*x]])))/(a*(1 + Sec[c + d*x]))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.94, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4752, 3042, 4307, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a\sec(c+dx)+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)}}{\sec(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{4307}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int \frac{a-a\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int \frac{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}\right)$$

$$\Big\downarrow \text{4274}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{a\int \frac{1}{\sqrt{\sec(c+dx)}}dx-a\int \sqrt{\sec(c+dx)}dx}{2a^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}\right)$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-a\sqrt{}}{2a^2}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-}{2a^2}\right.$$

$$\Big\downarrow \text{3119}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-a\sqrt{\cos(c+dx)}\sqrt{}}{2a^2}\right.$$

$$\Big\downarrow \text{3120}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{}}{2a^2}\right.$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*((2*a*Sqrt[Cos[c + d*x]]*Ellip
ticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (2*a*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(d*(a + a*Sec[c + d*x])))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(a*f*
(a + b*Csc[e + f*x]))), x] + Simp[d*((n - 1)/(a*b))   Int[(d*Csc[e + f*x])^
(n - 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ
[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(71) = 142$.

Time = 1.48 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.83

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+\mathrm{Elliptic}\right.\right.}{a\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}}$

input
```
int(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*
c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin
(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.63

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}\,dx$$
$$= \frac{\left(-i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\left(i\sqrt{2}\cos\left(dx+c\right.\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + (-I*sqrt(2)*cos(d*x +
c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c))) + (I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2
*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{1}{\sqrt{\cos\left(c+dx\right)}\sec\left(c+dx\right)+\sqrt{\cos\left(c+dx\right)}}\,dx}{a}$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(1/(sqrt(cos(c + d*x))*sec(c + d*x) + sqrt(cos(c + d*x))), x)/a
```

Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(a\sec{(dx+c)}+a)\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(a\sec{(dx+c)}+a)\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}\,dx = \int \frac{1}{\sqrt{\cos{(c+d\,x)}}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)\sec(dx+c)+\cos(dx+c)}dx}{a}$$

input
```
int(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)*sec(c + d*x) + cos(c + d*x)),x)/a
```

3.378 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx=\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}$$

output
```
EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+InverseJacobiAM(1/2*d*x+1/2*c,2^
(1/2))/a/d-sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.54 (sec) , antiderivative size = 263, normalized size of antiderivative = 3.76

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{2i\sqrt{2}e^{-i(c+dx)}\left(1+e^{2i(c+dx)}+\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)-e^{i(c+dx)}\left(-1+e^{2}\right.\right.}{d\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}}{a(1+\sec(c+dx)}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
```

output
```
(Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*(1 + E^((2*I)*(c + d*x)) + (-1 + E^((2
*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E
^((2*I)*(c + d*x))] - E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)
*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c
 + d*x])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x
)))/E^(I*(c + d*x))]) - (2*(Csc[c] + Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2
]))/(d*Sqrt[Cos[c + d*x]])))/(a*(1 + Sec[c + d*x]))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.96, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {3042, 4752, 3042, 4305, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sec(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{4305}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{\sec(c+dx)a+a}{2\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \ 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sec(c+dx)a+a}{\sqrt{\sec(c+dx)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \ 4274$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\int \frac{1}{\sqrt{\sec(c+dx)}}dx+a\int \sqrt{\sec(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \ 4258$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\text{co}}}{2a^2}\right.$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{}}{2a^2}\right.$$

$$\Big\downarrow \ 3119$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right.}{d}}{2a^2}\right.$$

$$\Big\downarrow \text{3120}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a^2}-\frac{\sin}{d}\right.$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*a*Sqrt[Cos[c + d*x]]*EllipticE[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/(2*a^2) - (Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(d*(a + a*Sec[c + d*x])))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4305
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Simp[d^2/(a*b)   Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
 - 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ
[a^2 - b^2, 0] && GtQ[n, 1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(71) = 142$.

Time = 0.67 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.86

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-\text{Elliptic}\right.\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}$

input
```
int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*
c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin
(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.63

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \frac{\left(-i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\left(i\sqrt{2}\cos(dx+c\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c
) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c))) + (-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2
*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \frac{\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sec(c+dx)+\cos^{\frac{3}{2}}(c+dx)}\,dx}{a}$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(1/(cos(c + d*x)**(3/2)*sec(c + d*x) + cos(c + d*x)**(3/2)), x)/a
```

Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{1}{\cos(c+d\,x)^{3/2}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2\sec(dx+c)+\cos(dx+c)^2}dx}{a}$$

input
```
int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**2*sec(c + d*x) + cos(c + d*x)**2),x)
/a
```

3.379 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 96

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = -\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad} + \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}$$

output

```
-3*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d-InverseJacobiAM(1/2*d*x+1/2*c
,2^(1/2))/a/d+3*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)-sin(d*x+c)/d/cos(d*x+c)^(3
/2)/(a+a*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.98 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.16

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$$
$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{(2\cos(\frac{1}{2}(c-dx))+\cos(\frac{1}{2}(3c+dx))+3\cos(\frac{1}{2}(c+3dx)))\csc(\frac{c}{2})\sec(\frac{c}{2})\sec(\frac{1}{2}(c+dx))}{2d\cos^{\frac{3}{2}}(c+dx)} - \frac{2i\sqrt{2}e^{-i(c+dx)}\left(3(1+e^{2i(c+d}\right.}{}\right.}{a}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```

output
```
(Cos[(c + d*x)/2]^2*(((2*Cos[(c - d*x)/2] + Cos[(3*c + d*x)/2] + 3*Cos[(c
+ 3*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2])/(2*d*Cos[c + d*x]^(3/2))
- ((2*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[
1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c +
d*x))] - E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*
Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x])/(d*E
^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c +
 d*x))])))/(a*(1 + Sec[c + d*x]))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.68, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4752, 3042, 4305, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)}\,dx$$
$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$\downarrow$ 4752

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sec(c+dx)a+a}dx$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx$$

$\downarrow$ 4305

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{1}{2}\sqrt{\sec(c+dx)}(a-3a\sec(c+dx))dx}{a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}\right)$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \sqrt{\sec(c+dx)}(a-3a\sec(c+dx))dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}\right)$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{a\int \sqrt{\sec(c+dx)}dx-3a\int \sec^{\frac{3}{2}}(c+dx)dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-3a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+d}{d(a\sec(c+dx)+a)}\right.$$

$\downarrow$ 4255

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{2a^2}-\frac{\sin(c+}{d(a\,\mathrm{s}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}-\frac{\sin}{d}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c}\right.}{2a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos}\right.}{2a^2}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-2\sqrt{\mathrm{co}}\right.}{2a^2}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{}}{}\right.}{2a^2}\right.$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-((Sec[c + d*x]^(3/2)*Sin[c + d*x])
/(d*(a + a*Sec[c + d*x]))) - ((2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/d - 3*a*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/
(2*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4305
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Simp[d^2/(a*b)   Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
 - 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ
[a^2 - b^2, 0] && GtQ[n, 1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(95) = 190$.

Time = 1.48 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.65

method	result
default	$-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(3\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-\mathrm{EllipticF}\left(\cos\right.\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}}$

input
```
int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-(cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+6*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-5*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a
/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.46

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$$
$$= \frac{2\,(3\,\cos(dx+c)+2)\sqrt{\cos(dx+c)}\sin(dx+c)+\left(i\sqrt{2}\cos(dx+c)^2+i\sqrt{2}\cos(dx+c)\right)\text{weierstrassPI}}{}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*(3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))*sin(d*x + c) + (I*sqrt(2)*c
os(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c)) + (-I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(2)*cos(d*x +
 c))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(I*sqrt
(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*(-I*sqrt(2)*cos(d
*x + c)^2 - I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*x + c)^2 + a*d*cos
(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{1}{\cos(c+d\,x)^{5/2}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\,(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3\sec(dx+c)+\cos(dx+c)^3}dx}{a}$$

input
```
int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**3*sec(c + d*x) + cos(c + d*x)**3),x)
/a
```

3.380 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 124

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{5\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad} + \frac{5\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}$$

output
```
3*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+5/3*InverseJacobiAM(1/2*d*x+1/
2*c,2^(1/2))/a/d+5/3*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)-3*sin(d*x+c)/a/d/cos(
d*x+c)^(1/2)-sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.24 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.73

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{(10\cos(\frac{1}{2}(c-dx))+8\cos(\frac{1}{2}(3c+dx))+4\cos(\frac{1}{2}(c+3dx))+5\cos(\frac{1}{2}(5c+3dx))+9\cos(\frac{1}{2}(3c+5dx)))\csc(\frac{c}{2})\sec(\frac{c}{2})\sec}{4d\cos^{\frac{5}{2}}(c+dx)}\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])),x]
```

output
```
(Cos[(c + d*x)/2]^2*(-1/4*((10*Cos[(c - d*x)/2] + 8*Cos[(3*c + d*x)/2] + 4
*Cos[(c + 3*d*x)/2] + 5*Cos[(5*c + 3*d*x)/2] + 9*Cos[(3*c + 5*d*x)/2])*Csc
[c/2]*Sec[c/2]*Sec[(c + d*x)/2])/(d*Cos[c + d*x]^(5/2)) + ((2*I)*Sqrt[2]*(
9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c +
d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 5*E^(I*(c
 + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F
1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x])/(d*E^(I*(c + d*x))*(
-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))])))/(3*a*
(1 + Sec[c + d*x]))
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.52, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4752, 3042, 4305, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a\sec(c+dx)+a)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{ 4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sec(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{ 4305}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{1}{2}\sec^{\frac{3}{2}}(c+dx)(3a-5a\sec(c+dx))dx}{a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \sec^{\frac{3}{2}}(c+dx)(3a-5a\sec(c+dx))dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3a-5a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \text{ 4274}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3a\int \sec^{\frac{3}{2}}(c+dx)dx-5a\int \sec^{\frac{5}{2}}(c+dx)dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx-5a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+}{d(a\sec(c+dx)+}\right.$$

$$\Big\downarrow \text{ 4255}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)-5a\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{}\right.}{2a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-5a\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\right.}{2a^2}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)-5a\left(\right.}{2a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)-}{2a^2}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)-5a\left(\frac{1}{3}\sqrt{\cos(c+\right.}{2a^2}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)-5a\left(\frac{2\sin(c+dx)\sec}{3d}\right.}{2a^2}\right.$$

input
```
Int[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-((Sec[c + d*x]^(5/2)*Sin[c + d*x])
/(d*(a + a*Sec[c + d*x]))) - (3*a*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d) -
 5*a*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)))/(2*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4305
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Simp[d^2/(a*b)   Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
 - 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ
[a^2 - b^2, 0] && GtQ[n, 1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(117) = 234$.

Time = 2.29 (sec) , antiderivative size = 413, normalized size of antiderivative = 3.33

method	result
default	$\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(10\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\ \text{EllipticF}\left(\cos\left(\right.\right.\right.}{}$

input
```
int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a/cos(1/2*d*
x+1/2*c)/sin(1/2*d*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)
^2+1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(10*cos(1/2*d*x
+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-18*cos(1/2*d*x+1/2*
c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-36*sin(1/2*d*x+1/2*c)^6-
5*cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)+9*cos(1/2*d*x+1/2*c)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*
d*x+1/2*c)^2)^(1/2)+44*sin(1/2*d*x+1/2*c)^4-11*sin(1/2*d*x+1/2*c)^2)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.08

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = -\frac{2\left(9\cos\left(dx+c\right)^2+4\cos\left(dx+c\right)-2\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+5\left(i\sqrt{2}\cos\left(dx+c\right)^3+i\sqrt{2}\cos\left(\right.\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/6*(2*(9*cos(d*x + c)^2 + 4*cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x
 + c) + 5*(I*sqrt(2)*cos(d*x + c)^3 + I*sqrt(2)*cos(d*x + c)^2)*weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x +
c)^3 - I*sqrt(2)*cos(d*x + c)^2)*weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c)) + 9*(-I*sqrt(2)*cos(d*x + c)^3 - I*sqrt(2)*cos(d*x + c)^2
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c))) + 9*(I*sqrt(2)*cos(d*x + c)^3 + I*sqrt(2)*cos(d*x + c)^2)*weiers
trassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{1}{(a\sec{(dx+c)}+a)\cos{(dx+c)}^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{1}{(a\sec{(dx+c)}+a)\cos{(dx+c)}^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{1}{\cos(c+d\,x)^{7/2}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^4\sec(dx+c)+\cos(dx+c)^4}dx}{a}$$

input
```
int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**4*sec(c + d*x) + cos(c + d*x)**4),x)
/a
```

3.381 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 160

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{56E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d} - \frac{5\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a^2d} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d} + \frac{56\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d} - \frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

output
```
56/5*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-5*InverseJacobiAM(1/2*d*x
+1/2*c,2^(1/2))/a^2/d-5*cos(d*x+c)^(1/2)*sin(d*x+c)/a^2/d+56/15*cos(d*x+c)
^(3/2)*sin(d*x+c)/a^2/d-3*cos(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))
-1/3*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.99 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\csc^3(c+dx)\left(-240-1186\cos(c+dx)+340\cos(2(c+dx))+207\cos(3(c+dx))-20\,\text{co}\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Sqrt[Cos[c + d*x]]*Csc[c + d*x]^3*(-240 - 1186*Cos[c + d*x] + 340*Cos[2*(
c + d*x)] + 207*Cos[3*(c + d*x)] - 20*Cos[4*(c + d*x)] + 3*Cos[5*(c + d*x)
] + 600*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*(Sin[c + d*x]^2)^
(3/2) + 1792*Cos[c + d*x]*Hypergeometric2F1[3/4, 5/2, 7/4, Cos[c + d*x]^2]
*(Sin[c + d*x]^2)^(3/2)))/(120*a^2*d)
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.47, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 4752, 3042, 4304, 27, 3042, 4508, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx$$

$$\Big\downarrow \text{4304}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{11a-7a\sec(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{11a-7a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{11a-7a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{4508}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{56a^2-45a^2\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx}{a^2}-\frac{18\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{56a^2-45a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx}{a^2}-\frac{18\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)\right.$$

$$\Big\downarrow \text{4274}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{56a^2\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx-45a^2\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{18\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+}{3d\sec^{\frac{3}{2}}(c+dx)(a\,s}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{56a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx-45a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{a^2}-\frac{18\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}}{6a^2}-\frac{}{3d\sec^{\frac{3}{2}}(c}\right.$$

$\downarrow$ 4256

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{56a^2\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-45a^2\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)}{a^2}-\frac{18\sin(}{d\sec^{\frac{3}{2}}(c+dx)(}}{6a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{56a^2\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-45a^2\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)}{a^2}-\frac{}{d\sec^{\frac{3}{2}}}}{6a^2}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{56a^2\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-45a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\right.}{a^2}}{6a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{56a^2\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-45a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.}{a^2}}{6a^2}\right.$$

$$\Big\downarrow \text{3119}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{56a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)-45a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{}{\sqrt{\sin(c}}\right.}{a^2}}{6a^2}\right.$$

$$\Big\downarrow \text{3120}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{56a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)-45a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{}\right.}{a^2}}{6a^2}\right.$$

input
```
Int[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/3*Sin[c + d*x]/(d*Sec[c + d*x]^(
3/2)*(a + a*Sec[c + d*x])^2) + ((-18*Sin[c + d*x])/(d*Sec[c + d*x]^(3/2)*(
1 + Sec[c + d*x])) + (56*a^2*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)))
 - 45*a^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/a^2)/(6*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 6.92 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.77

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(96\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}-352\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-150\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\right.}\right.}{30a^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right.}$

input
```
int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/30*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*cos(1/2*
d*x+1/2*c)^10-352*cos(1/2*d*x+1/2*c)^8+120*cos(1/2*d*x+1/2*c)^6-150*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-336*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))+266*cos(1/2*d*x+1/2*c)^4-135*cos(1/2*d*x+1/2*c)^2+5)/a
^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.80

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{2\left(6\cos\left(dx+c\right)^3-8\cos\left(dx+c\right)^2-94\cos\left(dx+c\right)-75\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-75\left(-i\sqrt{2}\cos\left(\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/30*(2*(6*cos(d*x + c)^3 - 8*cos(d*x + c)^2 - 94*cos(d*x + c) - 75)*sqrt(
cos(d*x + c))*sin(d*x + c) - 75*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*c
os(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c)) - 75*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqr
t(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 168*(-I*
sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 1
68*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx=\int \frac{{\cos\left(c+d\,x\right)}^{5/2}}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)^2}{\sec(dx+c)^2+2\sec(dx+c)+1}dx}{a^2}$$

input
```
int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x)
```

output
```
int((sqrt(cos(c + d*x))*cos(c + d*x)**2)/(sec(c + d*x)**2 + 2*sec(c + d*x)
 + 1),x)/a**2
```

3.382 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 138

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{10\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{10\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d}-\frac{7\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

output

```
-7*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+10/3*InverseJacobiAM(1/2*d*
x+1/2*c,2^(1/2))/a^2/d+10/3*cos(d*x+c)^(1/2)*sin(d*x+c)/a^2/d-7/3*cos(d*x+
c)^(1/2)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*cos(d*x+c)^(1/2)*sin(d*x+c)/d
/(a+a*sec(d*x+c))^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.62 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\csc^3(c+dx)\,(15+76\cos(c+dx)-24\cos(2(c+dx))-12\cos(3(c+dx))+\cos(4(c+dx}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Sqrt[Cos[c + d*x]]*Csc[c + d*x]^3*(15 + 76*Cos[c + d*x] - 24*Cos[2*(c + d
*x)] - 12*Cos[3*(c + d*x)] + Cos[4*(c + d*x)] - 40*Hypergeometric2F1[1/4,
1/2, 5/4, Cos[c + d*x]^2]*(Sin[c + d*x]^2)^(3/2) - 112*Cos[c + d*x]*Hyperg
eometric2F1[3/4, 5/2, 7/4, Cos[c + d*x]^2]*(Sin[c + d*x]^2)^(3/2)))/(12*a^
2*d)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.51, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4752, 3042, 4304, 27, 3042, 4508, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^2}dx$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx$$

$\downarrow$ 4304

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{9a-5a\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}\right)$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{9a-5a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{9a-5a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}\right)$$

$\downarrow$ 4508

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{3\left(10a^2-7a^2\sec(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\int \frac{10a^2-7a^2\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\int\frac{10a^2-7a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(10a^2\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx-7a^2\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin}{3d\sqrt{\sec(c+dx)}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(10a^2\int\frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx-7a^2\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{}{3d\sqrt{\sec}}\right.$$

$\downarrow$ 4256

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(10a^2\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-7a^2\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1}}{6a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(10a^2\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-7a^2\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx}}{6a^2}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(10a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-7a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{c}\right.}{a^2}}{6a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(10a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-7a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{a^2}}{6a^2}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(10a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-\frac{14a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\right.}{d}\right.}{a^2}}{6a^2}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(10a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)-\frac{14a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{d}\right.}{a^2}}{6a^2}\right.$$

input
```
Int[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/3*Sin[c + d*x]/(d*Sqrt[Sec[c + d
*x]]*(a + a*Sec[c + d*x])^2) + ((-14*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]*(
1 + Sec[c + d*x])) + (3*((-14*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/d + 10*a^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c +
d*x]]))))/a^2)/(6*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(127) = 254.

Time = 4.19 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.96

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(16\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^8+12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^6+20\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\right.}{6a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3\sqrt{-2\sin\left(\right.}}$

input
```
int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*cos(1/2*d
*x+1/2*c)^8+12*cos(1/2*d*x+1/2*c)^6+20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*
d*x+1/2*c)^3+42*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48*cos(1/2
*d*x+1/2*c)^4+21*cos(1/2*d*x+1/2*c)^2-1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.01

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{2\left(2\cos\left(dx+c\right)^2+13\cos\left(dx+c\right)+10\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-10\left(i\sqrt{2}\cos\left(dx+c\right)^2+2i\sqrt{2}\,\mathrm{co}\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(2*(2*cos(d*x + c)^2 + 13*cos(d*x + c) + 10)*sqrt(cos(d*x + c))*sin(d*
x + c) - 10*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(
2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 10*(-I*sqr
t(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 21*(I*sqrt(2)*cos(d*x + c)^2
 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*(-I*sqrt(2)*cos(d*x
+ c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c
)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(cos(c + d*x)**(3/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a*
*2
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos\left(c+d\,x\right)^{3/2}}{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^2}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)}{\sec(dx+c)^2+2\sec(dx+c)+1}dx}{a^2}$$

input
```
int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x)
```

output
```
int((sqrt(cos(c + d*x))*cos(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) +
1),x)/a**2
```

3.383 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 112

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d} - \frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}$$

output
```
4*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-5/3*InverseJacobiAM(1/2*d*x+
1/2*c,2^(1/2))/a^2/d-5/3*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)/(1+sec(d*x+c))-
1/3*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.51 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \frac{\sqrt{\cos(c+dx)}\csc^3(c+dx)\left(-6-46\cos(c+dx)+14\cos(2(c+dx))+6\cos(3(c+dx))+20\,\text{Hypergeor}\right.}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Sqrt[Cos[c + d*x]]*Csc[c + d*x]^3*(-6 - 46*Cos[c + d*x] + 14*Cos[2*(c + d
*x)] + 6*Cos[3*(c + d*x)] + 20*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*
x]^2]*(Sin[c + d*x]^2)^(3/2) + 64*Cos[c + d*x]*Hypergeometric2F1[3/4, 5/2,
 7/4, Cos[c + d*x]^2]*(Sin[c + d*x]^2)^(3/2)))/(12*a^2*d)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.60, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4752, 3042, 4304, 27, 3042, 4508, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx$$

$$\Big\downarrow \text{4304}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{7a-3a\sec(c+dx)}{2\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{7a-3a\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{7a-3a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{4508}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{12a^2-5a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{12a^2-5a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{4274}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{12a^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx-5a^2\int \sqrt{\sec(c+dx)}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+ \cdots}}{3d(a\sec(c+dx)+ \cdots}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{12a^2\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-5a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\mathrm{s}\cdots}}{3d(a\sec(c+ \cdots}\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{12a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{a^2}-\frac{10\sin(}{d(}}{6a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{12a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-}{6a^2}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{24a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{10\sin(c+}{d(\sec}}{6a^2}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{24a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{10a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}-\frac{10\sin(c+dx)}{d(\sec(c-}}{6a^2}\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/3*(Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^2) + (((24*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (10*a^2*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 - (10*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(d*(1 + Sec[c + d*x])))/(6*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
 - a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
 + b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(105) = 210$.

Time = 3.58 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.29

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^6+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\mathrm{co}}{6a^2\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\ \mathrm{c}}$

input
```
int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(1/2*d*
x+1/2*c)^6+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+24*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*cos(1/2*d*x+1/2*c)^4+15*cos(1/2*
d*x+1/2*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.39

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = -\frac{2\,(6\,\cos\left(dx+c\right)+5)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+5\left(-i\sqrt{2}\cos\left(dx+c\right)^2-2i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right.}{}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/6*(2*(6*cos(d*x + c) + 5)*sqrt(cos(d*x + c))*sin(d*x + c) + 5*(-I*sqrt(
2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(I*sqrt(2)*cos(d*x + c)^2 +
2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) + 12*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*
x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c))) + 12*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*c
os(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x
+ c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{\cos\left(c+dx\right)}}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^2}$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(sqrt(cos(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**
2
```

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{\cos(dx+c)}}{\sec(dx+c)^2+2\sec(dx+c)+1}dx}{a^2}$$

input
```
int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x)
```

output
```
int(sqrt(cos(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1),x)/a**2
```

3.384 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}\,dx = -\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d} + \frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}$$

output
```
-EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+2/3*InverseJacobiAM(1/2*d*x+1
/2*c,2^(1/2))/a^2/d+sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)/(1+sec(d*x+c))-1/3*s
in(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.37 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}\,dx = -\frac{2\sqrt{\cos(c+dx)}\csc^3(c+dx)\left(-2\cos(c+dx)+\cos(2(c+dx))+\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\cos^2(c+\right.\right.}{3a^2d}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

output
```
(-2*Sqrt[Cos[c + d*x]]*Csc[c + d*x]^3*(-2*Cos[c + d*x] + Cos[2*(c + d*x)]
+ Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*(Sin[c + d*x]^2)^(3/2)
+ 2*Cos[c + d*x]*Hypergeometric2F1[3/4, 5/2, 7/4, Cos[c + d*x]^2]*(Sin[c +
 d*x]^2)^(3/2)))/(3*a^2*d)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.65, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {3042, 4752, 3042, 4304, 27, 3042, 4507, 25, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)}}{(\sec(c+dx)a+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx$$

$$\Big\downarrow \text{4304}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{\sqrt{\sec(c+dx)}(5a-a\sec(c+dx))}{2(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sec(c+dx)}(5a-a\sec(c+dx))}{\sec(c+dx)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{4507}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int -\frac{3a^2-2a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\int \frac{3a^2-2a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\int \frac{3a^2-2a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \; 4274$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\int\frac{1}{\sqrt{\sec(c+dx)}}dx-2a^2\int\sqrt{\sec(c+dx)}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx}{3d(a\sec(c+dx)+a)^2}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-2a^2\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(}{3d(a\sec(c+dx}\right.$$

$$\Big\downarrow \; 4258$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^2}}{6a^2}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{a^2}}{6a^2}\right.$$

$$\Big\downarrow \; 3119$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\frac{6a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{}{\sqrt{\sin}}}{a^2}}{6a^2}\right.$$

$$\Big\downarrow \; 3120$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\frac{6a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}\right.}{d}}{a^2}}{6a^2}\right.$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/3*(Sec[c + d*x]^(3/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^2) + (-(((6*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (4*a^2*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2) + (6*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(d*(1 + Sec[c + d*x])))/(6*a^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(104) = 208$.

Time = 2.24 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.36

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\left(12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^6+4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\,c}{6a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+}}$

input
```
int(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d
*x+1/2*c)^6+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-20*cos(1/2*d*x+1/2*c)^4+9*cos(1/2*d*
x+1/2*c)^2-1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.46

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{2\,(3\,\cos{(dx+c)}+2)\sqrt{\cos{(dx+c)}}\sin{(dx+c)} - 2\left(i\sqrt{2}\cos{(dx+c)}^2 + 2i\sqrt{2}\cos{(dx+c)} + i\sqrt{2}\right)\text{wei}}{}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(2*(3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*(I*sqrt(2)
*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 2*(-I*sqrt(2)*cos(d*x + c)^2 - 2
*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c)) - 3*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x +
 c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c))) - 3*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(
d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c
) + a^2*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{1}{\sqrt{\cos(c+dx)}\sec^2(c+dx)+2\sqrt{\cos(c+dx)}\sec(c+dx)+\sqrt{\cos(c+dx)}}\,dx}{a^2}$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(1/(sqrt(cos(c + d*x))*sec(c + d*x)**2 + 2*sqrt(cos(c + d*x))*sec(
c + d*x) + sqrt(cos(c + d*x))), x)/a**2
```

Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\sqrt{\cos(c+d\,x)}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^2),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)\sec(dx+c)^2+2\cos(dx+c)\sec(dx+c)+\cos(dx+c)}dx}{a^2}$$

input
```
int(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)*sec(c + d*x)**2 + 2*cos(c + d*x)*sec(
c + d*x) + cos(c + d*x)),x)/a**2
```

3.385 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 57

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d} + \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}$$

output
```
1/3*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/a^2/d+1/3*sin(d*x+c)/d/cos(d*x+
c)^(3/2)/(a+a*sec(d*x+c))^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.25

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \frac{\sqrt{\cos(c+dx)}\csc(c+dx)\left(-\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}+\tan^2\left(\frac{1}{2}(c+d\right.\right.}{3a^2d}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Csc[c + d*x]*(-(Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c
 + d*x]^2]*Sqrt[Sin[c + d*x]^2]) + Tan[(c + d*x)/2]^2))/(3*a^2*d)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.72, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4752, 3042, 4302, 27, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(\sec(c+dx)a+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx$$

$$\Big\downarrow \text{4302}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{1}{2}\sqrt{\sec(c+dx)}dx}{3a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \sqrt{\sec(c+dx)}dx}{6a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{6a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{6a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{6a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{3120}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d) + (Sec[c + d*x]^(3/2)*Sin[c + d*x
])/(3*d*(a + a*Sec[c + d*x])^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4302
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] - Simp[d/(a*b*(2*m + 1))   Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*(a*(n - 1) - b*(m + n)*C
sc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && L
tQ[m, -1] && LtQ[1, n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(52) = 104$.

Time = 2.31 (sec) , antiderivative size = 188, normalized size of antiderivative = 3.30

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3+2\,\text{co}}{6a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$

input
```
int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+2*cos(1/2*d*x+1/2*c)^4-3*cos(1/2*d*x
+1/2*c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.63

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \frac{\left(-i\sqrt{2}\cos{(dx+c)}^2-2i\sqrt{2}\cos{(dx+c)}-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin{(dx+c)})}{6\left(a^2 \cdots\right.}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*((-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*cos(d
*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(cos(d*x + c))*sin(d*x + c))/(a^
2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sec^2(c+dx)+2\cos^{\frac{3}{2}}(c+dx)\sec(c+dx)+\cos^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(1/(cos(c + d*x)**(3/2)*sec(c + d*x)**2 + 2*cos(c + d*x)**(3/2)*se
c(c + d*x) + cos(c + d*x)**(3/2)), x)/a**2
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\cos(c+d\,x)^{3/2}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^2),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^2), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2\sec(dx+c)^2+2\cos(dx+c)^2\sec(dx+c)+\cos(dx+c)^2}dx}{a^2}$$

input
```
int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**2*sec(c + d*x)**2 + 2*cos(c + d*x)**
2*sec(c + d*x) + cos(c + d*x)**2),x)/a**2
```

3.386 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2} dx = \frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d} - \frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}$$

output
```
EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+2/3*InverseJacobiAM(1/2*d*x+1/
2*c,2^(1/2))/a^2/d-sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)/(1+sec(d*x+c))-1/3*si
n(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.42 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.86

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(-\frac{(7\cos(\frac{1}{2}(c-dx))+2\cos(\frac{1}{2}(3c+dx))+3\cos(\frac{1}{2}(c+3dx)))\csc(\frac{c}{2})\sec(\frac{c}{2})\sec^3(\frac{1}{2}(c+dx))}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{4i\sqrt{2}e^{-i(c+dx)}\left(3(1+e^{2}\right.}{}\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
(Cos[(c + d*x)/2]^4*(-1/2*((7*Cos[(c - d*x)/2] + 2*Cos[(3*c + d*x)/2] + 3*
Cos[(c + 3*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^3)/(d*Cos[c + d*x]^
(3/2)) + ((4*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c)
)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I
)*(c + d*x))] - 2*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c
+ d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d
*x]^2)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x))
)/E^(I*(c + d*x))])))/(3*a^2*(1 + Sec[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.65, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4752, 3042, 4303, 27, 3042, 4507, 25, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(\sec(c+dx)a+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx$$

$$\Big\downarrow \text{4303}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\sec(c+dx)}(a-5a\sec(c+dx))}{2(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\sec(c+dx)}(a-5a\sec(c+dx))}{\sec(c+dx)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a-5a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{4507}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int -\frac{2\sec(c+dx)a^2+3a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\int \frac{2\sec(c+dx)a^2+3a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\int\frac{2\csc\left(c+dx+\frac{\pi}{2}\right)a^2+3a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\int\frac{1}{\sqrt{\sec(c+dx)}}dx+2a^2\int\sqrt{\sec(c+dx)}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+}{3d(a\sec(c+dx)+a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2a^2\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec}{3d(a\sec(c+a}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+da}{a^2}}{6a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c}{a^2}}{6a^2}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}{d}}{a^2}}{6a^2}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E}{d}}{a^2}}{6a^2}\right.$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/3*(Sec[c + d*x]^(3/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^2) - (-(((6*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (4*a^2*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2) + (6*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(d*(1 + Sec[c + d*x])))/(6*a^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(104) = 208.

Time = 0.82 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.36

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\cos}{6a^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin}}$

input
```
int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*
x+1/2*c)^6-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))-16*cos(1/2*d*x+1/2*c)^4+3*cos(1/2*d*x
+1/2*c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.46

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = -\frac{2\left(3\cos\left(dx+c\right)+4\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+2\left(i\sqrt{2}\cos\left(dx+c\right)^{2}+2i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{v}}{}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/6*(2*(3*cos(d*x + c) + 4)*sqrt(cos(d*x + c))*sin(d*x + c) + 2*(I*sqrt(2
)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 2*(-I*sqrt(2)*cos(d*x + c)^2 -
2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) + 3*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x
 + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c))) + 3*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos
(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x +
c) + a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\cos(c+d\,x)^{5/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^2),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^2), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3\sec(dx+c)^2+2\cos(dx+c)^3\sec(dx+c)+\cos(dx+c)^3}dx}{a^2}$$

input
```
int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**3*sec(c + d*x)**2 + 2*cos(c + d*x)**
3*sec(c + d*x) + cos(c + d*x)**3),x)/a**2
```

3.387 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 136

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = -\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d} + \frac{4\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{5\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}$$

output
```
-4*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-5/3*InverseJacobiAM(1/2*d*x
+1/2*c,2^(1/2))/a^2/d+4*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)-5/3*sin(d*x+c)/a
^2/d/cos(d*x+c)^(3/2)/(1+sec(d*x+c))-1/3*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+
a*sec(d*x+c))^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.38 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.51

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{(29\cos(\frac{1}{2}(c-dx))+19\cos(\frac{1}{2}(3c+dx))+31\cos(\frac{1}{2}(c+3dx))+5\cos(\frac{1}{2}(5c+3dx))+12\cos(\frac{1}{2}(3c+5dx)))\csc(\frac{c}{2})\sec(\frac{c}{2})\,\mathrm{se}}{4d\cos^{\frac{5}{2}}(c+dx)}\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
(Cos[(c + d*x)/2]^4*(((29*Cos[(c - d*x)/2] + 19*Cos[(3*c + d*x)/2] + 31*Co
s[(c + 3*d*x)/2] + 5*Cos[(5*c + 3*d*x)/2] + 12*Cos[(3*c + 5*d*x)/2])*Csc[c
/2]*Sec[c/2]*Sec[(c + d*x)/2]^3)/(4*d*Cos[c + d*x]^(5/2)) - ((4*I)*Sqrt[2]
*(12*(1 + E^((2*I)*(c + d*x))) + 12*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(
c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 5*E^(
I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometr
ic2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x]^2)/(d*E^(I*(c + d
*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))])))
/(3*a^2*(1 + Sec[c + d*x])^2)
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.49, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 4752, 3042, 4303, 27, 3042, 4507, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$\Big\downarrow$ 4752

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(\sec(c+dx)a+a)^2}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx$$

$\Big\downarrow$ 4303

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)(3a-7a\sec(c+dx))}{2(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)(3a-7a\sec(c+dx))}{\sec(c+dx)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3a-7a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$\Big\downarrow$ 4507

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \sqrt{\sec(c+dx)}\left(5a^2-12a^2\sec(c+dx)\right)dx}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx}{3d(a\sec(c+dx)+a)}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5a^2-12a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}}{3d(a\sec(c+dx}\right.$$

$\Big\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{5a^2\int\sqrt{\sec(c+dx)}dx-12a^2\int\sec^{\frac{3}{2}}(c+dx)dx}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c\ldots}{3d(a\sec(c+dx)+\ldots}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{5a^2\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-12a^2\int\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\ldots}{3d(a\sec(c\ldots}\right.$$

$\downarrow$ 4255

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{5a^2\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx\ldots}{d(\sec(c+dx)+1)}}{6a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{5a^2\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(\ldots}{d(\sec(c+dx)+1\ldots}}{6a^2}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\ldots\right.}{a^2}}{6a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+\ldots}\right.}{a^2}}{6a^2}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{d}\right.}{a^2}}{6a^2}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{10a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E}{d}\right.}{a^2}}{6a^2}\right.$$

input
```
Int[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/3*(Sec[c + d*x]^(5/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^2) - ((10*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*
(1 + Sec[c + d*x])) + ((10*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/d - 12*a^2*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/a
^2)/(6*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(127) = 254$.

Time = 2.24 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.98

method	result
default	$-\frac{2\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(5\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-12\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{}$

input
```
int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)*cos(1/2*d*x+1/2*c)-48*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*sin(1/2*d*x+1/2*c)^6+86*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*sin(1/2*d*x+1/2*c)^4-37*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.34

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{2\left(12\cos(dx+c)^2+19\cos(dx+c)+6\right)\sqrt{\cos(dx+c)}\sin(dx+c)-5\left(-i\sqrt{2}\cos(dx+c)^3-2i\sqrt{2}\cos\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(2*(12*cos(d*x + c)^2 + 19*cos(d*x + c) + 6)*sqrt(cos(d*x + c))*sin(d*
x + c) - 5*(-I*sqrt(2)*cos(d*x + c)^3 - 2*I*sqrt(2)*cos(d*x + c)^2 - I*sqr
t(2)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c
)) - 5*(I*sqrt(2)*cos(d*x + c)^3 + 2*I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*
cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) -
12*(I*sqrt(2)*cos(d*x + c)^3 + 2*I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(
d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) - 12*(-I*sqrt(2)*cos(d*x + c)^3 - 2*I*sqrt(2)*cos(d*x +
c)^2 - I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*co
s(d*x + c)^2 + a^2*d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\cos(c+dx)^{7/2}\left(a+\frac{a}{\cos(c+dx)}\right)^2}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^2),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^2), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\left(a+a\sec(c+dx)\right)^2}\,dx$$
$$= \frac{\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^4\sec(dx+c)^2+2\cos(dx+c)^4\sec(dx+c)+\cos(dx+c)^4}dx}{a^2}$$

input
```
int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**4*sec(c + d*x)**2 + 2*cos(c + d*x)**
4*sec(c + d*x) + cos(c + d*x)**4),x)/a**2
```

3.388 $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 162

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{10\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d} + \frac{10\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{7\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{7\sin(c+dx)}{3a^2d\cos^{\frac{5}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}$$

output
```
7*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+10/3*InverseJacobiAM(1/2*d*x
+1/2*c,2^(1/2))/a^2/d+10/3*sin(d*x+c)/a^2/d/cos(d*x+c)^(3/2)-7*sin(d*x+c)/
a^2/d/cos(d*x+c)^(1/2)-7/3*sin(d*x+c)/a^2/d/cos(d*x+c)^(5/2)/(1+sec(d*x+c)
)-1/3*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.70 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.30

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(-\frac{(82\cos(\frac{1}{2}(c-dx))+65\cos(\frac{1}{2}(3c+dx))+68\cos(\frac{1}{2}(c+3dx))+37\cos(\frac{1}{2}(5c+3dx))+53\cos(\frac{1}{2}(3c+5dx))+10\cos(\frac{1}{2}(7c\cdots}{8d\cos^{\frac{7}{2}}(c+dx)}\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
(Cos[(c + d*x)/2]^4*(-1/8*((82*Cos[(c - d*x)/2] + 65*Cos[(3*c + d*x)/2] +
68*Cos[(c + 3*d*x)/2] + 37*Cos[(5*c + 3*d*x)/2] + 53*Cos[(3*c + 5*d*x)/2]
+ 10*Cos[(7*c + 5*d*x)/2] + 21*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec
[(c + d*x)/2]^3)/(d*Cos[c + d*x]^(7/2)) + ((4*I)*Sqrt[2]*(21*(1 + E^((2*I)
*(c + d*x))) + 21*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyperge
ometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 10*E^(I*(c + d*x))*(-1
+ E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5
/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x]^2)/(d*E^(I*(c + d*x))*(-1 + E^((2*
I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))])))/(3*a^2*(1 + Sec[
c + d*x])^2)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.43, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4752, 3042, 4303, 27, 3042, 4507, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a\sec(c+dx)+a)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(\sec(c+dx)a+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx$$

$$\Big\downarrow \text{4303}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)(5a-9a\sec(c+dx))}{2(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)(5a-9a\sec(c+dx))}{\sec(c+dx)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(5a-9a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{4507}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int 3\sec^{\frac{3}{2}}(c+dx)\left(7a^2-10a^2\sec(c+dx)\right)dx}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+a}{3d(a\sec(c+dx)+a}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\int \sec^{\frac{3}{2}}(c+dx)\left(7a^2-10a^2\sec(c+dx)\right)dx}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+a}{3d(a\sec(c+dx)+a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(7a^2-10a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec}{3d(a\sec(c+}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\left(7a^2\int \sec^{\frac{3}{2}}(c+dx)dx-10a^2\int \sec^{\frac{5}{2}}(c+dx)dx\right)}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}}{3d(a\sec(c+dx}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\left(7a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx-10a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx\right)}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+}{3d(a\,se}\right.$$

$\downarrow$ 4255

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\left(7a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)-10a^2\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)\right.}{a^2}}{6a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\left(7a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-10a^2\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}}{3d}\right.\right.}{a^2}}{6a^2}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\left(7a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)-10a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{se}\right.\right.}{a^2}}{6a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\left(7a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)-10a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\right.}{a^2}}{6a^2}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\left(7a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)-10a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{a^2}}{6a^2}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\left(7a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)-10a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\right.}{a^2}}{6a^2}\right.$$

input
```
Int[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/3*(Sec[c + d*x]^(7/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^2) - ((14*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*
(1 + Sec[c + d*x])) + (3*(7*a^2*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d) - 1
0*a^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d))))/a^2)/(6*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(149) = 298$.

Time = 3.09 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.55

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\frac{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{6\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{22\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}}{3}\right.}{}$

input
```
int(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(1/3*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3+6*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)-22
/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+14*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2/3*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1
/2*c)^2-1/2)^2+16*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*
x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.09

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = -\frac{2\left(21\cos\left(dx+c\right)^3+32\cos\left(dx+c\right)^2+8\cos\left(dx+c\right)-2\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+10\left(i\sqrt{2}\cos\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/6*(2*(21*cos(d*x + c)^3 + 32*cos(d*x + c)^2 + 8*cos(d*x + c) - 2)*sqrt(
cos(d*x + c))*sin(d*x + c) + 10*(I*sqrt(2)*cos(d*x + c)^4 + 2*I*sqrt(2)*co
s(d*x + c)^3 + I*sqrt(2)*cos(d*x + c)^2)*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 10*(-I*sqrt(2)*cos(d*x + c)^4 - 2*I*sqrt(2)*cos
(d*x + c)^3 - I*sqrt(2)*cos(d*x + c)^2)*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) + 21*(-I*sqrt(2)*cos(d*x + c)^4 - 2*I*sqrt(2)*cos(
d*x + c)^3 - I*sqrt(2)*cos(d*x + c)^2)*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*(I*sqrt(2)*cos(d*x + c
)^4 + 2*I*sqrt(2)*cos(d*x + c)^3 + I*sqrt(2)*cos(d*x + c)^2)*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2
*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(9/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(9/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\cos(c+d\,x)^{9/2}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int(1/(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^2),x)
```

output
```
int(1/(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^2), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^5\sec(dx+c)^2+2\cos(dx+c)^5\sec(dx+c)+\cos(dx+c)^5}dx}{a^2}$$

input
```
int(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**5*sec(c + d*x)**2 + 2*cos(c + d*x)**
5*sec(c + d*x) + cos(c + d*x)**5),x)/a**2
```

3.389 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 207

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3} dx = \frac{231E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{21\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d} - \frac{21\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d} + \frac{77\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{10a^3d} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{63\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{10d\,(a^3+a^3\sec(c+dx))}$$

output

```
231/10*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-21/2*InverseJacobiAM(1/
2*d*x+1/2*c,2^(1/2))/a^3/d-21/2*cos(d*x+c)^(1/2)*sin(d*x+c)/a^3/d+77/10*co
s(d*x+c)^(3/2)*sin(d*x+c)/a^3/d-1/5*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec
(d*x+c))^3-4/5*cos(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-63/10*co
s(d*x+c)^(3/2)*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.16 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.85

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\sqrt{\cos(c+dx)}\csc(c+dx)\left(-((614+2995\cos(c+dx)-766\cos(2(c+dx))-1139\cos(3(c+dx))+29\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Sqrt[Cos[c + d*x]]*Csc[c + d*x]*(-((614 + 2995*Cos[c + d*x] - 766*Cos[2*(
c + d*x)] - 1139*Cos[3*(c + d*x)] + 290*Cos[4*(c + d*x)] + 127*Cos[5*(c +
d*x)] - 10*Cos[6*(c + d*x)] + Cos[7*(c + d*x)])*Csc[c + d*x]^4) + 1680*Hyp
ergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 7040*
Cos[c + d*x]*Hypergeometric2F1[3/4, 7/2, 7/4, Cos[c + d*x]^2]*Sqrt[Sin[c +
 d*x]^2]))/(160*a^3*d)
```

Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.39, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4752, 3042, 4304, 27, 3042, 4508, 27, 3042, 4508, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx$$

$$\Big\downarrow \text{4304}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{3(5a-3a\sec(c+dx))}{2\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\int \frac{5a-3a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\int \frac{5a-3a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{4508}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{\int \frac{7\left(5a^2-4a^2\sec(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+d}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{7\int\frac{5a^2-4a^2\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{7\int\frac{5a^2-4a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{}{5d\sec^{\frac{3}{2}}(c+}\right.$$

$\downarrow$ 4508

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{7\left(\frac{\int\frac{5\left(11a^3-9a^3\sec(c+dx)\right)}{2\sec^{\frac{5}{2}}(c+dx)}dx}{a^2}-\frac{9a^2\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)}{10a^2}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{7\left(\frac{5\int\frac{11a^3-9a^3\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx}{2a^2}-\frac{9a^2\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)}{10a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{7\left(\frac{5\int\frac{11a^3-9a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx}{2a^2}-\frac{9a^2\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right.}{10a^2}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{7\left(\frac{5\left(11a^3\int\frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx-9a^3\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx\right)}{2a^2}-\frac{9a^2\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)}{3a^2}-\frac{8a\sin(c}{3d\sec^{\frac{3}{2}}(c+dx)(a}\right.}{10a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{7\left(\frac{5\left(11a^3\int\frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx-9a^3\int\frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx\right)}{2a^2}-\frac{9a^2\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)}{3a^2}-\frac{}{3d\sec}\right.}{10a^2}\right.$$

$\downarrow$ 4256

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{7\left(\frac{5\left(11a^3\left(\frac{3}{5}\int\frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-9a^3\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right)}{2a^2}-\frac{9a^2 s}{d\sec^{\frac{3}{2}}(c+dx}\right.}{3a^2}}{10a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{7\left(\frac{5\left(11a^3\left(\frac{3}{5}\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-9a^3\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right)}{2a^2}-\frac{}{d\,\mathrm{se}}\right.}{3a^2}}{10a^2}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{7\left(\frac{5\left(11a^3\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-9a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{}{\sqrt{}}\cdots\right.\right.}{2a^2}\cdots\right)}{3a^2}\cdots\right)}{10a^2\cdots}\cdots\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{7\left(\frac{5\left(11a^3\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-9a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\cdots\right.\right.}{2a^2}\cdots\right)}{3a^2}\cdots\right)}{1\cdots}\cdots\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{7\left(\frac{5\left(11a^3\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)-9a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{}{\sqrt{s}}\right.\right.}{2a^2}\right.}{3a^2}\right.}{10a}\right.$$

$\Big\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{7\left(\frac{5\left(11a^3\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)-9a^3\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{}}{}\right.\right.}{2a^2}\right.}{3a^2}\right.}{10a}\right.$$

input
```
Int[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^3,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/5*Sin[c + d*x]/(d*Sec[c + d*x]^(
3/2)*(a + a*Sec[c + d*x])^3) + (3*((-8*a*Sin[c + d*x])/(3*d*Sec[c + d*x]^(
3/2)*(a + a*Sec[c + d*x])^2) + (7*((-9*a^2*Sin[c + d*x])/(d*Sec[c + d*x]^(
3/2)*(a + a*Sec[c + d*x])) + (5*(11*a^3*((6*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*Sin[c + d*x])/(5*d*Sec[c + d
*x]^(3/2))) - 9*a^3*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]))))/(2*a^2
)))/(3*a^2)))/(10*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 11.20 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.43

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(64\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}-288\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}-76\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-210\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos(}\right.}{20a^{3}\,\mathrm{c}}$

input
```
int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(64*cos(1/2*
d*x+1/2*c)^12-288*cos(1/2*d*x+1/2*c)^10-76*cos(1/2*d*x+1/2*c)^8-210*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-462*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))+530*cos(1/2*d*x+1/2*c)^6-248*cos(1/2*d*x+1/2*c)^4+19*c
os(1/2*d*x+1/2*c)^2-1)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.76

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{2\left(4\,\cos\left(dx+c\right)^4-8\,\cos\left(dx+c\right)^3-147\,\cos\left(dx+c\right)^2-238\,\cos\left(dx+c\right)-105\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx\right.}{}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/20*(2*(4*cos(d*x + c)^4 - 8*cos(d*x + c)^3 - 147*cos(d*x + c)^2 - 238*co
s(d*x + c) - 105)*sqrt(cos(d*x + c))*sin(d*x + c) - 105*(-I*sqrt(2)*cos(d*
x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(
2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 105*(I*sqr
t(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c
) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) -
 231*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)
*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) - 231*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*s
qrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^
3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^3, x)
```

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\cos(c+d\,x)^{5/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^3,x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)^2}{\sec(dx+c)^3+3\sec(dx+c)^2+3\sec(dx+c)+1}dx}{a^3}$$

input
```
int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x)
```

output
```
int((sqrt(cos(c + d*x))*cos(c + d*x)**2)/(sec(c + d*x)**3 + 3*sec(c + d*x)
**2 + 3*sec(c + d*x) + 1),x)/a**3
```

3.390 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 181

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{11\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d}+\frac{11\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d}-\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad(a+a\sec(c+dx))^2}-\frac{119\sqrt{\cos(c+dx)}\sin(c+dx)}{30d\left(a^3+a^3\sec(c+dx)\right)}$$

output

```
-119/10*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+11/2*InverseJacobiAM(1
/2*d*x+1/2*c,2^(1/2))/a^3/d+11/2*cos(d*x+c)^(1/2)*sin(d*x+c)/a^3/d-1/5*cos
(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-2/3*cos(d*x+c)^(1/2)*sin(d*x
+c)/a/d/(a+a*sec(d*x+c))^2-119/30*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(a^3+a^3*s
ec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.41 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\csc(c+dx)\Big((511+2260\cos(c+dx)-559\cos(2(c+dx))-910\cos(3(c+dx))+245\cos}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Sqrt[Cos[c + d*x]]*Csc[c + d*x]*((511 + 2260*Cos[c + d*x] - 559*Cos[2*(c
+ d*x)] - 910*Cos[3*(c + d*x)] + 245*Cos[4*(c + d*x)] + 90*Cos[5*(c + d*x)
] - 5*Cos[6*(c + d*x)])*Csc[c + d*x]^4 - 1320*Hypergeometric2F1[1/4, 1/2,
5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] - 5440*Cos[c + d*x]*Hypergeometr
ic2F1[3/4, 7/2, 7/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]))/(240*a^3*d)
```

Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.43, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.826$, Rules used = {3042, 4752, 3042, 4304, 27, 3042, 4508, 3042, 4508, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a\sec(c+dx)+a)^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$
$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^3}dx$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx$$

$\downarrow$ 4304

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{13a-7a\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}\right)$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{13a-7a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{13a-7a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}\right)$$

$\downarrow$ 4508

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{69a^2-50a^2\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{69a^2-50a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c}{5d\sqrt{\sec(c+dx)}(a}$$

$\downarrow$ 4508

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int \frac{3\left(165a^3-119a^3\sec(c+dx)\right)}{2\sec^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{}{5d\sqrt{}}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\int \frac{165a^3-119a^3\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{}{5d\sqrt{s}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\int \frac{165a^3-119a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{}{5c}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\left(165a^3\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx-119a^3\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+}}{10a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\left(165a^3\int\frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx-119a^3\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{20a\,s}{3d\sqrt{\sec(c+dx}}}{10a^2}\right.$$

$\downarrow$ 4256

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\left(165a^3\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-119a^3\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-}{10a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\left(165a^3\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-119a^3\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}}{10a^2}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\left(165a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-119a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx}\right.}{2a^2}}{3a^2}}{10a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\left(165a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-119a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin(\cdots}{2a^2}}{3a^2}}{10a^2}\right.$$

$$\Big\downarrow \; 3119$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\left(165a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-\frac{238a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c\cdots\right.}{d}}{2a^2}}{3a^2}}{10a^2}\right.$$

$$\Big\downarrow \; 3120$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\left(165a^3\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)-\frac{238a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c\cdots\right.}{d}}{2a^2}}{3a^2}}{10a^2}\right.$$

input
```
Int[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^3,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/5*Sin[c + d*x]/(d*Sqrt[Sec[c + d
*x]]*(a + a*Sec[c + d*x])^3) + ((-20*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x
]]*(a + a*Sec[c + d*x])^2) + ((-119*a^2*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]
]*(a + a*Sec[c + d*x])) + (3*((-238*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + 165*a^3*((2*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[S
ec[c + d*x]]))))/(2*a^2))/(3*a^2))/(10*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 5.63 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.56

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(160\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+468\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+330\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \mathrm{Ellipt}\right.}{60a^{3}\sqrt{-2\sin(}}$

input
```
int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*cos(1/2
*d*x+1/2*c)^10+468*cos(1/2*d*x+1/2*c)^8+330*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos
(1/2*d*x+1/2*c)^5+714*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058
*cos(1/2*d*x+1/2*c)^6+474*cos(1/2*d*x+1/2*c)^4-47*cos(1/2*d*x+1/2*c)^2+3)/
a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.96

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{2\left(20\cos(dx+c)^3+237\cos(dx+c)^2+376\cos(dx+c)+165\right)\sqrt{\cos(dx+c)}\sin(dx+c)-165\left(i\sqrt{2}\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/60*(2*(20*cos(d*x + c)^3 + 237*cos(d*x + c)^2 + 376*cos(d*x + c) + 165)*
sqrt(cos(d*x + c))*sin(d*x + c) - 165*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt
(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 165*(-I*sqrt(2)*cos(d*x + c)^
3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 357*(I*sqrt(2)*co
s(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*s
qrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c))) - 357*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c
)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3
 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^3, x)
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\cos\left(c+d\,x\right)^{3/2}}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^3}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^3,x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)}{\sec(dx+c)^3+3\sec(dx+c)^2+3\sec(dx+c)+1}dx}{a^3}$$

input
```
int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x)
```

output
```
int((sqrt(cos(c + d*x))*cos(c + d*x))/(sec(c + d*x)**3 + 3*sec(c + d*x)**2
 + 3*sec(c + d*x) + 1),x)/a**3
```

3.391 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = \frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{13\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d} - \frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2} - \frac{13\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a^3+a^3\sec(c+dx))}$$

output
```
49/10*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-13/6*InverseJacobiAM(1/2
*d*x+1/2*c,2^(1/2))/a^3/d-1/5*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c
))^3-8/15*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2-13/6*sin(d*x+
c)/d/cos(d*x+c)^(1/2)/(a^3+a^3*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.32 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\csc(c+dx)\left(-((241+860\cos(c+dx)-164\cos(2(c+dx))-410\cos(3(c+dx))+115\,\mathrm{c}\right.}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Sqrt[Cos[c + d*x]]*Csc[c + d*x]*(-((241 + 860*Cos[c + d*x] - 164*Cos[2*(c
 + d*x)] - 410*Cos[3*(c + d*x)] + 115*Cos[4*(c + d*x)] + 30*Cos[5*(c + d*x
)])*Csc[c + d*x]^4) + 520*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]
*Sqrt[Sin[c + d*x]^2] + 2240*Cos[c + d*x]*Hypergeometric2F1[3/4, 7/2, 7/4,
 Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]))/(240*a^3*d)
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.49, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4752, 3042, 4304, 27, 3042, 4508, 3042, 4508, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx$$

$$\Big\downarrow \text{ 4304}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{11a-5a\sec(c+dx)}{2\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{11a-5a\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{11a-5a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{ 4508}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{41a^2-24a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{41a^2-24a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{ 4508}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int\frac{147a^3-65a^3\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{}}{5d(a\sec(c+}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int\frac{147a^3-65a^3\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{}}{5d(a\sec(c+}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int\frac{147a^3-65a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx}{5d(a\sec(}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{147a^3\int\frac{1}{\sqrt{\sec(c+dx)}}dx-65a^3\int\sqrt{\sec(c+dx)}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{147a^3\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-65a^3\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+}}{3d(a\sec(c+dx)+a)^2}}{10a^2}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{147a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec\ldots}}{d(a\sec(c+dx)+\ldots}}{3a^2}}{10a^2}\ldots\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{147a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{65a^2\sin(c\ldots}{d(a\sec\ldots}}{3a^2}}{10a^2}\ldots\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\frac{294a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{65a^2\sin(c+dx)\ldots}{d(a\sec(c+\ldots}}{3a^2}}{10a^2}\ldots\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\frac{294a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{130a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{2a^2}-\frac{65a^2\sin(c+d\ldots}{d(a\sec(c\ldots}}{3a^2}}{10a^2}\ldots\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^3,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/5*(Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^3) + ((-16*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(3*d*(a + a*Sec[c + d*x])^2) + (((294*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (130*a^3*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/(2*a^2) - (65*a^2*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3*a^2))/(10*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 4.31 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.74

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(348\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+130\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right.}{60a^{3}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}\sqrt{-2\sin(}}$

input
```
int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(348*cos(1/2*
d*x+1/2*c)^8+130*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+294*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*
c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-578*cos(1/2*d*x+1/2*c)^6+264*co
s(1/2*d*x+1/2*c)^4-37*cos(1/2*d*x+1/2*c)^2+3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.22

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = -\frac{2\left(87\cos\left(dx+c\right)^2+146\cos\left(dx+c\right)+65\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+65\left(-i\sqrt{2}\cos\left(dx+c\right)^3-3\right.}{}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/60*(2*(87*cos(d*x + c)^2 + 146*cos(d*x + c) + 65)*sqrt(cos(d*x + c))*si
n(d*x + c) + 65*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 -
3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + 65*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x
 + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c)) + 147*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqr
t(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 147*
(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d
*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)
^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sqrt{\cos\left(c+dx\right)}}{\sec^3\left(c+dx\right)+3\sec^2\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx}{a^3}$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(sqrt(cos(c + d*x))/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
 + d*x) + 1), x)/a**3
```

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^3, x)
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^3}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^3,x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sqrt{\cos(dx+c)}}{\sec(dx+c)^3+3\sec(dx+c)^2+3\sec(dx+c)+1}dx}{a^3}$$

input
```
int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x)
```

output
```
int(sqrt(cos(c + d*x))/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*
x) + 1),x)/a**3
```

3.392 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx = -\frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d} - \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} + \frac{2\sin(c+dx)}{5ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2} + \frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\left(a^3+a^3\sec(c+dx)\right)}$$

output
```
-9/10*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/2*InverseJacobiAM(1/2*
d*x+1/2*c,2^(1/2))/a^3/d-1/5*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)
)^3+2/5*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2+1/2*sin(d*x+c)/
d/cos(d*x+c)^(1/2)/(a^3+a^3*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.07 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\csc(c+dx)\left((259+120\cos(c+dx)+84\cos(2(c+dx))-280\cos(3(c+dx))+105\cos(4\right.}{}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Csc[c + d*x]*((259 + 120*Cos[c + d*x] + 84*Cos[2*(c +
d*x)] - 280*Cos[3*(c + d*x)] + 105*Cos[4*(c + d*x)])*Csc[c + d*x]^4 - 280*
Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] - 96
0*Cos[c + d*x]*Hypergeometric2F1[3/4, 7/2, 7/4, Cos[c + d*x]^2]*Sqrt[Sin[c
 + d*x]^2]))/(560*a^3*d)
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.49, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4752, 3042, 4304, 27, 3042, 4507, 25, 3042, 4508, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{ 4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)}}{(\sec(c+dx)a+a)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx$$

$$\Big\downarrow \text{ 4304}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{3\sqrt{\sec(c+dx)}(3a-a\sec(c+dx))}{2(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\int \frac{\sqrt{\sec(c+dx)}(3a-a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(3a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{ 4507}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{\int -\frac{2a^2-3a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}+\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\int \frac{2a^2-3a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\int\frac{2a^2-3a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+}{5d(a\sec(c+dx)+}\right.$$

$\big\downarrow$ 4508

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int\frac{9a^3-5a^3\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\,\mathrm{s}}{5d(a\sec(c+}\right.$$

$\big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int\frac{9a^3-5a^3\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\,\mathrm{s}}{5d(a\sec(c+}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int\frac{9a^3-5a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}-\frac{\sin(c+d}{5d(a\sec}\right.$$

$\big\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{9a^3\int\frac{1}{\sqrt{\sec(c+dx)}}dx-5a^3\int\sqrt{\sec(c+dx)}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}-\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{9a^3\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-5a^3\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+d}}{d(a\sec(c+dx)+a)}}{3a^2}\right.}{10a^2}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{9a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{}{\sqrt{c}}}{2a^2}}{3a^2}\right.}{10a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{9a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2a^2}}{3a^2}\right.}{10a^2}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\frac{18a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int -\frac{}{\sqrt{}}}{2a^2}}{3a^2}\right.}{10a^2}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\frac{18a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{10a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{Ell}}{d}}{2a^2}}{3a^2}\right.}{10a^2}\right.$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/5*(Sec[c + d*x]^(3/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^3) + (3*((4*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(3*d*(a + a*Sec[c + d*x])^2) - (((18*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (10*a^3*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/(2*a^2) - (5*a^2*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))))/(3*a^2)))/(10*a^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.74

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(36\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right.}{20a^{3}\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin}}$

input
```
int(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*
d*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)
^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-66*cos(1/2*d*x+1/2*c)^6+38*cos(1/
2*d*x+1/2*c)^4-9*cos(1/2*d*x+1/2*c)^2+1)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.22

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{2\left(9\cos\left(dx+c\right)^2+12\cos\left(dx+c\right)+5\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-5\left(i\sqrt{2}\cos\left(dx+c\right)^3+3i\sqrt{2}\cos\left(\right.\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/20*(2*(9*cos(d*x + c)^2 + 12*cos(d*x + c) + 5)*sqrt(cos(d*x + c))*sin(d*
x + c) - 5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sq
rt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) - 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^
2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) - 9*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(
d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 9*(-I*sqrt(2)
*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) -
I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3
*d*cos(d*x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\int \frac{1}{\sqrt{\cos(c+dx)}\sec^3(c+dx)+3\sqrt{\cos(c+dx)}\sec^2(c+dx)+3\sqrt{\cos(c+dx)}\sec(c+dx)+\sqrt{\cos(c+dx)}}\,dx}{a^3}$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(1/(sqrt(cos(c + d*x))*sec(c + d*x)**3 + 3*sqrt(cos(c + d*x))*sec(
c + d*x)**2 + 3*sqrt(cos(c + d*x))*sec(c + d*x) + sqrt(cos(c + d*x))), x)/
a**3
```

Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^3\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^3\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{\sqrt{\cos(c+d\,x)}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^3),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)\sec(dx+c)^3+3\cos(dx+c)\sec(dx+c)^2+3\cos(dx+c)\sec(dx+c)+\cos(dx+c)}dx}{a^3}$$

input
```
int(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)*sec(c + d*x)**3 + 3*cos(c + d*x)*sec(
c + d*x)**2 + 3*cos(c + d*x)*sec(c + d*x) + cos(c + d*x)),x)/a**3
```

3.393 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = -\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d} + \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2} + \frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a^3+a^3\sec(c+dx))}$$

output
```
-1/10*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/6*InverseJacobiAM(1/2*
d*x+1/2*c,2^(1/2))/a^3/d+1/5*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)
)^3-1/15*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2+1/6*sin(d*x+c)
/d/cos(d*x+c)^(1/2)/(a^3+a^3*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.07 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.94

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = -\frac{\sqrt{\cos(c+dx)}\csc(c+dx)\left((497-1160\cos(c+dx)+812\cos(2(c+dx))-280\cos(3(c+dx))+35\cos\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),x]
```

output
```
-1/1680*(Sqrt[Cos[c + d*x]]*Csc[c + d*x]*((497 - 1160*Cos[c + d*x] + 812*C
os[2*(c + d*x)] - 280*Cos[3*(c + d*x)] + 35*Cos[4*(c + d*x)])*Csc[c + d*x]
^4 + 280*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x
]^2] + 320*Cos[c + d*x]*Hypergeometric2F1[3/4, 7/2, 7/4, Cos[c + d*x]^2]*S
qrt[Sin[c + d*x]^2]))/(a^3*d)
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.49, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4752, 3042, 4302, 27, 3042, 4507, 3042, 4508, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(\sec(c+dx)a+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx$$

$$\Big\downarrow \text{4302}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sec(c+dx)}(3\sec(c+dx)a+a)}{2(\sec(c+dx)a+a)^2}dx}{5a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sec(c+dx)}(3\sec(c+dx)a+a)}{(\sec(c+dx)a+a)^2}dx}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(3\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{4507}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{6\sec(c+dx)a^2+a^2}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{6\csc\left(c+dx+\frac{\pi}{2}\right)a^2+a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{4508}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int -\frac{3a^3-5a^3\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(}{5d(a\sec(c+dx}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int\frac{3a^3-5a^3\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c}{5d(a\sec(c+dx)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int\frac{3a^3-5a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec}{5d(a\sec(c+}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{3a^3\int\frac{1}{\sqrt{\sec(c+dx)}}dx-5a^3\int\sqrt{\sec(c+dx)}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(}{5d(}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{3a^3\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-5a^3\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}}{2a^2}}{3a^2}}{10a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{}{\sqrt{\sin\left(c\right.}}}{2a^2}}{3a^2}}{10a^2}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\right.}}}{2a^2}}{3a^2}}{10a^2}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{10a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(\right.}{d}}{2a^2}}{3a^2}}{10a^2}\right.$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
5*d*(a + a*Sec[c + d*x])^3) + ((-2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d
*(a + a*Sec[c + d*x])^2) + (-1/2*((6*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (10*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (5*a^2*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(d*(a + a*Sec[c + d*x])))/(3*a^2))/(10*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4302
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] - Simp[d/(a*b*(2*m + 1))   Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*(a*(n - 1) - b*(m + n)*C
sc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && L
tQ[m, -1] && LtQ[1, n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 3.51 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.74

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right.}{60a^{3}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}\sqrt{-2\sin\left(\right.}}$

input
```
int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*
d*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^
5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*cos(1/2*d*x+1/2*c)^6-24*cos(1/2*
d*x+1/2*c)^4+17*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*
d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.22

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \frac{2\left(3\cos(dx+c)^2+14\cos(dx+c)+5\right)\sqrt{\cos(dx+c)}\sin(dx+c)-5\left(i\sqrt{2}\cos(dx+c)^3+3i\sqrt{2}\cos(\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/60*(2*(3*cos(d*x + c)^2 + 14*cos(d*x + c) + 5)*sqrt(cos(d*x + c))*sin(d*
x + c) - 5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sq
rt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) - 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^
2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) - 3*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(
d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*(-I*sqrt(2)
*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) -
I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3
*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{\cos(c+d\,x)^{3/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^3),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^3), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2\sec(dx+c)^3+3\cos(dx+c)^2\sec(dx+c)^2+3\cos(dx+c)^2\sec(dx+c)+\cos(dx+c)^2}dx}{a^3}$$

input
```
int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**2*sec(c + d*x)**3 + 3*cos(c + d*x)**
2*sec(c + d*x)**2 + 3*cos(c + d*x)**2*sec(c + d*x) + cos(c + d*x)**2),x)/a
**3
```

3.394 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d} - \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{4\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2} + \frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\left(a^3+a^3\sec(c+dx)\right)}$$

output
```
1/10*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/6*InverseJacobiAM(1/2*d
*x+1/2*c,2^(1/2))/a^3/d-1/5*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^3-4/15*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2+1/6*sin(d*x+c)/
d/cos(d*x+c)^(1/2)/(a^3+a^3*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.29 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.90

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = -\frac{\sqrt{\cos(c+dx)}\csc(c+dx)\left(\frac{1}{8}(-847+1440\cos(c+dx)-532\cos(2(c+dx))+35\cos(4(c+dx)))\csc^4\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),x]
```

output
```
-1/210*(Sqrt[Cos[c + d*x]]*Csc[c + d*x]*(((-847 + 1440*Cos[c + d*x] - 532*
Cos[2*(c + d*x)] + 35*Cos[4*(c + d*x)])*Csc[c + d*x]^4)/8 + 35*Hypergeomet
ric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] - 40*Cos[c + d*
x]*Hypergeometric2F1[3/4, 7/2, 7/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]))
/(a^3*d)
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.49, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4752, 3042, 4303, 27, 3042, 4507, 25, 3042, 4508, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(\sec(c+dx)a+a)^3}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx$$

$\Big\downarrow$ 4303

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\sec(c+dx)}(a-7a\sec(c+dx))}{2(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\sec(c+dx)}(a-7a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a-7a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$\Big\downarrow$ 4507

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int -\frac{9\sec(c+dx)a^2+4a^2}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}+\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$\Big\downarrow$ 25

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\int \frac{9\sec(c+dx)a^2+4a^2}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\int\frac{9\csc\left(c+dx+\frac{\pi}{2}\right)a^2+4a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx}{5d(a\sec(c+dx)+a)^3}\right.$$

$\downarrow$ 4508

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int\frac{5\sec(c+dx)a^3+3a^3}{2\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}}{5d(a\sec(c+dx}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int\frac{5\sec(c+dx)a^3+3a^3}{\sqrt{\sec(c+dx)}}dx}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}}{5d(a\sec(c+dx}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int\frac{5\csc\left(c+dx+\frac{\pi}{2}\right)a^3+3a^3}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{\sin(c+dx)\mathrm{s}}{5d(a\sec(c+}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{3a^3\int\frac{1}{\sqrt{\sec(c+dx)}}dx+5a^3\int\sqrt{\sec(c+dx)}dx}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{\sin}{5c}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{3a^3\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5a^3\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}\cdots}{3a^2}}{10a^2}\cdots\right.$$

$\Big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\mathrm{co}\cdots}}{2a^2}\cdots}{3a^2}}{10a^2}\cdots\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\cdots}{2a^2}\cdots}{3a^2}}{10a^2}\cdots\right.$$

$\Big\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\cdots}{d}}{2a^2}\cdots}{3a^2}}{10a^2}\cdots\right.$$

$\Big\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{\frac{10a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}\cdots}{2a^2}}{3a^2}}{10a^2}\cdots\right.$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/5*(Sec[c + d*x]^(3/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^3) - ((8*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3
*d*(a + a*Sec[c + d*x])^2) - (((6*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (10*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/(2*a^2) + (5*a^2*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3*a^2))/(10*a^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.74

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\cos\right.}{60a^{3}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}\sqrt{-2\sin\left(\frac{dx}{2}+\right.}}$

input
```
int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d
*x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-22*cos(1/2*d*x+1/2*c)^6+6*cos(1/2*d
*x+1/2*c)^4+7*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.22

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = -\frac{2\left(3\cos(dx+c)^2+4\cos(dx+c)-5\right)\sqrt{\cos(dx+c)}\sin(dx+c)+5\left(i\sqrt{2}\cos(dx+c)^3+3i\sqrt{2}\cos\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/60*(2*(3*cos(d*x + c)^2 + 4*cos(d*x + c) - 5)*sqrt(cos(d*x + c))*sin(d*
x + c) + 5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sq
rt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^
2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) + 3*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos
(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(I*sqrt(2)
*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) +
I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3
*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{\cos(c+d\,x)^{5/2}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^3),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^3), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3\sec(dx+c)^3+3\cos(dx+c)^3\sec(dx+c)^2+3\cos(dx+c)^3\sec(dx+c)+\cos(dx+c)^3}dx}{a^3}$$

input
```
int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**3*sec(c + d*x)**3 + 3*cos(c + d*x)**
3*sec(c + d*x)**2 + 3*cos(c + d*x)**3*sec(c + d*x) + cos(c + d*x)**3),x)/a
**3
```

3.395 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d} - \frac{\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{5ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} - \frac{9\sin(c+dx)}{10d\sqrt{\cos(c+dx)}\,(a^3+a^3\sec(c+dx))}$$

output
```
9/10*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/2*InverseJacobiAM(1/2*d
*x+1/2*c,2^(1/2))/a^3/d-1/5*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^3-2/5*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2-9/10*sin(d*x+c)/
d/cos(d*x+c)^(1/2)/(a^3+a^3*sec(d*x+c))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.35 (sec) , antiderivative size = 603, normalized size of antiderivative = 3.89

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx =$$

$$-\frac{2\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)\csc\left(\frac{c}{2}\right)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\arctan(\cot(c)))\right)\sec\left(\frac{c}{2}\right)\sec^3(c+dx)\sec(dx-\arctan(\text{co}}{d\sqrt{1+\text{co}}}$$

$$+\frac{\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-\frac{36\csc(c)}{5d}-\frac{36\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{dx}{2}\right)}{5d}-\frac{8\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{dx}{2}\right)}{5d}-\frac{2\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{dx}{2}\right)}{5d}\right.}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}$$

$$-\frac{9\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec^3(c+dx)\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sin(dx+\text{arcta}}{\sqrt{1-\cos(dx+\arctan(\tan(c)))}\sqrt{1+\cos(dx+\arctan(\tan(c)))}\sqrt{\cos(c)\cos(dx+\text{ar}}}\right.}{5d(a+a\sec(c+dx}$$

input
```
Integrate[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3),x]
```

output
```
(-2*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin
[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]^3*Sec[d*x - ArcTan[Cot[c]]
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin
[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 +
Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*((-36*Csc[c])/(5
*d) - (36*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/(5*d) - (8*Sec[c/2]*Se
c[c/2 + (d*x)/2]^3*Sin[(d*x)/2])/(5*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*
Sin[(d*x)/2])/(5*d) - (8*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(5*d) - (2*Sec[c/2
 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3
) - (9*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^3*((Hypergeomet
ricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[
Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + A
rcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*
Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^
2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 +
 Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5
*d*(a + a*Sec[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.49, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4752, 3042, 4303, 27, 3042, 4507, 3042, 4507, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(\sec(c+dx)a+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx$$

$$\Big\downarrow \text{4303}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{3\sec^{\frac{3}{2}}(c+dx)(a-3a\sec(c+dx))}{2(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\int \frac{\sec^{\frac{3}{2}}(c+dx)(a-3a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{4507}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\int\frac{\sqrt{\sec(c+dx)}\left(2a^2-7a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+}{5d(a\sec(c+dx)+}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(2a^2-7a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec}{5d(a\sec(c+}\right.$$

$$\Big\downarrow \text{4507}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\frac{\int-\frac{5\sec(c+dx)a^3+9a^3}{2\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+d}{5d(a\sec}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int\frac{5\sec(c+dx)a^3+9a^3}{\sqrt{\sec(c+dx)}}dx}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)}{5d(a\sec(c$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int\frac{5\csc\left(c+dx+\frac{\pi}{2}\right)a^3+9a^3}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+}{5d(a\sec$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{9a^3\int\frac{1}{\sqrt{\sec(c+dx)}}dx+5a^3\int\sqrt{\sec(c+dx)}dx}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{9a^3\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5a^3\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+}{3d(a\sec(c+dx)+a}\right.}{10a^2}$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+9a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos}}{2a^2}}{3a^2}}{10a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+9a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int}{2a^2}}{3a^2}}{10a^2}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{18a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}}{2a^2}}{3a^2}}{10a^2}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{10a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{18a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{d}}{2a^2}}{3a^2}}{10a^2}\right.$$

input
```
Int[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/5*(Sec[c + d*x]^(5/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^3) - (3*((4*a*Sec[c + d*x]^(3/2)*Sin[c + d*x])
/(3*d*(a + a*Sec[c + d*x])^2) + (-1/2*((18*a^3*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (10*a^3*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (9*a^2*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3*a^2)))/(10*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.73

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(36\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\operatorname{co}\right.}{20a^{3}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}\sqrt{-2\sin\left(\frac{dx}{2}+\right.}}$

input
```
int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*d
*x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^
5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-46*cos(1/2*d*x+1/2*c)^6+8*cos(1/2*
d*x+1/2*c)^4+cos(1/2*d*x+1/2*c)^2+1)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.22

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = -\frac{2\left(9\cos\left(dx+c\right)^2+22\cos\left(dx+c\right)+15\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+5\left(i\sqrt{2}\cos\left(dx+c\right)^3+3i\sqrt{2}\,c\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/20*(2*(9*cos(d*x + c)^2 + 22*cos(d*x + c) + 15)*sqrt(cos(d*x + c))*sin(
d*x + c) + 5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*
sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c
)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) + 9*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*c
os(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*(I*sqrt(
2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c)
+ I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a
^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^3),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^3), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^4\sec(dx+c)^3+3\cos(dx+c)^4\sec(dx+c)^2+3\cos(dx+c)^4\sec(dx+c)+\cos(dx+c)^4}dx}{a^3}$$

input
```
int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**4*sec(c + d*x)**3 + 3*cos(c + d*x)**
4*sec(c + d*x)**2 + 3*cos(c + d*x)**4*sec(c + d*x) + cos(c + d*x)**4),x)/a
**3
```

3.396 $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 181

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = -\frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{13\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d} + \frac{49\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{5d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2} - \frac{13\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)\left(a^3+a^3\sec(c+dx)\right)}$$

output
```
-49/10*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-13/6*InverseJacobiAM(1/
2*d*x+1/2*c,2^(1/2))/a^3/d+49/10*sin(d*x+c)/a^3/d/cos(d*x+c)^(1/2)-1/5*sin
(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3-8/15*sin(d*x+c)/a/d/cos(d*x+
c)^(5/2)/(a+a*sec(d*x+c))^2-13/6*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a^3+a^3*se
c(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.71 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.06

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{(1284\cos(\frac{1}{2}(c-dx))+921\cos(\frac{1}{2}(3c+dx))+1243\cos(\frac{1}{2}(c+3dx))+374\cos(\frac{1}{2}(5c+3dx))+670\cos(\frac{1}{2}(3c+5dx))+65\cos(}{16d\cos^{\frac{7}{2}}(c+dx)}\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^3),x]
```

output
```
(Cos[(c + d*x)/2]^6*(((1284*Cos[(c - d*x)/2] + 921*Cos[(3*c + d*x)/2] + 12
43*Cos[(c + 3*d*x)/2] + 374*Cos[(5*c + 3*d*x)/2] + 670*Cos[(3*c + 5*d*x)/2
] + 65*Cos[(7*c + 5*d*x)/2] + 147*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*
Sec[(c + d*x)/2]^5)/(16*d*Cos[c + d*x]^(7/2)) - ((4*I)*Sqrt[2]*(147*(1 + E
^((2*I)*(c + d*x))) + 147*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]
*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 65*E^(I*(c + d*
x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4
, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x]^3)/(d*E^(I*(c + d*x))*(-1
+ E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))])))/(15*a^3*
(1 + Sec[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.41, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.826$, Rules used = {3042, 4752, 3042, 4303, 27, 3042, 4507, 3042, 4507, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a\sec(c+dx)+a)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(\sec(c+dx)a+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx$$

$$\Big\downarrow \text{4303}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)(5a-11a\sec(c+dx))}{2(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)(5a-11a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(5a-11a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{4507}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(24a^2-41a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(24a^2-41a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}}{5d(a\sec(c+dx}\right.$$

$$\Big\downarrow \text{ 4507}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{\int \frac{1}{2}\sqrt{\sec(c+dx)}\left(65a^3-147a^3\sec(c+dx)\right)dx}{a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\right.$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{\int \sqrt{\sec(c+dx)}\left(65a^3-147a^3\sec(c+dx)\right)dx}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(65a^3-147a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}\right.$$

$$\Big\downarrow \text{ 4274}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{65a^3\int \sqrt{\sec(c+dx)}dx-147a^3\int \sec^{\frac{3}{2}}(c+dx)dx}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{65a^3\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-147a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(}{3d(a\sec(c+dx)+}}{10a^2}\right.$$

$\Big\downarrow$ 4255

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{65a^3\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+}{10a^2}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{65a^3\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}\right.$$

$\Big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c}\right.}{2a^2}}{3a^2}}{10a^2}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{}\right.}{2a^2}}{3a^2}}{10a^2}\right.$$

$\Big\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E(}{d}\right.}{2a^2}}{3a^2}}{10a^2}\right.$$

$$\Big\downarrow \text{3120}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{\frac{130a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+d}{d}\right.}{2a^2}}{3a^2}}{10a^2}\right.$$

input
```
Int[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^3),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/5*(Sec[c + d*x]^(7/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^3) - ((16*a*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
3*d*(a + a*Sec[c + d*x])^2) + ((65*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d
*(a + a*Sec[c + d*x])) + ((130*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/d - 147*a^3*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
d))/(2*a^2))/(3*a^2))/(10*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(164) = 328$.

Time = 3.50 (sec) , antiderivative size = 555, normalized size of antiderivative = 3.07

method	result
default	$-\frac{-2\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(65\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-147\,\mathrm{EllipticE}\left(\cos\right.\right.}{}$

input
```
int(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/60*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^4+4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(65
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(65*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)+588*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8-1634*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+1488*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-439*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(
1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 394, normalized size of antiderivative = 2.18

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \frac{2\left(147\cos\left(dx+c\right)^3+376\cos\left(dx+c\right)^2+295\cos\left(dx+c\right)+60\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-65\left(-i\sqrt{\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/60*(2*(147*cos(d*x + c)^3 + 376*cos(d*x + c)^2 + 295*cos(d*x + c) + 60)*
sqrt(cos(d*x + c))*sin(d*x + c) - 65*(-I*sqrt(2)*cos(d*x + c)^4 - 3*I*sqrt
(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(2)*cos(d*x + c))*
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 65*(I*sqrt(2)*
cos(d*x + c)^4 + 3*I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 +
 I*sqrt(2)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c)) - 147*(I*sqrt(2)*cos(d*x + c)^4 + 3*I*sqrt(2)*cos(d*x + c)^3 + 3*
I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 147*(-I*sqrt(
2)*cos(d*x + c)^4 - 3*I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^
2 - I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d
*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(9/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{\cos(c+d\,x)^{9/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int(1/(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^3),x)
```

output
```
int(1/(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^3), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^5\sec(dx+c)^3+3\cos(dx+c)^5\sec(dx+c)^2+3\cos(dx+c)^5\sec(dx+c)+\cos(dx+c)^5}dx}{a^3}$$

input
```
int(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**5*sec(c + d*x)**3 + 3*cos(c + d*x)**
5*sec(c + d*x)**2 + 3*cos(c + d*x)**5*sec(c + d*x) + cos(c + d*x)**5),x)/a
**3
```

3.397 $\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 207

$$\begin{aligned}\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx &= \frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} \\ &+ \frac{11\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d} \\ &+ \frac{11\sin(c+dx)}{2a^3d\cos^{\frac{3}{2}}(c+dx)} - \frac{119\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} \\ &- \frac{\sin(c+dx)}{5d\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3} \\ &- \frac{2\sin(c+dx)}{3ad\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2} \\ &- \frac{119\sin(c+dx)}{30d\cos^{\frac{5}{2}}(c+dx)\left(a^3+a^3\sec(c+dx)\right)}\end{aligned}$$

output
```
119/10*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+11/2*InverseJacobiAM(1/
2*d*x+1/2*c,2^(1/2))/a^3/d+11/2*sin(d*x+c)/a^3/d/cos(d*x+c)^(3/2)-119/10*s
in(d*x+c)/a^3/d/cos(d*x+c)^(1/2)-1/5*sin(d*x+c)/d/cos(d*x+c)^(9/2)/(a+a*se
c(d*x+c))^3-2/3*sin(d*x+c)/a/d/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2-119/30*
sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a^3+a^3*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.11 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.94

$$\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{(5134\cos\left(\frac{1}{2}(c-dx)\right)+4148\cos\left(\frac{1}{2}(3c+dx)\right)+4664\cos\left(\frac{1}{2}(c+3dx)\right)+2476\cos\left(\frac{1}{2}(5c+3dx)\right)+3340\cos\left(\frac{1}{2}(3c+5dx)\right)+9}{96d\,\mathrm{co}}\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^3),x]
```

output
```
(Cos[(c + d*x)/2]^6*(-1/96*((5134*Cos[(c - d*x)/2] + 4148*Cos[(3*c + d*x)/
2] + 4664*Cos[(c + 3*d*x)/2] + 2476*Cos[(5*c + 3*d*x)/2] + 3340*Cos[(3*c +
 5*d*x)/2] + 944*Cos[(7*c + 5*d*x)/2] + 1620*Cos[(5*c + 7*d*x)/2] + 165*Co
s[(9*c + 7*d*x)/2] + 357*Cos[(7*c + 9*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c +
d*x)/2]^5)/(d*Cos[c + d*x]^(9/2)) + ((4*I)*Sqrt[2]*(119*(1 + E^((2*I)*(c +
 d*x))) + 119*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 55*E^(I*(c + d*x))*(-1 + E^
((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4,
-E^((2*I)*(c + d*x))])*Sec[c + d*x]^3)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c
))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))])))/(5*a^3*(1 + Sec[c +
d*x])^3)
```

Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.37, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.826$, Rules used = {3042, 4752, 3042, 4303, 27, 3042, 4507, 3042, 4507, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a\sec(c+dx)+a)^3}dx$$

$\Big\downarrow$ 3042

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{11/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$\Big\downarrow$ 4752

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(\sec(c+dx)a+a)^3}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{11/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx$$

$\Big\downarrow$ 4303

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sec^{\frac{7}{2}}(c+dx)(7a-13a\sec(c+dx))}{2(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sec^{\frac{7}{2}}(c+dx)(7a-13a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(7a-13a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$\Big\downarrow$ 4507

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(50a^2-69a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}+\frac{20a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(50a^2-69a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{20a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}}{5d(a\sec(c+dx}\right.$$

$\downarrow$ 4507

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{\int \frac{3}{2}\sec^{\frac{3}{2}}(c+dx)\left(119a^3-165a^3\sec(c+dx)\right)dx}{a^2}+\frac{119a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{20a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{3\int \sec^{\frac{3}{2}}(c+dx)\left(119a^3-165a^3\sec(c+dx)\right)dx}{2a^2}+\frac{119a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{20a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{3\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(119a^3-165a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}+\frac{119a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{20a\sin(c+dx)\sec^{\frac{7}{2}}(c}{3d(a\sec(c+dx)+c}}{10a^2}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{3\left(119a^3\int \sec^{\frac{3}{2}}(c+dx)dx-165a^3\int \sec^{\frac{5}{2}}(c+dx)dx\right)}{2a^2}+\frac{119a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{20a\sin(c+dx)\sec^{\frac{7}{2}}(c+d}{3d(a\sec(c+dx)+a)^2}}{10a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{3\left(119a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx-165a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx\right)}{2a^2}+\frac{119a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{20a\sin(c+dx}{3d(a\sec(c}}{10a^2}\right.$$

$\downarrow$ 4255

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{3\left(119a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)-165a^3\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)\right)}{2a^2}+}{3a^2}}{10a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{3\left(119a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-165a^3\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+}{3d}\right.\right.}{2a^2}}{3a^2}}{10a^2}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{3\left(119a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)-165a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.\right.}{2a^2}}{3a^2}}{10a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\left(119a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)-165a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.\right.}{2a^2}}{3a^2}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\left(119a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)-165a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.\right.}{2a^2}}{3a^2}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{119a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{3\left(119a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)-16\right.}{2}}{3a^2}\right.$$

input
```
Int[1/(Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^3),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/5*(Sec[c + d*x]^(9/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^3) - ((20*a*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(
3*d*(a + a*Sec[c + d*x])^2) + ((119*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
d*(a + a*Sec[c + d*x])) + (3*(119*a^3*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
d) - 165*a^3*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d))))/(2*a^2))/(3*a
^2))/(10*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(186) = 372$.

Time = 4.09 (sec) , antiderivative size = 453, normalized size of antiderivative = 2.19

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\frac{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^5}+\frac{32\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{15\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{118\sqrt{-2\sin\left(\frac{dx}{2}\right.}}{5\cos}\right.}{}$

input
```
int(1/cos(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^3*(1/5*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5+32
/15*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)^3+118/5*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)-128/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+238/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-4/
3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(cos(1/2*d*x+1/2*c)^2-1/2)^2+48*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-
(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.00

$$\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = -\frac{2\left(357\cos(dx+c)^4+906\cos(dx+c)^3+695\cos(dx+c)^2+120\cos(dx+c)-20\right)\sqrt{\cos(dx+c)}\,\mathrm{s}}{}$$

input
```
integrate(1/cos(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/60*(2*(357*cos(d*x + c)^4 + 906*cos(d*x + c)^3 + 695*cos(d*x + c)^2 + 1
20*cos(d*x + c) - 20)*sqrt(cos(d*x + c))*sin(d*x + c) + 165*(I*sqrt(2)*cos
(d*x + c)^5 + 3*I*sqrt(2)*cos(d*x + c)^4 + 3*I*sqrt(2)*cos(d*x + c)^3 + I*
sqrt(2)*cos(d*x + c)^2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) + 165*(-I*sqrt(2)*cos(d*x + c)^5 - 3*I*sqrt(2)*cos(d*x + c)^4 - 3*
I*sqrt(2)*cos(d*x + c)^3 - I*sqrt(2)*cos(d*x + c)^2)*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)) + 357*(-I*sqrt(2)*cos(d*x + c)^5 - 3*
I*sqrt(2)*cos(d*x + c)^4 - 3*I*sqrt(2)*cos(d*x + c)^3 - I*sqrt(2)*cos(d*x
+ c)^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c))) + 357*(I*sqrt(2)*cos(d*x + c)^5 + 3*I*sqrt(2)*cos(d*x + c)
^4 + 3*I*sqrt(2)*cos(d*x + c)^3 + I*sqrt(2)*cos(d*x + c)^2)*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^3*
d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3*d
*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(11/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(11/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{\cos(c+d\,x)^{11/2}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int(1/(cos(c + d*x)^(11/2)*(a + a/cos(c + d*x))^3),x)
```

output
```
int(1/(cos(c + d*x)^(11/2)*(a + a/cos(c + d*x))^3), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^6\sec(dx+c)^3+3\cos(dx+c)^6\sec(dx+c)^2+3\cos(dx+c)^6\sec(dx+c)+\cos(dx+c)^6}dx}{a^3}$$

input
```
int(1/cos(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**6*sec(c + d*x)**3 + 3*cos(c + d*x)**
6*sec(c + d*x)**2 + 3*cos(c + d*x)**6*sec(c + d*x) + cos(c + d*x)**6),x)/a
**3
```

3.398 $\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 153

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{32a\sin(c+dx)}{35d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{16a\sqrt{\cos(c+dx)}\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{12a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2a\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}}$$

output
```
32/35*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+16/35*a*cos(d
*x+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+12/35*a*cos(d*x+c)^(3/2)*s
in(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/7*a*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+
a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.52

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$=\frac{\sqrt{\cos(c+dx)}\sqrt{a(1+\sec(c+dx))}(140\sin(c+dx)+42\sin(2(c+dx))+12\sin(3(c+dx))+5\sin(4(c-}{140d(1+\cos(c+dx))}$$

input
```
Integrate[Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]*(140*Sin[c + d*x] + 42*Sin[
2*(c + d*x)] + 12*Sin[3*(c + d*x)] + 5*Sin[4*(c + d*x)]))/(140*d*(1 + Cos[
c + d*x]))
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4752, 3042, 4292, 3042, 4292, 3042, 4292, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{ 4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \textbf{4292}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{6}{7}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{6}{7}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \textbf{4292}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{6}{7}\left(\frac{4}{5}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{}{7d\sec^{\frac{5}{2}}(c}\right.$$

$$\Big\downarrow \textbf{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{6}{7}\left(\frac{4}{5}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{}{7d}\right.$$

$$\Big\downarrow \textbf{4292}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{}{5d\,\mathrm{se}}\right.\right.$$

$$\Big\downarrow \textbf{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\right.\right.$$

$$\Big\downarrow \textbf{4291}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{6}{7}\left(\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{4}{5}\right.\right.$$

input
```
Int[Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*Sin[c + d*x])/(7*d*Sec[c + d*x
]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (6*((2*a*Sin[c + d*x])/(5*d*Sec[c + d*
x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (4*((2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[
c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (4*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(3*d*Sqrt[a + a*Sec[c + d*x]])))/5))/7)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.47

method	result	size
default	$\frac{2\sin(dx+c)\left(5\cos(dx+c)^3+6\cos(dx+c)^2+8\cos(dx+c)+16\right)\sqrt{\cos(dx+c)}\,\sqrt{a(1+\sec(dx+c))}}{35d(\cos(dx+c)+1)}$	72

input
```
int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/35/d*sin(d*x+c)*(5*cos(d*x+c)^3+6*cos(d*x+c)^2+8*cos(d*x+c)+16)*cos(d*x+
c)^(1/2)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.52

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\left(5\cos\left(dx+c\right)^{3}+6\cos\left(dx+c\right)^{2}+8\cos\left(dx+c\right)+16\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{35\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/35*(5*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 8*cos(d*x + c) + 16)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x
+ c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(129) = 258$.

Time = 0.19 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.92

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \frac{\sqrt{2}\left(105\cos\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+35\cos\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}dx\right.\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/280*sqrt(2)*(105*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2
*c)))*sin(7/2*d*x + 7/2*c) + 35*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(
7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 7*cos(2/7*arctan2(sin(7/2*d*x +
7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 105*cos(7/2*d*x + 7/
2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 35*cos
(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*
c))) - 7*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/
2*d*x + 7/2*c))) + 10*sin(7/2*d*x + 7/2*c) + 7*sin(5/7*arctan2(sin(7/2*d*x
 + 7/2*c), cos(7/2*d*x + 7/2*c))) + 35*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c))) + 105*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(
7/2*d*x + 7/2*c))))*sqrt(a)/d
```

Giac [F]

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \cos(c+d\,x)^{7/2}\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

input
```
int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$=\sqrt{a}\left(\int \sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)$$

input
```
int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**3,x)
```

3.399 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{16a\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{8a\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}}$$

output
```
16/15*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+8/15*a*cos(d*
x+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/5*a*cos(d*x+c)^(3/2)*sin(
d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.53

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{\sqrt{\cos(c+dx)}(19+8\cos(c+dx)+3\cos(2(c+dx)))\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{15d}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*(19 + 8*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sqrt[a*(1 +
 Sec[c + d*x])]*Tan[(c + d*x)/2])/(15*d)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 4752, 3042, 4292, 3042, 4292, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4292}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{5}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{5}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)$$

$\downarrow$ 4292

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{}{5d\sec^{\frac{3}{2}}(}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{}{5d}\right.$$

$\downarrow$ 4291

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{4}{5}\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{}{3d\sqrt{\sec(c}}\right.\right.$$

input
```
Int[Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*Sin[c + d*x])/(5*d*Sec[c + d*x
]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (4*((2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c
 + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (4*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(3*d*Sqrt[a + a*Sec[c + d*x]])))/5)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{2 \sin(dx+c)\left(3 \cos(dx+c)^{2}+4 \cos(dx+c)+8\right) \sqrt{\cos(dx+c)}\, \sqrt{a(1+\sec(dx+c))}}{15 d(\cos(dx+c)+1)}$	62

input
```
int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d*sin(d*x+c)*(3*cos(d*x+c)^2+4*cos(d*x+c)+8)*cos(d*x+c)^(1/2)*(a*(1+s
ec(d*x+c)))^(1/2)/(cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.60

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\left(3\cos\left(dx+c\right)^{2}+4\cos\left(dx+c\right)+8\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*cos(d*x + c)^2 + 4*cos(d*x + c) + 8)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(97) = 194.

Time = 0.19 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.77

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{\sqrt{2}\left(30\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\right.\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/60*sqrt(2)*(30*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))*sin(5/2*d*x + 5/2*c) + 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 30*cos(5/2*d*x + 5/2*c)*sin(4/5*arc
tan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*cos(5/2*d*x + 5/2*c)
*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 6*sin(5/2*
d*x + 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
))) + 30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sqr
t(a)/d
```

Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \cos\left(c+d\,x\right)^{5/2}\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \sqrt{a}\left(\int \sqrt{\sec\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^{2}dx\right)$$

input
```
int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**2,x)
```

3.400 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}$$

output
```
4/3*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2/3*a*cos(d*x+c
)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\sqrt{\cos(c+dx)}(2+\cos(c+dx))\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{3d}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*(2 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c
 + d*x)/2])/(3*d)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4752, 3042, 4292, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4292}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{3}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{4291}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)$$

input
```
Int[Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c
+ d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (4*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(3*d*Sqrt[a + a*Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{2\sqrt{\cos(dx+c)}\,(\cos(dx+c)+2)\sin(dx+c)\sqrt{a(1+\sec(dx+c))}}{d(3\cos(dx+c)+3)}$	52

input
```
int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*cos(d*x+c)^(1/2)*(cos(d*x+c)+2)*sin(d*x+c)/(3*cos(d*x+c)+3)*(a*(1+sec(
d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)+2)\sqrt{\cos(dx+c)}\sin(dx+c)}{3\,(d\cos(dx+c)+d)}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) + 2)*sqrt(cos(d*
x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.47

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \frac{\sqrt{2}\left(3\,\cos\left(\frac{2}{3}\,\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right)\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-3\,\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\sin\left(\frac{2}{3}\,\text{arct}\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(2)*(3*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)*sin(3/2*d*x + 3/2*c) - 3*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(3/2*d*x + 3/2*c) + 3*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)/d
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx=\int \sqrt{a\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx=\int \cos\left(c+d\,x\right)^{3/2}\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$=\sqrt{a}\left(\int\sqrt{\sec\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)dx\right)$$

input
```
int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x),x)
```

3.401 $\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 36

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

output
```
2*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\sqrt{\cos(c+dx)}\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/d
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 4752, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4291}$$

$$\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

method	result	size
default	$-\frac{2\sqrt{\cos(dx+c)}\,\sqrt{a(1+\sec(dx+c))}\,(\cot(dx+c)-\csc(dx+c))}{d}$	41
risch	$-\frac{2i\sqrt{\frac{a\left(\mathrm{e}^{i(dx+c)}+1\right)^{2}}{\mathrm{e}^{2i(dx+c)}+1}}\,\sqrt{\cos(dx+c)}\,\left(\mathrm{e}^{i(dx+c)}-1\right)}{\left(\mathrm{e}^{i(dx+c)}+1\right)d}$	69

input
```
int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*cos(d*x+c)^(1/2)*(a*(1+sec(d*x+c)))^(1/2)*(cot(d*x+c)-csc(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.36

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{d\cos(dx+c)+d}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/
(d*cos(d*x + c) + d)
```

Sympy [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a(\sec(c+dx)+1)}\sqrt{\cos(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*sqrt(cos(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\sqrt{2}\sqrt{a}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{d}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(2)*sqrt(a)*sin(1/2*d*x + 1/2*c)/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\sqrt{2}\sqrt{a}\mathrm{sgn}(\cos(dx+c))\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(2)*sqrt(a)*sgn(cos(d*x + c))*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x
+ 1/2*c)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{\cos\left(c+d\,x\right)}\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = \sqrt{a}\left(\int \sqrt{\sec\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}dx\right)$$

input
```
int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x)),x)
```

3.402 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 57

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \frac{2\sqrt{a}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}$$

output
```
2*a^(1/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1
/2)*sec(d*x+c)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = -\frac{2\arcsin\left(\sqrt{\sec(c+dx)}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

output
```
(-2*ArcSin[Sqrt[Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[
a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sqrt[1 - Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4752, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{4288}$$

$$-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{222}$$

$$\frac{2\sqrt{a}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

output
```
(2*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(47) = 94$.

Time = 2.78 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.05

method	result	size
default	$-\frac{\left(\arctan\left(\frac{-\cot(dx+c)+\csc(dx+c)+1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+\arctan\left(\frac{-\cot(dx+c)+\csc(dx+c)-1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\right)\sqrt{a(1+\sec(dx+c))}\,\sqrt{\cos(dx+c)}}{d(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$	117

input
```
int((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(arctan(1/2/(-1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1))+arc
tan(1/2*(-cot(d*x+c)+csc(d*x+c)-1)/(-1/(cos(d*x+c)+1))^(1/2)))*(a*(1+sec(d
*x+c)))^(1/2)*cos(d*x+c)^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.16

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \left[\frac{\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(dx+c)}\sin(dx+c)-7a\cos(dx+c)^2+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{2d}, \frac{\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\ldots}}{\ldots}\right)}{\ldots}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(
d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))/d, sqrt(-a)*arctan(2*
sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a))/d]
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\sqrt{a(\sec(c+dx)+1)}}{\sqrt{\cos(c+dx)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))/sqrt(cos(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(47) = 94$.

Time = 0.20 (sec) , antiderivative size = 241, normalized size of antiderivative = 4.23

$$\int \frac{\sqrt{a + a\sec(c + dx)}}{\sqrt{\cos(c + dx)}}\, dx = \frac{\sqrt{a}\left(\log\left(2\cos\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 2\sin\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 2\sqrt{2}\cos\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 2\sqrt{2}\sin\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 2\right)\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(a)*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(47) = 94$.

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.11

$$\int \frac{\sqrt{a + a\sec(c + dx)}}{\sqrt{\cos(c + dx)}}\, dx = \frac{a^{\frac{3}{2}}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\operatorname{sgn}(\cos(dx+c))}{d|a|}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
a^(3/2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c
) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))*sgn(c
os(d*x + c))/(d*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx=\sqrt{a}\left(\int \frac{\sqrt{\sec\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)}dx\right)$$

input
```
int((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)
```

output
```
sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x),x)
```

3.403 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 92

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{a}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{a\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

output
```
a^(1/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/d+a*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2
)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{a\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\left(\arcsin\left(\sqrt{1-\sec(c+dx)}\right)+\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\right)\sin(c}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

output
```
(a*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(ArcSin[Sqrt[1 - Sec[c + d*x]]] +
 Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Sin[c + d*x])/(d*Sqrt[1 - Sec[
c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 4752, 3042, 4290, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a \sec(c+dx)+a}}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a \csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{4290}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{4288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)$$

$$\Big\downarrow \text{222}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d
*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d
*Sqrt[a + a*Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.71

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left(\sin(dx+c)\sqrt{2}\sqrt{-\frac{2}{\cos(dx+c)+1}}-\cos(dx+c)\arctan\left(\frac{-\cot(dx+c)+\csc(dx+c)+1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-\cos(dx+c)\arctan\left(\frac{-\cot(dx+}{2\sqrt{-\frac{}{c}}}\right.\right.}{2d\sqrt{-\frac{1}{\cos(dx+c)+1}}\sqrt{\cos(dx+c)}\,(\cos(dx+c)+1)}$

input
```
int((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(a*(1+sec(d*x+c)))^(1/2)*(sin(d*x+c)*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/
2)-cos(d*x+c)*arctan(1/2/(-1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)
+1))-cos(d*x+c)*arctan(1/2*(-cot(d*x+c)+csc(d*x+c)-1)/(-1/(cos(d*x+c)+1))^
(1/2)))/(-1/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^(1/2)/(cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 325, normalized size of antiderivative = 3.53

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left[\frac{\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(dx+c)}\sin(dx+c)-7a\cos(dx-}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{4\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sq
rt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(
d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d
*x + c)^2)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/2*((cos(d*x + c)^2 +
 cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c)
 - 2*a)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{a\left(\sec(c+dx)+1\right)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))/cos(c + d*x)**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(78) = 156$.

Time = 0.23 (sec) , antiderivative size = 662, normalized size of antiderivative = 7.20

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2
*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2
*c) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*l
og(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2)
+ (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2
*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
 + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos
(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2
*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
- 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)
*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c)))
+ 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(78) = 156$.

Time = 0.25 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.78

$$\int \frac{\sqrt{a + a\sec(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)}\,dx = \frac{\sqrt{2}\left(\frac{\sqrt{2}a^{\frac{3}{2}}\log\left(\left|\frac{2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a}{2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a}\right|\right)}{|a|} + \frac{8\left(3\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan(\cdots}\right.\right.}{\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4-6\left(\sqrt{a}\cdots\right.}\right.}{4d}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(sqrt(2)*a^(3/2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a
)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)
*abs(a) - 6*a))/abs(a) + 8*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))^2*a^(3/2) - a^(5/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c
) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c
) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))*sgn(cos(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + a\sec(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{\sqrt{a + \frac{a}{\cos(c+d\,x)}}}{\cos\left(c + d\,x\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a + a\sec(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)}\,dx = \sqrt{a}\left(\int \frac{\sqrt{\sec(dx + c) + 1}\,\sqrt{\cos(dx + c)}}{\cos(dx + c)^2}dx\right)$$

input
```
int((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x)
```

output
```
sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**2,x)
```

3.404 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 136

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{3\sqrt{a}\,\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d} + \frac{a\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{3a\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

output
```
3/4*a^(1/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/d+1/2*a*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+
c))^(1/2)+3/4*a*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a + a\sec(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{a\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\left(3\arcsin\left(\sqrt{1-\sec(c+dx)}\right) + 2\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx) + 3\sqrt{-(}\right.}{4d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

output
```
(a*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(3*ArcSin[Sqrt[1 - Sec[c + d*x]]]
 + 2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + 3*Sqrt[-((-1 + Sec[c + d*
x])*Sec[c + d*x])])*Sin[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 +
Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 4752, 3042, 4290, 3042, 4290, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(c+dx)+a}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{4290}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{4}\int \sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\cdots\right.$$

$$\Big\downarrow \text{4290}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{4}\left(\frac{1}{2}\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin\cdots}{2d\cdots}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{4}\left(\frac{1}{2}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+\cdots}{d\sqrt{a\sec(c+dx)+a}\cdots}\right.\right.$$

$$\Big\downarrow \text{4288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{4}\left(\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a\sin(c\cdots}{2d\sqrt{\cdots}}\right.$$

$$\Big\downarrow \text{222}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{4}\left(\frac{\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\sec\cdots}{2d\sqrt{a\sec(c+\cdots}}\right.$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a*Sec[c + d*x]^(5/2)*Sin[c + d*x])
/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (3*((Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*
x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*
Sqrt[a + a*Sec[c + d*x]])))/4)
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.26

method	result
default	$\frac{\left(-3\cos(dx+c)^2\arctan\left(\frac{-\cot(dx+c)+\csc(dx+c)-1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-3\cos(dx+c)^2\arctan\left(\frac{-\cot(dx+c)+\csc(dx+c)+1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+\sin(dx+c)(2+3\cos(dx+c}{8d\cos(dx+c)^{\frac{3}{2}}(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*(-3*cos(d*x+c)^2*arctan(1/2*(-cot(d*x+c)+csc(d*x+c)-1)/(-1/(cos(d*x+
c)+1))^(1/2))-3*cos(d*x+c)^2*arctan(1/2/(-1/(cos(d*x+c)+1))^(1/2)*(-cot(d*
x+c)+csc(d*x+c)+1))+sin(d*x+c)*(2+3*cos(d*x+c))*2^(1/2)*(-2/(cos(d*x+c)+1)
)^(1/2))*(a*(1+sec(d*x+c)))^(1/2)/cos(d*x+c)^(3/2)/(cos(d*x+c)+1)/(-1/(cos
(d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.61

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$=\left[\frac{4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(3\cos(dx+c)+2)\sqrt{\cos(dx+c)}\sin(dx+c)+3\left(\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{a}\,\mathrm{lo}}{16\left(d\cos(dx+c)^3+d\cos(dx+\right.}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/16*(4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(3*cos(d*x + c) + 2)*sqrt
(cos(d*x + c))*sin(d*x + c) + 3*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*
log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 +
8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^3 + d*cos(d*x + c
)^2), 1/8*(2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(3*cos(d*x + c) + 2)*
sqrt(cos(d*x + c))*sin(d*x + c) + 3*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt
(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*
x + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + a\sec(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1264 vs. $2(112) = 224$.

Time = 0.24 (sec) , antiderivative size = 1264, normalized size of antiderivative = 9.29

$$\int \frac{\sqrt{a + a\sec(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
-1/16*(12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*
arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqr
t(2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(s
qrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(
d*x + c), cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*
d*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d
*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^
2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x
 + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt
(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4
*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)
^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*
d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(
sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 2) - 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*
d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4
*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*l...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(112) = 224$.

Time = 0.24 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.46

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{\sqrt{2}\left(\frac{3\sqrt{2}a^{\frac{3}{2}}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|} - \frac{8\left(5\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^6a^{\cdots}}{\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\cdots\right.\right.}\right.}{\cdots}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
1/16*sqrt(2)*(3*sqrt(2)*a^(3/2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt
(2)*abs(a) - 6*a))/abs(a) - 8*(5*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))^6*a^(3/2) + 19*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(5/2) - 17*(sqrt(a)*tan(1/2*d*x +
1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(7/2) + a^(9/2))/((sqrt(a
)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a
)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)
*sgn(cos(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\sqrt{a}\left(\int \frac{\sqrt{\sec\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{3}}dx\right)$$

input
```
int((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x)
```

output
```
sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**3,x)
```

3.405 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 161

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{208a^2\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{104a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{26a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}}$$

output
```
208/105*a^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+104/105*a
^2*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+26/35*a^2*cos(d*x+
c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/7*a^2*cos(d*x+c)^(5/2)*sin(
d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.45

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a\sqrt{\cos(c+dx)}(494+253\cos(c+dx)+78\cos(2(c+dx))+15\cos(3(c+dx)))\sqrt{a(1+\sec(}{210d}$$

input

```
Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

output

```
(a*Sqrt[Cos[c + d*x]]*(494 + 253*Cos[c + d*x] + 78*Cos[2*(c + d*x)] + 15*C
os[3*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(210*d)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4752, 3042, 4300, 27, 3042, 4292, 3042, 4292, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{7}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4300}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{7}a\int \frac{13\sqrt{\sec(c+dx)a+a}}{2\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{13}{7}a\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{13}{7}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{4292}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{13}{7}a\left(\frac{4}{5}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{}{7d\sec^{\frac{5}{2}}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{13}{7}a\left(\frac{4}{5}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\right.$$

$$\Big\downarrow \text{4292}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{13}{7}a\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{}{5d}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{13}{7}a\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)\right.\right.$$

$$\Big\downarrow \text{4291}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{13}{7}a\left(\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\right.\right.$$

input
```
Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sin[c + d*x])/(7*d*Sec[c + d
*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (13*a*((2*a*Sin[c + d*x])/(5*d*Sec[c
 + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (4*((2*a*Sin[c + d*x])/(3*d*Sqrt
[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (4*a*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])))/5))/7)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.45

method	result	size
default	$\frac{2a\sin(dx+c)\left(15\cos(dx+c)^{3}+39\cos(dx+c)^{2}+52\cos(dx+c)+104\right)\sqrt{\cos(dx+c)}\,\sqrt{a(1+\sec(dx+c))}}{105d(\cos(dx+c)+1)}$	73

input
```
int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/105/d*a*sin(d*x+c)*(15*cos(d*x+c)^3+39*cos(d*x+c)^2+52*cos(d*x+c)+104)*c
os(d*x+c)^(1/2)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.52

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx=\frac{2\left(15\,a\cos\left(dx+c\right)^{3}+39\,a\cos\left(dx+c\right)^{2}+52\,a\cos\left(dx+c\right)+104\,a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx\right.}}{105\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/105*(15*a*cos(d*x + c)^3 + 39*a*cos(d*x + c)^2 + 52*a*cos(d*x + c) + 104
*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c
)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(137) = 274$.

Time = 0.21 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.88

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/840*sqrt(2)*(735*a*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7
/2*c)))*sin(7/2*d*x + 7/2*c) + 175*a*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c),
 cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 63*a*cos(2/7*arctan2(sin(7/
2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 735*a*cos(7/
2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))
) - 175*a*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7
/2*d*x + 7/2*c))) - 63*a*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x
+ 7/2*c), cos(7/2*d*x + 7/2*c))) + 30*a*sin(7/2*d*x + 7/2*c) + 63*a*sin(5/
7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*a*sin(3/7*arc
tan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 735*a*sin(1/7*arctan2(
sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(a)/d
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.81

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{4\left(105\sqrt{2}a^5\text{sgn}(\cos(dx+c))+\left(140\sqrt{2}a^5\text{sgn}(\cos(dx+c))+19\left(2\sqrt{2}a^5\text{sgn}(\cos(dx+c))\right.\right.\right.}{105\left(a\tan\left(\right.\right.}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
4/105*(105*sqrt(2)*a^5*sgn(cos(d*x + c)) + (140*sqrt(2)*a^5*sgn(cos(d*x +
c)) + 19*(2*sqrt(2)*a^5*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 + 7*sqrt(
2)*a^5*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*
tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 + a)^(7/2)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int \cos(c+d\,x)^{7/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

input
```
int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \sqrt{a}\,a\left(\int \sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^3\sec(dx+c)\,dx + \int \sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)$$

input
```
int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**3*s
ec(c + d*x),x) + int(sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x
)**3,x))
```

3.406 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{8a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d}$$

output
```
8/5*a^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2/5*a*cos(d*x
+c)^(1/2)*(a+a*sec(d*x+c))^(1/2)*sin(d*x+c)/d+2/5*cos(d*x+c)^(3/2)*(a+a*se
c(d*x+c))^(3/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a\sqrt{\cos(c+dx)}(13+6\cos(c+dx)+\cos(2(c+dx)))\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{5d}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(a*Sqrt[Cos[c + d*x]]*(13 + 6*Cos[c + d*x] + Cos[2*(c + d*x)])*Sqrt[a*(1 +
 Sec[c + d*x])]*Tan[(c + d*x)/2])/(5*d)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4752, 3042, 4299, 3042, 4296, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4299}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{5}\int \frac{(\sec(c+dx)a+a)^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{5}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{4296}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{5}\left(\frac{4}{3}a\int\frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{5}\left(\frac{4}{3}a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\,\mathrm{si}}{}\right.$$

$$\Big\downarrow \text{4291}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{5}\left(\frac{8a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+}{}\right.$$

input
```
Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(a + a*Sec[c + d*x])^(3/2)*Sin[c
 + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (3*((8*a^2*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Sqrt[a + a*Sec[c + d*x]]*Sin[c
 + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/5)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4296
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)
*((d*Csc[e + f*x])^n/(f*m)), x] + Simp[b*((2*m - 1)/(d*m))   Int[(a + b*Csc
[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f
, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

rule 4299
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(m + 1))), x] + Simp[a*(m/(b*d*(m + 1)))   Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.53

method	result	size
default	$\frac{2a\sin(dx+c)\left(\cos(dx+c)^2+3\cos(dx+c)+6\right)\sqrt{\cos(dx+c)}\,\sqrt{a(1+\sec(dx+c))}}{5d(\cos(dx+c)+1)}$	61

input
```
int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/d*a*sin(d*x+c)*(cos(d*x+c)^2+3*cos(d*x+c)+6)*cos(d*x+c)^(1/2)*(a*(1+se
c(d*x+c)))^(1/2)/(cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.62

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2\left(a\cos\left(dx+c\right)^{2}+3\,a\cos\left(dx+c\right)+6\,a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{5\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/5*(a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 6*a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(98) = 196.

Time = 0.19 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.81

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{\sqrt{2}\left(20\,a\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,a\cos\left(\frac{2}{5}\,\text{arcta}\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/20*sqrt(2)*(20*a*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2
*c)))*sin(5/2*d*x + 5/2*c) + 5*a*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos
(5/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 20*a*cos(5/2*d*x + 5/2*c)*sin(4
/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*a*cos(5/2*d*x
+ 5/2*c)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 2*
a*sin(5/2*d*x + 5/2*c) + 5*a*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
*d*x + 5/2*c))) + 20*a*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x +
 5/2*c))))*sqrt(a)/d
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.87

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{4\left(5\sqrt{2}a^4\mathrm{sgn}(\cos(dx+c)) + \left(2\sqrt{2}a^4\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 5\sqrt{2}a^4\mathrm{sgn}(\cos(\right.\right.}{5\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{5}{2}}d}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
4/5*(5*sqrt(2)*a^4*sgn(cos(d*x + c)) + (2*sqrt(2)*a^4*sgn(cos(d*x + c))*ta
n(1/2*d*x + 1/2*c)^2 + 5*sqrt(2)*a^4*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*
c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 + a)^(5/2)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int \cos(c+d\,x)^{5/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \sqrt{a}\,a\left(\int \sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)\,dx + \int \sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)$$

input
```
int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**2*s
ec(c + d*x),x) + int(sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x
)**2,x))
```

3.407 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 79

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{8a^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d}$$

output
```
8/3*a^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2/3*a*cos(d*x
+c)^(1/2)*(a+a*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.63

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2a\sqrt{\cos(c+dx)}(5+\cos(c+dx))\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{3d}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(2*a*Sqrt[Cos[c + d*x]]*(5 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[
(c + d*x)/2])/(3*d)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 4752, 3042, 4296, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4296}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{3}a\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{3}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{4291}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((8*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Sqrt[a + a*Sec[c + d*x]]*Sin[c
+ d*x])/(3*d*Sqrt[Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4296
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)
*((d*Csc[e + f*x])^n/(f*m)), x] + Simp[b*((2*m - 1)/(d*m))   Int[(a + b*Csc
[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f
, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{2\sqrt{\cos(dx+c)}\,(\cos(dx+c)+5)\sin(dx+c)a\sqrt{a(1+\sec(dx+c))}}{d(3\cos(dx+c)+3)}$	53

input
```
int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*cos(d*x+c)^(1/2)*(cos(d*x+c)+5)*sin(d*x+c)/(3*cos(d*x+c)+3)*a*(a*(1+se
c(d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.77

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2\,(a\cos(dx+c)+5\,a)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sqrt{\cos(dx+c)}\sin(dx+c)}{3\,(d\cos(dx+c)+d)}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/3*(a*cos(d*x + c) + 5*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.48

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{\left(\sqrt{2}a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+9\,\sqrt{2}a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{3\,d}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/3*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*sq
rt(a)/d
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{4\left(2\,\sqrt{2}a^3\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,\sqrt{2}a^3\mathrm{sgn}(\cos(dx+c))\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{3\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\right)^{\frac{3}{2}}d}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
4/3*(2*sqrt(2)*a^3*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 + 3*sqrt(2)*a^
3*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 + a)^
(3/2)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int \cos\left(c+d\,x\right)^{3/2}\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

input
```
int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \sqrt{a}\,a\left(\int \sqrt{\sec\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)\sec\left(dx+c\right)dx + \int \sqrt{\sec\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)dx\right)$$

input
```
int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)*sec(
c + d*x),x) + int(sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x),x
))
```

3.408 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 96

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx = \frac{2a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

output
```
2*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1
/2)*sec(d*x+c)^(1/2)/d+2*a^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx = \frac{2a^2\left(\sqrt{1-\sec(c+dx)}+\arcsin\left(\sqrt{1-\sec(c+dx)}\right)\sqrt{\sec(c+dx)}\right)\sin(c+dx)}{d\sqrt{-1+\cos(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(2*a^2*(Sqrt[1 - Sec[c + d*x]] + ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c
 + d*x]])*Sin[c + d*x])/(d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x
])])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4752, 3042, 4300, 27, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^{3/2}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4300}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a\int \frac{1}{2}\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(a\int\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{4288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{2a\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)$$

$$\Big\downarrow \text{222}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}\right)$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c +
 d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(d*Sqrt[a + a*Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.52

method	result
default	$-\frac{a\sqrt{\cos(dx+c)}\,\sqrt{a(1+\sec(dx+c))}\left(\sqrt{2}\,\arctan\left(\frac{\cot(dx+c)-\csc(dx+c)-1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{2}{\cos(dx+c)+1}}+\sqrt{2}\,\arctan\left(\frac{\cot(dx+c)-\csc(dx+c)+1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right.\cdots}{2d}$

input
```
int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*a*cos(d*x+c)^(1/2)*(a*(1+sec(d*x+c)))^(1/2)*(2^(1/2)*arctan(1/2*(co
t(d*x+c)-csc(d*x+c)-1)/(-1/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2
)+2^(1/2)*arctan(1/2/(-1/(cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1))*
(-2/(cos(d*x+c)+1))^(1/2)+4*cot(d*x+c)-4*csc(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 298, normalized size of antiderivative = 3.10

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx = \left[\frac{4\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(a\cos\left(dx+c\right)+a\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{}}{}\right.}{2\left(d\cos\left(dx+c\right)+d\right)}\right.$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c) + (a*cos(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c
))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)
^2)))/(d*cos(d*x + c) + d), (2*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c) + (a*cos(d*x + c) + a)*sqrt(-a)*arctan(2*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
+ c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(82) = 164.

Time = 0.21 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.85

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx = \frac{\sqrt{2}\left(\sqrt{2}a\log\left(2\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\sqrt{2}\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\sqrt{2}\sin\left(\frac{1}{2}\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*(sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) - sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(
2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8*a*sin(1/2*d*x + 1/2*c
))*sqrt(a)/d
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.70

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx = \frac{\frac{2\sqrt{2}a^2\operatorname{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}+\frac{a^{\frac{5}{2}}\log\left(\left|\frac{2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a}{2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a}\right|\right)\operatorname{sgn}(\cos(dx+}{|a|}}{d}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
(2*sqrt(2)*a^2*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x +
 1/2*c)^2 + a) + a^(5/2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*
tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a)*ta
n(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs
(a) - 6*a))*sgn(cos(d*x + c))/abs(a))/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx = \int \sqrt{\cos\left(c+d\,x\right)}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

input
```
int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx = \sqrt{a}\,a\left(\int \sqrt{\sec\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}\,\sec\left(dx+c\right)dx + \int \sqrt{\sec\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}dx\right)$$

input
```
int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*sec(c + d*x),x) +
 int(sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x)),x))
```

3.409 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{3a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{a^2\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

output
```
3*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1
/2)*sec(d*x+c)^(1/2)/d+a^2*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^
(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = -\frac{a^2\left(-\sqrt{1-\sec(c+dx)}+\frac{3\arcsin\left(\sqrt{\sec(c+dx)}\right)}{\sqrt{\sec(c+dx)}}\right)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

output
```
-((a^2*(-Sqrt[1 - Sec[c + d*x]] + (3*ArcSin[Sqrt[Sec[c + d*x]]])/Sqrt[Sec[
c + d*x]])*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)*Sqrt[1 - Sec[c + d*x]]*Sqrt
[a*(1 + Sec[c + d*x])]))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 4752, 3042, 4301, 27, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{3/2}}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4301}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(a\int \frac{3}{2}\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{2}a\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{2}a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right.$$

$\downarrow$ 4288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{3a\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)$$

$\downarrow$ 222

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

input
```
Int[(a + a*Sec[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((3*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c +
 d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(d*Sqrt[a + a*Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 3.91 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.66

method	result
default	$\frac{a\sqrt{a(1+\sec(dx+c))}\left(\sin(dx+c)\sqrt{2}\,\sqrt{-\frac{2}{\cos(dx+c)+1}}-3\cos(dx+c)\arctan\left(\frac{-\cot(dx+c)+\csc(dx+c)+1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-3\cos(dx+c)\arctan\left(\frac{-\cot}{2}\right.\right.}{2d\sqrt{-\frac{1}{\cos(dx+c)+1}}\,\sqrt{\cos(dx+c)}\,(\cos(dx+c)+1)}$

input
```
int((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*a*(a*(1+sec(d*x+c)))^(1/2)*(sin(d*x+c)*2^(1/2)*(-2/(cos(d*x+c)+1))^(
1/2)-3*cos(d*x+c)*arctan(1/2/(-1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*
x+c)+1))-3*cos(d*x+c)*arctan(1/2*(-cot(d*x+c)+csc(d*x+c)-1)/(-1/(cos(d*x+c
)+1))^(1/2)))/(-1/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^(1/2)/(cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 337, normalized size of antiderivative = 3.55

$$\int \frac{(a + a\sec(c + dx))^{3/2}}{\sqrt{\cos(c + dx)}}\, dx = \left[\frac{4\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c) + 3\left(a\cos(dx+c)^2 + a\cos(d\right.}{4\left(d\cos(d\right.}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c) + 3*(a*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c
)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)
*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)
^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/2*(2*a*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*(a*
cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^
2 - a*cos(d*x + c) - 2*a))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F]

$$\int \frac{(a + a\sec(c + dx))^{3/2}}{\sqrt{\cos(c + dx)}}\, dx = \int \frac{(a(\sec(c + dx) + 1))^{\frac{3}{2}}}{\sqrt{\cos(c + dx)}}\, dx$$

input
```
integrate((a+a*sec(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)/sqrt(cos(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1143 vs. $2(81) = 162$.

Time = 0.20 (sec) , antiderivative size = 1143, normalized size of antiderivative = 12.03

$$\int \frac{(a + a\sec(c + dx))^{3/2}}{\sqrt{\cos(c + dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 + a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*
c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a
*sin(3/2*d*x + 3/2*c) - 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 2*(2*sqrt(2)*a*
sin(3/2*d*x + 3/2*c) - 2*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*
c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(81) = 162$.

Time = 0.28 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.72

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{2}a^{\frac{7}{2}}\left(\frac{3\sqrt{2}\log\left(\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\right)}{a|a|}+\frac{\cdots}{\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx\cdots\right.\right.}\right.}{\cdots}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*a^(7/2)*(3*sqrt(2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt
(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(
2)*abs(a) - 6*a))/(a*abs(a)) + 8*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a
*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)*a))*sgn(cos(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \sqrt{a}\,a\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\sec(dx+c)}{\cos(dx+c)}dx + \int \frac{\sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)$$

input
```
int((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x)
```

output
```
sqrt(a)*a*(int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*sec(c + d*x))/co
s(c + d*x),x) + int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*
x),x))
```

3.410 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 140

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{7a^{3/2}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d} + \frac{a^2\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{7a^2\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

output
```
7/4*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/d+1/2*a^2*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*
x+c))^(1/2)+7/4*a^2*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.76

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{a^2\left((2+7\cos(c+dx))\sqrt{1-\sec(c+dx)} + \frac{7\arcsin\left(\sqrt{1-\sec(c+dx)}\right)}{\sec^{\frac{3}{2}}(c+dx)}\right)\sin(c+dx}{4d\cos^{\frac{5}{2}}(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

output
```
(a^2*((2 + 7*Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] + (7*ArcSin[Sqrt[1 - Sec
[c + d*x]]])/Sec[c + d*x]^(3/2))*Sin[c + d*x])/(4*d*Cos[c + d*x]^(5/2)*Sqr
t[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4752, 3042, 4301, 27, 3042, 4290, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4301}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}a\int \frac{7}{2}\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7}{4}a\int \sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7}{4}a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+d}{2d\sqrt{a\sec(c+dx)+a}}\right.$$

$$\Big\downarrow \text{4290}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7}{4}a\left(\frac{1}{2}\int\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2 s}{2}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7}{4}a\left(\frac{1}{2}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+}{d\sqrt{a\sec(c+dx)+}}\right.\right.$$

$$\Big\downarrow \text{4288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7}{4}a\left(\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a^2\sin(}{2d\sqrt{}}\right.$$

$$\Big\downarrow \text{222}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}+\frac{7}{4}a\left(\frac{\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sec}{d\sqrt{a\sec(c+}}\right.\right.$$

input

```
Int[(a + a*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

output

```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x
])/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (7*a*((Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c
+ d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sec[c + d*x]^(3/2)*Sin[c + d*x])
/(d*Sqrt[a + a*Sec[c + d*x]])))/4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 3.70 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.23

method	result
default	$\frac{a\left(-7\cos(dx+c)^2\arctan\left(\frac{-\cot(dx+c)+\csc(dx+c)-1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-7\cos(dx+c)^2\arctan\left(\frac{-\cot(dx+c)+\csc(dx+c)+1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+\sin(dx+c)(7\cos(dx+c)\right.}{8d(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}\cos(dx+c)^{\frac{3}{2}}}$

input
```
int((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*a*(-7*cos(d*x+c)^2*arctan(1/2*(-cot(d*x+c)+csc(d*x+c)-1)/(-1/(cos(d*
x+c)+1))^(1/2))-7*cos(d*x+c)^2*arctan(1/2/(-1/(cos(d*x+c)+1))^(1/2)*(-cot(
d*x+c)+csc(d*x+c)+1))+sin(d*x+c)*(7*cos(d*x+c)+2)*2^(1/2)*(-2/(cos(d*x+c)+
1))^(1/2))*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/
2)/cos(d*x+c)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.64

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left[\frac{4\left(7\,a\cos\left(dx+c\right)+2\,a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+7\left(a\,\mathrm{co}\right.}{}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(4*(7*a*cos(d*x + c) + 2*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(cos(d*x + c))*sin(d*x + c) + 7*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*
sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x +
 c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^3 + d*cos
(d*x + c)^2), 1/8*(2*(7*a*cos(d*x + c) + 2*a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 7*(a*cos(d*x + c)^3 + a*cos(
d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) -
2*a)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a(\sec{(c+dx)}+1))^{\frac{3}{2}}}{\cos^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(3/2)/cos(d*x+c)**(3/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)/cos(c + d*x)**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2244 vs. $2(116) = 232$.

Time = 0.25 (sec) , antiderivative size = 2244, normalized size of antiderivative = 16.03

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
-1/16*(56*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 24*s
qrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(
4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*sqrt(2)*a*si
n(3/2*d*x + 3/2*c) + 28*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 4*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*s
in(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*
sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*(3*sqrt(2)*a*sin(3/2
*d*x + 3/2*c) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) - 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*si
n(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{%%{[-469762048,0]:[1,0,-2]%%},[14]%%%},0]:[1,0,%%%
{-1,[1]%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\sqrt{a}\,a\left(\int \frac{\sqrt{\sec\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}\,\sec\left(dx+c\right)}{\cos\left(dx+c\right)^{2}}dx+\int \frac{\sqrt{\sec\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{2}}dx\right)$$

input
```
int((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x)
```

output
```
sqrt(a)*a*(int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*sec(c + d*x))/co
s(c + d*x)**2,x) + int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c +
 d*x)**2,x))
```

3.411 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 180

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{11a^{3/2}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8d} + \frac{a^2\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{11a^2\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{11a^2\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

output
```
11/8*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/d+1/3*a^2*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*sec(d
*x+c))^(1/2)+11/12*a^2*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2
)+11/8*a^2*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.65

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{a^2\left((8+22\cos(c+dx)+33\cos^2(c+dx))\sqrt{1-\sec(c+dx)}+\frac{33\arcsin\left(\sqrt{1-s\ldots}\right.}{\sec^{\frac{5}{2}}(c+\ldots}\right.}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx)\ldots}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(5/2),x]
```

output
```
(a^2*((8 + 22*Cos[c + d*x] + 33*Cos[c + d*x]^2)*Sqrt[1 - Sec[c + d*x]] + (
33*ArcSin[Sqrt[1 - Sec[c + d*x]]])/Sec[c + d*x]^(5/2))*Sin[c + d*x])/(24*d
*Cos[c + d*x]^(7/2)*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3042, 4752, 3042, 4301, 27, 3042, 4290, 3042, 4290, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4301}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}a\int \frac{11}{2}\sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{11}{6}a\int \sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{11}{6}a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+}{3d\sqrt{a\sec(c+dx)+}}\right.$$

$$\Big\downarrow \text{4290}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{11}{6}a\left(\frac{3}{4}\int \sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2 s}{3}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{11}{6}a\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c}{2d\sqrt{a\sec(c+dx)}}\right.\right.$$

$$\Big\downarrow \text{4290}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{11}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{11}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}}{d\sqrt{a\sec(c+d}}\right.\right.\right.$$

$$\Big\downarrow \text{4288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{11}{6}a\left(\frac{3}{4}\left(\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a}{}\right.\right.$$

$$\Big\downarrow \text{222}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{11}{6}a\left(\frac{3}{4}\left(\frac{\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a\sec(}}\right.\right.\right.$$

input
```
Int[(a + a*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a^2*Sec[c + d*x]^(7/2)*Sin[c + d*x
])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (11*a*((a*Sec[c + d*x]^(5/2)*Sin[c + d
*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (3*((Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c
+ d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sec[c + d*x]^(3/2)*Sin[c + d*x])
/(d*Sqrt[a + a*Sec[c + d*x]])))/4))/6)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 3.73 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.01

method	result
default	$\frac{a\left(-33\cos(dx+c)^3\arctan\left(\frac{-\cot(dx+c)+\csc(dx+c)+1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-33\cos(dx+c)^3\arctan\left(\frac{-\cot(dx+c)+\csc(dx+c)-1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+\sin(dx+c)\left(33\cos(dx\right.\right.}{48d(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}\,\cos(dx+c)^{\frac{5}{2}}}$

input
```
int((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/d*a*(-33*cos(d*x+c)^3*arctan(1/2/(-1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+
c)+csc(d*x+c)+1))-33*cos(d*x+c)^3*arctan(1/2*(-cot(d*x+c)+csc(d*x+c)-1)/(-
1/(cos(d*x+c)+1))^(1/2))+sin(d*x+c)*(33*cos(d*x+c)^2+22*cos(d*x+c)+8)*2^(1
/2)*(-2/(cos(d*x+c)+1))^(1/2))*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/(-1
/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 391, normalized size of antiderivative = 2.17

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left[\frac{4\left(33\,a\cos\left(dx+c\right)^2+22\,a\cos\left(dx+c\right)+8\,a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/96*(4*(33*a*cos(d*x + c)^2 + 22*a*cos(d*x + c) + 8*a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 33*(a*cos(d*x + c
)^4 + a*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(
d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(
d*cos(d*x + c)^4 + d*cos(d*x + c)^3), 1/48*(2*(33*a*cos(d*x + c)^2 + 22*a*
cos(d*x + c) + 8*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c) + 33*(a*cos(d*x + c)^4 + a*cos(d*x + c)^3)*sqrt(-a)*arct
an(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^4
+ d*cos(d*x + c)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(3/2)/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2361 vs. 2(150) = 300.

Time = 0.28 (sec) , antiderivative size = 2361, normalized size of antiderivative = 13.12

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
-1/96*(132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*
sqrt(2)*a*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) +
3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c)
+ 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c
) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) - 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*
c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) - 132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x +
4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*
cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*
d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x
 + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*
c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(150) = 300$.

Time = 0.51 (sec) , antiderivative size = 472, normalized size of antiderivative = 2.62

$$\int \frac{(a + a\sec(c + dx))^{3/2}}{\cos^{\frac{5}{2}}(c + dx)}\, dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
1/48*(33*a^(3/2)*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))*sgn(cos(d*x + c)) - 33*a^(3/2)*
log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)
)^2 + a*(2*sqrt(2) - 3)))*sgn(cos(d*x + c)) + 4*(33*sqrt(2)*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*a^(5/2)*sgn(cos(d
*x + c)) - 303*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
+ 1/2*c)^2 + a))^8*a^(7/2)*sgn(cos(d*x + c)) + 2394*sqrt(2)*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*a^(9/2)*sgn(cos(d*
x + c)) - 1806*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
+ 1/2*c)^2 + a))^4*a^(11/2)*sgn(cos(d*x + c)) + 309*sqrt(2)*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(13/2)*sgn(cos(d
*x + c)) - 19*sqrt(2)*a^(15/2)*sgn(cos(d*x + c)))/((sqrt(a)*tan(1/2*d*x +
1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x +
1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\sqrt{a}\,a\left(\int \frac{\sqrt{\sec\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}\,\sec\left(dx+c\right)}{\cos\left(dx+c\right)^{3}}dx+\int \frac{\sqrt{\sec\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{3}}dx\right)$$

input
```
int((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x)
```

output
```
sqrt(a)*a*(int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*sec(c + d*x))/co
s(c + d*x)**3,x) + int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c +
 d*x)**3,x))
```

3.412 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 201

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{1168a^3\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{584a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} + \frac{146a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{38a^3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{9d}$$

output
```
1168/315*a^3*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+584/315*
a^3*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+146/105*a^3*cos(d
*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+38/63*a^3*cos(d*x+c)^(5/2)
*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/9*a^2*cos(d*x+c)^(7/2)*(a+a*sec(d*x
+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 5.57 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.45

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a^2\sqrt{\cos(c+dx)}(584+292\cos(c+dx)+219\cos^2(c+dx)+130\cos^3(c+dx)+35\cos^4(c-}{315d(1+\cos(c+dx))}$$

input
```
Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(2*a^2*Sqrt[Cos[c + d*x]]*(584 + 292*Cos[c + d*x] + 219*Cos[c + d*x]^2 + 1
30*Cos[c + d*x]^3 + 35*Cos[c + d*x]^4)*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c +
d*x])/(315*d*(1 + Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.16, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4752, 3042, 4300, 27, 3042, 4503, 3042, 4292, 3042, 4292, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{9}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{4300}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{9}a\int \frac{\sqrt{\sec(c+dx)a+a}(15\sec(c+dx)a+19a)}{2\sec^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+a}}{9d\sec^{\frac{7}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}a\int \frac{\sqrt{\sec(c+dx)a+a}(15\sec(c+dx)a+19a)}{\sec^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+a}}{9d\sec^{\frac{7}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(15\csc\left(c+dx+\frac{\pi}{2}\right)a+19a\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2a^2\sin(c+dx)\sqrt{}}{9d\sec^{\frac{7}{2}}}\right.$$

$$\Big\downarrow \text{4503}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}a\left(\frac{219}{7}a\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{38a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2 s}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}a\left(\frac{219}{7}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{38a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)\right.$$

$$\Big\downarrow \text{4292}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}a\left(\frac{219}{7}a\left(\frac{4}{5}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}a\left(\frac{219}{7}a\left(\frac{4}{5}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right.\right.\right.$$

$$\Big\downarrow \text{4292}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}a\left(\frac{219}{7}a\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)\right.\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}a\left(\frac{219}{7}a\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)}}\right.\right.\right.\right.$$

$\big\downarrow$ 4291

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{1}{9}a\left(\frac{38a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{21}{}\right.\right.$$

input
```
Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sqrt[a + a*Sec[c + d*x]]*Sin
[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (a*((38*a^2*Sin[c + d*x])/(7*d*Sec[c
 + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (219*a*((2*a*Sin[c + d*x])/(5*d*
Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (4*((2*a*Sin[c + d*x])/(3*d
*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (4*a*Sqrt[Sec[c + d*x]]*Si
n[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])))/5))/7))/9)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

rule 4503
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Simp
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n)   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[
e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a
*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.42

$$\frac{2a^2 \sin (dx+c) \left(35 \cos (dx+c)^4+130 \cos (dx+c)^3+219 \cos (dx+c)^2+292 \cos (dx+c)+584\right) \sqrt{\cos (}}{315d (\cos (dx+c)+1)}$$

input
```
int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2),x)
```

output
```
2/315/d*a^2*sin(d*x+c)*(35*cos(d*x+c)^4+130*cos(d*x+c)^3+219*cos(d*x+c)^2+
292*cos(d*x+c)+584)*cos(d*x+c)^(1/2)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+
1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.52

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2\left(35\,a^2\cos\left(dx+c\right)^4+130\,a^2\cos\left(dx+c\right)^3+219\,a^2\cos\left(dx+c\right)^2+292\,a^2\cos\left(dx+c\right)+58\right.}{315\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/315*(35*a^2*cos(d*x + c)^4 + 130*a^2*cos(d*x + c)^3 + 219*a^2*cos(d*x +
c)^2 + 292*a^2*cos(d*x + c) + 584*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(171) = 342$.

Time = 0.21 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.10

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/5040*sqrt(2)*(8190*a^2*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
 + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 2100*a^2*cos(2/3*arctan2(sin(9/2*d*x +
9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 756*a^2*cos(4/9*arct
an2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 22
5*a^2*cos(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2
*d*x + 9/2*c) - 8190*a^2*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x
+ 9/2*c), cos(9/2*d*x + 9/2*c))) - 2100*a^2*cos(9/2*d*x + 9/2*c)*sin(2/3*a
rctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 756*a^2*cos(9/2*d*x
+ 9/2*c)*sin(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 22
5*a^2*cos(9/2*d*x + 9/2*c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d
*x + 9/2*c))) + 70*a^2*sin(9/2*d*x + 9/2*c) + 225*a^2*sin(7/9*arctan2(sin(
9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 756*a^2*sin(5/9*arctan2(sin(9/2
*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 2100*a^2*sin(1/3*arctan2(sin(9/2*d
*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 8190*a^2*sin(1/9*arctan2(sin(9/2*d*x
 + 9/2*c), cos(9/2*d*x + 9/2*c))))*sqrt(a)/d
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.79

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{8\left(315\sqrt{2}a^7\mathrm{sgn}(\cos(dx+c)) + \left(630\sqrt{2}a^7\mathrm{sgn}(\cos(dx+c)) + 13\left(63\sqrt{2}a^7\mathrm{sgn}(\cos(dx+c)\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
8/315*(315*sqrt(2)*a^7*sgn(cos(d*x + c)) + (630*sqrt(2)*a^7*sgn(cos(d*x +
c)) + 13*(63*sqrt(2)*a^7*sgn(cos(d*x + c)) + 4*(2*sqrt(2)*a^7*sgn(cos(d*x
+ c))*tan(1/2*d*x + 1/2*c)^2 + 9*sqrt(2)*a^7*sgn(cos(d*x + c)))*tan(1/2*d*
x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x
+ 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 + a)^(9/2)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx=\int \cos\left(c+d\,x\right)^{9/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,dx$$

input
```
int(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx=\sqrt{a}\,a^2\left(\int \sqrt{\sec\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^4\sec\left(dx+c\right)^2dx+2\left(\int \sqrt{\sec\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^4\sec\left(dx+c\right)dx\right)+\int \sqrt{\sec\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^4dx\right)$$

input
```
int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**
4*sec(c + d*x)**2,x) + 2*int(sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*cos
(c + d*x)**4*sec(c + d*x),x) + int(sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x
))*cos(c + d*x)**4,x))
```

3.413 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 156

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{64a^3\sin(c+dx)}{21d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{16a^2\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{21d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{7d} + \frac{2\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{7d}$$

output

```
64/21*a^3*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+16/21*a^2*c
os(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2)*sin(d*x+c)/d+2/7*a*cos(d*x+c)^(3/2)
*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))
^(5/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 5.52 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.54

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx=\frac{a^2\sqrt{\cos(c+dx)}\sqrt{a(1+\sec(c+dx))}(392\sin(c+dx)+98\sin(2(c+dx))+3(8\sin(3(c+dx$$

input
```
Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^2*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]*(392*Sin[c + d*x] + 98*
Sin[2*(c + d*x)] + 3*(8*Sin[3*(c + d*x)] + Sin[4*(c + d*x)])))/(84*d*(1 +
Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4752, 3042, 4299, 3042, 4296, 3042, 4296, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{7}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \; 4299$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5}{7}\int \frac{(\sec(c+dx)a+a)^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d\sec^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5}{7}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d\sec^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \; 4296$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5}{7}\left(\frac{8}{5}a\int \frac{(\sec(c+dx)a+a)^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2\sin}{}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5}{7}\left(\frac{8}{5}a\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\right.$$

$$\Big\downarrow \; 4296$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5}{7}\left(\frac{8}{5}a\left(\frac{4}{3}a\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a\,\mathrm{si}}{}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5}{7}\left(\frac{8}{5}a\left(\frac{4}{3}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)\right.\right.$$

$$\Big\downarrow \; 4291$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5}{7}\left(\frac{8}{5}a\left(\frac{8a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a\,\mathrm{s}}{}\right.\right.$$

input
```
Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(a + a*Sec[c + d*x])^(5/2)*Sin[c
 + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (5*((2*a*(a + a*Sec[c + d*x])^(3/2)*Si
n[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (8*a*((8*a^2*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Sqrt[a + a*Sec[c + d*x]]*
Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/5))/7)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4296
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)
*((d*Csc[e + f*x])^n/(f*m)), x] + Simp[b*((2*m - 1)/(d*m))   Int[(a + b*Csc
[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f
, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

rule 4299
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(m + 1))), x] + Simp[a*(m/(b*d*(m + 1)))   Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.48

$$\frac{2a^2 \sin (dx+c) \left(3 \cos (dx+c)^3+12 \cos (dx+c)^2+23 \cos (dx+c)+46\right) \sqrt{\cos (dx+c)}\, \sqrt{a (1+\sec (dx+c))}}{21d (\cos (dx+c)+1)}$$

input
```
int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2),x)
```

output
```
2/21/d*a^2*sin(d*x+c)*(3*cos(d*x+c)^3+12*cos(d*x+c)^2+23*cos(d*x+c)+46)*co
s(d*x+c)^(1/2)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.59

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2\left(3a^2\cos(dx+c)^3+12a^2\cos(dx+c)^2+23a^2\cos(dx+c)+46a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{21\left(d\cos(dx+c)+d\right)}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/21*(3*a^2*cos(d*x + c)^3 + 12*a^2*cos(d*x + c)^2 + 23*a^2*cos(d*x + c) +
 46*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(132) = 264$.

Time = 0.20 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.07

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/168*sqrt(2)*(315*a^2*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
 7/2*c)))*sin(7/2*d*x + 7/2*c) + 77*a^2*cos(4/7*arctan2(sin(7/2*d*x + 7/2*
c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*a^2*cos(2/7*arctan2(s
in(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 315*a^2
*cos(7/2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
7/2*c))) - 77*a^2*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c))) - 21*a^2*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(si
n(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 6*a^2*sin(7/2*d*x + 7/2*c) +
21*a^2*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 77*a
^2*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 315*a^2*
sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(a)/d
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.83

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{8\left(21\sqrt{2}a^6\mathrm{sgn}(\cos(dx+c)) + \left(35\sqrt{2}a^6\mathrm{sgn}(\cos(dx+c)) + 4\left(2\sqrt{2}a^6\mathrm{sgn}(\cos(dx+c))\tan\right.\right.\right.}{21\left(a\tan\left(\frac{1}{2}c\right.\right.}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
8/21*(21*sqrt(2)*a^6*sgn(cos(d*x + c)) + (35*sqrt(2)*a^6*sgn(cos(d*x + c))
 + 4*(2*sqrt(2)*a^6*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 + 7*sqrt(2)*a
^6*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(
1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 + a)^(7/2)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \cos(c+d\,x)^{7/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}dx$$

input
```
int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx=\sqrt{a}\,a^2\left(\int \sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^3\sec(dx+c)^2\,dx+2\left(\int \sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^3\sec(dx+c)\,dx\right)+\int \sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)$$

input
```
int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**
3*sec(c + d*x)**2,x) + 2*int(sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*cos
(c + d*x)**3*sec(c + d*x),x) + int(sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x
))*cos(c + d*x)**3,x))
```

3.414 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 119

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{64a^3\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{16a^2\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d}$$

output
```
64/15*a^3*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+16/15*a^2*c
os(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2)*sin(d*x+c)/d+2/5*a*cos(d*x+c)^(3/2)
*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 5.46 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.54

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^2\sqrt{\cos(c+dx)}(89+28\cos(c+dx)+3\cos(2(c+dx)))\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx\right.}{15d}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^2*Sqrt[Cos[c + d*x]]*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sqrt[a
*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(15*d)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 4752, 3042, 4296, 3042, 4296, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4296}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a\int \frac{(\sec(c+dx)a+a)^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \ 4296$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a\left(\frac{4}{3}a\int\frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin(c$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a\left(\frac{4}{3}a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a}{}$$

$$\Big\downarrow \ 4291$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a\left(\frac{8a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin(}{}$$

input
```
Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*(a + a*Sec[c + d*x])^(3/2)*Sin
[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (8*a*((8*a^2*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Sqrt[a + a*Sec[c + d*x]]*S
in[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/5)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4296
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)
*((d*Csc[e + f*x])^n/(f*m)), x] + Simp[b*((2*m - 1)/(d*m))   Int[(a + b*Csc
[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f
, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.55

$$\frac{2a^2 \sin(dx+c) \left(3\cos(dx+c)^2 + 14\cos(dx+c) + 43\right) \sqrt{\cos(dx+c)}\, \sqrt{a\,(1+\sec(dx+c))}}{15d\,(\cos(dx+c)+1)}$$

input
```
int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x)
```

output
```
2/15/d*a^2*sin(d*x+c)*(3*cos(d*x+c)^2+14*cos(d*x+c)+43)*cos(d*x+c)^(1/2)*(
a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.66

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2\left(3\,a^2\cos(dx+c)^2 + 14\,a^2\cos(dx+c) + 43\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sqrt{\cos(dx+c)}\sin(dx+c)}{15\,(d\cos(dx+c)+d)}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/15*(3*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 43*a^2)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) +
 d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.50

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx=\frac{\left(3\sqrt{2}a^2\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+25\sqrt{2}a^2\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+150\sqrt{2}a^2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{30\,d}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/30*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x + 3/
2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.86

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{8\left(15\sqrt{2}a^5\mathrm{sgn}(\cos(dx+c)) + 4\left(2\sqrt{2}a^5\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 5\sqrt{2}a^5\mathrm{sgn}(\cos(dx+c))\right.\right.}{15\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{5}{2}}d}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
8/15*(15*sqrt(2)*a^5*sgn(cos(d*x + c)) + 4*(2*sqrt(2)*a^5*sgn(cos(d*x + c)
)*tan(1/2*d*x + 1/2*c)^2 + 5*sqrt(2)*a^5*sgn(cos(d*x + c)))*tan(1/2*d*x +
1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 + a)^(5/2)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \cos(c+d\,x)^{5/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}dx$$

input
```
int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \sqrt{a}\,a^2\left(\int \sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^2\,dx + 2\left(\int \sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)\,dx\right) + \int \sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)$$

input
```
int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**
2*sec(c + d*x)**2,x) + 2*int(sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*cos
(c + d*x)**2*sec(c + d*x),x) + int(sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x
))*cos(c + d*x)**2,x))
```

3.415 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 138

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a^{5/2}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{14a^3\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d}$$

output
```
2*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1
/2)*sec(d*x+c)^(1/2)/d+14/3*a^3*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x
+c))^(1/2)+2/3*a^2*cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.67

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a^3\left((8+\cos(c+dx))\sqrt{1-\sec(c+dx)}+3\arcsin\left(\sqrt{1-\sec(c+dx)}\right)\sqrt{\sec(c+dx)}\right)\sin(}{3d\sqrt{-1+\cos(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(2*a^3*((8 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] + 3*ArcSin[Sqrt[1 - Sec[
c + d*x]]]*Sqrt[Sec[c + d*x]])*Sin[c + d*x])/(3*d*Sqrt[-1 + Cos[c + d*x]]*
Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4752, 3042, 4300, 27, 3042, 4503, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4300}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{3}a\int \frac{\sqrt{\sec(c+dx)a+a}(3\sec(c+dx)a+7a)}{2\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}\right.$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}a\int \frac{\sqrt{\sec(c+dx)a+a}(3\sec(c+dx)a+7a)}{\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(3\csc\left(c+dx+\frac{\pi}{2}\right)a+7a\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a}}{3d\sqrt{\sec(}}\right.$$

$$\Big\downarrow \text{ 4503}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}a\left(3a\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{14a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}\right)+\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}a\left(3a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{14a^2\sin(c+dx)\sqrt{\sec}}{d\sqrt{a\sec(c+dx)}}\right.\right.$$

$$\Big\downarrow \text{ 4288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}a\left(\frac{14a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{6a\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\right.$$

$$\Big\downarrow \text{ 222}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}+\frac{1}{3}a\left(\frac{6a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{14a^2\text{si}}{d\sqrt{}}\right.\right.$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sqrt[a + a*Sec[c + d*x]]*Sin
[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (a*((6*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[
c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (14*a^2*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

rule 4503
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Simp
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n)   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[
e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a
*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.30

method	result
default	$\frac{a^2\left(\left(4\cos(dx+c)+32\right)\sin(dx+c)+\left(3\cos(dx+c)+3\right)\sqrt{2}\,\sqrt{-\frac{2}{\cos(dx+c)+1}}\,\arctan\left(\frac{-\cot(dx+c)+\csc(dx+c)-1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+\left(3\cos(dx+c)+3\right)\sqrt{2}}{6d\left(\cos(dx+c)+1\right)}$

input
```
int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/d*a^2*((4*cos(d*x+c)+32)*sin(d*x+c)+(3*cos(d*x+c)+3)*2^(1/2)*(-2/(cos(
d*x+c)+1))^(1/2)*arctan(1/2*(-cot(d*x+c)+csc(d*x+c)-1)/(-1/(cos(d*x+c)+1))
^(1/2))+(3*cos(d*x+c)+3)*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/2/(-1/
(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1)))*cos(d*x+c)^(1/2)*(a*(1+
sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.46

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{4\left(a^2\cos\left(dx+c\right)+8\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+3\left(a^2\cos\left(dx+c\right)+a^2\right)}{6\left(d\cos\left(dx+c\right.\right.}\right.$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(4*(a^2*cos(d*x + c) + 8*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sqrt(cos(d*x + c))*sin(d*x + c) + 3*(a^2*cos(d*x + c) + a^2)*sqrt(a)*log(
(a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos
(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)
/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c) + d), 1/3*(2*(a^2*cos
(d*x + c) + 8*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
c))*sin(d*x + c) + 3*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*arctan(2*sqrt(-a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*
cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(116) = 232$.

Time = 0.25 (sec) , antiderivative size = 593, normalized size of antiderivative = 4.30

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/12*sqrt(2)*(30*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))*sin(3/2*d*x + 3/2*c) - 30*a^2*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*log(2*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2
*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 3*sqrt(2)*a^
2*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2
)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqr
t(2)*a^2*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2
*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*
sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2)
+ 4*a^2*sin(3/2*d*x + 3/2*c) + 30*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2...
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.41

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{\frac{3a^{\frac{7}{2}}\log\left(\left|\frac{2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a}{2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a}\right|\right)\mathrm{sgn}(\cos(dx+c))}{|a|}+\frac{2\left(7\sqrt{2}a^4\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}d\right.\right.}{\left(a\,\mathrm{ta}\right.}}{3d}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/3*(3*a^(7/2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a)
)*sgn(cos(d*x + c))/abs(a) + 2*(7*sqrt(2)*a^4*sgn(cos(d*x + c))*tan(1/2*d*
x + 1/2*c)^2 + 9*sqrt(2)*a^4*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)/(a*ta
n(1/2*d*x + 1/2*c)^2 + a)^(3/2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \cos\left(c+d\,x\right)^{3/2}\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx=\sqrt{a}\,a^{2}\left(\int \sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)^{2}\,dx+2\left(\int \sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)\,dx\right)+\int \sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)$$

input
```
int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)*s
ec(c + d*x)**2,x) + 2*int(sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c
+ d*x)*sec(c + d*x),x) + int(sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*cos
(c + d*x),x))
```

3.416 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 132

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx = \frac{5a^{5/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
5*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1
/2)*sec(d*x+c)^(1/2)/d+a^3*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^
(1/2)+a^2*(a+a*sec(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.68

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^3\left(5\arcsin\left(\sqrt{1-\sec(c+dx)}\right)\sqrt{\sec(c+dx)}+\sqrt{1-\sec(c+dx)}(2+\sec(c+dx))\right)\sin(c}{d\sqrt{-1+\cos(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^3*(5*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]] + Sqrt[1 - Sec[c
 + d*x]]*(2 + Sec[c + d*x]))*Sin[c + d*x])/(d*Sqrt[-1 + Cos[c + d*x]]*Sqrt
[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4752, 3042, 4301, 27, 3042, 4503, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \ 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^{5/2}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \ 4301$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(a\int \frac{\sqrt{\sec(c+dx)a+a}(5\sec(c+dx)a+a)}{2\sqrt{\sec(c+dx)}}dx+\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\,\mathrm{se}}}{d}\right.$$

$$\Big\downarrow \ 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}a\int\frac{\sqrt{\sec(c+dx)a+a}(5\sec(c+dx)a+a)}{\sqrt{\sec(c+dx)}}dx+\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a}}{d}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(5\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a^2\sin(c+dx)\sqrt{\sec(}}{}\right.$$

$\big\downarrow$ 4503

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}a\left(5a\int\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}\right)+\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}a\left(5a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a^2\sin(c+dx)\sqrt{\sec(}}{d\sqrt{a\sec(c+dx)}}\right.\right.$$

$\big\downarrow$ 4288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}a\left(\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{10a\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\right.$$

$\big\downarrow$ 222

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{d}+\frac{1}{2}a\left(\frac{10a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx}{\sqrt{a\sec(c+dx)}}\right.}{d}\right.\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a^2*Sqrt[Sec[c + d*x]]*Sqrt[a + a*
Sec[c + d*x]]*Sin[c + d*x])/d + (a*((10*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d
*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(d*Sqrt[a + a*Sec[c + d*x]])))/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4503
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Simp
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n)   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[
e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a
*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 3.57 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.49

method	result
default	$-\frac{a^2\left((-8\cos(dx+c)-4)\sin(dx+c)+\sqrt{2}\,\sqrt{-\frac{2}{\cos(dx+c)+1}}\,\arctan\left(\frac{\cot(dx+c)-\csc(dx+c)-1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\left(5\cos(dx+c)^2+5\cos(dx+c)\right)+\sqrt{2}\,\sqrt{}\right.}{4d\sqrt{\cos(dx+c)}\,(\cos(dx+c)+1}$

input
```
int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d*a^2*((-8*cos(d*x+c)-4)*sin(d*x+c)+2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)
*arctan(1/2*(cot(d*x+c)-csc(d*x+c)-1)/(-1/(cos(d*x+c)+1))^(1/2))*(5*cos(d*
x+c)^2+5*cos(d*x+c))+2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/2/(-1/(cos
(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1))*(5*cos(d*x+c)^2+5*cos(d*x+c))
)*(a*(1+sec(d*x+c)))^(1/2)/cos(d*x+c)^(1/2)/(cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.83

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{4\left(2\,a^2\cos\left(dx+c\right)+a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+5\left(a^2\cos\left(dx+c\right)^2+a\right.}{4\left(d\cos\left(dx+c\right.\right.}\right.$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/4*(4*(2*a^2*cos(d*x + c) + a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sqrt(cos(d*x + c))*sin(d*x + c) + 5*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c
))*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*
x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^2 + d*
cos(d*x + c)), 1/2*(2*(2*a^2*cos(d*x + c) + a^2)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 5*(a^2*cos(d*x + c)^2 + a
^2*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x +
c) - 2*a)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11494 vs. 2(114) = 228.

Time = 0.29 (sec) , antiderivative size = 11494, normalized size of antiderivative = 87.08

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/4*(8*a^2*cos(1/2*d*x + 1/2*c)^4*sin(1/2*d*x + 1/2*c) + 16*a^2*cos(1/2*d*
x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^3 + 8*a^2*sin(1/2*d*x + 1/2*c)^5 + 5*(sq
rt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a
^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(1/2*d*x + 1/2*c)^4 + 10*(sq
rt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a
^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(1/2*d*x + 1/2*c)^2*sin(1/2*
d*x + 1/2*c)^2 + 5*(sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(114) = 228$.

Time = 0.53 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.37

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx = \frac{\frac{4\sqrt{2}a^3\operatorname{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}} + 5a^{\frac{5}{2}}\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/2*(4*sqrt(2)*a^3*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a) + 5*a^(5/2)*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))*sgn(cos(d*x + c))
 - 5*a^(5/2)*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))*sgn(cos(d*x + c)) + 4*(3*sqrt(2)*(s
qrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(7/2
)*sgn(cos(d*x + c)) - sqrt(2)*a^(9/2)*sgn(cos(d*x + c)))/((sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx=\int \sqrt{\cos\left(c+d\,x\right)}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,dx$$

input
```
int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx=\sqrt{a}\,a^{2}\left(\int \sqrt{\sec\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}\,\sec\left(dx+c\right)^{2}dx+2\left(\int \sqrt{\sec\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}\,\sec\left(dx+c\right)dx\right)+\int \sqrt{\sec\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}dx\right)$$

input
```
int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*sec(c + d*x)**
2,x) + 2*int(sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*sec(c + d*x),x) + i
nt(sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x)),x))
```

3.417 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 140

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx=\frac{19a^{5/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d}+\frac{9a^3\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
19/4*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/d+9/4*a^3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d
*x+c))^(1/2)+1/2*a^2*(a+a*sec(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.68

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx=\frac{a^2\sqrt{a(1+\sec(c+dx))}\left(-\frac{19\arcsin\left(\sqrt{\sec(c+dx)}\right)}{\sqrt{\sec(c+dx)}}+\sqrt{1-\sec(c+dx)}(11+2\text{se}\right.}{4d\sqrt{-1+\cos(c+dx)}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

output
```
(a^2*Sqrt[a*(1 + Sec[c + d*x])]*((-19*ArcSin[Sqrt[Sec[c + d*x]]])/Sqrt[Sec
[c + d*x]] + Sqrt[1 - Sec[c + d*x]]*(11 + 2*Sec[c + d*x]))*Tan[(c + d*x)/2
])/(4*d*Sqrt[-1 + Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4752, 3042, 4301, 27, 3042, 4504, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{5/2}}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4301}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}a\int \frac{1}{2}\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}(9\sec(c+dx)a+5a)dx+\frac{a^2\sin(c+dx)\,\text{se}}{}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}a\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}(9\sec(c+dx)a+5a)dx+\frac{a^2\sin(c+dx)\,\text{sec}}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(9\csc\left(c+dx+\frac{\pi}{2}\right)a+5a\right)dx$$

$\downarrow$ 4504

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}a\left(\frac{19}{2}a\int\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{9a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}a\left(\frac{19}{2}a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{9a^2\sin(c+dx)\sec^{\frac{3}{2}}}{d\sqrt{a\sec(c+dx)}}\right.\right.$$

$\downarrow$ 4288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}a\left(\frac{9a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{19a\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\right.$$

$\downarrow$ 222

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}+\frac{1}{4}a\left(\frac{19a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+}}\right.}{d}\right.\right.$$

input
```
Int[(a + a*Sec[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a^2*Sec[c + d*x]^(3/2)*Sqrt[a + a*
Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (a*((19*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c
 + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (9*a^2*Sec[c + d*x]^(3/2)*Sin[c +
d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4504
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Simp[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1))   Int[Sqrt[a + b*Csc[e + f*
x]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&
!LtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 3.63 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.24

method	result
default	$\frac{a^2\left(-19\cos(dx+c)^2\arctan\left(\frac{-\cot(dx+c)+\csc(dx+c)-1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-19\cos(dx+c)^2\arctan\left(\frac{-\cot(dx+c)+\csc(dx+c)+1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+\sin(dx+c)(11\cos(dx}{8d(\cos(dx+c)+1)\cos(dx+c)^{\frac{3}{2}}\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*a^2*(-19*cos(d*x+c)^2*arctan(1/2*(-cot(d*x+c)+csc(d*x+c)-1)/(-1/(cos
(d*x+c)+1))^(1/2))-19*cos(d*x+c)^2*arctan(1/2/(-1/(cos(d*x+c)+1))^(1/2)*(-
cot(d*x+c)+csc(d*x+c)+1))+sin(d*x+c)*(11*cos(d*x+c)+2)*2^(1/2)*(-2/(cos(d*
x+c)+1))^(1/2))*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/cos(d*x+c)^(3/2)/(
-1/(cos(d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.75

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \left[\frac{4\left(11\,a^2\cos\left(dx+c\right)+2\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+19\,(}{}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(4*(11*a^2*cos(d*x + c) + 2*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c) + 19*(a^2*cos(d*x + c)^3 + a^2*cos(d*
x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a
*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)
^3 + d*cos(d*x + c)^2), 1/8*(2*(11*a^2*cos(d*x + c) + 2*a^2)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 19*(a^2*cos(d
*x + c)^3 + a^2*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2
 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(c + dx))^{5/2}}{\sqrt{\cos(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2826 vs. $2(116) = 232$.

Time = 2.83 (sec) , antiderivative size = 2826, normalized size of antiderivative = 20.19

$$\int \frac{(a + a\sec(c + dx))^{5/2}}{\sqrt{\cos(c + dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
-1/16*(88*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) - 56*sqrt(2)*a
^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 28*sqrt(2)*a^2*sin(3/2*d*x + 3/
2*c) + 44*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*(a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sq
rt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x +
4*c)^2 - 76*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^
2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(
1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c
) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 - 19*a^2*log(2*cos(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(116) = 232.

Time = 0.39 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.41

$$\int \frac{(a + a\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{2}a^{\frac{11}{2}}\left(\frac{19\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{a^2|a|} + \frac{8\left(19\left(\sqrt{a}\tan\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
1/16*sqrt(2)*a^(11/2)*(19*sqrt(2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(s
qrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sq
rt(2)*abs(a) - 6*a))/(a^2*abs(a)) + 8*(19*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6 - 171*(sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a + 89*(sqrt(a)*tan(1/2*d*x + 1/2*
c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^2 - 9*a^3)/(((sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2*a^2))*sgn
(cos(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx=\sqrt{a}\,a^2\left(\left(\int \frac{\sqrt{\sec\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}\,\sec\left(dx+c\right)^2}{\cos\left(dx+c\right)}dx\right)\right.$$
$$+2\left(\int \frac{\sqrt{\sec\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}\,\sec\left(dx+c\right)}{\cos\left(dx+c\right)}dx\right)$$
$$\left.+\int \frac{\sqrt{\sec\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)}dx\right)$$

input
```
int((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x)
```

output
```
sqrt(a)*a**2*(int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*sec(c + d*x)*
*2)/cos(c + d*x),x) + 2*int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*sec
(c + d*x))/cos(c + d*x),x) + int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x)
))/cos(c + d*x),x))
```

3.418 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 180

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{25a^{5/2}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8d} + \frac{13a^3\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{25a^3\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)}$$

output
```
25/8*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/d+13/12*a^3*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec
(d*x+c))^(1/2)+25/8*a^3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/
2)+1/3*a^2*(a+a*sec(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.64

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{a^3\left(\frac{75\arcsin\left(\sqrt{1-\sec(c+dx)}\right)}{\sec^{\frac{3}{2}}(c+dx)}+\sqrt{1-\sec(c+dx)}(34+75\cos(c+dx)+8\sec(c \cdots\right.}{24d\cos^{\frac{5}{2}}(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))} \cdots}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

output
```
(a^3*((75*ArcSin[Sqrt[1 - Sec[c + d*x]]])/Sec[c + d*x]^(3/2) + Sqrt[1 - Se
c[c + d*x]]*(34 + 75*Cos[c + d*x] + 8*Sec[c + d*x]))*Sin[c + d*x])/(24*d*C
os[c + d*x]^(5/2)*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4752, 3042, 4301, 27, 3042, 4504, 3042, 4290, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4301}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}a\int\frac{1}{2}\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}(13\sec(c+dx)a+9a)dx+\frac{a^2\sin(c+dx)\,\mathrm{s}}{}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}a\int\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}(13\sec(c+dx)a+9a)dx+\frac{a^2\sin(c+dx)\,\mathrm{sec}}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}a\int\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(13\csc\left(c+dx+\frac{\pi}{2}\right)a+9a\right)\right.$$

$$\Big\downarrow \text{4504}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}a\left(\frac{75}{4}a\int\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{13a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}a\left(\frac{75}{4}a\int\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{13a^2\sin(c+dx)\,\mathrm{se}}{2d\sqrt{a\sec(c+}}\right.\right.$$

$$\Big\downarrow \text{4290}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}a\left(\frac{75}{4}a\left(\frac{1}{2}\int\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}a\left(\frac{75}{4}a\left(\frac{1}{2}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\,\mathrm{sec}}{d\sqrt{a\sec(c+}}\right.\right.\right.$$

$$\Big\downarrow \text{4288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}a\left(\frac{75}{4}a\left(\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\right.\right.$$

$\downarrow$ 222

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}a\left(\frac{13a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}+\frac{75}{4}a\left(\frac{\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c-}{d\sqrt{a}}\right.\right.\right.$$

input
```
Int[(a + a*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a^2*Sec[c + d*x]^(5/2)*Sqrt[a + a*
Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (a*((13*a^2*Sec[c + d*x]^(5/2)*Sin[c +
 d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (75*a*((Sqrt[a]*ArcSinh[(Sqrt[a]*T
an[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sec[c + d*x]^(3/2)*Sin[c +
d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4))/6)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4504
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Simp[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1))   Int[Sqrt[a + b*Csc[e + f*
x]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&
!LtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 3.75 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.02

method	result
default	$\frac{a^2\left(-75\cos(dx+c)^3\arctan\left(\frac{-\cot(dx+c)+\csc(dx+c)-1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-75\cos(dx+c)^3\arctan\left(\frac{-\cot(dx+c)+\csc(dx+c)+1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+\sin(dx+c)\left(75\cos(d\right.\right.}{48d(\cos(dx+c)+1)\cos(dx+c)^{\frac{5}{2}}\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/d*a^2*(-75*cos(d*x+c)^3*arctan(1/2*(-cot(d*x+c)+csc(d*x+c)-1)/(-1/(co
s(d*x+c)+1))^(1/2))-75*cos(d*x+c)^3*arctan(1/2/(-1/(cos(d*x+c)+1))^(1/2)*(
-cot(d*x+c)+csc(d*x+c)+1))+sin(d*x+c)*(75*cos(d*x+c)^2+34*cos(d*x+c)+8)*2^
(1/2)*(-2/(cos(d*x+c)+1))^(1/2))*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/c
os(d*x+c)^(5/2)/(-1/(cos(d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.28

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left[\frac{4\left(75\,a^2\cos\left(dx+c\right)^2+34\,a^2\cos\left(dx+c\right)+8\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+\right.}}{}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/96*(4*(75*a^2*cos(d*x + c)^2 + 34*a^2*cos(d*x + c) + 8*a^2)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 75*(a^2*cos
(d*x + c)^4 + a^2*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x
+ c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x +
 c)^2)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3), 1/48*(2*(75*a^2*cos(d*x +
c)^2 + 34*a^2*cos(d*x + c) + 8*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sqrt(cos(d*x + c))*sin(d*x + c) + 75*(a^2*cos(d*x + c)^4 + a^2*cos(d*x +
 c)^3)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a))
)/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(5/2)/cos(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3469 vs. 2(150) = 300.

Time = 0.39 (sec) , antiderivative size = 3469, normalized size of antiderivative = 19.27

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
1/96*(300*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))*sin(6*d*x + 6*c) - 28*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 28*sqrt(
2)*a^2*sin(3/2*d*x + 3/2*c) - 28*(sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - sqrt(
2)*a^2*sin(3/2*d*x + 3/2*c))*cos(6*d*x + 6*c) - 300*(sqrt(2)*a^2*sin(6*d*x
 + 6*c) + 3*sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))))*cos(11/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) - 12*(7*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x +
3/2*c) - 114*sqrt(2)*a^2*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 114*sqrt(2)*a^2*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + 75*sqrt(2)*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) - 456*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*sin(4/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 456*(sqrt(2)*a^2*sin(6*d*x + 6*c) +
3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
)*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*(7*sqr
t(2)*a^2*sin(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 75*sq
rt(2)*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*co
s(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 75*(a^2*co...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(150) = 300$.

Time = 0.58 (sec) , antiderivative size = 472, normalized size of antiderivative = 2.62

$$\int \frac{(a + a\sec(c + dx))^{5/2}}{\cos^{\frac{3}{2}}(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
1/48*(75*a^(5/2)*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))*sgn(cos(d*x + c)) - 75*a^(5/2)*
log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)
)^2 + a*(2*sqrt(2) - 3)))*sgn(cos(d*x + c)) + 4*(75*sqrt(2)*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*a^(7/2)*sgn(cos(d
*x + c)) - 1125*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a))^8*a^(9/2)*sgn(cos(d*x + c)) + 6174*sqrt(2)*(sqrt(a)*tan(
1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*a^(11/2)*sgn(cos(
d*x + c)) - 4314*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))^4*a^(13/2)*sgn(cos(d*x + c)) + 807*sqrt(2)*(sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(15/2)*sgn(cos
(d*x + c)) - 49*sqrt(2)*a^(17/2)*sgn(cos(d*x + c)))/((sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\sqrt{a}\,a^2\left(\int \frac{\sqrt{\sec\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}\,\sec\left(dx+c\right)^2}{\cos\left(dx+c\right)^2}dx+2\left(\int \frac{\sqrt{\sec\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}\,\sec\left(dx+c\right)}{\cos\left(dx+c\right)^2}dx\right)+\int \frac{\sqrt{\sec\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^2}dx\right)$$

input
```
int((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x)
```

output
```
sqrt(a)*a**2*(int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*sec(c + d*x)*
*2)/cos(c + d*x)**2,x) + 2*int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*
sec(c + d*x))/cos(c + d*x)**2,x) + int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c
+ d*x)))/cos(c + d*x)**2,x))
```

3.419 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 220

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{163a^{5/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64d} + \frac{17a^3\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\sin(c+dx)}{64d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)}$$

output
```
163/64*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+
c)^(1/2)*sec(d*x+c)^(1/2)/d+17/24*a^3*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*s
ec(d*x+c))^(1/2)+163/96*a^3*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^(1/2)+163/64*a^3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+1/4
*a^2*(a+a*sec(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(7/2)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.57

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{a^3\left(\frac{489\arcsin\left(\sqrt{1-\sec(c+dx)}\right)}{\sec^{\frac{5}{2}}(c+dx)}+\sqrt{1-\sec(c+dx)}(184+326\cos(c+dx)+489\right.}{192d\cos^{\frac{7}{2}}(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{a(1+}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]
```

output
```
(a^3*((489*ArcSin[Sqrt[1 - Sec[c + d*x]]])/Sec[c + d*x]^(5/2) + Sqrt[1 - S
ec[c + d*x]]*(184 + 326*Cos[c + d*x] + 489*Cos[c + d*x]^2 + 48*Sec[c + d*x
]))*Sin[c + d*x])/(192*d*Cos[c + d*x]^(7/2)*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*
(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4752, 3042, 4301, 27, 3042, 4504, 3042, 4290, 3042, 4290, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}dx$$

$\downarrow$ 4301

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}a\int \frac{1}{2}\sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}(17\sec(c+dx)a+13a)dx+\frac{a^2\sin(c+dx)}{}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}a\int \sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}(17\sec(c+dx)a+13a)dx+\frac{a^2\sin(c+dx)\,\mathrm{se}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(17\csc\left(c+dx+\frac{\pi}{2}\right)a+13a\right)\right.$$

$\downarrow$ 4504

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}a\left(\frac{163}{6}a\int \sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{17a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}\right)\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}a\left(\frac{163}{6}a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{17a^2\sin(c+dx)\,\mathrm{s}}{3d\sqrt{a\sec(c+}}\right.\right.$$

$\downarrow$ 4290

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\int \sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\,\mathrm{s}}{2d\sqrt{a\sec(c}}\right.\right.\right.$$

$\downarrow$ 4290

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right.\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(}}\right.\right.\right.\right.$$

$\downarrow$ 4288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\left(\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right.\right.\right.\right.$$

$\downarrow$ 222

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}a\left(\frac{17a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{163}{6}a\left(\frac{3}{4}\left(\frac{\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\,\mathrm{si}}{}\right.\right.\right.\right.$$

input
```
Int[(a + a*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a^2*Sec[c + d*x]^(7/2)*Sqrt[a + a*
Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (a*((17*a^2*Sec[c + d*x]^(7/2)*Sin[c +
 d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (163*a*((a*Sec[c + d*x]^(5/2)*Sin[
c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (3*((Sqrt[a]*ArcSinh[(Sqrt[a]*T
an[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sec[c + d*x]^(3/2)*Sin[c +
d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4))/6))/8)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4504
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Simp[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1))   Int[Sqrt[a + b*Csc[e + f*
x]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&
!LtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 3.84 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.88

method	result
default	$\frac{a^2\left(-489\cos(dx+c)^4\arctan\left(\frac{-\cot(dx+c)+\csc(dx+c)+1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-489\cos(dx+c)^4\arctan\left(\frac{-\cot(dx+c)+\csc(dx+c)-1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+\sin(dx+c)\left(489\cos\right.\right.}{384d(\cos(dx+c)+1)\cos(dx+c)^{\frac{7}{2}}\sqrt{-\frac{1}{\cos(dx+c)}}}$

input
```
int((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/384/d*a^2*(-489*cos(d*x+c)^4*arctan(1/2/(-1/(cos(d*x+c)+1))^(1/2)*(-cot(
d*x+c)+csc(d*x+c)+1))-489*cos(d*x+c)^4*arctan(1/2*(-cot(d*x+c)+csc(d*x+c)-
1)/(-1/(cos(d*x+c)+1))^(1/2))+sin(d*x+c)*(489*cos(d*x+c)^3+326*cos(d*x+c)^
2+184*cos(d*x+c)+48)*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2))*(a*(1+sec(d*x+c)))
^(1/2)/(cos(d*x+c)+1)/cos(d*x+c)^(7/2)/(-1/(cos(d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.99

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left[\frac{4\left(489\,a^2\cos\left(dx+c\right)^3+326\,a^2\cos\left(dx+c\right)^2+184\,a^2\cos\left(dx+c\right)+48\,a^2\right.}{}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/768*(4*(489*a^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 184*a^2*cos(d
*x + c) + 48*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c) + 489*(a^2*cos(d*x + c)^5 + a^2*cos(d*x + c)^4)*sqrt(a)*lo
g((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(c
os(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*
a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^
4), 1/384*(2*(489*a^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 184*a^2*co
s(d*x + c) + 48*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c) + 489*(a^2*cos(d*x + c)^5 + a^2*cos(d*x + c)^4)*sqrt(-a
)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x +
 c)^5 + d*cos(d*x + c)^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(5/2)/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3860 vs. $2(184) = 368$.

Time = 0.43 (sec) , antiderivative size = 3860, normalized size of antiderivative = 17.55

$$\int \frac{(a + a\sec(c + dx))^{5/2}}{\cos^{\frac{5}{2}}(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
-1/768*(1956*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c
) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(1
5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 652*(sqrt(2)*a^2*sin(8*
d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*
c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(13/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 6204*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(
6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x +
2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2060*(sqrt(2
)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*si
n(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 2060*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(
2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*s
in(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 62
04*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt
(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(5/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 652*(sqrt(2)*a^2*sin(8*d*x + 8*c)
+ 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt
(2)*a^2*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - 1956*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c)
 + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{%%{[2309237210123256509497344,0]:[1,0,-2]%%},[35]%
%%},0]:[1
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \sqrt{a}\,a^2\left(\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\sec(dx+c)^2}{\cos(dx+c)^3}dx\right)\right.$$
$$+2\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\sec(dx+c)}{\cos(dx+c)^3}dx\right)$$
$$\left.+\int \frac{\sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)$$

input
```
int((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x)
```

output
```
sqrt(a)*a**2*(int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*sec(c + d*x)*
*2)/cos(c + d*x)**3,x) + 2*int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*
sec(c + d*x))/cos(c + d*x)**3,x) + int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c
+ d*x)))/cos(c + d*x)**3,x))
```

3.420 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 189

$$\begin{aligned}\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx &= -\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} \\ &\quad+\frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} \\ &\quad-\frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}+\frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

output
```
-2^(1/2)*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(
d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(1/2)/d+26/15*sin(d*x+c
)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)-2/15*cos(d*x+c)^(1/2)*sin(d*x+
c)/d/(a+a*sec(d*x+c))^(1/2)+2/5*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x
+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.72

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)\left(15\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sec^{\frac{5}{2}}(c+dx)+2\sqrt{1-\sec(c+dx)}(3-\sec(c+dx)+13\sec^2$$
$$15d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Cos[c + d*x]^(3/2)*(15*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1
 - Sec[c + d*x]]]*Sec[c + d*x]^(5/2) + 2*Sqrt[1 - Sec[c + d*x]]*(3 - Sec[c
 + d*x] + 13*Sec[c + d*x]^2))*Sin[c + d*x])/(15*d*Sqrt[1 - Sec[c + d*x]]*S
qrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4752, 3042, 4310, 3042, 4510, 27, 3042, 4501, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \; 4310$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{a-4a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{5a}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{a-4a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}\right)$$

$$\Big\downarrow \; 4510$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2\int -\frac{13a^2-2a^2\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}+\frac{2a\sin(}{3d\sqrt{\sec(c+dx)}\sqrt{}}}{5a}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{13a^2-2a^2\sec(c}{\sqrt{\sec(c+dx)}\sqrt{\sec(c}}}{3a}}{5a}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{13a^2-2a^2\csc}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{c}}}{3a}}{5a}\right.$$

$$\Big\downarrow \; 4501$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)\sqrt{\sec(c}}{d\sqrt{a\sec(c+dx)+c}}}{}}{5a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)\sqrt{\sec(c}}{d\sqrt{a\sec(c+dx)+c}}}{}}{5a}\right.$$

$\downarrow$ 4295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{30a^2\int\frac{1}{2a-\frac{a^2\sin(c+}{\sec(c}}}{}}{}\right.$$

$\downarrow$ 219

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)\sqrt{\sec(c}}{d\sqrt{a\sec(c+dx)+c}}}{}}{5a}\right.$$

input
```
Int[Cos[c + d*x]^(5/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*Sin[c + d*x])/(5*d*Sec[c + d*x]^
(3/2)*Sqrt[a + a*Sec[c + d*x]]) - ((2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*
x]]*Sqrt[a + a*Sec[c + d*x]]) - ((-15*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[a]*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d + (26
*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(3*a))
/(5*a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4310
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a +
b*Csc[e + f*x]])), x] + Simp[1/(2*b*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*((a
 + b*(2*n + 1)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.67

method	result
default	$\frac{\left(\left(6\cos(dx+c)^2-2\cos(dx+c)+26\right)\sin(dx+c)+(-15\cos(dx+c)-15)\arctan\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{2}{\cos(dx+c)+1}}\right)\sqrt{\mathrm{co}}}{15da(\cos(dx+c)+1)}$

input
```
int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15/d/a*((6*cos(d*x+c)^2-2*cos(d*x+c)+26)*sin(d*x+c)+(-15*cos(d*x+c)-15)*
arctan(1/2*2^(1/2)/(-1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)))*(-2
/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^(1/2)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x
+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.76

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \left[\frac{4\left(3\cos\left(dx+c\right)^2-\cos\left(dx+c\right)+13\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\frac{15\sqrt{2}(a\cos(dx+c)+a)\log\left(\cdots\right.}{\cdots}}{30\left(ad\cos\left(dx+c\right)+ad\right)} \right.$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/30*(4*(3*cos(d*x + c)^2 - cos(d*x + c) + 13)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 15*sqrt(2)*(a*cos(d*x + c)
 + a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*
x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d), 1/15*(1
5*sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*arctan(1/2*sqrt(2)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))*sin(d*x + c)/(cos
(d*x + c) + 1)) + 2*(3*cos(d*x + c)^2 - cos(d*x + c) + 13)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c
) + a*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(156) = 312$.

Time = 0.20 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.89

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/60*sqrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))*sin(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c)*sin(4/5*arc
tan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*cos(5/2*d*x + 5/2*c)
*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 30*log(cos
(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arct
an2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin
(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 1) + 30*log(cos(1/5*arctan2(si
n(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x
 + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*
c), cos(5/2*d*x + 5/2*c))) + 1) + 6*sin(5/2*d*x + 5/2*c) - 5*sin(3/5*arcta
n2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 60*sin(1/5*arctan2(sin(5
/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))/(sqrt(a)*d)
```

Giac [A] (verification not implemented)

Time = 172.56 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.66

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\left(\frac{15\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{a}}+\frac{2\left(\left(\left(17a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+20a^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+15a^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{5}{2}}}\right)}{15\,d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/15*sqrt(2)*(15*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a)))/sqrt(a) + 2*((17*a^2*tan(1/2*d*x + 1/2*c)^2 + 20*a^2)*
tan(1/2*d*x + 1/2*c)^2 + 15*a^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2
*c)^2 + a)^(5/2))/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx=\int \frac{{\cos\left(c+d\,x\right)}^{5/2}}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^{2}}{\sec(dx+c)+1}dx\right)}{a}$$

input
```
int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**2)/(
sec(c + d*x) + 1),x))/a
```

3.421 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 151

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} - \frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}$$

output
```
2^(1/2)*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d
*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(1/2)/d-2/3*sin(d*x+c)/d
/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2/3*cos(d*x+c)^(1/2)*sin(d*x+c)/d
/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.77

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$=\frac{\sqrt{\cos(c+dx)}\left(2(1-\sec(c+dx))^{3/2}-3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sec^{\frac{3}{2}}(c+dx)\right)\sin(c+dx)}{3d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*(2*(1 - Sec[c + d*x])^(3/2) - 3*Sqrt[2]*ArcTan[(Sqrt[2
]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]^(3/2))*Sin[c +
d*x])/(3*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 4752, 3042, 4310, 3042, 4501, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a\sec(c+dx)+a}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$\Big\downarrow$ 4310

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{a-2a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{a-2a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}\right)$$

$\Big\downarrow$ 4501

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-3a\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{3a}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-3a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}\right)$$

$\Big\downarrow$ 4295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{6a\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{3a}\right)$$

$\Big\downarrow$ 219

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{3\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(}{\sqrt{2}\sqrt{}}\right.}{d}}{3a}\right.$$

input
```
Int[Cos[c + d*x]^(3/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*Sin[c + d*x])/(3*d*Sqrt[Sec[c +
d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((-3*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[a]*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d + (2
*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(3*a))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4310
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a +
b*Csc[e + f*x]])), x] + Simp[1/(2*b*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*((a
 + b*(2*n + 1)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.77

method	result
default	$\frac{\left(\left(2\cos(dx+c)-2\right)\sin(dx+c)+\left(3\cos(dx+c)+3\right)\sqrt{-\frac{2}{\cos(dx+c)+1}}\,\arctan\left(\frac{\sqrt{2}\,\left(-\cot(dx+c)+\csc(dx+c)\right)}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\right)\sqrt{\cos(dx+c)}\,\sqrt{a(1+\sec(}}{3da(\cos(dx+c)+1)}$

input
```
int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d/a*((2*cos(d*x+c)-2)*sin(d*x+c)+(3*cos(d*x+c)+3)*(-2/(cos(d*x+c)+1))^
(1/2)*arctan(1/2*2^(1/2)/(-1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)
)))*cos(d*x+c)^(1/2)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.05

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \left[\frac{4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-1)\sqrt{\cos(dx+c)}\sin(dx+c)+\frac{3\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(d\ldots}{\cos(\ldots}}}{\ldots}}{\cos(\ldots}\right.}{\sqrt{a}}}{6\,(ad\cos(dx+c)+ad)}\right.$$

$$-\frac{3\sqrt{2}(a\cos(dx+c)+a)\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\,(\cos(dx+c)+1)}\right)-2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(\ldots}{3\,(ad\cos(dx+c)+ad)}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 1)*sqrt(co
s(d*x + c))*sin(d*x + c) + 3*sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x +
c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c)
)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x +
 c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d), -1/3*(3*sqrt(2)*(a*cos(d*x +
c) + a)*sqrt(-1/a)*arctan(1/2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sqrt(-1/a)*sqrt(cos(d*x + c))*sin(d*x + c)/(cos(d*x + c) + 1)) - 2*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 1)*sqrt(cos(d*x + c))
*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)**(3/2)/sqrt(a*(sec(c + d*x) + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(124) = 248$.

Time = 0.20 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.87

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx =$$
$$-\frac{3\sqrt{2}\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3\sqrt{2}\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\sin\left(\frac{2}{3}\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/6*(3*sqrt(2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*log(cos(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))/(sqrt(a)*d)
```

Giac [A] (verification not implemented)

Time = 171.79 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.58

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\left(\frac{4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\right)^{\frac{3}{2}}}+\frac{3\,\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{\sqrt{a}}\right)}{3\,d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/3*sqrt(2)*(4*a*tan(1/2*d*x + 1/2*c)^3/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3
/2) + 3*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a)))/sqrt(a))/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^{3/2}}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)}{\sec(dx+c)+1}dx\right)}{a}$$

input
```
int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x))/(sec
(c + d*x) + 1),x))/a
```

3.422 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 113

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

output
```
-2^(1/2)*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(
d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(1/2)/d+2*sin(d*x+c)/d/
cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\left(2\sqrt{1-\sec(c+dx)}+\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sqrt{\sec(c+dx)}\right)\sin(c+dx)}{d\sqrt{-1+\cos(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
((2*Sqrt[1 - Sec[c + d*x]] + Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/S
qrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]])*Sin[c + d*x])/(d*Sqrt[-1 + Cos[
c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4752, 3042, 4299, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4299}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx\right)$$

$$\Big\downarrow \text{4295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{219}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}\right)$$

input
```
Int[Sqrt[Cos[c + d*x]]/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]])/(Sqrt[a]*d)
) + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4299
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(m + 1))), x] + Simp[a*(m/(b*d*(m + 1)))   Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{\sqrt{\cos(dx+c)}\,\sqrt{a(1+\sec(dx+c))}\left(\arctan\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{2}{\cos(dx+c)+1}}+2\cot(dx+c)-2\csc(dx+c)\right)}{da}$	96

input
```
int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d/a*cos(d*x+c)^(1/2)*(a*(1+sec(d*x+c)))^(1/2)*(arctan(1/2*2^(1/2)/(-1/(
cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)+2
*cot(d*x+c)-2*csc(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.57

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \left[\frac{\frac{\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}+4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+}}{2\left(ad\cos(dx+c)+ad\right)}\right.$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a)
- 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) + 4*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a
*d*cos(d*x + c) + a*d), (sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*arctan(1/
2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x
+ c))*sin(d*x + c)/(cos(d*x + c) + 1)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sqrt{\cos{(c+dx)}}}{\sqrt{a(\sec{(c+dx)}+1)}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(cos(c + d*x))/sqrt(a*(sec(c + d*x) + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\right.}{2\sqrt{a}d}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))/
(sqrt(a)*d)
```

Giac [A] (verification not implemented)

Time = 166.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\left(\frac{\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{\sqrt{a}}+\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}\right)}{d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*(log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a)))/sqrt(a) + 2*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/2*c)^2
 + a))/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx=\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\sec(dx+c)+1}dx\right)}{a}$$

input
```
int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x)))/(sec(c + d*x) + 1
),x))/a
```

3.423 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 56

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx=\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
2^(1/2)*arctanh(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*sec(d
*x+c))^(1/2))/a^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.70

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$$
$$=-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
-((Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqr
t[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]
*Sqrt[a*(1 + Sec[c + d*x])]))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4752, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4295}$$

$$-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
(Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
 + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.62

method	result	size
default	$-\frac{\sqrt{2}\,\sqrt{a(1+\sec(dx+c))}\,\arctan\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{\cos(dx+c)}}{da(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$	91

input
```
int(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d/a*2^(1/2)*(a*(1+sec(d*x+c)))^(1/2)*arctan(1/2*2^(1/2)/(-1/(cos(d*x+c)
+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)))*cos(d*x+c)^(1/2)/(cos(d*x+c)+1)/(-1/(
cos(d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 3.02

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \left[\frac{\sqrt{2}\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2\sqrt{a}d}, -\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}\sin(dx+c)}{2(\cos(dx+c)+1)}\right)}{d}\right]$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/
(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/(sqrt(a)*d), -sqrt(2)*sqrt(-1/a)*ar
ctan(1/2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(c
os(d*x + c))*sin(d*x + c)/(cos(d*x + c) + 1))/d]
```

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sqrt{\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(sec(c + d*x) + 1))*sqrt(cos(c + d*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.61

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\right.}{2\sqrt{a}d}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))/(sqrt(a)*d)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{\sqrt{a}d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-sqrt(2)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a)))/(sqrt(a)*d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)\sec(dx+c)+\cos(dx+c)}dx\right)}{a}$$

input
```
int(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)*sec
(c + d*x) + cos(c + d*x)),x))/a
```

3.424 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} - \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d}$$

output
```
2*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(
d*x+c)^(1/2)/a^(1/2)/d-2^(1/2)*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*
x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(
1/2)/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.81

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\left(-2\arcsin\left(\sqrt{\sec(c+dx)}\right)+\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
((-2*ArcSin[Sqrt[Sec[c + d*x]]] + Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x
]])/Sqrt[1 - Sec[c + d*x]]])*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.86, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 4752, 3042, 4308, 3042, 4288, 222, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4308}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx}{a}-\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{a}-\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx\right)$$

$$\Big\downarrow \text{4288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{2\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{ad}\right)$$

$$\Big\downarrow \text{222}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx\right)$$

$$\Big\downarrow \text{4295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{2\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}\right)$$

$$\Big\downarrow \text{219}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}\right)$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
((2*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d)
- (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt
[a + a*Sec[c + d*x]])])/(Sqrt[a]*d))*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4308
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d/b   Int[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[
e + f*x]], x], x] - Simp[a*(d/b)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.20

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\,\sqrt{\cos(dx+c)}\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-\arctan\left(\frac{-\cot(dx+c)+\csc(dx+c)+1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-\arctan\left(\frac{-\cot(d}{2\sqrt{}}\right.\right.}{da(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(a*(1+sec(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)*(2^(1/2)*arctan(1/2*2^(1/2
)/(-1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)))-arctan(1/2/(-1/(cos(
d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1))-arctan(1/2*(-cot(d*x+c)+csc(d
*x+c)-1)/(-1/(cos(d*x+c)+1))^(1/2)))/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1
/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.60

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$$
$$=\left[\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\cos(dx}{\cos(d}}\right.}{2\,ad}\right.$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(2)*sqrt(a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x +
 c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + sqrt(a)*log((a*cos(d*x +
 c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) -
2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x +
c)^3 + cos(d*x + c)^2)))/(a*d), (sqrt(2)*a*sqrt(-1/a)*arctan(1/2*sqrt(2)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))*sin(d
*x + c)/(cos(d*x + c) + 1)) + sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 -
 a*cos(d*x + c) - 2*a)))/(a*d)]
```

Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(sec(c + d*x) + 1))*cos(c + d*x)**(3/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(110) = 220$.

Time = 0.24 (sec) , antiderivative size = 476, normalized size of antiderivative = 3.53

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2
*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c))) + 1) - sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x +
c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sq
rt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arc
tan2(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c
), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2
*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c))) + 2) - log(2*cos(1/2*arctan2(sin(d*x
+ c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
- 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d
*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*si
n(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2))/(sqrt(a)*d)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.27

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{a}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-4\sqrt{2}|a|-6\,a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2+4\sqrt{2}|a|-6\,a\right|}\right)}{|a|}+\frac{\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2\right)}{\sqrt{a}}\right)}{2\,d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(sqrt(2)*sqrt(a)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a
)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)
*abs(a) - 6*a))/abs(a) + log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^2)/sqrt(a))/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^2\sec(dx+c)+\cos(dx+c)^2}dx\right)}{a}$$

input
```
int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**2*
sec(c + d*x) + cos(c + d*x)**2),x))/a
```

3.425 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 168

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

output
```
-arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/a^(1/2)/d+2^(1/2)*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x
+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(1
/2)/d+sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.15 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.86

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\left(\arcsin\left(\sqrt{1-\sec(c+dx)}\right)+2\arcsin\left(\sqrt{\sec(c+dx)}\right)-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{s}}{\sqrt{1-s}}\right.\right.}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(ArcSin[Sqrt[1 - Sec[c + d*x]]] + 2
*ArcSin[Sqrt[Sec[c + d*x]]] - Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/
Sqrt[1 - Sec[c + d*x]]] + Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Sin[c
 + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4752, 3042, 4309, 3042, 4511, 3042, 4288, 222, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{ 4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4309}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sec(c+dx)}(a-a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{2a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{4511}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx-\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx}{2a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c}{d\sqrt{a\sec(c+dx)}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{2a}+\frac{\sin(}{d\sqrt{}}\right.$$

$$\Big\downarrow \text{4288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{2\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{2a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c}{d\sqrt{a\sec(c+dx}}\right.$$

$$\Big\downarrow \text{222}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{2\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{2a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

$\Big\downarrow$ 4295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{4a\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{2\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{2a}+\text{si}\right.$$

$\Big\downarrow$ 219

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{2\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{2a}+\frac{\sin(c+dx)\text{sec}}{d\sqrt{a\sec(c+}}\right.$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((-2*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c
 + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[a
]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d)
/(2*a) + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4309
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(
f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[d^2/(b*(2*n - 3))   Int[(
d*Csc[e + f*x])^(n - 2)*((2*b*(n - 2) - a*Csc[e + f*x])/Sqrt[a + b*Csc[e +
f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n,
2] && IntegerQ[2*n]
```

rule 4511
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b -
a*B)/b   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Simp[B/b
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b
, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.23

method	result
default	$-\frac{\left(-\sin(dx+c)\sqrt{2}\,\sqrt{-\frac{2}{\cos(dx+c)+1}}+2\sqrt{2}\,\cos(dx+c)\arctan\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-\cos(dx+c)\arctan\left(\frac{-\cot(dx+c)+\csc(}{2\sqrt{-\frac{1}{\cos(dx+c}}}\right.\right.}{2d\sqrt{-\frac{1}{\cos(dx+c)+1}}\,\sqrt{\cos(dx+c)}\,(\cos(dx+c)+1)a}$

input
```
int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*(-sin(d*x+c)*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)+2*2^(1/2)*cos(d*x+c)
*arctan(1/2*2^(1/2)/(-1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)))-co
s(d*x+c)*arctan(1/2/(-1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1))-
cos(d*x+c)*arctan(1/2*(-cot(d*x+c)+csc(d*x+c)-1)/(-1/(cos(d*x+c)+1))^(1/2)
))/(-1/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^(1/2)/(cos(d*x+c)+1)/a*(a*(1+sec(d
*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 529, normalized size of antiderivative = 3.15

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \left[\frac{\left(\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3+4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(dx+c)}\sin(dx+c)-7a\cos(dx-}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{4} \right.$$

$$-\frac{2\sqrt{2}\left(a\cos\left(dx+c\right)^2+a\cos\left(dx+c\right)\right)\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(\cos(dx+c)+1\right)}\right)+\left(\cos\left(dx\right.\right.}{2\left(ad\cos\left(\right.\right.}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 + 4*sq
rt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(
d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d
*x + c)^2)) + 2*sqrt(2)*(a*cos(d*x + c)^2 + a*cos(d*x + c))*log(-(cos(d*x
+ c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x
 + c) + 1))/sqrt(a) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c)), -1/2*(2*sq
rt(2)*(a*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(-1/a)*arctan(1/2*sqrt(2)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))*sin(d*
x + c)/(cos(d*x + c) + 1)) + (cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*arct
an(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) - 2*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c
)^2 + a*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 876 vs. $2(139) = 278$.

Time $= 0.22$ (sec) , antiderivative size $= 876$, normalized size of antiderivative $= 5.21$

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2
*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2
*c) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*l
og(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2)
- (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2
*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
 + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos
(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2
*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
- 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 2*(sqrt(2)
*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2
*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(139) = 278$.

Time = 0.25 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.78

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx =$$

$$-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{a}\log\left(\left|\frac{2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-4\sqrt{2}|a|-6\,a}{2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2+4\sqrt{2}|a|-6\,a}\right|\right)}{|a|}+\frac{2\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)\right.}{\sqrt{a}}\right.}{4\,d\mathrm{sgn}\left(\cos\left(dx+c\right.\right.}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(sqrt(2)*sqrt(a)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2
)*abs(a) - 6*a))/abs(a) + 2*log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan
(1/2*d*x + 1/2*c)^2 + a))^2)/sqrt(a) - 8*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(a) - a^(3/2))/((sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))/(d*sgn(c
os(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^{3}\sec(dx+c)+\cos(dx+c)^{3}}dx\right)}{a}$$

input
```
int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**3*
sec(c + d*x) + cos(c + d*x)**3),x))/a
```

3.426 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 211

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{7\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4\sqrt{a}d} - \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

output
```
7/4*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*se
c(d*x+c)^(1/2)/a^(1/2)/d-2^(1/2)*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(
d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a
^(1/2)/d+1/2*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)-1/4*sin(
d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.84

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\left(-\arcsin\left(\sqrt{1-\sec(c+dx)}\right)-8\arcsin\left(\sqrt{\sec(c+dx)}\right)+4\sqrt{2}\arctan\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{4d\sqrt{1-\sec(c+dx)}\sqrt{}}$$

input
```
Integrate[1/(Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(-ArcSin[Sqrt[1 - Sec[c + d*x]]] -
8*ArcSin[Sqrt[Sec[c + d*x]]] + 4*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]
])/Sqrt[1 - Sec[c + d*x]]] + 2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) -
 Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Sin[c + d*x])/(4*d*Sqrt[1 - Se
c[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4752, 3042, 4309, 3042, 4509, 27, 3042, 4511, 3042, 4288, 222, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \textbf{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4309}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)(3a-a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{4a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{4509}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int -\frac{\sqrt{\sec(c+dx)}\left(a^2-7a^2\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)a+a}}dx}{a}-\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-7a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{2a}-\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2-7a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}-\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+
dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{4511}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{8a^2\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx-7a\int\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx}{2a}-\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a}+\frac{\sin(c}{2d\sqrt{}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{8a^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-7a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{2a}-\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a}\right.$$

$$\Big\downarrow \text{4288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{8a^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{14a\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{2a}-\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a}\right.$$

$$\Big\downarrow \text{222}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{8a^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{14a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{2a}-\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a}+\frac{\sin(}{2d}\right.$$

$$\Big\downarrow \text{4295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{-\frac{16a^2\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{14a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{2a}-}{4a}\right.$$

$$\Big\downarrow \text{219}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{\frac{8\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{14a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{2a}-\frac{a\sin(c+dx)\sec^{\frac{3}{2}}}{d\sqrt{a\sec(c+dx}}}{4a}\right.$$

input
```
Int[1/(Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
2*d*Sqrt[a + a*Sec[c + d*x]]) + (-1/2*((-14*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c
 + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (8*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[a
]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d)
/a - (a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(4*
a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4309
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(
f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[d^2/(b*(2*n - 3))   Int[(
d*Csc[e + f*x])^(n - 2)*((2*b*(n - 2) - a*Csc[e + f*x])/Sqrt[a + b*Csc[e +
f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n,
2] && IntegerQ[2*n]
```

rule 4509
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Simp[d/(b*(m + n))   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
 1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[n, 1]
```

rule 4511
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b -
a*B)/b   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Simp[B/b
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b
, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.05

method	result
default	$\frac{\left(8\sqrt{2}\,\cos(dx+c)^2 \arctan\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-7\cos(dx+c)^2 \arctan\left(\frac{-\cot(dx+c)+\csc(dx+c)-1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-7\cos(dx+c)^2 \arctan\left(\right.}{8da\cos(dx+c)^{\frac{3}{2}}(\cos(dx+c)+1)\sqrt{-\frac{}{\cos(}}}$

input
```
int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d/a*(8*2^(1/2)*cos(d*x+c)^2*arctan(1/2*2^(1/2)/(-1/(cos(d*x+c)+1))^(1/
2)*(-cot(d*x+c)+csc(d*x+c)))-7*cos(d*x+c)^2*arctan(1/2*(-cot(d*x+c)+csc(d*
x+c)-1)/(-1/(cos(d*x+c)+1))^(1/2))-7*cos(d*x+c)^2*arctan(1/2/(-1/(cos(d*x+
c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1))+sin(d*x+c)*(-cos(d*x+c)+2)*2^(1/2
)*(-2/(cos(d*x+c)+1))^(1/2))*(a*(1+sec(d*x+c)))^(1/2)/cos(d*x+c)^(3/2)/(co
s(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 559, normalized size of antiderivative = 2.65

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*(4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(
cos(d*x + c))*sin(d*x + c) - 7*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*l
og((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(
cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8
*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 8*sqrt(2)*(a*cos(d*x + c)^3 + a*c
os(d*x + c)^2)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c)^3 + a*
d*cos(d*x + c)^2), 1/8*(8*sqrt(2)*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sq
rt(-1/a)*arctan(1/2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-
1/a)*sqrt(cos(d*x + c))*sin(d*x + c)/(cos(d*x + c) + 1)) - 2*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x +
 c) + 7*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(
d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c
)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1646 vs. $2(172) = 344$.

Time = 0.26 (sec) , antiderivative size = 1646, normalized size of antiderivative = 7.80

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/16*(4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*ar
ctan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt
(2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(s
qrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(
d*x + c), cos(d*x + c))) - 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d
*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 7*(2*(2*cos(2*d*
x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2
 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x
+ 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(
2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*
d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^
2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d
*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*si
n(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(s
in(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(
d*x + c))) + 2) + 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d
*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*lo...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(172) = 344$.

Time = 0.30 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.80

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{\sqrt{2}\left(\frac{7\sqrt{2}\sqrt{a}\log\left(\left|\frac{2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-4\sqrt{2}|a|-6\,a}{2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2+4\sqrt{2}|a|-6\,a}\right|\right)}{|a|} + \frac{8\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2\right.}{\sqrt{a}}\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/16*sqrt(2)*(7*sqrt(2)*sqrt(a)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt
(2)*abs(a) - 6*a))/abs(a) + 8*log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a))^2)/sqrt(a) - 8*(17*(sqrt(a)*tan(1/2*d*x + 1/2*
c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(a) - 57*(sqrt(a)*tan(1/2*d
*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(3/2) + 19*(sqrt(a)*
tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(5/2) - 3*a
^(7/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
))^2*a + a^2)^2)/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^{4}\sec(dx+c)+\cos(dx+c)^{4}}dx\right)}{a}$$

input
```
int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**4*
sec(c + d*x) + cos(c + d*x)**4),x))/a
```

3.427 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 237

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{15\text{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{49\sin(c+dx)}{10ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{13\sqrt{\cos(c+dx)}\sin(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}} + \frac{9\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}}$$

output

```
-15/4*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d*x
+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/a^(3/2)/d-1/2*cos(d*
x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)+49/10*sin(d*x+c)/a/d/cos(d*
x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)-13/10*cos(d*x+c)^(1/2)*sin(d*x+c)/a/d/(a
+a*sec(d*x+c))^(1/2)+9/10*cos(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))
^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.64

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{75\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)+\sqrt{1}}{10d\sqrt{-1+\cos(c+}}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(75*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Co
s[(c + d*x)/2]^2*Sec[c + d*x]^(3/2)*Sin[c + d*x] + Sqrt[1 - Sec[c + d*x]]*
(4*(9 + Cos[c + d*x]^2)*Sin[c + d*x] - 2*Sin[2*(c + d*x)] + 49*Tan[c + d*x
]))/(10*d*Sqrt[-1 + Cos[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.08, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4752, 3042, 4304, 27, 3042, 4510, 27, 3042, 4510, 27, 3042, 4501, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$\big\downarrow$ 4304

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{3(3a-2a\sec(c+dx))}{2\sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}\right)$$

$\big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\int \frac{3a-2a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}\right)$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\int \frac{3a-2a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}\right)$$

$\big\downarrow$ 4510

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{2\int -\frac{13a^2-12a^2\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{5a}+\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c}{2d\sec^{\frac{3}{2}}(c+dx)(a}\right.$$

$\big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{13a^2-12a^2\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{5a}\right)}{4a^2}-\frac{\sin(c+}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{13a^2-12a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}\right)}{4a^2}-\frac{\mathrm{s}}{2d\sec^{\frac{3}{2}}(c+d}\right.$$

$\downarrow$ 4510

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2\int-\frac{49a^3-26a^3\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}+\frac{26a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}}{5a}\right.}{4a^2}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{49a^3-26a^3\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}}{5a}\right)}{4a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{49a^3-26a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+}}}{3a}}{5a}\right.}{4a^2}\right.$$

$\downarrow$ 4501

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{98a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-75a^3\int\ldots}{3a}}{5a}\ldots\right)}{4a^2}\ldots\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{98a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-75a^3\int\ldots}{3a}}{5a}\ldots\right)}{4a^2}\ldots\right.$$

$\downarrow$ 4295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{150a^3\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\ldots}{d}\ldots}{5a}\ldots\right)}{4a^2}\ldots\right.$$

$\downarrow$ 219

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{98a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{75\sqrt{2}a^{5/}}{}}{}}{5a}\right.}{4a^2}\right.$$

input
```
Int[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*Sin[c + d*x]/(d*Sec[c + d*x]^(
3/2)*(a + a*Sec[c + d*x])^(3/2)) + (3*((6*a*Sin[c + d*x])/(5*d*Sec[c + d*x
]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) - ((26*a^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c
 + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((-75*Sqrt[2]*a^(5/2)*ArcTanh[(Sqrt[a
]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d
+ (98*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(
3*a))/(5*a)))/(4*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.66

method	result
default	$\frac{\left(\left(8\cos(dx+c)^3-8\cos(dx+c)^2+72\cos(dx+c)+98\right)\sin(dx+c)+\left(-75\cos(dx+c)^2-150\cos(dx+c)-75\right)\arctan\left(\frac{\sqrt{2}\,(-\cot(dx+c)+c}{2\sqrt{-\frac{1}{\cos(dx+}}}\right.}{20d\,a^2\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)}$

input
```
int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/20/d/a^2*((8*cos(d*x+c)^3-8*cos(d*x+c)^2+72*cos(d*x+c)+98)*sin(d*x+c)+(-
75*cos(d*x+c)^2-150*cos(d*x+c)-75)*arctan(1/2*2^(1/2)/(-1/(cos(d*x+c)+1))^
(1/2)*(-cot(d*x+c)+csc(d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^(1/2
)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)^2+2*cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.72

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{75\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\,\mathrm{co}}{\mathrm{c}}}}{\mathrm{co}}\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/40*(75*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*co
s(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 +
 2*cos(d*x + c) + 1)) + 4*(4*cos(d*x + c)^3 - 4*cos(d*x + c)^2 + 36*cos(d*
x + c) + 49)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/20*(7
5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(1/2*sqrt(2
)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c)/(a*cos(d*x + c) + a)) + 2*(4*cos(d*x + c)^3 - 4*cos(d*x + c)^2 +
36*cos(d*x + c) + 49)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
 + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [A] (verification not implemented)

Time = 171.79 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.77

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\frac{\left(\left(\left(\frac{5\sqrt{2}a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{\mathrm{sgn}(\cos(dx+c))}+\frac{127\sqrt{2}a}{\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{175\sqrt{2}a}{\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{85}{\mathrm{sgn}(\mathrm{co}}}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\right)^{\frac{5}{2}}}}{20\,d}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/20*((((5*sqrt(2)*a*tan(1/2*d*x + 1/2*c)^2/sgn(cos(d*x + c)) + 127*sqrt(2
)*a/sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2 + 175*sqrt(2)*a/sgn(cos(d*x
+ c)))*tan(1/2*d*x + 1/2*c)^2 + 85*sqrt(2)*a/sgn(cos(d*x + c)))*tan(1/2*d*
x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(5/2) + 75*sqrt(2)*log(abs(-sqrt
(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(a^(3/2)*s
gn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^{5/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2}{\sec(dx+c)^2+2\sec(dx+c)+1}dx\right)}{a^2}$$

input
```
int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**2)/(
sec(c + d*x)**2 + 2*sec(c + d*x) + 1),x))/a**2
```

3.428 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 197

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{11\text{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{19\sin(c+dx)}{6ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{7\sqrt{\cos(c+dx)}\sin(c+dx)}{6ad\sqrt{a+a\sec(c+dx)}}$$

output
```
11/4*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d*x+
c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/a^(3/2)/d-1/2*cos(d*x
+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)-19/6*sin(d*x+c)/a/d/cos(d*x+
c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+7/6*cos(d*x+c)^(1/2)*sin(d*x+c)/a/d/(a+a*s
ec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.68

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\left((-12+4\cos(c+dx)-19\sec(c+dx))\sqrt{1-\sec(c+dx)}-33\sqrt{2}\arctan\left(\right.\right.}{6d\sqrt{-1+\cos(c+dx)}(a(1+\sec}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(((-12 + 4*Cos[c + d*x] - 19*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]] - 33*Sqr
t[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos[(c +
d*x)/2]^2*Sec[c + d*x]^(3/2))*Sin[c + d*x])/(6*d*Sqrt[-1 + Cos[c + d*x]]*(
a*(1 + Sec[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4752, 3042, 4304, 27, 3042, 4510, 27, 3042, 4501, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4304}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{7a-4a\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{7a-4a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{7a-4a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{4510}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\int -\frac{19a^2-14a^2\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}+\frac{14a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{14a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{19a^2-14a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{14a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{19a^2-14a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{\sin(c+}{2d\sqrt{\sec(c+dx)}(a\,\mathrm{se}}\right.$$

$\downarrow$ 4501

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{14a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{38a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-33a^2\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{}{2d\sqrt{\sec(c}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{14a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{38a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-33a^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{}{2d\sqrt{\mathrm{se}}}\right.$$

$\downarrow$ 4295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{14a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{66a^2\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{38a^2\sin}{d\sqrt{}}}{3a}}{4a^2}\right.$$

$\downarrow$ 219

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{14a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{38a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{33\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+d}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right.}{d}}{3a}}{4a^2}\right.$$

input
```
Int[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*Sin[c + d*x]/(d*Sqrt[Sec[c + d
*x]]*(a + a*Sec[c + d*x])^(3/2)) + ((14*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c +
d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((-33*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[a]*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d + (
38*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(3*a
))/(4*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.75

method	result
default	$\frac{\left(\left(8\cos(dx+c)^2-24\cos(dx+c)-38\right)\sin(dx+c)+\left(-33\cos(dx+c)^2-66\cos(dx+c)-33\right)\sqrt{-\frac{2}{\cos(dx+c)+1}}\arctan\left(\frac{\sqrt{2}\,(\cot(dx+c)-\mathrm{cs}}{2\sqrt{-\frac{1}{\cos(dx+}}}\right.\right.}{12d\,a^2\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)}$

input
```
int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/d/a^2*((8*cos(d*x+c)^2-24*cos(d*x+c)-38)*sin(d*x+c)+(-33*cos(d*x+c)^2
-66*cos(d*x+c)-33)*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(cot(d*x+c
)-csc(d*x+c))/(-1/(cos(d*x+c)+1))^(1/2)))*cos(d*x+c)^(1/2)*(a*(1+sec(d*x+c
)))^(1/2)/(cos(d*x+c)^2+2*cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.97

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{33\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos}{c}}}{\cos}\right.}{}\right.$$

$$-\frac{33\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\,(a\cos(dx+c)+a)}\right)-2\,(4\cos}{12\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+\right.}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/24*(33*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*co
s(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 +
 2*cos(d*x + c) + 1)) + 4*(4*cos(d*x + c)^2 - 12*cos(d*x + c) - 19)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*
cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/12*(33*sqrt(2)*(cos(d*x
 + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*
x + c) + a)) - 2*(4*cos(d*x + c)^2 - 12*cos(d*x + c) - 19)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x +
 c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)**(3/2)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33960 vs. $2(162) = 324$.

Time = 0.61 (sec) , antiderivative size = 33960, normalized size of antiderivative = 172.39

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/12*(4*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(
3/2*d*x + 3/2*c) - 9*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 64*(cos(3*d*x + 3*c)^2*sin(3/2*d
*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 9*(cos(3*d*x + 3*c
)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))))*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^4 + 4*sin(3/2*d*x + 3/2*c)^5 + 4*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c)
 + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 9*(cos(3*d*x + 3*c)^2 + sin(3
*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
))*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 64*(co
s(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3
/2*c) - 9*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(5/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c)))^4 + 4*(2*cos(3*d*x + 3*c)^2*cos(3/2*d*x + 3/2*c
)*sin(3/2*d*x + 3/2*c) + 2*cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)^2*sin(3/2
*d*x + 3/2*c) + 2*cos(3*d*x + 3*c)*cos(3/2*d*x + 3/2*c)*sin(3/2*d*x + 3/2*
c) + 8*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3
/2*d*x + 3/2*c) - 9*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arctan2(sin(3...
```

Giac [A] (verification not implemented)

Time = 171.42 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.77

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\frac{\left(\left(\frac{3\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{\mathrm{sgn}(\cos(dx+c))}+\frac{46\sqrt{2}}{\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{27\sqrt{2}}{\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\right)^{\frac{3}{2}}}+\frac{33\sqrt{2}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,d\right.}\right.\right.}{a^{\frac{3}{2}}\,\mathrm{sgn}(\cos(dx+c))}}{12\,d}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-1/12*(((3*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/sgn(cos(d*x + c)) + 46*sqrt(2)/s
gn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2 + 27*sqrt(2)/sgn(cos(d*x + c)))*t
an(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) + 33*sqrt(2)*log(
abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(
a^(3/2)*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^{3/2}}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)}{\sec(dx+c)^2+2\sec(dx+c)+1}dx\right)}{a^2}$$

input
```
int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x))/(sec
(c + d*x)**2 + 2*sec(c + d*x) + 1),x))/a**2
```

3.429 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{7\text{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{5\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

output
```
-7/4*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d*x+
c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/a^(3/2)/d-1/2*sin(d*x
+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2)+5/2*sin(d*x+c)/a/d/cos(d*x+c
)^(1/2)/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{\sec(c+dx)}\left(7\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)+(5}{2d\sqrt{-1+\cos(c+dx)}(a(1+}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(7*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1
- Sec[c + d*x]]]*Cos[(c + d*x)/2]^2*Sec[c + d*x] + (5 + 4*Cos[c + d*x])*Sq
rt[(-1 + Cos[c + d*x])*Sec[c + d*x]^2])*Sin[c + d*x])/(2*d*Sqrt[-1 + Cos[c
 + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4752, 3042, 4304, 27, 3042, 4501, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$\Big\downarrow$ 4304

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{5a-2a\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{5a-2a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{5a-2a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$\Big\downarrow$ 4501

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{10a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-7a\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{10a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-7a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$\Big\downarrow$ 4295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{14a\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{10a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}-\frac{\sin(c+}{2d(a\sec}\right.$$

$\Big\downarrow$ 219

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{10a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{7\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}-\frac{\sin(c+dx)\sqrt{\sec(c-}{2d(a\sec(c+dx)+}\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*(Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^(3/2)) + ((-7*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[a]
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]])/d +
 (10*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(4*a
^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.78

method	result
default	$-\frac{\sqrt{\cos(dx+c)}\,\sqrt{-a(-1-\sec(dx+c))}\left(-(1-\cos(dx+c))^{3}\csc(dx+c)^{3}-7\arctan\left(\frac{\sqrt{2}\,(\cot(dx+c)-\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{2}{\cos(dx+c)+1}}-9\,c\right.}{4d\,a^{2}}$

input
```
int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/a^2*cos(d*x+c)^(1/2)*(-a*(-1-sec(d*x+c)))^(1/2)*(-(1-cos(d*x+c))^3*
csc(d*x+c)^3-7*arctan(1/2*2^(1/2)*(cot(d*x+c)-csc(d*x+c))/(-1/(cos(d*x+c)+
1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)-9*csc(d*x+c)+9*cot(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.34

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{7\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(\cdots}{\cos\cdots}}}{\cos\cdots}\right.}{8\left(a^2d\,\mathrm{co}\cdots\right.}\right.$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(
d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(4*cos(d*x
 + c) + 5)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*
d*cos(d*x + c) + a^2*d), 1/4*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) +
 1)*sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c) + a)) + 2*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*(4*cos(d*x + c) + 5)*sqrt(cos(d*x + c))*sin
(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos(c+dx)}}{\left(a\left(\sec(c+dx)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sqrt(cos(c + d*x))/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7176 vs. $2(128) = 256$.

Time = 0.27 (sec) , antiderivative size = 7176, normalized size of antiderivative = 45.71

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(4*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2
*c)^4 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^4 + 4*(7*log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c
) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^4 + 70*(log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^2 + 7*(log(cos(1/2*d*x
+ 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1
))*sin(1/2*d*x + 1/2*c)^4 - 8*sin(1/2*d*x + 1/2*c)^5 + 28*(7*(log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))
*cos(3/2*d*x + 3/2*c)^3 + 4*(21*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 +...
```

Giac [A] (verification not implemented)

Time = 167.16 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\frac{\left(\frac{\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a\mathrm{sgn}(\cos(dx+c))}+\frac{9\sqrt{2}}{a\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}+\frac{7\sqrt{2}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\right.}\right.\right.}{a^{\frac{3}{2}}\mathrm{sgn}(\cos(dx+c))}}{4\,d}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/4*((sqrt(2)*tan(1/2*d*x + 1/2*c)^2/(a*sgn(cos(d*x + c))) + 9*sqrt(2)/(a*
sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
) + 7*sqrt(2)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x +
 1/2*c)^2 + a)))/(a^(3/2)*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\sec(dx+c)^2+2\sec(dx+c)+1}dx\right)}{a^2}$$

input
```
int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x)))/(sec(c + d*x)**2
+ 2*sec(c + d*x) + 1),x))/a**2
```

3.430 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 117

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = \frac{3\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}$$

output

```
3/4*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d*x+c
))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/a^(3/2)/d-1/2*sin(d*x+
c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\left(2\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}+3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)(1+\sec(c+dx))\right)\sin(c+dx)}{4ad\sqrt{-1+\cos(c+dx)}(1+\cos(c+dx))\sqrt{\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)),x]
```

output
```
-1/4*((2*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])] + 3*Sqrt[2]*ArcTan[(Sqr
t[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x]))*Sin[c
 + d*x])/(a*d*Sqrt[-1 + Cos[c + d*x]]*(1 + Cos[c + d*x])*Sqrt[Sec[c + d*x]
]*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4752, 3042, 4298, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)}}{(\sec(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4298}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{4a}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$\Big\downarrow$ 4295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2ad}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$\Big\downarrow$ 219

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((3*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*
x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*
d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4298
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[m/(a*(2*m + 1))   Int[(a + b*Csc[e +
 f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LtQ[m, -2^(-1)]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

method	result
default	$-\frac{\left((3\cos(dx+c)+3)\arctan\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+\sqrt{-\frac{2}{\cos(dx+c)+1}}\,\sin(dx+c)\right)\sqrt{2}\,\sqrt{a(1+\sec(dx+c))}\,\sqrt{\cos(dx+c)}}{4d\,a^{2}(\cos(dx+c)+1)^{2}\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/a^2*((3*cos(d*x+c)+3)*arctan(1/2*2^(1/2)/(-1/(cos(d*x+c)+1))^(1/2)*
(-cot(d*x+c)+csc(d*x+c)))+(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*2^(1/2)*(a
*(1+sec(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)/(cos(d*x+c)+1)^2/(-1/(cos(d*x+c)+1
))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.97

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{3\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c}{\cdots}\right.}{8\left(a\cdots\right.}\right.$$

$$-\frac{3\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(a\cos(dx+c)+a\right)}\right)+2\sqrt{\frac{a\cos(d\cdots}{\cos(\cdots}}}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(
d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*
d), -1/4*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(
1/2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c)/(a*cos(d*x + c) + a)) + 2*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*
d*cos(d*x + c) + a^2*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a\left(\sec(c+dx)+1\right)\right)^{\frac{3}{2}}\sqrt{\cos(c+dx)}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(1/((a*(sec(c + d*x) + 1))**(3/2)*sqrt(cos(c + d*x))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1031 vs. $2(94) = 188$.

Time = 0.23 (sec) , antiderivative size = 1031, normalized size of antiderivative = 8.81

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/4*(3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*
sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos
(d*x + c)^2 + 3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
+ 1))*sin(d*x + c)^2 + 2*(6*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/
2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 3*log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1
/2*c) + 1) - 2*sin(3/2*d*x + 3/2*c) + 2*sin(1/2*d*x + 1/2*c))*cos(2*d*x +
2*c) + 4*(3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 2*sin(1/2*d*x + 1/2*c))*cos(d*x + c) + 4
*(3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*...
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\frac{3\sqrt{2}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right|\right)}{a^{\frac{3}{2}}\operatorname{sgn}(\cos(dx+c))}+\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{2}\operatorname{sgn}(\cos(dx+c))}}{4\,d}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-1/4*(3*sqrt(2)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a)))/(a^(3/2)*sgn(cos(d*x + c))) + sqrt(2)*sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a^2*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)\sec(dx+c)^2+2\cos(dx+c)\sec(dx+c)+\cos(dx+c)}dx\right)}{a^2}$$

input
```
int(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)*sec
(c + d*x)**2 + 2*cos(c + d*x)*sec(c + d*x) + cos(c + d*x)),x))/a**2
```

3.431 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 117

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} + \frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}$$

output
```
1/4*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d*x+c
))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/a^(3/2)/d+1/2*sin(d*x+
c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 248 vs. 2(117) = 234.

Time = 0.61 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.12

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\left(-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)-\sqrt{2}\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(5/2)*(-(Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec
[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]) - Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c
+ d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos[c + d*x] + 2*ArcSin[Sqrt[1 - Sec[c +
d*x]]]*(1 + Cos[c + d*x]) + 2*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Cos[c + d*x]
) + Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + Cos[2*(c + d*x)]*Sqrt[1 -
Sec[c + d*x]]*Sec[c + d*x]^(3/2))*Sin[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]
]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4752, 3042, 4297, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(\sec(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4297}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{4a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$\Big\downarrow$ 4295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2ad}\right)$$

$\Big\downarrow$ 219

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) +
 (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4297
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[d*((m + 1)/(b*(2*m + 1)))
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ
[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m,
-2^(-1)] && IntegerQ[2*m]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.05

method	result
default	$-\frac{\left(-\sqrt{-\frac{2}{\cos(dx+c)+1}}\,\sin(dx+c)+\arctan\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)(\cos(dx+c)+1)\right)\sqrt{2}\,\sqrt{a(1+\sec(dx+c))}\,\sqrt{\cos(dx+c)}}{4d\,a^2(\cos(dx+c)+1)^2\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/a^2*(-(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+arctan(1/2*2^(1/2)/(-1/(
cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)))*(cos(d*x+c)+1))*2^(1/2)*(a*
(1+sec(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)/(cos(d*x+c)+1)^2/(-1/(cos(d*x+c)+1)
)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.96

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{\sqrt{2}\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-}{}\right.}{8\left(a^2d\right.}\right.$$

$$-\frac{\sqrt{2}\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\,(a\cos(dx+c)+a)}\right)-2\sqrt{\frac{a\cos(dx-}{\cos(dx}}}{4\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d*
x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*c
os(d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
 + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
, -1/4*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(1/2*
sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c)
)*sin(d*x + c)/(a*cos(d*x + c) + a)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*co
s(d*x + c) + a^2*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15721 vs. $2(94) = 188$.

Time = 0.71 (sec) , antiderivative size = 15721, normalized size of antiderivative = 134.37

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/4*(32*(cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c)*sin(3/2*
d*x + 3/2*c) + cos(d*x + c)*sin(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*si
n(d*x + c))*cos(3*d*x + 3*c)^2 + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)
 + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*
d*x + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*
sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*cos(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 96*(cos(3/2*d*x + 3/2*c
)*sin(3*d*x + 3*c) + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x
+ c) + 1)*sin(3/2*d*x + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3
*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x +
c))*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 32*(c
os(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*
c) + cos(d*x + c)*sin(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*sin(d*x + c)
)*sin(3*d*x + 3*c)^2 + 32*(6*cos(d*x + c) + 1)*cos(2*d*x + 2*c)*sin(3/2*d*
x + 3/2*c) + 96*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 96*sin(2*d*x + 2
*c)^2*sin(3/2*d*x + 3/2*c) + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 3
*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*d*x
+ 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(
3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*sin(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 96*(cos(3/2*d*x + 3/2*c)...
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\frac{\sqrt{2}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{a^{\frac{3}{2}}\operatorname{sgn}(\cos(dx+c))} - \frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2\operatorname{sgn}(\cos(dx+c))}}{4\,d}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-1/4*(sqrt(2)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x +
 1/2*c)^2 + a)))/(a^(3/2)*sgn(cos(d*x + c))) - sqrt(2)*sqrt(a*tan(1/2*d*x
+ 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a^2*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^2\sec(dx+c)^2+2\cos(dx+c)^2\sec(dx+c)+\cos(dx+c)^2}dx\right)}{a^2}$$

input
```
int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**2*
sec(c + d*x)**2 + 2*cos(c + d*x)**2*sec(c + d*x) + cos(c + d*x)**2),x))/a*
*2
```

3.432 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 174

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d} - \frac{5\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}$$

output
```
2*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(
d*x+c)^(1/2)/a^(3/2)/d-5/4*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)
*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)
/a^(3/2)/d-1/2*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.43

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx =$$

$$-\frac{\sqrt{\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\left(-5\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)-5\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos(c+dx)+2\,a}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

output
```
-1/4*(Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(5/2)*(-5*Sqrt[2]*ArcTan[(Sqrt[2]*Sq
rt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] - 5*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt
[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos[c + d*x] + 2*ArcSin[Sqrt[1 - S
ec[c + d*x]]]*(1 + Cos[c + d*x]) + 10*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Cos[
c + d*x]) + Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + Cos[2*(c + d*x)]*S
qrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2))*Sin[c + d*x])/(d*Sqrt[1 - Sec[c
+ d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.93, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4752, 3042, 4303, 27, 3042, 4511, 3042, 4288, 222, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(\sec(c+dx)a+a)^{3/2}}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$\Big\downarrow$ 4303

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\sec(c+dx)}(a-4a\sec(c+dx))}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\sec(c+dx)}(a-4a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a-4a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$\Big\downarrow$ 4511

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5a\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx-4\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx}{4a^2}-\frac{\sin(c+dx)\sec}{2d(a\sec(c+d}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-4\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{4a^2}-\right.$$

$\Big\downarrow$ 4288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{8\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}}{2d(a\sec(c+dx}\right.$$

$\downarrow$ 222

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{8\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right.$$

$\downarrow$ 4295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{-\frac{10a\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{8\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}-\right.$$

$\downarrow$ 219

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{5\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{8\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}-\frac{\sin(c+dx)\sec}{2d(a\sec(c+}\right.$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/4*((-8*Sqrt[a]*ArcSinh[(Sqrt[a]*
Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (5*Sqrt[2]*Sqrt[a]*ArcTanh[(S
qrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])
])/d)/a^2 - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3
/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4511
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b -
a*B)/b   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Simp[B/b
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b
, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 3.33 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.23

method	result
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\,\sqrt{\cos(dx+c)}\left(-5\sqrt{2}\,(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+(4\cos(dx+c)+4)\arctan\left(\frac{-\cot(}{2}\right.\right.}{4d\,a^2(\cos(dx+c)+1)^2\sqrt{-}}$

input
```
int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/a^2*(a*(1+sec(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)*(-5*2^(1/2)*(cos(d*x+
c)+1)*arctan(1/2*2^(1/2)/(-1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)
))+(4*cos(d*x+c)+4)*arctan(1/2*(-cot(d*x+c)+csc(d*x+c)-1)/(-1/(cos(d*x+c)+
1))^(1/2))+(4*cos(d*x+c)+4)*arctan(1/2/(-1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x
+c)+csc(d*x+c)+1))+sin(d*x+c)*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2))/(cos(d*x+
c)+1)^2/(-1/(cos(d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 558, normalized size of antiderivative = 3.21

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(
d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1)) + 4*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(
(a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos
(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)
/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos
(d*x + c) + a^2*d), 1/4*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*s
qrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c) + a)) + 4*(cos(d*x + c)^
2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos
(d*x + c) - 2*a)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
 + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2122 vs. $2(141) = 282$.

Time = 0.26 (sec) , antiderivative size = 2122, normalized size of antiderivative = 12.20

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/4*(4*(sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)
*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) +
 sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)
*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt
(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)
*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) +
 sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)
*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt
(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 2*(sqrt...
```

Giac [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.29

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\frac{5\sqrt{2}\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2\right)}{a^{\frac{3}{2}}\operatorname{sgn}(\cos(dx+c))} - \frac{2\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}}{a^2\operatorname{sgn}(c}}{}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/8*(5*sqrt(2)*log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^2)/(a^(3/2)*sgn(cos(d*x + c))) - 2*sqrt(2)*sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a^2*sgn(cos(d*x + c))) + 8*log(abs(
(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*
(2*sqrt(2) + 3)))/(a^(3/2)*sgn(cos(d*x + c))) - 8*log(abs((sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))
)/(a^(3/2)*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^3\sec(dx+c)^2+2\cos(dx+c)^3\sec(dx+c)+\cos(dx+c)^3}dx\right)}{a^2}$$

input
```
int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**3*
sec(c + d*x)**2 + 2*cos(c + d*x)**3*sec(c + d*x) + cos(c + d*x)**3),x))/a*
*2
```

3.433 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 214

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{3\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d} + \frac{9\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{3\sin(c+dx)}{2ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

output
```
-3*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/a^(3/2)/d+9/4*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c
)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(1/2
)/a^(3/2)/d-1/2*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2)+3/2*s
in(d*x+c)/a/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.27

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(6\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\,\mathrm{s}\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(6*Sqrt[1 - Sec[c + d*x]]*Sec[c + d
*x]^(3/2)*Sin[c + d*x] + 4*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c
 + d*x] - 9*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d
*x]]]*Tan[c + d*x] - 9*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1
- Sec[c + d*x]]]*Sec[c + d*x]*Tan[c + d*x] + 6*ArcSin[Sqrt[1 - Sec[c + d*x
]]]*(1 + Sec[c + d*x])*Tan[c + d*x] + 18*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + S
ec[c + d*x])*Tan[c + d*x]))/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*
x]))^(3/2))
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.95, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4752, 3042, 4303, 27, 3042, 4509, 25, 3042, 4511, 3042, 4288, 222, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(\sec(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4303}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{3\sec^{\frac{3}{2}}(c+dx)(a-2a\sec(c+dx))}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\int \frac{\sec^{\frac{3}{2}}(c+dx)(a-2a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{4509}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\int -\frac{\sqrt{\sec(c+dx)}\left(a^2-2a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+}{2d(a\sec(c+dx)+a}\right.$$

$$\Big\downarrow \text{25}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-2a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+}{2d(a\sec(c+dx)+}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2-2a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\text{se}}{2d(a\sec(c+}\right.$$

$\downarrow$ 4511

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(-\frac{3a^2\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx-2a\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(-\frac{3a^2\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-2a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}}{d\sqrt{a\sec(c+dx}}\right.}{4a^2}\right.$$

$\downarrow$ 4288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(-\frac{3a^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{4a\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(}{d\sqrt{a\sec(c+dx)}}\right.}{4a^2}\right.$$

$\downarrow$ 222

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(-\frac{3a^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{4a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}\right.$$

$\downarrow$ 4295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(-\frac{-\frac{6a^2\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{4a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}\right.}{4a^2}\right.$$

$\downarrow$ 219

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(-\frac{-\frac{3\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{4a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}-\frac{2a\sin(c+dx}{d\sqrt{a\sec}}\right.}{4a^2}\right.$$

input
```
Int[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*(Sec[c + d*x]^(5/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^(3/2)) - (3*(-(((-4*a^(3/2)*ArcSinh[(Sqrt[a]*T
an[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (3*Sqrt[2]*a^(3/2)*ArcTanh[(Sq
rt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]
)/d)/a) - (2*a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]
])))/(4*a^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4509
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Simp[d/(b*(m + n))   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
 1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[n, 1]
```

rule 4511
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b -
a*B)/b   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Simp[B/b
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b
, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.22

method	result
default	$-\frac{\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\left(9\cos(dx+c)^2+9\cos(dx+c)\right)+\arctan\left(\frac{-\cot(dx+c)+\csc(dx+c)-1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\left(-6\cos(dx+c)^2\right.}{4d\,a^2\sqrt{\cos(dx+c)}\,\left(\right.}$

input
```
int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/a^2*(2^(1/2)*arctan(1/2*2^(1/2)/(-1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x
+c)+csc(d*x+c)))*(9*cos(d*x+c)^2+9*cos(d*x+c))+arctan(1/2*(-cot(d*x+c)+csc
(d*x+c)-1)/(-1/(cos(d*x+c)+1))^(1/2))*(-6*cos(d*x+c)^2-6*cos(d*x+c))+arcta
n(1/2/(-1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1))*(-6*cos(d*x+c)
^2-6*cos(d*x+c))+sin(d*x+c)*(-2-3*cos(d*x+c))*2^(1/2)*(-2/(cos(d*x+c)+1))^
(1/2))*(a*(1+sec(d*x+c)))^(1/2)/cos(d*x+c)^(1/2)/(cos(d*x+c)^2+2*cos(d*x+c
)+1)/(-1/(cos(d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 622, normalized size of antiderivative = 2.91

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(9*sqrt(2)*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)
*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d
*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*(3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))*sin(d*x + c) + 6*(cos(d*x + c)^
3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 + 4*sqr
t(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d
*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*
x + c)^2)))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x
 + c)), -1/4*(9*sqrt(2)*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))
*sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c) + a)) - 2*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*(3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))*sin(d*
x + c) + 6*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*arc
tan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a^2*d*cos(d*x +
c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4934 vs. $2(175) = 350$.

Time = 0.44 (sec) , antiderivative size = 4934, normalized size of antiderivative = 23.06

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(12*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 8*(s
in(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - sin(3/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*
(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 12*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 3*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2
)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x
 + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*
d*x + 4*c) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + 2*
sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error index.cc index_gcd Error: Bad
 Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\cos{(c+d\,x)}^{7/2}\,{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^4\sec(dx+c)^2+2\cos(dx+c)^4\sec(dx+c)+\cos(dx+c)^4}dx\right)}{a^2}$$

input
```
int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**4*
sec(c + d*x)**2 + 2*cos(c + d*x)**4*sec(c + d*x) + cos(c + d*x)**4),x))/a*
*2
```

3.434 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 237

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{163\text{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{299\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{95\sqrt{\cos(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\sec(c+dx)}}$$

output
```
163/32*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d*
x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/a^(5/2)/d-1/4*cos(d
*x+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-17/16*cos(d*x+c)^(1/2)*sin
(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)-299/48*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2
)/(a+a*sec(d*x+c))^(1/2)+95/48*cos(d*x+c)^(1/2)*sin(d*x+c)/a^2/d/(a+a*sec(
d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.79 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.61

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx =$$
$$-\frac{\left(1956\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)+2\sqrt{1-\sec(c+dx)}(160-32\cos(c+dx}{96d\sqrt{-1+\cos(c+dx)}(a(1+\sec(c+dx)))^{5/2}}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
-1/96*((1956*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c +
d*x]]]*Cos[(c + d*x)/2]^4*Sec[c + d*x]^(5/2) + 2*Sqrt[1 - Sec[c + d*x]]*(1
60 - 32*Cos[c + d*x] + 503*Sec[c + d*x] + 299*Sec[c + d*x]^2))*Sin[c + d*x
])/(d*Sqrt[-1 + Cos[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.08, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4752, 3042, 4304, 27, 3042, 4508, 27, 3042, 4510, 27, 3042, 4501, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a\sec(c+dx)+a)^{5/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$
$$\Big\downarrow \text{4752}$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \; 4304$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{11a-6a\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{11a-6a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{11a-6a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \; 4508$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{95a^2-68a^2\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx}{4d\sqrt{\sec(c+dx)}(a\sec(c}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{95a^2-68a^2\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int\frac{95a^2-68a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(}{4d\sqrt{\sec(c+dx)}(}\right.$$

$\downarrow$ 4510

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\int-\frac{299a^3-190a^3\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}+\frac{190a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}}{4a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}}{8a^2}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{299a^3-190a^3\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{299a^3-190a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3}}}{8a^2}\right.$$

$\downarrow$ 4501

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{598a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-489a^3\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{17a\sin(}{2d\sqrt{\sec(c+dx)}(a\,s}}{8a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{598a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-489a^3\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{17a}{2d\sqrt{\sec(c+dx}}}{8a^2}\right.$$

$\downarrow$ 4295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{978a^3\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{598a^3\sin(c+}{d\sqrt{a\sec}}}{3a}}{4a^2}}{8a^2}\right.$$

$\downarrow$ 219

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{598a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{489\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{3a}}{4a^2}}{8a^2}\right.$$

input

```
Int[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

output

```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/4*Sin[c + d*x]/(d*Sqrt[Sec[c + d
*x]]*(a + a*Sec[c + d*x])^(5/2)) + ((-17*a*Sin[c + d*x])/(2*d*Sqrt[Sec[c +
 d*x]]*(a + a*Sec[c + d*x])^(3/2)) + ((190*a^2*Sin[c + d*x])/(3*d*Sqrt[Sec
[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((-489*Sqrt[2]*a^(5/2)*ArcTanh[(Sqr
t[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])
/d + (598*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]
))/(3*a))/(4*a^2))/(8*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.75

method	result
default	$\frac{\left(\left(64\cos(dx+c)^3-320\cos(dx+c)^2-1006\cos(dx+c)-598\right)\sin(dx+c)+\left(489\cos(dx+c)^3+1467\cos(dx+c)^2+1467\cos(dx+c)+489\right)\cdots}{96d\,a^3\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c\cdots\right.}$

input
```
int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/96/d/a^3*((64*cos(d*x+c)^3-320*cos(d*x+c)^2-1006*cos(d*x+c)-598)*sin(d*x
+c)+(489*cos(d*x+c)^3+1467*cos(d*x+c)^2+1467*cos(d*x+c)+489)*arctan(1/2*2^
(1/2)/(-1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)))*(-2/(cos(d*x+c)+
1))^(1/2))*cos(d*x+c)^(1/2)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)^3+3*cos(d
*x+c)^2+3*cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.92

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{489\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\,c\cdots}{\cdots}\right.\cdots}{\cdots}\right.$$

$$-\frac{489\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin\cdots}{2\,(a\cos(dx+c)+a)}\right.\cdots}{96\left(a^3d\cos(dx+c)^3+3a^3d\cdots\right.}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/192*(489*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) +
1)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3
*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(32*cos(d*x + c)^3 - 160*co
s(d*x + c)^2 - 503*cos(d*x + c) - 299)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(
d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/96*(489*sqrt(2)*(cos(d*x +
c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arctan(1/2*sqrt(2)*
sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c)/(a*cos(d*x + c) + a)) - 2*(32*cos(d*x + c)^3 - 160*cos(d*x + c)^2 -
 503*cos(d*x + c) - 299)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(
d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3
*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148823 vs. 2(196) = 392.

Time = 2.63 (sec) , antiderivative size = 148823, normalized size of antiderivative = 627.95

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/96*(32*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin
(3/2*d*x + 3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(11/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 41472*(cos(3*d*x + 3*c)^2*sin
(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 15*(cos(3*d*
x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^4 + 8192*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*
c)^2*sin(3/2*d*x + 3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 288*sin(3/2*d*x + 3/2*
c)^5 + 32*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*si
n(3/2*d*x + 3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(11/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 41472*(cos(3*d*x + 3*c)^2*si
n(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 15*(cos(3*d
*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))))*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))^4 + 8192*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3
*c)^2*sin(3/2*d*x + 3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^...
```

Giac [A] (verification not implemented)

Time = 173.23 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.81

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\frac{\left(\left(3\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a\mathrm{sgn}(\cos(dx+c))}-\frac{23\sqrt{2}}{a\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\frac{668\sqrt{2}}{a\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\frac{\cdots}{a\mathrm{sg}\cdots}}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\right)^{\frac{3}{2}}}}{96\,d}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/96*(((3*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/(a*sgn(cos(d*x + c))) - 23*sqr
t(2)/(a*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 - 668*sqrt(2)/(a*sgn(co
s(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 - 465*sqrt(2)/(a*sgn(cos(d*x + c))))*
tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) - 489*sqrt(2)*lo
g(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))
/(a^(5/2)*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^{3/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)}{\sec(dx+c)^3+3\sec(dx+c)^2+3\sec(dx+c)+1}dx\right)}{a^3}$$

input
```
int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x))/(sec
(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1),x))/a**3
```

3.435 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 197

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{75\text{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{13\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{49\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

output
```
-75/32*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d*
x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/a^(5/2)/d-1/4*sin(d
*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2)-13/16*sin(d*x+c)/a/d/cos(d
*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2)+49/16*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)
/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{150\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)+\sqrt{}}{16d\sqrt{-1+\cos(c+dx)}(a(}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(150*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*C
os[(c + d*x)/2]^4*Sec[c + d*x]^(5/2)*Sin[c + d*x] + Sqrt[1 - Sec[c + d*x]]
*(32*Sin[c + d*x] + (85 + 49*Sec[c + d*x])*Tan[c + d*x]))/(16*d*Sqrt[-1 +
Cos[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4752, 3042, 4304, 27, 3042, 4508, 27, 3042, 4501, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4304}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{9a-4a\sec(c+dx)}{2\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{9a-4a\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{9a-4a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \text{4508}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{49a^2-26a^2\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{13a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{49a^2-26a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{13a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{49a^2-26a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{13a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx}}{4d(a\sec(c+dx)+a)^{5/2}}\right.$$

$$\Big\downarrow \text{4501}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{98a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-75a^2\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{13a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{s}}{4d(a\sec(c+d}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{98a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-75a^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{13a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c}\right.$$

$$\Big\downarrow \text{4295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{150a^2\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{98a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}-\frac{13a\sin(c+dx}{2d(a\sec(c+}}{8a^2}\right.$$

$$\Big\downarrow \text{219}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{98a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{75\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}-\frac{13a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/4*(Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^(5/2)) + ((-13*a*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + ((-75*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt
[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/
d + (98*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))
/(4*a^2))/(8*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (warning: unable to verify)

Time = 2.32 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.73

method	result
default	$-\frac{\sqrt{\cos(dx+c)}\,\sqrt{-a(-1-\sec(dx+c))}\left(2(1-\cos(dx+c))^5\csc(dx+c)^5-17(1-\cos(dx+c))^3\csc(dx+c)^3-75\arctan\left(\frac{\sqrt{2}\,(\cot(dx+c)-}{2\sqrt{-\frac{}{\cos(dx}}}\right.\right.}{32d\,a^3}$

input
```
int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/d/a^3*cos(d*x+c)^(1/2)*(-a*(-1-sec(d*x+c)))^(1/2)*(2*(1-cos(d*x+c))^
5*csc(d*x+c)^5-17*(1-cos(d*x+c))^3*csc(d*x+c)^3-75*arctan(1/2*2^(1/2)*(cot
(d*x+c)-csc(d*x+c))/(-1/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)-8
3*csc(d*x+c)+83*cot(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 436, normalized size of antiderivative = 2.21

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{75\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos\cdots}{\cdots}\right.}{\cdots}\right.$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(32*cos(d*x + c)^2 + 85*cos(d
*x + c) + 49)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(
d*x + c) + a^3*d), 1/32*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3
*cos(d*x + c) + 1)*sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c) + a)
) + 2*(32*cos(d*x + c)^2 + 85*cos(d*x + c) + 49)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*
a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258456 vs. 2(162) = 324.

Time = 2.66 (sec) , antiderivative size = 258456, normalized size of antiderivative = 1311.96

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-1/32*(576*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2
*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x
 + 5/2*c)^6 + 14400*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*co
s(3/2*d*x + 3/2*c)^6 + 187500*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(
1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^6 +
 576*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2
*c)^6 + 5184*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1
/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*sin(3/2*d
*x + 3/2*c)^6 + 262500*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
 + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^4*sin(1/2*
d*x + 1/2*c)^2 + 77700*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*...
```

Giac [A] (verification not implemented)

Time = 168.87 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\frac{\left(\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^2\operatorname{sgn}(\cos(dx+c))}-\frac{17\sqrt{2}}{a^2\operatorname{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{83\sqrt{2}}{a^2\operatorname{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}-\frac{75\sqrt{2}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\right.\right.}{a^{\frac{5}{2}}\operatorname{sgn}(\cos(dx+c))}}{32\,d}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-1/32*(((2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/(a^2*sgn(cos(d*x + c))) - 17*sqr
t(2)/(a^2*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 - 83*sqrt(2)/(a^2*sgn
(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) -
 75*sqrt(2)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1
/2*c)^2 + a)))/(a^(5/2)*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\sec(dx+c)^3+3\sec(dx+c)^2+3\sec(dx+c)+1}dx\right)}{a^3}$$

input
```
int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x)))/(sec(c + d*x)**3
+ 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1),x))/a**3
```

3.436 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = \frac{19\text{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{9\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}$$

output
```
19/32*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d*x
+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/a^(5/2)/d-1/4*sin(d*
x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2)-9/16*sin(d*x+c)/a/d/cos(d*x
+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.89 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\left(9\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+38\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^4\left(\frac{1}{2}(c+\right.\right.}{16d\sqrt{1-\sec(c+dx)}(a(1+\sec(c+d}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)),x]
```

output
```
-1/16*(Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(9*Sqrt[1 - Sec[c + d*x]]*Sec
[c + d*x]^(3/2) + 38*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 -
Sec[c + d*x]]]*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2 + 13*Sqrt[-((-1 + Sec[c +
 d*x])*Sec[c + d*x])])*Sin[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec
[c + d*x]))^(5/2))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4752, 3042, 4304, 27, 3042, 4500, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)}}{(\sec(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4304}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{\sqrt{\sec(c+dx)}(7a-2a\sec(c+dx))}{2(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sec(c+dx)}(7a-2a\sec(c+dx))}{(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(7a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\Big\downarrow$ 4500

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{19}{4}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx-\frac{9a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{19}{4}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{9a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\Big\downarrow$ 4295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{19\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2d}-\frac{9a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+}{4d(a\sec}\right.$$

$\Big\downarrow$ 219

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{19\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\sqrt{a}d}-\frac{9a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right.$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/4*(Sec[c + d*x]^(3/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^(5/2)) + ((19*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*
x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*Sqrt[a]*
d) - (9*a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)
))/(8*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4500
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] + Simp[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1))   Int[(a + b*Csc[e +
 f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}
, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[
m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.04

method	result
default	$\frac{\left(\left(19\cos(dx+c)^2+38\cos(dx+c)+19\right)\arctan\left(\frac{\sqrt{2}\,(\cot(dx+c)-\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+(-13\cos(dx+c)-9)\sin(dx+c)\sqrt{-\frac{2}{\cos(dx+c)+1}}\right)\sqrt{\cos}}{32d\,a^3\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/d/a^3*((19*cos(d*x+c)^2+38*cos(d*x+c)+19)*arctan(1/2*2^(1/2)*(cot(d*x
+c)-csc(d*x+c))/(-1/(cos(d*x+c)+1))^(1/2))+(-13*cos(d*x+c)-9)*sin(d*x+c)*(
-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^(1/2)*2^(1/2)*(a*(1+sec(d*x+c)))^(1/2
)/(cos(d*x+c)^3+3*cos(d*x+c)^2+3*cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.65

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{19\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\right.}{}$$

$$-\frac{19\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(}{2\,(a\cos(dx+c)+a)}\right.}{32\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\,c\right.}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*(13*cos(d*x + c) + 9)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*c
os(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1
/32*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c) + a)) + 2*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*(13*cos(d*x + c) + 9)*sqrt(cos(d*x + c))*sin(d*x
 + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
c) + a^3*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3049 vs. 2(128) = 256.

Time = 0.50 (sec) , antiderivative size = 3049, normalized size of antiderivative = 19.42

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/32*(19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
2*sin(1/2*d*x + 1/2*c) + 1))*cos(4*d*x + 4*c)^2 + 304*(log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*
cos(3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 304*(log
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
+ 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
 + 1/2*c) + 1))*cos(d*x + c)^2 + 19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(4*d*x + 4*c)^2
 + 304*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*
sin(1/2*d*x + 1/2*c) + 1))*sin(3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*si
n(2*d*x + 2*c)^2 + 304*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*...
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3\mathrm{sgn}(\cos(dx+c))}-\frac{11\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)}{3}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/32*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2
/(a^3*sgn(cos(d*x + c))) - 11*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d*x
 + 1/2*c) - 19*sqrt(2)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a)))/(a^(5/2)*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)\sec(dx+c)^3+3\cos(dx+c)\sec(dx+c)^2+3\cos(dx+c)\sec(dx+c)+c}\right.}{a^3}$$

input
```
int(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)*sec
(c + d*x)**3 + 3*cos(c + d*x)*sec(c + d*x)**2 + 3*cos(c + d*x)*sec(c + d*x
) + cos(c + d*x)),x))/a**3
```

3.437 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \frac{5\text{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{5\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}$$

output
```
5/32*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d*x+
c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/a^(5/2)/d-1/4*sin(d*x
+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2)+5/16*sin(d*x+c)/a/d/cos(d*x+
c)^(3/2)/(a+a*sec(d*x+c))^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.59 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.43

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx) - \frac{5(1}{}\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-8*Sec[c + d*x]^(5/2)*Sin[c + d*x]
 - (5*(1 + Sec[c + d*x])*(2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[
c + d*x] - (1 + Sec[c + d*x])*(2*ArcSin[Sqrt[1 - Sec[c + d*x]]] + 2*ArcSin
[Sqrt[Sec[c + d*x]]] - Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1
- Sec[c + d*x]]] + 2*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Tan[c + d*
x]))/Sqrt[1 - Sec[c + d*x]]))/(32*d*(a*(1 + Sec[c + d*x]))^(5/2))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4752, 3042, 4298, 3042, 4297, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(\sec(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4298}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(\sec(c+dx)a+a)^{3/2}}dx}{8a}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\Big\downarrow$ 4297

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5\left(\frac{\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{4a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5\left(\frac{\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\Big\downarrow$ 4295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5\left(\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2ad}\right)}{8a}-\frac{\sin(c+}{4d(a\sec(}\right.$$

$\Big\downarrow$ 219

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx}{4d(a\sec(c+dx)+a)^{5/2}}\right.$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/4*(Sec[c + d*x]^(5/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^(5/2)) + (5*(ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]/(2*Sqrt[2]*a^(3/2)*d)
 + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2))))/(8
*a))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4297
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[d*((m + 1)/(b*(2*m + 1)))
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ
[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m,
-2^(-1)] && IntegerQ[2*m]
```

rule 4298
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[m/(a*(2*m + 1))   Int[(a + b*Csc[e +
 f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LtQ[m, -2^(-1)]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.04

method	result
default	$\frac{\left(\left(5\cos(dx+c)^2+10\cos(dx+c)+5\right)\arctan\left(\frac{\sqrt{2}\,(\cot(dx+c)-\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+(5\cos(dx+c)+1)\sin(dx+c)\sqrt{-\frac{2}{\cos(dx+c)+1}}\right)\sqrt{\cos(dx+}}{32d\,a^3\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/d/a^3*((5*cos(d*x+c)^2+10*cos(d*x+c)+5)*arctan(1/2*2^(1/2)*(cot(d*x+c
)-csc(d*x+c))/(-1/(cos(d*x+c)+1))^(1/2))+(5*cos(d*x+c)+1)*sin(d*x+c)*(-2/(
cos(d*x+c)+1))^(1/2))*cos(d*x+c)^(1/2)*2^(1/2)*(a*(1+sec(d*x+c)))^(1/2)/(c
os(d*x+c)^3+3*cos(d*x+c)^2+3*cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.65

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{5\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{\cdots}}{\cdots}\right.$$

$$-\frac{5\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(\cdots}{2\,(a\cos(dx+c)+a)}\cdots\right.}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos\cdots\right.}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)
/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*(5*cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos
(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/3
2*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt
(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c) + a)) - 2*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*(5*cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x + c
))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) +
 a^3*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2875 vs. 2(128) = 256.

Time = 0.42 (sec) , antiderivative size = 2875, normalized size of antiderivative = 18.31

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/32*(4*(3*sin(3/2*d*x + 3/2*c) + 5*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) - 3*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 40*(2
*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))
*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(2*sin(
3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(
5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(3*sin(3/2*d
*x + 3/2*c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*(3*s
in(3/2*d*x + 3/2*c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))))*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 5*(16*cos(3*d*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c)))^2 + 12*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c...
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.83

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3\mathrm{sgn}(\cos(dx+c))}-\frac{3\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{5\sqrt{2}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{a^{\frac{5}{2}}\,\mathrm{sgn}(\cos(dx+c))}}{32\,d}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-1/32*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^
2/(a^3*sgn(cos(d*x + c))) - 3*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d*x
 + 1/2*c) + 5*sqrt(2)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a)))/(a^(5/2)*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^2\sec(dx+c)^3+3\cos(dx+c)^2\sec(dx+c)^2+3\cos(dx+c)^2\sec(dx+c)\cdots}\right.}{a^3}$$

input
```
int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**2*
sec(c + d*x)**3 + 3*cos(c + d*x)**2*sec(c + d*x)**2 + 3*cos(c + d*x)**2*se
c(c + d*x) + cos(c + d*x)**2),x))/a**3
```

3.438 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \frac{3\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} + \frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{3\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}$$

output
```
3/32*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*sec(d*x+
c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/a^(5/2)/d+1/4*sin(d*x
+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2)+3/16*sin(d*x+c)/a/d/cos(d*x+
c)^(3/2)/(a+a*sec(d*x+c))^(3/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 328 vs. 2(157) = 314.

Time = 0.59 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.09

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(6\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\,s\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(6*Sqrt[1 - Sec[c + d*x]]*Sec[c + d
*x]^(3/2)*Sin[c + d*x] + 14*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[
c + d*x] - 3*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c +
d*x]]]*Tan[c + d*x] - 6*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1
 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[c + d*x] - 3*Sqrt[2]*ArcTan[(Sqrt[2]*Sq
rt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]^2*Tan[c + d*x] + 6*
ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan[c + d*x] + 6*ArcSi
n[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan[c + d*x]))/(32*d*Sqrt[1 - S
ec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4752, 3042, 4297, 3042, 4297, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(\sec(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4297}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(\sec(c+dx)a+a)^{3/2}}dx}{8a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\Big\downarrow$ 4297

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{4a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\Big\downarrow$ 4295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2ad}\right)}{8a}+\frac{\sin(c+\ldots}{4d(a\sec\ldots}\right)$$

$\Big\downarrow$ 219

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx}{4d(a\sec(c+dx)+a)^{5/2}}\right.$$

input

```
Int[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

output

```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
4*d*(a + a*Sec[c + d*x])^(5/2)) + (3*(ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) +
(Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2))))/(8*a)
)
```

Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4297
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[d*((m + 1)/(b*(2*m + 1)))
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ
[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m,
-2^(-1)] && IntegerQ[2*m]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.04

method	result
default	$\frac{\left(\left(-3\cos(dx+c)^{2}-6\cos(dx+c)-3\right)\arctan\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+(3\cos(dx+c)+7)\sin(dx+c)\sqrt{-\frac{2}{\cos(dx+c)+1}}\right)\sqrt{\cos(d}}{32d\,a^{3}\left(\cos(dx+c)^{3}+3\cos(dx+c)^{2}+3\cos(dx+c)+1\right)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/d/a^3*((-3*cos(d*x+c)^2-6*cos(d*x+c)-3)*arctan(1/2*2^(1/2)/(-1/(cos(d
*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)))+(3*cos(d*x+c)+7)*sin(d*x+c)*(-2/
(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^(1/2)*2^(1/2)*(a*(1+sec(d*x+c)))^(1/2)/(
cos(d*x+c)^3+3*cos(d*x+c)^2+3*cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.65

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{3\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{\cdots}}{\cdots}\right.$$

$$-\frac{3\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(d\cdots}{2\left(a\cos(dx+c)+a\right)}\cdots\right.}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos\cdots\right.}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)
/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*(3*cos(d*x + c) + 7)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos
(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/3
2*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt
(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c) + a)) - 2*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*(3*cos(d*x + c) + 7)*sqrt(cos(d*x + c))*sin(d*x + c
))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) +
 a^3*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84332 vs. $2(128) = 256$.

Time = 13.97 (sec) , antiderivative size = 84332, normalized size of antiderivative = 537.15

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/32*(512*((2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(
5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 2*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c
) + (2*cos(2*d*x + 2*c) + cos(d*x + c))*sin(5/2*d*x + 5/2*c) + cos(4*d*x +
 4*c)*sin(5/2*d*x + 5/2*c) + 2*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(
5*d*x + 5*c)^2 + 2560*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x +
 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*s
in(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x
 + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(
5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x
+ 3*c)*sin(5/2*d*x + 5/2*c))*cos(8/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
*d*x + 5/2*c)))^2 + 10240*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d
*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*
c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2
*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*
sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*
d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos
(5/2*d*x + 5/2*c)))^2 + 10240*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5
/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x +
5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*c
os(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x ...
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.83

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\,\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3\mathrm{sgn}(\cos(dx+c))}+\frac{5\,\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\mathrm{t}}{32}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/32*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2
/(a^3*sgn(cos(d*x + c))) + 5*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d*x
+ 1/2*c) - 3*sqrt(2)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a)))/(a^(5/2)*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^3\sec(dx+c)^3+3\cos(dx+c)^3\sec(dx+c)^2+3\cos(dx+c)^3\sec(dx+c)\cdots}\right.}{a^3}$$

input
```
int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x)
```

output

```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**3*
sec(c + d*x)**3 + 3*cos(c + d*x)**3*sec(c + d*x)**2 + 3*cos(c + d*x)**3*se
c(c + d*x) + cos(c + d*x)**3),x))/a**3
```

3.439 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 214

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \frac{2\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d} - \frac{43\text{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{11\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}$$

output
```
2*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(
d*x+c)^(1/2)/a^(5/2)/d-43/32*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+
c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(1/
2)/a^(5/2)/d-1/4*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2)-11/1
6*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.53

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-22\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-22*Sqrt[1 - Sec[c + d*x]]*Sec[c +
 d*x]^(3/2)*Sin[c + d*x] - 30*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Si
n[c + d*x] + 43*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c
 + d*x]]]*Tan[c + d*x] + 86*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sq
rt[1 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[c + d*x] + 43*Sqrt[2]*ArcTan[(Sqrt[
2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]^2*Tan[c + d*x]
 - 22*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan[c + d*x] - 8
6*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan[c + d*x]))/(32*d*Sqr
t[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.97, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4752, 3042, 4303, 27, 3042, 4507, 27, 3042, 4511, 3042, 4288, 222, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(\sec(c+dx)a+a)^{5/2}}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$\Big\downarrow$ 4303

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)(3a-8a\sec(c+dx))}{2(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)(3a-8a\sec(c+dx))}{(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3a-8a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\Big\downarrow$ 4507

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{\sqrt{\sec(c+dx)}\left(11a^2-32a^2\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/}}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{\sqrt{\sec(c+dx)}\left(11a^2-32a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/}}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(11a^2-32a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(}{4d(a\sec(c+dx)}\right.$$

$\downarrow$ 4511

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{43a^2\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx-32a\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin}{4d}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{43a^2\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-32a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c-}{2d(a\sec(c+dx)+a)^3}}{8a^2}\right.$$

$\downarrow$ 4288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{43a^2\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{64a\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+}{2d(a\sec(c+dx)+a)^{3/}}}{8a^2}\right.$$

$\downarrow$ 222

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{43a^2\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{64a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\text{si}}{4}\right.$$

$$\Big\downarrow \ 4295$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{-\frac{86a^2\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{64a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}+}{8a^2}\right.$$

$$\Big\downarrow \ 219$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{43\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{64a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}+\frac{11a\sin(c+dx)\text{se}}{2d(a\sec(c+dx}}{8a^2}\right.$$

input
```
Int[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/4*(Sec[c + d*x]^(5/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^(5/2)) - (((-64*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c
 + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (43*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[
a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d
)/(4*a^2) + (11*a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x
])^(3/2)))/(8*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4511
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b -
a*B)/b   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Simp[B/b
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b
, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 3.72 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.29

method	result
default	$\frac{\sqrt{\cos(dx+c)}\left(\left(43\cos(dx+c)^2+86\cos(dx+c)+43\right)\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+\left(-32\cos(dx+c)^2-64\cos(dx+c)-3\right.}{32d\,a}$

input
```
int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/d/a^3*cos(d*x+c)^(1/2)*((43*cos(d*x+c)^2+86*cos(d*x+c)+43)*2^(1/2)*ar
ctan(1/2*2^(1/2)/(-1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)))+(-32*
cos(d*x+c)^2-64*cos(d*x+c)-32)*arctan(1/2*(-cot(d*x+c)+csc(d*x+c)-1)/(-1/(
cos(d*x+c)+1))^(1/2))+(-32*cos(d*x+c)^2-64*cos(d*x+c)-32)*arctan(1/2/(-1/(
cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1))+sin(d*x+c)*(-11*cos(d*x+c
)-15)*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2))*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x
+c)^3+3*cos(d*x+c)^2+3*cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 646, normalized size of antiderivative = 3.02

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*(11*cos(d*x + c) + 15)*sqrt(cos(d*x + c))*sin(d*x + c) + 32*(co
s(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*log((a*cos(d
*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c
) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*
x + c)^3 + cos(d*x + c)^2)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^
2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(43*sqrt(2)*(cos(d*x + c)^3 + 3*co
s(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*c
os(d*x + c) + a)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(11*cos(d*x
+ c) + 15)*sqrt(cos(d*x + c))*sin(d*x + c) + 32*(cos(d*x + c)^3 + 3*cos(d*
x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2
- a*cos(d*x + c) - 2*a)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 +
 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4988 vs. $2(175) = 350$.

Time = 0.42 (sec) , antiderivative size = 4988, normalized size of antiderivative = 23.31

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/32*(44*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 16*(
19*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 19*sin(3/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 11*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 76*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - 76*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - 44*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c))*cos(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*cos(4*d*x + 4*c)^2
 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c)
 + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos
(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +...
```

Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.20

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx =$$

$$-\frac{2\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3\mathrm{sgn}(\cos(dx+c))}+\frac{13\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{43\sqrt{2}\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.\right.}{a^{\frac{5}{2}}\mathrm{sgn}(c}}{}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-1/64*(2*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c
)^2/(a^3*sgn(cos(d*x + c))) + 13*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*
d*x + 1/2*c) - 43*sqrt(2)*log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))^2)/(a^(5/2)*sgn(cos(d*x + c))) - 64*log(abs((sqrt(
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqr
t(2) + 3)))/(a^(5/2)*sgn(cos(d*x + c))) + 64*log(abs((sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(a^
(5/2)*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^4\sec(dx+c)^3+3\cos(dx+c)^4\sec(dx+c)^2+3\cos(dx+c)^4\sec(dx+c)\cdots}\right.}{a^3}$$

input
```
int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**4*
sec(c + d*x)**3 + 3*cos(c + d*x)**4*sec(c + d*x)**2 + 3*cos(c + d*x)**4*se
c(c + d*x) + cos(c + d*x)**4),x))/a**3
```

3.440 $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 254

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{5\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d} + \frac{115\text{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{15\sin(c+dx)}{16ad\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{35\sin(c+dx)}{16a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

output
```
-5*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/a^(5/2)/d+115/32*arctanh(1/2*a^(1/2)*sec(d*x+c)^(1/2)*sin(d*
x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(
1/2)/a^(5/2)/d-1/4*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2)-15
/16*sin(d*x+c)/a/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2)+35/16*sin(d*x+c
)/a^2/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 2.84 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.69

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\left(\frac{1}{1+\cos(c+dx)}\right)^{3/2}\left(230\sqrt{2}\text{arcsinh}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\cos^4\left(\frac{1}{2}(c+\right.\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

output
```
(((1 + Cos[c + d*x])^(-1))^(3/2)*(230*Sqrt[2]*ArcSinh[Tan[(c + d*x)/2]]*Co
s[(c + d*x)/2]^4*Cos[c + d*x] - 320*ArcTanh[Tan[(c + d*x)/2]/Sqrt[(1 + Cos
[c + d*x])^(-1)]]*Cos[(c + d*x)/2]^4*Cos[c + d*x] + (Sqrt[(1 + Cos[c + d*x
])^(-1)]*(67 + 110*Cos[c + d*x] + 35*Cos[2*(c + d*x)])*Sin[c + d*x])/2))/(
16*a^2*d*Cos[c + d*x]^(3/2)*Sqrt[a*(1 + Sec[c + d*x])])
```

Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.99, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 4752, 3042, 4303, 27, 3042, 4507, 27, 3042, 4509, 25, 3042, 4511, 3042, 4288, 222, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(\sec(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4303}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{5\sec^{\frac{5}{2}}(c+dx)(a-2a\sec(c+dx))}{2(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\int \frac{\sec^{\frac{5}{2}}(c+dx)(a-2a\sec(c+dx))}{(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \text{4507}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(9a^2-14a^2\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+}{4d(a\sec(c+dx)+a}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(9a^2-14a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+}{4d(a\sec(c+dx)+a}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(9a^2-14a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\,\mathrm{s}}{4d(a\sec(c+}\right.$$

$\Big\downarrow$ 4509

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{\frac{\int -\frac{\sqrt{\sec(c+dx)}\left(7a^3-16a^3\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}\right.$$

$\Big\downarrow$ 25

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(7a^3-16a^3\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(7a^3-16a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c-}{2d(a\sec(c+dx)+a)}\right.}{8a^2}\right.$$

$\downarrow$ 4511

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{-\frac{23a^3\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx-16a^2\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}+\frac{3a\sin}{2d(a}\right.}{8a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{-\frac{23a^3\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-16a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+d}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}\right.}{8a^2}\right.$$

$\downarrow$ 4288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(-\frac{-\frac{23a^3\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{32a^2\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{a}}{4a^2}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(}{d\sqrt{a\sec(c+dx)+}}\right.}{8a^2}\right.$$

$\downarrow$ 222

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{-\frac{23a^3\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{32a^{5/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}+\frac{3}{}\right.}{8a^2}\right.$$

$\downarrow$ 4295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(-\frac{-\frac{46a^3\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{32a^{5/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}}{4a^2}\right.}{8a^2}\right.$$

$$\Big\downarrow \text{219}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{-\frac{\frac{23\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{32a^{5/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}-\frac{14a^2\sin(c+dx)\ldots}{d\sqrt{a\sec(\ldots}}}{4a^2}\ldots\right)}{8a^2}\ldots\right.$$

input
```
Int[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/4*(Sec[c + d*x]^(7/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^(5/2)) - (5*((3*a*Sec[c + d*x]^(5/2)*Sin[c + d
*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + (-(((-32*a^(5/2)*ArcSinh[(Sqrt[a]*
Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (23*Sqrt[2]*a^(5/2)*ArcTanh[(
Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]]
)])/d)/a) - (14*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c +
 d*x]]))/(4*a^2)))/(8*a^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4509
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Simp[d/(b*(m + n))   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
 1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[n, 1]
```

rule 4511
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b -
a*B)/b   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Simp[B/b
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b
, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 3.56 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.23

method	result
default	$\frac{\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(\cot(dx+c)-\csc(dx+c))}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\left(115\cos(dx+c)^3+230\cos(dx+c)^2+115\cos(dx+c)\right)+\arctan\left(\frac{\cot(dx+c)-\csc(dx+c)-1}{2\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\left(\right.}{}$

input
```
int(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/d/a^3*(2^(1/2)*arctan(1/2*2^(1/2)*(cot(d*x+c)-csc(d*x+c))/(-1/(cos(d*
x+c)+1))^(1/2))*(115*cos(d*x+c)^3+230*cos(d*x+c)^2+115*cos(d*x+c))+arctan(
1/2*(cot(d*x+c)-csc(d*x+c)-1)/(-1/(cos(d*x+c)+1))^(1/2))*(-80*cos(d*x+c)^3
-160*cos(d*x+c)^2-80*cos(d*x+c))+arctan(1/2/(-1/(cos(d*x+c)+1))^(1/2)*(cot
(d*x+c)-csc(d*x+c)+1))*(-80*cos(d*x+c)^3-160*cos(d*x+c)^2-80*cos(d*x+c))+s
in(d*x+c)*(35*cos(d*x+c)^2+55*cos(d*x+c)+16)*2^(1/2)*(-2/(cos(d*x+c)+1))^(
1/2))*(a*(1+sec(d*x+c)))^(1/2)/cos(d*x+c)^(1/2)/(cos(d*x+c)^3+3*cos(d*x+c)
^2+3*cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 710, normalized size of antiderivative = 2.80

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(115*sqrt(2)*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 +
 cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(
d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(35*cos(d*x + c
)^2 + 55*cos(d*x + c) + 16)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c) + 80*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(
d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 + 4*sqrt(a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*s
in(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))
)/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2
+ a^3*d*cos(d*x + c)), -1/32*(115*sqrt(2)*(cos(d*x + c)^4 + 3*cos(d*x + c)
^3 + 3*cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(
a*cos(d*x + c) + a)) - 2*(35*cos(d*x + c)^2 + 55*cos(d*x + c) + 16)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 80*(co
s(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a
)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a^3*d*cos(d
*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*
x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(9/2)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9048 vs. $2(209) = 418$.

Time = 2.25 (sec) , antiderivative size = 9048, normalized size of antiderivative = 35.62

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-1/32*(140*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 4
*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*sin(3/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - 16*(75*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 24*sin(7/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 24*sin(5/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 75*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) - 35*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*
cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 300*(sin(6*d*x + 6*
c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 8*sin(3/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 96*(sin
(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 8*sin(3/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 32*(24*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 75*sin(3
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 35*sin(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - 96*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*
c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/4*ar...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error index.cc index_gcd Error: Bad
 Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^{9/2}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^5\sec(dx+c)^3+3\cos(dx+c)^5\sec(dx+c)^2+3\cos(dx+c)^5\sec(dx+c)}\right.}{a^3}$$

input
```
int(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**5*
sec(c + d*x)**3 + 3*cos(c + d*x)**5*sec(c + d*x)**2 + 3*cos(c + d*x)**5*se
c(c + d*x) + cos(c + d*x)**5),x))/a**3
```

3.441 $\int (d\cos(e+fx))^n(a+a\sec(e+fx))^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 244

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^3\,dx = -\frac{a^3(7-4n)(d\cos(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{f(2-n)n\sqrt{\sin^2(e+fx)}} - \frac{a^3(1-4n)\cos(e+fx)(d\cos(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{f(1-n)(1+n)\sqrt{\sin^2(e+fx)}} + \frac{a^3(5-2n)(d\cos(e+fx))^n\tan(e+fx)}{f(1-n)(2-n)} + \frac{(d\cos(e+fx))^n\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{f(2-n)}$$

output
```
-a^3*(7-4*n)*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(f*x+e)^
2)*sin(f*x+e)/f/(2-n)/n/(sin(f*x+e)^2)^(1/2)-a^3*(1-4*n)*cos(f*x+e)*(d*cos
(f*x+e))^n*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)
/f/(1-n)/(1+n)/(sin(f*x+e)^2)^(1/2)+a^3*(5-2*n)*(d*cos(f*x+e))^n*tan(f*x+e
)/f/(1-n)/(2-n)+(d*cos(f*x+e))^n*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/f/(2-n)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.64

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^3\,dx$$
$$=\frac{a^3d(d\cos(e+fx))^{-1+n}\left(-n(-7+4n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n),\frac{n}{2},\cos^2(e+fx)\right)+(-2+n)\right.}{}$$

input
```
Integrate[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x])^3,x]
```

output
```
(a^3*d*(d*Cos[e + f*x])^(-1 + n)*(-(n*(-7 + 4*n)*Hypergeometric2F1[1/2, (-
2 + n)/2, n/2, Cos[e + f*x]^2]) + (-2 + n)*(-((-1 + 4*n)*Cos[e + f*x]*Hype
rgeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[e + f*x]^2]) + (3*n + (-1 +
n)*Cos[e + f*x])*Sqrt[Sin[e + f*x]^2]))*Tan[e + f*x])/(f*(-2 + n)*(-1 + n)
*n*Sqrt[Sin[e + f*x]^2])
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4752, 3042, 4301, 3042, 4485, 3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^3(d\cos(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4752}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int (d\sec(e+fx))^{-n}(\sec(e+fx)a+a)^3dx$$

$$\Big\downarrow \text{3042}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^3dx$$

$\downarrow$ 4301

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{a\int(d\sec(e+fx))^{-n}(\sec(e+fx)a+a)(2a(1-n)+a(5-2n)\sec(e+fx))dx}{2-n}+\frac{\tan(e+fx)\left(a^3\sec(e\right.}{}\right.$$

$\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{a\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)\left(2a(1-n)+a(5-2n)\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx}{2-n}+\frac{\tan}{}\right.$$

$\downarrow$ 4485

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{a\left(\frac{\int(d\sec(e+fx))^{-n}\left((1-4n)(2-n)a^2+(7-4n)(1-n)\sec(e+fx)a^2\right)dx}{1-n}+\frac{a^2(5-2n)\tan(e+fx)(d\sec(e+fx))^{-n}}{f(1-n)}\right)}{2-n}+\frac{\tan(e+f}{}\right.$$

$\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{a\left(\frac{\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left((1-4n)(2-n)a^2+(7-4n)(1-n)\csc\left(e+fx+\frac{\pi}{2}\right)a^2\right)dx}{1-n}+\frac{a^2(5-2n)\tan(e+fx)(d\sec(e+fx))^{-n}}{f(1-n)}\right)}{2-n}+\frac{\tan}{}\right.$$

$\downarrow$ 4274

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{a\left(\frac{\frac{a^2(7-4n)(1-n)\int(d\sec(e+fx))^{1-n}dx}{d}+a^2(1-4n)(2-n)\int(d\sec(e+fx))^{-n}dx}{1-n}+\frac{a^2(5-2n)\tan(e+fx)(d\sec(e+fx))^{-n}}{f(1-n)}\right)}{2-n}+\frac{\tan(}{}\right.$$

$\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{a\left(\frac{\frac{a^2(7-4n)(1-n)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{1-n}dx}{d}+a^2(1-4n)(2-n)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}dx}{1-n}+\frac{a^2(5-2n)\tan(e+fx)(d\sec(e+fx))^{-n}}{f(1-n)}\right)}{2-n}\right.$$

$\downarrow$ 4259

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{a\left(\frac{\frac{a^2(7-4n)(1-n)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\cos(e+fx)}{d}\right)^{n-1}dx}{d}+a^2(1-4n)(2-n)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\cos(e+fx}{d}\right.}{1-n}\right)}{2-n}\right.$$

$\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{a\left(\frac{\frac{a^2(7-4n)(1-n)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{n-1}dx}{d}+a^2(1-4n)(2-n)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\sin(}{}\right.}{1-n}\right)}{2-n}\right.$$

$\downarrow$ 3122

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)(d\sec(e+fx))^{-n}}{f(2-n)}+\frac{a\left(-\frac{a^2d(1-4n)(2-n)\sin(e+fx)(d\sec(e+fx))^{-n-1}\text{Hypergeometr}}{f(n+1)\sqrt{\sin^2(e+fx)}}\right.}{}\right.$$

input
```
Int[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x])^3,x]
```

output
```
(d*Cos[e + f*x])^n*(d*Sec[e + f*x])^n*(((a^3 + a^3*Sec[e + f*x])*Tan[e + f
*x])/(f*(2 - n)*(d*Sec[e + f*x])^n) + (a*((-((a^2*d*(1 - 4*n)*(2 - n)*Hype
rgeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^
(-1 - n)*Sin[e + f*x])/(f*(1 + n)*Sqrt[Sin[e + f*x]^2])) - (a^2*(7 - 4*n)*
(1 - n)*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e + f*x]^2]*Sin[e + f*x
])/(f*n*(d*Sec[e + f*x])^n*Sqrt[Sin[e + f*x]^2]))/(1 - n) + (a^2*(5 - 2*n)
*Tan[e + f*x])/(f*(1 - n)*(d*Sec[e + f*x])^n)))/(2 - n))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [F]

$$\int (d\cos (fx+e))^n (a+a\sec (fx+e))^3 \, dx$$

input
```
int((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^3,x)
```

output
```
int((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^3,x)
```

Fricas [F]

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^3 \, dx = \int (a\sec (fx+e)+a)^3(d\cos (fx+e))^n \, dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
integral((a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) +
 a^3)*(d*cos(f*x + e))^n, x)
```

Sympy [F]

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^3\,dx = a^3\left(\int (d\cos(e+fx))^n\,dx + \int 3(d\cos(e+fx))^n\sec(e+fx)\,dx + \int 3(d\cos(e+fx))^n\sec^2(e+fx)\,dx + \int (d\cos(e+fx))^n\sec^3(e+fx)\,dx\right)$$

input
```
integrate((d*cos(f*x+e))**n*(a+a*sec(f*x+e))**3,x)
```

output
```
a**3*(Integral((d*cos(e + f*x))**n, x) + Integral(3*(d*cos(e + f*x))**n*se
c(e + f*x), x) + Integral(3*(d*cos(e + f*x))**n*sec(e + f*x)**2, x) + Inte
gral((d*cos(e + f*x))**n*sec(e + f*x)**3, x))
```

Maxima [F]

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^3\,dx = \int (a\sec(fx+e)+a)^3(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^3*(d*cos(f*x + e))^n, x)
```

Giac [F]

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^3\,dx=\int (a\sec(fx+e)+a)^3(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^3*(d*cos(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^3\,dx=\int (d\cos(e+fx))^n\left(a+\frac{a}{\cos(e+fx)}\right)^3\,dx$$

input
```
int((d*cos(e + f*x))^n*(a + a/cos(e + f*x))^3,x)
```

output
```
int((d*cos(e + f*x))^n*(a + a/cos(e + f*x))^3, x)
```

Reduce [F]

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^3\,dx=d^n a^3\left(\int \cos(fx+e)^n\,dx+\int \cos(fx+e)^n\sec(fx+e)^3\,dx+3\left(\int \cos(fx+e)^n\sec(fx+e)^2\,dx\right)+3\left(\int \cos(fx+e)^n\sec(fx+e)\,dx\right)\right)$$

input
```
int((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^3,x)
```

output
```
d**n*a**3*(int(cos(e + f*x)**n,x) + int(cos(e + f*x)**n*sec(e + f*x)**3,x)
 + 3*int(cos(e + f*x)**n*sec(e + f*x)**2,x) + 3*int(cos(e + f*x)**n*sec(e
+ f*x),x))
```

3.442 $\int (d\cos(e+fx))^n(a+a\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 179

$$\begin{aligned}\int (d\cos(e+fx))^n(a+a\sec(e+fx))^2\,dx &= -\frac{2a^2(d\cos(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}}\\ &\quad-\frac{a^2(1-2n)\cos(e+fx)(d\cos(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{f(1-n)(1+n)\sqrt{\sin^2(e+fx)}}\\ &\quad+\frac{a^2(d\cos(e+fx))^n\tan(e+fx)}{f(1-n)}\end{aligned}$$

output
```
-2*a^2*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(f*x+e)^2)*sin
(f*x+e)/f/n/(sin(f*x+e)^2)^(1/2)-a^2*(1-2*n)*cos(f*x+e)*(d*cos(f*x+e))^n*h
ypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/f/(1-n)/(1+
n)/(sin(f*x+e)^2)^(1/2)+a^2*(d*cos(f*x+e))^n*tan(f*x+e)/f/(1-n)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.70

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^2\,dx$$
$$=\frac{a^2(d\cos(e+fx))^n\left((1-2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(e+fx)\right)+(-1+n)\left(-2\cos\right.\right.}{f(-1+n)n\sqrt{\sin^2(e}}$$

input
```
Integrate[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x])^2,x]
```

output
```
(a^2*(d*Cos[e + f*x])^n*((1 - 2*n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 +
 n)/2, Cos[e + f*x]^2] + (-1 + n)*(-2*Cos[e + f*x]*Hypergeometric2F1[1/2,
n/2, (2 + n)/2, Cos[e + f*x]^2] + Sqrt[Sin[e + f*x]^2]))*Tan[e + f*x])/(f*
(-1 + n)*n*Sqrt[Sin[e + f*x]^2])
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.565$, Rules used = {3042, 4752, 3042, 4275, 3042, 4259, 3042, 3122, 4534, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^2(d\cos(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4752}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int (d\sec(e+fx))^{-n}(\sec(e+fx)a+a)^2dx$$

$$\Big\downarrow \text{3042}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2dx$$

$\Big\downarrow$ 4275

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\int(d\sec(e+fx))^{-n}\left(\sec^2(e+fx)a^2+a^2\right)dx+\frac{2a^2\int(d\sec(e+fx))^{1-n}dx}{d}\right)$$

$\Big\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{2a^2\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{1-n}dx}{d}+\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx\right)$$

$\Big\downarrow$ 4259

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+\frac{2a^2\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\Big(}{d}\right.$$

$\Big\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+\frac{2a^2\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\Big(}{d}\right.$$

$\Big\downarrow$ 3122

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx-\frac{2a^2\sin(e+fx)(d\sec(e+fx))^{-n}\text{Hyperg}}{fn\sqrt{\sin^2}}\right.$$

$\Big\downarrow$ 4534

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{a^2(1-2n)\int(d\sec(e+fx))^{-n}dx}{1-n}-\frac{2a^2\sin(e+fx)(d\sec(e+fx))^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+2}{2},\cos\right.}{fn\sqrt{\sin^2(e+fx)}}\right.$$

$\Big\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{a^2(1-2n)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}dx}{1-n}-\frac{2a^2\sin(e+fx)(d\sec(e+fx))^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\right.}{fn\sqrt{\sin^2(e+fx)}}\right.$$

$$\Big\downarrow \text{4259}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{a^2(1-2n)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\cos(e+fx)}{d}\right)^n dx}{1-n}-\frac{2a^2\sin(e+fx)(d\sec(e+fx))^{-n}\text{Hype}}{fn\sqrt{\sin}}\right.$$

$$\Big\downarrow \text{3042}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{a^2(1-2n)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\sin(e+fx+\frac{\pi}{2})}{d}\right)^n dx}{1-n}-\frac{2a^2\sin(e+fx)(d\sec(e+fx))^{-n}\text{Hy}}{fn\sqrt{}}\right.$$

$$\Big\downarrow \text{3122}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(-\frac{a^2d(1-2n)\sin(e+fx)(d\sec(e+fx))^{-n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(e+fx)\right)}{f(1-n)(n+1)\sqrt{\sin^2(e+fx)}}-\frac{2a^2\sin}{}\right.$$

input
```
Int[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x])^2,x]
```

output
```
(d*Cos[e + f*x])^n*(d*Sec[e + f*x])^n*(-((a^2*d*(1 - 2*n)*Hypergeometric2F
1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 - n)*Sin
[e + f*x])/(f*(1 - n)*(1 + n)*Sqrt[Sin[e + f*x]^2])) - (2*a^2*Hypergeometr
ic2F1[1/2, n/2, (2 + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*n*(d*Sec[e + f
*x])^n*Sqrt[Sin[e + f*x]^2]) + (a^2*Tan[e + f*x])/(f*(1 - n)*(d*Sec[e + f*
x])^n))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [F]

$$\int (d\cos(fx+e))^n (a+a\sec(fx+e))^2 \, dx$$

input
```
int((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^2,x)
```

output
```
int((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^2,x)
```

Fricas [F]

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^2\,dx=\int (a\sec(fx+e)+a)^2(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*(d*cos(f*x + e))^
n, x)
```

Sympy [F]

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^2\,dx=a^2\left(\int (d\cos(e+fx))^n\,dx+\int 2(d\cos(e+fx))^n\sec(e+fx)\,dx+\int (d\cos(e+fx))^n\sec^2(e+fx)\,dx\right)$$

input
```
integrate((d*cos(f*x+e))**n*(a+a*sec(f*x+e))**2,x)
```

output
```
a**2*(Integral((d*cos(e + f*x))**n, x) + Integral(2*(d*cos(e + f*x))**n*se
c(e + f*x), x) + Integral((d*cos(e + f*x))**n*sec(e + f*x)**2, x))
```

Maxima [F]

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^2\,dx=\int (a\sec(fx+e)+a)^2(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^2*(d*cos(f*x + e))^n, x)
```

Giac [F]

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^2\,dx = \int (a\sec(fx+e)+a)^2(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^2*(d*cos(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^2\,dx = \int (d\,\cos(e+f\,x))^n\left(a+\frac{a}{\cos(e+f\,x)}\right)^2 dx$$

input
```
int((d*cos(e + f*x))^n*(a + a/cos(e + f*x))^2,x)
```

output
```
int((d*cos(e + f*x))^n*(a + a/cos(e + f*x))^2, x)
```

Reduce [F]

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^2\,dx = d^n a^2\left(\int \cos(fx+e)^n\,dx + \int \cos(fx+e)^n\sec(fx+e)^2\,dx + 2\left(\int \cos(fx+e)^n\sec(fx+e)\,dx\right)\right)$$

input
```
int((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^2,x)
```

output
```
d**n*a**2*(int(cos(e + f*x)**n,x) + int(cos(e + f*x)**n*sec(e + f*x)**2,x)
 + 2*int(cos(e + f*x)**n*sec(e + f*x),x))
```

3.443 $\int (d\cos(e+fx))^n(a+a\sec(e+fx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 132

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))\,dx = -\frac{a(d\cos(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}} - \frac{a(d\cos(e+fx))^{1+n} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{df(1+n)\sqrt{\sin^2(e+fx)}}$$

output
```
-a*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(f*x+e)^2)*sin(f*x
+e)/f/n/(sin(f*x+e)^2)^(1/2)-a*(d*cos(f*x+e))^(1+n)*hypergeom([1/2, 1/2+1/
2*n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/d/f/(1+n)/(sin(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.80

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))\,dx = -\frac{a(d\cos(e+fx))^n \left((1+n)\csc(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(e+fx)\right)+n\cot(e+fx)\right.}{fn(1+n)}$$

input
```
Integrate[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x]),x]
```

output
```
-((a*(d*Cos[e + f*x])^n*((1 + n)*Csc[e + f*x]*Hypergeometric2F1[1/2, n/2,
(2 + n)/2, Cos[e + f*x]^2] + n*Cot[e + f*x]*Hypergeometric2F1[1/2, (1 + n)
/2, (3 + n)/2, Cos[e + f*x]^2])*Sqrt[Sin[e + f*x]^2])/(f*n*(1 + n)))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4713, 3042, 2030, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)(d\cos(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4713}$$

$$\int \sec(e+fx)(a\cos(e+fx)+a)(d\cos(e+fx))^n dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+a\right)\left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\sin\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$d\int \left(d\sin\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{n-1}\left(\sin\left(\frac{1}{2}(2e+\pi)+fx\right)a+a\right)dx$$

$$\Big\downarrow \text{3227}$$

$$d\left(a\int (d\cos(e+fx))^{n-1}dx+\frac{a\int (d\cos(e+fx))^n dx}{d}\right)$$

$$\Big\downarrow \text{3042}$$

$$d\left(a\int \left(d\sin \left(e+fx+\frac{\pi}{2}\right)\right)^{n-1}dx+\frac{a\int \left(d\sin \left(e+fx+\frac{\pi}{2}\right)\right)^{n}dx}{d}\right)$$

$\Big\downarrow$ 3122

$$d\left(-\frac{a\sin(e+fx)(d\cos(e+fx))^{n+1}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(e+fx)\right)}{d^2f(n+1)\sqrt{\sin^2(e+fx)}}-\frac{a\sin(e+fx)(d\cos(e+\right.$$

input
```
Int[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x]),x]
```

output
```
d*(-((a*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e +
f*x]^2]*Sin[e + f*x])/(d*f*n*Sqrt[Sin[e + f*x]^2])) - (a*(d*Cos[e + f*x])^
(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]*Sin[e
 + f*x])/(d^2*f*(1 + n)*Sqrt[Sin[e + f*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Maple [F]

$$\int (d\cos(fx+e))^n (a+a\sec(fx+e))\,dx$$

input
```
int((d*cos(f*x+e))^n*(a+a*sec(f*x+e)),x)
```

output
```
int((d*cos(f*x+e))^n*(a+a*sec(f*x+e)),x)
```

Fricas [F]

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))\,dx = \int (a\sec(fx+e)+a)(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

Sympy [F]

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))\,dx = a\left(\int (d\cos(e+fx))^n\,dx + \int (d\cos(e+fx))^n\sec(e+fx)\,dx\right)$$

input
```
integrate((d*cos(f*x+e))**n*(a+a*sec(f*x+e)),x)
```

output
```
a*(Integral((d*cos(e + f*x))**n, x) + Integral((d*cos(e + f*x))**n*sec(e +
 f*x), x))
```

Maxima [F]

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))\,dx = \int (a\sec(fx+e)+a)(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

Giac [F]

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))\,dx = \int (a\sec(fx+e)+a)(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))\,dx = \int (d\cos(e+fx))^n\left(a+\frac{a}{\cos(e+fx)}\right)\,dx$$

input
```
int((d*cos(e + f*x))^n*(a + a/cos(e + f*x)),x)
```

output
```
int((d*cos(e + f*x))^n*(a + a/cos(e + f*x)), x)
```

Reduce [F]

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))\,dx = d^n a\left(\int \cos(fx+e)^n\,dx + \int \cos(fx+e)^n \sec(fx+e)\,dx\right)$$

input
```
int((d*cos(f*x+e))^n*(a+a*sec(f*x+e)),x)
```

output
```
d**n*a*(int(cos(e + f*x)**n,x) + int(cos(e + f*x)**n*sec(e + f*x),x))
```

3.444 $\int \frac{(d\cos(e+fx))^n}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 178

$$\int \frac{(d\cos(e+fx))^n}{a+a\sec(e+fx)}\,dx = \frac{(d\cos(e+fx))^n\sin(e+fx)}{f(a+a\sec(e+fx))} - \frac{\cos(e+fx)(d\cos(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{af\sqrt{\sin^2(e+fx)}} + \frac{(1+n)\cos^2(e+fx)(d\cos(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{af(2+n)\sqrt{\sin^2(e+fx)}}$$

output
```
(d*cos(f*x+e))^n*sin(f*x+e)/f/(a+a*sec(f*x+e))-cos(f*x+e)*(d*cos(f*x+e))^n
*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/a/f/(sin(
f*x+e)^2)^(1/2)+(1+n)*cos(f*x+e)^2*(d*cos(f*x+e))^n*hypergeom([1/2, 1+1/2*
n],[2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/a/f/(2+n)/(sin(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.93

$$\int \frac{(d\cos(e+fx))^n}{a+a\sec(e+fx)}\,dx = \frac{\cos(e+fx)(d\cos(e+fx))^n\sin(e+fx)\left(-2(2+n)\cos^2\left(\frac{1}{2}(e+fx)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\right.\right.}{af(2+n}$$

input
```
Integrate[(d*Cos[e + f*x])^n/(a + a*Sec[e + f*x]),x]
```

output
```
(Cos[e + f*x]*(d*Cos[e + f*x])^n*Sin[e + f*x]*(-2*(2 + n)*Cos[(e + f*x)/2]
^2*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2] + 2*(1 + n
)*Cos[(e + f*x)/2]^2*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n
)/2, Cos[e + f*x]^2] + (2 + n)*Sqrt[Sin[e + f*x]^2]))/(a*f*(2 + n)*(1 + Co
s[e + f*x])*Sqrt[Sin[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4752, 3042, 4307, 3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\cos(e+fx))^n}{a\sec(e+fx)+a}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n}{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{ 4752}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int \frac{(d\sec(e+fx))^{-n}}{\sec(e+fx)a+a}dx$$

$$\Big\downarrow \; 3042$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \; 4307$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{d\tan(e+fx)(d\sec(e+fx))^{-n-1}}{f(a\sec(e+fx)+a)}-\frac{d(n+1)\int(d\sec(e+fx))^{-n-1}(a-a\sec(e+fx))dx}{a^2}\right)$$

$$\Big\downarrow \; 3042$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{d\tan(e+fx)(d\sec(e+fx))^{-n-1}}{f(a\sec(e+fx)+a)}-\frac{d(n+1)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n-1}\left(a-a\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx}{a^2}\right)$$

$$\Big\downarrow \; 4274$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{d\tan(e+fx)(d\sec(e+fx))^{-n-1}}{f(a\sec(e+fx)+a)}-\frac{d(n+1)\left(a\int(d\sec(e+fx))^{-n-1}dx-\frac{a\int(d\sec(e+fx))^{-n}dx}{d}\right)}{a^2}\right)$$

$$\Big\downarrow \; 3042$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{d\tan(e+fx)(d\sec(e+fx))^{-n-1}}{f(a\sec(e+fx)+a)}-\frac{d(n+1)\left(a\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n-1}dx-\frac{a\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}dx}{d}\right.}{a^2}\right.$$

$$\Big\downarrow \; 4259$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{d\tan(e+fx)(d\sec(e+fx))^{-n-1}}{f(a\sec(e+fx)+a)}-\frac{d(n+1)\left(a\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\cos(e+fx)}{d}\right)^{n+1}dx-\right.}{a^2}\right.$$

$$\Big\downarrow \; 3042$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{d\tan(e+fx)(d\sec(e+fx))^{-n-1}}{f(a\sec(e+fx)+a)}-\frac{d(n+1)\left(a\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\sin(e+fx+\frac{\pi}{2})}{d}\right)^{n+1}d\right.}{a^2}\right.$$

$\Big\downarrow$ 3122

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{d\tan(e+fx)(d\sec(e+fx))^{-n-1}}{f(a\sec(e+fx)+a)}-\frac{d(n+1)\left(\frac{a\sin(e+fx)(d\sec(e+fx))^{-n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(e\right.}{f(n+1)\sqrt{\sin^2(e+fx)}}\right.}{}\right.$$

input
```
Int[(d*Cos[e + f*x])^n/(a + a*Sec[e + f*x]),x]
```

output
```
(d*Cos[e + f*x])^n*(d*Sec[e + f*x])^n*(-((d*(1 + n)*(-((a*d*Hypergeometric
2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-2 - n)*S
in[e + f*x])/(f*(2 + n)*Sqrt[Sin[e + f*x]^2])) + (a*Hypergeometric2F1[1/2,
 (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 - n)*Sin[e + f
*x])/(f*(1 + n)*Sqrt[Sin[e + f*x]^2])))/a^2) + (d*(d*Sec[e + f*x])^(-1 - n
)*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(a*f*
(a + b*Csc[e + f*x]))), x] + Simp[d*((n - 1)/(a*b))   Int[(d*Csc[e + f*x])^
(n - 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ
[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [F]

$$\int \frac{(d\cos(fx+e))^n}{a+a\sec(fx+e)}dx$$

input
```
int((d*cos(f*x+e))^n/(a+a*sec(f*x+e)),x)
```

output
```
int((d*cos(f*x+e))^n/(a+a*sec(f*x+e)),x)
```

Fricas [F]

$$\int \frac{(d\cos(e+fx))^n}{a+a\sec(e+fx)}\,dx=\int \frac{(d\cos(fx+e))^n}{a\sec(fx+e)+a}\,dx$$

input
```
integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral((d*cos(f*x + e))^n/(a*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{(d\cos(e+fx))^n}{a+a\sec(e+fx)}\,dx=\frac{\int \frac{(d\cos(e+fx))^n}{\sec(e+fx)+1}\,dx}{a}$$

input
```
integrate((d*cos(f*x+e))**n/(a+a*sec(f*x+e)),x)
```

output
```
Integral((d*cos(e + f*x))**n/(sec(e + f*x) + 1), x)/a
```

Maxima [F]

$$\int \frac{(d\cos(e+fx))^n}{a+a\sec(e+fx)}\,dx=\int \frac{(d\cos(fx+e))^n}{a\sec(fx+e)+a}\,dx$$

input
```
integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*cos(f*x + e))^n/(a*sec(f*x + e) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d\cos(e+fx))^n}{a+a\sec(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[0,1,0]%%%} / %%%{2,[0,0,1]%%%} Error: Bad Argument Val
ue
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\cos(e+fx))^n}{a+a\sec(e+fx)}\,dx = \int \frac{(d\,\cos(e+f\,x))^n}{a+\frac{a}{\cos(e+f\,x)}}\,dx$$

input
```
int((d*cos(e + f*x))^n/(a + a/cos(e + f*x)),x)
```

output
```
int((d*cos(e + f*x))^n/(a + a/cos(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(d\cos(e+fx))^n}{a+a\sec(e+fx)}\,dx = \frac{d^n\left(\int \frac{\cos(fx+e)^n}{\sec(fx+e)+1}dx\right)}{a}$$

input
```
int((d*cos(f*x+e))^n/(a+a*sec(f*x+e)),x)
```

output
```
(d**n*int(cos(e + f*x)**n/(sec(e + f*x) + 1),x))/a
```

3.445 $\int \frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 215

$$\int \frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx = \frac{2(2+n)(d\cos(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{3a^2f\sqrt{\sin^2(e+fx)}} - \frac{(3+2n)\cos(e+fx)(d\cos(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{3a^2f\sqrt{\sin^2(e+fx)}} - \frac{2(2+n)(d\cos(e+fx))^n\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{(d\cos(e+fx))^n\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

output
```
2/3*(2+n)*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(f*x+e)^2)*
sin(f*x+e)/a^2/f/(sin(f*x+e)^2)^(1/2)-1/3*(3+2*n)*cos(f*x+e)*(d*cos(f*x+e)
)^n*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/a^2/f/
(sin(f*x+e)^2)^(1/2)-2/3*(2+n)*(d*cos(f*x+e))^n*tan(f*x+e)/a^2/f/(1+sec(f*
x+e))-1/3*(d*cos(f*x+e))^n*tan(f*x+e)/f/(a+a*sec(f*x+e))^2
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.04

$$\int \frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx = -\frac{(d\cos(e+fx))^{3+n}\sin(e+fx)}{3d^3f(a+a\cos(e+fx))^2}$$
$$-\frac{\left(d(2+n)(4+n)(3+2n)(d\cos(e+fx))^{3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^2(e+fx)\right)-2(1+\cdots\right.}{3a^2d^4f(3+n)(4+n)\sqrt{\cdots}}$$
$$+\frac{2(1+n)\cos^3(e+fx)(d\cos(e+fx))^n\tan\left(\frac{1}{2}(e+fx)\right)}{3a^2f}$$

input
```
Integrate[(d*Cos[e + f*x])^n/(a + a*Sec[e + f*x])^2,x]
```

output
```
-1/3*((d*Cos[e + f*x])^(3 + n)*Sin[e + f*x])/(d^3*f*(a + a*Cos[e + f*x])^2
) - ((d*(2 + n)*(4 + n)*(3 + 2*n)*(d*Cos[e + f*x])^(3 + n)*Hypergeometric2
F1[1/2, (3 + n)/2, (5 + n)/2, Cos[e + f*x]^2] - 2*(1 + n)*(3 + n)^2*(d*Cos
[e + f*x])^(4 + n)*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[e + f*
x]^2])*Sin[e + f*x])/(3*a^2*d^4*f*(3 + n)*(4 + n)*Sqrt[Sin[e + f*x]^2]) +
(2*(1 + n)*Cos[e + f*x]^3*(d*Cos[e + f*x])^n*Tan[(e + f*x)/2])/(3*a^2*f)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4752, 3042, 4304, 25, 3042, 4508, 3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\cos(e+fx))^n}{(a\sec(e+fx)+a)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4752}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int\frac{(d\sec(e+fx))^{-n}}{(\sec(e+fx)a+a)^2}dx$$

$\Big\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int\frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}dx$$

$\Big\downarrow$ 4304

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(-\frac{\int-\frac{(d\sec(e+fx))^{-n}(a(n+3)-a(n+1)\sec(e+fx))}{\sec(e+fx)a+a}dx}{3a^2}-\frac{\tan(e+fx)(d\sec(e+fx))^{-n}}{3f(a\sec(e+fx)+a)^2}\right)$$

$\Big\downarrow$ 25

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\int\frac{(d\sec(e+fx))^{-n}(a(n+3)-a(n+1)\sec(e+fx))}{\sec(e+fx)a+a}dx}{3a^2}-\frac{\tan(e+fx)(d\sec(e+fx))^{-n}}{3f(a\sec(e+fx)+a)^2}\right)$$

$\Big\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\int\frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(a(n+3)-a(n+1)\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{\tan(e+fx)(d\sec(e+fx))^{-n}}{3f(a\sec(e+fx)+a)^2}\right)$$

$\Big\downarrow$ 4508

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\frac{\int(d\sec(e+fx))^{-n}\left(a^2(n+1)(2n+3)-2a^2n(n+2)\sec(e+fx)\right)dx}{a^2}-\frac{2(n+2)\tan(e+fx)(d\sec(e+fx))^{-n}}{f(\sec(e+fx)+1)}}{3a^2}-\frac{\tan(e+fx)(d\sec(e}{3f(a\sec(e+fx)}\right.$$

$\Big\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\frac{\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(a^2(n+1)(2n+3)-2a^2n(n+2)\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx}{a^2}-\frac{2(n+2)\tan(e+fx)(d\sec(e+fx))^{-n}}{f(\sec(e+fx)+1)}}{3a^2}-\frac{\tan(e+fx)(}{3f(a\sec(e}\right.$$

$\Big\downarrow$ 4274

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\frac{a^2(n+1)(2n+3)\int(d\sec(e+fx))^{-n}dx-\frac{2a^2n(n+2)\int(d\sec(e+fx))^{1-n}dx}{d}}{a^2}-\frac{2(n+2)\tan(e+fx)(d\sec(e+fx))^{-n}}{f(\sec(e+fx)+1)}}{3a^2}-\frac{\tan(e+fx)(d}{3f(a\sec(e}\right.$$

$\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\frac{a^2(n+1)(2n+3)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}dx-\frac{2a^2n(n+2)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{1-n}dx}{d}}{a^2}-\frac{2(n+2)\tan(e+fx)(d\sec(e+fx))^{-n}}{f(\sec(e+fx)+1)}}{3a^2}-\frac{\tan(e+}{3f(}\right.$$

$\downarrow$ 4259

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\frac{a^2(n+1)(2n+3)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\cos(e+fx)}{d}\right)^n dx-\frac{2a^2n(n+2)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\cos(e+fx)}{d}\right)^{n-1}dx}{d}}{a^2}}{3a^2}\right.$$

$\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\frac{a^2(n+1)(2n+3)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^n dx-\frac{2a^2n(n+2)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right.}{d}}{a^2}}{3a^2}\right.$$

$\downarrow$ 3122

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\frac{\frac{2a^2(n+2)\sin(e+fx)(d\sec(e+fx))^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+2}{2},\cos^2(e+fx)\right)}{f\sqrt{\sin^2(e+fx)}}-\frac{a^2d(2n+3)\sin(e+fx)(d\sec(e+fx))^{-n-1}\text{Hypergeometric2F1}\left(\frac{1}{2}\right.}{f\sqrt{\sin^2(e+fx)}}}{a^2}}{3a^2}\right.$$

input
```
Int[(d*Cos[e + f*x])^n/(a + a*Sec[e + f*x])^2,x]
```

output
```
(d*Cos[e + f*x])^n*(d*Sec[e + f*x])^n*(-1/3*Tan[e + f*x]/(f*(d*Sec[e + f*x
])^n*(a + a*Sec[e + f*x])^2) + ((-((a^2*d*(3 + 2*n)*Hypergeometric2F1[1/2,
 (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 - n)*Sin[e + f
*x])/(f*Sqrt[Sin[e + f*x]^2])) + (2*a^2*(2 + n)*Hypergeometric2F1[1/2, n/2
, (2 + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(d*Sec[e + f*x])^n*Sqrt[Sin[
e + f*x]^2]))/a^2 - (2*(2 + n)*Tan[e + f*x])/(f*(d*Sec[e + f*x])^n*(1 + Se
c[e + f*x])))/(3*a^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [F]

$$\int \frac{(d\cos(fx+e))^n}{(a+a\sec(fx+e))^2}dx$$

input
```
int((d*cos(f*x+e))^n/(a+a*sec(f*x+e))^2,x)
```

output
```
int((d*cos(f*x+e))^n/(a+a*sec(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx = \int \frac{(d\cos(fx+e))^n}{(a\sec(fx+e)+a)^2}\,dx$$

input
```
integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((d*cos(f*x + e))^n/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2
), x)
```

Sympy [F]

$$\int \frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx = \frac{\int \frac{(d\cos(e+fx))^n}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx}{a^2}$$

input
```
integrate((d*cos(f*x+e))**n/(a+a*sec(f*x+e))**2,x)
```

output
```
Integral((d*cos(e + f*x))**n/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)/a*
*2
```

Maxima [F]

$$\int \frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx = \int \frac{(d\cos(fx+e))^n}{(a\sec(fx+e)+a)^2}\,dx$$

input
```
integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*cos(f*x + e))^n/(a*sec(f*x + e) + a)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,2,0]%%%}+%%%{-3,[0,1,0,0]%%%} / %%%{4,[0,0,0,2]%%%}
 Error: B
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx = \int \frac{(d\,\cos(e+f\,x))^n}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int((d*cos(e + f*x))^n/(a + a/cos(e + f*x))^2,x)
```

output
```
int((d*cos(e + f*x))^n/(a + a/cos(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx = \frac{d^n\left(\int \frac{\cos(fx+e)^n}{\sec(fx+e)^2+2\sec(fx+e)+1}dx\right)}{a^2}$$

input
```
int((d*cos(f*x+e))^n/(a+a*sec(f*x+e))^2,x)
```

output
```
(d**n*int(cos(e + f*x)**n/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),x))/a**2
```

3.446 $\int \sec^4(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 85

$$\int \sec^4(c+dx)(a+b\sec(c+dx))\,dx = \frac{3b\text{arctanh}(\sin(c+dx))}{8d} + \frac{a\tan(c+dx)}{d} + \frac{3b\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{a\tan^3(c+dx)}{3d}$$

output
```
3/8*b*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+3/8*b*sec(d*x+c)*tan(d*x+c)/d+1
/4*b*sec(d*x+c)^3*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.69

$$\int \sec^4(c+dx)(a+b\sec(c+dx))\,dx = \frac{9b\text{arctanh}(\sin(c+dx)) + \tan(c+dx)\left(9b\sec(c+dx) + 6b\sec^3(c+dx) + 8a(3+\tan^2(c+dx))\right)}{24d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

output
```
(9*b*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(9*b*Sec[c + d*x] + 6*b*Sec[c +
d*x]^3 + 8*a*(3 + Tan[c + d*x]^2)))/(24*d)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {3042, 4274, 3042, 4254, 2009, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\sec(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{ 4274}$$

$$a\int \sec^4(c+dx)dx+b\int \sec^5(c+dx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx+b\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \text{ 4254}$$

$$b\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx-\frac{a\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$b\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{ 4255}$$

$$b\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$b\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$\Big\downarrow$ 4255

$$b\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$\Big\downarrow$ 3042

$$b\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$\Big\downarrow$ 4257

$$b\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

input
```
Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

output
```
-((a*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/d) + b*((Sec[c + d*x]^3*Tan[c + d
*x])/(4*d) + (3*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])
/(2*d)))/4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{-a\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+b\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
default	$\frac{-a\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+b\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
parts	$-\frac{a\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{b\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
risch	$-\frac{i\left(9b\,e^{7i(dx+c)}+33b\,e^{5i(dx+c)}-48\,e^{4i(dx+c)}a-33b\,e^{3i(dx+c)}-64\,e^{2i(dx+c)}a-9b\,e^{i(dx+c)}-16a\right)}{12d\left(e^{2i(dx+c)}+1\right)^4}-\frac{3b\ln\left(e^{i(dx+c)}-i\right)}{8d}$
norman	$\frac{-\frac{(8a-5b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}+\frac{(8a+5b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{(40a-9b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12d}+\frac{(40a+9b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{12d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}-\frac{3b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8d}$
parallelrisch	$\frac{-18\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+18\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{12d(3+\cos(4dx+4c)+4\cos(2dx+2c))}$

input
```
int(sec(d*x+c)^4*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+b*(-(-1/4*sec(d*x+c)^3-3/8*sec(
d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.16

$$\int \sec^4(c+dx)(a+b\sec(c+dx))\,dx = \frac{9\,b\cos(dx+c)^4\log(\sin(dx+c)+1) - 9\,b\cos(dx+c)^4\log(-\sin(dx+c)+1) + 2\,(16\,a\cos(dx+c)^3 + \cdots}{48\,d\cos(dx+c)^4}$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/48*(9*b*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 9*b*cos(d*x + c)^4*log(-s
in(d*x + c) + 1) + 2*(16*a*cos(d*x + c)^3 + 9*b*cos(d*x + c)^2 + 8*a*cos(d
*x + c) + 6*b)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\sec^4(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**4*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*sec(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

$$\int \sec^4(c+dx)(a+b\sec(c+dx))\,dx$$
$$=\frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)a-3b\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\mathrm{si}\right.}{48d}$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a - 3*b*(2*(3*sin(d*x + c)^3 -
5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x +
c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(77) = 154.

Time = 0.13 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.93

$$\int \sec^4(c+dx)(a+b\sec(c+dx))\,dx$$
$$=\frac{9b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-9b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(24a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-15b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-40\right.}{}}{24d}$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/24*(9*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*b*log(abs(tan(1/2*d*x + 1
/2*c) - 1)) - 2*(24*a*tan(1/2*d*x + 1/2*c)^7 - 15*b*tan(1/2*d*x + 1/2*c)^7
 - 40*a*tan(1/2*d*x + 1/2*c)^5 - 9*b*tan(1/2*d*x + 1/2*c)^5 + 40*a*tan(1/2
*d*x + 1/2*c)^3 - 9*b*tan(1/2*d*x + 1/2*c)^3 - 24*a*tan(1/2*d*x + 1/2*c) -
 15*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 12.55 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.79

$$\int \sec^4(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{\left(\frac{5b}{4}-2a\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(\frac{10a}{3}+\frac{3b}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(\frac{3b}{4}-\frac{10a}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\left(2a+\frac{5b}{4}\right)\tan\left(\frac{c}{2}+\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}$$
$$+\frac{3b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{4d}$$

input
```
int((a + b/cos(c + d*x))/cos(c + d*x)^4,x)
```

output
```
(tan(c/2 + (d*x)/2)*(2*a + (5*b)/4) - tan(c/2 + (d*x)/2)^7*(2*a - (5*b)/4)
 - tan(c/2 + (d*x)/2)^3*((10*a)/3 - (3*b)/4) + tan(c/2 + (d*x)/2)^5*((10*a
)/3 + (3*b)/4))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*ta
n(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (3*b*atanh(tan(c/2 + (d*
x)/2)))/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.38

$$\int \sec^4(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{-16\cos(dx+c)\sin(dx+c)^3a+24\cos(dx+c)\sin(dx+c)a-9\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^4b}{}$$

input
```
int(sec(d*x+c)^4*(a+b*sec(d*x+c)),x)
```

output
```
( - 16*cos(c + d*x)*sin(c + d*x)**3*a + 24*cos(c + d*x)*sin(c + d*x)*a - 9
*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*b + 18*log(tan((c + d*x)/2) - 1
)*sin(c + d*x)**2*b - 9*log(tan((c + d*x)/2) - 1)*b + 9*log(tan((c + d*x)/
2) + 1)*sin(c + d*x)**4*b - 18*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*b
 + 9*log(tan((c + d*x)/2) + 1)*b - 9*sin(c + d*x)**3*b + 15*sin(c + d*x)*b
)/(24*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.447 $\int \sec^3(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \sec^3(c+dx)(a+b\sec(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{2d}+\frac{b\tan(c+dx)}{d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}+\frac{b\tan^3(c+dx)}{3d}$$

output
```
1/2*a*arctanh(sin(d*x+c))/d+b*tan(d*x+c)/d+1/2*a*sec(d*x+c)*tan(d*x+c)/d+1
/3*b*tan(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \sec^3(c+dx)(a+b\sec(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{2d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}+\frac{b\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

input
```
Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (b
*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 4274, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\sec(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \sec^3(c+dx)dx+b\int \sec^4(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+b\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{4254}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{b\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{b\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{b\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{b\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4257}$$

$$a\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{b\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

input
```
Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

output
```
a*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)) - (b*(
-Tan[c + d*x] - Tan[c + d*x]^3/3))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-b\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
default	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-b\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
parts	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}-\frac{b\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
risch	$-\frac{i\left(3a\,e^{5i(dx+c)}-12\,e^{2i(dx+c)}b-3\,e^{i(dx+c)}a-4b\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{2d}$
norman	$\frac{\frac{(a-2b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}+\frac{4b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{(a+2b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$
parallelrisch	$\frac{-9\left(\frac{\cos(3dx+3c)}{3}+\cos(dx+c)\right)a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+9\left(\frac{\cos(3dx+3c)}{3}+\cos(dx+c)\right)a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+6a\sin(2dx+}{6d(\cos(3dx+3c)+3\cos(dx+c))}$

input
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-b*(-2/3-1
/3*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.40

$$\int \sec^3(c+dx)(a+b\sec(c+dx))\,dx$$
$$=\frac{3\,a\cos(dx+c)^3\log(\sin(dx+c)+1)-3\,a\cos(dx+c)^3\log(-\sin(dx+c)+1)+2\left(4\,b\cos(dx+c)^2+\right.}{12\,d\cos(dx+c)^3}$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(3*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*cos(d*x + c)^3*log(-s
in(d*x + c) + 1) + 2*(4*b*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 2*b)*sin(d*x
 + c))/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\sec^3(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*sec(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11

$$\int \sec^3(c+dx)(a+b\sec(c+dx))\,dx = \frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)b-3a\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)}{12d}$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*b - 3*a*(2*sin(d*x + c)/(sin(d*x
 + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(57) = 114$.

Time = 0.14 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.94

$$\int \sec^3(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4\,b\tan\cdots}{\left(\tan\left(\frac{1}{2}\cdots\right.\right.}}{6\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(3*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*log(abs(tan(1/2*d*x + 1/
2*c) - 1)) + 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 6*b*tan(1/2*d*x + 1/2*c)^5 +
4*b*tan(1/2*d*x + 1/2*c)^3 - 3*a*tan(1/2*d*x + 1/2*c) - 6*b*tan(1/2*d*x +
1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 11.80 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.73

$$\int \sec^3(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{(a-2\,b)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+(-a-2\,b)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$
$$+\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

input
```
int((a + b/cos(c + d*x))/cos(c + d*x)^3,x)
```

output
```
(tan(c/2 + (d*x)/2)^5*(a - 2*b) - tan(c/2 + (d*x)/2)*(a + 2*b) + (4*b*tan(
c/2 + (d*x)/2)^3)/3)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 +
 tan(c/2 + (d*x)/2)^6 - 1)) + (a*atanh(tan(c/2 + (d*x)/2)))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.56

$$\int \sec^3(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{-3\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2 a+3\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a+3\cos(dx}{}$$

input
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c)),x)
```

output
```
( - 3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a + 3*cos(c +
 d*x)*log(tan((c + d*x)/2) - 1)*a + 3*cos(c + d*x)*log(tan((c + d*x)/2) +
1)*sin(c + d*x)**2*a - 3*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a - 3*cos(
c + d*x)*sin(c + d*x)*a + 4*sin(c + d*x)**3*b - 6*sin(c + d*x)*b)/(6*cos(c
 + d*x)*d*(sin(c + d*x)**2 - 1))
```

3.448 $\int \sec^2(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \sec^2(c+dx)(a+b\sec(c+dx))\,dx=\frac{b\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
1/2*b*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+1/2*b*sec(d*x+c)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a+b\sec(c+dx))\,dx=\frac{b\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec(c+dx)\tan(c+dx)}{2d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

output
```
(b*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (b*Sec[c + d*x]*Tan
[c + d*x])/(2*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 4274, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\sec(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \sec^2(c+dx)dx+b\int \sec^3(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+b\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{4254}$$

$$b\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{a\int 1d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{24}$$

$$b\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{a\tan(c+dx)}{d}$$

$$\Big\downarrow \text{4255}$$

$$b\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a\tan(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a\tan(c+dx)}{d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{a\tan(c+dx)}{d}+b\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)$$

input
```
Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

output
```
(a*Tan[c + d*x])/d + b*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c
+ d*x])/(2*d))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{a\tan(dx+c)+b\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{a\tan(dx+c)+b\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
parts	$\frac{a\tan(dx+c)}{d}+\frac{b\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
parallelrisch	$\frac{-b(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+b(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2a\sin(2dx+2c)+2b\sin(dx+c)}{2d(1+\cos(2dx+2c))}$
norman	$\frac{\frac{(2a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{(2a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$
risch	$-\frac{i\left(b\,e^{3i(dx+c)}-2\,e^{2i(dx+c)}a-b\,e^{i(dx+c)}-2a\right)}{d\left(e^{2i(dx+c)}+1\right)^{2}}+\frac{b\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{b\ln\left(e^{i(dx+c)}-i\right)}{2d}$

input
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*tan(d*x+c)+b*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c
))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.57

$$\int \sec^2(c+dx)(a+b\sec(c+dx))\,dx$$
$$=\frac{b\cos(dx+c)^2\log(\sin(dx+c)+1)-b\cos(dx+c)^2\log(-\sin(dx+c)+1)+2\,(2\,a\cos(dx+c)+b)\,\mathrm{si}}{4\,d\cos(dx+c)^2}$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(b*cos(d*x + c)^2*log(sin(d*x + c) + 1) - b*cos(d*x + c)^2*log(-sin(d*
x + c) + 1) + 2*(2*a*cos(d*x + c) + b)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \sec^2(c + dx)(a + b\sec(c + dx))\,dx = \int (a + b\sec(c + dx))\sec^2(c + dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int \sec^2(c + dx)(a + b\sec(c + dx))\,dx = -\frac{b\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx + c) + 1) + \log(\sin(dx + c) - 1)\right) - 4\,a\tan(dx + c)}{4\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/4*(b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log
(sin(d*x + c) - 1)) - 4*a*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(43) = 86.

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.28

$$\int \sec^2(c + dx)(a + b\sec(c + dx))\,dx = \frac{b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 2\,a\tan\left(\frac{1}{2}\,d\cdots\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b*log(abs(tan(1/2*d*x + 1/2*c)
 - 1)) - 2*(2*a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 - 2*a*ta
n(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^
2)/d
```

Mupad [B] (verification not implemented)

Time = 10.64 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.81

$$\int \sec^2(c+dx)(a+b\sec(c+dx))\,dx$$
$$=\frac{b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(2\,a-b)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(2\,a+b)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

input
```
int((a + b/cos(c + d*x))/cos(c + d*x)^2,x)
```

output
```
(b*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)^3*(2*a - b) - tan(c/
2 + (d*x)/2)*(2*a + b))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2
+ 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.47

$$\int \sec^2(c+dx)(a+b\sec(c+dx))\,dx$$
$$=\frac{-2\cos(dx+c)\sin(dx+c)\,a-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2\,b+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b+\log\left(\mathrm{ta}\right.}{2d\left(\sin(dx+c)^2-1\right)}$$

input
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c)),x)
```

output
```
( - 2*cos(c + d*x)*sin(c + d*x)*a - log(tan((c + d*x)/2) - 1)*sin(c + d*x)
**2*b + log(tan((c + d*x)/2) - 1)*b + log(tan((c + d*x)/2) + 1)*sin(c + d*
x)**2*b - log(tan((c + d*x)/2) + 1)*b - sin(c + d*x)*b)/(2*d*(sin(c + d*x)
**2 - 1))
```

3.449 $\int \sec(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 24

$$\int \sec(c+dx)(a+b\sec(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{d} + \frac{b\tan(c+dx)}{d}$$

output
```
a*arctanh(sin(d*x+c))/d+b*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \sec(c+dx)(a+b\sec(c+dx))\,dx = \frac{a\coth^{-1}(\sin(c+dx))}{d} + \frac{b\tan(c+dx)}{d}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x]),x]
```

output
```
(a*ArcCoth[Sin[c + d*x]])/d + (b*Tan[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\sec(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \sec(c+dx)dx+b\int \sec^2(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+b\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{4254}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{b\int 1d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{24}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{b\tan(c+dx)}{d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{b\tan(c+dx)}{d}$$

input
```
Int[Sec[c + d*x]*(a + b*Sec[c + d*x]),x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/d + (b*Tan[c + d*x])/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\frac{a\ln(\sec(dx+c)+\tan(dx+c))+b\tan(dx+c)}{d}$	30
default	$\frac{a\ln(\sec(dx+c)+\tan(dx+c))+b\tan(dx+c)}{d}$	30
parts	$\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b\tan(dx+c)}{d}$	32
risch	$\frac{2ib}{d\left(e^{2i(dx+c)}+1\right)}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{d}$	59
parallelrisch	$\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)-a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+b\sin(dx+c)}{\cos(dx+c)d}$	63
norman	$-\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	67

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*ln(sec(d*x+c)+tan(d*x+c))+b*tan(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(24) = 48$.

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.50

$$\int \sec(c+dx)(a+b\sec(c+dx))\,dx = \frac{a\cos(dx+c)\log(\sin(dx+c)+1) - a\cos(dx+c)\log(-\sin(dx+c)+1) + 2b\sin(dx+c)}{2d\cos(dx+c)}$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x +
c) + 1) + 2*b*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [A] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \sec(c+dx)(a+b\sec(c+dx))\,dx = \begin{cases} \frac{a\log(\tan(c+dx)+\sec(c+dx))+b\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x(a+b\sec(c))\sec(c) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c)),x)
```

output
```
Piecewise(((a*log(tan(c + d*x) + sec(c + d*x)) + b*tan(c + d*x))/d, Ne(d,
0)), (x*(a + b*sec(c))*sec(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \sec(c+dx)(a+b\sec(c+dx))\,dx = \frac{a\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+b\tan\left(dx+c\right)}{d}$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
(a*log(sec(d*x + c) + tan(d*x + c)) + b*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(24) = 48.

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int \sec(c+dx)(a+b\sec(c+dx))\,dx = \frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
(a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1
)) - 2*b*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

Mupad [B] (verification not implemented)

Time = 10.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int \sec(c+dx)(a+b\sec(c+dx))\,dx = \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + b/cos(c + d*x))/cos(c + d*x),x)
```

output
```
(2*a*atanh(tan(c/2 + (d*x)/2)))/d - (2*b*tan(c/2 + (d*x)/2))/(d*(tan(c/2 +
 (d*x)/2)^2 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.58

$$\int \sec(c+dx)(a+b\sec(c+dx))\,dx$$
$$=\frac{-\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a+\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a+\sin(dx+c)\,b}{\cos(dx+c)\,d}$$

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c)),x)
```

output
```
( - cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a + cos(c + d*x)*log(tan((c + d
*x)/2) + 1)*a + sin(c + d*x)*b)/(cos(c + d*x)*d)
```

3.450 $\int (a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 10, antiderivative size = 16

$$\int (a+b\sec(c+dx))\,dx = ax+\frac{b\text{arctanh}(\sin(c+dx))}{d}$$

output
```
a*x+b*arctanh(sin(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))\,dx = ax+\frac{b\coth^{-1}(\sin(c+dx))}{d}$$

input
```
Integrate[a + b*Sec[c + d*x],x]
```

output
```
a*x + (b*ArcCoth[Sin[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(c+dx))\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$ax+\frac{b\text{arctanh}(\sin(c+dx))}{d}$$

input
```
Int[a + b*Sec[c + d*x],x]
```

output
```
a*x + (b*ArcTanh[Sin[c + d*x]])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

method	result	size
default	$ax+\frac{b\ln(\sec(dx+c)+\tan(dx+c))}{d}$	24
parts	$ax+\frac{b\ln(\sec(dx+c)+\tan(dx+c))}{d}$	24
derivativedivides	$\frac{(dx+c)a+b\ln(\sec(dx+c)+\tan(dx+c))}{d}$	29
parallelrisch	$\frac{b\left(\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\right)}{d}+ax$	37
norman	$ax+\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	40
risch	$ax-\frac{b\ln(\mathrm{e}^{i(dx+c)}-i)}{d}+\frac{b\ln(\mathrm{e}^{i(dx+c)}+i)}{d}$	42

input
```
int(a+b*sec(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*x+b/d*ln(sec(d*x+c)+tan(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(16) = 32.

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int (a+b\sec(c+dx))\,dx=\frac{2\,adx+b\log\left(\sin\left(dx+c\right)+1\right)-b\log\left(-\sin\left(dx+c\right)+1\right)}{2\,d}$$

input
```
integrate(a+b*sec(d*x+c),x, algorithm="fricas")
```

output
```
1/2*(2*a*d*x + b*log(sin(d*x + c) + 1) - b*log(-sin(d*x + c) + 1))/d
```

Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.56

$$\int (a+b\sec(c+dx))\,dx = ax + b\left(\begin{cases} \frac{\log(\tan(c+dx)+\sec(c+dx))}{d} & \text{for } d \neq 0 \\ \frac{x(\tan(c)\sec(c)+\sec^2(c))}{\tan(c)+\sec(c)} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+b*sec(d*x+c),x)
```

output
```
a*x + b*Piecewise((log(tan(c + d*x) + sec(c + d*x))/d, Ne(d, 0)), (x*(tan(
c)*sec(c) + sec(c)**2)/(tan(c) + sec(c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int (a+b\sec(c+dx))\,dx = ax + \frac{b\log(\sec(dx+c)+\tan(dx+c))}{d}$$

input
```
integrate(a+b*sec(d*x+c),x, algorithm="maxima")
```

output
```
a*x + b*log(sec(d*x + c) + tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(16) = 32.

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.06

$$\int (a+b\sec(c+dx))\,dx = ax + \frac{b\left(\log\left(\left|\frac{1}{\sin(dx+c)}+\sin(dx+c)+2\right|\right)-\log\left(\left|\frac{1}{\sin(dx+c)}+\sin(dx+c)-2\right|\right)\right)}{4\,d}$$

input
```
integrate(a+b*sec(d*x+c),x, algorithm="giac")
```

output
```
a*x + 1/4*b*(log(abs(1/sin(d*x + c) + sin(d*x + c) + 2)) - log(abs(1/sin(d
*x + c) + sin(d*x + c) - 2)))/d
```

Mupad [B] (verification not implemented)

Time = 10.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.56

$$\int (a + b\sec(c + dx))\,dx = \frac{2\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

input
```
int(a + b/cos(c + d*x),x)
```

output
```
(2*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*b*atanh(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.38

$$\int (a + b\sec(c + dx))\,dx = \frac{-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b + \log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)b + adx}{d}$$

input
```
int(a+b*sec(d*x+c),x)
```

output
```
( - log(tan((c + d*x)/2) - 1)*b + log(tan((c + d*x)/2) + 1)*b + a*d*x)/d
```

3.451 $\int \cos(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \cos(c+dx)(a+b\sec(c+dx))\,dx = bx + \frac{a\sin(c+dx)}{d}$$

output
```
b*x+a*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \cos(c+dx)(a+b\sec(c+dx))\,dx = bx + \frac{a\cos(dx)\sin(c)}{d} + \frac{a\cos(c)\sin(dx)}{d}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x]),x]
```

output
```
b*x + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 4274, 24, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\sec(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \cos(c+dx)dx+b\int 1dx$$

$$\Big\downarrow \text{24}$$

$$a\int \cos(c+dx)dx+bx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)dx+bx$$

$$\Big\downarrow \text{3117}$$

$$\frac{a\sin(c+dx)}{d}+bx$$

input
```
Int[Cos[c + d*x]*(a + b*Sec[c + d*x]),x]
```

output
```
b*x + (a*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
risch	$bx+\frac{a\sin(dx+c)}{d}$	16
parallelrisch	$\frac{bdx+a\sin(dx+c)}{d}$	18
derivativedivides	$\frac{a\sin(dx+c)+b(dx+c)}{d}$	21
default	$\frac{a\sin(dx+c)+b(dx+c)}{d}$	21
norman	$\frac{bx+bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}$	50

input
```
int(cos(d*x+c)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
b*x+a*sin(d*x+c)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cos(c+dx)(a+b\sec(c+dx))\,dx = \frac{bdx+a\sin(dx+c)}{d}$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
(b*d*x + a*sin(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cos(c+dx)(a+b\sec(c+dx))\,dx = a\left(\begin{cases} x\cos(c) & \text{for } d=0 \\ \frac{\sin(c+dx)}{d} & \text{otherwise} \end{cases}\right)+bx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c)),x)
```

output
```
a*Piecewise((x*cos(c), Eq(d, 0)), (sin(c + d*x)/d, True)) + b*x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cos(c+dx)(a+b\sec(c+dx))\,dx = \frac{(dx+c)b+a\sin(dx+c)}{d}$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
((d*x + c)*b + a*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(15) = 30$.

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \cos(c+dx)(a+b\sec(c+dx))\,dx = \frac{(dx+c)b + \frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
((d*x + c)*b + 2*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1))/d
```

Mupad [B] (verification not implemented)

Time = 10.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cos(c+dx)(a+b\sec(c+dx))\,dx = \frac{a\,\sin\left(c+d\,x\right)+b\,d\,x}{d}$$

input
```
int(cos(c + d*x)*(a + b/cos(c + d*x)),x)
```

output
```
(a*sin(c + d*x) + b*d*x)/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cos(c+dx)(a+b\sec(c+dx))\,dx = \frac{\sin\left(dx+c\right)a+bdx}{d}$$

input
```
int(cos(d*x+c)*(a+b*sec(d*x+c)),x)
```

output
```
(sin(c + d*x)*a + b*d*x)/d
```

3.452 $\int \cos^2(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 38

$$\int \cos^2(c+dx)(a+b\sec(c+dx))\,dx=\frac{ax}{2}+\frac{b\sin(c+dx)}{d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
1/2*a*x+b*sin(d*x+c)/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \cos^2(c+dx)(a+b\sec(c+dx))\,dx=\frac{4b\sin(c+dx)+a(2(c+dx)+\sin(2(c+dx)))}{4d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

output
```
(4*b*Sin[c + d*x] + a*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 4274, 3042, 3115, 24, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\sec(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \cos^2(c+dx)dx+b\int \cos(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+b\int \sin\left(c+dx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+b\int \sin\left(c+dx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{24}$$

$$b\int \sin\left(c+dx+\frac{\pi}{2}\right)dx+a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)$$

$$\Big\downarrow \text{3117}$$

$$a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)+\frac{b\sin(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

output
```
(b*Sin[c + d*x])/d + a*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{ax}{2}+\frac{b\sin(dx+c)}{d}+\frac{a\sin(2dx+2c)}{4d}$	32
parallelrisch	$\frac{2adx+4b\sin(dx+c)+a\sin(2dx+2c)}{4d}$	32
derivativedivides	$\frac{a\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b\sin(dx+c)}{d}$	38
default	$\frac{a\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b\sin(dx+c)}{d}$	38
norman	$\frac{ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\frac{(a+2b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{ax}{2}+\frac{ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{2}-\frac{(a-2b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}}$	90

input
```
int(cos(d*x+c)^2*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x+b*sin(d*x+c)/d+1/4*a/d*sin(2*d*x+2*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \cos^2(c+dx)(a+b\sec(c+dx))\,dx=\frac{adx+(a\cos(dx+c)+2\,b)\sin(dx+c)}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(a*d*x + (a*cos(d*x + c) + 2*b)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^2(c+dx)(a+b\sec(c+dx))\,dx=\int (a+b\sec(c+dx))\cos^2(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**2*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*cos(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \cos^2(c+dx)(a+b\sec(c+dx))\,dx = \frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a+4\,b\sin(dx+c)}{4\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a + 4*b*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(34) = 68$.

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.16

$$\int \cos^2(c+dx)(a+b\sec(c+dx))\,dx = \frac{(dx+c)a - \frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/2*((d*x + c)*a - 2*(a*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c)^
3 - a*tan(1/2*d*x + 1/2*c) - 2*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*
c)^2 + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 10.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \cos^2(c+dx)(a+b\sec(c+dx))\,dx=\frac{a\,x}{2}+\frac{a\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{b\,\sin\left(c+d\,x\right)}{d}$$

input
```
int(cos(c + d*x)^2*(a + b/cos(c + d*x)),x)
```

output
```
(a*x)/2 + (a*sin(2*c + 2*d*x))/(4*d) + (b*sin(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \cos^2(c+dx)(a+b\sec(c+dx))\,dx=\frac{\cos\left(dx+c\right)\sin\left(dx+c\right)a+2\sin\left(dx+c\right)b+adx}{2d}$$

input
```
int(cos(d*x+c)^2*(a+b*sec(d*x+c)),x)
```

output
```
(cos(c + d*x)*sin(c + d*x)*a + 2*sin(c + d*x)*b + a*d*x)/(2*d)
```

3.453 $\int \cos^3(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \cos^3(c+dx)(a+b\sec(c+dx))\,dx = \frac{bx}{2}+\frac{a\sin(c+dx)}{d}+\frac{b\cos(c+dx)\sin(c+dx)}{2d}-\frac{a\sin^3(c+dx)}{3d}$$

output
```
1/2*b*x+a*sin(d*x+c)/d+1/2*b*cos(d*x+c)*sin(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int \cos^3(c+dx)(a+b\sec(c+dx))\,dx = \frac{b(c+dx)}{2d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}+\frac{b\sin(2(c+dx))}{4d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

output
```
(b*(c + d*x))/(2*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (b*S
in[2*(c + d*x)])/(4*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 4274, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+b\sec(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \cos^3(c+dx)dx+b\int \cos^2(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx+b\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3113}$$

$$b\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{a\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$b\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3115}$$

$$b\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{24}$$

$$b\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

input
```
Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

output
```
b*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (a*(-Sin[c + d*x] + Sin[c +
d*x]^3/3))/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

method	result
parallelrisch	$\frac{6bdx+9a\sin(dx+c)+a\sin(3dx+3c)+3b\sin(2dx+2c)}{12d}$
risch	$\frac{bx}{2}+\frac{3a\sin(dx+c)}{4d}+\frac{a\sin(3dx+3c)}{12d}+\frac{b\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{\frac{a\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+b\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{\frac{a\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+b\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
norman	$\frac{\frac{(2a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}+\frac{(2a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{bx}{2}+\frac{4a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{3bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2}+\frac{3bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2}+\frac{bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{2}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}$

input
```
int(cos(d*x+c)^3*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/12*(6*b*d*x+9*a*sin(d*x+c)+a*sin(3*d*x+3*c)+3*b*sin(2*d*x+2*c))/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \cos^3(c+dx)(a+b\sec(c+dx))\,dx = \frac{3\,bdx+\left(2\,a\cos\left(dx+c\right)^2+3\,b\cos\left(dx+c\right)+4\,a\right)\sin\left(dx+c\right)}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(3*b*d*x + (2*a*cos(d*x + c)^2 + 3*b*cos(d*x + c) + 4*a)*sin(d*x + c))
/d
```

Sympy [F]

$$\int \cos^3(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec{(c+dx)})\cos^3{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**3*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*cos(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \cos^3(c+dx)(a+b\sec(c+dx))\,dx = -\frac{4\left(\sin{(dx+c)}^3-3\sin{(dx+c)}\right)a-3\left(2\,dx+2\,c+\sin{(2\,dx+2\,c)}\right)b}{12\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*a - 3*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*b)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(48) = 96.

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.81

$$\int \cos^3(c+dx)(a+b\sec(c+dx))\,dx = \frac{3\,(dx+c)b+\frac{2\left(6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(3*(d*x + c)*b + 2*(6*a*tan(1/2*d*x + 1/2*c)^5 - 3*b*tan(1/2*d*x + 1/2
*c)^5 + 4*a*tan(1/2*d*x + 1/2*c)^3 + 6*a*tan(1/2*d*x + 1/2*c) + 3*b*tan(1/
2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 10.44 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \cos^3(c+dx)(a+b\sec(c+dx))\,dx = \frac{b\,x}{2} + \frac{2\,a\,\sin\left(c+d\,x\right)}{3\,d} + \frac{b\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d} + \frac{a\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}$$

input
```
int(cos(c + d*x)^3*(a + b/cos(c + d*x)),x)
```

output
```
(b*x)/2 + (2*a*sin(c + d*x))/(3*d) + (b*cos(c + d*x)*sin(c + d*x))/(2*d) +
 (a*cos(c + d*x)^2*sin(c + d*x))/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \cos^3(c+dx)(a+b\sec(c+dx))\,dx = \frac{3\cos\left(dx+c\right)\sin\left(dx+c\right)b - 2\sin\left(dx+c\right)^3 a + 6\sin\left(dx+c\right)a + 3bdx}{6d}$$

input
```
int(cos(d*x+c)^3*(a+b*sec(d*x+c)),x)
```

output
```
(3*cos(c + d*x)*sin(c + d*x)*b - 2*sin(c + d*x)**3*a + 6*sin(c + d*x)*a +
3*b*d*x)/(6*d)
```

3.454 $\int \cos^4(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \cos^4(c+dx)(a+b\sec(c+dx))\,dx=\frac{3ax}{8}+\frac{b\sin(c+dx)}{d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{b\sin^3(c+dx)}{3d}$$

output
```
3/8*a*x+b*sin(d*x+c)/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c)^3*si
n(d*x+c)/d-1/3*b*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \cos^4(c+dx)(a+b\sec(c+dx))\,dx=\frac{3a(c+dx)}{8d}+\frac{b\sin(c+dx)}{d}-\frac{b\sin^3(c+dx)}{3d}+\frac{a\sin(2(c+dx))}{4d}+\frac{a\sin(4(c+dx))}{32d}$$

input
```
Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

output
```
(3*a*(c + d*x))/(8*d) + (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*d) + (a
*Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)])/(32*d)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 4274, 3042, 3113, 2009, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+b\sec(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \cos^4(c+dx)dx+b\int \cos^3(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx+b\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3113}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{b\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{b\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{b\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2 dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{b\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$\Big\downarrow$ 3115

$$a\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{b\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$\Big\downarrow$ 24

$$a\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{b\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

input
```
Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

output
```
-((b*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/d) + a*((Cos[c + d*x]^3*Sin[c + d
*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

method	result
parallelrisch	$\frac{36adx+72b\sin(dx+c)+3a\sin(4dx+4c)+8b\sin(3dx+3c)+24a\sin(2dx+2c)}{96d}$
derivativedivides	$\frac{a\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{b\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}}{d}$
default	$\frac{a\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{b\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}}{d}$
risch	$\frac{3ax}{8}+\frac{3b\sin(dx+c)}{4d}+\frac{a\sin(4dx+4c)}{32d}+\frac{b\sin(3dx+3c)}{12d}+\frac{a\sin(2dx+2c)}{4d}$
norman	$\frac{\frac{3ax}{8}+\frac{3ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2}+\frac{9ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4}+\frac{3ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{2}+\frac{3ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{8}-\frac{(5a-8b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}+\frac{(5a+8b)\tan\left(\frac{d}{}\right.}{4d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}$

input
```
int(cos(d*x+c)^4*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/96*(36*a*d*x+72*b*sin(d*x+c)+3*a*sin(4*d*x+4*c)+8*b*sin(3*d*x+3*c)+24*a*
sin(2*d*x+2*c))/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

$$\int \cos^4(c+dx)(a+b\sec(c+dx))\,dx = \frac{9\,adx+\left(6\,a\cos\left(dx+c\right)^3+8\,b\cos\left(dx+c\right)^2+9\,a\cos\left(dx+c\right)+16\,b\right)\sin\left(dx+c\right)}{24\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/24*(9*a*d*x + (6*a*cos(d*x + c)^3 + 8*b*cos(d*x + c)^2 + 9*a*cos(d*x + c
) + 16*b)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^4(c+dx)(a+b\sec(c+dx))\,dx = \int \left(a+b\sec\left(c+dx\right)\right)\cos^4\left(c+dx\right)\,dx$$

input
```
integrate(cos(d*x+c)**4*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*cos(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

$$\int \cos^4(c+dx)(a+b\sec(c+dx))\,dx = \frac{3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a-32\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)b}{96\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a - 32*(si
n(d*x + c)^3 - 3*sin(d*x + c))*b)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(68) = 136$.

Time = 0.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.84

$$\int \cos^4(c+dx)(a+b\sec(c+dx))\,dx = \frac{9\,(dx+c)a - \frac{2\left(15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 24\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 40\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 40\,b\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/24*(9*(d*x + c)*a - 2*(15*a*tan(1/2*d*x + 1/2*c)^7 - 24*b*tan(1/2*d*x +
1/2*c)^7 - 9*a*tan(1/2*d*x + 1/2*c)^5 - 40*b*tan(1/2*d*x + 1/2*c)^5 + 9*a*
tan(1/2*d*x + 1/2*c)^3 - 40*b*tan(1/2*d*x + 1/2*c)^3 - 15*a*tan(1/2*d*x +
1/2*c) - 24*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 10.45 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int \cos^4(c+dx)(a+b\sec(c+dx))\,dx = \frac{3\,a\,x}{8} + \frac{2\,b\,\sin\left(c+d\,x\right)}{3\,d} + \frac{3\,a\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{8\,d} + \frac{a\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{4\,d} + \frac{b\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}$$

input
```
int(cos(c + d*x)^4*(a + b/cos(c + d*x)),x)
```

output
```
(3*a*x)/8 + (2*b*sin(c + d*x))/(3*d) + (3*a*cos(c + d*x)*sin(c + d*x))/(8*
d) + (a*cos(c + d*x)^3*sin(c + d*x))/(4*d) + (b*cos(c + d*x)^2*sin(c + d*x
))/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.83

$$\int \cos^4(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{-6\cos(dx+c)\sin(dx+c)^3 a + 15\cos(dx+c)\sin(dx+c) a - 8\sin(dx+c)^3 b + 24\sin(dx+c) b + 9a}{24d}$$

input
```
int(cos(d*x+c)^4*(a+b*sec(d*x+c)),x)
```

output
```
( - 6*cos(c + d*x)*sin(c + d*x)**3*a + 15*cos(c + d*x)*sin(c + d*x)*a - 8*
sin(c + d*x)**3*b + 24*sin(c + d*x)*b + 9*a*d*x)/(24*d)
```

3.455 $\int \cos^5(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 92

$$\int \cos^5(c+dx)(a+b\sec(c+dx))\,dx=\frac{3bx}{8}+\frac{a\sin(c+dx)}{d}+\frac{3b\cos(c+dx)\sin(c+dx)}{8d}+\frac{b\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin^5(c+dx)}{5d}$$

output
```
3/8*b*x+a*sin(d*x+c)/d+3/8*b*cos(d*x+c)*sin(d*x+c)/d+1/4*b*cos(d*x+c)^3*si
n(d*x+c)/d-2/3*a*sin(d*x+c)^3/d+1/5*a*sin(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.97

$$\int \cos^5(c+dx)(a+b\sec(c+dx))\,dx=\frac{3b(c+dx)}{8d}+\frac{a\sin(c+dx)}{d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin^5(c+dx)}{5d}+\frac{b\sin(2(c+dx))}{4d}+\frac{b\sin(4(c+dx))}{32d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x]),x]
```

output
```
(3*b*(c + d*x))/(8*d) + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/(3*d) +
(a*Sin[c + d*x]^5)/(5*d) + (b*Sin[2*(c + d*x)])/(4*d) + (b*Sin[4*(c + d*x)
])/(32*d)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {3042, 4274, 3042, 3113, 2009, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+b\sec(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \cos^5(c+dx)dx+b\int \cos^4(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^5dx+b\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3113}$$

$$b\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{a\int \left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$b\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3115}$$

$$b\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3115}$$

$$b\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{24}$$

$$b\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

input
```
Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x]),x]
```

output
```
-((a*(-Sin[c + d*x] + (2*Sin[c + d*x]^3)/3 - Sin[c + d*x]^5/5))/d) + b*((C
os[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/
(2*d)))/4)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.75

method	result
parallelrisch	$\frac{180bdx+300a\sin(dx+c)+6a\sin(5dx+5c)+15b\sin(4dx+4c)+50a\sin(3dx+3c)+120b\sin(2dx+2c)}{480d}$
derivativedivides	$\frac{\frac{a\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+b\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
default	$\frac{\frac{a\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+b\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
risch	$\frac{3bx}{8}+\frac{5a\sin(dx+c)}{8d}+\frac{a\sin(5dx+5c)}{80d}+\frac{b\sin(4dx+4c)}{32d}+\frac{5a\sin(3dx+3c)}{48d}+\frac{b\sin(2dx+2c)}{4d}$
norman	$\frac{\frac{3bx}{8}+\frac{116a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{15d}+\frac{15bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8}+\frac{15bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4}+\frac{15bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{4}+\frac{15bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{8}+\frac{3bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{8}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}$

input
```
int(cos(d*x+c)^5*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/480*(180*b*d*x+300*a*sin(d*x+c)+6*a*sin(5*d*x+5*c)+15*b*sin(4*d*x+4*c)+5
0*a*sin(3*d*x+3*c)+120*b*sin(2*d*x+2*c))/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.70

$$\int \cos^5(c+dx)(a+b\sec(c+dx))\,dx$$
$$=\frac{45\,bdx+\left(24\,a\cos\left(dx+c\right)^4+30\,b\cos\left(dx+c\right)^3+32\,a\cos\left(dx+c\right)^2+45\,b\cos\left(dx+c\right)+64\,a\right)\sin\left(dx+\right.}{120\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/120*(45*b*d*x + (24*a*cos(d*x + c)^4 + 30*b*cos(d*x + c)^3 + 32*a*cos(d*
x + c)^2 + 45*b*cos(d*x + c) + 64*a)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^5(c+dx)(a+b\sec(c+dx))\,dx=\int(a+b\sec(c+dx))\cos^5(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**5*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*cos(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.75

$$\int \cos^5(c+dx)(a+b\sec(c+dx))\,dx$$
$$=\frac{32\left(3\sin\left(dx+c\right)^5-10\sin\left(dx+c\right)^3+15\sin\left(dx+c\right)\right)a+15\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\sin\left(2\right.\right.}{480\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a + 15*
(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*b)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.67

$$\int \cos^5(c+dx)(a+b\sec(c+dx))\,dx$$
$$=\frac{45\,(dx+c)b+\frac{2\left(120\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-75\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+160\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-30\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+464\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+160\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{120\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/120*(45*(d*x + c)*b + 2*(120*a*tan(1/2*d*x + 1/2*c)^9 - 75*b*tan(1/2*d*x
 + 1/2*c)^9 + 160*a*tan(1/2*d*x + 1/2*c)^7 - 30*b*tan(1/2*d*x + 1/2*c)^7 +
 464*a*tan(1/2*d*x + 1/2*c)^5 + 160*a*tan(1/2*d*x + 1/2*c)^3 + 30*b*tan(1/
2*d*x + 1/2*c)^3 + 120*a*tan(1/2*d*x + 1/2*c) + 75*b*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 14.40 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.23

$$\int \cos^5(c+dx)(a+b\sec(c+dx))\,dx = \frac{3\,b\,x}{8} + \frac{\left(2\,a-\frac{5\,b}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{8\,a}{3}-\frac{b}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\frac{116\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}+\left(\frac{8\,a}{3}+\frac{b}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(2\,a\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^5}$$

input

```
int(cos(c + d*x)^5*(a + b/cos(c + d*x)),x)
```

output

```
(3*b*x)/8 + (tan(c/2 + (d*x)/2)*(2*a + (5*b)/4) + tan(c/2 + (d*x)/2)^3*((8
*a)/3 + b/2) + tan(c/2 + (d*x)/2)^9*(2*a - (5*b)/4) + tan(c/2 + (d*x)/2)^7
*((8*a)/3 - b/2) + (116*a*tan(c/2 + (d*x)/2)^5)/15)/(d*(tan(c/2 + (d*x)/2)
^2 + 1)^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.80

$$\int \cos^5(c+dx)(a+b\sec(c+dx))\,dx = \frac{-30\cos(dx+c)\sin(dx+c)^3 b+75\cos(dx+c)\sin(dx+c)\,b+24\sin(dx+c)^5 a-80\sin(dx+c)^3 a+}{120d}$$

input

```
int(cos(d*x+c)^5*(a+b*sec(d*x+c)),x)
```

output

```
( - 30*cos(c + d*x)*sin(c + d*x)**3*b + 75*cos(c + d*x)*sin(c + d*x)*b + 2
4*sin(c + d*x)**5*a - 80*sin(c + d*x)**3*a + 120*sin(c + d*x)*a + 45*b*d*x
)/(120*d)
```

3.456 $\int \sec^4(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 135

$$\begin{aligned}\int \sec^4(c+dx)(a+b\sec(c+dx))^2\,dx &= \frac{3ab\operatorname{arctanh}(\sin(c+dx))}{4d}\\ &\quad+\frac{(5a^2+4b^2)\tan(c+dx)}{5d}\\ &\quad+\frac{3ab\sec(c+dx)\tan(c+dx)}{4d}\\ &\quad+\frac{ab\sec^3(c+dx)\tan(c+dx)}{2d}\\ &\quad+\frac{b^2\sec^4(c+dx)\tan(c+dx)}{5d}\\ &\quad+\frac{(5a^2+4b^2)\tan^3(c+dx)}{15d}\end{aligned}$$

output
```
3/4*a*b*arctanh(sin(d*x+c))/d+1/5*(5*a^2+4*b^2)*tan(d*x+c)/d+3/4*a*b*sec(d
*x+c)*tan(d*x+c)/d+1/2*a*b*sec(d*x+c)^3*tan(d*x+c)/d+1/5*b^2*sec(d*x+c)^4*
tan(d*x+c)/d+1/15*(5*a^2+4*b^2)*tan(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.67

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$=\frac{45ab\text{arctanh}(\sin(c+dx))+\tan(c+dx)\left(60(a^2+b^2)+45ab\sec(c+dx)+30ab\sec^3(c+dx)+20(a^2+\right.}{60d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

output
```
(45*a*b*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(60*(a^2 + b^2) + 45*a*b*Sec[
c + d*x] + 30*a*b*Sec[c + d*x]^3 + 20*(a^2 + 2*b^2)*Tan[c + d*x]^2 + 12*b^
2*Tan[c + d*x]^4))/(60*d)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.95, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.571$, Rules used = {3042, 4275, 3042, 4255, 3042, 4255, 3042, 4257, 4534, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$
$$\Big\downarrow \text{4275}$$
$$\int \sec^4(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int \sec^5(c+dx)dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \text{4255}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{4255}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{4257}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{4534}$$

$$\frac{1}{5}\left(5a^2+4b^2\right)\int \sec^4(c+dx)dx+2ab\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{b^2\tan(c+dx)\sec^4(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(5a^2+4b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx+$$
$$2ab\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+$$
$$\frac{b^2\tan(c+dx)\sec^4(c+dx)}{5d}$$

$\Big\downarrow$ 4254

$$-\frac{\left(5a^2+4b^2\right)\int\left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{5d}+$$
$$2ab\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+$$
$$\frac{b^2\tan(c+dx)\sec^4(c+dx)}{5d}$$

$\Big\downarrow$ 2009

$$-\frac{\left(5a^2+4b^2\right)\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{5d}+$$
$$2ab\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+$$
$$\frac{b^2\tan(c+dx)\sec^4(c+dx)}{5d}$$

input
```
Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

output
```
(b^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) - ((5*a^2 + 4*b^2)*(-Tan[c + d*x]
- Tan[c + d*x]^3/3))/(5*d) + 2*a*b*((Sec[c + d*x]^3*Tan[c + d*x])/(4*d) +
(3*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{-a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+2ab\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-b^2\left(\right.}{d}$
default	$\frac{-a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+2ab\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-b^2\left(\right.}{d}$
parts	$-\frac{a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}-\frac{b^2\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}+\frac{2ab\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3}{}\right.\right.}{}$
risch	$-\frac{i\left(45ab\,e^{9i(dx+c)}+210ab\,e^{7i(dx+c)}-120a^2e^{6i(dx+c)}-280a^2e^{4i(dx+c)}-320\,e^{4i(dx+c)}b^2-210ab\,e^{3i(dx+c)}-200a^2e^{2i(d}\right.}{30d\left(e^{2i(dx+c)}+1\right)^5}$
norman	$\frac{-\frac{4(25a^2+29b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{15d}-\frac{(4a^2-5ab+4b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{2d}-\frac{(4a^2+5ab+4b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}+\frac{(16a^2-3ab+8b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{3d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}$
parallelrisch	$\frac{-450a\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3dx+3c)}{2}+\cos(dx+c)\right)b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+450a\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3dx+3c)}{2}+\cos(dx+c)\right)b}{60d(\cos(5d}$

input
```
int(sec(d*x+c)^4*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+2*a*b*(-(-1/4*sec(d*x+c)^3-3/
8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-b^2*(-8/15-1/5*sec
(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$=\frac{45\,ab\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-45\,ab\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+2\left(45\,ab\cos\left(dx\right.\right.}{120\,d\cos}$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/120*(45*a*b*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 45*a*b*cos(d*x + c)^5
*log(-sin(d*x + c) + 1) + 2*(45*a*b*cos(d*x + c)^3 + 8*(5*a^2 + 4*b^2)*cos
(d*x + c)^4 + 30*a*b*cos(d*x + c) + 4*(5*a^2 + 4*b^2)*cos(d*x + c)^2 + 12*
b^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2\sec^4(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**4*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*sec(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.98

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{40\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^2+8\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)b^2-15}{120\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/120*(40*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 + 8*(3*tan(d*x + c)^5 + 10
*tan(d*x + c)^3 + 15*tan(d*x + c))*b^2 - 15*a*b*(2*(3*sin(d*x + c)^3 - 5*s
in(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
+ 1) + 3*log(sin(d*x + c) - 1)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(123) = 246.

Time = 0.16 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.01

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{45\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-45\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(60\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-75\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/60*(45*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*a*b*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) - 2*(60*a^2*tan(1/2*d*x + 1/2*c)^9 - 75*a*b*tan(1/2*d*x
+ 1/2*c)^9 + 60*b^2*tan(1/2*d*x + 1/2*c)^9 - 160*a^2*tan(1/2*d*x + 1/2*c)^
7 + 30*a*b*tan(1/2*d*x + 1/2*c)^7 - 80*b^2*tan(1/2*d*x + 1/2*c)^7 + 200*a^
2*tan(1/2*d*x + 1/2*c)^5 + 232*b^2*tan(1/2*d*x + 1/2*c)^5 - 160*a^2*tan(1/
2*d*x + 1/2*c)^3 - 30*a*b*tan(1/2*d*x + 1/2*c)^3 - 80*b^2*tan(1/2*d*x + 1/
2*c)^3 + 60*a^2*tan(1/2*d*x + 1/2*c) + 75*a*b*tan(1/2*d*x + 1/2*c) + 60*b^
2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 13.33 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.64

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{3\,a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}$$
$$-\frac{\left(2\,a^2-\frac{5\,a\,b}{2}+2\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(-\frac{16\,a^2}{3}+a\,b-\frac{8\,b^2}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{20\,a^2}{3}+\frac{116\,b^2}{15}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\,\mathrm{ta}\right.}$$

input
```
int((a + b/cos(c + d*x))^2/cos(c + d*x)^4,x)
```

output
```
(3*a*b*atanh(tan(c/2 + (d*x)/2)))/(2*d) - (tan(c/2 + (d*x)/2)^5*((20*a^2)/
3 + (116*b^2)/15) + tan(c/2 + (d*x)/2)^9*(2*a^2 - (5*a*b)/2 + 2*b^2) - tan
(c/2 + (d*x)/2)^3*(a*b + (16*a^2)/3 + (8*b^2)/3) - tan(c/2 + (d*x)/2)^7*((
16*a^2)/3 - a*b + (8*b^2)/3) + tan(c/2 + (d*x)/2)*((5*a*b)/2 + 2*a^2 + 2*b
^2))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (
d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.28

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{-45\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^4ab+90\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+\cdots}{}$$

input
```
int(sec(d*x+c)^4*(a+b*sec(d*x+c))^2,x)
```

output
```
( - 45*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a*b + 90*cos
(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a*b - 45*cos(c + d*x)*
log(tan((c + d*x)/2) - 1)*a*b + 45*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*
sin(c + d*x)**4*a*b - 90*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*
x)**2*a*b + 45*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a*b - 45*cos(c + d*x
)*sin(c + d*x)**3*a*b + 75*cos(c + d*x)*sin(c + d*x)*a*b + 40*sin(c + d*x)
**5*a**2 + 32*sin(c + d*x)**5*b**2 - 100*sin(c + d*x)**3*a**2 - 80*sin(c +
 d*x)**3*b**2 + 60*sin(c + d*x)*a**2 + 60*sin(c + d*x)*b**2)/(60*cos(c + d
*x)*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.457 $\int \sec^3(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^2\,dx=\frac{(4a^2+3b^2)\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{2ab\tan(c+dx)}{d}+\frac{(4a^2+3b^2)\sec(c+dx)\tan(c+dx)}{8d}+\frac{b^2\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{2ab\tan^3(c+dx)}{3d}$$

output
```
1/8*(4*a^2+3*b^2)*arctanh(sin(d*x+c))/d+2*a*b*tan(d*x+c)/d+1/8*(4*a^2+3*b^
2)*sec(d*x+c)*tan(d*x+c)/d+1/4*b^2*sec(d*x+c)^3*tan(d*x+c)/d+2/3*a*b*tan(d
*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.75

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{3(4a^2+3b^2)\operatorname{arctanh}(\sin(c+dx))+\tan(c+dx)\left(3(4a^2+3b^2)\sec(c+dx)+6b^2\sec^3(c+dx)+16ab(3\right.}{24d}$$

input
```
Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

output
```
(3*(4*a^2 + 3*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*(4*a^2 + 3*b^2)
*Sec[c + d*x] + 6*b^2*Sec[c + d*x]^3 + 16*a*b*(3 + Tan[c + d*x]^2)))/(24*d
)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4275, 3042, 4254, 2009, 4534, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$
$$\Big\downarrow \; 4275$$
$$\int \sec^3(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int \sec^4(c+dx)dx$$
$$\Big\downarrow \; 3042$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \; 4254$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx-\frac{2ab\int\left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \; 2009$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx-\frac{2ab\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \; 4534$$

$$\frac{1}{4}\left(4a^2+3b^2\right)\int \sec^3(c+dx)dx-\frac{2ab\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}+\frac{b^2\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(4a^2+3b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{2ab\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}+\frac{b^2\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \; 4255$$

$$\frac{1}{4}\left(4a^2+3b^2\right)\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{2ab\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}+\frac{b^2\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(4a^2+3b^2\right)\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{2ab\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}+\frac{b^2\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \; 4257$$

$$\frac{1}{4}\left(4a^2+3b^2\right)\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{2ab\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}+\frac{b^2\tan(c+dx)\sec^3(c+dx)}{4d}$$

input
```
Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

output
```
(b^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((4*a^2 + 3*b^2)*(ArcTanh[Sin[c
+ d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4 - (2*a*b*(-Tan[c + d
*x] - Tan[c + d*x]^3/3))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-2ab\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+b^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx}{8}\right.\right.}{d}$
default	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-2ab\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+b^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx}{8}\right.\right.}{d}$
parts	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{b^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+}{8}\right.}{d}$
parallelrisch	$\frac{-48\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\left(a^2+\frac{3b^2}{4}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+48\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\left(a^2+\frac{3b^2}{4}\right)\mathrm{l}}{24d(3+\cos(4dx+4c)}$
norman	$\frac{\frac{\left(4a^2-16ab+5b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}+\frac{\left(4a^2+16ab+5b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{\left(12a^2-80ab-9b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{12d}-\frac{\left(12a^2+80ab-9b^2\right)\tan\left(\frac{d}{2}\right.}{12d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}$
risch	$-\frac{i\left(12a^2e^{7i(dx+c)}+9b^2e^{7i(dx+c)}+12a^2e^{5i(dx+c)}+33b^2e^{5i(dx+c)}-96ab\,e^{4i(dx+c)}-12a^2e^{3i(dx+c)}-33b^2e^{3i(dx+c)}-128\right.}{12d\left(e^{2i(dx+c)}+1\right)^4}$

input
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-2*a*b*(
-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+b^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))
*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.21

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$=\frac{3\left(4a^2+3b^2\right)\cos(dx+c)^4\log(\sin(dx+c)+1)-3\left(4a^2+3b^2\right)\cos(dx+c)^4\log(-\sin(dx+c)+1)+}{48\,d\cos(dx+}$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/48*(3*(4*a^2 + 3*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*a^2 +
3*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(32*a*b*cos(d*x + c)^3 +
16*a*b*cos(d*x + c) + 3*(4*a^2 + 3*b^2)*cos(d*x + c)^2 + 6*b^2)*sin(d*x +
c))/(d*cos(d*x + c)^4)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^2\,dx=\int(a+b\sec(c+dx))^2\sec^3(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*sec(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.31

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$=\frac{32\left(\tan(dx+c)^3+3\tan(dx+c)\right)ab-3b^2\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\right.}{48d}$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b - 3*b^2*(2*(3*sin(d*x + c)^
3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*
x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a^2*(2*sin(d*x + c)/(sin(d*x +
 c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(102) = 204$.

Time = 0.16 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.35

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{3\,(4\,a^2+3\,b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(4\,a^2+3\,b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(12\,a^2\tan\left(\frac{1}{2}\,dx+\cdots\right.\right.}{\cdots}}{\cdots}$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/24*(3*(4*a^2 + 3*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*a^2 + 3*
b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(12*a^2*tan(1/2*d*x + 1/2*c)^7
 - 48*a*b*tan(1/2*d*x + 1/2*c)^7 + 15*b^2*tan(1/2*d*x + 1/2*c)^7 - 12*a^2*
tan(1/2*d*x + 1/2*c)^5 + 80*a*b*tan(1/2*d*x + 1/2*c)^5 + 9*b^2*tan(1/2*d*x
 + 1/2*c)^5 - 12*a^2*tan(1/2*d*x + 1/2*c)^3 - 80*a*b*tan(1/2*d*x + 1/2*c)^
3 + 9*b^2*tan(1/2*d*x + 1/2*c)^3 + 12*a^2*tan(1/2*d*x + 1/2*c) + 48*a*b*ta
n(1/2*d*x + 1/2*c) + 15*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
- 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 13.07 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.67

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2+\frac{3\,b^2}{4}\right)}{d}$$
$$+\frac{\left(a^2-4\,a\,b+\frac{5\,b^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(-a^2+\frac{20\,a\,b}{3}+\frac{3\,b^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-a^2-\frac{20\,a\,b}{3}+\frac{3\,b^2}{4}\right)\,\tan\left(\frac{c}{2}\cdots\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,\tan\left(\frac{c}{2}+\frac{d}{\cdots}\right.\right.}$$

input
```
int((a + b/cos(c + d*x))^2/cos(c + d*x)^3,x)
```

output
```
(atanh(tan(c/2 + (d*x)/2))*(a^2 + (3*b^2)/4))/d + (tan(c/2 + (d*x)/2)^5*((
20*a*b)/3 - a^2 + (3*b^2)/4) + tan(c/2 + (d*x)/2)*(4*a*b + a^2 + (5*b^2)/4
) + tan(c/2 + (d*x)/2)^7*(a^2 - 4*a*b + (5*b^2)/4) - tan(c/2 + (d*x)/2)^3*
((20*a*b)/3 + a^2 - (3*b^2)/4))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (
d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 378, normalized size of antiderivative = 3.44

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{-32\cos(dx+c)\sin(dx+c)^3ab+48\cos(dx+c)\sin(dx+c)ab-12\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)}{}$$

input
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c))^2,x)
```

output
```
( - 32*cos(c + d*x)*sin(c + d*x)**3*a*b + 48*cos(c + d*x)*sin(c + d*x)*a*b
 - 12*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a**2 - 9*log(tan((c + d*x)
/2) - 1)*sin(c + d*x)**4*b**2 + 24*log(tan((c + d*x)/2) - 1)*sin(c + d*x)*
*2*a**2 + 18*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b**2 - 12*log(tan((
c + d*x)/2) - 1)*a**2 - 9*log(tan((c + d*x)/2) - 1)*b**2 + 12*log(tan((c +
 d*x)/2) + 1)*sin(c + d*x)**4*a**2 + 9*log(tan((c + d*x)/2) + 1)*sin(c + d
*x)**4*b**2 - 24*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**2 - 18*log(t
an((c + d*x)/2) + 1)*sin(c + d*x)**2*b**2 + 12*log(tan((c + d*x)/2) + 1)*a
**2 + 9*log(tan((c + d*x)/2) + 1)*b**2 - 12*sin(c + d*x)**3*a**2 - 9*sin(c
 + d*x)**3*b**2 + 12*sin(c + d*x)*a**2 + 15*sin(c + d*x)*b**2)/(24*d*(sin(
c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.458 $\int \sec^2(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{ab\text{arctanh}(\sin(c+dx))}{d} + \frac{(3a^2+2b^2)\tan(c+dx)}{3d} + \frac{ab\sec(c+dx)\tan(c+dx)}{d} + \frac{b^2\sec^2(c+dx)\tan(c+dx)}{3d}$$

output
```
a*b*arctanh(sin(d*x+c))/d+1/3*(3*a^2+2*b^2)*tan(d*x+c)/d+a*b*sec(d*x+c)*ta
n(d*x+c)/d+1/3*b^2*sec(d*x+c)^2*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{ab\text{arctanh}(\sin(c+dx))}{d} + \frac{a^2\tan(c+dx)}{d} + \frac{ab\sec(c+dx)\tan(c+dx)}{d} + \frac{b^2\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

output
```
(a*b*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d + (a*b*Sec[c + d*x]*T
an[c + d*x])/d + (b^2*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {3042, 4275, 3042, 4255, 3042, 4257, 4534, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$\int \sec^2(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int \sec^3(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{4255}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{4257}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{4534}$$

$$\frac{1}{3}\left(3a^2+2b^2\right)\int \sec^2(c+dx)dx+2ab\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{b^2\tan(c+dx)\sec^2(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(3a^2+2b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+2ab\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{b^2\tan(c+dx)\sec^2(c+dx)}{3d}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\left(3a^2+2b^2\right)\int 1d(-\tan(c+dx))}{3d}+2ab\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{b^2\tan(c+dx)\sec^2(c+dx)}{3d}$$

$$\Big\downarrow \text{24}$$

$$\frac{\left(3a^2+2b^2\right)\tan(c+dx)}{3d}+2ab\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{b^2\tan(c+dx)\sec^2(c+dx)}{3d}$$

input
```
Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

output
```
((3*a^2 + 2*b^2)*Tan[c + d*x])/(3*d) + (b^2*Sec[c + d*x]^2*Tan[c + d*x])/(
3*d) + 2*a*b*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2
*d))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{a^2\tan(dx+c)+2ab\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-b^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
default	$\frac{a^2\tan(dx+c)+2ab\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-b^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
parts	$\frac{a^2\tan(dx+c)}{d}-\frac{b^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{ab\sec(dx+c)\tan(dx+c)}{d}+\frac{ab\ln(\sec(dx+c)+\tan(dx+c))}{d}$
norman	$\frac{\frac{4(3a^2+b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{2(a^2-ab+b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}-\frac{2(a^2+ab+b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}+\frac{ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{ab\ln(}{}$
risch	$-\frac{2i(3ab\,e^{5i(dx+c)}-3a^2e^{4i(dx+c)}-6a^2e^{2i(dx+c)}-6\,e^{2i(dx+c)}b^2-3ab\,e^{i(dx+c)}-3a^2-2b^2)}{3d(e^{2i(dx+c)}+1)^3}+\frac{ab\ln(e^{i(dx+c)}+i)}{d}-\frac{a}{}$
parallelrisch	$\frac{-9\left(\frac{\cos(3dx+3c)}{3}+\cos(dx+c)\right)ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+9\left(\frac{\cos(3dx+3c)}{3}+\cos(dx+c)\right)ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+(3a^2+2b^2}{3d(\cos(3dx+3c)+3\cos(dx+c))}$

input
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*tan(d*x+c)+2*a*b*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan
(d*x+c)))-b^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.25

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$=\frac{3\,ab\cos(dx+c)^3\log(\sin(dx+c)+1)-3\,ab\cos(dx+c)^3\log(-\sin(dx+c)+1)+2\,(3\,ab\cos(dx+c)}{6\,d\cos(dx+c)^3}$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(3*a*b*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*b*cos(d*x + c)^3*log
(-sin(d*x + c) + 1) + 2*(3*a*b*cos(d*x + c) + (3*a^2 + 2*b^2)*cos(d*x + c)
^2 + b^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{2\left(\tan(dx+c)^3+3\tan(dx+c)\right)b^2-3\,ab\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)}{6\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/6*(2*(tan(d*x + c)^3 + 3*tan(d*x + c))*b^2 - 3*a*b*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a^2*t
an(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(76) = 152$.

Time = 0.15 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.22

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 +}{}}{3\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/3*(3*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*b*log(abs(tan(1/2*d*x
+ 1/2*c) - 1)) - 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*a*b*tan(1/2*d*x + 1/2
*c)^5 + 3*b^2*tan(1/2*d*x + 1/2*c)^5 - 6*a^2*tan(1/2*d*x + 1/2*c)^3 - 2*b^
2*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c) + 3*a*b*tan(1/2*d*x
+ 1/2*c) + 3*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 12.58 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.76

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{2\,a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$
$$- \frac{\left(2\,a^2-2\,a\,b+2\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 + \left(-4\,a^2-\frac{4\,b^2}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 + \left(2\,a^2+2\,a\,b+2\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 - 3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 - 1\right)}$$

input
```
int((a + b/cos(c + d*x))^2/cos(c + d*x)^2,x)
```

output
```
(2*a*b*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)^5*(2*a^2 - 2*a*b
 + 2*b^2) - tan(c/2 + (d*x)/2)^3*(4*a^2 + (4*b^2)/3) + tan(c/2 + (d*x)/2)*
(2*a*b + 2*a^2 + 2*b^2))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)
^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.42

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$= \frac{-3\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2ab+3\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)ab+3\cos(}{}$$

input
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c))^2,x)
```

output
```
( - 3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a*b + 3*cos(c
 + d*x)*log(tan((c + d*x)/2) - 1)*a*b + 3*cos(c + d*x)*log(tan((c + d*x)/2
) + 1)*sin(c + d*x)**2*a*b - 3*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a*b
- 3*cos(c + d*x)*sin(c + d*x)*a*b + 3*sin(c + d*x)**3*a**2 + 2*sin(c + d*x
)**3*b**2 - 3*sin(c + d*x)*a**2 - 3*sin(c + d*x)*b**2)/(3*cos(c + d*x)*d*(
sin(c + d*x)**2 - 1))
```

3.459 $\int \sec(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \sec(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{(2a^2+b^2)\,\text{arctanh}(\sin(c+dx))}{2d} + \frac{2ab\tan(c+dx)}{d} + \frac{b^2\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
1/2*(2*a^2+b^2)*arctanh(sin(d*x+c))/d+2*a*b*tan(d*x+c)/d+1/2*b^2*sec(d*x+c
)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \sec(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{2a^2\coth^{-1}(\sin(c+dx)) + b(b\,\text{arctanh}(\sin(c+dx)) + (4a+b\sec(c+dx))\tan(c+dx))}{2d}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

output
```
(2*a^2*ArcCoth[Sin[c + d*x]] + b*(b*ArcTanh[Sin[c + d*x]] + (4*a + b*Sec[c
 + d*x])*Tan[c + d*x]))/(2*d)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 4275, 3042, 4254, 24, 4534, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$\int \sec(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int \sec^2(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{4254}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx-\frac{2ab\int 1d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{24}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+\frac{2ab\tan(c+dx)}{d}$$

$$\Big\downarrow \text{4534}$$

$$\frac{1}{2}(2a^2+b^2)\int \sec(c+dx)dx+\frac{2ab\tan(c+dx)}{d}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(2a^2+b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{2ab\tan(c+dx)}{d}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\left(2a^2+b^2\right)\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{2ab\tan(c+dx)}{d}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2d}$$

input
```
Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

output
```
((2*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a*b*Tan[c + d*x])/d + (b^
2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))+2ab\tan(dx+c)+b^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))+2ab\tan(dx+c)+b^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
parts	$\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{2ab\tan(dx+c)}{d}$
parallelrisch	$\frac{-(1+\cos(2dx+2c))\left(a^2+\frac{b^2}{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(1+\cos(2dx+2c))\left(a^2+\frac{b^2}{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2ab\sin(2dx+2c}{d(1+\cos(2dx+2c))}$
norman	$\frac{\frac{b(4a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{b(4a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-\frac{(2a^2+b^2)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{(2a^2+b^2)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$
risch	$-\frac{ib\left(b\,e^{3i(dx+c)}-4\,e^{2i(dx+c)}a-b\,e^{i(dx+c)}-4a\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{a^2\ln\left(e^{i(dx+c)}-i\right)}{d}-\frac{\ln\left(e^{i(dx+c)}-i\right)b^2}{2d}+\frac{a^2\ln\left(e^{i(dx+c)}+i\right)}{d}+$

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*ln(sec(d*x+c)+tan(d*x+c))+2*a*b*tan(d*x+c)+b^2*(1/2*sec(d*x+c)*ta
n(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.58

$$\int \sec(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$=\frac{(2\,a^2+b^2)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-(2\,a^2+b^2)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\,(4\,a}{4\,d\cos\left(dx+c\right)^2}$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/4*((2*a^2 + b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^2 + b^2)*co
s(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(4*a*b*cos(d*x + c) + b^2)*sin(d*x
 + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^2\,dx = \int (a+b\sec{(c+dx)})^2\sec{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*sec(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int \sec(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{b^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log{(\sin{(dx+c)}+1)} + \log{(\sin{(dx+c)}-1)}\right) - 4a^2\log{(\sec{(dx+c)}+\tan{(dx+c}}{4d}$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/4*(b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)) - 4*a^2*log(sec(d*x + c) + tan(d*x + c)) - 8*a*b*tan
(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(55) = 110$.

Time = 0.14 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.19

$$\int \sec(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{(2\,a^2+b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(2\,a^2+b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b^2\,\mathrm{t}}{}}{2\,d}$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*((2*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (2*a^2 + b^2)*log(
abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(4*a*b*tan(1/2*d*x + 1/2*c)^3 - b^2*tan
(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c) - b^2*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 11.56 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.68

$$\int \sec(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,(2\,a^2+b^2)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3(4\,a\,b-b^2)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(b^2+4\,a\,b)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

input
```
int((a + b/cos(c + d*x))^2/cos(c + d*x),x)
```

output
```
(atanh(tan(c/2 + (d*x)/2))*(2*a^2 + b^2))/d - (tan(c/2 + (d*x)/2)^3*(4*a*b
 - b^2) - tan(c/2 + (d*x)/2)*(4*a*b + b^2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*t
an(c/2 + (d*x)/2)^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.58

$$\int \sec(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{-4\cos(dx+c)\sin(dx+c)\,ab - 2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2 a^2 - \log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx}{}$$

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^2,x)
```

output
```
( - 4*cos(c + d*x)*sin(c + d*x)*a*b - 2*log(tan((c + d*x)/2) - 1)*sin(c +
d*x)**2*a**2 - log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b**2 + 2*log(tan(
(c + d*x)/2) - 1)*a**2 + log(tan((c + d*x)/2) - 1)*b**2 + 2*log(tan((c + d
*x)/2) + 1)*sin(c + d*x)**2*a**2 + log(tan((c + d*x)/2) + 1)*sin(c + d*x)*
*2*b**2 - 2*log(tan((c + d*x)/2) + 1)*a**2 - log(tan((c + d*x)/2) + 1)*b**
2 - sin(c + d*x)*b**2)/(2*d*(sin(c + d*x)**2 - 1))
```

3.460 $\int (a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 33

$$\int (a+b\sec(c+dx))^2\,dx = a^2x + \frac{2ab\text{arctanh}(\sin(c+dx))}{d} + \frac{b^2\tan(c+dx)}{d}$$

output
```
a^2*x+2*a*b*arctanh(sin(d*x+c))/d+b^2*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int (a+b\sec(c+dx))^2\,dx = \frac{a^2dx + 2ab\coth^{-1}(\sin(c+dx)) + b^2\tan(c+dx)}{d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^2,x]
```

output
```
(a^2*d*x + 2*a*b*ArcCoth[Sin[c + d*x]] + b^2*Tan[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4260, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4260}$$

$$2ab\int \sec(c+dx)dx+b^2\int \sec^2(c+dx)dx+a^2x$$

$$\Big\downarrow \text{3042}$$

$$2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+b^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+a^2x$$

$$\Big\downarrow \text{4254}$$

$$2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{b^2\int 1d(-\tan(c+dx))}{d}+a^2x$$

$$\Big\downarrow \text{24}$$

$$2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+a^2x+\frac{b^2\tan(c+dx)}{d}$$

$$\Big\downarrow \text{4257}$$

$$a^2x+\frac{2ab\text{arctanh}(\sin(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

input
```
Int[(a + b*Sec[c + d*x])^2,x]
```

output
```
a^2*x + (2*a*b*ArcTanh[Sin[c + d*x]])/d + (b^2*Tan[c + d*x])/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4260
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Simp[2*a*b   Int[Csc[c + d*x], x], x] + Simp[b^2   Int[Csc[c + d*x]^2, x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

method	result	size
parts	$a^{2}x+\frac{b^{2}\tan\left(dx+c\right)}{d}+\frac{2ab\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}$	41
derivativedivides	$\frac{a^{2}\left(dx+c\right)+2ab\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+b^{2}\tan\left(dx+c\right)}{d}$	43
default	$\frac{a^{2}\left(dx+c\right)+2ab\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+b^{2}\tan\left(dx+c\right)}{d}$	43
risch	$a^{2}x+\frac{2ib^{2}}{d\left(e^{2i\left(dx+c\right)}+1\right)}+\frac{2ab\ln\left(e^{i\left(dx+c\right)}+i\right)}{d}-\frac{2ab\ln\left(e^{i\left(dx+c\right)}-i\right)}{d}$	69
parallelrisch	$\frac{2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos\left(dx+c\right)-2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos\left(dx+c\right)+a^{2}dx\cos\left(dx+c\right)+b^{2}\sin\left(dx+c\right)}{\cos\left(dx+c\right)d}$	80
norman	$\frac{a^{2}x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-a^{2}x-\frac{2b^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}-\frac{2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}+\frac{2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	96

input
```
int((a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+b^2*tan(d*x+c)/d+2*a*b/d*ln(sec(d*x+c)+tan(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(33) = 66$.

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.24

$$\int (a + b\sec(c + dx))^2\,dx = \frac{a^2 dx \cos(dx + c) + ab\cos(dx + c)\log(\sin(dx + c) + 1) - ab\cos(dx + c)\log(-\sin(dx + c) + 1) + b^2\,\mathrm{si}}{d\cos(dx + c)}$$

input
```
integrate((a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
(a^2*d*x*cos(d*x + c) + a*b*cos(d*x + c)*log(sin(d*x + c) + 1) - a*b*cos(d
*x + c)*log(-sin(d*x + c) + 1) + b^2*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int (a + b\sec(c + dx))^2\,dx = \int (a + b\sec(c + dx))^2\,dx$$

input
```
integrate((a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

$$\int (a+b\sec(c+dx))^2\,dx=a^2x+\frac{2\,ab\log(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b^2\tan(dx+c)}{d}$$

input
```
integrate((a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
a^2*x + 2*a*b*log(sec(d*x + c) + tan(d*x + c))/d + b^2*tan(d*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(33) = 66.

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.33

$$\int (a+b\sec(c+dx))^2\,dx$$
$$=\frac{(dx+c)a^2+2\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

input
```
integrate((a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
((d*x + c)*a^2 + 2*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a*b*log(abs(
tan(1/2*d*x + 1/2*c) - 1)) - 2*b^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2
*c)^2 - 1))/d
```

Mupad [B] (verification not implemented)

Time = 10.52 (sec) , antiderivative size = 181, normalized size of antiderivative = 5.48

$$\int (a+b\sec(c+dx))^2\,dx = \frac{2\,a^2\,\mathrm{atan}\left(\frac{64\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^6+256\,a^4\,b^2}+\frac{256\,a^4\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^6+256\,a^4\,b^2}\right)}{d} - \frac{2\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)} + \frac{4\,a\,b\,\mathrm{atanh}\left(\frac{128\,a^5\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128\,a^5\,b+512\,a^3\,b^3}+\frac{512\,a^3\,b^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128\,a^5\,b+512\,a^3\,b^3}\right)}{d}$$

input
```
int((a + b/cos(c + d*x))^2,x)
```

output
```
(2*a^2*atan((64*a^6*tan(c/2 + (d*x)/2))/(64*a^6 + 256*a^4*b^2) + (256*a^4*
b^2*tan(c/2 + (d*x)/2))/(64*a^6 + 256*a^4*b^2)))/d - (2*b^2*tan(c/2 + (d*x
)/2))/(d*(tan(c/2 + (d*x)/2)^2 - 1)) + (4*a*b*atanh((128*a^5*b*tan(c/2 + (
d*x)/2))/(128*a^5*b + 512*a^3*b^3) + (512*a^3*b^3*tan(c/2 + (d*x)/2))/(128
*a^5*b + 512*a^3*b^3)))/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.39

$$\int (a+b\sec(c+dx))^2\,dx = \frac{-2\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)ab+2\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)ab+\cos(dx+c)\,a^2dx+}{\cos(dx+c)\,d}$$

input
```
int((a+b*sec(d*x+c))^2,x)
```

output
```
( - 2*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a*b + 2*cos(c + d*x)*log(tan(
(c + d*x)/2) + 1)*a*b + cos(c + d*x)*a**2*d*x + sin(c + d*x)*b**2)/(cos(c
+ d*x)*d)
```

3.461 $\int \cos(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 33

$$\int \cos(c+dx)(a+b\sec(c+dx))^2\,dx=2abx+\frac{b^2\text{arctanh}(\sin(c+dx))}{d}+\frac{a^2\sin(c+dx)}{d}$$

output
```
2*a*b*x+b^2*arctanh(sin(d*x+c))/d+a^2*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \cos(c+dx)(a+b\sec(c+dx))^2\,dx=2abx+\frac{b^2\coth^{-1}(\sin(c+dx))}{d}+\frac{a^2\cos(dx)\sin(c)}{d}+\frac{a^2\cos(c)\sin(dx)}{d}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

output
```
2*a*b*x + (b^2*ArcCoth[Sin[c + d*x]])/d + (a^2*Cos[d*x]*Sin[c])/d + (a^2*C
os[c]*Sin[d*x])/d
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3042, 4275, 24, 3042, 4533, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 4275}$$

$$\int \cos(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int 1dx$$

$$\Big\downarrow \text{ 24}$$

$$\int \cos(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx+2abx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+2abx$$

$$\Big\downarrow \text{ 4533}$$

$$b^2\int \sec(c+dx)dx+\frac{a^2\sin(c+dx)}{d}+2abx$$

$$\Big\downarrow \text{ 3042}$$

$$b^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{a^2\sin(c+dx)}{d}+2abx$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{a^2\sin(c+dx)}{d}+2abx+\frac{b^2\text{arctanh}(\sin(c+dx))}{d}$$

input
```
Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

output
```
2*a*b*x + (b^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{a^2\sin(dx+c)+2ab(dx+c)+b^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$
default	$\frac{a^2\sin(dx+c)+2ab(dx+c)+b^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$
parallelrisch	$\frac{2abxd+b^2\left(\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\right)+a^2\sin(dx+c)}{d}$
risch	$2abx-\frac{ia^2\mathrm{e}^{i(dx+c)}}{2d}+\frac{ia^2\mathrm{e}^{-i(dx+c)}}{2d}+\frac{\ln(\mathrm{e}^{i(dx+c)}+i)b^2}{d}-\frac{\ln(\mathrm{e}^{i(dx+c)}-i)b^2}{d}$
norman	$\frac{-2abx-\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}+2abx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{b^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{b^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right.}{d}$

input
```
int(cos(d*x+c)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*sin(d*x+c)+2*a*b*(d*x+c)+b^2*ln(sec(d*x+c)+tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.58

$$\int \cos(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{4\,abdx+b^2\log\left(\sin\left(dx+c\right)+1\right)-b^2\log\left(-\sin\left(dx+c\right)+1\right)+2\,a^2\sin\left(dx+c\right)}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(4*a*b*d*x + b^2*log(sin(d*x + c) + 1) - b^2*log(-sin(d*x + c) + 1) +
2*a^2*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos(c+dx)(a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2\cos(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*cos(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int \cos(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{4(dx+c)ab + b^2(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 2a^2\sin(dx+c)}{2d}$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*(4*(d*x + c)*a*b + b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
 + 2*a^2*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(33) = 66.

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.36

$$\int \cos(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{2(dx+c)ab + b^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - b^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) + \frac{2a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}}{d}$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
(2*(d*x + c)*a*b + b^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b^2*log(abs(ta
n(1/2*d*x + 1/2*c) - 1)) + 2*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c
)^2 + 1))/d
```

Mupad [B] (verification not implemented)

Time = 10.75 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.21

$$\int \cos(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{a^2\,\sin\left(c+d\,x\right)}{d} + \frac{2\,b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{4\,a\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

input
```
int(cos(c + d*x)*(a + b/cos(c + d*x))^2,x)
```

output
```
(a^2*sin(c + d*x))/d + (2*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))
)/d + (4*a*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.79

$$\int \cos(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b^2+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)b^2+\sin\left(dx+c\right)a^2+2abc+2abdx}{d}$$

input
```
int(cos(d*x+c)*(a+b*sec(d*x+c))^2,x)
```

output
```
( - log(tan((c + d*x)/2) - 1)*b**2 + log(tan((c + d*x)/2) + 1)*b**2 + sin(
c + d*x)*a**2 + 2*a*b*c + 2*a*b*d*x)/d
```

3.462 $\int \cos^2(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 50

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{1}{2}\left(a^2+2b^2\right)x+\frac{2ab\sin(c+dx)}{d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
1/2*(a^2+2*b^2)*x+2*a*b*sin(d*x+c)/d+1/2*a^2*cos(d*x+c)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{2\left(a^2+2b^2\right)(c+dx)+8ab\sin(c+dx)+a^2\sin(2(c+dx))}{4d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

output
```
(2*(a^2 + 2*b^2)*(c + d*x) + 8*a*b*Sin[c + d*x] + a^2*Sin[2*(c + d*x)])/(4
*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4275, 3042, 3117, 4533, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \ 4275$$

$$\int \cos^2(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int \cos(c+dx)dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx+2ab\int \sin\left(c+dx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \ 3117$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{2ab\sin(c+dx)}{d}$$

$$\Big\downarrow \ 4533$$

$$\frac{1}{2}\left(a^2+2b^2\right)\int 1dx+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{2ab\sin(c+dx)}{d}$$

$$\Big\downarrow \ 24$$

$$\frac{1}{2}x\left(a^2+2b^2\right)+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{2ab\sin(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

output
```
((a^2 + 2*b^2)*x)/2 + (2*a*b*Sin[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d
*x])/(2*d)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

method	result
risch	$\frac{a^2 x}{2}+x\,b^2+\frac{2ab\sin(dx+c)}{d}+\frac{a^2\sin(2dx+2c)}{4d}$
parallelrisch	$\frac{a^2\sin(2dx+2c)+8ab\sin(dx+c)+2(a^2+2b^2)xd}{4d}$
derivativedivides	$\frac{a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\sin(dx+c)+b^2(dx+c)}{d}$
default	$\frac{a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\sin(dx+c)+b^2(dx+c)}{d}$
norman	$\frac{\left(-\frac{a^2}{2}-b^2\right)x+\left(-\frac{a^2}{2}-b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(\frac{a^2}{2}+b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\left(\frac{a^2}{2}+b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}-}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$

input
```
int(cos(d*x+c)^2*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x+x*b^2+2*a*b*sin(d*x+c)/d+1/4*a^2/d*sin(2*d*x+2*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^2\,dx=\frac{(a^2+2\,b^2)dx+(a^2\cos(dx+c)+4\,ab)\sin(dx+c)}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*((a^2 + 2*b^2)*d*x + (a^2*cos(d*x + c) + 4*a*b)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^2\,dx = \int (a+b\sec{(c+dx)})^2\cos^2{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*cos(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{(2\,dx+2\,c+\sin{(2\,dx+2\,c)})a^2+4\,(dx+c)b^2+8\,ab\sin{(dx+c)}}{4\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^2 + 4*(d*x + c)*b^2 + 8*a*b*sin(d*
x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(46) = 92.

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.92

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{(a^2+2\,b^2)(dx+c)-\frac{2\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*((a^2 + 2*b^2)*(d*x + c) - 2*(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1
/2*d*x + 1/2*c)^3 - a^2*tan(1/2*d*x + 1/2*c) - 4*a*b*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 10.71 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{a^2\,x}{2}+b^2\,x+\frac{a^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{2\,a\,b\,\sin\left(c+d\,x\right)}{d}$$

input
```
int(cos(c + d*x)^2*(a + b/cos(c + d*x))^2,x)
```

output
```
(a^2*x)/2 + b^2*x + (a^2*sin(2*c + 2*d*x))/(4*d) + (2*a*b*sin(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{\cos\left(dx+c\right)\sin\left(dx+c\right)a^2+4\sin\left(dx+c\right)ab+a^2dx+2b^2dx}{2d}$$

input
```
int(cos(d*x+c)^2*(a+b*sec(d*x+c))^2,x)
```

output
```
(cos(c + d*x)*sin(c + d*x)*a**2 + 4*sin(c + d*x)*a*b + a**2*d*x + 2*b**2*d
*x)/(2*d)
```

3.463 $\int \cos^3(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^2\,dx = abx+\frac{(a^2+b^2)\sin(c+dx)}{d}+\frac{ab\cos(c+dx)\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{3d}$$

output
```
a*b*x+(a^2+b^2)*sin(d*x+c)/d+a*b*cos(d*x+c)*sin(d*x+c)/d-1/3*a^2*sin(d*x+c
)^3/d
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{3(3a^2+4b^2)\sin(c+dx)+a(12b(c+dx)+6b\sin(2(c+dx))+a\sin(3(c+dx)))}{12d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

output
```
(3*(3*a^2 + 4*b^2)*Sin[c + d*x] + a*(12*b*(c + d*x) + 6*b*Sin[2*(c + d*x)]
 + a*Sin[3*(c + d*x)]))/(12*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4275, 3042, 3115, 24, 4532, 3042, 3492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{4275}$$

$$\int \cos^3(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int \cos^2(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+2ab\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3115}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+2ab\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{24}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+2ab\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)$$

$$\Big\downarrow \text{4532}$$

$$\int \cos(c+dx)\left(b^2+a^2\cos^2(c+dx)\right)dx+2ab\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(b^2+a^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)$$

$$\Big\downarrow \text{3492}$$

$$2ab\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{\int\left(-\sin^2(c+dx)a^2+a^2+b^2\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$2ab\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{\frac{1}{3}a^2\sin^3(c+dx)-\left(a^2+b^2\right)\sin(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

output
```
2*a*b*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (-((a^2 + b^2)*Sin[c + d
*x]) + (a^2*Sin[c + d*x]^3)/3)/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4532
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05

method	result
parallelrisch	$\frac{12abxd+9a^2\sin(dx+c)+12b^2\sin(dx+c)+a^2\sin(3dx+3c)+6ab\sin(2dx+2c)}{12d}$
derivativedivides	$\frac{\frac{a^2\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+2ab\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b^2\sin(dx+c)}{d}$
default	$\frac{\frac{a^2\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+2ab\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b^2\sin(dx+c)}{d}$
risch	$abx+\frac{3a^2\sin(dx+c)}{4d}+\frac{\sin(dx+c)b^2}{d}+\frac{a^2\sin(3dx+3c)}{12d}+\frac{ab\sin(2dx+2c)}{2d}$
norman	$\frac{abx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8-abx-\frac{2\left(a^2-3ab-3b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{3d}+\frac{2\left(a^2-ab+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}-\frac{2\left(a^2+ab+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2\left(a^2+\right.}{}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$

input
```
int(cos(d*x+c)^3*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*(12*a*b*x*d+9*a^2*sin(d*x+c)+12*b^2*sin(d*x+c)+a^2*sin(3*d*x+3*c)+6*a
*b*sin(2*d*x+2*c))/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{3\,abdx + \left(a^2\cos\left(dx+c\right)^2 + 3\,ab\cos\left(dx+c\right) + 2\,a^2 + 3\,b^2\right)\sin\left(dx+c\right)}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3*(3*a*b*d*x + (a^2*cos(d*x + c)^2 + 3*a*b*cos(d*x + c) + 2*a^2 + 3*b^2)
*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^2\,dx = \int \left(a+b\sec\left(c+dx\right)\right)^2\cos^3\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*cos(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{2\left(\sin\left(dx+c\right)^3 - 3\,\sin\left(dx+c\right)\right)a^2 - 3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)ab - 6\,b^2\sin\left(dx+c\right)}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/6*(2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 - 3*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*a*b - 6*b^2*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(56) = 112$.

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.64

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{3\,(dx+c)ab + \frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a^2\tan}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/3*(3*(d*x + c)*a*b + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*a*b*tan(1/2*d*x
 + 1/2*c)^5 + 3*b^2*tan(1/2*d*x + 1/2*c)^5 + 2*a^2*tan(1/2*d*x + 1/2*c)^3
+ 6*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c) + 3*a*b*tan(1/
2*d*x + 1/2*c) + 3*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^
3)/d
```

Mupad [B] (verification not implemented)

Time = 10.85 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.24

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{2\,a^2\,\sin\left(c+d\,x\right)}{3\,d}+\frac{b^2\,\sin\left(c+d\,x\right)}{d}+a\,b\,x+\frac{a^2\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a\,b\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{d}$$

input
```
int(cos(c + d*x)^3*(a + b/cos(c + d*x))^2,x)
```

output
```
(2*a^2*sin(c + d*x))/(3*d) + (b^2*sin(c + d*x))/d + a*b*x + (a^2*cos(c + d
*x)^2*sin(c + d*x))/(3*d) + (a*b*cos(c + d*x)*sin(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$=\frac{3\cos(dx+c)\sin(dx+c)\,ab-\sin(dx+c)^3\,a^2+3\sin(dx+c)\,a^2+3\sin(dx+c)\,b^2+3abdx}{3d}$$

input
```
int(cos(d*x+c)^3*(a+b*sec(d*x+c))^2,x)
```

output
```
(3*cos(c + d*x)*sin(c + d*x)*a*b - sin(c + d*x)**3*a**2 + 3*sin(c + d*x)*a
**2 + 3*sin(c + d*x)*b**2 + 3*a*b*d*x)/(3*d)
```

3.464 $\int \cos^4(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^2\,dx=\frac{1}{8}\left(3a^2+4b^2\right)x+\frac{2ab\sin(c+dx)}{d}+\frac{(3a^2+4b^2)\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{2ab\sin^3(c+dx)}{3d}$$

output
```
1/8*(3*a^2+4*b^2)*x+2*a*b*sin(d*x+c)/d+1/8*(3*a^2+4*b^2)*cos(d*x+c)*sin(d*
x+c)/d+1/4*a^2*cos(d*x+c)^3*sin(d*x+c)/d-2/3*a*b*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.85

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^2\,dx=\frac{36a^2c+48b^2c+36a^2dx+48b^2dx+192ab\sin(c+dx)-64ab\sin^3(c+dx)+24(a^2+b^2)\sin(2(c+dx))}{96d}$$

input
```
Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

output
```
(36*a^2*c + 48*b^2*c + 36*a^2*d*x + 48*b^2*d*x + 192*a*b*Sin[c + d*x] - 64
*a*b*Sin[c + d*x]^3 + 24*(a^2 + b^2)*Sin[2*(c + d*x)] + 3*a^2*Sin[4*(c + d
*x)])/(96*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4275, 3042, 3113, 2009, 4533, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{4275}$$

$$\int \cos^4(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int \cos^3(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx+2ab\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3113}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx-\frac{2ab\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx-\frac{2ab\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \ 4533$$

$$\frac{1}{4}\left(3a^2+4b^2\right)\int \cos^2(c+dx)dx+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}-\frac{2ab\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{4}\left(3a^2+4b^2\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}-\frac{2ab\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \ 3115$$

$$\frac{1}{4}\left(3a^2+4b^2\right)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}-\frac{2ab\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \ 24$$

$$\frac{1}{4}\left(3a^2+4b^2\right)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}-\frac{2ab\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

input
```
Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

output
```
(a^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + ((3*a^2 + 4*b^2)*(x/2 + (Cos[c +
 d*x]*Sin[c + d*x])/(2*d)))/4 - (2*a*b*(-Sin[c + d*x] + Sin[c + d*x]^3/3))
/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{24(a^2+b^2)\sin(2dx+2c)+16ab\sin(3dx+3c)+3a^2\sin(4dx+4c)+144ab\sin(dx+c)+36\left(a^2+\frac{4b^2}{3}\right)xd}{96d}$
derivativedivides	$\frac{a^2\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2ab\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+b^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a^2\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2ab\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+b^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
risch	$\frac{3a^2x}{8}+\frac{x\,b^2}{2}+\frac{3ab\sin(dx+c)}{2d}+\frac{a^2\sin(4dx+4c)}{32d}+\frac{ab\sin(3dx+3c)}{6d}+\frac{a^2\sin(2dx+2c)}{4d}+\frac{\sin(2dx+2c)b^2}{4d}$
norman	$\left(-\frac{3a^2}{8}-\frac{b^2}{2}\right)x+\left(-\frac{9a^2}{8}-\frac{3b^2}{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(-\frac{3a^2}{4}-b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\left(\frac{3a^2}{4}+b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+\left(\frac{3a^2}{8}+\frac{b^2}{2}\right)$

input
```
int(cos(d*x+c)^4*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/96*(24*(a^2+b^2)*sin(2*d*x+2*c)+16*a*b*sin(3*d*x+3*c)+3*a^2*sin(4*d*x+4*
c)+144*a*b*sin(d*x+c)+36*(a^2+4/3*b^2)*x*d)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.76

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{3\,(3\,a^2+4\,b^2)dx+\left(6\,a^2\cos\left(dx+c\right)^3+16\,ab\cos\left(dx+c\right)^2+32\,ab+3\,(3\,a^2+4\,b^2)\cos\left(dx+c\right)\right)\sin\left(dx\right.}{24\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/24*(3*(3*a^2 + 4*b^2)*d*x + (6*a^2*cos(d*x + c)^3 + 16*a*b*cos(d*x + c)^
2 + 32*a*b + 3*(3*a^2 + 4*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2\cos^4(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*cos(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.81

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{3\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^2-64\,(\sin(dx+c)^3-3\,\sin(dx+c))ab+24\,(2\,d}{96\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2 - 64*(
sin(d*x + c)^3 - 3*sin(d*x + c))*a*b + 24*(2*d*x + 2*c + sin(2*d*x + 2*c))
*b^2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(93) = 186.

Time = 0.14 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.22

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{3\,(3\,a^2+4\,b^2)(dx+c)-\frac{2\left(15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-48\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-80\,ab\tan\left(\frac{1}{2}\right.}{}}{}$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/24*(3*(3*a^2 + 4*b^2)*(d*x + c) - 2*(15*a^2*tan(1/2*d*x + 1/2*c)^7 - 48*
a*b*tan(1/2*d*x + 1/2*c)^7 + 12*b^2*tan(1/2*d*x + 1/2*c)^7 - 9*a^2*tan(1/2
*d*x + 1/2*c)^5 - 80*a*b*tan(1/2*d*x + 1/2*c)^5 + 12*b^2*tan(1/2*d*x + 1/2
*c)^5 + 9*a^2*tan(1/2*d*x + 1/2*c)^3 - 80*a*b*tan(1/2*d*x + 1/2*c)^3 - 12*
b^2*tan(1/2*d*x + 1/2*c)^3 - 15*a^2*tan(1/2*d*x + 1/2*c) - 48*a*b*tan(1/2*
d*x + 1/2*c) - 12*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4
)/d
```

Mupad [B] (verification not implemented)

Time = 10.90 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{3\,a^2\,x}{8}+\frac{b^2\,x}{2}+\frac{a^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{a^2\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{b^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{3\,a\,b\,\sin\left(c+d\,x\right)}{2\,d}+\frac{a\,b\,\sin\left(3\,c+3\,d\,x\right)}{6\,d}$$

input
```
int(cos(c + d*x)^4*(a + b/cos(c + d*x))^2,x)
```

output
```
(3*a^2*x)/8 + (b^2*x)/2 + (a^2*sin(2*c + 2*d*x))/(4*d) + (a^2*sin(4*c + 4*
d*x))/(32*d) + (b^2*sin(2*c + 2*d*x))/(4*d) + (3*a*b*sin(c + d*x))/(2*d) +
 (a*b*sin(3*c + 3*d*x))/(6*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.94

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{-6\cos\left(dx+c\right)\sin\left(dx+c\right)^3a^2+15\cos\left(dx+c\right)\sin\left(dx+c\right)a^2+12\cos\left(dx+c\right)\sin\left(dx+c\right)b^2-16\sin}{24d}$$

input
```
int(cos(d*x+c)^4*(a+b*sec(d*x+c))^2,x)
```

output
```
( - 6*cos(c + d*x)*sin(c + d*x)**3*a**2 + 15*cos(c + d*x)*sin(c + d*x)*a**
2 + 12*cos(c + d*x)*sin(c + d*x)*b**2 - 16*sin(c + d*x)**3*a*b + 48*sin(c
+ d*x)*a*b + 9*a**2*d*x + 12*b**2*d*x)/(24*d)
```

3.465 $\int \cos^5(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{3abx}{4} + \frac{(a^2+b^2)\sin(c+dx)}{d} + \frac{3ab\cos(c+dx)\sin(c+dx)}{4d} + \frac{ab\cos^3(c+dx)\sin(c+dx)}{2d} - \frac{(2a^2+b^2)\sin^3(c+dx)}{3d} + \frac{a^2\sin^5(c+dx)}{5d}$$

output
```
3/4*a*b*x+(a^2+b^2)*sin(d*x+c)/d+3/4*a*b*cos(d*x+c)*sin(d*x+c)/d+1/2*a*b*c
os(d*x+c)^3*sin(d*x+c)/d-1/3*(2*a^2+b^2)*sin(d*x+c)^3/d+1/5*a^2*sin(d*x+c)
^5/d
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.77

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{240(a^2+b^2)\sin(c+dx)-80(2a^2+b^2)\sin^3(c+dx)+48a^2\sin^5(c+dx)+15ab(12(c+dx)+8\sin(2(c+$$

input
```
Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^2,x]
```

output
```
(240*(a^2 + b^2)*Sin[c + d*x] - 80*(2*a^2 + b^2)*Sin[c + d*x]^3 + 48*a^2*S
in[c + d*x]^5 + 15*a*b*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x
)]))/(240*d)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4275, 3042, 3115, 3042, 3115, 24, 4532, 3042, 3492, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{ 4275}$$

$$\int \cos^5(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int \cos^4(c+dx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+2ab\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3115}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+2ab\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+2ab\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{3115}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+ 2ab\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{24}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+ 2ab\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{4532}$$

$$\int \cos^3(c+dx)\left(b^2+a^2\cos^2(c+dx)\right)dx+ 2ab\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(b^2+a^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+ 2ab\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{3492}$$

$$2ab\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)- \frac{\int \left(1-\sin^2(c+dx)\right)\left(-\sin^2(c+dx)a^2+a^2+b^2\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{290}$$

$$2ab\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{\int\left(a^2\sin^4(c+dx)-\left(2a^2+b^2\right)\sin^2(c+dx)+a^2\left(\frac{b^2}{a^2}+1\right)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$2ab\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{\frac{1}{3}\left(2a^2+b^2\right)\sin^3(c+dx)-\left(a^2+b^2\right)\sin(c+dx)-\frac{1}{5}a^2\sin^5(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^2,x]
```

output
```
-((-((a^2 + b^2)*Sin[c + d*x]) + ((2*a^2 + b^2)*Sin[c + d*x]^3)/3 - (a^2*S
in[c + d*x]^5)/5)/d) + 2*a*b*((Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x/
2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4532
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{\frac{a^2\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+2ab\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{b^2\left(2+\cos(dx+c)^2\right)}{3}}{d}$
default	$\frac{\frac{a^2\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+2ab\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{b^2\left(2+\cos(dx+c)^2\right)}{3}}{d}$
parallelrisch	$\frac{180abxd+3a^2\sin(5dx+5c)+15ab\sin(4dx+4c)+25a^2\sin(3dx+3c)+20b^2\sin(3dx+3c)+120ab\sin(2dx+2c)+150a^2\sin(}{240d}$
risch	$\frac{3abx}{4}+\frac{5a^2\sin(dx+c)}{8d}+\frac{3\sin(dx+c)b^2}{4d}+\frac{a^2\sin(5dx+5c)}{80d}+\frac{ab\sin(4dx+4c)}{16d}+\frac{5a^2\sin(3dx+3c)}{48d}+\frac{\sin(3dx+}{12d}$
norman	$\frac{-\frac{3abx}{4}-\frac{\left(4a^2-9ab+20b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{6d}+\frac{\left(4a^2-5ab+4b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{2d}-\frac{\left(4a^2+5ab+4b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}+\frac{\left(4a^2+9ab+20b^2\right)}{6d}}{}$

input
```
int(cos(d*x+c)^5*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+2*a*b*(1/4*(co
s(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*b^2*(2+cos(d*x+c)
^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{45\,abdx + \left(12\,a^2\cos\left(dx+c\right)^4 + 30\,ab\cos\left(dx+c\right)^3 + 45\,ab\cos\left(dx+c\right) + 4\left(4\,a^2+5\,b^2\right)\cos\left(dx+c\right)^2 + \right.}{60\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/60*(45*a*b*d*x + (12*a^2*cos(d*x + c)^4 + 30*a*b*cos(d*x + c)^3 + 45*a*b
*cos(d*x + c) + 4*(4*a^2 + 5*b^2)*cos(d*x + c)^2 + 32*a^2 + 40*b^2)*sin(d*
x + c))/d
```

Sympy [F]

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^2\,dx = \int \left(a+b\sec\left(c+dx\right)\right)^2\cos^5\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*cos(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{16\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^2+15\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\sin(\right.}{240\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/240*(16*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^2 + 1
5*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a*b - 80*(sin(d*
x + c)^3 - 3*sin(d*x + c))*b^2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(101) = 202.

Time = 0.13 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.23

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{45\,(dx+c)ab+\frac{2\left(60\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-75\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+60\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+80\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-30\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+\right.}{}}{}$$

input
```
integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/60*(45*(d*x + c)*a*b + 2*(60*a^2*tan(1/2*d*x + 1/2*c)^9 - 75*a*b*tan(1/2
*d*x + 1/2*c)^9 + 60*b^2*tan(1/2*d*x + 1/2*c)^9 + 80*a^2*tan(1/2*d*x + 1/2
*c)^7 - 30*a*b*tan(1/2*d*x + 1/2*c)^7 + 160*b^2*tan(1/2*d*x + 1/2*c)^7 + 2
32*a^2*tan(1/2*d*x + 1/2*c)^5 + 200*b^2*tan(1/2*d*x + 1/2*c)^5 + 80*a^2*ta
n(1/2*d*x + 1/2*c)^3 + 30*a*b*tan(1/2*d*x + 1/2*c)^3 + 160*b^2*tan(1/2*d*x
 + 1/2*c)^3 + 60*a^2*tan(1/2*d*x + 1/2*c) + 75*a*b*tan(1/2*d*x + 1/2*c) +
60*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 10.68 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.05

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{5\,a^2\,\sin\left(c+d\,x\right)}{8\,d}+\frac{3\,b^2\,\sin\left(c+d\,x\right)}{4\,d}+\frac{3\,a\,b\,x}{4}+\frac{5\,a^2\,\sin\left(3\,c+3\,d\,x\right)}{48\,d}+\frac{a^2\,\sin\left(5\,c+5\,d\,x\right)}{80\,d}+\frac{b^2\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{a\,b\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}+\frac{a\,b\,\sin\left(4\,c+4\,d\,x\right)}{16\,d}$$

input
```
int(cos(c + d*x)^5*(a + b/cos(c + d*x))^2,x)
```

output
```
(5*a^2*sin(c + d*x))/(8*d) + (3*b^2*sin(c + d*x))/(4*d) + (3*a*b*x)/4 + (5
*a^2*sin(3*c + 3*d*x))/(48*d) + (a^2*sin(5*c + 5*d*x))/(80*d) + (b^2*sin(3
*c + 3*d*x))/(12*d) + (a*b*sin(2*c + 2*d*x))/(2*d) + (a*b*sin(4*c + 4*d*x)
)/(16*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{-30\cos\left(dx+c\right)\sin\left(dx+c\right)^3ab+75\cos\left(dx+c\right)\sin\left(dx+c\right)ab+12\sin\left(dx+c\right)^5a^2-40\sin\left(dx+c\right)^3}{60d}$$

input
```
int(cos(d*x+c)^5*(a+b*sec(d*x+c))^2,x)
```

output
```
( - 30*cos(c + d*x)*sin(c + d*x)**3*a*b + 75*cos(c + d*x)*sin(c + d*x)*a*b
 + 12*sin(c + d*x)**5*a**2 - 40*sin(c + d*x)**3*a**2 - 20*sin(c + d*x)**3*
b**2 + 60*sin(c + d*x)*a**2 + 60*sin(c + d*x)*b**2 + 45*a*b*d*x)/(60*d)
```

3.466 $\int \sec^3(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 189

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{a(4a^2+9b^2)\operatorname{arctanh}(\sin(c+dx))}{8d} - \frac{(3a^4-52a^2b^2-16b^4)\tan(c+dx)}{30bd} - \frac{a(6a^2-71b^2)\sec(c+dx)\tan(c+dx)}{120d} - \frac{(3a^2-16b^2)\,(a+b\sec(c+dx))^2\tan(c+dx)}{60bd} - \frac{a(a+b\sec(c+dx))^3\tan(c+dx)}{20bd} + \frac{(a+b\sec(c+dx))^4\tan(c+dx)}{5bd}$$

output
```
1/8*a*(4*a^2+9*b^2)*arctanh(sin(d*x+c))/d-1/30*(3*a^4-52*a^2*b^2-16*b^4)*t
an(d*x+c)/b/d-1/120*a*(6*a^2-71*b^2)*sec(d*x+c)*tan(d*x+c)/d-1/60*(3*a^2-1
6*b^2)*(a+b*sec(d*x+c))^2*tan(d*x+c)/b/d-1/20*a*(a+b*sec(d*x+c))^3*tan(d*x
+c)/b/d+1/5*(a+b*sec(d*x+c))^4*tan(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.63

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$=\frac{15a(4a^2+9b^2)\operatorname{arctanh}(\sin(c+dx))+\tan(c+dx)\left(15a(4a^2+9b^2)\sec(c+dx)+90ab^2\sec^3(c+dx)+8\right.}{120d}$$

input
```
Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
```

output
```
(15*a*(4*a^2 + 9*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*a*(4*a^2 +
9*b^2)*Sec[c + d*x] + 90*a*b^2*Sec[c + d*x]^3 + 8*b*(15*(3*a^2 + b^2) + 5*
(3*a^2 + 2*b^2)*Tan[c + d*x]^2 + 3*b^2*Tan[c + d*x]^4)))/(120*d)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.667$, Rules used = {3042, 4327, 3042, 4490, 3042, 4490, 3042, 4485, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4327}$$

$$\frac{\int \sec(c+dx)(4b-a\sec(c+dx))(a+b\sec(c+dx))^3dx}{5b}+\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(4b-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3dx}{5b}+\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5bd}$$

$\downarrow$ 4490

$$\frac{\frac{1}{4}\int \sec(c+dx)(a+b\sec(c+dx))^2\left(13ab-\left(3a^2-16b^2\right)\sec(c+dx)\right)dx-\frac{a\tan(c+dx)(a+b\sec(c+dx))^3}{4d}}{5b}+\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5bd}$$

$\downarrow$ 3042

$$\frac{\frac{1}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(13ab+\left(16b^2-3a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{a\tan(c+dx)(a+b\sec(c+d}{4d}}{5b}$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5bd}$$

$\downarrow$ 4490

$$\frac{\frac{1}{4}\left(\frac{1}{3}\int \sec(c+dx)(a+b\sec(c+dx))\left(b\left(33a^2+32b^2\right)-a\left(6a^2-71b^2\right)\sec(c+dx)\right)dx-\frac{\left(3a^2-16b^2\right)\tan(c+dx)(a+b\,\mathrm{s}}{3d}\right.}{5b}$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5bd}$$

$\downarrow$ 3042

$$\frac{\frac{1}{4}\left(\frac{1}{3}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(b\left(33a^2+32b^2\right)-a\left(6a^2-71b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{(3a^2-16}{}\right.}{5b}$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5bd}$$

$\downarrow$ 4485

$$\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \sec(c+dx)\left(15ab\left(4a^2+9b^2\right)-4\left(3a^4-52b^2a^2-16b^4\right)\sec(c+dx)\right)dx-\frac{ab\left(6a^2-71b^2\right)\tan(c+dx)\sec(c+dx)}{2d}\right.\right.}{5b}$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5bd}$$

$\downarrow$ 3042

$$\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(15ab\left(4a^2+9b^2\right)-4\left(3a^4-52b^2a^2-16b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{ab\left(6a^2-71b^2\right)\tan(c+d}{2d}}{5b}}{\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5bd}}$$

$$\Big\downarrow \; 4274$$

$$\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(15ab\left(4a^2+9b^2\right)\int \sec(c+dx)dx-4\left(3a^4-52a^2b^2-16b^4\right)\int \sec^2(c+dx)dx\right)-\frac{ab\left(6a^2-71b^2\right)\tan(c+dx)\sec(}{2d}}{5b}}{\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5bd}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(15ab\left(4a^2+9b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-4\left(3a^4-52a^2b^2-16b^4\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)-\frac{ab\left(6a^2-71b^2\right)}{}}{5b}}{\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5bd}}$$

$$\Big\downarrow \; 4254$$

$$\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(15ab\left(4a^2+9b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{4\left(3a^4-52a^2b^2-16b^4\right)\int 1d(-\tan(c+dx))}{d}\right)-\frac{ab\left(6a^2-71b^2\right)\tan(c+dx)\sec(c+}{2d}}{5b}}{\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5bd}}$$

$$\Big\downarrow \; 24$$

$$\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(15ab\left(4a^2+9b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{4\left(3a^4-52a^2b^2-16b^4\right)\tan(c+dx)}{d}\right)-\frac{ab\left(6a^2-71b^2\right)\tan(c+dx)\sec(c+dx)}{2d}\right)-}{5b}}{\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5bd}}$$

$$\Big\downarrow \; 4257$$

$$\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(\frac{15ab\left(4a^2+9b^2\right)\text{arctanh}(\sin(c+dx))}{d}-\frac{4\left(3a^4-52a^2b^2-16b^4\right)\tan(c+dx)}{d}\right)-\frac{ab\left(6a^2-71b^2\right)\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{(3a^2-1}{}}{5b}}{\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5bd}}$$

input
```
Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
```

output
```
((a + b*Sec[c + d*x])^4*Tan[c + d*x])/(5*b*d) + (-1/4*(a*(a + b*Sec[c + d*
x])^3*Tan[c + d*x])/d + (-1/3*((3*a^2 - 16*b^2)*(a + b*Sec[c + d*x])^2*Tan
[c + d*x])/d + (-1/2*(a*b*(6*a^2 - 71*b^2)*Sec[c + d*x]*Tan[c + d*x])/d +
((15*a*b*(4*a^2 + 9*b^2)*ArcTanh[Sin[c + d*x]])/d - (4*(3*a^4 - 52*a^2*b^2
 - 16*b^4)*Tan[c + d*x])/d)/2)/3)/4)/(5*b)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4327
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 -
b^2, 0] &&  !LtQ[m, -1]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-3a^2b\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+3a\,b^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\,\mathrm{se}}{}\right.\right.}{d}$
default	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-3a^2b\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+3a\,b^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\,\mathrm{se}}{}\right.\right.}{d}$
parts	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}-\frac{b^3\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}+\frac{3a\,b^2\left(-\left(\right.\right.}{}$
parallelrisch	$-600\left(a^2+\frac{9b^2}{4}\right)a\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3dx+3c)}{2}+\cos(dx+c)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+600\left(a^2+\frac{9b^2}{4}\right)a\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3d}{2}\right.$
norman	$\frac{\frac{\left(4a^3-24a^2b+15a\,b^2-8b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{4d}-\frac{\left(4a^3+24a^2b+15a\,b^2+8b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{\left(12a^3-96a^2b+9a\,b^2-16b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{6d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}$
risch	$-\frac{i\left(60a^3\mathrm{e}^{9i(dx+c)}+135a\,b^2\mathrm{e}^{9i(dx+c)}+120a^3\mathrm{e}^{7i(dx+c)}+630a\,b^2\mathrm{e}^{7i(dx+c)}-720a^2b\,\mathrm{e}^{6i(dx+c)}-1680a^2b\,\mathrm{e}^{4i(dx+c)}-640b\right.}{60}$

input
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-3*a^2*b
*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+3*a*b^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d
*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-b^3*(-8/15-1/5*sec(d*x+c)
^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.90

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{15\,(4\,a^3+9\,ab^2)\cos{(dx+c)}^5\log{(\sin{(dx+c)}+1)}-15\,(4\,a^3+9\,ab^2)\cos{(dx+c)}^5\log{(-\sin{(dx+c)}+}$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/240*(15*(4*a^3 + 9*a*b^2)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(4*a
^3 + 9*a*b^2)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(16*(15*a^2*b + 4*
b^3)*cos(d*x + c)^4 + 90*a*b^2*cos(d*x + c) + 15*(4*a^3 + 9*a*b^2)*cos(d*x
 + c)^3 + 24*b^3 + 8*(15*a^2*b + 4*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*c
os(d*x + c)^5)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^3\,dx = \int (a+b\sec{(c+dx)})^3\sec^3{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**3,x)
```

output
```
Integral((a + b*sec(c + d*x))**3*sec(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.96

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$= \frac{240\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^2b+16\left(3\,\tan\left(dx+c\right)^5+10\,\tan\left(dx+c\right)^3+15\,\tan\left(dx+c\right)\right)b^3-}{}$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/240*(240*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2*b + 16*(3*tan(d*x + c)^5
+ 10*tan(d*x + c)^3 + 15*tan(d*x + c))*b^3 - 45*a*b^2*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
 + c) + 1) + 3*log(sin(d*x + c) - 1)) - 60*a^3*(2*sin(d*x + c)/(sin(d*x +
c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(177) = 354.

Time = 0.17 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.94

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$= \frac{15\left(4\,a^3+9\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(4\,a^3+9\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(60\,a^3\tan(\cdots\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/120*(15*(4*a^3 + 9*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*a^3
 + 9*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(60*a^3*tan(1/2*d*x + 1
/2*c)^9 - 360*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 225*a*b^2*tan(1/2*d*x + 1/2*c
)^9 - 120*b^3*tan(1/2*d*x + 1/2*c)^9 - 120*a^3*tan(1/2*d*x + 1/2*c)^7 + 96
0*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 90*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 160*b^3
*tan(1/2*d*x + 1/2*c)^7 - 1200*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 464*b^3*tan(
1/2*d*x + 1/2*c)^5 + 120*a^3*tan(1/2*d*x + 1/2*c)^3 + 960*a^2*b*tan(1/2*d*
x + 1/2*c)^3 + 90*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 160*b^3*tan(1/2*d*x + 1/2
*c)^3 - 60*a^3*tan(1/2*d*x + 1/2*c) - 360*a^2*b*tan(1/2*d*x + 1/2*c) - 225
*a*b^2*tan(1/2*d*x + 1/2*c) - 120*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
 1/2*c)^2 - 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 14.33 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.37

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(a^3+\frac{9\,a\,b^2}{4}\right)}{d} - \frac{\left(-a^3+6\,a^2\,b-\frac{15\,a\,b^2}{4}+2\,b^3\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\left(2\,a^3-16\,a^2\,b+\frac{3\,a\,b^2}{2}-\frac{8\,b^3}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(20\,a^2\,b\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+10\right.}$$

input
```
int((a + b/cos(c + d*x))^3/cos(c + d*x)^3,x)
```

output
```
(atanh(tan(c/2 + (d*x)/2))*((9*a*b^2)/4 + a^3))/d - (tan(c/2 + (d*x)/2)^7*
((3*a*b^2)/2 - 16*a^2*b + 2*a^3 - (8*b^3)/3) - tan(c/2 + (d*x)/2)^3*((3*a*
b^2)/2 + 16*a^2*b + 2*a^3 + (8*b^3)/3) + tan(c/2 + (d*x)/2)*((15*a*b^2)/4
+ 6*a^2*b + a^3 + 2*b^3) + tan(c/2 + (d*x)/2)^5*(20*a^2*b + (116*b^3)/15)
- tan(c/2 + (d*x)/2)^9*((15*a*b^2)/4 - 6*a^2*b + a^3 - 2*b^3))/(d*(5*tan(c
/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*ta
n(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 533, normalized size of antiderivative = 2.82

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^3\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c))^3,x)
```

output
```
( - 60*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a**3 - 135*c
os(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a*b**2 + 120*cos(c +
 d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**3 + 270*cos(c + d*x)*lo
g(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a*b**2 - 60*cos(c + d*x)*log(tan((
c + d*x)/2) - 1)*a**3 - 135*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a*b**2
+ 60*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a**3 + 135*cos
(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a*b**2 - 120*cos(c + d
*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**3 - 270*cos(c + d*x)*log(
tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a*b**2 + 60*cos(c + d*x)*log(tan((c
+ d*x)/2) + 1)*a**3 + 135*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a*b**2 -
60*cos(c + d*x)*sin(c + d*x)**3*a**3 - 135*cos(c + d*x)*sin(c + d*x)**3*a*
b**2 + 60*cos(c + d*x)*sin(c + d*x)*a**3 + 225*cos(c + d*x)*sin(c + d*x)*a
*b**2 + 240*sin(c + d*x)**5*a**2*b + 64*sin(c + d*x)**5*b**3 - 600*sin(c +
 d*x)**3*a**2*b - 160*sin(c + d*x)**3*b**3 + 360*sin(c + d*x)*a**2*b + 120
*sin(c + d*x)*b**3)/(120*cos(c + d*x)*d*(sin(c + d*x)**4 - 2*sin(c + d*x)*
*2 + 1))
```

3.467 $\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 130

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{3b(4a^2+b^2)\operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{a(a^2+4b^2)\tan(c+dx)}{2d} + \frac{b(2a^2+3b^2)\sec(c+dx)\tan(c+dx)}{8d} + \frac{a(a+b\sec(c+dx))^2\tan(c+dx)}{4d} + \frac{(a+b\sec(c+dx))^3\tan(c+dx)}{4d}$$

output

```
3/8*b*(4*a^2+b^2)*arctanh(sin(d*x+c))/d+1/2*a*(a^2+4*b^2)*tan(d*x+c)/d+1/8
*b*(2*a^2+3*b^2)*sec(d*x+c)*tan(d*x+c)/d+1/4*a*(a+b*sec(d*x+c))^2*tan(d*x+
c)/d+1/4*(a+b*sec(d*x+c))^3*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.69

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{3b(4a^2+b^2)\,\text{arctanh}(\sin(c+dx))+\tan(c+dx)\,(3b(4a^2+b^2)\sec(c+dx)+2b^3\sec^3(c+dx)+8a(a^2+}{8d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```

output
```
(3*b*(4*a^2 + b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*b*(4*a^2 + b^2)
*Sec[c + d*x] + 2*b^3*Sec[c + d*x]^3 + 8*a*(a^2 + 3*b^2 + b^2*Tan[c + d*x]
^2)))/(8*d)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4322, 3042, 4490, 3042, 4485, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{ 4322}$$

$$\frac{3}{4}\int \sec(c+dx)(b+a\sec(c+dx))(a+b\sec(c+dx))^2dx+\frac{\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx+\frac{\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$\Big\downarrow$ 4490

$$\frac{3}{4}\left(\frac{1}{3}\int \sec(c+dx)(a+b\sec(c+dx))\left(5ab+\left(2a^2+3b^2\right)\sec(c+dx)\right)dx+\frac{a\tan(c+dx)(a+b\sec(c+dx))^2}{3d}\right)+\frac{\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$\Big\downarrow$ 3042

$$\frac{3}{4}\left(\frac{1}{3}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(5ab+\left(2a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{a\tan(c+dx)(\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$\Big\downarrow$ 4485

$$\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \sec(c+dx)\left(3b\left(4a^2+b^2\right)+4a\left(a^2+4b^2\right)\sec(c+dx)\right)dx+\frac{b\left(2a^2+3b^2\right)\tan(c+dx)\sec(c+dx)}{2d}\right)\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$\Big\downarrow$ 3042

$$\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(3b\left(4a^2+b^2\right)+4a\left(a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{b\left(2a^2+3b^2\right)\tan(c+dx)\text{se}}{2d}\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$\Big\downarrow$ 4274

$$\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(4a\left(a^2+4b^2\right)\int \sec^2(c+dx)dx+3b\left(4a^2+b^2\right)\int \sec(c+dx)dx\right)+\frac{b\left(2a^2+3b^2\right)\tan(c+dx)\sec(c+}{2d}\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$\Big\downarrow$ 3042

$$\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(3b(4a^2+b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+4a(a^2+4b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)+\frac{b(2a^2+3b^2)\tan(c+\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$$\Big\downarrow \text{4254}$$

$$\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(3b(4a^2+b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{4a(a^2+4b^2)\int 1d(-\tan(c+dx))}{d}\right)+\frac{b(2a^2+3b^2)\tan(c+dx}{2d}\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$$\Big\downarrow \text{24}$$

$$\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(3b(4a^2+b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{4a(a^2+4b^2)\tan(c+dx)}{d}\right)+\frac{b(2a^2+3b^2)\tan(c+dx)\sec(c+}{2d}\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(\frac{3b(4a^2+b^2)\text{arctanh}(\sin(c+dx))}{d}+\frac{4a(a^2+4b^2)\tan(c+dx)}{d}\right)+\frac{b(2a^2+3b^2)\tan(c+dx)\sec(c+}{2d}\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

input
```
Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```

output
```
((a + b*Sec[c + d*x])^3*Tan[c + d*x])/(4*d) + (3*((a*(a + b*Sec[c + d*x])^
2*Tan[c + d*x])/(3*d) + ((b*(2*a^2 + 3*b^2)*Sec[c + d*x]*Tan[c + d*x])/(2*
d) + ((3*b*(4*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/d + (4*a*(a^2 + 4*b^2)*Tan
[c + d*x])/d)/2)/3))/4
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4322
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[m/(m + 1)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc
[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{a^3\tan(dx+c)+3a^2b\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-3a\,b^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+b^3\left(-\left(-\right.\right.}{d}$
default	$\frac{a^3\tan(dx+c)+3a^2b\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-3a\,b^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+b^3\left(-\left(-\right.\right.}{d}$
parts	$\frac{a^3\tan(dx+c)}{d}+\frac{b^3\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}+\frac{3a^2b\left(\frac{\sec(dx+c)\tan(\right.}{2}}{}$
parallelrisch	$\frac{-24\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)b\left(a^2+\frac{b^2}{4}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+24\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)b\left(a^2+\frac{b^2}{4}\right)\ln}{4d(3+\cos(4d}$
norman	$\frac{-\frac{\left(8a^3-12a^2b+24a\,b^2-5b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}+\frac{\left(8a^3+12a^2b+24a\,b^2+5b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{\left(24a^3-12a^2b+40a\,b^2+3b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}$
risch	$-\frac{i\left(12a^2b\,e^{7i(dx+c)}+3b^3e^{7i(dx+c)}-8a^3e^{6i(dx+c)}+12a^2b\,e^{5i(dx+c)}+11b^3e^{5i(dx+c)}-24a^3e^{4i(dx+c)}-48a\,b^2e^{4i(dx+c)}-\right.}{4d\left(e^{2i(dx+c)}\right.}$

input
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*tan(d*x+c)+3*a^2*b*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+t
an(d*x+c)))-3*a*b^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+b^3*(-(-1/4*sec(d*x
+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.08

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$=\frac{3\,(4\,a^2b+b^3)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\,(4\,a^2b+b^3)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+}{16\,d\cos\left(dx\right.}$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/16*(3*(4*a^2*b + b^3)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*a^2*b
+ b^3)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*a*b^2*cos(d*x + c) + 8
*(a^3 + 2*a*b^2)*cos(d*x + c)^3 + 2*b^3 + 3*(4*a^2*b + b^3)*cos(d*x + c)^2
)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx=\int\left(a+b\sec\left(c+dx\right)\right)^3\sec^2\left(c+dx\right)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**3,x)
```

output
```
Integral((a + b*sec(c + d*x))**3*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.22

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$=\frac{16\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)ab^2-b^3\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/16*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^2 - b^3*(2*(3*sin(d*x + c)^
3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*
x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a^2*b*(2*sin(d*x + c)/(sin(d*x
 + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 16*a^3*tan
(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(120) = 240$.

Time = 0.17 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.54

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$= \frac{3\left(4a^2b+b^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3\left(4a^2b+b^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(8a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/8*(3*(4*a^2*b + b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*a^2*b + b
^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(8*a^3*tan(1/2*d*x + 1/2*c)^7 -
 12*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 5*b^3
*tan(1/2*d*x + 1/2*c)^7 - 24*a^3*tan(1/2*d*x + 1/2*c)^5 + 12*a^2*b*tan(1/2
*d*x + 1/2*c)^5 - 40*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3*b^3*tan(1/2*d*x + 1/
2*c)^5 + 24*a^3*tan(1/2*d*x + 1/2*c)^3 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^3 +
 40*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 3*b^3*tan(1/2*d*x + 1/2*c)^3 - 8*a^3*ta
n(1/2*d*x + 1/2*c) - 12*a^2*b*tan(1/2*d*x + 1/2*c) - 24*a*b^2*tan(1/2*d*x
+ 1/2*c) - 5*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 14.36 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.74

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{3\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(4\,a^2+b^2\right)}{4\,d} - \frac{\left(2\,a^3-3\,a^2\,b+6\,a\,b^2-\frac{5\,b^3}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(-6\,a^3+3\,a^2\,b-10\,a\,b^2-\frac{3\,b^3}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(6\,a^3\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,\tan\left(\frac{c}{2}\right.\right.}$$

input
```
int((a + b/cos(c + d*x))^3/cos(c + d*x)^2,x)
```

output
```
(3*b*atanh(tan(c/2 + (d*x)/2))*(4*a^2 + b^2))/(4*d) - (tan(c/2 + (d*x)/2)^
7*(6*a*b^2 - 3*a^2*b + 2*a^3 - (5*b^3)/4) + tan(c/2 + (d*x)/2)^3*(10*a*b^2
 + 3*a^2*b + 6*a^3 - (3*b^3)/4) - tan(c/2 + (d*x)/2)^5*(10*a*b^2 - 3*a^2*b
 + 6*a^3 + (3*b^3)/4) - tan(c/2 + (d*x)/2)*(6*a*b^2 + 3*a^2*b + 2*a^3 + (5
*b^3)/4))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2
+ (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 426, normalized size of antiderivative = 3.28

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{-8\cos(dx+c)\sin(dx+c)^3\,a^3-16\cos(dx+c)\sin(dx+c)^3\,a\,b^2+8\cos(dx+c)\sin(dx+c)\,a^3+24\,\mathrm{co}}{}$$

input
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c))^3,x)
```

output
```
( - 8*cos(c + d*x)*sin(c + d*x)**3*a**3 - 16*cos(c + d*x)*sin(c + d*x)**3*
a*b**2 + 8*cos(c + d*x)*sin(c + d*x)*a**3 + 24*cos(c + d*x)*sin(c + d*x)*a
*b**2 - 12*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a**2*b - 3*log(tan((c
 + d*x)/2) - 1)*sin(c + d*x)**4*b**3 + 24*log(tan((c + d*x)/2) - 1)*sin(c
+ d*x)**2*a**2*b + 6*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b**3 - 12*l
og(tan((c + d*x)/2) - 1)*a**2*b - 3*log(tan((c + d*x)/2) - 1)*b**3 + 12*lo
g(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a**2*b + 3*log(tan((c + d*x)/2) +
1)*sin(c + d*x)**4*b**3 - 24*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**
2*b - 6*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*b**3 + 12*log(tan((c + d
*x)/2) + 1)*a**2*b + 3*log(tan((c + d*x)/2) + 1)*b**3 - 12*sin(c + d*x)**3
*a**2*b - 3*sin(c + d*x)**3*b**3 + 12*sin(c + d*x)*a**2*b + 5*sin(c + d*x)
*b**3)/(8*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.468 $\int \sec(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 99

$$\int \sec(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{a(2a^2+3b^2)\operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{2b(4a^2+b^2)\tan(c+dx)}{3d} + \frac{5ab^2\sec(c+dx)\tan(c+dx)}{6d} + \frac{b(a+b\sec(c+dx))^2\tan(c+dx)}{3d}$$

output
```
1/2*a*(2*a^2+3*b^2)*arctanh(sin(d*x+c))/d+2/3*b*(4*a^2+b^2)*tan(d*x+c)/d+5
/6*a*b^2*sec(d*x+c)*tan(d*x+c)/d+1/3*b*(a+b*sec(d*x+c))^2*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.76

$$\int \sec(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{6a^3\coth^{-1}(\sin(c+dx)) + b(9ab\operatorname{arctanh}(\sin(c+dx)) + \tan(c+dx)\left(18a^2+6b^2+9ab\sec(c+dx)+2b^2\right.}{6d}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^3,x]
```

output
```
(6*a^3*ArcCoth[Sin[c + d*x]] + b*(9*a*b*ArcTanh[Sin[c + d*x]] + Tan[c + d*
x]*(18*a^2 + 6*b^2 + 9*a*b*Sec[c + d*x] + 2*b^2*Tan[c + d*x]^2)))/(6*d)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 4317, 3042, 4485, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4317}$$

$$\frac{1}{3}\int \sec(c+dx)(a+b\sec(c+dx))\left(3a^2+5b\sec(c+dx)a+2b^2\right)dx+\frac{b\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(3a^2+5b\csc\left(c+dx+\frac{\pi}{2}\right)a+2b^2\right)dx+\frac{b\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \text{4485}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \sec(c+dx)\left(3a\left(2a^2+3b^2\right)+4b\left(4a^2+b^2\right)\sec(c+dx)\right)dx+\frac{5ab^2\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{b\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(3a(2a^2+3b^2)+4b(4a^2+b^2)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{5ab^2\tan(c+dx)\sec(c+dx)}{2d}\right)$$
$$\frac{b\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$\Big\downarrow$ 4274

$$\frac{1}{3}\left(\frac{1}{2}\left(4b(4a^2+b^2)\int \sec^2(c+dx)dx+3a(2a^2+3b^2)\int \sec(c+dx)dx\right)+\frac{5ab^2\tan(c+dx)\sec(c+dx)}{2d}\right)+$$
$$\frac{b\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{1}{2}\left(3a(2a^2+3b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+4b(4a^2+b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)+\frac{5ab^2\tan(c+dx)\sec(c}{2d}\right.$$
$$\frac{b\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$\Big\downarrow$ 4254

$$\frac{1}{3}\left(\frac{1}{2}\left(3a(2a^2+3b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{4b(4a^2+b^2)\int 1d(-\tan(c+dx))}{d}\right)+\frac{5ab^2\tan(c+dx)\sec(c+d}{2d}\right.$$
$$\frac{b\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$\Big\downarrow$ 24

$$\frac{1}{3}\left(\frac{1}{2}\left(3a(2a^2+3b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{4b(4a^2+b^2)\tan(c+dx)}{d}\right)+\frac{5ab^2\tan(c+dx)\sec(c+dx)}{2d}\right)+$$
$$\frac{b\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$\Big\downarrow$ 4257

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{3a(2a^2+3b^2)\text{arctanh}(\sin(c+dx))}{d}+\frac{4b(4a^2+b^2)\tan(c+dx)}{d}\right)+\frac{5ab^2\tan(c+dx)\sec(c+dx)}{2d}\right)+$$
$$\frac{b\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

input
```
Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^3,x]
```

output
```
(b*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(3*d) + ((5*a*b^2*Sec[c + d*x]*Tan
[c + d*x])/(2*d) + ((3*a*(2*a^2 + 3*b^2)*ArcTanh[Sin[c + d*x]])/d + (4*b*(
4*a^2 + b^2)*Tan[c + d*x])/d)/2)/3
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4317
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Simp[1/m   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*m + a*b*(2*m - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ
[a^2 - b^2, 0] && GtQ[m, 1] && IntegerQ[2*m]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))+3a^2b\tan(dx+c)+3a\,b^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-b^3\left(-\frac{2}{3}-\frac{\sec(d}{}\right.}{d}$
default	$\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))+3a^2b\tan(dx+c)+3a\,b^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-b^3\left(-\frac{2}{3}-\frac{\sec(d}{}\right.}{d}$
parts	$\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))}{d}-\frac{b^3\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{3a\,b^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(d}{2}\right.}{d}$
parallelrisch	$\frac{-3\left(\frac{\cos(3dx+3c)}{3}+\cos(dx+c)\right)a\left(a^2+\frac{3b^2}{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+3\left(\frac{\cos(3dx+3c)}{3}+\cos(dx+c)\right)a\left(a^2+\frac{3b^2}{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{d(\cos(3dx+3c)+3\cos(dx+c))}$
norman	$\frac{\frac{4b(9a^2+b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{b(6a^2-3ab+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}-\frac{b(6a^2+3ab+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}-\frac{a(2a^2+3b^2)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{2d}$
risch	$-\frac{ib\left(9ab\,e^{5i(dx+c)}-18a^2e^{4i(dx+c)}-36a^2e^{2i(dx+c)}-12\,e^{2i(dx+c)}b^2-9ab\,e^{i(dx+c)}-18a^2-4b^2\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}+\frac{a^3\ln\left(e^{i(dx+c)}+i\right)}{d}$

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^2*b*tan(d*x+c)+3*a*b^2*(1/2*sec(d*x
+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-b^3*(-2/3-1/3*sec(d*x+c)^2)*
tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.27

$$\int \sec(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{3\left(2a^3+3ab^2\right)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\left(2a^3+3ab^2\right)\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)}{12\,d\cos\left(dx+c\right)^3}$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/12*(3*(2*a^3 + 3*a*b^2)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*a^3
+ 3*a*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(9*a*b^2*cos(d*x + c)
 + 2*b^3 + 2*(9*a^2*b + 2*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x +
c)^3)
```

Sympy [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^3\,dx = \int \left(a+b\sec\left(c+dx\right)\right)^3\sec\left(c+dx\right)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))**3,x)
```

output
```
Integral((a + b*sec(c + d*x))**3*sec(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.07

$$\int \sec(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{4\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)b^3-9\,ab^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{12\,d}$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*b^3 - 9*a*b^2*(2*sin(d*x + c)/(s
in(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a
^3*log(sec(d*x + c) + tan(d*x + c)) + 36*a^2*b*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(91) = 182$.

Time = 0.16 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.07

$$\int \sec(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{3\left(2a^3+3ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3\left(2a^3+3ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(18a^2b\tan\left(\frac{1}{2}c\right.\right.}{}}{}$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/6*(3*(2*a^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*a^3 + 3
*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(18*a^2*b*tan(1/2*d*x + 1/2
*c)^5 - 9*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*b^3*tan(1/2*d*x + 1/2*c)^5 - 36
*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 4*b^3*tan(1/2*d*x + 1/2*c)^3 + 18*a^2*b*ta
n(1/2*d*x + 1/2*c) + 9*a*b^2*tan(1/2*d*x + 1/2*c) + 6*b^3*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 12.82 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.59

$$\int \sec(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,\left(2a^3+3ab^2\right)}{d}$$
$$-\frac{\left(6a^2b-3ab^2+2b^3\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(-12a^2b-\frac{4b^3}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\left(6a^2b+3ab^2+2b^3\right)\tan\left(\frac{c}{2}\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

input
```
int((a + b/cos(c + d*x))^3/cos(c + d*x),x)
```

output
```
(atanh(tan(c/2 + (d*x)/2))*(3*a*b^2 + 2*a^3))/d - (tan(c/2 + (d*x)/2)^5*(6
*a^2*b - 3*a*b^2 + 2*b^3) - tan(c/2 + (d*x)/2)^3*(12*a^2*b + (4*b^3)/3) +
tan(c/2 + (d*x)/2)*(3*a*b^2 + 6*a^2*b + 2*b^3))/(d*(3*tan(c/2 + (d*x)/2)^2
 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.17

$$\int \sec(c+dx)(a+b\sec(c+dx))^3 \, dx = \frac{-6\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2 a^3 - 9\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)}{}$$

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^3,x)
```

output
```
( - 6*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**3 - 9*cos(
c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a*b**2 + 6*cos(c + d*x)
*log(tan((c + d*x)/2) - 1)*a**3 + 9*cos(c + d*x)*log(tan((c + d*x)/2) - 1)
*a*b**2 + 6*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**3 +
9*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a*b**2 - 6*cos(c
+ d*x)*log(tan((c + d*x)/2) + 1)*a**3 - 9*cos(c + d*x)*log(tan((c + d*x)/2
) + 1)*a*b**2 - 9*cos(c + d*x)*sin(c + d*x)*a*b**2 + 18*sin(c + d*x)**3*a*
*2*b + 4*sin(c + d*x)**3*b**3 - 18*sin(c + d*x)*a**2*b - 6*sin(c + d*x)*b*
*3)/(6*cos(c + d*x)*d*(sin(c + d*x)**2 - 1))
```

3.469 $\int (a+b\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 73

$$\int (a+b\sec(c+dx))^3\,dx = a^3x+\frac{b(6a^2+b^2)\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{5ab^2\tan(c+dx)}{2d}+\frac{b^2(a+b\sec(c+dx))\tan(c+dx)}{2d}$$

output
```
a^3*x+1/2*b*(6*a^2+b^2)*arctanh(sin(d*x+c))/d+5/2*a*b^2*tan(d*x+c)/d+1/2*b
^2*(a+b*sec(d*x+c))*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int (a+b\sec(c+dx))^3\,dx = \frac{2a^3dx+6a^2b\coth^{-1}(\sin(c+dx))+b^3\operatorname{arctanh}(\sin(c+dx))+6ab^2\tan(c+dx)+b^3\sec(c+dx)\tan(c+}{2d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^3,x]
```

output
```
(2*a^3*d*x + 6*a^2*b*ArcCoth[Sin[c + d*x]] + b^3*ArcTanh[Sin[c + d*x]] + 6
*a*b^2*Tan[c + d*x] + b^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4269, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \ 4269$$

$$\frac{1}{2}\int \left(2a^3+5b^2\sec^2(c+dx)a+b\left(6a^2+b^2\right)\sec(c+dx)\right)dx+\frac{b^2\tan(c+dx)(a+b\sec(c+dx))}{2d}$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(2a^3x+\frac{b\left(6a^2+b^2\right)\text{arctanh}(\sin(c+dx))}{d}+\frac{5ab^2\tan(c+dx)}{d}\right)+\frac{b^2\tan(c+dx)(a+b\sec(c+dx))}{2d}$$

input
```
Int[(a + b*Sec[c + d*x])^3,x]
```

output
```
(b^2*(a + b*Sec[c + d*x])*Tan[c + d*x])/(2*d) + (2*a^3*x + (b*(6*a^2 + b^2
)*ArcTanh[Sin[c + d*x]])/d + (5*a*b^2*Tan[c + d*x])/d)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4269
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[1/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*
a^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{a^3(dx+c)+3a^2b\ln(\sec(dx+c)+\tan(dx+c))+3a\,b^2\tan(dx+c)+b^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{a^3(dx+c)+3a^2b\ln(\sec(dx+c)+\tan(dx+c))+3a\,b^2\tan(dx+c)+b^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
parts	$a^3x+\frac{b^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{3a^2b\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{3a\,b^2\tan(dx+c)}{d}$
parallelrisch	$\frac{-3(1+\cos(2dx+2c))b\left(a^2+\frac{b^2}{6}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+3(1+\cos(2dx+2c))b\left(a^2+\frac{b^2}{6}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+a^3xd\cos(2}{d(1+\cos(2dx+2c))}$
risch	$a^3x-\frac{ib^2\left(b\,e^{3i(dx+c)}-6\,e^{2i(dx+c)}a-b\,e^{i(dx+c)}-6a\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{3b\ln\left(e^{i(dx+c)}-i\right)a^2}{d}-\frac{b^3\ln\left(e^{i(dx+c)}-i\right)}{2d}+\frac{3b\ln(e^{i(dx-}{d}$
norman	$\frac{a^3x+a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\frac{b^2(b+6a)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-2a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{b^2(6a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-\frac{b(6a^2+b^2)\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{2d}$

input
```
int((a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(d*x+c)+3*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+3*a*b^2*tan(d*x+c)+b^3*
(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.53

$$\int (a+b\sec(c+dx))^3\,dx$$
$$=\frac{4\,a^3dx\cos\left(dx+c\right)^2+\left(6\,a^2b+b^3\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(6\,a^2b+b^3\right)\cos\left(dx+c\right)^2\log\left(-s\right.}{4\,d\cos\left(dx+c\right)^2}$$

input
```
integrate((a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4*(4*a^3*d*x*cos(d*x + c)^2 + (6*a^2*b + b^3)*cos(d*x + c)^2*log(sin(d*x
 + c) + 1) - (6*a^2*b + b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(6*
a*b^2*cos(d*x + c) + b^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int (a+b\sec(c+dx))^3\,dx=\int (a+b\sec\left(c+dx\right))^3\;dx$$

input
```
integrate((a+b*sec(d*x+c))**3,x)
```

output
```
Integral((a + b*sec(c + d*x))**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.27

$$\int (a+b\sec(c+dx))^3\,dx$$
$$=a^3x-\frac{b^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{4\,d}$$
$$+\frac{3\,a^2b\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{3\,ab^2\tan\left(dx+c\right)}{d}$$

input
```
integrate((a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
a^3*x - 1/4*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) +
1) + log(sin(d*x + c) - 1))/d + 3*a^2*b*log(sec(d*x + c) + tan(d*x + c))/d
 + 3*a*b^2*tan(d*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(67) = 134.

Time = 0.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.99

$$\int (a + b\sec(c + dx))^3 \, dx$$

$$= \frac{2\,(dx + c)a^3 + (6\,a^2b + b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - (6\,a^2b + b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\,(6\,a}{}}{2\,d}$$

input
```
integrate((a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(2*(d*x + c)*a^3 + (6*a^2*b + b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1))
- (6*a^2*b + b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*a*b^2*tan(1/2*
d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d*x + 1/2*c)
 - b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 10.58 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.86

$$\int (a+b\sec(c+dx))^3\,dx = \frac{2\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{b^3\,\sin\left(c+d\,x\right)}{2\,d\cos\left(c+d\,x\right)^2} + \frac{6\,a^2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{3\,a\,b^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}$$

input
```
int((a + b/cos(c + d*x))^3,x)
```

output
```
(2*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^3*atanh(sin(c/2
 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^3*sin(c + d*x))/(2*d*cos(c + d*x)^
2) + (6*a^2*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (3*a*b^2*s
in(c + d*x))/(d*cos(c + d*x))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.27

$$\int (a+b\sec(c+dx))^3\,dx = \frac{-6\cos\left(dx+c\right)\sin\left(dx+c\right)a\,b^2-6\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2a^2b-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(\right.}{}$$

input
```
int((a+b*sec(d*x+c))^3,x)
```

output
```
( - 6*cos(c + d*x)*sin(c + d*x)*a*b**2 - 6*log(tan((c + d*x)/2) - 1)*sin(c
 + d*x)**2*a**2*b - log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b**3 + 6*log
(tan((c + d*x)/2) - 1)*a**2*b + log(tan((c + d*x)/2) - 1)*b**3 + 6*log(tan
((c + d*x)/2) + 1)*sin(c + d*x)**2*a**2*b + log(tan((c + d*x)/2) + 1)*sin(
c + d*x)**2*b**3 - 6*log(tan((c + d*x)/2) + 1)*a**2*b - log(tan((c + d*x)/
2) + 1)*b**3 + 2*sin(c + d*x)**2*a**3*d*x - sin(c + d*x)*b**3 - 2*a**3*d*x
)/(2*d*(sin(c + d*x)**2 - 1))
```

3.470 $\int \cos(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \cos(c+dx)(a+b\sec(c+dx))^3\,dx = 3a^2bx + \frac{3ab^2\text{arctanh}(\sin(c+dx))}{d} + \frac{a(a^2-b^2)\sin(c+dx)}{d} + \frac{b^2(a+b\sec(c+dx))\sin(c+dx)}{d}$$

output
```
3*a^2*b*x+3*a*b^2*arctanh(sin(d*x+c))/d+a*(a^2-b^2)*sin(d*x+c)/d+b^2*(a+b*
sec(d*x+c))*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \cos(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{3ab\left(ac+adx-b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+}{d}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^3,x]
```

output
```
(3*a*b*(a*c + a*d*x - b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + b*Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]]) + a^3*Sin[c + d*x] + b^3*Tan[c + d*x]
)/d
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 4329, 3042, 4535, 24, 3042, 4533, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4329}$$

$$\int \cos(c+dx)\left(3b\sec(c+dx)a^2+3b^2\sec^2(c+dx)a+\left(a^2-b^2\right)a\right)dx+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{3b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+3b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a+\left(a^2-b^2\right)a}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))}{d}$$

$$\Big\downarrow \text{4535}$$

$$\int \cos(c+dx)\left(3ab^2\sec^2(c+dx)+a\left(a^2-b^2\right)\right)dx+3a^2b\int 1dx+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))}{d}$$

$$\Big\downarrow \text{24}$$

$$\int \cos(c+dx)\left(3ab^2\sec^2(c+dx)+a\left(a^2-b^2\right)\right)dx+3a^2bx+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{3ab^2 \csc\left(c+dx+\frac{\pi}{2}\right)^2 + a(a^2-b^2)}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx + 3a^2bx + \frac{b^2\sin(c+dx)(a+b\sec(c+dx))}{d}$$

$$\Big\downarrow \text{4533}$$

$$3ab^2\int \sec(c+dx)dx + \frac{a(a^2-b^2)\sin(c+dx)}{d} + 3a^2bx + \frac{b^2\sin(c+dx)(a+b\sec(c+dx))}{d}$$

$$\Big\downarrow \text{3042}$$

$$3ab^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx + \frac{a(a^2-b^2)\sin(c+dx)}{d} + 3a^2bx + \frac{b^2\sin(c+dx)(a+b\sec(c+dx))}{d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{a(a^2-b^2)\sin(c+dx)}{d} + 3a^2bx + \frac{3ab^2\text{arctanh}(\sin(c+dx))}{d} + \frac{b^2\sin(c+dx)(a+b\sec(c+dx))}{d}$$

input
```
Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^3,x]
```

output
```
3*a^2*b*x + (3*a*b^2*ArcTanh[Sin[c + d*x]])/d + (a*(a^2 - b^2)*Sin[c + d*x
])/d + (b^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{a^3 \sin(dx+c)+3a^2b(dx+c)+3a\,b^2\ln(\sec(dx+c)+\tan(dx+c))+b^3\tan(dx+c)}{d}$
default	$\frac{a^3 \sin(dx+c)+3a^2b(dx+c)+3a\,b^2\ln(\sec(dx+c)+\tan(dx+c))+b^3\tan(dx+c)}{d}$
parallelrisch	$\frac{6a^2bxd\cos(dx+c)+6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a\,b^2\cos(dx+c)-6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a\,b^2\cos(dx+c)+a^3\sin(2dx+2c)+2b^3}{2\cos(dx+c)d}$
risch	$3a^2bx-\frac{ia^3e^{i(dx+c)}}{2d}+\frac{ia^3e^{-i(dx+c)}}{2d}+\frac{2ib^3}{d(e^{2i(dx+c)}+1)}-\frac{3a\ln(e^{i(dx+c)}-i)b^2}{d}+\frac{3a\ln(e^{i(dx+c)}+i)b^2}{d}$
norman	$\frac{3a^2bx-\frac{4a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}+\frac{2(a^3-b^3)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}+\frac{2(a^3+b^3)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-3a^2bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-3a^2bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$

input
```
int(cos(d*x+c)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*sin(d*x+c)+3*a^2*b*(d*x+c)+3*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+b^3*
tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.40

$$\int \cos(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{6\,a^2bdx\cos\left(dx+c\right)+3\,ab^2\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-3\,ab^2\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1}{2\,d\cos\left(dx+c\right)}$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(6*a^2*b*d*x*cos(d*x + c) + 3*a*b^2*cos(d*x + c)*log(sin(d*x + c) + 1)
 - 3*a*b^2*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(a^3*cos(d*x + c) + b^3
)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \cos(c+dx)(a+b\sec(c+dx))^3\,dx = \int \left(a+b\sec\left(c+dx\right)\right)^3\cos\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))**3,x)
```

output
```
Integral((a + b*sec(c + d*x))**3*cos(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

$$\int \cos(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{6\,(dx+c)a^2b + 3\,ab^2(\log\left(\sin\left(dx+c\right)+1\right) - \log\left(\sin\left(dx+c\right)-1\right)) + 2\,a^3\sin\left(dx+c\right) + 2\,b^3\tan\left(dx+\right.}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(6*(d*x + c)*a^2*b + 3*a*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
 - 1)) + 2*a^3*sin(d*x + c) + 2*b^3*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.96

$$\int \cos(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{3\,(dx+c)a^2b + 3\,ab^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 3\,ab^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3\right.}{}}{d}$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
(3*(d*x + c)*a^2*b + 3*a*b^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*b^2*
log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(a^3*tan(1/2*d*x + 1/2*c)^3 - b^3*t
an(1/2*d*x + 1/2*c)^3 - a^3*tan(1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

Mupad [B] (verification not implemented)

Time = 10.64 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.45

$$\int \cos(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{a^3\,\sin(c+d\,x)}{d}+\frac{b^3\,\sin(c+d\,x)}{d\,\cos(c+d\,x)} + \frac{6\,a^2\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{6\,a\,b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

input
```
int(cos(c + d*x)*(a + b/cos(c + d*x))^3,x)
```

output
```
(a^3*sin(c + d*x))/d + (b^3*sin(c + d*x))/(d*cos(c + d*x)) + (6*a^2*b*atan
(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (6*a*b^2*atanh(sin(c/2 + (d*x
)/2)/cos(c/2 + (d*x)/2)))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.70

$$\int \cos(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{-3\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a\,b^2+3\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a\,b^2+\cos(dx+c)\sin(}{\cos(dx+c)\,d}$$

input
```
int(cos(d*x+c)*(a+b*sec(d*x+c))^3,x)
```

output
```
( - 3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a*b**2 + 3*cos(c + d*x)*log(t
an((c + d*x)/2) + 1)*a*b**2 + cos(c + d*x)*sin(c + d*x)*a**3 + 3*cos(c + d
*x)*a**2*b*c + 3*cos(c + d*x)*a**2*b*d*x + sin(c + d*x)*b**3)/(cos(c + d*x
)*d)
```

3.471 $\int \cos^2(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{1}{2}a\left(a^2+6b^2\right)x+\frac{b^3\text{arctanh}(\sin(c+dx))}{d}+\frac{5a^2b\sin(c+dx)}{2d}+\frac{a^2\cos(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{2d}$$

output
```
1/2*a*(a^2+6*b^2)*x+b^3*arctanh(sin(d*x+c))/d+5/2*a^2*b*sin(d*x+c)/d+1/2*a
^2*cos(d*x+c)*(a+b*sec(d*x+c))*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.33

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{2a\left(a^2+6b^2\right)(c+dx)-4b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c\right.\right.}{4d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```

output
```
(2*a*(a^2 + 6*b^2)*(c + d*x) - 4*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]] + 4*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 12*a^2*b*Sin[c + d*
x] + a^3*Sin[2*(c + d*x)])/(4*d)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4328, 3042, 4535, 24, 3042, 4533, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4328}$$

$$\frac{1}{2}\int \cos(c+dx)\left(2\sec^2(c+dx)b^3+5a^2b+a\left(a^2+6b^2\right)\sec(c+dx)\right)dx+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{2\csc\left(c+dx+\frac{\pi}{2}\right)^2b^3+5a^2b+a\left(a^2+6b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))}{2d}$$

$$\Big\downarrow \text{4535}$$

$$\frac{1}{2}\left(\int \cos(c+dx)\left(2\sec^2(c+dx)b^3+5a^2b\right)dx+a\left(a^2+6b^2\right)\int 1dx\right)+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))}{2d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{2}\left(\int \cos(c+dx)\left(2\sec^2(c+dx)b^3+5a^2b\right)dx+ax\left(a^2+6b^2\right)\right)+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))}{2d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\left(\int \frac{2\csc\left(c+dx+\frac{\pi}{2}\right)^2b^3+5a^2b}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+ax\left(a^2+6b^2\right)\right)+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))}{2d}$$

$$\Big\downarrow \text{ 4533}$$

$$\frac{1}{2}\left(2b^3\int \sec(c+dx)dx+ax\left(a^2+6b^2\right)+\frac{5a^2b\sin(c+dx)}{d}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))}{2d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\left(2b^3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+ax\left(a^2+6b^2\right)+\frac{5a^2b\sin(c+dx)}{d}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))}{2d}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{1}{2}\left(ax\left(a^2+6b^2\right)+\frac{5a^2b\sin(c+dx)}{d}+\frac{2b^3\text{arctanh}(\sin(c+dx))}{d}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))}{2d}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```

output
```
(a^2*Cos[c + d*x]*(a + b*Sec[c + d*x])*Sin[c + d*x])/(2*d) + (a*(a^2 + 6*b
^2)*x + (2*b^3*ArcTanh[Sin[c + d*x]])/d + (5*a^2*b*Sin[c + d*x])/d)/2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{a^{3}\left(\frac{\sin\left(dx+c\right)\cos\left(dx+c\right)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^{2}b\sin\left(dx+c\right)+3a\,b^{2}\left(dx+c\right)+b^{3}\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}$
default	$\frac{a^{3}\left(\frac{\sin\left(dx+c\right)\cos\left(dx+c\right)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^{2}b\sin\left(dx+c\right)+3a\,b^{2}\left(dx+c\right)+b^{3}\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}$
parallelrisch	$\frac{2a^{3}xd+12a\,b^{2}dx+4b^{3}\left(\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\right)+a^{3}\sin\left(2dx+2c\right)+12a^{2}b\sin\left(dx+c\right)}{4d}$
risch	$\frac{a^{3}x}{2}+3a\,b^{2}x-\frac{3ia^{2}b\,{\mathrm e}^{i\left(dx+c\right)}}{2d}+\frac{3ia^{2}b\,{\mathrm e}^{-i\left(dx+c\right)}}{2d}+\frac{b^{3}\ln\left({\mathrm e}^{i\left(dx+c\right)}+i\right)}{d}-\frac{b^{3}\ln\left({\mathrm e}^{i\left(dx+c\right)}-i\right)}{d}+\frac{a^{3}\sin\left(2dx+2\right.}{4d}$
norman	$\frac{\left(\frac{1}{2}a^{3}+3a\,b^{2}\right)x+\left(\frac{1}{2}a^{3}+3a\,b^{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+\left(-a^{3}-6a\,b^{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\frac{a^{2}\left(a+6b\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{a^{2}\left(a-6b\right)\tan\left(\frac{dx}{2}+\right.}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}}$

input
```
int(cos(d*x+c)^2*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+3*a^2*b*sin(d*x+c)+3*a*
b^2*(d*x+c)+b^3*ln(sec(d*x+c)+tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{b^3\log\left(\sin\left(dx+c\right)+1\right)-b^3\log\left(-\sin\left(dx+c\right)+1\right)+(a^3+6\,ab^2)dx+(a^3\cos\left(dx+c\right)+6\,a^2b)\sin\left(dx\right.}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(b^3*log(sin(d*x + c) + 1) - b^3*log(-sin(d*x + c) + 1) + (a^3 + 6*a*b
^2)*d*x + (a^3*cos(d*x + c) + 6*a^2*b)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^3\,dx = \int (a+b\sec(c+dx))^3\cos^2(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**3,x)
```

output
```
Integral((a + b*sec(c + d*x))**3*cos(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.96

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^3+12\,(dx+c)ab^2+2\,b^3(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+}{4\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 + 12*(d*x + c)*a*b^2 + 2*b^3*(lo
g(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*a^2*b*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.73

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{2\,b^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,b^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+(a^3+6\,ab^2)(dx+c)-\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\right.\right.}{}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(2*b^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*b^3*log(abs(tan(1/2*d*x
+ 1/2*c) - 1)) + (a^3 + 6*a*b^2)*(d*x + c) - 2*(a^3*tan(1/2*d*x + 1/2*c)^3
 - 6*a^2*b*tan(1/2*d*x + 1/2*c)^3 - a^3*tan(1/2*d*x + 1/2*c) - 6*a^2*b*tan
(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 10.50 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.56

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^3\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{3\,a^2\,b\,\sin\left(c+d\,x\right)}{d}+\frac{6\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

input
```
int(cos(c + d*x)^2*(a + b/cos(c + d*x))^3,x)
```

output
```
(a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*b^3*atanh(sin(c/2
 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (a^3*sin(2*c + 2*d*x))/(4*d) + (3*a^2
*b*sin(c + d*x))/d + (6*a*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))
/d
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.19

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{\cos(dx+c)\sin(dx+c)\,a^3-2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b^3+2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)b^3+6\sin(dx+c)\,a^2b}{2d}$$

input
```
int(cos(d*x+c)^2*(a+b*sec(d*x+c))^3,x)
```

output
```
(cos(c + d*x)*sin(c + d*x)*a**3 - 2*log(tan((c + d*x)/2) - 1)*b**3 + 2*log
(tan((c + d*x)/2) + 1)*b**3 + 6*sin(c + d*x)*a**2*b + a**3*c + a**3*d*x +
6*a*b**2*c + 6*a*b**2*d*x)/(2*d)
```

3.472 $\int \cos^3(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{1}{2}b\left(3a^2+2b^2\right)x+\frac{a(2a^2+9b^2)\sin(c+dx)}{3d}+\frac{7a^2b\cos(c+dx)\sin(c+dx)}{6d}+\frac{a^2\cos^2(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{3d}$$

output
```
1/2*b*(3*a^2+2*b^2)*x+1/3*a*(2*a^2+9*b^2)*sin(d*x+c)/d+7/6*a^2*b*cos(d*x+c
)*sin(d*x+c)/d+1/3*a^2*cos(d*x+c)^2*(a+b*sec(d*x+c))*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.80

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{18a^2bc+12b^3c+18a^2bdx+12b^3dx+9a(a^2+4b^2)\sin(c+dx)+9a^2b\sin(2(c+dx))+a^3\sin(3(c+dx)}{12d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
```

output
```
(18*a^2*b*c + 12*b^3*c + 18*a^2*b*d*x + 12*b^3*d*x + 9*a*(a^2 + 4*b^2)*Sin
[c + d*x] + 9*a^2*b*Sin[2*(c + d*x)] + a^3*Sin[3*(c + d*x)])/(12*d)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4328, 3042, 4535, 3042, 3117, 4533, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{4328}$$

$$\frac{1}{3}\int \cos^2(c+dx)\left(7ba^2+\left(2a^2+9b^2\right)\sec(c+dx)a+b\left(a^2+3b^2\right)\sec^2(c+dx)\right)dx+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{7ba^2+\left(2a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))}{3d}$$

$$\Big\downarrow \text{4535}$$

$$\frac{1}{3}\left(a\left(2a^2+9b^2\right)\int \cos(c+dx)dx+\int \cos^2(c+dx)\left(7ba^2+b\left(a^2+3b^2\right)\sec^2(c+dx)\right)dx\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(a(2a^2+9b^2)\int \sin\left(c+dx+\frac{\pi}{2}\right)dx+\int \frac{7ba^2+b(a^2+3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))}{3d}$$

$$\Big\downarrow \text{3117}$$

$$\frac{1}{3}\left(\int \frac{7ba^2+b(a^2+3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a(2a^2+9b^2)\sin(c+dx)}{d}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))}{3d}$$

$$\Big\downarrow \text{4533}$$

$$\frac{1}{3}\left(\frac{3}{2}b(3a^2+2b^2)\int 1dx+\frac{a(2a^2+9b^2)\sin(c+dx)}{d}+\frac{7a^2b\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))}{3d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{3}\left(\frac{a(2a^2+9b^2)\sin(c+dx)}{d}+\frac{3}{2}bx(3a^2+2b^2)+\frac{7a^2b\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))}{3d}$$

input
```
Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
```

output
```
(a^2*Cos[c + d*x]^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(3*d) + ((3*b*(3*a^
2 + 2*b^2)*x)/2 + (a*(2*a^2 + 9*b^2)*Sin[c + d*x])/d + (7*a^2*b*Cos[c + d*
x]*Sin[c + d*x])/(2*d))/3
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.67

method	result
parallelrisch	$\frac{9a^2b\sin(2dx+2c)+a^3\sin(3dx+3c)+9(a^3+4a\,b^2)\sin(dx+c)+18\left(a^2+\frac{2b^2}{3}\right)xdb}{12d}$
derivativedivides	$\frac{\frac{a^3\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+3a^2b\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a\,b^2\sin(dx+c)+b^3(dx+c)}{d}$
default	$\frac{\frac{a^3\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+3a^2b\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a\,b^2\sin(dx+c)+b^3(dx+c)}{d}$
risch	$\frac{3a^2bx}{2}+b^3x+\frac{3a^3\sin(dx+c)}{4d}+\frac{3\sin(dx+c)a\,b^2}{d}+\frac{a^3\sin(3dx+3c)}{12d}+\frac{3a^2b\sin(2dx+2c)}{4d}$
norman	$(\frac{3}{2}a^2b+b^3)x+(\frac{3}{2}a^2b+b^3)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+(\frac{3}{2}a^2b+b^3)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+(\frac{3}{2}a^2b+b^3)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(-3a^2b-2b^3)x\,t$

input
```
int(cos(d*x+c)^3*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*(9*a^2*b*sin(2*d*x+2*c)+a^3*sin(3*d*x+3*c)+9*(a^3+4*a*b^2)*sin(d*x+c)
+18*(a^2+2/3*b^2)*x*d*b)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.66

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{3\left(3\,a^2b+2\,b^3\right)dx+\left(2\,a^3\cos\left(dx+c\right)^2+9\,a^2b\cos\left(dx+c\right)+4\,a^3+18\,ab^2\right)\sin\left(dx+c\right)}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/6*(3*(3*a^2*b + 2*b^3)*d*x + (2*a^3*cos(d*x + c)^2 + 9*a^2*b*cos(d*x + c
) + 4*a^3 + 18*a*b^2)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^3\,dx=\int (a+b\sec{(c+dx)})^3\cos^3{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**3,x)
```

output
```
Integral((a + b*sec(c + d*x))**3*cos(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.73

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^3\,dx = -\frac{4\left(\sin{(dx+c)}^3-3\sin{(dx+c)}\right)a^3-9\left(2\,dx+2\,c+\sin{(2\,dx+2\,c)}\right)a^2b-12\,(dx+c)b^3-36\,ab^2\sin{(}}{12\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 - 9*(2*d*x + 2*c + sin(2*d*
x + 2*c))*a^2*b - 12*(d*x + c)*b^3 - 36*a*b^2*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.70

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{3\left(3\,a^2b+2\,b^3\right)(dx+c)+\frac{2\left(6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-9\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+18\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+36\,ab^2\tan\left(\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/6*(3*(3*a^2*b + 2*b^3)*(d*x + c) + 2*(6*a^3*tan(1/2*d*x + 1/2*c)^5 - 9*a
^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 4*a^3*tan(
1/2*d*x + 1/2*c)^3 + 36*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^3*tan(1/2*d*x +
 1/2*c) + 9*a^2*b*tan(1/2*d*x + 1/2*c) + 18*a*b^2*tan(1/2*d*x + 1/2*c))/(t
an(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 10.64 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.77

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^3\,dx = b^3\,x+\frac{3\,a^3\,\sin\left(c+d\,x\right)}{4\,d}+\frac{a^3\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{3\,a^2\,b\,x}{2}+\frac{3\,a^2\,b\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{3\,a\,b^2\,\sin\left(c+d\,x\right)}{d}$$

input
```
int(cos(c + d*x)^3*(a + b/cos(c + d*x))^3,x)
```

output
```
b^3*x + (3*a^3*sin(c + d*x))/(4*d) + (a^3*sin(3*c + 3*d*x))/(12*d) + (3*a^
2*b*x)/2 + (3*a^2*b*sin(2*c + 2*d*x))/(4*d) + (3*a*b^2*sin(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.75

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{9\cos\left(dx+c\right)\sin\left(dx+c\right)a^2b-2\sin\left(dx+c\right)^3a^3+6\sin\left(dx+c\right)a^3+18\sin\left(dx+c\right)a\,b^2+9a^2bdx+6b}{6d}$$

input
```
int(cos(d*x+c)^3*(a+b*sec(d*x+c))^3,x)
```

output
```
(9*cos(c + d*x)*sin(c + d*x)*a**2*b - 2*sin(c + d*x)**3*a**3 + 6*sin(c + d
*x)*a**3 + 18*sin(c + d*x)*a*b**2 + 9*a**2*b*d*x + 6*b**3*d*x)/(6*d)
```

3.473 $\int \cos^4(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\begin{aligned}\int \cos^4(c+dx)(a+b\sec(c+dx))^3\,dx &= \frac{3}{8}a\left(a^2+4b^2\right)x+\frac{b(11a^2+4b^2)\sin(c+dx)}{4d}\\ &\quad+\frac{3a(a^2+4b^2)\cos(c+dx)\sin(c+dx)}{8d}\\ &\quad+\frac{a^2\cos^3(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{4d}\\ &\quad-\frac{3a^2b\sin^3(c+dx)}{4d}\end{aligned}$$

output
```
3/8*a*(a^2+4*b^2)*x+1/4*b*(11*a^2+4*b^2)*sin(d*x+c)/d+3/8*a*(a^2+4*b^2)*co
s(d*x+c)*sin(d*x+c)/d+1/4*a^2*cos(d*x+c)^3*(a+b*sec(d*x+c))*sin(d*x+c)/d-3
/4*a^2*b*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.81

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$=\frac{8b(9a^2+4b^2)\sin(c+dx)+a(12a^2c+48b^2c+12a^2dx+48b^2dx+8(a^2+3b^2)\sin(2(c+dx))+8ab\sin(}{32d}$$

input
```
Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3,x]
```

output
```
(8*b*(9*a^2 + 4*b^2)*Sin[c + d*x] + a*(12*a^2*c + 48*b^2*c + 12*a^2*d*x +
48*b^2*d*x + 8*(a^2 + 3*b^2)*Sin[2*(c + d*x)] + 8*a*b*Sin[3*(c + d*x)] + a
^2*Sin[4*(c + d*x)]))/(32*d)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4328, 3042, 4535, 3042, 3115, 24, 4532, 3042, 3492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{4328}$$

$$\frac{1}{4}\int \cos^3(c+dx)\left(9ba^2+3\left(a^2+4b^2\right)\sec(c+dx)a+2b\left(a^2+2b^2\right)\sec^2(c+dx)\right)dx+\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{9ba^2+3\left(a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+2b\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))}{4d}$$

$$\Big\downarrow \text{4535}$$

$$\frac{1}{4}\left(3a\left(a^2+4b^2\right)\int \cos^2(c+dx)dx+\int \cos^3(c+dx)\left(9ba^2+2b\left(a^2+2b^2\right)\sec^2(c+dx)\right)dx\right)+\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(3a\left(a^2+4b^2\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\int \frac{9ba^2+2b\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx\right)+\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))}{4d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{4}\left(\int \frac{9ba^2+2b\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+3a\left(a^2+4b^2\right)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)\right)+\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))}{4d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{4}\left(\int \frac{9ba^2+2b\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+3a\left(a^2+4b^2\right)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)+\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))}{4d}$$

$$\Big\downarrow \text{4532}$$

$$\frac{1}{4}\left(\int \cos(c+dx)\left(9a^2b\cos^2(c+dx)+2b\left(a^2+2b^2\right)\right)dx+3a\left(a^2+4b^2\right)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)+\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(9a^2b\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b(a^2+2b^2)\right)dx+3a(a^2+4b^2)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))}{4d}$$

$$\Big\downarrow \text{3492}$$

$$\frac{1}{4}\left(3a(a^2+4b^2)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{\int (b(11a^2+4b^2)-9a^2b\sin^2(c+dx))\,d(-\sin(c+dx))}{d}\right)+$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))}{4d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(3a(a^2+4b^2)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{3a^2b\sin^3(c+dx)-b(11a^2+4b^2)\sin(c+dx)}{d}\right)+$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))}{4d}$$

input
```
Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3,x]
```

output
```
(a^2*Cos[c + d*x]^3*(a + b*Sec[c + d*x])*Sin[c + d*x])/(4*d) + (3*a*(a^2 +
 4*b^2)*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (-(b*(11*a^2 + 4*b^2)*
Sin[c + d*x]) + 3*a^2*b*Sin[c + d*x]^3)/d)/4
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4532
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.73

method	result
parallelrisch	$\frac{8(a^3+3a\,b^2)\sin(2dx+2c)+8a^2b\sin(3dx+3c)+a^3\sin(4dx+4c)+8(9a^2b+4b^3)\sin(dx+c)+12a(a^2+4b^2)xd}{32d}$
derivativedivides	$\frac{a^3\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a^2b\left(2+\cos(dx+c)^2\right)\sin(dx+c)+3a\,b^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}\right.}{d}$
default	$\frac{a^3\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a^2b\left(2+\cos(dx+c)^2\right)\sin(dx+c)+3a\,b^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}\right.}{d}$
risch	$\frac{3a^3x}{8}+\frac{3a\,b^2x}{2}+\frac{9a^2b\sin(dx+c)}{4d}+\frac{\sin(dx+c)b^3}{d}+\frac{a^3\sin(4dx+4c)}{32d}+\frac{a^2b\sin(3dx+3c)}{4d}+\frac{a^3\sin(2dx+2c)}{4d}+$
norman	$\frac{(\frac{3}{8}a^3+\frac{3}{2}a\,b^2)x+(-\frac{3}{2}a^3-6a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+(-\frac{3}{8}a^3-\frac{3}{2}a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+(-\frac{3}{8}a^3-\frac{3}{2}a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+(\frac{3}{4}}{}$

input
```
int(cos(d*x+c)^4*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/32*(8*(a^3+3*a*b^2)*sin(2*d*x+2*c)+8*a^2*b*sin(3*d*x+3*c)+a^3*sin(4*d*x+
4*c)+8*(9*a^2*b+4*b^3)*sin(d*x+c)+12*a*(a^2+4*b^2)*x*d)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.68

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$=\frac{3\left(a^3+4\,ab^2\right)dx+\left(2\,a^3\cos\left(dx+c\right)^3+8\,a^2b\cos\left(dx+c\right)^2+16\,a^2b+8\,b^3+3\left(a^3+4\,ab^2\right)\cos\left(dx+c\right)\right)\mathrm{s}}{8\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/8*(3*(a^3 + 4*a*b^2)*d*x + (2*a^3*cos(d*x + c)^3 + 8*a^2*b*cos(d*x + c)^
2 + 16*a^2*b + 8*b^3 + 3*(a^3 + 4*a*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.77

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^3-32\,(\sin(dx+c)^3-3\,\sin(dx+c))a^2b+24\,(2\,d\cdots}{32\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/32*((12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3 - 32*(si
n(d*x + c)^3 - 3*sin(d*x + c))*a^2*b + 24*(2*d*x + 2*c + sin(2*d*x + 2*c))
*a*b^2 + 32*b^3*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. 2(113) = 226.

Time = 0.14 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.41

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{3\,(a^3+4\,ab^2)(dx+c)-\frac{2\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-24\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+12\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-8\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-3\,a^3\tan\left(\frac{1}{2}\cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/8*(3*(a^3 + 4*a*b^2)*(d*x + c) - 2*(5*a^3*tan(1/2*d*x + 1/2*c)^7 - 24*a^
2*b*tan(1/2*d*x + 1/2*c)^7 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 8*b^3*tan(1
/2*d*x + 1/2*c)^7 - 3*a^3*tan(1/2*d*x + 1/2*c)^5 - 40*a^2*b*tan(1/2*d*x +
1/2*c)^5 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 24*b^3*tan(1/2*d*x + 1/2*c)^5
 + 3*a^3*tan(1/2*d*x + 1/2*c)^3 - 40*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 12*a*b
^2*tan(1/2*d*x + 1/2*c)^3 - 24*b^3*tan(1/2*d*x + 1/2*c)^3 - 5*a^3*tan(1/2*
d*x + 1/2*c) - 24*a^2*b*tan(1/2*d*x + 1/2*c) - 12*a*b^2*tan(1/2*d*x + 1/2*
c) - 8*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 13.21 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.03

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{\left(-\frac{5a^3}{4}+6a^2b-3ab^2+2b^3\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(\frac{3a^3}{4}+10a^2b-3ab^2+6b^3\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(-\frac{3a^3}{4}+\cdots\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\cdots\right.\right.} + \frac{3a\,\mathrm{atan}\left(\frac{3a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^2+4b^2)}{4\left(\frac{3a^3}{4}+3ab^2\right)}\right)(a^2+4b^2)}{4d}$$

input
```
int(cos(c + d*x)^4*(a + b/cos(c + d*x))^3,x)
```

output
```
(tan(c/2 + (d*x)/2)^3*(3*a*b^2 + 10*a^2*b - (3*a^3)/4 + 6*b^3) - tan(c/2 +
 (d*x)/2)^7*(3*a*b^2 - 6*a^2*b + (5*a^3)/4 - 2*b^3) + tan(c/2 + (d*x)/2)^5
*(10*a^2*b - 3*a*b^2 + (3*a^3)/4 + 6*b^3) + tan(c/2 + (d*x)/2)*(3*a*b^2 +
6*a^2*b + (5*a^3)/4 + 2*b^3))/(d*(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*
x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (3*a*atan(
(3*a*tan(c/2 + (d*x)/2)*(a^2 + 4*b^2))/(4*(3*a*b^2 + (3*a^3)/4)))*(a^2 + 4
*b^2))/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.91

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{-2\cos(dx+c)\sin(dx+c)^3a^3+5\cos(dx+c)\sin(dx+c)a^3+12\cos(dx+c)\sin(dx+c)a\,b^2-8\sin}{8d}$$

input
```
int(cos(d*x+c)^4*(a+b*sec(d*x+c))^3,x)
```

output
```
( - 2*cos(c + d*x)*sin(c + d*x)**3*a**3 + 5*cos(c + d*x)*sin(c + d*x)*a**3
 + 12*cos(c + d*x)*sin(c + d*x)*a*b**2 - 8*sin(c + d*x)**3*a**2*b + 24*sin
(c + d*x)*a**2*b + 8*sin(c + d*x)*b**3 + 3*a**3*d*x + 12*a*b**2*d*x)/(8*d)
```

3.474 $\int \cos^5(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 160

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{1}{8}b\left(9a^2+4b^2\right)x+\frac{a(4a^2+15b^2)\sin(c+dx)}{5d} + \frac{b(9a^2+4b^2)\cos(c+dx)\sin(c+dx)}{8d} + \frac{11a^2b\cos^3(c+dx)\sin(c+dx)}{20d} + \frac{a^2\cos^4(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{5d} - \frac{a(4a^2+15b^2)\sin^3(c+dx)}{15d}$$

output
```
1/8*b*(9*a^2+4*b^2)*x+1/5*a*(4*a^2+15*b^2)*sin(d*x+c)/d+1/8*b*(9*a^2+4*b^2
)*cos(d*x+c)*sin(d*x+c)/d+11/20*a^2*b*cos(d*x+c)^3*sin(d*x+c)/d+1/5*a^2*co
s(d*x+c)^4*(a+b*sec(d*x+c))*sin(d*x+c)/d-1/15*a*(4*a^2+15*b^2)*sin(d*x+c)^
3/d
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.81

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{540a^2bc+240b^3c+540a^2bdx+240b^3dx+60a(5a^2+18b^2)\sin(c+dx)+120(3a^2b+b^3)\sin(2(c+dx))}{480d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3,x]
```

output
```
(540*a^2*b*c + 240*b^3*c + 540*a^2*b*d*x + 240*b^3*d*x + 60*a*(5*a^2 + 18*
b^2)*Sin[c + d*x] + 120*(3*a^2*b + b^3)*Sin[2*(c + d*x)] + 50*a^3*Sin[3*(c
 + d*x)] + 120*a*b^2*Sin[3*(c + d*x)] + 45*a^2*b*Sin[4*(c + d*x)] + 6*a^3*
Sin[5*(c + d*x)])/(480*d)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.90, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {3042, 4328, 3042, 4535, 3042, 3113, 2009, 4533, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{4328}$$

$$\frac{1}{5}\int \cos^4(c+dx)\left(11ba^2+\left(4a^2+15b^2\right)\sec(c+dx)a+b\left(3a^2+5b^2\right)\sec^2(c+dx)\right)dx+\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{11ba^2+\left(4a^2+15b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(3a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\Big\downarrow \text{4535}$$

$$\frac{1}{5}\left(a\left(4a^2+15b^2\right)\int \cos^3(c+dx)dx+\int \cos^4(c+dx)\left(11ba^2+b\left(3a^2+5b^2\right)\sec^2(c+dx)\right)dx\right)+\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(a\left(4a^2+15b^2\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx+\int \frac{11ba^2+b\left(3a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx\right)+\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\Big\downarrow \text{3113}$$

$$\frac{1}{5}\left(\int \frac{11ba^2+b\left(3a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx-\frac{a\left(4a^2+15b^2\right)\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}\right)+\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}\left(\int \frac{11ba^2+b\left(3a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx-\frac{a\left(4a^2+15b^2\right)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\Big\downarrow \text{4533}$$

$$\frac{1}{5}\left(\frac{5}{4}b\left(9a^2+4b^2\right)\int \cos^2(c+dx)dx-\frac{a\left(4a^2+15b^2\right)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{11a^2b\sin(c+dx)\cos^3(c+}{4d}\right.$$
$$\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{5}{4}b(9a^2+4b^2)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2 dx-\frac{a(4a^2+15b^2)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{11a^2b\sin(c+dx)\text{c}}{4d}\right.$$
$$\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))}{5d}$$

$\Big\downarrow$ 3115

$$\frac{1}{5}\left(\frac{5}{4}b(9a^2+4b^2)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{a(4a^2+15b^2)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{11a^2b\,\text{s}}{}\right.$$
$$\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))}{5d}$$

$\Big\downarrow$ 24

$$\frac{1}{5}\left(-\frac{a(4a^2+15b^2)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{5}{4}b(9a^2+4b^2)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)+\frac{11a^2b\sin}{}\right.$$
$$\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))}{5d}$$

input
```
Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3,x]
```

output
```
(a^2*Cos[c + d*x]^4*(a + b*Sec[c + d*x])*Sin[c + d*x])/(5*d) + ((11*a^2*b*
Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (5*b*(9*a^2 + 4*b^2)*(x/2 + (Cos[c +
d*x]*Sin[c + d*x])/(2*d)))/4 - (a*(4*a^2 + 15*b^2)*(-Sin[c + d*x] + Sin[c
+ d*x]^3/3))/d)/5
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.71

method	result
parallelrisch	$\frac{(360a^2b+120b^3)\sin(2dx+2c)+(50a^3+120a\,b^2)\sin(3dx+3c)+45a^2b\sin(4dx+4c)+6a^3\sin(5dx+5c)+(300a^3+1080a\,b}{480d}$
derivativedivides	$\frac{\frac{a^3\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+3a^2b\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a\,b^2\left(2+\cos(dx+\right.}{d}$
default	$\frac{\frac{a^3\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+3a^2b\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a\,b^2\left(2+\cos(dx+\right.}{d}$
risch	$\frac{9a^2bx}{8}+\frac{b^3x}{2}+\frac{5a^3\sin(dx+c)}{8d}+\frac{9\sin(dx+c)a\,b^2}{4d}+\frac{a^3\sin(5dx+5c)}{80d}+\frac{3a^2b\sin(4dx+4c)}{32d}+\frac{5a^3\sin(3dx+3c)}{48d}$
norman	$\left(\frac{9}{8}a^2b+\frac{1}{2}b^3\right)x+\left(-\frac{45}{8}a^2b-\frac{5}{2}b^3\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+\left(-\frac{45}{8}a^2b-\frac{5}{2}b^3\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+\left(\frac{9}{8}a^2b+\frac{1}{2}b^3\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\left(\frac{9}{8}a^2\right.$

input
```
int(cos(d*x+c)^5*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/480*((360*a^2*b+120*b^3)*sin(2*d*x+2*c)+(50*a^3+120*a*b^2)*sin(3*d*x+3*c
)+45*a^2*b*sin(4*d*x+4*c)+6*a^3*sin(5*d*x+5*c)+(300*a^3+1080*a*b^2)*sin(d*
x+c)+540*x*d*b*(a^2+4/9*b^2))/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.69

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$=\frac{15\,(9\,a^2b+4\,b^3)dx+\left(24\,a^3\cos(dx+c)^4+90\,a^2b\cos(dx+c)^3+64\,a^3+240\,ab^2+8\,(4\,a^3+15\,ab^2)\cos\right.}{120\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/120*(15*(9*a^2*b + 4*b^3)*d*x + (24*a^3*cos(d*x + c)^4 + 90*a^2*b*cos(d*
x + c)^3 + 64*a^3 + 240*a*b^2 + 8*(4*a^3 + 15*a*b^2)*cos(d*x + c)^2 + 15*(
9*a^2*b + 4*b^3)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.74

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^3+45\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\sin(\right.}{480\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^3 + 4
5*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2*b - 480*(sin
(d*x + c)^3 - 3*sin(d*x + c))*a*b^2 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))
*b^3)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(148) = 296$.

Time = 0.16 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.08

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{15\,(9\,a^2b+4\,b^3)(dx+c) + \frac{2\left(120\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-225\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+360\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-60\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+16\cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/120*(15*(9*a^2*b + 4*b^3)*(d*x + c) + 2*(120*a^3*tan(1/2*d*x + 1/2*c)^9
- 225*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 360*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 60
*b^3*tan(1/2*d*x + 1/2*c)^9 + 160*a^3*tan(1/2*d*x + 1/2*c)^7 - 90*a^2*b*ta
n(1/2*d*x + 1/2*c)^7 + 960*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 120*b^3*tan(1/2*
d*x + 1/2*c)^7 + 464*a^3*tan(1/2*d*x + 1/2*c)^5 + 1200*a*b^2*tan(1/2*d*x +
 1/2*c)^5 + 160*a^3*tan(1/2*d*x + 1/2*c)^3 + 90*a^2*b*tan(1/2*d*x + 1/2*c)
^3 + 960*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*b^3*tan(1/2*d*x + 1/2*c)^3 + 1
20*a^3*tan(1/2*d*x + 1/2*c) + 225*a^2*b*tan(1/2*d*x + 1/2*c) + 360*a*b^2*t
an(1/2*d*x + 1/2*c) + 60*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
 + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 14.17 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.79

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{\left(2\,a^3-\frac{15\,a^2\,b}{4}+6\,a\,b^2-b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{8\,a^3}{3}-\frac{3\,a^2\,b}{2}+16\,a\,b^2-2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{116\,a^3}{15}+20\cdots\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,\tan\left(\cdots\right.\right.}$$
$$+\frac{b\,\mathrm{atan}\left(\frac{b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(9\,a^2+4\,b^2\right)}{4\,\left(\frac{9\,a^2\,b}{4}+b^3\right)}\right)\,\left(9\,a^2+4\,b^2\right)}{4\,d}$$

input
```
int(cos(c + d*x)^5*(a + b/cos(c + d*x))^3,x)
```

output
```
(tan(c/2 + (d*x)/2)^3*(16*a*b^2 + (3*a^2*b)/2 + (8*a^3)/3 + 2*b^3) + tan(c
/2 + (d*x)/2)^9*(6*a*b^2 - (15*a^2*b)/4 + 2*a^3 - b^3) + tan(c/2 + (d*x)/2
)^7*(16*a*b^2 - (3*a^2*b)/2 + (8*a^3)/3 - 2*b^3) + tan(c/2 + (d*x)/2)*(6*a
*b^2 + (15*a^2*b)/4 + 2*a^3 + b^3) + tan(c/2 + (d*x)/2)^5*(20*a*b^2 + (116
*a^3)/15))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c
/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) + (
b*atan((b*tan(c/2 + (d*x)/2)*(9*a^2 + 4*b^2))/(4*((9*a^2*b)/4 + b^3)))*(9*
a^2 + 4*b^2))/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.87

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{-90\cos(dx+c)\sin(dx+c)^3 a^2b + 225\cos(dx+c)\sin(dx+c)a^2b + 60\cos(dx+c)\sin(dx+c)b^3 + 2}{}$$

input
```
int(cos(d*x+c)^5*(a+b*sec(d*x+c))^3,x)
```

output
```
( - 90*cos(c + d*x)*sin(c + d*x)**3*a**2*b + 225*cos(c + d*x)*sin(c + d*x)
*a**2*b + 60*cos(c + d*x)*sin(c + d*x)*b**3 + 24*sin(c + d*x)**5*a**3 - 80
*sin(c + d*x)**3*a**3 - 120*sin(c + d*x)**3*a*b**2 + 120*sin(c + d*x)*a**3
 + 360*sin(c + d*x)*a*b**2 + 135*a**2*b*d*x + 60*b**3*d*x)/(120*d)
```

3.475 $\int \cos^6(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 185

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{1}{16}a\left(5a^2+18b^2\right)x+\frac{b(17a^2+6b^2)\sin(c+dx)}{6d}+\frac{a(5a^2+18b^2)\cos(c+dx)\sin(c+dx)}{16d}+\frac{a(5a^2+18b^2)\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{a^2\cos^5(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{6d}-\frac{b(5a^2+b^2)\sin^3(c+dx)}{3d}+\frac{13a^2b\sin^5(c+dx)}{30d}$$

output
```
1/16*a*(5*a^2+18*b^2)*x+1/6*b*(17*a^2+6*b^2)*sin(d*x+c)/d+1/16*a*(5*a^2+18
*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/24*a*(5*a^2+18*b^2)*cos(d*x+c)^3*sin(d*x+c
)/d+1/6*a^2*cos(d*x+c)^5*(a+b*sec(d*x+c))*sin(d*x+c)/d-1/3*b*(5*a^2+b^2)*s
in(d*x+c)^3/d+13/30*a^2*b*sin(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.86

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{300a^3c+1080ab^2c+300a^3dx+1080ab^2dx+360b(5a^2+2b^2)\sin(c+dx)+45(5a^3+16ab^2)\sin(2(c+}{}$$

input
```
Integrate[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^3,x]
```

output
```
(300*a^3*c + 1080*a*b^2*c + 300*a^3*d*x + 1080*a*b^2*d*x + 360*b*(5*a^2 +
2*b^2)*Sin[c + d*x] + 45*(5*a^3 + 16*a*b^2)*Sin[2*(c + d*x)] + 300*a^2*b*S
in[3*(c + d*x)] + 80*b^3*Sin[3*(c + d*x)] + 45*a^3*Sin[4*(c + d*x)] + 90*a
*b^2*Sin[4*(c + d*x)] + 36*a^2*b*Sin[5*(c + d*x)] + 5*a^3*Sin[6*(c + d*x)]
)/(960*d)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.89, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4328, 3042, 4535, 3042, 3115, 3042, 3115, 24, 4532, 3042, 3492, 27, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{4328}$$

$$\frac{1}{6}\int \cos^5(c+dx)\left(13ba^2+\left(5a^2+18b^2\right)\sec(c+dx)a+2b\left(2a^2+3b^2\right)\sec^2(c+dx)\right)dx+\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

$\Big\downarrow$ 3042

$$\frac{1}{6}\int \frac{13ba^2+\left(5a^2+18b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+2b\left(2a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

$\Big\downarrow$ 4535

$$\frac{1}{6}\left(a\left(5a^2+18b^2\right)\int \cos^4(c+dx)dx+\int \cos^5(c+dx)\left(13ba^2+2b\left(2a^2+3b^2\right)\sec^2(c+dx)\right)dx\right)+\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

$\Big\downarrow$ 3042

$$\frac{1}{6}\left(a\left(5a^2+18b^2\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx+\int \frac{13ba^2+2b\left(2a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx\right)+\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

$\Big\downarrow$ 3115

$$\frac{1}{6}\left(\int \frac{13ba^2+2b\left(2a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+a\left(5a^2+18b^2\right)\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+}{4d}\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

$\Big\downarrow$ 3042

$$\frac{1}{6}\left(\int \frac{13ba^2+2b\left(2a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+a\left(5a^2+18b^2\right)\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\text{co}}{4d}\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

$\Big\downarrow$ 3115

$$\frac{1}{6}\left(\int \frac{13ba^2+2b(2a^2+3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+a(5a^2+18b^2)\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin}{}\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

$\downarrow$ 24

$$\frac{1}{6}\left(\int \frac{13ba^2+2b(2a^2+3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+a(5a^2+18b^2)\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\,c}{2d}\right.\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

$\downarrow$ 4532

$$\frac{1}{6}\left(\int \cos^3(c+dx)\left(13a^2b\cos^2(c+dx)+2b(2a^2+3b^2)\right)dx+a(5a^2+18b^2)\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin}{}\right.\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(13a^2b\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b(2a^2+3b^2)\right)dx+a(5a^2+18b^2)\left(\frac{\sin(c+dx)\cos^3(c+}{4d}\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

$\downarrow$ 3492

$$\frac{1}{6}\left(a(5a^2+18b^2)\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{\int b(1-\sin^2(c+dx))\,(-13}{}\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

$\downarrow$ 27

$$\frac{1}{6}\left(a(5a^2+18b^2)\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{b\int(1-\sin^2(c+dx))\,(-13}{}\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

$\downarrow$ 290

$$\frac{1}{6}\left(a(5a^2+18b^2)\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{b\int\left(13a^2\sin^4(c+dx)-6\right.}{}\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}\left(a(5a^2+18b^2)\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{b(2(5a^2+b^2)\sin^3(c+dx)}{}\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

input
```
Int[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^3,x]
```

output
```
(a^2*Cos[c + d*x]^5*(a + b*Sec[c + d*x])*Sin[c + d*x])/(6*d) + (-((b*(-((1
7*a^2 + 6*b^2)*Sin[c + d*x]) + 2*(5*a^2 + b^2)*Sin[c + d*x]^3 - (13*a^2*Si
n[c + d*x]^5)/5))/d) + a*(5*a^2 + 18*b^2)*((Cos[c + d*x]^3*Sin[c + d*x])/(
4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/6
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4532
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.73

method	result
parallelrisch	$\frac{(225a^3+720a\,b^2)\sin(2dx+2c)+(300a^2b+80b^3)\sin(3dx+3c)+(45a^3+90a\,b^2)\sin(4dx+4c)+36a^2b\sin(5dx+5c)+5a^3\,\mathrm{s}}{960d}$
derivativedivides	$\frac{a^3\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{3a^2b\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+3c}{d}$
default	$\frac{a^3\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{3a^2b\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+3c}{d}$
risch	$\frac{5a^3x}{16}+\frac{9a\,b^2x}{8}+\frac{15a^2b\sin(dx+c)}{8d}+\frac{3\sin(dx+c)b^3}{4d}+\frac{a^3\sin(6dx+6c)}{192d}+\frac{3a^2b\sin(5dx+5c)}{80d}+\frac{3a^3\sin(4dx+4c}{64d}$
norman	$\frac{(\frac{5}{16}a^3+\frac{9}{8}a\,b^2)x+(-\frac{25}{8}a^3-\frac{45}{4}a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+(-\frac{5}{4}a^3-\frac{9}{2}a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+(-\frac{5}{4}a^3-\frac{9}{2}a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{}$

input
```
int(cos(d*x+c)^6*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/960*((225*a^3+720*a*b^2)*sin(2*d*x+2*c)+(300*a^2*b+80*b^3)*sin(3*d*x+3*c
)+(45*a^3+90*a*b^2)*sin(4*d*x+4*c)+36*a^2*b*sin(5*d*x+5*c)+5*a^3*sin(6*d*x
+6*c)+(1800*a^2*b+720*b^3)*sin(d*x+c)+300*x*a*(a^2+18/5*b^2)*d)/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.71

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$=\frac{15\,(5\,a^3+18\,ab^2)dx+\left(40\,a^3\cos\left(dx+c\right)^5+144\,a^2b\cos\left(dx+c\right)^4+10\,(5\,a^3+18\,ab^2)\cos\left(dx+c\right)^3+38\right.}{240\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/240*(15*(5*a^3 + 18*a*b^2)*d*x + (40*a^3*cos(d*x + c)^5 + 144*a^2*b*cos(
d*x + c)^4 + 10*(5*a^3 + 18*a*b^2)*cos(d*x + c)^3 + 384*a^2*b + 160*b^3 +
16*(12*a^2*b + 5*b^3)*cos(d*x + c)^2 + 15*(5*a^3 + 18*a*b^2)*cos(d*x + c))
*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**6*(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.78

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^3\,dx =$$
$$-\frac{5\left(4\sin\left(2\,dx+2\,c\right)^3-60\,dx-60\,c-9\sin\left(4\,dx+4\,c\right)-48\sin\left(2\,dx+2\,c\right)\right)a^3-192\left(3\sin\left(dx+c\right)\right.}{}$$

input
```
integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/960*(5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*
sin(2*d*x + 2*c))*a^3 - 192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin
(d*x + c))*a^2*b - 90*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*
c))*a*b^2 + 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*b^3)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(171) = 342$.

Time = 0.16 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.33

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{15\,(5\,a^3+18\,ab^2)(dx+c) - \frac{2\left(165\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} - 720\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} + 450\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} - 240\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}\right.}{}}{}$$

input
```
integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/240*(15*(5*a^3 + 18*a*b^2)*(d*x + c) - 2*(165*a^3*tan(1/2*d*x + 1/2*c)^1
1 - 720*a^2*b*tan(1/2*d*x + 1/2*c)^11 + 450*a*b^2*tan(1/2*d*x + 1/2*c)^11
- 240*b^3*tan(1/2*d*x + 1/2*c)^11 - 25*a^3*tan(1/2*d*x + 1/2*c)^9 - 1680*a
^2*b*tan(1/2*d*x + 1/2*c)^9 + 630*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 880*b^3*t
an(1/2*d*x + 1/2*c)^9 + 450*a^3*tan(1/2*d*x + 1/2*c)^7 - 3744*a^2*b*tan(1/
2*d*x + 1/2*c)^7 + 180*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 1440*b^3*tan(1/2*d*x
 + 1/2*c)^7 - 450*a^3*tan(1/2*d*x + 1/2*c)^5 - 3744*a^2*b*tan(1/2*d*x + 1/
2*c)^5 - 180*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 1440*b^3*tan(1/2*d*x + 1/2*c)^
5 + 25*a^3*tan(1/2*d*x + 1/2*c)^3 - 1680*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 63
0*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 880*b^3*tan(1/2*d*x + 1/2*c)^3 - 165*a^3*
tan(1/2*d*x + 1/2*c) - 720*a^2*b*tan(1/2*d*x + 1/2*c) - 450*a*b^2*tan(1/2*
d*x + 1/2*c) - 240*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^
6)/d
```

Mupad [B] (verification not implemented)

Time = 13.45 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.89

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{\left(-\frac{11\,a^3}{8}+6\,a^2\,b-\frac{15\,a\,b^2}{4}+2\,b^3\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{5\,a^3}{24}+14\,a^2\,b-\frac{21\,a\,b^2}{4}+\frac{22\,b^3}{3}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(-\frac{15\,a}{4}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}\right.}$$
$$+ \frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(5\,a^2+18\,b^2)}{8\,\left(\frac{5\,a^3}{8}+\frac{9\,a\,b^2}{4}\right)}\right)\,(5\,a^2+18\,b^2)}{8\,d}$$

input
```
int(cos(c + d*x)^6*(a + b/cos(c + d*x))^3,x)
```

output
```
(tan(c/2 + (d*x)/2)^3*((21*a*b^2)/4 + 14*a^2*b - (5*a^3)/24 + (22*b^3)/3)
- tan(c/2 + (d*x)/2)^11*((15*a*b^2)/4 - 6*a^2*b + (11*a^3)/8 - 2*b^3) + ta
n(c/2 + (d*x)/2)^9*(14*a^2*b - (21*a*b^2)/4 + (5*a^3)/24 + (22*b^3)/3) + t
an(c/2 + (d*x)/2)^5*((3*a*b^2)/2 + (156*a^2*b)/5 + (15*a^3)/4 + 12*b^3) -
tan(c/2 + (d*x)/2)^7*((3*a*b^2)/2 - (156*a^2*b)/5 + (15*a^3)/4 - 12*b^3) +
 tan(c/2 + (d*x)/2)*((15*a*b^2)/4 + 6*a^2*b + (11*a^3)/8 + 2*b^3))/(d*(6*t
an(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 +
15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12
+ 1)) + (a*atan((a*tan(c/2 + (d*x)/2)*(5*a^2 + 18*b^2))/(8*((9*a*b^2)/4 +
(5*a^3)/8)))*(5*a^2 + 18*b^2))/(8*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.96

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{40\cos(dx+c)\sin(dx+c)^5 a^3 - 130\cos(dx+c)\sin(dx+c)^3 a^3 - 180\cos(dx+c)\sin(dx+c)^3 a b^2 + \cdots}{}$$

input
```
int(cos(d*x+c)^6*(a+b*sec(d*x+c))^3,x)
```

output
```
(40*cos(c + d*x)*sin(c + d*x)**5*a**3 - 130*cos(c + d*x)*sin(c + d*x)**3*a
**3 - 180*cos(c + d*x)*sin(c + d*x)**3*a*b**2 + 165*cos(c + d*x)*sin(c + d
*x)*a**3 + 450*cos(c + d*x)*sin(c + d*x)*a*b**2 + 144*sin(c + d*x)**5*a**2
*b - 480*sin(c + d*x)**3*a**2*b - 80*sin(c + d*x)**3*b**3 + 720*sin(c + d*
x)*a**2*b + 240*sin(c + d*x)*b**3 + 75*a**3*d*x + 270*a*b**2*d*x)/(240*d)
```

3.476 $\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 244

$$\begin{aligned}
&\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx \\
&= \frac{(8a^4+36a^2b^2+5b^4)\operatorname{arctanh}(\sin(c+dx))}{16d} - \frac{a(4a^4-121a^2b^2-128b^4)\tan(c+dx)}{60bd} \\
&\quad - \frac{(8a^4-178a^2b^2-75b^4)\sec(c+dx)\tan(c+dx)}{240d} \\
&\quad - \frac{a(4a^2-53b^2)(a+b\sec(c+dx))^2\tan(c+dx)}{120bd} \\
&\quad - \frac{(4a^2-25b^2)(a+b\sec(c+dx))^3\tan(c+dx)}{120bd} \\
&\quad - \frac{a(a+b\sec(c+dx))^4\tan(c+dx)}{30bd} + \frac{(a+b\sec(c+dx))^5\tan(c+dx)}{6bd}
\end{aligned}$$

output
```
1/16*(8*a^4+36*a^2*b^2+5*b^4)*arctanh(sin(d*x+c))/d-1/60*a*(4*a^4-121*a^2*
b^2-128*b^4)*tan(d*x+c)/b/d-1/240*(8*a^4-178*a^2*b^2-75*b^4)*sec(d*x+c)*ta
n(d*x+c)/d-1/120*a*(4*a^2-53*b^2)*(a+b*sec(d*x+c))^2*tan(d*x+c)/b/d-1/120*
(4*a^2-25*b^2)*(a+b*sec(d*x+c))^3*tan(d*x+c)/b/d-1/30*a*(a+b*sec(d*x+c))^4
*tan(d*x+c)/b/d+1/6*(a+b*sec(d*x+c))^5*tan(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.63

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{15(8a^4+36a^2b^2+5b^4)\operatorname{arctanh}(\sin(c+dx))+\tan(c+dx)\left(15(8a^4+36a^2b^2+5b^4)\sec(c+dx)+10b^2(3\right.}{}$$

input
```
Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^4,x]
```

output
```
(15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*
(8*a^4 + 36*a^2*b^2 + 5*b^4)*Sec[c + d*x] + 10*b^2*(36*a^2 + 5*b^2)*Sec[c
+ d*x]^3 + 40*b^4*Sec[c + d*x]^5 + 64*a*b*(15*(a^2 + b^2) + 5*(a^2 + 2*b^2
)*Tan[c + d*x]^2 + 3*b^2*Tan[c + d*x]^4)))/(240*d)
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.06, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$, Rules used = {3042, 4327, 3042, 4490, 3042, 4490, 27, 3042, 4490, 3042, 4485, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4dx$$

$$\Big\downarrow \; 4327$$

$$\frac{\int \sec(c+dx)(5b-a\sec(c+dx))(a+b\sec(c+dx))^4dx}{6b}+\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \csc \left(c+dx+\frac{\pi}{2}\right) \left(5b-a \csc \left(c+dx+\frac{\pi}{2}\right)\right) \left(a+b \csc \left(c+dx+\frac{\pi}{2}\right)\right)^4dx}{6b}+\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}$$

$\Big\downarrow$ 4490

$$\frac{\frac{1}{5}\int \sec(c+dx)(a+b\sec(c+dx))^3 \left(21ab-\left(4a^2-25b^2\right)\sec(c+dx)\right)dx-\frac{a\tan(c+dx)(a+b\sec(c+dx))^4}{5d}}{6b}+\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{5}\int \csc \left(c+dx+\frac{\pi}{2}\right) \left(a+b \csc \left(c+dx+\frac{\pi}{2}\right)\right)^3 \left(21ab+\left(25b^2-4a^2\right)\csc \left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{a\tan(c+dx)(a+b\sec(c+d}{5d}}{6b}$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}$$

$\Big\downarrow$ 4490

$$\frac{\frac{1}{5}\left(\frac{1}{4}\int 3\sec(c+dx)(a+b\sec(c+dx))^2 \left(b\left(24a^2+25b^2\right)-a\left(4a^2-53b^2\right)\sec(c+dx)\right)dx-\frac{\left(4a^2-25b^2\right)\tan(c+dx)(a+}{4d}\right.}{6b}$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{5}\left(\frac{3}{4}\int \sec(c+dx)(a+b\sec(c+dx))^2 \left(b\left(24a^2+25b^2\right)-a\left(4a^2-53b^2\right)\sec(c+dx)\right)dx-\frac{\left(4a^2-25b^2\right)\tan(c+dx)(a+b}{4d}\right.}{6b}$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{5}\left(\frac{3}{4}\int \csc \left(c+dx+\frac{\pi}{2}\right) \left(a+b \csc \left(c+dx+\frac{\pi}{2}\right)\right)^2 \left(b\left(24a^2+25b^2\right)-a\left(4a^2-53b^2\right)\csc \left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{(4a^2-}{}\right.}{6b}$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}$$

$\Big\downarrow$ 4490

$$\frac{\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\int \sec(c+dx)(a+b\sec(c+dx))\left(ab(64a^2+181b^2)-(8a^4-178b^2a^2-75b^4)\sec(c+dx)\right)dx-\frac{a(4a^2-53b}{6b}\right.\right.}{\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(ab(64a^2+181b^2)+(-8a^4+178b^2a^2+75b^4)\csc\left(c+dx+\right.\right.\right.\right.}{6b}$$

$$\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}$$

$\Big\downarrow$ 4485

$$\frac{\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \sec(c+dx)\left(15b(8a^4+36b^2a^2+5b^4)-4a(4a^4-121b^2a^2-128b^4)\sec(c+dx)\right)dx-\frac{b(8a^4-178a^2b^2-}{}\right.\right.\right.}{\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(15b(8a^4+36b^2a^2+5b^4)-4a(4a^4-121b^2a^2-128b^4)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{b(8a}{}\right.\right.\right.}{\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}}$$

$\Big\downarrow$ 4274

$$\frac{\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(15b(8a^4+36a^2b^2+5b^4)\int \sec(c+dx)dx-4a(4a^4-121a^2b^2-128b^4)\int \sec^2(c+dx)dx\right)-\frac{b(8a^4-178}{}\right.\right.\right.}{\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(15b(8a^4+36a^2b^2+5b^4)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-4a(4a^4-121a^2b^2-128b^4)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)\right.\right.\right.}{\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}}$$

$\Big\downarrow$ 4254

$$\frac{\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(\frac{4a(4a^4-121a^2b^2-128b^4)\int 1d(-\tan(c+dx))}{d}+15b(8a^4+36a^2b^2+5b^4)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx\right)-\frac{b(8a^4-178a^2b}{}\right.\right.\right.}{}$$

$$\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}$$

$\downarrow$ 24

$$\frac{\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(15b(8a^4+36a^2b^2+5b^4)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{4a(4a^4-121a^2b^2-128b^4)\tan(c+dx)}{d}\right)-\frac{b(8a^4-178a^2b^2-75b^4)}{2d}\right.\right.\right.}{}$$

$$\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}$$

$\downarrow$ 4257

$$\frac{\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(\frac{15b(8a^4+36a^2b^2+5b^4)\text{arctanh}(\sin(c+dx))}{d}-\frac{4a(4a^4-121a^2b^2-128b^4)\tan(c+dx)}{d}\right)-\frac{b(8a^4-178a^2b^2-75b^4)\tan(c+dx)s}{2d}\right.\right.\right.}{6b}$$

$$\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}$$

input
```
Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^4,x]
```

output
```
((a + b*Sec[c + d*x])^5*Tan[c + d*x])/(6*b*d) + (-1/5*(a*(a + b*Sec[c + d*
x])^4*Tan[c + d*x])/d + (-1/4*((4*a^2 - 25*b^2)*(a + b*Sec[c + d*x])^3*Tan
[c + d*x])/d + (3*(-1/3*(a*(4*a^2 - 53*b^2)*(a + b*Sec[c + d*x])^2*Tan[c +
 d*x])/d + (-1/2*(b*(8*a^4 - 178*a^2*b^2 - 75*b^4)*Sec[c + d*x]*Tan[c + d*
x])/d + ((15*b*(8*a^4 + 36*a^2*b^2 + 5*b^4)*ArcTanh[Sin[c + d*x]])/d - (4*
a*(4*a^4 - 121*a^2*b^2 - 128*b^4)*Tan[c + d*x])/d)/2)/3))/4)/5)/(6*b)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4327
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 -
b^2, 0] &&  !LtQ[m, -1]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 3.17 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-4b\,a^3\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+6a^2b^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\,\mathrm{s}}{}\right.\right.}{}$
default	$\frac{a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-4b\,a^3\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+6a^2b^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\,\mathrm{s}}{}\right.\right.}{}$
parts	$\frac{a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{b^4\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5}{}\right.}{d}$
parallelrisch	$\frac{-1800\left(a^4+\frac{9}{2}a^2b^2+\frac{5}{8}b^4\right)\left(\frac{\cos(6dx+6c)}{15}+\frac{2\cos(4dx+4c)}{5}+\cos(2dx+2c)+\frac{2}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+1800\left(a^4+\frac{9}{2}a^2b^2+\frac{5}{8}b^4\right.}{}$
norman	$\frac{\left(8a^4-64b\,a^3+60a^2b^2-64a\,b^3+11b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{8d}+\frac{\left(8a^4+64b\,a^3+60a^2b^2+64a\,b^3+11b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{\left(40a^4-960b\,a^3+60a^2\right.}{}$
risch	$-\frac{i\left(-640b\,a^3+360a^4\mathrm{e}^{9i(dx+c)}+120a^4\mathrm{e}^{11i(dx+c)}+425b^4\mathrm{e}^{9i(dx+c)}-425b^4\mathrm{e}^{3i(dx+c)}-360a^4\mathrm{e}^{3i(dx+c)}-75b^4\mathrm{e}^{i(dx+c)}-1\right.}{}$

input
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-4*b*a^3
*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+6*a^2*b^2*(-(-1/4*sec(d*x+c)^3-3/8*sec
(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-4*a*b^3*(-8/15-1/5*sec(
d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+b^4*(-(-1/6*sec(d*x+c)^5-5/24*sec(d
*x+c)^3-5/16*sec(d*x+c))*tan(d*x+c)+5/16*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.89

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$=\frac{15\left(8a^4+36a^2b^2+5b^4\right)\cos(dx+c)^6\log(\sin(dx+c)+1)-15\left(8a^4+36a^2b^2+5b^4\right)\cos(dx+c)^6\log}{}$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/480*(15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*cos(d*x + c)^6*log(sin(d*x + c) + 1
) - 15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*cos(d*x + c)^6*log(-sin(d*x + c) + 1)
+ 2*(128*(5*a^3*b + 4*a*b^3)*cos(d*x + c)^5 + 192*a*b^3*cos(d*x + c) + 15*
(8*a^4 + 36*a^2*b^2 + 5*b^4)*cos(d*x + c)^4 + 40*b^4 + 64*(5*a^3*b + 4*a*b
^3)*cos(d*x + c)^3 + 10*(36*a^2*b^2 + 5*b^4)*cos(d*x + c)^2)*sin(d*x + c))
/(d*cos(d*x + c)^6)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx = \int (a+b\sec(c+dx))^4\sec^3(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**4,x)
```

output
```
Integral((a + b*sec(c + d*x))**4*sec(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.13

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{640\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^3b+128\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)ab^3}{}$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/480*(640*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3*b + 128*(3*tan(d*x + c)^5
 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a*b^3 - 5*b^4*(2*(15*sin(d*x + c)^
5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^
4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c)
 - 1)) - 180*a^2*b^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^
4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) -
 1)) - 120*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1
) + log(sin(d*x + c) - 1)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(230) = 460$.

Time = 0.19 (sec) , antiderivative size = 592, normalized size of antiderivative = 2.43

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/240*(15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1))
- 15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(
120*a^4*tan(1/2*d*x + 1/2*c)^11 - 960*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 900*
a^2*b^2*tan(1/2*d*x + 1/2*c)^11 - 960*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 165*
b^4*tan(1/2*d*x + 1/2*c)^11 - 360*a^4*tan(1/2*d*x + 1/2*c)^9 + 3520*a^3*b*
tan(1/2*d*x + 1/2*c)^9 - 1260*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 2240*a*b^3*
tan(1/2*d*x + 1/2*c)^9 + 25*b^4*tan(1/2*d*x + 1/2*c)^9 + 240*a^4*tan(1/2*d
*x + 1/2*c)^7 - 5760*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 360*a^2*b^2*tan(1/2*d*
x + 1/2*c)^7 - 4992*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 450*b^4*tan(1/2*d*x + 1
/2*c)^7 + 240*a^4*tan(1/2*d*x + 1/2*c)^5 + 5760*a^3*b*tan(1/2*d*x + 1/2*c)
^5 + 360*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 4992*a*b^3*tan(1/2*d*x + 1/2*c)^
5 + 450*b^4*tan(1/2*d*x + 1/2*c)^5 - 360*a^4*tan(1/2*d*x + 1/2*c)^3 - 3520
*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 1260*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 2240
*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 25*b^4*tan(1/2*d*x + 1/2*c)^3 + 120*a^4*ta
n(1/2*d*x + 1/2*c) + 960*a^3*b*tan(1/2*d*x + 1/2*c) + 900*a^2*b^2*tan(1/2*
d*x + 1/2*c) + 960*a*b^3*tan(1/2*d*x + 1/2*c) + 165*b^4*tan(1/2*d*x + 1/2*
c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

Mupad [B] (verification not implemented)

Time = 14.87 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.52

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(a^4+\frac{9a^2b^2}{2}+\frac{5b^4}{8}\right)}{d} + \frac{\left(a^4-8a^3b+\frac{15a^2b^2}{2}-8ab^3+\frac{11b^4}{8}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+\left(-3a^4+\frac{88a^3b}{3}-\frac{21a^2b^2}{2}+\frac{56ab^3}{3}+\frac{5b^4}{24}\right)\tan\left(\frac{c}{2}\right.}{}$$

input
```
int((a + b/cos(c + d*x))^4/cos(c + d*x)^3,x)
```

output
```
(atanh(tan(c/2 + (d*x)/2))*(a^4 + (5*b^4)/8 + (9*a^2*b^2)/2))/d + (tan(c/2
 + (d*x)/2)^9*((56*a*b^3)/3 + (88*a^3*b)/3 - 3*a^4 + (5*b^4)/24 - (21*a^2*
b^2)/2) - tan(c/2 + (d*x)/2)^3*((56*a*b^3)/3 + (88*a^3*b)/3 + 3*a^4 - (5*b
^4)/24 + (21*a^2*b^2)/2) + tan(c/2 + (d*x)/2)^5*((208*a*b^3)/5 + 48*a^3*b
+ 2*a^4 + (15*b^4)/4 + 3*a^2*b^2) + tan(c/2 + (d*x)/2)^7*(2*a^4 - 48*a^3*b
 - (208*a*b^3)/5 + (15*b^4)/4 + 3*a^2*b^2) + tan(c/2 + (d*x)/2)*(8*a*b^3 +
 8*a^3*b + a^4 + (11*b^4)/8 + (15*a^2*b^2)/2) + tan(c/2 + (d*x)/2)^11*(a^4
 - 8*a^3*b - 8*a*b^3 + (11*b^4)/8 + (15*a^2*b^2)/2))/(d*(15*tan(c/2 + (d*x
)/2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (
d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 850, normalized size of antiderivative = 3.48

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c))^4,x)
```

output
```
( - 640*cos(c + d*x)*sin(c + d*x)**5*a**3*b - 512*cos(c + d*x)*sin(c + d*x
)**5*a*b**3 + 1600*cos(c + d*x)*sin(c + d*x)**3*a**3*b + 1280*cos(c + d*x)
*sin(c + d*x)**3*a*b**3 - 960*cos(c + d*x)*sin(c + d*x)*a**3*b - 960*cos(c
 + d*x)*sin(c + d*x)*a*b**3 - 120*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**
6*a**4 - 540*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**6*a**2*b**2 - 75*log(
tan((c + d*x)/2) - 1)*sin(c + d*x)**6*b**4 + 360*log(tan((c + d*x)/2) - 1)
*sin(c + d*x)**4*a**4 + 1620*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a**
2*b**2 + 225*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*b**4 - 360*log(tan(
(c + d*x)/2) - 1)*sin(c + d*x)**2*a**4 - 1620*log(tan((c + d*x)/2) - 1)*si
n(c + d*x)**2*a**2*b**2 - 225*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b*
*4 + 120*log(tan((c + d*x)/2) - 1)*a**4 + 540*log(tan((c + d*x)/2) - 1)*a*
*2*b**2 + 75*log(tan((c + d*x)/2) - 1)*b**4 + 120*log(tan((c + d*x)/2) + 1
)*sin(c + d*x)**6*a**4 + 540*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**6*a**
2*b**2 + 75*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**6*b**4 - 360*log(tan((
c + d*x)/2) + 1)*sin(c + d*x)**4*a**4 - 1620*log(tan((c + d*x)/2) + 1)*sin
(c + d*x)**4*a**2*b**2 - 225*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*b**
4 + 360*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**4 + 1620*log(tan((c +
 d*x)/2) + 1)*sin(c + d*x)**2*a**2*b**2 + 225*log(tan((c + d*x)/2) + 1)*si
n(c + d*x)**2*b**4 - 120*log(tan((c + d*x)/2) + 1)*a**4 - 540*log(tan((c +
 d*x)/2) + 1)*a**2*b**2 - 75*log(tan((c + d*x)/2) + 1)*b**4 - 120*sin(c...
```

3.477 $\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 179

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{ab(4a^2+3b^2)\operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{2(3a^4+28a^2b^2+4b^4)\tan(c+dx)}{15d} + \frac{ab(6a^2+29b^2)\sec(c+dx)\tan(c+dx)}{30d} + \frac{(3a^2+4b^2)(a+b\sec(c+dx))^2\tan(c+dx)}{15d} + \frac{a(a+b\sec(c+dx))^3\tan(c+dx)}{5d} + \frac{(a+b\sec(c+dx))^4\tan(c+dx)}{5d}$$

output

```
1/2*a*b*(4*a^2+3*b^2)*arctanh(sin(d*x+c))/d+2/15*(3*a^4+28*a^2*b^2+4*b^4)*
tan(d*x+c)/d+1/30*a*b*(6*a^2+29*b^2)*sec(d*x+c)*tan(d*x+c)/d+1/15*(3*a^2+4
*b^2)*(a+b*sec(d*x+c))^2*tan(d*x+c)/d+1/5*a*(a+b*sec(d*x+c))^3*tan(d*x+c)/
d+1/5*(a+b*sec(d*x+c))^4*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.70

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$=\frac{15ab(4a^2+3b^2)\,\text{arctanh}(\sin(c+dx))+\tan(c+dx)\left(30(a^4+6a^2b^2+b^4)+15ab(4a^2+3b^2)\sec(c+dx)\right.}{30d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^4,x]
```

output
```
(15*a*b*(4*a^2 + 3*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(30*(a^4 + 6*
a^2*b^2 + b^4) + 15*a*b*(4*a^2 + 3*b^2)*Sec[c + d*x] + 30*a*b^3*Sec[c + d*
x]^3 + 20*b^2*(3*a^2 + b^2)*Tan[c + d*x]^2 + 6*b^4*Tan[c + d*x]^4))/(30*d)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4322, 3042, 4490, 3042, 4490, 3042, 4485, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4dx$$

$$\Big\downarrow \; 4322$$

$$\frac{4}{5}\int \sec(c+dx)(b+a\sec(c+dx))(a+b\sec(c+dx))^3dx+\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{4}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3dx+\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5d}$$

$\Big\downarrow$ 4490

$$\frac{4}{5}\left(\frac{1}{4}\int \sec(c+dx)(a+b\sec(c+dx))^2\left(7ab+\left(3a^2+4b^2\right)\sec(c+dx)\right)dx+\frac{a\tan(c+dx)(a+b\sec(c+dx))^3}{4d}\right)+\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5d}$$

$\Big\downarrow$ 3042

$$\frac{4}{5}\left(\frac{1}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(7ab+\left(3a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{a\tan(c+dx)(\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5d}$$

$\Big\downarrow$ 4490

$$\frac{4}{5}\left(\frac{1}{4}\left(\frac{1}{3}\int \sec(c+dx)(a+b\sec(c+dx))\left(b\left(27a^2+8b^2\right)+a\left(6a^2+29b^2\right)\sec(c+dx)\right)dx+\frac{\left(3a^2+4b^2\right)\tan(c+\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5d}$$

$\Big\downarrow$ 3042

$$\frac{4}{5}\left(\frac{1}{4}\left(\frac{1}{3}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(b\left(27a^2+8b^2\right)+a\left(6a^2+29b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5d}$$

$\Big\downarrow$ 4485

$$\frac{4}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \sec(c+dx)\left(15ab\left(4a^2+3b^2\right)+4\left(3a^4+28b^2a^2+4b^4\right)\sec(c+dx)\right)dx+\frac{ab\left(6a^2+29b^2\right)\tan(c+}{2d}\right.\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5d}$$

$\Big\downarrow$ 3042

$$\frac{4}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(15ab(4a^2+3b^2)+4(3a^4+28b^2a^2+4b^4)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{ab(6a^2+2\right.\right.\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5d}$$

$\downarrow$ 4274

$$\frac{4}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(15ab(4a^2+3b^2)\int \sec(c+dx)dx+4(3a^4+28a^2b^2+4b^4)\int \sec^2(c+dx)dx\right)+\frac{ab(6a^2+29b^2)\,\mathrm{t}}{}\right.\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5d}$$

$\downarrow$ 3042

$$\frac{4}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(15ab(4a^2+3b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+4(3a^4+28a^2b^2+4b^4)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)+\frac{ab(}{}\right.\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5d}$$

$\downarrow$ 4254

$$\frac{4}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(15ab(4a^2+3b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{4(3a^4+28a^2b^2+4b^4)\int 1d(-\tan(c+dx))}{d}\right)+\frac{ab(6a^2}{}\right.\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5d}$$

$\downarrow$ 24

$$\frac{4}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(15ab(4a^2+3b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{4(3a^4+28a^2b^2+4b^4)\tan(c+dx)}{d}\right)+\frac{ab(6a^2+29b^2)}{}\right.\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5d}$$

$\downarrow$ 4257

$$\frac{4}{5}\left(\frac{1}{4}\left(\frac{(3a^2+4b^2)\tan(c+dx)(a+b\sec(c+dx))^2}{3d}+\frac{1}{3}\left(\frac{ab(6a^2+29b^2)\tan(c+dx)\sec(c+dx)}{2d}+\frac{1}{2}\left(\frac{15ab(4a^2}{}\right.\right.\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5d}$$

input
```
Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^4,x]
```

output
```
((a + b*Sec[c + d*x])^4*Tan[c + d*x])/(5*d) + (4*((a*(a + b*Sec[c + d*x])^
3*Tan[c + d*x])/(4*d) + (((3*a^2 + 4*b^2)*(a + b*Sec[c + d*x])^2*Tan[c + d
*x])/(3*d) + ((a*b*(6*a^2 + 29*b^2)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + ((1
5*a*b*(4*a^2 + 3*b^2)*ArcTanh[Sin[c + d*x]])/d + (4*(3*a^4 + 28*a^2*b^2 +
4*b^4)*Tan[c + d*x])/d)/2)/3)/4))/5
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4322
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[m/(m + 1)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc
[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{a^4\tan(dx+c)+4b\,a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-6a^2b^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+4a\,b^3\left(-\right.}{}$
default	$\frac{a^4\tan(dx+c)+4b\,a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-6a^2b^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+4a\,b^3\left(-\right.}{}$
parts	$\frac{a^4\tan(dx+c)}{d}-\frac{b^4\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}+\frac{2b\,a^3\sec(dx+c)\tan(dx+c)}{d}+\frac{2b\,a^3\ln(\sec(dx\right.}{}$
parallelrisch	$\frac{-600a\left(a^2+\frac{3b^2}{4}\right)\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3dx+3c)}{2}+\cos(dx+c)\right)b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+600a\left(a^2+\frac{3b^2}{4}\right)\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3\right.}{}$
norman	$\frac{-\frac{4\left(45a^4+150a^2b^2+29b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{15d}-\frac{\left(2a^4-4b\,a^3+12a^2b^2-5a\,b^3+2b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{d}-\frac{\left(2a^4+4b\,a^3+12a^2b^2+5a\,b^3+2b^4\right)\mathrm{t}}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\right.\right.}$
risch	$\frac{i\left(-60b\,a^3e^{9i(dx+c)}-45a\,b^3e^{9i(dx+c)}+30a^4e^{8i(dx+c)}-120a^3b\,e^{7i(dx+c)}-210a\,b^3e^{7i(dx+c)}+120a^4e^{6i(dx+c)}+360a^2b^2e\right.}{}$

input
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*tan(d*x+c)+4*b*a^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+t
an(d*x+c)))-6*a^2*b^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+4*a*b^3*(-(-1/4*s
ec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-b^4*
(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.02

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{15\,(4\,a^3b+3\,ab^3)\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\,(4\,a^3b+3\,ab^3)\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)\right.}{}$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/60*(15*(4*a^3*b + 3*a*b^3)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(4*
a^3*b + 3*a*b^3)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(30*a*b^3*cos(d
*x + c) + 2*(15*a^4 + 60*a^2*b^2 + 8*b^4)*cos(d*x + c)^4 + 6*b^4 + 15*(4*a
^3*b + 3*a*b^3)*cos(d*x + c)^3 + 4*(15*a^2*b^2 + 2*b^4)*cos(d*x + c)^2)*si
n(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx = \int \left(a+b\sec\left(c+dx\right)\right)^4\sec^2\left(c+dx\right)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**4,x)
```

output
```
Integral((a + b*sec(c + d*x))**4*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.09

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx$$

$$= \frac{120\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^2b^2+4\left(3\,\tan\left(dx+c\right)^5+10\,\tan\left(dx+c\right)^3+15\,\tan\left(dx+c\right)\right)b^4-}{}$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/60*(120*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2*b^2 + 4*(3*tan(d*x + c)^5
+ 10*tan(d*x + c)^3 + 15*tan(d*x + c))*b^4 - 15*a*b^3*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
 + c) + 1) + 3*log(sin(d*x + c) - 1)) - 60*a^3*b*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 60*a^4*tan(
d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(167) = 334.

Time = 0.19 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.58

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/30*(15*(4*a^3*b + 3*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*a^
3*b + 3*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(30*a^4*tan(1/2*d*x
+ 1/2*c)^9 - 60*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 180*a^2*b^2*tan(1/2*d*x + 1
/2*c)^9 - 75*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 30*b^4*tan(1/2*d*x + 1/2*c)^9
- 120*a^4*tan(1/2*d*x + 1/2*c)^7 + 120*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 480*
a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 30*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 40*b^4*
tan(1/2*d*x + 1/2*c)^7 + 180*a^4*tan(1/2*d*x + 1/2*c)^5 + 600*a^2*b^2*tan(
1/2*d*x + 1/2*c)^5 + 116*b^4*tan(1/2*d*x + 1/2*c)^5 - 120*a^4*tan(1/2*d*x
+ 1/2*c)^3 - 120*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 480*a^2*b^2*tan(1/2*d*x +
1/2*c)^3 - 30*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 40*b^4*tan(1/2*d*x + 1/2*c)^3
 + 30*a^4*tan(1/2*d*x + 1/2*c) + 60*a^3*b*tan(1/2*d*x + 1/2*c) + 180*a^2*b
^2*tan(1/2*d*x + 1/2*c) + 75*a*b^3*tan(1/2*d*x + 1/2*c) + 30*b^4*tan(1/2*d
*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 14.20 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.70

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,\left(4\,a^3\,b+3\,a\,b^3\right)}{d} - \frac{\left(2\,a^4-4\,a^3\,b+12\,a^2\,b^2-5\,a\,b^3+2\,b^4\right)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\left(-8\,a^4+8\,a^3\,b-32\,a^2\,b^2+2\,a\,b^3-\frac{8\,b^4}{3}\right)\,\tan}{d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\right.}$$

input
```
int((a + b/cos(c + d*x))^4/cos(c + d*x)^2,x)
```

output
```
(atanh(tan(c/2 + (d*x)/2))*(3*a*b^3 + 4*a^3*b))/d - (tan(c/2 + (d*x)/2)^5*
(12*a^4 + (116*b^4)/15 + 40*a^2*b^2) + tan(c/2 + (d*x)/2)^9*(2*a^4 - 4*a^3
*b - 5*a*b^3 + 2*b^4 + 12*a^2*b^2) - tan(c/2 + (d*x)/2)^3*(2*a*b^3 + 8*a^3
*b + 8*a^4 + (8*b^4)/3 + 32*a^2*b^2) - tan(c/2 + (d*x)/2)^7*(8*a^4 - 8*a^3
*b - 2*a*b^3 + (8*b^4)/3 + 32*a^2*b^2) + tan(c/2 + (d*x)/2)*(5*a*b^3 + 4*a
^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/
2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/
2 + (d*x)/2)^10 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 584, normalized size of antiderivative = 3.26

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c))^4,x)
```

output
```
( - 60*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a**3*b - 45*
cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a*b**3 + 120*cos(c
+ d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**3*b + 90*cos(c + d*x)*
log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a*b**3 - 60*cos(c + d*x)*log(tan
((c + d*x)/2) - 1)*a**3*b - 45*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a*b*
*3 + 60*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a**3*b + 45
*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a*b**3 - 120*cos(c
 + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**3*b - 90*cos(c + d*x)
*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a*b**3 + 60*cos(c + d*x)*log(ta
n((c + d*x)/2) + 1)*a**3*b + 45*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a*b
**3 - 60*cos(c + d*x)*sin(c + d*x)**3*a**3*b - 45*cos(c + d*x)*sin(c + d*x
)**3*a*b**3 + 60*cos(c + d*x)*sin(c + d*x)*a**3*b + 75*cos(c + d*x)*sin(c
+ d*x)*a*b**3 + 30*sin(c + d*x)**5*a**4 + 120*sin(c + d*x)**5*a**2*b**2 +
16*sin(c + d*x)**5*b**4 - 60*sin(c + d*x)**3*a**4 - 300*sin(c + d*x)**3*a*
*2*b**2 - 40*sin(c + d*x)**3*b**4 + 30*sin(c + d*x)*a**4 + 180*sin(c + d*x
)*a**2*b**2 + 30*sin(c + d*x)*b**4)/(30*cos(c + d*x)*d*(sin(c + d*x)**4 -
2*sin(c + d*x)**2 + 1))
```

3.478 $\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal result

Integrand size = 19, antiderivative size = 146

$$\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{(8a^4+24a^2b^2+3b^4)\,\text{arctanh}(\sin(c+dx))}{8d} + \frac{ab(19a^2+16b^2)\tan(c+dx)}{6d} + \frac{b^2(26a^2+9b^2)\sec(c+dx)\tan(c+dx)}{24d} + \frac{7ab(a+b\sec(c+dx))^2\tan(c+dx)}{12d} + \frac{b(a+b\sec(c+dx))^3\tan(c+dx)}{4d}$$

output
```
1/8*(8*a^4+24*a^2*b^2+3*b^4)*arctanh(sin(d*x+c))/d+1/6*a*b*(19*a^2+16*b^2)
*tan(d*x+c)/d+1/24*b^2*(26*a^2+9*b^2)*sec(d*x+c)*tan(d*x+c)/d+7/12*a*b*(a+
b*sec(d*x+c))^2*tan(d*x+c)/d+1/4*b*(a+b*sec(d*x+c))^3*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.73

$$\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$=\frac{24a^4\coth^{-1}(\sin(c+dx))+b(9b(8a^2+b^2)\operatorname{arctanh}(\sin(c+dx))+\tan(c+dx)\,(9b(8a^2+b^2)\sec(c+dx)}{24d}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^4,x]
```

output
```
(24*a^4*ArcCoth[Sin[c + d*x]] + b*(9*b*(8*a^2 + b^2)*ArcTanh[Sin[c + d*x]]
 + Tan[c + d*x]*(9*b*(8*a^2 + b^2)*Sec[c + d*x] + 6*b^3*Sec[c + d*x]^3 + 3
2*a*(3*(a^2 + b^2) + b^2*Tan[c + d*x]^2))))/(24*d)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {3042, 4317, 3042, 4490, 3042, 4485, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4dx$$

$$\Big\downarrow \text{4317}$$

$$\frac{1}{4}\int \sec(c+dx)(a+b\sec(c+dx))^2\left(4a^2+7b\sec(c+dx)a+3b^2\right)dx+\frac{b\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(4a^2+7b\csc\left(c+dx+\frac{\pi}{2}\right)a+3b^2\right)dx+\frac{b\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$\downarrow$ 4490

$$\frac{1}{4}\left(\frac{1}{3}\int \sec(c+dx)(a+b\sec(c+dx))\left(a\left(12a^2+23b^2\right)+b\left(26a^2+9b^2\right)\sec(c+dx)\right)dx+\frac{7ab\tan(c+dx)(a+b}{3d}\right.$$
$$\frac{b\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a\left(12a^2+23b^2\right)+b\left(26a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\right.$$
$$\frac{b\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$\downarrow$ 4485

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \sec(c+dx)\left(3\left(8a^4+24b^2a^2+3b^4\right)+4ab\left(19a^2+16b^2\right)\sec(c+dx)\right)dx+\frac{b^2\left(26a^2+9b^2\right)\tan(c+dx}{2d}\right.\right.$$
$$\frac{b\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(3\left(8a^4+24b^2a^2+3b^4\right)+4ab\left(19a^2+16b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{b^2\left(26a^2+9b\right.}{}\right.\right.$$
$$\frac{b\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$\downarrow$ 4274

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(4ab\left(19a^2+16b^2\right)\int \sec^2(c+dx)dx+3\left(8a^4+24a^2b^2+3b^4\right)\int \sec(c+dx)dx\right)+\frac{b^2\left(26a^2+9b^2\right)\tan}{2}\right.\right.$$
$$\frac{b\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(4ab(19a^2+16b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+3(8a^4+24a^2b^2+3b^4)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx\right)+\frac{b^2(26\right.\right.$$
$$\frac{b\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$$\Big\downarrow \; 4254$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(3(8a^4+24a^2b^2+3b^4)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{4ab(19a^2+16b^2)\int 1d(-\tan(c+dx))}{d}\right)+\frac{b^2(26a^2-}{}\right.\right.$$
$$\frac{b\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$$\Big\downarrow \; 24$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(3(8a^4+24a^2b^2+3b^4)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{4ab(19a^2+16b^2)\tan(c+dx)}{d}\right)+\frac{b^2(26a^2+9b^2)\,\mathrm{ta}}{}\right.\right.$$
$$\frac{b\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$$\Big\downarrow \; 4257$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{b^2(26a^2+9b^2)\tan(c+dx)\sec(c+dx)}{2d}+\frac{1}{2}\left(\frac{4ab(19a^2+16b^2)\tan(c+dx)}{d}+\frac{3(8a^4+24a^2b^2+3b^4)\,\mathrm{arc}}{d}\right.\right.\right.$$
$$\frac{b\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

input
```
Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^4,x]
```

output
```
(b*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(4*d) + ((7*a*b*(a + b*Sec[c + d*x
])^2*Tan[c + d*x])/(3*d) + ((b^2*(26*a^2 + 9*b^2)*Sec[c + d*x]*Tan[c + d*x
])/(2*d) + ((3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*ArcTanh[Sin[c + d*x]])/d + (4*
a*b*(19*a^2 + 16*b^2)*Tan[c + d*x])/d)/2)/3)/4
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4317
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Simp[1/m   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*m + a*b*(2*m - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ
[a^2 - b^2, 0] && GtQ[m, 1] && IntegerQ[2*m]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{a^4 \ln(\sec(dx+c)+\tan(dx+c))+4a^3b\tan(dx+c)+6a^2b^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-4a\,b^3\left(-\frac{2}{3}-\frac{s}{}\right.}{d}$
default	$\frac{a^4 \ln(\sec(dx+c)+\tan(dx+c))+4a^3b\tan(dx+c)+6a^2b^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-4a\,b^3\left(-\frac{2}{3}-\frac{s}{}\right.}{d}$
parts	$\frac{a^4 \ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b^4\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}-\frac{4a\,b}{}$
parallelrisch	$-96\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\left(a^4+3a^2b^2+\frac{3}{8}b^4\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+96\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\left(a^4+\right.$
norman	$\frac{-\frac{b\left(32a^3-24a^2b+32a\,b^2-5b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}+\frac{b\left(32a^3+24a^2b+32a\,b^2+5b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{b\left(288a^3-72a^2b+160a\,b^2+9b^3\right)\tan(}{12d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}$
risch	$-\frac{ib\left(72a^2b\,e^{7i(dx+c)}+9b^3e^{7i(dx+c)}-96a^3e^{6i(dx+c)}+72a^2b\,e^{5i(dx+c)}+33b^3e^{5i(dx+c)}-288a^3e^{4i(dx+c)}-192a\,b^2e^{4i(dx+}\right.}{12d\left(e^{2i(}\right.}$

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*ln(sec(d*x+c)+tan(d*x+c))+4*a^3*b*tan(d*x+c)+6*a^2*b^2*(1/2*sec(d
*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-4*a*b^3*(-2/3-1/3*sec(d*x+
c)^2)*tan(d*x+c)+b^4*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*l
n(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.12

$$\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{3\left(8\,a^4+24\,a^2b^2+3\,b^4\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(8\,a^4+24\,a^2b^2+3\,b^4\right)\cos\left(dx+c\right)^4\log\left(-\right.}{}$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/48*(3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1)
- 3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2
*(32*a*b^3*cos(d*x + c) + 6*b^4 + 32*(3*a^3*b + 2*a*b^3)*cos(d*x + c)^3 +
9*(8*a^2*b^2 + b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx = \int \left(a+b\sec\left(c+dx\right)\right)^4\sec\left(c+dx\right)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))**4,x)
```

output
```
Integral((a + b*sec(c + d*x))**4*sec(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.23

$$\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{64\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)ab^3-3\,b^4\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\right.}{}$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/48*(64*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^3 - 3*b^4*(2*(3*sin(d*x + c
)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(
d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 72*a^2*b^2*(2*sin(d*x + c)/(sin
(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*a^4
*log(sec(d*x + c) + tan(d*x + c)) + 192*a^3*b*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(136) = 272$.

Time = 0.18 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.47

$$\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx$$

$$= \frac{3\,(8\,a^4+24\,a^2b^2+3\,b^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(8\,a^4+24\,a^2b^2+3\,b^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right.\right.}{}$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/24*(3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) -
3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(96*
a^3*b*tan(1/2*d*x + 1/2*c)^7 - 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 96*a*b^
3*tan(1/2*d*x + 1/2*c)^7 - 15*b^4*tan(1/2*d*x + 1/2*c)^7 - 288*a^3*b*tan(1
/2*d*x + 1/2*c)^5 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 160*a*b^3*tan(1/2*
d*x + 1/2*c)^5 - 9*b^4*tan(1/2*d*x + 1/2*c)^5 + 288*a^3*b*tan(1/2*d*x + 1/
2*c)^3 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 160*a*b^3*tan(1/2*d*x + 1/2*c
)^3 - 9*b^4*tan(1/2*d*x + 1/2*c)^3 - 96*a^3*b*tan(1/2*d*x + 1/2*c) - 72*a^
2*b^2*tan(1/2*d*x + 1/2*c) - 96*a*b^3*tan(1/2*d*x + 1/2*c) - 15*b^4*tan(1/
2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 13.63 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.68

$$\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{\left(-8a^3b+6a^2b^2-8ab^3+\frac{5b^4}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(24a^3b-6a^2b^2+\frac{40ab^3}{3}+\frac{3b^4}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(-\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\right.} + \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(2a^4+6a^2b^2+\frac{3b^4}{4}\right)}{d}$$

input
```
int((a + b/cos(c + d*x))^4/cos(c + d*x),x)
```

output
```
(tan(c/2 + (d*x)/2)*(8*a*b^3 + 8*a^3*b + (5*b^4)/4 + 6*a^2*b^2) - tan(c/2
+ (d*x)/2)^7*(8*a*b^3 + 8*a^3*b - (5*b^4)/4 - 6*a^2*b^2) - tan(c/2 + (d*x)
/2)^3*((40*a*b^3)/3 + 24*a^3*b - (3*b^4)/4 + 6*a^2*b^2) + tan(c/2 + (d*x)/
2)^5*((40*a*b^3)/3 + 24*a^3*b + (3*b^4)/4 - 6*a^2*b^2))/(d*(6*tan(c/2 + (d
*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*
x)/2)^8 + 1)) + (atanh(tan(c/2 + (d*x)/2))*(2*a^4 + (3*b^4)/4 + 6*a^2*b^2)
)/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 578, normalized size of antiderivative = 3.96

$$\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^4,x)
```

output

```
( - 96*cos(c + d*x)*sin(c + d*x)**3*a**3*b - 64*cos(c + d*x)*sin(c + d*x)*
*3*a*b**3 + 96*cos(c + d*x)*sin(c + d*x)*a**3*b + 96*cos(c + d*x)*sin(c +
d*x)*a*b**3 - 24*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a**4 - 72*log(t
an((c + d*x)/2) - 1)*sin(c + d*x)**4*a**2*b**2 - 9*log(tan((c + d*x)/2) -
1)*sin(c + d*x)**4*b**4 + 48*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**
4 + 144*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**2*b**2 + 18*log(tan((
c + d*x)/2) - 1)*sin(c + d*x)**2*b**4 - 24*log(tan((c + d*x)/2) - 1)*a**4
- 72*log(tan((c + d*x)/2) - 1)*a**2*b**2 - 9*log(tan((c + d*x)/2) - 1)*b**
4 + 24*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a**4 + 72*log(tan((c + d*
x)/2) + 1)*sin(c + d*x)**4*a**2*b**2 + 9*log(tan((c + d*x)/2) + 1)*sin(c +
 d*x)**4*b**4 - 48*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**4 - 144*lo
g(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**2*b**2 - 18*log(tan((c + d*x)/2
) + 1)*sin(c + d*x)**2*b**4 + 24*log(tan((c + d*x)/2) + 1)*a**4 + 72*log(t
an((c + d*x)/2) + 1)*a**2*b**2 + 9*log(tan((c + d*x)/2) + 1)*b**4 - 72*sin
(c + d*x)**3*a**2*b**2 - 9*sin(c + d*x)**3*b**4 + 72*sin(c + d*x)*a**2*b**
2 + 15*sin(c + d*x)*b**4)/(24*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.479 $\int (a+b\sec(c+dx))^4\,dx$

Optimal result

Integrand size = 12, antiderivative size = 107

$$\int (a+b\sec(c+dx))^4\,dx = a^4x+\frac{2ab(2a^2+b^2)\,\text{arctanh}(\sin(c+dx))}{d} + \frac{b^2(17a^2+2b^2)\tan(c+dx)}{3d}+\frac{4ab^3\sec(c+dx)\tan(c+dx)}{3d} + \frac{b^2(a+b\sec(c+dx))^2\tan(c+dx)}{3d}$$

output
```
a^4*x+2*a*b*(2*a^2+b^2)*arctanh(sin(d*x+c))/d+1/3*b^2*(17*a^2+2*b^2)*tan(d
*x+c)/d+4/3*a*b^3*sec(d*x+c)*tan(d*x+c)/d+1/3*b^2*(a+b*sec(d*x+c))^2*tan(d
*x+c)/d
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.90

$$\int (a+b\sec(c+dx))^4\,dx = \frac{3a^4dx+12a^3b\coth^{-1}(\sin(c+dx))+6ab^3\text{arctanh}(\sin(c+dx))+18a^2b^2\tan(c+dx)+3b^4\tan(c+dx)\cdots}{3d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^4,x]
```

output
```
(3*a^4*d*x + 12*a^3*b*ArcCoth[Sin[c + d*x]] + 6*a*b^3*ArcTanh[Sin[c + d*x]
] + 18*a^2*b^2*Tan[c + d*x] + 3*b^4*Tan[c + d*x] + 6*a*b^3*Sec[c + d*x]*Ta
n[c + d*x] + b^4*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 4269, 3042, 4536, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(c+dx))^4 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4 dx$$

$$\Big\downarrow \text{4269}$$

$$\frac{1}{3}\int (a+b\sec(c+dx))\left(3a^3+8b^2\sec^2(c+dx)a+b\left(9a^2+2b^2\right)\sec(c+dx)\right)dx+\frac{b^2\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(3a^3+8b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a+b\left(9a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{b^2\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \text{4536}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \left(6a^4+12b\left(2a^2+b^2\right)\sec(c+dx)a+2b^2\left(17a^2+2b^2\right)\sec^2(c+dx)\right)dx+\frac{4ab^3\tan(c+dx)\sec(c+dx)}{d}\right)+\frac{b^2\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(6a^4x+\frac{12ab(2a^2+b^2)\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{2b^2(17a^2+2b^2)\tan(c+dx)}{d}\right)+\frac{4ab^3\tan(c+dx)\sec(c+\right.$$
$$\frac{b^2\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

input
```
Int[(a + b*Sec[c + d*x])^4,x]
```

output
```
(b^2*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(3*d) + ((4*a*b^3*Sec[c + d*x]*T
an[c + d*x])/d + (6*a^4*x + (12*a*b*(2*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/d
 + (2*b^2*(17*a^2 + 2*b^2)*Tan[c + d*x])/d)/2)/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4269
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[1/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*
a^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

rule 4536
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Simp[1/2   Int[Simp[2*A*a + (2*B*a + b*(2*
A + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, e, f, A, B, C}, x]
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{a^4(dx+c)+4b\,a^3\ln(\sec(dx+c)+\tan(dx+c))+6a^2b^2\tan(dx+c)+4a\,b^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{a^4(dx+c)+4b\,a^3\ln(\sec(dx+c)+\tan(dx+c))+6a^2b^2\tan(dx+c)+4a\,b^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
parts	$a^4x-\frac{b^4\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{4b\,a^3\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{6a^2b^2\tan(dx+c)}{d}+\frac{2a\,b^3\sec(dx}{}$
risch	$a^4x-\frac{4ib^2(3ab\,e^{5i(dx+c)}-9a^2e^{4i(dx+c)}-18a^2e^{2i(dx+c)}-3\,e^{2i(dx+c)}b^2-3ab\,e^{i(dx+c)}-9a^2-b^2)}{3d(e^{2i(dx+c)}+1)^3}+\frac{4a^3b\ln(e^{i(dx+c}}{d}$
parallelrisch	$\frac{-36a\left(a^2+\frac{b^2}{2}\right)b\left(\frac{\cos(3dx+3c)}{3}+\cos(dx+c)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+36a\left(a^2+\frac{b^2}{2}\right)b\left(\frac{\cos(3dx+3c)}{3}+\cos(dx+c)\right)\ln\left(\tan\left(\frac{d}{2}\right.\right.}{3d(\cos(3dx}$
norman	$\frac{a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6-a^4x+3a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-3a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\frac{4b^2(18a^2+b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{2b^2(6a^2-2ab+b^2)\tan(}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}$

input
```
int((a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*(d*x+c)+4*b*a^3*ln(sec(d*x+c)+tan(d*x+c))+6*a^2*b^2*tan(d*x+c)+4*
a*b^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-b^4*(-2/3-
1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.29

$$\int (a+b\sec(c+dx))^4\,dx$$
$$=\frac{3\,a^4dx\cos(dx+c)^3+3\left(2\,a^3b+ab^3\right)\cos(dx+c)^3\log(\sin(dx+c)+1)-3\left(2\,a^3b+ab^3\right)\cos(dx+c)^3\,\mathrm{lo}}{3\,d\cos(dx+c}$$

input
```
integrate((a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/3*(3*a^4*d*x*cos(d*x + c)^3 + 3*(2*a^3*b + a*b^3)*cos(d*x + c)^3*log(sin
(d*x + c) + 1) - 3*(2*a^3*b + a*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1)
 + (6*a*b^3*cos(d*x + c) + b^4 + 2*(9*a^2*b^2 + b^4)*cos(d*x + c)^2)*sin(d
*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int (a+b\sec(c+dx))^4\,dx=\int (a+b\sec{(c+dx)})^4\,dx$$

input
```
integrate((a+b*sec(d*x+c))**4,x)
```

output
```
Integral((a + b*sec(c + d*x))**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.13

$$\begin{aligned}\int (a+b\sec(c+dx))^4\,dx &= a^4x+\frac{\left(\tan{(dx+c)}^3+3\,\tan{(dx+c)}\right)b^4}{3\,d}\\&\quad-\frac{ab^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log{(\sin{(dx+c)}+1)}+\log{(\sin{(dx+c)}-1)}\right)}{d}\\&\quad+\frac{4\,a^3b\log{(\sec{(dx+c)}+\tan{(dx+c)})}}{d}+\frac{6\,a^2b^2\tan{(dx+c)}}{d}\end{aligned}$$

input
```
integrate((a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
a^4*x + 1/3*(tan(d*x + c)^3 + 3*tan(d*x + c))*b^4/d - a*b^3*(2*sin(d*x + c
)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1))/d
+ 4*a^3*b*log(sec(d*x + c) + tan(d*x + c))/d + 6*a^2*b^2*tan(d*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(101) = 202$.

Time = 0.14 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.07

$$\int (a + b\sec(c + dx))^4\,dx$$

$$= \frac{3\,(dx + c)a^4 + 6\,(2\,a^3b + ab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 6\,(2\,a^3b + ab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) -}{}$$

input
```
integrate((a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(3*(d*x + c)*a^4 + 6*(2*a^3*b + a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) +
1)) - 6*(2*a^3*b + a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(18*a^2*b
^2*tan(1/2*d*x + 1/2*c)^5 - 6*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*b^4*tan(1/2
*d*x + 1/2*c)^5 - 36*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 2*b^4*tan(1/2*d*x +
1/2*c)^3 + 18*a^2*b^2*tan(1/2*d*x + 1/2*c) + 6*a*b^3*tan(1/2*d*x + 1/2*c)
+ 3*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 10.32 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.73

$$\int (a + b\sec(c + dx))^4\,dx = \frac{2\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{2\,b^4\,\sin\left(c+d\,x\right)}{3\,d\,\cos\left(c+d\,x\right)} + \frac{b^4\,\sin\left(c+d\,x\right)}{3\,d\,{\cos\left(c+d\,x\right)}^3} + \frac{4\,a\,b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{8\,a^3\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{2\,a\,b^3\,\sin\left(c+d\,x\right)}{d\,{\cos\left(c+d\,x\right)}^2} + \frac{6\,a^2\,b^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}$$

input
```
int((a + b/cos(c + d*x))^4,x)
```

output
```
(2*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*b^4*sin(c + d*x
))/(3*d*cos(c + d*x)) + (b^4*sin(c + d*x))/(3*d*cos(c + d*x)^3) + (4*a*b^3
*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (8*a^3*b*atanh(sin(c/2
+ (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*a*b^3*sin(c + d*x))/(d*cos(c + d*x)
^2) + (6*a^2*b^2*sin(c + d*x))/(d*cos(c + d*x))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 356, normalized size of antiderivative = 3.33

$$\int (a + b\sec(c + dx))^4\, dx$$
$$= \frac{-12\cos(dx + c)\log\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)\sin(dx + c)^2 a^3 b - 6\cos(dx + c)\log\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)\sin(dx +}{}$$

input
```
int((a+b*sec(d*x+c))^4,x)
```

output
```
( - 12*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**3*b - 6*c
os(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a*b**3 + 12*cos(c +
d*x)*log(tan((c + d*x)/2) - 1)*a**3*b + 6*cos(c + d*x)*log(tan((c + d*x)/2
) - 1)*a*b**3 + 12*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*
a**3*b + 6*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a*b**3 -
 12*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a**3*b - 6*cos(c + d*x)*log(tan
((c + d*x)/2) + 1)*a*b**3 + 3*cos(c + d*x)*sin(c + d*x)**2*a**4*d*x - 6*co
s(c + d*x)*sin(c + d*x)*a*b**3 - 3*cos(c + d*x)*a**4*d*x + 18*sin(c + d*x)
**3*a**2*b**2 + 2*sin(c + d*x)**3*b**4 - 18*sin(c + d*x)*a**2*b**2 - 3*sin
(c + d*x)*b**4)/(3*cos(c + d*x)*d*(sin(c + d*x)**2 - 1))
```

3.480 $\int \cos(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal result

Integrand size = 19, antiderivative size = 104

$$\int \cos(c+dx)(a+b\sec(c+dx))^4\,dx = 4a^3bx+\frac{b^2(12a^2+b^2)\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{a^2(2a^2-b^2)\sin(c+dx)}{2d}+\frac{b^2(a+b\sec(c+dx))^2\sin(c+dx)}{2d}+\frac{3ab^3\tan(c+dx)}{d}$$

output
```
4*a^3*b*x+1/2*b^2*(12*a^2+b^2)*arctanh(sin(d*x+c))/d+1/2*a^2*(2*a^2-b^2)*s
in(d*x+c)/d+1/2*b^2*(a+b*sec(d*x+c))^2*sin(d*x+c)/d+3*a*b^3*tan(d*x+c)/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 280 vs. 2(104) = 208.

Time = 1.23 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.69

$$\int \cos(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{\sec^2(c+dx)\left(8a^3bc+8a^3bdx-12a^2b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-b^4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^4,x]
```

output
```
(Sec[c + d*x]^2*(8*a^3*b*c + 8*a^3*b*d*x - 12*a^2*b^2*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]] - b^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*a
^2*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + b^4*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]] + b*Cos[2*(c + d*x)]*(8*a^3*(c + d*x) - b*(12*a^2 + b
^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + b*(12*a^2 + b^2)*Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]]) + (a^4 + 2*b^4)*Sin[c + d*x] + 8*a*b^3*Sin
[2*(c + d*x)] + a^4*Sin[3*(c + d*x)]))/(4*d)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {3042, 4329, 3042, 4564, 3042, 4535, 24, 3042, 4533, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\sec(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4329}$$

$$\frac{1}{2}\int \cos(c+dx)(a+b\sec(c+dx))\left(6ab^2\sec^2(c+dx)+b\left(6a^2+b^2\right)\sec(c+dx)+a\left(2a^2-b^2\right)\right)dx+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(6ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\left(6a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+a\left(2a^2-b^2\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \text{4564}$$

$$\frac{1}{2}\left(\int \cos(c+dx)\left(8b\sec(c+dx)a^3+\left(2a^2-b^2\right)a^2+b^2\left(12a^2+b^2\right)\sec^2(c+dx)\right)dx+\frac{6ab^3\tan(c+dx)}{d}\right)+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \frac{8b\csc\left(c+dx+\frac{\pi}{2}\right)a^3+\left(2a^2-b^2\right)a^2+b^2\left(12a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{6ab^3\tan(c+dx)}{d}\right)+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \text{4535}$$

$$\frac{1}{2}\left(8a^3b\int 1dx+\int \cos(c+dx)\left(\left(2a^2-b^2\right)a^2+b^2\left(12a^2+b^2\right)\sec^2(c+dx)\right)dx+\frac{6ab^3\tan(c+dx)}{d}\right)+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{2}\left(\int \cos(c+dx)\left(\left(2a^2-b^2\right)a^2+b^2\left(12a^2+b^2\right)\sec^2(c+dx)\right)dx+8a^3bx+\frac{6ab^3\tan(c+dx)}{d}\right)+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \frac{\left(2a^2-b^2\right)a^2+b^2\left(12a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+8a^3bx+\frac{6ab^3\tan(c+dx)}{d}\right)+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \text{4533}$$

$$\frac{1}{2}\left(b^2\left(12a^2+b^2\right)\int \sec(c+dx)dx+8a^3bx+\frac{a^2\left(2a^2-b^2\right)\sin(c+dx)}{d}+\frac{6ab^3\tan(c+dx)}{d}\right)+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(b^2(12a^2+b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+8a^3bx+\frac{a^2(2a^2-b^2)\sin(c+dx)}{d}+\frac{6ab^3\tan(c+dx)}{d}\right)+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$\big\downarrow$ 4257

$$\frac{1}{2}\left(8a^3bx+\frac{b^2(12a^2+b^2)\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{a^2(2a^2-b^2)\sin(c+dx)}{d}+\frac{6ab^3\tan(c+dx)}{d}\right)+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))^2}{2d}$$

input
```
Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^4,x]
```

output
```
(b^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(2*d) + (8*a^3*b*x + (b^2*(12*a^
2 + b^2)*ArcTanh[Sin[c + d*x]])/d + (a^2*(2*a^2 - b^2)*Sin[c + d*x])/d + (
6*a*b^3*Tan[c + d*x])/d)/2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4564
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Simp[1/(n + 2)   Int[(d*Csc[e + f*x])^n*Simp[A*a*(n +
2) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
&&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{a^4\sin(dx+c)+4ba^3(dx+c)+6a^2b^2\ln(\sec(dx+c)+\tan(dx+c))+4ab^3\tan(dx+c)+b^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)}{}\right.}{d}$
default	$\frac{a^4\sin(dx+c)+4ba^3(dx+c)+6a^2b^2\ln(\sec(dx+c)+\tan(dx+c))+4ab^3\tan(dx+c)+b^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)}{}\right.}{d}$
parallelrisch	$\frac{-12(1+\cos(2dx+2c))\left(a^2+\frac{b^2}{12}\right)b^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+12(1+\cos(2dx+2c))\left(a^2+\frac{b^2}{12}\right)b^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+8a^3bx}{2d(1+\cos(2dx+2c))}$
risch	$4a^3bx-\frac{ia^4e^{i(dx+c)}}{2d}+\frac{ia^4e^{-i(dx+c)}}{2d}-\frac{ib^3\left(b\,e^{3i(dx+c)}-8\,e^{2i(dx+c)}a-b\,e^{i(dx+c)}-8a\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{6\ln\left(e^{i(dx+c)}+i\right)a^2b^2}{d}$
norman	$\frac{\frac{(2a^4-8ab^3+b^4)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}+\frac{(6a^4+8ab^3-b^4)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}-4a^3bx-\frac{(2a^4+8ab^3+b^4)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{(6a^4-8ab^3-b^4)\tan}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$

input
```
int(cos(d*x+c)*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*sin(d*x+c)+4*b*a^3*(d*x+c)+6*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))+4*
a*b^3*tan(d*x+c)+b^4*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+
c))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.25

$$\int\cos(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$=\frac{16\,a^3bdx\cos(dx+c)^2+(12\,a^2b^2+b^4)\cos(dx+c)^2\log(\sin(dx+c)+1)-(12\,a^2b^2+b^4)\cos(dx+c)^2\,\mathrm{l}}{4\,d\cos(dx+c)^2}$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/4*(16*a^3*b*d*x*cos(d*x + c)^2 + (12*a^2*b^2 + b^4)*cos(d*x + c)^2*log(s
in(d*x + c) + 1) - (12*a^2*b^2 + b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1
) + 2*(2*a^4*cos(d*x + c)^2 + 8*a*b^3*cos(d*x + c) + b^4)*sin(d*x + c))/(d
*cos(d*x + c)^2)
```

Sympy [F]

$$\int \cos(c+dx)(a+b\sec(c+dx))^4\,dx = \int (a+b\sec(c+dx))^4\cos(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))**4,x)
```

output
```
Integral((a + b*sec(c + d*x))**4*cos(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.11

$$\int \cos(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{16\,(dx+c)a^3b - b^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + 12\,a^2b^2(\log(\sin(dx}{4\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/4*(16*(d*x + c)*a^3*b - b^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(s
in(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^2*b^2*(log(sin(d*x + c) +
 1) - log(sin(d*x + c) - 1)) + 4*a^4*sin(d*x + c) + 16*a*b^3*tan(d*x + c))
/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.72

$$\int \cos(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{8\,(dx+c)a^3b + \frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1} + (12\,a^2b^2+b^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - (12\,a^2b^2+b^4)\log\left(\left|\tan\left(\frac{1}{2}\right.\right.\right.}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/2*(8*(d*x + c)*a^3*b + 4*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^
2 + 1) + (12*a^2*b^2 + b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (12*a^2*b
^2 + b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(8*a*b^3*tan(1/2*d*x + 1/
2*c)^3 - b^4*tan(1/2*d*x + 1/2*c)^3 - 8*a*b^3*tan(1/2*d*x + 1/2*c) - b^4*t
an(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 10.20 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.46

$$\int \cos(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{a^4\,\sin\left(c+d\,x\right)}{d} + \frac{b^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{b^4\,\sin\left(c+d\,x\right)}{2\,d\cos\left(c+d\,x\right)^2} + \frac{12\,a^2\,b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{8\,a^3\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{4\,a\,b^3\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}$$

input
```
int(cos(c + d*x)*(a + b/cos(c + d*x))^4,x)
```

output
```
(a^4*sin(c + d*x))/d + (b^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/
d + (b^4*sin(c + d*x))/(2*d*cos(c + d*x)^2) + (12*a^2*b^2*atanh(sin(c/2 +
(d*x)/2)/cos(c/2 + (d*x)/2)))/d + (8*a^3*b*atan(sin(c/2 + (d*x)/2)/cos(c/2
 + (d*x)/2)))/d + (4*a*b^3*sin(c + d*x))/(d*cos(c + d*x))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.84

$$\int \cos(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{-8\cos(dx+c)\sin(dx+c)\,a\,b^3 - 12\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2\,a^2b^2 - \log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\text{si}}{}$$

input
```
int(cos(d*x+c)*(a+b*sec(d*x+c))^4,x)
```

output
```
( - 8*cos(c + d*x)*sin(c + d*x)*a*b**3 - 12*log(tan((c + d*x)/2) - 1)*sin(
c + d*x)**2*a**2*b**2 - log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b**4 + 1
2*log(tan((c + d*x)/2) - 1)*a**2*b**2 + log(tan((c + d*x)/2) - 1)*b**4 + 1
2*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**2*b**2 + log(tan((c + d*x)/
2) + 1)*sin(c + d*x)**2*b**4 - 12*log(tan((c + d*x)/2) + 1)*a**2*b**2 - lo
g(tan((c + d*x)/2) + 1)*b**4 + 2*sin(c + d*x)**3*a**4 + 8*sin(c + d*x)**2*
a**3*b*c + 8*sin(c + d*x)**2*a**3*b*d*x - 2*sin(c + d*x)*a**4 - sin(c + d*
x)*b**4 - 8*a**3*b*c - 8*a**3*b*d*x)/(2*d*(sin(c + d*x)**2 - 1))
```

3.481 $\int \cos^2(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{1}{2}a^2\left(a^2+12b^2\right)x+\frac{4ab^3\operatorname{arctanh}(\sin(c+dx))}{d} + \frac{3a^3b\sin(c+dx)}{d} + \frac{a^2\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d} - \frac{b^2(a^2-2b^2)\tan(c+dx)}{2d}$$

output
```
1/2*a^2*(a^2+12*b^2)*x+4*a*b^3*arctanh(sin(d*x+c))/d+3*a^3*b*sin(d*x+c)/d+
1/2*a^2*cos(d*x+c)*(a+b*sec(d*x+c))^2*sin(d*x+c)/d-1/2*b^2*(a^2-2*b^2)*tan
(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.10

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{2a\left(a(a^2+12b^2)(c+dx)-8b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+8b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\right.\right.}{4d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^4,x]
```

output
```
(2*a*(a*(a^2 + 12*b^2)*(c + d*x) - 8*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] + 8*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 16*a^3*b*Sin[c
 + d*x] + a^4*Sin[2*(c + d*x)] + 4*b^4*Tan[c + d*x])/(4*d)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4328, 3042, 4564, 3042, 4535, 24, 3042, 4533, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4328}$$

$$\frac{1}{2}\int \cos(c+dx)(a+b\sec(c+dx))\left(6ba^2+\left(a^2+6b^2\right)\sec(c+dx)a-b\left(a^2-2b^2\right)\sec^2(c+dx)\right)dx+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(6ba^2+\left(a^2+6b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a-b\left(a^2-2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \text{ 4564}$$

$$\frac{1}{2}\left(\int \cos(c+dx)\left(6ba^3+\left(a^2+12b^2\right)\sec(c+dx)a^2+8b^3\sec^2(c+dx)a\right)dx-\frac{b^2\left(a^2-2b^2\right)\tan(c+dx)}{d}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\left(\int \frac{6ba^3+\left(a^2+12b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^2+8b^3\csc\left(c+dx+\frac{\pi}{2}\right)^2a}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{b^2\left(a^2-2b^2\right)\tan(c+dx)}{d}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \text{ 4535}$$

$$\frac{1}{2}\left(\int \cos(c+dx)\left(6ba^3+8b^3\sec^2(c+dx)a\right)dx+a^2\left(a^2+12b^2\right)\int 1dx-\frac{b^2\left(a^2-2b^2\right)\tan(c+dx)}{d}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{1}{2}\left(\int \cos(c+dx)\left(6ba^3+8b^3\sec^2(c+dx)a\right)dx-\frac{b^2\left(a^2-2b^2\right)\tan(c+dx)}{d}+a^2x\left(a^2+12b^2\right)\right)+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\left(\int \frac{6ba^3+8b^3\csc\left(c+dx+\frac{\pi}{2}\right)^2a}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{b^2\left(a^2-2b^2\right)\tan(c+dx)}{d}+a^2x\left(a^2+12b^2\right)\right)+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \text{ 4533}$$

$$\frac{1}{2}\left(8ab^3\int \sec(c+dx)dx+\frac{6a^3b\sin(c+dx)}{d}-\frac{b^2(a^2-2b^2)\tan(c+dx)}{d}+a^2x(a^2+12b^2)\right)+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\left(8ab^3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{6a^3b\sin(c+dx)}{d}-\frac{b^2(a^2-2b^2)\tan(c+dx)}{d}+a^2x(a^2+12b^2)\right)+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \; 4257$$

$$\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}+\frac{1}{2}\left(\frac{6a^3b\sin(c+dx)}{d}-\frac{b^2(a^2-2b^2)\tan(c+dx)}{d}+a^2x(a^2+12b^2)+\frac{8ab^3\text{arctanh}(\sin(c+dx))}{d}\right)$$

input
```
Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^4,x]
```

output
```
(a^2*Cos[c + d*x]*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(2*d) + (a^2*(a^2 +
 12*b^2)*x + (8*a*b^3*ArcTanh[Sin[c + d*x]])/d + (6*a^3*b*Sin[c + d*x])/d
- (b^2*(a^2 - 2*b^2)*Tan[c + d*x])/d)/2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4564
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Simp[1/(n + 2)   Int[(d*Csc[e + f*x])^n*Simp[A*a*(n +
2) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
&&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{a^{4}\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4b\,a^{3}\sin(dx+c)+6a^{2}b^{2}(dx+c)+4a\,b^{3}\ln(\sec(dx+c)+\tan(dx+c))+b^{4}\tan(dx+c)}{d}$
default	$\frac{a^{4}\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4b\,a^{3}\sin(dx+c)+6a^{2}b^{2}(dx+c)+4a\,b^{3}\ln(\sec(dx+c)+\tan(dx+c))+b^{4}\tan(dx+c)}{d}$
parallelrisch	$\frac{-32\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a\,b^{3}\cos(dx+c)+32\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a\,b^{3}\cos(dx+c)+16b\,a^{3}\sin(2dx+2c)+a^{4}\sin(3dx+3c)}{8d\cos(dx+c)}$
risch	$\frac{a^{4}x}{2}+6a^{2}b^{2}x-\frac{ia^{4}e^{2i(dx+c)}}{8d}-\frac{2ib\,a^{3}e^{i(dx+c)}}{d}+\frac{2ib\,a^{3}e^{-i(dx+c)}}{d}+\frac{ia^{4}e^{-2i(dx+c)}}{8d}+\frac{2ib^{4}}{d\left(e^{2i(dx+c)}+1\right)}-$
norman	$\frac{\left(-\frac{1}{2}a^{4}-6a^{2}b^{2}\right)x+\left(-\frac{1}{2}a^{4}-6a^{2}b^{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+\left(\frac{1}{2}a^{4}+6a^{2}b^{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\left(\frac{1}{2}a^{4}+6a^{2}b^{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(-}{}$

input
```
int(cos(d*x+c)^2*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+4*b*a^3*sin(d*x+c)+6*a^
2*b^2*(d*x+c)+4*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+b^4*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.07

$$\int \cos^{2}(c+dx)(a+b\sec(c+dx))^{4}\,dx$$
$$=\frac{4\,ab^{3}\cos(dx+c)\log(\sin(dx+c)+1)-4\,ab^{3}\cos(dx+c)\log(-\sin(dx+c)+1)+(a^{4}+12\,a^{2}b^{2})dx\cos}{2\,d\cos(dx+c)}$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/2*(4*a*b^3*cos(d*x + c)*log(sin(d*x + c) + 1) - 4*a*b^3*cos(d*x + c)*log
(-sin(d*x + c) + 1) + (a^4 + 12*a^2*b^2)*d*x*cos(d*x + c) + (a^4*cos(d*x +
 c)^2 + 8*a^3*b*cos(d*x + c) + 2*b^4)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^4\,dx=\int (a+b\sec(c+dx))^4\cos^2(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**4,x)
```

output
```
Integral((a + b*sec(c + d*x))**4*cos(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^4+24\,(dx+c)a^2b^2+8\,ab^3(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))}{4\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 24*(d*x + c)*a^2*b^2 + 8*a*b^3
*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 16*a^3*b*sin(d*x + c) +
 4*b^4*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.57

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{8\,ab^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-8\,ab^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{4\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+(a^4+12\,a^2b^2)(}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/2*(8*a*b^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 8*a*b^3*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) - 4*b^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 -
1) + (a^4 + 12*a^2*b^2)*(d*x + c) - 2*(a^4*tan(1/2*d*x + 1/2*c)^3 - 8*a^3*
b*tan(1/2*d*x + 1/2*c)^3 - a^4*tan(1/2*d*x + 1/2*c) - 8*a^3*b*tan(1/2*d*x
+ 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 10.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.39

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{b^4\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{12\,a^2\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^4\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}+\frac{4\,a^3\,b\,\sin\left(c+d\,x\right)}{d}+\frac{8\,a\,b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

input
```
int(cos(c + d*x)^2*(a + b/cos(c + d*x))^4,x)
```

output
```
(a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^4*sin(c + d*x))/(
d*cos(c + d*x)) + (12*a^2*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))
/d + (a^4*cos(c + d*x)*sin(c + d*x))/(2*d) + (4*a^3*b*sin(c + d*x))/d + (8
*a*b^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.56

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{-8\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a\,b^3+8\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a\,b^3+8\cos(dx+c)\sin}{}$$

input
```
int(cos(d*x+c)^2*(a+b*sec(d*x+c))^4,x)
```

output
```
( - 8*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a*b**3 + 8*cos(c + d*x)*log(t
an((c + d*x)/2) + 1)*a*b**3 + 8*cos(c + d*x)*sin(c + d*x)*a**3*b + cos(c +
 d*x)*a**4*c + cos(c + d*x)*a**4*d*x + 12*cos(c + d*x)*a**2*b**2*c + 12*co
s(c + d*x)*a**2*b**2*d*x - sin(c + d*x)**3*a**4 + sin(c + d*x)*a**4 + 2*si
n(c + d*x)*b**4)/(2*cos(c + d*x)*d)
```

3.482 $\int \cos^3(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 115

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^4\,dx = 2ab(a^2+2b^2)\,x+\frac{b^4\text{arctanh}(\sin(c+dx))}{d}+\frac{a^2(2a^2+17b^2)\sin(c+dx)}{3d}+\frac{4a^3b\cos(c+dx)\sin(c+dx)}{3d}+\frac{a^2\cos^2(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{3d}$$

output

```
2*a*b*(a^2+2*b^2)*x+b^4*arctanh(sin(d*x+c))/d+1/3*a^2*(2*a^2+17*b^2)*sin(d
*x+c)/d+4/3*a^3*b*cos(d*x+c)*sin(d*x+c)/d+1/3*a^2*cos(d*x+c)^2*(a+b*sec(d*
x+c))^2*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.11

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{24ab(a^2+2b^2)(c+dx)-12b^4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12b^4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\right.}{12d}$$

input

```
Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^4,x]
```

output
```
(24*a*b*(a^2 + 2*b^2)*(c + d*x) - 12*b^4*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] + 12*b^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 9*a^2*(a^2 + 8
*b^2)*Sin[c + d*x] + 12*a^3*b*Sin[2*(c + d*x)] + a^4*Sin[3*(c + d*x)])/(12
*d)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4328, 3042, 4562, 27, 3042, 4535, 24, 3042, 4533, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^4\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \ 4328$$

$$\frac{1}{3}\int \cos^2(c+dx)(a+b\sec(c+dx))\left(3\sec^2(c+dx)b^3+8a^2b+a\left(2a^2+9b^2\right)\sec(c+dx)\right)dx+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{3}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(3\csc\left(c+dx+\frac{\pi}{2}\right)^2b^3+8a^2b+a(2a^2+9b^2)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \ 4562$$

$$\frac{1}{3}\left(\frac{4a^3b\sin(c+dx)\cos(c+dx)}{d}-\frac{1}{2}\int -2\cos(c+dx)\left(3\sec^2(c+dx)b^4+6a\left(a^2+2b^2\right)\sec(c+dx)b+a^2\left(2a^2+\right.\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\int \cos(c+dx)\left(3\sec^2(c+dx)b^4+6a\left(a^2+2b^2\right)\sec(c+dx)b+a^2\left(2a^2+17b^2\right)\right)dx+\frac{4a^3b\sin(c+dx)\cos(c+\right.$$
$$\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\int \frac{3\csc\left(c+dx+\frac{\pi}{2}\right)^2b^4+6a\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)b+a^2\left(2a^2+17b^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{4a^3b\sin(c+dx)\cos(c+d\right.$$
$$\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \text{4535}$$

$$\frac{1}{3}\left(6ab\left(a^2+2b^2\right)\int 1dx+\int \cos(c+dx)\left(3\sec^2(c+dx)b^4+a^2\left(2a^2+17b^2\right)\right)dx+\frac{4a^3b\sin(c+dx)\cos(c+dx)}{d}\right.$$
$$\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{3}\left(\int \cos(c+dx)\left(3\sec^2(c+dx)b^4+a^2\left(2a^2+17b^2\right)\right)dx+\frac{4a^3b\sin(c+dx)\cos(c+dx)}{d}+6abx\left(a^2+2b^2\right)\right)+$$
$$\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\int \frac{3\csc\left(c+dx+\frac{\pi}{2}\right)^2b^4+a^2\left(2a^2+17b^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{4a^3b\sin(c+dx)\cos(c+dx)}{d}+6abx\left(a^2+2b^2\right)\right)+$$
$$\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \text{4533}$$

$$\frac{1}{3}\left(3b^4\int \sec(c+dx)dx+\frac{4a^3b\sin(c+dx)\cos(c+dx)}{d}+\frac{a^2\left(2a^2+17b^2\right)\sin(c+dx)}{d}+6abx\left(a^2+2b^2\right)\right)+$$
$$\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(3b^4\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{4a^3b\sin(c+dx)\cos(c+dx)}{d}+\frac{a^2(2a^2+17b^2)\sin(c+dx)}{d}+6abx(a^2+2b^2)\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^2}{3d}+\frac{1}{3}\left(\frac{4a^3b\sin(c+dx)\cos(c+dx)}{d}+\frac{a^2(2a^2+17b^2)\sin(c+dx)}{d}+6abx(a^2+2b^2)+\frac{3b^4\text{arctanh}(\sin(c+dx))}{d}\right)$$

input
```
Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^4,x]
```

output
```
(a^2*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(3*d) + (6*a*b*(a
^2 + 2*b^2)*x + (3*b^4*ArcTanh[Sin[c + d*x]])/d + (a^2*(2*a^2 + 17*b^2)*Si
n[c + d*x])/d + (4*a^3*b*Cos[c + d*x]*Sin[c + d*x])/d)/3
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4562
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Si
mp[1/(d*n)   Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*
b) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{\frac{a^4(2+\cos(dx+c)^2)\sin(dx+c)}{3}+4b\,a^3\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+6a^2b^2\sin(dx+c)+4a\,b^3(dx+c)+b^4\ln(\sec(dx+c)+\text{ta}}{d}$
default	$\frac{\frac{a^4(2+\cos(dx+c)^2)\sin(dx+c)}{3}+4b\,a^3\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+6a^2b^2\sin(dx+c)+4a\,b^3(dx+c)+b^4\ln(\sec(dx+c)+\text{ta}}{d}$
parallelrisch	$\frac{24a^3bxd+48a\,b^3dx+12b^4\left(\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\right)+12b\,a^3\sin(2dx+2c)+a^4\sin(3dx+3c)+72a^2b^2}{12d}$
risch	$2a^3bx+4a\,b^3x-\frac{3ia^4e^{i(dx+c)}}{8d}-\frac{3ie^{i(dx+c)}a^2b^2}{d}+\frac{3ia^4e^{-i(dx+c)}}{8d}+\frac{3ie^{-i(dx+c)}a^2b^2}{d}+\frac{\ln(e^{i(dx+c)}+i)b}{d}$
norman	$\frac{(-2b\,a^3-4a\,b^3)x+(-6b\,a^3-12a\,b^3)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+(2b\,a^3+4a\,b^3)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}+(6b\,a^3+12a\,b^3)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{}$

input
```
int(cos(d*x+c)^3*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*a^4*(2+cos(d*x+c)^2)*sin(d*x+c)+4*b*a^3*(1/2*sin(d*x+c)*cos(d*x+c
)+1/2*d*x+1/2*c)+6*a^2*b^2*sin(d*x+c)+4*a*b^3*(d*x+c)+b^4*ln(sec(d*x+c)+ta
n(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.85

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{3\,b^4\log\left(\sin\left(dx+c\right)+1\right)-3\,b^4\log\left(-\sin\left(dx+c\right)+1\right)+12\,(a^3b+2\,ab^3)dx+2\,\left(a^4\cos\left(dx+c\right)^2+6\,a^3\right.}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/6*(3*b^4*log(sin(d*x + c) + 1) - 3*b^4*log(-sin(d*x + c) + 1) + 12*(a^3*
b + 2*a*b^3)*d*x + 2*(a^4*cos(d*x + c)^2 + 6*a^3*b*cos(d*x + c) + 2*a^4 +
18*a^2*b^2)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^4\,dx = \int (a+b\sec(c+dx))^4\cos^3(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**4,x)
```

output
```
Integral((a + b*sec(c + d*x))**4*cos(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^4\,dx = -\frac{2\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^4-6\,(2\,dx+2\,c+\sin(2\,dx+2\,c))a^3b-24\,(dx+c)ab^3-3\,b^4(\log(\ldots}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/6*(2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 - 6*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*a^3*b - 24*(d*x + c)*a*b^3 - 3*b^4*(log(sin(d*x + c) + 1) - log(s
in(d*x + c) - 1)) - 36*a^2*b^2*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.84

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{3\,b^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,b^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+6\,(a^3b+2\,ab^3)(dx+c)+\frac{2\left(3\,a^4\tan(\ldots\right.}{\ldots}}{\ldots}$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(3*b^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*b^4*log(abs(tan(1/2*d*x
+ 1/2*c) - 1)) + 6*(a^3*b + 2*a*b^3)*(d*x + c) + 2*(3*a^4*tan(1/2*d*x + 1/
2*c)^5 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 18*a^2*b^2*tan(1/2*d*x + 1/2*c)^
5 + 2*a^4*tan(1/2*d*x + 1/2*c)^3 + 36*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*a
^4*tan(1/2*d*x + 1/2*c) + 6*a^3*b*tan(1/2*d*x + 1/2*c) + 18*a^2*b^2*tan(1/
2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 10.37 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.37

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{3\,a^4\,\sin\left(c+d\,x\right)}{4\,d}+\frac{2\,b^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^4\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{a^3\,b\,\sin\left(2\,c+2\,d\,x\right)}{d}+\frac{6\,a^2\,b^2\,\sin\left(c+d\,x\right)}{d}+\frac{8\,a\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{4\,a^3\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

input
```
int(cos(c + d*x)^3*(a + b/cos(c + d*x))^4,x)
```

output
```
(3*a^4*sin(c + d*x))/(4*d) + (2*b^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*
x)/2)))/d + (a^4*sin(3*c + 3*d*x))/(12*d) + (a^3*b*sin(2*c + 2*d*x))/d + (
6*a^2*b^2*sin(c + d*x))/d + (8*a*b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*
x)/2)))/d + (4*a^3*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.10

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$=\frac{6\cos(dx+c)\sin(dx+c)\,a^3b-3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b^4+3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)b^4-\sin(dx+c)^3\,a}{3d}$$

input
```
int(cos(d*x+c)^3*(a+b*sec(d*x+c))^4,x)
```

output
```
(6*cos(c + d*x)*sin(c + d*x)*a**3*b - 3*log(tan((c + d*x)/2) - 1)*b**4 + 3
*log(tan((c + d*x)/2) + 1)*b**4 - sin(c + d*x)**3*a**4 + 3*sin(c + d*x)*a*
*4 + 18*sin(c + d*x)*a**2*b**2 + 6*a**3*b*c + 6*a**3*b*d*x + 12*a*b**3*c +
 12*a*b**3*d*x)/(3*d)
```

3.483 $\int \cos^4(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 145

$$\begin{aligned}\int \cos^4(c+dx)(a+b\sec(c+dx))^4\,dx &= \frac{1}{8}\left(3a^4+24a^2b^2+8b^4\right)x+\frac{4ab(2a^2+3b^2)\sin(c+dx)}{3d}\\ &\quad+\frac{a^2(3a^2+22b^2)\cos(c+dx)\sin(c+dx)}{8d}+\frac{5a^3b\cos^2(c+dx)\sin(c+dx)}{6d}\\ &\quad+\frac{a^2\cos^3(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{4d}\end{aligned}$$

output
```
1/8*(3*a^4+24*a^2*b^2+8*b^4)*x+4/3*a*b*(2*a^2+3*b^2)*sin(d*x+c)/d+1/8*a^2*
(3*a^2+22*b^2)*cos(d*x+c)*sin(d*x+c)/d+5/6*a^3*b*cos(d*x+c)^2*sin(d*x+c)/d
+1/4*a^2*cos(d*x+c)^3*(a+b*sec(d*x+c))^2*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.72

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$=\frac{12(3a^4+24a^2b^2+8b^4)\,(c+dx)+96ab(3a^2+4b^2)\sin(c+dx)+24a^2(a^2+6b^2)\sin(2(c+dx))+32a^3b\,\mathrm{s}}{96d}$$

input
```
Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^4,x]
```

output
```
(12*(3*a^4 + 24*a^2*b^2 + 8*b^4)*(c + d*x) + 96*a*b*(3*a^2 + 4*b^2)*Sin[c
+ d*x] + 24*a^2*(a^2 + 6*b^2)*Sin[2*(c + d*x)] + 32*a^3*b*Sin[3*(c + d*x)]
 + 3*a^4*Sin[4*(c + d*x)])/(96*d)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {3042, 4328, 3042, 4562, 25, 3042, 4535, 3042, 3117, 4533, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{4328}$$

$$\frac{1}{4}\int \cos^3(c+dx)(a+b\sec(c+dx))\left(10ba^2+3\left(a^2+4b^2\right)\sec(c+dx)a+b\left(a^2+4b^2\right)\sec^2(c+dx)\right)dx+\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^2}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(10ba^2+3\left(a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^2}{4d}$$

$\downarrow$ 4562

$$\frac{1}{4}\left(\frac{10a^3b\sin(c+dx)\cos^2(c+dx)}{3d}-\frac{1}{3}\int -\cos^2(c+dx)\left(3\left(3a^2+22b^2\right)a^2+16b\left(2a^2+3b^2\right)\sec(c+dx)a+3b^2\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^2}{4d}$$

$\downarrow$ 25

$$\frac{1}{4}\left(\frac{1}{3}\int \cos^2(c+dx)\left(3\left(3a^2+22b^2\right)a^2+16b\left(2a^2+3b^2\right)\sec(c+dx)a+3b^2\left(a^2+4b^2\right)\sec^2(c+dx)\right)dx+\frac{10a^3b}{}\right.$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^2}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\int \frac{3\left(3a^2+22b^2\right)a^2+16b\left(2a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+3b^2\left(a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{10a^3b\sin(}{}\right.$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^2}{4d}$$

$\downarrow$ 4535

$$\frac{1}{4}\left(\frac{1}{3}\left(16ab\left(2a^2+3b^2\right)\int\cos(c+dx)dx+\int\cos^2(c+dx)\left(3\left(3a^2+22b^2\right)a^2+3b^2\left(a^2+4b^2\right)\sec^2(c+dx)\right)dx\right)\right.$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^2}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\left(16ab\left(2a^2+3b^2\right)\int\sin\left(c+dx+\frac{\pi}{2}\right)dx+\int\frac{3\left(3a^2+22b^2\right)a^2+3b^2\left(a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx\right)+\right.$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^2}{4d}$$

$\downarrow$ 3117

$$\frac{1}{4}\left(\frac{1}{3}\left(\int \frac{3\left(3a^2+22b^2\right)a^2+3b^2\left(a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{16ab\left(2a^2+3b^2\right)\sin(c+dx)}{d}\right)+\frac{10a^3b\sin(c+\right.$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^2}{4d}$$

$\Big\downarrow$ 4533

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\left(3a^4+24a^2b^2+8b^4\right)\int 1dx+\frac{16ab\left(2a^2+3b^2\right)\sin(c+dx)}{d}+\frac{3a^2\left(3a^2+22b^2\right)\sin(c+dx)\cos(c+dx)}{2d}\right)\right.$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^2}{4d}$$

$\Big\downarrow$ 24

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^2}{4d}+$$
$$\frac{1}{4}\left(\frac{10a^3b\sin(c+dx)\cos^2(c+dx)}{3d}+\frac{1}{3}\left(\frac{16ab\left(2a^2+3b^2\right)\sin(c+dx)}{d}+\frac{3a^2\left(3a^2+22b^2\right)\sin(c+dx)\cos(c+dx)}{2d}\right.\right.$$

input
```
Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^4,x]
```

output
```
(a^2*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(4*d) + ((10*a^3*
b*Cos[c + d*x]^2*Sin[c + d*x])/(3*d) + ((3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*x)
/2 + (16*a*b*(2*a^2 + 3*b^2)*Sin[c + d*x])/d + (3*a^2*(3*a^2 + 22*b^2)*Cos
[c + d*x]*Sin[c + d*x])/(2*d))/3)/4
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4562
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Si
mp[1/(d*n)   Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*
b) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.70

method	result
parallelrisch	$\frac{24(a^4+6a^2b^2)\sin(2dx+2c)+32b\,a^3\sin(3dx+3c)+3a^4\sin(4dx+4c)+96(3b\,a^3+4a\,b^3)\sin(dx+c)+36xd(a^4+8a^2b^2+\frac{8}{3}b}{96d}$
derivativedivides	$\frac{a^4\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{4b\,a^3\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+6a^2b^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}\right.}{d}$
default	$\frac{a^4\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{4b\,a^3\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+6a^2b^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}\right.}{d}$
risch	$\frac{3a^4x}{8}+3a^2b^2x+x\,b^4+\frac{3a^3b\sin(dx+c)}{d}+\frac{4\sin(dx+c)a\,b^3}{d}+\frac{a^4\sin(4dx+4c)}{32d}+\frac{b\,a^3\sin(3dx+3c)}{3d}+\sin($
norman	$\left(-\frac{3}{8}a^4-3a^2b^2-b^4\right)x+\left(-\frac{9}{8}a^4-9a^2b^2-3b^4\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+\left(-\frac{9}{8}a^4-9a^2b^2-3b^4\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+\left(-\frac{3}{8}a^4-3a^2b^2-b\right.$

input
```
int(cos(d*x+c)^4*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/96*(24*(a^4+6*a^2*b^2)*sin(2*d*x+2*c)+32*b*a^3*sin(3*d*x+3*c)+3*a^4*sin(
4*d*x+4*c)+96*(3*a^3*b+4*a*b^3)*sin(d*x+c)+36*x*d*(a^4+8*a^2*b^2+8/3*b^4))
/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.66

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$=\frac{3\left(3\,a^4+24\,a^2b^2+8\,b^4\right)dx+\left(6\,a^4\cos\left(dx+c\right)^3+32\,a^3b\cos\left(dx+c\right)^2+64\,a^3b+96\,ab^3+9\left(a^4+8\,a^2b^2\right)\right.}{24\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/24*(3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*d*x + (6*a^4*cos(d*x + c)^3 + 32*a^3*
b*cos(d*x + c)^2 + 64*a^3*b + 96*a*b^3 + 9*(a^4 + 8*a^2*b^2)*cos(d*x + c))
*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.75

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{3\,(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right))a^4-128\,\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^3b+144}{96\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4 - 128*
(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3*b + 144*(2*d*x + 2*c + sin(2*d*x + 2
*c))*a^2*b^2 + 96*(d*x + c)*b^4 + 384*a*b^3*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(135) = 270$.

Time = 0.15 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.19

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{3\,(3\,a^4+24\,a^2b^2+8\,b^4)(dx+c) - \frac{2\left(15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-96\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+72\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-96\,ab^3\tan\left(\frac{1}{2}\,dx+\right.\right.}{}}{}$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/24*(3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*(d*x + c) - 2*(15*a^4*tan(1/2*d*x + 1
/2*c)^7 - 96*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c
)^7 - 96*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 9*a^4*tan(1/2*d*x + 1/2*c)^5 - 160
*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 288*a*
b^3*tan(1/2*d*x + 1/2*c)^5 + 9*a^4*tan(1/2*d*x + 1/2*c)^3 - 160*a^3*b*tan(
1/2*d*x + 1/2*c)^3 - 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 288*a*b^3*tan(1/2
*d*x + 1/2*c)^3 - 15*a^4*tan(1/2*d*x + 1/2*c) - 96*a^3*b*tan(1/2*d*x + 1/2
*c) - 72*a^2*b^2*tan(1/2*d*x + 1/2*c) - 96*a*b^3*tan(1/2*d*x + 1/2*c))/(ta
n(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 10.18 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.85

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{3\,a^4\,x}{8}+b^4\,x+3\,a^2\,b^2\,x+\frac{a^4\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{a^4\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{a^3\,b\,\sin\left(3\,c+3\,d\,x\right)}{3\,d}+\frac{3\,a^2\,b^2\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}+\frac{4\,a\,b^3\,\sin\left(c+d\,x\right)}{d}+\frac{3\,a^3\,b\,\sin\left(c+d\,x\right)}{d}$$

input
```
int(cos(c + d*x)^4*(a + b/cos(c + d*x))^4,x)
```

output
```
(3*a^4*x)/8 + b^4*x + 3*a^2*b^2*x + (a^4*sin(2*c + 2*d*x))/(4*d) + (a^4*si
n(4*c + 4*d*x))/(32*d) + (a^3*b*sin(3*c + 3*d*x))/(3*d) + (3*a^2*b^2*sin(2
*c + 2*d*x))/(2*d) + (4*a*b^3*sin(c + d*x))/d + (3*a^3*b*sin(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.86

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{-6\cos(dx+c)\sin(dx+c)^3 a^4 + 15\cos(dx+c)\sin(dx+c)a^4 + 72\cos(dx+c)\sin(dx+c)a^2b^2 - 32\,\mathrm{s}}{24d}$$

input
```
int(cos(d*x+c)^4*(a+b*sec(d*x+c))^4,x)
```

output
```
( - 6*cos(c + d*x)*sin(c + d*x)**3*a**4 + 15*cos(c + d*x)*sin(c + d*x)*a**
4 + 72*cos(c + d*x)*sin(c + d*x)*a**2*b**2 - 32*sin(c + d*x)**3*a**3*b + 9
6*sin(c + d*x)*a**3*b + 96*sin(c + d*x)*a*b**3 + 9*a**4*d*x + 72*a**2*b**2
*d*x + 24*b**4*d*x)/(24*d)
```

3.484 $\int \cos^5(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 173

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{1}{2}ab(3a^2+4b^2)\,x+\frac{(4a^4+29a^2b^2+5b^4)\sin(c+dx)}{5d} + \frac{ab(3a^2+4b^2)\cos(c+dx)\sin(c+dx)}{2d}+\frac{3a^3b\cos^3(c+dx)\sin(c+dx)}{5d} + \frac{a^2\cos^4(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{5d}-\frac{a^2(4a^2+27b^2)\sin^3(c+dx)}{15d}$$

output
```
1/2*a*b*(3*a^2+4*b^2)*x+1/5*(4*a^4+29*a^2*b^2+5*b^4)*sin(d*x+c)/d+1/2*a*b*
(3*a^2+4*b^2)*cos(d*x+c)*sin(d*x+c)/d+3/5*a^3*b*cos(d*x+c)^3*sin(d*x+c)/d+
1/5*a^2*cos(d*x+c)^4*(a+b*sec(d*x+c))^2*sin(d*x+c)/d-1/15*a^2*(4*a^2+27*b^
2)*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.77

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$=\frac{30(5a^4+36a^2b^2+8b^4)\sin(c+dx)+a(360a^2bc+480b^3c+360a^2bdx+480b^3dx+240b(a^2+b^2)\sin(2(c}{240d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4,x]
```

output
```
(30*(5*a^4 + 36*a^2*b^2 + 8*b^4)*Sin[c + d*x] + a*(360*a^2*b*c + 480*b^3*c
 + 360*a^2*b*d*x + 480*b^3*d*x + 240*b*(a^2 + b^2)*Sin[2*(c + d*x)] + 5*(5
*a^3 + 24*a*b^2)*Sin[3*(c + d*x)] + 30*a^2*b*Sin[4*(c + d*x)] + 3*a^3*Sin[
5*(c + d*x)]))/(240*d)
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.95, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4328, 3042, 4562, 27, 3042, 4535, 3042, 3115, 24, 4532, 3042, 3492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{4328}$$

$$\frac{1}{5}\int \cos^4(c+dx)(a+b\sec(c+dx))\left(12ba^2+\left(4a^2+15b^2\right)\sec(c+dx)a+b\left(2a^2+5b^2\right)\sec^2(c+dx)\right)dx+\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(12ba^2+\left(4a^2+15b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(2a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{5d}$$

$$\Big\downarrow \; 4562$$

$$\frac{1}{5}\left(\frac{3a^3b\sin(c+dx)\cos^3(c+dx)}{d}-\frac{1}{4}\int -4\cos^3(c+dx)\left(\left(4a^2+27b^2\right)a^2+5b\left(3a^2+4b^2\right)\sec(c+dx)a+b^2\left(2a^2\right.\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{5d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}\left(\int \cos^3(c+dx)\left(\left(4a^2+27b^2\right)a^2+5b\left(3a^2+4b^2\right)\sec(c+dx)a+b^2\left(2a^2+5b^2\right)\sec^2(c+dx)\right)dx+\frac{3a^3b\sin(c\ldots}{}\right.$$
$$\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(\int \frac{\left(4a^2+27b^2\right)a^2+5b\left(3a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b^2\left(2a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{3a^3b\sin(c+d\ldots}{\ldots}\right.$$
$$\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{5d}$$

$$\Big\downarrow \; 4535$$

$$\frac{1}{5}\left(5ab\left(3a^2+4b^2\right)\int \cos^2(c+dx)dx+\int \cos^3(c+dx)\left(\left(4a^2+27b^2\right)a^2+b^2\left(2a^2+5b^2\right)\sec^2(c+dx)\right)dx+\frac{3a^3\ldots}{}\right.$$
$$\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(5ab\left(3a^2+4b^2\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\int \frac{\left(4a^2+27b^2\right)a^2+b^2\left(2a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{3a^3b\sin\ldots}{}\right.$$
$$\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{5d}$$

$$\Big\downarrow \; 3115$$

$$\frac{1}{5}\left(\int \frac{\left(4a^2+27b^2\right)a^2+b^2\left(2a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+5ab\left(3a^2+4b^2\right)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{5d}$$

$\Big\downarrow$ 24

$$\frac{1}{5}\left(\int \frac{\left(4a^2+27b^2\right)a^2+b^2\left(2a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{3a^3b\sin(c+dx)\cos^3(c+dx)}{d}+5ab\left(3a^2+4b^2\right)\left(\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{5d}$$

$\Big\downarrow$ 4532

$$\frac{1}{5}\left(\int \cos(c+dx)\left(\left(2a^2+5b^2\right)b^2+a^2\left(4a^2+27b^2\right)\cos^2(c+dx)\right)dx+\frac{3a^3b\sin(c+dx)\cos^3(c+dx)}{d}+5ab\left(3a^2\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{5d}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(\left(2a^2+5b^2\right)b^2+a^2\left(4a^2+27b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+\frac{3a^3b\sin(c+dx)\cos^3(c+dx)}{d}\right.$$
$$\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{5d}$$

$\Big\downarrow$ 3492

$$\frac{1}{5}\left(-\frac{\int\left(4a^4+29b^2a^2-\left(4a^2+27b^2\right)\sin^2(c+dx)a^2+5b^4\right)d(-\sin(c+dx))}{d}+\frac{3a^3b\sin(c+dx)\cos^3(c+dx)}{d}+5\right.$$
$$\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{5d}$$

$\Big\downarrow$ 2009

$$\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{5d}+$$
$$\frac{1}{5}\left(\frac{3a^3b\sin(c+dx)\cos^3(c+dx)}{d}+5ab\left(3a^2+4b^2\right)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{\frac{1}{3}a^2\left(4a^2+27b^2\right)\sin^3(c+\right.\right.$$

input
```
Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4,x]
```

output
```
(a^2*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(5*d) + ((3*a^3*b
*Cos[c + d*x]^3*Sin[c + d*x])/d + 5*a*b*(3*a^2 + 4*b^2)*(x/2 + (Cos[c + d*
x]*Sin[c + d*x])/(2*d)) - (-((4*a^4 + 29*a^2*b^2 + 5*b^4)*Sin[c + d*x]) +
(a^2*(4*a^2 + 27*b^2)*Sin[c + d*x]^3)/3)/d)/5
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4532
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4562
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Si
mp[1/(d*n)   Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*
b) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.72

method	result
parallelrisch	$\frac{240(b\,a^3+a\,b^3)\sin(2dx+2c)+5(5a^4+24a^2b^2)\sin(3dx+3c)+30b\,a^3\sin(4dx+4c)+3a^4\sin(5dx+5c)+30(5a^4+36a^2b^2+}{240d}$
derivativedivides	$\frac{\frac{a^4\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+4b\,a^3\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2a^2b^2\left(2+\cos(dx\right.}{d}$
default	$\frac{\frac{a^4\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+4b\,a^3\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2a^2b^2\left(2+\cos(dx\right.}{d}$
risch	$\frac{3a^3bx}{2}+2a\,b^3x+\frac{5a^4\sin(dx+c)}{8d}+\frac{9\sin(dx+c)a^2b^2}{2d}+\frac{\sin(dx+c)b^4}{d}+\frac{a^4\sin(5dx+5c)}{80d}+\frac{b\,a^3\sin(4dx+4c)}{8d}$
norman	$\frac{\left(-\frac{3}{2}b\,a^3-2a\,b^3\right)x+\left(\frac{3}{2}b\,a^3+2a\,b^3\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{16}+\left(-9b\,a^3-12a\,b^3\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+\left(-3b\,a^3-4a\,b^3\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{}$

input
```
int(cos(d*x+c)^5*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/240*(240*(a^3*b+a*b^3)*sin(2*d*x+2*c)+5*(5*a^4+24*a^2*b^2)*sin(3*d*x+3*c
)+30*b*a^3*sin(4*d*x+4*c)+3*a^4*sin(5*d*x+5*c)+30*(5*a^4+36*a^2*b^2+8*b^4)
*sin(d*x+c)+360*x*a*(a^2+4/3*b^2)*d*b)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.70

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$=\frac{15\left(3\,a^3b+4\,ab^3\right)dx+\left(6\,a^4\cos\left(dx+c\right)^4+30\,a^3b\cos\left(dx+c\right)^3+16\,a^4+120\,a^2b^2+30\,b^4+4\left(2\,a^4+15\right.\right.}{30\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/30*(15*(3*a^3*b + 4*a*b^3)*d*x + (6*a^4*cos(d*x + c)^4 + 30*a^3*b*cos(d*
x + c)^3 + 16*a^4 + 120*a^2*b^2 + 30*b^4 + 4*(2*a^4 + 15*a^2*b^2)*cos(d*x
+ c)^2 + 15*(3*a^3*b + 4*a*b^3)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.77

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{8\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^4+15\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\sin(2\right.}{}$$

input
```
integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/120*(8*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4 + 15
*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3*b - 240*(sin(
d*x + c)^3 - 3*sin(d*x + c))*a^2*b^2 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c)
)*a*b^3 + 120*b^4*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. 2(161) = 322.

Time = 0.16 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.46

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{15\left(3\,a^3b+4\,ab^3\right)(dx+c)+\frac{2\left(30\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-75\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+180\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-60\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+3\right.}{}}{}$$

input
```
integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/30*(15*(3*a^3*b + 4*a*b^3)*(d*x + c) + 2*(30*a^4*tan(1/2*d*x + 1/2*c)^9
- 75*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 180*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 6
0*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 30*b^4*tan(1/2*d*x + 1/2*c)^9 + 40*a^4*ta
n(1/2*d*x + 1/2*c)^7 - 30*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 480*a^2*b^2*tan(1
/2*d*x + 1/2*c)^7 - 120*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 120*b^4*tan(1/2*d*x
 + 1/2*c)^7 + 116*a^4*tan(1/2*d*x + 1/2*c)^5 + 600*a^2*b^2*tan(1/2*d*x + 1
/2*c)^5 + 180*b^4*tan(1/2*d*x + 1/2*c)^5 + 40*a^4*tan(1/2*d*x + 1/2*c)^3 +
 30*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 480*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 12
0*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 120*b^4*tan(1/2*d*x + 1/2*c)^3 + 30*a^4*t
an(1/2*d*x + 1/2*c) + 75*a^3*b*tan(1/2*d*x + 1/2*c) + 180*a^2*b^2*tan(1/2*
d*x + 1/2*c) + 60*a*b^3*tan(1/2*d*x + 1/2*c) + 30*b^4*tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 13.96 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.91

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{\left(2\,a^4-5\,a^3\,b+12\,a^2\,b^2-4\,a\,b^3+2\,b^4\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{8\,a^4}{3}-2\,a^3\,b+32\,a^2\,b^2-8\,a\,b^3+8\,b^4\right)\,\tan\left(\frac{c}{2}+\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+5\right.} + \frac{a\,b\,\mathrm{atan}\left(\frac{a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\,a^2+4\,b^2\right)}{3\,a^3\,b+4\,a\,b^3}\right)\,\left(3\,a^2+4\,b^2\right)}{d}$$

input
```
int(cos(c + d*x)^5*(a + b/cos(c + d*x))^4,x)
```

output
```
(tan(c/2 + (d*x)/2)^5*((116*a^4)/15 + 12*b^4 + 40*a^2*b^2) + tan(c/2 + (d*
x)/2)^9*(2*a^4 - 5*a^3*b - 4*a*b^3 + 2*b^4 + 12*a^2*b^2) + tan(c/2 + (d*x)
/2)^3*(8*a*b^3 + 2*a^3*b + (8*a^4)/3 + 8*b^4 + 32*a^2*b^2) + tan(c/2 + (d*
x)/2)^7*((8*a^4)/3 - 2*a^3*b - 8*a*b^3 + 8*b^4 + 32*a^2*b^2) + tan(c/2 + (
d*x)/2)*(4*a*b^3 + 5*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2))/(d*(5*tan(c/2 +
(d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2
 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) + (a*b*atan((a*b*tan(c/2 + (d*
x)/2)*(3*a^2 + 4*b^2))/(4*a*b^3 + 3*a^3*b))*(3*a^2 + 4*b^2))/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.90

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{-30\cos(dx+c)\sin(dx+c)^3 a^3b + 75\cos(dx+c)\sin(dx+c)a^3b + 60\cos(dx+c)\sin(dx+c)ab^3 + 6}{}$$

input
```
int(cos(d*x+c)^5*(a+b*sec(d*x+c))^4,x)
```

output
```
( - 30*cos(c + d*x)*sin(c + d*x)**3*a**3*b + 75*cos(c + d*x)*sin(c + d*x)*
a**3*b + 60*cos(c + d*x)*sin(c + d*x)*a*b**3 + 6*sin(c + d*x)**5*a**4 - 20
*sin(c + d*x)**3*a**4 - 60*sin(c + d*x)**3*a**2*b**2 + 30*sin(c + d*x)*a**
4 + 180*sin(c + d*x)*a**2*b**2 + 30*sin(c + d*x)*b**4 + 45*a**3*b*d*x + 60
*a*b**3*d*x)/(30*d)
```

3.485 $\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 213

$$\begin{aligned}&\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx\\&=\frac{1}{16}\left(5a^4+36a^2b^2+8b^4\right)x+\frac{4ab(4a^2+5b^2)\sin(c+dx)}{5d}\\&+\frac{(5a^4+36a^2b^2+8b^4)\cos(c+dx)\sin(c+dx)}{16d}\\&+\frac{a^2(5a^2+32b^2)\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{7a^3b\cos^4(c+dx)\sin(c+dx)}{15d}\\&+\frac{a^2\cos^5(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{6d}-\frac{4ab(4a^2+5b^2)\sin^3(c+dx)}{15d}\end{aligned}$$

output
```
1/16*(5*a^4+36*a^2*b^2+8*b^4)*x+4/5*a*b*(4*a^2+5*b^2)*sin(d*x+c)/d+1/16*(5
*a^4+36*a^2*b^2+8*b^4)*cos(d*x+c)*sin(d*x+c)/d+1/24*a^2*(5*a^2+32*b^2)*cos
(d*x+c)^3*sin(d*x+c)/d+7/15*a^3*b*cos(d*x+c)^4*sin(d*x+c)/d+1/6*a^2*cos(d*
x+c)^5*(a+b*sec(d*x+c))^2*sin(d*x+c)/d-4/15*a*b*(4*a^2+5*b^2)*sin(d*x+c)^3
/d
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.73

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$=\frac{60(5a^4+36a^2b^2+8b^4)(c+dx)+480ab(5a^2+6b^2)\sin(c+dx)+15(15a^4+96a^2b^2+16b^4)\sin(2(c+dx$$

input
```
Integrate[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4,x]
```

output
```
(60*(5*a^4 + 36*a^2*b^2 + 8*b^4)*(c + d*x) + 480*a*b*(5*a^2 + 6*b^2)*Sin[c
 + d*x] + 15*(15*a^4 + 96*a^2*b^2 + 16*b^4)*Sin[2*(c + d*x)] + 80*a*b*(5*a
^2 + 4*b^2)*Sin[3*(c + d*x)] + 45*a^2*(a^2 + 4*b^2)*Sin[4*(c + d*x)] + 48*
a^3*b*Sin[5*(c + d*x)] + 5*a^4*Sin[6*(c + d*x)])/(960*d)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.91, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4328, 3042, 4562, 25, 3042, 4535, 3042, 3113, 2009, 4533, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{4328}$$

$$\frac{1}{6}\int \cos^5(c+dx)(a+b\sec(c+dx))\left(14ba^2+\left(5a^2+18b^2\right)\sec(c+dx)a+3b\left(a^2+2b^2\right)\sec^2(c+dx)\right)dx+\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))^2}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{6}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(14ba^2+\left(5a^2+18b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+3b\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+}{}$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))^2}{6d}$$

$$\Big\downarrow \text{4562}$$

$$\frac{1}{6}\left(\frac{14a^3b\sin(c+dx)\cos^4(c+dx)}{5d}-\frac{1}{5}\int -\cos^4(c+dx)\left(5\left(5a^2+32b^2\right)a^2+24b\left(4a^2+5b^2\right)\sec(c+dx)a+15b^2\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))^2}{6d}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{6}\left(\frac{1}{5}\int \cos^4(c+dx)\left(5\left(5a^2+32b^2\right)a^2+24b\left(4a^2+5b^2\right)\sec(c+dx)a+15b^2\left(a^2+2b^2\right)\sec^2(c+dx)\right)dx+\frac{14a^3}{}\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))^2}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{1}{5}\int \frac{5\left(5a^2+32b^2\right)a^2+24b\left(4a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+15b^2\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{14a^3b\sin}{}\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))^2}{6d}$$

$$\Big\downarrow \text{4535}$$

$$\frac{1}{6}\left(\frac{1}{5}\left(24ab\left(4a^2+5b^2\right)\int \cos^3(c+dx)dx+\int \cos^4(c+dx)\left(5\left(5a^2+32b^2\right)a^2+15b^2\left(a^2+2b^2\right)\sec^2(c+dx)\right)dx\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))^2}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{1}{5}\left(24ab\left(4a^2+5b^2\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx+\int \frac{5\left(5a^2+32b^2\right)a^2+15b^2\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx\right)\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))^2}{6d}$$

$$\Big\downarrow \text{3113}$$

$$\frac{1}{6}\left(\frac{1}{5}\left(\int \frac{5(5a^2+32b^2)a^2+15b^2(a^2+2b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx-\frac{24ab(4a^2+5b^2)\int\left(1-\sin^2(c+dx)\right)d(-\sin\right.\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))^2}{6d}$$

$\Big\downarrow$ 2009

$$\frac{1}{6}\left(\frac{1}{5}\left(\int \frac{5(5a^2+32b^2)a^2+15b^2(a^2+2b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx-\frac{24ab(4a^2+5b^2)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+d\right.}{d}\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))^2}{6d}$$

$\Big\downarrow$ 4533

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{15}{4}(5a^4+36a^2b^2+8b^4)\int\cos^2(c+dx)dx-\frac{24ab(4a^2+5b^2)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{5a^2(5a^2-}{}\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))^2}{6d}$$

$\Big\downarrow$ 3042

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{15}{4}(5a^4+36a^2b^2+8b^4)\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{24ab(4a^2+5b^2)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{5a^2}{}\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))^2}{6d}$$

$\Big\downarrow$ 3115

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{15}{4}(5a^4+36a^2b^2+8b^4)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{24ab(4a^2+5b^2)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+\right.}{d}\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))^2}{6d}$$

$\Big\downarrow$ 24

$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))^2}{6d}+$$
$$\frac{1}{6}\left(\frac{14a^3b\sin(c+dx)\cos^4(c+dx)}{5d}+\frac{1}{5}\left(-\frac{24ab(4a^2+5b^2)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{5a^2(5a^2+32b^2)\sin}{}\right.\right.$$

input
```
Int[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4,x]
```

output
```
(a^2*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(6*d) + ((14*a^3*
b*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) + ((5*a^2*(5*a^2 + 32*b^2)*Cos[c + d*
x]^3*Sin[c + d*x])/(4*d) + (15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*(x/2 + (Cos[c
+ d*x]*Sin[c + d*x])/(2*d)))/4 - (24*a*b*(4*a^2 + 5*b^2)*(-Sin[c + d*x] +
Sin[c + d*x]^3/3))/d)/5)/6
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4562
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Si
mp[1/(d*n)   Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*
b) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 2.53 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.72

method	result
parallelrisch	$\frac{(225a^4+1440a^2b^2+240b^4)\sin(2dx+2c)+(400b\,a^3+320a\,b^3)\sin(3dx+3c)+(45a^4+180a^2b^2)\sin(4dx+4c)+48b\,a^3\sin(\ldots}{960d}$
derivativedivides	$\frac{a^4\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{4b\,a^3\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+6\ldots}{}$
default	$\frac{a^4\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{4b\,a^3\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+6\ldots}{}$
risch	$\frac{5a^4x}{16}+\frac{9a^2b^2x}{4}+\frac{x\,b^4}{2}+\frac{5a^3b\sin(dx+c)}{2d}+\frac{3\sin(dx+c)a\,b^3}{d}+\frac{a^4\sin(6dx+6c)}{192d}+\frac{b\,a^3\sin(5dx+5c)}{20d}+\frac{3a^4\sin\ldots}{}$

input
```
int(cos(d*x+c)^6*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/960*((225*a^4+1440*a^2*b^2+240*b^4)*sin(2*d*x+2*c)+(400*a^3*b+320*a*b^3)
*sin(3*d*x+3*c)+(45*a^4+180*a^2*b^2)*sin(4*d*x+4*c)+48*b*a^3*sin(5*d*x+5*c
)+5*a^4*sin(6*d*x+6*c)+(2400*a^3*b+2880*a*b^3)*sin(d*x+c)+300*x*d*(a^4+36/
5*a^2*b^2+8/5*b^4))/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.70

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$=\frac{15\,(5\,a^4+36\,a^2b^2+8\,b^4)dx+\left(40\,a^4\cos(dx+c)^5+192\,a^3b\cos(dx+c)^4+512\,a^3b+640\,ab^3+10\,(5\,a^4\ldots\right.}{}$$

input
```
integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/240*(15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*d*x + (40*a^4*cos(d*x + c)^5 + 192*
a^3*b*cos(d*x + c)^4 + 512*a^3*b + 640*a*b^3 + 10*(5*a^4 + 36*a^2*b^2)*cos
(d*x + c)^3 + 64*(4*a^3*b + 5*a*b^3)*cos(d*x + c)^2 + 15*(5*a^4 + 36*a^2*b
^2 + 8*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**6*(a+b*sec(d*x+c))**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.80

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx = -\frac{5\left(4\sin\left(2\,dx+2\,c\right)^3-60\,dx-60\,c-9\,\sin\left(4\,dx+4\,c\right)-48\,\sin\left(2\,dx+2\,c\right)\right)a^4-256\left(3\,\sin\left(dx+c\right)\right.}{}$$

input
```
integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/960*(5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*
sin(2*d*x + 2*c))*a^4 - 256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin
(d*x + c))*a^3*b - 180*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2
*c))*a^2*b^2 + 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*a*b^3 - 240*(2*d*x +
 2*c + sin(2*d*x + 2*c))*b^4)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(199) = 398.

Time = 0.17 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.58

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/240*(15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*(d*x + c) - 2*(165*a^4*tan(1/2*d*x
+ 1/2*c)^11 - 960*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 900*a^2*b^2*tan(1/2*d*x
+ 1/2*c)^11 - 960*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 120*b^4*tan(1/2*d*x + 1/
2*c)^11 - 25*a^4*tan(1/2*d*x + 1/2*c)^9 - 2240*a^3*b*tan(1/2*d*x + 1/2*c)^
9 + 1260*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 3520*a*b^3*tan(1/2*d*x + 1/2*c)^
9 + 360*b^4*tan(1/2*d*x + 1/2*c)^9 + 450*a^4*tan(1/2*d*x + 1/2*c)^7 - 4992
*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 360*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 5760*
a*b^3*tan(1/2*d*x + 1/2*c)^7 + 240*b^4*tan(1/2*d*x + 1/2*c)^7 - 450*a^4*ta
n(1/2*d*x + 1/2*c)^5 - 4992*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 360*a^2*b^2*tan
(1/2*d*x + 1/2*c)^5 - 5760*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 240*b^4*tan(1/2*
d*x + 1/2*c)^5 + 25*a^4*tan(1/2*d*x + 1/2*c)^3 - 2240*a^3*b*tan(1/2*d*x +
1/2*c)^3 - 1260*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 3520*a*b^3*tan(1/2*d*x +
1/2*c)^3 - 360*b^4*tan(1/2*d*x + 1/2*c)^3 - 165*a^4*tan(1/2*d*x + 1/2*c) -
 960*a^3*b*tan(1/2*d*x + 1/2*c) - 900*a^2*b^2*tan(1/2*d*x + 1/2*c) - 960*a
*b^3*tan(1/2*d*x + 1/2*c) - 120*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1
/2*c)^2 + 1)^6)/d
```

Mupad [B] (verification not implemented)

Time = 10.68 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{5\,a^4\,x}{16}+\frac{b^4\,x}{2}+\frac{9\,a^2\,b^2\,x}{4}+\frac{15\,a^4\,\sin\left(2\,c+2\,d\,x\right)}{64\,d}+\frac{3\,a^4\,\sin\left(4\,c+4\,d\,x\right)}{64\,d}+\frac{a^4\,\sin\left(6\,c+6\,d\,x\right)}{192\,d}+\frac{b^4\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{a\,b^3\,\sin\left(3\,c+3\,d\,x\right)}{3\,d}+\frac{5\,a^3\,b\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{a^3\,b\,\sin\left(5\,c+5\,d\,x\right)}{20\,d}+\frac{3\,a^2\,b^2\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}+\frac{3\,a^2\,b^2\,\sin\left(4\,c+4\,d\,x\right)}{16\,d}+\frac{3\,a\,b^3\,\sin\left(c+d\,x\right)}{d}+\frac{5\,a^3\,b\,\sin\left(c+d\,x\right)}{2\,d}$$

input
```
int(cos(c + d*x)^6*(a + b/cos(c + d*x))^4,x)
```

output
```
(5*a^4*x)/16 + (b^4*x)/2 + (9*a^2*b^2*x)/4 + (15*a^4*sin(2*c + 2*d*x))/(64
*d) + (3*a^4*sin(4*c + 4*d*x))/(64*d) + (a^4*sin(6*c + 6*d*x))/(192*d) + (
b^4*sin(2*c + 2*d*x))/(4*d) + (a*b^3*sin(3*c + 3*d*x))/(3*d) + (5*a^3*b*si
n(3*c + 3*d*x))/(12*d) + (a^3*b*sin(5*c + 5*d*x))/(20*d) + (3*a^2*b^2*sin(
2*c + 2*d*x))/(2*d) + (3*a^2*b^2*sin(4*c + 4*d*x))/(16*d) + (3*a*b^3*sin(c
 + d*x))/d + (5*a^3*b*sin(c + d*x))/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.99

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx$$

$$= \frac{40\cos(dx+c)\sin(dx+c)^5 a^4 - 130\cos(dx+c)\sin(dx+c)^3 a^4 - 360\cos(dx+c)\sin(dx+c)^3 a^2b^2 +}{}$$

input
```
int(cos(d*x+c)^6*(a+b*sec(d*x+c))^4,x)
```

output
```
(40*cos(c + d*x)*sin(c + d*x)**5*a**4 - 130*cos(c + d*x)*sin(c + d*x)**3*a
**4 - 360*cos(c + d*x)*sin(c + d*x)**3*a**2*b**2 + 165*cos(c + d*x)*sin(c
+ d*x)*a**4 + 900*cos(c + d*x)*sin(c + d*x)*a**2*b**2 + 120*cos(c + d*x)*s
in(c + d*x)*b**4 + 192*sin(c + d*x)**5*a**3*b - 640*sin(c + d*x)**3*a**3*b
 - 320*sin(c + d*x)**3*a*b**3 + 960*sin(c + d*x)*a**3*b + 960*sin(c + d*x)
*a*b**3 + 75*a**4*d*x + 540*a**2*b**2*d*x + 120*b**4*d*x)/(240*d)
```

3.486 $\int (a+b\sec(c+dx))^5\,dx$

Optimal result

Integrand size = 12, antiderivative size = 158

$$\int (a+b\sec(c+dx))^5\,dx = a^5x+\frac{b(40a^4+40a^2b^2+3b^4)\operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{ab^2(53a^2+20b^2)\tan(c+dx)}{6d} + \frac{b^3(58a^2+9b^2)\sec(c+dx)\tan(c+dx)}{24d} + \frac{11ab^2(a+b\sec(c+dx))^2\tan(c+dx)}{12d} + \frac{b^2(a+b\sec(c+dx))^3\tan(c+dx)}{4d}$$

output
```
a^5*x+1/8*b*(40*a^4+40*a^2*b^2+3*b^4)*arctanh(sin(d*x+c))/d+1/6*a*b^2*(53*
a^2+20*b^2)*tan(d*x+c)/d+1/24*b^3*(58*a^2+9*b^2)*sec(d*x+c)*tan(d*x+c)/d+1
1/12*a*b^2*(a+b*sec(d*x+c))^2*tan(d*x+c)/d+1/4*b^2*(a+b*sec(d*x+c))^3*tan(
d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.93

$$\int (a+b\sec(c+dx))^5\,dx$$
$$=\frac{24a^5dx+120a^4b\coth^{-1}(\sin(c+dx))+3b^3(40a^2+3b^2)\,\text{arctanh}(\sin(c+dx))+240a^3b^2\tan(c+dx)+1}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^5,x]
```

output
```
(24*a^5*d*x + 120*a^4*b*ArcCoth[Sin[c + d*x]] + 3*b^3*(40*a^2 + 3*b^2)*Arc
Tanh[Sin[c + d*x]] + 240*a^3*b^2*Tan[c + d*x] + 120*a*b^4*Tan[c + d*x] + 1
20*a^2*b^3*Sec[c + d*x]*Tan[c + d*x] + 9*b^5*Sec[c + d*x]*Tan[c + d*x] + 6
*b^5*Sec[c + d*x]^3*Tan[c + d*x] + 40*a*b^4*Tan[c + d*x]^3)/(24*d)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3042, 4269, 3042, 4544, 3042, 4536, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^5dx$$

$$\Big\downarrow \text{4269}$$

$$\frac{1}{4}\int (a+b\sec(c+dx))^2\left(4a^3+11b^2\sec^2(c+dx)a+3b\left(4a^2+b^2\right)\sec(c+dx)\right)dx+\frac{b^2\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(4a^3+11b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2 a+3b(4a^2+b^2)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{b^2\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$\Big\downarrow$ 4544

$$\frac{1}{4}\left(\frac{1}{3}\int (a+b\sec(c+dx))\left(12a^4+b\left(48a^2+31b^2\right)\sec(c+dx)a+b^2\left(58a^2+9b^2\right)\sec^2(c+dx)\right)dx+\frac{11ab^2\tan(c\right.$$
$$\frac{b^2\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\int \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(12a^4+b\left(48a^2+31b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b^2\left(58a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right.\right.\right.$$
$$\frac{b^2\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$\Big\downarrow$ 4536

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \left(24a^5+4b^2\left(53a^2+20b^2\right)\sec^2(c+dx)a+3b\left(40a^4+40b^2a^2+3b^4\right)\sec(c+dx)\right)dx+\frac{b^3\left(58a^2+9b^2\right)}{}\right.\right.$$
$$\frac{b^2\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$\Big\downarrow$ 2009

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{b^3\left(58a^2+9b^2\right)\tan(c+dx)\sec(c+dx)}{2d}+\frac{1}{2}\left(24a^5x+\frac{4ab^2\left(53a^2+20b^2\right)\tan(c+dx)}{d}+\frac{3b\left(40a^4+40a^2b\right.}{}\right.\right.\right.$$
$$\frac{b^2\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

input
```
Int[(a + b*Sec[c + d*x])^5,x]
```

output
```
(b^2*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(4*d) + ((11*a*b^2*(a + b*Sec[c
+ d*x])^2*Tan[c + d*x])/(3*d) + ((b^3*(58*a^2 + 9*b^2)*Sec[c + d*x]*Tan[c
+ d*x])/(2*d) + (24*a^5*x + (3*b*(40*a^4 + 40*a^2*b^2 + 3*b^4)*ArcTanh[Sin
[c + d*x]])/d + (4*a*b^2*(53*a^2 + 20*b^2)*Tan[c + d*x])/d)/2)/3)/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4269
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[1/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*
a^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

rule 4536
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Simp[1/2   Int[Simp[2*A*a + (2*B*a + b*(2*
A + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, e, f, A, B, C}, x]
```

rule 4544
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> Simp[(-C)*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[(
a + b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m
)*Csc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{a^5(dx+c)+5a^4b\ln(\sec(dx+c)+\tan(dx+c))+10b^2a^3\tan(dx+c)+10a^2b^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right.}{d}$
default	$\frac{a^5(dx+c)+5a^4b\ln(\sec(dx+c)+\tan(dx+c))+10b^2a^3\tan(dx+c)+10a^2b^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right.}{d}$
parts	$a^5x+\frac{b^5\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}-\frac{5ab^4\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(}{d}$
parallelrisch	$\frac{-480\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\left(a^4+a^2b^2+\frac{3}{40}b^4\right)b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+480\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)(}{}$
norman	$\frac{a^5x+a^5x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8-4a^5x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+6a^5x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4-4a^5x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6-\frac{5b^2\left(16a^3-8a^2b+8ab^2-b^3\right)\tan}{4d}}{}$
risch	$a^5x-\frac{ib^2\left(120a^2b\,e^{7i(dx+c)}+9b^3e^{7i(dx+c)}-240a^3e^{6i(dx+c)}+120a^2b\,e^{5i(dx+c)}+33b^3e^{5i(dx+c)}-720a^3e^{4i(dx+c)}-240\right.}{}$

input
```
int((a+b*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^5*(d*x+c)+5*a^4*b*ln(sec(d*x+c)+tan(d*x+c))+10*b^2*a^3*tan(d*x+c)+1
0*a^2*b^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-5*a*b^
4*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+b^5*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+
c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.16

$$\int (a+b\sec(c+dx))^5\,dx$$
$$=\frac{48\,a^5dx\cos(dx+c)^4+3\left(40\,a^4b+40\,a^2b^3+3\,b^5\right)\cos(dx+c)^4\log(\sin(dx+c)+1)-3\left(40\,a^4b+40\,a^2\right.}{}$$

input
```
integrate((a+b*sec(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/48*(48*a^5*d*x*cos(d*x + c)^4 + 3*(40*a^4*b + 40*a^2*b^3 + 3*b^5)*cos(d*
x + c)^4*log(sin(d*x + c) + 1) - 3*(40*a^4*b + 40*a^2*b^3 + 3*b^5)*cos(d*x
 + c)^4*log(-sin(d*x + c) + 1) + 2*(40*a*b^4*cos(d*x + c) + 6*b^5 + 80*(3*
a^3*b^2 + a*b^4)*cos(d*x + c)^3 + 3*(40*a^2*b^3 + 3*b^5)*cos(d*x + c)^2)*s
in(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]

$$\int (a+b\sec(c+dx))^5\,dx=\int (a+b\sec(c+dx))^5\;dx$$

input
```
integrate((a+b*sec(d*x+c))**5,x)
```

output
```
Integral((a + b*sec(c + d*x))**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.25

$$\begin{aligned}&\int (a+b\sec(c+dx))^5\,dx\\&=a^5x+\frac{5\left(\tan(dx+c)^3+3\tan(dx+c)\right)ab^4}{3d}\\&\quad-\frac{b^5\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)}{16d}\\&\quad-\frac{5a^2b^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)}{2d}\\&\quad+\frac{5a^4b\log(\sec(dx+c)+\tan(dx+c))}{d}+\frac{10a^3b^2\tan(dx+c)}{d}\end{aligned}$$

input
```
integrate((a+b*sec(d*x+c))^5,x, algorithm="maxima")
```

output
```
a^5*x + 5/3*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^4/d - 1/16*b^5*(2*(3*sin
(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*
log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))/d - 5/2*a^2*b^3*(2*sin(d*
x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1
))/d + 5*a^4*b*log(sec(d*x + c) + tan(d*x + c))/d + 10*a^3*b^2*tan(d*x + c
)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(148) = 296$.

Time = 0.15 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.41

$$\int (a + b\sec(c + dx))^5 \, dx$$

$$= \frac{24\,(dx + c)a^5 + 3\,(40\,a^4b + 40\,a^2b^3 + 3\,b^5)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3\,(40\,a^4b + 40\,a^2b^3 + 3\,b^5)\log\left(\right|}{}$$

input
```
integrate((a+b*sec(d*x+c))^5,x, algorithm="giac")
```

output
```
1/24*(24*(d*x + c)*a^5 + 3*(40*a^4*b + 40*a^2*b^3 + 3*b^5)*log(abs(tan(1/2
*d*x + 1/2*c) + 1)) - 3*(40*a^4*b + 40*a^2*b^3 + 3*b^5)*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) - 2*(240*a^3*b^2*tan(1/2*d*x + 1/2*c)^7 - 120*a^2*b^3*tan
(1/2*d*x + 1/2*c)^7 + 120*a*b^4*tan(1/2*d*x + 1/2*c)^7 - 15*b^5*tan(1/2*d*
x + 1/2*c)^7 - 720*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 120*a^2*b^3*tan(1/2*d*
x + 1/2*c)^5 - 200*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 9*b^5*tan(1/2*d*x + 1/2*
c)^5 + 720*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*a^2*b^3*tan(1/2*d*x + 1/2*
c)^3 + 200*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 9*b^5*tan(1/2*d*x + 1/2*c)^3 - 2
40*a^3*b^2*tan(1/2*d*x + 1/2*c) - 120*a^2*b^3*tan(1/2*d*x + 1/2*c) - 120*a
*b^4*tan(1/2*d*x + 1/2*c) - 15*b^5*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/
2*c)^2 - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 11.13 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.73

$$\int (a+b\sec(c+dx))^5\,dx=\frac{2\,a^5\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{3\,b^5\,\sin\left(c+d\,x\right)}{8\,d\cos\left(c+d\,x\right)^2}+\frac{b^5\,\sin\left(c+d\,x\right)}{4\,d\cos\left(c+d\,x\right)^4}+\frac{10\,a\,b^4\,\sin\left(c+d\,x\right)}{3\,d\,\cos\left(c+d\,x\right)}+\frac{5\,a\,b^4\,\sin\left(c+d\,x\right)}{3\,d\cos\left(c+d\,x\right)^3}+\frac{10\,a^3\,b^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{5\,a^2\,b^3\,\sin\left(c+d\,x\right)}{d\cos\left(c+d\,x\right)^2}-\frac{b^5\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,3\mathrm{i}}{4\,d}-\frac{a^2\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,10\mathrm{i}}{d}-\frac{a^4\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,10\mathrm{i}}{d}$$

input
```
int((a + b/cos(c + d*x))^5,x)
```

output
```
(2*a^5*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (b^5*atan((sin(c/2
 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*3i)/(4*d) + (3*b^5*sin(c + d*x))/(8*d*
cos(c + d*x)^2) + (b^5*sin(c + d*x))/(4*d*cos(c + d*x)^4) - (a^2*b^3*atan(
(sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*10i)/d - (a^4*b*atan((sin(c/2
+ (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*10i)/d + (10*a*b^4*sin(c + d*x))/(3*d*c
os(c + d*x)) + (5*a*b^4*sin(c + d*x))/(3*d*cos(c + d*x)^3) + (10*a^3*b^2*s
in(c + d*x))/(d*cos(c + d*x)) + (5*a^2*b^3*sin(c + d*x))/(d*cos(c + d*x)^2
)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 625, normalized size of antiderivative = 3.96

$$\int (a+b\sec(c+dx))^5\,dx=\text{Too large to display}$$

input
```
int((a+b*sec(d*x+c))^5,x)
```

output

```
( - 240*cos(c + d*x)*sin(c + d*x)**3*a**3*b**2 - 80*cos(c + d*x)*sin(c + d
*x)**3*a*b**4 + 240*cos(c + d*x)*sin(c + d*x)*a**3*b**2 + 120*cos(c + d*x)
*sin(c + d*x)*a*b**4 - 120*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a**4*
b - 120*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a**2*b**3 - 9*log(tan((c
 + d*x)/2) - 1)*sin(c + d*x)**4*b**5 + 240*log(tan((c + d*x)/2) - 1)*sin(c
 + d*x)**2*a**4*b + 240*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**2*b**
3 + 18*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b**5 - 120*log(tan((c + d
*x)/2) - 1)*a**4*b - 120*log(tan((c + d*x)/2) - 1)*a**2*b**3 - 9*log(tan((
c + d*x)/2) - 1)*b**5 + 120*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a**4
*b + 120*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a**2*b**3 + 9*log(tan((
c + d*x)/2) + 1)*sin(c + d*x)**4*b**5 - 240*log(tan((c + d*x)/2) + 1)*sin(
c + d*x)**2*a**4*b - 240*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**2*b*
*3 - 18*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*b**5 + 120*log(tan((c +
d*x)/2) + 1)*a**4*b + 120*log(tan((c + d*x)/2) + 1)*a**2*b**3 + 9*log(tan(
(c + d*x)/2) + 1)*b**5 + 24*sin(c + d*x)**4*a**5*d*x - 120*sin(c + d*x)**3
*a**2*b**3 - 9*sin(c + d*x)**3*b**5 - 48*sin(c + d*x)**2*a**5*d*x + 120*si
n(c + d*x)*a**2*b**3 + 15*sin(c + d*x)*b**5 + 24*a**5*d*x)/(24*d*(sin(c +
d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.487 $\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 157

$$\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{a(2a^2+b^2)\operatorname{arctanh}(\sin(c+dx))}{2b^4d} + \frac{2a^4\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^4\sqrt{a+b}d} + \frac{(3a^2+2b^2)\tan(c+dx)}{3b^3d} - \frac{a\sec(c+dx)\tan(c+dx)}{2b^2d} + \frac{\sec^2(c+dx)\tan(c+dx)}{3bd}$$

output
```
-1/2*a*(2*a^2+b^2)*arctanh(sin(d*x+c))/b^4/d+2*a^4*arctanh((a-b)^(1/2)*tan
(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(1/2)/b^4/(a+b)^(1/2)/d+1/3*(3*a^2+2*b^
2)*tan(d*x+c)/b^3/d-1/2*a*sec(d*x+c)*tan(d*x+c)/b^2/d+1/3*sec(d*x+c)^2*tan
(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 2.05 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.64

$$\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{-\frac{24a^4\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{1}{2}\sec^3(c+dx)\left(9a(2a^2+b^2)\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\right.\right.\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

output
```
((-24*a^4*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 -
 b^2] + (Sec[c + d*x]^3*(9*a*(2*a^2 + b^2)*Cos[c + d*x]*(Log[Cos[(c + d*x)
/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*a*
(2*a^2 + b^2)*Cos[3*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
 Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 4*b*(3*a^2 + 4*b^2 - 3*a*b*Co
s[c + d*x] + (3*a^2 + 2*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/2)/(12*b^4*d
)
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.11, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {3042, 4338, 3042, 4580, 25, 3042, 4570, 27, 3042, 4486, 3042, 4257, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4338}$$

$$\frac{\int \frac{\sec^2(c+dx)(-3a\sec^2(c+dx)+2b\sec(c+dx)+2a)}{a+b\sec(c+dx)}dx}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(-3a\csc\left(c+dx+\frac{\pi}{2}\right)^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)+2a\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

$\Big\downarrow$ 4580

$$\frac{\frac{\int -\frac{\sec(c+dx)\left(3a^2-b\sec(c+dx)a-2\left(3a^2+2b^2\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{2b}-\frac{3a\tan(c+dx)\sec(c+dx)}{2bd}}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

$\Big\downarrow$ 25

$$\frac{-\frac{\int \frac{\sec(c+dx)\left(3a^2-b\sec(c+dx)a-2\left(3a^2+2b^2\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{2b}-\frac{3a\tan(c+dx)\sec(c+dx)}{2bd}}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a-2\left(3a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2b}-\frac{3a\tan(c+dx)\sec(c+dx)}{2bd}}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

$\Big\downarrow$ 4570

$$\frac{-\frac{\frac{\int \frac{3\sec(c+dx)\left(ba^2+\left(2a^2+b^2\right)\sec(c+dx)a\right)}{a+b\sec(c+dx)}dx}{b}-\frac{2\left(3a^2+2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{3a\tan(c+dx)\sec(c+dx)}{2bd}}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

$\Big\downarrow$ 27

$$\frac{-\frac{\frac{3\int \frac{\sec(c+dx)\left(ba^2+\left(2a^2+b^2\right)\sec(c+dx)a\right)}{a+b\sec(c+dx)}dx}{b}-\frac{2\left(3a^2+2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{3a\tan(c+dx)\sec(c+dx)}{2bd}}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

$\Big\downarrow$ 3042

$$-\frac{-\frac{\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(ba^2+\left(2a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{2\left(3a^2+2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{3a\tan(c+dx)\sec(c+dx)}{2bd}}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

$\Big\downarrow$ 4486

$$-\frac{-\frac{\frac{3\left(\frac{a\left(2a^2+b^2\right)\int \sec(c+dx)dx}{b}-\frac{2a^4\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{b}\right)}{b}-\frac{2\left(3a^2+2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{3a\tan(c+dx)\sec(c+dx)}{2bd}}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

$\Big\downarrow$ 3042

$$-\frac{-\frac{\frac{3\left(\frac{a\left(2a^2+b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b}-\frac{2a^4\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)}{b}-\frac{2\left(3a^2+2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{3a\tan(c+dx)\sec(c+dx)}{2bd}}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

$\Big\downarrow$ 4257

$$-\frac{-\frac{\frac{3\left(\frac{a\left(2a^2+b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a^4\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)}{b}-\frac{2\left(3a^2+2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{3a\tan(c+dx)\sec(c+dx)}{2bd}}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

$\Big\downarrow$ 4318

$$-\frac{-\frac{\frac{3\left(\frac{a\left(2a^2+b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a^4\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b^2}\right)}{b}-\frac{2\left(3a^2+2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{3a\tan(c+dx)\sec(c+dx)}{2bd}}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{\frac{3\left(\frac{a(2a^2+b^2)\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a^4\int\frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b^2}\right)}{b}-\frac{2(3a^2+2b^2)\tan(c+dx)}{bd}}{2b}-\frac{3a\tan(c+dx)\sec(c+dx)}{2bd}}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

$\Big\downarrow$ 3138

$$\frac{-\frac{\frac{3\left(\frac{a(2a^2+b^2)\text{arctanh}(\sin(c+dx))}{bd}-\frac{4a^4\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{b^2d}\right)}{b}-\frac{2(3a^2+2b^2)\tan(c+dx)}{bd}}{2b}-\frac{3a\tan(c+dx)\sec(c+dx)}{2bd}}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

$\Big\downarrow$ 221

$$\frac{-\frac{\frac{3\left(\frac{a(2a^2+b^2)\text{arctanh}(\sin(c+dx))}{bd}-\frac{4a^4\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}\right)}{b}-\frac{2(3a^2+2b^2)\tan(c+dx)}{bd}}{2b}-\frac{3a\tan(c+dx)\sec(c+dx)}{2bd}}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

input
```
Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

output
```
(Sec[c + d*x]^2*Tan[c + d*x])/(3*b*d) + ((-3*a*Sec[c + d*x]*Tan[c + d*x])/
(2*b*d) - ((3*((a*(2*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(b*d) - (4*a^4*ArcT
anh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b
]*d)))/b - (2*(3*a^2 + 2*b^2)*Tan[c + d*x])/(b*d))/(2*b))/(3*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4338
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 3)/(b*f*
(n - 2))), x] + Simp[d^3/(b*(n - 2))   Int[(d*Csc[e + f*x])^(n - 3)*(Simp[a
*(n - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x]/(a + b*Csc
[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && Gt
Q[n, 3]
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

rule 4580
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(-C)*Csc[e + f*x]*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m +
1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[Csc[e + f*x]*(a + b*Csc[e
+ f*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*
(m + 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &
& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.61

method	result
derivativedivides	$\frac{-\frac{1}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{a+b}{2b^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{2a^{2}+ab+2b^{2}}{2b^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\left(2a^{2}+b^{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^{4}}-\frac{1}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1 \cdots\right.}}{d}$
default	$\frac{-\frac{1}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{a+b}{2b^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{2a^{2}+ab+2b^{2}}{2b^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\left(2a^{2}+b^{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^{4}}-\frac{1}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1 \cdots\right.}}{d}$
risch	$\frac{i\left(3ab\,{\mathrm e}^{5i(dx+c)}+6a^{2}{\mathrm e}^{4i(dx+c)}+12a^{2}{\mathrm e}^{2i(dx+c)}+12\,{\mathrm e}^{2i(dx+c)}b^{2}-3ab\,{\mathrm e}^{i(dx+c)}+6a^{2}+4b^{2}\right)}{3b^{3}d\left({\mathrm e}^{2i(dx+c)}+1\right)^{3}}+\frac{a^{4}\ln\left({\mathrm e}^{i(dx+c)}+\frac{ia^{2}-ib^{2}+\cdots}{a\sqrt{a \cdots}}\right.}{\sqrt{a^{2}-b^{2}}\,d\,b^{4}}$

input
```
int(sec(d*x+c)^5/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3/b/(tan(1/2*d*x+1/2*c)-1)^3-1/2*(a+b)/b^2/(tan(1/2*d*x+1/2*c)-1)^
2-1/2*(2*a^2+a*b+2*b^2)/b^3/(tan(1/2*d*x+1/2*c)-1)+1/2*a*(2*a^2+b^2)/b^4*l
n(tan(1/2*d*x+1/2*c)-1)-1/3/b/(tan(1/2*d*x+1/2*c)+1)^3-1/2*(-a-b)/b^2/(tan
(1/2*d*x+1/2*c)+1)^2-1/2*(2*a^2+a*b+2*b^2)/b^3/(tan(1/2*d*x+1/2*c)+1)-1/2*
a*(2*a^2+b^2)/b^4*ln(tan(1/2*d*x+1/2*c)+1)+2*a^4/b^4/((a+b)*(a-b))^(1/2)*a
rctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 557, normalized size of antiderivative = 3.55

$$\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx = \left[\frac{6\sqrt{a^2-b^2}a^4\cos(dx+c)^3\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)-3\,(\right.$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[1/12*(6*sqrt(a^2 - b^2)*a^4*cos(d*x + c)^3*log((2*a*b*cos(d*x + c) - (a^2
 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x
+ c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 3*(
2*a^5 - a^3*b^2 - a*b^4)*cos(d*x + c)^3*log(sin(d*x + c) + 1) + 3*(2*a^5 -
 a^3*b^2 - a*b^4)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*a^2*b^3 - 2
*b^5 + 2*(3*a^4*b - a^2*b^3 - 2*b^5)*cos(d*x + c)^2 - 3*(a^3*b^2 - a*b^4)*
cos(d*x + c))*sin(d*x + c))/((a^2*b^4 - b^6)*d*cos(d*x + c)^3), 1/12*(12*s
qrt(-a^2 + b^2)*a^4*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 -
b^2)*sin(d*x + c)))*cos(d*x + c)^3 - 3*(2*a^5 - a^3*b^2 - a*b^4)*cos(d*x +
 c)^3*log(sin(d*x + c) + 1) + 3*(2*a^5 - a^3*b^2 - a*b^4)*cos(d*x + c)^3*l
og(-sin(d*x + c) + 1) + 2*(2*a^2*b^3 - 2*b^5 + 2*(3*a^4*b - a^2*b^3 - 2*b^
5)*cos(d*x + c)^2 - 3*(a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*
b^4 - b^6)*d*cos(d*x + c)^3)]
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx=\int \frac{\sec^5{(c+dx)}}{a+b\sec{(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**5/(a + b*sec(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(140) = 280$.

Time = 0.17 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.82

$$\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$=\frac{\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a^4}{\sqrt{-a^2+b^2}b^4}-\frac{3(2a^3+ab^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^4}+\frac{3(2a^3+ab^2}{}}{}$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan
(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^4/(sqrt(-
a^2 + b^2)*b^4) - 3*(2*a^3 + a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4
 + 3*(2*a^3 + a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 - 2*(6*a^2*tan
(1/2*d*x + 1/2*c)^5 + 3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*b^2*tan(1/2*d*x + 1
/2*c)^5 - 12*a^2*tan(1/2*d*x + 1/2*c)^3 - 4*b^2*tan(1/2*d*x + 1/2*c)^3 + 6
*a^2*tan(1/2*d*x + 1/2*c) - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*b^2*tan(1/2*d*x
 + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*b^3))/d
```

Mupad [B] (verification not implemented)

Time = 12.05 (sec) , antiderivative size = 1021, normalized size of antiderivative = 6.50

$$\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))),x)
```

output
```
-((9*a^4*cos(c + d*x)*atanh((8*a^6*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^8*
sin(c/2 + (d*x)/2) + b^6*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 8*a^7*b*sin(c/2
+ (d*x)/2) - 2*a*b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^5*b*sin(c/2 + (d
*x)/2)*(a^2 - b^2) + 5*a^2*b^4*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^3*b^3*
sin(c/2 + (d*x)/2)*(a^2 - b^2) + 8*a^4*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2))
/(b*cos(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(4*a^4*(a^2 - b^2) + b^4*(a^2 - b
^2) + 2*a^5*b - 4*a^6 + 2*a^3*b^3 + 4*a^2*b^2*(a^2 - b^2) - a*b^3*(a^2 - b
^2) - 2*a^3*b*(a^2 - b^2)))))/2 - (3*b^3*sin(c + d*x)*(a^2 - b^2)^(1/2))/2
 - (b^3*sin(3*c + 3*d*x)*(a^2 - b^2)^(1/2))/2 + (3*a^4*cos(3*c + 3*d*x)*at
anh((8*a^6*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^8*sin(c/2 + (d*x)/2) + b^6
*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 8*a^7*b*sin(c/2 + (d*x)/2) - 2*a*b^5*sin
(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^5*b*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 5*a
^2*b^4*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^3*b^3*sin(c/2 + (d*x)/2)*(a^2
- b^2) + 8*a^4*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2))/(b*cos(c/2 + (d*x)/2)*(
a^2 - b^2)^(1/2)*(4*a^4*(a^2 - b^2) + b^4*(a^2 - b^2) + 2*a^5*b - 4*a^6 +
2*a^3*b^3 + 4*a^2*b^2*(a^2 - b^2) - a*b^3*(a^2 - b^2) - 2*a^3*b*(a^2 - b^2
)))))/2 + (3*a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*
d*x)*(a^2 - b^2)^(1/2))/2 - (3*a^2*b*sin(c + d*x)*(a^2 - b^2)^(1/2))/4 + (
9*a^3*cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*(a^2 - b^2
)^(1/2))/2 + (3*a*b^2*sin(2*c + 2*d*x)*(a^2 - b^2)^(1/2))/4 - (3*a^2*b*...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 623, normalized size of antiderivative = 3.97

$$\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^5/(a+b*sec(d*x+c)),x)
```

output
```
(12*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sq
rt( - a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**2*a**4 - 12*sqrt( - a**2 +
b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))
*cos(c + d*x)*a**4 + 6*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)
**2*a**5 - 3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**3*b
**2 - 3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a*b**4 - 6*
cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a**5 + 3*cos(c + d*x)*log(tan((c +
d*x)/2) - 1)*a**3*b**2 + 3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a*b**4 -
 6*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**5 + 3*cos(c +
 d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**3*b**2 + 3*cos(c + d*x)
*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a*b**4 + 6*cos(c + d*x)*log(tan
((c + d*x)/2) + 1)*a**5 - 3*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a**3*b*
*2 - 3*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a*b**4 + 3*cos(c + d*x)*sin(
c + d*x)*a**3*b**2 - 3*cos(c + d*x)*sin(c + d*x)*a*b**4 + 6*sin(c + d*x)**
3*a**4*b - 2*sin(c + d*x)**3*a**2*b**3 - 4*sin(c + d*x)**3*b**5 - 6*sin(c
+ d*x)*a**4*b + 6*sin(c + d*x)*b**5)/(6*cos(c + d*x)*b**4*d*(sin(c + d*x)*
*2*a**2 - sin(c + d*x)**2*b**2 - a**2 + b**2))
```

3.488 $\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 119

$$\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{(2a^2+b^2)\operatorname{arctanh}(\sin(c+dx))}{2b^3d} - \frac{2a^3\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^3\sqrt{a+b}\,d} - \frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd}$$

output
```
1/2*(2*a^2+b^2)*arctanh(sin(d*x+c))/b^3/d-2*a^3*arctanh((a-b)^(1/2)*tan(1/
2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(1/2)/b^3/(a+b)^(1/2)/d-a*tan(d*x+c)/b^2/d
+1/2*sec(d*x+c)*tan(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.00

$$\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$= \frac{\frac{8a^3\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 4a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) - 2b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \cdots\right.}{\cdots}$$

input
```
Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

output
```
((8*a^3*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b
^2] - 4*a^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2*b^2*Log[Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]] + 4*a^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]] + 2*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + b^2/(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])^2 - b^2/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 4*
a*b*Tan[c + d*x])/(4*b^3*d)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4338, 3042, 4570, 3042, 4486, 3042, 4257, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4338}$$

$$\frac{\int \frac{\sec(c+dx)\left(-2a\sec^2(c+dx)+b\sec(c+dx)+a\right)}{a+b\sec(c+dx)}dx}{2b}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(-2a\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2b}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

$$\Big\downarrow \text{4570}$$

$$\frac{\frac{\int \frac{\sec(c+dx)\left(ab+\left(2a^2+b^2\right)\sec(c+dx)\right)}{a+b\sec(c+dx)}dx}{b}-\frac{2a\tan(c+dx)}{bd}}{2b}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(ab+\left(2a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{2a\tan(c+dx)}{bd}}{2b}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

$$\Big\downarrow \text{4486}$$

$$\frac{\frac{\frac{\left(2a^2+b^2\right)\int\sec(c+dx)dx}{b}-\frac{2a^3\int\frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{b}}{b}-\frac{2a\tan(c+dx)}{bd}}{2b}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\left(2a^2+b^2\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{b}-\frac{2a^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{b}-\frac{2a\tan(c+dx)}{bd}}{2b}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{\frac{\left(2a^2+b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{b}-\frac{2a\tan(c+dx)}{bd}}{2b}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

$$\Big\downarrow \text{4318}$$

$$\frac{\frac{\frac{\left(2a^2+b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a^3\int\frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b^2}}{b}-\frac{2a\tan(c+dx)}{bd}}{2b}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\left(2a^2+b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a^3\int\frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b^2}}{b}-\frac{2a\tan(c+dx)}{bd}}{2b}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{\frac{\left(2a^2+b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{4a^3\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{b^2d}}{b}-\frac{2a\tan(c+dx)}{bd}}{2b}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\frac{(2a^2+b^2)\text{arctanh}(\sin(c+dx))}{bd}-\frac{4a^3\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}}{b}-\frac{2a\tan(c+dx)}{bd}}{2b}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

input
```
Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

output
```
(Sec[c + d*x]*Tan[c + d*x])/(2*b*d) + ((((2*a^2 + b^2)*ArcTanh[Sin[c + d*x
]])/(b*d) - (4*a^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(S
qrt[a - b]*b*Sqrt[a + b]*d))/b - (2*a*Tan[c + d*x])/(b*d))/(2*b)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4338
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 3)/(b*f*
(n - 2))), x] + Simp[d^3/(b*(n - 2))   Int[(d*Csc[e + f*x])^(n - 3)*(Simp[a
*(n - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x]/(a + b*Csc
[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && Gt
Q[n, 3]
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.61

method	result
derivativedivides	$\frac{-\frac{2a^3 \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^3\sqrt{(a+b)(a-b)}}+\frac{1}{2b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-2a-b}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-2a^2-b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^3}-\frac{}{2b\left(\tan\right.}}{d}$
default	$\frac{-\frac{2a^3 \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^3\sqrt{(a+b)(a-b)}}+\frac{1}{2b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-2a-b}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-2a^2-b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^3}-\frac{}{2b\left(\tan\right.}}{d}$
risch	$-\frac{i\left(b\,e^{3i(dx+c)}+2\,e^{2i(dx+c)}a-b\,e^{i(dx+c)}+2a\right)}{d\,b^2\left(e^{2i(dx+c)}+1\right)^2}+\frac{\ln\left(e^{i(dx+c)}+i\right)a^2}{d\,b^3}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{2db}+\frac{a^3\ln\left(e^{i(dx+c)}+\frac{-ia^2+ib}{a\sqrt{}}\right.}{\sqrt{a^2-b^2}\,d\,b^3}$

input
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a^3/b^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b
)*(a-b))^(1/2))+1/2/b/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(-2*a-b)/b^2/(tan(1/2*d
*x+1/2*c)-1)+1/2/b^3*(-2*a^2-b^2)*ln(tan(1/2*d*x+1/2*c)-1)-1/2/b/(tan(1/2*
d*x+1/2*c)+1)^2-1/2*(-2*a-b)/b^2/(tan(1/2*d*x+1/2*c)+1)+1/2*(2*a^2+b^2)/b^
3*ln(tan(1/2*d*x+1/2*c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(106) = 212$.

Time = 0.20 (sec) , antiderivative size = 485, normalized size of antiderivative = 4.08

$$\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[\frac{2\sqrt{a^2-b^2}a^3\cos(dx+c)^2\log\left(\frac{2ab\cos(dx+c)-(a^2-2b^2)\cos(dx+c)^2-2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2a^2-b^2}{a^2\cos(dx+c)^2+2ab\cos(dx+c)+b^2}\right)+(2}{}\right.$$
$$-\frac{4\sqrt{-a^2+b^2}a^3\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)\cos(dx+c)^2-(2a^4-a^2b^2-b^4)\cos(dx+c)^2\log(\sin}{}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(a^2 - b^2)*a^3*cos(d*x + c)^2*log((2*a*b*cos(d*x + c) - (a^2
- 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x +
 c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (2*a
^4 - a^2*b^2 - b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^4 - a^2*b^
2 - b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(a^2*b^2 - b^4 - 2*(a^3
*b - a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^2*b^3 - b^5)*d*cos(d*x + c)^2)
, -1/4*(4*sqrt(-a^2 + b^2)*a^3*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) +
a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^2 - (2*a^4 - a^2*b^2 - b^4)*co
s(d*x + c)^2*log(sin(d*x + c) + 1) + (2*a^4 - a^2*b^2 - b^4)*cos(d*x + c)^
2*log(-sin(d*x + c) + 1) - 2*(a^2*b^2 - b^4 - 2*(a^3*b - a*b^3)*cos(d*x +
c))*sin(d*x + c))/((a^2*b^3 - b^5)*d*cos(d*x + c)^2)]
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.77

$$\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a^3}{\sqrt{-a^2+b^2}\,b^3} - \frac{(2a^2+b^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^3} + \frac{(2a^2+b^2)\log}{}}{2d}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan
(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^3/(sqrt(-
a^2 + b^2)*b^3) - (2*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 + (
2*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^3 - 2*(2*a*tan(1/2*d*x +
 1/2*c)^3 + b*tan(1/2*d*x + 1/2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c) + b*tan(1/
2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*b^2))/d
```

Mupad [B] (verification not implemented)

Time = 10.73 (sec) , antiderivative size = 1002, normalized size of antiderivative = 8.42

$$\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))),x)
```

output
```
sin(c + d*x)/(2*b*d*(cos(2*c + 2*d*x)/2 + 1/2)) + atanh(sin(c/2 + (d*x)/2)
/cos(c/2 + (d*x)/2))/(2*b*d*(cos(2*c + 2*d*x)/2 + 1/2)) + (a^2*atanh(sin(c
/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^3*d*(cos(2*c + 2*d*x)/2 + 1/2)) + (a
tanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x))/(2*b*d*(cos(
2*c + 2*d*x)/2 + 1/2)) - (a*sin(2*c + 2*d*x))/(2*b^2*d*(cos(2*c + 2*d*x)/2
 + 1/2)) - (a^3*atan(((8*a^6*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^8*sin(c/
2 + (d*x)/2) + b^6*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 8*a^7*b*sin(c/2 + (d*x
)/2) - 2*a*b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^5*b*sin(c/2 + (d*x)/2)
*(a^2 - b^2) + 5*a^2*b^4*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^3*b^3*sin(c/
2 + (d*x)/2)*(a^2 - b^2) + 8*a^4*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2))*1i)/(
b*cos(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(4*a^4*(a^2 - b^2) + b^4*(a^2 - b^2
) + 2*a^5*b - 4*a^6 + 2*a^3*b^3 + 4*a^2*b^2*(a^2 - b^2) - a*b^3*(a^2 - b^2
) - 2*a^3*b*(a^2 - b^2))))*1i)/(b^3*d*(a^2 - b^2)^(1/2)*(cos(2*c + 2*d*x)/
2 + 1/2)) + (a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*
d*x))/(b^3*d*(cos(2*c + 2*d*x)/2 + 1/2)) - (a^3*cos(2*c + 2*d*x)*atan(((8*
a^6*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^8*sin(c/2 + (d*x)/2) + b^6*sin(c/
2 + (d*x)/2)*(a^2 - b^2) + 8*a^7*b*sin(c/2 + (d*x)/2) - 2*a*b^5*sin(c/2 +
(d*x)/2)*(a^2 - b^2) - 8*a^5*b*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 5*a^2*b^4*
sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^3*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)
+ 8*a^4*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2))*1i)/(b*cos(c/2 + (d*x)/2)*(...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 479, normalized size of antiderivative = 4.03

$$\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{-4\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{-a^2+b^2}}\right)\sin(dx+c)^2 a^3+4\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{-a^2+b^2}}\right.}{}$$

input

```
int(sec(d*x+c)^4/(a+b*sec(d*x+c)),x)
```

output

```
( - 4*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/
sqrt( - a**2 + b**2))*sin(c + d*x)**2*a**3 + 4*sqrt( - a**2 + b**2)*atan((
tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*a**3 + 2*co
s(c + d*x)*sin(c + d*x)*a**3*b - 2*cos(c + d*x)*sin(c + d*x)*a*b**3 - 2*lo
g(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**4 + log(tan((c + d*x)/2) - 1)*s
in(c + d*x)**2*a**2*b**2 + log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b**4
+ 2*log(tan((c + d*x)/2) - 1)*a**4 - log(tan((c + d*x)/2) - 1)*a**2*b**2 -
 log(tan((c + d*x)/2) - 1)*b**4 + 2*log(tan((c + d*x)/2) + 1)*sin(c + d*x)
**2*a**4 - log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**2*b**2 - log(tan((
c + d*x)/2) + 1)*sin(c + d*x)**2*b**4 - 2*log(tan((c + d*x)/2) + 1)*a**4 +
 log(tan((c + d*x)/2) + 1)*a**2*b**2 + log(tan((c + d*x)/2) + 1)*b**4 - si
n(c + d*x)*a**2*b**2 + sin(c + d*x)*b**4)/(2*b**3*d*(sin(c + d*x)**2*a**2
- sin(c + d*x)**2*b**2 - a**2 + b**2))
```

3.489 $\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 85

$$\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{a\operatorname{arctanh}(\sin(c+dx))}{b^2d} + \frac{2a^2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^2\sqrt{a+b}\,d} + \frac{\tan(c+dx)}{bd}$$

output
```
-a*arctanh(sin(d*x+c))/b^2/d+2*a^2*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/
(a+b)^(1/2))/(a-b)^(1/2)/b^2/(a+b)^(1/2)/d+tan(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.35

$$\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{-\frac{2a^2\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + a\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\right.\right.}{b^2d}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

output
```
((-2*a^2*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 -
b^2] + a*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]]) + b*Tan[c + d*x])/(b^2*d)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4277, 3042, 4276, 3042, 4257, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4277}$$

$$\frac{\tan(c+dx)}{bd}-\frac{a\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{bd}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{4276}$$

$$\frac{\tan(c+dx)}{bd}-\frac{a\left(\frac{\int \sec(c+dx)dx}{b}-\frac{a\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{bd}-\frac{a\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\tan(c+dx)}{bd}-\frac{a\left(\frac{\text{arctanh}(\sin(c+dx))}{bd}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{4318}$$

$$\frac{\tan(c+dx)}{bd}-\frac{a\left(\frac{\text{arctanh}(\sin(c+dx))}{bd}-\frac{a\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b^2}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{bd}-\frac{a\left(\frac{\text{arctanh}(\sin(c+dx))}{bd}-\frac{a\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b^2}\right)}{b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\tan(c+dx)}{bd}-\frac{a\left(\frac{\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{b^2d}\right)}{b}$$

$$\Big\downarrow \text{221}$$

$$\frac{\tan(c+dx)}{bd}-\frac{a\left(\frac{\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}\right)}{b}$$

input

```
Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

output

```
-((a*(ArcTanh[Sin[c + d*x]]/(b*d) - (2*a*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*d)))/b) + Tan[c + d*x]/(b*d
)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4276
```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[1/b   Int[Csc[e + f*x], x], x] - Simp[a/b   Int[Csc[e + f*x]/(
a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 4277
```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[-Cot[e + f*x]/(b*f), x] - Simp[a/b   Int[Csc[e + f*x]^2/(a + b
*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.45

method	result
derivativedivides	$\frac{-\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}+\frac{2a^2\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^2\sqrt{(a+b)(a-b)}}-\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}}{d}$
default	$\frac{-\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}+\frac{2a^2\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^2\sqrt{(a+b)(a-b)}}-\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}}{d}$
risch	$\frac{2i}{db\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{b^2d}+\frac{a^2\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d\,b^2}-\frac{a^2\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^2-ib^2-b\sqrt{a^2-}}{a\sqrt{a^2-b^2}}\right.}{\sqrt{a^2-b^2}\,d\,b^2}$

input
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b/(tan(1/2*d*x+1/2*c)+1)-a/b^2*ln(tan(1/2*d*x+1/2*c)+1)+2*a^2/b^2/
((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-
1/b/(tan(1/2*d*x+1/2*c)-1)+a/b^2*ln(tan(1/2*d*x+1/2*c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(76) = 152.

Time = 0.13 (sec) , antiderivative size = 392, normalized size of antiderivative = 4.61

$$\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$=\left[\frac{\sqrt{a^2-b^2}a^2\cos(dx+c)\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)-(a^3-}{2\,(a^2b^2}\right.$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a^2 - b^2)*a^2*cos(d*x + c)*log((2*a*b*cos(d*x + c) - (a^2 - 2*
b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c)
+ 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (a^3 - a
*b^2)*cos(d*x + c)*log(sin(d*x + c) + 1) + (a^3 - a*b^2)*cos(d*x + c)*log(
-sin(d*x + c) + 1) + 2*(a^2*b - b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d*cos(
d*x + c)), 1/2*(2*sqrt(-a^2 + b^2)*a^2*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x
 + c) + a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*x + c) - (a^3 - a*b^2)*cos(d*
x + c)*log(sin(d*x + c) + 1) + (a^3 - a*b^2)*cos(d*x + c)*log(-sin(d*x + c
) + 1) + 2*(a^2*b - b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d*cos(d*x + c))]
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*sec(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.79

$$\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a^2}{\sqrt{-a^2+b^2}b^2} - \frac{a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^2} + \frac{a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right.}{b^2}}{d}$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^2/(sqrt(-a^2 +
 b^2)*b^2) - a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^2 + a*log(abs(tan(1/2*
d*x + 1/2*c) - 1))/b^2 - 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 -
 1)*b))/d
```

Mupad [B] (verification not implemented)

Time = 9.99 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.40

$$\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\tan(c+d\,x)}{b\,d} - \frac{2\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^2\,d} - \frac{a^2\,\mathrm{atan}\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}}\right)\,2\mathrm{i}}{b^2\,d\,\sqrt{a^2-b^2}}$$

input
```
int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))),x)
```

output
```
tan(c + d*x)/(b*d) - (2*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b
^2*d) - (a^2*atan((a*sin(c/2 + (d*x)/2)*1i - b*sin(c/2 + (d*x)/2)*1i)/(cos
(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)))*2i)/(b^2*d*(a^2 - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.35

$$\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{2\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{-a^2+b^2}}\right)\cos\left(dx+c\right)a^2+\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a^3-\cos}{}$$

input
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c)),x)
```

output
```
(2*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqr
t( - a**2 + b**2))*cos(c + d*x)*a**2 + cos(c + d*x)*log(tan((c + d*x)/2) -
 1)*a**3 - cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a*b**2 - cos(c + d*x)*lo
g(tan((c + d*x)/2) + 1)*a**3 + cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a*b*
*2 + sin(c + d*x)*a**2*b - sin(c + d*x)*b**3)/(cos(c + d*x)*b**2*d*(a**2 -
 b**2))
```

3.490 $\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 68

$$\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{bd} - \frac{2a\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}d}$$

output
```
arctanh(sin(d*x+c))/b/d-2*a*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(
1/2))/(a-b)^(1/2)/b/(a+b)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.50

$$\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{\frac{2a\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + \log\left(\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c\right.\right.}{bd}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

output
```
((2*a*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2
] - Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]])/(b*d)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 4276, 3042, 4257, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4276}$$

$$\frac{\int \sec(c+dx)dx}{b}-\frac{a\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\text{arctanh}(\sin(c+dx))}{bd}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{4318}$$

$$\frac{\text{arctanh}(\sin(c+dx))}{bd}-\frac{a\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\operatorname{arctanh}(\sin(c+dx))}{bd}-\frac{a\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b^2}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\operatorname{arctanh}(\sin(c+dx))}{bd}-\frac{2a\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{b^2d}$$

$$\Big\downarrow \text{221}$$

$$\frac{\operatorname{arctanh}(\sin(c+dx))}{bd}-\frac{2a\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

output
```
ArcTanh[Sin[c + d*x]]/(b*d) - (2*a*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/
Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*d)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4276
```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[1/b   Int[Csc[e + f*x], x], x] - Simp[a/b   Int[Csc[e + f*x]/(
a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}-\frac{2a\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b\sqrt{(a+b)(a-b)}}}{d}$
default	$\frac{-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}-\frac{2a\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b\sqrt{(a+b)(a-b)}}}{d}$
risch	$\frac{a\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,db}-\frac{a\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,db}-\frac{\ln\left(e^{i(dx+c)}-i\right)}{db}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{db}$

input
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b*ln(tan(1/2*d*x+1/2*c)-1)+1/b*ln(tan(1/2*d*x+1/2*c)+1)-2/b*a/((a+
b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 290, normalized size of antiderivative = 4.26

$$\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[\frac{\sqrt{a^2-b^2}a\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)+(a^2-b^2)\log(\sin(\cdots}{2\,(a^2b-b^3)d}\right.$$
$$-\frac{2\sqrt{-a^2+b^2}a\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)-(a^2-b^2)\log(\sin(dx+c)+1)+(a^2-b^2)\log(-\sin\cdots}{2\,(a^2b-b^3)d}$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a^2 - b^2)*a*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x +
c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/
(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (a^2 - b^2)*log(sin(d*x
 + c) + 1) - (a^2 - b^2)*log(-sin(d*x + c) + 1))/((a^2*b - b^3)*d), -1/2*(
2*sqrt(-a^2 + b^2)*a*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 -
 b^2)*sin(d*x + c))) - (a^2 - b^2)*log(sin(d*x + c) + 1) + (a^2 - b^2)*log
(-sin(d*x + c) + 1))/((a^2*b - b^3)*d)]
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx=\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(59) = 118$.

Time = 0.16 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.76

$$\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a}{\sqrt{-a^2+b^2}\,b}-\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b}+\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b}}{d}$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
-(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2
*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a/(sqrt(-a^2 +
b^2)*b) - log(abs(tan(1/2*d*x + 1/2*c) + 1))/b + log(abs(tan(1/2*d*x + 1/2
*c) - 1))/b)/d
```

Mupad [B] (verification not implemented)

Time = 10.12 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.74

$$\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d} + \frac{2\,a\,\mathrm{atanh}\left(\frac{2\,a^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-b^2\right)-2\,a^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+b^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-b^2\right)+2\,a^3\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,a\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-b^2\right)}{b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}\,\left(a\,b-b^2\right)}\right)}{b\,d\,\sqrt{a^2-b^2}}$$

input
```
int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))),x)
```

output
```
(2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b*d) + (2*a*atanh((2*a^2
*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 2*a^4*sin(c/2 + (d*x)/2) + b^2*sin(c/2 +
 (d*x)/2)*(a^2 - b^2) + 2*a^3*b*sin(c/2 + (d*x)/2) - 2*a*b*sin(c/2 + (d*x)
/2)*(a^2 - b^2))/(b*cos(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(a*b - b^2))))/(b
*d*(a^2 - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.00

$$\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{-2\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{-a^2+b^2}}\right)a-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a^2+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b^2+}{bd\left(a^2-b^2\right)}$$

input
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c)),x)
```

output
```
( - 2*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/
sqrt( - a**2 + b**2))*a - log(tan((c + d*x)/2) - 1)*a**2 + log(tan((c + d*
x)/2) - 1)*b**2 + log(tan((c + d*x)/2) + 1)*a**2 - log(tan((c + d*x)/2) +
1)*b**2)/(b*d*(a**2 - b**2))
```

3.491 $\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}d}$$

output
```
2*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(1/2)/(a+b)^(1
/2)/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x]),x]
```

output
```
(-2*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]
*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4318}$$

$$\frac{\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{2\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[Sec[c + d*x]/(a + b*Sec[c + d*x]),x]
```

output
```
(2*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[
a + b]*d)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{2 \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{d\sqrt{(a+b)(a-b)}}$	44
default	$\frac{2 \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{d\sqrt{(a+b)(a-b)}}$	44
risch	$\frac{\ln\left(e^{i(dx+c)}+\frac{i a^2-i b^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\, d}-\frac{\ln\left(e^{i(dx+c)}+\frac{-i a^2+i b^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\, d}$	137

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/d/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 185, normalized size of antiderivative = 3.78

$$\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$=\left[\frac{\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)}{2\,\sqrt{a^2-b^2}d},\ \frac{\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b}{(a^2-b^2)}\right.}{(a^2-b^2)d}\right.$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[1/2*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 -
 b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2
 + 2*a*b*cos(d*x + c) + b^2))/(sqrt(a^2 - b^2)*d), sqrt(-a^2 + b^2)*arctan
(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))/((a^2
- b^2)*d)]
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx=\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)/(a + b*sec(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.57

$$\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}d}$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
-2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*
x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*
d)
```

Mupad [B] (verification not implemented)

Time = 9.76 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a-b}}{\sqrt{a+b}}\right)}{d\,\sqrt{a+b}\,\sqrt{a-b}}$$

input
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))),x)
```

output
```
(2*atanh((tan(c/2 + (d*x)/2)*(a - b)^(1/2))/(a + b)^(1/2)))/(d*(a + b)^(1/
2)*(a - b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.31

$$\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{2\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{-a^2+b^2}}\right)}{d\,(a^2-b^2)}$$

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c)),x)
```

output
```
(2*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqr
t( - a**2 + b**2)))/(d*(a**2 - b**2))
```

3.492 $\int \frac{1}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 59

$$\int \frac{1}{a+b\sec(c+dx)}\,dx = \frac{x}{a} - \frac{2b\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}d}$$

output
```
x/a-2*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/(a-b)^(1/2)/
(a+b)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \frac{1}{a+b\sec(c+dx)}\,dx = \frac{\frac{c}{d} + x + \frac{2b\,\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}}{a}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(-1),x]
```

output
```
(c/d + x + (2*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(Sqr
t[a^2 - b^2]*d))/a
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 4270, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4270}$$

$$\frac{x}{a}-\frac{\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x}{a}-\frac{\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a}$$

$$\Big\downarrow \text{3138}$$

$$\frac{x}{a}-\frac{2\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}$$

$$\Big\downarrow \text{221}$$

$$\frac{x}{a}-\frac{2b\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[(a + b*Sec[c + d*x])^(-1),x]
```

output
```
x/a - (2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a
- b]*Sqrt[a + b]*d)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4270
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Simp[1/a   Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2b\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}}{d}$	65
default	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2b\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}}{d}$	65
risch	$\frac{x}{a}+\frac{b\ln\left(\mathrm{e}^{i(dx+c)}+\frac{-ia^2+ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,da}-\frac{b\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,da}$	150

input
```
int(1/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/a*arctan(tan(1/2*d*x+1/2*c))-2/a*b/((a+b)*(a-b))^(1/2)*arctanh((a-b
)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 230, normalized size of antiderivative = 3.90

$$\int \frac{1}{a+b\sec(c+dx)}\,dx = \left[\frac{2\,(a^2-b^2)dx+\sqrt{a^2-b^2}b\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)}{2\,(a^3-ab^2)d},\ \frac{(a^2}{}\right.$$

input
```
integrate(1/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[1/2*(2*(a^2 - b^2)*d*x + sqrt(a^2 - b^2)*b*log((2*a*b*cos(d*x + c) - (a^2
 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x
+ c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)))/((a^
3 - a*b^2)*d), ((a^2 - b^2)*d*x - sqrt(-a^2 + b^2)*b*arctan(-sqrt(-a^2 + b
^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))))/((a^3 - a*b^2)*d)]
```

Sympy [F]

$$\int \frac{1}{a+b\sec(c+dx)}\,dx = \int \frac{1}{a+b\sec\left(c+dx\right)}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c)),x)
```

output
```
Integral(1/(a + b*sec(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(50) = 100$.

Time = 0.12 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.69

$$\int \frac{1}{a+b\sec(c+dx)}\,dx = \frac{\frac{\left(\sqrt{-a^2+b^2}(a-2b)|-a+b|-\sqrt{-a^2+b^2}|a||-a+b|\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-\frac{b+\sqrt{(a+b)(a-b)+b^2}}{a-b}}}\right)\right)}{(a^2-2ab+b^2)a^2+(a^2b-2ab^2+b^3)|a|}+\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-\frac{b-\sqrt{(a+b)(a-b)+b^2}}{a-b}}}\right)\right)(a-2b+|a|)}{a^2-b|a|}}{d}$$

input
```
integrate(1/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
((sqrt(-a^2 + b^2)*(a - 2*b)*abs(-a + b) - sqrt(-a^2 + b^2)*abs(a)*abs(-a
+ b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt
(-(b + sqrt((a + b)*(a - b) + b^2))/(a - b))))/((a^2 - 2*a*b + b^2)*a^2 +
(a^2*b - 2*a*b^2 + b^3)*abs(a)) + (pi*floor(1/2*(d*x + c)/pi + 1/2) + arct
an(tan(1/2*d*x + 1/2*c)/sqrt(-(b - sqrt((a + b)*(a - b) + b^2))/(a - b))))
*(a - 2*b + abs(a))/(a^2 - b*abs(a)))/d
```

Mupad [B] (verification not implemented)

Time = 9.99 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.15

$$\int \frac{1}{a+b\sec(c+dx)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a\,d} + \frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-b^2\right)+2\,b^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-b^2\right)-2\,a\,b^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,a\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-b^2\right)}{a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}\,\left(a\,b-a^2\right)}\right)}{a\,d\,\sqrt{a^2-b^2}}$$

input
```
int(1/(a + b/cos(c + d*x)),x)
```

output
```
(2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a*d) + (2*b*atanh((2*b^4*
sin(c/2 + (d*x)/2) + a^2*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 2*b^2*sin(c/2 +
(d*x)/2)*(a^2 - b^2) - 2*a*b^3*sin(c/2 + (d*x)/2) - 2*a*b*sin(c/2 + (d*x)/
2)*(a^2 - b^2))/(a*cos(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(a*b - a^2))))/(a*
d*(a^2 - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.41

$$\int \frac{1}{a+b\sec(c+dx)}\,dx = \frac{-2\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{-a^2+b^2}}\right)b+a^2dx-b^2dx}{ad\,(a^2-b^2)}$$

input
```
int(1/(a+b*sec(d*x+c)),x)
```

output
```
( - 2*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/
sqrt( - a**2 + b**2))*b + a**2*d*x - b**2*d*x)/(a*d*(a**2 - b**2))
```

3.493 $\int \frac{\cos(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \frac{\cos(c+dx)}{a+b\sec(c+dx)}\,dx=-\frac{bx}{a^2}+\frac{2b^2\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}d}+\frac{\sin(c+dx)}{ad}$$

output
```
-b*x/a^2+2*b^2*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^2/(a-
b)^(1/2)/(a+b)^(1/2)/d+sin(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{\cos(c+dx)}{a+b\sec(c+dx)}\,dx=\frac{-b(c+dx)-\frac{2b^2\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+a\sin(c+dx)}{a^2d}$$

input
```
Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x]),x]
```

output
```
(-(b*(c + d*x)) - (2*b^2*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]])/Sqrt[a^2 - b^2] + a*Sin[c + d*x])/(a^2*d)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {3042, 4340, 25, 27, 3042, 4270, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4340}$$

$$\frac{\int -\frac{b}{a+b\sec(c+dx)}dx}{a}+\frac{\sin(c+dx)}{ad}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sin(c+dx)}{ad}-\frac{\int \frac{b}{a+b\sec(c+dx)}dx}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sin(c+dx)}{ad}-\frac{b\int \frac{1}{a+b\sec(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)}{ad}-\frac{b\int \frac{1}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}$$

$$\Big\downarrow \text{4270}$$

$$\frac{\sin(c+dx)}{ad}-\frac{b\left(\frac{x}{a}-\frac{\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)}{ad}-\frac{b\left(\frac{x}{a}-\frac{\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a}\right)}{a}$$

$\Big\downarrow$ 3138

$$\frac{\sin(c+dx)}{ad}-\frac{b\left(\frac{x}{a}-\frac{2\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}\right)}{a}$$

$\Big\downarrow$ 221

$$\frac{\sin(c+dx)}{ad}-\frac{b\left(\frac{x}{a}-\frac{2b\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a}$$

input
```
Int[Cos[c + d*x]/(a + b*Sec[c + d*x]),x]
```

output
```
-((b*(x/a - (2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*S
qrt[a - b]*Sqrt[a + b]*d)))/a) + Sin[c + d*x]/(a*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4270
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Simp[1/a   Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[a^2 - b^2, 0]
```

rule 4340
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Sim
p[1/(a*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n
- a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+b\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^2}+\frac{2b^2\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^2\sqrt{(a+b)(a-b)}}}{d}$
default	$\frac{-\frac{2\left(-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+b\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^2}+\frac{2b^2\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^2\sqrt{(a+b)(a-b)}}}{d}$
risch	$-\frac{bx}{a^2}-\frac{i\mathrm{e}^{i(dx+c)}}{2ad}+\frac{i\mathrm{e}^{-i(dx+c)}}{2ad}+\frac{b^2\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d\,a^2}-\frac{b^2\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^2-ib^2-b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d\,a^2}$

input
```
int(cos(d*x+c)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/a^2*(-a*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+b*arctan(tan(1
/2*d*x+1/2*c)))+2*b^2/a^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/
2*c)/((a+b)*(a-b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 277, normalized size of antiderivative = 3.64

$$\int \frac{\cos(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[\frac{\sqrt{a^2-b^2}b^2\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)-2\,(a^2b-b^3)dx+2}{2\,(a^4-a^2b^2)d}\right.$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a^2 - b^2)*b^2*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
+ c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2
)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(a^2*b - b^3)*d*x +
 2*(a^3 - a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d), (sqrt(-a^2 + b^2)*b^2*
arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))
- (a^2*b - b^3)*d*x + (a^3 - a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d)]
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cos(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)/(a + b*sec(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.66

$$\int \frac{\cos(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b^2}{\sqrt{-a^2+b^2}a^2}-\frac{(dx+c)b}{a^2}+\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)a}}{d}$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b^2/(sqrt(-a^2 +
 b^2)*a^2) - (d*x + c)*b/a^2 + 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*
c)^2 + 1)*a))/d
```

Mupad [B] (verification not implemented)

Time = 10.17 (sec) , antiderivative size = 395, normalized size of antiderivative = 5.20

$$\int \frac{\cos(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{a^3\sin(c+dx)}{d(a^4-a^2b^2)} + \frac{2b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d(a^4-a^2b^2)} - \frac{ab^2\sin(c+dx)}{d(a^4-a^2b^2)} - \frac{2a^2b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d(a^4-a^2b^2)}$$
$$+ \frac{b^2\,\mathrm{atan}\left(\frac{-a^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}\,1\mathrm{i}+b^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)(a^2-b^2)^{3/2}\,2\mathrm{i}+b^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}\,2\mathrm{i}-a^2b^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}\,3\mathrm{i}+a^3b^2\sin\ldots}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^6-2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^4b^2+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b^4}\ldots}{d(a^4-a^2b^2)}$$

input
```
int(cos(c + d*x)/(a + b/cos(c + d*x)),x)
```

output
```
(a^3*sin(c + d*x))/(d*(a^4 - a^2*b^2)) + (2*b^3*atan(sin(c/2 + (d*x)/2)/co
s(c/2 + (d*x)/2)))/(d*(a^4 - a^2*b^2)) - (a*b^2*sin(c + d*x))/(d*(a^4 - a^
2*b^2)) + (b^2*atan((b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(3/2)*2i - a^5*sin
(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*1i + b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^
(1/2)*2i - a^2*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*3i + a^3*b^2*sin(c
/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*1i + a^4*b*sin(c/2 + (d*x)/2)*(a^2 - b^2)^
(1/2)*1i)/(a^6*cos(c/2 + (d*x)/2) + a^2*b^4*cos(c/2 + (d*x)/2) - 2*a^4*b^2
*cos(c/2 + (d*x)/2)))*(a^2 - b^2)^(1/2)*2i)/(d*(a^4 - a^2*b^2)) - (2*a^2*b
*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(d*(a^4 - a^2*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.58

$$\int \frac{\cos(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{2\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{-a^2+b^2}}\right)b^2+\sin(dx+c)\,a^3-\sin(dx+c)\,ab^2-a^2bc-a^2bdx+b^3c-\ldots}{a^2d(a^2-b^2)}$$

input
```
int(cos(d*x+c)/(a+b*sec(d*x+c)),x)
```

output
```
(2*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqr
t( - a**2 + b**2))*b**2 + sin(c + d*x)*a**3 - sin(c + d*x)*a*b**2 - a**2*b
*c - a**2*b*d*x + b**3*c + b**3*d*x)/(a**2*d*(a**2 - b**2))
```

3.494 $\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx=\frac{(a^2+2b^2)\,x}{2a^3}-\frac{2b^3\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}d}-\frac{b\sin(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2ad}$$

output
```
1/2*(a^2+2*b^2)*x/a^3-2*b^3*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(
1/2))/a^3/(a-b)^(1/2)/(a+b)^(1/2)/d-b*sin(d*x+c)/a^2/d+1/2*cos(d*x+c)*sin(
d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.88

$$\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx=\frac{2(a^2+2b^2)\,(c+dx)+\frac{8b^3\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-4ab\sin(c+dx)+a^2\sin(2(c+dx))}{4a^3d}$$

input
```
Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

output
```
(2*(a^2 + 2*b^2)*(c + d*x) + (8*b^3*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sq
rt[a^2 - b^2]])/Sqrt[a^2 - b^2] - 4*a*b*Sin[c + d*x] + a^2*Sin[2*(c + d*x)
])/(4*a^3*d)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4340, 25, 3042, 4592, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4340}$$

$$\frac{\int -\frac{\cos(c+dx)\left(-b\sec^2(c+dx)-a\sec(c+dx)+2b\right)}{a+b\sec(c+dx)}dx}{2a}+\frac{\sin(c+dx)\cos(c+dx)}{2ad}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\int \frac{\cos(c+dx)\left(-b\sec^2(c+dx)-a\sec(c+dx)+2b\right)}{a+b\sec(c+dx)}dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\int \frac{-b\csc\left(c+dx+\frac{\pi}{2}\right)^2-a\csc\left(c+dx+\frac{\pi}{2}\right)+2b}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a}$$

$$\Big\downarrow \text{4592}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b\sin(c+dx)}{ad}-\frac{\int \frac{a^2+b\sec(c+dx)a+2b^2}{a+b\sec(c+dx)}dx}{a}}{2a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b\sin(c+dx)}{ad}-\frac{\int \frac{a^2+b\csc\left(c+dx+\frac{\pi}{2}\right)a+2b^2}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2a}$$

$$\Big\downarrow \ 4407$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b\sin(c+dx)}{ad}-\frac{\frac{x(a^2+2b^2)}{a}-\frac{2b^3\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}}{a}}{2a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b\sin(c+dx)}{ad}-\frac{\frac{x(a^2+2b^2)}{a}-\frac{2b^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{a}}{2a}$$

$$\Big\downarrow \ 4318$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b\sin(c+dx)}{ad}-\frac{\frac{x(a^2+2b^2)}{a}-\frac{2b^2\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a}}{a}}{2a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b\sin(c+dx)}{ad}-\frac{\frac{x(a^2+2b^2)}{a}-\frac{2b^2\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a}}{a}}{2a}$$

$$\Big\downarrow \ 3138$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b\sin(c+dx)}{ad}-\frac{\frac{x(a^2+2b^2)}{a}-\frac{4b^2\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}}{a}}{2a}$$

$$\Big\downarrow \ 221$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b\sin(c+dx)}{ad}-\frac{\frac{x(a^2+2b^2)}{a}-\frac{4b^3\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a}}{2a}$$

input

```
Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

output
```
(Cos[c + d*x]*Sin[c + d*x])/(2*a*d) - (-((((a^2 + 2*b^2)*x)/a - (4*b^3*Arc
Tanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a +
b]*d))/a) + (2*b*Sin[c + d*x])/(a*d))/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4340
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Sim
p[1/(a*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n
- a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{\frac{\frac{2\left(\left(-\frac{1}{2}a^2-ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(\frac{1}{2}a^2-ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\left(a^2+2b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}-\frac{2b^3\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^3\sqrt{(a+b)(a-b)}}}{d}$
default	$\frac{\frac{\frac{2\left(\left(-\frac{1}{2}a^2-ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(\frac{1}{2}a^2-ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\left(a^2+2b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}-\frac{2b^3\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^3\sqrt{(a+b)(a-b)}}}{d}$
risch	$\frac{x}{2a}+\frac{x\,b^2}{a^3}+\frac{ib\,e^{i(dx+c)}}{2a^2d}-\frac{ib\,e^{-i(dx+c)}}{2a^2d}+\frac{b^3\ln\left(e^{i(dx+c)}+\frac{-ia^2+ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d\,a^3}-\frac{b^3\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+b}{a\sqrt{a^2}}\right.}{\sqrt{a^2-b^2}\,d\,a^3}$

input
```
int(cos(d*x+c)^2/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/a^3*(((-1/2*a^2-a*b)*tan(1/2*d*x+1/2*c)^3+(1/2*a^2-a*b)*tan(1/2*d*x
+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(a^2+2*b^2)*arctan(tan(1/2*d*x+1/2
*c)))-2*b^3/a^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b
)*(a-b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 334, normalized size of antiderivative = 3.04

$$\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[\frac{\sqrt{a^2-b^2}b^3\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)+(a^4+a^2b^2-2\,b^4)}{2\,(a^5-a^3b^2)d}\right.$$
$$-\frac{2\,\sqrt{-a^2+b^2}b^3\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)-(a^4+a^2b^2-2\,b^4)dx+(2\,a^3b-2\,ab^3-(a^4-a^2b^2)}{2\,(a^5-a^3b^2)d}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a^2 - b^2)*b^3*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
+ c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2
)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (a^4 + a^2*b^2 - 2*b^
4)*d*x - (2*a^3*b - 2*a*b^3 - (a^4 - a^2*b^2)*cos(d*x + c))*sin(d*x + c))/
((a^5 - a^3*b^2)*d), -1/2*(2*sqrt(-a^2 + b^2)*b^3*arctan(-sqrt(-a^2 + b^2)
*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (a^4 + a^2*b^2 - 2*b^4
)*d*x + (2*a^3*b - 2*a*b^3 - (a^4 - a^2*b^2)*cos(d*x + c))*sin(d*x + c))/(
(a^5 - a^3*b^2)*d)]
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cos^2{(c+dx)}}{a+b\sec{(c+dx)}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+b*sec(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.62

$$\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b^3}{\sqrt{-a^2+b^2}\,a^3}-\frac{(a^2+2b^2)(dx+c)}{a^3}+\frac{2\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2b\tan(\cdots\right.}{\left(\tan\cdots\right.}}{2d}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan
(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b^3/(sqrt(-
a^2 + b^2)*a^3) - (a^2 + 2*b^2)*(d*x + c)/a^3 + 2*(a*tan(1/2*d*x + 1/2*c)^
3 + 2*b*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) + 2*b*tan(1/2*d*x
+ 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2))/d
```

Mupad [B] (verification not implemented)

Time = 10.66 (sec) , antiderivative size = 592, normalized size of antiderivative = 5.38

$$\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{a\left(\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+\frac{\sin(2\,c+2\,d\,x)}{4}\right)}{d\,\left(a^2-b^2\right)} - \frac{b\,\sin\left(c+d\,x\right)}{d\,\left(a^2-b^2\right)} + \frac{b^2\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-\frac{b^2\,\sin(2\,c+2\,d\,x)}{4}}{a\,d\,\left(a^2-b^2\right)} + \frac{b^3\,\sin\left(c+d\,x\right)}{a^2\,d\,\left(a^2-b^2\right)} - \frac{2\,b^4\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a^3\,d\,\left(a^2-b^2\right)} - \frac{b^3\operatorname{atan}\left(\frac{\left(8\,b^7\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,{\left(a^2-b^2\right)}^{3/2}-a^9\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}+8\,b^9\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}-8\,a^2\,b^7\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}-3\,a^4\,b^5\,\sin \cdots}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a\,b^2-a^3\right)\,\left(4\,b^5\,\left(a^2-b^2\right)+2\,a\,b^6-a^7+4\,b^7 \cdots}\right)}{a^3\,d \cdots}$$

input

```
int(cos(c + d*x)^2/(a + b/cos(c + d*x)),x)
```

output

```
(a*(atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + sin(2*c + 2*d*x)/4))/(d*
(a^2 - b^2)) - (b*sin(c + d*x))/(d*(a^2 - b^2)) + (b^2*atan(sin(c/2 + (d*x
)/2)/cos(c/2 + (d*x)/2)) - (b^2*sin(2*c + 2*d*x))/4)/(a*d*(a^2 - b^2)) - (
b^3*atan(((8*b^7*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(3/2) - a^9*sin(c/2 + (d*x
)/2)*(a^2 - b^2)^(1/2) + 8*b^9*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 8*a^
2*b^7*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 3*a^4*b^5*sin(c/2 + (d*x)/2)*
(a^2 - b^2)^(1/2) + 3*a^5*b^4*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 2*a^6
*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 2*a^7*b^2*sin(c/2 + (d*x)/2)*(
a^2 - b^2)^(1/2) + a^8*b*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))*1i)/(cos(c/
2 + (d*x)/2)*(a*b^2 - a^3)*(4*b^5*(a^2 - b^2) + 2*a*b^6 - a^7 + 4*b^7 - 2*
a^2*b^5 + a^3*b^4 - 2*a^4*b^3 - 2*a^5*b^2 + 2*a^2*b^3*(a^2 - b^2) + 2*a*b^
4*(a^2 - b^2))))*2i)/(a^3*d*(a^2 - b^2)^(1/2)) + (b^3*sin(c + d*x))/(a^2*d
*(a^2 - b^2)) - (2*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^3*d
*(a^2 - b^2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.58

$$\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{-4\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{-a^2+b^2}}\right)b^3+\cos(dx+c)\sin(dx+c)\,a^4-\cos(dx+c)\sin(dx+c)\,\cdots}{2a^3d\,(a^2-b^2\cdots}$$

input
```
int(cos(d*x+c)^2/(a+b*sec(d*x+c)),x)
```

output
```
( - 4*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/
sqrt( - a**2 + b**2))*b**3 + cos(c + d*x)*sin(c + d*x)*a**4 - cos(c + d*x)
*sin(c + d*x)*a**2*b**2 - 2*sin(c + d*x)*a**3*b + 2*sin(c + d*x)*a*b**3 +
a**4*c + a**4*d*x + a**2*b**2*c + a**2*b**2*d*x - 2*b**4*c - 2*b**4*d*x)/(
2*a**3*d*(a**2 - b**2))
```

3.495 $\int \frac{\cos^3(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 148

$$\int \frac{\cos^3(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{b(a^2+2b^2)\,x}{2a^4}+\frac{2b^4\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}d}+\frac{(2a^2+3b^2)\sin(c+dx)}{3a^3d}-\frac{b\cos(c+dx)\sin(c+dx)}{2a^2d}+\frac{\cos^2(c+dx)\sin(c+dx)}{3ad}$$

output
```
-1/2*b*(a^2+2*b^2)*x/a^4+2*b^4*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b
)^(1/2))/a^4/(a-b)^(1/2)/(a+b)^(1/2)/d+1/3*(2*a^2+3*b^2)*sin(d*x+c)/a^3/d-
1/2*b*cos(d*x+c)*sin(d*x+c)/a^2/d+1/3*cos(d*x+c)^2*sin(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.82

$$\int \frac{\cos^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{-6b(a^2+2b^2)(c+dx)-\frac{24b^4\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+3a(3a^2+4b^2)\sin(c+dx)-3a^2b\sin(2(c+dx}{12a^4d}$$

input
```
Integrate[Cos[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

output
```
(-6*b*(a^2 + 2*b^2)*(c + d*x) - (24*b^4*ArcTanh[((-a + b)*Tan[(c + d*x)/2]
)/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 3*a*(3*a^2 + 4*b^2)*Sin[c + d*x] - 3
*a^2*b*Sin[2*(c + d*x)] + a^3*Sin[3*(c + d*x)])/(12*a^4*d)
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4340, 25, 3042, 4592, 3042, 4592, 27, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4340}$$

$$\frac{\int -\frac{\cos^2(c+dx)\left(-2b\sec^2(c+dx)-2a\sec(c+dx)+3b\right)}{a+b\sec(c+dx)}dx}{3a}+\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\int \frac{\cos^2(c+dx)\left(-2b\sec^2(c+dx)-2a\sec(c+dx)+3b\right)}{a+b\sec(c+dx)}dx}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\int \frac{-2b\csc\left(c+dx+\frac{\pi}{2}\right)^2-2a\csc\left(c+dx+\frac{\pi}{2}\right)+3b}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3a}$$

$$\Big\downarrow \text{4592}$$

$$\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\int\frac{\cos(c+dx)\left(-3b^2\sec^2(c+dx)+ab\sec(c+dx)+2\left(2a^2+3b^2\right)\right)}{a+b\sec(c+dx)}dx}{2a}}{3a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\int\frac{-3b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+ab\csc\left(c+dx+\frac{\pi}{2}\right)+2\left(2a^2+3b^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a}}{3a}$$

$$\Big\downarrow \; 4592$$

$$\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2\left(2a^2+3b^2\right)\sin(c+dx)}{ad}-\frac{\int\frac{3\left(a\sec(c+dx)b^2+\left(a^2+2b^2\right)b\right)}{a+b\sec(c+dx)}dx}{a}}{2a}}{3a}$$

$$\Big\downarrow \; 27$$

$$\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2\left(2a^2+3b^2\right)\sin(c+dx)}{ad}-\frac{3\int\frac{a\sec(c+dx)b^2+\left(a^2+2b^2\right)b}{a+b\sec(c+dx)}dx}{a}}{2a}}{3a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2\left(2a^2+3b^2\right)\sin(c+dx)}{ad}-\frac{3\int\frac{a\csc\left(c+dx+\frac{\pi}{2}\right)b^2+\left(a^2+2b^2\right)b}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2a}}{3a}$$

$$\Big\downarrow \; 4407$$

$$\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2\left(2a^2+3b^2\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{bx\left(a^2+2b^2\right)}{a}-\frac{2b^4\int\frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}\right)}{a}}{2a}}{3a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2\left(2a^2+3b^2\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{bx\left(a^2+2b^2\right)}{a}-\frac{2b^4\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}\right)}{a}}{2a}}{3a}$$

$$\Big\downarrow \; 4318$$

$$\frac{\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2(2a^2+3b^2)\sin(c+dx)}{ad}-\frac{3\left(\frac{bx(a^2+2b^2)}{a}-\frac{2b^3\int\frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a}\right)}{a}}{2a}}{3a}}{}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2(2a^2+3b^2)\sin(c+dx)}{ad}-\frac{3\left(\frac{bx(a^2+2b^2)}{a}-\frac{2b^3\int\frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a}\right)}{a}}{2a}}{3a}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2(2a^2+3b^2)\sin(c+dx)}{ad}-\frac{3\left(\frac{bx(a^2+2b^2)}{a}-\frac{4b^3\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}\right)}{a}}{2a}}{3a}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2(2a^2+3b^2)\sin(c+dx)}{ad}-\frac{3\left(\frac{bx(a^2+2b^2)}{a}-\frac{4b^4\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a}}{2a}}{3a}$$

input
```
Int[Cos[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

output
```
(Cos[c + d*x]^2*Sin[c + d*x])/(3*a*d) - ((3*b*Cos[c + d*x]*Sin[c + d*x])/(
2*a*d) - ((-3*((b*(a^2 + 2*b^2)*x)/a - (4*b^4*ArcTanh[(Sqrt[a - b]*Tan[(c
+ d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d)))/a + (2*(2*a^2 + 3
*b^2)*Sin[c + d*x])/(a*d))/(2*a))/(3*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4340
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Sim
p[1/(a*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n
- a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{-\frac{2\left(\frac{\left(-a^3-\frac{1}{2}a^2b-ab^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(-\frac{2}{3}a^3-2ab^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-a^3-ab^2+\frac{1}{2}a^2b\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}+\frac{b\left(a^2+2b^2\right)\arctan\left(\tan\right.}{2}\right.}{a^4}}{d}$
default	$\frac{-\frac{2\left(\frac{\left(-a^3-\frac{1}{2}a^2b-ab^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(-\frac{2}{3}a^3-2ab^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-a^3-ab^2+\frac{1}{2}a^2b\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}+\frac{b\left(a^2+2b^2\right)\arctan\left(\tan\right.}{2}\right.}{a^4}}{d}$
risch	$-\frac{bx}{2a^2}-\frac{b^3x}{a^4}-\frac{3ie^{i(dx+c)}}{8ad}-\frac{ie^{i(dx+c)}b^2}{2a^3d}+\frac{3ie^{-i(dx+c)}}{8ad}+\frac{ie^{-i(dx+c)}b^2}{2a^3d}+\frac{b^4\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b}}{a\sqrt{a^2-b^2}}\right.}{\sqrt{a^2-b^2}\,d\,a^4}$

input
```
int(cos(d*x+c)^3/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/a^4*(((-a^3-1/2*a^2*b-a*b^2)*tan(1/2*d*x+1/2*c)^5+(-2/3*a^3-2*a*b^
2)*tan(1/2*d*x+1/2*c)^3+(-a^3-a*b^2+1/2*a^2*b)*tan(1/2*d*x+1/2*c))/(1+tan(
1/2*d*x+1/2*c)^2)^3+1/2*b*(a^2+2*b^2)*arctan(tan(1/2*d*x+1/2*c)))+2*b^4/a^
4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)
))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.71

$$\int \frac{\cos^3(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[\frac{3\sqrt{a^2-b^2}b^4\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)-3\,(a^4b+a^2b^3-}{6\,(a^6-}\right.$$

input
```
integrate(cos(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(a^2 - b^2)*b^4*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*
x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b
^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 3*(a^4*b + a^2*b^3
- 2*b^5)*d*x + (4*a^5 + 2*a^3*b^2 - 6*a*b^4 + 2*(a^5 - a^3*b^2)*cos(d*x +
c)^2 - 3*(a^4*b - a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d)
, 1/6*(6*sqrt(-a^2 + b^2)*b^4*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a
)/((a^2 - b^2)*sin(d*x + c))) - 3*(a^4*b + a^2*b^3 - 2*b^5)*d*x + (4*a^5 +
 2*a^3*b^2 - 6*a*b^4 + 2*(a^5 - a^3*b^2)*cos(d*x + c)^2 - 3*(a^4*b - a^2*b
^3)*cos(d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+b*sec(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.68

$$\int \frac{\cos^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b^4}{\sqrt{-a^2+b^2}\,a^4} - \frac{3\,(a^2b+2\,b^3)(dx+c)}{a^4} + \frac{2\left(6\,a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3\,ab\,t\right.}{}}{}$$

input
```
integrate(cos(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan
(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b^4/(sqrt(-
a^2 + b^2)*a^4) - 3*(a^2*b + 2*b^3)*(d*x + c)/a^4 + 2*(6*a^2*tan(1/2*d*x +
 1/2*c)^5 + 3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*b^2*tan(1/2*d*x + 1/2*c)^5 +
4*a^2*tan(1/2*d*x + 1/2*c)^3 + 12*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*tan(1
/2*d*x + 1/2*c) - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*b^2*tan(1/2*d*x + 1/2*c))
/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^3))/d
```

Mupad [B] (verification not implemented)

Time = 10.94 (sec) , antiderivative size = 654, normalized size of antiderivative = 4.42

$$\int \frac{\cos^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{b^2\sin(c+dx)}{4}-\frac{b^2\sin(3c+3dx)}{12}}{a\,d\,(a^2-b^2)} - \frac{b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)+\frac{b\sin(2c+2dx)}{4}}{d\,(a^2-b^2)} + \frac{a\left(\frac{3\sin(c+dx)}{4}+\frac{\sin(3c+3dx)}{12}\right)}{d\,(a^2-b^2)} - \frac{b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)-\frac{b^3\sin(2c+2dx)}{4}}{a^2\,d\,(a^2-b^2)} - \frac{b^4\sin(c+dx)}{a^3\,d\,(a^2-b^2)} + \frac{2\,b^5\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{a^4\,d\,(a^2-b^2)} + \frac{b^4\,\mathrm{atan}\left(\frac{\left(8\,b^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)(a^2-b^2)^{3/2}-a^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}+8\,b^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}-8\,a^2\,b^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}-3\,a^4\,b^5\sin\left(\right.\right.}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)(a\,b^2-a^3)\left(4\,b^5\,(a^2-b^2)+2\,a\,b^6-a^7+4\,b^7\right.}\right.}{a^4\,d}$$

input

```
int(cos(c + d*x)^3/(a + b/cos(c + d*x)),x)
```

output

```
((b^2*sin(c + d*x))/4 - (b^2*sin(3*c + 3*d*x))/12)/(a*d*(a^2 - b^2)) - (b*
atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + (b*sin(2*c + 2*d*x))/4)/(d*(
a^2 - b^2)) + (a*((3*sin(c + d*x))/4 + sin(3*c + 3*d*x)/12))/(d*(a^2 - b^2
)) - (b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - (b^3*sin(2*c + 2*d
*x))/4)/(a^2*d*(a^2 - b^2)) + (b^4*atan(((8*b^7*sin(c/2 + (d*x)/2)*(a^2 -
b^2)^(3/2) - a^9*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 8*b^9*sin(c/2 + (d
*x)/2)*(a^2 - b^2)^(1/2) - 8*a^2*b^7*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)
- 3*a^4*b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 3*a^5*b^4*sin(c/2 + (d*
x)/2)*(a^2 - b^2)^(1/2) + 2*a^6*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) -
 2*a^7*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + a^8*b*sin(c/2 + (d*x)/2)
*(a^2 - b^2)^(1/2))*1i)/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3)*(4*b^5*(a^2 - b^
2) + 2*a*b^6 - a^7 + 4*b^7 - 2*a^2*b^5 + a^3*b^4 - 2*a^4*b^3 - 2*a^5*b^2 +
 2*a^2*b^3*(a^2 - b^2) + 2*a*b^4*(a^2 - b^2))))*2i)/(a^4*d*(a^2 - b^2)^(1/
2)) - (b^4*sin(c + d*x))/(a^3*d*(a^2 - b^2)) + (2*b^5*atan(sin(c/2 + (d*x)
/2)/cos(c/2 + (d*x)/2)))/(a^4*d*(a^2 - b^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.42

$$\int \frac{\cos^3(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{12\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{-a^2+b^2}}\right)b^4-3\cos(dx+c)\sin(dx+c)\,a^4b+3\cos(dx+c)\sin(dx+}{}$$

input
```
int(cos(d*x+c)^3/(a+b*sec(d*x+c)),x)
```

output
```
(12*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sq
rt( - a**2 + b**2))*b**4 - 3*cos(c + d*x)*sin(c + d*x)*a**4*b + 3*cos(c +
d*x)*sin(c + d*x)*a**2*b**3 - 2*sin(c + d*x)**3*a**5 + 2*sin(c + d*x)**3*a
**3*b**2 + 6*sin(c + d*x)*a**5 - 6*sin(c + d*x)*a*b**4 - 3*a**4*b*c - 3*a*
*4*b*d*x - 3*a**2*b**3*c - 3*a**2*b**3*d*x + 6*b**5*c + 6*b**5*d*x)/(6*a**
4*d*(a**2 - b**2))
```

3.496 $\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 193

$$\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{(3a^4+4a^2b^2+8b^4)\,x}{8a^5} - \frac{2b^5\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5\sqrt{a-b}\sqrt{a+b}d} - \frac{b(2a^2+3b^2)\sin(c+dx)}{3a^4d} + \frac{(3a^2+4b^2)\cos(c+dx)\sin(c+dx)}{8a^3d} - \frac{b\cos^2(c+dx)\sin(c+dx)}{3a^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4ad}$$

output
```
1/8*(3*a^4+4*a^2*b^2+8*b^4)*x/a^5-2*b^5*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/
2*c)/(a+b)^(1/2))/a^5/(a-b)^(1/2)/(a+b)^(1/2)/d-1/3*b*(2*a^2+3*b^2)*sin(d*
x+c)/a^4/d+1/8*(3*a^2+4*b^2)*cos(d*x+c)*sin(d*x+c)/a^3/d-1/3*b*cos(d*x+c)^
2*sin(d*x+c)/a^2/d+1/4*cos(d*x+c)^3*sin(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.79

$$\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{12(3a^4+4a^2b^2+8b^4)(c+dx)+\frac{192b^5\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-24ab(3a^2+4b^2)\sin(c+dx)+24a^2(a}{96a^5d}$$

input
```
Integrate[Cos[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

output
```
(12*(3*a^4 + 4*a^2*b^2 + 8*b^4)*(c + d*x) + (192*b^5*ArcTanh[((-a + b)*Tan
[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - 24*a*b*(3*a^2 + 4*b^2)*
Sin[c + d*x] + 24*a^2*(a^2 + b^2)*Sin[2*(c + d*x)] - 8*a^3*b*Sin[3*(c + d*
x)] + 3*a^4*Sin[4*(c + d*x)])/(96*a^5*d)
```

Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.13, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.810, Rules used = {3042, 4340, 25, 3042, 4592, 3042, 4592, 3042, 4592, 27, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$
$$\Big\downarrow \; 4340$$
$$\frac{\int -\frac{\cos^3(c+dx)(-3b\sec^2(c+dx)-3a\sec(c+dx)+4b)}{a+b\sec(c+dx)}dx}{4a}+\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}$$

$$\Big\downarrow \; 25$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\int \frac{\cos^3(c+dx)(-3b\sec^2(c+dx)-3a\sec(c+dx)+4b)}{a+b\sec(c+dx)}dx}{4a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\int \frac{-3b\csc\left(c+dx+\frac{\pi}{2}\right)^2-3a\csc\left(c+dx+\frac{\pi}{2}\right)+4b}{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{4a}$$

$$\Big\downarrow \; 4592$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\frac{4b\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\int \frac{\cos^2(c+dx)\left(-8b^2\sec^2(c+dx)+ab\sec(c+dx)+3\left(3a^2+4b^2\right)\right)}{a+b\sec(c+dx)}dx}{3a}}{4a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\frac{4b\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\int \frac{-8b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+ab\csc\left(c+dx+\frac{\pi}{2}\right)+3\left(3a^2+4b^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3a}}{4a}$$

$$\Big\downarrow \; 4592$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\frac{4b\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3\left(3a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\int \frac{\cos(c+dx)\left(-3b\left(3a^2+4b^2\right)\sec^2(c+dx)-a\left(9a^2-4b^2\right)\sec(c+dx)+8b\left(2a^2+3b^2\right)\right)}{a+b\sec(c+dx)}dx}{2a}}{3a}}{4a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\frac{4b\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3\left(3a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\int \frac{-3b\left(3a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-a\left(9a^2-4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+8b\left(2a^2+3b^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a}}{3a}}{4a}$$

$$\Big\downarrow \; 4592$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\frac{4b\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3\left(3a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{8b\left(2a^2+3b^2\right)\sin(c+dx)}{ad}-\frac{\int \frac{3\left(3a^4+4b^2a^2+b\left(3a^2+4b^2\right)\sec(c+dx)a+8b^4\right)}{a+b\sec(c+dx)}dx}{a}}{2a}}{3a}}{4a}$$

$\big\downarrow$ 27

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\frac{4b\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3(3a^2+4b^2)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{8b(2a^2+3b^2)\sin(c+dx)}{ad}-\frac{3\int\frac{3a^4+4b^2a^2+b(3a^2+4b^2)\sec(c+dx)a+8b^4}{a+b\sec(c+dx)}dx}{a}}{2a}}{3a}}{4a}$$

$\big\downarrow$ 3042

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\frac{4b\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3(3a^2+4b^2)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{8b(2a^2+3b^2)\sin(c+dx)}{ad}-\frac{3\int\frac{3a^4+4b^2a^2+b(3a^2+4b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a+8b^4}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2a}}{3a}}{4a}$$

$\big\downarrow$ 4407

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\frac{4b\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3(3a^2+4b^2)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{8b(2a^2+3b^2)\sin(c+dx)}{ad}-\frac{3\left(\frac{x(3a^4+4a^2b^2+8b^4)}{a}-\frac{8b^5\int\frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}\right)}{a}}{2a}}{3a}}{4a}$$

$\big\downarrow$ 3042

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\frac{4b\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3(3a^2+4b^2)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{8b(2a^2+3b^2)\sin(c+dx)}{ad}-\frac{3\left(\frac{x(3a^4+4a^2b^2+8b^4)}{a}-\frac{8b^5\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}\right)}{a}}{2a}}{3a}}{4a}$$

$\big\downarrow$ 4318

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\frac{4b\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3(3a^2+4b^2)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{8b(2a^2+3b^2)\sin(c+dx)}{ad}-\frac{3\left(\frac{x(3a^4+4a^2b^2+8b^4)}{a}-\frac{8b^4\int\frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a}\right)}{a}}{2a}}{3a}}{4a}$$

$\big\downarrow$ 3042

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\frac{4b\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3\left(3a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{8b\left(2a^2+3b^2\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{x\left(3a^4+4a^2b^2+8b^4\right)}{a}-\frac{8b^4\int\frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a}\right)}{a}}{2a}}{3a}}{4a}$$

$$\Big\downarrow \; 3138$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\frac{4b\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3\left(3a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{8b\left(2a^2+3b^2\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{x\left(3a^4+4a^2b^2+8b^4\right)}{a}-\frac{16b^4\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}\right)}{a}}{2a}}{3a}}{4a}$$

$$\Big\downarrow \; 221$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\frac{4b\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3\left(3a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{8b\left(2a^2+3b^2\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{x\left(3a^4+4a^2b^2+8b^4\right)}{a}-\frac{16b^5\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a}}{2a}}{3a}}{4a}$$

input
```
Int[Cos[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

output
```
(Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d) - ((4*b*Cos[c + d*x]^2*Sin[c + d*x])
/(3*a*d) - ((3*(3*a^2 + 4*b^2)*Cos[c + d*x]*Sin[c + d*x])/(2*a*d) - ((-3*(
((3*a^4 + 4*a^2*b^2 + 8*b^4)*x)/a - (16*b^5*ArcTanh[(Sqrt[a - b]*Tan[(c +
d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d)))/a + (8*b*(2*a^2 + 3
*b^2)*Sin[c + d*x])/(a*d))/(2*a))/(3*a))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4340
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Sim
p[1/(a*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n
- a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.33

method	result
derivativedivides	$-\frac{2b^5 \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^5\sqrt{(a+b)(a-b)}}+\frac{2\left(\left(-\frac{5}{8}a^4-b\,a^3-\frac{1}{2}a^2b^2-a\,b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7+\left(\frac{3}{8}a^4-\frac{5}{3}b\,a^3-3a\,b^3-\frac{1}{2}a^2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots\right.}{\left(1+\tan\left(\cdots\right.\right.}$
default	$-\frac{2b^5 \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^5\sqrt{(a+b)(a-b)}}+\frac{2\left(\left(-\frac{5}{8}a^4-b\,a^3-\frac{1}{2}a^2b^2-a\,b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7+\left(\frac{3}{8}a^4-\frac{5}{3}b\,a^3-3a\,b^3-\frac{1}{2}a^2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots\right.}{\left(1+\tan\left(\cdots\right.\right.}$
risch	$\frac{3x}{8a}+\frac{x\,b^2}{2a^3}+\frac{x\,b^4}{a^5}+\frac{3ib\,e^{i(dx+c)}}{8a^2d}+\frac{ib^3e^{i(dx+c)}}{2d\,a^4}-\frac{3ib\,e^{-i(dx+c)}}{8a^2d}-\frac{ib^3e^{-i(dx+c)}}{2d\,a^4}+\frac{b^5\ln\left(e^{i(dx+c)}+\frac{-ia^2+ib^2\cdots}{a\sqrt{\cdots}}\right.}{\sqrt{a^2-b^2}\,d\,a^5\cdots}$

input
```
int(cos(d*x+c)^4/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*b^5/a^5/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b
)*(a-b))^(1/2))+2/a^5*(((-5/8*a^4-b*a^3-1/2*a^2*b^2-a*b^3)*tan(1/2*d*x+1/2
*c)^7+(3/8*a^4-5/3*b*a^3-3*a*b^3-1/2*a^2*b^2)*tan(1/2*d*x+1/2*c)^5+(-3/8*a
^4+1/2*a^2*b^2-5/3*b*a^3-3*a*b^3)*tan(1/2*d*x+1/2*c)^3+(5/8*a^4+1/2*a^2*b^
2-b*a^3-a*b^3)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^4+1/8*(3*a^4+4
*a^2*b^2+8*b^4)*arctan(tan(1/2*d*x+1/2*c))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 482, normalized size of antiderivative = 2.50

$$\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[\frac{12\sqrt{a^2-b^2}b^5\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)+3\,(3\,a^6+a^4b^2-\right.$$
$$-\frac{24\sqrt{-a^2+b^2}b^5\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)-3\,(3\,a^6+a^4b^2+4\,a^2b^4-8\,b^6)dx+(16\,a^5b+8\,a^3b^3}{}$$

input
```
integrate(cos(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[1/24*(12*sqrt(a^2 - b^2)*b^5*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(
d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 -
 b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 3*(3*a^6 + a^4*b^
2 + 4*a^2*b^4 - 8*b^6)*d*x - (16*a^5*b + 8*a^3*b^3 - 24*a*b^5 - 6*(a^6 - a
^4*b^2)*cos(d*x + c)^3 + 8*(a^5*b - a^3*b^3)*cos(d*x + c)^2 - 3*(3*a^6 + a
^4*b^2 - 4*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^7 - a^5*b^2)*d), -1/24
*(24*sqrt(-a^2 + b^2)*b^5*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((
a^2 - b^2)*sin(d*x + c))) - 3*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*d*x +
(16*a^5*b + 8*a^3*b^3 - 24*a*b^5 - 6*(a^6 - a^4*b^2)*cos(d*x + c)^3 + 8*(a
^5*b - a^3*b^3)*cos(d*x + c)^2 - 3*(3*a^6 + a^4*b^2 - 4*a^2*b^4)*cos(d*x +
 c))*sin(d*x + c))/((a^7 - a^5*b^2)*d)]
```

Sympy [F]

$$\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cos^4{(c+dx)}}{a+b\sec{(c+dx)}}\,dx$$

input
```
integrate(cos(d*x+c)**4/(a+b*sec(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(174) = 348$.

Time = 0.15 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.04

$$\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\frac{48\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b^5}{\sqrt{-a^2+b^2}\,a^5}-\frac{3\left(3a^4+4a^2b^2+8b^4\right)(dx+c)}{a^5}+\frac{2\left(15a^3\tan\left(\frac{1}{2}dx-\right.\right.}{}}{}$$

input
```
integrate(cos(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/24*(48*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*t
an(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b^5/(sqrt
(-a^2 + b^2)*a^5) - 3*(3*a^4 + 4*a^2*b^2 + 8*b^4)*(d*x + c)/a^5 + 2*(15*a^
3*tan(1/2*d*x + 1/2*c)^7 + 24*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 12*a*b^2*tan(
1/2*d*x + 1/2*c)^7 + 24*b^3*tan(1/2*d*x + 1/2*c)^7 - 9*a^3*tan(1/2*d*x + 1
/2*c)^5 + 40*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^
5 + 72*b^3*tan(1/2*d*x + 1/2*c)^5 + 9*a^3*tan(1/2*d*x + 1/2*c)^3 + 40*a^2*
b*tan(1/2*d*x + 1/2*c)^3 - 12*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 72*b^3*tan(1/
2*d*x + 1/2*c)^3 - 15*a^3*tan(1/2*d*x + 1/2*c) + 24*a^2*b*tan(1/2*d*x + 1/
2*c) - 12*a*b^2*tan(1/2*d*x + 1/2*c) + 24*b^3*tan(1/2*d*x + 1/2*c))/((tan(
1/2*d*x + 1/2*c)^2 + 1)^4*a^4))/d
```

Mupad [B] (verification not implemented)

Time = 12.10 (sec) , antiderivative size = 2678, normalized size of antiderivative = 13.88

$$\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^4/(a + b/cos(c + d*x)),x)
```

output
```
(b^5*atan(((b^5*(a^2 - b^2)^(1/2)*((tan(c/2 + (d*x)/2)*(256*a*b^10 - 27*a^
10*b + 9*a^11 - 128*b^11 - 256*a^2*b^9 + 256*a^3*b^8 - 256*a^4*b^7 + 256*a
^5*b^6 - 216*a^6*b^5 + 136*a^7*b^4 - 81*a^8*b^3 + 51*a^9*b^2))/(2*a^8) + (
b^5*((12*a^16 - 12*a^15*b + 32*a^10*b^6 - 48*a^11*b^5 + 16*a^12*b^4 - 4*a^
13*b^3 + 4*a^14*b^2)/a^12 - (b^5*tan(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(128
*a^12*b + 128*a^10*b^3 - 256*a^11*b^2))/(2*a^8*(a^7 - a^5*b^2)))*(a^2 - b^
2)^(1/2))/(a^7 - a^5*b^2))*1i)/(a^7 - a^5*b^2) + (b^5*(a^2 - b^2)^(1/2)*((
tan(c/2 + (d*x)/2)*(256*a*b^10 - 27*a^10*b + 9*a^11 - 128*b^11 - 256*a^2*b
^9 + 256*a^3*b^8 - 256*a^4*b^7 + 256*a^5*b^6 - 216*a^6*b^5 + 136*a^7*b^4 -
 81*a^8*b^3 + 51*a^9*b^2))/(2*a^8) - (b^5*((12*a^16 - 12*a^15*b + 32*a^10*
b^6 - 48*a^11*b^5 + 16*a^12*b^4 - 4*a^13*b^3 + 4*a^14*b^2)/a^12 + (b^5*tan
(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(128*a^12*b + 128*a^10*b^3 - 256*a^11*b^
2))/(2*a^8*(a^7 - a^5*b^2)))*(a^2 - b^2)^(1/2))/(a^7 - a^5*b^2))*1i)/(a^7
- a^5*b^2))/((96*a*b^13 - 64*b^14 - 96*a^2*b^12 + 104*a^3*b^11 - 104*a^4*b
^10 + 88*a^5*b^9 - 48*a^6*b^8 + 33*a^7*b^7 - 18*a^8*b^6 + 9*a^9*b^5)/a^12
- (b^5*(a^2 - b^2)^(1/2)*((tan(c/2 + (d*x)/2)*(256*a*b^10 - 27*a^10*b + 9*
a^11 - 128*b^11 - 256*a^2*b^9 + 256*a^3*b^8 - 256*a^4*b^7 + 256*a^5*b^6 -
216*a^6*b^5 + 136*a^7*b^4 - 81*a^8*b^3 + 51*a^9*b^2))/(2*a^8) + (b^5*((12*
a^16 - 12*a^15*b + 32*a^10*b^6 - 48*a^11*b^5 + 16*a^12*b^4 - 4*a^13*b^3 +
4*a^14*b^2)/a^12 - (b^5*tan(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(128*a^12*...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.50

$$\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{-48\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{-a^2+b^2}}\right)b^5-6\cos(dx+c)\sin(dx+c)^3a^6+6\cos(dx+c)\sin(dx}{}$$

input
```
int(cos(d*x+c)^4/(a+b*sec(d*x+c)),x)
```

output
```
( - 48*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)
/sqrt( - a**2 + b**2))*b**5 - 6*cos(c + d*x)*sin(c + d*x)**3*a**6 + 6*cos(
c + d*x)*sin(c + d*x)**3*a**4*b**2 + 15*cos(c + d*x)*sin(c + d*x)*a**6 - 3
*cos(c + d*x)*sin(c + d*x)*a**4*b**2 - 12*cos(c + d*x)*sin(c + d*x)*a**2*b
**4 + 8*sin(c + d*x)**3*a**5*b - 8*sin(c + d*x)**3*a**3*b**3 - 24*sin(c +
d*x)*a**5*b + 24*sin(c + d*x)*a*b**5 + 9*a**6*c + 9*a**6*d*x + 3*a**4*b**2
*c + 3*a**4*b**2*d*x + 12*a**2*b**4*c + 12*a**2*b**4*d*x - 24*b**6*c - 24*
b**6*d*x)/(24*a**5*d*(a**2 - b**2))
```

3.497 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 222

$$\begin{aligned}\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = {} & \frac{(6a^2+b^2)\operatorname{arctanh}(\sin(c+dx))}{2b^4d} \\ & - \frac{2a^3(3a^2-4b^2)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^4(a+b)^{3/2}d} \\ & - \frac{a(3a^2-2b^2)\tan(c+dx)}{b^3(a^2-b^2)\,d} \\ & + \frac{(3a^2-b^2)\sec(c+dx)\tan(c+dx)}{2b^2(a^2-b^2)\,d} \\ & - \frac{a^2\sec^2(c+dx)\tan(c+dx)}{b(a^2-b^2)\,d(a+b\sec(c+dx))}\end{aligned}$$

output
```
1/2*(6*a^2+b^2)*arctanh(sin(d*x+c))/b^4/d-2*a^3*(3*a^2-4*b^2)*arctanh((a-b
)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/b^4/(a+b)^(3/2)/d-a*(3
*a^2-2*b^2)*tan(d*x+c)/b^3/(a^2-b^2)/d+1/2*(3*a^2-b^2)*sec(d*x+c)*tan(d*x+
c)/b^2/(a^2-b^2)/d-a^2*sec(d*x+c)^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+
c))
```

Mathematica [A] (verified)

Time = 5.32 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.28

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{8a^3(3a^2-4b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} - 12a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) - 2b^2\log\left(\cos\left(\frac{1}{2}(c\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

output
```
((8*a^3*(3*a^2 - 4*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2
]])/(a^2 - b^2)^(3/2) - 12*a^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
2*b^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*a^2*Log[Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]] + 2*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] +
b^2/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - b^2/(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^2 + (4*a^4*b*Sin[c + d*x])/((-a + b)*(a + b)*(b + a*Cos[c +
d*x])) - 8*a*b*Tan[c + d*x])/(4*b^4*d)
```

Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.05, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4332, 3042, 4580, 25, 3042, 4570, 3042, 4486, 3042, 4257, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4332}$$

$$-\frac{\int \frac{\sec^2(c+dx)\left(2a^2-b\sec(c+dx)a-(3a^2-b^2)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(2a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a+(b^2-3a^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{4580}$$

$$-\frac{\frac{\int -\frac{\sec(c+dx)\left(-2a\left(3a^2-2b^2\right)\sec^2(c+dx)-b\left(a^2+b^2\right)\sec(c+dx)+a\left(3a^2-b^2\right)\right)}{a+b\sec(c+dx)}dx}{2b}-\frac{(3a^2-b^2)\tan(c+dx)\sec(c+dx)}{2bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-\frac{\int \frac{\sec(c+dx)\left(-2a\left(3a^2-2b^2\right)\sec^2(c+dx)-b\left(a^2+b^2\right)\sec(c+dx)+a\left(3a^2-b^2\right)\right)}{a+b\sec(c+dx)}dx}{2b}-\frac{(3a^2-b^2)\tan(c+dx)\sec(c+dx)}{2bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(-2a\left(3a^2-2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-b\left(a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+a\left(3a^2-b^2\right)\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2b}-\frac{(3a^2-b^2)\tan(c+dx)\sec(c+dx)}{2bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{4570}$$

$$-\frac{-\frac{\frac{\int \frac{\sec(c+dx)\left(ab\left(3a^2-b^2\right)+\left(a^2-b^2\right)\left(6a^2+b^2\right)\sec(c+dx)\right)}{a+b\sec(c+dx)}dx}{b}-\frac{2a\left(3a^2-2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{(3a^2-b^2)\tan(c+dx)\sec(c+dx)}{2bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(ab\left(3a^2-b^2\right)+\left(a^2-b^2\right)\left(6a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{2a\left(3a^2-2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{\left(3a^2-b^2\right)\tan(c+dx)\sec(c+dx)}{2bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \; 4486$$

$$-\frac{-\frac{\frac{\frac{\left(a^2-b^2\right)\left(6a^2+b^2\right)\int\sec(c+dx)dx}{b}-2a^3\left(\frac{3a^2}{b}-4b\right)\int\frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{b}-\frac{2a\left(3a^2-2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{\left(3a^2-b^2\right)\tan(c+dx)\sec(c+dx)}{2bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{\frac{\frac{\left(a^2-b^2\right)\left(6a^2+b^2\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{b}-2a^3\left(\frac{3a^2}{b}-4b\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{2a\left(3a^2-2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{\left(3a^2-b^2\right)\tan(c+dx)\sec(c+dx)}{2bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \; 4257$$

$$-\frac{-\frac{\frac{\frac{\left(a^2-b^2\right)\left(6a^2+b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-2a^3\left(\frac{3a^2}{b}-4b\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{2a\left(3a^2-2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{\left(3a^2-b^2\right)\tan(c+dx)\sec(c+dx)}{2bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \; 4318$$

$$-\frac{-\frac{\frac{\frac{\left(a^2-b^2\right)\left(6a^2+b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a^3\left(\frac{3a^2}{b}-4b\right)\int\frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b}}{b}-\frac{2a\left(3a^2-2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{\left(3a^2-b^2\right)\tan(c+dx)\sec(c+dx)}{2bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{\frac{(a^2-b^2)(6a^2+b^2)\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a^3\left(\frac{3a^2}{b}-4b\right)\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b}}{2b}-\frac{2a(3a^2-2b^2)\tan(c+dx)}{bd}}{b\,(a^2-b^2)} - \frac{(3a^2-b^2)\tan(c+dx)\sec(c+dx)}{2bd}$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\,(a^2-b^2)\,(a+b\sec(c+dx))}$$

$\downarrow$ 3138

$$-\frac{-\frac{\frac{(a^2-b^2)(6a^2+b^2)\text{arctanh}(\sin(c+dx))}{bd}-\frac{4a^3\left(\frac{3a^2}{b}-4b\right)\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}}{2b}-\frac{2a(3a^2-2b^2)\tan(c+dx)}{bd}}{b\,(a^2-b^2)} - \frac{(3a^2-b^2)\,\text{t}}{}$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\,(a^2-b^2)\,(a+b\sec(c+dx))}$$

$\downarrow$ 221

$$-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\,(a^2-b^2)\,(a+b\sec(c+dx))} -$$

$$\frac{-\frac{(3a^2-b^2)\tan(c+dx)\sec(c+dx)}{2bd} - \frac{\frac{(a^2-b^2)(6a^2+b^2)\text{arctanh}(\sin(c+dx))}{bd}-\frac{4a^3\left(\frac{3a^2}{b}-4b\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}}{2b} - \frac{2a(3a^2-2b^2)\,\text{tan}}{bd}}{b\,(a^2-b^2)}$$

input
```
Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

output
```
-((a^2*Sec[c + d*x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
) - (-1/2*((3*a^2 - b^2)*Sec[c + d*x]*Tan[c + d*x])/(b*d) - ((((a^2 - b^2)
*(6*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(b*d) - (4*a^3*((3*a^2)/b - 4*b)*Arc
Tanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]
*d))/b - (2*a*(3*a^2 - 2*b^2)*Tan[c + d*x])/(b*d))/(2*b))/(b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

rule 4580
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(-C)*Csc[e + f*x]*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m +
1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[Csc[e + f*x]*(a + b*Csc[e
+ f*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*
(m + 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &
& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{\frac{1}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-4a-b}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-6a^2-b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^4}-\frac{1}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-4a-b}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\cdots\right.}}{}$
default	$\frac{\frac{1}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-4a-b}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-6a^2-b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^4}-\frac{1}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-4a-b}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\cdots\right.}}{}$
risch	$-\frac{i\left(-3a^3b\,e^{5i(dx+c)}+a\,b^3e^{5i(dx+c)}-6a^4e^{4i(dx+c)}+2a^2b^2e^{4i(dx+c)}+2b^4e^{4i(dx+c)}-12b\,a^3e^{3i(dx+c)}+8a\,b^3e^{3i(dx+c)}-\cdots\right.}{d\,b^3\left(e^{2i(dx+c)}+1\right)^2\left(-a^2+b^2\right)\left(e^{2i(dx+c)}a\cdots\right.}$

input
```
int(sec(d*x+c)^5/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/b^2/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(-4*a-b)/b^3/(tan(1/2*d*x+1/2*c)
-1)+1/2/b^4*(-6*a^2-b^2)*ln(tan(1/2*d*x+1/2*c)-1)-1/2/b^2/(tan(1/2*d*x+1/2
*c)+1)^2-1/2*(-4*a-b)/b^3/(tan(1/2*d*x+1/2*c)+1)+1/2*(6*a^2+b^2)/b^4*ln(ta
n(1/2*d*x+1/2*c)+1)+2*a^3/b^4*(a*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d
*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-(3*a^2-4*b^2)/(a+b)/(a-b)/((a+b)
*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(209) = 418$.

Time = 0.51 (sec) , antiderivative size = 909, normalized size of antiderivative = 4.09

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[1/4*(2*((3*a^6 - 4*a^4*b^2)*cos(d*x + c)^3 + (3*a^5*b - 4*a^3*b^3)*cos(d*
x + c)^2)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
+ c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2
)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + ((6*a^7 - 11*a^5*b^2
+ 4*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (6*a^6*b - 11*a^4*b^3 + 4*a^2*b^5 +
b^7)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - ((6*a^7 - 11*a^5*b^2 + 4*a^3*
b^4 + a*b^6)*cos(d*x + c)^3 + (6*a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + b^7)*cos
(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(a^4*b^3 - 2*a^2*b^5 + b^7 - 2*(3*
a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^2 - 3*(a^5*b^2 - 2*a^3*b^4 + a
*b^6)*cos(d*x + c))*sin(d*x + c))/((a^5*b^4 - 2*a^3*b^6 + a*b^8)*d*cos(d*x
 + c)^3 + (a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x + c)^2), -1/4*(4*((3*a^6 -
 4*a^4*b^2)*cos(d*x + c)^3 + (3*a^5*b - 4*a^3*b^3)*cos(d*x + c)^2)*sqrt(-a
^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d
*x + c))) - ((6*a^7 - 11*a^5*b^2 + 4*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (6*
a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + b^7)*cos(d*x + c)^2)*log(sin(d*x + c) + 1
) + ((6*a^7 - 11*a^5*b^2 + 4*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (6*a^6*b -
11*a^4*b^3 + 4*a^2*b^5 + b^7)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(
a^4*b^3 - 2*a^2*b^5 + b^7 - 2*(3*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cos(d*x +
c)^2 - 3*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^5*b
^4 - 2*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3 + (a^4*b^5 - 2*a^2*b^7 + b^9)*...
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx=\int \frac{\sec^5{(c+dx)}}{(a+b\sec{(c+dx)})^2}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**5/(a + b*sec(c + d*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.35

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^2b^3-b^5)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)} - \frac{4\,(3\,a^5-4\,a^3b^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\right.}{\sqrt{-a^2+b^2}}\right.\right.}{(a^2b^4-b^6)\sqrt{-a^2+b^2}}}{}$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(4*a^4*tan(1/2*d*x + 1/2*c)/((a^2*b^3 - b^5)*(a*tan(1/2*d*x + 1/2*c)^2
 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) - 4*(3*a^5 - 4*a^3*b^2)*(pi*floor(1/
2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) -
b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^2*b^4 - b^6)*sqrt(-a^2 + b^
2)) + (6*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - (6*a^2 + b^2)
*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 + 2*(4*a*tan(1/2*d*x + 1/2*c)^3 +
b*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) + b*tan(1/2*d*x + 1/2*
c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*b^3))/d
```

Mupad [B] (verification not implemented)

Time = 16.21 (sec) , antiderivative size = 3685, normalized size of antiderivative = 16.60

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^2),x)
```

output
```
- ((tan(c/2 + (d*x)/2)^5*(3*a*b^3 - 3*a^3*b + 6*a^4 + b^4 - 5*a^2*b^2))/((
a*b^3 - b^4)*(a + b)) + (2*tan(c/2 + (d*x)/2)^3*(b^4 - 6*a^4 + 3*a^2*b^2))
/(b*(a*b^2 - b^3)*(a + b)) + (tan(c/2 + (d*x)/2)*(3*a^3*b - 3*a*b^3 + 6*a^
4 + b^4 - 5*a^2*b^2))/(b^3*(a + b)*(a - b)))/(d*(a + b - tan(c/2 + (d*x)/2
)^2*(3*a + b) - tan(c/2 + (d*x)/2)^6*(a - b) + tan(c/2 + (d*x)/2)^4*(3*a -
 b))) - (atan((((6*a^2 + b^2)*((8*tan(c/2 + (d*x)/2)*(72*a^10 - 72*a^9*b -
 2*a*b^9 + b^10 + 11*a^2*b^8 - 20*a^3*b^7 + 23*a^4*b^6 - 26*a^5*b^5 + 17*a
^6*b^4 + 120*a^7*b^3 - 120*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (
(6*a^2 + b^2)*((8*(2*b^15 + 6*a^2*b^13 - 16*a^3*b^12 - 14*a^4*b^11 + 28*a^
5*b^10 + 6*a^6*b^9 - 12*a^7*b^8))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) - (
4*tan(c/2 + (d*x)/2)*(6*a^2 + b^2)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 +
16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/(b^4*(a*b^8 + b^9 - a^2*b^7 - a^3*b^
6))))/(2*b^4))*1i)/(2*b^4) + ((6*a^2 + b^2)*((8*tan(c/2 + (d*x)/2)*(72*a^1
0 - 72*a^9*b - 2*a*b^9 + b^10 + 11*a^2*b^8 - 20*a^3*b^7 + 23*a^4*b^6 - 26*
a^5*b^5 + 17*a^6*b^4 + 120*a^7*b^3 - 120*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7
- a^3*b^6) + ((6*a^2 + b^2)*((8*(2*b^15 + 6*a^2*b^13 - 16*a^3*b^12 - 14*a^
4*b^11 + 28*a^5*b^10 + 6*a^6*b^9 - 12*a^7*b^8))/(a*b^11 + b^12 - a^2*b^10
- a^3*b^9) + (4*tan(c/2 + (d*x)/2)*(6*a^2 + b^2)*(8*a*b^13 - 8*a^2*b^12 -
16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/(b^4*(a*b^8 + b^9 - a^
2*b^7 - a^3*b^6))))/(2*b^4))*1i)/(2*b^4))/((16*(108*a^11 - 54*a^10*b + ...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1637, normalized size of antiderivative = 7.37

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^5/(a+b*sec(d*x+c))^2,x)
```

output
```
( - 12*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)
/sqrt( - a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**2*a**6 + 16*sqrt( - a**2
 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**
2))*cos(c + d*x)*sin(c + d*x)**2*a**4*b**2 + 12*sqrt( - a**2 + b**2)*atan(
(tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*
x)*a**6 - 16*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)
/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*a**4*b**2 - 12*sqrt( - a**2 + b*
*2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*s
in(c + d*x)**2*a**5*b + 16*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a -
 tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*sin(c + d*x)**2*a**3*b**3 + 12*
sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(
- a**2 + b**2))*a**5*b - 16*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a
- tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*a**3*b**3 - 6*cos(c + d*x)*log
(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**7 + 11*cos(c + d*x)*log(tan((c +
 d*x)/2) - 1)*sin(c + d*x)**2*a**5*b**2 - 4*cos(c + d*x)*log(tan((c + d*x)
/2) - 1)*sin(c + d*x)**2*a**3*b**4 - cos(c + d*x)*log(tan((c + d*x)/2) - 1
)*sin(c + d*x)**2*a*b**6 + 6*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a**7 -
 11*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a**5*b**2 + 4*cos(c + d*x)*log(
tan((c + d*x)/2) - 1)*a**3*b**4 + cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a
*b**6 + 6*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**7 -...
```

3.498 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 164

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{2a\text{arctanh}(\sin(c+dx))}{b^3d} + \frac{2a^2(2a^2-3b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^3(a+b)^{3/2}d} + \frac{(2a^2-b^2)\tan(c+dx)}{b^2(a^2-b^2)\,d} - \frac{a^2\sec(c+dx)\tan(c+dx)}{b(a^2-b^2)\,d(a+b\sec(c+dx))}$$

output

```
-2*a*arctanh(sin(d*x+c))/b^3/d+2*a^2*(2*a^2-3*b^2)*arctanh((a-b)^(1/2)*tan
(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/b^3/(a+b)^(3/2)/d+(2*a^2-b^2)*tan
(d*x+c)/b^2/(a^2-b^2)/d-a^2*sec(d*x+c)*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d
*x+c))
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.99

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{-\frac{2a^2(2a^2-3b^2)\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + 2a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) - 2a\log\left(\cos\left(\frac{1}{2}(c+\right.\right.}{b^3d}$$

input
```
Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

output
```
((-2*a^2*(2*a^2 - 3*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]])/(a^2 - b^2)^(3/2) + 2*a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2*
a*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^3*b*Sin[c + d*x])/((a - b)
*(a + b)*(b + a*Cos[c + d*x])) + b*Tan[c + d*x])/(b^3*d)
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.12, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4332, 3042, 4570, 3042, 4486, 3042, 4257, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4332}$$

$$-\frac{\int \frac{\sec(c+dx)(a^2-b\sec(c+dx)a-(2a^2-b^2)\sec^2(c+dx))}{a+b\sec(c+dx)}dx}{b(a^2-b^2)} - \frac{a^2\tan(c+dx)\sec(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(b^2-2a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{4570}$$

$$-\frac{\frac{\int \frac{\sec(c+dx)\left(ba^2+2\left(a^2-b^2\right)\sec(c+dx)a\right)}{a+b\sec(c+dx)}dx}{b}-\frac{\left(2a^2-b^2\right)\tan(c+dx)}{bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(ba^2+2\left(a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(2a^2-b^2\right)\tan(c+dx)}{bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{4486}$$

$$-\frac{\frac{\frac{2a\left(a^2-b^2\right)\int \sec(c+dx)dx}{b}-a^2\left(\frac{2a^2}{b}-3b\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{b}-\frac{\left(2a^2-b^2\right)\tan(c+dx)}{bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{2a\left(a^2-b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b}-a^2\left(\frac{2a^2}{b}-3b\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(2a^2-b^2\right)\tan(c+dx)}{bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\frac{\frac{2a\left(a^2-b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-a^2\left(\frac{2a^2}{b}-3b\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(2a^2-b^2\right)\tan(c+dx)}{bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{4318}$$

$$-\frac{\frac{\frac{2a(a^2-b^2)\text{arctanh}(\sin(c+dx))}{bd}-\frac{a^2\left(\frac{2a^2}{b}-3b\right)\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b}}{b}-\frac{(2a^2-b^2)\tan(c+dx)}{bd}}{b(a^2-b^2)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{2a(a^2-b^2)\text{arctanh}(\sin(c+dx))}{bd}-\frac{a^2\left(\frac{2a^2}{b}-3b\right)\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b}}{b}-\frac{(2a^2-b^2)\tan(c+dx)}{bd}}{b(a^2-b^2)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3138}$$

$$-\frac{\frac{\frac{2a(a^2-b^2)\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a^2\left(\frac{2a^2}{b}-3b\right)\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}}{b}-\frac{(2a^2-b^2)\tan(c+dx)}{bd}}{b(a^2-b^2)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{\frac{2a(a^2-b^2)\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a^2\left(\frac{2a^2}{b}-3b\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}}{b}-\frac{(2a^2-b^2)\tan(c+dx)}{bd}}{b(a^2-b^2)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

input
```
Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

output
```
-((a^2*Sec[c + d*x]*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])))
- (((2*a*(a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(b*d) - (2*a^2*((2*a^2)/b - 3*
b)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[
a + b]*d))/b - ((2*a^2 - b^2)*Tan[c + d*x])/(b*d))/(b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{-\frac{1}{b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^3}+\frac{2a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^3}-\frac{1}{b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2a^2\left(\frac{ab\tan(}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right.}\right.}{}}{d}$
default	$\frac{-\frac{1}{b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^3}+\frac{2a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^3}-\frac{1}{b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2a^2\left(\frac{ab\tan(}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right.}\right.}{}}{d}$
risch	$\frac{2i\left(-a^2b\,e^{3i(dx+c)}-2a^3e^{2i(dx+c)}+a\,b^2e^{2i(dx+c)}-3a^2b\,e^{i(dx+c)}+2b^3e^{i(dx+c)}-2a^3+a\,b^2\right)}{\left(-a^2+b^2\right)d\,b^2\left(e^{2i(dx+c)}a+2b\,e^{i(dx+c)}+a\right)\left(e^{2i(dx+c)}+1\right)}+\frac{2a\ln\left(e^{i(dx+c)}-i\right)}{b^3d}+-\frac{2a}{}$

input
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^2/(tan(1/2*d*x+1/2*c)+1)-2*a/b^3*ln(tan(1/2*d*x+1/2*c)+1)+2*a/b^
3*ln(tan(1/2*d*x+1/2*c)-1)-1/b^2/(tan(1/2*d*x+1/2*c)-1)-2*a^2/b^3*(a*b/(a^
2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a
-b)-(2*a^2-3*b^2)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*
x+1/2*c)/((a+b)*(a-b))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(155) = 310$.

Time = 0.30 (sec) , antiderivative size = 760, normalized size of antiderivative = 4.63

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[1/2*(((2*a^5 - 3*a^3*b^2)*cos(d*x + c)^2 + (2*a^4*b - 3*a^2*b^3)*cos(d*x
+ c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)
^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a
^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*((a^6 - 2*a^4*b^2 + a^2
*b^4)*cos(d*x + c)^2 + (a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x + c))*log(sin(d
*x + c) + 1) + 2*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(d*x + c)^2 + (a^5*b - 2*
a^3*b^3 + a*b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(a^4*b^2 - 2*a^2
*b^4 + b^6 + (2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(d*x + c))*sin(d*x + c))/((a
^5*b^3 - 2*a^3*b^5 + a*b^7)*d*cos(d*x + c)^2 + (a^4*b^4 - 2*a^2*b^6 + b^8)
*d*cos(d*x + c)), (((2*a^5 - 3*a^3*b^2)*cos(d*x + c)^2 + (2*a^4*b - 3*a^2*
b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x +
c) + a)/((a^2 - b^2)*sin(d*x + c))) - ((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(d*x
 + c)^2 + (a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x + c))*log(sin(d*x + c) + 1)
+ ((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(d*x + c)^2 + (a^5*b - 2*a^3*b^3 + a*b^5
)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (a^4*b^2 - 2*a^2*b^4 + b^6 + (2*a
^5*b - 3*a^3*b^3 + a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^5*b^3 - 2*a^3*b^
5 + a*b^7)*d*cos(d*x + c)^2 + (a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*x + c))]
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sec^4{(c+dx)}}{(a+b\sec{(c+dx)})^2}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(155) = 310$.

Time = 0.16 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.02

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{2\left(\frac{(2a^4-3a^2b^2)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^2b^3-b^5)\sqrt{-a^2+b^2}} - \frac{2a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}{}\right.}{}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
2*((2*a^4 - 3*a^2*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) +
 arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2
)))/((a^2*b^3 - b^5)*sqrt(-a^2 + b^2)) - (2*a^3*tan(1/2*d*x + 1/2*c)^3 - a
^2*b*tan(1/2*d*x + 1/2*c)^3 - a*b^2*tan(1/2*d*x + 1/2*c)^3 + b^3*tan(1/2*d
*x + 1/2*c)^3 - 2*a^3*tan(1/2*d*x + 1/2*c) - a^2*b*tan(1/2*d*x + 1/2*c) +
a*b^2*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x + 1
/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)*(
a^2*b^2 - b^4)) - a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 + a*log(abs(tan
(1/2*d*x + 1/2*c) - 1))/b^3)/d
```

Mupad [B] (verification not implemented)

Time = 15.37 (sec) , antiderivative size = 3159, normalized size of antiderivative = 19.26

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^2),x)
```

output
```
((2*tan(c/2 + (d*x)/2)^3*(a*b^2 + a^2*b - 2*a^3 - b^3))/(b^2*(a + b)*(a -
b)) - (2*tan(c/2 + (d*x)/2)*(a*b^2 - a^2*b - 2*a^3 + b^3))/(b^2*(a + b)*(a
 - b)))/(d*(a + b + tan(c/2 + (d*x)/2)^4*(a - b) - 2*a*tan(c/2 + (d*x)/2)^
2)) + (a*atan(((a*((32*tan(c/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8
*a^3*b^5 + 5*a^4*b^4 + 16*a^5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 -
a^3*b^4) - (2*a*((32*(2*a*b^11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*
b^7 - 2*a^6*b^6))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (64*a*tan(c/2 + (d*x
)/2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^
6))/(b^3*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4))))/b^3)*2i)/b^3 + (a*((32*tan(c
/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b^4 + 16*a^
5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) + (2*a*((32*(2*a*b^
11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^6*b^6))/(a*b^8 + b
^9 - a^2*b^7 - a^3*b^6) + (64*a*tan(c/2 + (d*x)/2)*(2*a*b^11 - 2*a^2*b^10
- 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/(b^3*(a*b^6 + b^7 - a^2*
b^5 - a^3*b^4))))/b^3)*2i)/b^3)/((64*(8*a^8 - 4*a^7*b + 12*a^4*b^4 + 6*a^5
*b^3 - 20*a^6*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (2*a*((32*tan(c/2
+ (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b^4 + 16*a^5*b
^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) - (2*a*((32*(2*a*b^11
- 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^6*b^6))/(a*b^8 + b^9
- a^2*b^7 - a^3*b^6) - (64*a*tan(c/2 + (d*x)/2)*(2*a*b^11 - 2*a^2*b^10 ...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 977, normalized size of antiderivative = 5.96

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c))^2,x)
```

output
```
(4*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqr
t( - a**2 + b**2))*cos(c + d*x)*a**4*b - 6*sqrt( - a**2 + b**2)*atan((tan(
(c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*a*
*2*b**3 - 4*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/
2)*b)/sqrt( - a**2 + b**2))*sin(c + d*x)**2*a**5 + 6*sqrt( - a**2 + b**2)*
atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*sin(c
 + d*x)**2*a**3*b**2 + 4*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - t
an((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*a**5 - 6*sqrt( - a**2 + b**2)*ata
n((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*a**3*b**
2 + 2*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a**5*b - 4*cos(c + d*x)*log(t
an((c + d*x)/2) - 1)*a**3*b**3 + 2*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*
a*b**5 - 2*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a**5*b + 4*cos(c + d*x)*
log(tan((c + d*x)/2) + 1)*a**3*b**3 - 2*cos(c + d*x)*log(tan((c + d*x)/2)
+ 1)*a*b**5 + 2*cos(c + d*x)*sin(c + d*x)*a**5*b - 3*cos(c + d*x)*sin(c +
d*x)*a**3*b**3 + cos(c + d*x)*sin(c + d*x)*a*b**5 - 2*log(tan((c + d*x)/2)
 - 1)*sin(c + d*x)**2*a**6 + 4*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a
**4*b**2 - 2*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**2*b**4 + 2*log(t
an((c + d*x)/2) - 1)*a**6 - 4*log(tan((c + d*x)/2) - 1)*a**4*b**2 + 2*log(
tan((c + d*x)/2) - 1)*a**2*b**4 + 2*log(tan((c + d*x)/2) + 1)*sin(c + d*x)
**2*a**6 - 4*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**4*b**2 + 2*lo...
```

3.499 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 117

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{b^2d} - \frac{2a(a^2-2b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^2(a+b)^{3/2}d} - \frac{a^2\tan(c+dx)}{b\,(a^2-b^2)\,d(a+b\sec(c+dx))}$$

output
```
arctanh(sin(d*x+c))/b^2/d-2*a*(a^2-2*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+
1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/b^2/(a+b)^(3/2)/d-a^2*tan(d*x+c)/b/(a^2-b^
2)/d/(a+b*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.25

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$=\frac{\frac{2a(a^2-2b^2)\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\text{s}\right.}{b^2d}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

output
```
((2*a*(a^2 - 2*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(a^2 - b^2)^(3/2) - Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]] + (a^2*b*Sin[c + d*x])/((-a + b)*(a + b)*(b
+ a*Cos[c + d*x])))/(b^2*d)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.21, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {3042, 4326, 25, 3042, 4486, 3042, 4257, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \text{4326}$$
$$-\frac{\int -\frac{\sec(c+dx)\left(ab+(a^2-b^2)\sec(c+dx)\right)}{a+b\sec(c+dx)}dx}{b\,(a^2-b^2)}-\frac{a^2\tan(c+dx)}{bd\,(a^2-b^2)\,(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sec(c+dx)\left(ab+\left(a^2-b^2\right)\sec(c+dx)\right)}{a+b\sec(c+dx)}dx}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(ab+\left(a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{4486}$$

$$\frac{\frac{\left(a^2-b^2\right)\int \sec(c+dx)dx}{b}-\frac{a\left(a^2-2b^2\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{b}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\left(a^2-b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b}-\frac{a\left(a^2-2b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{\left(a^2-b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{a\left(a^2-2b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{4318}$$

$$\frac{\frac{\left(a^2-b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{a\left(a^2-2b^2\right)\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b^2}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\left(a^2-b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{a\left(a^2-2b^2\right)\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b^2}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{\left(a^2-b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a\left(a^2-2b^2\right)\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{b^2d}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{(a^2-b^2)\operatorname{arctanh}(\sin(c+dx))}{bd}-\frac{2a(a^2-2b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}}{b(a^2-b^2)}-\frac{a^2\tan(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

input
```
Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

output
```
(((a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(b*d) - (2*a*(a^2 - 2*b^2)*ArcTanh[(S
qrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*d))/
(b*(a^2 - b^2)) - (a^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4326
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-a^2)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m
 + 1)*(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]
*(a + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e
+ f*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.42

method	result
derivativedivides	$\frac{-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{2a\left(\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}-\frac{\left(a^2-2b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}\right)}{b^2}+\cdots}{d}$
default	$\frac{-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{2a\left(\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}-\frac{\left(a^2-2b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}\right)}{b^2}+\cdots}{d}$
risch	$-\frac{2ia\left(b\,e^{i(dx+c)}+a\right)}{(a^2-b^2)db\left(e^{2i(dx+c)}a+2b\,e^{i(dx+c)}+a\right)}+\frac{a^3\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)d\,b^2}-\frac{2a\ln\left(e^{i(dx+c)}-\cdots\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)d\cdots}$

input
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^2*ln(tan(1/2*d*x+1/2*c)-1)+2*a/b^2*(a*b/(a^2-b^2)*tan(1/2*d*x+1/
2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-(a^2-2*b^2)/(a+b)
/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^
(1/2)))+1/b^2*ln(tan(1/2*d*x+1/2*c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(108) = 216$.

Time = 0.30 (sec) , antiderivative size = 596, normalized size of antiderivative = 5.09

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \left[\frac{(a^3b-2\,ab^3+(a^4-2\,a^2b^2)\cos(dx+c))\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}(b\cos(dx+c)+}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right.}{}\right.$$
$$-\frac{2\,(a^3b-2\,ab^3+(a^4-2\,a^2b^2)\cos(dx+c))\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)-(a^4b-2\,a^2b^3}{}$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[1/2*((a^3*b - 2*a*b^3 + (a^4 - 2*a^2*b^2)*cos(d*x + c))*sqrt(a^2 - b^2)*l
og((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*
(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a
*b*cos(d*x + c) + b^2)) + (a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*
b^4)*cos(d*x + c))*log(sin(d*x + c) + 1) - (a^4*b - 2*a^2*b^3 + b^5 + (a^5
 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(a^4*b - a^
2*b^3)*sin(d*x + c))/((a^5*b^2 - 2*a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^4*
b^3 - 2*a^2*b^5 + b^7)*d), -1/2*(2*(a^3*b - 2*a*b^3 + (a^4 - 2*a^2*b^2)*co
s(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)
/((a^2 - b^2)*sin(d*x + c))) - (a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a^3*b^2
 + a*b^4)*cos(d*x + c))*log(sin(d*x + c) + 1) + (a^4*b - 2*a^2*b^3 + b^5 +
 (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(a^4*b
 - a^2*b^3)*sin(d*x + c))/((a^5*b^2 - 2*a^3*b^4 + a*b^6)*d*cos(d*x + c) +
(a^4*b^3 - 2*a^2*b^5 + b^7)*d)]
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sec^3{(c+dx)}}{(a+b\sec{(c+dx)})^2}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.74

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{2a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{(a^2b-b^3)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a-b\right)} - \frac{2(a^3-2ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\right.}{\sqrt{-a^2+b^2}}\right.\right.}{(a^2b^2-b^4)\sqrt{-a^2+b^2}}}{d}$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
(2*a^2*tan(1/2*d*x + 1/2*c)/((a^2*b - b^3)*(a*tan(1/2*d*x + 1/2*c)^2 - b*t
an(1/2*d*x + 1/2*c)^2 - a - b)) - 2*(a^3 - 2*a*b^2)*(pi*floor(1/2*(d*x + c
)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*
d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^2*b^2 - b^4)*sqrt(-a^2 + b^2)) + log(
abs(tan(1/2*d*x + 1/2*c) + 1))/b^2 - log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^
2)/d
```

Mupad [B] (verification not implemented)

Time = 15.67 (sec) , antiderivative size = 2848, normalized size of antiderivative = 24.34

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^2),x)
```

output
```
- (atan(((((((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5 +
b^6 - a^2*b^4 - a^3*b^3) - (32*tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4
*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4 + b^5 - a^2*b^3
 - a^3*b^2)))/b^2 - (32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5 + b^
6 + 3*a^2*b^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))
*1i)/b^2 - ((((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5
+ b^6 - a^2*b^4 - a^3*b^3) + (32*tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 -
 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4 + b^5 - a^2*b
^3 - a^3*b^2)))/b^2 + (32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5 +
b^6 + 3*a^2*b^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2
))*1i)/b^2)/((((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5
 + b^6 - a^2*b^4 - a^3*b^3) - (32*tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8
- 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4 + b^5 - a^2*
b^3 - a^3*b^2)))/b^2 - (32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5 +
 b^6 + 3*a^2*b^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^
2))/b^2 - (64*(2*a*b^4 - a^4*b + a^5 + 2*a^2*b^3 - 3*a^3*b^2))/(a*b^5 + b^
6 - a^2*b^4 - a^3*b^3) + (((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*
b^4))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (32*tan(c/2 + (d*x)/2)*(2*a*b^9
- 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4
+ b^5 - a^2*b^3 - a^3*b^2)))/b^2 + (32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 568, normalized size of antiderivative = 4.85

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c))^2,x)
```

output
```
( - 2*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/
sqrt( - a**2 + b**2))*cos(c + d*x)*a**4 + 4*sqrt( - a**2 + b**2)*atan((tan
((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*a
**2*b**2 - 2*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)
/2)*b)/sqrt( - a**2 + b**2))*a**3*b + 4*sqrt( - a**2 + b**2)*atan((tan((c
+ d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*a*b**3 - cos(c + d
*x)*log(tan((c + d*x)/2) - 1)*a**5 + 2*cos(c + d*x)*log(tan((c + d*x)/2) -
 1)*a**3*b**2 - cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a*b**4 + cos(c + d*
x)*log(tan((c + d*x)/2) + 1)*a**5 - 2*cos(c + d*x)*log(tan((c + d*x)/2) +
1)*a**3*b**2 + cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a*b**4 - log(tan((c
+ d*x)/2) - 1)*a**4*b + 2*log(tan((c + d*x)/2) - 1)*a**2*b**3 - log(tan((c
 + d*x)/2) - 1)*b**5 + log(tan((c + d*x)/2) + 1)*a**4*b - 2*log(tan((c + d
*x)/2) + 1)*a**2*b**3 + log(tan((c + d*x)/2) + 1)*b**5 - sin(c + d*x)*a**4
*b + sin(c + d*x)*a**2*b**3)/(b**2*d*(cos(c + d*x)*a**5 - 2*cos(c + d*x)*a
**3*b**2 + cos(c + d*x)*a*b**4 + a**4*b - 2*a**2*b**3 + b**5))
```

3.500 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 85

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{2b\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d} + \frac{a\tan(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))}$$

output
```
-2*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)
^(3/2)/d+a*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{2b\,\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + \frac{a\sin(c+dx)}{(a-b)(a+b)(b+a\cos(c+dx))}}{d}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

output
```
((2*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3
/2) + (a*Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c + d*x])))/d
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4323, 25, 27, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{ 4323}$$

$$\frac{\int -\frac{b\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}+\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int \frac{b\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{b\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{ 4318}$$

$$\frac{a\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int\frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a^2-b^2}$$

$\Big\downarrow$ 3042

$$\frac{a\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int\frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a^2-b^2}$$

$\Big\downarrow$ 3138

$$\frac{a\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}-\frac{2\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{d(a^2-b^2)}$$

$\Big\downarrow$ 221

$$\frac{a\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}-\frac{2b\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}$$

input
```
Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

output
```
(-2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sq
rt[a + b]*(a^2 - b^2)*d) + (a*Tan[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c +
d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4323
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[a*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(
a^2 - b^2))), x] - Simp[1/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^{2}-b^{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b-a-b\right)}-\frac{2b\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{d}$
default	$\frac{-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^{2}-b^{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b-a-b\right)}-\frac{2b\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{d}$
risch	$\frac{2i\left(b\,e^{i(dx+c)}+a\right)}{d(a^{2}-b^{2})\left(e^{2i(dx+c)}a+2b\,e^{i(dx+c)}+a\right)}+\frac{b\ln\left(e^{i(dx+c)}+\frac{-ia^{2}+ib^{2}+b\sqrt{a^{2}-b^{2}}}{a\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,(a+b)(a-b)d}-\frac{b\ln\left(e^{i(dx+c)}+\frac{ia^{2}-ib^{2}+b\sqrt{a^{2}-b^{2}}}{a\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,(a+b)(a-b)d}$

input
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x
+1/2*c)^2*b-a-b)-2*b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2
*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 329, normalized size of antiderivative = 3.87

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \left[-\frac{(ab\cos(dx+c)+b^2)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)}{2\left((a^5-2\,a^3b^2+ab^4)d\cos(dx+c)+(a^4b-2\,a^2b^3+b^5)d\right)}\right.$$
$$\left.-\frac{(ab\cos(dx+c)+b^2)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)-(a^3-ab^2)\sin(dx+c)}{(a^5-2\,a^3b^2+ab^4)d\cos(dx+c)+(a^4b-2\,a^2b^3+b^5)d}\right]$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/2*((a*b*cos(d*x + c) + b^2)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) -
(a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(
d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) -
 2*(a^3 - a*b^2)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c) +
 (a^4*b - 2*a^2*b^3 + b^5)*d), -((a*b*cos(d*x + c) + b^2)*sqrt(-a^2 + b^2)
*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))
 - (a^3 - a*b^2)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c) +
 (a^4*b - 2*a^2*b^3 + b^5)*d)]
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.76

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b}{(a^2-b^2)\sqrt{-a^2+b^2}}+\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a-b\right)(a^2-}\right)}{d}$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2
*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b/((a^2 - b^2)*
sqrt(-a^2 + b^2)) + a*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*
tan(1/2*d*x + 1/2*c)^2 - a - b)*(a^2 - b^2)))/d
```

Mupad [B] (verification not implemented)

Time = 10.36 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,(a+b)\,(a-b)\,\left((b-a)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a+b\right)} - \frac{2\,b\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a-b}}{\sqrt{a+b}}\right)}{d\,(a+b)^{3/2}\,(a-b)^{3/2}}$$

input
```
int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^2),x)
```

output
```
(2*a*tan(c/2 + (d*x)/2))/(d*(a + b)*(a - b)*(a + b - tan(c/2 + (d*x)/2)^2*
(a - b))) - (2*b*atanh((tan(c/2 + (d*x)/2)*(a - b)^(1/2))/(a + b)^(1/2)))/
(d*(a + b)^(3/2)*(a - b)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.26

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{-2\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{-a^2+b^2}}\right)\cos(dx+c)\,ab-2\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{-a^2+b^2}}\right)}{d\left(\cos(dx+c)\,a^5-2\cos(dx+c)\,a^3b^2+\cos(dx+c)\,a\,b^4+a^4b-2a^2b^3\right)}$$

input
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c))^2,x)
```

output
```
( - 2*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/
sqrt( - a**2 + b**2))*cos(c + d*x)*a*b - 2*sqrt( - a**2 + b**2)*atan((tan(
(c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*b**2 + sin(c +
d*x)*a**3 - sin(c + d*x)*a*b**2)/(d*(cos(c + d*x)*a**5 - 2*cos(c + d*x)*a*
*3*b**2 + cos(c + d*x)*a*b**4 + a**4*b - 2*a**2*b**3 + b**5))
```

3.501 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 86

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{2a\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d} - \frac{b\tan(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))}$$

output
```
2*a*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^
(3/2)/d-b*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{-\frac{2a\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + \frac{b\sin(c+dx)}{(-a+b)(a+b)(b+a\cos(c+dx))}}{d}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

output
```
((-2*a*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(
3/2) + (b*Sin[c + d*x])/((-a + b)*(a + b)*(b + a*Cos[c + d*x])))/d
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 4320, 25, 27, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4320}$$

$$-\frac{\int -\frac{a\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}-\frac{b\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}-\frac{b\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}-\frac{b\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{b\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{4318}$$

$$\frac{a\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b(a^2-b^2)}-\frac{b\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 3042

$$\frac{a\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b(a^2-b^2)}-\frac{b\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 3138

$$\frac{2a\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd(a^2-b^2)}-\frac{b\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 221

$$\frac{2a\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}\,(a^2-b^2)}-\frac{b\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}$$

input
```
Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

output
```
(2*a*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqr
t[a + b]*(a^2 - b^2)*d) - (b*Tan[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d
*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4320
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*
Csc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ
[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.37

method	result
derivativedivides	$\frac{\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}+\frac{2a\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{d}$
default	$\frac{\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}+\frac{2a\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{d}$
risch	$-\frac{2ib\left(b\,e^{i(dx+c)}+a\right)}{a\left(a^2-b^2\right)d\left(e^{2i(dx+c)}a+2b\,e^{i(dx+c)}+a\right)}+\frac{a\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}-\frac{a\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}$

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)+2*a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*
d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 332, normalized size of antiderivative = 3.86

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \left[-\frac{(a^2\cos(dx+c)+ab)\sqrt{a^2-b^2}\log\left(\frac{2ab\cos(dx+c)-(a^2-2b^2)\cos(dx+c)^2-2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2a^2-b^2}{a^2\cos(dx+c)^2+2ab\cos(dx+c)+b^2}\right.}{2\left((a^5-2a^3b^2+ab^4)d\cos(dx+c)+(a^4b-2a^2b^3+b^5)d\right)}\right.$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/2*((a^2*cos(d*x + c) + a*b)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) -
(a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(
d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) +
 2*(a^2*b - b^3)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c) +
 (a^4*b - 2*a^2*b^3 + b^5)*d), ((a^2*cos(d*x + c) + a*b)*sqrt(-a^2 + b^2)*
arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))
- (a^2*b - b^3)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c) +
(a^4*b - 2*a^2*b^3 + b^5)*d)]
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.74

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a}{(a^2-b^2)\sqrt{-a^2+b^2}}-\frac{b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a-b\right)(a^2-b^2)}\right)}{d}$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a/((a^2 - b^2)*sq
rt(-a^2 + b^2)) - b*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*ta
n(1/2*d*x + 1/2*c)^2 - a - b)*(a^2 - b^2)))/d
```

Mupad [B] (verification not implemented)

Time = 10.44 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.07

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{2\,a\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a-b}}{\sqrt{a+b}}\right)}{d\,(a+b)^{3/2}\,(a-b)^{3/2}} - \frac{2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,(a+b)\,(a-b)\,\left((b-a)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a+b\right)}$$

input
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^2),x)
```

output
```
(2*a*atanh((tan(c/2 + (d*x)/2)*(a - b)^(1/2))/(a + b)^(1/2)))/(d*(a + b)^(
3/2)*(a - b)^(3/2)) - (2*b*tan(c/2 + (d*x)/2))/(d*(a + b)*(a - b)*(a + b -
 tan(c/2 + (d*x)/2)^2*(a - b)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.23

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{2\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{-a^2+b^2}}\right)\cos(dx+c)\,a^2+2\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{-a^2+b^2}}\right)a}{d\left(\cos(dx+c)\,a^5-2\cos(dx+c)\,a^3b^2+\cos(dx+c)\,a\,b^4+a^4b-2a^2b^3\right.}$$

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^2,x)
```

output
```
(2*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqr
t( - a**2 + b**2))*cos(c + d*x)*a**2 + 2*sqrt( - a**2 + b**2)*atan((tan((c
 + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*a*b - sin(c + d*x
)*a**2*b + sin(c + d*x)*b**3)/(d*(cos(c + d*x)*a**5 - 2*cos(c + d*x)*a**3*
b**2 + cos(c + d*x)*a*b**4 + a**4*b - 2*a**2*b**3 + b**5))
```

3.502 $\int \frac{1}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 109

$$\int \frac{1}{(a+b\sec(c+dx))^2}\,dx=\frac{x}{a^2}-\frac{2b(2a^2-b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d}+\frac{b^2\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))}$$

output
```
x/a^2-2*b*(2*a^2-b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/
a^2/(a-b)^(3/2)/(a+b)^(3/2)/d+b^2*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c)
)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.27

$$\int \frac{1}{(a+b\sec(c+dx))^2}\,dx$$
$$=\frac{-\frac{2b(-2a^2+b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{a(a^2-b^2)(c+dx)\cos(c+dx)+b\left((a^2-b^2)(c+dx)+ab\sin(c+dx)\right)}{b+a\cos(c+dx)}}{a^2(a-b)(a+b)d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(-2),x]
```

output
```
((-2*b*(-2*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
)/Sqrt[a^2 - b^2] + (a*(a^2 - b^2)*(c + d*x)*Cos[c + d*x] + b*((a^2 - b^2)
*(c + d*x) + a*b*Sin[c + d*x]))/(b + a*Cos[c + d*x]))/(a^2*(a - b)*(a + b)
*d)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.23, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3042, 4272, 25, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4272}$$

$$\frac{b^2\tan(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int -\frac{a^2-b\sec(c+dx)a-b^2}{a+b\sec(c+dx)}dx}{a(a^2-b^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a^2-b\sec(c+dx)a-b^2}{a+b\sec(c+dx)}dx}{a(a^2-b^2)}+\frac{b^2\tan(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a-b^2}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a(a^2-b^2)}+\frac{b^2\tan(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{4407}$$

$$\frac{\frac{x(a^2-b^2)}{a}-\frac{b(2a^2-b^2)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$\Big\downarrow$ 3042

$$\frac{\frac{x(a^2-b^2)}{a}-\frac{b(2a^2-b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$\Big\downarrow$ 4318

$$\frac{\frac{x(a^2-b^2)}{a}-\frac{(2a^2-b^2)\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a}}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$\Big\downarrow$ 3042

$$\frac{\frac{x(a^2-b^2)}{a}-\frac{(2a^2-b^2)\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a}}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$\Big\downarrow$ 3138

$$\frac{\frac{x(a^2-b^2)}{a}-\frac{2(2a^2-b^2)\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$\Big\downarrow$ 221

$$\frac{\frac{x(a^2-b^2)}{a}-\frac{2b(2a^2-b^2)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

input
```
Int[(a + b*Sec[c + d*x])^(-2),x]
```

output
```
(((a^2 - b^2)*x)/a - (2*b*(2*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)
/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d))/(a*(a^2 - b^2)) + (b^2*T
an[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{\frac{2b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b-a-b\right)}-\frac{\left(2a^2-b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}\right)}{a^2}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}}{d}$
default	$\frac{\frac{2b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b-a-b\right)}-\frac{\left(2a^2-b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}\right)}{a^2}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}}{d}$
risch	$\frac{x}{a^2}+\frac{2ib^2\left(b\,e^{i(dx+c)}+a\right)}{a^2(a^2-b^2)d\left(e^{2i(dx+c)}a+2b\,e^{i(dx+c)}+a\right)}+\frac{2b\ln\left(e^{i(dx+c)}+\frac{-ia^2+ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}-\frac{b^3\ln\left(e^{i(dx+c)}+\frac{-ia^2+\cdots}{\cdots}\right.}{\sqrt{a^2-b^2}\,(a+b)(\cdots}$

input
```
int(1/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*b/a^2*(-a*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-ta
n(1/2*d*x+1/2*c)^2*b-a-b)-(2*a^2-b^2)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arct
anh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))+2/a^2*arctan(tan(1/2*d*
x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(100) = 200.

Time = 0.13 (sec) , antiderivative size = 484, normalized size of antiderivative = 4.44

$$\int \frac{1}{(a+b\sec(c+dx))^2}\,dx$$
$$=\left[\frac{2\left(a^5-2\,a^3b^2+ab^4\right)dx\cos\left(dx+c\right)+2\left(a^4b-2\,a^2b^3+b^5\right)dx+\left(2\,a^2b^2-b^4+\left(2\,a^3b-ab^3\right)\cos\left(dx+c\cdots\right.\right.}{2\left(\left(a^7-2\,a^5b^2+a^3b^4\right)d\cos\cdots\right.}\right.$$

input
```
integrate(1/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[1/2*(2*(a^5 - 2*a^3*b^2 + a*b^4)*d*x*cos(d*x + c) + 2*(a^4*b - 2*a^2*b^3
+ b^5)*d*x + (2*a^2*b^2 - b^4 + (2*a^3*b - a*b^3)*cos(d*x + c))*sqrt(a^2 -
 b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2
- b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^
2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(a^3*b^2 - a*b^4)*sin(d*x + c))/((a^7 -
 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), (
(a^5 - 2*a^3*b^2 + a*b^4)*d*x*cos(d*x + c) + (a^4*b - 2*a^2*b^3 + b^5)*d*x
 - (2*a^2*b^2 - b^4 + (2*a^3*b - a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arc
tan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (
a^3*b^2 - a*b^4)*sin(d*x + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c)
 + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)]
```

Sympy [F]

$$\int \frac{1}{(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{\left(a+b\sec\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**(-2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.64

$$\int \frac{1}{(a+b\sec(c+dx))^2}\,dx = -\frac{\frac{2\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^3-ab^2)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)}+\frac{2\,(2\,a^2b-b^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\right.}{\sqrt{-a^2+b^2}}\right.\right.}{(a^4-a^2b^2)\sqrt{-a^2+b^2}}}{d}$$

input
```
integrate(1/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-(2*b^2*tan(1/2*d*x + 1/2*c)/((a^3 - a*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - b*
tan(1/2*d*x + 1/2*c)^2 - a - b)) + 2*(2*a^2*b - b^3)*(pi*floor(1/2*(d*x +
c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2
*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^4 - a^2*b^2)*sqrt(-a^2 + b^2)) - (d*
x + c)/a^2)/d
```

Mupad [B] (verification not implemented)

Time = 15.59 (sec) , antiderivative size = 2886, normalized size of antiderivative = 26.48

$$\int \frac{1}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b/cos(c + d*x))^2,x)
```

output
```
(2*atan((((((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^2))/(a^5*b +
a^6 - a^3*b^3 - a^4*b^2) - (tan(c/2 + (d*x)/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^
5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2)*32i)/(a^2*(a^4*b + a^5 - a^2*b^
3 - a^3*b^2)))*1i)/a^2 + (32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*b - 2*a*b^5 +
 2*b^6 - 5*a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b + a^5 - a^2*b^3 - a^3*
b^2))/a^2 - ((((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^2))/(a^5*b
 + a^6 - a^3*b^3 - a^4*b^2) + (tan(c/2 + (d*x)/2)*(2*a^9*b - 2*a^4*b^6 + 2
*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2)*32i)/(a^2*(a^4*b + a^5 - a^2
*b^3 - a^3*b^2)))*1i)/a^2 - (32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*b - 2*a*b^
5 + 2*b^6 - 5*a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b + a^5 - a^2*b^3 - a
^3*b^2))/a^2)/((64*(2*a^4*b - a*b^4 + b^5 - 3*a^2*b^3 + 2*a^3*b^2))/(a^5*b
 + a^6 - a^3*b^3 - a^4*b^2) + (((((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3
 + a^7*b^2))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) - (tan(c/2 + (d*x)/2)*(2*a^
9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2)*32i)/(a^2
*(a^4*b + a^5 - a^2*b^3 - a^3*b^2)))*1i)/a^2 + (32*tan(c/2 + (d*x)/2)*(a^6
 - 2*a^5*b - 2*a*b^5 + 2*b^6 - 5*a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b
+ a^5 - a^2*b^3 - a^3*b^2))*1i)/a^2 + (((((32*(2*a^8*b - a^9 + a^4*b^5 - 3
*a^6*b^3 + a^7*b^2))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (tan(c/2 + (d*x)/
2)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2)*3
2i)/(a^2*(a^4*b + a^5 - a^2*b^3 - a^3*b^2)))*1i)/a^2 - (32*tan(c/2 + (d...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 380, normalized size of antiderivative = 3.49

$$\int \frac{1}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{-4\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{-a^2+b^2}}\right)\cos(dx+c)\,a^3b+2\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{-a^2+b^2}}\right.}{}$$

input
```
int(1/(a+b*sec(d*x+c))^2,x)
```

output
```
( - 4*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/
sqrt( - a**2 + b**2))*cos(c + d*x)*a**3*b + 2*sqrt( - a**2 + b**2)*atan((t
an((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)
*a*b**3 - 4*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/
2)*b)/sqrt( - a**2 + b**2))*a**2*b**2 + 2*sqrt( - a**2 + b**2)*atan((tan((
c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*b**4 + cos(c + d
*x)*a**5*d*x - 2*cos(c + d*x)*a**3*b**2*d*x + cos(c + d*x)*a*b**4*d*x + si
n(c + d*x)*a**3*b**2 - sin(c + d*x)*a*b**4 + a**4*b*d*x - 2*a**2*b**3*d*x
+ b**5*d*x)/(a**2*d*(cos(c + d*x)*a**5 - 2*cos(c + d*x)*a**3*b**2 + cos(c
+ d*x)*a*b**4 + a**4*b - 2*a**2*b**3 + b**5))
```

3.503 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 146

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2} \, dx = -\frac{2bx}{a^3} + \frac{2b^2(3a^2-2b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{3/2}(a+b)^{3/2}d} + \frac{(a^2-2b^2)\sin(c+dx)}{a^2\,(a^2-b^2)\,d} + \frac{b^2\sin(c+dx)}{a\,(a^2-b^2)\,d(a+b\sec(c+dx))}$$

output
```
-2*b*x/a^3+2*b^2*(3*a^2-2*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b
)^(1/2))/a^3/(a-b)^(3/2)/(a+b)^(3/2)/d+(a^2-2*b^2)*sin(d*x+c)/a^2/(a^2-b^2
)/d+b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.18

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2} \, dx = \frac{\frac{4b^2(-3a^2+2b^2)\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{-4ab(a^2-b^2)(c+dx)\cos(c+dx)+2ab(a^2-2b^2)\sin(c+dx)+(a^2-b^2)(-4b^2(c+dx)+a^2 s}{b+a\cos(c+dx)}}{2a^3(a-b)(a+b)d}$$

input
```
Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

output
```
((4*b^2*(-3*a^2 + 2*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]])/Sqrt[a^2 - b^2] + (-4*a*b*(a^2 - b^2)*(c + d*x)*Cos[c + d*x] + 2*a*b*
(a^2 - 2*b^2)*Sin[c + d*x] + (a^2 - b^2)*(-4*b^2*(c + d*x) + a^2*Sin[2*(c
+ d*x)]))/(b + a*Cos[c + d*x]))/(2*a^3*(a - b)*(a + b)*d)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {3042, 4334, 25, 3042, 4592, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4334}$$

$$\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int -\frac{\cos(c+dx)\left(a^2-b\sec(c+dx)a-2b^2+b^2\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\cos(c+dx)\left(a^2-b\sec(c+dx)a-2b^2+b^2\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a-2b^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{4592}$$

$$\frac{\frac{(a^2-2b^2)\sin(c+dx)}{ad}-\frac{\int \frac{2b(a^2-b^2)-ab^2\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}}{a(a^2-b^2)}+\frac{b^2\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}$$

$\downarrow$ 3042

$$\frac{\frac{(a^2-2b^2)\sin(c+dx)}{ad}-\frac{\int \frac{2b(a^2-b^2)-ab^2\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{a(a^2-b^2)}+\frac{b^2\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}$$

$\downarrow$ 4407

$$\frac{\frac{(a^2-2b^2)\sin(c+dx)}{ad}-\frac{\frac{2bx(a^2-b^2)}{a}-\frac{b^2(3a^2-2b^2)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}}{a}}{a(a^2-b^2)}+\frac{b^2\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}$$

$\downarrow$ 3042

$$\frac{\frac{(a^2-2b^2)\sin(c+dx)}{ad}-\frac{\frac{2bx(a^2-b^2)}{a}-\frac{b^2(3a^2-2b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{a}}{a(a^2-b^2)}+\frac{b^2\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}$$

$\downarrow$ 4318

$$\frac{\frac{(a^2-2b^2)\sin(c+dx)}{ad}-\frac{\frac{2bx(a^2-b^2)}{a}-\frac{b(3a^2-2b^2)\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a}}{a}}{a(a^2-b^2)}+\frac{b^2\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}$$

$\downarrow$ 3042

$$\frac{\frac{(a^2-2b^2)\sin(c+dx)}{ad}-\frac{\frac{2bx(a^2-b^2)}{a}-\frac{b(3a^2-2b^2)\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a}}{a}}{a(a^2-b^2)}+\frac{b^2\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}$$

$\downarrow$ 3138

$$\frac{\frac{(a^2-2b^2)\sin(c+dx)}{ad}-\frac{\frac{2bx(a^2-b^2)}{a}-\frac{2b(3a^2-2b^2)\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}}{a}}{a(a^2-b^2)}+\frac{b^2\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}$$

$\downarrow$ 221

$$\frac{\frac{(a^2-2b^2)\sin(c+dx)}{ad}-\frac{\frac{2bx(a^2-b^2)}{a}-\frac{2b^2(3a^2-2b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

input

```
Int[Cos[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

output

```
(b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])) + (-(((2*b*(a^2
- b^2)*x)/a - (2*b^2*(3*a^2 - 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d))/a) + ((a^2 - 2*b^2)*Sin[c +
 d*x])/(a*d))/(a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+2b\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^3}-\frac{2b^2\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}-\frac{\left(3a^2-2b^2\right)\mathrm{arc}\cdots}{(a+b)(\cdots}\cdots\right.}{a^3}\cdots}{d}$
default	$\frac{-\frac{2\left(-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+2b\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^3}-\frac{2b^2\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}-\frac{\left(3a^2-2b^2\right)\mathrm{arc}\cdots}{(a+b)(\cdots}\cdots\right.}{a^3}\cdots}{d}$
risch	$-\frac{2bx}{a^3}-\frac{ie^{i(dx+c)}}{2a^2d}+\frac{ie^{-i(dx+c)}}{2a^2d}-\frac{2ib^3\left(b\,e^{i(dx+c)}+a\right)}{a^3\left(a^2-b^2\right)d\left(e^{2i(dx+c)}a+2b\,e^{i(dx+c)}+a\right)}+\frac{3b^2\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2\cdots}}{a\sqrt{a^2-b^2}}\cdots\right.}{\sqrt{a^2-b^2}\,(a+b)(a-b)da}\cdots$

input
```
int(cos(d*x+c)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/a^3*(-a*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+2*b*arctan(tan
(1/2*d*x+1/2*c)))-2*b^2/a^3*(-a*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*
x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-(3*a^2-2*b^2)/(a+b)/(a-b)/((a+b)*
(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 565, normalized size of antiderivative = 3.87

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \left[-\frac{4\,(a^5b-2\,a^3b^3+ab^5)dx\cos{(dx+c)}+4\,(a^4b^2-2\,a^2b^4+b^6)dx-(3\,a^2b^3-2\,b^5+(3\,a^3b^2-2\,ab^4)\cos}{} \right.$$
$$-\frac{2\,(a^5b-2\,a^3b^3+ab^5)dx\cos{(dx+c)}+2\,(a^4b^2-2\,a^2b^4+b^6)dx-(3\,a^2b^3-2\,b^5+(3\,a^3b^2-2\,ab^4)\cos}{(a^8-2\,a^6b^2+a^4b}$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/2*(4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x*cos(d*x + c) + 4*(a^4*b^2 - 2*a^2
*b^4 + b^6)*d*x - (3*a^2*b^3 - 2*b^5 + (3*a^3*b^2 - 2*a*b^4)*cos(d*x + c))
*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 +
2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*co
s(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(a^5*b - 3*a^3*b^3 + 2*a*b^5
 + (a^6 - 2*a^4*b^2 + a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 - 2*a^6*b
^2 + a^4*b^4)*d*cos(d*x + c) + (a^7*b - 2*a^5*b^3 + a^3*b^5)*d), -(2*(a^5*
b - 2*a^3*b^3 + a*b^5)*d*x*cos(d*x + c) + 2*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*
x - (3*a^2*b^3 - 2*b^5 + (3*a^3*b^2 - 2*a*b^4)*cos(d*x + c))*sqrt(-a^2 + b
^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c
))) - (a^5*b - 3*a^3*b^3 + 2*a*b^5 + (a^6 - 2*a^4*b^2 + a^2*b^4)*cos(d*x +
 c))*sin(d*x + c))/((a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x + c) + (a^7*b -
2*a^5*b^3 + a^3*b^5)*d)]
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(cos(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 837 vs. $2(137) = 274$.

Time = 0.21 (sec) , antiderivative size = 837, normalized size of antiderivative = 5.73

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-((2*a^7*b - 5*a^6*b^2 - 4*a^5*b^3 + 9*a^4*b^4 + 2*a^3*b^5 - 4*a^2*b^6 - 2
*a^2*b*abs(-a^5 + a^3*b^2) - a*b^2*abs(-a^5 + a^3*b^2) + 2*b^3*abs(-a^5 +
a^3*b^2))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/
sqrt(-(a^4*b - a^2*b^3 + sqrt((a^5 + a^4*b - a^3*b^2 - a^2*b^3)*(a^5 - a^4
*b - a^3*b^2 + a^2*b^3) + (a^4*b - a^2*b^3)^2))/(a^5 - a^4*b - a^3*b^2 + a
^2*b^3))))/(a^4*b*abs(-a^5 + a^3*b^2) - a^2*b^3*abs(-a^5 + a^3*b^2) + (a^5
 - a^3*b^2)^2) + ((2*a^2*b + a*b^2 - 2*b^3)*sqrt(-a^2 + b^2)*abs(-a^5 + a^
3*b^2)*abs(-a + b) + (2*a^7*b - 5*a^6*b^2 - 4*a^5*b^3 + 9*a^4*b^4 + 2*a^3*
b^5 - 4*a^2*b^6)*sqrt(-a^2 + b^2)*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi
+ 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(a^4*b - a^2*b^3 - sqrt((a^5 +
a^4*b - a^3*b^2 - a^2*b^3)*(a^5 - a^4*b - a^3*b^2 + a^2*b^3) + (a^4*b - a^
2*b^3)^2))/(a^5 - a^4*b - a^3*b^2 + a^2*b^3))))/((a^5 - a^3*b^2)^2*(a^2 -
2*a*b + b^2) - (a^6*b - 2*a^5*b^2 + 2*a^3*b^4 - a^2*b^5)*abs(-a^5 + a^3*b^
2)) - 2*(a^3*tan(1/2*d*x + 1/2*c)^3 - a^2*b*tan(1/2*d*x + 1/2*c)^3 - a*b^2
*tan(1/2*d*x + 1/2*c)^3 + 2*b^3*tan(1/2*d*x + 1/2*c)^3 - a^3*tan(1/2*d*x +
 1/2*c) - a^2*b*tan(1/2*d*x + 1/2*c) + a*b^2*tan(1/2*d*x + 1/2*c) + 2*b^3*
tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^
4 - 2*b*tan(1/2*d*x + 1/2*c)^2 - a - b)*(a^4 - a^2*b^2)))/d
```

Mupad [B] (verification not implemented)

Time = 15.65 (sec) , antiderivative size = 3169, normalized size of antiderivative = 21.71

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)/(a + b/cos(c + d*x))^2,x)
```

output
```
((2*tan(c/2 + (d*x)/2)^3*(a*b^2 + a^2*b - a^3 - 2*b^3))/(a^2*(a + b)*(a -
b)) - (2*tan(c/2 + (d*x)/2)*(a*b^2 - a^2*b - a^3 + 2*b^3))/(a^2*(a + b)*(a
 - b)))/(d*(a + b - tan(c/2 + (d*x)/2)^4*(a - b) + 2*b*tan(c/2 + (d*x)/2)^
2)) - (4*b*atan(((2*b*((32*tan(c/2 + (d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^
6 + 16*a^3*b^5 + 5*a^4*b^4 - 8*a^5*b^3 + 4*a^6*b^2))/(a^6*b + a^7 - a^4*b^
3 - a^5*b^2) + (b*((32*(2*a^11*b - 2*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9
*b^3 - 3*a^10*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (b*tan(c/2 + (d*x)
/2)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2
)*64i)/(a^3*(a^6*b + a^7 - a^4*b^3 - a^5*b^2)))*2i)/a^3))/a^3 + (2*b*((32*
tan(c/2 + (d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3*b^5 + 5*a^4*b^4
- 8*a^5*b^3 + 4*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) - (b*((32*(2*a
^11*b - 2*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 - 3*a^10*b^2))/(a^8*b
+ a^9 - a^6*b^3 - a^7*b^2) + (b*tan(c/2 + (d*x)/2)*(2*a^11*b - 2*a^6*b^6 +
 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2)*64i)/(a^3*(a^6*b + a^7 -
a^4*b^3 - a^5*b^2)))*2i)/a^3))/a^3)/((64*(8*b^8 - 4*a*b^7 - 20*a^2*b^6 + 6
*a^3*b^5 + 12*a^4*b^4))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (b*((32*tan(c/
2 + (d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3*b^5 + 5*a^4*b^4 - 8*a^
5*b^3 + 4*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) + (b*((32*(2*a^11*b
- 2*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 - 3*a^10*b^2))/(a^8*b + a^9
- a^6*b^3 - a^7*b^2) - (b*tan(c/2 + (d*x)/2)*(2*a^11*b - 2*a^6*b^6 + 2*...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 521, normalized size of antiderivative = 3.57

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{6\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{-a^2+b^2}}\right)\cos\left(dx+c\right)a^3b^2-4\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{-a^2+b^2}}\right.}{}$$

input
```
int(cos(d*x+c)/(a+b*sec(d*x+c))^2,x)
```

output
```
(6*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqr
t( - a**2 + b**2))*cos(c + d*x)*a**3*b**2 - 4*sqrt( - a**2 + b**2)*atan((t
an((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)
*a*b**4 + 6*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/
2)*b)/sqrt( - a**2 + b**2))*a**2*b**3 - 4*sqrt( - a**2 + b**2)*atan((tan((
c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*b**5 + cos(c + d
*x)*sin(c + d*x)*a**6 - 2*cos(c + d*x)*sin(c + d*x)*a**4*b**2 + cos(c + d*
x)*sin(c + d*x)*a**2*b**4 - 2*cos(c + d*x)*a**5*b*c - 2*cos(c + d*x)*a**5*
b*d*x + 4*cos(c + d*x)*a**3*b**3*c + 4*cos(c + d*x)*a**3*b**3*d*x - 2*cos(
c + d*x)*a*b**5*c - 2*cos(c + d*x)*a*b**5*d*x + sin(c + d*x)*a**5*b - 3*si
n(c + d*x)*a**3*b**3 + 2*sin(c + d*x)*a*b**5 - 2*a**4*b**2*c - 2*a**4*b**2
*d*x + 4*a**2*b**4*c + 4*a**2*b**4*d*x - 2*b**6*c - 2*b**6*d*x)/(a**3*d*(c
os(c + d*x)*a**5 - 2*cos(c + d*x)*a**3*b**2 + cos(c + d*x)*a*b**4 + a**4*b
 - 2*a**2*b**3 + b**5))
```

3.504 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 208

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{(a^2+6b^2)\,x}{2a^4} - \frac{2b^3(4a^2-3b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}d} - \frac{b(2a^2-3b^2)\sin(c+dx)}{a^3\,(a^2-b^2)\,d} + \frac{(a^2-3b^2)\cos(c+dx)\sin(c+dx)}{2a^2\,(a^2-b^2)\,d} + \frac{b^2\cos(c+dx)\sin(c+dx)}{a\,(a^2-b^2)\,d(a+b\sec(c+dx))}$$

output
```
1/2*(a^2+6*b^2)*x/a^4-2*b^3*(4*a^2-3*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+
1/2*c)/(a+b)^(1/2))/a^4/(a-b)^(3/2)/(a+b)^(3/2)/d-b*(2*a^2-3*b^2)*sin(d*x+
c)/a^3/(a^2-b^2)/d+1/2*(a^2-3*b^2)*cos(d*x+c)*sin(d*x+c)/a^2/(a^2-b^2)/d+b
^2*cos(d*x+c)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.69

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{2(a^2+6b^2)(c+dx) - \frac{8b^3(-4a^2+3b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} - 8ab\sin(c+dx) + \frac{4ab^4\sin(c+dx)}{(a-b)(a+b)(b+a\cos(c+dx))}}{4a^4d}$$

input
```
Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

output
```
(2*(a^2 + 6*b^2)*(c + d*x) - (8*b^3*(-4*a^2 + 3*b^2)*ArcTanh[((-a + b)*Tan
[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - 8*a*b*Sin[c + d*x] +
(4*a*b^4*Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c + d*x])) + a^2*Sin[2*
(c + d*x)])/(4*a^4*d)
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4334, 25, 3042, 4592, 3042, 4592, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4334}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{ad\,(a^2-b^2)\,(a+b\sec(c+dx))} - \frac{\int -\frac{\cos^2(c+dx)\left(a^2-b\sec(c+dx)a-3b^2+2b^2\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{a\,(a^2-b^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\cos^2(c+dx)\left(a^2-b\sec(c+dx)a-3b^2+2b^2\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a-3b^2+2b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{4592}$$

$$\frac{\frac{\left(a^2-3b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\int \frac{\cos(c+dx)\left(-b\left(a^2-3b^2\right)\sec^2(c+dx)-a\left(a^2+b^2\right)\sec(c+dx)+2b\left(2a^2-3b^2\right)\right)}{a+b\sec(c+dx)}dx}{2a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\left(a^2-3b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\int \frac{-b\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-a\left(a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+2b\left(2a^2-3b^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{4592}$$

$$\frac{\frac{\left(a^2-3b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b\left(2a^2-3b^2\right)\sin(c+dx)}{ad}-\frac{\int \frac{a^4+5b^2a^2+b\left(a^2-3b^2\right)\sec(c+dx)a-6b^4}{a+b\sec(c+dx)}dx}{a}}{2a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\left(a^2-3b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b\left(2a^2-3b^2\right)\sin(c+dx)}{ad}-\frac{\int \frac{a^4+5b^2a^2+b\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a-6b^4}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{4407}$$

$$\frac{\frac{(a^2-3b^2)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b(2a^2-3b^2)\sin(c+dx)}{ad}-\frac{\frac{x(a^4+5a^2b^2-6b^4)}{a}-2b^3\left(4a-\frac{3b^2}{a}\right)\int\frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}}{2a}}{a(a^2-b^2)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(a^2-3b^2)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b(2a^2-3b^2)\sin(c+dx)}{ad}-\frac{\frac{x(a^4+5a^2b^2-6b^4)}{a}-2b^3\left(4a-\frac{3b^2}{a}\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2a}}{a(a^2-b^2)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 4318

$$\frac{\frac{(a^2-3b^2)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b(2a^2-3b^2)\sin(c+dx)}{ad}-\frac{\frac{x(a^4+5a^2b^2-6b^4)}{a}-2b^2\left(4a-\frac{3b^2}{a}\right)\int\frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a}}{2a}}{a(a^2-b^2)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(a^2-3b^2)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b(2a^2-3b^2)\sin(c+dx)}{ad}-\frac{\frac{x(a^4+5a^2b^2-6b^4)}{a}-2b^2\left(4a-\frac{3b^2}{a}\right)\int\frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a}}{2a}}{a(a^2-b^2)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 3138

$$\frac{\frac{(a^2-3b^2)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b(2a^2-3b^2)\sin(c+dx)}{ad}-\frac{x(a^4+5a^2b^2-6b^4)}{a}-\frac{\frac{4b^2\left(4a-\frac{3b^2}{a}\right)\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{d}}{a}}{2a}}{a(a^2-b^2)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 221

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}+\frac{\frac{(a^2-3b^2)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b(2a^2-3b^2)\sin(c+dx)}{ad}-\frac{\frac{x(a^4+5a^2b^2-6b^4)}{a}-\frac{4b^3\left(4a-\frac{3b^2}{a}\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}}{a}}{2a}}{a(a^2-b^2)}$$

input
```
Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

output
```
(b^2*Cos[c + d*x]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])) + (
((a^2 - 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(2*a*d) - (-((((a^4 + 5*a^2*b^2
- 6*b^4)*x)/a - (4*b^3*(4*a - (3*b^2)/a)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*d))/a) + (2*b*(2*a^2 - 3*b^2)
*Sin[c + d*x])/(a*d))/(2*a))/(a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{\frac{\frac{2\left(\left(-\frac{1}{2}a^2-2ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(\frac{1}{2}a^2-2ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\left(a^2+6b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^4}+\frac{2b^3\left(-\frac{ab\tan\left(\right.}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a\right.}\right.}{}}{d}$
default	$\frac{\frac{\frac{2\left(\left(-\frac{1}{2}a^2-2ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(\frac{1}{2}a^2-2ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\left(a^2+6b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^4}+\frac{2b^3\left(-\frac{ab\tan\left(\right.}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a\right.}\right.}{}}{d}$
risch	$\frac{x}{2a^2}+\frac{3xb^2}{a^4}-\frac{ie^{2i(dx+c)}}{8a^2d}+\frac{ib\,e^{i(dx+c)}}{a^3d}-\frac{ib\,e^{-i(dx+c)}}{a^3d}+\frac{ie^{-2i(dx+c)}}{8a^2d}+\frac{2ib^4\left(b\,e^{i(dx+c)}+a\right)}{a^4(a^2-b^2)d\left(e^{2i(dx+c)}a+2b\,e^{i(dx+c)}\right.}$

input
```
int(cos(d*x+c)^2/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/a^4*(((-1/2*a^2-2*a*b)*tan(1/2*d*x+1/2*c)^3+(1/2*a^2-2*a*b)*tan(1/2
*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(a^2+6*b^2)*arctan(tan(1/2*d*x
+1/2*c)))+2*b^3/a^4*(-a*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)
^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-(4*a^2-3*b^2)/(a+b)/(a-b)/((a+b)*(a-b))^(
1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 660, normalized size of antiderivative = 3.17

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[1/2*((a^7 + 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*d*x*cos(d*x + c) + (a^6*b +
 4*a^4*b^3 - 11*a^2*b^5 + 6*b^7)*d*x + (4*a^2*b^4 - 3*b^6 + (4*a^3*b^3 - 3
*a*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b
^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) +
 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (4*a^5*b^
2 - 10*a^3*b^4 + 6*a*b^6 - (a^7 - 2*a^5*b^2 + a^3*b^4)*cos(d*x + c)^2 + 3*
(a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^9 - 2*a^7*b^
2 + a^5*b^4)*d*cos(d*x + c) + (a^8*b - 2*a^6*b^3 + a^4*b^5)*d), 1/2*((a^7
+ 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*d*x*cos(d*x + c) + (a^6*b + 4*a^4*b^3
- 11*a^2*b^5 + 6*b^7)*d*x - 2*(4*a^2*b^4 - 3*b^6 + (4*a^3*b^3 - 3*a*b^5)*c
os(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a
)/((a^2 - b^2)*sin(d*x + c))) - (4*a^5*b^2 - 10*a^3*b^4 + 6*a*b^6 - (a^7 -
 2*a^5*b^2 + a^3*b^4)*cos(d*x + c)^2 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*cos
(d*x + c))*sin(d*x + c))/((a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos(d*x + c) + (a^
8*b - 2*a^6*b^3 + a^4*b^5)*d)]
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2{(c+dx)}}{(a+b\sec{(c+dx)})^2}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(cos(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.27

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx =$$

$$-\frac{\frac{4\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^5-a^3b^2)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)}+\frac{4\,(4\,a^2b^3-3\,b^5)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan}{\sqrt{-a^2+b^2}}\right.\right.}{(a^6-a^4b^2)\sqrt{-a^2+b^2}}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/2*(4*b^4*tan(1/2*d*x + 1/2*c)/((a^5 - a^3*b^2)*(a*tan(1/2*d*x + 1/2*c)^
2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) + 4*(4*a^2*b^3 - 3*b^5)*(pi*floor(1
/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) -
 b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^6 - a^4*b^2)*sqrt(-a^2 + b
^2)) - (a^2 + 6*b^2)*(d*x + c)/a^4 + 2*(a*tan(1/2*d*x + 1/2*c)^3 + 4*b*tan
(1/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) + 4*b*tan(1/2*d*x + 1/2*c))/(
(tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3))/d
```

Mupad [B] (verification not implemented)

Time = 16.62 (sec) , antiderivative size = 3738, normalized size of antiderivative = 17.97

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^2/(a + b/cos(c + d*x))^2,x)
```

output
```
(atan(((((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 72*a*b^9 + 72*b^10 - 120*
a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - 26*a^5*b^5 + 23*a^6*b^4 - 20*a^7*b^3
+ 11*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + ((a^2*1i + b^2*6i)*((8*
(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^10*b^5 - 14*a^11*b^4 - 16*a^12*b^3
 + 6*a^13*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (4*tan(c/2 + (d*x)/
2)*(a^2*1i + b^2*6i)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*
a^11*b^3 - 8*a^12*b^2))/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2))))/(2*a^4))
*(a^2*1i + b^2*6i)*1i)/(2*a^4) + (((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b -
 72*a*b^9 + 72*b^10 - 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - 26*a^5*b^5
+ 23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2)
 - ((a^2*1i + b^2*6i)*((8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^10*b^5 -
 14*a^11*b^4 - 16*a^12*b^3 + 6*a^13*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*
b^2) + (4*tan(c/2 + (d*x)/2)*(a^2*1i + b^2*6i)*(8*a^13*b - 8*a^8*b^6 + 8*a
^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/(a^4*(a^8*b + a^9 - a^6*
b^3 - a^7*b^2))))/(2*a^4))*(a^2*1i + b^2*6i)*1i)/(2*a^4))/((16*(108*b^11 -
 54*a*b^10 - 216*a^2*b^9 + 81*a^3*b^8 + 63*a^4*b^7 - 9*a^5*b^6 + 41*a^6*b^
5 - 4*a^7*b^4 + 4*a^8*b^3))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (((8*ta
n(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 72*a*b^9 + 72*b^10 - 120*a^2*b^8 + 120*
a^3*b^7 + 17*a^4*b^6 - 26*a^5*b^5 + 23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2))
/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + ((a^2*1i + b^2*6i)*((8*(2*a^15 - 1...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 623, normalized size of antiderivative = 3.00

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{-11a^2b^5c + 6b^7dx + 12\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{-a^2+b^2}}\right)b^6 + 2\sin(dx+c)^3a^5b^2 - 6\sin(dx+c}{}$$

input
```
int(cos(d*x+c)^2/(a+b*sec(d*x+c))^2,x)
```

output
```
( - 16*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)
/sqrt( - a**2 + b**2))*cos(c + d*x)*a**3*b**3 + 12*sqrt( - a**2 + b**2)*at
an((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c +
 d*x)*a*b**5 - 16*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c +
 d*x)/2)*b)/sqrt( - a**2 + b**2))*a**2*b**4 + 12*sqrt( - a**2 + b**2)*atan
((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*b**6 - 3*
cos(c + d*x)*sin(c + d*x)*a**6*b + 6*cos(c + d*x)*sin(c + d*x)*a**4*b**3 -
 3*cos(c + d*x)*sin(c + d*x)*a**2*b**5 + cos(c + d*x)*a**7*c + cos(c + d*x
)*a**7*d*x + 4*cos(c + d*x)*a**5*b**2*c + 4*cos(c + d*x)*a**5*b**2*d*x - 1
1*cos(c + d*x)*a**3*b**4*c - 11*cos(c + d*x)*a**3*b**4*d*x + 6*cos(c + d*x
)*a*b**6*c + 6*cos(c + d*x)*a*b**6*d*x - sin(c + d*x)**3*a**7 + 2*sin(c +
d*x)**3*a**5*b**2 - sin(c + d*x)**3*a**3*b**4 + sin(c + d*x)*a**7 - 6*sin(
c + d*x)*a**5*b**2 + 11*sin(c + d*x)*a**3*b**4 - 6*sin(c + d*x)*a*b**6 + a
**6*b*c + a**6*b*d*x + 4*a**4*b**3*c + 4*a**4*b**3*d*x - 11*a**2*b**5*c -
11*a**2*b**5*d*x + 6*b**7*c + 6*b**7*d*x)/(2*a**4*d*(cos(c + d*x)*a**5 - 2
*cos(c + d*x)*a**3*b**2 + cos(c + d*x)*a*b**4 + a**4*b - 2*a**2*b**3 + b**
5))
```

3.505 $\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 261

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = & -\frac{b(a^2+4b^2)\,x}{a^5}+\frac{2b^4(5a^2-4b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{3/2}(a+b)^{3/2}d}\\ & +\frac{(2a^4+7a^2b^2-12b^4)\sin(c+dx)}{3a^4\left(a^2-b^2\right)d}\\ & -\frac{b(a^2-2b^2)\cos(c+dx)\sin(c+dx)}{a^3\left(a^2-b^2\right)d}\\ & +\frac{(a^2-4b^2)\cos^2(c+dx)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d}\\ & +\frac{b^2\cos^2(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))}\end{aligned}$$

output
```
-b*(a^2+4*b^2)*x/a^5+2*b^4*(5*a^2-4*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1
/2*c)/(a+b)^(1/2))/a^5/(a-b)^(3/2)/(a+b)^(3/2)/d+1/3*(2*a^4+7*a^2*b^2-12*b
^4)*sin(d*x+c)/a^4/(a^2-b^2)/d-b*(a^2-2*b^2)*cos(d*x+c)*sin(d*x+c)/a^3/(a^
2-b^2)/d+1/3*(a^2-4*b^2)*cos(d*x+c)^2*sin(d*x+c)/a^2/(a^2-b^2)/d+b^2*cos(d
*x+c)^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.67

$$\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{-12b(-ia+2b)(ia+2b)(c+dx)+\frac{24b^4(-5a^2+4b^2)\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+9a(a^2+4b^2)\sin(c+dx)-}{12a^5d}$$

input
```
Integrate[Cos[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

output
```
(-12*b*((-I)*a + 2*b)*(I*a + 2*b)*(c + d*x) + (24*b^4*(-5*a^2 + 4*b^2)*Arc
Tanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + 9*a
*(a^2 + 4*b^2)*Sin[c + d*x] + (12*a*b^5*Sin[c + d*x])/((-a + b)*(a + b)*(b
 + a*Cos[c + d*x])) - 6*a^2*b*Sin[2*(c + d*x)] + a^3*Sin[3*(c + d*x)])/(12
*a^5*d)
```

Rubi [A] (verified)

Time = 2.04 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.03, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 4334, 25, 3042, 4592, 3042, 4592, 27, 3042, 4592, 27, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4334}$$

$$\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}-\frac{\int -\frac{\cos^3(c+dx)\left(a^2-b\sec(c+dx)a-4b^2+3b^2\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\cos^3(c+dx)\left(a^2-b\sec(c+dx)a-4b^2+3b^2\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a-4b^2+3b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \ 4592$$

$$\frac{\frac{\left(a^2-4b^2\right)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\int \frac{\cos^2(c+dx)\left(-2b\left(a^2-4b^2\right)\sec^2(c+dx)-a\left(2a^2+b^2\right)\sec(c+dx)+6b\left(a^2-2b^2\right)\right)}{a+b\sec(c+dx)}dx}{3a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\left(a^2-4b^2\right)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\int \frac{-2b\left(a^2-4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-a\left(2a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+6b\left(a^2-2b^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \ 4592$$

$$\frac{\frac{\left(a^2-4b^2\right)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\int \frac{2\cos(c+dx)\left(2a^4+7b^2a^2-b\left(a^2+2b^2\right)\sec(c+dx)a-12b^4-3b^2\left(a^2-2b^2\right)\sec^2(c+d\cdots\right.}{a+b\sec(c+dx)}}{2a}}{3a}}{a\left(a^2-b^2\right)}$$
$$\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\left(a^2-4b^2\right)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\int \frac{\cos(c+dx)\left(2a^4+7b^2a^2-b\left(a^2+2b^2\right)\sec(c+dx)a-12b^4-3b^2\left(a^2-2b^2\right)\sec^2(c+dx)\cdots\right.}{a+b\sec(c+dx)}}{a}}{3a}}{a\left(a^2-b^2\right)}$$
$$\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(a^2-4b^2)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b(a^2-2b^2)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\int\frac{2a^4+7b^2a^2-b(a^2+2b^2)\csc(c+dx+\frac{\pi}{2})a-12b^4-3b^2(a^2-2b^2)\csc(c+dx+\frac{\pi}{2})^2}{\csc(c+dx+\frac{\pi}{2})(a+b\csc(c+dx+\frac{\pi}{2}))}dx}{a}}{3a}}{a(a^2-b^2)}$$

$$\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 4592

$$\frac{\frac{(a^2-4b^2)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b(a^2-2b^2)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{(2a^4+7a^2b^2-12b^4)\sin(c+dx)}{ad}-\frac{\int\frac{3(a(a^2-2b^2)\sec(c+dx)b^2+(a^4+3b^2a^2-4b^4)b}{a+b\sec(c+dx)}}{a}}{a}}{3a}}{a(a^2-b^2)}$$

$$\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 27

$$\frac{\frac{(a^2-4b^2)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b(a^2-2b^2)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{(2a^4+7a^2b^2-12b^4)\sin(c+dx)}{ad}-\frac{3\int\frac{a(a^2-2b^2)\sec(c+dx)b^2+(a^4+3b^2a^2-4b^4)b}{a+b\sec(c+dx)}d}{a}}{a}}{3a}}{a(a^2-b^2)}$$

$$\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(a^2-4b^2)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b(a^2-2b^2)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{(2a^4+7a^2b^2-12b^4)\sin(c+dx)}{ad}-\frac{3\int\frac{a(a^2-2b^2)\csc(c+dx+\frac{\pi}{2})b^2+(a^4+3b^2a^2-4b}{a+b\csc(c+dx+\frac{\pi}{2})}}{a}}{a}}{3a}}{a(a^2-b^2)}$$

$$\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 4407

$$\frac{\frac{(a^2-4b^2)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b(a^2-2b^2)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{(2a^4+7a^2b^2-12b^4)\sin(c+dx)}{ad}-\frac{3\left(\frac{bx(a^4+3a^2b^2-4b^4)}{a}-\frac{b^4(5a^2-4b^2)\int\frac{\sec(c}{a+b\sec}}{a}\right.}{a}}{a}}{3a}}{a(a^2-b^2)}$$

$$\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\left(a^2-4b^2\right)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{\left(2a^4+7a^2b^2-12b^4\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{bx\left(a^4+3a^2b^2-4b^4\right)}{a}-\frac{b^4\left(5a^2-4b^2\right)\int\frac{\csc(c\cdots}{a+b\csc\cdots}}{a}\cdots\right)}{a}}{a}}{3a}}{a\left(a^2-b^2\right)}$$
$$\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$\Big\downarrow$ 4318

$$\frac{\frac{\left(a^2-4b^2\right)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{\left(2a^4+7a^2b^2-12b^4\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{bx\left(a^4+3a^2b^2-4b^4\right)}{a}-\frac{b^3\left(5a^2-4b^2\right)\int\frac{1}{\frac{a\cos(c\cdots}{b}\cdots}}{a}\cdots\right)}{a}}{a}}{3a}}{a\left(a^2-b^2\right)}$$
$$\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\left(a^2-4b^2\right)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{\left(2a^4+7a^2b^2-12b^4\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{bx\left(a^4+3a^2b^2-4b^4\right)}{a}-\frac{b^3\left(5a^2-4b^2\right)\int\frac{1}{\frac{a\sin\left(c\cdots\right.}{\cdots}}}{a}\cdots\right)}{a}}{a}}{3a}}{a\left(a^2-b^2\right)}$$
$$\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$\Big\downarrow$ 3138

$$\frac{\frac{\left(a^2-4b^2\right)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{\left(2a^4+7a^2b^2-12b^4\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{bx\left(a^4+3a^2b^2-4b^4\right)}{a}-\frac{2b^3\left(5a^2-4b^2\right)\int\frac{1}{\left(1-\frac{a}{b}\cdots\right.}}{a}\cdots\right)}{a}}{a}}{3a}}{a\left(a^2-b^2\right)}$$
$$\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$\Big\downarrow$ 221

$$\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}+$$

$$\frac{\frac{\left(a^2-4b^2\right)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{\left(2a^4+7a^2b^2-12b^4\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{bx\left(a^4+3a^2b^2-4b^4\right)}{a}-\frac{2b^4\left(5a^2-4b^2\right)\operatorname{arctan}}{ad\sqrt{}}\right.}{a}}{a}}{3a}}{a\left(a^2-b^2\right)}$$

input
```
Int[Cos[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

output
```
(b^2*Cos[c + d*x]^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])) +
 (((a^2 - 4*b^2)*Cos[c + d*x]^2*Sin[c + d*x])/(3*a*d) - ((3*b*(a^2 - 2*b^2
)*Cos[c + d*x]*Sin[c + d*x])/(a*d) - ((-3*((b*(a^4 + 3*a^2*b^2 - 4*b^4)*x)
/a - (2*b^4*(5*a^2 - 4*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
+ b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d)))/a + ((2*a^4 + 7*a^2*b^2 - 12*b^4)*S
in[c + d*x])/(a*d))/a)/(3*a))/(a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.01

method	result
derivativedivides	$-\frac{-\frac{2b^4\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}-\frac{\left(5a^2-4b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}\right)}{a^5}-\frac{2\left(\frac{\left(-a^3-a^2b-3a b\right.}{}\right.}{}}{d}$
default	$-\frac{-\frac{2b^4\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}-\frac{\left(5a^2-4b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}\right)}{a^5}-\frac{2\left(\frac{\left(-a^3-a^2b-3a b\right.}{}\right.}{}}{d}$
risch	$-\frac{bx}{a^3}-\frac{4b^3x}{a^5}+\frac{ib\,e^{2i(dx+c)}}{4a^3d}-\frac{3ie^{i(dx+c)}}{8a^2d}-\frac{3ie^{i(dx+c)}b^2}{2d\,a^4}+\frac{3ie^{-i(dx+c)}}{8a^2d}+\frac{3ie^{-i(dx+c)}b^2}{2d\,a^4}-\frac{ib\,e^{-2i(dx+c)}}{4a^3d}$

input
```
int(cos(d*x+c)^3/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*b^4/a^5*(-a*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a
-tan(1/2*d*x+1/2*c)^2*b-a-b)-(5*a^2-4*b^2)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)
*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))-2/a^5*(((-a^3-a^2*
b-3*a*b^2)*tan(1/2*d*x+1/2*c)^5+(-2/3*a^3-6*a*b^2)*tan(1/2*d*x+1/2*c)^3+(-
a^3+a^2*b-3*a*b^2)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^3+b*(a^2+4
*b^2)*arctan(tan(1/2*d*x+1/2*c))))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 757, normalized size of antiderivative = 2.90

$$\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/6*(6*(a^7*b + 2*a^5*b^3 - 7*a^3*b^5 + 4*a*b^7)*d*x*cos(d*x + c) + 6*(a
^6*b^2 + 2*a^4*b^4 - 7*a^2*b^6 + 4*b^8)*d*x - 3*(5*a^2*b^5 - 4*b^7 + (5*a^
3*b^4 - 4*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (
a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d
*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) -
2*(2*a^7*b + 5*a^5*b^3 - 19*a^3*b^5 + 12*a*b^7 + (a^8 - 2*a^6*b^2 + a^4*b^
4)*cos(d*x + c)^3 - 2*(a^7*b - 2*a^5*b^3 + a^3*b^5)*cos(d*x + c)^2 + 2*(a^
8 + a^6*b^2 - 5*a^4*b^4 + 3*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^10 -
2*a^8*b^2 + a^6*b^4)*d*cos(d*x + c) + (a^9*b - 2*a^7*b^3 + a^5*b^5)*d), -1
/3*(3*(a^7*b + 2*a^5*b^3 - 7*a^3*b^5 + 4*a*b^7)*d*x*cos(d*x + c) + 3*(a^6*
b^2 + 2*a^4*b^4 - 7*a^2*b^6 + 4*b^8)*d*x - 3*(5*a^2*b^5 - 4*b^7 + (5*a^3*b
^4 - 4*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*c
os(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (2*a^7*b + 5*a^5*b^3 - 19*a
^3*b^5 + 12*a*b^7 + (a^8 - 2*a^6*b^2 + a^4*b^4)*cos(d*x + c)^3 - 2*(a^7*b
- 2*a^5*b^3 + a^3*b^5)*cos(d*x + c)^2 + 2*(a^8 + a^6*b^2 - 5*a^4*b^4 + 3*a
^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^10 - 2*a^8*b^2 + a^6*b^4)*d*cos(d*
x + c) + (a^9*b - 2*a^7*b^3 + a^5*b^5)*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.28

$$\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{6\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^6-a^4b^2)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)} + \frac{6\,(5\,a^2b^4-4\,b^6)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\right.}{\sqrt{-a^2+b^2}}\right.\right.}{(a^7-a^5b^2)\sqrt{-a^2+b^2}}}{}$$

input
```
integrate(cos(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/3*(6*b^5*tan(1/2*d*x + 1/2*c)/((a^6 - a^4*b^2)*(a*tan(1/2*d*x + 1/2*c)^2
 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) + 6*(5*a^2*b^4 - 4*b^6)*(pi*floor(1/
2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) -
b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^7 - a^5*b^2)*sqrt(-a^2 + b^
2)) - 3*(a^2*b + 4*b^3)*(d*x + c)/a^5 + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 +
3*a*b*tan(1/2*d*x + 1/2*c)^5 + 9*b^2*tan(1/2*d*x + 1/2*c)^5 + 2*a^2*tan(1/
2*d*x + 1/2*c)^3 + 18*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2
*c) - 3*a*b*tan(1/2*d*x + 1/2*c) + 9*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d
*x + 1/2*c)^2 + 1)^3*a^4))/d
```

Mupad [B] (verification not implemented)

Time = 16.98 (sec) , antiderivative size = 3839, normalized size of antiderivative = 14.71

$$\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^3/(a + b/cos(c + d*x))^2,x)
```

output
```
- ((2*tan(c/2 + (d*x)/2)^7*(a^5 - 2*a*b^4 + 4*b^5 - 3*a^2*b^3 + a^3*b^2))/
(a^4*(a + b)*(a - b)) + (2*tan(c/2 + (d*x)/2)^3*(6*a*b^4 - 8*a^4*b + a^5 +
 36*b^5 - 19*a^2*b^3 - 7*a^3*b^2))/(3*a^4*(a + b)*(a - b)) - (2*tan(c/2 +
(d*x)/2)^5*(6*a*b^4 + 8*a^4*b + a^5 - 36*b^5 + 19*a^2*b^3 - 7*a^3*b^2))/(3
*a^4*(a + b)*(a - b)) - (2*tan(c/2 + (d*x)/2)*(a^5 - 2*a*b^4 - 4*b^5 + 3*a
^2*b^3 + a^3*b^2))/(a^4*(a + b)*(a - b)))/(d*(a + b - tan(c/2 + (d*x)/2)^8
*(a - b) + tan(c/2 + (d*x)/2)^2*(2*a + 4*b) - tan(c/2 + (d*x)/2)^6*(2*a -
4*b) + 6*b*tan(c/2 + (d*x)/2)^4)) - (2*b*atan(((b*(a^2 + 4*b^2)*((32*tan(c
/2 + (d*x)/2)*(32*b^12 - 32*a*b^11 - 48*a^2*b^10 + 48*a^3*b^9 + 2*a^4*b^8
- 2*a^5*b^7 + 7*a^6*b^6 - 12*a^7*b^5 + 7*a^8*b^4 - 2*a^9*b^3 + a^10*b^2))/
(a^10*b + a^11 - a^8*b^3 - a^9*b^2) + (b*(a^2 + 4*b^2)*((32*(a^17*b - 4*a^
10*b^8 + 2*a^11*b^7 + 9*a^12*b^6 - 4*a^13*b^5 - 5*a^14*b^4 + a^15*b^3))/(a
^14*b + a^15 - a^12*b^3 - a^13*b^2) - (b*tan(c/2 + (d*x)/2)*(a^2 + 4*b^2)*
(2*a^15*b - 2*a^10*b^6 + 2*a^11*b^5 + 4*a^12*b^4 - 4*a^13*b^3 - 2*a^14*b^2
)*32i)/(a^5*(a^10*b + a^11 - a^8*b^3 - a^9*b^2)))*1i)/a^5))/a^5 + (b*(a^2
+ 4*b^2)*((32*tan(c/2 + (d*x)/2)*(32*b^12 - 32*a*b^11 - 48*a^2*b^10 + 48*a
^3*b^9 + 2*a^4*b^8 - 2*a^5*b^7 + 7*a^6*b^6 - 12*a^7*b^5 + 7*a^8*b^4 - 2*a^
9*b^3 + a^10*b^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^2) - (b*(a^2 + 4*b^2)*
((32*(a^17*b - 4*a^10*b^8 + 2*a^11*b^7 + 9*a^12*b^6 - 4*a^13*b^5 - 5*a^14*
b^4 + a^15*b^3))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) + (b*tan(c/2 + (...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 686, normalized size of antiderivative = 2.63

$$\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(cos(d*x+c)^3/(a+b*sec(d*x+c))^2,x)
```

output
```
(30*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sq
rt( - a**2 + b**2))*cos(c + d*x)*a**3*b**4 - 24*sqrt( - a**2 + b**2)*atan(
(tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*
x)*a*b**6 + 30*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*
x)/2)*b)/sqrt( - a**2 + b**2))*a**2*b**5 - 24*sqrt( - a**2 + b**2)*atan((t
an((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*b**7 - cos(c
 + d*x)*sin(c + d*x)**3*a**8 + 2*cos(c + d*x)*sin(c + d*x)**3*a**6*b**2 -
cos(c + d*x)*sin(c + d*x)**3*a**4*b**4 + 3*cos(c + d*x)*sin(c + d*x)*a**8
- 9*cos(c + d*x)*sin(c + d*x)*a**4*b**4 + 6*cos(c + d*x)*sin(c + d*x)*a**2
*b**6 - 3*cos(c + d*x)*a**7*b*c - 3*cos(c + d*x)*a**7*b*d*x - 6*cos(c + d*
x)*a**5*b**3*c - 6*cos(c + d*x)*a**5*b**3*d*x + 21*cos(c + d*x)*a**3*b**5*
c + 21*cos(c + d*x)*a**3*b**5*d*x - 12*cos(c + d*x)*a*b**7*c - 12*cos(c +
d*x)*a*b**7*d*x + 2*sin(c + d*x)**3*a**7*b - 4*sin(c + d*x)**3*a**5*b**3 +
 2*sin(c + d*x)**3*a**3*b**5 + 9*sin(c + d*x)*a**5*b**3 - 21*sin(c + d*x)*
a**3*b**5 + 12*sin(c + d*x)*a*b**7 - 3*a**6*b**2*c - 3*a**6*b**2*d*x - 6*a
**4*b**4*c - 6*a**4*b**4*d*x + 21*a**2*b**6*c + 21*a**2*b**6*d*x - 12*b**8
*c - 12*b**8*d*x)/(3*a**5*d*(cos(c + d*x)*a**5 - 2*cos(c + d*x)*a**3*b**2
+ cos(c + d*x)*a*b**4 + a**4*b - 2*a**2*b**3 + b**5))
```

3.506 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 230

$$\begin{aligned}\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = &-\frac{3a\operatorname{arctanh}(\sin(c+dx))}{b^4d} \\ &+\frac{3a^2(2a^4-5a^2b^2+4b^4)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^4(a+b)^{5/2}d} \\ &+\frac{(3a^2-2b^2)\tan(c+dx)}{2b^3(a^2-b^2)\,d} \\ &-\frac{a^2\sec^2(c+dx)\tan(c+dx)}{2b(a^2-b^2)\,d(a+b\sec(c+dx))^2} \\ &+\frac{3a^3(a^2-2b^2)\tan(c+dx)}{2b^3(a^2-b^2)^2\,d(a+b\sec(c+dx))}\end{aligned}$$

output

```
-3*a*arctanh(sin(d*x+c))/b^4/d+3*a^2*(2*a^4-5*a^2*b^2+4*b^4)*arctanh((a-b)
^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/b^4/(a+b)^(5/2)/d+1/2*(
3*a^2-2*b^2)*tan(d*x+c)/b^3/(a^2-b^2)/d-1/2*a^2*sec(d*x+c)^2*tan(d*x+c)/b/
(a^2-b^2)/d/(a+b*sec(d*x+c))^2+3/2*a^3*(a^2-2*b^2)*tan(d*x+c)/b^3/(a^2-b^2
)^2/d/(a+b*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 3.55 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.89

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{-\frac{6a^2(2a^4-5a^2b^2+4b^4)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + 6a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) - 6a\log\left(\cos\left(\right.\right.}{2b^4d}$$

input
```
Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
```

output
```
((-6*a^2*(2*a^4 - 5*a^2*b^2 + 4*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/S
qrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + 6*a*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] - 6*a*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^3*b*(5*a^2*b -
 8*b^3 + a*(4*a^2 - 7*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^
2*(b + a*Cos[c + d*x])^2) + 2*b*Tan[c + d*x])/(2*b^4*d)
```

Rubi [A] (verified)

Time = 1.85 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.16, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4332, 3042, 4578, 3042, 4570, 27, 3042, 4486, 3042, 4257, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4332}$$

$$-\frac{\int \frac{\sec^2(c+dx)\left(2a^2-2b\sec(c+dx)a-\left(3a^2-2b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\big\downarrow$ 3042

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(2a^2-2b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(2b^2-3a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\big\downarrow$ 4578

$$-\frac{\frac{\int \frac{\sec(c+dx)\left(3b\left(a^2-2b^2\right)a^2+\left(3a^2-4b^2\right)\left(a^2-b^2\right)\sec(c+dx)a-b\left(3a^2-2b^2\right)\left(a^2-b^2\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{b^2(a^2-b^2)}-\frac{3a^3\left(a^2-2b^2\right)\tan(c+dx)}{b^2d(a^2-b^2)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\big\downarrow$ 3042

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3b\left(a^2-2b^2\right)a^2+\left(3a^2-4b^2\right)\left(a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a-b\left(3a^2-2b^2\right)\left(a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2(a^2-b^2)}-\frac{3a^3\left(a^2-2b^2\right)\tan(c+dx)}{b^2d(a^2-b^2)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\big\downarrow$ 4570

$$-\frac{\frac{\frac{\int \frac{3\sec(c+dx)\left(a^2\left(a^2-2b^2\right)b^2+2a\left(a^2-b^2\right)^2\sec(c+dx)b\right)}{a+b\sec(c+dx)}dx}{b}-\frac{\left(3a^2-2b^2\right)\left(a^2-b^2\right)\tan(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{3a^3\left(a^2-2b^2\right)\tan(c+dx)}{b^2d(a^2-b^2)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\big\downarrow$ 27

$$-\frac{\frac{\frac{3\int \frac{\sec(c+dx)\left(a^2\left(a^2-2b^2\right)b^2+2a\left(a^2-b^2\right)^2\sec(c+dx)b\right)}{a+b\sec(c+dx)}dx}{b}-\frac{\left(3a^2-2b^2\right)\left(a^2-b^2\right)\tan(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{3a^3\left(a^2-2b^2\right)\tan(c+dx)}{b^2d(a^2-b^2)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a^2\left(a^2-2b^2\right)b^2+2a\left(a^2-b^2\right)^2\csc\left(c+dx+\frac{\pi}{2}\right)b\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(3a^2-2b^2\right)\left(a^2-b^2\right)\tan(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{3a^3\left(a^2-2b^2\right)\tan(c+dx)}{b^2d(a^2-b^2)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{4486}$$

$$-\frac{\frac{\frac{3\left(2a\left(a^2-b^2\right)^2\int \sec(c+dx)dx-a^2\left(2a^4-5a^2b^2+4b^4\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx\right)}{b}-\frac{\left(3a^2-2b^2\right)\left(a^2-b^2\right)\tan(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{3a^3\left(a^2-2b^2\right)\tan(c+dx)}{b^2d(a^2-b^2)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{3\left(2a\left(a^2-b^2\right)^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-a^2\left(2a^4-5a^2b^2+4b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{b}-\frac{\left(3a^2-2b^2\right)\left(a^2-b^2\right)\tan(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{3a^3\left(a^2-2b^2\right)\tan(c+dx)}{b^2d(a^2-b^2)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\frac{\frac{3\left(\frac{2a\left(a^2-b^2\right)^2\text{arctanh}(\sin(c+dx))}{d}-a^2\left(2a^4-5a^2b^2+4b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{b}-\frac{\left(3a^2-2b^2\right)\left(a^2-b^2\right)\tan(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{3a^3\left(a^2-2b^2\right)\tan(c+dx)}{b^2d(a^2-b^2)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{4318}$$

$$-\frac{\frac{\frac{3\left(\frac{2a\left(a^2-b^2\right)^2\text{arctanh}(\sin(c+dx))}{d}-\frac{a^2\left(2a^4-5a^2b^2+4b^4\right)\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b}\right)}{b}-\frac{\left(3a^2-2b^2\right)\left(a^2-b^2\right)\tan(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{3a^3\left(a^2-2b^2\right)\tan(c+dx)}{b^2d(a^2-b^2)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{\frac{3\left(\frac{2a\left(a^2-b^2\right)^2\text{arctanh}(\sin(c+dx))}{d}-\frac{a^2\left(2a^4-5a^2b^2+4b^4\right)\int\frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b}\right)}{b}-\frac{\left(3a^2-2b^2\right)\left(a^2-b^2\right)\tan(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{3a^3\left(a^2-2b^2\right)\tan(c+dx)}{b^2d(a^2-b^2)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}$$
$$\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \textbf{3138}$$

$$-\frac{\frac{3\left(\frac{2a\left(a^2-b^2\right)^2\text{arctanh}(\sin(c+dx))}{d}-\frac{2a^2\left(2a^4-5a^2b^2+4b^4\right)\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}\right)}{b}-\frac{\left(3a^2-2b^2\right)\left(a^2-b^2\right)\tan(c+dx)}{d}}{b^2(a^2-b^2)}-\ldots}{2b\left(a^2-b^2\right)}$$
$$\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \textbf{221}$$

$$-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-$$
$$\frac{\frac{3\left(\frac{2a\left(a^2-b^2\right)^2\text{arctanh}(\sin(c+dx))}{d}-\frac{2a^2\left(2a^4-5a^2b^2+4b^4\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}\right)}{b}-\frac{\left(3a^2-2b^2\right)\left(a^2-b^2\right)\tan(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{3a^3\left(a^2-2b^2\right)\ldots}{b^2d(a^2-b^2)(a+\ldots}}{2b\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
```

output
```
-1/2*(a^2*Sec[c + d*x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x
])^2) - ((-3*a^3*(a^2 - 2*b^2)*Tan[c + d*x])/(b^2*(a^2 - b^2)*d*(a + b*Sec
[c + d*x])) + ((3*((2*a*(a^2 - b^2)^2*ArcTanh[Sin[c + d*x]])/d - (2*a^2*(2
*a^4 - 5*a^2*b^2 + 4*b^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a +
b]])/(Sqrt[a - b]*Sqrt[a + b]*d)))/b - ((3*a^2 - 2*b^2)*(a^2 - b^2)*Tan[c
+ d*x])/d)/(b^2*(a^2 - b^2)))/(2*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

rule 4578
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x
])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - Simp[1/(b^2*(m + 1)*(a^2 - b^
2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*((-a)*(b
*B - a*C) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
 b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{-\frac{1}{b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^4}-\frac{2a^2\left(\frac{\frac{\left(4a^2-ab-8b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}-\frac{\left(4a^2+ab-8b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}-\frac{3\left(2a\right.}{}\right.}{b^4}}{d}$
default	$\frac{-\frac{1}{b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^4}-\frac{2a^2\left(\frac{\frac{\left(4a^2-ab-8b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}-\frac{\left(4a^2+ab-8b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}-\frac{3\left(2a\right.}{}\right.}{b^4}}{d}$
risch	$\frac{i\left(3a^5b\,e^{5i(dx+c)}-6a^3b^3e^{5i(dx+c)}+6a^6e^{4i(dx+c)}-3a^4b^2e^{4i(dx+c)}-12a^2b^4e^{4i(dx+c)}+24a^5b\,e^{3i(dx+c)}-44a^3b^3e^{3i(dx+c}\right.}{\left(-a^2+b^2\right)^2}$

input
```
int(sec(d*x+c)^5/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^3/(tan(1/2*d*x+1/2*c)+1)-3*a/b^4*ln(tan(1/2*d*x+1/2*c)+1)-2*a^2/
b^4*((1/2*(4*a^2-a*b-8*b^2)*a*b/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3
-1/2*(4*a^2+a*b-8*b^2)*a*b/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(
1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2-3/2*(2*a^4-5*a^2*b^2+4*b^
4)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c
)/((a+b)*(a-b))^(1/2)))-1/b^3/(tan(1/2*d*x+1/2*c)-1)+3*a/b^4*ln(tan(1/2*d*
x+1/2*c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(215) = 430$.

Time = 0.66 (sec) , antiderivative size = 1354, normalized size of antiderivative = 5.89

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/4*(3*((2*a^8 - 5*a^6*b^2 + 4*a^4*b^4)*cos(d*x + c)^3 + 2*(2*a^7*b - 5*a
^5*b^3 + 4*a^3*b^5)*cos(d*x + c)^2 + (2*a^6*b^2 - 5*a^4*b^4 + 4*a^2*b^6)*c
os(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d
*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 -
b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 6*((a^9 - 3*a^7*b^
2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x + c)^3 + 2*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5
 - a^2*b^7)*cos(d*x + c)^2 + (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cos
(d*x + c))*log(sin(d*x + c) + 1) + 6*((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b
^6)*cos(d*x + c)^3 + 2*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(d*x +
 c)^2 + (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(d*x + c))*log(-sin(d
*x + c) + 1) + 2*(2*a^6*b^3 - 6*a^4*b^5 + 6*a^2*b^7 - 2*b^9 + (6*a^8*b - 1
7*a^6*b^3 + 13*a^4*b^5 - 2*a^2*b^7)*cos(d*x + c)^2 + (9*a^7*b^2 - 25*a^5*b
^4 + 20*a^3*b^6 - 4*a*b^8)*cos(d*x + c))*sin(d*x + c))/((a^8*b^4 - 3*a^6*b
^6 + 3*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^3 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a
^3*b^9 - a*b^11)*d*cos(d*x + c)^2 + (a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^
12)*d*cos(d*x + c)), 1/2*(3*((2*a^8 - 5*a^6*b^2 + 4*a^4*b^4)*cos(d*x + c)^
3 + 2*(2*a^7*b - 5*a^5*b^3 + 4*a^3*b^5)*cos(d*x + c)^2 + (2*a^6*b^2 - 5*a^
4*b^4 + 4*a^2*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)
*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - 3*((a^9 - 3*a^7*b^2 +
3*a^5*b^4 - a^3*b^6)*cos(d*x + c)^3 + 2*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 ...
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sec^5{(c+dx)}}{(a+b\sec{(c+dx)})^3}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**5/(a + b*sec(c + d*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.67

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{\frac{3\,(2\,a^6-5\,a^4b^2+4\,a^2b^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^4b^4-2\,a^2b^6+b^8)\sqrt{-a^2+b^2}}+\frac{4\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,a^5b\tan\left(\frac{1}{2}\,dx\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-(3*(2*a^6 - 5*a^4*b^2 + 4*a^2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(
2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt
(-a^2 + b^2)))/((a^4*b^4 - 2*a^2*b^6 + b^8)*sqrt(-a^2 + b^2)) + (4*a^6*tan
(1/2*d*x + 1/2*c)^3 - 5*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 7*a^4*b^2*tan(1/2*d
*x + 1/2*c)^3 + 8*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 4*a^6*tan(1/2*d*x + 1/2
*c) - 5*a^5*b*tan(1/2*d*x + 1/2*c) + 7*a^4*b^2*tan(1/2*d*x + 1/2*c) + 8*a^
3*b^3*tan(1/2*d*x + 1/2*c))/((a^4*b^3 - 2*a^2*b^5 + b^7)*(a*tan(1/2*d*x +
1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) + 3*a*log(abs(tan(1/2*d*x
+ 1/2*c) + 1))/b^4 - 3*a*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 + 2*tan(1/
2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*b^3))/d
```

Mupad [B] (verification not implemented)

Time = 17.69 (sec) , antiderivative size = 5332, normalized size of antiderivative = 23.18

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^3),x)
```

output
```
((tan(c/2 + (d*x)/2)^5*(2*a*b^4 - 3*a^4*b + 6*a^5 - 2*b^5 + 4*a^2*b^3 - 12
*a^3*b^2))/((a*b^3 - b^4)*(a + b)^2) + (tan(c/2 + (d*x)/2)*(2*a*b^4 + 3*a^
4*b + 6*a^5 + 2*b^5 - 4*a^2*b^3 - 12*a^3*b^2))/((a + b)*(b^5 - 2*a*b^4 + a
^2*b^3)) - (2*tan(c/2 + (d*x)/2)^3*(6*a^6 - 2*b^6 + 6*a^2*b^4 - 13*a^4*b^2
))/(b*(a*b^2 - b^3)*(a + b)^2*(a - b)))/(d*(2*a*b - tan(c/2 + (d*x)/2)^2*(
2*a*b + 3*a^2 - b^2) - tan(c/2 + (d*x)/2)^6*(a^2 - 2*a*b + b^2) + a^2 + b^
2 - tan(c/2 + (d*x)/2)^4*(2*a*b - 3*a^2 + b^2))) + (a*atan(((a*((8*tan(c/2
 + (d*x)/2)*(72*a^12 - 72*a^11*b + 36*a^2*b^10 - 72*a^3*b^9 + 36*a^4*b^8 +
 288*a^5*b^7 - 288*a^6*b^6 - 432*a^7*b^5 + 441*a^8*b^4 + 288*a^9*b^3 - 288
*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b
^8 - a^6*b^7 - a^7*b^6) - (3*a*((24*(4*a*b^17 - 8*a^2*b^16 - 12*a^3*b^15 +
 26*a^4*b^14 + 14*a^5*b^13 - 32*a^6*b^12 - 8*a^7*b^11 + 18*a^8*b^10 + 2*a^
9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12
 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) - (24*a*tan(c/2 + (d*x)/2)*(8*a*b^17 -
 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^6*b^12 - 32*a
^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8))/(b^4*(a*b^12 + b^13 - 3*a
^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6))))/b^4)*
3i)/b^4 + (a*((8*tan(c/2 + (d*x)/2)*(72*a^12 - 72*a^11*b + 36*a^2*b^10 - 7
2*a^3*b^9 + 36*a^4*b^8 + 288*a^5*b^7 - 288*a^6*b^6 - 432*a^7*b^5 + 441*a^8
*b^4 + 288*a^9*b^3 - 288*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 2086, normalized size of antiderivative = 9.07

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^5/(a+b*sec(d*x+c))^3,x)
```

output
```
(12*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sq
rt( - a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**2*a**8 - 30*sqrt( - a**2 +
b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))
*cos(c + d*x)*sin(c + d*x)**2*a**6*b**2 + 24*sqrt( - a**2 + b**2)*atan((ta
n((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*
sin(c + d*x)**2*a**4*b**4 - 12*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)
*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*a**8 + 18*sqrt
( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a*
*2 + b**2))*cos(c + d*x)*a**6*b**2 + 6*sqrt( - a**2 + b**2)*atan((tan((c +
 d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*a**4*b
**4 - 24*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*
b)/sqrt( - a**2 + b**2))*cos(c + d*x)*a**2*b**6 + 24*sqrt( - a**2 + b**2)*
atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*sin(c
 + d*x)**2*a**7*b - 60*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan
((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*sin(c + d*x)**2*a**5*b**3 + 48*sqrt
( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a*
*2 + b**2))*sin(c + d*x)**2*a**3*b**5 - 24*sqrt( - a**2 + b**2)*atan((tan(
(c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*a**7*b + 60*sqr
t( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a
**2 + b**2))*a**5*b**3 - 48*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)...
```

3.507 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 188

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{b^3d} - \frac{a(2a^4-5a^2b^2+6b^4)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^3(a+b)^{5/2}d} - \frac{a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a^2(2a^2-5b^2)\tan(c+dx)}{2b^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))}$$

output
```
arctanh(sin(d*x+c))/b^3/d-a*(2*a^4-5*a^2*b^2+6*b^4)*arctanh((a-b)^(1/2)*ta
n(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/b^3/(a+b)^(5/2)/d-1/2*a^2*sec(d*
x+c)*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^2-1/2*a^2*(2*a^2-5*b^2)*tan
(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.03

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$=\frac{\frac{2a(2a^4-5a^2b^2+6b^4)\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}-2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\log\left(\cos\left(\frac{1}{2}(c-\right.\right.}{2b^3d}$$

input
```
Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```

output
```
((2*a*(2*a^4 - 5*a^2*b^2 + 6*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt
[a^2 - b^2]])/(a^2 - b^2)^(5/2) - 2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] + 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - (a^2*b*(3*b*(a^2 - 2*b^2
) + a*(2*a^2 - 5*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b
+ a*Cos[c + d*x])^2))/(2*b^3*d)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.24, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4332, 3042, 4568, 3042, 4486, 3042, 4257, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$
$$\Big\downarrow \; 4332$$
$$-\frac{\int \frac{\sec(c+dx)\left(a^2-2b\sec(c+dx)a-2\left(a^2-b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a^2-2b\csc\left(c+dx+\frac{\pi}{2}\right)a-2\left(a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{4568}$$

$$-\frac{\frac{a^2\left(2a^2-5b^2\right)\tan(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int \frac{\sec(c+dx)\left(2\sec(c+dx)\left(a^2-b^2\right)^2+ab\left(a^2-4b^2\right)\right)}{a+b\sec(c+dx)}dx}{b(a^2-b^2)}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{a^2\left(2a^2-5b^2\right)\tan(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2\csc\left(c+dx+\frac{\pi}{2}\right)\left(a^2-b^2\right)^2+ab\left(a^2-4b^2\right)\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b(a^2-b^2)}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{4486}$$

$$-\frac{\frac{a^2\left(2a^2-5b^2\right)\tan(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\frac{2\left(a^2-b^2\right)^2\int\sec(c+dx)dx}{b}-\frac{a\left(2a^4-5a^2b^2+6b^4\right)\int\frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{b}}{b(a^2-b^2)}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{a^2\left(2a^2-5b^2\right)\tan(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\frac{2\left(a^2-b^2\right)^2\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{b}-\frac{a\left(2a^4-5a^2b^2+6b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{b(a^2-b^2)}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\frac{a^2\left(2a^2-5b^2\right)\tan(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\frac{2\left(a^2-b^2\right)^2\text{arctanh}(\sin(c+dx))}{bd}-\frac{a\left(2a^4-5a^2b^2+6b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{b(a^2-b^2)}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{4318}$$

$$-\frac{\frac{a^2(2a^2-5b^2)\tan(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\frac{2(a^2-b^2)^2\text{arctanh}(\sin(c+dx))}{bd}-\frac{a(2a^4-5a^2b^2+6b^4)\int\frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b^2}}{b(a^2-b^2)}}{2b(a^2-b^2)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{a^2(2a^2-5b^2)\tan(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\frac{2(a^2-b^2)^2\text{arctanh}(\sin(c+dx))}{bd}-\frac{a(2a^4-5a^2b^2+6b^4)\int\frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b^2}}{b(a^2-b^2)}}{2b(a^2-b^2)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{3138}$$

$$-\frac{\frac{a^2(2a^2-5b^2)\tan(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\frac{2(a^2-b^2)^2\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a(2a^4-5a^2b^2+6b^4)\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{b^2d}}{b(a^2-b^2)}}{2b(a^2-b^2)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{221}$$

$$-\frac{a^2\tan(c+dx)\sec(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\frac{a^2(2a^2-5b^2)\tan(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\frac{2(a^2-b^2)^2\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a(2a^4-5a^2b^2+6b^4)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}}{b(a^2-b^2)}}{2b(a^2-b^2)}$$

input

```
Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```

output

```
-1/2*(a^2*Sec[c + d*x]*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])
^2) - (-(((2*(a^2 - b^2)^2*ArcTanh[Sin[c + d*x]])/(b*d) - (2*a*(2*a^4 - 5*
a^2*b^2 + 6*b^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqr
t[a - b]*b*Sqrt[a + b]*d))/(b*(a^2 - b^2))) + (a^2*(2*a^2 - 5*b^2)*Tan[c +
 d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])))/(2*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

rule 4568
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cot[e + f*x]*((a + b*Csc[e + f*x]
)^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))
 Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m
+ 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x],
x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^
2, 0]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^3}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^3}+\frac{2a\left(\frac{\frac{\left(2a^2-ab-6b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}-\frac{\left(2a^2+ab-6b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}-\frac{\left(2a^4-5a\right.}{}\right.}{b^3}}{d}$
default	$\frac{-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^3}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^3}+\frac{2a\left(\frac{\frac{\left(2a^2-ab-6b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}-\frac{\left(2a^2+ab-6b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}-\frac{\left(2a^4-5a\right.}{}\right.}{b^3}}{d}$
risch	$\frac{ia\left(-b\,a^3e^{3i(dx+c)}+4a\,b^3e^{3i(dx+c)}-2a^4e^{2i(dx+c)}+a^2b^2e^{2i(dx+c)}+10b^4e^{2i(dx+c)}-7a^3b\,e^{i(dx+c)}+16b^3a\,e^{i(dx+c)}-2a^4\right.}{\left(-a^2+b^2\right)^2d\,b^2\left(e^{2i(dx+c)}a+2b\,e^{i(dx+c)}+a\right)^2}$

input
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^3*ln(tan(1/2*d*x+1/2*c)-1)+1/b^3*ln(tan(1/2*d*x+1/2*c)+1)+2*a/b^
3*((1/2*(2*a^2-a*b-6*b^2)*a*b/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1
/2*(2*a^2+a*b-6*b^2)*a*b/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/
2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2-1/2*(2*a^4-5*a^2*b^2+6*b^4)
/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/
((a+b)*(a-b))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(175) = 350$.

Time = 0.66 (sec) , antiderivative size = 1153, normalized size of antiderivative = 6.13

$$\int \frac{\sec^4(c + dx)}{(a + b\sec(c + dx))^3} \, dx = \text{Too large to display}$$

input

```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output

```
[1/4*((2*a^5*b^2 - 5*a^3*b^4 + 6*a*b^6 + (2*a^7 - 5*a^5*b^2 + 6*a^3*b^4)*c
os(d*x + c)^2 + 2*(2*a^6*b - 5*a^4*b^3 + 6*a^2*b^5)*cos(d*x + c))*sqrt(a^2
 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^
2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c
)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^
8 + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*cos(d*x + c)^2 + 2*(a^7*b - 3*
a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) - 2*(a^6*
b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6
)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c))
*log(-sin(d*x + c) + 1) - 2*(3*a^6*b^2 - 9*a^4*b^4 + 6*a^2*b^6 + (2*a^7*b
- 7*a^5*b^3 + 5*a^3*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 3*a^6*b^5
 + 3*a^4*b^7 - a^2*b^9)*d*cos(d*x + c)^2 + 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*
b^8 - a*b^10)*d*cos(d*x + c) + (a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*d)
, -1/2*((2*a^5*b^2 - 5*a^3*b^4 + 6*a*b^6 + (2*a^7 - 5*a^5*b^2 + 6*a^3*b^4)
*cos(d*x + c)^2 + 2*(2*a^6*b - 5*a^4*b^3 + 6*a^2*b^5)*cos(d*x + c))*sqrt(-
a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(
d*x + c))) - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + (a^8 - 3*a^6*b^2 + 3
*a^4*b^4 - a^2*b^6)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*
b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^
6 - b^8 + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*cos(d*x + c)^2 + 2*(a...
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx=\int \frac{\sec^4{(c+dx)}}{(a+b\sec{(c+dx)})^3}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.85

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\frac{(2a^5-5a^3b^2+6ab^4)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^4b^3-2a^2b^5+b^7)\sqrt{-a^2+b^2}}+\frac{2a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{}}{}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
((2*a^5 - 5*a^3*b^2 + 6*a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a -
 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2
 + b^2)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*sqrt(-a^2 + b^2)) + (2*a^5*tan(1/2*
d*x + 1/2*c)^3 - 3*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 5*a^3*b^2*tan(1/2*d*x +
1/2*c)^3 + 6*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*a^5*tan(1/2*d*x + 1/2*c) -
 3*a^4*b*tan(1/2*d*x + 1/2*c) + 5*a^3*b^2*tan(1/2*d*x + 1/2*c) + 6*a^2*b^3
*tan(1/2*d*x + 1/2*c))/((a^4*b^2 - 2*a^2*b^4 + b^6)*(a*tan(1/2*d*x + 1/2*c
)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) + log(abs(tan(1/2*d*x + 1/2*c)
+ 1))/b^3 - log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^3)/d
```

Mupad [B] (verification not implemented)

Time = 18.46 (sec) , antiderivative size = 5078, normalized size of antiderivative = 27.01

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^3),x)
```

output
```
- (atan((((((8*(12*a*b^14 - 4*b^15 + 8*a^2*b^13 - 34*a^3*b^12 - 6*a^4*b^11
 + 36*a^5*b^10 + 4*a^6*b^9 - 18*a^7*b^8 - 2*a^8*b^7 + 4*a^9*b^6))/(a*b^12
+ b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b
^6) - (8*tan(c/2 + (d*x)/2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*
b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8
*a^10*b^6))/(b^3*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^
5*b^6 - a^6*b^5 - a^7*b^4)))/b^3 - (8*tan(c/2 + (d*x)/2)*(8*a^10 - 8*a^9*b
 - 8*a*b^9 + 4*b^10 + 24*a^2*b^8 + 32*a^3*b^7 - 52*a^4*b^6 - 48*a^5*b^5 +
57*a^6*b^4 + 32*a^7*b^3 - 32*a^8*b^2))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*
b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4))*1i)/b^3 - ((((8*(12*a*b^
14 - 4*b^15 + 8*a^2*b^13 - 34*a^3*b^12 - 6*a^4*b^11 + 36*a^5*b^10 + 4*a^6*
b^9 - 18*a^7*b^8 - 2*a^8*b^7 + 4*a^9*b^6))/(a*b^12 + b^13 - 3*a^2*b^11 - 3
*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (8*tan(c/2 + (d*x
)/2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48
*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6))/(b^3*(a*b^1
0 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b
^4)))/b^3 + (8*tan(c/2 + (d*x)/2)*(8*a^10 - 8*a^9*b - 8*a*b^9 + 4*b^10 + 2
4*a^2*b^8 + 32*a^3*b^7 - 52*a^4*b^6 - 48*a^5*b^5 + 57*a^6*b^4 + 32*a^7*b^3
 - 32*a^8*b^2))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5
*b^6 - a^6*b^5 - a^7*b^4))*1i)/b^3)/((((8*(12*a*b^14 - 4*b^15 + 8*a^2*b...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1406, normalized size of antiderivative = 7.48

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c))^3,x)
```

output
```
( - 8*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/
sqrt( - a**2 + b**2))*cos(c + d*x)*a**6*b + 20*sqrt( - a**2 + b**2)*atan((
tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x
)*a**4*b**3 - 24*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c +
d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*a**2*b**5 + 4*sqrt( - a**2 +
 b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2)
)*sin(c + d*x)**2*a**7 - 10*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a
- tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*sin(c + d*x)**2*a**5*b**2 + 12
*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(
 - a**2 + b**2))*sin(c + d*x)**2*a**3*b**4 - 4*sqrt( - a**2 + b**2)*atan((
tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*a**7 + 6*sq
rt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( -
a**2 + b**2))*a**5*b**2 - 2*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a
- tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*a**3*b**4 - 12*sqrt( - a**2 +
b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))
*a*b**6 - 4*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a**7*b + 12*cos(c + d*x
)*log(tan((c + d*x)/2) - 1)*a**5*b**3 - 12*cos(c + d*x)*log(tan((c + d*x)/
2) - 1)*a**3*b**5 + 4*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a*b**7 + 4*co
s(c + d*x)*log(tan((c + d*x)/2) + 1)*a**7*b - 12*cos(c + d*x)*log(tan((c +
 d*x)/2) + 1)*a**5*b**3 + 12*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a**...
```

3.508 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 149

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{(a^2+2b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{a^2\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a(a^2-4b^2)\tan(c+dx)}{2b\left(a^2-b^2\right)^2d(a+b\sec(c+dx))}$$

output
```
(a^2+2*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2
)/(a+b)^(5/2)/d-1/2*a^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^2+1/2*a*
(a^2-4*b^2)*tan(d*x+c)/b/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.76

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{-\frac{2(a^2+2b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{a(a^2-4b^2-3ab\cos(c+dx))\sin(c+dx)}{(a-b)^2(a+b)^2(b+a\cos(c+dx))^2}}{2d}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

output
```
((-2*(a^2 + 2*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^2 - b^2)^(5/2) + (a*(a^2 - 4*b^2 - 3*a*b*Cos[c + d*x])*Sin[c + d*x])/((a
 - b)^2*(a + b)^2*(b + a*Cos[c + d*x])^2))/(2*d)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.17, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4326, 25, 3042, 4491, 25, 27, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \; 4326$$

$$-\frac{\int -\frac{\sec(c+dx)\left(2ab+(a^2-2b^2)\sec(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\sec(c+dx)(2ab+(a^2-2b^2)\sec(c+dx))}{(a+b\sec(c+dx))^2}dx}{2b(a^2-b^2)}-\frac{a^2\tan(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2ab+(a^2-2b^2)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2b(a^2-b^2)}-\frac{a^2\tan(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{ 4491}$$

$$\frac{\frac{a(a^2-4b^2)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int -\frac{b(a^2+2b^2)\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}}{2b(a^2-b^2)}-\frac{a^2\tan(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{b(a^2+2b^2)\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}+\frac{a(a^2-4b^2)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2b(a^2-b^2)}-\frac{a^2\tan(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{b(a^2+2b^2)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}+\frac{a(a^2-4b^2)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2b(a^2-b^2)}-\frac{a^2\tan(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{b(a^2+2b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{a(a^2-4b^2)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2b(a^2-b^2)}-\frac{a^2\tan(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{ 4318}$$

$$\frac{\frac{(a^2+2b^2)\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a^2-b^2}+\frac{a(a^2-4b^2)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2b(a^2-b^2)}-\frac{a^2\tan(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{(a^2+2b^2)\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a^2-b^2}+\frac{a(a^2-4b^2)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2b(a^2-b^2)}-\frac{a^2\tan(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{ 3138}$$

$$\frac{\frac{2(a^2+2b^2)\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{d(a^2-b^2)}+\frac{a(a^2-4b^2)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2b(a^2+2b^2)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}+\frac{a(a^2-4b^2)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

input
```
Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

output
```
-1/2*(a^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((2*b*(
a^2 + 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a
- b]*Sqrt[a + b]*(a^2 - b^2)*d) + (a*(a^2 - 4*b^2)*Tan[c + d*x])/((a^2 - b
^2)*d*(a + b*Sec[c + d*x])))/(2*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4326
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-a^2)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m
 + 1)*(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]
*(a + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e
+ f*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

rule 4491
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1
/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp
[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; Free
Q[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m
, -1]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{(4b+a)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2(a-b)\left(a^{2}+2ab+b^{2}\right)}-\frac{(a-4b)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^{2}-2ab+b^{2}\right)}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b-a-b\right)^{2}}+\frac{\left(a^{2}+2b^{2}\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\left(a^{4}-2a^{2}b^{2}+b^{4}\right)\sqrt{(a+b)(a-b)}}}{d}$
default	$\frac{-\frac{2\left(-\frac{(4b+a)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2(a-b)\left(a^{2}+2ab+b^{2}\right)}-\frac{(a-4b)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^{2}-2ab+b^{2}\right)}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b-a-b\right)^{2}}+\frac{\left(a^{2}+2b^{2}\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\left(a^{4}-2a^{2}b^{2}+b^{4}\right)\sqrt{(a+b)(a-b)}}}{d}$
risch	$-\frac{i\left(a^{3}e^{3i(dx+c)}+2a\,b^{2}e^{3i(dx+c)}+3a^{2}b\,e^{2i(dx+c)}+6b^{3}e^{2i(dx+c)}-a^{3}e^{i(dx+c)}+10b^{2}a\,e^{i(dx+c)}+3a^{2}b\right)}{d\left(-a^{2}+b^{2}\right)^{2}\left(e^{2i(dx+c)}a+2b\,e^{i(dx+c)}+a\right)^{2}}+\frac{a^{2}\ln\left(e^{i(dx+c}\right.}{2\sqrt{a^{2}-}}$

input
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-1/2*(4*b+a)*a/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(a-
4*b)*a/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-t
an(1/2*d*x+1/2*c)^2*b-a-b)^2+(a^2+2*b^2)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))
^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 594, normalized size of antiderivative = 3.99

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \left[\frac{\left(a^{2}b^{2}+2\,b^{4}+\left(a^{4}+2\,a^{2}b^{2}\right)\cos\left(dx+c\right)^{2}+2\left(a^{3}b+2\,ab^{3}\right)\cos\left(dx+c\right)\right)\sqrt{a^{2}-b^{2}}\log\left(\frac{2\,ab\cos(dx+c)-(a^{2}-2}{}\right.}{4\left(\left(a^{8}-3\,a^{6}b^{2}+3\,a^{4}b^{4}-a^{2}b^{6}\right)d\cos\left(dx+c\right)^{2}+2\left(a^{7}b-\right.\right.}\right.$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/4*((a^2*b^2 + 2*b^4 + (a^4 + 2*a^2*b^2)*cos(d*x + c)^2 + 2*(a^3*b + 2*a
*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2
)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2
*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(a^5 - 5*
a^3*b^2 + 4*a*b^4 - 3*(a^4*b - a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^8
- 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3
 + 3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 -
b^8)*d), 1/2*((a^2*b^2 + 2*b^4 + (a^4 + 2*a^2*b^2)*cos(d*x + c)^2 + 2*(a^3
*b + 2*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*c
os(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (a^5 - 5*a^3*b^2 + 4*a*b^4
- 3*(a^4*b - a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^
4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b
^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d)]
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sec^3{(c+dx)}}{(a+b\sec{(c+dx)})^3}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.70

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx =$$
$$-\frac{\frac{\left(\pi\left\lfloor \frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)(a^2+2b^2)}{(a^4-2a^2b^2+b^4)\sqrt{-a^2+b^2}} - \frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-4ab^2\cdots}{(a^4-2a^2b^2+b^4)\cdots}}{d}$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x
 + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*(a^2 + 2*b^2)/((a^4
 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) - (a^3*tan(1/2*d*x + 1/2*c)^3 + 3*a^
2*b*tan(1/2*d*x + 1/2*c)^3 - 4*a*b^2*tan(1/2*d*x + 1/2*c)^3 + a^3*tan(1/2*
d*x + 1/2*c) - 3*a^2*b*tan(1/2*d*x + 1/2*c) - 4*a*b^2*tan(1/2*d*x + 1/2*c)
)/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2
*c)^2 - a - b)^2))/d
```

Mupad [B] (verification not implemented)

Time = 12.50 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.37

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(a^2+4ba)}{(a+b)^2(a-b)} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(4ab-a^2)}{(a+b)(a^2-2ab+b^2)}}{d\left(2ab-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(2a^2-2b^2)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(a^2-2ab+b^2)+a^2+b^2\right)} + \frac{\operatorname{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(2a-2b)(a^2-2ab+b^2)}{2\sqrt{a+b}\,(a-b)^{5/2}}\right)(a^2+2b^2)}{d\,(a+b)^{5/2}\,(a-b)^{5/2}}$$

input
```
int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^3),x)
```

output
```
((tan(c/2 + (d*x)/2)^3*(4*a*b + a^2))/((a + b)^2*(a - b)) - (tan(c/2 + (d*
x)/2)*(4*a*b - a^2))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(2*a*b - tan(c/2 +
(d*x)/2)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^
2 + b^2)) + (atanh((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a^2 - 2*a*b + b^2))/(2
*(a + b)^(1/2)*(a - b)^(5/2)))*(a^2 + 2*b^2))/(d*(a + b)^(5/2)*(a - b)^(5/
2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 626, normalized size of antiderivative = 4.20

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c))^3,x)
```

output

```
(4*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqr
t( - a**2 + b**2))*cos(c + d*x)*a**3*b + 8*sqrt( - a**2 + b**2)*atan((tan(
(c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*a*
b**3 - 2*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*
b)/sqrt( - a**2 + b**2))*sin(c + d*x)**2*a**4 - 4*sqrt( - a**2 + b**2)*ata
n((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*sin(c +
d*x)**2*a**2*b**2 + 2*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan(
(c + d*x)/2)*b)/sqrt( - a**2 + b**2))*a**4 + 6*sqrt( - a**2 + b**2)*atan((
tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*a**2*b**2 +
 4*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqr
t( - a**2 + b**2))*b**4 - 3*cos(c + d*x)*sin(c + d*x)*a**4*b + 3*cos(c + d
*x)*sin(c + d*x)*a**2*b**3 + sin(c + d*x)*a**5 - 5*sin(c + d*x)*a**3*b**2
+ 4*sin(c + d*x)*a*b**4)/(2*d*(2*cos(c + d*x)*a**7*b - 6*cos(c + d*x)*a**5
*b**3 + 6*cos(c + d*x)*a**3*b**5 - 2*cos(c + d*x)*a*b**7 - sin(c + d*x)**2
*a**8 + 3*sin(c + d*x)**2*a**6*b**2 - 3*sin(c + d*x)**2*a**4*b**4 + sin(c
+ d*x)**2*a**2*b**6 + a**8 - 2*a**6*b**2 + 2*a**2*b**6 - b**8))
```

3.509 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 134

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{3ab\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{a\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(a^2+2b^2\right)\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))}$$

output

```
-3*a*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/(a+
b)^(5/2)/d+1/2*a*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^2+1/2*(a^2+2*b^2)
*tan(d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.86

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\frac{6ab\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{(b(a^2+2b^2)+a(2a^2+b^2)\cos(c+dx))\sin(c+dx)}{(b+a\cos(c+dx))^2}}{2(a-b)^2(a+b)^2d}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

output
```
((6*a*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b
^2] + ((b*(a^2 + 2*b^2) + a*(2*a^2 + b^2)*Cos[c + d*x])*Sin[c + d*x])/(b +
 a*Cos[c + d*x])^2)/(2*(a - b)^2*(a + b)^2*d)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.19, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {3042, 4323, 25, 3042, 4491, 27, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{ 4323}$$

$$\frac{\int -\frac{\sec(c+dx)(2b-a\sec(c+dx))}{(a+b\sec(c+dx))^2}dx}{2\left(a^2-b^2\right)}+\frac{a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\int\frac{\sec(c+dx)(2b-a\sec(c+dx))}{(a+b\sec(c+dx))^2}dx}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2b-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 4491$$

$$\frac{a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{-\frac{\int-\frac{3ab\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}-\frac{\left(a^2+2b^2\right)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{3ab\int\frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}-\frac{\left(a^2+2b^2\right)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{3ab\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{\left(a^2+2b^2\right)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 4318$$

$$\frac{a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{3a\int\frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a^2-b^2}-\frac{\left(a^2+2b^2\right)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{3a\int\frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a^2-b^2}-\frac{\left(a^2+2b^2\right)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3138$$

$$\frac{a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{6a\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{d(a^2-b^2)}-\frac{\left(a^2+2b^2\right)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 221$$

$$\frac{a\tan(c+dx)}{2d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}-\frac{\frac{6ab\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}-\frac{(a^2+2b^2)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

output
```
(a*Tan[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) - ((6*a*b*ArcTan
h[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*(a
^2 - b^2)*d) - ((a^2 + 2*b^2)*Tan[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c +
d*x])))/(2*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4323
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[a*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(
a^2 - b^2))), x] - Simp[1/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

rule 4491
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1
/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp
[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; Free
Q[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m
, -1]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.46

method	result
derivativedivides	$\frac{\frac{-\frac{(2a^2+ab+2b^2)\tan(\frac{dx}{2}+\frac{c}{2})^3}{(a-b)(a^2+2ab+b^2)}+\frac{(2a^2-ab+2b^2)\tan(\frac{dx}{2}+\frac{c}{2})}{(a+b)(a^2-2ab+b^2)}}{\left(\tan(\frac{dx}{2}+\frac{c}{2})^2a-\tan(\frac{dx}{2}+\frac{c}{2})^2b-a-b\right)^2}-\frac{3ab\,\mathrm{arctanh}\left(\frac{(a-b)\tan(\frac{dx}{2}+\frac{c}{2})}{\sqrt{(a+b)(a-b)}}\right)}{(a^4-2a^2b^2+b^4)\sqrt{(a+b)(a-b)}}}{d}$
default	$\frac{\frac{-\frac{(2a^2+ab+2b^2)\tan(\frac{dx}{2}+\frac{c}{2})^3}{(a-b)(a^2+2ab+b^2)}+\frac{(2a^2-ab+2b^2)\tan(\frac{dx}{2}+\frac{c}{2})}{(a+b)(a^2-2ab+b^2)}}{\left(\tan(\frac{dx}{2}+\frac{c}{2})^2a-\tan(\frac{dx}{2}+\frac{c}{2})^2b-a-b\right)^2}-\frac{3ab\,\mathrm{arctanh}\left(\frac{(a-b)\tan(\frac{dx}{2}+\frac{c}{2})}{\sqrt{(a+b)(a-b)}}\right)}{(a^4-2a^2b^2+b^4)\sqrt{(a+b)(a-b)}}}{d}$
risch	$\frac{i(3ba^3e^{3i(dx+c)}+2a^4e^{2i(dx+c)}+5a^2b^2e^{2i(dx+c)}+2b^4e^{2i(dx+c)}+5a^3b\,e^{i(dx+c)}+4b^3a\,e^{i(dx+c)}+2a^4+a^2b^2)}{a(-a^2+b^2)^2d(e^{2i(dx+c)}a+2b\,e^{i(dx+c)}+a)^2}+\frac{3ab\ln(}{2}$

input
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(-1/2*(2*a^2+a*b+2*b^2)/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+
1/2*(2*a^2-a*b+2*b^2)/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/2*d
*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2-3*a*b/(a^4-2*a^2*b^2+b^4)/((a+
b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(121) = 242$.

Time = 0.12 (sec) , antiderivative size = 565, normalized size of antiderivative = 4.22

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$= \left[\frac{3\left(a^3b\cos(dx+c)^2+2a^2b^2\cos(dx+c)+ab^3\right)\sqrt{a^2-b^2}\log\left(\frac{2ab\cos(dx+c)-(a^2-2b^2)\cos(dx+c)^2-2\sqrt{a^2-b^2}(b\cos}{a^2\cos(dx+c)^2+2ab\cos(dx+}\right.}{4\left((a^8-3a^6b^2+3a^4b^4-a^2b^6)d\cos(dx+c)^2+2(a^7b-3a^5b^3+3\right.}\right.$$

$$-\frac{3\left(a^3b\cos(dx+c)^2+2a^2b^2\cos(dx+c)+ab^3\right)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)-(a^4b+a}{2\left((a^8-3a^6b^2+3a^4b^4-a^2b^6)d\cos(dx+c)^2+2(a^7b-3a^5b^3+3a^3b^5-ab^7)d\cos(\right.}$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/4*(3*(a^3*b*cos(d*x + c)^2 + 2*a^2*b^2*cos(d*x + c) + a*b^3)*sqrt(a^2 -
 b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2
- b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^
2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(a^4*b + a^2*b^3 - 2*b^5 + (2*a^5 - a^3
*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4*b^4 -
a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*co
s(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d), -1/2*(3*(a^3*b*co
s(d*x + c)^2 + 2*a^2*b^2*cos(d*x + c) + a*b^3)*sqrt(-a^2 + b^2)*arctan(-sq
rt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (a^4*b +
 a^2*b^3 - 2*b^5 + (2*a^5 - a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/(
(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^
5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b
^6 - b^8)*d)]
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(121) = 242$.

Time = 0.19 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.07

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)ab}{(a^4-2a^2b^2+b^4)\sqrt{-a^2+b^2}} - \frac{2a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+ab^2\tan\left(\frac{1}{2}dx+\right.}{(a^4-2}}{d}$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
(3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*
x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a*b/((a^4 - 2*a^2*
b^2 + b^4)*sqrt(-a^2 + b^2)) - (2*a^3*tan(1/2*d*x + 1/2*c)^3 - a^2*b*tan(1
/2*d*x + 1/2*c)^3 + a*b^2*tan(1/2*d*x + 1/2*c)^3 - 2*b^3*tan(1/2*d*x + 1/2
*c)^3 - 2*a^3*tan(1/2*d*x + 1/2*c) - a^2*b*tan(1/2*d*x + 1/2*c) - a*b^2*ta
n(1/2*d*x + 1/2*c) - 2*b^3*tan(1/2*d*x + 1/2*c))/((a^4 - 2*a^2*b^2 + b^4)*
(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2))/d
```

Mupad [B] (verification not implemented)

Time = 12.58 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.57

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(2a^2+ab+2b^2\right)}{(a+b)^2\,(a-b)}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(2a^2-ab+2b^2\right)}{(a+b)\,(a^2-2ab+b^2)}}{d\left(2ab-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(2a^2-2b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(a^2-2ab+b^2\right)+a^2+b^2\right)} - \frac{3ab\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(2a-2b)\left(a^2-2ab+b^2\right)}{2\sqrt{a+b}\,(a-b)^{5/2}}\right)}{d\,(a+b)^{5/2}\,(a-b)^{5/2}}$$

input
```
int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^3),x)
```

output
```
- ((tan(c/2 + (d*x)/2)^3*(a*b + 2*a^2 + 2*b^2))/((a + b)^2*(a - b)) - (tan
(c/2 + (d*x)/2)*(2*a^2 - a*b + 2*b^2))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(
2*a*b - tan(c/2 + (d*x)/2)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 -
 2*a*b + b^2) + a^2 + b^2)) - (3*a*b*atanh((tan(c/2 + (d*x)/2)*(2*a - 2*b)
*(a^2 - 2*a*b + b^2))/(2*(a + b)^(1/2)*(a - b)^(5/2))))/(d*(a + b)^(5/2)*(
a - b)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 467, normalized size of antiderivative = 3.49

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{-12\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{-a^2+b^2}}\right)\cos\left(dx+c\right)a^2b^2+6\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{\sqrt{-a^2+b^2}}\right.}{2d\left(2\cos\left(dx-\right.\right.}$$

input
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c))^3,x)
```

output
```
( - 12*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)
/sqrt( - a**2 + b**2))*cos(c + d*x)*a**2*b**2 + 6*sqrt( - a**2 + b**2)*ata
n((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*sin(c +
d*x)**2*a**3*b - 6*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c
+ d*x)/2)*b)/sqrt( - a**2 + b**2))*a**3*b - 6*sqrt( - a**2 + b**2)*atan((t
an((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*a*b**3 + 2*c
os(c + d*x)*sin(c + d*x)*a**5 - cos(c + d*x)*sin(c + d*x)*a**3*b**2 - cos(
c + d*x)*sin(c + d*x)*a*b**4 + sin(c + d*x)*a**4*b + sin(c + d*x)*a**2*b**
3 - 2*sin(c + d*x)*b**5)/(2*d*(2*cos(c + d*x)*a**7*b - 6*cos(c + d*x)*a**5
*b**3 + 6*cos(c + d*x)*a**3*b**5 - 2*cos(c + d*x)*a*b**7 - sin(c + d*x)**2
*a**8 + 3*sin(c + d*x)**2*a**6*b**2 - 3*sin(c + d*x)**2*a**4*b**4 + sin(c
+ d*x)**2*a**2*b**6 + a**8 - 2*a**6*b**2 + 2*a**2*b**6 - b**8))
```

3.510 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 133

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{(2a^2+b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{b\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{3ab\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))}$$

output
```
(2*a^2+b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2
)/(a+b)^(5/2)/d-1/2*b*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^2-3/2*a*b*ta
n(d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.86

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{-\frac{2(2a^2+b^2)\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{b(-3ab+(-4a^2+b^2)\cos(c+dx))\sin(c+dx)}{(b+a\cos(c+dx))^2}}{2(a-b)^2(a+b)^2d}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

output
```
((-2*(2*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/S
qrt[a^2 - b^2] + (b*(-3*a*b + (-4*a^2 + b^2)*Cos[c + d*x])*Sin[c + d*x])/(
b + a*Cos[c + d*x])^2)/(2*(a - b)^2*(a + b)^2*d)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.20, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {3042, 4320, 25, 3042, 4491, 25, 27, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{ 4320}$$

$$-\frac{\int -\frac{\sec(c+dx)(2a-b\sec(c+dx))}{(a+b\sec(c+dx))^2}dx}{2(a^2-b^2)}-\frac{b\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\sec(c+dx)(2a-b\sec(c+dx))}{(a+b\sec(c+dx))^2}dx}{2(a^2-b^2)}-\frac{b\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2a-b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2(a^2-b^2)}-\frac{b\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{ 4491}$$

$$\frac{-\frac{\int -\frac{(2a^2+b^2)\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}-\frac{3ab\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{b\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{(2a^2+b^2)\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}-\frac{3ab\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{b\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(2a^2+b^2)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}-\frac{3ab\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{b\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(2a^2+b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{3ab\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{b\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{4318}$$

$$\frac{\frac{(2a^2+b^2)\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b(a^2-b^2)}-\frac{3ab\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{b\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(2a^2+b^2)\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b(a^2-b^2)}-\frac{3ab\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{b\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{2(2a^2+b^2)\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd(a^2-b^2)}-\frac{3ab\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{b\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2(2a^2+b^2)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}-\frac{3ab\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{b\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

input
```
Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

output
```
-1/2*(b*Tan[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((2*(2*a^2
+ b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*S
qrt[a + b]*(a^2 - b^2)*d) - (3*a*b*Tan[c + d*x])/((a^2 - b^2)*d*(a + b*Sec
[c + d*x])))/(2*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4320
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*
Csc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ
[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 4491
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1
/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp
[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; Free
Q[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m
, -1]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{(4a+b)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2(a-b)\left(a^{2}+2ab+b^{2}\right)}+\frac{(4a-b)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^{2}-2ab+b^{2}\right)}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b-a-b\right)^{2}}+\frac{\left(2a^{2}+b^{2}\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\left(a^{4}-2a^{2}b^{2}+b^{4}\right)\sqrt{(a+b)(a-b)}}}{d}$
default	$\frac{-\frac{2\left(-\frac{(4a+b)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2(a-b)\left(a^{2}+2ab+b^{2}\right)}+\frac{(4a-b)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^{2}-2ab+b^{2}\right)}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b-a-b\right)^{2}}+\frac{\left(2a^{2}+b^{2}\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\left(a^{4}-2a^{2}b^{2}+b^{4}\right)\sqrt{(a+b)(a-b)}}}{d}$
risch	$\frac{ib\left(-5b\,a^{3}\mathrm{e}^{3i(dx+c)}+2a\,b^{3}\mathrm{e}^{3i(dx+c)}-4a^{4}\mathrm{e}^{2i(dx+c)}-7a^{2}b^{2}\mathrm{e}^{2i(dx+c)}+2b^{4}\mathrm{e}^{2i(dx+c)}-11a^{3}b\,\mathrm{e}^{i(dx+c)}+2b^{3}a\,\mathrm{e}^{i(dx+c)}-4a^{4}\right.}{a^{2}\left(-a^{2}+b^{2}\right)^{2}d\left(\mathrm{e}^{2i(dx+c)}a+2b\,\mathrm{e}^{i(dx+c)}+a\right)^{2}}$

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-1/2*(4*a+b)*b/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/2*(4*
a-b)*b/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-t
an(1/2*d*x+1/2*c)^2*b-a-b)^2+(2*a^2+b^2)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))
^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(120) = 240$.

Time = 0.12 (sec) , antiderivative size = 595, normalized size of antiderivative = 4.47

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \left[\frac{\left(2\,a^2b^2 + b^4 + \left(2\,a^4 + a^2b^2\right)\cos\left(dx+c\right)^2 + 2\left(2\,a^3b + ab^3\right)\cos\left(dx+c\right)\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c) - (a^2-2}{}\right.}{4\left(\left(a^8 - 3\,a^6b^2 + 3\,a^4b^4 - a^2b^6\right)d\cos\left(dx+c\right)^2 + 2\left(a^7b - \right.\right.}\right.$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/4*((2*a^2*b^2 + b^4 + (2*a^4 + a^2*b^2)*cos(d*x + c)^2 + 2*(2*a^3*b + a
*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2
)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2
*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(3*a^3*b^
2 - 3*a*b^4 + (4*a^4*b - 5*a^2*b^3 + b^5)*cos(d*x + c))*sin(d*x + c))/((a^
8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b
^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6
- b^8)*d), 1/2*((2*a^2*b^2 + b^4 + (2*a^4 + a^2*b^2)*cos(d*x + c)^2 + 2*(2
*a^3*b + a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b
*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (3*a^3*b^2 - 3*a*b^4 + (4
*a^4*b - 5*a^2*b^3 + b^5)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a^6*b^2 +
3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 -
 a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d)]
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sec\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)/(a + b*sec(c + d*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(120) = 240$.

Time = 0.19 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.91

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)(2a^2+b^2)}{(a^4-2a^2b^2+b^4)\sqrt{-a^2+b^2}} - \frac{4a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-b^3\,\mathrm{t}}{(a^4-2a^2b^2+b^4)(}}{d}$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x
 + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*(2*a^2 + b^2)/((a^4
 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) - (4*a^2*b*tan(1/2*d*x + 1/2*c)^3 -
3*a*b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 4*a^2*b*tan(
1/2*d*x + 1/2*c) - 3*a*b^2*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*x + 1/2*c)
)/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2
*c)^2 - a - b)^2))/d
```

Mupad [B] (verification not implemented)

Time = 12.43 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.53

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(b^2+4\,a\,b)}{(a+b)^2\,(a-b)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(4\,a\,b-b^2)}{(a+b)\,(a^2-2\,a\,b+b^2)}}{d\,\left(2\,a\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(2\,a^2-2\,b^2)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,(a^2-2\,a\,b+b^2)+a^2+b^2\right)} + \frac{\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(2\,a-2\,b)\,(a^2-2\,a\,b+b^2)}{2\,\sqrt{a+b}\,(a-b)^{5/2}}\right)\,(2\,a^2+b^2)}{d\,(a+b)^{5/2}\,(a-b)^{5/2}}$$

input
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^3),x)
```

output
```
((tan(c/2 + (d*x)/2)^3*(4*a*b + b^2))/((a + b)^2*(a - b)) - (tan(c/2 + (d*
x)/2)*(4*a*b - b^2))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(2*a*b - tan(c/2 +
(d*x)/2)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^
2 + b^2)) + (atanh((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a^2 - 2*a*b + b^2))/(2
*(a + b)^(1/2)*(a - b)^(5/2)))*(2*a^2 + b^2))/(d*(a + b)^(5/2)*(a - b)^(5/
2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 633, normalized size of antiderivative = 4.76

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^3,x)
```

output
```
(8*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqr
t( - a**2 + b**2))*cos(c + d*x)*a**3*b + 4*sqrt( - a**2 + b**2)*atan((tan(
(c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*a*
b**3 - 4*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*
b)/sqrt( - a**2 + b**2))*sin(c + d*x)**2*a**4 - 2*sqrt( - a**2 + b**2)*ata
n((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*sin(c +
d*x)**2*a**2*b**2 + 4*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan(
(c + d*x)/2)*b)/sqrt( - a**2 + b**2))*a**4 + 6*sqrt( - a**2 + b**2)*atan((
tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*a**2*b**2 +
 2*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqr
t( - a**2 + b**2))*b**4 - 4*cos(c + d*x)*sin(c + d*x)*a**4*b + 5*cos(c + d
*x)*sin(c + d*x)*a**2*b**3 - cos(c + d*x)*sin(c + d*x)*b**5 - 3*sin(c + d*
x)*a**3*b**2 + 3*sin(c + d*x)*a*b**4)/(2*d*(2*cos(c + d*x)*a**7*b - 6*cos(
c + d*x)*a**5*b**3 + 6*cos(c + d*x)*a**3*b**5 - 2*cos(c + d*x)*a*b**7 - si
n(c + d*x)**2*a**8 + 3*sin(c + d*x)**2*a**6*b**2 - 3*sin(c + d*x)**2*a**4*
b**4 + sin(c + d*x)**2*a**2*b**6 + a**8 - 2*a**6*b**2 + 2*a**2*b**6 - b**8
))
```

3.511 $\int \frac{1}{(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 173

$$\int \frac{1}{(a+b\sec(c+dx))^3}\,dx = \frac{x}{a^3} - \frac{b(6a^4-5a^2b^2+2b^4)\,\mathrm{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} + \frac{b^2\tan(c+dx)}{2a\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{b^2(5a^2-2b^2)\tan(c+dx)}{2a^2\,(a^2-b^2)^2\,d(a+b\sec(c+dx))}$$

output
```
x/a^3-b*(6*a^4-5*a^2*b^2+2*b^4)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+
b)^(1/2))/a^3/(a-b)^(5/2)/(a+b)^(5/2)/d+1/2*b^2*tan(d*x+c)/a/(a^2-b^2)/d/(
a+b*sec(d*x+c))^2+1/2*b^2*(5*a^2-2*b^2)*tan(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*
sec(d*x+c))
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.18

$$\int \frac{1}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{(b+a\cos(c+dx))\sec^3(c+dx)\left(2(c+dx)(b+a\cos(c+dx))^2+\frac{2b(6a^4-5a^2b^2+2b^4)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+d}{\sqrt{a^2-b^2}}\right.}{(a^2-b^2)^{5/2}}\right.}{2a^3d(a+b\sec(c+dx))^3}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(-3),x]
```

output
```
((b + a*Cos[c + d*x])*Sec[c + d*x]^3*(2*(c + d*x)*(b + a*Cos[c + d*x])^2 +
 (2*b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt
[a^2 - b^2]]*(b + a*Cos[c + d*x])^2)/(a^2 - b^2)^(5/2) + (a*b^3*Sin[c + d*
x])/((-a + b)*(a + b)) + (3*a*b^2*(2*a^2 - b^2)*(b + a*Cos[c + d*x])*Sin[c
 + d*x])/((a - b)^2*(a + b)^2)))/(2*a^3*d*(a + b*Sec[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.25, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 4272, 25, 3042, 4548, 25, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec(c+dx))^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$
$$\Big\downarrow \text{4272}$$

$$\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\int -\frac{b^2\sec^2(c+dx)-2ab\sec(c+dx)+2\left(a^2-b^2\right)}{(a+b\sec(c+dx))^2}dx}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\int \frac{b^2\sec^2(c+dx)-2ab\sec(c+dx)+2\left(a^2-b^2\right)}{(a+b\sec(c+dx))^2}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\int \frac{b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2-2ab\csc\left(c+dx+\frac{\pi}{2}\right)+2\left(a^2-b^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \textbf{4548}$$

$$\frac{\frac{b^2\left(5a^2-2b^2\right)\tan(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int -\frac{2\left(a^2-b^2\right)^2-ab\left(4a^2-b^2\right)\sec(c+dx)}{a+b\sec(c+dx)}dx}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\frac{\int \frac{2\left(a^2-b^2\right)^2-ab\left(4a^2-b^2\right)\sec(c+dx)}{a+b\sec(c+dx)}dx}{a(a^2-b^2)}+\frac{b^2\left(5a^2-2b^2\right)\tan(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{\int \frac{2\left(a^2-b^2\right)^2-ab\left(4a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a(a^2-b^2)}+\frac{b^2\left(5a^2-2b^2\right)\tan(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \textbf{4407}$$

$$\frac{\frac{\frac{2x\left(a^2-b^2\right)^2}{a}-\frac{b\left(6a^4-5a^2b^2+2b^4\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}}{a(a^2-b^2)}+\frac{b^2\left(5a^2-2b^2\right)\tan(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{\frac{2x\left(a^2-b^2\right)^2}{a}-\frac{b\left(6a^4-5a^2b^2+2b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{a(a^2-b^2)}+\frac{b^2\left(5a^2-2b^2\right)\tan(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{4318}$$

$$\frac{\frac{\frac{2x(a^2-b^2)^2}{a}-\frac{(6a^4-5a^2b^2+2b^4)\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a}}{a(a^2-b^2)}+\frac{b^2(5a^2-2b^2)\tan(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+\frac{b^2\tan(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{2x(a^2-b^2)^2}{a}-\frac{(6a^4-5a^2b^2+2b^4)\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a}}{a(a^2-b^2)}+\frac{b^2(5a^2-2b^2)\tan(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+\frac{b^2\tan(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{\frac{2x(a^2-b^2)^2}{a}-\frac{2(6a^4-5a^2b^2+2b^4)\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}}{a(a^2-b^2)}+\frac{b^2(5a^2-2b^2)\tan(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+\frac{b^2\tan(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{b^2\tan(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}+\frac{\frac{b^2(5a^2-2b^2)\tan(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}+\frac{\frac{2x(a^2-b^2)^2}{a}-\frac{2b(6a^4-5a^2b^2+2b^4)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a(a^2-b^2)}}{2a(a^2-b^2)}$$

input

```
Int[(a + b*Sec[c + d*x])^(-3),x]
```

output

```
(b^2*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (((2*(a^2
- b^2)^2*x)/a - (2*b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d))/(a*(a^2 - b^2))
 + (b^2*(5*a^2 - 2*b^2)*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]
)))/(2*a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.37

method	result
derivativedivides	$\frac{\frac{2b\left(\frac{-\frac{(6a^2+ab-2b^2)ab\tan(\frac{dx}{2}+\frac{c}{2})^3}{2(a-b)(a^2+2ab+b^2)}+\frac{(6a^2-ab-2b^2)ab\tan(\frac{dx}{2}+\frac{c}{2})}{2(a+b)(a^2-2ab+b^2)}}{\left(\tan(\frac{dx}{2}+\frac{c}{2})^2a-\tan(\frac{dx}{2}+\frac{c}{2})^2b-a-b\right)^2}-\frac{(6a^4-5a^2b^2+2b^4)\operatorname{arctanh}\left(\frac{(a-b)\tan(\frac{dx}{2}+\frac{c}{2})}{\sqrt{(a+b)(a-b)}}\right)}{2(a^4-2a^2b^2+b^4)\sqrt{(a+b)(a-b)}}\right)}{a^3}+}{d}$
default	$\frac{\frac{2b\left(\frac{-\frac{(6a^2+ab-2b^2)ab\tan(\frac{dx}{2}+\frac{c}{2})^3}{2(a-b)(a^2+2ab+b^2)}+\frac{(6a^2-ab-2b^2)ab\tan(\frac{dx}{2}+\frac{c}{2})}{2(a+b)(a^2-2ab+b^2)}}{\left(\tan(\frac{dx}{2}+\frac{c}{2})^2a-\tan(\frac{dx}{2}+\frac{c}{2})^2b-a-b\right)^2}-\frac{(6a^4-5a^2b^2+2b^4)\operatorname{arctanh}\left(\frac{(a-b)\tan(\frac{dx}{2}+\frac{c}{2})}{\sqrt{(a+b)(a-b)}}\right)}{2(a^4-2a^2b^2+b^4)\sqrt{(a+b)(a-b)}}\right)}{a^3}+}{d}$
risch	$\frac{x}{a^3}-\frac{ib^2(-7ba^3e^{3i(dx+c)}+4ab^3e^{3i(dx+c)}-6a^4e^{2i(dx+c)}-9a^2b^2e^{2i(dx+c)}+6b^4e^{2i(dx+c)}-17a^3be^{i(dx+c)}+8b^3ae^{i(dx}}{a^3(-a^2+b^2)^2d(e^{2i(dx+c)}a+2be^{i(dx+c)}+a)^2}$

input
```
int(1/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/a^3*b*((-1/2*(6*a^2+a*b-2*b^2)*a*b/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*
x+1/2*c)^3+1/2*(6*a^2-a*b-2*b^2)*a*b/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2
*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2-1/2*(6*a^4-5*a^
2*b^2+2*b^4)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2
*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))+2/a^3*arctan(tan(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(160) = 320$.

Time = 0.14 (sec) , antiderivative size = 919, normalized size of antiderivative = 5.31

$$\int \frac{1}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/4*(4*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x*cos(d*x + c)^2 + 8*(a^
7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x*cos(d*x + c) + 4*(a^6*b^2 - 3*a^4
*b^4 + 3*a^2*b^6 - b^8)*d*x + (6*a^4*b^3 - 5*a^2*b^5 + 2*b^7 + (6*a^6*b -
5*a^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^2 + 2*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6
)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*co
s(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2
 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(5*a^5*b^3 -
7*a^3*b^5 + 2*a*b^7 + 3*(2*a^6*b^2 - 3*a^4*b^4 + a^2*b^6)*cos(d*x + c))*si
n(d*x + c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cos(d*x + c)^2 + 2
*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cos(d*x + c) + (a^9*b^2 - 3*
a^7*b^4 + 3*a^5*b^6 - a^3*b^8)*d), 1/2*(2*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a
^2*b^6)*d*x*cos(d*x + c)^2 + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x
*cos(d*x + c) + 2*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*x - (6*a^4*b^3
 - 5*a^2*b^5 + 2*b^7 + (6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^2 +
2*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(
-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (5*a^
5*b^3 - 7*a^3*b^5 + 2*a*b^7 + 3*(2*a^6*b^2 - 3*a^4*b^4 + a^2*b^6)*cos(d*x
+ c))*sin(d*x + c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cos(d*x +
c)^2 + 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cos(d*x + c) + (a^9*
b^2 - 3*a^7*b^4 + 3*a^5*b^6 - a^3*b^8)*d)]
```

Sympy [F]

$$\int \frac{1}{(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{(a+b\sec{(c+dx)})^3}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral((a + b*sec(c + d*x))**(-3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\sec(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(160) = 320.

Time = 0.14 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.86

$$\int \frac{1}{(a+b\sec(c+dx))^3}\,dx = \frac{\frac{(6a^4b-5a^2b^3+2b^5)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^7-2a^5b^2+a^3b^4)\sqrt{-a^2+b^2}} - \frac{6a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5a^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}{}}{}$$

input
```
integrate(1/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
((6*a^4*b - 5*a^2*b^3 + 2*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a -
 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2
 + b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(-a^2 + b^2)) - (6*a^3*b^2*tan(
1/2*d*x + 1/2*c)^3 - 5*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 3*a*b^4*tan(1/2*d*
x + 1/2*c)^3 + 2*b^5*tan(1/2*d*x + 1/2*c)^3 - 6*a^3*b^2*tan(1/2*d*x + 1/2*
c) - 5*a^2*b^3*tan(1/2*d*x + 1/2*c) + 3*a*b^4*tan(1/2*d*x + 1/2*c) + 2*b^5
*tan(1/2*d*x + 1/2*c))/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*d*x + 1/2*c
)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) + (d*x + c)/a^3)/d
```

Mupad [B] (verification not implemented)

Time = 18.31 (sec) , antiderivative size = 5090, normalized size of antiderivative = 29.42

$$\int \frac{1}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b/cos(c + d*x))^3,x)
```

output
```
(2*atan((((((8*(12*a^14*b - 4*a^15 + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 +
4*a^9*b^6 + 36*a^10*b^5 - 6*a^11*b^4 - 34*a^12*b^3 + 8*a^13*b^2))/(a^12*b
+ a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b
^2) - (tan(c/2 + (d*x)/2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8
- 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a
^14*b^2)*8i)/(a^3*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b
^4 - 3*a^8*b^3 - 3*a^9*b^2)))*1i)/a^3 + (8*tan(c/2 + (d*x)/2)*(4*a^10 - 8*
a^9*b - 8*a*b^9 + 8*b^10 - 32*a^2*b^8 + 32*a^3*b^7 + 57*a^4*b^6 - 48*a^5*b
^5 - 52*a^6*b^4 + 32*a^7*b^3 + 24*a^8*b^2))/(a^10*b + a^11 - a^4*b^7 - a^5
*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2))/a^3 - ((((8*(12*a^1
4*b - 4*a^15 + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 + 4*a^9*b^6 + 36*a^10*b^
5 - 6*a^11*b^4 - 34*a^12*b^3 + 8*a^13*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7
*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (tan(c/2 + (d*x)
/2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10
*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b^2)*8i)/(a^3*(a^1
0*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9
*b^2)))*1i)/a^3 - (8*tan(c/2 + (d*x)/2)*(4*a^10 - 8*a^9*b - 8*a*b^9 + 8*b^
10 - 32*a^2*b^8 + 32*a^3*b^7 + 57*a^4*b^6 - 48*a^5*b^5 - 52*a^6*b^4 + 32*a
^7*b^3 + 24*a^8*b^2))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a
^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2))/a^3)/((((((8*(12*a^14*b - 4*a^15 + 4*a...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1004, normalized size of antiderivative = 5.80

$$\int \frac{1}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b*sec(d*x+c))^3,x)
```

output
```
( - 24*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)
/sqrt( - a**2 + b**2))*cos(c + d*x)*a**5*b**2 + 20*sqrt( - a**2 + b**2)*at
an((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c +
 d*x)*a**3*b**4 - 8*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c
 + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*a*b**6 + 12*sqrt( - a**2
+ b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2
))*sin(c + d*x)**2*a**6*b - 10*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)
*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*sin(c + d*x)**2*a**4*b**3 +
 4*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqr
t( - a**2 + b**2))*sin(c + d*x)**2*a**2*b**5 - 12*sqrt( - a**2 + b**2)*ata
n((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*a**6*b -
 2*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqr
t( - a**2 + b**2))*a**4*b**3 + 6*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/
2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*a**2*b**5 - 4*sqrt( - a**
2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b*
*2))*b**7 + 6*cos(c + d*x)*sin(c + d*x)*a**6*b**2 - 9*cos(c + d*x)*sin(c +
 d*x)*a**4*b**4 + 3*cos(c + d*x)*sin(c + d*x)*a**2*b**6 + 4*cos(c + d*x)*a
**7*b*d*x - 12*cos(c + d*x)*a**5*b**3*d*x + 12*cos(c + d*x)*a**3*b**5*d*x
- 4*cos(c + d*x)*a*b**7*d*x - 2*sin(c + d*x)**2*a**8*d*x + 6*sin(c + d*x)*
*2*a**6*b**2*d*x - 6*sin(c + d*x)**2*a**4*b**4*d*x + 2*sin(c + d*x)**2*...
```

3.512 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 223

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{3bx}{a^4}+\frac{3b^2(4a^4-5a^2b^2+2b^4)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{5/2}(a+b)^{5/2}d}+\frac{(2a^4-11a^2b^2+6b^4)\sin(c+dx)}{2a^3(a^2-b^2)^2d}+\frac{b^2\sin(c+dx)}{2a(a^2-b^2)\,d(a+b\sec(c+dx))^2}+\frac{3b^2(2a^2-b^2)\sin(c+dx)}{2a^2(a^2-b^2)^2\,d(a+b\sec(c+dx))}$$

output
```
-3*b*x/a^4+3*b^2*(4*a^4-5*a^2*b^2+2*b^4)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1
/2*c)/(a+b)^(1/2))/a^4/(a-b)^(5/2)/(a+b)^(5/2)/d+1/2*(2*a^4-11*a^2*b^2+6*b
^4)*sin(d*x+c)/a^3/(a^2-b^2)^2/d+1/2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec
(d*x+c))^2+3/2*b^2*(2*a^2-b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c
))
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.03

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$= \frac{(b+a\cos(c+dx))\sec^3(c+dx)\left(-6b(c+dx)(b+a\cos(c+dx))^2 - \frac{6b^2(4a^4-5a^2b^2+2b^4)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{\cdots}{2}\right)}{\sqrt{a^2-\cdots}}\right)}{(a^2-b^2)^{5/2}} \cdots\right.}{2a^4d(a+b\cdots}$$

input
```
Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

output
```
((b + a*Cos[c + d*x])*Sec[c + d*x]^3*(-6*b*(c + d*x)*(b + a*Cos[c + d*x])^
2 - (6*b^2*(4*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])
/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2)/(a^2 - b^2)^(5/2) + (a*b^4*Sin[c
 + d*x])/((a - b)*(a + b)) - (a*b^3*(8*a^2 - 5*b^2)*(b + a*Cos[c + d*x])*S
in[c + d*x])/((a - b)^2*(a + b)^2) + 2*a*(b + a*Cos[c + d*x])^2*Sin[c + d*
x]))/(2*a^4*d*(a + b*Sec[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.16, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.842$, Rules used = {3042, 4334, 25, 3042, 4588, 25, 3042, 4592, 27, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4334}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\int -\frac{\cos(c+dx)\left(2a^2-2b\sec(c+dx)a-3b^2+2b^2\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\cos(c+dx)\left(2a^2-2b\sec(c+dx)a-3b^2+2b^2\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2a^2-2b\csc\left(c+dx+\frac{\pi}{2}\right)a-3b^2+2b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{4588}$$

$$\frac{\frac{3b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int -\frac{\cos(c+dx)\left(2a^4-11b^2a^2-b\left(4a^2-b^2\right)\sec(c+dx)a+6b^4+3b^2\left(2a^2-b^2\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\cos(c+dx)\left(2a^4-11b^2a^2-b\left(4a^2-b^2\right)\sec(c+dx)a+6b^4+3b^2\left(2a^2-b^2\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{a(a^2-b^2)}+\frac{3b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{2a^4-11b^2a^2-b\left(4a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+6b^4+3b^2\left(2a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a(a^2-b^2)}+\frac{3b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{4592}$$

$$\frac{\frac{\frac{\left(2a^4-11a^2b^2+6b^4\right)\sin(c+dx)}{ad}-\frac{\int \frac{3\left(2b\left(a^2-b^2\right)^2-ab^2\left(2a^2-b^2\right)\sec(c+dx)\right)}{a+b\sec(c+dx)}dx}{a}}{a(a^2-b^2)}+\frac{3b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{\left(2a^4-11a^2b^2+6b^4\right)\sin(c+dx)}{ad}-\frac{3\int \frac{2b\left(a^2-b^2\right)^2-ab^2\left(2a^2-b^2\right)\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}}{a(a^2-b^2)}+\frac{3b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{\left(2a^4-11a^2b^2+6b^4\right)\sin(c+dx)}{ad}-\frac{3\int \frac{2b\left(a^2-b^2\right)^2-ab^2\left(2a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{a(a^2-b^2)}+\frac{3b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \ 4407$$

$$\frac{\frac{\frac{\left(2a^4-11a^2b^2+6b^4\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{2bx\left(a^2-b^2\right)^2}{a}-\frac{b^2\left(4a^4-5a^2b^2+2b^4\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}\right)}{a}}{a(a^2-b^2)}+\frac{3b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{\left(2a^4-11a^2b^2+6b^4\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{2bx\left(a^2-b^2\right)^2}{a}-\frac{b^2\left(4a^4-5a^2b^2+2b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}\right)}{a}}{a(a^2-b^2)}+\frac{3b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \ 4318$$

$$\frac{\frac{\frac{\left(2a^4-11a^2b^2+6b^4\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{2bx\left(a^2-b^2\right)^2}{a}-\frac{b\left(4a^4-5a^2b^2+2b^4\right)\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a}\right)}{a}}{a(a^2-b^2)}+\frac{3b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\left(2a^4-11a^2b^2+6b^4\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{2bx\left(a^2-b^2\right)^2}{a}-\frac{b\left(4a^4-5a^2b^2+2b^4\right)\int\frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a}\right)}{a}}{a(a^2-b^2)}+\frac{3b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{\left(2a^4-11a^2b^2+6b^4\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{2bx\left(a^2-b^2\right)^2}{a}-\frac{2b\left(4a^4-5a^2b^2+2b^4\right)\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}\right)}{a}}{a(a^2-b^2)}+\frac{3b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{\frac{3b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}+\frac{\frac{\left(2a^4-11a^2b^2+6b^4\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{2bx\left(a^2-b^2\right)^2}{a}-\frac{2b^2\left(4a^4-5a^2b^2+2b^4\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a}}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}$$

input
```
Int[Cos[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

output
```
(b^2*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((3*b^2*(2
*a^2 - b^2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])) + ((-3*((
2*b*(a^2 - b^2)^2*x)/a - (2*b^2*(4*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[(Sqrt[
a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d)))/a +
 ((2*a^4 - 11*a^2*b^2 + 6*b^4)*Sin[c + d*x])/(a*d))/(a*(a^2 - b^2)))/(2*a*
(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{-\frac{2b^2\left(\frac{-\frac{(8a^2+ab-4b^2)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)(a^2+2ab+b^2)}+\frac{(8a^2-ab-4b^2)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)(a^2-2ab+b^2)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}-\frac{3(4a^4-5a^2b^2+2b^4)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{2(a^4-2a^2b^2+b^4)\sqrt{(a+b)(a-b)}}\cdots\right)}{a^4}\cdots}{d}$
default	$\frac{-\frac{2b^2\left(\frac{-\frac{(8a^2+ab-4b^2)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)(a^2+2ab+b^2)}+\frac{(8a^2-ab-4b^2)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)(a^2-2ab+b^2)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}-\frac{3(4a^4-5a^2b^2+2b^4)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{2(a^4-2a^2b^2+b^4)\sqrt{(a+b)(a-b)}}\cdots\right)}{a^4}\cdots}{d}$
risch	$-\frac{3bx}{a^4}-\frac{ie^{i(dx+c)}}{2d\,a^3}+\frac{ie^{-i(dx+c)}}{2d\,a^3}+\frac{ib^3\left(-9b\,a^3e^{3i(dx+c)}+6a\,b^3e^{3i(dx+c)}-8a^4e^{2i(dx+c)}-11a^2b^2e^{2i(dx+c)}+10b^4e^{2\cdots}\right)}{a^4(-a^2+b^2)^2d\left(e^{2i(dx+c)}a+2b\,e^{i(\cdots}\right)}$

input
```
int(cos(d*x+c)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*b^2/a^4*((-1/2*(8*a^2+a*b-4*b^2)*a*b/(a-b)/(a^2+2*a*b+b^2)*tan(1/2
*d*x+1/2*c)^3+1/2*(8*a^2-a*b-4*b^2)*a*b/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+
1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2-3/2*(4*a^4-5
*a^2*b^2+2*b^4)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(
1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))-2/a^4*(-a*tan(1/2*d*x+1/2*c)/(1+tan(1
/2*d*x+1/2*c)^2)+3*b*arctan(tan(1/2*d*x+1/2*c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(208) = 416$.

Time = 0.16 (sec) , antiderivative size = 1037, normalized size of antiderivative = 4.65

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
[-1/4*(12*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d*x*cos(d*x + c)^2 + 2
4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*d*x*cos(d*x + c) + 12*(a^6*b^3
 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*d*x - 3*(4*a^4*b^4 - 5*a^2*b^6 + 2*b^8 + (
4*a^6*b^2 - 5*a^4*b^4 + 2*a^2*b^6)*cos(d*x + c)^2 + 2*(4*a^5*b^3 - 5*a^3*b
^5 + 2*a*b^7)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2
 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x
+ c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(
2*a^7*b^2 - 13*a^5*b^4 + 17*a^3*b^6 - 6*a*b^8 + 2*(a^9 - 3*a^7*b^2 + 3*a^5
*b^4 - a^3*b^6)*cos(d*x + c)^2 + (4*a^8*b - 20*a^6*b^3 + 25*a^4*b^5 - 9*a^
2*b^7)*cos(d*x + c))*sin(d*x + c))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b
^6)*d*cos(d*x + c)^2 + 2*(a^11*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5*b^7)*d*cos(
d*x + c) + (a^10*b^2 - 3*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*d), -1/2*(6*(a^8*b
 - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d*x*cos(d*x + c)^2 + 12*(a^7*b^2 - 3*a
^5*b^4 + 3*a^3*b^6 - a*b^8)*d*x*cos(d*x + c) + 6*(a^6*b^3 - 3*a^4*b^5 + 3*
a^2*b^7 - b^9)*d*x - 3*(4*a^4*b^4 - 5*a^2*b^6 + 2*b^8 + (4*a^6*b^2 - 5*a^4
*b^4 + 2*a^2*b^6)*cos(d*x + c)^2 + 2*(4*a^5*b^3 - 5*a^3*b^5 + 2*a*b^7)*cos
(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/
((a^2 - b^2)*sin(d*x + c))) - (2*a^7*b^2 - 13*a^5*b^4 + 17*a^3*b^6 - 6*a*b
^8 + 2*(a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x + c)^2 + (4*a^8*b -
 20*a^6*b^3 + 25*a^4*b^5 - 9*a^2*b^7)*cos(d*x + c))*sin(d*x + c))/((a^1...
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(cos(c + d*x)/(a + b*sec(c + d*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.60

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{\frac{3\,(4\,a^4b^2-5\,a^2b^4+2\,b^6)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^8-2\,a^6b^2+a^4b^4)\sqrt{-a^2+b^2}} - \frac{8\,a^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,a^2b^4\tan\left(\frac{1}{2}\right.}{}}{}$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-(3*(4*a^4*b^2 - 5*a^2*b^4 + 2*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(
2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt
(-a^2 + b^2)))/((a^8 - 2*a^6*b^2 + a^4*b^4)*sqrt(-a^2 + b^2)) - (8*a^3*b^3
*tan(1/2*d*x + 1/2*c)^3 - 7*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 - 5*a*b^5*tan(1
/2*d*x + 1/2*c)^3 + 4*b^6*tan(1/2*d*x + 1/2*c)^3 - 8*a^3*b^3*tan(1/2*d*x +
 1/2*c) - 7*a^2*b^4*tan(1/2*d*x + 1/2*c) + 5*a*b^5*tan(1/2*d*x + 1/2*c) +
4*b^6*tan(1/2*d*x + 1/2*c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*(a*tan(1/2*d*x +
1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) + 3*(d*x + c)*b/a^4 - 2*ta
n(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^3))/d
```

Mupad [B] (verification not implemented)

Time = 17.69 (sec) , antiderivative size = 5338, normalized size of antiderivative = 23.94

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3} dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)/(a + b/cos(c + d*x))^3,x)
```

output
```
((tan(c/2 + (d*x)/2)*(3*a*b^4 + 2*a^4*b + 2*a^5 + 6*b^5 - 12*a^2*b^3 - 4*a
^3*b^2))/((a + b)*(a^5 - 2*a^4*b + a^3*b^2)) - (tan(c/2 + (d*x)/2)^5*(3*a*
b^4 - 2*a^4*b + 2*a^5 - 6*b^5 + 12*a^2*b^3 - 4*a^3*b^2))/((a^3*b - a^4)*(a
 + b)^2) + (2*tan(c/2 + (d*x)/2)^3*(2*a^6 - 6*b^6 + 13*a^2*b^4 - 6*a^4*b^2
))/(a*(a^2*b - a^3)*(a + b)^2*(a - b)))/(d*(2*a*b + tan(c/2 + (d*x)/2)^2*(
2*a*b - a^2 + 3*b^2) + tan(c/2 + (d*x)/2)^6*(a^2 - 2*a*b + b^2) + a^2 + b^
2 - tan(c/2 + (d*x)/2)^4*(2*a*b + a^2 - 3*b^2))) - (6*b*atan(((3*b*((8*tan
(c/2 + (d*x)/2)*(72*b^12 - 72*a*b^11 - 288*a^2*b^10 + 288*a^3*b^9 + 441*a^
4*b^8 - 432*a^5*b^7 - 288*a^6*b^6 + 288*a^7*b^5 + 36*a^8*b^4 - 72*a^9*b^3
+ 36*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4
 - 3*a^10*b^3 - 3*a^11*b^2) + (b*((24*(4*a^17*b - 4*a^8*b^10 + 2*a^9*b^9 +
 18*a^10*b^8 - 8*a^11*b^7 - 32*a^12*b^6 + 14*a^13*b^5 + 26*a^14*b^4 - 12*a
^15*b^3 - 8*a^16*b^2))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 +
3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) - (b*tan(c/2 + (d*x)/2)*(8*a^17*b -
8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^13
*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2)*24i)/(a^4*(a^12*b + a^13 -
a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)))*3i)
/a^4))/a^4 + (3*b*((8*tan(c/2 + (d*x)/2)*(72*b^12 - 72*a*b^11 - 288*a^2*b^
10 + 288*a^3*b^9 + 441*a^4*b^8 - 432*a^5*b^7 - 288*a^6*b^6 + 288*a^7*b^5 +
 36*a^8*b^4 - 72*a^9*b^3 + 36*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1271, normalized size of antiderivative = 5.70

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3} dx = \text{Too large to display}$$

input
```
int(cos(d*x+c)/(a+b*sec(d*x+c))^3,x)
```

output
```
(48*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sq
rt( - a**2 + b**2))*cos(c + d*x)*a**5*b**3 - 60*sqrt( - a**2 + b**2)*atan(
(tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*
x)*a**3*b**5 + 24*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c +
 d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*a*b**7 - 24*sqrt( - a**2 +
b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))
*sin(c + d*x)**2*a**6*b**2 + 30*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2
)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*sin(c + d*x)**2*a**4*b**4
- 12*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/s
qrt( - a**2 + b**2))*sin(c + d*x)**2*a**2*b**6 + 24*sqrt( - a**2 + b**2)*a
tan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*a**6*b
**2 - 6*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b
)/sqrt( - a**2 + b**2))*a**4*b**4 - 18*sqrt( - a**2 + b**2)*atan((tan((c +
 d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*a**2*b**6 + 12*sqrt
( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a*
*2 + b**2))*b**8 + 4*cos(c + d*x)*sin(c + d*x)*a**8*b - 20*cos(c + d*x)*si
n(c + d*x)*a**6*b**3 + 25*cos(c + d*x)*sin(c + d*x)*a**4*b**5 - 9*cos(c +
d*x)*sin(c + d*x)*a**2*b**7 - 12*cos(c + d*x)*a**7*b**2*c - 12*cos(c + d*x
)*a**7*b**2*d*x + 36*cos(c + d*x)*a**5*b**4*c + 36*cos(c + d*x)*a**5*b**4*
d*x - 36*cos(c + d*x)*a**3*b**6*c - 36*cos(c + d*x)*a**3*b**6*d*x + 12*...
```

3.513 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 296

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{(a^2+12b^2)\,x}{2a^5} - \frac{b^3(20a^4-29a^2b^2+12b^4)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{5/2}(a+b)^{5/2}d} - \frac{3b(2a^4-7a^2b^2+4b^4)\sin(c+dx)}{2a^4\left(a^2-b^2\right)^2 d} + \frac{(a^4-10a^2b^2+6b^4)\cos(c+dx)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2 d} + \frac{b^2\cos(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2(7a^2-4b^2)\cos(c+dx)\sin(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))}$$

output
```
1/2*(a^2+12*b^2)*x/a^5-b^3*(20*a^4-29*a^2*b^2+12*b^4)*arctanh((a-b)^(1/2)*
tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^5/(a-b)^(5/2)/(a+b)^(5/2)/d-3/2*b*(2*a^4
-7*a^2*b^2+4*b^4)*sin(d*x+c)/a^4/(a^2-b^2)^2/d+1/2*(a^4-10*a^2*b^2+6*b^4)*
cos(d*x+c)*sin(d*x+c)/a^3/(a^2-b^2)^2/d+1/2*b^2*cos(d*x+c)*sin(d*x+c)/a/(a
^2-b^2)/d/(a+b*sec(d*x+c))^2+1/2*b^2*(7*a^2-4*b^2)*cos(d*x+c)*sin(d*x+c)/a
^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.67

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$=\frac{2(a^2+12b^2)(c+dx)+\frac{4b^3(20a^4-29a^2b^2+12b^4)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}-12ab\sin(c+dx)+\frac{2ab^5\sin(}{(-a+b)(a+b)(b+}}{4a^5d}$$

input
```
Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

output
```
(2*(a^2 + 12*b^2)*(c + d*x) + (4*b^3*(20*a^4 - 29*a^2*b^2 + 12*b^4)*ArcTan
h[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) - 12*a*b
*Sin[c + d*x] + (2*a*b^5*Sin[c + d*x])/((-a + b)*(a + b)*(b + a*Cos[c + d*
x])^2) + (2*a*b^4*(10*a^2 - 7*b^2)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b +
 a*Cos[c + d*x])) + a^2*Sin[2*(c + d*x)])/(4*a^5*d)
```

Rubi [A] (verified)

Time = 2.30 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.08, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 4334, 25, 3042, 4588, 25, 3042, 4592, 27, 3042, 4592, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \ 4334$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\int -\frac{\cos^2(c+dx)\left(3b^2\sec^2(c+dx)-2ab\sec(c+dx)+2\left(a^2-2b^2\right)\right)}{(a+b\sec(c+dx))^2}dx}{2a\left(a^2-b^2\right)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{\cos^2(c+dx)\left(3b^2\sec^2(c+dx)-2ab\sec(c+dx)+2\left(a^2-2b^2\right)\right)}{(a+b\sec(c+dx))^2}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{3b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2-2ab\csc\left(c+dx+\frac{\pi}{2}\right)+2\left(a^2-2b^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 4588

$$\frac{\frac{b^2\left(7a^2-4b^2\right)\sin(c+dx)\cos(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int -\frac{\cos^2(c+dx)\left(2b^2\left(7a^2-4b^2\right)\sec^2(c+dx)-ab\left(4a^2-b^2\right)\sec(c+dx)+2\left(a^4-10b^2a^2+6b^4\right)\right)}{a+b\sec(c+dx)}dx}{a\left(a^2-b^2\right)}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{\cos^2(c+dx)\left(2b^2\left(7a^2-4b^2\right)\sec^2(c+dx)-ab\left(4a^2-b^2\right)\sec(c+dx)+2\left(a^4-10b^2a^2+6b^4\right)\right)}{a+b\sec(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\left(7a^2-4b^2\right)\sin(c+dx)\cos(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{2b^2\left(7a^2-4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-ab\left(4a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+2\left(a^4-10b^2a^2+6b^4\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\left(7a^2-4b^2\right)\sin(c+dx)\cos(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 4592

$$\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\int \frac{2\cos(c+dx)\left(-b\left(a^4-10b^2a^2+6b^4\right)\sec^2(c+dx)-a\left(a^4+4b^2a^2-2b^4\right)\sec(c+dx)+3b\left(2a^4-7b^2a^2+4b^4\right)\right)}{a+b\sec(c+dx)}dx}{2a}}{a(a^2-b^2)}+\frac{b^2(7}{a}$$
$$2a\left(a^2-b^2\right)$$
$$\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 27

$$\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\int \frac{\cos(c+dx)\left(-b\left(a^4-10b^2a^2+6b^4\right)\sec^2(c+dx)-a\left(a^4+4b^2a^2-2b^4\right)\sec(c+dx)+3b\left(2a^4-7b^2a^2+4b^4\right)\right)}{a+b\sec(c+dx)}dx}{a}}{a(a^2-b^2)}+\frac{b^2(7a}{ad}$$
$$2a\left(a^2-b^2\right)$$
$$\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\int \frac{-b\left(a^4-10b^2a^2+6b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-a\left(a^4+4b^2a^2-2b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)+3b\left(2a^4-7b^2a^2+4b^4\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}}{a(a^2-b^2)}+\frac{b^2(7a^2-}{ad(a^2}$$
$$2a\left(a^2-b^2\right)$$
$$\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 4592

$$\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\sin(c+dx)}{ad}-\frac{\int \frac{\left(a^2+12b^2\right)\left(a^2-b^2\right)^2+ab\left(a^4-10b^2a^2+6b^4\right)\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}}{a}}{a(a^2-b^2)}+\frac{b^2\left(7a^2-4b^2\right)\text{si}}{ad(a^2-b^2)(}$$
$$2a\left(a^2-b^2\right)$$
$$\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\sin(c+dx)}{ad}-\frac{\int \frac{\left(a^2+12b^2\right)\left(a^2-b^2\right)^2+ab\left(a^4-10b^2a^2+6b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{a}}{a(a^2-b^2)}+\frac{b^2\left(7a^2-4b\right.}{ad(a^2-}$$
$$2a\left(a^2-b^2\right)$$
$$\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 4407

$$\frac{\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\sin(c+dx)}{ad}-\frac{\frac{x\left(a^2-b^2\right)^2\left(a^2+12b^2\right)}{a}-\frac{b^3\left(20a^4-29a^2b^2+12b^4\right)\int\frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}}{a}}{a}}{a(a^2-b^2)}+\frac{b^2(7a}{ac}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\sin(c+dx)}{ad}-\frac{\frac{x\left(a^2-b^2\right)^2\left(a^2+12b^2\right)}{a}-\frac{b^3\left(20a^4-29a^2b^2+12b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{a}}{a}}{a(a^2-b^2)}+\frac{b^2}{}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$\Big\downarrow$ 4318

$$\frac{\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\sin(c+dx)}{ad}-\frac{\frac{x\left(a^2-b^2\right)^2\left(a^2+12b^2\right)}{a}-\frac{b^2\left(20a^4-29a^2b^2+12b^4\right)\int\frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a}}{a}}{a}}{a(a^2-b^2)}+\frac{b^2(7}{a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\sin(c+dx)}{ad}-\frac{\frac{x\left(a^2-b^2\right)^2\left(a^2+12b^2\right)}{a}-\frac{b^2\left(20a^4-29a^2b^2+12b^4\right)\int\frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a}}{a}}{a}}{a(a^2-b^2)}+\frac{b}{}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$\Big\downarrow$ 3138

$$\frac{\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\sin(c+dx)}{ad}-\frac{\frac{x\left(a^2-b^2\right)^2\left(a^2+12b^2\right)}{a}-\frac{\frac{2b^2\left(20a^4-29a^2b^2+12b^4\right)\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+}{b}}}{ad}}{a}}{a}}{a}}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$$\Big\downarrow \; 221$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{\frac{b^2\left(7a^2-4b^2\right)\sin(c+dx)\cos(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}+\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\sin(c+dx)}{ad}-\frac{\frac{x\left(a^2-b^2\right)^2\left(a^2+12b^2\right)}{a}-\frac{2b^3\left(20a^4-29\cdots\right.}{\cdots}}{a}}{a(a^2-b^2)}}{\cdots}}{2a\left(a^2-b^2\right)}$$

input
```
Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

output
```
(b^2*Cos[c + d*x]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2)
 + ((b^2*(7*a^2 - 4*b^2)*Cos[c + d*x]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a +
b*Sec[c + d*x])) + (((a^4 - 10*a^2*b^2 + 6*b^4)*Cos[c + d*x]*Sin[c + d*x])
/(a*d) - (-((((a^2 - b^2)^2*(a^2 + 12*b^2)*x)/a - (2*b^3*(20*a^4 - 29*a^2*
b^2 + 12*b^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt
[a - b]*Sqrt[a + b]*d))/a) + (3*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Sin[c + d*x]
)/(a*d))/a)/(a*(a^2 - b^2)))/(2*a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\frac{2\left(\frac{\left(-\frac{1}{2}a^2-3ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-3ab+\frac{1}{2}a^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\left(a^2+12b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^5}+\frac{2b^3\left(\frac{-\frac{\left(10a^2+ab-6b^2\right)ab\tan\ldots}{2(a-b)\left(a^2+2ab\ldots\right)}}{\left(\tan\left(\frac{dx}{2}+\ldots\right.\right.}\ldots\right.}{\ldots}}{d}$
default	$\frac{\frac{2\left(\frac{\left(-\frac{1}{2}a^2-3ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-3ab+\frac{1}{2}a^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\left(a^2+12b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^5}+\frac{2b^3\left(\frac{-\frac{\left(10a^2+ab-6b^2\right)ab\tan\ldots}{2(a-b)\left(a^2+2ab\ldots\right)}}{\left(\tan\left(\frac{dx}{2}+\ldots\right.\right.}\ldots\right.}{\ldots}}{d}$
risch	$\frac{x}{2a^3}+\frac{6xb^2}{a^5}-\frac{ie^{2i(dx+c)}}{8d\,a^3}+\frac{3ib\,e^{i(dx+c)}}{2d\,a^4}-\frac{3ib\,e^{-i(dx+c)}}{2d\,a^4}+\frac{ie^{-2i(dx+c)}}{8d\,a^3}-\frac{ib^4\left(-11b\,a^3e^{3i(dx+c)}+8a\,b^3e^{3i(d\ldots}\right.}{\ldots}$

input
```
int(cos(d*x+c)^2/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/a^5*(((-1/2*a^2-3*a*b)*tan(1/2*d*x+1/2*c)^3+(-3*a*b+1/2*a^2)*tan(1/
2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(a^2+12*b^2)*arctan(tan(1/2*d
*x+1/2*c)))+2*b^3/a^5*((-1/2*(10*a^2+a*b-6*b^2)*a*b/(a-b)/(a^2+2*a*b+b^2)*
tan(1/2*d*x+1/2*c)^3+1/2*(10*a^2-a*b-6*b^2)*a*b/(a+b)/(a^2-2*a*b+b^2)*tan(
1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2-1/2*
(20*a^4-29*a^2*b^2+12*b^4)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh
((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1158, normalized size of antiderivative = 3.91

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/4*(2*(a^10 + 9*a^8*b^2 - 33*a^6*b^4 + 35*a^4*b^6 - 12*a^2*b^8)*d*x*cos(
d*x + c)^2 + 4*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*d*
x*cos(d*x + c) + 2*(a^8*b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^8 - 12*b^1
0)*d*x + (20*a^4*b^5 - 29*a^2*b^7 + 12*b^9 + (20*a^6*b^3 - 29*a^4*b^5 + 12
*a^2*b^7)*cos(d*x + c)^2 + 2*(20*a^5*b^4 - 29*a^3*b^6 + 12*a*b^8)*cos(d*x
+ c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)
^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a
^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(6*a^7*b^3 - 27*a^5*b^5
 + 33*a^3*b^7 - 12*a*b^9 - (a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*cos(d*
x + c)^3 + 4*(a^9*b - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*cos(d*x + c)^2 + (1
1*a^8*b^2 - 43*a^6*b^4 + 50*a^4*b^6 - 18*a^2*b^8)*cos(d*x + c))*sin(d*x +
c))/((a^13 - 3*a^11*b^2 + 3*a^9*b^4 - a^7*b^6)*d*cos(d*x + c)^2 + 2*(a^12*
b - 3*a^10*b^3 + 3*a^8*b^5 - a^6*b^7)*d*cos(d*x + c) + (a^11*b^2 - 3*a^9*b
^4 + 3*a^7*b^6 - a^5*b^8)*d), 1/2*((a^10 + 9*a^8*b^2 - 33*a^6*b^4 + 35*a^4
*b^6 - 12*a^2*b^8)*d*x*cos(d*x + c)^2 + 2*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5
+ 35*a^3*b^7 - 12*a*b^9)*d*x*cos(d*x + c) + (a^8*b^2 + 9*a^6*b^4 - 33*a^4*
b^6 + 35*a^2*b^8 - 12*b^10)*d*x - (20*a^4*b^5 - 29*a^2*b^7 + 12*b^9 + (20*
a^6*b^3 - 29*a^4*b^5 + 12*a^2*b^7)*cos(d*x + c)^2 + 2*(20*a^5*b^4 - 29*a^3
*b^6 + 12*a*b^8)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(
b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (6*a^7*b^3 - 27*a^5*b...
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(cos(c + d*x)**2/(a + b*sec(c + d*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1723 vs. 2(277) = 554.

Time = 0.37 (sec) , antiderivative size = 1723, normalized size of antiderivative = 5.82

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2*(((a^6 - a^5*b + 10*a^4*b^2 + 10*a^3*b^3 - 23*a^2*b^4 - 6*a*b^5 + 12*
b^6)*sqrt(-a^2 + b^2)*abs(a^9 - 2*a^7*b^2 + a^5*b^4)*abs(-a + b) - (a^15 -
 a^14*b + 8*a^13*b^2 - 28*a^12*b^3 - 42*a^11*b^4 + 111*a^10*b^5 + 68*a^9*b
^6 - 158*a^8*b^7 - 47*a^7*b^8 + 100*a^6*b^9 + 12*a^5*b^10 - 24*a^4*b^11)*s
qrt(-a^2 + b^2)*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(ta
n(1/2*d*x + 1/2*c)/sqrt(-(a^8*b - 2*a^6*b^3 + a^4*b^5 + sqrt((a^9 + a^8*b
- 2*a^7*b^2 - 2*a^6*b^3 + a^5*b^4 + a^4*b^5)*(a^9 - a^8*b - 2*a^7*b^2 + 2*
a^6*b^3 + a^5*b^4 - a^4*b^5) + (a^8*b - 2*a^6*b^3 + a^4*b^5)^2))/(a^9 - a^
8*b - 2*a^7*b^2 + 2*a^6*b^3 + a^5*b^4 - a^4*b^5))))/((a^9 - 2*a^7*b^2 + a^
5*b^4)^2*(a^2 - 2*a*b + b^2) + (a^10*b - 2*a^9*b^2 - a^8*b^3 + 4*a^7*b^4 -
 a^6*b^5 - 2*a^5*b^6 + a^4*b^7)*abs(a^9 - 2*a^7*b^2 + a^5*b^4)) + (a^15 -
a^14*b + 8*a^13*b^2 - 28*a^12*b^3 - 42*a^11*b^4 + 111*a^10*b^5 + 68*a^9*b^
6 - 158*a^8*b^7 - 47*a^7*b^8 + 100*a^6*b^9 + 12*a^5*b^10 - 24*a^4*b^11 + a
^6*abs(a^9 - 2*a^7*b^2 + a^5*b^4) - a^5*b*abs(a^9 - 2*a^7*b^2 + a^5*b^4) +
 10*a^4*b^2*abs(a^9 - 2*a^7*b^2 + a^5*b^4) + 10*a^3*b^3*abs(a^9 - 2*a^7*b^
2 + a^5*b^4) - 23*a^2*b^4*abs(a^9 - 2*a^7*b^2 + a^5*b^4) - 6*a*b^5*abs(a^9
 - 2*a^7*b^2 + a^5*b^4) + 12*b^6*abs(a^9 - 2*a^7*b^2 + a^5*b^4))*(pi*floor
(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(a^8*b - 2*a^
6*b^3 + a^4*b^5 - sqrt((a^9 + a^8*b - 2*a^7*b^2 - 2*a^6*b^3 + a^5*b^4 + a^
4*b^5)*(a^9 - a^8*b - 2*a^7*b^2 + 2*a^6*b^3 + a^5*b^4 - a^4*b^5) + (a^8...
```

Mupad [B] (verification not implemented)

Time = 18.10 (sec) , antiderivative size = 5950, normalized size of antiderivative = 20.10

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^2/(a + b/cos(c + d*x))^3,x)
```

output
```
(atan(((((8*tan(c/2 + (d*x)/2)*(a^14 - 2*a^13*b - 288*a*b^13 + 288*b^14 -
1104*a^2*b^12 + 1104*a^3*b^11 + 1538*a^4*b^10 - 1538*a^5*b^9 - 827*a^6*b^8
 + 872*a^7*b^7 + 18*a^8*b^6 - 108*a^9*b^5 + 74*a^10*b^4 - 40*a^11*b^3 + 21
*a^12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 -
 3*a^12*b^3 - 3*a^13*b^2) + ((a^2*1i + b^2*12i)*((4*(4*a^21 - 48*a^10*b^11
 + 24*a^11*b^10 + 212*a^12*b^9 - 100*a^13*b^8 - 360*a^14*b^7 + 164*a^15*b^
6 + 276*a^16*b^5 - 120*a^17*b^4 - 80*a^18*b^3 + 28*a^19*b^2))/(a^18*b + a^
19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b
^2) - (4*tan(c/2 + (d*x)/2)*(a^2*1i + b^2*12i)*(8*a^19*b - 8*a^10*b^10 + 8
*a^11*b^9 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^1
6*b^4 - 32*a^17*b^3 - 8*a^18*b^2))/(a^5*(a^14*b + a^15 - a^8*b^7 - a^9*b^6
 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2))))/(2*a^5))*(a^2*1i
+ b^2*12i)*1i)/(2*a^5) + (((8*tan(c/2 + (d*x)/2)*(a^14 - 2*a^13*b - 288*a*
b^13 + 288*b^14 - 1104*a^2*b^12 + 1104*a^3*b^11 + 1538*a^4*b^10 - 1538*a^5
*b^9 - 827*a^6*b^8 + 872*a^7*b^7 + 18*a^8*b^6 - 108*a^9*b^5 + 74*a^10*b^4
- 40*a^11*b^3 + 21*a^12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*
b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2) - ((a^2*1i + b^2*12i)*((4*(4*a
^21 - 48*a^10*b^11 + 24*a^11*b^10 + 212*a^12*b^9 - 100*a^13*b^8 - 360*a^14
*b^7 + 164*a^15*b^6 + 276*a^16*b^5 - 120*a^17*b^4 - 80*a^18*b^3 + 28*a^19*
b^2))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - ...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1469, normalized size of antiderivative = 4.96

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(cos(d*x+c)^2/(a+b*sec(d*x+c))^3,x)
```

output

```
( - 80*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)
/sqrt( - a**2 + b**2))*cos(c + d*x)*a**5*b**4 + 116*sqrt( - a**2 + b**2)*a
tan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c
+ d*x)*a**3*b**6 - 48*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan(
(c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*a*b**8 + 40*sqrt( - a**
2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b*
*2))*sin(c + d*x)**2*a**6*b**3 - 58*sqrt( - a**2 + b**2)*atan((tan((c + d*
x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*sin(c + d*x)**2*a**4*b
**5 + 24*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*
b)/sqrt( - a**2 + b**2))*sin(c + d*x)**2*a**2*b**7 - 40*sqrt( - a**2 + b**
2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*a*
*6*b**3 + 18*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)
/2)*b)/sqrt( - a**2 + b**2))*a**4*b**5 + 34*sqrt( - a**2 + b**2)*atan((tan
((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*a**2*b**7 - 24
*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(
 - a**2 + b**2))*b**9 - cos(c + d*x)*sin(c + d*x)**3*a**10 + 3*cos(c + d*x
)*sin(c + d*x)**3*a**8*b**2 - 3*cos(c + d*x)*sin(c + d*x)**3*a**6*b**4 + c
os(c + d*x)*sin(c + d*x)**3*a**4*b**6 + cos(c + d*x)*sin(c + d*x)*a**10 -
14*cos(c + d*x)*sin(c + d*x)*a**8*b**2 + 46*cos(c + d*x)*sin(c + d*x)*a**6
*b**4 - 51*cos(c + d*x)*sin(c + d*x)*a**4*b**6 + 18*cos(c + d*x)*sin(c ...
```

3.514 $\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 316

$$\begin{aligned}\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx &= -\frac{4a\operatorname{arctanh}(\sin(c+dx))}{b^5d} \\ &\quad+\frac{a^2(8a^6-28a^4b^2+35a^2b^4-20b^6)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}b^5(a+b)^{7/2}d} \\ &\quad+\frac{(12a^4-23a^2b^2+6b^4)\tan(c+dx)}{6b^4(a^2-b^2)^2d}-\frac{a^2\sec^3(c+dx)\tan(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^3} \\ &\quad-\frac{a^2(4a^2-9b^2)\sec^2(c+dx)\tan(c+dx)}{6b^2(a^2-b^2)^2d(a+b\sec(c+dx))^2}+\frac{a^3(4a^4-11a^2b^2+12b^4)\tan(c+dx)}{2b^4(a^2-b^2)^3d(a+b\sec(c+dx))}\end{aligned}$$

output
```
-4*a*arctanh(sin(d*x+c))/b^5/d+a^2*(8*a^6-28*a^4*b^2+35*a^2*b^4-20*b^6)*ar
ctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/b^5/(a+b)^(7
/2)/d+1/6*(12*a^4-23*a^2*b^2+6*b^4)*tan(d*x+c)/b^4/(a^2-b^2)^2/d-1/3*a^2*s
ec(d*x+c)^3*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^3-1/6*a^2*(4*a^2-9*b
^2)*sec(d*x+c)^2*tan(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2+1/2*a^3*(
4*a^4-11*a^2*b^2+12*b^4)*tan(d*x+c)/b^4/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 6.65 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.32

$$\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$
$$= -\frac{a^2(-8a^6+28a^4b^2-35a^2b^4+20b^6)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5\sqrt{a^2-b^2}\,(-a^2+b^2)^3\,d}$$
$$+\frac{4a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{b^5d}$$
$$-\frac{4a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{b^5d}$$
$$+\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{b^4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{b^4d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$
$$-\frac{a^3\sin(c+dx)}{3b^2(-a+b)(a+b)d(b+a\cos(c+dx))^3}$$
$$+\frac{6a^5\sin(c+dx)-11a^3b^2\sin(c+dx)}{6b^3(-a+b)^2(a+b)^2d(b+a\cos(c+dx))^2}$$
$$+\frac{-18a^7\sin(c+dx)+50a^5b^2\sin(c+dx)-47a^3b^4\sin(c+dx)}{6b^4(-a+b)^3(a+b)^3d(b+a\cos(c+dx))}$$

input
```
Integrate[Sec[c + d*x]^6/(a + b*Sec[c + d*x])^4,x]
```

output
```
-((a^2*(-8*a^6 + 28*a^4*b^2 - 35*a^2*b^4 + 20*b^6)*ArcTanh[((-a + b)*Tan[(
c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^5*Sqrt[a^2 - b^2]*(-a^2 + b^2)^3*d)) + (
4*a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(b^5*d) - (4*a*Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]])/(b^5*d) + Sin[(c + d*x)/2]/(b^4*d*(Cos[(c + d
*x)/2] - Sin[(c + d*x)/2])) + Sin[(c + d*x)/2]/(b^4*d*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])) - (a^3*Sin[c + d*x])/(3*b^2*(-a + b)*(a + b)*d*(b + a*C
os[c + d*x])^3) + (6*a^5*Sin[c + d*x] - 11*a^3*b^2*Sin[c + d*x])/(6*b^3*(-
a + b)^2*(a + b)^2*d*(b + a*Cos[c + d*x])^2) + (-18*a^7*Sin[c + d*x] + 50*
a^5*b^2*Sin[c + d*x] - 47*a^3*b^4*Sin[c + d*x])/(6*b^4*(-a + b)^3*(a + b)^
3*d*(b + a*Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 2.61 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.18, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 4332, 3042, 4586, 25, 3042, 4578, 3042, 4570, 27, 3042, 4486, 3042, 4257, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^6}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \ 4332$$

$$-\frac{\int \frac{\sec^3(c+dx)\left(3a^2-3b\sec(c+dx)a-\left(4a^2-3b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}dx}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(3a^2-3b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(3b^2-4a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \ 4586$$

$$-\frac{\frac{a^2\left(4a^2-9b^2\right)\tan(c+dx)\sec^2(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\int -\frac{\sec^2(c+dx)\left(2\left(4a^2-9b^2\right)a^2-2b\left(a^2-6b^2\right)\sec(c+dx)a-\left(12a^4-23b^2a^2+6b^4\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{\int \frac{\sec^2(c+dx)\left(2\left(4a^2-9b^2\right)a^2-2b\left(a^2-6b^2\right)\sec(c+dx)a-\left(12a^4-23b^2a^2+6b^4\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2b(a^2-b^2)}+\frac{a^2\left(4a^2-9b^2\right)\tan(c+dx)\sec^2(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(2\left(4a^2-9b^2\right)a^2-2b\left(a^2-6b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(-12a^4+23b^2a^2-6b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2b(a^2-b^2)}+\frac{a^2\left(4a^2-9b^2\right)\tan(c+dx)\sec^2(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{4578}$$

$$-\frac{\frac{\frac{\int \frac{\sec(c+dx)\left(3b\left(4a^4-11b^2a^2+12b^4\right)a^2+\left(12a^6-37b^2a^4+43b^4a^2-18b^6\right)\sec(c+dx)a-b\left(a^2-b^2\right)\left(12a^4-23b^2a^2+6b^4\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(4a^4-11a^2b^2+12\ldots\right.}{b^2d\left(a^2-b^2\right)(a+b\ldots}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3b\left(4a^4-11b^2a^2+12b^4\right)a^2+\left(12a^6-37b^2a^4+43b^4a^2-18b^6\right)\csc\left(c+dx+\frac{\pi}{2}\right)a-b\left(a^2-b^2\right)\left(12a^4-23b^2a^2+6b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(4\ldots\right.}{b^2\ldots}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{4570}$$

$$-\frac{\frac{\frac{\frac{\int \frac{3\sec(c+dx)\left(8ab\sec(c+dx)\left(a^2-b^2\right)^3+a^2b^2\left(4a^4-11b^2a^2+12b^4\right)\right)}{a+b\sec(c+dx)}dx}{b}-\frac{\left(a^2-b^2\right)\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{d}}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(4a^4-11a^2b^2+12b^4\right)\tan(c+dx)}{b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2b(a^2-b^2)}\ldots}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\frac{\frac{3\int \frac{\sec(c+dx)\left(8ab\sec(c+dx)\left(a^2-b^2\right)^3+a^2b^2\left(4a^4-11b^2a^2+12b^4\right)\right)}{a+b\sec(c+dx)}dx}{b}-\frac{\left(a^2-b^2\right)\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{d}}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(4a^4-11a^2b^2+12b^4\right)\tan(c+dx)}{b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(8ab\csc\left(c+dx+\frac{\pi}{2}\right)\left(a^2-b^2\right)^3+a^2b^2\left(4a^4-11b^2a^2+12b^4\right)\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(a^2-b^2\right)\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{d}}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(4a^4-11a^2b^2+12b^4\right)\tan}{b^2d\left(a^2-b^2\right)(a+b\sec(c+}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 4486

$$-\frac{\frac{\frac{\frac{3\left(8a\left(a^2-b^2\right)^3\int \sec(c+dx)dx-a^2\left(8a^6-28a^4b^2+35a^2b^4-20b^6\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx\right)}{b}-\frac{\left(a^2-b^2\right)\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{d}}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(4a^4-11a^2b^2+12b^4}{b^2d\left(a^2-b^2\right)(a+b\sec}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{\frac{3\left(8a\left(a^2-b^2\right)^3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-a^2\left(8a^6-28a^4b^2+35a^2b^4-20b^6\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{b}-\frac{\left(a^2-b^2\right)\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{d}}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(4a^4-11a}{b^2d\left(a^2-b\right.}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 4257

$$-\frac{\frac{\frac{3\left(\frac{8a\left(a^2-b^2\right)^3\text{arctanh}(\sin(c+dx))}{d}-a^2\left(8a^6-28a^4b^2+35a^2b^4-20b^6\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{b}-\frac{\left(a^2-b^2\right)\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{d}}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(4a^4-\right.}{b^2d\left(a\right.}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 4318$$

$$-\frac{\frac{\frac{3\left(\frac{8a\left(a^2-b^2\right)^3\text{arctanh}(\sin(c+dx))}{d}-\frac{a^2\left(8a^6-28a^4b^2+35a^2b^4-20b^6\right)\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b}\right)}{b}-\frac{\left(a^2-b^2\right)\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{d}}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(4a^4-11\right.}{b^2d\left(a^2-\right.}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{3\left(\frac{8a\left(a^2-b^2\right)^3\text{arctanh}(\sin(c+dx))}{d}-\frac{a^2\left(8a^6-28a^4b^2+35a^2b^4-20b^6\right)\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b}\right)}{b}-\frac{\left(a^2-b^2\right)\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{d}}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(4a^4\right.}{b^2d\left(\right.}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 3138$$

$$-\frac{\frac{\frac{3\left(\frac{8a\left(a^2-b^2\right)^3\text{arctanh}(\sin(c+dx))}{d}-\frac{2a^2\left(8a^6-28a^4b^2+35a^2b^4-20b^6\right)\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}\right)}{b}-\frac{\left(a^2-b^2\right)\left(12a^4-23a^2b^2+6\right.}{d}}{b^2\left(a^2-b^2\right)}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 221$$

$$-\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^3}-\frac{\frac{a^2(4a^2-9b^2)\tan(c+dx)\sec^2(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}+\frac{\frac{\frac{3\left(\frac{8a(a^2-b^2)^3\text{arctanh}(\sin(c+dx))}{d}-\frac{2a^2(8a^6-28a^4b^2+35a^2b^4-20b^6)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}\right)}{b}}{b^2(a^2-b^2)}}{2b(a^2-b^2)}}{3b(a^2-b^2)}$$

input
```
Int[Sec[c + d*x]^6/(a + b*Sec[c + d*x])^4,x]
```

output
```
-1/3*(a^2*Sec[c + d*x]^3*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x
])^3) - ((a^2*(4*a^2 - 9*b^2)*Sec[c + d*x]^2*Tan[c + d*x])/(2*b*(a^2 - b^2
)*d*(a + b*Sec[c + d*x])^2) + ((-3*a^3*(4*a^4 - 11*a^2*b^2 + 12*b^4)*Tan[c
 + d*x])/(b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])) + ((3*((8*a*(a^2 - b^2)^
3*ArcTanh[Sin[c + d*x]])/d - (2*a^2*(8*a^6 - 28*a^4*b^2 + 35*a^2*b^4 - 20*
b^6)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqr
t[a + b]*d)))/b - ((a^2 - b^2)*(12*a^4 - 23*a^2*b^2 + 6*b^4)*Tan[c + d*x])
/d)/(b^2*(a^2 - b^2)))/(2*b*(a^2 - b^2)))/(3*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

rule 4578

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x
])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - Simp[1/(b^2*(m + 1)*(a^2 - b^
2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*((-a)*(b
*B - a*C) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
 b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
]
```

rule 4586

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Simp[d/(b*(a^2 - b^2)*(m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
 - b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n
+ b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C
}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{-\frac{1}{b^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{4a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^5}-\frac{2a^2\left(\frac{\left(6a^4-2b\,a^3-18a^2b^2+5a\,b^3+20b^4\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}-\frac{2\left(9a^4-29a^2b^2+30\cdots\right.}{3\left(a^2+2ab+b^2\cdots\right.}\cdots\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\cdots\right.}}{\cdots}$
default	$\frac{-\frac{1}{b^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{4a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^5}-\frac{2a^2\left(\frac{\left(6a^4-2b\,a^3-18a^2b^2+5a\,b^3+20b^4\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}-\frac{2\left(9a^4-29a^2b^2+30\cdots\right.}{3\left(a^2+2ab+b^2\cdots\right.}\cdots\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\cdots\right.}}{\cdots}$
risch	Expression too large to display

input
```
int(sec(d*x+c)^6/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^4/(tan(1/2*d*x+1/2*c)-1)+4*a/b^5*ln(tan(1/2*d*x+1/2*c)-1)-2*a^2/
b^5*((1/2*(6*a^4-2*a^3*b-18*a^2*b^2+5*a*b^3+20*b^4)*a*b/(a-b)/(a^3+3*a^2*b
+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-2/3*(9*a^4-29*a^2*b^2+30*b^4)*a*b/(a^2+
2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/2*(6*a^4+2*a^3*b-18*a^2*
b^2-5*a*b^3+20*b^4)*a*b/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)
)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3-1/2*(8*a^6-28*a^4*
b^2+35*a^2*b^4-20*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*a
rctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))-1/b^4/(tan(1/2*d*x+1
/2*c)+1)-4*a/b^5*ln(tan(1/2*d*x+1/2*c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1000 vs. $2(299) = 598$.

Time = 1.54 (sec) , antiderivative size = 2058, normalized size of antiderivative = 6.51

$$\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^6/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
[1/12*(3*((8*a^11 - 28*a^9*b^2 + 35*a^7*b^4 - 20*a^5*b^6)*cos(d*x + c)^4 +
 3*(8*a^10*b - 28*a^8*b^3 + 35*a^6*b^5 - 20*a^4*b^7)*cos(d*x + c)^3 + 3*(8
*a^9*b^2 - 28*a^7*b^4 + 35*a^5*b^6 - 20*a^3*b^8)*cos(d*x + c)^2 + (8*a^8*b
^3 - 28*a^6*b^5 + 35*a^4*b^7 - 20*a^2*b^9)*cos(d*x + c))*sqrt(a^2 - b^2)*l
og((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*
(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a
*b*cos(d*x + c) + b^2)) - 24*((a^12 - 4*a^10*b^2 + 6*a^8*b^4 - 4*a^6*b^6 +
 a^4*b^8)*cos(d*x + c)^4 + 3*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 +
 a^3*b^9)*cos(d*x + c)^3 + 3*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*b^8
 + a^2*b^10)*cos(d*x + c)^2 + (a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9
 + a*b^11)*cos(d*x + c))*log(sin(d*x + c) + 1) + 24*((a^12 - 4*a^10*b^2 +
6*a^8*b^4 - 4*a^6*b^6 + a^4*b^8)*cos(d*x + c)^4 + 3*(a^11*b - 4*a^9*b^3 +
6*a^7*b^5 - 4*a^5*b^7 + a^3*b^9)*cos(d*x + c)^3 + 3*(a^10*b^2 - 4*a^8*b^4
+ 6*a^6*b^6 - 4*a^4*b^8 + a^2*b^10)*cos(d*x + c)^2 + (a^9*b^3 - 4*a^7*b^5
+ 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2
*(6*a^8*b^4 - 24*a^6*b^6 + 36*a^4*b^8 - 24*a^2*b^10 + 6*b^12 + (24*a^11*b
- 92*a^9*b^3 + 133*a^7*b^5 - 71*a^5*b^7 + 6*a^3*b^9)*cos(d*x + c)^3 + 3*(2
0*a^10*b^2 - 77*a^8*b^4 + 110*a^6*b^6 - 59*a^4*b^8 + 6*a^2*b^10)*cos(d*x +
 c)^2 + (44*a^9*b^3 - 169*a^7*b^5 + 239*a^5*b^7 - 132*a^3*b^9 + 18*a*b^11)
*cos(d*x + c))*sin(d*x + c))/((a^11*b^5 - 4*a^9*b^7 + 6*a^7*b^9 - 4*a^5...
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx=\int \frac{\sec^6{(c+dx)}}{(a+b\sec{(c+dx)})^4}\,dx$$

input
```
integrate(sec(d*x+c)**6/(a+b*sec(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**6/(a + b*sec(c + d*x))**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^6/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 592, normalized size of antiderivative = 1.87

$$\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^6/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/3*(3*(8*a^8 - 28*a^6*b^2 + 35*a^4*b^4 - 20*a^2*b^6)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2
*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11
)*sqrt(-a^2 + b^2)) + (18*a^9*tan(1/2*d*x + 1/2*c)^5 - 42*a^8*b*tan(1/2*d*
x + 1/2*c)^5 - 24*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 + 117*a^6*b^3*tan(1/2*d*x
 + 1/2*c)^5 - 24*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 - 105*a^4*b^5*tan(1/2*d*x
+ 1/2*c)^5 + 60*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 - 36*a^9*tan(1/2*d*x + 1/2*
c)^3 + 152*a^7*b^2*tan(1/2*d*x + 1/2*c)^3 - 236*a^5*b^4*tan(1/2*d*x + 1/2*
c)^3 + 120*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 + 18*a^9*tan(1/2*d*x + 1/2*c) +
42*a^8*b*tan(1/2*d*x + 1/2*c) - 24*a^7*b^2*tan(1/2*d*x + 1/2*c) - 117*a^6*
b^3*tan(1/2*d*x + 1/2*c) - 24*a^5*b^4*tan(1/2*d*x + 1/2*c) + 105*a^4*b^5*t
an(1/2*d*x + 1/2*c) + 60*a^3*b^6*tan(1/2*d*x + 1/2*c))/((a^6*b^4 - 3*a^4*b
^6 + 3*a^2*b^8 - b^10)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^
2 - a - b)^3) + 12*a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^5 - 12*a*log(abs
(tan(1/2*d*x + 1/2*c) - 1))/b^5 + 6*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1
/2*c)^2 - 1)*b^4))/d
```

Mupad [B] (verification not implemented)

Time = 19.42 (sec) , antiderivative size = 7476, normalized size of antiderivative = 23.66

$$\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^6*(a + b/cos(c + d*x))^4),x)
```

output
```
((tan(c/2 + (d*x)/2)^3*(12*a^7*b - 72*a^8 - 18*b^8 + 72*a^2*b^6 + 60*a^3*b
^5 - 273*a^4*b^4 - 47*a^5*b^3 + 236*a^6*b^2))/(3*b^4*(a + b)^2*(a - b)^3)
+ (tan(c/2 + (d*x)/2)^5*(12*a^7*b + 72*a^8 + 18*b^8 - 72*a^2*b^6 + 60*a^3*
b^5 + 273*a^4*b^4 - 47*a^5*b^3 - 236*a^6*b^2))/(3*b^4*(a + b)^3*(a - b)^2)
 - (tan(c/2 + (d*x)/2)*(2*a*b^6 - 4*a^6*b - 8*a^7 + 2*b^7 - 6*a^2*b^5 - 26
*a^3*b^4 + 11*a^4*b^3 + 24*a^5*b^2))/(b^4*(a + b)*(a - b)^3) + (tan(c/2 +
(d*x)/2)^7*(2*a*b^6 + 4*a^6*b - 8*a^7 - 2*b^7 + 6*a^2*b^5 - 26*a^3*b^4 - 1
1*a^4*b^3 + 24*a^5*b^2))/(b^4*(a + b)^3*(a - b)))/(d*(3*a*b^2 + 3*a^2*b -
tan(c/2 + (d*x)/2)^4*(6*a*b^2 - 6*a^3) - tan(c/2 + (d*x)/2)^2*(6*a^2*b + 4
*a^3 - 2*b^3) - tan(c/2 + (d*x)/2)^6*(4*a^3 - 6*a^2*b + 2*b^3) + a^3 + b^3
 + tan(c/2 + (d*x)/2)^8*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) + (a*atan(((a*((
8*tan(c/2 + (d*x)/2)*(128*a^16 - 128*a^15*b + 64*a^2*b^14 - 128*a^3*b^13 +
 80*a^4*b^12 + 768*a^5*b^11 - 824*a^6*b^10 - 1920*a^7*b^9 + 2025*a^8*b^8 +
 2560*a^9*b^7 - 2600*a^10*b^6 - 1920*a^11*b^5 + 1920*a^12*b^4 + 768*a^13*b
^3 - 768*a^14*b^2))/(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15
 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^1
0*b^9 - a^11*b^8) - (4*a*((16*(8*a*b^23 - 20*a^2*b^22 - 36*a^3*b^21 + 95*a
^4*b^20 + 73*a^5*b^19 - 193*a^6*b^18 - 87*a^7*b^17 + 217*a^8*b^16 + 63*a^9
*b^15 - 143*a^10*b^14 - 25*a^11*b^13 + 52*a^12*b^12 + 4*a^13*b^11 - 8*a^14
*b^10))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a^4*b^19 + 10*a^5...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 3632, normalized size of antiderivative = 11.49

$$\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^6/(a+b*sec(d*x+c))^4,x)
```

output
```
(144*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/s
qrt( - a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**2*a**10*b - 504*sqrt( - a*
*2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b
**2))*cos(c + d*x)*sin(c + d*x)**2*a**8*b**3 + 630*sqrt( - a**2 + b**2)*at
an((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c +
 d*x)*sin(c + d*x)**2*a**6*b**5 - 360*sqrt( - a**2 + b**2)*atan((tan((c +
d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*sin(c +
 d*x)**2*a**4*b**7 - 144*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - t
an((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*a**10*b + 456*sqrt(
- a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2
 + b**2))*cos(c + d*x)*a**8*b**3 - 462*sqrt( - a**2 + b**2)*atan((tan((c +
 d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*a**6*b
**5 + 150*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)
*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*a**4*b**7 + 120*sqrt( - a**2 + b**2
)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos
(c + d*x)*a**2*b**9 - 48*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - t
an((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*sin(c + d*x)**4*a**11 + 168*sqrt(
 - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**
2 + b**2))*sin(c + d*x)**4*a**9*b**2 - 210*sqrt( - a**2 + b**2)*atan((tan(
(c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*sin(c + d*x)...
```

3.515 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 259

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \frac{\text{arctanh}(\sin(c+dx))}{b^4d} - \frac{a(2a^6-7a^4b^2+8a^2b^4-8b^6)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}b^4(a+b)^{7/2}d} - \frac{a^2\sec^2(c+dx)\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{a^3(3a^2-8b^2)\tan(c+dx)}{6b^3\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} - \frac{a^2(9a^4-28a^2b^2+34b^4)\tan(c+dx)}{6b^3\,(a^2-b^2)^3\,d(a+b\sec(c+dx))}$$

output
```
arctanh(sin(d*x+c))/b^4/d-a*(2*a^6-7*a^4*b^2+8*a^2*b^4-8*b^6)*arctanh((a-b
)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/b^4/(a+b)^(7/2)/d-1/3*
a^2*sec(d*x+c)^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^3+1/6*a^3*(3*a^
2-8*b^2)*tan(d*x+c)/b^3/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2-1/6*a^2*(9*a^4-28
*a^2*b^2+34*b^4)*tan(d*x+c)/b^3/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 3.81 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.97

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$
$$= \frac{\frac{6a(2a^6-7a^4b^2+8a^2b^4-8b^6)\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} - 6\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+6\log\left(\cos\right.}{}$$

input
```
Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^4,x]
```

output
```
((6*a*(2*a^6 - 7*a^4*b^2 + 8*a^2*b^4 - 8*b^6)*ArcTanh[((-a + b)*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2) - 6*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] + 6*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - (a^2*b*(11*a
^4*b^2 - 32*a^2*b^4 + 36*b^6 + 15*a*b*(a^4 - 3*a^2*b^2 + 4*b^4)*Cos[c + d*
x] + a^2*(6*a^4 - 17*a^2*b^2 + 26*b^4)*Cos[c + d*x]^2)*Sin[c + d*x])/((a -
 b)^3*(a + b)^3*(b + a*Cos[c + d*x])^3))/(6*b^4*d)
```

Rubi [A] (verified)

Time = 1.89 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.21, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4332, 3042, 4578, 3042, 4568, 27, 3042, 4486, 3042, 4257, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$
$$\Big\downarrow \text{4332}$$

$$-\frac{\int \frac{\sec^2(c+dx)(2a^2-3b\sec(c+dx)a-3(a^2-b^2)\sec^2(c+dx))}{(a+b\sec(c+dx))^3}dx}{3b(a^2-b^2)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^3}$$

$\downarrow$ 3042

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(2a^2-3b\csc\left(c+dx+\frac{\pi}{2}\right)a-3(a^2-b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3b(a^2-b^2)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^3}$$

$\downarrow$ 4578

$$-\frac{\frac{\int \frac{\sec(c+dx)\left(2b\left(3a^2-8b^2\right)a^2+\left(3a^4-10b^2a^2+12b^4\right)\sec(c+dx)a-6b\left(a^2-b^2\right)^2\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2b^2(a^2-b^2)}-\frac{a^3\left(3a^2-8b^2\right)\tan(c+dx)}{2b^2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3b(a^2-b^2)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^3}$$

$\downarrow$ 3042

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2b\left(3a^2-8b^2\right)a^2+\left(3a^4-10b^2a^2+12b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)a-6b\left(a^2-b^2\right)^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2b^2(a^2-b^2)}-\frac{a^3\left(3a^2-8b^2\right)\tan(c+dx)}{2b^2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3b(a^2-b^2)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^3}$$

$\downarrow$ 4568

$$-\frac{\frac{\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int \frac{3\sec(c+dx)\left(2b\sec(c+dx)\left(a^2-b^2\right)^3+ab^2\left(a^4-2b^2a^2+6b^4\right)\right)}{a+b\sec(c+dx)}dx}{b\left(a^2-b^2\right)}}{2b^2(a^2-b^2)}-\frac{a^3\left(3a^2-8b^2\right)\tan(c+dx)}{2b^2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3b(a^2-b^2)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^3}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{3\int\frac{\sec(c+dx)\left(2b\sec(c+dx)\left(a^2-b^2\right)^3+ab^2\left(a^4-2b^2a^2+6b^4\right)\right)}{a+b\sec(c+dx)}dx}{b\left(a^2-b^2\right)}}{2b^2(a^2-b^2)}-\frac{a^3\left(3a^2-8b^2\right)\tan(c+dx)}{2b^2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2b\csc\left(c+dx+\frac{\pi}{2}\right)\left(a^2-b^2\right)^3+ab^2\left(a^4-2b^2a^2+6b^4\right)\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b\left(a^2-b^2\right)}}{2b^2(a^2-b^2)}-\frac{a^3\left(3a^2-8b^2\right)\tan(c+dx)}{2b^2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 4486

$$-\frac{\frac{\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{3\left(2\left(a^2-b^2\right)^3\int\sec(c+dx)dx-a\left(2a^6-7a^4b^2+8a^2b^4-8b^6\right)\int\frac{\sec(c+dx)}{a+b\sec(c+dx)}dx\right)}{b\left(a^2-b^2\right)}}{2b^2(a^2-b^2)}-\frac{a^3\left(3a^2-8b^2\right)\tan(c+dx)}{2b^2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{3\left(2\left(a^2-b^2\right)^3\int\csc\left(c+dx+\frac{\pi}{2}\right)dx-a\left(2a^6-7a^4b^2+8a^2b^4-8b^6\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{b\left(a^2-b^2\right)}}{2b^2(a^2-b^2)}-\frac{a^3\left(3a^2-8b^2\right)\tan(c+dx)}{2b^2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 4257

$$-\frac{\frac{\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{3\left(\frac{2\left(a^2-b^2\right)^3\text{arctanh}(\sin(c+dx))}{d}-a\left(2a^6-7a^4b^2+8a^2b^4-8b^6\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{b\left(a^2-b^2\right)}}{2b^2(a^2-b^2)}-\frac{a^3\left(3a^2-8b^2\right)\tan(c+dx)}{2b^2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 4318$$

$$-\frac{\frac{\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{3\left(\frac{2\left(a^2-b^2\right)^3\text{arctanh}(\sin(c+dx))}{d}-\frac{a\left(2a^6-7a^4b^2+8a^2b^4-8b^6\right)\int\frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b}\right)}{b\left(a^2-b^2\right)}}{2b^2(a^2-b^2)}-\frac{a^3(3a^2-8b^2)\tan(\cdots}{2b^2d(a^2-b^2)(a+b\sec\cdots}}{3b\left(a^2-b^2\right)}$$
$$\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{3\left(\frac{2\left(a^2-b^2\right)^3\text{arctanh}(\sin(c+dx))}{d}-\frac{a\left(2a^6-7a^4b^2+8a^2b^4-8b^6\right)\int\frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b}\right)}{b\left(a^2-b^2\right)}}{2b^2(a^2-b^2)}-\frac{a^3(3a^2-8b^2)\cdots}{2b^2d(a^2-b^2)(a+\cdots}}{3b\left(a^2-b^2\right)}$$
$$\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 3138$$

$$-\frac{\frac{\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{3\left(\frac{2\left(a^2-b^2\right)^3\text{arctanh}(\sin(c+dx))}{d}-\frac{2a\left(2a^6-7a^4b^2+8a^2b^4-8b^6\right)\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)\cdots}{bd}\right.\cdots}{b\left(a^2-b^2\right)}}{2b^2(a^2-b^2)}\cdots}{3b\left(a^2-b^2\right)}$$
$$\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 221$$

$$-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-$$
$$\frac{\frac{\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{3\left(\frac{2\left(a^2-b^2\right)^3\text{arctanh}(\sin(c+dx))}{d}-\frac{2a\left(2a^6-7a^4b^2+8a^2b^4-8b^6\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}\right)}{b\left(a^2-b^2\right)}}{2b^2(a^2-b^2)}-\frac{a^3(3\cdots}{2b^2d(a^2\cdots}}{3b\left(a^2-b^2\right)}$$

input

```
Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^4,x]
```

output
```
-1/3*(a^2*Sec[c + d*x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x
])^3) - (-1/2*(a^3*(3*a^2 - 8*b^2)*Tan[c + d*x])/(b^2*(a^2 - b^2)*d*(a + b
*Sec[c + d*x])^2) + ((-3*((2*(a^2 - b^2)^3*ArcTanh[Sin[c + d*x]])/d - (2*a
*(2*a^6 - 7*a^4*b^2 + 8*a^2*b^4 - 8*b^6)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*d)))/(b*(a^2 - b^2)) + (a^2*(
9*a^4 - 28*a^2*b^2 + 34*b^4)*Tan[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d
*x])))/(2*b^2*(a^2 - b^2)))/(3*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

rule 4568
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cot[e + f*x]*((a + b*Csc[e + f*x]
)^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))
 Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m
+ 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x],
x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^
2, 0]
```

rule 4578
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x
])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - Simp[1/(b^2*(m + 1)*(a^2 - b^
2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*((-a)*(b
*B - a*C) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
 b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
]
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.48

method	result
derivativedivides	$\frac{-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^4}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^4}+\frac{2a\left(\frac{\left(2a^4-b\,a^3-6a^2b^2+4a\,b^3+12b^4\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}-\frac{2\left(3a^4-11a^2b^2+18b^4\right)ab}{3\left(a^2+2ab+b^2\right)\left(a^2-\right.}\right.}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\right.\right.}}{d}$
default	$\frac{-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^4}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^4}+\frac{2a\left(\frac{\left(2a^4-b\,a^3-6a^2b^2+4a\,b^3+12b^4\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}-\frac{2\left(3a^4-11a^2b^2+18b^4\right)ab}{3\left(a^2+2ab+b^2\right)\left(a^2-\right.}\right.}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\right.\right.}}{d}$
risch	Expression too large to display

input
```
int(sec(d*x+c)^5/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^4*ln(tan(1/2*d*x+1/2*c)-1)+1/b^4*ln(tan(1/2*d*x+1/2*c)+1)+2/b^4*
a*((1/2*(2*a^4-a^3*b-6*a^2*b^2+4*a*b^3+12*b^4)*a*b/(a-b)/(a^3+3*a^2*b+3*a*
b^2+b^3)*tan(1/2*d*x+1/2*c)^5-2/3*(3*a^4-11*a^2*b^2+18*b^4)*a*b/(a^2+2*a*b
+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/2*(2*a^4+a^3*b-6*a^2*b^2-4*a*
b^3+12*b^4)*a*b/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1
/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3-1/2*(2*a^6-7*a^4*b^2+8*a^2
*b^4-8*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b
)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. 2(244) = 488.

Time = 1.55 (sec) , antiderivative size = 1822, normalized size of antiderivative = 7.03

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
[1/12*(3*(2*a^7*b^3 - 7*a^5*b^5 + 8*a^3*b^7 - 8*a*b^9 + (2*a^10 - 7*a^8*b^
2 + 8*a^6*b^4 - 8*a^4*b^6)*cos(d*x + c)^3 + 3*(2*a^9*b - 7*a^7*b^3 + 8*a^5
*b^5 - 8*a^3*b^7)*cos(d*x + c)^2 + 3*(2*a^8*b^2 - 7*a^6*b^4 + 8*a^4*b^6 -
8*a^2*b^8)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 -
2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c
) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 6*(a^8
*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11 + (a^11 - 4*a^9*b^2 + 6*a^
7*b^4 - 4*a^5*b^6 + a^3*b^8)*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^
6*b^5 - 4*a^4*b^7 + a^2*b^9)*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a
^5*b^6 - 4*a^3*b^8 + a*b^10)*cos(d*x + c))*log(sin(d*x + c) + 1) - 6*(a^8*
b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11 + (a^11 - 4*a^9*b^2 + 6*a^7
*b^4 - 4*a^5*b^6 + a^3*b^8)*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6
*b^5 - 4*a^4*b^7 + a^2*b^9)*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^
5*b^6 - 4*a^3*b^8 + a*b^10)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(11*a
^8*b^3 - 43*a^6*b^5 + 68*a^4*b^7 - 36*a^2*b^9 + (6*a^10*b - 23*a^8*b^3 + 4
3*a^6*b^5 - 26*a^4*b^7)*cos(d*x + c)^2 + 15*(a^9*b^2 - 4*a^7*b^4 + 7*a^5*b
^6 - 4*a^3*b^8)*cos(d*x + c))*sin(d*x + c))/((a^11*b^4 - 4*a^9*b^6 + 6*a^7
*b^8 - 4*a^5*b^10 + a^3*b^12)*d*cos(d*x + c)^3 + 3*(a^10*b^5 - 4*a^8*b^7 +
 6*a^6*b^9 - 4*a^4*b^11 + a^2*b^13)*d*cos(d*x + c)^2 + 3*(a^9*b^6 - 4*a^7*
b^8 + 6*a^5*b^10 - 4*a^3*b^12 + a*b^14)*d*cos(d*x + c) + (a^8*b^7 - 4*a...
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx=\int \frac{\sec^5{(c+dx)}}{(a+b\sec{(c+dx)})^4}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**5/(a + b*sec(c + d*x))**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(244) = 488$.

Time = 0.22 (sec) , antiderivative size = 559, normalized size of antiderivative = 2.16

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(3*(2*a^7 - 7*a^5*b^2 + 8*a^3*b^4 - 8*a*b^6)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
 1/2*c))/sqrt(-a^2 + b^2)))/((a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*sqrt
(-a^2 + b^2)) + (6*a^8*tan(1/2*d*x + 1/2*c)^5 - 15*a^7*b*tan(1/2*d*x + 1/2
*c)^5 - 6*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 + 45*a^5*b^3*tan(1/2*d*x + 1/2*c)
^5 - 6*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 60*a^3*b^5*tan(1/2*d*x + 1/2*c)^5
+ 36*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 - 12*a^8*tan(1/2*d*x + 1/2*c)^3 + 56*a
^6*b^2*tan(1/2*d*x + 1/2*c)^3 - 116*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 + 72*a^
2*b^6*tan(1/2*d*x + 1/2*c)^3 + 6*a^8*tan(1/2*d*x + 1/2*c) + 15*a^7*b*tan(1
/2*d*x + 1/2*c) - 6*a^6*b^2*tan(1/2*d*x + 1/2*c) - 45*a^5*b^3*tan(1/2*d*x
+ 1/2*c) - 6*a^4*b^4*tan(1/2*d*x + 1/2*c) + 60*a^3*b^5*tan(1/2*d*x + 1/2*c
) + 36*a^2*b^6*tan(1/2*d*x + 1/2*c))/((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b
^9)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3) + 3*l
og(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 3*log(abs(tan(1/2*d*x + 1/2*c) - 1
))/b^4)/d
```

Mupad [B] (verification not implemented)

Time = 21.72 (sec) , antiderivative size = 7222, normalized size of antiderivative = 27.88

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4} \, dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^4),x)
```

output
```
- ((tan(c/2 + (d*x)/2)^5*(2*a^6 - a^5*b + 12*a^2*b^4 + 4*a^3*b^3 - 6*a^4*b
^2))/((a*b^3 - b^4)*(a + b)^3) - (4*tan(c/2 + (d*x)/2)^3*(3*a^6 + 18*a^2*b
^4 - 11*a^4*b^2))/(3*(a + b)^2*(b^5 - 2*a*b^4 + a^2*b^3)) + (tan(c/2 + (d*
x)/2)*(a^5*b + 2*a^6 + 12*a^2*b^4 - 4*a^3*b^3 - 6*a^4*b^2))/((a + b)*(3*a*
b^5 - b^6 - 3*a^2*b^4 + a^3*b^3)))/(d*(tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a
^2*b - 3*a^3 + 3*b^3) - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 -
3*b^3) + 3*a*b^2 + 3*a^2*b + a^3 + b^3 - tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3
*a^2*b + a^3 - b^3))) - (atan((((((8*(16*a*b^20 - 4*b^21 + 12*a^2*b^19 - 6
4*a^3*b^18 - 20*a^4*b^17 + 110*a^5*b^16 + 30*a^6*b^15 - 110*a^7*b^14 - 30*
a^8*b^13 + 70*a^9*b^12 + 14*a^10*b^11 - 26*a^11*b^10 - 2*a^12*b^9 + 4*a^13
*b^8))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^1
5 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11
*b^9) - (8*tan(c/2 + (d*x)/2)*(8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^
4*b^18 + 120*a^5*b^17 - 120*a^6*b^16 - 160*a^7*b^15 + 160*a^8*b^14 + 120*a
^9*b^13 - 120*a^10*b^12 - 48*a^11*b^11 + 48*a^12*b^10 + 8*a^13*b^9 - 8*a^1
4*b^8))/(b^4*(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a
^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a
^11*b^6)))/b^4 - (8*tan(c/2 + (d*x)/2)*(8*a^14 - 8*a^13*b - 8*a*b^13 + 4*b
^14 + 44*a^2*b^12 + 48*a^3*b^11 - 92*a^4*b^10 - 120*a^5*b^9 + 156*a^6*b^8
+ 160*a^7*b^7 - 164*a^8*b^6 - 120*a^9*b^5 + 117*a^10*b^4 + 48*a^11*b^3 ...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 2767, normalized size of antiderivative = 10.68

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^5/(a+b*sec(d*x+c))^4,x)
```

output
```
( - 12*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)
/sqrt( - a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**2*a**10 + 42*sqrt( - a**
2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b*
*2))*cos(c + d*x)*sin(c + d*x)**2*a**8*b**2 - 48*sqrt( - a**2 + b**2)*atan
((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d
*x)*sin(c + d*x)**2*a**6*b**4 + 48*sqrt( - a**2 + b**2)*atan((tan((c + d*x
)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*sin(c + d*
x)**2*a**4*b**6 + 12*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((
c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*a**10 - 6*sqrt( - a**2 +
 b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2)
)*cos(c + d*x)*a**8*b**2 - 78*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*
a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*a**6*b**4 + 96*
sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(
- a**2 + b**2))*cos(c + d*x)*a**4*b**6 - 144*sqrt( - a**2 + b**2)*atan((ta
n((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*
a**2*b**8 - 36*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*
x)/2)*b)/sqrt( - a**2 + b**2))*sin(c + d*x)**2*a**9*b + 126*sqrt( - a**2 +
 b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2)
)*sin(c + d*x)**2*a**7*b**3 - 144*sqrt( - a**2 + b**2)*atan((tan((c + d*x)
/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*sin(c + d*x)**2*a**5*...
```

3.516 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 222

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx = -\frac{b(3a^2+2b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{a^2\sec(c+dx)\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} - \frac{a^2(2a^2-7b^2)\tan(c+dx)}{6b^2\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} + \frac{a(2a^4-5a^2b^2+18b^4)\tan(c+dx)}{6b^2\,(a^2-b^2)^3\,d(a+b\sec(c+dx))}$$

output
```
-b*(3*a^2+2*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)
^(7/2)/(a+b)^(7/2)/d-1/3*a^2*sec(d*x+c)*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(
d*x+c))^3-1/6*a^2*(2*a^2-7*b^2)*tan(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*x+
c))^2+1/6*a*(2*a^4-5*a^2*b^2+18*b^4)*tan(d*x+c)/b^2/(a^2-b^2)^3/d/(a+b*sec
(d*x+c))
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.71

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$
$$=\frac{\frac{6b(3a^2+2b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}}+\frac{a(2a^4-5a^2b^2+18b^4+3ab(a^2+9b^2)\cos(c+dx)+a^2(4a^2+11b^2)\cos^2(c+dx))\sin(c+dx)}{(a-b)^3(a+b)^3(b+a\cos(c+dx))^3}}{6d}$$

input
```
Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^4,x]
```

output
```
((6*b*(3*a^2 + 2*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
)/(a^2 - b^2)^(7/2) + (a*(2*a^4 - 5*a^2*b^2 + 18*b^4 + 3*a*b*(a^2 + 9*b^2)
*Cos[c + d*x] + a^2*(4*a^2 + 11*b^2)*Cos[c + d*x]^2)*Sin[c + d*x])/((a - b
)^3*(a + b)^3*(b + a*Cos[c + d*x])^3))/(6*d)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.20, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4332, 3042, 4568, 3042, 4491, 27, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$
$$\Big\downarrow \text{4332}$$
$$-\frac{\int \frac{\sec(c+dx)\left(a^2-3b\sec(c+dx)a-\left(2a^2-3b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}dx}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a^2-3b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(3b^2-2a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{4568}$$

$$-\frac{\frac{a^2\left(2a^2-7b^2\right)\tan(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\int \frac{\sec(c+dx)\left(2ab\left(a^2-6b^2\right)+\left(2a^4-3b^2a^2+6b^4\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{a^2\left(2a^2-7b^2\right)\tan(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2ab\left(a^2-6b^2\right)+\left(2a^4-3b^2a^2+6b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{4491}$$

$$-\frac{\frac{a^2\left(2a^2-7b^2\right)\tan(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\frac{a\left(2a^4-5a^2b^2+18b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int \frac{3b^3\left(3a^2+2b^2\right)\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{a^2\left(2a^2-7b^2\right)\tan(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\frac{a\left(2a^4-5a^2b^2+18b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{3b^3\left(3a^2+2b^2\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{a^2(2a^2-7b^2)\tan(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\frac{a\left(2a^4-5a^2b^2+18b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{3b^3\left(3a^2+2b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{2b(a^2-b^2)}}{3b\,(a^2-b^2)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{3bd\,(a^2-b^2)\,(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 4318$$

$$-\frac{\frac{a^2(2a^2-7b^2)\tan(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\frac{a\left(2a^4-5a^2b^2+18b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{3b^2\left(3a^2+2b^2\right)\int\frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a^2-b^2}}{2b(a^2-b^2)}}{3b\,(a^2-b^2)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{3bd\,(a^2-b^2)\,(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{a^2(2a^2-7b^2)\tan(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\frac{a\left(2a^4-5a^2b^2+18b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{3b^2\left(3a^2+2b^2\right)\int\frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a^2-b^2}}{2b(a^2-b^2)}}{3b\,(a^2-b^2)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{3bd\,(a^2-b^2)\,(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 3138$$

$$-\frac{\frac{a^2(2a^2-7b^2)\tan(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\frac{a\left(2a^4-5a^2b^2+18b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{6b^2\left(3a^2+2b^2\right)\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{d\left(a^2-b^2\right)}}{2b(a^2-b^2)}}{3b\,(a^2-b^2)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{3bd\,(a^2-b^2)\,(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 221$$

$$-\frac{a^2\tan(c+dx)\sec(c+dx)}{3bd\,(a^2-b^2)\,(a+b\sec(c+dx))^3}-\frac{\frac{a^2(2a^2-7b^2)\tan(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\frac{a\left(2a^4-5a^2b^2+18b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{6b^3\left(3a^2+2b^2\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}}{2b(a^2-b^2)}}{3b\,(a^2-b^2)}$$

input
```
Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^4,x]
```

output
```
-1/3*(a^2*Sec[c + d*x]*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])
^3) - ((a^2*(2*a^2 - 7*b^2)*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c
+ d*x])^2) - ((-6*b^3*(3*a^2 + 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2
])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2)*d) + (a*(2*a^4 - 5*a
^2*b^2 + 18*b^4)*Tan[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x])))/(2*b*
(a^2 - b^2)))/(3*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4491
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1
/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp
[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; Free
Q[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m
, -1]
```

rule 4568
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cot[e + f*x]*((a + b*Csc[e + f*x]
)^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))
 Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m
+ 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x],
x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^
2, 0]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{\frac{-\frac{(2a^2+3ab+6b^2)a\tan(\frac{dx}{2}+\frac{c}{2})^5}{(a-b)(a^3+3a^2b+3a\,b^2+b^3)}+\frac{4(a^2+9b^2)a\tan(\frac{dx}{2}+\frac{c}{2})^3}{3(a^2+2ab+b^2)(a^2-2ab+b^2)}-\frac{(2a^2-3ab+6b^2)a\tan(\frac{dx}{2}+\frac{c}{2})}{(a+b)(a^3-3a^2b+3a\,b^2-b^3)}}{(\tan(\frac{dx}{2}+\frac{c}{2})^2a-\tan(\frac{dx}{2}+\frac{c}{2})^2b-a-b)^3}-\frac{b(3a^2+2b^2)\operatorname{arctanh}(\frac{(a-}{\sqrt{}}}{(a^6-3a^4b^2+3a^2b^4-b^6)}}{d}$
default	$\frac{\frac{-\frac{(2a^2+3ab+6b^2)a\tan(\frac{dx}{2}+\frac{c}{2})^5}{(a-b)(a^3+3a^2b+3a\,b^2+b^3)}+\frac{4(a^2+9b^2)a\tan(\frac{dx}{2}+\frac{c}{2})^3}{3(a^2+2ab+b^2)(a^2-2ab+b^2)}-\frac{(2a^2-3ab+6b^2)a\tan(\frac{dx}{2}+\frac{c}{2})}{(a+b)(a^3-3a^2b+3a\,b^2-b^3)}}{(\tan(\frac{dx}{2}+\frac{c}{2})^2a-\tan(\frac{dx}{2}+\frac{c}{2})^2b-a-b)^3}-\frac{b(3a^2+2b^2)\operatorname{arctanh}(\frac{(a-}{\sqrt{}}}{(a^6-3a^4b^2+3a^2b^4-b^6)}}{d}$
risch	$-\frac{i(9a^4b\,e^{5i(dx+c)}+6a^2b^3e^{5i(dx+c)}+45a^3b^2e^{4i(dx+c)}+30a\,b^4e^{4i(dx+c)}+24a^4b\,e^{3i(dx+c)}+82a^2b^3e^{3i(dx+c)}+44b^5e^{3i(d}}{3(-a^2+b^2)^3d(e^{2i(dx+c)}a-}$

input
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(-1/2*(2*a^2+3*a*b+6*b^2)*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2
*d*x+1/2*c)^5+2/3*(a^2+9*b^2)*a/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*
x+1/2*c)^3-1/2*(2*a^2-3*a*b+6*b^2)*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1
/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3-b*(3*
a^2+2*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)
*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. 2(207) = 414.

Time = 0.15 (sec) , antiderivative size = 903, normalized size of antiderivative = 4.07

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
[-1/12*(3*(3*a^2*b^4 + 2*b^6 + (3*a^5*b + 2*a^3*b^3)*cos(d*x + c)^3 + 3*(3
*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c)^2 + 3*(3*a^3*b^3 + 2*a*b^5)*cos(d*x + c
))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2
+ 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*
cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(2*a^7 - 7*a^5*b^2 + 23*a^
3*b^4 - 18*a*b^6 + (4*a^7 + 7*a^5*b^2 - 11*a^3*b^4)*cos(d*x + c)^2 + 3*(a^
6*b + 8*a^4*b^3 - 9*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^
2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*
b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a
^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6
*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d), -1/6*(3*(3*a^2*b^4 + 2*b^6 + (3*a
^5*b + 2*a^3*b^3)*cos(d*x + c)^3 + 3*(3*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c)^
2 + 3*(3*a^3*b^3 + 2*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a
^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (2*a^7 - 7*a^
5*b^2 + 23*a^3*b^4 - 18*a*b^6 + (4*a^7 + 7*a^5*b^2 - 11*a^3*b^4)*cos(d*x +
 c)^2 + 3*(a^6*b + 8*a^4*b^3 - 9*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^
11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^
10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(
a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^
8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx=\int \frac{\sec^4{(c+dx)}}{(a+b\sec{(c+dx)})^4}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.82

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx = -\frac{\frac{3\,(3\,a^2b+2\,b^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{-a^2+b^2}}+\frac{6\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{}}{}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/3*(3*(3*a^2*b + 2*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b
) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 +
b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) + (6*a^5*tan
(1/2*d*x + 1/2*c)^5 - 3*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 6*a^3*b^2*tan(1/2*d
*x + 1/2*c)^5 - 27*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 18*a*b^4*tan(1/2*d*x +
 1/2*c)^5 - 4*a^5*tan(1/2*d*x + 1/2*c)^3 - 32*a^3*b^2*tan(1/2*d*x + 1/2*c)
^3 + 36*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 6*a^5*tan(1/2*d*x + 1/2*c) + 3*a^4*
b*tan(1/2*d*x + 1/2*c) + 6*a^3*b^2*tan(1/2*d*x + 1/2*c) + 27*a^2*b^3*tan(1
/2*d*x + 1/2*c) + 18*a*b^4*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2
*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^
3))/d
```

Mupad [B] (verification not implemented)

Time = 13.51 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.70

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$
$$= \frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(2a^3+3a^2b+6ab^2)}{(a+b)^3(a-b)} - \frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(a^3+9ab^2)}{3(a+b)^2(a^2-2ab+b^2)} + \frac{\tan\left(\frac{c}{2}+\right.}{(a+b)}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(-3a^3-3a^2b+3ab^2+3b^3) - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(-3a^3+3a^2b+3ab^2-3b^3)+3ab^2\right.}$$
$$- \frac{b\,\mathrm{atanh}\left(\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(3a^2+2b^2)(2a-2b)(a^3-3a^2b+3ab^2-b^3)}{2(3a^2b+2b^3)\sqrt{a+b}\,(a-b)^{7/2}}\right)(3a^2+2b^2)}{d\,(a+b)^{7/2}(a-b)^{7/2}}$$

input
```
int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^4),x)
```

output
```
((tan(c/2 + (d*x)/2)^5*(6*a*b^2 + 3*a^2*b + 2*a^3))/((a + b)^3*(a - b)) -
(4*tan(c/2 + (d*x)/2)^3*(9*a*b^2 + a^3))/(3*(a + b)^2*(a^2 - 2*a*b + b^2))
 + (tan(c/2 + (d*x)/2)*(6*a*b^2 - 3*a^2*b + 2*a^3))/((a + b)*(3*a*b^2 - 3*
a^2*b + a^3 - b^3)))/(d*(tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 +
 3*b^3) - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) + 3*a*b
^2 + 3*a^2*b + a^3 + b^3 - tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 -
 b^3))) - (b*atanh((b*tan(c/2 + (d*x)/2)*(3*a^2 + 2*b^2)*(2*a - 2*b)*(3*a*
b^2 - 3*a^2*b + a^3 - b^3))/(2*(3*a^2*b + 2*b^3)*(a + b)^(1/2)*(a - b)^(7/
2)))*(3*a^2 + 2*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1065, normalized size of antiderivative = 4.80

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c))^4,x)
```

output
```
( - 18*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)
/sqrt( - a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**2*a**5*b - 12*sqrt( - a*
*2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b
**2))*cos(c + d*x)*sin(c + d*x)**2*a**3*b**3 + 18*sqrt( - a**2 + b**2)*ata
n((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c +
d*x)*a**5*b + 66*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c +
d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*a**3*b**3 + 36*sqrt( - a**2
+ b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2
))*cos(c + d*x)*a*b**5 - 54*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a
- tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*sin(c + d*x)**2*a**4*b**2 - 36
*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(
 - a**2 + b**2))*sin(c + d*x)**2*a**2*b**4 + 54*sqrt( - a**2 + b**2)*atan(
(tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*a**4*b**2
+ 54*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/s
qrt( - a**2 + b**2))*a**2*b**4 + 12*sqrt( - a**2 + b**2)*atan((tan((c + d*
x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*b**6 - 3*cos(c + d*x)*
sin(c + d*x)*a**6*b - 24*cos(c + d*x)*sin(c + d*x)*a**4*b**3 + 27*cos(c +
d*x)*sin(c + d*x)*a**2*b**5 + 4*sin(c + d*x)**3*a**7 + 7*sin(c + d*x)**3*a
**5*b**2 - 11*sin(c + d*x)**3*a**3*b**4 - 6*sin(c + d*x)*a**7 - 12*sin(c +
 d*x)*a**3*b**4 + 18*sin(c + d*x)*a*b**6)/(6*d*(cos(c + d*x)*sin(c + d*...
```

3.517 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 206

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \frac{a(a^2+4b^2)\,\mathrm{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{a^2\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{a(a^2-6b^2)\tan(c+dx)}{6b\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} + \frac{(a^4-10a^2b^2-6b^4)\tan(c+dx)}{6b\,(a^2-b^2)^3\,d(a+b\sec(c+dx))}$$

output
```
a*(a^2+4*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7
/2)/(a+b)^(7/2)/d-1/3*a^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^3+1/6*
a*(a^2-6*b^2)*tan(d*x+c)/b/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2+1/6*(a^4-10*a^
2*b^2-6*b^4)*tan(d*x+c)/b/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.80

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \frac{-\frac{6a(a^2+4b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \frac{(b(a^4-10a^2b^2-6b^4)+3a(a^4-9a^2b^2-2b^4)\cos(c+dx)-a^2b(13a^2+2b^2)\cos^2(c+dx))\sin(}{(a-b)^3(a+b)^3(b+a\cos(c+dx))^3}}{6d}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^4,x]
```

output
```
((-6*a*(a^2 + 4*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])
/(a^2 - b^2)^(7/2) + ((b*(a^4 - 10*a^2*b^2 - 6*b^4) + 3*a*(a^4 - 9*a^2*b^2
 - 2*b^4)*Cos[c + d*x] - a^2*b*(13*a^2 + 2*b^2)*Cos[c + d*x]^2)*Sin[c + d*
x])/((a - b)^3*(a + b)^3*(b + a*Cos[c + d*x])^3))/(6*d)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.19, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4326, 25, 3042, 4491, 25, 3042, 4491, 27, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \ 4326$$

$$-\frac{\int -\frac{\sec(c+dx)\left(3ab+\left(a^2-3b^2\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^3}dx}{3b\left(a^2-b^2\right)} - \frac{a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\sec(c+dx)\left(3ab+\left(a^2-3b^2\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^3}dx}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3ab+\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 4491$$

$$\frac{\frac{a\left(a^2-6b^2\right)\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\int -\frac{\sec(c+dx)\left(2b\left(2a^2+3b^2\right)+a\left(a^2-6b^2\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2\left(a^2-b^2\right)}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\sec(c+dx)\left(2b\left(2a^2+3b^2\right)+a\left(a^2-6b^2\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2\left(a^2-b^2\right)}+\frac{a\left(a^2-6b^2\right)\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2b\left(2a^2+3b^2\right)+a\left(a^2-6b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2\left(a^2-b^2\right)}+\frac{a\left(a^2-6b^2\right)\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 4491$$

$$\frac{\frac{\frac{\left(a^4-10a^2b^2-6b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int -\frac{3ab\left(a^2+4b^2\right)\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}}{2\left(a^2-b^2\right)}+\frac{a\left(a^2-6b^2\right)\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3ab(a^2+4b^2)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}+\frac{(a^4-10a^2b^2-6b^4)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2(a^2-b^2)}+\frac{a(a^2-6b^2)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3b(a^2-b^2)}-\frac{a^2\tan(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{3ab(a^2+4b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{(a^4-10a^2b^2-6b^4)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2(a^2-b^2)}+\frac{a(a^2-6b^2)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3b(a^2-b^2)}-\frac{a^2\tan(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{4318}$$

$$\frac{\frac{\frac{3a(a^2+4b^2)\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a^2-b^2}+\frac{(a^4-10a^2b^2-6b^4)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2(a^2-b^2)}+\frac{a(a^2-6b^2)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3b(a^2-b^2)}-\frac{a^2\tan(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{3a(a^2+4b^2)\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a^2-b^2}+\frac{(a^4-10a^2b^2-6b^4)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2(a^2-b^2)}+\frac{a(a^2-6b^2)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3b(a^2-b^2)}-\frac{a^2\tan(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{\frac{6a(a^2+4b^2)\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{d(a^2-b^2)}+\frac{(a^4-10a^2b^2-6b^4)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2(a^2-b^2)}+\frac{a(a^2-6b^2)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3b(a^2-b^2)}-\frac{a^2\tan(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{a(a^2-6b^2)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}+\frac{\frac{6ab\left(a^2+4b^2\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}+\frac{\left(a^4-10a^2b^2-6b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

input
```
Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^4,x]
```

output
```
-1/3*(a^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + ((a*(a^
2 - 6*b^2)*Tan[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((6*a*
b*(a^2 + 4*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt
[a - b]*Sqrt[a + b]*(a^2 - b^2)*d) + ((a^4 - 10*a^2*b^2 - 6*b^4)*Tan[c + d
*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x])))/(2*(a^2 - b^2)))/(3*b*(a^2 - b^
2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4326
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-a^2)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m
 + 1)*(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]
*(a + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e
+ f*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

rule 4491
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1
/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp
[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; Free
Q[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m
, -1]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.43

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{\left(a^3+6a^2b+2a\,b^2+2b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}+\frac{2\left(7a^2+3b^2\right)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3\left(a^2+2ab+b^2\right)\left(a^2-2ab+b^2\right)}+\frac{\left(a^3-6a^2b+2a\,b^2-2b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^3-3a^2b+3a\,b^2-b^3\right)}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^3}+\frac{a\left(a^2+4b\cdots\right.}{\left(a^6-3\cdots\right.}}{d}$
default	$\frac{-\frac{2\left(-\frac{\left(a^3+6a^2b+2a\,b^2+2b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}+\frac{2\left(7a^2+3b^2\right)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3\left(a^2+2ab+b^2\right)\left(a^2-2ab+b^2\right)}+\frac{\left(a^3-6a^2b+2a\,b^2-2b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^3-3a^2b+3a\,b^2-b^3\right)}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^3}+\frac{a\left(a^2+4b\cdots\right.}{\left(a^6-3\cdots\right.}}{d}$
risch	$\frac{i(3a^6e^{5i(dx+c)}+12a^4b^2e^{5i(dx+c)}+15a^5b\,e^{4i(dx+c)}+60a^3b^3e^{4i(dx+c)}+78a^4b^2e^{3i(dx+c)}+64a^2b^4e^{3i(dx+c)}+8b^6e^{3i(dx+\cdots}{3a(-a^2+b^2)^3d(e^{2i(dx\cdots}$

input
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-1/2*(a^3+6*a^2*b+2*a*b^2+2*b^3)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*
tan(1/2*d*x+1/2*c)^5+2/3*(7*a^2+3*b^2)*b/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*t
an(1/2*d*x+1/2*c)^3+1/2*(a^3-6*a^2*b+2*a*b^2-2*b^3)/(a+b)/(a^3-3*a^2*b+3*a
*b^2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2
*b-a-b)^3+a*(a^2+4*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*
arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(191) = 382$.

Time = 0.15 (sec) , antiderivative size = 902, normalized size of antiderivative = 4.38

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
[-1/12*(3*(a^3*b^3 + 4*a*b^5 + (a^6 + 4*a^4*b^2)*cos(d*x + c)^3 + 3*(a^5*b
 + 4*a^3*b^3)*cos(d*x + c)^2 + 3*(a^4*b^2 + 4*a^2*b^4)*cos(d*x + c))*sqrt(
a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt
(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x
+ c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(a^6*b - 11*a^4*b^3 + 4*a^2*b^5 +
6*b^7 - (13*a^6*b - 11*a^4*b^3 - 2*a^2*b^5)*cos(d*x + c)^2 + 3*(a^7 - 10*a
^5*b^2 + 7*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b
^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8
*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*
a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^
6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d), 1/6*(3*(a^3*b^3 + 4*a*b^5 + (a^6
 + 4*a^4*b^2)*cos(d*x + c)^3 + 3*(a^5*b + 4*a^3*b^3)*cos(d*x + c)^2 + 3*(a
^4*b^2 + 4*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2
)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (a^6*b - 11*a^4*b^3 +
 4*a^2*b^5 + 6*b^7 - (13*a^6*b - 11*a^4*b^3 - 2*a^2*b^5)*cos(d*x + c)^2 +
3*(a^7 - 10*a^5*b^2 + 7*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a
^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a
^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*
(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a
^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. 2(191) = 382.

Time = 0.20 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.09

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx = -\frac{\frac{3\,(a^3+4\,ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{-a^2+b^2}} - \frac{3\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-}{}}{}$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/3*(3*(a^3 + 4*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) +
 arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)
))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) - (3*a^5*tan(1/2
*d*x + 1/2*c)^5 + 12*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 27*a^3*b^2*tan(1/2*d*x
 + 1/2*c)^5 + 12*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 6*a*b^4*tan(1/2*d*x + 1/
2*c)^5 + 6*b^5*tan(1/2*d*x + 1/2*c)^5 - 28*a^4*b*tan(1/2*d*x + 1/2*c)^3 +
16*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 12*b^5*tan(1/2*d*x + 1/2*c)^3 - 3*a^5*
tan(1/2*d*x + 1/2*c) + 12*a^4*b*tan(1/2*d*x + 1/2*c) + 27*a^3*b^2*tan(1/2*
d*x + 1/2*c) + 12*a^2*b^3*tan(1/2*d*x + 1/2*c) + 6*a*b^4*tan(1/2*d*x + 1/2
*c) + 6*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*
tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d
```

Mupad [B] (verification not implemented)

Time = 13.57 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.84

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$
$$= \frac{\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(7\,a^2\,b+3\,b^3\right)}{3\,{\left(a+b\right)}^2\,\left(a^2-2\,a\,b+b^2\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(a^3+6\,a^2\,b+2\,a\,b^2+2\,b^3\right)}{{\left(a+b\right)}^3\,\left(a-b\right)}+\frac{\tan\left(\frac{c}{2}+\right.}{(a+}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(-3\,a^3-3\,a^2\,b+3\,a\,b^2+3\,b^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(-3\,a^3+3\,a^2\,b+3\,a\,b^2-3\,b^3\right)+3\,a\,b^{}\right.}$$
$$+\frac{a\,\mathrm{atanh}\left(\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2+4\,b^2\right)\,\left(2\,a-2\,b\right)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{2\,\sqrt{a+b}\,{\left(a-b\right)}^{7/2}\,\left(a^3+4\,a\,b^2\right)}\right)\,\left(a^2+4\,b^2\right)}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}$$

input
```
int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^4),x)
```

output
```
((4*tan(c/2 + (d*x)/2)^3*(7*a^2*b + 3*b^3))/(3*(a + b)^2*(a^2 - 2*a*b + b^
2)) - (tan(c/2 + (d*x)/2)^5*(2*a*b^2 + 6*a^2*b + a^3 + 2*b^3))/((a + b)^3*
(a - b)) + (tan(c/2 + (d*x)/2)*(2*a*b^2 - 6*a^2*b + a^3 - 2*b^3))/((a + b)
*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))/(d*(tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a
^2*b - 3*a^3 + 3*b^3) - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 -
3*b^3) + 3*a*b^2 + 3*a^2*b + a^3 + b^3 - tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3
*a^2*b + a^3 - b^3))) + (a*atanh((a*tan(c/2 + (d*x)/2)*(a^2 + 4*b^2)*(2*a
- 2*b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(2*(a + b)^(1/2)*(a - b)^(7/2)*(4*
a*b^2 + a^3)))*(a^2 + 4*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1080, normalized size of antiderivative = 5.24

$$\int \frac{\sec^3(c + dx)}{(a + b\sec(c + dx))^4} \, dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c))^4,x)
```

output

```
(6*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqr
t( - a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**2*a**6 + 24*sqrt( - a**2 + b
**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*
cos(c + d*x)*sin(c + d*x)**2*a**4*b**2 - 6*sqrt( - a**2 + b**2)*atan((tan(
(c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*a*
*6 - 42*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b
)/sqrt( - a**2 + b**2))*cos(c + d*x)*a**4*b**2 - 72*sqrt( - a**2 + b**2)*a
tan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c
+ d*x)*a**2*b**4 + 18*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan(
(c + d*x)/2)*b)/sqrt( - a**2 + b**2))*sin(c + d*x)**2*a**5*b + 72*sqrt( -
a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 +
 b**2))*sin(c + d*x)**2*a**3*b**3 - 18*sqrt( - a**2 + b**2)*atan((tan((c +
 d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*a**5*b - 78*sqrt( -
 a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2
+ b**2))*a**3*b**3 - 24*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - ta
n((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*a*b**5 - 3*cos(c + d*x)*sin(c + d*
x)*a**7 + 30*cos(c + d*x)*sin(c + d*x)*a**5*b**2 - 21*cos(c + d*x)*sin(c +
 d*x)*a**3*b**4 - 6*cos(c + d*x)*sin(c + d*x)*a*b**6 - 13*sin(c + d*x)**3*
a**6*b + 11*sin(c + d*x)**3*a**4*b**3 + 2*sin(c + d*x)**3*a**2*b**5 + 12*s
in(c + d*x)*a**6*b - 6*sin(c + d*x)*a**2*b**5 - 6*sin(c + d*x)*b**7)/(6...
```

3.518 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 192

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = -\frac{b(4a^2+b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} + \frac{a\tan(c+dx)}{3\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{(2a^2+3b^2)\tan(c+dx)}{6\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} + \frac{a(2a^2+13b^2)\tan(c+dx)}{6\,(a^2-b^2)^3\,d(a+b\sec(c+dx))}$$

output
```
-b*(4*a^2+b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(
7/2)/(a+b)^(7/2)/d+1/3*a*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^3+1/6*(2*
a^2+3*b^2)*tan(d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2+1/6*a*(2*a^2+13*b^2
)*tan(d*x+c)/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.85

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$
$$=\frac{\frac{6b(4a^2+b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}}+\frac{(2a^3b^2+13ab^4-3b(-2a^4-9a^2b^2+b^4)\cos(c+dx)+a(6a^4+10a^2b^2-b^4)\cos^2(c+dx))\sin(c+dx)}{(a-b)^3(a+b)^3(b+a\cos(c+dx))^3}}{6d}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^4,x]
```

output
```
((6*b*(4*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(a^2 - b^2)^(7/2) + ((2*a^3*b^2 + 13*a*b^4 - 3*b*(-2*a^4 - 9*a^2*b^2 + b^4
)*Cos[c + d*x] + a*(6*a^4 + 10*a^2*b^2 - b^4)*Cos[c + d*x]^2)*Sin[c + d*x]
)/((a - b)^3*(a + b)^3*(b + a*Cos[c + d*x])^3))/(6*d)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.21, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4323, 25, 3042, 4491, 25, 3042, 4491, 27, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$
$$\Big\downarrow \ 4323$$
$$\frac{\int -\frac{\sec(c+dx)(3b-2a\sec(c+dx))}{(a+b\sec(c+dx))^3}dx}{3(a^2-b^2)}+\frac{a\tan(c+dx)}{3d(a^2-b^2)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \ 25$$

$$\frac{a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{\int\frac{\sec(c+dx)(3b-2a\sec(c+dx))}{(a+b\sec(c+dx))^3}dx}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 3042$$

$$\frac{a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3b-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 4491$$

$$\frac{a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{-\frac{\int-\frac{\sec(c+dx)\left(10ab-\left(2a^2+3b^2\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2(a^2-b^2)}-\frac{\left(2a^2+3b^2\right)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{\frac{\int\frac{\sec(c+dx)\left(10ab-\left(2a^2+3b^2\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2(a^2-b^2)}-\frac{\left(2a^2+3b^2\right)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 3042$$

$$\frac{a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{\frac{\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(10ab+\left(-2a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2(a^2-b^2)}-\frac{\left(2a^2+3b^2\right)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 4491$$

$$\frac{a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{-\frac{-\frac{\int-\frac{3b\left(4a^2+b^2\right)\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}-\frac{a\left(2a^2+13b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{\left(2a^2+3b^2\right)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{a\tan(c+dx)}{3d(a^2-b^2)(a+b\sec(c+dx))^3}-\frac{\frac{\frac{3b(4a^2+b^2)\int\frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}-\frac{a(2a^2+13b^2)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{(2a^2+3b^2)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3(a^2-b^2)}$$

$\Big\downarrow$ 3042

$$\frac{a\tan(c+dx)}{3d(a^2-b^2)(a+b\sec(c+dx))^3}-\frac{\frac{\frac{3b(4a^2+b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{a(2a^2+13b^2)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{(2a^2+3b^2)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3(a^2-b^2)}$$

$\Big\downarrow$ 4318

$$\frac{a\tan(c+dx)}{3d(a^2-b^2)(a+b\sec(c+dx))^3}-\frac{\frac{\frac{3(4a^2+b^2)\int\frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a^2-b^2}-\frac{a(2a^2+13b^2)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{(2a^2+3b^2)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3(a^2-b^2)}$$

$\Big\downarrow$ 3042

$$\frac{a\tan(c+dx)}{3d(a^2-b^2)(a+b\sec(c+dx))^3}-\frac{\frac{\frac{3(4a^2+b^2)\int\frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a^2-b^2}-\frac{a(2a^2+13b^2)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{(2a^2+3b^2)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3(a^2-b^2)}$$

$\Big\downarrow$ 3138

$$\frac{a\tan(c+dx)}{3d(a^2-b^2)(a+b\sec(c+dx))^3}-\frac{\frac{\frac{6(4a^2+b^2)\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{d(a^2-b^2)}-\frac{a(2a^2+13b^2)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{(2a^2+3b^2)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3(a^2-b^2)}$$

$\Big\downarrow$ 221

$$\frac{a\tan(c+dx)}{3d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^3}-\frac{\frac{\frac{6b\left(4a^2+b^2\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}-\frac{a\left(2a^2+13b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{(2a^2+3b^2)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^4,x]
```

output
```
(a*Tan[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) - (-1/2*((2*a^2
+ 3*b^2)*Tan[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((6*b*(4*a
^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b
]*Sqrt[a + b]*(a^2 - b^2)*d) - (a*(2*a^2 + 13*b^2)*Tan[c + d*x])/((a^2 - b
^2)*d*(a + b*Sec[c + d*x])))/(2*(a^2 - b^2)))/(3*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4323
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[a*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(
a^2 - b^2))), x] - Simp[1/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

rule 4491
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1
/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp
[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; Free
Q[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m
, -1]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.55

method	result
derivativedivides	$\frac{\frac{-\frac{\left(2a^3+2a^2b+6a\,b^2+b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}+\frac{4\left(3a^2+7b^2\right)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3\left(a^2+2ab+b^2\right)\left(a^2-2ab+b^2\right)}-\frac{\left(2a^3-2a^2b+6a\,b^2-b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)\left(a^3-3a^2b+3a\,b^2-b^3\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^3}-\frac{b\left(4a^2+b^2\right)\text{arct}}{\left(a^6-3a^4b^2+\right.}}{d}$
default	$\frac{\frac{-\frac{\left(2a^3+2a^2b+6a\,b^2+b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}+\frac{4\left(3a^2+7b^2\right)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3\left(a^2+2ab+b^2\right)\left(a^2-2ab+b^2\right)}-\frac{\left(2a^3-2a^2b+6a\,b^2-b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)\left(a^3-3a^2b+3a\,b^2-b^3\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^3}-\frac{b\left(4a^2+b^2\right)\text{arct}}{\left(a^6-3a^4b^2+\right.}}{d}$
risch	$\frac{i\left(-12a^6b\,e^{5i(dx+c)}-3a^4b^3e^{5i(dx+c)}-6a^7e^{4i(dx+c)}-42a^5b^2e^{4i(dx+c)}-33a^3b^4e^{4i(dx+c)}+6a\,b^6e^{4i(dx+c)}-36a^6b\,e^{3i(dx\right.}}{}$

input
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(-1/2*(2*a^3+2*a^2*b+6*a*b^2+b^3)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*t
an(1/2*d*x+1/2*c)^5+2/3*(3*a^2+7*b^2)*a/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*ta
n(1/2*d*x+1/2*c)^3-1/2*(2*a^3-2*a^2*b+6*a*b^2-b^3)/(a+b)/(a^3-3*a^2*b+3*a*
b^2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*
b-a-b)^3-b*(4*a^2+b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*a
rctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(177) = 354$.

Time = 0.15 (sec) , antiderivative size = 901, normalized size of antiderivative = 4.69

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
[-1/12*(3*(4*a^2*b^4 + b^6 + (4*a^5*b + a^3*b^3)*cos(d*x + c)^3 + 3*(4*a^4
*b^2 + a^2*b^4)*cos(d*x + c)^2 + 3*(4*a^3*b^3 + a*b^5)*cos(d*x + c))*sqrt(
a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt
(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x
+ c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(2*a^5*b^2 + 11*a^3*b^4 - 13*a*b^6
 + (6*a^7 + 4*a^5*b^2 - 11*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + 3*(2*a^6*b +
7*a^4*b^3 - 10*a^2*b^5 + b^7)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b
^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8
*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*
a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^
6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d), -1/6*(3*(4*a^2*b^4 + b^6 + (4*a^
5*b + a^3*b^3)*cos(d*x + c)^3 + 3*(4*a^4*b^2 + a^2*b^4)*cos(d*x + c)^2 + 3
*(4*a^3*b^3 + a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^
2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (2*a^5*b^2 + 11*a^3*
b^4 - 13*a*b^6 + (6*a^7 + 4*a^5*b^2 - 11*a^3*b^4 + a*b^6)*cos(d*x + c)^2 +
 3*(2*a^6*b + 7*a^4*b^3 - 10*a^2*b^5 + b^7)*cos(d*x + c))*sin(d*x + c))/((
a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(
a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3
*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (
a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \int \frac{\sec^2{(c+dx)}}{(a+b\sec{(c+dx)})^4}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. 2(177) = 354.

Time = 0.21 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.24

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = -\frac{\frac{3\,(4\,a^2b+b^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{-a^2+b^2}}+\frac{6\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}{}}{}$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/3*(3*(4*a^2*b + b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b)
+ arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^
2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) + (6*a^5*tan(1
/2*d*x + 1/2*c)^5 - 6*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 12*a^3*b^2*tan(1/2*d*
x + 1/2*c)^5 - 27*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 12*a*b^4*tan(1/2*d*x +
1/2*c)^5 + 3*b^5*tan(1/2*d*x + 1/2*c)^5 - 12*a^5*tan(1/2*d*x + 1/2*c)^3 -
16*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 28*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 6*a^
5*tan(1/2*d*x + 1/2*c) + 6*a^4*b*tan(1/2*d*x + 1/2*c) + 12*a^3*b^2*tan(1/2
*d*x + 1/2*c) + 27*a^2*b^3*tan(1/2*d*x + 1/2*c) + 12*a*b^4*tan(1/2*d*x + 1
/2*c) - 3*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(
a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d
```

Mupad [B] (verification not implemented)

Time = 13.49 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.99

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$
$$= \frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 (2a^3+2a^2b+6ab^2+b^3)}{(a+b)^3(a-b)} - \frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 (3a^3+7ab^2)}{3(a+b)^2(a^2-2ab+b^2)} + \frac{\tan\left(\frac{c}{2}+\right.}{(a+}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(-3a^3-3a^2b+3ab^2+3b^3) - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(-3a^3+3a^2b+3ab^2-3b^3) + 3ab^2\right.}$$
$$- \frac{b\,\mathrm{atanh}\left(\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(4a^2+b^2)(2a-2b)(a^3-3a^2b+3ab^2-b^3)}{2\sqrt{a+b}\,(a-b)^{7/2}(4a^2b+b^3)}\right)(4a^2+b^2)}{d\,(a+b)^{7/2}(a-b)^{7/2}}$$

input
```
int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^4),x)
```

output
```
((tan(c/2 + (d*x)/2)^5*(6*a*b^2 + 2*a^2*b + 2*a^3 + b^3))/((a + b)^3*(a -
b)) - (4*tan(c/2 + (d*x)/2)^3*(7*a*b^2 + 3*a^3))/(3*(a + b)^2*(a^2 - 2*a*b
 + b^2)) + (tan(c/2 + (d*x)/2)*(6*a*b^2 - 2*a^2*b + 2*a^3 - b^3))/((a + b)
*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))/(d*(tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a
^2*b - 3*a^3 + 3*b^3) - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 -
3*b^3) + 3*a*b^2 + 3*a^2*b + a^3 + b^3 - tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3
*a^2*b + a^3 - b^3))) - (b*atanh((b*tan(c/2 + (d*x)/2)*(4*a^2 + b^2)*(2*a
- 2*b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(2*(a + b)^(1/2)*(a - b)^(7/2)*(4*
a^2*b + b^3)))*(4*a^2 + b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1095, normalized size of antiderivative = 5.70

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c))^4,x)
```

output
```
( - 24*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)
/sqrt( - a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**2*a**5*b - 6*sqrt( - a**
2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b*
*2))*cos(c + d*x)*sin(c + d*x)**2*a**3*b**3 + 24*sqrt( - a**2 + b**2)*atan
((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d
*x)*a**5*b + 78*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d
*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*a**3*b**3 + 18*sqrt( - a**2 +
 b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2)
)*cos(c + d*x)*a*b**5 - 72*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a -
 tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*sin(c + d*x)**2*a**4*b**2 - 18*
sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(
- a**2 + b**2))*sin(c + d*x)**2*a**2*b**4 + 72*sqrt( - a**2 + b**2)*atan((
tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*a**4*b**2 +
 42*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sq
rt( - a**2 + b**2))*a**2*b**4 + 6*sqrt( - a**2 + b**2)*atan((tan((c + d*x)
/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*b**6 - 6*cos(c + d*x)*si
n(c + d*x)*a**6*b - 21*cos(c + d*x)*sin(c + d*x)*a**4*b**3 + 30*cos(c + d*
x)*sin(c + d*x)*a**2*b**5 - 3*cos(c + d*x)*sin(c + d*x)*b**7 + 6*sin(c + d
*x)**3*a**7 + 4*sin(c + d*x)**3*a**5*b**2 - 11*sin(c + d*x)**3*a**3*b**4 +
 sin(c + d*x)**3*a*b**6 - 6*sin(c + d*x)*a**7 - 6*sin(c + d*x)*a**5*b**...
```

3.519 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 184

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \frac{a(2a^2+3b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{b\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{5ab\tan(c+dx)}{6\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2} - \frac{b(11a^2+4b^2)\tan(c+dx)}{6\left(a^2-b^2\right)^3d(a+b\sec(c+dx))}$$

output
```
a*(2*a^2+3*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^
(7/2)/(a+b)^(7/2)/d-1/3*b*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^3-5/6*a*
b*tan(d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2-1/6*b*(11*a^2+4*b^2)*tan(d*x
+c)/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.89

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx = -\frac{\frac{12a(2a^2+3b^2)\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{b(18a^4+17a^2b^2+10b^4+6ab(9a^2+b^2)\cos(c+dx)+(18a^4-5a^2b^2+2b^4)\cos(2(c+dx)))}{(b+a\cos(c+dx))^3}}{12(a-b)^3(a+b)^3d}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^4,x]
```

output
```
-1/12*((12*a*(2*a^2 + 3*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2
- b^2]])/Sqrt[a^2 - b^2] + (b*(18*a^4 + 17*a^2*b^2 + 10*b^4 + 6*a*b*(9*a^2
 + b^2)*Cos[c + d*x] + (18*a^4 - 5*a^2*b^2 + 2*b^4)*Cos[2*(c + d*x)])*Sin[
c + d*x])/(b + a*Cos[c + d*x])^3)/((a - b)^3*(a + b)^3*d)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.23, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {3042, 4320, 25, 3042, 4491, 25, 3042, 4491, 27, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{ 4320}$$

$$-\frac{\int -\frac{\sec(c+dx)(3a-2b\sec(c+dx))}{(a+b\sec(c+dx))^3}dx}{3(a^2-b^2)} - \frac{b\tan(c+dx)}{3d(a^2-b^2)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\sec(c+dx)(3a-2b\sec(c+dx))}{(a+b\sec(c+dx))^3}dx}{3(a^2-b^2)}-\frac{b\tan(c+dx)}{3d(a^2-b^2)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3a-2b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3(a^2-b^2)}-\frac{b\tan(c+dx)}{3d(a^2-b^2)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \ 4491$$

$$\frac{-\frac{\int -\frac{\sec(c+dx)\left(2\left(3a^2+2b^2\right)-5ab\sec(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2(a^2-b^2)}-\frac{5ab\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3(a^2-b^2)}-\frac{b\tan(c+dx)}{3d(a^2-b^2)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{\sec(c+dx)\left(2\left(3a^2+2b^2\right)-5ab\sec(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2(a^2-b^2)}-\frac{5ab\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3(a^2-b^2)}-\frac{b\tan(c+dx)}{3d(a^2-b^2)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2\left(3a^2+2b^2\right)-5ab\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2(a^2-b^2)}-\frac{5ab\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3(a^2-b^2)}-\frac{b\tan(c+dx)}{3d(a^2-b^2)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \ 4491$$

$$\frac{\frac{-\frac{\int -\frac{3a\left(2a^2+3b^2\right)\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}-\frac{b\left(11a^2+4b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{5ab\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3(a^2-b^2)}-\frac{b\tan(c+dx)}{3d(a^2-b^2)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{3a\left(2a^2+3b^2\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}-\frac{b\left(11a^2+4b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{5ab\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\frac{3a\left(2a^2+3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{b\left(11a^2+4b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{5ab\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{ 4318}$$

$$\frac{\frac{\frac{3a\left(2a^2+3b^2\right)\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b\left(a^2-b^2\right)}-\frac{b\left(11a^2+4b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{5ab\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\frac{3a\left(2a^2+3b^2\right)\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b\left(a^2-b^2\right)}-\frac{b\left(11a^2+4b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{5ab\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{ 3138}$$

$$\frac{\frac{\frac{6a\left(2a^2+3b^2\right)\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd\left(a^2-b^2\right)}-\frac{b\left(11a^2+4b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{5ab\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{\frac{6a(2a^2+3b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}-\frac{b(11a^2+4b^2)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{5ab\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3(a^2-b^2)}-\frac{b\tan(c+dx)}{3d(a^2-b^2)(a+b\sec(c+dx))^3}$$

input
```
Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^4,x]
```

output
```
-1/3*(b*Tan[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + ((-5*a*b*Ta
n[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((6*a*(2*a^2 + 3*b^
2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[
a + b]*(a^2 - b^2)*d) - (b*(11*a^2 + 4*b^2)*Tan[c + d*x])/((a^2 - b^2)*d*(
a + b*Sec[c + d*x])))/(2*(a^2 - b^2)))/(3*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4320
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*
Csc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ
[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 4491
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1
/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp
[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; Free
Q[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m
, -1]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.54

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{\left(6a^2+3ab+2b^2\right)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}+\frac{2\left(9a^2+b^2\right)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3\left(a^2+2ab+b^2\right)\left(a^2-2ab+b^2\right)}-\frac{\left(6a^2-3ab+2b^2\right)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^3-3a^2b+3a\,b^2-b^3\right)}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^3}+\frac{a\left(2a^2+3b^2\right)\operatorname{arctanh}}{\left(a^6-3a^4b^2+3a^2b\right.}}{d}$
default	$\frac{-\frac{2\left(-\frac{\left(6a^2+3ab+2b^2\right)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}+\frac{2\left(9a^2+b^2\right)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3\left(a^2+2ab+b^2\right)\left(a^2-2ab+b^2\right)}-\frac{\left(6a^2-3ab+2b^2\right)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^3-3a^2b+3a\,b^2-b^3\right)}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^3}+\frac{a\left(2a^2+3b^2\right)\operatorname{arctanh}}{\left(a^6-3a^4b^2+3a^2b\right.}}{d}$
risch	$\frac{ib\left(27a^6b\,e^{5i(dx+c)}-18a^4b^3e^{5i(dx+c)}+6a^2b^5e^{5i(dx+c)}+18a^7e^{4i(dx+c)}+81a^5b^2e^{4i(dx+c)}-36a^3b^4e^{4i(dx+c)}+12a\,b^6e^{4i(\right.}{}$

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-1/2*(6*a^2+3*a*b+2*b^2)*b/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/
2*d*x+1/2*c)^5+2/3*(9*a^2+b^2)*b/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d
*x+1/2*c)^3-1/2*(6*a^2-3*a*b+2*b^2)*b/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(
1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3+a*(2
*a^2+3*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b
)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(169) = 338$.

Time = 0.16 (sec) , antiderivative size = 905, normalized size of antiderivative = 4.92

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
[-1/12*(3*(2*a^3*b^3 + 3*a*b^5 + (2*a^6 + 3*a^4*b^2)*cos(d*x + c)^3 + 3*(2
*a^5*b + 3*a^3*b^3)*cos(d*x + c)^2 + 3*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c
))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2
- 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*
cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(11*a^4*b^3 - 7*a^2*b^5 -
4*b^7 + (18*a^6*b - 23*a^4*b^3 + 7*a^2*b^5 - 2*b^7)*cos(d*x + c)^2 + 3*(9*
a^5*b^2 - 8*a^3*b^4 - a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^
2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*
b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a
^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6
*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d), 1/6*(3*(2*a^3*b^3 + 3*a*b^5 + (2*
a^6 + 3*a^4*b^2)*cos(d*x + c)^3 + 3*(2*a^5*b + 3*a^3*b^3)*cos(d*x + c)^2 +
 3*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^
2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (11*a^4*b^3 -
7*a^2*b^5 - 4*b^7 + (18*a^6*b - 23*a^4*b^3 + 7*a^2*b^5 - 2*b^7)*cos(d*x +
c)^2 + 3*(9*a^5*b^2 - 8*a^3*b^4 - a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^1
1 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^1
0*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a
^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8
*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx=\int \frac{\sec{(c+dx)}}{(a+b\sec{(c+dx)})^4}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)/(a + b*sec(c + d*x))**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(169)=338$.

Time = 0.19 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.19

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx= -\frac{\frac{3\,(2\,a^3+3\,ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{-a^2+b^2}}-\frac{18\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-27\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}}{}$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/3*(3*(2*a^3 + 3*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b)
 + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^
2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) - (18*a^4*b*ta
n(1/2*d*x + 1/2*c)^5 - 27*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*a^2*b^3*tan(1
/2*d*x + 1/2*c)^5 - 3*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*b^5*tan(1/2*d*x + 1
/2*c)^5 - 36*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 32*a^2*b^3*tan(1/2*d*x + 1/2*c
)^3 + 4*b^5*tan(1/2*d*x + 1/2*c)^3 + 18*a^4*b*tan(1/2*d*x + 1/2*c) + 27*a^
3*b^2*tan(1/2*d*x + 1/2*c) + 6*a^2*b^3*tan(1/2*d*x + 1/2*c) + 3*a*b^4*tan(
1/2*d*x + 1/2*c) + 6*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3)
)/d
```

Mupad [B] (verification not implemented)

Time = 13.35 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.05

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \frac{a\,\mathrm{atanh}\left(\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2+3\,b^2\right)\,\left(2\,a-2\,b\right)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{2\,\left(2\,a^3+3\,a\,b^2\right)\,\sqrt{a+b}\,{\left(a-b\right)}^{7/2}}\right)\,\left(2\,a^2+3\,b^2\right)}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}} - \frac{\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(6\,a^2\,b+3\,a\,b^2+2\,b^3\right)}{{\left(a+b\right)}^3\,\left(a-b\right)} - \frac{4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(9\,a^2\,b+b^3\right)}{3\,{\left(a+b\right)}^2\,\left(a^2-2\,a\,b+b^2\right)} + \frac{\tan\left(\frac{c}{2}\right.}{(a+}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(-3\,a^3-3\,a^2\,b+3\,a\,b^2+3\,b^3\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(-3\,a^3+3\,a^2\,b+3\,a\,b^2-3\,b^3\right)+3\,a\right.}$$

input
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^4),x)
```

output
```
(a*atanh((a*tan(c/2 + (d*x)/2)*(2*a^2 + 3*b^2)*(2*a - 2*b)*(3*a*b^2 - 3*a^
2*b + a^3 - b^3))/(2*(3*a*b^2 + 2*a^3)*(a + b)^(1/2)*(a - b)^(7/2)))*(2*a^
2 + 3*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2)) - ((tan(c/2 + (d*x)/2)^5*(3*a*
b^2 + 6*a^2*b + 2*b^3))/((a + b)^3*(a - b)) - (4*tan(c/2 + (d*x)/2)^3*(9*a
^2*b + b^3))/(3*(a + b)^2*(a^2 - 2*a*b + b^2)) + (tan(c/2 + (d*x)/2)*(6*a^
2*b - 3*a*b^2 + 2*b^3))/((a + b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))/(d*(tan
(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) - tan(c/2 + (d*x)/2)
^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) + 3*a*b^2 + 3*a^2*b + a^3 + b^3 - t
an(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1076, normalized size of antiderivative = 5.85

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^4,x)
```

output
```
(12*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sq
rt( - a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**2*a**6 + 18*sqrt( - a**2 +
b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))
*cos(c + d*x)*sin(c + d*x)**2*a**4*b**2 - 12*sqrt( - a**2 + b**2)*atan((ta
n((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*
a**6 - 54*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)
*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*a**4*b**2 - 54*sqrt( - a**2 + b**2)
*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(
c + d*x)*a**2*b**4 + 36*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - ta
n((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*sin(c + d*x)**2*a**5*b + 54*sqrt(
- a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2
 + b**2))*sin(c + d*x)**2*a**3*b**3 - 36*sqrt( - a**2 + b**2)*atan((tan((c
 + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*a**5*b - 66*sqrt(
 - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**
2 + b**2))*a**3*b**3 - 18*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a -
tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*a*b**5 + 27*cos(c + d*x)*sin(c +
 d*x)*a**5*b**2 - 24*cos(c + d*x)*sin(c + d*x)*a**3*b**4 - 3*cos(c + d*x)*
sin(c + d*x)*a*b**6 - 18*sin(c + d*x)**3*a**6*b + 23*sin(c + d*x)**3*a**4*
b**3 - 7*sin(c + d*x)**3*a**2*b**5 + 2*sin(c + d*x)**3*b**7 + 18*sin(c + d
*x)*a**6*b - 12*sin(c + d*x)*a**4*b**3 - 6*sin(c + d*x)*b**7)/(6*d*(cos...
```

3.520 $\int \frac{1}{(a+b\sec(c+dx))^4} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 242

$$\int \frac{1}{(a+b\sec(c+dx))^4}\,dx = \frac{x}{a^4} - \frac{b(8a^6-8a^4b^2+7a^2b^4-2b^6)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{7/2}(a+b)^{7/2}d} + \frac{b^2\tan(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{b^2(8a^2-3b^2)\tan(c+dx)}{6a^2\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} + \frac{b^2(26a^4-17a^2b^2+6b^4)\tan(c+dx)}{6a^3\,(a^2-b^2)^3\,d(a+b\sec(c+dx))}$$

output
```
x/a^4-b*(8*a^6-8*a^4*b^2+7*a^2*b^4-2*b^6)*arctanh((a-b)^(1/2)*tan(1/2*d*x+
1/2*c)/(a+b)^(1/2))/a^4/(a-b)^(7/2)/(a+b)^(7/2)/d+1/3*b^2*tan(d*x+c)/a/(a^
2-b^2)/d/(a+b*sec(d*x+c))^3+1/6*b^2*(8*a^2-3*b^2)*tan(d*x+c)/a^2/(a^2-b^2)
^2/d/(a+b*sec(d*x+c))^2+1/6*b^2*(26*a^4-17*a^2*b^2+6*b^4)*tan(d*x+c)/a^3/(
a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.11

$$\int \frac{1}{(a+b\sec(c+dx))^4}\,dx = \frac{(b+a\cos(c+dx))\sec^4(c+dx)\left(6(c+dx)(b+a\cos(c+dx))^3-\frac{6b(-8a^6+8a^4b^2-7a^2b^4+2b^6)\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))^3}{(a^2-b^2)^{7/2}}+\dots\right)}{6a^4d(\dots}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(-4),x]
```

output
```
((b + a*Cos[c + d*x])*Sec[c + d*x]^4*(6*(c + d*x)*(b + a*Cos[c + d*x])^3 -
 (6*b*(-8*a^6 + 8*a^4*b^2 - 7*a^2*b^4 + 2*b^6)*ArcTanh[((-a + b)*Tan[(c +
d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^3)/(a^2 - b^2)^(7/2) + (2*a
*b^4*Sin[c + d*x])/((a - b)*(a + b)) - (a*b^3*(12*a^2 - 7*b^2)*(b + a*Cos[
c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2) + (a*b^2*(36*a^4 - 32*a^2*b^
2 + 11*b^4)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/((a - b)^3*(a + b)^3)))/(
6*a^4*d*(a + b*Sec[c + d*x])^4)
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.26, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {3042, 4272, 25, 3042, 4548, 25, 3042, 4548, 27, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec(c+dx))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{4272}$$

$$\frac{b^2\tan(c+dx)}{3ad\,(a^2-b^2)\,(a+b\sec(c+dx))^3}-\frac{\int -\frac{2b^2\sec^2(c+dx)-3ab\sec(c+dx)+3(a^2-b^2)}{(a+b\sec(c+dx))^3}dx}{3a\,(a^2-b^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2b^2\sec^2(c+dx)-3ab\sec(c+dx)+3(a^2-b^2)}{(a+b\sec(c+dx))^3}dx}{3a\,(a^2-b^2)}+\frac{b^2\tan(c+dx)}{3ad\,(a^2-b^2)\,(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2-3ab\csc\left(c+dx+\frac{\pi}{2}\right)+3(a^2-b^2)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3a\,(a^2-b^2)}+\frac{b^2\tan(c+dx)}{3ad\,(a^2-b^2)\,(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{4548}$$

$$\frac{\frac{b^2(8a^2-3b^2)\tan(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\int -\frac{6\left(a^2-b^2\right)^2+b^2\left(8a^2-3b^2\right)\sec^2(c+dx)-2ab\left(6a^2-b^2\right)\sec(c+dx)}{(a+b\sec(c+dx))^2}dx}{2a(a^2-b^2)}}{3a\,(a^2-b^2)}+\frac{b^2\tan(c+dx)}{3ad\,(a^2-b^2)\,(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{6\left(a^2-b^2\right)^2+b^2\left(8a^2-3b^2\right)\sec^2(c+dx)-2ab\left(6a^2-b^2\right)\sec(c+dx)}{(a+b\sec(c+dx))^2}dx}{2a(a^2-b^2)}+\frac{b^2(8a^2-3b^2)\tan(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}}{3a\,(a^2-b^2)}+\frac{b^2\tan(c+dx)}{3ad\,(a^2-b^2)\,(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{6\left(a^2-b^2\right)^2+b^2\left(8a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-2ab\left(6a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2a(a^2-b^2)}+\frac{b^2(8a^2-3b^2)\tan(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}}{3a\,(a^2-b^2)}+\frac{b^2\tan(c+dx)}{3ad\,(a^2-b^2)\,(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{4548}$$

$$\frac{\frac{\frac{b^2\left(26a^4-17a^2b^2+6b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int -\frac{3\left(2\left(a^2-b^2\right)^3-ab\left(6a^4-2b^2a^2+b^4\right)\sec(c+dx)\right)}{a+b\sec(c+dx)}dx}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}+\frac{b^2\left(8a^2-3b^2\right)\tan(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\frac{3\int \frac{2\left(a^2-b^2\right)^3-ab\left(6a^4-2b^2a^2+b^4\right)\sec(c+dx)}{a+b\sec(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\left(26a^4-17a^2b^2+6b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+\frac{b^2\left(8a^2-3b^2\right)\tan(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\frac{3\int \frac{2\left(a^2-b^2\right)^3-ab\left(6a^4-2b^2a^2+b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\left(26a^4-17a^2b^2+6b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+\frac{b^2\left(8a^2-3b^2\right)\tan(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{ 4407}$$

$$\frac{\frac{\frac{3\left(\frac{2x\left(a^2-b^2\right)^3}{a}-\frac{b\left(8a^6-8a^4b^2+7a^2b^4-2b^6\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}\right)}{a\left(a^2-b^2\right)}+\frac{b^2\left(26a^4-17a^2b^2+6b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+\frac{b^2\left(8a^2-3b^2\right)\tan(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\frac{3\left(\frac{2x\left(a^2-b^2\right)^3}{a}-\frac{b\left(8a^6-8a^4b^2+7a^2b^4-2b^6\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}\right)}{a\left(a^2-b^2\right)}+\frac{b^2\left(26a^4-17a^2b^2+6b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+\frac{b^2\left(8a^2-3b^2\right)\tan(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 4318

$$\frac{\frac{\frac{3\left(\frac{2x\left(a^2-b^2\right)^3}{a}-\frac{\left(8a^6-8a^4b^2+7a^2b^4-2b^6\right)\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a}\right)}{a\left(a^2-b^2\right)}+\frac{b^2\left(26a^4-17a^2b^2+6b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+\frac{b^2(8a^2-3b^2)\tan(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{3\left(\frac{2x\left(a^2-b^2\right)^3}{a}-\frac{\left(8a^6-8a^4b^2+7a^2b^4-2b^6\right)\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a}\right)}{a\left(a^2-b^2\right)}+\frac{b^2\left(26a^4-17a^2b^2+6b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+\frac{b^2(8a^2-3b^2)\tan(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 3138

$$\frac{\frac{\frac{3\left(\frac{2x\left(a^2-b^2\right)^3}{a}-\frac{2\left(8a^6-8a^4b^2+7a^2b^4-2b^6\right)\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}\right)}{a\left(a^2-b^2\right)}+\frac{b^2\left(26a^4-17a^2b^2+6b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+\frac{b^2(8a^2-3b^2)\tan(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 221

$$\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}+\frac{\frac{b^2(8a^2-3b^2)\tan(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}+\frac{\frac{b^2\left(26a^4-17a^2b^2+6b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{3\left(\frac{2x\left(a^2-b^2\right)^3}{a}-\frac{2b\left(8a^6-8a^4b^2+7a^2b^4-2b^6\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

input

```
Int[(a + b*Sec[c + d*x])^(-4),x]
```

output
```
(b^2*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + ((b^2*(8*a
^2 - 3*b^2)*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((3
*((2*(a^2 - b^2)^3*x)/a - (2*b*(8*a^6 - 8*a^4*b^2 + 7*a^2*b^4 - 2*b^6)*Arc
Tanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a +
b]*d)))/(a*(a^2 - b^2)) + (b^2*(26*a^4 - 17*a^2*b^2 + 6*b^4)*Tan[c + d*x])
/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])))/(2*a*(a^2 - b^2)))/(3*a*(a^2 - b^
2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.51

method	result
derivativedivides	$\frac{\frac{2b\left(\frac{-\frac{\left(12a^4+4b\,a^3-6a^2b^2-a\,b^3+2b^4\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}+\frac{2\left(18a^4-11a^2b^2+3b^4\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3\left(a^2+2ab+b^2\right)\left(a^2-2ab+b^2\right)}-\frac{\left(12a^4-4b\,a^3-6a^2b^2+a\,b^3+2b^4\right)\cdots}{2(a+b)\left(a^3-3a^2b+3a\cdots\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^3}\cdots\right)}{a^4}\cdots}{d}$
default	$\frac{\frac{2b\left(\frac{-\frac{\left(12a^4+4b\,a^3-6a^2b^2-a\,b^3+2b^4\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}+\frac{2\left(18a^4-11a^2b^2+3b^4\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3\left(a^2+2ab+b^2\right)\left(a^2-2ab+b^2\right)}-\frac{\left(12a^4-4b\,a^3-6a^2b^2+a\,b^3+2b^4\right)\cdots}{2(a+b)\left(a^3-3a^2b+3a\cdots\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^3}\cdots\right)}{a^4}\cdots}{d}$
risch	Expression too large to display

input
```
int(1/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*b/a^4*((-1/2*(12*a^4+4*a^3*b-6*a^2*b^2-a*b^3+2*b^4)*a*b/(a-b)/(a^3+
3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/3*(18*a^4-11*a^2*b^2+3*b^4)*a*
b/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(12*a^4-4*a^3*b
-6*a^2*b^2+a*b^3+2*b^4)*a*b/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/
2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3-1/2*(8*a^6-8*a
^4*b^2+7*a^2*b^4-2*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*
arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))+2/a^4*arctan(tan(1/
2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(227) = 454$.

Time = 0.20 (sec) , antiderivative size = 1456, normalized size of antiderivative = 6.02

$$\int \frac{1}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
[1/12*(12*(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*x*cos(d*x
 + c)^3 + 36*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*x*co
s(d*x + c)^2 + 36*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d
*x*cos(d*x + c) + 12*(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*
d*x + 3*(8*a^6*b^4 - 8*a^4*b^6 + 7*a^2*b^8 - 2*b^10 + (8*a^9*b - 8*a^7*b^3
 + 7*a^5*b^5 - 2*a^3*b^7)*cos(d*x + c)^3 + 3*(8*a^8*b^2 - 8*a^6*b^4 + 7*a^
4*b^6 - 2*a^2*b^8)*cos(d*x + c)^2 + 3*(8*a^7*b^3 - 8*a^5*b^5 + 7*a^3*b^7 -
 2*a*b^9)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2
*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c)
 + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(26*a
^7*b^4 - 43*a^5*b^6 + 23*a^3*b^8 - 6*a*b^10 + (36*a^9*b^2 - 68*a^7*b^4 + 4
3*a^5*b^6 - 11*a^3*b^8)*cos(d*x + c)^2 + 15*(4*a^8*b^3 - 7*a^6*b^5 + 4*a^4
*b^7 - a^2*b^9)*cos(d*x + c))*sin(d*x + c))/((a^15 - 4*a^13*b^2 + 6*a^11*b
^4 - 4*a^9*b^6 + a^7*b^8)*d*cos(d*x + c)^3 + 3*(a^14*b - 4*a^12*b^3 + 6*a^
10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c)^2 + 3*(a^13*b^2 - 4*a^11*b^4
+ 6*a^9*b^6 - 4*a^7*b^8 + a^5*b^10)*d*cos(d*x + c) + (a^12*b^3 - 4*a^10*b^
5 + 6*a^8*b^7 - 4*a^6*b^9 + a^4*b^11)*d), 1/6*(6*(a^11 - 4*a^9*b^2 + 6*a^7
*b^4 - 4*a^5*b^6 + a^3*b^8)*d*x*cos(d*x + c)^3 + 18*(a^10*b - 4*a^8*b^3 +
6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*x*cos(d*x + c)^2 + 18*(a^9*b^2 - 4*a^7*
b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*x*cos(d*x + c) + 6*(a^8*b^3 - 4...
```

Sympy [F]

$$\int \frac{1}{(a+b\sec(c+dx))^4}\,dx=\int \frac{1}{\left(a+b\sec\left(c+dx\right)\right)^4}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**4,x)
```

output
```
Integral((a + b*sec(c + d*x))**(-4), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\sec(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(227) = 454$.

Time = 0.15 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.20

$$\int \frac{1}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(3*(8*a^6*b - 8*a^4*b^3 + 7*a^2*b^5 - 2*b^7)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
 1/2*c))/sqrt(-a^2 + b^2)))/((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*sqrt
(-a^2 + b^2)) - (36*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 - 60*a^5*b^3*tan(1/2*d*
x + 1/2*c)^5 - 6*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 + 45*a^3*b^5*tan(1/2*d*x +
 1/2*c)^5 - 6*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 - 15*a*b^7*tan(1/2*d*x + 1/2*
c)^5 + 6*b^8*tan(1/2*d*x + 1/2*c)^5 - 72*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 +
116*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 - 56*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 + 1
2*b^8*tan(1/2*d*x + 1/2*c)^3 + 36*a^6*b^2*tan(1/2*d*x + 1/2*c) + 60*a^5*b^
3*tan(1/2*d*x + 1/2*c) - 6*a^4*b^4*tan(1/2*d*x + 1/2*c) - 45*a^3*b^5*tan(1
/2*d*x + 1/2*c) - 6*a^2*b^6*tan(1/2*d*x + 1/2*c) + 15*a*b^7*tan(1/2*d*x +
1/2*c) + 6*b^8*tan(1/2*d*x + 1/2*c))/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b
^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3) + 3*(
d*x + c)/a^4)/d
```

Mupad [B] (verification not implemented)

Time = 21.95 (sec) , antiderivative size = 7234, normalized size of antiderivative = 29.89

$$\int \frac{1}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b/cos(c + d*x))^4,x)
```

output
```
(2*atan((((((8*(16*a^20*b - 4*a^21 + 4*a^8*b^13 - 2*a^9*b^12 - 26*a^10*b^1
1 + 14*a^11*b^10 + 70*a^12*b^9 - 30*a^13*b^8 - 110*a^14*b^7 + 30*a^15*b^6
+ 110*a^16*b^5 - 20*a^17*b^4 - 64*a^18*b^3 + 12*a^19*b^2))/(a^19*b + a^20
- a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b
^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) - (tan(c/2 + (d*
x)/2)*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 -
120*a^12*b^10 + 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6
+ 120*a^17*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2)*8i)/(a^4*(a^16*b
+ a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^
11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2)))*1i)/a^4 +
(8*tan(c/2 + (d*x)/2)*(4*a^14 - 8*a^13*b - 8*a*b^13 + 8*b^14 - 48*a^2*b^12
 + 48*a^3*b^11 + 117*a^4*b^10 - 120*a^5*b^9 - 164*a^6*b^8 + 160*a^7*b^7 +
156*a^8*b^6 - 120*a^9*b^5 - 92*a^10*b^4 + 48*a^11*b^3 + 44*a^12*b^2))/(a^1
6*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 1
0*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2))/a^4 - (
(((8*(16*a^20*b - 4*a^21 + 4*a^8*b^13 - 2*a^9*b^12 - 26*a^10*b^11 + 14*a^1
1*b^10 + 70*a^12*b^9 - 30*a^13*b^8 - 110*a^14*b^7 + 30*a^15*b^6 + 110*a^16
*b^5 - 20*a^17*b^4 - 64*a^18*b^3 + 12*a^19*b^2))/(a^19*b + a^20 - a^9*b^11
 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^
15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) + (tan(c/2 + (d*x)/2)*(...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1981, normalized size of antiderivative = 8.19

$$\int \frac{1}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b*sec(d*x+c))^4,x)
```

output
```
( - 48*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)
/sqrt( - a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**2*a**9*b + 48*sqrt( - a*
*2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b
**2))*cos(c + d*x)*sin(c + d*x)**2*a**7*b**3 - 42*sqrt( - a**2 + b**2)*ata
n((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c +
d*x)*sin(c + d*x)**2*a**5*b**5 + 12*sqrt( - a**2 + b**2)*atan((tan((c + d*
x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*sin(c + d
*x)**2*a**3*b**7 + 48*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan(
(c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*a**9*b + 96*sqrt( - a**
2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b*
*2))*cos(c + d*x)*a**7*b**3 - 102*sqrt( - a**2 + b**2)*atan((tan((c + d*x)
/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*a**5*b**5 +
 114*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/s
qrt( - a**2 + b**2))*cos(c + d*x)*a**3*b**7 - 36*sqrt( - a**2 + b**2)*atan
((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d
*x)*a*b**9 - 144*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c +
d*x)/2)*b)/sqrt( - a**2 + b**2))*sin(c + d*x)**2*a**8*b**2 + 144*sqrt( - a
**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 +
b**2))*sin(c + d*x)**2*a**6*b**4 - 126*sqrt( - a**2 + b**2)*atan((tan((c +
 d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*sin(c + d*x)**2*...
```

3.521 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 299

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx &= -\frac{4bx}{a^5}+\frac{b^2(20a^6-35a^4b^2+28a^2b^4-8b^6)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{7/2}(a+b)^{7/2}d}\\ &\quad+\frac{(6a^6-65a^4b^2+68a^2b^4-24b^6)\sin(c+dx)}{6a^4\left(a^2-b^2\right)^3d}+\frac{b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3}\\ &\quad+\frac{b^2(9a^2-4b^2)\sin(c+dx)}{6a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2}+\frac{b^2(12a^4-11a^2b^2+4b^4)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^3d(a+b\sec(c+dx))}\end{aligned}$$

output
```
-4*b*x/a^5+b^2*(20*a^6-35*a^4*b^2+28*a^2*b^4-8*b^6)*arctanh((a-b)^(1/2)*ta
n(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^5/(a-b)^(7/2)/(a+b)^(7/2)/d+1/6*(6*a^6-65*
a^4*b^2+68*a^2*b^4-24*b^6)*sin(d*x+c)/a^4/(a^2-b^2)^3/d+1/3*b^2*sin(d*x+c)
/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^3+1/6*b^2*(9*a^2-4*b^2)*sin(d*x+c)/a^2/(a^
2-b^2)^2/d/(a+b*sec(d*x+c))^2+1/2*b^2*(12*a^4-11*a^2*b^2+4*b^4)*sin(d*x+c)
/a^3/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.98

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

$$= \frac{(b+a\cos(c+dx))\sec^4(c+dx)\left(-24b(c+dx)(b+a\cos(c+dx))^3+\frac{6b^2(-20a^6+35a^4b^2-28a^2b^4+8b^6)\operatorname{arctanh}}{(a^2-b^2}\right.}{}$$

input
```
Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x])^4,x]
```

output
```
((b + a*Cos[c + d*x])*Sec[c + d*x]^4*(-24*b*(c + d*x)*(b + a*Cos[c + d*x])
^3 + (6*b^2*(-20*a^6 + 35*a^4*b^2 - 28*a^2*b^4 + 8*b^6)*ArcTanh[((-a + b)*
Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^3)/(a^2 - b^2)^(7/
2) + (2*a*b^5*Sin[c + d*x])/((-a + b)*(a + b)) + (5*a*b^4*(3*a^2 - 2*b^2)*
(b + a*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2) - (a*b^3*(60*a^4
- 71*a^2*b^2 + 26*b^4)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/((a - b)^3*(a
+ b)^3) + 6*a*(b + a*Cos[c + d*x])^3*Sin[c + d*x]))/(6*a^5*d*(a + b*Sec[c
+ d*x])^4)
```

Rubi [A] (verified)

Time = 2.37 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.19, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4334, 25, 3042, 4588, 25, 3042, 4588, 25, 3042, 4592, 27, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{4334}$$

$$\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{\int -\frac{\cos(c+dx)\left(3a^2-3b\sec(c+dx)a-4b^2+3b^2\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}dx}{3a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\cos(c+dx)\left(3a^2-3b\sec(c+dx)a-4b^2+3b^2\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}dx}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3a^2-3b\csc\left(c+dx+\frac{\pi}{2}\right)a-4b^2+3b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{4588}$$

$$\frac{\frac{b^2\left(9a^2-4b^2\right)\sin(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\int -\frac{\cos(c+dx)\left(6a^4-23b^2a^2-2b\left(6a^2-b^2\right)\sec(c+dx)a+12b^4+2b^2\left(9a^2-4b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\cos(c+dx)\left(6a^4-23b^2a^2-2b\left(6a^2-b^2\right)\sec(c+dx)a+12b^4+2b^2\left(9a^2-4b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2a(a^2-b^2)}+\frac{b^2\left(9a^2-4b^2\right)\sin(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{6a^4-23b^2a^2-2b\left(6a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+12b^4+2b^2\left(9a^2-4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2a(a^2-b^2)}+\frac{b^2\left(9a^2-4b^2\right)\sin(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{4588}$$

$$\frac{\frac{\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int -\frac{\cos(c+dx)\left(6a^6-65b^2a^4+68b^4a^2-b\left(18a^4-7b^2a^2+4b^4\right)\sec(c+dx)a-24b^6+3b^2\left(12a^4-11b^2a^2+4b^4\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\int \frac{\cos(c+dx)\left(6a^6-65b^2a^4+68b^4a^2-b\left(18a^4-7b^2a^2+4b^4\right)\sec(c+dx)a-24b^6+3b^2\left(12a^4-11b^2a^2+4b^4\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\int \frac{6a^6-65b^2a^4+68b^4a^2-b\left(18a^4-7b^2a^2+4b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)a-24b^6+3b^2\left(12a^4-11b^2a^2+4b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a\left(a^2-b^2\right)}+\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+\frac{}{2a\cdots}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 4592$$

$$\frac{\frac{\frac{\frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\sin(c+dx)}{ad}-\frac{\int \frac{3\left(8b\left(a^2-b^2\right)^3-ab^2\left(12a^4-11b^2a^2+4b^4\right)\sec(c+dx)\right)}{a+b\sec(c+dx)}dx}{a}}{a\left(a^2-b^2\right)}+\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+\frac{b^2\left(9a^2-4\cdots\right.}{2ad(a^2-b^2)(\cdots}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\sin(c+dx)}{ad}-\frac{3\int \frac{8b\left(a^2-b^2\right)^3-ab^2\left(12a^4-11b^2a^2+4b^4\right)\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}}{a\left(a^2-b^2\right)}+\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+\frac{b^2\left(9a^2-4b^2\cdots\right.}{2ad(a^2-b^2)(a+\cdots}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\sin(c+dx)}{ad}-\frac{3\int\frac{8b\left(a^2-b^2\right)^3-ab^2\left(12a^4-11b^2a^2+4b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{a\left(a^2-b^2\right)}+\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+\frac{b^2\left(9a^2-\cdots\right.}{2ad(a^2-b^2\cdots}}{3a\left(a^2-b^2\right)}$$
$$\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{4407}$$

$$\frac{\frac{\frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{8bx\left(a^2-b^2\right)^3}{a}-\frac{b^2\left(20a^6-35a^4b^2+28a^2b^4-8b^6\right)\int\frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}\right)}{a}}{a\left(a^2-b^2\right)}+\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+\cdots}{3a\left(a^2-b^2\right)}$$
$$\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{8bx\left(a^2-b^2\right)^3}{a}-\frac{b^2\left(20a^6-35a^4b^2+28a^2b^4-8b^6\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}\right)}{a}}{a\left(a^2-b^2\right)}+\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\sin(c+dx\cdots}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}\cdots}{3a\left(a^2-b^2\right)}$$
$$\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{4318}$$

$$\frac{\frac{\frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{8bx\left(a^2-b^2\right)^3}{a}-\frac{b\left(20a^6-35a^4b^2+28a^2b^4-8b^6\right)\int\frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a}\right)}{a}}{a\left(a^2-b^2\right)}+\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+\cdots}{3a\left(a^2-b^2\right)}$$
$$\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{8bx\left(a^2-b^2\right)^3}{a}-\frac{b\left(20a^6-35a^4b^2+28a^2b^4-8b^6\right)\int\frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a}\right)}{a}}{a\left(a^2-b^2\right)}+\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{ 3138}$$

$$\frac{\frac{\frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{8bx\left(a^2-b^2\right)^3}{a}-\frac{2b\left(20a^6-35a^4b^2+28a^2b^4-8b^6\right)\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}\right)}{a}}{a\left(a^2-b^2\right)}+\frac{3b^2\left(12a^4\right.}{ad\left(a^2\right.}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}+$$

$$\frac{\frac{b^2\left(9a^2-4b^2\right)\sin(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}+\frac{\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{8bx\left(a^2-b^2\right)^3}{a}-\frac{2b^2\left(20a^6-35a^4b^2+2\right.}{}\right.}{}}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

input
```
Int[Cos[c + d*x]/(a + b*Sec[c + d*x])^4,x]
```

output
```
(b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + ((b^2*(9*a
^2 - 4*b^2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((3
*b^2*(12*a^4 - 11*a^2*b^2 + 4*b^4)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*S
ec[c + d*x])) + ((-3*((8*b*(a^2 - b^2)^3*x)/a - (2*b^2*(20*a^6 - 35*a^4*b^
2 + 28*a^2*b^4 - 8*b^6)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]
])/(a*Sqrt[a - b]*Sqrt[a + b]*d)))/a + ((6*a^6 - 65*a^4*b^2 + 68*a^2*b^4 -
 24*b^6)*Sin[c + d*x])/(a*d))/(a*(a^2 - b^2)))/(2*a*(a^2 - b^2)))/(3*a*(a^
2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.33

method	result
derivativedivides	$-\frac{2\left(-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+4b\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^5}-\frac{2b^2\left(\frac{-\frac{\left(20a^4+5ba^3-18a^2b^2-2ab^3+6b^4\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3ab^2+b^3\right)}+\frac{2\left(30a^4-29a^2\right.}{3\left(a^2+2a\right.}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a\right.}$
default	$-\frac{2\left(-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+4b\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^5}-\frac{2b^2\left(\frac{-\frac{\left(20a^4+5ba^3-18a^2b^2-2ab^3+6b^4\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3ab^2+b^3\right)}+\frac{2\left(30a^4-29a^2\right.}{3\left(a^2+2a\right.}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a\right.}$
risch	Expression too large to display

input
```
int(cos(d*x+c)/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/a^5*(-a*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+4*b*arctan(tan
(1/2*d*x+1/2*c)))-2/a^5*b^2*((-1/2*(20*a^4+5*a^3*b-18*a^2*b^2-2*a*b^3+6*b^
4)*a*b/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/3*(30*a^4-29
*a^2*b^2+9*b^4)*a*b/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1
/2*(20*a^4-5*a^3*b-18*a^2*b^2+2*a*b^3+6*b^4)*a*b/(a+b)/(a^3-3*a^2*b+3*a*b^
2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-
a-b)^3-1/2*(20*a^6-35*a^4*b^2+28*a^2*b^4-8*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b
^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/
2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 773 vs. 2(282) = 564.

Time = 0.21 (sec) , antiderivative size = 1603, normalized size of antiderivative = 5.36

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
[-1/12*(48*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*b^9)*d*x*cos(
d*x + c)^3 + 144*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*b^8 + a^2*b^10)
*d*x*cos(d*x + c)^2 + 144*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a
*b^11)*d*x*cos(d*x + c) + 48*(a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10
 + b^12)*d*x - 3*(20*a^6*b^5 - 35*a^4*b^7 + 28*a^2*b^9 - 8*b^11 + (20*a^9*
b^2 - 35*a^7*b^4 + 28*a^5*b^6 - 8*a^3*b^8)*cos(d*x + c)^3 + 3*(20*a^8*b^3
- 35*a^6*b^5 + 28*a^4*b^7 - 8*a^2*b^9)*cos(d*x + c)^2 + 3*(20*a^7*b^4 - 35
*a^5*b^6 + 28*a^3*b^8 - 8*a*b^10)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b
*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*
x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*
x + c) + b^2)) - 2*(6*a^9*b^3 - 71*a^7*b^5 + 133*a^5*b^7 - 92*a^3*b^9 + 24
*a*b^11 + 6*(a^12 - 4*a^10*b^2 + 6*a^8*b^4 - 4*a^6*b^6 + a^4*b^8)*cos(d*x
+ c)^3 + (18*a^11*b - 132*a^9*b^3 + 239*a^7*b^5 - 169*a^5*b^7 + 44*a^3*b^9
)*cos(d*x + c)^2 + 3*(6*a^10*b^2 - 59*a^8*b^4 + 110*a^6*b^6 - 77*a^4*b^8 +
 20*a^2*b^10)*cos(d*x + c))*sin(d*x + c))/((a^16 - 4*a^14*b^2 + 6*a^12*b^4
 - 4*a^10*b^6 + a^8*b^8)*d*cos(d*x + c)^3 + 3*(a^15*b - 4*a^13*b^3 + 6*a^1
1*b^5 - 4*a^9*b^7 + a^7*b^9)*d*cos(d*x + c)^2 + 3*(a^14*b^2 - 4*a^12*b^4 +
 6*a^10*b^6 - 4*a^8*b^8 + a^6*b^10)*d*cos(d*x + c) + (a^13*b^3 - 4*a^11*b^
5 + 6*a^9*b^7 - 4*a^7*b^9 + a^5*b^11)*d), -1/6*(24*(a^11*b - 4*a^9*b^3 + 6
*a^7*b^5 - 4*a^5*b^7 + a^3*b^9)*d*x*cos(d*x + c)^3 + 72*(a^10*b^2 - 4*a...
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))**4,x)
```

output
```
Integral(cos(c + d*x)/(a + b*sec(c + d*x))**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.89

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/3*(3*(20*a^6*b^2 - 35*a^4*b^4 + 28*a^2*b^6 - 8*b^8)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2
*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6
)*sqrt(-a^2 + b^2)) - (60*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 - 105*a^5*b^4*tan
(1/2*d*x + 1/2*c)^5 - 24*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 + 117*a^3*b^6*tan(
1/2*d*x + 1/2*c)^5 - 24*a^2*b^7*tan(1/2*d*x + 1/2*c)^5 - 42*a*b^8*tan(1/2*
d*x + 1/2*c)^5 + 18*b^9*tan(1/2*d*x + 1/2*c)^5 - 120*a^6*b^3*tan(1/2*d*x +
 1/2*c)^3 + 236*a^4*b^5*tan(1/2*d*x + 1/2*c)^3 - 152*a^2*b^7*tan(1/2*d*x +
 1/2*c)^3 + 36*b^9*tan(1/2*d*x + 1/2*c)^3 + 60*a^6*b^3*tan(1/2*d*x + 1/2*c
) + 105*a^5*b^4*tan(1/2*d*x + 1/2*c) - 24*a^4*b^5*tan(1/2*d*x + 1/2*c) - 1
17*a^3*b^6*tan(1/2*d*x + 1/2*c) - 24*a^2*b^7*tan(1/2*d*x + 1/2*c) + 42*a*b
^8*tan(1/2*d*x + 1/2*c) + 18*b^9*tan(1/2*d*x + 1/2*c))/((a^10 - 3*a^8*b^2
+ 3*a^6*b^4 - a^4*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^
2 - a - b)^3) + 12*(d*x + c)*b/a^5 - 6*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x
+ 1/2*c)^2 + 1)*a^4))/d
```

Mupad [B] (verification not implemented)

Time = 19.15 (sec) , antiderivative size = 7534, normalized size of antiderivative = 25.20

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)/(a + b/cos(c + d*x))^4,x)
```

output
```
- ((tan(c/2 + (d*x)/2)^7*(4*a*b^6 + 2*a^6*b - 2*a^7 - 8*b^7 + 24*a^2*b^5 -
 11*a^3*b^4 - 26*a^4*b^3 + 6*a^5*b^2))/((a^4*b - a^5)*(a + b)^3) + (tan(c/
2 + (d*x)/2)^3*(12*a*b^7 - 18*a^8 - 72*b^8 + 236*a^2*b^6 - 47*a^3*b^5 - 27
3*a^4*b^4 + 60*a^5*b^3 + 72*a^6*b^2))/(3*(a + b)^2*(3*a^6*b - a^7 + a^4*b^
3 - 3*a^5*b^2)) - (tan(c/2 + (d*x)/2)*(4*a*b^6 - 2*a^6*b - 2*a^7 + 8*b^7 -
 24*a^2*b^5 - 11*a^3*b^4 + 26*a^4*b^3 + 6*a^5*b^2))/((a + b)*(3*a^6*b - a^
7 + a^4*b^3 - 3*a^5*b^2)) + (tan(c/2 + (d*x)/2)^5*(12*a*b^7 + 18*a^8 + 72*
b^8 - 236*a^2*b^6 - 47*a^3*b^5 + 273*a^4*b^4 + 60*a^5*b^3 - 72*a^6*b^2))/(
3*(a^4*b - a^5)*(a + b)^3*(a - b)))/(d*(3*a*b^2 + 3*a^2*b - tan(c/2 + (d*x
)/2)^4*(6*a^2*b - 6*b^3) + tan(c/2 + (d*x)/2)^2*(6*a*b^2 - 2*a^3 + 4*b^3)
+ tan(c/2 + (d*x)/2)^6*(2*a^3 - 6*a*b^2 + 4*b^3) + a^3 + b^3 - tan(c/2 + (
d*x)/2)^8*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) - (8*b*atan(((4*b*((8*tan(c/2
+ (d*x)/2)*(128*b^16 - 128*a*b^15 - 768*a^2*b^14 + 768*a^3*b^13 + 1920*a^4
*b^12 - 1920*a^5*b^11 - 2600*a^6*b^10 + 2560*a^7*b^9 + 2025*a^8*b^8 - 1920
*a^9*b^7 - 824*a^10*b^6 + 768*a^11*b^5 + 80*a^12*b^4 - 128*a^13*b^3 + 64*a
^14*b^2))/(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 -
 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^
17*b^2) + (b*((16*(8*a^23*b - 8*a^10*b^14 + 4*a^11*b^13 + 52*a^12*b^12 - 2
5*a^13*b^11 - 143*a^14*b^10 + 63*a^15*b^9 + 217*a^16*b^8 - 87*a^17*b^7 - 1
93*a^18*b^6 + 73*a^19*b^5 + 95*a^20*b^4 - 36*a^21*b^3 - 20*a^22*b^2))/(...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 2510, normalized size of antiderivative = 8.39

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(cos(d*x+c)/(a+b*sec(d*x+c))^4,x)
```

output

```
(120*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/s
qrt( - a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**2*a**9*b**2 - 210*sqrt( -
a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 +
 b**2))*cos(c + d*x)*sin(c + d*x)**2*a**7*b**4 + 168*sqrt( - a**2 + b**2)*
atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c
 + d*x)*sin(c + d*x)**2*a**5*b**6 - 48*sqrt( - a**2 + b**2)*atan((tan((c +
 d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*sin(c
+ d*x)**2*a**3*b**8 - 120*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a -
tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*a**9*b**2 - 150*sqr
t( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a
**2 + b**2))*cos(c + d*x)*a**7*b**4 + 462*sqrt( - a**2 + b**2)*atan((tan((
c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*a**
5*b**6 - 456*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)
/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*a**3*b**8 + 144*sqrt( - a**2 + b
**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*
cos(c + d*x)*a*b**10 + 360*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a -
 tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*sin(c + d*x)**2*a**8*b**3 - 630
*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(
 - a**2 + b**2))*sin(c + d*x)**2*a**6*b**5 + 504*sqrt( - a**2 + b**2)*atan
((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*sin(c ...
```

3.522 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 387

$$\begin{aligned}
\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx
&= \frac{(a^2+20b^2)\,x}{2a^6} - \frac{b^3(40a^6-84a^4b^2+69a^2b^4-20b^6)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^6(a-b)^{7/2}(a+b)^{7/2}d} \\
&\quad - \frac{b(24a^6-146a^4b^2+167a^2b^4-60b^6)\sin(c+dx)}{6a^5\left(a^2-b^2\right)^3 d} \\
&\quad + \frac{(a^6-23a^4b^2+27a^2b^4-10b^6)\cos(c+dx)\sin(c+dx)}{2a^4\left(a^2-b^2\right)^3 d} \\
&\quad + \frac{b^2\cos(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{5b^2(2a^2-b^2)\cos(c+dx)\sin(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} \\
&\quad + \frac{b^2(48a^4-53a^2b^2+20b^4)\cos(c+dx)\sin(c+dx)}{6a^3\left(a^2-b^2\right)^3 d(a+b\sec(c+dx))}
\end{aligned}$$

output
```
1/2*(a^2+20*b^2)*x/a^6-b^3*(40*a^6-84*a^4*b^2+69*a^2*b^4-20*b^6)*arctanh((
a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^6/(a-b)^(7/2)/(a+b)^(7/2)/d-1
/6*b*(24*a^6-146*a^4*b^2+167*a^2*b^4-60*b^6)*sin(d*x+c)/a^5/(a^2-b^2)^3/d+
1/2*(a^6-23*a^4*b^2+27*a^2*b^4-10*b^6)*cos(d*x+c)*sin(d*x+c)/a^4/(a^2-b^2)
^3/d+1/3*b^2*cos(d*x+c)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^3+5/6*b^
2*(2*a^2-b^2)*cos(d*x+c)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2+1
/6*b^2*(48*a^4-53*a^2*b^2+20*b^4)*cos(d*x+c)*sin(d*x+c)/a^3/(a^2-b^2)^3/d/
(a+b*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 5.66 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.68

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

$$= \frac{6(a^2+20b^2)(c+dx) - \frac{12b^3(-40a^6+84a^4b^2-69a^2b^4+20b^6)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} - 48ab\sin(c+dx) + \frac{\cdots}{(a-b\cdots}}{12a^6d}$$

input
```
Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^4,x]
```

output
```
(6*(a^2 + 20*b^2)*(c + d*x) - (12*b^3*(-40*a^6 + 84*a^4*b^2 - 69*a^2*b^4 +
 20*b^6)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)
^(7/2) - 48*a*b*Sin[c + d*x] + (4*a*b^6*Sin[c + d*x])/((a - b)*(a + b)*(b
+ a*Cos[c + d*x])^3) + (2*a*b^5*(-18*a^2 + 13*b^2)*Sin[c + d*x])/((a - b)^
2*(a + b)^2*(b + a*Cos[c + d*x])^2) + (2*a*b^4*(90*a^4 - 122*a^2*b^2 + 47*
b^4)*Sin[c + d*x])/((a - b)^3*(a + b)^3*(b + a*Cos[c + d*x])) + 3*a^2*Sin[
2*(c + d*x)])/(12*a^6*d)
```

Rubi [A] (verified)

Time = 3.12 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.11, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.048, Rules used = {3042, 4334, 25, 3042, 4588, 25, 3042, 4588, 25, 3042, 4592, 27, 3042, 4592, 27, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{ 4334}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{\int -\frac{\cos^2(c+dx)\left(3a^2-3b\sec(c+dx)a-5b^2+4b^2\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}dx}{3a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\cos^2(c+dx)\left(3a^2-3b\sec(c+dx)a-5b^2+4b^2\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}dx}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{3a^2-3b\csc\left(c+dx+\frac{\pi}{2}\right)a-5b^2+4b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{ 4588}$$

$$\frac{\frac{5b^2\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\int -\frac{\cos^2(c+dx)\left(15b^2\left(2a^2-b^2\right)\sec^2(c+dx)-2ab\left(6a^2-b^2\right)\sec(c+dx)+2\left(3a^4-18b^2a^2+10b^4\right)\right)}{(a+b\sec(c+dx))^2}dx}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{\cos^2(c+dx)\left(15b^2\left(2a^2-b^2\right)\sec^2(c+dx)-2ab\left(6a^2-b^2\right)\sec(c+dx)+2\left(3a^4-18b^2a^2+10b^4\right)\right)}{(a+b\sec(c+dx))^2}dx}{2a(a^2-b^2)}+\frac{5b^2(2a^2-b^2)\sin(c+dx)\cos(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{15b^2\left(2a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-2ab\left(6a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+2\left(3a^4-18b^2a^2+10b^4\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2a(a^2-b^2)}+\frac{5b^2(2a^2-b^2)\sin(c+dx)\cos(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 4588

$$\frac{\frac{\frac{b^2\left(48a^4-53a^2b^2+20b^4\right)\sin(c+dx)\cos(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int -\frac{\cos^2(c+dx)\left(2b^2\left(48a^4-53b^2a^2+20b^4\right)\sec^2(c+dx)-ab\left(18a^4-8b^2a^2+5b^4\right)\sec(c+dx)+6\left(a^6-23b^2a^4+27b^4a^2-10b^6\right)\right)}{a+b\sec(c+dx)}dx}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{\cos^2(c+dx)\left(2b^2\left(48a^4-53b^2a^2+20b^4\right)\sec^2(c+dx)-ab\left(18a^4-8b^2a^2+5b^4\right)\sec(c+dx)+6\left(a^6-23b^2a^4+27b^4a^2-10b^6\right)\right)}{a+b\sec(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\left(48a^4-53a^2b^2+20b^4\right)\sin(c+dx)\cos(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{2b^2\left(48a^4-53b^2a^2+20b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-ab\left(18a^4-8b^2a^2+5b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)+6\left(a^6-23b^2a^4+27b^4a^2-10b^6\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\left(48a^4-53a^2b^2+20b^4\right)\sin(c+dx)\cos(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 4592

$$\frac{3\left(a^6-23a^4b^2+27a^2b^4-10b^6\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\int\frac{2\cos(c+dx)\left(-3b\left(a^6-23b^2a^4+27b^4a^2-10b^6\right)\sec^2(c+dx)-a\left(3a^6+27b^2a^4-25b^4a^2+10b^6\right)\sec(c+dx)+b}{a+b\sec(c+dx)}}{2a}$$

$$a\left(a^2-b^2\right)$$

$$2a(a^2-b^2)$$

$$3a\left(a^2-b^2\right)$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 27

$$\frac{3\left(a^6-23a^4b^2+27a^2b^4-10b^6\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\int\frac{\cos(c+dx)\left(-3b\left(a^6-23b^2a^4+27b^4a^2-10b^6\right)\sec^2(c+dx)-a\left(3a^6+27b^2a^4-25b^4a^2+10b^6\right)\sec(c+dx)+b\left(\right.}{a+b\sec(c+dx)}}{a}$$

$$a\left(a^2-b^2\right)$$

$$2a(a^2-b^2)$$

$$3a\left(a^2-b^2\right)$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 3042

$$\frac{3\left(a^6-23a^4b^2+27a^2b^4-10b^6\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\int\frac{-3b\left(a^6-23b^2a^4+27b^4a^2-10b^6\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-a\left(3a^6+27b^2a^4-25b^4a^2+10b^6\right)\csc\left(c+dx+\frac{\pi}{2}\right)+b\left(24\right.}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}}{a}$$

$$a\left(a^2-b^2\right)$$

$$2a(a^2-b^2)$$

$$3a\left(a^2-b^2\right)$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 4592

$$\frac{3\left(a^6-23a^4b^2+27a^2b^4-10b^6\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{b\left(24a^6-146a^4b^2+167a^2b^4-60b^6\right)\sin(c+dx)}{ad}-\frac{\int\frac{3\left(\left(a^2+20b^2\right)\left(a^2-b^2\right)^3+ab\left(a^6-23b^2a^4+27b^4a^2-10b^6\right.\right.}{a+b\sec(c+dx)}}{a}}{a}$$

$$a\left(a^2-b^2\right)$$

$$2a(a^2-b^2)$$

$$3a\left(a^2-b^2\right)$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 27

$$\frac{\frac{\frac{3\left(a^6-23a^4b^2+27a^2b^4-10b^6\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{b\left(24a^6-146a^4b^2+167a^2b^4-60b^6\right)\sin(c+dx)}{ad}-\frac{3\int\frac{\left(a^2+20b^2\right)\left(a^2-b^2\right)^3+ab\left(a^6-23b^2a^4+27b^4a^2-10b^6\right)}{a+b\sec(c+dx)}}{a}}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^3}$$

$\big\downarrow$ 3042

$$\frac{\frac{\frac{3\left(a^6-23a^4b^2+27a^2b^4-10b^6\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{b\left(24a^6-146a^4b^2+167a^2b^4-60b^6\right)\sin(c+dx)}{ad}-\frac{3\int\frac{\left(a^2+20b^2\right)\left(a^2-b^2\right)^3+ab\left(a^6-23b^2a^4+27b^4a^2-10b^6\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{a}}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^3}$$

$\big\downarrow$ 4407

$$\frac{\frac{\frac{3\left(a^6-23a^4b^2+27a^2b^4-10b^6\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{b\left(24a^6-146a^4b^2+167a^2b^4-60b^6\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{x\left(a^2-b^2\right)^3\left(a^2+20b^2\right)}{a}-\frac{b^3\left(40a^6-84a^4b^2+69a^2b^4-2\right.}{a}\right.}{a}}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^3}$$

$\big\downarrow$ 3042

$$\frac{\frac{\frac{3\left(a^6-23a^4b^2+27a^2b^4-10b^6\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{b\left(24a^6-146a^4b^2+167a^2b^4-60b^6\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{x\left(a^2-b^2\right)^3\left(a^2+20b^2\right)}{a}-\frac{b^3\left(40a^6-84a^4b^2+69a^2b^4-2\right.}{a}\right.}{a}}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^3}$$

$\big\downarrow$ 4318

$$\frac{\frac{\frac{3\left(a^6-23a^4b^2+27a^2b^4-10b^6\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{b\left(24a^6-146a^4b^2+167a^2b^4-60b^6\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{x\left(a^2-b^2\right)^3\left(a^2+20b^2\right)}{a}-\frac{\frac{b^2\left(40a^6-84a^4b^2+69a^2b^4-2\cdots\right.}{a}}{a}\cdots\right.}{a}}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}\cdots}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\big\downarrow$ 3042

$$\frac{\frac{\frac{3\left(a^6-23a^4b^2+27a^2b^4-10b^6\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{b\left(24a^6-146a^4b^2+167a^2b^4-60b^6\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{x\left(a^2-b^2\right)^3\left(a^2+20b^2\right)}{a}-\frac{b^2\left(40a^6-84a^4b^2+69a^2b^4-2\cdots\right.}{\cdots}\cdots\right.}{a}}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}\cdots}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\big\downarrow$ 3138

$$\frac{\frac{\frac{3\left(a^6-23a^4b^2+27a^2b^4-10b^6\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{b\left(24a^6-146a^4b^2+167a^2b^4-60b^6\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{x\left(a^2-b^2\right)^3\left(a^2+20b^2\right)}{a}-\frac{2b^2\left(40a^6-84a^4b^2+69a^2b^4-\cdots\right.}{\cdots}\cdots\right.}{a}}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}\cdots}{3a\left(a^2-b^2\right)\cdots}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\big\downarrow$ 221

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}+$$

$$\frac{\frac{5b^2(2a^2-b^2)\sin(c+dx)\cos(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}+\frac{\frac{b^2\left(48a^4-53a^2b^2+20b^4\right)\sin(c+dx)\cos(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\frac{3\left(a^6-23a^4b^2+27a^2b^4-10b^6\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{b\left(24a^6-146a^4\cdots\right.}{\cdots}\cdots}{\cdots}}{\cdots}}{3a\left(a^2-b^2\right)}$$

input
```
Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^4,x]
```

output
```
(b^2*Cos[c + d*x]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3)
 + ((5*b^2*(2*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a
+ b*Sec[c + d*x])^2) + ((b^2*(48*a^4 - 53*a^2*b^2 + 20*b^4)*Cos[c + d*x]*S
in[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])) + ((3*(a^6 - 23*a^4*b^
2 + 27*a^2*b^4 - 10*b^6)*Cos[c + d*x]*Sin[c + d*x])/(a*d) - ((-3*(((a^2 -
b^2)^3*(a^2 + 20*b^2)*x)/a - (2*b^3*(40*a^6 - 84*a^4*b^2 + 69*a^2*b^4 - 20
*b^6)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*
Sqrt[a + b]*d)))/a + (b*(24*a^6 - 146*a^4*b^2 + 167*a^2*b^4 - 60*b^6)*Sin[
c + d*x])/(a*d))/a)/(a*(a^2 - b^2)))/(2*a*(a^2 - b^2)))/(3*a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{\frac{2\left(\left(-\frac{1}{2}a^2-4ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(\frac{1}{2}a^2-4ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\left(a^2+20b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^6}+\frac{2b^3\left(-\frac{\left(30a^4+6b\,a^3-34a^2b^2-\cdots\right.}{2(a-b)\left(a^3+\cdots\right.}\cdots\right.}{\cdots}$
default	$\frac{\frac{2\left(\left(-\frac{1}{2}a^2-4ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(\frac{1}{2}a^2-4ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\left(a^2+20b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^6}+\frac{2b^3\left(-\frac{\left(30a^4+6b\,a^3-34a^2b^2-\cdots\right.}{2(a-b)\left(a^3+\cdots\right.}\cdots\right.}{\cdots}$
risch	Expression too large to display

input
```
int(cos(d*x+c)^2/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/a^6*(((-1/2*a^2-4*a*b)*tan(1/2*d*x+1/2*c)^3+(1/2*a^2-4*a*b)*tan(1/2
*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(a^2+20*b^2)*arctan(tan(1/2*d*
x+1/2*c)))+2*b^3/a^6*((-1/2*(30*a^4+6*a^3*b-34*a^2*b^2-3*a*b^3+12*b^4)*a*b
/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/3*(45*a^4-53*a^2*b
^2+18*b^4)*a*b/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(3
0*a^4-6*a^3*b-34*a^2*b^2+3*a*b^3+12*b^4)*a*b/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^
3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)
^3-1/2*(40*a^6-84*a^4*b^2+69*a^2*b^4-20*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)
/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 855 vs. 2(366) = 732.

Time = 0.26 (sec) , antiderivative size = 1767, normalized size of antiderivative = 4.57

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
[1/12*(6*(a^13 + 16*a^11*b^2 - 74*a^9*b^4 + 116*a^7*b^6 - 79*a^5*b^8 + 20*
a^3*b^10)*d*x*cos(d*x + c)^3 + 18*(a^12*b + 16*a^10*b^3 - 74*a^8*b^5 + 116
*a^6*b^7 - 79*a^4*b^9 + 20*a^2*b^11)*d*x*cos(d*x + c)^2 + 18*(a^11*b^2 + 1
6*a^9*b^4 - 74*a^7*b^6 + 116*a^5*b^8 - 79*a^3*b^10 + 20*a*b^12)*d*x*cos(d*
x + c) + 6*(a^10*b^3 + 16*a^8*b^5 - 74*a^6*b^7 + 116*a^4*b^9 - 79*a^2*b^11
 + 20*b^13)*d*x + 3*(40*a^6*b^6 - 84*a^4*b^8 + 69*a^2*b^10 - 20*b^12 + (40
*a^9*b^3 - 84*a^7*b^5 + 69*a^5*b^7 - 20*a^3*b^9)*cos(d*x + c)^3 + 3*(40*a^
8*b^4 - 84*a^6*b^6 + 69*a^4*b^8 - 20*a^2*b^10)*cos(d*x + c)^2 + 3*(40*a^7*
b^5 - 84*a^5*b^7 + 69*a^3*b^9 - 20*a*b^11)*cos(d*x + c))*sqrt(a^2 - b^2)*l
og((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*
(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a
*b*cos(d*x + c) + b^2)) - 2*(24*a^9*b^4 - 170*a^7*b^6 + 313*a^5*b^8 - 227*
a^3*b^10 + 60*a*b^12 - 3*(a^13 - 4*a^11*b^2 + 6*a^9*b^4 - 4*a^7*b^6 + a^5*
b^8)*cos(d*x + c)^4 + 15*(a^12*b - 4*a^10*b^3 + 6*a^8*b^5 - 4*a^6*b^7 + a^
4*b^9)*cos(d*x + c)^3 + (63*a^11*b^2 - 342*a^9*b^4 + 590*a^7*b^6 - 421*a^5
*b^8 + 110*a^3*b^10)*cos(d*x + c)^2 + 3*(23*a^10*b^3 - 146*a^8*b^5 + 263*a
^6*b^7 - 190*a^4*b^9 + 50*a^2*b^11)*cos(d*x + c))*sin(d*x + c))/((a^17 - 4
*a^15*b^2 + 6*a^13*b^4 - 4*a^11*b^6 + a^9*b^8)*d*cos(d*x + c)^3 + 3*(a^16*
b - 4*a^14*b^3 + 6*a^12*b^5 - 4*a^10*b^7 + a^8*b^9)*d*cos(d*x + c)^2 + 3*(
a^15*b^2 - 4*a^13*b^4 + 6*a^11*b^6 - 4*a^9*b^8 + a^7*b^10)*d*cos(d*x + ...
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \int \frac{\cos^2{(c+dx)}}{(a+b\sec{(c+dx)})^4}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**4,x)
```

output
```
Integral(cos(c + d*x)**2/(a + b*sec(c + d*x))**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.59

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/6*(6*(40*a^6*b^3 - 84*a^4*b^5 + 69*a^2*b^7 - 20*b^9)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2
*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^
6)*sqrt(-a^2 + b^2)) - 2*(90*a^6*b^4*tan(1/2*d*x + 1/2*c)^5 - 162*a^5*b^5*
tan(1/2*d*x + 1/2*c)^5 - 48*a^4*b^6*tan(1/2*d*x + 1/2*c)^5 + 213*a^3*b^7*t
an(1/2*d*x + 1/2*c)^5 - 48*a^2*b^8*tan(1/2*d*x + 1/2*c)^5 - 81*a*b^9*tan(1
/2*d*x + 1/2*c)^5 + 36*b^10*tan(1/2*d*x + 1/2*c)^5 - 180*a^6*b^4*tan(1/2*d
*x + 1/2*c)^3 + 392*a^4*b^6*tan(1/2*d*x + 1/2*c)^3 - 284*a^2*b^8*tan(1/2*d
*x + 1/2*c)^3 + 72*b^10*tan(1/2*d*x + 1/2*c)^3 + 90*a^6*b^4*tan(1/2*d*x +
1/2*c) + 162*a^5*b^5*tan(1/2*d*x + 1/2*c) - 48*a^4*b^6*tan(1/2*d*x + 1/2*c
) - 213*a^3*b^7*tan(1/2*d*x + 1/2*c) - 48*a^2*b^8*tan(1/2*d*x + 1/2*c) + 8
1*a*b^9*tan(1/2*d*x + 1/2*c) + 36*b^10*tan(1/2*d*x + 1/2*c))/((a^11 - 3*a^
9*b^2 + 3*a^7*b^4 - a^5*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1
/2*c)^2 - a - b)^3) + 3*(a^2 + 20*b^2)*(d*x + c)/a^6 - 6*(a*tan(1/2*d*x +
1/2*c)^3 + 8*b*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) + 8*b*tan(1
/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^5))/d
```

Mupad [B] (verification not implemented)

Time = 18.98 (sec) , antiderivative size = 8133, normalized size of antiderivative = 21.02

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^2/(a + b/cos(c + d*x))^4,x)
```

output
```
((tan(c/2 + (d*x)/2)^9*(7*a^7*b - 10*a*b^7 + a^8 + 20*b^8 - 59*a^2*b^6 + 2
7*a^3*b^5 + 57*a^4*b^4 - 21*a^5*b^3 - 11*a^6*b^2))/(a^5*(a + b)^3*(a - b))
 + (2*tan(c/2 + (d*x)/2)^3*(30*a*b^8 + 21*a^8*b - 6*a^9 + 120*b^9 - 364*a^
2*b^7 - 71*a^3*b^6 + 369*a^4*b^5 + 45*a^5*b^4 - 111*a^6*b^3 - 3*a^7*b^2))/
(3*a^5*(a + b)^2*(a - b)^3) - (2*tan(c/2 + (d*x)/2)^7*(21*a^8*b - 30*a*b^8
 + 6*a^9 + 120*b^9 - 364*a^2*b^7 + 71*a^3*b^6 + 369*a^4*b^5 - 45*a^5*b^4 -
 111*a^6*b^3 + 3*a^7*b^2))/(3*a^5*(a + b)^3*(a - b)^2) + (2*tan(c/2 + (d*x
)/2)^5*(9*a^10 + 180*b^10 - 611*a^2*b^8 + 740*a^4*b^6 - 324*a^6*b^4 + 36*a
^8*b^2))/(3*a^5*(a + b)^3*(a - b)^3) + (tan(c/2 + (d*x)/2)*(10*a*b^7 - 7*a
^7*b + a^8 + 20*b^8 - 59*a^2*b^6 - 27*a^3*b^5 + 57*a^4*b^4 + 21*a^5*b^3 -
11*a^6*b^2))/(a^5*(a + b)*(a - b)^3))/(d*(tan(c/2 + (d*x)/2)^2*(9*a*b^2 +
3*a^2*b - a^3 + 5*b^3) + tan(c/2 + (d*x)/2)^4*(6*a*b^2 - 6*a^2*b - 2*a^3 +
 10*b^3) - tan(c/2 + (d*x)/2)^6*(6*a*b^2 + 6*a^2*b - 2*a^3 - 10*b^3) + 3*a
*b^2 + 3*a^2*b + a^3 + b^3 - tan(c/2 + (d*x)/2)^10*(3*a*b^2 - 3*a^2*b + a^
3 - b^3) + tan(c/2 + (d*x)/2)^8*(3*a^2*b - 9*a*b^2 + a^3 + 5*b^3))) - (ata
n((((((((4*(4*a^27 - 80*a^12*b^15 + 40*a^13*b^14 + 516*a^14*b^13 - 248*a^15
*b^12 - 1404*a^16*b^11 + 640*a^17*b^10 + 2076*a^18*b^9 - 896*a^19*b^8 - 17
64*a^20*b^7 + 724*a^21*b^6 + 816*a^22*b^5 - 316*a^23*b^4 - 160*a^24*b^3 +
52*a^25*b^2))/(a^25*b + a^26 - a^15*b^11 - a^16*b^10 + 5*a^17*b^9 + 5*a^18
*b^8 - 10*a^19*b^7 - 10*a^20*b^6 + 10*a^21*b^5 + 10*a^22*b^4 - 5*a^23*b...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 2737, normalized size of antiderivative = 7.07

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(cos(d*x+c)^2/(a+b*sec(d*x+c))^4,x)
```

output

```
( - 240*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b
)/sqrt( - a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**2*a**9*b**3 + 504*sqrt(
 - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**
2 + b**2))*cos(c + d*x)*sin(c + d*x)**2*a**7*b**5 - 414*sqrt( - a**2 + b**
2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*co
s(c + d*x)*sin(c + d*x)**2*a**5*b**7 + 120*sqrt( - a**2 + b**2)*atan((tan(
(c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*si
n(c + d*x)**2*a**3*b**9 + 240*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*
a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*a**9*b**3 + 216
*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(
 - a**2 + b**2))*cos(c + d*x)*a**7*b**5 - 1098*sqrt( - a**2 + b**2)*atan((
tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x
)*a**5*b**7 + 1122*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c
+ d*x)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x)*a**3*b**9 - 360*sqrt( - a*
*2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b
**2))*cos(c + d*x)*a*b**11 - 720*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/
2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2))*sin(c + d*x)**2*a**8*b**4
 + 1512*sqrt( - a**2 + b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b
)/sqrt( - a**2 + b**2))*sin(c + d*x)**2*a**6*b**6 - 1242*sqrt( - a**2 + b*
*2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt( - a**2 + b**2)...
```

3.523 $\int \frac{1}{3+5\sec(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{1}{3+5\sec(c+dx)}\,dx=-\frac{x}{12}+\frac{5\arctan\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{6d}$$

output
```
-1/12*x+5/6*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{1}{3+5\sec(c+dx)}\,dx=\frac{2(c+dx)+5\arctan\left(2\cot\left(\frac{1}{2}(c+dx)\right)\right)}{6d}$$

input
```
Integrate[(3 + 5*Sec[c + d*x])^(-1),x]
```

output
```
(2*(c + d*x) + 5*ArcTan[2*Cot[(c + d*x)/2]])/(6*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4270, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{5\sec(c+dx)+3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{5\csc\left(c+dx+\frac{\pi}{2}\right)+3}dx$$
$$\Big\downarrow \text{4270}$$
$$\frac{x}{3}-\frac{1}{3}\int \frac{1}{\frac{3}{5}\cos(c+dx)+1}dx$$
$$\Big\downarrow \text{3042}$$
$$\frac{x}{3}-\frac{1}{3}\int \frac{1}{\frac{3}{5}\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx$$
$$\Big\downarrow \text{3136}$$
$$\frac{1}{3}\left(\frac{5\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{2d}-\frac{5x}{4}\right)+\frac{x}{3}$$

input
```
Int[(3 + 5*Sec[c + d*x])^(-1),x]
```

output
```
x/3 + ((-5*x)/4 + (5*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(2*d))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 4270
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Simp[1/a   Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{-\frac{5\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{6}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}}{d}$	32
default	$\frac{-\frac{5\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{6}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}}{d}$	32
risch	$\frac{x}{3}+\frac{5i\ln\left(e^{i(dx+c)}+\frac{1}{3}\right)}{12d}-\frac{5i\ln\left(e^{i(dx+c)}+3\right)}{12d}$	41
parallelrisch	$\frac{4dx-5i\left(-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2i\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2i\right)\right)}{12d}$	43

input
```
int(1/(3+5*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-5/6*arctan(1/2*tan(1/2*d*x+1/2*c))+2/3*arctan(tan(1/2*d*x+1/2*c)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{3+5\sec(c+dx)}\,dx=\frac{4\,dx+5\,\arctan\left(\frac{5\,\cos(dx+c)+3}{4\,\sin(dx+c)}\right)}{12\,d}$$

input
```
integrate(1/(3+5*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(4*d*x + 5*arctan(1/4*(5*cos(d*x + c) + 3)/sin(d*x + c)))/d
```

Sympy [F]

$$\int \frac{1}{3+5\sec(c+dx)}\,dx=\int \frac{1}{5\sec{(c+dx)}+3}\,dx$$

input
```
integrate(1/(3+5*sec(d*x+c)),x)
```

output
```
Integral(1/(5*sec(c + d*x) + 3), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int \frac{1}{3+5\sec(c+dx)}\,dx=\frac{4\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)-5\,\arctan\left(\frac{\sin(dx+c)}{2\,(\cos(dx+c)+1)}\right)}{6\,d}$$

input
```
integrate(1/(3+5*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/6*(4*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) - 5*arctan(1/2*sin(d*x + c)
/(cos(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{1}{3+5\sec(c+dx)}\,dx = -\frac{dx+c-10\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{12\,d}$$

input
```
integrate(1/(3+5*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/12*(d*x + c - 10*arctan(sin(d*x + c)/(cos(d*x + c) + 3)))/d
```

Mupad [B] (verification not implemented)

Time = 10.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{1}{3+5\sec(c+dx)}\,dx = \frac{x}{3} - \frac{5\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{6\,d}$$

input
```
int(1/(5/cos(c + d*x) + 3),x)
```

output
```
x/3 - (5*atan(tan(c/2 + (d*x)/2)/2))/(6*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1}{3+5\sec(c+dx)}\,dx = \frac{-5atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)+2dx}{6d}$$

input
```
int(1/(3+5*sec(d*x+c)),x)
```

output
```
( - 5*atan(tan((c + d*x)/2)/2) + 2*d*x)/(6*d)
```

3.524 $\int \frac{1}{(3+5\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{1}{(3+5\sec(c+dx))^2}\,dx=\frac{29x}{576}+\frac{35\arctan\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{288d}-\frac{25\tan(c+dx)}{48d(3+5\sec(c+dx))}$$

output
```
29/576*x+35/288*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d-25/48*tan(d*x+c)/d/(3+
5*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.30

$$\int \frac{1}{(3+5\sec(c+dx))^2}\,dx = \frac{160(c+dx)+96(c+dx)\cos(c+dx)+35\arctan\left(2\cot\left(\frac{1}{2}(c+dx)\right)\right)(5+3\cos(c+dx))-150\sin(c+dx)}{288d(5+3\cos(c+dx))}$$

input
```
Integrate[(3 + 5*Sec[c + d*x])^(-2),x]
```

output
```
(160*(c + d*x) + 96*(c + d*x)*Cos[c + d*x] + 35*ArcTan[2*Cot[(c + d*x)/2]]
*(5 + 3*Cos[c + d*x]) - 150*Sin[c + d*x])/(288*d*(5 + 3*Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.27, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4272, 3042, 4407, 3042, 4318, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5\sec(c+dx)+3)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(5\csc\left(c+dx+\frac{\pi}{2}\right)+3\right)^2}dx$$

$$\Big\downarrow \text{ 4272}$$

$$\frac{1}{48}\int \frac{15\sec(c+dx)+16}{5\sec(c+dx)+3}dx-\frac{25\tan(c+dx)}{48d(5\sec(c+dx)+3)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{48}\int \frac{15\csc\left(c+dx+\frac{\pi}{2}\right)+16}{5\csc\left(c+dx+\frac{\pi}{2}\right)+3}dx-\frac{25\tan(c+dx)}{48d(5\sec(c+dx)+3)}$$

$$\Big\downarrow \text{ 4407}$$

$$\frac{1}{48}\left(\frac{16x}{3}-\frac{35}{3}\int \frac{\sec(c+dx)}{5\sec(c+dx)+3}dx\right)-\frac{25\tan(c+dx)}{48d(5\sec(c+dx)+3)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{48}\left(\frac{16x}{3}-\frac{35}{3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{5\csc\left(c+dx+\frac{\pi}{2}\right)+3}dx\right)-\frac{25\tan(c+dx)}{48d(5\sec(c+dx)+3)}$$

$$\Big\downarrow \text{ 4318}$$

$$\frac{1}{48}\left(\frac{16x}{3}-\frac{7}{3}\int \frac{1}{\frac{3}{5}\cos(c+dx)+1}dx\right)-\frac{25\tan(c+dx)}{48d(5\sec(c+dx)+3)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{48}\left(\frac{16x}{3}-\frac{7}{3}\int \frac{1}{\frac{3}{5}\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx\right)-\frac{25\tan(c+dx)}{48d(5\sec(c+dx)+3)}$$

$$\Big\downarrow \text{3136}$$

$$\frac{1}{48}\left(\frac{16x}{3}-\frac{7}{3}\left(\frac{5x}{4}-\frac{5\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{2d}\right)\right)-\frac{25\tan(c+dx)}{48d(5\sec(c+dx)+3)}$$

input
```
Int[(3 + 5*Sec[c + d*x])^(-2),x]
```

output
```
((16*x)/3 - (7*((5*x)/4 - (5*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(2*d
)))/3)/48 - (25*Tan[c + d*x])/(48*d*(3 + 5*Sec[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+4\right)}-\frac{35\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{288}}{d}$
default	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+4\right)}-\frac{35\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{288}}{d}$
risch	$\frac{x}{9}-\frac{25i\left(5\,\mathrm{e}^{i(dx+c)}+3\right)}{72d\left(3\,\mathrm{e}^{2i(dx+c)}+10\,\mathrm{e}^{i(dx+c)}+3\right)}-\frac{35i\ln\left(\mathrm{e}^{i(dx+c)}+3\right)}{576d}+\frac{35i\ln\left(\mathrm{e}^{i(dx+c)}+\frac{1}{3}\right)}{576d}$
parallelrisch	$\frac{(175i+105i\cos(dx+c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2i\right)+(-105i\cos(dx+c)-175i)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2i\right)+192dx\cos(dx+c)+320}{1728d\cos(dx+c)+2880d}$

input
```
int(1/(3+5*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/9*arctan(tan(1/2*d*x+1/2*c))-25/48*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+
1/2*c)^2+4)-35/288*arctan(1/2*tan(1/2*d*x+1/2*c)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.30

$$\int \frac{1}{(3+5\sec(c+dx))^2}\,dx$$
$$=\frac{192\,dx\cos\left(dx+c\right)+320\,dx+35\left(3\,\cos\left(dx+c\right)+5\right)\arctan\left(\frac{5\,\cos(dx+c)+3}{4\,\sin(dx+c)}\right)-300\,\sin\left(dx+c\right)}{576\left(3\,d\cos\left(dx+c\right)+5\,d\right)}$$

input
```
integrate(1/(3+5*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/576*(192*d*x*cos(d*x + c) + 320*d*x + 35*(3*cos(d*x + c) + 5)*arctan(1/4
*(5*cos(d*x + c) + 3)/sin(d*x + c)) - 300*sin(d*x + c))/(3*d*cos(d*x + c)
+ 5*d)
```

Sympy [F]

$$\int \frac{1}{(3+5\sec(c+dx))^2}\,dx = \int \frac{1}{(5\sec{(c+dx)}+3)^2}\,dx$$

input
```
integrate(1/(3+5*sec(d*x+c))**2,x)
```

output
```
Integral((5*sec(c + d*x) + 3)**(-2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.57

$$\int \frac{1}{(3+5\sec(c+dx))^2}\,dx = -\frac{\frac{150\,\sin(dx+c)}{\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+4\right)(\cos(dx+c)+1)} - 64\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + 35\,\arctan\left(\frac{\sin(dx+c)}{2\,(\cos(dx+c)+1)}\right)}{288\,d}$$

input
```
integrate(1/(3+5*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/288*(150*sin(d*x + c)/((sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4)*(cos(d
*x + c) + 1)) - 64*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 35*arctan(1/2
*sin(d*x + c)/(cos(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.05

$$\int \frac{1}{(3+5\sec(c+dx))^2}\,dx = \frac{29\,dx + 29\,c - \frac{300\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4} + 70\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{576\,d}$$

input
```
integrate(1/(3+5*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/576*(29*d*x + 29*c - 300*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 +
4) + 70*arctan(sin(d*x + c)/(cos(d*x + c) + 3)))/d
```

Mupad [B] (verification not implemented)

Time = 10.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \frac{1}{(3+5\sec(c+dx))^2}\,dx = \frac{x}{9} - \frac{\frac{35\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{288} + \frac{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{48\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+4\right)}}{d}$$

input
```
int(1/(5/cos(c + d*x) + 3)^2,x)
```

output
```
x/9 - ((35*atan(tan(c/2 + (d*x)/2)/2))/288 + (25*tan(c/2 + (d*x)/2))/(48*(
tan(c/2 + (d*x)/2)^2 + 4)))/d
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.32

$$\int \frac{1}{(3+5\sec(c+dx))^2}\,dx$$

$$= \frac{-105atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)\cos(dx+c) - 175atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right) + 96\cos(dx+c)\,dx - 150\sin(dx+c) + 16}{288d\,(3\cos(dx+c)+5)}$$

input
```
int(1/(3+5*sec(d*x+c))^2,x)
```

output
```
( - 105*atan(tan((c + d*x)/2)/2)*cos(c + d*x) - 175*atan(tan((c + d*x)/2)/
2) + 96*cos(c + d*x)*d*x - 150*sin(c + d*x) + 160*d*x)/(288*d*(3*cos(c + d
*x) + 5))
```

3.525 $\int \frac{1}{(3+5\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 81

$$\int \frac{1}{(3+5\sec(c+dx))^3}\,dx = -\frac{1007x}{55296} + \frac{3055\arctan\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{27648d} - \frac{25\tan(c+dx)}{96d(3+5\sec(c+dx))^2} - \frac{125\tan(c+dx)}{4608d(3+5\sec(c+dx))}$$

output
```
-1007/55296*x+3055/27648*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d-25/96*tan(d*x
+c)/d/(3+5*sec(d*x+c))^2-125/4608*tan(d*x+c)/d/(3+5*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.33

$$\int \frac{1}{(3+5\sec(c+dx))^3}\,dx = \frac{30208c + 30208dx + 30720(c+dx)\cos(c+dx) + 3055\arctan\left(2\cot\left(\frac{1}{2}(c+dx)\right)\right)(5+3\cos(c+dx))^2 + \cdots}{27648d(5+3\cos(c+\cdots}$$

input
```
Integrate[(3 + 5*Sec[c + d*x])^(-3),x]
```

output
```
(30208*c + 30208*d*x + 30720*(c + d*x)*Cos[c + d*x] + 3055*ArcTan[2*Cot[(c
 + d*x)/2]]*(5 + 3*Cos[c + d*x])^2 + 4608*c*Cos[2*(c + d*x)] + 4608*d*x*Co
s[2*(c + d*x)] - 3750*Sin[c + d*x] - 4725*Sin[2*(c + d*x)])/(27648*d*(5 +
3*Cos[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {3042, 4272, 3042, 4548, 3042, 4407, 3042, 4318, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5\sec(c+dx)+3)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(5\csc\left(c+dx+\frac{\pi}{2}\right)+3\right)^3}dx$$

$$\Big\downarrow \text{4272}$$

$$\frac{1}{96}\int \frac{-25\sec^2(c+dx)+30\sec(c+dx)+32}{(5\sec(c+dx)+3)^2}dx-\frac{25\tan(c+dx)}{96d(5\sec(c+dx)+3)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{96}\int \frac{-25\csc\left(c+dx+\frac{\pi}{2}\right)^2+30\csc\left(c+dx+\frac{\pi}{2}\right)+32}{\left(5\csc\left(c+dx+\frac{\pi}{2}\right)+3\right)^2}dx-\frac{25\tan(c+dx)}{96d(5\sec(c+dx)+3)^2}$$

$$\Big\downarrow \text{4548}$$

$$\frac{1}{96}\left(\frac{1}{48}\int \frac{512-165\sec(c+dx)}{5\sec(c+dx)+3}dx-\frac{125\tan(c+dx)}{48d(5\sec(c+dx)+3)}\right)-\frac{25\tan(c+dx)}{96d(5\sec(c+dx)+3)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{96}\left(\frac{1}{48}\int \frac{512-165\csc\left(c+dx+\frac{\pi}{2}\right)}{5\csc\left(c+dx+\frac{\pi}{2}\right)+3}dx-\frac{125\tan(c+dx)}{48d(5\sec(c+dx)+3)}\right)-\frac{25\tan(c+dx)}{96d(5\sec(c+dx)+3)^2}$$

$$\Big\downarrow \text{4407}$$

$$\frac{1}{96}\left(\frac{1}{48}\left(\frac{512x}{3}-\frac{3055}{3}\int\frac{\sec(c+dx)}{5\sec(c+dx)+3}dx\right)-\frac{125\tan(c+dx)}{48d(5\sec(c+dx)+3)}\right)-\frac{25\tan(c+dx)}{96d(5\sec(c+dx)+3)^2}$$

$\Big\downarrow$ 3042

$$\frac{1}{96}\left(\frac{1}{48}\left(\frac{512x}{3}-\frac{3055}{3}\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{5\csc\left(c+dx+\frac{\pi}{2}\right)+3}dx\right)-\frac{125\tan(c+dx)}{48d(5\sec(c+dx)+3)}\right)-\frac{25\tan(c+dx)}{96d(5\sec(c+dx)+3)^2}$$

$\Big\downarrow$ 4318

$$\frac{1}{96}\left(\frac{1}{48}\left(\frac{512x}{3}-\frac{611}{3}\int\frac{1}{\frac{3}{5}\cos(c+dx)+1}dx\right)-\frac{125\tan(c+dx)}{48d(5\sec(c+dx)+3)}\right)-\frac{25\tan(c+dx)}{96d(5\sec(c+dx)+3)^2}$$

$\Big\downarrow$ 3042

$$\frac{1}{96}\left(\frac{1}{48}\left(\frac{512x}{3}-\frac{611}{3}\int\frac{1}{\frac{3}{5}\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx\right)-\frac{125\tan(c+dx)}{48d(5\sec(c+dx)+3)}\right)-\frac{25\tan(c+dx)}{96d(5\sec(c+dx)+3)^2}$$

$\Big\downarrow$ 3136

$$\frac{1}{96}\left(\frac{1}{48}\left(\frac{512x}{3}-\frac{611}{3}\left(\frac{5x}{4}-\frac{5\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{2d}\right)\right)-\frac{125\tan(c+dx)}{48d(5\sec(c+dx)+3)}\right)-\frac{25\tan(c+dx)}{96d(5\sec(c+dx)+3)^2}$$

input
```
Int[(3 + 5*Sec[c + d*x])^(-3),x]
```

output
```
(-25*Tan[c + d*x])/(96*d*(3 + 5*Sec[c + d*x])^2) + (((512*x)/3 - (611*((5*
x)/4 - (5*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(2*d)))/3)/48 - (125*Ta
n[c + d*x])/(48*d*(3 + 5*Sec[c + d*x])))/96
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{-\frac{5\left(-\frac{285\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{128}+\frac{165\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32}\right)}{108\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+4\right)^2}-\frac{3055\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{27648}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{27}}{d}$
default	$\frac{-\frac{5\left(-\frac{285\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{128}+\frac{165\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32}\right)}{108\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+4\right)^2}-\frac{3055\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{27648}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{27}}{d}$
risch	$\frac{x}{27}-\frac{25i\left(185\,e^{3i(dx+c)}+413\,e^{2i(dx+c)}+235\,e^{i(dx+c)}+63\right)}{2304d\left(3\,e^{2i(dx+c)}+10\,e^{i(dx+c)}+3\right)^2}+\frac{3055i\ln\left(e^{i(dx+c)}+\frac{1}{3}\right)}{55296d}-\frac{3055i\ln\left(e^{i(dx+c)}+3\right)}{55296d}$
parallelrisch	$\frac{3055i(9\cos(2dx+2c)+59+60\cos(dx+c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2i\right)+3055i(-9\cos(2dx+2c)-59-60\cos(dx+c))\ln\left(\tan\left(\frac{d}{2}\right.\right.}{55296d(9\cos(2dx+2c)+59}$

input
```
int(1/(3+5*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-5/108*(-285/128*tan(1/2*d*x+1/2*c)^3+165/32*tan(1/2*d*x+1/2*c))/(tan
(1/2*d*x+1/2*c)^2+4)^2-3055/27648*arctan(1/2*tan(1/2*d*x+1/2*c))+2/27*arct
an(tan(1/2*d*x+1/2*c)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.43

$$\int \frac{1}{(3+5\sec(c+dx))^3}\,dx$$
$$=\frac{18432\,dx\cos(dx+c)^2+61440\,dx\cos(dx+c)+51200\,dx+3055\left(9\cos(dx+c)^2+30\cos(dx+c)+2\right.}{55296\left(9\,d\cos(dx+c)^2+30\,d\cos(dx+c)+\right.}$$

input
```
integrate(1/(3+5*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/55296*(18432*d*x*cos(d*x + c)^2 + 61440*d*x*cos(d*x + c) + 51200*d*x + 3
055*(9*cos(d*x + c)^2 + 30*cos(d*x + c) + 25)*arctan(1/4*(5*cos(d*x + c) +
 3)/sin(d*x + c)) - 300*(63*cos(d*x + c) + 25)*sin(d*x + c))/(9*d*cos(d*x
+ c)^2 + 30*d*cos(d*x + c) + 25*d)
```

Sympy [F]

$$\int \frac{1}{(3+5\sec(c+dx))^3}\,dx = \int \frac{1}{(5\sec{(c+dx)}+3)^3}\,dx$$

input
```
integrate(1/(3+5*sec(d*x+c))**3,x)
```

output
```
Integral((5*sec(c + d*x) + 3)**(-3), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.62

$$\int \frac{1}{(3+5\sec(c+dx))^3}\,dx = -\frac{\frac{150\left(\frac{44\sin(dx+c)}{\cos(dx+c)+1}-\frac{19\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\frac{8\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+16}-2048\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+3055\arctan\left(\frac{\sin(dx+c)}{2(\cos(dx+c)+1)}\right)}{27648\,d}$$

input
```
integrate(1/(3+5*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/27648*(150*(44*sin(d*x + c)/(cos(d*x + c) + 1) - 19*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3)/(8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/
(cos(d*x + c) + 1)^4 + 16) - 2048*arctan(sin(d*x + c)/(cos(d*x + c) + 1))
+ 3055*arctan(1/2*sin(d*x + c)/(cos(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{1}{(3+5\sec(c+dx))^3}\,dx = -\frac{1007\,dx + 1007\,c - \frac{300\left(19\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 44\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\right)^2} - 6110\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{55296\,d}$$

input
```
integrate(1/(3+5*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/55296*(1007*d*x + 1007*c - 300*(19*tan(1/2*d*x + 1/2*c)^3 - 44*tan(1/2*
d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 4)^2 - 6110*arctan(sin(d*x + c)/(c
os(d*x + c) + 3)))/d
```

Mupad [B] (verification not implemented)

Time = 10.34 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int \frac{1}{(3+5\sec(c+dx))^3}\,dx = \frac{x}{27} - \frac{3055\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{27648\,d} - \frac{\frac{275\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{1152} - \frac{475\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4608}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + 16\right)}$$

input
```
int(1/(5/cos(c + d*x) + 3)^3,x)
```

output
```
x/27 - (3055*atan(tan(c/2 + (d*x)/2)/2))/(27648*d) - ((275*tan(c/2 + (d*x)
/2))/1152 - (475*tan(c/2 + (d*x)/2)^3)/4608)/(d*(8*tan(c/2 + (d*x)/2)^2 +
tan(c/2 + (d*x)/2)^4 + 16))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.63

$$\int \frac{1}{(3+5\sec(c+dx))^3}\,dx$$
$$= \frac{-91650atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)\cos\left(dx+c\right)+27495atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)\sin\left(dx+c\right)^2-103870atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right.}{27648d\left(30\cos\left(dx\right.\right.}$$

input
```
int(1/(3+5*sec(d*x+c))^3,x)
```

output
```
( - 91650*atan(tan((c + d*x)/2)/2)*cos(c + d*x) + 27495*atan(tan((c + d*x)
/2)/2)*sin(c + d*x)**2 - 103870*atan(tan((c + d*x)/2)/2) - 9450*cos(c + d*
x)*sin(c + d*x) + 30720*cos(c + d*x)*d*x - 9216*sin(c + d*x)**2*d*x - 3750
*sin(c + d*x) + 34816*d*x)/(27648*d*(30*cos(c + d*x) - 9*sin(c + d*x)**2 +
 34))
```

3.526 $\int \frac{1}{(3+5\sec(c+dx))^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 106

$$\int \frac{1}{(3+5\sec(c+dx))^4}\,dx = \frac{21553x}{2654208}+\frac{11215\arctan\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{1327104d}-\frac{25\tan(c+dx)}{144d(3+5\sec(c+dx))^3}-\frac{25\tan(c+dx)}{4608d(3+5\sec(c+dx))^2}-\frac{16925\tan(c+dx)}{221184d(3+5\sec(c+dx))}$$

output
```
21553/2654208*x+11215/1327104*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d-25/144*t
an(d*x+c)/d/(3+5*sec(d*x+c))^3-25/4608*tan(d*x+c)/d/(3+5*sec(d*x+c))^2-169
25/221184*tan(d*x+c)/d/(3+5*sec(d*x+c))
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.33

$$\int \frac{1}{(3+5\sec(c+dx))^4}\,dx$$
$$= \frac{6307840c+6307840dx+8036352(c+dx)\cos(c+dx)+22430\arctan\left(2\cot\left(\frac{1}{2}(c+dx)\right)\right)(5+3\cos(c+$$

input
```
Integrate[(3 + 5*Sec[c + d*x])^(-4),x]
```

output
```
(6307840*c + 6307840*d*x + 8036352*(c + d*x)*Cos[c + d*x] + 22430*ArcTan[2
*Cot[(c + d*x)/2]]*(5 + 3*Cos[c + d*x])^3 + 2211840*c*Cos[2*(c + d*x)] + 2
211840*d*x*Cos[2*(c + d*x)] + 221184*c*Cos[3*(c + d*x)] + 221184*d*x*Cos[3
*(c + d*x)] - 5660475*Sin[c + d*x] - 3082500*Sin[2*(c + d*x)] - 582975*Sin
[3*(c + d*x)])/(2654208*d*(5 + 3*Cos[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.24, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 4272, 3042, 4548, 27, 3042, 4548, 3042, 4407, 3042, 4318, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5\sec(c+dx)+3)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(5\csc\left(c+dx+\frac{\pi}{2}\right)+3\right)^4}dx$$

$$\Big\downarrow \text{4272}$$

$$\frac{1}{144}\int \frac{-50\sec^2(c+dx)+45\sec(c+dx)+48}{(5\sec(c+dx)+3)^3}dx-\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{144}\int \frac{-50\csc\left(c+dx+\frac{\pi}{2}\right)^2+45\csc\left(c+dx+\frac{\pi}{2}\right)+48}{\left(5\csc\left(c+dx+\frac{\pi}{2}\right)+3\right)^3}dx-\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}$$

$$\Big\downarrow \text{4548}$$

$$\frac{1}{144}\left(\frac{1}{96}\int \frac{3\left(-25\sec^2(c+dx)-290\sec(c+dx)+512\right)}{(5\sec(c+dx)+3)^2}dx-\frac{25\tan(c+dx)}{32d(5\sec(c+dx)+3)^2}\right)-\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}$$

$$\Big\downarrow 27$$

$$\frac{1}{144}\left(\frac{1}{32}\int \frac{-25\sec^2(c+dx)-290\sec(c+dx)+512}{(5\sec(c+dx)+3)^2}dx-\frac{25\tan(c+dx)}{32d(5\sec(c+dx)+3)^2}\right)-\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}$$

$$\Big\downarrow 3042$$

$$\frac{1}{144}\left(\frac{1}{32}\int \frac{-25\csc\left(c+dx+\frac{\pi}{2}\right)^2-290\csc\left(c+dx+\frac{\pi}{2}\right)+512}{\left(5\csc\left(c+dx+\frac{\pi}{2}\right)+3\right)^2}dx-\frac{25\tan(c+dx)}{32d(5\sec(c+dx)+3)^2}\right)-\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}$$

$$\Big\downarrow 4548$$

$$\frac{1}{144}\left(\frac{1}{32}\left(\frac{1}{48}\int \frac{9915\sec(c+dx)+8192}{5\sec(c+dx)+3}dx-\frac{16925\tan(c+dx)}{48d(5\sec(c+dx)+3)}\right)-\frac{25\tan(c+dx)}{32d(5\sec(c+dx)+3)^2}\right)-\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}$$

$$\Big\downarrow 3042$$

$$\frac{1}{144}\left(\frac{1}{32}\left(\frac{1}{48}\int \frac{9915\csc\left(c+dx+\frac{\pi}{2}\right)+8192}{5\csc\left(c+dx+\frac{\pi}{2}\right)+3}dx-\frac{16925\tan(c+dx)}{48d(5\sec(c+dx)+3)}\right)-\frac{25\tan(c+dx)}{32d(5\sec(c+dx)+3)^2}\right)-\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}$$

$$\Big\downarrow 4407$$

$$\frac{1}{144}\left(\frac{1}{32}\left(\frac{1}{48}\left(\frac{8192x}{3}-\frac{11215}{3}\int \frac{\sec(c+dx)}{5\sec(c+dx)+3}dx\right)-\frac{16925\tan(c+dx)}{48d(5\sec(c+dx)+3)}\right)-\frac{25\tan(c+dx)}{32d(5\sec(c+dx)+3)^2}\right)-\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}$$

$$\Big\downarrow 3042$$

$$\frac{1}{144}\left(\frac{1}{32}\left(\frac{1}{48}\left(\frac{8192x}{3}-\frac{11215}{3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{5\csc\left(c+dx+\frac{\pi}{2}\right)+3}dx\right)-\frac{16925\tan(c+dx)}{48d(5\sec(c+dx)+3)}\right)-\frac{25\tan(c+dx)}{32d(5\sec(c+dx)+\right.\right.$$
$$\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}$$

$$\Big\downarrow 4318$$

$$\frac{1}{144}\left(\frac{1}{32}\left(\frac{1}{48}\left(\frac{8192x}{3}-\frac{2243}{3}\int \frac{1}{\frac{3}{5}\cos(c+dx)+1}dx\right)-\frac{16925\tan(c+dx)}{48d(5\sec(c+dx)+3)}\right)-\frac{25\tan(c+dx)}{32d(5\sec(c+dx)+3)^2}\right)-\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{144}\left(\frac{1}{32}\left(\frac{1}{48}\left(\frac{8192x}{3}-\frac{2243}{3}\int \frac{1}{\frac{3}{5}\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx\right)-\frac{16925\tan(c+dx)}{48d(5\sec(c+dx)+3)}\right)-\frac{25\tan(c+dx)}{32d(5\sec(c+dx)+3)^2}\right)-\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}$$

$$\Big\downarrow \text{3136}$$

$$\frac{1}{144}\left(\frac{1}{32}\left(\frac{1}{48}\left(\frac{8192x}{3}-\frac{2243}{3}\left(\frac{5x}{4}-\frac{5\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{2d}\right)\right)-\frac{16925\tan(c+dx)}{48d(5\sec(c+dx)+3)}\right)-\frac{25\tan(c+dx)}{32d(5\sec(c+dx)+3)^2}\right)-\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}$$

input
```
Int[(3 + 5*Sec[c + d*x])^(-4),x]
```

output
```
(-25*Tan[c + d*x])/(144*d*(3 + 5*Sec[c + d*x])^3) + ((-25*Tan[c + d*x])/(3
2*d*(3 + 5*Sec[c + d*x])^2) + (((8192*x)/3 - (2243*((5*x)/4 - (5*ArcTan[Si
n[c + d*x]/(3 + Cos[c + d*x])])/(2*d)))/3)/48 - (16925*Tan[c + d*x])/(48*d
*(3 + 5*Sec[c + d*x])))/32)/144
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{\frac{-\frac{25925\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{221184}-\frac{3575\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{6912}-\frac{17675\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{13824}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+4\right)^3}-\frac{11215\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{1327104}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{81}}{d}$
default	$\frac{\frac{-\frac{25925\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{221184}-\frac{3575\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{6912}-\frac{17675\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{13824}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+4\right)^3}-\frac{11215\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{1327104}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{81}}{d}$
risch	$\frac{x}{81}-\frac{25i\left(164835\,e^{5i(dx+c)}+931257\,e^{4i(dx+c)}+1995070\,e^{3i(dx+c)}+1610514\,e^{2i(dx+c)}+534735\,e^{i(dx+c)}+69957\right)}{995328d\left(3\,e^{2i(dx+c)}+10\,e^{i(dx+c)}+3\right)^3}-$
parallelrisch	$\frac{11215i(27\cos(3dx+3c)+981\cos(dx+c)+270\cos(2dx+2c)+770)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2i\right)+11215i(-27\cos(3dx+3c)-98}{}$

input
```
int(1/(3+5*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(5/648*(-15555/1024*tan(1/2*d*x+1/2*c)^5-2145/32*tan(1/2*d*x+1/2*c)^3-
10605/64*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2+4)^3-11215/1327104*arct
an(1/2*tan(1/2*d*x+1/2*c))+2/81*arctan(tan(1/2*d*x+1/2*c)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.50

$$\int \frac{1}{(3+5\sec(c+dx))^4}\,dx$$

$$=\frac{884736\,dx\cos(dx+c)^3+4423680\,dx\cos(dx+c)^2+7372800\,dx\cos(dx+c)+4096000\,dx+11215\,(27}{2654208\,(27\,d}$$

input
```
integrate(1/(3+5*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/2654208*(884736*d*x*cos(d*x + c)^3 + 4423680*d*x*cos(d*x + c)^2 + 737280
0*d*x*cos(d*x + c) + 4096000*d*x + 11215*(27*cos(d*x + c)^3 + 135*cos(d*x
+ c)^2 + 225*cos(d*x + c) + 125)*arctan(1/4*(5*cos(d*x + c) + 3)/sin(d*x +
 c)) - 300*(7773*cos(d*x + c)^2 + 20550*cos(d*x + c) + 16925)*sin(d*x + c)
)/(27*d*cos(d*x + c)^3 + 135*d*cos(d*x + c)^2 + 225*d*cos(d*x + c) + 125*d
)
```

Sympy [F]

$$\int \frac{1}{(3+5\sec(c+dx))^4}\,dx = \int \frac{1}{(5\sec(c+dx)+3)^4}\,dx$$

input
```
integrate(1/(3+5*sec(d*x+c))**4,x)
```

output
```
Integral((5*sec(c + d*x) + 3)**(-4), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.61

$$\int \frac{1}{(3+5\sec(c+dx))^4}\,dx = -\frac{\frac{150\left(\frac{11312\sin(dx+c)}{\cos(dx+c)+1}+\frac{4576\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1037\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{12\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+64}-32768\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+11215\arctan\left(\frac{\sin(dx+c)}{2(\cos(dx+c)+1)}\right)}{1327104\,d}$$

input
```
integrate(1/(3+5*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/1327104*(150*(11312*sin(d*x + c)/(cos(d*x + c) + 1) + 4576*sin(d*x + c)
^3/(cos(d*x + c) + 1)^3 + 1037*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(48*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 12*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 64) - 32768*arctan(sin(d*x + c)/(
cos(d*x + c) + 1)) + 11215*arctan(1/2*sin(d*x + c)/(cos(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.83

$$\int \frac{1}{(3+5\sec(c+dx))^4}\,dx = \frac{21553\,dx+21553\,c-\frac{300\left(1037\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4576\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+11312\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\right)^3}+22430\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{2654208\,d}$$

input
```
integrate(1/(3+5*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/2654208*(21553*d*x + 21553*c - 300*(1037*tan(1/2*d*x + 1/2*c)^5 + 4576*t
an(1/2*d*x + 1/2*c)^3 + 11312*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^
2 + 4)^3 + 22430*arctan(sin(d*x + c)/(cos(d*x + c) + 3)))/d
```

Mupad [B] (verification not implemented)

Time = 10.88 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.99

$$\int \frac{1}{(3+5\sec(c+dx))^4}\,dx = \frac{x}{81}-\frac{11215\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{1327104\,d}-\frac{\frac{25925\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{221184}+\frac{3575\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6912}+\frac{17675\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{13824}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+12\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+48\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+64\right)}$$

input
```
int(1/(5/cos(c + d*x) + 3)^4,x)
```

output
```
x/81 - (11215*atan(tan(c/2 + (d*x)/2)/2))/(1327104*d) - ((17675*tan(c/2 +
(d*x)/2))/13824 + (3575*tan(c/2 + (d*x)/2)^3)/6912 + (25925*tan(c/2 + (d*x
)/2)^5)/221184)/(d*(48*tan(c/2 + (d*x)/2)^2 + 12*tan(c/2 + (d*x)/2)^4 + ta
n(c/2 + (d*x)/2)^6 + 64))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.92

$$\int \frac{1}{(3+5\sec(c+dx))^4}\,dx$$
$$= \frac{-302805atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)\cos(dx+c)\sin(dx+c)^2+2826180atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)\cos(dx+c)-151402}{}$$

input
```
int(1/(3+5*sec(d*x+c))^4,x)
```

output
```
( - 302805*atan(tan((c + d*x)/2)/2)*cos(c + d*x)*sin(c + d*x)**2 + 2826180
*atan(tan((c + d*x)/2)/2)*cos(c + d*x) - 1514025*atan(tan((c + d*x)/2)/2)*
sin(c + d*x)**2 + 2915900*atan(tan((c + d*x)/2)/2) + 442368*cos(c + d*x)*s
in(c + d*x)**2*d*x + 3082500*cos(c + d*x)*sin(c + d*x) - 4128768*cos(c + d
*x)*d*x - 1165950*sin(c + d*x)**3 + 2211840*sin(c + d*x)**2*d*x + 3704700*
sin(c + d*x) - 4259840*d*x)/(1327104*d*(27*cos(c + d*x)*sin(c + d*x)**2 -
252*cos(c + d*x) + 135*sin(c + d*x)**2 - 260))
```

3.527 $\int \frac{1}{5+3\sec(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 70

$$\int \frac{1}{5+3\sec(c+dx)}\,dx=\frac{x}{5}+\frac{3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20d}-\frac{3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20d}$$

output
```
1/5*x+3/20*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-3/20*ln(2*cos(1/2
*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.99

$$\int \frac{1}{5+3\sec(c+dx)}\,dx=\frac{4(c+dx)+3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20d}$$

input
```
Integrate[(5 + 3*Sec[c + d*x])^(-1),x]
```

output
```
(4*(c + d*x) + 3*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 3*Log[2*Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]])/(20*d)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.40, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 4270, 3042, 3138, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3\sec(c+dx)+5}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{3\csc\left(c+dx+\frac{\pi}{2}\right)+5}dx$$
$$\Big\downarrow \text{ 4270}$$
$$\frac{x}{5}-\frac{1}{5}\int \frac{1}{\frac{5}{3}\cos(c+dx)+1}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{x}{5}-\frac{1}{5}\int \frac{1}{\frac{5}{3}\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx$$
$$\Big\downarrow \text{ 3138}$$
$$\frac{x}{5}-\frac{2\int \frac{1}{\frac{8}{3}-\frac{2}{3}\tan^2\left(\frac{1}{2}(c+dx)\right)}d\tan\left(\frac{1}{2}(c+dx)\right)}{5d}$$
$$\Big\downarrow \text{ 219}$$
$$\frac{x}{5}-\frac{3\text{arctanh}\left(\frac{1}{2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{10d}$$

input
```
Int[(5 + 3*Sec[c + d*x])^(-1),x]
```

output
```
x/5 - (3*ArcTanh[Tan[(c + d*x)/2]/2])/(10*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4270
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Simp[1/a   Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.56

method	result	size
norman	$\frac{x}{5}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{20d}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{20d}$	39
parallelrisch	$\frac{4dx+3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)-3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{20d}$	39
risch	$\frac{x}{5}+\frac{3\ln\left(\mathrm{e}^{i(dx+c)}+\frac{3}{5}-\frac{4i}{5}\right)}{20d}-\frac{3\ln\left(\mathrm{e}^{i(dx+c)}+\frac{3}{5}+\frac{4i}{5}\right)}{20d}$	43
derivativedivides	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{20}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{20}}{d}$	46
default	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{20}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{20}}{d}$	46

input
```
int(1/(5+3*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/5*x+3/20/d*ln(tan(1/2*d*x+1/2*c)-2)-3/20/d*ln(tan(1/2*d*x+1/2*c)+2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.74

$$\int \frac{1}{5+3\sec(c+dx)}\,dx = \frac{8\,dx-3\,\log\left(\frac{3}{2}\,\cos\left(dx+c\right)+2\,\sin\left(dx+c\right)+\frac{5}{2}\right)+3\,\log\left(\frac{3}{2}\,\cos\left(dx+c\right)-2\,\sin\left(dx+c\right)+\frac{5}{2}\right)}{40\,d}$$

input
```
integrate(1/(5+3*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/40*(8*d*x - 3*log(3/2*cos(d*x + c) + 2*sin(d*x + c) + 5/2) + 3*log(3/2*c
os(d*x + c) - 2*sin(d*x + c) + 5/2))/d
```

Sympy [F]

$$\int \frac{1}{5+3\sec(c+dx)}\,dx = \int \frac{1}{3\sec\left(c+dx\right)+5}\,dx$$

input
```
integrate(1/(5+3*sec(d*x+c)),x)
```

output
```
Integral(1/(3*sec(c + d*x) + 5), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{1}{5+3\sec(c+dx)}\,dx = \frac{8\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) - 3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+2\right) + 3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-2\right)}{20\,d}$$

input
```
integrate(1/(5+3*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/20*(8*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) - 3*log(sin(d*x + c)/(cos(
d*x + c) + 1) + 2) + 3*log(sin(d*x + c)/(cos(d*x + c) + 1) - 2))/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int \frac{1}{5+3\sec(c+dx)}\,dx = \frac{4\,dx + 4\,c - 3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right) + 3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right|\right)}{20\,d}$$

input
```
integrate(1/(5+3*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/20*(4*d*x + 4*c - 3*log(abs(tan(1/2*d*x + 1/2*c) + 2)) + 3*log(abs(tan(1
/2*d*x + 1/2*c) - 2)))/d
```

Mupad [B] (verification not implemented)

Time = 10.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.30

$$\int \frac{1}{5+3\sec(c+dx)}\,dx=\frac{x}{5}-\frac{3\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{10\,d}$$

input
```
int(1/(3/cos(c + d*x) + 5),x)
```

output
```
x/5 - (3*atanh(tan(c/2 + (d*x)/2)/2))/(10*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.54

$$\int \frac{1}{5+3\sec(c+dx)}\,dx=\frac{3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)-3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)+4dx}{20d}$$

input
```
int(1/(5+3*sec(d*x+c)),x)
```

output
```
(3*log(tan((c + d*x)/2) - 2) - 3*log(tan((c + d*x)/2) + 2) + 4*d*x)/(20*d)
```

3.528 $\int \frac{1}{(5+3\sec(c+dx))^2}\, dx$

Optimal result

Integrand size = 12, antiderivative size = 95

$$\int \frac{1}{(5+3\sec(c+dx))^2}\, dx = \frac{x}{25} + \frac{123\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{1600d} - \frac{123\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{1600d} + \frac{9\tan(c+dx)}{80d(5+3\sec(c+dx))}$$

output
```
1/25*x+123/1600*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-123/1600*ln(
2*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d+9/80*tan(d*x+c)/d/(5+3*sec(d*x+
c))
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.71

$$\int \frac{1}{(5+3\sec(c+dx))^2}\, dx$$
$$= \frac{5\cos(c+dx)\left(64(c+dx)+123\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-123\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\cdots\right.\right.}{}$$

input
```
Integrate[(5 + 3*Sec[c + d*x])^(-2),x]
```

output
```
(5*Cos[c + d*x]*(64*(c + d*x) + 123*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] - 123*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*(64*c + 64*d*x
+ 123*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 123*Log[2*Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]] + 60*Sin[c + d*x]))/(1600*d*(3 + 5*Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.61, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3042, 4272, 25, 3042, 4407, 3042, 4318, 3042, 3138, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3\sec(c+dx)+5)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(3\csc\left(c+dx+\frac{\pi}{2}\right)+5\right)^2}dx$$

$$\Big\downarrow \text{4272}$$

$$\frac{9\tan(c+dx)}{80d(3\sec(c+dx)+5)}-\frac{1}{80}\int -\frac{16-15\sec(c+dx)}{3\sec(c+dx)+5}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{80}\int \frac{16-15\sec(c+dx)}{3\sec(c+dx)+5}dx+\frac{9\tan(c+dx)}{80d(3\sec(c+dx)+5)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{80}\int \frac{16-15\csc\left(c+dx+\frac{\pi}{2}\right)}{3\csc\left(c+dx+\frac{\pi}{2}\right)+5}dx+\frac{9\tan(c+dx)}{80d(3\sec(c+dx)+5)}$$

$$\Big\downarrow \text{4407}$$

$$\frac{1}{80}\left(\frac{16x}{5}-\frac{123}{5}\int \frac{\sec(c+dx)}{3\sec(c+dx)+5}dx\right)+\frac{9\tan(c+dx)}{80d(3\sec(c+dx)+5)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{80}\left(\frac{16x}{5}-\frac{123}{5}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{3\csc\left(c+dx+\frac{\pi}{2}\right)+5}dx\right)+\frac{9\tan(c+dx)}{80d(3\sec(c+dx)+5)}$$

$$\Big\downarrow \text{4318}$$

$$\frac{1}{80}\left(\frac{16x}{5}-\frac{41}{5}\int \frac{1}{\frac{5}{3}\cos(c+dx)+1}dx\right)+\frac{9\tan(c+dx)}{80d(3\sec(c+dx)+5)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{80}\left(\frac{16x}{5}-\frac{41}{5}\int \frac{1}{\frac{5}{3}\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx\right)+\frac{9\tan(c+dx)}{80d(3\sec(c+dx)+5)}$$

$$\Big\downarrow \text{3138}$$

$$\frac{1}{80}\left(\frac{16x}{5}-\frac{82\int \frac{1}{\frac{8}{3}-\frac{2}{3}\tan^2\left(\frac{1}{2}(c+dx)\right)}d\tan\left(\frac{1}{2}(c+dx)\right)}{5d}\right)+\frac{9\tan(c+dx)}{80d(3\sec(c+dx)+5)}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{80}\left(\frac{16x}{5}-\frac{123\text{arctanh}\left(\frac{1}{2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{10d}\right)+\frac{9\tan(c+dx)}{80d(3\sec(c+dx)+5)}$$

input
```
Int[(5 + 3*Sec[c + d*x])^(-2),x]
```

output
```
((16*x)/5 - (123*ArcTanh[Tan[(c + d*x)/2]/2])/(10*d))/80 + (9*Tan[c + d*x]
)/(80*d*(5 + 3*Sec[c + d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{-\frac{9}{160\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}+\frac{123\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{1600}-\frac{9}{160\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}-\frac{123\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{1600}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{25}}{d}$
default	$\frac{-\frac{9}{160\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}+\frac{123\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{1600}-\frac{9}{160\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}-\frac{123\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{1600}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{25}}{d}$
norman	$\frac{-\frac{4x}{25}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{80d}+\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{25}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-4}+\frac{123\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{1600d}-\frac{123\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{1600d}$
risch	$\frac{x}{25}+\frac{9i\left(3\,\mathrm{e}^{i(dx+c)}+5\right)}{200d\left(5\,\mathrm{e}^{2i(dx+c)}+6\,\mathrm{e}^{i(dx+c)}+5\right)}+\frac{123\ln\left(\mathrm{e}^{i(dx+c)}+\frac{3}{5}-\frac{4i}{5}\right)}{1600d}-\frac{123\ln\left(\mathrm{e}^{i(dx+c)}+\frac{3}{5}+\frac{4i}{5}\right)}{1600d}$
parallelrisch	$\frac{192dx+369\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)-369\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)+320dx\cos(dx+c)+615\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)\cos(dx+c)-6}{8000d\cos(dx+c)+4800d}$

input
```
int(1/(5+3*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-9/160/(tan(1/2*d*x+1/2*c)-2)+123/1600*ln(tan(1/2*d*x+1/2*c)-2)-9/160
/(tan(1/2*d*x+1/2*c)+2)-123/1600*ln(tan(1/2*d*x+1/2*c)+2)+2/25*arctan(tan(
1/2*d*x+1/2*c)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.07

$$\int \frac{1}{(5+3\sec(c+dx))^2}\,dx$$
$$=\frac{640\,dx\cos\left(dx+c\right)+384\,dx-123\left(5\,\cos\left(dx+c\right)+3\right)\log\left(\frac{3}{2}\,\cos\left(dx+c\right)+2\,\sin\left(dx+c\right)+\frac{5}{2}\right)+123\,(}{3200\left(5\,d\cos\left(dx+c\right)+3\,c\right.}$$

input
```
integrate(1/(5+3*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3200*(640*d*x*cos(d*x + c) + 384*d*x - 123*(5*cos(d*x + c) + 3)*log(3/2*
cos(d*x + c) + 2*sin(d*x + c) + 5/2) + 123*(5*cos(d*x + c) + 3)*log(3/2*co
s(d*x + c) - 2*sin(d*x + c) + 5/2) + 360*sin(d*x + c))/(5*d*cos(d*x + c) +
 3*d)
```

Sympy [F]

$$\int \frac{1}{(5+3\sec(c+dx))^2}\,dx = \int \frac{1}{\left(3\sec\left(c+dx\right)+5\right)^2}\,dx$$

input
```
integrate(1/(5+3*sec(d*x+c))**2,x)
```

output
```
Integral((3*sec(c + d*x) + 5)**(-2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.17

$$\int \frac{1}{(5+3\sec(c+dx))^2}\,dx = -\frac{\frac{180\sin(dx+c)}{\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-4\right)(\cos(dx+c)+1)} - 128\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + 123\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+2\right) - 123\log\left(\frac{\sin(dx+}{\cos(dx+c}\right.}{1600\,d}$$

input
```
integrate(1/(5+3*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/1600*(180*sin(d*x + c)/((sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4)*(cos(
d*x + c) + 1)) - 128*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 123*log(sin
(d*x + c)/(cos(d*x + c) + 1) + 2) - 123*log(sin(d*x + c)/(cos(d*x + c) + 1
) - 2))/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.73

$$\int \frac{1}{(5+3\sec(c+dx))^2}\,dx = \frac{64\,dx + 64\,c - \frac{180\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4} - 123\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right) + 123\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right|\right)}{1600\,d}$$

input
```
integrate(1/(5+3*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/1600*(64*d*x + 64*c - 180*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 -
 4) - 123*log(abs(tan(1/2*d*x + 1/2*c) + 2)) + 123*log(abs(tan(1/2*d*x + 1
/2*c) - 2)))/d
```

Mupad [B] (verification not implemented)

Time = 10.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.55

$$\int \frac{1}{(5+3\sec(c+dx))^2}\,dx = \frac{x}{25} - \frac{\frac{123\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{800} + \frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{80\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\right)}}{d}$$

input
```
int(1/(3/cos(c + d*x) + 5)^2,x)
```

output
```
x/25 - ((123*atanh(tan(c/2 + (d*x)/2)/2))/800 + (9*tan(c/2 + (d*x)/2))/(80
*(tan(c/2 + (d*x)/2)^2 - 4)))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.36

$$\int \frac{1}{(5+3\sec(c+dx))^2}\,dx$$
$$= \frac{123\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 - 492\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right) - 123\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)\tan\left(\frac{c}{2}\right.}{1600d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right.}$$

input
```
int(1/(5+3*sec(d*x+c))^2,x)
```

output
```
(123*log(tan((c + d*x)/2) - 2)*tan((c + d*x)/2)**2 - 492*log(tan((c + d*x)
/2) - 2) - 123*log(tan((c + d*x)/2) + 2)*tan((c + d*x)/2)**2 + 492*log(tan
((c + d*x)/2) + 2) + 64*tan((c + d*x)/2)**2*d*x - 180*tan((c + d*x)/2) - 2
56*d*x)/(1600*d*(tan((c + d*x)/2)**2 - 4))
```

3.529 $\int \frac{1}{(5+3\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 120

$$\int \frac{1}{(5+3\sec(c+dx))^3}\,dx = \frac{x}{125}+\frac{8361\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{256000d} - \frac{8361\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{256000d} + \frac{9\tan(c+dx)}{160d(5+3\sec(c+dx))^2}+\frac{963\tan(c+dx)}{12800d(5+3\sec(c+dx))}$$

output
```
1/125*x+8361/256000*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-8361/256
000*ln(2*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d+9/160*tan(d*x+c)/d/(5+3*
sec(d*x+c))^2+963/12800*tan(d*x+c)/d/(5+3*sec(d*x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 241 vs. 2(120) = 240.

Time = 0.38 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.01

$$\int \frac{1}{(5+3\sec(c+dx))^3}\,dx = \frac{88064c+88064dx+359523\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+60\cos(c+dx)\left(2048(c+dx)+\right.}{}$$

input
```
Integrate[(5 + 3*Sec[c + d*x])^(-3),x]
```

output
```
(88064*c + 88064*d*x + 359523*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
 60*Cos[c + d*x]*(2048*(c + d*x) + 8361*Log[2*Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] - 8361*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 25*Cos[2*(c
+ d*x)]*(2048*(c + d*x) + 8361*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
- 8361*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 359523*Log[2*Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]] + 115560*Sin[c + d*x] + 110700*Sin[2*(c + d*x
)])/(512000*d*(3 + 5*Cos[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.73, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.083, Rules used = {3042, 4272, 25, 3042, 4548, 25, 3042, 4407, 3042, 4318, 3042, 3138, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3\sec(c+dx)+5)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(3\csc\left(c+dx+\frac{\pi}{2}\right)+5\right)^3}dx$$

$$\Big\downarrow \text{4272}$$

$$\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}-\frac{1}{160}\int -\frac{9\sec^2(c+dx)-30\sec(c+dx)+32}{(3\sec(c+dx)+5)^2}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{160}\int \frac{9\sec^2(c+dx)-30\sec(c+dx)+32}{(3\sec(c+dx)+5)^2}dx+\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{160}\int \frac{9\csc\left(c+dx+\frac{\pi}{2}\right)^2-30\csc\left(c+dx+\frac{\pi}{2}\right)+32}{\left(3\csc\left(c+dx+\frac{\pi}{2}\right)+5\right)^2}dx+\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}$$

$$\Big\downarrow \text{4548}$$

$$\frac{1}{160}\left(\frac{963\tan(c+dx)}{80d(3\sec(c+dx)+5)}-\frac{1}{80}\int -\frac{512-1365\sec(c+dx)}{3\sec(c+dx)+5}dx\right)+\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{160}\left(\frac{1}{80}\int \frac{512-1365\sec(c+dx)}{3\sec(c+dx)+5}dx+\frac{963\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{160}\left(\frac{1}{80}\int \frac{512-1365\csc\left(c+dx+\frac{\pi}{2}\right)}{3\csc\left(c+dx+\frac{\pi}{2}\right)+5}dx+\frac{963\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}$$

$$\Big\downarrow \text{4407}$$

$$\frac{1}{160}\left(\frac{1}{80}\left(\frac{512x}{5}-\frac{8361}{5}\int \frac{\sec(c+dx)}{3\sec(c+dx)+5}dx\right)+\frac{963\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{160}\left(\frac{1}{80}\left(\frac{512x}{5}-\frac{8361}{5}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{3\csc\left(c+dx+\frac{\pi}{2}\right)+5}dx\right)+\frac{963\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}$$

$$\Big\downarrow \text{4318}$$

$$\frac{1}{160}\left(\frac{1}{80}\left(\frac{512x}{5}-\frac{2787}{5}\int \frac{1}{\frac{5}{3}\cos(c+dx)+1}dx\right)+\frac{963\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{160}\left(\frac{1}{80}\left(\frac{512x}{5}-\frac{2787}{5}\int \frac{1}{\frac{5}{3}\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx\right)+\frac{963\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}$$

$$\Big\downarrow \text{3138}$$

$$\frac{1}{160}\left(\frac{1}{80}\left(\frac{512x}{5}-\frac{5574\int \frac{1}{\frac{8}{3}-\frac{2}{3}\tan^2\left(\frac{1}{2}(c+dx)\right)}d\tan\left(\frac{1}{2}(c+dx)\right)}{5d}\right)+\frac{963\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{160}\left(\frac{1}{80}\left(\frac{512x}{5}-\frac{8361\text{arctanh}\left(\frac{1}{2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{10d}\right)+\frac{963\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}$$

input
```
Int[(5 + 3*Sec[c + d*x])^(-3),x]
```

output
```
(9*Tan[c + d*x])/(160*d*(5 + 3*Sec[c + d*x])^2) + (((512*x)/5 - (8361*ArcT
anh[Tan[(c + d*x)/2]/2])/(10*d))/80 + (963*Tan[c + d*x])/(80*d*(5 + 3*Sec[
c + d*x])))/160
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{-\frac{27}{2560\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^2}-\frac{1323}{25600\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}+\frac{8361\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{256000}+\frac{27}{2560\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^2}-\frac{1323}{25600\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{d}$
default	$\frac{-\frac{27}{2560\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^2}-\frac{1323}{25600\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}+\frac{8361\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{256000}+\frac{27}{2560\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^2}-\frac{1323}{25600\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{d}$
risch	$\frac{x}{125}+\frac{27i\left(695\,\mathrm{e}^{3i(dx+c)}+1763\,\mathrm{e}^{2i(dx+c)}+1765\,\mathrm{e}^{i(dx+c)}+1025\right)}{32000d\left(5\,\mathrm{e}^{2i(dx+c)}+6\,\mathrm{e}^{i(dx+c)}+5\right)^2}-\frac{8361\ln\left(\mathrm{e}^{i(dx+c)}+\frac{3}{5}+\frac{4i}{5}\right)}{256000d}+\frac{8361\ln\left(\mathrm{e}^{i(dx+c)}+\right.}{256000d}$
norman	$\frac{\frac{16x}{125}+\frac{1053\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3200d}-\frac{1323\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12800d}-\frac{8x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{125}+\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{125}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-4\right)^2}+\frac{8361\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{256000d}-\frac{8361\ln}{}$
parallelrisch	$\frac{(501660\cos(dx+c)+209025\cos(2dx+2c)+359523)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)+(-501660\cos(dx+c)-209025\cos(2dx+2c)-}{256000(43+25\cos}$

input
```
int(1/(5+3*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-27/2560/(tan(1/2*d*x+1/2*c)-2)^2-1323/25600/(tan(1/2*d*x+1/2*c)-2)+8
361/256000*ln(tan(1/2*d*x+1/2*c)-2)+27/2560/(tan(1/2*d*x+1/2*c)+2)^2-1323/
25600/(tan(1/2*d*x+1/2*c)+2)-8361/256000*ln(tan(1/2*d*x+1/2*c)+2)+2/125*ar
ctan(tan(1/2*d*x+1/2*c)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.29

$$\int \frac{1}{(5+3\sec(c+dx))^3}\,dx$$
$$= \frac{102400\,dx\cos\left(dx+c\right)^2+122880\,dx\cos\left(dx+c\right)+36864\,dx-8361\left(25\,\cos\left(dx+c\right)^2+30\,\cos\left(dx+c\right)\right.}{}$$

input
```
integrate(1/(5+3*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/512000*(102400*d*x*cos(d*x + c)^2 + 122880*d*x*cos(d*x + c) + 36864*d*x
- 8361*(25*cos(d*x + c)^2 + 30*cos(d*x + c) + 9)*log(3/2*cos(d*x + c) + 2*
sin(d*x + c) + 5/2) + 8361*(25*cos(d*x + c)^2 + 30*cos(d*x + c) + 9)*log(3
/2*cos(d*x + c) - 2*sin(d*x + c) + 5/2) + 1080*(205*cos(d*x + c) + 107)*si
n(d*x + c))/(25*d*cos(d*x + c)^2 + 30*d*cos(d*x + c) + 9*d)
```

Sympy [F]

$$\int \frac{1}{(5+3\sec(c+dx))^3}\,dx=\int \frac{1}{(3\sec{(c+dx)}+5)^3}\,dx$$

input
```
integrate(1/(5+3*sec(d*x+c))**3,x)
```

output
```
Integral((3*sec(c + d*x) + 5)**(-3), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.29

$$\int \frac{1}{(5+3\sec(c+dx))^3}\,dx = -\frac{\frac{540\left(\frac{156\sin(dx+c)}{\cos(dx+c)+1}-\frac{49\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\frac{8\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-16}-4096\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+8361\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+2\right)-8361\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right.}{256000\,d}$$

input
```
integrate(1/(5+3*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/256000*(540*(156*sin(d*x + c)/(cos(d*x + c) + 1) - 49*sin(d*x + c)^3/(c
os(d*x + c) + 1)^3)/(8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 - 16) - 4096*arctan(sin(d*x + c)/(cos(d*x + c) + 1)
) + 8361*log(sin(d*x + c)/(cos(d*x + c) + 1) + 2) - 8361*log(sin(d*x + c)/
(cos(d*x + c) + 1) - 2))/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

$$\int \frac{1}{(5+3\sec(c+dx))^3}\,dx = \frac{2048\,dx+2048\,c-\frac{540\left(49\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-156\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\right)^2}-8361\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right)+8361\,\log\left(\right.}{256000\,d}$$

input
```
integrate(1/(5+3*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/256000*(2048*d*x + 2048*c - 540*(49*tan(1/2*d*x + 1/2*c)^3 - 156*tan(1/2
*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 4)^2 - 8361*log(abs(tan(1/2*d*x +
 1/2*c) + 2)) + 8361*log(abs(tan(1/2*d*x + 1/2*c) - 2)))/d
```

Mupad [B] (verification not implemented)

Time = 9.92 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.65

$$\int \frac{1}{(5+3\sec(c+dx))^3}\,dx = \frac{x}{125}-\frac{8361\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{128000\,d}+\frac{\frac{1053\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3200}-\frac{1323\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12800}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+16\right)}$$

input
```
int(1/(3/cos(c + d*x) + 5)^3,x)
```

output
```
x/125 - (8361*atanh(tan(c/2 + (d*x)/2)/2))/(128000*d) + ((1053*tan(c/2 + (
d*x)/2))/3200 - (1323*tan(c/2 + (d*x)/2)^3)/12800)/(d*(tan(c/2 + (d*x)/2)^
4 - 8*tan(c/2 + (d*x)/2)^2 + 16))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.57

$$\int \frac{1}{(5+3\sec(c+dx))^3}\,dx$$
$$= \frac{250830\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)-250830\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)+110700\cos(dx}{}$$

input
```
int(1/(5+3*sec(d*x+c))^3,x)
```

output
```
(250830*cos(c + d*x)*log(tan((c + d*x)/2) - 2) - 250830*cos(c + d*x)*log(t
an((c + d*x)/2) + 2) + 110700*cos(c + d*x)*sin(c + d*x) + 61440*cos(c + d*
x)*d*x - 209025*log(tan((c + d*x)/2) - 2)*sin(c + d*x)**2 + 284274*log(tan
((c + d*x)/2) - 2) + 209025*log(tan((c + d*x)/2) + 2)*sin(c + d*x)**2 - 28
4274*log(tan((c + d*x)/2) + 2) - 51200*sin(c + d*x)**2*d*x + 57780*sin(c +
 d*x) + 69632*d*x)/(256000*d*(30*cos(c + d*x) - 25*sin(c + d*x)**2 + 34))
```

3.530 $\int \frac{1}{(5+3\sec(c+dx))^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 145

$$\begin{aligned}\int \frac{1}{(5+3\sec(c+dx))^4}\,dx = {} & \frac{x}{625}+\frac{278151\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20480000d}\\ & -\frac{278151\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20480000d}\\ & +\frac{3\tan(c+dx)}{80d(5+3\sec(c+dx))^3}+\frac{519\tan(c+dx)}{12800d(5+3\sec(c+dx))^2}\\ & +\frac{38733\tan(c+dx)}{1024000d(5+3\sec(c+dx))}\end{aligned}$$

output
```
1/625*x+278151/20480000*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-2781
51/20480000*ln(2*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d+3/80*tan(d*x+c)/
d/(5+3*sec(d*x+c))^3+519/12800*tan(d*x+c)/d/(5+3*sec(d*x+c))^2+38733/10240
00*tan(d*x+c)/d/(5+3*sec(d*x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 344 vs. $2(145) = 290$.

Time = 0.59 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.37

$$\int \frac{1}{(5+3\sec(c+dx))^4}\,dx$$
$$= \frac{18284544c + 18284544dx + 4096000c\cos(3(c+dx)) + 4096000dx\cos(3(c+dx)) + 155208258\log\left(2\cos\right.}{}$$

input
```
Integrate[(5 + 3*Sec[c + d*x])^(-4),x]
```

output
```
(18284544*c + 18284544*d*x + 4096000*c*Cos[3*(c + d*x)] + 4096000*d*x*Cos[
3*(c + d*x)] + 155208258*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3476
8875*Cos[3*(c + d*x)]*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 915*Cos
[c + d*x]*(32768*(c + d*x) + 278151*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] - 278151*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 450*Cos[2*(c +
 d*x)]*(32768*(c + d*x) + 278151*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
] - 278151*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 155208258*Log[2*C
os[(c + d*x)/2] + Sin[(c + d*x)/2]] - 34768875*Cos[3*(c + d*x)]*Log[2*Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]] + 52174260*Sin[c + d*x] + 51462000*Sin[2*
(c + d*x)] + 24286500*Sin[3*(c + d*x)])/(81920000*d*(3 + 5*Cos[c + d*x])^3
)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.81, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {3042, 4272, 27, 3042, 4548, 25, 3042, 4548, 25, 3042, 4407, 3042, 4318, 3042, 3138, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3\sec(c+dx)+5)^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(3\csc\left(c+dx+\frac{\pi}{2}\right)+5\right)^4}dx$$

$$\Big\downarrow \text{4272}$$

$$\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}-\frac{1}{240}\int -\frac{3\left(6\sec^2(c+dx)-15\sec(c+dx)+16\right)}{(3\sec(c+dx)+5)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{80}\int \frac{6\sec^2(c+dx)-15\sec(c+dx)+16}{(3\sec(c+dx)+5)^3}dx+\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{80}\int \frac{6\csc\left(c+dx+\frac{\pi}{2}\right)^2-15\csc\left(c+dx+\frac{\pi}{2}\right)+16}{\left(3\csc\left(c+dx+\frac{\pi}{2}\right)+5\right)^3}dx+\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}$$

$$\Big\downarrow \text{4548}$$

$$\frac{1}{80}\left(\frac{519\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}-\frac{1}{160}\int -\frac{519\sec^2(c+dx)-1410\sec(c+dx)+512}{(3\sec(c+dx)+5)^2}dx\right)+\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{80}\left(\frac{1}{160}\int \frac{519\sec^2(c+dx)-1410\sec(c+dx)+512}{(3\sec(c+dx)+5)^2}dx+\frac{519\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}\right)+\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{80}\left(\frac{1}{160}\int \frac{519\csc\left(c+dx+\frac{\pi}{2}\right)^2-1410\csc\left(c+dx+\frac{\pi}{2}\right)+512}{\left(3\csc\left(c+dx+\frac{\pi}{2}\right)+5\right)^2}dx+\frac{519\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}\right)+\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}$$

$$\Big\downarrow \text{4548}$$

$$\frac{1}{80}\left(\frac{1}{160}\left(\frac{38733\tan(c+dx)}{80d(3\sec(c+dx)+5)}-\frac{1}{80}\int -\frac{8192-50715\sec(c+dx)}{3\sec(c+dx)+5}dx\right)+\frac{519\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}\right)+\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{80}\left(\frac{1}{160}\left(\frac{1}{80}\int \frac{8192-50715\sec(c+dx)}{3\sec(c+dx)+5}dx+\frac{38733\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{519\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}\right)+\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{80}\left(\frac{1}{160}\left(\frac{1}{80}\int \frac{8192-50715\csc\left(c+dx+\frac{\pi}{2}\right)}{3\csc\left(c+dx+\frac{\pi}{2}\right)+5}dx+\frac{38733\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{519\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}\right)+\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}$$

$$\Big\downarrow \ 4407$$

$$\frac{1}{80}\left(\frac{1}{160}\left(\frac{1}{80}\left(\frac{8192x}{5}-\frac{278151}{5}\int \frac{\sec(c+dx)}{3\sec(c+dx)+5}dx\right)+\frac{38733\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{519\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}\right)+\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{80}\left(\frac{1}{160}\left(\frac{1}{80}\left(\frac{8192x}{5}-\frac{278151}{5}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{3\csc\left(c+dx+\frac{\pi}{2}\right)+5}dx\right)+\frac{38733\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{519\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}\right)+\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}$$

$$\Big\downarrow \ 4318$$

$$\frac{1}{80}\left(\frac{1}{160}\left(\frac{1}{80}\left(\frac{8192x}{5}-\frac{92717}{5}\int \frac{1}{\frac{5}{3}\cos(c+dx)+1}dx\right)+\frac{38733\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{519\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}\right)+\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{80}\left(\frac{1}{160}\left(\frac{1}{80}\left(\frac{8192x}{5}-\frac{92717}{5}\int \frac{1}{\frac{5}{3}\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx\right)+\frac{38733\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{519\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}\right)+\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}$$

$$\Big\downarrow \; 3138$$

$$\frac{1}{80}\left(\frac{1}{160}\left(\frac{1}{80}\left(\frac{8192x}{5}-\frac{185434\int \frac{1}{\frac{8}{3}-\frac{2}{3}\tan^2\left(\frac{1}{2}(c+dx)\right)}d\tan\left(\frac{1}{2}(c+dx)\right)}{5d}\right)+\frac{38733\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{519\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}\right)+\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{80}\left(\frac{1}{160}\left(\frac{1}{80}\left(\frac{8192x}{5}-\frac{278151\text{arctanh}\left(\frac{1}{2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{10d}\right)+\frac{38733\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{519\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}\right)+\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}$$

input
```
Int[(5 + 3*Sec[c + d*x])^(-4),x]
```

output
```
(3*Tan[c + d*x])/(80*d*(5 + 3*Sec[c + d*x])^3) + ((519*Tan[c + d*x])/(160*
d*(5 + 3*Sec[c + d*x])^2) + (((8192*x)/5 - (278151*ArcTanh[Tan[(c + d*x)/2
]/2])/(10*d))/80 + (38733*Tan[c + d*x])/(80*d*(5 + 3*Sec[c + d*x])))/160)/
80
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.91

method	result
risch	$\frac{x}{625}+\frac{27i\left(166525\,\mathrm{e}^{5i(dx+c)}+581495\,\mathrm{e}^{4i(dx+c)}+1003842\,\mathrm{e}^{3i(dx+c)}+1064590\,\mathrm{e}^{2i(dx+c)}+643025\,\mathrm{e}^{i(dx+c)}+224875\right)}{2560000d\left(5\,\mathrm{e}^{2i(dx+c)}+6\,\mathrm{e}^{i(dx+c)}+5\right)^{3}}-$
derivativedivides	$\frac{-\frac{27}{10240\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^{3}}+\frac{1431}{102400\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^{2}}-\frac{69093}{2048000\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}-\frac{278151\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{20480000}+\frac{2\arctan\left(\tan\right.}{62}}{d}$
default	$\frac{-\frac{27}{10240\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^{3}}+\frac{1431}{102400\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^{2}}-\frac{69093}{2048000\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}-\frac{278151\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{20480000}+\frac{2\arctan\left(\tan\right.}{62}}{d}$
norman	$\frac{-\frac{64x}{625}-\frac{44523\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64000d}+\frac{13527\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{32000d}-\frac{69093\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{1024000d}+\frac{48x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{625}-\frac{12x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{625}+\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{625}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-4\right)^{3}}$
parallelrisch	$\frac{(254508165\cos(dx+c)+125167950\cos(2dx+2c)+34768875\cos(3dx+3c)+155208258)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)+(-25450}{}$

input
```
int(1/(5+3*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/625*x+27/2560000*I*(166525*exp(5*I*(d*x+c))+581495*exp(4*I*(d*x+c))+1003
842*exp(3*I*(d*x+c))+1064590*exp(2*I*(d*x+c))+643025*exp(I*(d*x+c))+224875
)/d/(5*exp(2*I*(d*x+c))+6*exp(I*(d*x+c))+5)^3-278151/20480000/d*ln(exp(I*(
d*x+c))+3/5+4/5*I)+278151/20480000/d*ln(exp(I*(d*x+c))+3/5-4/5*I)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.43

$$\int \frac{1}{(5+3\sec(c+dx))^4}\,dx$$
$$=\frac{8192000\,dx\cos{(dx+c)}^{3}+14745600\,dx\cos{(dx+c)}^{2}+8847360\,dx\cos{(dx+c)}+1769472\,dx-278151}{}$$

input
```
integrate(1/(5+3*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/40960000*(8192000*d*x*cos(d*x + c)^3 + 14745600*d*x*cos(d*x + c)^2 + 884
7360*d*x*cos(d*x + c) + 1769472*d*x - 278151*(125*cos(d*x + c)^3 + 225*cos
(d*x + c)^2 + 135*cos(d*x + c) + 27)*log(3/2*cos(d*x + c) + 2*sin(d*x + c)
 + 5/2) + 278151*(125*cos(d*x + c)^3 + 225*cos(d*x + c)^2 + 135*cos(d*x +
c) + 27)*log(3/2*cos(d*x + c) - 2*sin(d*x + c) + 5/2) + 1080*(44975*cos(d*
x + c)^2 + 47650*cos(d*x + c) + 12911)*sin(d*x + c))/(125*d*cos(d*x + c)^3
 + 225*d*cos(d*x + c)^2 + 135*d*cos(d*x + c) + 27*d)
```

Sympy [F]

$$\int \frac{1}{(5+3\sec(c+dx))^4}\,dx = \int \frac{1}{(3\sec{(c+dx)}+5)^4}\,dx$$

input
```
integrate(1/(5+3*sec(d*x+c))**4,x)
```

output
```
Integral((3*sec(c + d*x) + 5)**(-4), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.34

$$\int \frac{1}{(5+3\sec(c+dx))^4}\,dx = -\frac{\frac{540\left(\frac{26384\sin(dx+c)}{\cos(dx+c)+1}-\frac{16032\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2559\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{12\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-64}-65536\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+278151\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+\right.}{20480000\,d}$$

input
```
integrate(1/(5+3*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/20480000*(540*(26384*sin(d*x + c)/(cos(d*x + c) + 1) - 16032*sin(d*x +
c)^3/(cos(d*x + c) + 1)^3 + 2559*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(48*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 12*sin(d*x + c)^4/(cos(d*x + c) + 1)
^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 64) - 65536*arctan(sin(d*x + c)
/(cos(d*x + c) + 1)) + 278151*log(sin(d*x + c)/(cos(d*x + c) + 1) + 2) - 2
78151*log(sin(d*x + c)/(cos(d*x + c) + 1) - 2))/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.68

$$\int \frac{1}{(5+3\sec(c+dx))^4}\,dx = \frac{32768\,dx + 32768\,c - \frac{540\left(2559\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-16032\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+26384\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\right)^3} - 278151\,\log\left(\left|\tan\left(\frac{1}{2}\,dx\right.\right.\right.}{20480000\,d}$$

input
```
integrate(1/(5+3*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/20480000*(32768*d*x + 32768*c - 540*(2559*tan(1/2*d*x + 1/2*c)^5 - 16032
*tan(1/2*d*x + 1/2*c)^3 + 26384*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c
)^2 - 4)^3 - 278151*log(abs(tan(1/2*d*x + 1/2*c) + 2)) + 278151*log(abs(ta
n(1/2*d*x + 1/2*c) - 2)))/d
```

Mupad [B] (verification not implemented)

Time = 10.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.72

$$\int \frac{1}{(5+3\sec(c+dx))^4}\,dx = \frac{x}{625} - \frac{278151\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{10240000\,d} - \frac{\frac{69093\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{1024000} - \frac{13527\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{32000} + \frac{44523\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64000}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 - 12\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 48\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 - 64\right)}$$

input
```
int(1/(3/cos(c + d*x) + 5)^4,x)
```

output
```
x/625 - (278151*atanh(tan(c/2 + (d*x)/2)/2))/(10240000*d) - ((44523*tan(c/
2 + (d*x)/2))/64000 - (13527*tan(c/2 + (d*x)/2)^3)/32000 + (69093*tan(c/2
+ (d*x)/2)^5)/1024000)/(d*(48*tan(c/2 + (d*x)/2)^2 - 12*tan(c/2 + (d*x)/2)
^4 + tan(c/2 + (d*x)/2)^6 - 64))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.99

$$\int \frac{1}{(5+3\sec(c+dx))^4}\,dx = \frac{34768875\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)\sin(dx+c)^2-72319260\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{}$$

input
```
int(1/(5+3*sec(d*x+c))^4,x)
```

output
```
(34768875*cos(c + d*x)*log(tan((c + d*x)/2) - 2)*sin(c + d*x)**2 - 7231926
0*cos(c + d*x)*log(tan((c + d*x)/2) - 2) - 34768875*cos(c + d*x)*log(tan((
c + d*x)/2) + 2)*sin(c + d*x)**2 + 72319260*cos(c + d*x)*log(tan((c + d*x)
/2) + 2) + 4096000*cos(c + d*x)*sin(c + d*x)**2*d*x - 25731000*cos(c + d*x
)*sin(c + d*x) - 8519680*cos(c + d*x)*d*x + 62583975*log(tan((c + d*x)/2)
- 2)*sin(c + d*x)**2 - 70094052*log(tan((c + d*x)/2) - 2) - 62583975*log(t
an((c + d*x)/2) + 2)*sin(c + d*x)**2 + 70094052*log(tan((c + d*x)/2) + 2)
+ 24286500*sin(c + d*x)**3 + 7372800*sin(c + d*x)**2*d*x - 31258440*sin(c
+ d*x) - 8257536*d*x)/(20480000*d*(125*cos(c + d*x)*sin(c + d*x)**2 - 260*
cos(c + d*x) + 225*sin(c + d*x)**2 - 252))
```

3.531 $\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 292

$$\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \frac{2(a-b)\sqrt{a+b}(2a^2-9b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{15b^3d} + \frac{2(a-b)\sqrt{a+b}(2a+9b)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{15b^2d} - \frac{4a\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15bd} + \frac{2(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5bd}$$

output
```
2/15*(a-b)*(a+b)^(1/2)*(2*a^2-9*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))
^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b
*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d+2/15*(a-b)*(a+b)^(1/2)*(2*a+9*b)*cot(d*
x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*
(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d-4/15*a*(
a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b/d+2/5*(a+b*sec(d*x+c))^(3/2)*tan(d*x+c)
/b/d
```

Mathematica [A] (warning: unable to verify)

Time = 27.52 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.25

$$\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\left((-2a^2+9b^2)\sin(c+dx)+\frac{\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(2(2a^3+2a^2b-9ab^2-9b^3\right.}{}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[a + b*Sec[c + d*x]]*((-2*a^2 + 9*b^2)*Sin[c + d*x] + (Sqrt[Sec[(c
+ d*x)/2]^2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(2*(2*a^3 + 2*a^2*b -
 9*a*b^2 - 9*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c
+ d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]],
(a - b)/(a + b)] + 2*b*(-2*a^2 + 7*a*b + 9*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipt
icF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (2*a^2 - 9*b^2)*Cos[c + d
*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/((b + a*Cos
[c + d*x])*Sec[c + d*x]^(3/2)) + b*(a + 3*b*Sec[c + d*x])*Tan[c + d*x]))/(
15*b^2*d)
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4327, 27, 3042, 4490, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4327}$$

$$\frac{2\int \frac{1}{2}\sec(c+dx)(3b-2a\sec(c+dx))\sqrt{a+b\sec(c+dx)}dx}{5b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sec(c+dx)(3b-2a\sec(c+dx))\sqrt{a+b\sec(c+dx)}dx}{5b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(3b-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{5b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{4490}$$

$$\frac{\frac{2}{3}\int \frac{\sec(c+dx)\left(7ab-\left(2a^2-9b^2\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx-\frac{4a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}}{5b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{3}\int \frac{\sec(c+dx)\left(7ab-\left(2a^2-9b^2\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx-\frac{4a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}}{5b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(7ab+\left(9b^2-2a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{4a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}}{5b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{4493}$$

$$\frac{\frac{1}{3}\left((a-b)(2a+9b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-\left(2a^2-9b^2\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx\right)-\frac{4a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}}{5b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{3}\left((a-b)(2a+9b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\left(2a^2-9b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-\frac{4a\tan(c+dx)\sqrt{a+b\sec(c}{3d}}{5b}$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \; 4319$$

$$\frac{\frac{1}{3}\left(\frac{2(a-b)\sqrt{a+b}(2a+9b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-\left(2a^2-9b^2\right)\int \frac{\csc(c+}{}\right.}{5b}$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \; 4492$$

$$\frac{\frac{1}{3}\left(\frac{2(a-b)\sqrt{a+b}\left(2a^2-9b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}+\frac{2(a-b)\sqrt{a+b}(2a+9b)\cot(c+}{}\right.}{5b}$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5bd}$$

input
```
Int[Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*b*d) + (((2*(a - b)*Sqrt[a
+ b]*(2*a^2 - 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) + (2*(a - b)*Sqrt[a + b]*(2*a
 + 9*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Se
c[c + d*x]))/(a - b))])/(b*d))/3 - (4*a*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d
*x])/(3*d))/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4327
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 -
b^2, 0] &&  !LtQ[m, -1]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 859 vs. $2(262) = 524$.

Time = 12.66 (sec) , antiderivative size = 860, normalized size of antiderivative = 2.95

method	result	size
default	Expression too large to display	860

input
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15/d/b^2*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+b*cos(d*x+
c)+b)*(2*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*EllipticE(cot(d*x+c)-cs
c(d*x+c),((a-b)/(a+b))^(1/2))+2*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*
b*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+9*(-cos(d*x+c)^2-2*
cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*a*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^
(1/2))+9*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^3*EllipticE(cot(d*x+c)-c
sc(d*x+c),((a-b)/(a+b))^(1/2))+2*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^
2*b*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+7*(cos(d*x+c)^2+2
*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*a*b^2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))
^(1/2))+9*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^3*EllipticF(cot(d*x+c)-c
sc(d*x+c),((a-b)/(a+b))^(1/2))+2*a^3*cos(d*x+c)*sin(d*x+c)+sin(d*x+c)*(1-c
os(d*x+c))*a^2*b+(-9*cos(d*x+c)^2-4*cos(d*x+c)-4)*a*b^2*tan(d*x+c)+3*b^3*(
-3*sin(d*x+c)-tan(d*x+c)-sec(d*x+c)*tan(d*x+c)))
```

Fricas [F]

$$\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3, x)
```

Sympy [F]

$$\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+b\sec(c+dx)}\sec^3(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**3, x)
```

Maxima [F]

$$\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3, x)
```

Giac [F]

$$\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\cos(c+d\,x)^3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^3,x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^3, x)
```

Reduce [F]

$$\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)^3\,dx$$

input
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**3,x)
```

3.532 $\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 241

$$\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = -\frac{2a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3b^2d} - \frac{2(a-b)\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3bd} + \frac{2\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d}$$

output
```
-2/3*a*(a-b)*(a+b)^(1/2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)
^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+
c))/(a-b))^(1/2)/b^2/d-2/3*(a-b)*(a+b)^(1/2)*cot(d*x+c)*EllipticF((a+b*sec
(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(
1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+2/3*(a+b*sec(d*x+c))^(1/2)*tan(d*
x+c)/d
```

Mathematica [A] (verified)

Time = 15.58 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.22

$$\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx =$$

$$-\frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+b\sec(c+dx)}\left(2a(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+\right.\right.\right.\right.}{}$$

$$+\frac{\sqrt{a+b\sec(c+dx)}\left(\frac{2a\sin(c+dx)}{3b}+\frac{2}{3}\tan(c+dx)\right)}{d}$$

input
```
Integrate[Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(-2*Cos[(c + d*x)/2]^2*Sqrt[a + b*Sec[c + d*x]]*(2*a*(a + b)*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] +
 a*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))
/(3*b*d*(b + a*Cos[c + d*x])) + (Sqrt[a + b*Sec[c + d*x]]*((2*a*Sin[c + d*
x])/(3*b) + (2*Tan[c + d*x])/3))/d
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4322, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow \text{4322}$$

$$\frac{1}{3}\int \frac{\sec(c+dx)(b+a\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}dx+\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\downarrow \text{4493}$$

$$\frac{1}{3}\left(a\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx-(a-b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\downarrow \text{3042}$$

$$\frac{1}{3}\left(a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\downarrow \text{4319}$$

$$\frac{1}{3}\left(a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}}{bd}\right.$$
$$\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\downarrow \text{4492}$$

$$\frac{1}{3}\left(-\frac{2a(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2(a-b)\sqrt{}}{}\right.$$
$$\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

input
```
Int[Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
((-2*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*(a - b)*Sqrt[a + b
]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
d*x]))/(a - b))])/(b*d))/3 + (2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*
d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4322
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[m/(m + 1)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc
[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. 2(215) = 430.

Time = 10.72 (sec) , antiderivative size = 503, normalized size of antiderivative = 2.09

method	result
default	$-\frac{2\sqrt{a+b\sec(dx+c)}\left(\left(-\cos(dx+c)^{2}-2\cos(dx+c)-1\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\,a^{2}\operatorname{EllipticE}\left(\cot(dx+c)-\csc(dx+c),\right.$

input
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d/b*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+b*cos(d*x+c)+
b)*((-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*EllipticE(cot(d*x+c)-csc(d*
x+c),((a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b*Ellip
ticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(cos(d*x+c)^2+2*cos(d*x+c)
+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*a*b*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(cos(
d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b
)/(a+b))^(1/2))-a^2*cos(d*x+c)*sin(d*x+c)+sin(d*x+c)*(-cos(d*x+c)-2)*a*b+b
^2*(-sin(d*x+c)-tan(d*x+c)))
```

Fricas [F]

$$\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2, x)
```

Sympy [F]

$$\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+b\sec(c+dx)}\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**2, x)
```

Maxima [F]

$$\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2, x)
```

Giac [F]

$$\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx=\int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx=\int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\cos(c+d\,x)^2}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^2,x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx=\int \sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)^2\,dx$$

input
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**2,x)
```

3.533 $\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 209

$$\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = -\frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{bd} + \frac{2(a-b)\sqrt{a+b}\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{bd}$$

output
```
-2*(a-b)*(a+b)^(1/2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/
2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/
(a-b))^(1/2)/b/d+2*(a-b)*(a+b)^(1/2)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))
^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b
*(1+sec(d*x+c))/(a-b))^(1/2)/b/d
```

Mathematica [A] (warning: unable to verify)

Time = 8.39 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00

$$\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\left(\sin(c+dx) - \frac{\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(\frac{(a+b)\left(E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)-\text{Ellipti}\right.}{\sqrt{1}}\right.}{(b+a\cos(c+dx))}\right.}{d}$$

input
```
Integrate[Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[a + b*Sec[c + d*x]]*(Sin[c + d*x] - (Sqrt[Sec[(c + d*x)/2]^2]*Sqrt
[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((a + b)*(EllipticE[ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)] - EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)])*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))])/Sqrt[(1 + S
ec[c + d*x])^(-1)] + (b + a*Cos[c + d*x])*Tan[(c + d*x)/2]))/((b + a*Cos[c
 + d*x])*Sec[c + d*x]^(3/2))))/d
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4316, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4316$$

$$(a-b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+b\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4319}$$

$$b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}$$

$$\Big\downarrow \text{4492}$$

$$\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}$$

input
```
Int[Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (2*(a - b)*Sqrt[a + b]*Cot
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/
(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x])
)/(a - b))])/(b*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4316
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(a - b)   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x]
+ Simp[b   Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]),
x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(191) = 382$.

Time = 8.06 (sec) , antiderivative size = 463, normalized size of antiderivative = 2.22

method	result
default	$-\frac{(\cos(dx+c)+1)^2\left(\left(-(1-\cos(dx+c))^3\csc(dx+c)^3+\csc(dx+c)-\cot(dx+c)\right)a+\left((1-\cos(dx+c))^3\csc(dx+c)^3+\csc(dx+c)-\cot(dx\right.\right.}{}$

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d/(b+a*cos(d*x+c))*(cos(d*x+c)+1)^2*((-(1-cos(d*x+c))^3*csc(d*x+c)^3+
csc(d*x+c)-cot(d*x+c))*a+((1-cos(d*x+c))^3*csc(d*x+c)^3+csc(d*x+c)-cot(d*x
+c))*b-2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a-2*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*b+2*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a+2*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(
cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*b)*((1-cos(d*x+c))^2*csc(d*x+c)
^2-1)*(a+b*sec(d*x+c))^(1/2)*sec(d*x+c)
```

Fricas [F]

$$\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c), x)
```

Sympy [F]

$$\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+b\sec(c+dx)}\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c), x)
```

Giac [F]

$$\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\cos\left(c+d\,x\right)}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x), x)
```

Reduce [F]

$$\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)\,dx$$

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*sec(c + d*x),x)
```

3.534 $\int \sqrt{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 125

$$\int \sqrt{a+b\sec(c+dx)}\,dx = -\frac{2\cot(c+dx)\,\text{EllipticPi}\left(\frac{a}{a+b},\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right),\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(1+\sec(c+dx))}{a+b\sec(c+dx)}}(a+b\sec(c}{\sqrt{a+b}d}$$

output
```
-2*cot(d*x+c)*EllipticPi((a+b)^(1/2)/(a+b*sec(d*x+c))^(1/2),a/(a+b),((a-b)
/(a+b))^(1/2))*(-b*(1-sec(d*x+c))/(a+b*sec(d*x+c)))^(1/2)*(b*(1+sec(d*x+c)
)/(a+b*sec(d*x+c)))^(1/2)*(a+b*sec(d*x+c))/(a+b)^(1/2)/d
```

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.21

$$\int \sqrt{a+b\sec(c+dx)}\,dx = \frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\left((-a+b)\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{a-b}{a+b}\right)+\right.}{d(b+a\cos(c+dx))}$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*((-a + b)*EllipticF[ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)])*Sqrt[a + b*Sec[c + d*x]])/(d*(b + a*Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4267}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4267$$

$$-\frac{2\cot(c+dx)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(\sec(c+dx)+1)}{a+b\sec(c+dx)}}(a+b\sec(c+dx))\,\text{EllipticPi}\left(\frac{a}{a+b},\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right),\frac{a-b}{a+b}\right)}{d\sqrt{a+b}}$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(-2*Cot[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sec[c
 + d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sec[c + d*x]))/(a + b*Sec[c + d
*x]))]*Sqrt[(b*(1 + Sec[c + d*x]))/(a + b*Sec[c + d*x])]*(a + b*Sec[c + d*
x]))/(Sqrt[a + b]*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4267
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*((a + b
*Csc[c + d*x])/(d*Rt[a + b, 2]*Cot[c + d*x]))*Sqrt[b*((1 + Csc[c + d*x])/(a
 + b*Csc[c + d*x]))]*Sqrt[(-b)*((1 - Csc[c + d*x])/(a + b*Csc[c + d*x]))]*E
llipticPi[a/(a + b), ArcSin[Rt[a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)
/(a + b)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 5.75 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.46

method	result
default	$-\frac{2\left(2a\,\text{EllipticPi}\left(\cot(dx+c)-\csc(dx+c),-1,\sqrt{\frac{a-b}{a+b}}\right)-\text{EllipticF}\left(\cot(dx+c)-\csc(dx+c),\sqrt{\frac{a-b}{a+b}}\right)a+\text{EllipticF}\left(\cot(dx+c)-\csc(dx\right.\right.}{d(b+a\cos(dx+c))}$

input
```
int((a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(2*a*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))-Ellipti
cF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a+EllipticF(cot(d*x+c)-csc(d
*x+c),((a-b)/(a+b))^(1/2))*b)*(cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(a+b*sec(d*x+c))^(1/2
)/(b+a*cos(d*x+c))
```

Fricas [F]

$$\int \sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]

$$\int \sqrt{a + b\sec(c + dx)}\, dx = \int \sqrt{a + b\sec{(c + dx)}}\, dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{a + b\sec(c + dx)}\, dx = \int \sqrt{b\sec{(dx + c)} + a}\, dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \sqrt{a + b\sec(c + dx)}\, dx = \int \sqrt{b\sec{(dx + c)} + a}\, dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec(c+dx)}\,dx=\int \sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sec(c+dx)}\,dx=\int \sqrt{\sec\left(dx+c\right)b+a}dx$$

input
```
int((a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a),x)
```

3.535 $\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 330

$$\begin{aligned}
&\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,dx \\
&= \frac{(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{bd} \\
&+ \frac{\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d} \\
&- \frac{b\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ad} \\
&+ \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d}
\end{aligned}$$

output
```
(a-b)*(a+b)^(1/2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),
((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-
b))^(1/2)/b/d+(a+b)^(1/2)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b
)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x
+c))/(a-b))^(1/2)/d-b*(a+b)^(1/2)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(
1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/
2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (warning: unable to verify)

Time = 9.97 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.13

$$\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$
$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)}\sqrt{1+\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\left(2(a+b)\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\right.}{}$$

input
```
Integrate[Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^4]*Sqrt[1 + Sec[c +
 d*x]]*Sqrt[a + b*Sec[c + d*x]]*(2*(a + b)*Sqrt[(b + a*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)] - 4*b*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elliptic
F[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 4*b*Sqrt[(b + a*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]],
(a - b)/(a + b)] + a*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2
]*Sin[(3*(c + d*x))/2] - a*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c +
d*x)/2] + 2*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]))/(2*
d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4344, 3042, 4547, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4344}$$

$$\frac{1}{2}\int \frac{b-b\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{b-b\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4547}$$

$$\frac{1}{2}\left(\int \frac{\sec(c+dx)b+b}{\sqrt{a+b\sec(c+dx)}}dx-b\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)b+b}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4409}$$

$$\frac{1}{2}\left(-b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx+b\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(b\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)\right.}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}\right.+\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4271}$$

$$\frac{1}{2}\left(b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\sqrt{a+b}\cot(c+dx)\sqrt{\cdot}}{}\right.$$
$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 4319

$$\frac{1}{2}\left(-b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{Elli}}{d}\right.$$
$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 4492

$$\frac{1}{2}\left(\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}+\frac{2(a-b)\sqrt{a}}{}\right.$$
$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

input
```
Int[Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
((2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (2*Sqrt[a + b]*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b
))])/d - (2*b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d))/2 + (Sqrt[a +
 b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4344
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*Csc[e
 + f*x])^n/(f*n)), x] - Simp[1/(2*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*(Simp
[b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x]/Sqrt[a + b*C
sc[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] &&
LeQ[n, -1] && IntegerQ[2*n]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4547
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Simp[C   Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f
*x]]), x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 3.73 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.43

method	result
default	$\frac{\left(\left(-2\cos(dx+c)^{2}-4\cos(dx+c)-2\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\,b\,\mathrm{EllipticPi}\left(\cot(dx+c)-\csc(dx+c),-1,\sqrt{\frac{a-b}{a+b}}\right)+\left(-\cos(\right.\right.}{}$

input
```
int(cos(d*x+c)*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*((-2*cos(d*x+c)^2-4*cos(d*x+c)-2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b*EllipticPi(cot(d*x+c)-csc(
d*x+c),-1,((a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*El
lipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*
x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*b*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(2*
cos(d*x+c)^2+4*cos(d*x+c)+2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b*EllipticF(cot(d*x+c)-csc(d*x+c),((a
-b)/(a+b))^(1/2))+sin(d*x+c)*cos(d*x+c)^2*a+sin(d*x+c)*cos(d*x+c)*b)*(a+b*
sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+b*cos(d*x+c)+b)
```

Fricas [F]

$$\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c), x)
```

Sympy [F]

$$\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+b\sec(c+dx)}\cos(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))*cos(c + d*x), x)
```

Maxima [F]

$$\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \cos\left(c+d\,x\right)\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)*(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)*(a + b/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{\sec\left(dx+c\right)b+a}\,\cos\left(dx+c\right)dx$$

input
```
int(cos(d*x+c)*(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*cos(c + d*x),x)
```

3.536 $\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 396

$$
\begin{aligned}
&\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx \\
&= \frac{(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4ad} \\
&+ \frac{\sqrt{a+b}(2a+b)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4ad} \\
&- \frac{\sqrt{a+b}(4a^2-b^2)\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4a^2d} \\
&+ \frac{b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4ad} + \frac{\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d}
\end{aligned}
$$

output
```
1/4*(a-b)*(a+b)^(1/2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1
/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))
/(a-b))^(1/2)/a/d+1/4*(a+b)^(1/2)*(2*a+b)*cot(d*x+c)*EllipticF((a+b*sec(d*
x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2
)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d-1/4*(a+b)^(1/2)*(4*a^2-b^2)*cot(d*x+
c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/
2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+1
/4*b*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/a/d+1/2*cos(d*x+c)*(a+b*sec(d*x+c))
^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 13.81 (sec) , antiderivative size = 668, normalized size of antiderivative = 1.69

$$\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \frac{\sqrt{a+b\sec(c+dx)}\sin(2(c+dx))}{4d}$$
$$-\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a+b\sec(c+dx)}\Big(-8b(a+b)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E(\text{arc}$$

input
```
Integrate[Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(Sqrt[a + b*Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) - (Sec[(c + d*x)/2]*Sqrt
[a + b*Sec[c + d*x]]*(-8*b*(a + b)*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 16*a*(2*a - b)*Cos[
(c + d*x)/2]^3*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d
*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a
- b)/(a + b)] - 48*a^2*Cos[(c + d*x)/2]*EllipticPi[-1, ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c +
d*x])/((a + b)*(1 + Sec[c + d*x]))] + 12*b^2*Cos[(c + d*x)/2]*EllipticPi[-
1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)
]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] - 16*a^2*Cos[(3*
(c + d*x))/2]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sq
rt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c
+ d*x]))] + 4*b^2*Cos[(3*(c + d*x))/2]*EllipticPi[-1, ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c + d
*x])/((a + b)*(1 + Sec[c + d*x]))] - 2*a*b*Sin[(c + d*x)/2] + 2*b^2*Sin[(c
 + d*x)/2] + a*b*Sin[(3*(c + d*x))/2] - 2*b^2*Sin[(3*(c + d*x))/2] - a*b*S
in[(5*(c + d*x))/2]))/(16*a*d*(b + a*Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 394, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4344, 3042, 4592, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4344}$$

$$\frac{1}{4}\int \frac{\cos(c+dx)\left(b\sec^2(c+dx)+2a\sec(c+dx)+b\right)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{b\csc\left(c+dx+\frac{\pi}{2}\right)^2+2a\csc\left(c+dx+\frac{\pi}{2}\right)+b}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{4592}$$

$$\frac{1}{4}\left(\frac{b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{\int -\frac{4a^2+2b\sec(c+dx)a-b^2-b^2\sec^2(c+dx)}{2\sqrt{a+b\sec(c+dx)}}dx}{a}\right)+\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{\int \frac{4a^2+2b\sec(c+dx)a-b^2-b^2\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{2a}+\frac{b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}\right)+\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{4}\left(\frac{\int \frac{4a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a-b^2-b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}\right)+\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \ 4546$$

$$\frac{1}{4}\left(\frac{\int \frac{4a^2-b^2+(b^2+2ab)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-b^2\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx}{2a}+\frac{b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}\right)+\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{4}\left(\frac{\int \frac{4a^2-b^2+(b^2+2ab)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}\right)+\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \ 4409$$

$$\frac{1}{4}\left(\frac{(4a^2-b^2)\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx+b^2\left(-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+b(2a+b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{2a}+\frac{b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}\right)+\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{4}\left(\frac{\left(4a^2-b^2\right)\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b^2\left(-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+b(2a+b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}\right.$$

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 4271

$$\frac{1}{4}\left(\frac{b^2\left(-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+b(2a+b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(4a^2-b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+}{a+b}}}{}}{2a}\right.$$

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 4319

$$\frac{1}{4}\left(\frac{b^2\left(-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-\frac{2\sqrt{a+b}\left(4a^2-b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\text{arcs}\right.}{ad}}{2a}\right.$$

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 4492

$$\frac{1}{4}\left(\frac{-\frac{2\sqrt{a+b}\left(4a^2-b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}+\frac{2\sqrt{a+b}(2a+b)\cot(c}{}}{}\right.$$

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

input
```
Int[Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (((2*(a - b)*
Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
+ Sec[c + d*x]))/(a - b))])/d + (2*Sqrt[a + b]*(2*a + b)*Cot[c + d*x]*Elli
pticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[
(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d
 - (2*Sqrt[a + b]*(4*a^2 - b^2)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d))/(2*a) +
 (b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(a*d))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4344
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*Csc[e
 + f*x])^n/(f*n)), x] - Simp[1/(2*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*(Simp
[b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x]/Sqrt[a + b*C
sc[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] &&
LeQ[n, -1] && IntegerQ[2*n]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 720 vs. $2(355) = 710$.

Time = 6.04 (sec) , antiderivative size = 721, normalized size of antiderivative = 1.82

method	result
default	$\left(\left(-8\cos(dx+c)^2-16\cos(dx+c)-8\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\,a^2\,\mathrm{EllipticPi}\left(\cot(dx+c)-\csc(dx+c),-1,\sqrt{\frac{a-b}{a+b}}\right)+\left(2\,\mathrm{co}\right.\right.$

input
```
int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a*((-8*cos(d*x+c)^2-16*cos(d*x+c)-8)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*EllipticPi(cot(d*x+
c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))+(2*cos(d*x+c)^2+4*cos(d*x+c)+2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*b^2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))+(-cos(d*x
+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(
a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^2*EllipticE(cot(d*x+
c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(4*cos(d*x+c)^2+8*cos(d*x+c)+4)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*a^2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(-2*cos(d*x+c)^2
-4*cos(d*x+c)-2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*a*b*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))
^(1/2))+sin(d*x+c)*cos(d*x+c)^2*(2*cos(d*x+c)+2)*a^2+sin(d*x+c)*cos(d*x+c)
*(2+3*cos(d*x+c))*a*b+b^2*cos(d*x+c)*sin(d*x+c))*(a+b*sec(d*x+c))^(1/2)/(c
os(d*x+c)^2*a+a*cos(d*x+c)+b*cos(d*x+c)+b)
```

Fricas [F]

$$\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^2, x)
```

Sympy [F]

$$\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx=\int \sqrt{a+b\sec{(c+dx)}}\cos^2{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))*cos(c + d*x)**2, x)
```

Maxima [F]

$$\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx=\int \sqrt{b\sec{(dx+c)}+a}\cos{(dx+c)}^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^2, x)
```

Giac [F]

$$\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx=\int \sqrt{b\sec{(dx+c)}+a}\cos{(dx+c)}^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx=\int \cos\left(c+d\,x\right)^2\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^2*(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^2*(a + b/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx=\int \sqrt{\sec\left(dx+c\right)b+a}\,\cos\left(dx+c\right)^2dx$$

input
```
int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*cos(c + d*x)**2,x)
```

3.537 $\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 405

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = -\frac{2(a-b)\sqrt{a+b}(8a^4+33a^2b^2+147b^4)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1\cdots}{\cdots}}}{315b^4d}$$
$$-\frac{2(a-b)\sqrt{a+b}(8a^3+6a^2b+39ab^2-147b^3)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-s\cdots}{\cdots}}}{315b^3d}$$
$$+\frac{2a(8a^2+39b^2)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{315b^2d}$$
$$+\frac{2(8a^2+49b^2)(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{315b^2d}$$
$$-\frac{8a(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{63b^2d}+\frac{2\sec(c+dx)(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{9bd}$$

output
```
-2/315*(a-b)*(a+b)^(1/2)*(8*a^4+33*a^2*b^2+147*b^4)*cot(d*x+c)*EllipticE((
a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(
a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d-2/315*(a-b)*(a+b)^(1/2)*
(8*a^3+6*a^2*b+39*a*b^2-147*b^3)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/
2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+
sec(d*x+c))/(a-b))^(1/2)/b^3/d+2/315*a*(8*a^2+39*b^2)*(a+b*sec(d*x+c))^(1/
2)*tan(d*x+c)/b^2/d+2/315*(8*a^2+49*b^2)*(a+b*sec(d*x+c))^(3/2)*tan(d*x+c)
/b^2/d-8/63*a*(a+b*sec(d*x+c))^(5/2)*tan(d*x+c)/b^2/d+2/9*sec(d*x+c)*(a+b*
sec(d*x+c))^(5/2)*tan(d*x+c)/b/d
```

Mathematica [A] (warning: unable to verify)

Time = 14.03 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.36

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx =$$

$$-\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\left(2(8a^5+8a^4b+33a^3b^2+33a^2b^3+147ab^4+147b\right.}{}$$

$$+\frac{\cos(c+dx)(a+b\sec(c+dx))^{3/2}\left(\frac{2(8a^4+33a^2b^2+147b^4)\sin(c+dx)}{315b^3}+\frac{2\sec^2(c+dx)(3a^2\sin(c+dx)+49b^2\sin(c+dx))}{315b}+\frac{8\sec(c\right.}{d(b+a\cos(c+}$$

input
```
Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(2*(8
*a^5 + 8*a^4*b + 33*a^3*b^2 + 33*a^2*b^3 + 147*a*b^4 + 147*b^5)*Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(8*a^
4 + 2*a^3*b + 33*a^2*b^2 + 186*a*b^3 + 147*b^4)*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipt
icF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (8*a^4 + 33*a^2*b^2 + 147
*b^4)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2
]))/(315*b^3*d*(b + a*Cos[c + d*x])^2*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x
]^(3/2)) + (Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*((2*(8*a^4 + 33*a^2*b^
2 + 147*b^4)*Sin[c + d*x])/(315*b^3) + (2*Sec[c + d*x]^2*(3*a^2*Sin[c + d*
x] + 49*b^2*Sin[c + d*x]))/(315*b) + (8*Sec[c + d*x]*(-(a^3*Sin[c + d*x])
+ 22*a*b^2*Sin[c + d*x]))/(315*b^2) + (20*a*Sec[c + d*x]^2*Tan[c + d*x])/6
3 + (2*b*Sec[c + d*x]^3*Tan[c + d*x])/9))/(d*(b + a*Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.04, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4352, 27, 3042, 4570, 27, 3042, 4490, 27, 3042, 4490, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4352}$$

$$\frac{2\int \frac{1}{2}\sec(c+dx)(a+b\sec(c+dx))^{3/2}\left(-4a\sec^2(c+dx)+7b\sec(c+dx)+2a\right)dx}{9b}+\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\left(-4a\sec^2(c+dx)+7b\sec(c+dx)+2a\right)dx}{9b}+\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(-4a\csc\left(c+dx+\frac{\pi}{2}\right)^2+7b\csc\left(c+dx+\frac{\pi}{2}\right)+2a\right)dx}{9b}+\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 4570$$

$$\frac{\frac{2\int -\frac{1}{2}\sec(c+dx)(a+b\sec(c+dx))^{3/2}\left(6ab-\left(8a^2+49b^2\right)\sec(c+dx)\right)dx}{7b}-\frac{8a\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}}{9b}+\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\left(6ab-\left(8a^2+49b^2\right)\sec(c+dx)\right)dx}{7b}-\frac{8a\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}}{9b}+\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(6ab+\left(-8a^2-49b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{7b}-\frac{8a\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}}{9b}+\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 4490$$

$$\frac{-\frac{\frac{2}{5}\int \frac{3}{2}\sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(b\left(2a^2-49b^2\right)-a\left(8a^2+39b^2\right)\sec(c+dx)\right)dx-\frac{2\left(8a^2+49b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}}{7b}-\frac{8a\tan(c+dx)(a+b}{7bd}}{9b}$$
$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{3}{5}\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(b(2a^2-49b^2)-a(8a^2+39b^2)\sec(c+dx)\right)dx-\frac{2(8a^2+49b^2)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}}{7b}-\frac{8a\tan(c+dx)(a+b\sec}{7bd}}{9b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{3}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b(2a^2-49b^2)-a(8a^2+39b^2)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{2(8a^2+49b^2)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}}{7b}-8a\tan(}{9b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$\downarrow$ 4490

$$-\frac{\frac{\frac{3}{5}\left(\frac{2}{3}\int -\frac{\sec(c+dx)\left(2ab\left(a^2+93b^2\right)+\left(8a^4+33b^2a^2+147b^4\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx-\frac{2a\left(8a^2+39b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)-\frac{2\left(8a^2+49b^2\right)\tan(c+dx)(a+b\sec}{5d}}{7b}}{9b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{3}{5}\left(-\frac{1}{3}\int \frac{\sec(c+dx)\left(2ab\left(a^2+93b^2\right)+\left(8a^4+33b^2a^2+147b^4\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx-\frac{2a\left(8a^2+39b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)-\frac{2\left(8a^2+49b^2\right)\tan(c+dx)(a+b\sec}{5d}}{7b}}{9b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{3}{5}\left(-\frac{1}{3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2ab\left(a^2+93b^2\right)+\left(8a^4+33b^2a^2+147b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2a\left(8a^2+39b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)-\frac{2\left(8a^2+49b^2\right)\tan(c+d}{5d}}{7b}}{9b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$\downarrow$ 4493

$$-\frac{\frac{3}{5}\left(\frac{1}{3}\left((a-b)\left(8a^3+6a^2b+39ab^2-147b^3\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-\left(8a^4+33a^2b^2+147b^4\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx\right)-\frac{2a\left(8a^2+39b^2\right)\tan(c+dx)\sqrt{a}}{3d}}{7b}}{9b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$\big\downarrow$ 3042

$$-\frac{\frac{3}{5}\left(\frac{1}{3}\left((a-b)\left(8a^3+6a^2b+39ab^2-147b^3\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\left(8a^4+33a^2b^2+147b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-\frac{2a\left(8a^2+39b^2\right.}{}}{7b}}{9b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$\big\downarrow$ 4319

$$-\frac{\frac{3}{5}\left(\frac{1}{3}\left(\frac{2(a-b)\sqrt{a+b}\left(8a^3+6a^2b+39ab^2-147b^3\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-\left(8a^4+33a^2b\right.\right.\right.}{7b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$\big\downarrow$ 4492

$$-\frac{\frac{3}{5}\left(\frac{1}{3}\left(\frac{2(a-b)\sqrt{a+b}\left(8a^4+33a^2b^2+147b^4\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}+\frac{2(a-b)\sqrt{a+b}\left(8a^3+6a^2b+\right.}{}\right.\right.}{}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

input

```
Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(9*b*d) + ((-8*a*
(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*b*d) - ((-2*(8*a^2 + 49*b^2)*(
a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d) + (3*(((2*(a - b)*Sqrt[a + b
]*(8*a^4 + 33*a^2*b^2 + 147*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) + (2*(a - b)*Sqrt
[a + b]*(8*a^3 + 6*a^2*b + 39*a*b^2 - 147*b^3)*Cot[c + d*x]*EllipticF[ArcS
in[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d))/3 -
(2*a*(8*a^2 + 39*b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)))/5)/(7
*b))/(9*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4352
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + n - 1))), x] + Simp[d^3/(b*(m + n -
1))   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 3)*Simp[a*(n - 3) +
b*(m + n - 2)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 3] && (IntegerQ[n] ||
 IntegersQ[2*m, 2*n]) &&  !IGtQ[m, 2]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1405 vs. $2(367) = 734$.

Time = 24.37 (sec) , antiderivative size = 1406, normalized size of antiderivative = 3.47

method	result	size
default	Expression too large to display	1406

input
```
int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/315/d/b^3*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+b*cos(d*x+
c)+b)*(8*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5*EllipticE(cot(d*x+c)-cs
c(d*x+c),((a-b)/(a+b))^(1/2))+8*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*
b*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+33*(cos(d*x+c)^2+2*
cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*a^3*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b)
)^(1/2))+33*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3*EllipticE(cot(d*
x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+147*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*a*b^4*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+147*(cos(d
*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^5*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)
/(a+b))^(1/2))+8*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b*EllipticF(co
t(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+2*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*a^3*b^2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+33...
```

Fricas [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c)^5 + a*sec(d*x + c)^4)*sqrt(b*sec(d*x + c) + a), x
)
```

Sympy [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (a+b\sec(c+dx))^{\frac{3}{2}}\sec^4(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**4*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**4, x)
```

Maxima [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^4, x)
```

Giac [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\cos\left(c+d\,x\right)^4}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^4,x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^4, x)
```

Reduce [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\left(\int \sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^5dx\right)b+\left(\int \sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^4dx\right)a$$

input
```
int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**5,x)*b + int(sqrt(sec(c + d*x)*
b + a)*sec(c + d*x)**4,x)*a
```

3.538 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 342

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{4a(a-b)\sqrt{a+b}(3a^2-41b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\ldots}}{105b^3d} + \frac{2(a-b)\sqrt{a+b}(6a^2+57ab-25b^2)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{\ldots}}{105b^2d} - \frac{2(6a^2-25b^2)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{105bd} - \frac{4a(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{35bd} + \frac{2(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{7bd}$$

output
```
4/105*a*(a-b)*(a+b)^(1/2)*(3*a^2-41*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x
+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)
*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d+2/105*(a-b)*(a+b)^(1/2)*(6*a^2+57*a
*b-25*b^2)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/
(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/
2)/b^2/d-2/105*(6*a^2-25*b^2)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b/d-4/35*a
*(a+b*sec(d*x+c))^(3/2)*tan(d*x+c)/b/d+2/7*(a+b*sec(d*x+c))^(5/2)*tan(d*x+
c)/b/d
```

Mathematica [A] (warning: unable to verify)

Time = 11.65 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.38

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{4\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\left(2a(3a^3+3a^2b-41ab^2-41b^3)\sqrt{\frac{\cos}{1+c}}\right.}{}$$

$$+\frac{\cos(c+dx)(a+b\sec(c+dx))^{3/2}\left(-\frac{4a(3a^2-41b^2)\sin(c+dx)}{105b^2}+\frac{2\sec(c+dx)(3a^2\sin(c+dx)+25b^2\sin(c+dx))}{105b}+\frac{16}{35}a\sec(c\right.}{d(b+a\cos(c+dx))}$$

input
```
Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(2*a*(
3*a^3 + 3*a^2*b - 41*a*b^2 - 41*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]
*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[
Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(-6*a^3 + 51*a^2*b + 82*a*b^2 + 25
*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)] + a*(3*a^2 - 41*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/
2]^2*Tan[(c + d*x)/2]))/(105*b^2*d*(b + a*Cos[c + d*x])^2*Sqrt[Sec[(c + d*
x)/2]^2]*Sec[c + d*x]^(3/2)) + (Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*((
-4*a*(3*a^2 - 41*b^2)*Sin[c + d*x])/(105*b^2) + (2*Sec[c + d*x]*(3*a^2*Sin
[c + d*x] + 25*b^2*Sin[c + d*x]))/(105*b) + (16*a*Sec[c + d*x]*Tan[c + d*x
])/35 + (2*b*Sec[c + d*x]^2*Tan[c + d*x])/7))/(d*(b + a*Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4327, 27, 3042, 4490, 27, 3042, 4490, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$\Big\downarrow$ 3042

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$\Big\downarrow$ 4327

$$\frac{2\int \frac{1}{2}\sec(c+dx)(5b-2a\sec(c+dx))(a+b\sec(c+dx))^{3/2}dx}{7b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}$$

$\Big\downarrow$ 27

$$\frac{\int \sec(c+dx)(5b-2a\sec(c+dx))(a+b\sec(c+dx))^{3/2}dx}{7b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}$$

$\Big\downarrow$ 3042

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(5b-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{7b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}$$

$\Big\downarrow$ 4490

$$\frac{\frac{2}{5}\int \frac{1}{2}\sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(19ab-\left(6a^2-25b^2\right)\sec(c+dx)\right)dx-\frac{4a\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}}{7b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{5}\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(19ab-\left(6a^2-25b^2\right)\sec(c+dx)\right)dx-\frac{4a\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}}{7b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{1}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}\left(19ab+\left(25b^2-6a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{4a\tan(c+dx)(a+b\sec(c+dx)}{5d}}{7b}}{\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}}$$

$$\Big\downarrow \; 4490$$

$$\frac{\frac{\frac{1}{5}\left(\frac{2}{3}\int \frac{\sec(c+dx)\left(b\left(51a^2+25b^2\right)-2a\left(3a^2-41b^2\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx-\frac{2\left(6a^2-25b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)-\frac{4a\tan(c+dx)(a+b\sec(c+}{5d}}{7b}}{\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{1}{5}\left(\frac{1}{3}\int \frac{\sec(c+dx)\left(b\left(51a^2+25b^2\right)-2a\left(3a^2-41b^2\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx-\frac{2\left(6a^2-25b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)-\frac{4a\tan(c+dx)(a+b\sec(c+}{5d}}{7b}}{\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{1}{5}\left(\frac{1}{3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(b\left(51a^2+25b^2\right)-2a\left(3a^2-41b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\left(6a^2-25b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)-\frac{4a\tan(c+dx)(a+}{5c}}{7b}}{\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}}$$

$$\Big\downarrow \; 4493$$

$$\frac{\frac{\frac{1}{5}\left(\frac{1}{3}\left((a-b)\left(6a^2+57ab-25b^2\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-2a\left(3a^2-41b^2\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx\right)-\frac{2\left(6a^2-25b^2\right)\text{ta}}{}\right.}{7b}}{\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{1}{5}\left(\frac{1}{3}\left((a-b)\left(6a^2+57ab-25b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-2a\left(3a^2-41b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-2\right.}{7b}}{\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}}$$

$$\downarrow \text{4319}$$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(\frac{2(a-b)\sqrt{a+b}(6a^2+57ab-25b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-2a(3a^2\right.\right.}{7b}$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}$$

$$\downarrow \text{4492}$$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(\frac{2(a-b)\sqrt{a+b}(6a^2+57ab-25b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}+\frac{4a(a-b)\sqrt{}}{}\right.\right.}{}$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}$$

input
```
Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*b*d) + ((-4*a*(a + b*Sec[c
 + d*x])^(3/2)*Tan[c + d*x])/(5*d) + (((4*a*(a - b)*Sqrt[a + b]*(3*a^2 - 41
*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))])/(b^2*d) + (2*(a - b)*Sqrt[a + b]*(6*a^2 + 57*a*b - 2
5*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/(b*d))/3 - (2*(6*a^2 - 25*b^2)*Sqrt[a + b*Sec[c + d
*x]]*Tan[c + d*x])/(3*d))/5)/(7*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4327
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 -
b^2, 0] &&  !LtQ[m, -1]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(308) = 616$.

Time = 21.10 (sec) , antiderivative size = 1028, normalized size of antiderivative = 3.01

method	result	size
default	Expression too large to display	1028

input
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105/d/b^2*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+b*cos(d*x
+c)+b)*(6*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^4*EllipticE(cot(d*x+c)-c
sc(d*x+c),((a-b)/(a+b))^(1/2))+6*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3
*b*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+82*(-cos(d*x+c)^2-
2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*a^2*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+
b))^(1/2))+82*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^3*EllipticE(cot(d
*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+6*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*a^3*b*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+51*(cos(d*
x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b^2*EllipticF(cot(d*x+c)-csc(d*x+c),((a
-b)/(a+b))^(1/2))+82*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^3*EllipticF
(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+25*(cos(d*x+c)^2+2*cos(d*x+c)+
1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*b^4*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+6*a...
```

Fricas [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c)^4 + a*sec(d*x + c)^3)*sqrt(b*sec(d*x + c) + a), x
)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int (a+b\sec(c+dx))^{\frac{3}{2}}\sec^3(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**3, x)
```

Maxima [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x)
```

Giac [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\cos(c+d\,x)^3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^3,x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^3, x)
```

Reduce [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \left(\int \sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)^4\,dx\right)b + \left(\int \sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)^3\,dx\right)a$$

input
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**4,x)*b + int(sqrt(sec(c + d*x)*
b + a)*sec(c + d*x)**3,x)*a
```

3.539 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 282

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = -\frac{2(a-b)\sqrt{a+b}(a^2+3b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{5b^2d} - \frac{2(a-3b)(a-b)\sqrt{a+b}\cot(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{5bd} + \frac{2a\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5d} + \frac{2(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d}$$

output
```
-2/5*(a-b)*(a+b)^(1/2)*(a^2+3*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(
1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(
1+sec(d*x+c))/(a-b))^(1/2)/b^2/d-2/5*(a-3*b)*(a-b)*(a+b)^(1/2)*cot(d*x+c)*
EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-se
c(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+2/5*a*(a+b*sec(
d*x+c))^(1/2)*tan(d*x+c)/d+2/5*(a+b*sec(d*x+c))^(3/2)*tan(d*x+c)/d
```

Mathematica [A] (warning: unable to verify)

Time = 10.89 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.45

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx =$$

$$-\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\left(2(a^3+a^2b+3ab^2+3b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(}{(a+b)(1+\mathrm{co}}}\right.}{}$$

$$+\frac{\cos(c+dx)(a+b\sec(c+dx))^{3/2}\left(\frac{2(a^2+3b^2)\sin(c+dx)}{5b}+\frac{4}{5}a\tan(c+dx)+\frac{2}{5}b\sec(c+dx)\tan(c+dx)\right)}{d(b+a\cos(c+dx))}$$

input
```
Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(2*(a
^3 + a^2*b + 3*a*b^2 + 3*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(
b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)] - 2*b*(a^2 + 4*a*b + 3*b^2)*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (a^2 + 3*b^2)*Co
s[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(5*b
*d*(b + a*Cos[c + d*x])^2*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(3/2)) + (
Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*((2*(a^2 + 3*b^2)*Sin[c + d*x])/(5
*b) + (4*a*Tan[c + d*x])/5 + (2*b*Sec[c + d*x]*Tan[c + d*x])/5))/(d*(b + a
*Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3042, 4322, 3042, 4490, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4322}$$

$$\frac{3}{5}\int \sec(c+dx)(b+a\sec(c+dx))\sqrt{a+b\sec(c+dx)}dx+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{4490}$$

$$\frac{3}{5}\left(\frac{2}{3}\int \frac{\sec(c+dx)\left(4ab+\left(a^2+3b^2\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx+\frac{2a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{5}\left(\frac{1}{3}\int \frac{\sec(c+dx)\left(4ab+\left(a^2+3b^2\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{2a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}\left(\frac{1}{3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(4ab+\left(a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{4493}$$

$$\frac{3}{5}\left(\frac{1}{3}\left(\left(a^2+3b^2\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx-(a-3b)(a-b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{2a\tan(c+\right.$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}\left(\frac{1}{3}\left(\left(a^2+3b^2\right)\int \frac{\csc \left(c+dx+\frac{\pi}{2}\right) \left(\csc \left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc \left(c+dx+\frac{\pi}{2}\right)}}dx-(a-3b)(a-b)\int \frac{\csc \left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc \left(c+dx+\frac{\pi}{2}\right)}}dx\right.\right.$$
$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$\Big\downarrow$ 4319

$$\frac{3}{5}\left(\frac{1}{3}\left(\left(a^2+3b^2\right)\int \frac{\csc \left(c+dx+\frac{\pi}{2}\right) \left(\csc \left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc \left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-3b)(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\cdots}{\cdots}\right.\right.$$
$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$\Big\downarrow$ 4492

$$\frac{3}{5}\left(\frac{1}{3}\left(-\frac{2(a-b)\sqrt{a+b}\left(a^2+3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}\cdots\right.\right.$$
$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

input
```
Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d) + (3*(((-2*(a - b)*Sqrt[
a + b]*(a^2 + 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*(a - 3*b)*(a - b)*Sqrt[a
 + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))])/(b*d))/3 + (2*a*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x
])/(3*d)))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4322
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[m/(m + 1)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc
[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 851 vs. $2(252) = 504$.

Time = 15.67 (sec) , antiderivative size = 852, normalized size of antiderivative = 3.02

method	result	size
default	Expression too large to display	852

input
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/d/b*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+b*cos(d*x+c)+b
)*((cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*EllipticE(cot(d*x+c)-csc(d*x+
c),((a-b)/(a+b))^(1/2))+(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b*Ellipt
icE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(3*cos(d*x+c)^2+6*cos(d*x+c
)+3)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*a*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(3
*cos(d*x+c)^2+6*cos(d*x+c)+3)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^3*EllipticE(cot(d*x+c)-csc(d*x+c),
((a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b*Elliptic
F(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(-4*cos(d*x+c)^2-8*cos(d*x+c)
-4)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*a*b^2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(-3
*cos(d*x+c)^2-6*cos(d*x+c)-3)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^3*EllipticF(cot(d*x+c)-csc(d*x+c),
((a-b)/(a+b))^(1/2))+a^3*cos(d*x+c)*sin(d*x+c)+sin(d*x+c)*(3+2*cos(d*x+c))
*a^2*b+(3*cos(d*x+c)^2+3*cos(d*x+c)+3)*a*b^2*tan(d*x+c)+b^3*(3*sin(d*x+c)+
tan(d*x+c)+sec(d*x+c)*tan(d*x+c)))
```

Fricas [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c)^3 + a*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x
)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (a+b\sec(c+dx))^{\frac{3}{2}}\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**2, x)
```

Maxima [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x)
```

Giac [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\cos(c+d\,x)^2}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^2,x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\left(\int \sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)^3\,dx\right)b+\left(\int \sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)^2\,dx\right)a$$

input
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**3,x)*b + int(sqrt(sec(c + d*x)*
b + a)*sec(c + d*x)**2,x)*a
```

3.540 $\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 249

$$\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = -\frac{8a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3bd} + \frac{2(a-b)(3a-b)\sqrt{a+b}\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3bd} + \frac{2b\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d}$$

output
```
-8/3*a*(a-b)*(a+b)^(1/2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)
^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+
c))/(a-b))^(1/2)/b/d+2/3*(a-b)*(3*a-b)*(a+b)^(1/2)*cot(d*x+c)*EllipticF((a
+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a
+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+2/3*b*(a+b*sec(d*x+c))^(1/2
)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 7.51 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.22

$$\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\,dx =$$

$$-\frac{2\sqrt{a+b\sec(c+dx)}\left(8a(a+b)\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx\right)\right.\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*Sqrt[a + b*Sec[c + d*x]]*(8*a*(a + b)*Cos[(c + d*x)/2]^2*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*(3*a^2 + 4
*a*b + b^2)*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[
(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)] - 5*a*b*Sin[c + d*x] - 2*a^2*Sin[2*(c + d*x)
] + 4*a*b*Cos[c + d*x]*Tan[(c + d*x)/2] + 4*a^2*Cos[c + d*x]^2*Tan[(c + d*
x)/2] - b^2*Tan[c + d*x]))/(3*d*(b + a*Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4317, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4317}$$

$$\frac{2}{3}\int \frac{\sec(c+dx)\left(3a^2+4b\sec(c+dx)a+b^2\right)}{2\sqrt{a+b\sec(c+dx)}}dx+\frac{2b\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{\sec(c+dx)\left(3a^2+4b\sec(c+dx)a+b^2\right)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{2b\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3a^2+4b\csc\left(c+dx+\frac{\pi}{2}\right)a+b^2\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{4493}$$

$$\frac{1}{3}\left((a-b)(3a-b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+4ab\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{2b\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left((a-b)(3a-b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+4ab\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2b\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{4319}$$

$$\frac{1}{3}\left(4ab\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)(3a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b}{}}}{bd}\right.$$
$$\frac{2b\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{4492}$$

$$\frac{1}{3}\left(\frac{2(a-b)(3a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}\right.$$
$$\frac{2b\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

input
```
Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
((-8*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (2*(a - b)*(3*a - b)*Sq
rt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
Sec[c + d*x]))/(a - b))])/(b*d))/3 + (2*b*Sqrt[a + b*Sec[c + d*x]]*Tan[c +
 d*x])/(3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4317
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Simp[1/m   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*m + a*b*(2*m - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ
[a^2 - b^2, 0] && GtQ[m, 1] && IntegerQ[2*m]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(223) = 446$.

Time = 12.27 (sec) , antiderivative size = 598, normalized size of antiderivative = 2.40

method	result
default	$\frac{2\sqrt{a+b\sec(dx+c)}\left(\left(4\cos(dx+c)^2+8\cos(dx+c)+4\right)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,a^2\,\mathrm{EllipticE}\left(\cot(dx+c)-\csc(dx+c),\sqrt{\frac{a}{a}}\right.\right.}{}$

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+b*cos(d*x+c)+b)*
((4*cos(d*x+c)^2+8*cos(d*x+c)+4)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*EllipticE(cot(d*x+c)-csc(d*x+
c),((a-b)/(a+b))^(1/2))+(4*cos(d*x+c)^2+8*cos(d*x+c)+4)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b*Ellipt
icE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(-3*cos(d*x+c)^2-6*cos(d*x+
c)-3)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*a^2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(-4
*cos(d*x+c)^2-8*cos(d*x+c)-4)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b*EllipticF(cot(d*x+c)-csc(d*x+c),
((a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^2*EllipticF(
cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+4*a^2*cos(d*x+c)*sin(d*x+c)+sin
(d*x+c)*(cos(d*x+c)+5)*a*b+b^2*(sin(d*x+c)+tan(d*x+c)))
```

Fricas [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c)^2 + a*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int (a+b\sec(c+dx))^{\frac{3}{2}}\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

Giac [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\cos(c+d\,x)}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x), x)
```

Reduce [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \left(\int \sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^{2}dx\right)b + \left(\int \sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)dx\right)a$$

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**2,x)*b + int(sqrt(sec(c + d*x)*
b + a)*sec(c + d*x),x)*a
```

3.541 $\int (a+b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 309

$$\int (a+b\sec(c+dx))^{3/2}\,dx = -\frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d} + \frac{2(2a-b)\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d} - \frac{2a\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d}$$

output
```
-2*(a-b)*(a+b)^(1/2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/
2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/
(a-b))^(1/2)/d+2*(2*a-b)*(a+b)^(1/2)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))
^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b
*(1+sec(d*x+c))/(a-b))^(1/2)/d-2*a*(a+b)^(1/2)*cot(d*x+c)*EllipticPi((a+b*
sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c
))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 684 vs. $2(309) = 618$.

Time = 16.59 (sec) , antiderivative size = 684, normalized size of antiderivative = 2.21

$$\int (a + b\sec(c + dx))^{3/2}\,dx = \frac{2b\cos(c+dx)(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{d(b+a\cos(c+dx))} + \frac{2(a+b\sec(c+dx))^{3/2}\left(ab\tan\left(\frac{1}{2}(c+dx)\right)+b^2\tan\left(\frac{1}{2}(c+dx)\right)-2ab\tan^3\left(\frac{1}{2}(c+dx)\right)+ab\tan^5\left(\frac{1}{2}(c+\right.\right.}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*b*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(d*(b + a*Cos[c
 + d*x])) + (2*(a + b*Sec[c + d*x])^(3/2)*(a*b*Tan[(c + d*x)/2] + b^2*Tan[
(c + d*x)/2] - 2*a*b*Tan[(c + d*x)/2]^3 + a*b*Tan[(c + d*x)/2]^5 - b^2*Tan
[(c + d*x)/2]^5 - 2*a^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + b*(a +
b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c +
d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b
*Tan[(c + d*x)/2]^2)/(a + b)] + (a^2 - 2*a*b - b^2)*EllipticF[ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c +
d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
 b)]))/(d*(b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[(1 - Tan[(c +
 d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)
*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c +
d*x)/2]^2)])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4268, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\,dx$$

$$\Big\downarrow \; 4268$$

$$\int \frac{a^2+(2a-b)b\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+b^2\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{a^2+(2a-b)b\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4409$$

$$a^2\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx+b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(2a-b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$a^2\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(2a-b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4271$$

$$b^2 \int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx + b(2a-b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx - \frac{2a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}$$

$$\Big\downarrow \text{4319}$$

$$b^2 \int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx + \frac{2(2a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d} - \frac{2a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}$$

$$\Big\downarrow \text{4492}$$

$$\frac{2(2a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d} - \frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d} - \frac{2a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}$$

input
```
Int[(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*(2*a - b)*Sqrt[a + b]*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
 - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/d - (2*a*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4268
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(3/2), x_Symbol] :> Int[(a^2 + b
*(2*a - b)*Csc[c + d*x])/Sqrt[a + b*Csc[c + d*x]], x] + Simp[b^2   Int[Csc[
c + d*x]*((1 + Csc[c + d*x])/Sqrt[a + b*Csc[c + d*x]]), x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 634 vs. 2(282) = 564.

Time = 8.13 (sec) , antiderivative size = 635, normalized size of antiderivative = 2.06

method	result
default	$\frac{(\cos(dx+c)+1)^2\left(\left((1-\cos(dx+c))^3\csc(dx+c)^3-\csc(dx+c)+\cot(dx+c)\right)ab+\left(-(1-\cos(dx+c))^3\csc(dx+c)^3-\csc(dx+c)+\cot(dx+\right.\right.}{}$

input
```
int((a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d/(b+a*cos(d*x+c))*(cos(d*x+c)+1)^2*(((1-cos(d*x+c))^3*csc(d*x+c)^3-cs
c(d*x+c)+cot(d*x+c))*a*b+(-(1-cos(d*x+c))^3*csc(d*x+c)^3-csc(d*x+c)+cot(d*
x+c))*b^2-2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a
^2+4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a*b+2*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*b^2-2*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a*b-2*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*b^2+4*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(
d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2)*((1-cos(d*x+c))^2*csc(d*x+c
)^2-1)*(a+b*sec(d*x+c))^(1/2)*sec(d*x+c)
```

Fricas [F]

$$\int (a+b\sec(c+dx))^{3/2}\,dx=\int (b\sec(dx+c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(3/2), x)
```

Sympy [F]

$$\int (a+b\sec(c+dx))^{3/2}\,dx=\int (a+b\sec{(c+dx)})^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int (a+b\sec(c+dx))^{3/2}\,dx=\int (b\sec{(dx+c)}+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int (a+b\sec(c+dx))^{3/2}\,dx=\int (b\sec{(dx+c)}+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^{3/2}\,dx=\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int (a+b\sec(c+dx))^{3/2}\,dx=\left(\int \sqrt{\sec\left(dx+c\right)b+a}dx\right)a+\left(\int \sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)dx\right)b$$

input
```
int((a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a),x)*a + int(sqrt(sec(c + d*x)*b + a)*sec(c + d
*x),x)*b
```

3.542 $\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 334

$$\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{bd}$$
$$+\frac{\sqrt{a+b}(a+2b)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d}$$
$$-\frac{3b\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d}$$
$$+\frac{a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
a*(a-b)*(a+b)^(1/2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2
),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(
a-b))^(1/2)/b/d+(a+b)^(1/2)*(a+2*b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^
(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*
(1+sec(d*x+c))/(a-b))^(1/2)/d-3*b*(a+b)^(1/2)*cot(d*x+c)*EllipticPi((a+b*s
ec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c)
)/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d+a*(a+b*sec(d*x+c))^(1/2)*
sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 8.50 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.99

$$\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\cos(c+dx)(a+b\sec(c+dx))^{3/2}\left(2a(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\right.}{}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(Cos[(c + d*x)/2]^2*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*(2*a*(a + b)*S
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] -
4*(2*a - b)*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*
x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)] + 12*a*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)] + a*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2]))/(d*(b + a*Cos[c + d*x])^2)
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4351, 25, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4351}$$

$$\frac{a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}-\frac{1}{2}\int -\frac{2\sec(c+dx)b^2-a\sec^2(c+dx)b+3ab}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{2\sec(c+dx)b^2-a\sec^2(c+dx)b+3ab}{\sqrt{a+b\sec(c+dx)}}dx+\frac{a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{2\csc\left(c+dx+\frac{\pi}{2}\right)b^2-a\csc\left(c+dx+\frac{\pi}{2}\right)^2b+3ab}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4546}$$

$$\frac{1}{2}\left(\int \frac{3ab+\left(2b^2+ab\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-ab\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \frac{3ab+\left(2b^2+ab\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-ab\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4409}$$

$$\frac{1}{2}\left(-ab\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+3ab\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx+b(a+2b)\int \frac{\sec(c+}{\sqrt{a+b\sec(}}\right.$$
$$\frac{a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(3ab\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(a+2b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-ab\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c\right.\right.}{\sqrt{a+b\csc\left(c+\right.}}\right.$$
$$\frac{a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4271}$$

$$\frac{1}{2}\left(b(a+2b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-ab\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6b\sqrt{a+b}\cot(c\right.$$
$$\frac{a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4319}$$

$$\frac{1}{2}\left(-ab\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}(a+2b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+}{a-}}}{d}\right.$$
$$\frac{a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4492}$$

$$\frac{1}{2}\left(\frac{2\sqrt{a+b}(a+2b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}+\frac{2a(}{}\right.$$
$$\frac{a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

input
```
Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
((2*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
 d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (2*Sqrt[a + b]*(a + 2*b)
*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
 b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d
*x]))/(a - b))])/d - (6*b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, A
rcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d)/2 + (
a*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4351
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*C
sc[e + f*x])^n/(f*n)), x] + Simp[1/(2*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/
Sqrt[a + b*Csc[e + f*x]])*Simp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[
e + f*x] + a*b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f
}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 8.38 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.73

method	result
default	$\frac{\left(\left(-6\cos(dx+c)^2-12\cos(dx+c)-6\right)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,ab\,\mathrm{EllipticPi}\left(\cot(dx+c)-\csc(dx+c),-1,\sqrt{\frac{a-b}{a+b}}\right)+\left(-\mathrm{co}\right.\right.}{}$

input
```
int(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*((-6*cos(d*x+c)^2-12*cos(d*x+c)-6)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b*EllipticPi(cot(d*x+c)-c
sc(d*x+c),-1,((a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a
^2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*c
os(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*a*b*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/
2))+(4*cos(d*x+c)^2+8*cos(d*x+c)+4)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b*EllipticF(cot(d*x+c)-csc(d
*x+c),((a-b)/(a+b))^(1/2))+(-2*cos(d*x+c)^2-4*cos(d*x+c)-2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^2*El
lipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+a^2*cos(d*x+c)^2*sin(d*
x+c)+a*b*cos(d*x+c)*sin(d*x+c))*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*c
os(d*x+c)+b*cos(d*x+c)+b)
```

Fricas [F]

$$\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c)*sec(d*x + c) + a*cos(d*x + c))*sqrt(b*sec(d*x + c
) + a), x)
```

Sympy [F]

$$\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (a+b\sec(c+dx))^{\frac{3}{2}}\cos(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)*cos(c + d*x), x)
```

Maxima [F]

$$\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int \cos(c+d\,x)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

input
```
int(cos(c + d*x)*(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)*(a + b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \left(\int \sqrt{\sec(dx+c)\,b+a}\,\cos(dx+c)\sec(dx+c)\,dx\right)b + \left(\int \sqrt{\sec(dx+c)\,b+a}\,\cos(dx+c)\,dx\right)a$$

input
```
int(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x),x)*b + int(sqrt(sec
(c + d*x)*b + a)*cos(c + d*x),x)*a
```

3.543 $\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 390

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{5(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4d} + \frac{\sqrt{a+b}(2a+5b)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4d} - \frac{\sqrt{a+b}(4a^2+3b^2)\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4ad} + \frac{5b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{a\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d}$$

output
```
5/4*(a-b)*(a+b)^(1/2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1
/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))
/(a-b))^(1/2)/d+1/4*(a+b)^(1/2)*(2*a+5*b)*cot(d*x+c)*EllipticF((a+b*sec(d*
x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2
)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d-1/4*(a+b)^(1/2)*(4*a^2+3*b^2)*cot(d*x+
c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/
2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+5/4
*b*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d+1/2*a*cos(d*x+c)*(a+b*sec(d*x+c))^(
1/2)*sin(d*x+c)/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 891 vs. $2(390) = 780$.

Time = 16.26 (sec) , antiderivative size = 891, normalized size of antiderivative = 2.28

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(a*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Sin[2*(c + d*x)])/(4*d*(b + a*C
os[c + d*x])) - ((a + b*Sec[c + d*x])^(3/2)*(5*a*b*Tan[(c + d*x)/2] + 5*b^
2*Tan[(c + d*x)/2] - 10*a*b*Tan[(c + d*x)/2]^3 + 5*a*b*Tan[(c + d*x)/2]^5
- 5*b^2*Tan[(c + d*x)/2]^5 + 8*a^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d
*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*b^2*EllipticPi[-1, ArcSin[Ta
n[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
 - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*a^2*EllipticP
i[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x
)/2]^2)/(a + b)] + 6*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/
(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*T
an[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 5*b*(a + b)*EllipticE
[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(
1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*
x)/2]^2)/(a + b)] - 2*(2*a^2 - a*b + 4*b^2)*EllipticF[ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^
2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(
4*d*(b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[(1 - Tan[(c + d*x)/
2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sq...
```

Rubi [A] (verified)

Time = 1.67 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {3042, 4351, 25, 3042, 4592, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \ 4351$$

$$\frac{a\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}-\frac{1}{4}\int -\frac{\cos(c+dx)\left(ab\sec^2(c+dx)+2\left(a^2+2b^2\right)\sec(c+dx)+5ab\right)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \ 25$$

$$\frac{1}{4}\int \frac{\cos(c+dx)\left(ab\sec^2(c+dx)+2\left(a^2+2b^2\right)\sec(c+dx)+5ab\right)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{a\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{4}\int \frac{ab\csc\left(c+dx+\frac{\pi}{2}\right)^2+2\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+5ab}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \ 4592$$

$$\frac{1}{4}\left(\frac{5b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int -\frac{2b\sec(c+dx)a^2-5b^2\sec^2(c+dx)a+\left(4a^2+3b^2\right)a}{2\sqrt{a+b\sec(c+dx)}}dx}{a}\right)+\frac{a\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}\left(\frac{\int \frac{2b\sec(c+dx)a^2-5b^2\sec^2(c+dx)a+(4a^2+3b^2)a}{\sqrt{a+b\sec(c+dx)}}dx}{2a}+\frac{5b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}\right)+\frac{a\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(\frac{\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2-5b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a+(4a^2+3b^2)a}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{5b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}\right)+\frac{a\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 4546$$

$$\frac{1}{4}\left(\frac{\int \frac{a(4a^2+3b^2)+(2ba^2+5b^2a)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-5ab^2\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx}{2a}+\frac{5b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}\right)+\frac{a\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(\frac{\int \frac{a(4a^2+3b^2)+(2ba^2+5b^2a)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-5ab^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{5b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}\right)+\frac{a\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 4409$$

$$\frac{1}{4}\left(\frac{a(4a^2+3b^2)\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx-5ab^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+ab(2a+5b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{2a}+\frac{5}{}\right.$$

$$\frac{a\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(\frac{a\left(4a^2+3b^2\right)\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-5ab^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+ab(2a+5b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}\right.}{2a}\right.$$

$$\frac{a\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 4271

$$\frac{1}{4}\left(\frac{-5ab^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+ab(2a+5b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(4a^2+3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(\ldots}{a+\ldots}}}{}}{2a}\right.$$

$$\frac{a\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 4319

$$\frac{1}{4}\left(\frac{-5ab^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(4a^2+3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\text{arcsi}\ldots\right.}{d}}{2a}\right.$$

$$\frac{a\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 4492

$$\frac{1}{4}\left(\frac{-\frac{2\sqrt{a+b}\left(4a^2+3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}+\frac{2a\sqrt{a+b}(2a+5b)\text{co}\ldots}{}}{}\right.$$

$$\frac{a\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(a*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (((10*a*(a
- b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*a*Sqrt[a + b]*(2*a + 5*b)*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
- b))])/d - (2*Sqrt[a + b]*(4*a^2 + 3*b^2)*Cot[c + d*x]*EllipticPi[(a + b)
/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d)/
(2*a) + (5*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d)/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4351
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*C
sc[e + f*x])^n/(f*n)), x] + Simp[1/(2*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/
Sqrt[a + b*Csc[e + f*x]])*Simp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[
e + f*x] + a*b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f
}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. 2(349) = 698.

Time = 6.95 (sec) , antiderivative size = 819, normalized size of antiderivative = 2.10

method	result	size
default	Expression too large to display	819

input
```
int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*((-8*cos(d*x+c)^2-16*cos(d*x+c)-8)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*EllipticPi(cot(d*x+c)
-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))+(-6*cos(d*x+c)^2-12*cos(d*x+c)-6)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*b^2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))+(-5*cos(d
*x+c)^2-10*cos(d*x+c)-5)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b
)/(a+b))^(1/2))+(-5*cos(d*x+c)^2-10*cos(d*x+c)-5)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^2*EllipticE(co
t(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(4*cos(d*x+c)^2+8*cos(d*x+c)+4)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*a^2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(-2*cos(d*
x+c)^2-4*cos(d*x+c)-2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/
(a+b))^(1/2))+(8*cos(d*x+c)^2+16*cos(d*x+c)+8)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^2*EllipticF(cot(d
*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+sin(d*x+c)*cos(d*x+c)^2*(2*cos(d*x+c
)+2)*a^2+sin(d*x+c)*cos(d*x+c)*(7*cos(d*x+c)+2)*a*b+5*b^2*cos(d*x+c)*sin(d
*x+c))*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+b*cos(d*x+c)+b)
```

Fricas [F]

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c)^2*sec(d*x + c) + a*cos(d*x + c)^2)*sqrt(b*sec(d*x
 + c) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x)
```

Giac [F]

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int \cos\left(c+d\,x\right)^2\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

input
```
int(cos(c + d*x)^2*(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^2*(a + b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\left(\int \sqrt{\sec\left(dx+c\right)b+a}\,\cos\left(dx+c\right)^2\sec\left(dx+c\right)dx\right)b+\left(\int \sqrt{\sec\left(dx+c\right)b+a}\,\cos\left(dx+c\right)^2dx\right)a$$

input
```
int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*cos(c + d*x)**2*sec(c + d*x),x)*b + int(sqrt(
sec(c + d*x)*b + a)*cos(c + d*x)**2,x)*a
```

3.544 $\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 463

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx =$$

$$-\frac{2a(a-b)\sqrt{a+b}(8a^4+51a^2b^2+741b^4)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(}{}}}{693b^4d}$$

$$-\frac{2(a-b)\sqrt{a+b}(8a^4+6a^3b+57a^2b^2-606ab^3+135b^4)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a-}{a-}\right.}{693b^3d}$$

$$+\frac{2(8a^4+57a^2b^2+135b^4)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{693b^2d}$$

$$+\frac{2a(8a^2+67b^2)(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{693b^2d}$$

$$+\frac{2(8a^2+81b^2)(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{693b^2d}$$

$$-\frac{8a(a+b\sec(c+dx))^{7/2}\tan(c+dx)}{99b^2d}+\frac{2\sec(c+dx)(a+b\sec(c+dx))^{7/2}\tan(c+dx)}{11bd}$$

output
```
-2/693*a*(a-b)*(a+b)^(1/2)*(8*a^4+51*a^2*b^2+741*b^4)*cot(d*x+c)*EllipticE
((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))
/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d-2/693*(a-b)*(a+b)^(1/2
)*(8*a^4+6*a^3*b+57*a^2*b^2-606*a*b^3+135*b^4)*cot(d*x+c)*EllipticF((a+b*s
ec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))
^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d+2/693*(8*a^4+57*a^2*b^2+135*b
^4)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^2/d+2/693*a*(8*a^2+67*b^2)*(a+b*se
c(d*x+c))^(3/2)*tan(d*x+c)/b^2/d+2/693*(8*a^2+81*b^2)*(a+b*sec(d*x+c))^(5/
2)*tan(d*x+c)/b^2/d-8/99*a*(a+b*sec(d*x+c))^(7/2)*tan(d*x+c)/b^2/d+2/11*se
c(d*x+c)*(a+b*sec(d*x+c))^(7/2)*tan(d*x+c)/b/d
```

Mathematica [A] (warning: unable to verify)

Time = 13.79 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.33

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx =$$

$$-\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\left(2a(8a^5+8a^4b+51a^3b^2+51a^2b^3+741ab^4+741\right.}{}$$

$$+\frac{\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{2a(8a^4+51a^2b^2+741b^4)\sin(c+dx)}{693b^3}+\frac{2}{693}\sec^3(c+dx)\left(113a^2\sin(c+dx)+81\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(2*a*
(8*a^5 + 8*a^4*b + 51*a^3*b^2 + 51*a^2*b^3 + 741*a*b^4 + 741*b^5)*Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(8*
a^5 + 2*a^4*b + 51*a^3*b^2 + 663*a^2*b^3 + 741*a*b^4 + 135*b^5)*Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + a*(8*a^4
+ 51*a^2*b^2 + 741*b^4)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2]))/(693*b^3*d*(b + a*Cos[c + d*x])^3*Sqrt[Sec[(c + d*x)
/2]^2]*Sec[c + d*x]^(5/2)) + (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((
2*a*(8*a^4 + 51*a^2*b^2 + 741*b^4)*Sin[c + d*x])/(693*b^3) + (2*Sec[c + d*
x]^3*(113*a^2*Sin[c + d*x] + 81*b^2*Sin[c + d*x]))/693 + (2*Sec[c + d*x]^2
*(3*a^3*Sin[c + d*x] + 229*a*b^2*Sin[c + d*x]))/(693*b) + (2*Sec[c + d*x]*
(-4*a^4*Sin[c + d*x] + 205*a^2*b^2*Sin[c + d*x] + 135*b^4*Sin[c + d*x]))/(
693*b^2) + (46*a*b*Sec[c + d*x]^3*Tan[c + d*x])/99 + (2*b^2*Sec[c + d*x]^4
*Tan[c + d*x])/11))/(d*(b + a*Cos[c + d*x])^2)
```

Rubi [A] (verified)

Time = 2.26 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.04, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4352, 27, 3042, 4570, 27, 3042, 4490, 27, 3042, 4490, 27, 3042, 4490, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{ 4352}$$

$$\frac{2\int \frac{1}{2}\sec(c+dx)(a+b\sec(c+dx))^{5/2}\left(-4a\sec^2(c+dx)+9b\sec(c+dx)+2a\right)dx}{11b}+\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\left(-4a\sec^2(c+dx)+9b\sec(c+dx)+2a\right)dx}{11b}+\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(-4a\csc\left(c+dx+\frac{\pi}{2}\right)^2+9b\csc\left(c+dx+\frac{\pi}{2}\right)+2a\right)dx}{11b}+\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \; 4570$$

$$\frac{\frac{2\int -\frac{1}{2}\sec(c+dx)(a+b\sec(c+dx))^{5/2}\left(10ab-\left(8a^2+81b^2\right)\sec(c+dx)\right)dx}{9b}-\frac{8a\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}}{11b}+\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\left(10ab-\left(8a^2+81b^2\right)\sec(c+dx)\right)dx}{9b}-\frac{8a\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}}{11b}+\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(10ab+\left(-8a^2-81b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{9b}-\frac{8a\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}}{11b}+\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \; 4490$$

$$\frac{-\frac{\frac{2}{7}\int \frac{5}{2}\sec(c+dx)(a+b\sec(c+dx))^{3/2}\left(3b\left(2a^2-27b^2\right)-a\left(8a^2+67b^2\right)\sec(c+dx)\right)dx-\frac{2\left(8a^2+81b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}}{9b}-\frac{8a\tan(c+dx)(}{}}{11b}+\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{-\frac{\frac{5}{7}\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\left(3b\left(2a^2-27b^2\right)-a\left(8a^2+67b^2\right)\sec(c+dx)\right)dx-\frac{2\left(8a^2+81b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}}{9b}-\frac{8a\tan(c+dx)(a-}{9}}{11b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$\downarrow$ 3042

$$-\frac{\frac{-\frac{\frac{5}{7}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(3b\left(2a^2-27b^2\right)-a\left(8a^2+67b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{2\left(8a^2+81b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}}{9b}-8a\,t}{11b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$\downarrow$ 4490

$$-\frac{\frac{-\frac{\frac{5}{7}\left(\frac{2}{5}\int \frac{3}{2}\sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(2ab\left(a^2-101b^2\right)-\left(8a^4+57b^2a^2+135b^4\right)\sec(c+dx)\right)dx-\frac{2a\left(8a^2+67b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}\right)-\frac{2(8}{}}{9b}}{11b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$\downarrow$ 27

$$-\frac{\frac{-\frac{\frac{5}{7}\left(\frac{3}{5}\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(2ab\left(a^2-101b^2\right)-\left(8a^4+57b^2a^2+135b^4\right)\sec(c+dx)\right)dx-\frac{2a\left(8a^2+67b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}\right)-\frac{2(8a^2}{}}{9b}}{11b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$\downarrow$ 3042

$$-\frac{\frac{-\frac{\frac{5}{7}\left(\frac{3}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}\left(2ab\left(a^2-101b^2\right)+\left(-8a^4-57b^2a^2-135b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{2a\left(8a^2+67b^2\right)\tan(c+dx)(a+b\sec(c+dx}{5d}}{9b}}{11b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$\downarrow$ 4490

$$-\frac{\frac{5}{7}\left(\frac{3}{5}\left(\frac{\frac{2}{3}\int -\frac{\sec(c+dx)\left(b\left(2a^4+663b^2a^2+135b^4\right)+a\left(8a^4+51b^2a^2+741b^4\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx-\frac{2\left(8a^4+57a^2b^2+135b^4\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}}{9b}\right)-\frac{2a\left(8a^2+\right.}{}\right.}{11b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$\Big\downarrow$ 27

$$-\frac{\frac{5}{7}\left(\frac{3}{5}\left(\frac{-\frac{1}{3}\int \frac{\sec(c+dx)\left(b\left(2a^4+663b^2a^2+135b^4\right)+a\left(8a^4+51b^2a^2+741b^4\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx-\frac{2\left(8a^4+57a^2b^2+135b^4\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}}{9b}\right)-\frac{2a\left(8a^2+\right.}{}\right.}{11b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{5}{7}\left(\frac{3}{5}\left(\frac{-\frac{1}{3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(b\left(2a^4+663b^2a^2+135b^4\right)+a\left(8a^4+51b^2a^2+741b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\left(8a^4+57a^2b^2+135b^4\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}}{9b}\right)-\right.}{11b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$\Big\downarrow$ 4493

$$-\frac{\frac{5}{7}\left(\frac{3}{5}\left(\frac{\frac{1}{3}\left((a-b)(8a^4+6a^3b+57a^2b^2-606ab^3+135b^4)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-a(8a^4+51a^2b^2+741b^4)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx\right)-\frac{2\left(8a^4+57a^2\right.}{}}{9b}\right.\right.}{}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{5}{7}\left(\frac{3}{5}\left(\frac{\frac{1}{3}\left((a-b)(8a^4+6a^3b+57a^2b^2-606ab^3+135b^4)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-a(8a^4+51a^2b^2+741b^4)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{9b}\right.\right.}{}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$\Big\downarrow$ 4319

$$-\frac{\frac{5}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{2(a-b)\sqrt{a+b}\left(8a^4+6a^3b+57a^2b^2-606ab^3+135b^4\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd} -\right.\right.\right.}{}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$\Big\downarrow$ 4492

$$-\frac{\frac{5}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{2a(a-b)\sqrt{a+b}\left(8a^4+51a^2b^2+741b^4\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}+\frac{2(a-b)\sqrt{a+b}\left(8a^4+\right.}{}\right.\right.\right.}{}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

input
```
Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(2*Sec[c + d*x]*(a + b*Sec[c + d*x])^(7/2)*Tan[c + d*x])/(11*b*d) + ((-8*a
*(a + b*Sec[c + d*x])^(7/2)*Tan[c + d*x])/(9*b*d) - ((-2*(8*a^2 + 81*b^2)*
(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*d) + (5*((-2*a*(8*a^2 + 67*b^2
)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d) + (3*(((2*a*(a - b)*Sqrt[
a + b]*(8*a^4 + 51*a^2*b^2 + 741*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) + (2*(a - b)
*Sqrt[a + b]*(8*a^4 + 6*a^3*b + 57*a^2*b^2 - 606*a*b^3 + 135*b^4)*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
- b))])/(b*d))/3 - (2*(8*a^4 + 57*a^2*b^2 + 135*b^4)*Sqrt[a + b*Sec[c + d*
x]]*Tan[c + d*x])/(3*d)))/5))/7)/(9*b))/(11*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4352
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + n - 1))), x] + Simp[d^3/(b*(m + n -
1))   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 3)*Simp[a*(n - 3) +
b*(m + n - 2)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 3] && (IntegerQ[n] ||
 IntegersQ[2*m, 2*n]) &&  !IGtQ[m, 2]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1589 vs. $2(421) = 842$.

Time = 73.26 (sec) , antiderivative size = 1590, normalized size of antiderivative = 3.43

method	result	size
default	Expression too large to display	1590

input
```
int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/693/d/b^3*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+b*cos(d*x
+c)+b)*(8*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^6*EllipticE(cot(d*x+c)-
csc(d*x+c),((a-b)/(a+b))^(1/2))+8*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a
^5*b*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+51*(-cos(d*x+c)^
2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*a^4*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(
a+b))^(1/2))+51*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3*b^3*EllipticE(c
ot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+741*(-cos(d*x+c)^2-2*cos(d*x+c)-
1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*a^2*b^4*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+74
1*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^5*EllipticE(cot(d*x+c)-csc(d*
x+c),((a-b)/(a+b))^(1/2))+8*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^5*b*El
lipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+2*(cos(d*x+c)^2+2*cos(d
*x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*a^4*b^2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^...
```

Fricas [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx=\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^6 + 2*a*b*sec(d*x + c)^5 + a^2*sec(d*x + c)^4)*
sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**4*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\cos\left(c+d\,x\right)^4}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^4,x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^4, x)
```

Reduce [F]

$$\begin{aligned}\int \sec^4(c+dx)(a & \\ +b\sec(c+dx))^{5/2}\,dx &= \left(\int \sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^6 dx\right)b^2 \\ &+ 2\left(\int \sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^5 dx\right)ab \\ &+ \left(\int \sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^4 dx\right)a^2\end{aligned}$$

input
```
int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**6,x)*b**2 + 2*int(sqrt(sec(c +
d*x)*b + a)*sec(c + d*x)**5,x)*a*b + int(sqrt(sec(c + d*x)*b + a)*sec(c +
d*x)**4,x)*a**2
```

3.545 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 399

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{2(a-b)\sqrt{a+b}(10a^4-279a^2b^2-147b^4)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{se}}{}}}{315b^3d}$$
$$+\frac{2(a-b)\sqrt{a+b}(10a^3+165a^2b-114ab^2+147b^3)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{}}{315b^2d}$$
$$-\frac{4a(5a^2-57b^2)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{315bd}$$
$$-\frac{2(10a^2-49b^2)(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{315bd}$$
$$-\frac{4a(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{63bd}+\frac{2(a+b\sec(c+dx))^{7/2}\tan(c+dx)}{9bd}$$

output
```
2/315*(a-b)*(a+b)^(1/2)*(10*a^4-279*a^2*b^2-147*b^4)*cot(d*x+c)*EllipticE(
(a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/
(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d+2/315*(a-b)*(a+b)^(1/2)
*(10*a^3+165*a^2*b-114*a*b^2+147*b^3)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c)
)^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-
b*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d-4/315*a*(5*a^2-57*b^2)*(a+b*sec(d*x+c)
)^(1/2)*tan(d*x+c)/b/d-2/315*(10*a^2-49*b^2)*(a+b*sec(d*x+c))^(3/2)*tan(d*
x+c)/b/d-4/63*a*(a+b*sec(d*x+c))^(5/2)*tan(d*x+c)/b/d+2/9*(a+b*sec(d*x+c))
^(7/2)*tan(d*x+c)/b/d
```

Mathematica [A] (warning: unable to verify)

Time = 13.14 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.38

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\left(2(10a^5+10a^4b-279a^3b^2-279a^2b^3\right.}{}$$

$$+\frac{\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{2(-10a^4+279a^2b^2+147b^4)\sin(c+dx)}{315b^2}+\frac{2}{315}\sec^2(c+dx)\left(75a^2\sin(c+dx)+49\right.\right.}{d(b}$$

input
```
Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(2*(10
*a^5 + 10*a^4*b - 279*a^3*b^2 - 279*a^2*b^3 - 147*a*b^4 - 147*b^5)*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos
[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(-
10*a^4 + 155*a^3*b + 279*a^2*b^2 + 261*a*b^3 + 147*b^4)*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (10*a^4 - 279*a^2
*b^2 - 147*b^4)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2]))/(315*b^2*d*(b + a*Cos[c + d*x])^3*Sqrt[Sec[(c + d*x)/2]^2]*S
ec[c + d*x]^(5/2)) + (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((2*(-10*a
^4 + 279*a^2*b^2 + 147*b^4)*Sin[c + d*x])/(315*b^2) + (2*Sec[c + d*x]^2*(7
5*a^2*Sin[c + d*x] + 49*b^2*Sin[c + d*x]))/315 + (2*Sec[c + d*x]*(5*a^3*Si
n[c + d*x] + 163*a*b^2*Sin[c + d*x]))/(315*b) + (38*a*b*Sec[c + d*x]^2*Tan
[c + d*x])/63 + (2*b^2*Sec[c + d*x]^3*Tan[c + d*x])/9))/(d*(b + a*Cos[c +
d*x])^2)
```

Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.03, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4327, 27, 3042, 4490, 27, 3042, 4490, 27, 3042, 4490, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{ 4327}$$

$$\frac{2\int \frac{1}{2}\sec(c+dx)(7b-2a\sec(c+dx))(a+b\sec(c+dx))^{5/2}dx}{9b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \sec(c+dx)(7b-2a\sec(c+dx))(a+b\sec(c+dx))^{5/2}dx}{9b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(7b-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{9b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{4490}$$

$$\frac{\frac{2}{7}\int \frac{1}{2}\sec(c+dx)(a+b\sec(c+dx))^{3/2}\left(39ab-\left(10a^2-49b^2\right)\sec(c+dx)\right)dx-\frac{4a\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}}{9b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{7}\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\left(39ab-\left(10a^2-49b^2\right)\sec(c+dx)\right)dx-\frac{4a\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}}{9b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{7}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(39ab+\left(49b^2-10a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{4a\tan(c+dx)(a+b\sec}{7d}}{9b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{4490}$$

$$\frac{\frac{1}{7}\left(\frac{2}{5}\int \frac{3}{2}\sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(b\left(55a^2+49b^2\right)-2a\left(5a^2-57b^2\right)\sec(c+dx)\right)dx-\frac{2\left(10a^2-49b^2\right)\tan(c+dx)}{5d}\right.}{9b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{7}\left(\frac{3}{5}\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(b\left(55a^2+49b^2\right)-2a\left(5a^2-57b^2\right)\sec(c+dx)\right)dx-\frac{2\left(10a^2-49b^2\right)\tan(c+dx)(a}{5d}}{9b}$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}$$

$\downarrow$ 3042

$$\frac{\frac{1}{7}\left(\frac{3}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b\left(55a^2+49b^2\right)-2a\left(5a^2-57b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{2(10a^2}{}}{9b}$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}$$

$\downarrow$ 4490

$$\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{2}{3}\int \frac{\sec(c+dx)\left(ab\left(155a^2+261b^2\right)-\left(10a^4-279b^2a^2-147b^4\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx-\frac{4a\left(5a^2-57b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)-\frac{2(10a^2}{}}{9b}$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}$$

$\downarrow$ 27

$$\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\int \frac{\sec(c+dx)\left(ab\left(155a^2+261b^2\right)-\left(10a^4-279b^2a^2-147b^4\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx-\frac{4a\left(5a^2-57b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)-\frac{2(10a^2}{}}{9b}$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}$$

$\downarrow$ 3042

$$\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(ab\left(155a^2+261b^2\right)+\left(-10a^4+279b^2a^2+147b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{4a\left(5a^2-57b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)}{9b}$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}$$

$\downarrow$ 4493

$$\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left((a-b)\left(10a^3+165a^2b-114ab^2+147b^3\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-\left(10a^4-279a^2b^2-147b^4\right)\int \frac{\sec(c+dx)(\sec}{\sqrt{a+b\sec}}\right.\right.\right.$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left((a-b)\left(10a^3+165a^2b-114ab^2+147b^3\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\left(10a^4-279a^2b^2-147b^4\right)\int \frac{\csc(c+dx}{\sqrt{}}\right.\right.\right.}{\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}}$$

$$\Big\downarrow \text{4319}$$

$$\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{2(a-b)\sqrt{a+b}(10a^3+165a^2b-114ab^2+147b^3)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a}{a}\right.}{bd}\right.\right.\right.}{\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}}$$

$$\Big\downarrow \text{4492}$$

$$\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{2(a-b)\sqrt{a+b}(10a^4-279a^2b^2-147b^4)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}+\frac{2(a-b)}{}\right.\right.\right.}{\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}}$$

input
```
Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(2*(a + b*Sec[c + d*x])^(7/2)*Tan[c + d*x])/(9*b*d) + ((-4*a*(a + b*Sec[c
+ d*x])^(5/2)*Tan[c + d*x])/(7*d) + ((-2*(10*a^2 - 49*b^2)*(a + b*Sec[c +
d*x])^(3/2)*Tan[c + d*x])/(5*d) + (3*(((2*(a - b)*Sqrt[a + b]*(10*a^4 - 27
9*a^2*b^2 - 147*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) + (2*(a - b)*Sqrt[a + b]*(10*
a^3 + 165*a^2*b - 114*a*b^2 + 147*b^3)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d))/3 - (4*a*(5*
a^2 - 57*b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)))/5)/7)/(9*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4327
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 -
b^2, 0] &&  !LtQ[m, -1]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1405 vs. $2(361) = 722$.

Time = 40.95 (sec) , antiderivative size = 1406, normalized size of antiderivative = 3.52

method	result	size
default	Expression too large to display	1406

input
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/315/d/b^2*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+b*cos(d*x+
c)+b)*(10*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^5*EllipticE(cot(d*x+c)-
csc(d*x+c),((a-b)/(a+b))^(1/2))+10*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
a^4*b*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+279*(cos(d*x+c)
^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/
(a+b))^(1/2))+279*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3*EllipticE(
cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+147*(cos(d*x+c)^2+2*cos(d*x+c)+
1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*a*b^4*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+147*
(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^5*EllipticE(cot(d*x+c)-csc(d*x+c),
((a-b)/(a+b))^(1/2))+10*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^4*b*Ellipt
icF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+155*(-cos(d*x+c)^2-2*cos(d*
x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*a^3*b^2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(...
```

Fricas [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^5 + 2*a*b*sec(d*x + c)^4 + a^2*sec(d*x + c)^3)*
sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx=\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\cos(c+d\,x)^3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^3,x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^3, x)
```

Reduce [F]

$$\begin{aligned}\int \sec^3(c+dx)(a&\\ +b\sec(c+dx))^{5/2}\,dx&=\left(\int \sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)^5\,dx\right)b^2\\ &+2\left(\int \sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)^4\,dx\right)ab\\ &+\left(\int \sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)^3\,dx\right)a^2\end{aligned}$$

input
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x)
```

output

```
int(sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**5,x)*b**2 + 2*int(sqrt(sec(c +
d*x)*b + a)*sec(c + d*x)**4,x)*a*b + int(sqrt(sec(c + d*x)*b + a)*sec(c +
d*x)**3,x)*a**2
```

3.546 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 333

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx =$$

$$-\frac{2a(a-b)\sqrt{a+b}(3a^2+29b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{21b^2d}$$

$$-\frac{2(a-b)\sqrt{a+b}(3a^2-24ab+5b^2)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{\cdots}}{21bd}$$

$$+\frac{2(3a^2+5b^2)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{21d}$$

$$+\frac{2a(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{7d}+\frac{2(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{7d}$$

output
```
-2/21*a*(a-b)*(a+b)^(1/2)*(3*a^2+29*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x
+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)
*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d-2/21*(a-b)*(a+b)^(1/2)*(3*a^2-24*a*
b+5*b^2)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a
-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)
/b/d+2/21*(3*a^2+5*b^2)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/d+2/7*a*(a+b*sec
(d*x+c))^(3/2)*tan(d*x+c)/d+2/7*(a+b*sec(d*x+c))^(5/2)*tan(d*x+c)/d
```

Mathematica [A] (warning: unable to verify)

Time = 11.34 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.42

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx =$$

$$-\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\left(2a(3a^3+3a^2b+29ab^2+29b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b}{(a+}}\right.}{}$$

$$+\frac{\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{2a(3a^2+29b^2)\sin(c+dx)}{21b}+\frac{2}{21}\sec(c+dx)\left(9a^2\sin(c+dx)+5b^2\sin(c+dx\right.\right.}{d(b+a\cos(c+dx))^2}$$

input
```
Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(2*a*
(3*a^3 + 3*a^2*b + 29*a*b^2 + 29*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(3*a^3 + 27*a^2*b + 29*a*b^2 +
5*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)] + a*(3*a^2 + 29*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2]))/(21*b*d*(b + a*Cos[c + d*x])^3*Sqrt[Sec[(c + d*x)
/2]^2]*Sec[c + d*x]^(5/2)) + (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((
2*a*(3*a^2 + 29*b^2)*Sin[c + d*x])/(21*b) + (2*Sec[c + d*x]*(9*a^2*Sin[c +
 d*x] + 5*b^2*Sin[c + d*x]))/21 + (6*a*b*Sec[c + d*x]*Tan[c + d*x])/7 + (2
*b^2*Sec[c + d*x]^2*Tan[c + d*x])/7))/(d*(b + a*Cos[c + d*x])^2)
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4322, 3042, 4490, 27, 3042, 4490, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$\Big\downarrow$ 3042

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$\Big\downarrow$ 4322

$$\frac{5}{7}\int \sec(c+dx)(b+a\sec(c+dx))(a+b\sec(c+dx))^{3/2}dx+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}$$

$\Big\downarrow$ 3042

$$\frac{5}{7}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}$$

$\Big\downarrow$ 4490

$$\frac{5}{7}\left(\frac{2}{5}\int \frac{1}{2}\sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(8ab+\left(3a^2+5b^2\right)\sec(c+dx)\right)dx+\frac{2a\tan(c+dx)(a+b\sec(c+dx))}{5d}\right.$$
$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}$$

$\Big\downarrow$ 27

$$\frac{5}{7}\left(\frac{1}{5}\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(8ab+\left(3a^2+5b^2\right)\sec(c+dx)\right)dx+\frac{2a\tan(c+dx)(a+b\sec(c+dx))^{3/}}{5d}\right.$$
$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}$$

$\Big\downarrow$ 3042

$$\frac{5}{7}\left(\frac{1}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}\left(8ab+\left(3a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2a\tan(c+dx)(}{}\right.$$
$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}$$

$\Big\downarrow$ 4490

$$\frac{5}{7}\left(\frac{1}{5}\left(\frac{2}{3}\int \frac{\sec(c+dx)\left(b\left(27a^2+5b^2\right)+a\left(3a^2+29b^2\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx+\frac{2\left(3a^2+5b^2\right)\tan(c+dx)\sqrt{a+b\sec(c}{3d}\right.\right.$$
$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}$$

$\Big\downarrow$ 27

$$\frac{5}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{\sec(c+dx)\left(b\left(27a^2+5b^2\right)+a\left(3a^2+29b^2\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{2\left(3a^2+5b^2\right)\tan(c+dx)\sqrt{a+b\sec(c}{3d}\right.\right.$$
$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}$$

$\Big\downarrow$ 3042

$$\frac{5}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(b\left(27a^2+5b^2\right)+a\left(3a^2+29b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(3a^2+5b^2\right)\tan(c+dx)\sqrt{}}{3d}\right.\right.$$
$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}$$

$\Big\downarrow$ 4493

$$\frac{5}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(a\left(3a^2+29b^2\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx-(a-b)\left(3a^2-24ab+5b^2\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}d\right.\right.\right.$$
$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}$$

$\Big\downarrow$ 3042

$$\frac{5}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(a\left(3a^2+29b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)\left(3a^2-24ab+5b^2\right)\int \frac{\csc(}{\sqrt{a+b}}\right.\right.\right.$$
$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}$$

$\Big\downarrow$ 4319

$$\frac{5}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(a\left(3a^2+29b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\left(3a^2-24ab+5b^2\right)\cot}{}\right.\right.\right.$$
$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{4492}$$

$$\frac{5}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(-\frac{2(a-b)\sqrt{a+b}(3a^2-24ab+5b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\ldots}}{}\right.\right.}{bd}\right.\right.\right.$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}$$

input
```
Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(2*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*d) + (5*((2*a*(a + b*Sec[c
+ d*x])^(3/2)*Tan[c + d*x])/(5*d) + (((-2*a*(a - b)*Sqrt[a + b]*(3*a^2 + 2
9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/(b^2*d) - (2*(a - b)*Sqrt[a + b]*(3*a^2 - 24*a*b +
5*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/(b*d))/3 + (2*(3*a^2 + 5*b^2)*Sqrt[a + b*Sec[c + d*
x]]*Tan[c + d*x])/(3*d))/5))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4322
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[m/(m + 1)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc
[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(299) = 598$.

Time = 23.66 (sec) , antiderivative size = 1029, normalized size of antiderivative = 3.09

method	result	size
default	Expression too large to display	1029

input
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/d/b*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+b*cos(d*x+c)+
b)*(3*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^4*EllipticE(cot(d*x+c)-csc(d
*x+c),((a-b)/(a+b))^(1/2))+3*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3*b*E
llipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+29*(cos(d*x+c)^2+2*cos
(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*a^2*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(
1/2))+29*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^3*EllipticE(cot(d*x+c)-
csc(d*x+c),((a-b)/(a+b))^(1/2))+3*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a
^3*b*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+27*(-cos(d*x+c)^
2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*a^2*b^2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(
a+b))^(1/2))+29*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^3*EllipticF(cot
(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+5*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*b^4*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+3*a^4*c...
```

Fricas [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx=\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^4 + 2*a*b*sec(d*x + c)^3 + a^2*sec(d*x + c)^2)*
sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (a+b\sec(c+dx))^{\frac{5}{2}}\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(5/2)*sec(c + d*x)**2, x)
```

Maxima [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x)
```

Giac [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\cos\left(c+d\,x\right)^2}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^2,x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx=\left(\int \sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^4dx\right)b^2+2\left(\int \sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^3dx\right)ab+\left(\int \sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^2dx\right)a^2$$

input
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**4,x)*b**2 + 2*int(sqrt(sec(c +
d*x)*b + a)*sec(c + d*x)**3,x)*a*b + int(sqrt(sec(c + d*x)*b + a)*sec(c +
d*x)**2,x)*a**2
```

3.547 $\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 296

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = -\frac{2(a-b)\sqrt{a+b}(23a^2+9b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{15bd} + \frac{2(a-b)\sqrt{a+b}(15a^2-8ab+9b^2)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{15bd} + \frac{16ab\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15d} + \frac{2b(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d}$$

output
```
-2/15*(a-b)*(a+b)^(1/2)*(23*a^2+9*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c
))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(
-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+2/15*(a-b)*(a+b)^(1/2)*(15*a^2-8*a*b+9*
b^2)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))
^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d
+16/15*a*b*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/d+2/5*b*(a+b*sec(d*x+c))^(3/2
)*tan(d*x+c)/d
```

Mathematica [A] (warning: unable to verify)

Time = 12.79 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.49

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx =$$
$$-\frac{2(a+b\sec(c+dx))^{5/2}\left(-2(23a^3+23a^2b+9ab^2+9b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}\right.\right.\right.\right.}{15d(b}$$
$$+\frac{\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{2}{15}(23a^2+9b^2)\sin(c+dx)+\frac{22}{15}ab\tan(c+dx)+\frac{2}{5}b^2\sec(c+dx)\tan(c\right.}{d(b+a\cos(c+dx))^2}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*(a + b*Sec[c + d*x])^(5/2)*(-2*(23*a^3 + 23*a^2*b + 9*a*b^2 + 9*b^3)*S
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] +
2*(15*a^3 + 23*a^2*b + 17*a*b^2 + 9*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - (23*a^2 + 9*b^2)*Cos[c + d*x]*(b
 + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*d*(b + a*Cos[
c + d*x])^3*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)
/2]^2*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)) + (Cos[c + d*x]^2*(a + b*Se
c[c + d*x])^(5/2)*((2*(23*a^2 + 9*b^2)*Sin[c + d*x])/15 + (22*a*b*Tan[c +
d*x])/15 + (2*b^2*Sec[c + d*x]*Tan[c + d*x])/5))/(d*(b + a*Cos[c + d*x])^2
)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4317, 27, 3042, 4490, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$\Big\downarrow$ 3042

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$\Big\downarrow$ 4317

$$\frac{2}{5}\int \frac{1}{2}\sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(5a^2+8b\sec(c+dx)a+3b^2\right)dx+\frac{2b\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$\Big\downarrow$ 27

$$\frac{1}{5}\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(5a^2+8b\sec(c+dx)a+3b^2\right)dx+\frac{2b\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5a^2+8b\csc\left(c+dx+\frac{\pi}{2}\right)a+3b^2\right)dx+\frac{2b\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$\Big\downarrow$ 4490

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{\sec(c+dx)\left(a\left(15a^2+17b^2\right)+b\left(23a^2+9b^2\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx+\frac{16ab\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)+\frac{2b\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$\Big\downarrow$ 27

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{\sec(c+dx)\left(a\left(15a^2+17b^2\right)+b\left(23a^2+9b^2\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{16ab\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)+\frac{2b\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a(15a^2+17b^2)+b(23a^2+9b^2)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{16ab\tan(c+dx)\sqrt{a+b\sec(c}{3d}\right.$$
$$\frac{2b\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$\downarrow$ 4493

$$\frac{1}{5}\left(\frac{1}{3}\left((a-b)\left(15a^2-8ab+9b^2\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+b(23a^2+9b^2)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx\right)\right.$$
$$\frac{2b\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left((a-b)\left(15a^2-8ab+9b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(23a^2+9b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+d\right.\right.}{\sqrt{a+b\csc\left(c+dx\right.}}\right.\right.$$
$$\frac{2b\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$\downarrow$ 4319

$$\frac{1}{5}\left(\frac{1}{3}\left(b(23a^2+9b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}(15a^2-8ab+9b^2)\cot(c+}{}\right.\right.$$
$$\frac{2b\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$\downarrow$ 4492

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{2(a-b)\sqrt{a+b}(15a^2-8ab+9b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\,\mathrm{se}}}{\sqrt{a}}\right.\right.}{bd}\right.\right.$$
$$\frac{2b\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

input
```
Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(2*b*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d) + (((-2*(a - b)*Sqrt[a
 + b]*(23*a^2 + 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (2*(a - b)*Sqrt[a + b]*(15*
a^2 - 8*a*b + 9*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d))/3 + (16*a*b*Sqrt[a + b*Sec[c +
 d*x]]*Tan[c + d*x])/(3*d))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4317
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Simp[1/m   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*m + a*b*(2*m - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ
[a^2 - b^2, 0] && GtQ[m, 1] && IntegerQ[2*m]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 957 vs. $2(266) = 532$.

Time = 16.92 (sec) , antiderivative size = 958, normalized size of antiderivative = 3.24

method	result	size
default	Expression too large to display	958

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15/d*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+b*cos(d*x+c)+b
)*(23*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*EllipticE(cot(d*x+c)-csc(
d*x+c),((a-b)/(a+b))^(1/2))+23*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*
b*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+9*(-cos(d*x+c)^2-2*
cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*a*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^
(1/2))+9*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^3*EllipticE(cot(d*x+c)-c
sc(d*x+c),((a-b)/(a+b))^(1/2))+15*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^
3*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+23*(cos(d*x+c)^2+2*
cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*a^2*b*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^
(1/2))+17*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2*EllipticF(cot(d*x+c)
-csc(d*x+c),((a-b)/(a+b))^(1/2))+9*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b
^3*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))-23*a^3*cos(d*x+...
```

Fricas [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx=\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^3 + 2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c))*sq
rt(b*sec(d*x + c) + a), x)
```

Sympy [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (a+b\sec(c+dx))^{\frac{5}{2}}\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(5/2)*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

Giac [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\cos\left(c+d\,x\right)}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x), x)
```

Reduce [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \left(\int \sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^{3}dx\right)b^{2} + 2\left(\int \sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^{2}dx\right)ab + \left(\int \sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)dx\right)a^{2}$$

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**3,x)*b**2 + 2*int(sqrt(sec(c +
d*x)*b + a)*sec(c + d*x)**2,x)*a*b + int(sqrt(sec(c + d*x)*b + a)*sec(c +
d*x),x)*a**2
```

3.548 $\int (a+b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 352

$$\int (a+b\sec(c+dx))^{5/2}\,dx = -\frac{14a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3d}$$
$$+\frac{2\sqrt{a+b}(9a^2-7ab+b^2)\cot(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3d}$$
$$-\frac{2a^2\sqrt{a+b}\cot(c+dx)\,\mathrm{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d}$$
$$+\frac{2b^2\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d}$$

output
```
-14/3*a*(a-b)*(a+b)^(1/2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b
)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x
+c))/(a-b))^(1/2)/d+2/3*(a+b)^(1/2)*(9*a^2-7*a*b+b^2)*cot(d*x+c)*EllipticF
((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))
/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d-2*a^2*(a+b)^(1/2)*cot(d*x+
c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/
2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d+2/3*b
^2*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 10.43 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.20

$$\int (a+b\sec(c+dx))^{5/2}\,dx =$$

$$-\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(28ab(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\right.\right.}{}$$

$$+\frac{\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{14}{3}ab\sin(c+dx)+\frac{2}{3}b^2\tan(c+dx)\right)}{d(b+a\cos(c+dx))^2}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
-1/3*(Cos[(c + d*x)/2]^2*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(28*a*b
*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)] + 4*(3*a^3 - 9*a^2*b - 7*a*b^2 - b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c
 + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elliptic
F[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 24*a^3*Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 14*a*b*Cos[c +
 d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(d*(b + a
*Cos[c + d*x])^3) + (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((14*a*b*Si
n[c + d*x])/3 + (2*b^2*Tan[c + d*x])/3))/(d*(b + a*Cos[c + d*x])^2)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {3042, 4269, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4269}$$

$$\frac{2}{3}\int \frac{3a^3+7b^2\sec^2(c+dx)a+b(9a^2+b^2)\sec(c+dx)}{2\sqrt{a+b\sec(c+dx)}}dx+\frac{2b^2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{3a^3+7b^2\sec^2(c+dx)a+b(9a^2+b^2)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{2b^2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{3a^3+7b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a+b(9a^2+b^2)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b^2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{4546}$$

$$\frac{1}{3}\left(\int \frac{3a^3+(b(9a^2+b^2)-7ab^2)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+7ab^2\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{2b^2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\int \frac{3a^3+(b(9a^2+b^2)-7ab^2)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+7ab^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2b^2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{4409}$$

$$\frac{1}{3}\left(3a^3\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx+b(9a^2-7ab+b^2)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+7ab^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(\right.\right.}{\sqrt{a+b\csc\left(c+\right.}}\right.$$

$$\frac{2b^2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(3a^3\int\frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\left(9a^2-7ab+b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+7ab^2\int\frac{\csc\left(c+dx-\right.}{\sqrt{a-}}\right.$$

$$\frac{2b^2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{4271}$$

$$\frac{1}{3}\left(b\left(9a^2-7ab+b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+7ab^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6a^2}{}\right.$$

$$\frac{2b^2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{4319}$$

$$\frac{1}{3}\left(7ab^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}\left(9a^2-7ab+b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{}}{d}\right.$$

$$\frac{2b^2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{4492}$$

$$\frac{1}{3}\left(\frac{2\sqrt{a+b}\left(9a^2-7ab+b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right.}{d}\right.$$

$$\frac{2b^2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

input
```
Int[(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
((-14*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*Sqrt[a + b]*(9*a^2 - 7*
a*b + b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
 b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[c + d*x]))/(a - b))])/d - (6*a^2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[
(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b)
)])/d)/3 + (2*b^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4269
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[1/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*
a^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 807 vs. $2(317) = 634$.

Time = 11.78 (sec) , antiderivative size = 808, normalized size of antiderivative = 2.30

method	result
default	$\frac{2\sqrt{a+b\sec(dx+c)}\left(6\left(-\cos(dx+c)^2-2\cos(dx+c)-1\right)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,a^3\,\mathrm{EllipticPi}\left(\cot(dx+c)-\csc(dx+c),-\right.\right.}{}$

input
```
int((a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+b*cos(d*x+c)+b)*
(6*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3*EllipticPi(cot(d*x+c)-csc(d*
x+c),-1,((a-b)/(a+b))^(1/2))+7*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b
*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+7*(cos(d*x+c)^2+2*co
s(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*a*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1
/2))+3*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*EllipticF(cot(d*x+c)-csc(
d*x+c),((a-b)/(a+b))^(1/2))+9*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b
*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+7*(-cos(d*x+c)^2-2*c
os(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*a*b^2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(
1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^3*EllipticF(cot(d*x+c)-csc(
d*x+c),((a-b)/(a+b))^(1/2))+7*a^2*b*cos(d*x+c)*sin(d*x+c)+sin(d*x+c)*(cos(
d*x+c)+8)*a*b^2+b^3*(sin(d*x+c)+tan(d*x+c)))
```

Fricas [F]

$$\int (a+b\sec(c+dx))^{5/2}\,dx=\int (b\sec(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x +
c) + a), x)
```

Sympy [F]

$$\int (a+b\sec(c+dx))^{5/2}\,dx=\int (a+b\sec(c+dx))^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int (a+b\sec(c+dx))^{5/2}\,dx=\int (b\sec(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int (a+b\sec(c+dx))^{5/2}\,dx=\int (b\sec(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^{5/2}\,dx=\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{5/2}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int (a+b\sec(c+dx))^{5/2}\,dx=\left(\int \sqrt{\sec\left(dx+c\right)b+a}dx\right)a^2+\left(\int \sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^2dx\right)b^2+2\left(\int \sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)dx\right)ab$$

input
```
int((a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a),x)*a**2 + int(sqrt(sec(c + d*x)*b + a)*sec(c
+ d*x)**2,x)*b**2 + 2*int(sqrt(sec(c + d*x)*b + a)*sec(c + d*x),x)*a*b
```

3.549 $\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 353

$$\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{(a-b)\sqrt{a+b}(a^2-2b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{se}}{}}}{bd}$$
$$+\frac{\sqrt{a+b}(a^2+6ab-2b^2)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+d}{a-b}}}{d}$$
$$-\frac{5ab\sqrt{a+b}\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d}$$
$$+\frac{a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
(a-b)*(a+b)^(1/2)*(a^2-2*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/
(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec
(d*x+c))/(a-b))^(1/2)/b/d+(a+b)^(1/2)*(a^2+6*a*b-2*b^2)*cot(d*x+c)*Ellipti
cF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c
))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d-5*a*b*(a+b)^(1/2)*cot(d*
x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(
1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d+a^2
*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (warning: unable to verify)

Time = 16.58 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.29

$$\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{2b^2\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\sin(c+dx)}{d(b+a\cos(c+dx))^2} + \frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\left(2(a^3+a^2b-2ab^2-2b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+}{(a+b)(1+\cos(}}\right.}{}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(2*b^2*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(d*(b + a*C
os[c + d*x])^2) + (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*
x])^(5/2)*(2*(a^3 + a^2*b - 2*a*b^2 - 2*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE
[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 4*b*(-3*a^2 + 3*a*b + b^2)*S
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] +
20*a^2*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)] + ((a^2 - 2*b^2)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^3*(-Si
n[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/2))/(d*(b + a*Cos[c + d*x])^3*Sqrt
[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4328, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 4328}$$

$$\int \frac{5ba^2+6b^2\sec(c+dx)a-b\left(a^2-2b^2\right)\sec^2(c+dx)}{2\sqrt{a+b\sec(c+dx)}}dx+\frac{a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\int \frac{5ba^2+6b^2\sec(c+dx)a-b\left(a^2-2b^2\right)\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\int \frac{5ba^2+6b^2\csc\left(c+dx+\frac{\pi}{2}\right)a-b\left(a^2-2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{ 4546}$$

$$\frac{1}{2}\left(\int \frac{5ba^2+\left(6ab^2+\left(a^2-2b^2\right)b\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-b\left(a^2-2b^2\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\left(\int \frac{5ba^2+\left(6ab^2+\left(a^2-2b^2\right)b\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b\left(a^2-2b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}\right.$$
$$\frac{a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{ 4409}$$

$$\frac{1}{2}\left(-b\left(a^2-2b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\left(a^2+6ab-2b^2\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\right.$$
$$\frac{a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\left(b(a^2+6ab-2b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b(a^2-2b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right.$$
$$\frac{a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 4271

$$\frac{1}{2}\left(b(a^2+6ab-2b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b(a^2-2b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right.$$
$$\frac{a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 4319

$$\frac{1}{2}\left(-b(a^2-2b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}(a^2+6ab-2b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(}{a+}}}{}\right.$$
$$\frac{a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 4492

$$\frac{1}{2}\left(\frac{2\sqrt{a+b}(a^2+6ab-2b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right.}{d}\right.$$
$$\frac{a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

input
```
Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
((2*(a - b)*Sqrt[a + b]*(a^2 - 2*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (2*Sqrt[a +
b]*(a^2 + 6*a*b - 2*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d - (10*a*b*Sqrt[a + b]*Cot[c + d
*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
 d*x]))/(a - b))])/d)/2 + (a^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 917 vs. $2(324) = 648$.

Time = 16.81 (sec) , antiderivative size = 918, normalized size of antiderivative = 2.60

method	result	size
default	Expression too large to display	918

input
```
int(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(10*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b*EllipticPi(cot(d*x+c)
-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*a^3*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2
*cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*a^2*b*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))
^(1/2))+2*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^2*EllipticE(cot(d*x+c)
-csc(d*x+c),((a-b)/(a+b))^(1/2))+2*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b
^3*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+6*(cos(d*x+c)^2+2*
cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*a^2*b*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^
(1/2))+6*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^2*EllipticF(cot(d*x+c)
-csc(d*x+c),((a-b)/(a+b))^(1/2))+2*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
b^3*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+a^3*cos(d*x+c)^2*
sin(d*x+c)+a^2*b*cos(d*x+c)*sin(d*x+c)+2*a*b^2*cos(d*x+c)*sin(d*x+c)+2*...
```

Fricas [F]

$$\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx=\int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*cos(d*x + c)*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)*sec(d*x + c
) + a^2*cos(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx=\int \cos\left(c+d\,x\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

input
```
int(cos(c + d*x)*(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)*(a + b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx=\left(\int \sqrt{\sec\left(dx+c\right) b+a}\,\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}dx\right) b^{2}+2\left(\int \sqrt{\sec\left(dx+c\right) b+a}\,\cos\left(dx+c\right)\sec\left(dx+c\right)dx\right) a b+\left(\int \sqrt{\sec\left(dx+c\right) b+a}\,\cos\left(dx+c\right)dx\right) a^{2}$$

input
```
int(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x)**2,x)*b**2 + 2*int(
sqrt(sec(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x),x)*a*b + int(sqrt(sec(c
 + d*x)*b + a)*cos(c + d*x),x)*a**2
```

3.550 $\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 399

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{9a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4d} + \frac{\sqrt{a+b}(2a^2+9ab+8b^2)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4d} - \frac{\sqrt{a+b}(4a^2+15b^2)\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4d} + \frac{9ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{a^2\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d}$$

output
```
9/4*a*(a-b)*(a+b)^(1/2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^
(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c
))/(a-b))^(1/2)/d+1/4*(a+b)^(1/2)*(2*a^2+9*a*b+8*b^2)*cot(d*x+c)*EllipticF
((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))
/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d-1/4*(a+b)^(1/2)*(4*a^2+15*
b^2)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+
b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^
(1/2)/d+9/4*a*b*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d+1/2*a^2*cos(d*x+c)*(a+
b*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (warning: unable to verify)

Time = 13.51 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.16

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{a^2\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\sin(2(c+dx))}{4d(b+a\cos(c+dx))^2} + \frac{(a+b\sec(c+dx))^{5/2}\left(-18ab(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)+4\right.}{}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(a^2*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*Sin[2*(c + d*x)])/(4*d*(b +
 a*Cos[c + d*x])^2) + ((a + b*Sec[c + d*x])^(5/2)*(-18*a*b*(a + b)*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos
[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 4*(2*a
^3 - a^2*b + 12*a*b^2 - 4*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[
(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)] - 4*a*(4*a^2 + 15*b^2)*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 9*a*b*Cos[c + d
*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(4*d*(b + a
*Cos[c + d*x])^3*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c +
 d*x)/2]^2*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2))
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4328, 27, 3042, 4592, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \; 4328$$

$$\frac{1}{2}\int \frac{\cos(c+dx)\left(9ba^2+2\left(a^2+6b^2\right)\sec(c+dx)a+b\left(a^2+4b^2\right)\sec^2(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}\int \frac{\cos(c+dx)\left(9ba^2+2\left(a^2+6b^2\right)\sec(c+dx)a+b\left(a^2+4b^2\right)\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\int \frac{9ba^2+2\left(a^2+6b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 4592$$

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int -\frac{-9b^2\sec^2(c+dx)a^2+\left(4a^2+15b^2\right)a^2+2b\left(a^2+4b^2\right)\sec(c+dx)a}{2\sqrt{a+b\sec(c+dx)}}dx}{a}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}\left(\frac{\int \frac{-9b^2\sec^2(c+dx)a^2+\left(4a^2+15b^2\right)a^2+2b\left(a^2+4b^2\right)\sec(c+dx)a}{\sqrt{a+b\sec(c+dx)}}dx}{2a}+\frac{9ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(\frac{\int \frac{-9b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+(4a^2+15b^2)a^2+2b(a^2+4b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{9ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}\right)+$$
$$\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{4546}$$

$$\frac{1}{4}\left(\frac{\int \frac{(4a^2+15b^2)a^2+(9a^2b^2+2a(a^2+4b^2)b)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-9a^2b^2\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx}{2a}+\frac{9ab\sin(c+dx)\sqrt{a+b\sec(c}{d}\right.$$
$$\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{\int \frac{(4a^2+15b^2)a^2+(9a^2b^2+2a(a^2+4b^2)b)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-9a^2b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{9ab\sin(c+dx)\sqrt{}}{d}\right.$$
$$\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{4409}$$

$$\frac{1}{4}\left(\frac{-9a^2b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a^2(4a^2+15b^2)\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx+ab(2a^2+9ab+8b^2)\int \frac{\sec(c}{\sqrt{a+b\,\text{se}}}}{2a}\right.$$
$$\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{a^2(4a^2+15b^2)\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-9a^2b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+ab(2a^2+9ab+8b^2)\int \frac{\csc(}{\sqrt{a+b}}}{2a}\right.$$
$$\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{4271}$$

$$\frac{1}{4}\left(\frac{-9a^2b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+ab\left(2a^2+9ab+8b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2a\sqrt{a+b}\left(4a^2+15b^2\right)\cot(c+}{}}{2a}\right.$$

$$\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 4319

$$\frac{1}{4}\left(\frac{-9a^2b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sqrt{a+b}\left(2a^2+9ab+8b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\text{arc}\right.}{d}}{}\right.$$

$$\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 4492

$$\frac{1}{4}\left(\frac{\frac{2a\sqrt{a+b}\left(2a^2+9ab+8b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}-\frac{2a\sqrt{a+b}\left(4a^2+15b^2\right)}{}}{}\right.$$

$$\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(a^2*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (((18*a^2
*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*a*Sqrt[a + b]*(2*a^2 + 9*a*b +
 8*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Se
c[c + d*x]))/(a - b))])/d - (2*a*Sqrt[a + b]*(4*a^2 + 15*b^2)*Cot[c + d*x]
*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*
x]))/(a - b))])/d)/(2*a) + (9*a*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 928 vs. $2(358) = 716$.

Time = 37.28 (sec) , antiderivative size = 929, normalized size of antiderivative = 2.33

method	result	size
default	Expression too large to display	929

input
```
int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*((-8*cos(d*x+c)^2-16*cos(d*x+c)-8)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3*EllipticPi(cot(d*x+c)
-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))+(-30*cos(d*x+c)^2-60*cos(d*x+c)-30)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*a*b^2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))+(-9*c
os(d*x+c)^2-18*cos(d*x+c)-9)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b*EllipticE(cot(d*x+c)-csc(d*x+c)
,((a-b)/(a+b))^(1/2))+(-9*cos(d*x+c)^2-18*cos(d*x+c)-9)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^2*Elli
pticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(4*cos(d*x+c)^2+8*cos(d*x
+c)+4)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*a^3*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(-
2*cos(d*x+c)^2-4*cos(d*x+c)-2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b*EllipticF(cot(d*x+c)-csc(d*x+
c),((a-b)/(a+b))^(1/2))+(24*cos(d*x+c)^2+48*cos(d*x+c)+24)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^2*E
llipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(-8*cos(d*x+c)^2-16*co
s(d*x+c)-8)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*b^3*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2
))+sin(d*x+c)*cos(d*x+c)^2*(2*cos(d*x+c)+2)*a^3+sin(d*x+c)*cos(d*x+c)*(...
```

Fricas [F]

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*cos(d*x + c)^2*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)^2*sec(d*x
 + c) + a^2*cos(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

Giac [F]

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx=\int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx=\int \cos(c+d\,x)^2\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)^2*(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^2*(a + b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx=\left(\int \sqrt{\sec(dx+c)\,b+a}\,\cos(dx+c)^2\sec(dx+c)^2\,dx\right)b^2+2\left(\int \sqrt{\sec(dx+c)\,b+a}\,\cos(dx+c)^2\sec(dx+c)\,dx\right)ab+\left(\int \sqrt{\sec(dx+c)\,b+a}\,\cos(dx+c)^2\,dx\right)a^2$$

input
```
int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*cos(c + d*x)**2*sec(c + d*x)**2,x)*b**2 + 2*i
nt(sqrt(sec(c + d*x)*b + a)*cos(c + d*x)**2*sec(c + d*x),x)*a*b + int(sqrt
(sec(c + d*x)*b + a)*cos(c + d*x)**2,x)*a**2
```

3.551 $\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 460

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{(a-b)\sqrt{a+b}(16a^2+33b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(}{}}}{24bd}$$

$$+\frac{\sqrt{a+b}(16a^2+26ab+33b^2)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+se}{a}}}{24d}$$

$$-\frac{5b\sqrt{a+b}(4a^2+b^2)\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+}{a-b}}}{8ad}$$

$$+\frac{(16a^2+33b^2)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d}$$

$$+\frac{13ab\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{12d}$$

$$+\frac{a^2\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d}$$

output
```
1/24*(a-b)*(a+b)^(1/2)*(16*a^2+33*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c
))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(
-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+1/24*(a+b)^(1/2)*(16*a^2+26*a*b+33*b^2)
*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d-5/8*b
*(a+b)^(1/2)*(4*a^2+b^2)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b
)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1
+sec(d*x+c))/(a-b))^(1/2)/a/d+1/24*(16*a^2+33*b^2)*(a+b*sec(d*x+c))^(1/2)*
sin(d*x+c)/d+13/12*a*b*cos(d*x+c)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d+1/3*
a^2*cos(d*x+c)^2*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1018 vs. $2(460) = 920$.

Time = 15.02 (sec) , antiderivative size = 1018, normalized size of antiderivative = 2.21

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((a^2*Sin[c + d*x])/12 + (13*a*
b*Sin[2*(c + d*x)])/24 + (a^2*Sin[3*(c + d*x)])/12))/(d*(b + a*Cos[c + d*x
])^2) + ((a + b*Sec[c + d*x])^(5/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(1
6*a^3*Tan[(c + d*x)/2] + 16*a^2*b*Tan[(c + d*x)/2] + 33*a*b^2*Tan[(c + d*x
)/2] + 33*b^3*Tan[(c + d*x)/2] - 32*a^3*Tan[(c + d*x)/2]^3 - 66*a*b^2*Tan[
(c + d*x)/2]^3 + 16*a^3*Tan[(c + d*x)/2]^5 - 16*a^2*b*Tan[(c + d*x)/2]^5 +
 33*a*b^2*Tan[(c + d*x)/2]^5 - 33*b^3*Tan[(c + d*x)/2]^5 + 120*a^2*b*Ellip
ticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)
] + 30*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[
1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*
x)/2]^2)/(a + b)] + 120*a^2*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a
- b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
- a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*b^3*EllipticP
i[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x
)/2]^2)/(a + b)] + (16*a^3 + 16*a^2*b + 33*a*b^2 + 33*b^3)*EllipticE[ArcSi
n[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Ta
n[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^
2)/(a + b)] - 2*b*(38*a^2 - 13*a*b + 24*b^2)*EllipticF[ArcSin[Tan[(c + ...
```

Rubi [A] (verified)

Time = 2.31 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.04, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4328, 27, 3042, 4592, 27, 3042, 4592, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \; 4328$$

$$\frac{1}{3}\int \frac{\cos^2(c+dx)\left(13ba^2+2\left(2a^2+9b^2\right)\sec(c+dx)a+3b\left(a^2+2b^2\right)\sec^2(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{6}\int \frac{\cos^2(c+dx)\left(13ba^2+2\left(2a^2+9b^2\right)\sec(c+dx)a+3b\left(a^2+2b^2\right)\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}\int \frac{13ba^2+2\left(2a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+3b\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \; 4592$$

$$\frac{1}{6}\left(\frac{13ab\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}-\frac{\int -\frac{\cos(c+dx)\left(13b^2\sec^2(c+dx)a^2+\left(16a^2+33b^2\right)a^2+2b\left(19a^2+12b^2\right)\sec(c+dx)a\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{2a}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{6}\left(\frac{\int \frac{\cos(c+dx)\left(13b^2\sec^2(c+dx)a^2+\left(16a^2+33b^2\right)a^2+2b\left(19a^2+12b^2\right)\sec(c+dx)a\right)}{\sqrt{a+b\sec(c+dx)}}dx}{4a}+\frac{13ab\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}\left(\frac{\int \frac{13b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+\left(16a^2+33b^2\right)a^2+2b\left(19a^2+12b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{4a}+\frac{13ab\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{4592}$$

$$\frac{1}{6}\left(\frac{\frac{a(16a^2+33b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int -\frac{26b^2\sec(c+dx)a^3-b\left(16a^2+33b^2\right)\sec^2(c+dx)a^2+15b\left(4a^2+b^2\right)a^2}{2\sqrt{a+b\sec(c+dx)}}dx}{a}}{4a}+\frac{13ab\sin(c+dx)\cdots}{\cdots}\right.$$

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(\frac{\frac{\int \frac{26b^2\sec(c+dx)a^3-b\left(16a^2+33b^2\right)\sec^2(c+dx)a^2+15b\left(4a^2+b^2\right)a^2}{\sqrt{a+b\sec(c+dx)}}dx}{2a}+\frac{a(16a^2+33b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}}{4a}+\frac{13ab\sin(c+dx)\cos\cdots}{\cdots}\right.$$

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{\frac{\int \frac{26b^2\csc\left(c+dx+\frac{\pi}{2}\right)a^3-b\left(16a^2+33b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+15b\left(4a^2+b^2\right)a^2}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{a(16a^2+33b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}}{4a}+\frac{13ab\sin(c+\cdots}{\cdots}\right.$$

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{4546}$$

$$\frac{1}{6}\left(\frac{\frac{\int \frac{15b\left(4a^2+b^2\right)a^2+\left(26b^2a^3+b\left(16a^2+33b^2\right)a^2\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-a^2b(16a^2+33b^2)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx}{2a}+\frac{a(16a^2+33b^2)\sin(c+dx)\sqrt{a+b\,\mathrm{s}\cdots}}{d}}{4a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{\frac{\int \frac{15b(4a^2+b^2)a^2+(26b^2a^3+b(16a^2+33b^2)a^2)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-a^2b(16a^2+33b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{a(16a^2+33b^2)\sin(c}{}}{4a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{4409}$$

$$\frac{1}{6}\left(\frac{\frac{a^2(-b)(16a^2+33b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+15a^2b(4a^2+b^2)\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx+a^2b(16a^2+26ab+33b^2)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{2a}}{4a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{\frac{15a^2b(4a^2+b^2)\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a^2b(16a^2+26ab+33b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-a^2b(16a^2+33b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+\right.}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}}{2a}}{4a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{4271}$$

$$\frac{1}{6}\left(\frac{\frac{a^2b(16a^2+26ab+33b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-a^2b(16a^2+33b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{30ab\sqrt{a+b}(4a^2+b^2)\cot(c+dx)\sqrt{\frac{b(1-}{}}}{}}{2a}}{4a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{4319}$$

$$\frac{1}{6}\left(\frac{a^2(-b)\left(16a^2+33b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sqrt{a+b}\left(16a^2+26ab+33b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\right.}{d}}{}\right.$$

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\downarrow$ 4492

$$\frac{1}{6}\left(\frac{\frac{2a^2\sqrt{a+b}\left(16a^2+26ab+33b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}+\frac{2a^2(a-b)\sqrt{a+b}\left(16a^2+33b^2\right)}{}}{}\right.$$

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

input

```
Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2),x]
```

output

```
(a^2*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d) + ((13*a*
b*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (((2*a^2*(a
- b)*Sqrt[a + b]*(16*a^2 + 33*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (2*a^2*Sqrt[a +
 b]*(16*a^2 + 26*a*b + 33*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Se
c[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d - (30*a*b*Sqrt[a + b]*(4*
a^2 + b^2)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d)/(2*a) + (a*(16*a^2 + 33*b^2)*Sqr
t[a + b*Sec[c + d*x]]*Sin[c + d*x])/d)/(4*a))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1074 vs. $2(415) = 830$.

Time = 124.14 (sec) , antiderivative size = 1075, normalized size of antiderivative = 2.34

method	result	size
default	Expression too large to display	1075

input
```
int(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/d*((-120*cos(d*x+c)^2-240*cos(d*x+c)-120)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b*EllipticPi(co
t(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))+(-30*cos(d*x+c)^2-60*cos(d*x+c
)-30)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*b^3*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))
+(-16*cos(d*x+c)^2-32*cos(d*x+c)-16)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*EllipticE(cot(d*x+c)-csc(
d*x+c),((a-b)/(a+b))^(1/2))+(-16*cos(d*x+c)^2-32*cos(d*x+c)-16)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^
2*b*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(-33*cos(d*x+c)^2
-66*cos(d*x+c)-33)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*a*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a
+b))^(1/2))+(-33*cos(d*x+c)^2-66*cos(d*x+c)-33)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^3*EllipticE(cot(
d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(76*cos(d*x+c)^2+152*cos(d*x+c)+76)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*a^2*b*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(-26*c
os(d*x+c)^2-52*cos(d*x+c)-26)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2*EllipticF(cot(d*x+c)-csc(d*x+c
),((a-b)/(a+b))^(1/2))+(48*cos(d*x+c)^2+96*cos(d*x+c)+48)*(cos(d*x+c)/(...
```

Fricas [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)
```

Giac [F]

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx=\int \cos\left(c+d\,x\right)^3\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)^3*(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^3*(a + b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx=\left(\int \sqrt{\sec\left(dx+c\right)b+a}\,\cos\left(dx+c\right)^3\sec\left(dx+c\right)^2dx\right)b^2+2\left(\int \sqrt{\sec\left(dx+c\right)b+a}\,\cos\left(dx+c\right)^3\sec\left(dx+c\right)dx\right)ab+\left(\int \sqrt{\sec\left(dx+c\right)b+a}\,\cos\left(dx+c\right)^3dx\right)a^2$$

input
```
int(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*cos(c + d*x)**3*sec(c + d*x)**2,x)*b**2 + 2*i
nt(sqrt(sec(c + d*x)*b + a)*cos(c + d*x)**3*sec(c + d*x),x)*a*b + int(sqrt
(sec(c + d*x)*b + a)*cos(c + d*x)**3,x)*a**2
```

3.552 $\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 530

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{(a-b)\sqrt{a+b}(284a^2+15b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\cdots}}{192ad}$$
$$+\frac{\sqrt{a+b}(72a^3+284a^2b+118ab^2+15b^3)\cot(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+d\cdots}{a+b}}}{192ad}$$
$$-\frac{\sqrt{a+b}(48a^4+120a^2b^2-5b^4)\cot(c+dx)\,\mathrm{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{\cdots}}{64a^2d}$$
$$+\frac{b(284a^2+15b^2)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{192ad}$$
$$+\frac{(36a^2+59b^2)\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{96d}$$
$$+\frac{17ab\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d}$$
$$+\frac{a^2\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d}$$

output
```
1/192*(a-b)*(a+b)^(1/2)*(284*a^2+15*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x
+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)
*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+1/192*(a+b)^(1/2)*(72*a^3+284*a^2*b+1
18*a*b^2+15*b^3)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(
(a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b
))^(1/2)/a/d-1/64*(a+b)^(1/2)*(48*a^4+120*a^2*b^2-5*b^4)*cot(d*x+c)*Ellipt
icPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(b*(1
-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+1/192*b*(2
84*a^2+15*b^2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/a/d+1/96*(36*a^2+59*b^2)*
cos(d*x+c)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d+17/24*a*b*cos(d*x+c)^2*(a+b
*sec(d*x+c))^(1/2)*sin(d*x+c)/d+1/4*a^2*cos(d*x+c)^3*(a+b*sec(d*x+c))^(1/2
)*sin(d*x+c)/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1274 vs. $2(530) = 1060$.

Time = 14.21 (sec) , antiderivative size = 1274, normalized size of antiderivative = 2.40

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((17*a*b*Sin[c + d*x])/96 + ((4
8*a^2 + 59*b^2)*Sin[2*(c + d*x)])/192 + (17*a*b*Sin[3*(c + d*x)])/96 + (a^
2*Sin[4*(c + d*x)])/32))/(d*(b + a*Cos[c + d*x])^2) + ((a + b*Sec[c + d*x]
)^(5/2)*(-284*a^3*b*Tan[(c + d*x)/2] - 284*a^2*b^2*Tan[(c + d*x)/2] - 15*a
*b^3*Tan[(c + d*x)/2] - 15*b^4*Tan[(c + d*x)/2] + 568*a^3*b*Tan[(c + d*x)/
2]^3 + 30*a*b^3*Tan[(c + d*x)/2]^3 - 284*a^3*b*Tan[(c + d*x)/2]^5 + 284*a^
2*b^2*Tan[(c + d*x)/2]^5 - 15*a*b^3*Tan[(c + d*x)/2]^5 + 15*b^4*Tan[(c + d
*x)/2]^5 - 288*a^4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan
[(c + d*x)/2]^2)/(a + b)] - 720*a^2*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(
c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*b^4*EllipticPi[-1, Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[
(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 288*a^4*E
llipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^
2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[
(c + d*x)/2]^2)/(a + b)] - 720*a^2*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*b^4*
EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)...
```

Rubi [A] (verified)

Time = 2.95 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.03, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4328, 27, 3042, 4592, 27, 3042, 4592, 27, 3042, 4592, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \ 4328$$

$$\frac{1}{4}\int \frac{\cos^3(c+dx)\left(17ba^2+6\left(a^2+4b^2\right)\sec(c+dx)a+b\left(5a^2+8b^2\right)\sec^2(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx+\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{8}\int \frac{\cos^3(c+dx)\left(17ba^2+6\left(a^2+4b^2\right)\sec(c+dx)a+b\left(5a^2+8b^2\right)\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{8}\int \frac{17ba^2+6\left(a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(5a^2+8b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$$\Big\downarrow \text{ 4592}$$

$$\frac{1}{8}\left(\frac{17ab\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}-\frac{\int -\frac{\cos^2(c+dx)\left(51b^2\sec^2(c+dx)a^2+\left(36a^2+59b^2\right)a^2+2b\left(49a^2+24b^2\right)\text{ s}\right.}{2\sqrt{a+b\sec(c+dx)}}}{3a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{8}\left(\frac{\int \frac{\cos^2(c+dx)\left(51b^2\sec^2(c+dx)a^2+\left(36a^2+59b^2\right)a^2+2b\left(49a^2+24b^2\right)\sec(c+dx)a\right)}{\sqrt{a+b\sec(c+dx)}}dx}{6a}+\frac{17ab\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\,\text{se}}}{3d}\right.$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{8}\left(\frac{\int \frac{51b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+\left(36a^2+59b^2\right)a^2+2b\left(49a^2+24b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{6a}+\frac{17ab\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c}}{3d}\right.$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$$\Big\downarrow \text{4592}$$

$$\frac{1}{8}\left(\frac{\frac{a(36a^2+59b^2)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}-\frac{\int -\frac{\cos(c+dx)\left(2\left(36a^2+161b^2\right)\sec(c+dx)a^3+b\left(36a^2+59b^2\right)\sec^2(c+dx)a^2+b\left(284a^2+15b^2\right.\right.}{2\sqrt{a+b\sec(c+dx)}}}{2a}}{6a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{\frac{\int \frac{\cos(c+dx)\left(2\left(36a^2+161b^2\right)\sec(c+dx)a^3+b\left(36a^2+59b^2\right)\sec^2(c+dx)a^2+b\left(284a^2+15b^2\right)a^2\right)}{\sqrt{a+b\sec(c+dx)}}dx}{4a}+\frac{a(36a^2+59b^2)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(}}{2d}}{6a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\left(\frac{\frac{\int \frac{2\left(36a^2+161b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^3+b\left(36a^2+59b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+b\left(284a^2+15b^2\right)a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{4a}+\frac{a(36a^2+59b^2)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+}}{2d}}{6a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$$\Big\downarrow \text{4592}$$

$$\frac{1}{8}\left(\frac{\frac{\frac{ab\left(284a^2+15b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int -\frac{2b\left(36a^2+59b^2\right)\sec(c+dx)a^3-b^2\left(284a^2+15b^2\right)\sec^2(c+dx)a^2+3\left(48a^4+120b^2a^2-5b^4\right)a^2}{2\sqrt{a+b\sec(c+dx)}}dx}{a}}{4a}+\frac{a(36a}{}}{6a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{\frac{\frac{\int \frac{2b(36a^2+59b^2)\sec(c+dx)a^3-b^2(284a^2+15b^2)\sec^2(c+dx)a^2+3(48a^4+120b^2a^2-5b^4)a^2}{\sqrt{a+b\sec(c+dx)}}dx}{2a}+\frac{ab(284a^2+15b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}}{4a}+\frac{a(36a^2}{}}{6a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\left(\frac{\frac{\frac{\int \frac{2b(36a^2+59b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a^3-b^2(284a^2+15b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+3(48a^4+120b^2a^2-5b^4)a^2}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{ab(284a^2+15b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}}{4a}+}{6a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$$\Big\downarrow \text{4546}$$

$$\frac{1}{8}\left(\frac{\frac{\frac{\int \frac{3(48a^4+120b^2a^2-5b^4)a^2+(2b(36a^2+59b^2)a^3+b^2(284a^2+15b^2)a^2)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-a^2b^2(284a^2+15b^2)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx}{2a}+\frac{ab(284a^2+15b}{}}{4a}}{6a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\left(\frac{\frac{\frac{\int \frac{3(48a^4+120b^2a^2-5b^4)a^2+(2b(36a^2+59b^2)a^3+b^2(284a^2+15b^2)a^2)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-a^2b^2(284a^2+15b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+}{4a}}{6a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$$\Big\downarrow \text{4409}$$

$$\frac{1}{8}\left(\frac{\frac{\frac{a^2\left(-b^2\right)\left(284a^2+15b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^2\left(48a^4+120a^2b^2-5b^4\right)\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx+a^2b\left(72a^3+284a^2b+118ab^2+15b^3\right)\int \frac{}{\sqrt{}}}{2a}}{4a}}{6a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$\big\downarrow$ 3042

$$\frac{1}{8}\left(\frac{\frac{\frac{-a^2b^2\left(284a^2+15b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^2\left(48a^4+120a^2b^2-5b^4\right)\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a^2b\left(72a^3+284a^2b+118ab^2+15b^3\right)\int}{2a}}{4a}}{6a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$\big\downarrow$ 4271

$$\frac{1}{8}\left(\frac{\frac{-a^2b^2\left(284a^2+15b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a^2b\left(72a^3+284a^2b+118ab^2+15b^3\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6a\sqrt{a+b}\left(48a^4+120a^2b^2-5b\right.}{}}{2a}}{4a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$\big\downarrow$ 4319

$$\frac{1}{8}\left(\frac{a^2\left(-b^2\right)\left(284a^2+15b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6a\sqrt{a+b}\left(48a^4+120a^2b^2-5b^4\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}}{d}}{}\right.$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$\big\downarrow$ 4492

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}+$$

$$\frac{1}{8}\left(\frac{\frac{a(36a^2+59b^2)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}+\frac{ab\left(284a^2+15b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}+\frac{2a^2(a-b)\sqrt{a+b}\left(284a^2+15b^2\right)\cot(c+dx)\sqrt{\frac{b}{}}}{}}{}\right.$$

input
```
Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(a^2*Cos[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + ((17*a*
b*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d) + ((a*(36*a^
2 + 59*b^2)*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + ((
(2*a^2*(a - b)*Sqrt[a + b]*(284*a^2 + 15*b^2)*Cot[c + d*x]*EllipticE[ArcSi
n[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*a^2*S
qrt[a + b]*(72*a^3 + 284*a^2*b + 118*a*b^2 + 15*b^3)*Cot[c + d*x]*Elliptic
F[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d - (
6*a*Sqrt[a + b]*(48*a^4 + 120*a^2*b^2 - 5*b^4)*Cot[c + d*x]*EllipticPi[(a
+ b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqr
t[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/d)/(2*a) + (a*b*(284*a^2 + 15*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])
/d)/(4*a))/(6*a))/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1345 vs. $2(481) = 962$.

Time = 392.75 (sec) , antiderivative size = 1346, normalized size of antiderivative = 2.54

method	result	size
default	Expression too large to display	1346

input
```
int(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/192/d/a*((-288*cos(d*x+c)^2-576*cos(d*x+c)-288)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*EllipticPi(c
ot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))+(-720*cos(d*x+c)^2-1440*cos(d
*x+c)-720)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*a^2*b^2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b
))^(1/2))+(30*cos(d*x+c)^2+60*cos(d*x+c)+30)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4*EllipticPi(cot(d*
x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))+(-284*cos(d*x+c)^2-568*cos(d*x+c)-
284)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*a^3*b*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(-
284*cos(d*x+c)^2-568*cos(d*x+c)-284)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2*EllipticE(cot(d*x+c)-
csc(d*x+c),((a-b)/(a+b))^(1/2))+(-15*cos(d*x+c)^2-30*cos(d*x+c)-15)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*a*b^3*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(-15*cos(d*x+
c)^2-30*cos(d*x+c)-15)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/
(a+b))^(1/2))+(144*cos(d*x+c)^2+288*cos(d*x+c)+144)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*EllipticF(
cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(-72*cos(d*x+c)^2-144*cos(d*...
```

Fricas [F]

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*cos(d*x + c)^4*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)^4*sec(d*x
 + c) + a^2*cos(d*x + c)^4)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^4, x)
```

Giac [F]

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx=\int \cos\left(c+d\,x\right)^4\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)^4*(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^4*(a + b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\begin{aligned}\int \cos^4(c+dx)(a&\\ +b\sec(c+dx))^{5/2}\,dx&=\left(\int \sqrt{\sec\left(dx+c\right)b+a}\,\cos\left(dx+c\right)^4\sec\left(dx+c\right)^2dx\right)b^2\\ &+2\left(\int \sqrt{\sec\left(dx+c\right)b+a}\,\cos\left(dx+c\right)^4\sec\left(dx+c\right)dx\right)ab\\ &+\left(\int \sqrt{\sec\left(dx+c\right)b+a}\,\cos\left(dx+c\right)^4dx\right)a^2\end{aligned}$$

input
```
int(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*cos(c + d*x)**4*sec(c + d*x)**2,x)*b**2 + 2*i
nt(sqrt(sec(c + d*x)*b + a)*cos(c + d*x)**4*sec(c + d*x),x)*a*b + int(sqrt
(sec(c + d*x)*b + a)*cos(c + d*x)**4,x)*a**2
```

3.553 $\int (a+b\sec(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 403

$$\int (a+b\sec(c+dx))^{7/2}\,dx = -\frac{2(a-b)\sqrt{a+b}(58a^2+9b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{15d}$$
$$+\frac{2\sqrt{a+b}(60a^3-58a^2b+22ab^2-9b^3)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{15d}$$
$$-\frac{2a^3\sqrt{a+b}\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d}$$
$$+\frac{26ab^2\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15d}+\frac{2b^2(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d}$$

output
```
-2/15*(a-b)*(a+b)^(1/2)*(58*a^2+9*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c
))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(
-b*(1+sec(d*x+c))/(a-b))^(1/2)/d+2/15*(a+b)^(1/2)*(60*a^3-58*a^2*b+22*a*b^
2-9*b^3)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a
-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)
/d-2*a^3*(a+b)^(1/2)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1
/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec
(d*x+c))/(a-b))^(1/2)/d+26/15*a*b^2*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/d+2/
5*b^2*(a+b*sec(d*x+c))^(3/2)*tan(d*x+c)/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 873 vs. $2(403) = 806$.

Time = 13.54 (sec) , antiderivative size = 873, normalized size of antiderivative = 2.17

$$\int (a+b\sec(c+dx))^{7/2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(7/2),x]
```

output
```
(2*(a + b*Sec[c + d*x])^(7/2)*(58*a^3*b*Tan[(c + d*x)/2] + 58*a^2*b^2*Tan[
(c + d*x)/2] + 9*a*b^3*Tan[(c + d*x)/2] + 9*b^4*Tan[(c + d*x)/2] - 116*a^3
*b*Tan[(c + d*x)/2]^3 - 18*a*b^3*Tan[(c + d*x)/2]^3 + 58*a^3*b*Tan[(c + d*
x)/2]^5 - 58*a^2*b^2*Tan[(c + d*x)/2]^5 + 9*a*b^3*Tan[(c + d*x)/2]^5 - 9*b
^4*Tan[(c + d*x)/2]^5 - 30*a^4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a
 - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/
2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*a^4*EllipticPi[-1, ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2
]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] +
 b*(58*a^3 + 58*a^2*b + 9*a*b^2 + 9*b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]
], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (15*
a^4 - 60*a^3*b - 58*a^2*b^2 - 22*a*b^3 - 9*b^4)*EllipticF[ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)
/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
))/(15*d*(b + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/2)*Sqrt[(1 - Tan[(c +
d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d
*x)/2]^2)]) + (Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(7/2)*((2*b*(58*a^2 + 9
*b^2)*Sin[c + d*x])/15 + (32*a*b^2*Tan[c + d*x])/15 + (2*b^3*Sec[c + d*...
```

Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4269, 27, 3042, 4544, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx$$

$$\Big\downarrow \text{4269}$$

$$\frac{2}{5}\int \frac{1}{2}\sqrt{a+b\sec(c+dx)}\left(5a^3+13b^2\sec^2(c+dx)a+3b\left(5a^2+b^2\right)\sec(c+dx)\right)dx+\frac{2b^2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \sqrt{a+b\sec(c+dx)}\left(5a^3+13b^2\sec^2(c+dx)a+3b\left(5a^2+b^2\right)\sec(c+dx)\right)dx+\frac{2b^2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5a^3+13b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a+3b\left(5a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2b^2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{4544}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{15a^4+2b\left(30a^2+11b^2\right)\sec(c+dx)a+b^2\left(58a^2+9b^2\right)\sec^2(c+dx)}{2\sqrt{a+b\sec(c+dx)}}dx+\frac{26ab^2\tan(c+dx)\sqrt{a+b\sec(c}{3d}\right.$$
$$\frac{2b^2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{15a^4+2b(30a^2+11b^2)\sec(c+dx)a+b^2(58a^2+9b^2)\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{26ab^2\tan(c+dx)\sqrt{a+b\sec(c}{3d}\right.$$
$$\frac{2b^2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{15a^4+2b(30a^2+11b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a+b^2(58a^2+9b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{26ab^2\tan(c+dx)\sqrt{}}{3a}\right.$$
$$\frac{2b^2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$\downarrow$ 4546

$$\frac{1}{5}\left(\frac{1}{3}\left(b^2(58a^2+9b^2)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx+\int \frac{15a^4+\left(2ab(30a^2+11b^2)-b^2(58a^2+9b^2)\right)\sec(}{\sqrt{a+b\sec(c+dx)}}\right.\right.$$
$$\frac{2b^2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(b^2(58a^2+9b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{15a^4+\left(2ab(30a^2+11b^2)-b^2(58a^2\right.}{\sqrt{a+b\csc(c+dx}}\right.\right.$$
$$\frac{2b^2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$\downarrow$ 4409

$$\frac{1}{5}\left(\frac{1}{3}\left(15a^4\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx+b^2(58a^2+9b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(60a^3\right.\right.$$
$$\frac{2b^2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(15a^4\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b^2(58a^2+9b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(6\right.\right.$$
$$\frac{2b^2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$\downarrow$ 4271

$$\frac{1}{5}\left(\frac{1}{3}\left(b^2\left(58a^2+9b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\left(60a^3-58a^2b+22ab^2-9b^3\right)\int \frac{\csc}{\sqrt{a+}}\right.\right.$$
$$\frac{2b^2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$\downarrow$ 4319

$$\frac{1}{5}\left(\frac{1}{3}\left(b^2\left(58a^2+9b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{30a^3\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{}}{}\right.\right.$$
$$\frac{2b^2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$\downarrow$ 4492

$$\frac{1}{5}\left(\frac{1}{3}\left(-\frac{30a^3\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}\right.\right.$$
$$\frac{2b^2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

input

```
Int[(a + b*Sec[c + d*x])^(7/2),x]
```

output

```
(2*b^2*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d) + (((-2*(a - b)*Sqrt
[a + b]*(58*a^2 + 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*Sqrt[a + b]*(60*a^3 - 58*a
^2*b + 22*a*b^2 - 9*b^3)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d - (30*a^3*Sqrt[a + b]*Cot[c + d
*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
 d*x]))/(a - b))])/d)/3 + (26*a*b^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])
/(3*d))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4269
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[1/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*
a^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4544
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> Simp[(-C)*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[(
a + b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m
)*Csc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1165 vs. $2(364) = 728$.

Time = 15.88 (sec) , antiderivative size = 1166, normalized size of antiderivative = 2.89

method	result	size
default	Expression too large to display	1166

input
```
int((a+b*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+b*cos(d*x+c)+b)
*(30*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^4*EllipticPi(cot(d*x+c)-csc(
d*x+c),-1,((a-b)/(a+b))^(1/2))+58*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^
3*b*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+58*(cos(d*x+c)^2+
2*cos(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*a^2*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+
b))^(1/2))+9*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^3*EllipticE(cot(d*x
+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+9*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*b^4*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+15*(cos(d*x+c)^
2+2*cos(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*a^4*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b)
)^(1/2))+60*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3*b*EllipticF(cot(d*x
+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+58*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*a^2*b^2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+22*(-c...
```

Fricas [F]

$$\int (a+b\sec(c+dx))^{7/2}\,dx=\int (b\sec(dx+c)+a)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
integral((b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x +
c) + a^3)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\sec(c+dx))^{7/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(7/2), x)
```

Giac [F]

$$\int (a+b\sec(c+dx))^{7/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^{7/2}\,dx=\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{7/2}\,dx$$

input
```
int((a + b/cos(c + d*x))^(7/2),x)
```

output
```
int((a + b/cos(c + d*x))^(7/2), x)
```

Reduce [F]

$$\begin{aligned}\int (a+b\sec(c+dx))^{7/2}\,dx &= \left(\int \sqrt{\sec\left(dx+c\right)b+a}dx\right)a^3\\ &+\left(\int \sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^3dx\right)b^3\\ &+3\left(\int \sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^2dx\right)a\,b^2\\ &+3\left(\int \sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)dx\right)a^2b\end{aligned}$$

input
```
int((a+b*sec(d*x+c))^(7/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a),x)*a**3 + int(sqrt(sec(c + d*x)*b + a)*sec(c
+ d*x)**3,x)*b**3 + 3*int(sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**2,x)*a*b*
*2 + 3*int(sqrt(sec(c + d*x)*b + a)*sec(c + d*x),x)*a**2*b
```

3.554 $\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 359

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{8a(a-b)\sqrt{a+b}(12a^2+11b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx}{a-b}}}{105b^5d}$$
$$+ \frac{2\sqrt{a+b}(48a^3-12a^2b+44ab^2+25b^3)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c}{a+b}}}{105b^4d}$$
$$+ \frac{2(24a^2+25b^2)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{105b^3d}$$
$$- \frac{12a\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{35b^2d}$$
$$+ \frac{2\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{7bd}$$

output
```
8/105*a*(a-b)*(a+b)^(1/2)*(12*a^2+11*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*
x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2
)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^5/d+2/105*(a+b)^(1/2)*(48*a^3-12*a^2*b
+44*a*b^2+25*b^3)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),
((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-
b))^(1/2)/b^4/d+2/105*(24*a^2+25*b^2)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^
3/d-12/35*a*sec(d*x+c)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^2/d+2/7*sec(d*x
+c)^2*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b/d
```

Mathematica [A] (warning: unable to verify)

Time = 11.78 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.29

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{4\sqrt{\sec(c+dx)}\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\Big(4a(12a^3+12a^2b+11ab^2+11b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos}{(a+b)(1+c}}}{}$$

$$+\frac{(b+a\cos(c+dx))\sec(c+dx)\left(-\frac{8a(12a^2+11b^2)\sin(c+dx)}{105b^4}+\frac{2\sec(c+dx)(24a^2\sin(c+dx)+25b^2\sin(c+dx))}{105b^3}-\frac{12a\sec(c+}{}\right.}{d\sqrt{a+b\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^5/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(4*Sqrt[Sec[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(4*a*(12*a^3 +
 12*a^2*b + 11*a*b^2 + 11*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[
(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)] + b*(-48*a^3 - 12*a^2*b - 44*a*b^2 + 25*b^3)
*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
+ 2*a*(12*a^2 + 11*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2]))/(105*b^4*d*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[a + b*Sec[c
 + d*x]]) + ((b + a*Cos[c + d*x])*Sec[c + d*x]*((-8*a*(12*a^2 + 11*b^2)*Si
n[c + d*x])/(105*b^4) + (2*Sec[c + d*x]*(24*a^2*Sin[c + d*x] + 25*b^2*Sin[
c + d*x]))/(105*b^3) - (12*a*Sec[c + d*x]*Tan[c + d*x])/(35*b^2) + (2*Sec[
c + d*x]^2*Tan[c + d*x])/(7*b)))/(d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {3042, 4347, 3042, 4580, 27, 3042, 4570, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4347}$$

$$\frac{\int \frac{\sec^2(c+dx)(-6a\sec^2(c+dx)+5b\sec(c+dx)+4a)}{\sqrt{a+b\sec(c+dx)}}dx}{7b}+\frac{2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(-6a\csc\left(c+dx+\frac{\pi}{2}\right)^2+5b\csc\left(c+dx+\frac{\pi}{2}\right)+4a\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{7b}+\frac{2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{7bd}$$

$$\Big\downarrow \text{4580}$$

$$\frac{\frac{2\int -\frac{\sec(c+dx)\left(12a^2-2b\sec(c+dx)a-\left(24a^2+25b^2\right)\sec^2(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{5b}-\frac{12a\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}}{7b}+\frac{2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{7bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{\sec(c+dx)\left(12a^2-2b\sec(c+dx)a-\left(24a^2+25b^2\right)\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{5b}-\frac{12a\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}}{7b}+\frac{2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{7bd}$$

$\downarrow$ 3042

$$\frac{-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(12a^2-2b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(-24a^2-25b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b}-\frac{12a\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}}{7b}+\frac{2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{7bd}$$

$\downarrow$ 4570

$$\frac{-\frac{\frac{2\int \frac{\sec(c+dx)\left(b\left(12a^2-25b^2\right)+4a\left(12a^2+11b^2\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{3b}-\frac{2\left(24a^2+25b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{5b}-\frac{12a\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}}{7b}+\frac{2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{7bd}$$

$\downarrow$ 27

$$\frac{-\frac{\frac{\int \frac{\sec(c+dx)\left(b\left(12a^2-25b^2\right)+4a\left(12a^2+11b^2\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{3b}-\frac{2\left(24a^2+25b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{5b}-\frac{12a\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}}{7b}+\frac{2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{7bd}$$

$\downarrow$ 3042

$$\frac{-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(b\left(12a^2-25b^2\right)+4a\left(12a^2+11b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}-\frac{2\left(24a^2+25b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{5b}-\frac{12a\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}}{7b}+\frac{2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{7bd}$$

$\downarrow$ 4493

$$\frac{-\frac{\frac{4a\left(12a^2+11b^2\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx-\left(48a^3-12a^2b+44ab^2+25b^3\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{3b}-\frac{2\left(24a^2+25b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{5b}-\frac{12a\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}}{7b}+\frac{2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{7bd}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{4a(12a^2+11b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-(48a^3-12a^2b+44ab^2+25b^3)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}-\frac{2(24a^2+25b^2)\tan(c+dx)\sqrt{a+b\sec(c+dx}}{3bd}}{5b}}{7b}$$

$$\frac{2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{7bd}$$

$\Big\downarrow$ 4319

$$-\frac{\frac{\frac{4a(12a^2+11b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}(48a^3-12a^2b+44ab^2+25b^3)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}(\text{arc}}{bd}}{3b}}{5b}}{7b}$$

$$\frac{2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{7bd}$$

$\Big\downarrow$ 4492

$$-\frac{\frac{-\frac{8a(a-b)\sqrt{a+b}(12a^2+11b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2\sqrt{a+b}(48a^3-12a^2b+44ab^2+25b^3)\cot}{}}{3b}}{5b}$$

$$\frac{2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{7bd}$$

input
```
Int[Sec[c + d*x]^5/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(7*b*d) + ((-12*a
*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(5*b*d) - (((-8*a*(a
- b)*Sqrt[a + b]*(12*a^2 + 11*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a + b
]*(48*a^3 - 12*a^2*b + 44*a*b^2 + 25*b^3)*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d))/(3*b) - (
2*(24*a^2 + 25*b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*b*d))/(5*b))
/(7*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4347
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*(S
qrt[a + b*Csc[e + f*x]]/(b*f*(2*n - 3))), x] + Simp[d^3/(b*(2*n - 3))   Int
[((d*Csc[e + f*x])^(n - 3)/Sqrt[a + b*Csc[e + f*x]])*Simp[2*a*(n - 3) + b*(
2*n - 5)*Csc[e + f*x] - 2*a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

rule 4580
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(-C)*Csc[e + f*x]*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m +
1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[Csc[e + f*x]*(a + b*Csc[e
+ f*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*
(m + 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &
& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1030 vs. $2(325) = 650$.

Time = 22.67 (sec) , antiderivative size = 1031, normalized size of antiderivative = 2.87

method	result	size
default	Expression too large to display	1031

input
```
int(sec(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/105/d/b^4*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+b*cos(d*x+
c)+b)*(48*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^4*EllipticE(cot(d*x+c)-
csc(d*x+c),((a-b)/(a+b))^(1/2))+48*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
a^3*b*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+44*(-cos(d*x+c)
^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/
(a+b))^(1/2))+44*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^3*EllipticE(co
t(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+48*(cos(d*x+c)^2+2*cos(d*x+c)+1)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*a^3*b*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+12*(cos
(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b^2*EllipticF(cot(d*x+c)-csc(d*x+c),
((a-b)/(a+b))^(1/2))+44*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^3*Ellipt
icF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+25*(-cos(d*x+c)^2-2*cos(d*x
+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*b^4*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))...
```

Fricas [F]

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^5}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^5/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^5{(c+dx)}}{\sqrt{a+b\sec{(c+dx)}}}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**5/sqrt(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec{(dx+c)}^5}{\sqrt{b\sec{(dx+c)}+a}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^5/sqrt(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec{(dx+c)}^5}{\sqrt{b\sec{(dx+c)}+a}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^5/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^5\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^5}{\sec\left(dx+c\right)b+a}dx$$

input
```
int(sec(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**5)/(sec(c + d*x)*b + a),x)
```

3.555 $\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 301

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx =$$

$$-\frac{2(a-b)\sqrt{a+b}(8a^2+9b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{15b^4d}$$

$$-\frac{2\sqrt{a+b}(8a^2-2ab+9b^2)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+s}{}}}{15b^3d}$$

$$-\frac{8a\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15b^2d}+\frac{2\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5bd}$$

output

```
-2/15*(a-b)*(a+b)^(1/2)*(8*a^2+9*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c)
)^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-
b*(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d-2/15*(a+b)^(1/2)*(8*a^2-2*a*b+9*b^2)*c
ot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2)
)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d-8/1
5*a*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^2/d+2/5*sec(d*x+c)*(a+b*sec(d*x+c)
)^(1/2)*tan(d*x+c)/b/d
```

Mathematica [A] (warning: unable to verify)

Time = 9.38 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.21

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{2\sqrt{\sec(c+dx)}\left(-\frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2\left(8a^3+8a^2b+9ab^2+9b^3\right)E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{\frac{a+b\sec(c+}{(a+b)(1+\sec(c}}}{}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^4/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[Sec[c + d*x]]*(-((Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(8*a^3
+ 8*a^2*b + 9*a*b^2 + 9*b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(
1 + Sec[c + d*x]))] - 2*b*(8*a^2 + 2*a*b + 9*b^2)*EllipticF[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec
[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] + (8*a^2 + 9*b^2)*Cos[c + d*x]*(b
 + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/Sqrt[Sec[(c + d*x
)/2]^2]) + (b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*((8*a^2 + 9*b^2)*Sin[c
+ d*x] + b*(-4*a + 3*b*Sec[c + d*x])*Tan[c + d*x])))/(15*b^3*d*Sqrt[a + b*
Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4347, 3042, 4570, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$\Big\downarrow \ 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4347}$$

$$\frac{\int \frac{\sec(c+dx)\left(-4a\sec^2(c+dx)+3b\sec(c+dx)+2a\right)}{\sqrt{a+b\sec(c+dx)}}dx}{5b}+\frac{2\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(-4a\csc\left(c+dx+\frac{\pi}{2}\right)^2+3b\csc\left(c+dx+\frac{\pi}{2}\right)+2a\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b}+\frac{2\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}$$

$$\Big\downarrow \text{4570}$$

$$\frac{\frac{2\int \frac{\sec(c+dx)\left(2ab+\left(8a^2+9b^2\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{3b}-\frac{8a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{5b}+\frac{2\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\sec(c+dx)\left(2ab+\left(8a^2+9b^2\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{3b}-\frac{8a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{5b}+\frac{2\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2ab+\left(8a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}-\frac{8a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{5b}+\frac{2\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}$$

$$\Big\downarrow \text{4493}$$

$$\frac{\frac{\left(8a^2+9b^2\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx-\left(8a^2-2ab+9b^2\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{3b}-\frac{8a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{5b}+\frac{2\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(8a^2+9b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-(8a^2-2ab+9b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}-\frac{8a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{5b}+\frac{2\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}$$

$\Big\downarrow$ 4319

$$\frac{\frac{(8a^2+9b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(8a^2-2ab+9b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right.\right.}{bd}}{3b}}{5b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}$$

$\Big\downarrow$ 4492

$$\frac{\frac{-\frac{2\sqrt{a+b}\left(8a^2-2ab+9b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-\frac{2(a-b)\sqrt{a+b}\left(8a^2+9b^2\right)\cot(c+dx)\sqrt{\frac{b}{}}}{}}{3b}}{5b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}$$

input
```
Int[Sec[c + d*x]^4/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(5*b*d) + (((-2*(a
 - b)*Sqrt[a + b]*(8*a^2 + 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a + b]*
(8*a^2 - 2*a*b + 9*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d))/(3*b) - (8*a*Sqrt[a + b*Sec
[c + d*x]]*Tan[c + d*x])/(3*b*d))/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4347
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*(S
qrt[a + b*Csc[e + f*x]]/(b*f*(2*n - 3))), x] + Simp[d^3/(b*(2*n - 3))   Int
[((d*Csc[e + f*x])^(n - 3)/Sqrt[a + b*Csc[e + f*x]])*Simp[2*a*(n - 3) + b*(
2*n - 5)*Csc[e + f*x] - 2*a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 857 vs. $2(271) = 542$.

Time = 16.87 (sec) , antiderivative size = 858, normalized size of antiderivative = 2.85

method	result	size
default	Expression too large to display	858

input
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d/b^3*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+b*cos(d*x+c
)+b)*(8*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*EllipticE(cot(d*x+c)-csc
(d*x+c),((a-b)/(a+b))^(1/2))+8*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b
*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+9*(cos(d*x+c)^2+2*co
s(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*a*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1
/2))+9*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^3*EllipticE(cot(d*x+c)-csc(
d*x+c),((a-b)/(a+b))^(1/2))+8*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b
*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+2*(-cos(d*x+c)^2-2*c
os(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*a*b^2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(
1/2))+9*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^3*EllipticF(cot(d*x+c)-cs
c(d*x+c),((a-b)/(a+b))^(1/2))+8*a^3*cos(d*x+c)*sin(d*x+c)+4*sin(d*x+c)*(1-
cos(d*x+c))*a^2*b+(9*cos(d*x+c)^2-cos(d*x+c)-1)*a*b^2*tan(d*x+c)+3*b^3*(3*
sin(d*x+c)+tan(d*x+c)+sec(d*x+c)*tan(d*x+c)))
```

Fricas [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^4}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^4\left(c+dx\right)}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**4/sqrt(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^4}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^4}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)^4\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)^4}{\sec(dx+c)\,b+a}dx$$

input
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**4)/(sec(c + d*x)*b + a),x)
```

3.556 $\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 244

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{4a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3b^3d} + \frac{2\sqrt{a+b}(2a+b)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3b^2d} + \frac{2\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3bd}$$

output
```
4/3*a*(a-b)*(a+b)^(1/2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^
(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c
))/(a-b))^(1/2)/b^3/d+2/3*(a+b)^(1/2)*(2*a+b)*cot(d*x+c)*EllipticF((a+b*se
c(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^
(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d+2/3*(a+b*sec(d*x+c))^(1/2)*tan
(d*x+c)/b/d
```

Mathematica [A] (warning: unable to verify)

Time = 11.08 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.40

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{4\sqrt{\sec(c+dx)}\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2a(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}\right.\right.\right.\right.}{}$$
$$+\frac{(b+a\cos(c+dx))\sec(c+dx)\left(-\frac{4a\sin(c+dx)}{3b^2}+\frac{2\tan(c+dx)}{3b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(4*Sqrt[Sec[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*a*(a + b)*S
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] -
(2*a - b)*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)] + a*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2]))/(3*b^2*d*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[a + b*Sec[c + d*x]]) +
 ((b + a*Cos[c + d*x])*Sec[c + d*x]*((-4*a*Sin[c + d*x])/(3*b^2) + (2*Tan[
c + d*x])/(3*b)))/(d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 4327, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4327}$$

$$\frac{2\int \frac{\sec(c+dx)(b-2a\sec(c+dx))}{2\sqrt{a+b\sec(c+dx)}}dx}{3b}+\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sec(c+dx)(b-2a\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}dx}{3b}+\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(b-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}+\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}$$

$$\Big\downarrow \text{4493}$$

$$\frac{(2a+b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-2a\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx}{3b}+\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(2a+b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-2a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}+\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}$$

$$\Big\downarrow \text{4319}$$

$$\frac{\frac{2\sqrt{a+b}(2a+b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-2a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)\right.}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}}{3b}$$
$$\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}$$

$$\Big\downarrow \text{4492}$$

$$\frac{\frac{4a(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}+\frac{2\sqrt{a+b}(2a+b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{}}{}}{3b}$$
$$\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}$$

input
```
Int[Sec[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
((4*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
 d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) + (2*Sqrt[a + b]*(2*a +
b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
 d*x]))/(a - b))])/(b*d))/(3*b) + (2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]
)/(3*b*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4327
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 -
b^2, 0] &&  !LtQ[m, -1]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. 2(218) = 436.

Time = 13.66 (sec) , antiderivative size = 501, normalized size of antiderivative = 2.05

method	result
default	$\frac{2\sqrt{a+b\sec(dx+c)}\left(\left(-2\cos(dx+c)^2-4\cos(dx+c)-2\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\,a^2\operatorname{EllipticE}\left(\cot(dx+c)-\csc(dx+c),\sqrt{\right.\right.}{}$

input
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/b^2*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+b*cos(d*x+c)
+b)*((-2*cos(d*x+c)^2-4*cos(d*x+c)-2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*EllipticE(cot(d*x+c)-csc
(d*x+c),((a-b)/(a+b))^(1/2))+(-2*cos(d*x+c)^2-4*cos(d*x+c)-2)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b*
EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(2*cos(d*x+c)^2+4*cos
(d*x+c)+2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*a*b*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2)
)+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^2*EllipticF(cot(d*x+c)-csc(d*x+
c),((a-b)/(a+b))^(1/2))-2*a^2*cos(d*x+c)*sin(d*x+c)+sin(d*x+c)*(cos(d*x+c)
-1)*a*b+b^2*(sin(d*x+c)+tan(d*x+c)))
```

Fricas [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^3}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**3/sqrt(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^3}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^3}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^3\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)^3}{\sec(dx+c)\,b+a}dx$$

input
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**3)/(sec(c + d*x)*b + a),x)
```

3.557 $\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 204

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b^2d} - \frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{bd}$$

output
```
-2*(a-b)*(a+b)^(1/2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/
2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/
(a-b))^(1/2)/b^2/d-2*(a+b)^(1/2)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/
2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+
sec(d*x+c))/(a-b))^(1/2)/b/d
```

Mathematica [A] (verified)

Time = 13.13 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.17

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{(1+\cos(c+dx))\left(-2(a+b)\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\sec(c+dx)\sqrt{\frac{1}{1+\sec(c+dx)}}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
((1 + Cos[c + d*x])*(-2*(a + b)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[c +
 d*x]*Sqrt[(1 + Sec[c + d*x])^(-1)] + 2*b*EllipticF[ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)]*Sec[c + d*x]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a +
b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] - (b + a*Cos[c + d*x])*Sec[(
c + d*x)/2]^2*Tan[(c + d*x)/2]) + 2*(b + a*Cos[c + d*x])*Tan[c + d*x])/(b*
d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4324, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4324}$$

$$\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx-\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4319}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}$$

$$\Big\downarrow \text{4492}$$

$$-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}$$

input
```
Int[Sec[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a + b]*Cot[c + d
*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/(b*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4324
```
Int[csc[(e_.) + (f_.)*(x_)]^2/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x
_Symbol] :> -Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x] + Int[Csc[e + f*
x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, e, f},
 x] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Maple [A] (verified)

Time = 8.39 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.78

method	result
default	$\frac{2\left(\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,a\,\mathrm{EllipticE}\left(\cot(dx+c)-\csc(dx+c),\sqrt{\frac{a-b}{a+b}}\right)+\left(\cos(dx+c)^2+\right.\right.}{}$

input
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/b*((cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*EllipticE(cot(d*x+c)-csc(d*
x+c),((a-b)/(a+b))^(1/2))+(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b*Elliptic
E(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*b*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+sin(d*x+c
)*a*cos(d*x+c)+b*sin(d*x+c))*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(
d*x+c)+b*cos(d*x+c)+b)
```

Fricas [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\sec\left(dx+c\right)^2}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\sec^2\left(c+dx\right)}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**2/sqrt(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^2}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^2}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)^2\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)^2}{\sec(dx+c)\,b+a}dx$$

input
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**2)/(sec(c + d*x)*b + a),x)
```

3.558 $\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 99

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{bd}$$

output
```
2*(a+b)^(1/2)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+
b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^
(1/2)/b/d
```

Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.94

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{a-b}{a+b}\right)}{d\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{a+b\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSi
n[Tan[(c + d*x)/2]], (a - b)/(a + b)])/(d*Sqrt[Cos[c + d*x]/(1 + Cos[c + d
*x])]*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 4319}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4319}$$

$$\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}$$

input
```
Int[Sec[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt
[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(b*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 6.32 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.15

method	result	size
default	$-\frac{2(\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\,\operatorname{EllipticF}\left(\cot(dx+c)-\csc(dx+c),\sqrt{\frac{a-b}{a+b}}\right)\sqrt{a+b\sec(dx+c)}}{d(b+a\cos(dx+c))}$	114

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^
(1/2))*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))
```

Fricas [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\sec\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\sec\left(c+dx\right)}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\sec\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\sec\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)}{\sec(dx+c)\,b+a}dx$$

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sec(c + d*x))/(sec(c + d*x)*b + a),x)
```

3.559 $\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 106

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ad}$$

output
```
-2*(a+b)^(1/2)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a
+b)/a,((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c
))/(a-b))^(1/2)/a/d
```

Mathematica [A] (verified)

Time = 2.03 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\left(\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{a-b}{a+b}\right)-2\,\text{Ellipt}\right.}{d\sqrt{a+b\sec(c+dx)}}$$

input
```
Integrate[1/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(-4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*(EllipticF[ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)] - 2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)])*Sec[c + d*x])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{3042, 4271\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4271}$$

$$-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}$$

input
```
Int[1/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(-2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 4.62 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.41

method	result
default	$-\frac{2\left(2\,\mathrm{EllipticPi}\left(\cot(dx+c)-\csc(dx+c),-1,\sqrt{\frac{a-b}{a+b}}\right)-\mathrm{EllipticF}\left(\cot(dx+c)-\csc(dx+c),\sqrt{\frac{a-b}{a+b}}\right)\right)(\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\cdots}}{d(b+a\cos(dx+c))}$

input
```
int(1/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))-EllipticF
(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2)))*(cos(d*x+c)+1)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(a+b*s
ec(d*x+c))^(1/2)/(b+a*cos(d*x+c))
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(1/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{a+b\sec{(c+dx)}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\sec{(dx+c)}+a}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\sec{(dx+c)}+a}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(1/(a + b/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)\,b+a}dx$$

input
```
int(1/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)/(sec(c + d*x)*b + a),x)
```

3.560 $\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 338

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{abd} + \frac{\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ad} + \frac{b\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a^2d} + \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{ad}$$

output
```
(a-b)*(a+b)^(1/2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),
((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-
b))^(1/2)/a/b/d+(a+b)^(1/2)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a
+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d
*x+c))/(a-b))^(1/2)/a/d+b*(a+b)^(1/2)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c
))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))
^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+(a+b*sec(d*x+c))^(1/2)*sin(d*
x+c)/a/d
```

Mathematica [A] (warning: unable to verify)

Time = 5.73 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.71

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{2\sqrt{\frac{\cos(c+dx)}{(1+\cos(c+dx))^2}}\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left((a+b)E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{(b+a\cos(c+dx}{}}\right.}{}$$

input
```
Integrate[Cos[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])^2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x
])^(3/2)*((a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqr
t[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - 2*b*EllipticPi[-1,
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[
(c + d*x)/2]^2)/(a + b)] + (b + a*Cos[c + d*x])*Sqrt[Cos[c + d*x]*Sec[(c +
 d*x)/2]^2]*Tan[(c + d*x)/2]))/(a*d*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c +
d*x]])
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4348, 3042, 4547, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4348}$$

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{b\int\frac{\sec^2(c+dx)+1}{\sqrt{a+b\sec(c+dx)}}dx}{2a}$$

$\downarrow$ 3042

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}$$

$\downarrow$ 4547

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{b\left(\int\frac{1-\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\int\frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx\right)}{2a}$$

$\downarrow$ 3042

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{b\left(\int\frac{1-\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a}$$

$\downarrow$ 4409

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{b\left(\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int\frac{1}{\sqrt{a+b\sec(c+dx)}}dx-\int\frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx\right)}{2a}$$

$\downarrow$ 3042

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{b\left(\int\frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a}$$

$\downarrow$ 4271

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{b\left(-\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{Elliptic}}{ad}\right.}{2a}$$

$\downarrow$ 4319

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{b\left(\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}\right.}{2a}$$

$\downarrow$ 4492

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{b\left(-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-}\right.}{}$$

input
```
Int[Cos[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
-1/2*(b*((-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a + b]*
Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*
x]))/(a - b))])/(b*d) - (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a,
ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d)))
/a + (Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(a*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4348
```
Int[1/(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]),
 x_Symbol] :> Simp[(-Cos[e + f*x])*(Sqrt[a + b*Csc[e + f*x]]/(a*f)), x] - S
imp[b/(2*a)   Int[(1 + Csc[e + f*x]^2)/Sqrt[a + b*Csc[e + f*x]], x], x] /;
FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4547
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Simp[C   Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f
*x]]), x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 7.37 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.11

method	result
default	$\frac{\left(\left(2\cos(dx+c)^2+4\cos(dx+c)+2\right)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\, b\, \mathrm{EllipticPi}\left(\cot(dx+c)-\csc(dx+c),-1,\sqrt{\frac{a-b}{a+b}}\right)+\left(-\cos(dx\right.\right.}{}$

input
```
int(cos(d*x+c)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*((2*cos(d*x+c)^2+4*cos(d*x+c)+2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b*EllipticPi(cot(d*x+c)-csc
(d*x+c),-1,((a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*E
llipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d
*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*b*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+si
n(d*x+c)*cos(d*x+c)^2*a+sin(d*x+c)*cos(d*x+c)*b)*(a+b*sec(d*x+c))^(1/2)/(c
os(d*x+c)^2*a+a*cos(d*x+c)+b*cos(d*x+c)+b)
```

Fricas [F]

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\, dx = \int \frac{\cos\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)+a}}\, dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(cos(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\cos\left(c+dx\right)}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\cos\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\cos\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(cos(c + d*x)/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)/(a + b/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)b+a}\,\cos\left(dx+c\right)}{\sec\left(dx+c\right)b+a}dx$$

input
```
int(cos(d*x+c)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*cos(c + d*x))/(sec(c + d*x)*b + a),x)
```

3.561 $\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 401

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{3(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4a^2d}$$
$$+\frac{(2a-3b)\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4a^2d}$$
$$-\frac{\sqrt{a+b}(4a^2+3b^2)\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4a^3d}$$
$$-\frac{3b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4a^2d}+\frac{\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2ad}$$

output
```
-3/4*(a-b)*(a+b)^(1/2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(
1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c)
)/(a-b))^(1/2)/a^2/d+1/4*(2*a-3*b)*(a+b)^(1/2)*cot(d*x+c)*EllipticF((a+b*s
ec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))
^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d-1/4*(a+b)^(1/2)*(4*a^2+3*b^2)
*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(
a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2
)/a^3/d-3/4*b*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/a^2/d+1/2*cos(d*x+c)*(a+b*
sec(d*x+c))^(1/2)*sin(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 12.34 (sec) , antiderivative size = 682, normalized size of antiderivative = 1.70

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{(b+a\cos(c+dx))\sec(c+dx)\sin(2(c+dx))}{4ad\sqrt{a+b\sec(c+dx)}} - \frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(24b(a+b)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\right.\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
((b + a*Cos[c + d*x])*Sec[c + d*x]*Sin[2*(c + d*x)])/(4*a*d*Sqrt[a + b*Sec
[c + d*x]]) - (Sec[(c + d*x)/2]*Sec[c + d*x]*(24*b*(a + b)*Cos[(c + d*x)/2
]^3*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)] + 16*a*(2*a - b)*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]
)]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSi
n[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 48*a^2*Cos[(c + d*x)/2]*EllipticPi
[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-
1)]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] - 36*b^2*Cos[(
c + d*x)/2]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt
[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c +
d*x]))] - 16*a^2*Cos[(3*(c + d*x))/2]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/
2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c + d*
x])/((a + b)*(1 + Sec[c + d*x]))] - 12*b^2*Cos[(3*(c + d*x))/2]*EllipticPi
[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-
1)]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] + 6*a*b*Sin[(c
 + d*x)/2] - 6*b^2*Sin[(c + d*x)/2] - 3*a*b*Sin[(3*(c + d*x))/2] + 6*b^2*S
in[(3*(c + d*x))/2] + 3*a*b*Sin[(5*(c + d*x))/2]))/(16*a^2*d*Sqrt[a + b*Se
c[c + d*x]])
```

Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4350, 3042, 4592, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4350}$$

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2ad}-\frac{\int \frac{\cos(c+dx)\left(-b\sec^2(c+dx)-2a\sec(c+dx)+3b\right)}{\sqrt{a+b\sec(c+dx)}}dx}{4a}$$

$\big\downarrow$ 3042

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2ad}-\frac{\int \frac{-b\csc\left(c+dx+\frac{\pi}{2}\right)^2-2a\csc\left(c+dx+\frac{\pi}{2}\right)+3b}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{4a}$$

$\big\downarrow$ 4592

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2ad}-\frac{\frac{3b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{\int \frac{4a^2+2b\sec(c+dx)a+3b^2+3b^2\sec^2(c+dx)}{2\sqrt{a+b\sec(c+dx)}}dx}{a}}{4a}$$

$\big\downarrow$ 27

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2ad}-\frac{\frac{3b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{\int \frac{4a^2+2b\sec(c+dx)a+3b^2+3b^2\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{2a}}{4a}$$

$\big\downarrow$ 3042

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2ad}-\frac{\frac{3b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{\int \frac{4a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a+3b^2+3b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}}{4a}$$

$\big\downarrow$ 4546

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2ad}-\frac{\frac{3b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{\int \frac{4a^2+3b^2+\left(2ab-3b^2\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+3b^2\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx}{2a}}{4a}$$

$\big\downarrow$ 3042

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2ad}-\frac{\frac{3b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{\int \frac{4a^2+3b^2+\left(2ab-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}}{4a}$$

$\big\downarrow$ 4409

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2ad}-\frac{\frac{3b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{(4a^2+3b^2)\int\frac{1}{\sqrt{a+b\sec(c+dx)}}dx+3b^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(2a-3b)\int\frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{2a}}{4a}$$

$\downarrow$ 3042

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2ad}-\frac{\frac{3b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{(4a^2+3b^2)\int\frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(2a-3b)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}}{2a}}{4a}$$

$\downarrow$ 4271

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2ad}-\frac{\frac{3b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{3b^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(2a-3b)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(4a^2+3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-}{}}}{}}{2a}}{4a}$$

$\downarrow$ 4319

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2ad}-\frac{\frac{3b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{3b^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(4a^2+3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{Elliptic}}{ad}}{}}{4a}$$

$\downarrow$ 4492

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2ad}-\frac{\frac{3b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{-\frac{2\sqrt{a+b}\left(4a^2+3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}+}{}}{}$$

input
```
Int[Cos[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*a*d) - (-1/2*((-6*
(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*(2*a - 3*b)*Sqrt[a + b]*Cot[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/d - (2*Sqrt[a + b]*(4*a^2 + 3*b^2)*Cot[c + d*x]*EllipticPi[(a + b
)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a
*d))/a + (3*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(a*d))/(4*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4350
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1)*(Sqrt[a +
 b*Csc[e + f*x]]/(a*d*f*n)), x] + Simp[1/(2*a*d*n)   Int[((d*Csc[e + f*x])^
(n + 1)/Sqrt[a + b*Csc[e + f*x]])*Simp[(-b)*(2*n + 1) + 2*a*(n + 1)*Csc[e +
 f*x] + b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 721 vs. 2(360) = 720.

Time = 8.51 (sec) , antiderivative size = 722, normalized size of antiderivative = 1.80

method	result
default	$\frac{\left(\left(-8\cos(dx+c)^2-16\cos(dx+c)-8\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\,a^2\,\text{EllipticPi}\left(\cot(dx+c)-\csc(dx+c),-1,\sqrt{\frac{a-b}{a+b}}\right)+\left(-6\cos\right.\right.}{}$

input
```
int(cos(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^2*((-8*cos(d*x+c)^2-16*cos(d*x+c)-8)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*EllipticPi(cot(d*
x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))+(-6*cos(d*x+c)^2-12*cos(d*x+c)-6)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*b^2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))+(3*co
s(d*x+c)^2+6*cos(d*x+c)+3)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b*EllipticE(cot(d*x+c)-csc(d*x+c),((a
-b)/(a+b))^(1/2))+(3*cos(d*x+c)^2+6*cos(d*x+c)+3)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^2*EllipticE(co
t(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(4*cos(d*x+c)^2+8*cos(d*x+c)+4)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*a^2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(-2*cos(d*
x+c)^2-4*cos(d*x+c)-2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/
(a+b))^(1/2))+sin(d*x+c)*cos(d*x+c)^2*(2*cos(d*x+c)+2)*a^2+sin(d*x+c)*cos(
d*x+c)*(-cos(d*x+c)+2)*a*b-3*b^2*cos(d*x+c)*sin(d*x+c))*(a+b*sec(d*x+c))^(
1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+b*cos(d*x+c)+b)
```

Fricas [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^2}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(cos(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos^2{(c+dx)}}{\sqrt{a+b\sec{(c+dx)}}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)**2/sqrt(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos{(dx+c)}^2}{\sqrt{b\sec{(dx+c)}+a}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos{(dx+c)}^2}{\sqrt{b\sec{(dx+c)}+a}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos(c+d\,x)^2}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(cos(c + d*x)^2/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^2/(a + b/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}\,\cos(dx+c)^2}{\sec(dx+c)\,b+a}dx$$

input
```
int(cos(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*cos(c + d*x)**2)/(sec(c + d*x)*b + a),x)
```

3.562 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 399

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2(16a^4-8a^2b^2-3b^4)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{5b^5\sqrt{a+b}d} - \frac{2(4a+3b)(4a^2+b^2)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{5b^4\sqrt{a+b}d} - \frac{2a^2\sec^2(c+dx)\tan(c+dx)}{b(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} - \frac{2a(8a^2-3b^2)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5b^3(a^2-b^2)d} + \frac{2(6a^2-b^2)\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5b^2(a^2-b^2)d}$$

output
```
-2/5*(16*a^4-8*a^2*b^2-3*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/
(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec
(d*x+c))/(a-b))^(1/2)/b^5/(a+b)^(1/2)/d-2/5*(4*a+3*b)*(4*a^2+b^2)*cot(d*x+
c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1
-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^4/(a+b)^(1/2)/
d-2*a^2*sec(d*x+c)^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)-2/5*a
*(8*a^2-3*b^2)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^3/(a^2-b^2)/d+2/5*(6*a^
2-b^2)*sec(d*x+c)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^2/(a^2-b^2)/d
```

Mathematica [A] (warning: unable to verify)

Time = 10.98 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.14

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\left(\frac{2(4a^2+b^2)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2(4a^3+4a^2b-3a\right.}{}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
((b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*((2*(4*a^2 + b^2)*Sqrt[Cos[(c + d
*x)/2]^2*Sec[c + d*x]]*(2*(4*a^3 + 4*a^2*b - 3*a*b^2 - 3*b^3)*EllipticE[Ar
cSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqr
t[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] + 2*b*(-4*a^2 - a*b +
 3*b^2)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec
[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] +
 (4*a^2 - 3*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[
(c + d*x)/2]))/(b^4*(-a^2 + b^2)*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Sec[c
 + d*x]]*((-8*a^4*b + 5*a^2*b^3 + 3*b^5)*Sin[c + d*x] + (-8*a^5 + 4*a^3*b^
2 + (3*a*b^4)/2)*Sin[2*(c + d*x)] + b^2*(-a^2 + b^2)*(b - 2*a*Cos[c + d*x]
)*Sec[c + d*x]*Tan[c + d*x]))/(-(a^2*b^4) + b^6)))/(5*d*(a + b*Sec[c + d*x
])^(3/2))
```

Rubi [A] (verified)

Time = 1.79 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4332, 27, 3042, 4580, 27, 3042, 4570, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4332}$$

$$-\frac{2\int \frac{\sec^2(c+dx)\left(4a^2-b\sec(c+dx)a-\left(6a^2-b^2\right)\sec^2(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sec^2(c+dx)\left(4a^2-b\sec(c+dx)a-\left(6a^2-b^2\right)\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(4a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(b^2-6a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{4580}$$

$$-\frac{\frac{2\int -\frac{\sec(c+dx)\left(-3a\left(8a^2-3b^2\right)\sec^2(c+dx)-b\left(2a^2+3b^2\right)\sec(c+dx)+2a\left(6a^2-b^2\right)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{5b}-\frac{2\left(6a^2-b^2\right)\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{\sec(c+dx)\left(-3a\left(8a^2-3b^2\right)\sec^2(c+dx)-b\left(2a^2+3b^2\right)\sec(c+dx)+2a\left(6a^2-b^2\right)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{5b}-\frac{2\left(6a^2-b^2\right)\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(-3a\left(8a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-b\left(2a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+2a\left(6a^2-b^2\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b}-\frac{2\left(6a^2-b^2\right)\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 4570$$

$$-\frac{-\frac{\frac{2\int \frac{3\sec(c+dx)\left(ab\left(4a^2+b^2\right)+\left(16a^4-8b^2a^2-3b^4\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{3b}-\frac{2a\left(8a^2-3b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{bd}}{5b}-\frac{2\left(6a^2-b^2\right)\tan(c+dx)\sec(c+dx)\sqrt{a}}{5bd}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\frac{\int \frac{\sec(c+dx)\left(ab\left(4a^2+b^2\right)+\left(16a^4-8b^2a^2-3b^4\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{b}-\frac{2a\left(8a^2-3b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{bd}}{5b}-\frac{2\left(6a^2-b^2\right)\tan(c+dx)\sec(c+dx)\sqrt{a+b}}{5bd}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(ab\left(4a^2+b^2\right)+\left(16a^4-8b^2a^2-3b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2a\left(8a^2-3b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{bd}}{5b}-\frac{2\left(6a^2-b^2\right)\tan(c+dx)\sec(c+d}{5bd}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 4493$$

$$-\frac{-\frac{\frac{\left(16a^4-8a^2b^2-3b^4\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx-(a-b)(4a+3b)\left(4a^2+b^2\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{b}-\frac{2a\left(8a^2-3b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{bd}}{5b}-2\left(6a^2-\right.}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{\frac{\left(16a^4-8a^2b^2-3b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)(4a+3b)\left(4a^2+b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2a\left(8a^2-3b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+d}}{bd}}{5b}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 4319$$

$$-\frac{-\frac{\frac{\left(16a^4-8a^2b^2-3b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}(4a+3b)\left(4a^2+b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(a\right.}{bd}}{b}}{5b}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 4492$$

$$-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-$$

$$-\frac{2\left(6a^2-b^2\right)\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}-\frac{-\frac{2(a-b)\sqrt{a+b}(4a+3b)\left(4a^2+b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\text{arcs}\right.}{bd}}{}$$

input
```
Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*a^2*Sec[c + d*x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d
*x]]) - ((-2*(6*a^2 - b^2)*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d
*x])/(5*b*d) - (((-2*(a - b)*Sqrt[a + b]*(16*a^4 - 8*a^2*b^2 - 3*b^4)*Cot[
c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))
/(a - b))])/(b^2*d) - (2*(a - b)*Sqrt[a + b]*(4*a + 3*b)*(4*a^2 + b^2)*Cot
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/
(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x])
)/(a - b))])/(b*d))/b - (2*a*(8*a^2 - 3*b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[
c + d*x])/(b*d))/(5*b))/(b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

rule 4580
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(-C)*Csc[e + f*x]*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m +
1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[Csc[e + f*x]*(a + b*Csc[e
+ f*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*
(m + 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &
& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1356 vs. $2(367) = 734$.

Time = 20.22 (sec) , antiderivative size = 1357, normalized size of antiderivative = 3.40

method	result	size
default	Expression too large to display	1357

input
```
int(sec(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/d/(a-b)/(a+b)/b^4*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)
+b*cos(d*x+c)+b)*(16*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^5*EllipticE(
cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+16*(-cos(d*x+c)^2-2*cos(d*x+c)-
1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*a^4*b*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+8*(c
os(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2*EllipticE(cot(d*x+c)-csc(d*x+c
),((a-b)/(a+b))^(1/2))+8*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3*Ell
ipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+3*(cos(d*x+c)^2+2*cos(d*
x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*a*b^4*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))
+3*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^5*EllipticE(cot(d*x+c)-csc(d*x+
c),((a-b)/(a+b))^(1/2))+16*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^4*b*Ell
ipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+4*(cos(d*x+c)^2+2*cos(d*
x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*a^3*b^2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(...
```

Fricas [F]

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec{(dx+c)}^5}{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^5/(b^2*sec(d*x + c)^2 + 2*a
*b*sec(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^5{(c+dx)}}{(a+b\sec{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**5/(a + b*sec(c + d*x))**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec{(dx+c)}^5}{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^5/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^5\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^5}{\sec\left(dx+c\right)^2 b^2+2\sec\left(dx+c\right)ab+a^2}dx$$

input
```
int(sec(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**5)/(sec(c + d*x)**2*b**2 + 2*s
ec(c + d*x)*a*b + a**2),x)
```

3.563 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 325

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2a(8a^2-5b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\cdots}}{3b^4\sqrt{a+b}d}$$
$$+\frac{2(2a+b)(4a+b)\cot(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3b^3\sqrt{a+b}d}$$
$$-\frac{2a^2\sec(c+dx)\tan(c+dx)}{b(a^2-b^2)d\sqrt{a+b\sec(c+dx)}}+\frac{2(4a^2-b^2)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3b^2(a^2-b^2)d}$$

output
```
2/3*a*(8*a^2-5*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2
),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(
a-b))^(1/2)/b^4/(a+b)^(1/2)/d+2/3*(2*a+b)*(4*a+b)*cot(d*x+c)*EllipticF((a+
b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+
b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/(a+b)^(1/2)/d-2*a^2*sec(d*x+
c)*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+2/3*(4*a^2-b^2)*(a+b*se
c(d*x+c))^(1/2)*tan(d*x+c)/b^2/(a^2-b^2)/d
```

Mathematica [A] (warning: unable to verify)

Time = 11.54 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.45

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx =$$
$$-\frac{2(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2a(8a^3+8a^2b-5ab^2-5b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\right.}{}$$
$$+\frac{(b+a\cos(c+dx))^2\sec^2(c+dx)\left(-\frac{2a(-8a^2+5b^2)\sin(c+dx)}{3b^3(-a^2+b^2)}-\frac{2a^3\sin(c+dx)}{b^2(-a^2+b^2)(b+a\cos(c+dx))}+\frac{2\tan(c+dx)}{3b^2}\right)}{d(a+b\sec(c+dx))^{3/2}}$$

input
```
Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
+ d*x]]*(2*a*(8*a^3 + 8*a^2*b - 5*a*b^2 - 5*b^3)*Sqrt[Cos[c + d*x]/(1 + Co
s[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellip
ticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(8*a^3 + 2*a^2*b - 5
*a*b^2 + b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*
x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)] + a*(8*a^2 - 5*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c
+ d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^3*(-a^2 + b^2)*d*Sqrt[Sec[(c + d*x)/2]
^2]*(a + b*Sec[c + d*x])^(3/2)) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*(
(-2*a*(-8*a^2 + 5*b^2)*Sin[c + d*x])/(3*b^3*(-a^2 + b^2)) - (2*a^3*Sin[c +
 d*x])/(b^2*(-a^2 + b^2)*(b + a*Cos[c + d*x])) + (2*Tan[c + d*x])/(3*b^2))
)/(d*(a + b*Sec[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4332, 27, 3042, 4570, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4332}$$

$$-\frac{2\int \frac{\sec(c+dx)(2a^2-b\sec(c+dx)a-(4a^2-b^2)\sec^2(c+dx))}{2\sqrt{a+b\sec(c+dx)}}dx}{b(a^2-b^2)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sec(c+dx)(2a^2-b\sec(c+dx)a-(4a^2-b^2)\sec^2(c+dx))}{\sqrt{a+b\sec(c+dx)}}dx}{b(a^2-b^2)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a+(b^2-4a^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b(a^2-b^2)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{4570}$$

$$-\frac{\frac{2\int \frac{\sec(c+dx)\left(b\left(2a^2+b^2\right)+a\left(8a^2-5b^2\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{3b}-\frac{2(4a^2-b^2)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{b(a^2-b^2)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{\sec(c+dx)\left(b\left(2a^2+b^2\right)+a\left(8a^2-5b^2\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{3b}-\frac{2(4a^2-b^2)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{b(a^2-b^2)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(b\left(2a^2+b^2\right)+a\left(8a^2-5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}-\frac{2\left(4a^2-b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{4493}$$

$$-\frac{\frac{a\left(8a^2-5b^2\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx-(a-b)(2a+b)(4a+b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{3b}-\frac{2\left(4a^2-b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{a\left(8a^2-5b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)(2a+b)(4a+b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}-\frac{2\left(4a^2-b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{4319}$$

$$-\frac{\frac{a\left(8a^2-5b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}(2a+b)(4a+b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{}}\right.\right.}{bd}}{3b}}{b\left(a^2-b^2\right)}$$

$$-\frac{2a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{4492}$$

$$-\frac{\frac{-\frac{2a(a-b)\sqrt{a+b}\left(8a^2-5b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2(a-b)\sqrt{a+b}(2a+b)(4a+b)\cot(c+dx)\sqrt{b(}}{}}{3b}}{b\left(a^2-b^2\right)}$$

$$-\frac{2a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*a^2*Sec[c + d*x]*Tan[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x
]]) - (((-2*a*(a - b)*Sqrt[a + b]*(8*a^2 - 5*b^2)*Cot[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d)
- (2*(a - b)*Sqrt[a + b]*(2*a + b)*(4*a + b)*Cot[c + d*x]*EllipticF[ArcSin
[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d))/(3*b)
- (2*(4*a^2 - b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*b*d))/(b*(a^2
 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 982 vs. $2(297) = 594$.

Time = 14.70 (sec) , antiderivative size = 983, normalized size of antiderivative = 3.02

method	result	size
default	Expression too large to display	983

input
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d/(a-b)/(a+b)/b^3*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)
+b*cos(d*x+c)+b)*(8*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^4*EllipticE(co
t(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+8*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*a^3*b*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+5*(-cos(
d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),(
(a-b)/(a+b))^(1/2))+5*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^3*Ellipti
cE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+8*(-cos(d*x+c)^2-2*cos(d*x+c
)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*a^3*b*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+2*
(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b^2*EllipticF(cot(d*x+c)-csc(d*
x+c),((a-b)/(a+b))^(1/2))+5*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^3*El
lipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*
x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*b^4*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2)...
```

Fricas [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^4}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^4/(b^2*sec(d*x + c)^2 + 2*a
*b*sec(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^4{(c+dx)}}{(a+b\sec{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec{(dx+c)}^4}{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^4\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^{4}}{\sec\left(dx+c\right)^{2}b^{2}+2\sec\left(dx+c\right)ab+a^{2}}dx$$

input
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**4)/(sec(c + d*x)**2*b**2 + 2*s
ec(c + d*x)*a*b + a**2),x)
```

3.564 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 257

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2(2a^2-b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b^3\sqrt{a+b}d}$$
$$-\frac{2(2a+b)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b^2\sqrt{a+b}d}$$
$$-\frac{2a^2\tan(c+dx)}{b(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}$$

output
```
-2*(2*a^2-b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a
+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))
^(1/2)/b^3/(a+b)^(1/2)/d-2*(2*a+b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(
1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(
1+sec(d*x+c))/(a-b))^(1/2)/b^2/(a+b)^(1/2)/d-2*a^2*tan(d*x+c)/b/(a^2-b^2)/
d/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 9.86 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.54

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx =$$
$$-\frac{2(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\left((-a^2b+b^3+(-2a^3+ab^2)\cos(c+dx))\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\sec(c+\right.}{}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*((-(a^2*b) + b^3 + (-2*a^3 + a
*b^2)*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Sec[c + d*x]]*Sin[c + d*
x] + Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(2*a^3 + 2*a^2*b - a*b^2 - b
^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)] + 2*b*(-2*a^2 - a*b + b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(
b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)] + (2*a^2 - b^2)*Cos[c + d*x]*(b + a*Cos[c + d
*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])))/(b^2*(a^2 - b^2)*d*Sqrt[Sec[(c
 + d*x)/2]^2]*(a + b*Sec[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4326, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4326}$$

$$-\frac{2\int -\frac{\sec(c+dx)\left(ab+\left(2a^2-b^2\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sec(c+dx)\left(ab+\left(2a^2-b^2\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(ab+\left(2a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 4493}$$

$$\frac{\left(2a^2-b^2\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx-(a-b)(2a+b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\left(2a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)(2a+b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 4319}$$

$$\frac{\left(2a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}(2a+b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\right.\right.}{bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 4492}$$

$$-\frac{-\frac{2(a-b)\sqrt{a+b}(2a^2-b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2(a-b)\sqrt{a+b}(2a+b)\cot(c+dx)\sqrt{}}{}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\tan(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
((-2*(a - b)*Sqrt[a + b]*(2*a^2 - b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*(a - b
)*Sqrt[a + b]*(2*a + b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d))/(b*(a^2 - b^2)) - (2*a^2*Ta
n[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4326
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-a^2)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m
 + 1)*(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]
*(a + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e
+ f*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 809 vs. $2(237) = 474$.

Time = 11.65 (sec) , antiderivative size = 810, normalized size of antiderivative = 3.15

method	result
default	$\frac{2\left(\left((1-\cos(dx+c))^3\csc(dx+c)^3+\csc(dx+c)-\cot(dx+c)\right)b^3+\left(2(1-\cos(dx+c))^3\csc(dx+c)^3-2\csc(dx+c)+2\cot(dx+c)\right)a^3-2a^2b}{}$

input
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/b^2/(a+b)/(a-b)*(((1-cos(d*x+c))^3*csc(d*x+c)^3+csc(d*x+c)-cot(d*x+c))
*b^3+(2*(1-cos(d*x+c))^3*csc(d*x+c)^3-2*csc(d*x+c)+2*cot(d*x+c))*a^3-2*a^2
*b*(1-cos(d*x+c))^3*csc(d*x+c)^3+(-(1-cos(d*x+c))^3*csc(d*x+c)^3+csc(d*x+c
)-cot(d*x+c))*b^2*a-2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b)
)^(1/2))*b^3-4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2)
)*a^3+2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*b^3+2
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+4*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+2*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-4*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b)*(a+b*sec(d*x+c))^(1/
2)/(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)
```

Fricas [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\sec\left(dx+c\right)^3}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3/(b^2*sec(d*x + c)^2 + 2*a
*b*sec(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^3}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^3}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^3\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)^3}{\sec(dx+c)^2\,b^2+2\sec(dx+c)\,ab+a^2}dx$$

input
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**3)/(sec(c + d*x)**2*b**2 + 2*s
ec(c + d*x)*a*b + a**2),x)
```

3.565 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 237

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2a\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx}{a-b}}}{b^2\sqrt{a+b}d}$$
$$+\frac{2\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b\sqrt{a+b}d}$$
$$+\frac{2a\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}$$

output
```
2*a*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^
(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/
(a+b)^(1/2)/d+2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((
a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b)
)^(1/2)/b/(a+b)^(1/2)/d+2*a*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 7.29 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.05

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(4a(a+b)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\right.}{}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(Sec[(c + d*x)/2]*Sec[c + d*x]*(4*a*(a + b)*Cos[(c + d*x)/2]^3*Sqrt[(b + a
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)] - 4*b*(a + b)*Cos[(c
 + d*x)/2]^3*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellip
ticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-
1)] + a*(a - b)*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2])))/(b*(a^2 - b^2)
*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4323, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4323}$$

$$\frac{2\int -\frac{\sec(c+dx)(b+a\sec(c+dx))}{2\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}+\frac{2a\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \ 27$$

$$\frac{2a\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{\sec(c+dx)(b+a\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{2a\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}$$

$$\Big\downarrow \ 4493$$

$$\frac{2a\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{a\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx-(a-b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{2a\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}$$

$$\Big\downarrow \ 4319$$

$$\frac{2a\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a}{a}\right.}{bd}}{a^2-b^2}$$

$$\Big\downarrow \ 4492$$

$$\frac{2a\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{2a(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{}}{a^2-b^2}$$

input
```
Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
-(((-2*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[
c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*(a - b)*Sqrt[a +
 b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
+ d*x]))/(a - b))])/(b*d))/(a^2 - b^2)) + (2*a*Tan[c + d*x])/((a^2 - b^2)*
d*Sqrt[a + b*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4323
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[a*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(
a^2 - b^2))), x] - Simp[1/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. 2(217) = 434.

Time = 7.47 (sec) , antiderivative size = 490, normalized size of antiderivative = 2.07

method	result
default	$2\left(\left(\left(1-\cos(dx+c)\right)^3\csc(dx+c)^3-\csc(dx+c)+\cot(dx+c)\right)ab+\left(-\left(1-\cos(dx+c)\right)^3\csc(dx+c)^3+\csc(dx+c)-\cot(dx+c)\right)a^2-2\sqrt{\frac{\mathrm{co}}{\cos}}$

input
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/b/(a+b)/(a-b)*(((1-cos(d*x+c))^3*csc(d*x+c)^3-csc(d*x+c)+cot(d*x+c))*a
*b+(-(1-cos(d*x+c))^3*csc(d*x+c)^3+csc(d*x+c)-cot(d*x+c))*a^2-2*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a*b-2*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF
(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*b^2+2*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*
x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^2+2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-cs
c(d*x+c),((a-b)/(a+b))^(1/2))*a*b)*(a+b*sec(d*x+c))^(1/2)/(a*(1-cos(d*x+c)
)^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)
```

Fricas [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\sec\left(dx+c\right)^2}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2/(b^2*sec(d*x + c)^2 + 2*a
*b*sec(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\sec^2\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\sec\left(dx+c\right)^2}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^2\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)^2}{\sec(dx+c)^2\,b^2+2\sec(dx+c)\,ab+a^2}dx$$

input
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**2)/(sec(c + d*x)**2*b**2 + 2*s
ec(c + d*x)*a*b + a**2),x)
```

3.566 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 236

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b\sqrt{a+b}d} + \frac{2\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b\sqrt{a+b}d} - \frac{2b\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}$$

output

```
-2*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(
1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/(a+
b)^(1/2)/d+2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b
)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(
1/2)/b/(a+b)^(1/2)/d-2*b*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 7.31 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.03

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx =$$
$$-\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(4(a+b)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{a-b}{a+b}\right.}{}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
-((Sec[(c + d*x)/2]*Sec[c + d*x]*(4*(a + b)*Cos[(c + d*x)/2]^3*Sqrt[(b + a
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)] - 4*(a + b)*Cos[(c +
 d*x)/2]^3*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipti
cF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)
] + (a - b)*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2])))/((a^2 - b^2)*d*Sqr
t[a + b*Sec[c + d*x]]))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4320, 27, 3042, 4316, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4320}$$

$$-\frac{2\int -\frac{1}{2}\sec(c+dx)\sqrt{a+b\sec(c+dx)}dx}{a^2-b^2}-\frac{2b\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 27

$$\frac{\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}dx}{a^2-b^2}-\frac{2b\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{2b\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 4316

$$\frac{(a-b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+b\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}-\frac{2b\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{(a-b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2b\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 4319

$$\frac{b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a}{a}\right.}{bd}}{a^2-b^2}$$

$$\frac{2b\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 4492

$$\frac{\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+}{a+b}}}{}}{a^2-b^2}$$

$$\frac{2b\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
((-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (2*(a - b)*Sqrt[a + b]*Co
t[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)
/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]
))/(a - b))])/(b*d))/(a^2 - b^2) - (2*b*Tan[c + d*x])/((a^2 - b^2)*d*Sqrt[
a + b*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4316
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(a - b)   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x]
+ Simp[b   Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]),
x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4320
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*
Csc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ
[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(216) = 432$.

Time = 3.60 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.03

method	result
default	$-\frac{2\left(\left(-(1-\cos(dx+c))^3\csc(dx+c)^3+\csc(dx+c)-\cot(dx+c)\right)a+\left((1-\cos(dx+c))^3\csc(dx+c)^3-\csc(dx+c)+\cot(dx+c)\right)b-2\sqrt{\frac{\mathrm{co}}{\mathrm{cos}}}}{}$

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/(a-b)/(a+b)*((-(1-cos(d*x+c))^3*csc(d*x+c)^3+csc(d*x+c)-cot(d*x+c))*a
+((1-cos(d*x+c))^3*csc(d*x+c)^3-csc(d*x+c)+cot(d*x+c))*b-2*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a-2*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*
x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*b+2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(
d*x+c),((a-b)/(a+b))^(1/2))*a+2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((
a-b)/(a+b))^(1/2))*b)*(a+b*sec(d*x+c))^(1/2)/(a*(1-cos(d*x+c))^2*csc(d*x+c
)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)
```

Fricas [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)/(b^2*sec(d*x + c)^2 + 2*a*b
*sec(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos(c+d\,x)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)}{\sec(dx+c)^2\,b^2+2\sec(dx+c)\,ab+a^2}dx$$

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sec(c + d*x))/(sec(c + d*x)**2*b**2 + 2*sec(
c + d*x)*a*b + a**2),x)
```

3.567 $\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 347

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d}$$
$$-\frac{2\cot(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d}$$
$$-\frac{2\sqrt{a+b}\cot(c+dx)\,\mathrm{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a^2d}$$
$$+\frac{2b^2\tan(c+dx)}{a(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}$$

output
```
2*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1
/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/(a+b
)^(1/2)/d-2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)
/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1
/2)/a/(a+b)^(1/2)/d-2*(a+b)^(1/2)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(
1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/
2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+2*b^2*tan(d*x+c)/a/(a^2-b^2)/d/(a
+b*sec(d*x+c))^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 751 vs. $2(347) = 694$.

Time = 12.71 (sec) , antiderivative size = 751, normalized size of antiderivative = 2.16

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(-3/2),x]
```

output
```
((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*b*Sin[c + d*x])/(a*(-a^2 + b^2)
) + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*(b + a*Cos[c + d*x]))))/(d*(a + b*
Sec[c + d*x])^(3/2)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]*Sec[c + d*x]
^2*(-8*b*(a + b)*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*
Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[T
an[(c + d*x)/2]], (a - b)/(a + b)] + 8*a*(a + b)*Cos[(c + d*x)/2]^3*Sqrt[C
os[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 12*a^
2*Cos[(c + d*x)/2]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + S
ec[c + d*x]))] + 12*b^2*Cos[(c + d*x)/2]*EllipticPi[-1, ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c +
 d*x])/((a + b)*(1 + Sec[c + d*x]))] - 4*a^2*Cos[(3*(c + d*x))/2]*Elliptic
Pi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^
(-1)]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] + 4*b^2*Cos[
(3*(c + d*x))/2]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec
[c + d*x]))] - 2*a*b*Sin[(c + d*x)/2] + 2*b^2*Sin[(c + d*x)/2] + a*b*Sin[(
3*(c + d*x))/2] - 2*b^2*Sin[(3*(c + d*x))/2] - a*b*Sin[(5*(c + d*x))/2]))/
(2*a*(-a^2 + b^2)*d*(a + b*Sec[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {3042, 4272, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4272}$$

$$\frac{2b^2\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{a^2-b\sec(c+dx)a-b^2-b^2\sec^2(c+dx)}{2\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a^2-b\sec(c+dx)a-b^2-b^2\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}+\frac{2b^2\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a-b^2-b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{2b^2\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{4546}$$

$$\frac{\int \frac{a^2-b^2+(b^2-ab)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-b^2\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}+\frac{2b^2\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a^2-b^2+(b^2-ab)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{2b^2\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{4409}$$

$$\frac{(a^2-b^2)\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx+b^2\left(-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-b(a-b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}+\frac{2b^2\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2-b^2)\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b^2\left(-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-b(a-b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{2b^2\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{4271}$$

$$\frac{b^2\left(-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-b(a-b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}(a^2-b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\cdots}}{\cdots}}{a(a^2-b^2)}$$

$$\frac{2b^2\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{4319}$$

$$\frac{b^2\left(-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-\frac{2\sqrt{a+b}(a^2-b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{\cdots}}{\cdots}\right.\right.}{ad}}{a(a^2-b^2)}$$

$$\frac{2b^2\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{4492}$$

$$\frac{-\frac{2\sqrt{a+b}(a^2-b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2b^2\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

input
```
Int[(a + b*Sec[c + d*x])^(-3/2),x]
```

output
```
((2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d - (2*(a - b)*Sqrt[a + b]*Cot[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/d - (2*Sqrt[a + b]*(a^2 - b^2)*Cot[c + d*x]*EllipticPi[(a + b)/a,
 ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d))
/(a*(a^2 - b^2)) + (2*b^2*Tan[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c
+ d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(318) = 636$.

Time = 6.65 (sec) , antiderivative size = 654, normalized size of antiderivative = 1.88

method	result
default	$\frac{2\left(\left(-(1-\cos(dx+c))^3\csc(dx+c)^3+\csc(dx+c)-\cot(dx+c)\right)ab+\left((1-\cos(dx+c))^3\csc(dx+c)^3-\csc(dx+c)+\cot(dx+c)\right)b^2-2\sqrt{\frac{\mathrm{co}}{\mathrm{cos}}}\right.}{}$

input
```
int(1/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/a/(a+b)/(a-b)*((-(1-cos(d*x+c))^3*csc(d*x+c)^3+csc(d*x+c)-cot(d*x+c))*
a*b+((1-cos(d*x+c))^3*csc(d*x+c)^3-csc(d*x+c)+cot(d*x+c))*b^2-2*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^2-2*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF
(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a*b+2*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*
x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a*b+2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-cs
c(d*x+c),((a-b)/(a+b))^(1/2))*b^2+4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+
c),-1,((a-b)/(a+b))^(1/2))*a^2-4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),
-1,((a-b)/(a+b))^(1/2))*b^2)*(a+b*sec(d*x+c))^(1/2)/(a*(1-cos(d*x+c))^2*cs
c(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)
```

Fricas [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c)
 + a^2), x)
```

Sympy [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(1/(a + b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^2\,b^2+2\sec(dx+c)\,ab+a^2}dx$$

input
```
int(1/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)/(sec(c + d*x)**2*b**2 + 2*sec(c + d*x)*a*b +
a**2),x)
```

3.568 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 396

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{(a^2-3b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1}{}}}{a^2b\sqrt{a+b}d}$$
$$+\frac{(a+3b)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a^2\sqrt{a+b}d}$$
$$+\frac{3b\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a^3d}$$
$$+\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}+\frac{b(a^2-3b^2)\tan(c+dx)}{a^2(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}$$

output
```
(a^2-3*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)
/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1
/2)/a^2/b/(a+b)^(1/2)/d+(a+3*b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2
)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+s
ec(d*x+c))/(a-b))^(1/2)/a^2/(a+b)^(1/2)/d+3*b*(a+b)^(1/2)*cot(d*x+c)*Ellip
ticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(b*(
1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d+sin(d*x+c
)/a/d/(a+b*sec(d*x+c))^(1/2)+b*(a^2-3*b^2)*tan(d*x+c)/a^2/(a^2-b^2)/d/(a+b
*sec(d*x+c))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1069 vs. $2(396) = 792$.

Time = 13.66 (sec) , antiderivative size = 1069, normalized size of antiderivative = 2.70

$$\int \frac{\cos(c + dx)}{(a + b\sec(c + dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((-2*b^2*Sin[c + d*x])/(a^2*(-a^2 +
 b^2)) - (2*b^3*Sin[c + d*x])/(a^2*(a^2 - b^2)*(b + a*Cos[c + d*x]))))/(d*
(a + b*Sec[c + d*x])^(3/2)) - ((b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/
2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
+ b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(a^3*Tan[(c + d*x)/2] +
a^2*b*Tan[(c + d*x)/2] - 3*a*b^2*Tan[(c + d*x)/2] - 3*b^3*Tan[(c + d*x)/2]
 - 2*a^3*Tan[(c + d*x)/2]^3 + 6*a*b^2*Tan[(c + d*x)/2]^3 + a^3*Tan[(c + d*
x)/2]^5 - a^2*b*Tan[(c + d*x)/2]^5 - 3*a*b^2*Tan[(c + d*x)/2]^5 + 3*b^3*Ta
n[(c + d*x)/2]^5 - 6*a^2*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*b^3*EllipticPi[-1, ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan
[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a^2*b*EllipticPi[-1,
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan
[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2
)/(a + b)] + 6*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c
+ d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (a^3 + a^2*b - 3*a*b^2 - 3*
b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^...
```

Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 4333, 27, 3042, 4549, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4333}$$

$$\frac{\int -\frac{3b-b\sec^2(c+dx)}{2(a+b\sec(c+dx))^{3/2}}dx}{a}+\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{3b-b\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{3b-b\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{2a}$$

$$\Big\downarrow \text{4549}$$

$$\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{2\int -\frac{-2a\sec(c+dx)b^2+\left(a^2-3b^2\right)\sec^2(c+dx)b+3\left(a^2-b^2\right)b}{2\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}-\frac{2b(a^2-3b^2)\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{2a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{\int \frac{-2a\sec(c+dx)b^2+\left(a^2-3b^2\right)\sec^2(c+dx)b+3\left(a^2-b^2\right)b}{\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}-\frac{2b(a^2-3b^2)\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{2a}$$

$\Big\downarrow$ 3042

$$\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{\int \frac{-2a\csc\left(c+dx+\frac{\pi}{2}\right)b^2+\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2b+3\left(a^2-b^2\right)b}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}-\frac{2b(a^2-3b^2)\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{2a}$$

$\Big\downarrow$ 4546

$$\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{b\left(a^2-3b^2\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx+\int \frac{3b\left(a^2-b^2\right)+\left(-2ab^2-\left(a^2-3b^2\right)b\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}-\frac{2b(a^2-3b^2)\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{2a}$$

$\Big\downarrow$ 3042

$$\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{b\left(a^2-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{3b\left(a^2-b^2\right)+\left(-2ab^2-\left(a^2-3b^2\right)b\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}-\frac{2b(a^2-3b^2)\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{2a}$$

$\Big\downarrow$ 4409

$$\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{b\left(a^2-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+3b\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx-b(a-b)(a+3b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}-\frac{2b(a^2-3b^2)\tan(c+d}{ad(a^2-b^2)\sqrt{a+b\sec(c-}}{2a}$$

$\Big\downarrow$ 3042

$$\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{3b\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\left(a^2-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b(a-b)(a+3b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}-\frac{2b(a^2-3b^2)}{ad(a^2-b^2)\sqrt{a}}}{2a}$$

$\Big\downarrow$ 4271

$$\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{b(a^2-3b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b(a-b)(a+3b)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6b\sqrt{a+b}\left(a^2-b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+d}{a-b}}}{ad}}{a(a^2-b^2)}}{2a}$$

$\Big\downarrow$ 4319

$$\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{b(a^2-3b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6b\sqrt{a+b}\left(a^2-b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+d}}{\sqrt{a+b}}\right.\right.}{ad}}{a(a^2-b^2)}$$

$\Big\downarrow$ 4492

$$\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{2(a-b)\sqrt{a+b}\left(a^2-3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}-\frac{6b\sqrt{a+b}\left(a^2-b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{}}{}$$

input
```
Int[Cos[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Sin[c + d*x]/(a*d*Sqrt[a + b*Sec[c + d*x]]) - (((-2*(a - b)*Sqrt[a + b]*(a
^2 - 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
+ Sec[c + d*x]))/(a - b))])/(b*d) - (2*(a - b)*Sqrt[a + b]*(a + 3*b)*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
 - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/d - (6*b*Sqrt[a + b]*(a^2 - b^2)*Cot[c + d*x]*EllipticPi[(a + b
)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a
*d))/(a*(a^2 - b^2)) - (2*b*(a^2 - 3*b^2)*Tan[c + d*x])/(a*(a^2 - b^2)*d*S
qrt[a + b*Sec[c + d*x]]))/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4333
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*
Csc[e + f*x])^n/(a*f*n)), x] - Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[b*(m + n + 1) - a*(n + 1)*Csc[e + f*x] - b*(m
 + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m + 1/2, 0] && ILtQ[n, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4549
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 + a^2*C)*Cot[e + f*x]*((a + b*Csc[
e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 -
 b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*
(A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m
] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 948 vs. $2(365) = 730$.

Time = 9.22 (sec) , antiderivative size = 949, normalized size of antiderivative = 2.40

method	result	size
default	Expression too large to display	949

input
```
int(cos(d*x+c)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2/(a+b)/(a-b)*((6*cos(d*x+c)^2+12*cos(d*x+c)+6)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b*Ellipt
icPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))+(-6*cos(d*x+c)^2-12*cos
(d*x+c)-6)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*b^3*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(
1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*EllipticE(cot(d*x+c)-csc(
d*x+c),((a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b*E
llipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(3*cos(d*x+c)^2+6*cos(
d*x+c)+3)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*a*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2
))+(3*cos(d*x+c)^2+6*cos(d*x+c)+3)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^3*EllipticE(cot(d*x+c)-csc(d*
x+c),((a-b)/(a+b))^(1/2))+(-2*cos(d*x+c)^2-4*cos(d*x+c)-2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b*E
llipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(-2*cos(d*x+c)^2-4*cos
(d*x+c)-2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*a*b^2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/
2))+a^3*cos(d*x+c)^2*sin(d*x+c)+a^2*b*cos(d*x+c)*sin(d*x+c)+sin(d*x+c)*...
```

Fricas [F]

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)/(b^2*sec(d*x + c)^2 + 2*a*b
*sec(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)/(a + b*sec(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\cos\left(c+d\,x\right)}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

input
```
int(cos(c + d*x)/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)/(a + b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)b+a}\,\cos\left(dx+c\right)}{\sec\left(dx+c\right)^{2}b^{2}+2\sec\left(dx+c\right)ab+a^{2}}dx$$

input
```
int(cos(d*x+c)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*cos(c + d*x))/(sec(c + d*x)**2*b**2 + 2*sec(
c + d*x)*a*b + a**2),x)
```

3.569 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 470

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{(7a^2-15b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4a^3\sqrt{a+b}d}$$
$$+\frac{(2a^2-5ab-15b^2)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4a^3\sqrt{a+b}d}$$
$$-\frac{\sqrt{a+b}(4a^2+15b^2)\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c-}{a-b}}}{4a^4d}$$
$$-\frac{5b\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}}+\frac{\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}-\frac{b^2(7a^2-15b^2)\tan(c+dx)}{4a^3(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}$$

output
```
-1/4*(7*a^2-15*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2
),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(
a-b))^(1/2)/a^3/(a+b)^(1/2)/d+1/4*(2*a^2-5*a*b-15*b^2)*cot(d*x+c)*Elliptic
F((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c)
)/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/(a+b)^(1/2)/d-1/4*(a+b)
^(1/2)*(4*a^2+15*b^2)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(
1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+se
c(d*x+c))/(a-b))^(1/2)/a^4/d-5/4*b*sin(d*x+c)/a^2/d/(a+b*sec(d*x+c))^(1/2)
+1/2*cos(d*x+c)*sin(d*x+c)/a/d/(a+b*sec(d*x+c))^(1/2)-1/4*b^2*(7*a^2-15*b^
2)*tan(d*x+c)/a^3/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1330 vs. $2(470) = 940$.

Time = 12.89 (sec) , antiderivative size = 1330, normalized size of antiderivative = 2.83

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*b^3*Sin[c + d*x])/(a^3*(-a^2 +
b^2)) + (2*b^4*Sin[c + d*x])/(a^3*(a^2 - b^2)*(b + a*Cos[c + d*x])) + Sin[
2*(c + d*x)]/(4*a^2)))/(d*(a + b*Sec[c + d*x])^(3/2)) + ((b + a*Cos[c + d*
x])^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
 + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(-7*a^3*b*Tan[(c + d*x)/2] - 7*a^2
*b^2*Tan[(c + d*x)/2] + 15*a*b^3*Tan[(c + d*x)/2] + 15*b^4*Tan[(c + d*x)/2
] + 14*a^3*b*Tan[(c + d*x)/2]^3 - 30*a*b^3*Tan[(c + d*x)/2]^3 - 7*a^3*b*Ta
n[(c + d*x)/2]^5 + 7*a^2*b^2*Tan[(c + d*x)/2]^5 + 15*a*b^3*Tan[(c + d*x)/2
]^5 - 15*b^4*Tan[(c + d*x)/2]^5 + 8*a^4*EllipticPi[-1, ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(
c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 22*a^2*b^2*EllipticPi[-1,
 ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*S
qrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*b^
4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(
c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/
(a + b)] + 8*a^4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c +
d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 22*a^2*b^2*EllipticPi[-1, Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c
 + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2...
```

Rubi [A] (verified)

Time = 2.25 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4333, 27, 3042, 4592, 27, 3042, 4548, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4333}$$

$$\frac{\int -\frac{\cos(c+dx)(-3b\sec^2(c+dx)-2a\sec(c+dx)+5b)}{2(a+b\sec(c+dx))^{3/2}}dx}{2a}+\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{\cos(c+dx)(-3b\sec^2(c+dx)-2a\sec(c+dx)+5b)}{(a+b\sec(c+dx))^{3/2}}dx}{4a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{-3b\csc\left(c+dx+\frac{\pi}{2}\right)^2-2a\csc\left(c+dx+\frac{\pi}{2}\right)+5b}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{4a}$$

$$\Big\downarrow \text{4592}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{5b\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{4a^2+6b\sec(c+dx)a+15b^2-5b^2\sec^2(c+dx)}{2(a+b\sec(c+dx))^{3/2}}dx}{a}}{4a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{5b\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{4a^2+6b\sec(c+dx)a+15b^2-5b^2\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx}{2a}}{4a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{5b\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{4a^2+6b\csc\left(c+dx+\frac{\pi}{2}\right)a+15b^2-5b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{2a}}{4a}$$

$$\Big\downarrow \text{4548}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{5b\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{2\int -\frac{b^2\left(7a^2-15b^2\right)\sec^2(c+dx)+2ab\left(a^2-5b^2\right)\sec(c+dx)+\left(a^2-b^2\right)\left(4a^2+15b^2\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{2b^2\left(7a^2-15b^2\right)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{2a}}{4a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{5b\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{\int\frac{4a^4+11b^2a^2+2b\left(a^2-5b^2\right)\sec(c+dx)a-15b^4+b^2\left(7a^2-15b^2\right)\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{2b^2\left(7a^2-15b^2\right)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{2a}}{4a}$$

$\downarrow$ 3042

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{5b\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{\int\frac{4a^4+11b^2a^2+2b\left(a^2-5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a-15b^4+b^2\left(7a^2-15b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}-\frac{2b^2\left(7a^2-15b^2\right)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{2a}}{4a}$$

$\downarrow$ 4546

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{5b\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{b^2\left(7a^2-15b^2\right)\int\frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx+\int\frac{4a^4+11b^2a^2-15b^4+\left(2ab\left(a^2-5b^2\right)-b^2\left(7a^2-15b^2\right)\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{2b^2\left(7a^2-15b^2\right.}{ad\left(a^2-b^2\right)\sqrt{a}}}{2a}}{4a}$$

$\downarrow$ 3042

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{5b\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{b^2\left(7a^2-15b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int\frac{4a^4+11b^2a^2-15b^4+\left(2ab\left(a^2-5b^2\right)-b^2\left(7a^2-15b^2\right)\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}-}{2a}}{4a}$$

$\downarrow$ 4409

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{5b\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{b^2\left(7a^2-15b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(a-b)\left(2a^2-5ab-15b^2\right)\int\frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\left(4a^4+11a^2b^2-15b^4\right)\int\frac{}{\sqrt{a+}}}{a\left(a^2-b^2\right)}}{2a}}{4a}$$

$\downarrow$ 3042

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}-$$

$$\frac{\frac{5b\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{b^2\left(7a^2-15b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(a-b)\left(2a^2-5ab-15b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(4a^4+11a^2b^2-15b^4\right)\int}{a\left(a^2-b^2\right)}}{2a}}{4a}$$

$\Big\downarrow$ 4271

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}-$$

$$\frac{\frac{5b\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{b^2\left(7a^2-15b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(a-b)\left(2a^2-5ab-15b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(4a^4+11a^2b^2-1\right.}{}}{a\left(a^2-b^2\right)}}{4a}$$

$\Big\downarrow$ 4319

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}-$$

$$\frac{5b\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{b^2\left(7a^2-15b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}\left(2a^2-5ab-15b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)}{a-b}}}{d}}{}$$

$\Big\downarrow$ 4492

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}-$$

$$\frac{5b\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{2(a-b)\sqrt{a+b}\left(2a^2-5ab-15b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}-\frac{2(a-b)}{}}{}$$

input
```
Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(Cos[c + d*x]*Sin[c + d*x])/(2*a*d*Sqrt[a + b*Sec[c + d*x]]) - ((5*b*Sin[c
 + d*x])/(a*d*Sqrt[a + b*Sec[c + d*x]]) - (((-2*(a - b)*Sqrt[a + b]*(7*a^2
 - 15*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
 b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[c + d*x]))/(a - b))])/d + (2*(a - b)*Sqrt[a + b]*(2*a^2 - 5*a*b - 15*
b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/d - (2*Sqrt[a + b]*(4*a^4 + 11*a^2*b^2 - 15*b^4)*Cot[
c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Se
c[c + d*x]))/(a - b))])/(a*d))/(a*(a^2 - b^2)) - (2*b^2*(7*a^2 - 15*b^2)*T
an[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))/(2*a))/(4*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4333
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*
Csc[e + f*x])^n/(a*f*n)), x] - Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[b*(m + n + 1) - a*(n + 1)*Csc[e + f*x] - b*(m
 + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m + 1/2, 0] && ILtQ[n, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1317 vs. $2(425) = 850$.

Time = 10.47 (sec) , antiderivative size = 1318, normalized size of antiderivative = 2.80

method	result	size
default	Expression too large to display	1318

input
```
int(cos(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^3/(a+b)/(a-b)*((-8*cos(d*x+c)^2-16*cos(d*x+c)-8)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*Ellip
ticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))+(-22*cos(d*x+c)^2-44*c
os(d*x+c)-22)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*a^2*b^2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(
a+b))^(1/2))+(30*cos(d*x+c)^2+60*cos(d*x+c)+30)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4*EllipticPi(cot
(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))+(7*cos(d*x+c)^2+14*cos(d*x+c)+7
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*a^3*b*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(7*co
s(d*x+c)^2+14*cos(d*x+c)+7)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2*EllipticE(cot(d*x+c)-csc(d*x+c
),((a-b)/(a+b))^(1/2))+(-15*cos(d*x+c)^2-30*cos(d*x+c)-15)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3*E
llipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(-15*cos(d*x+c)^2-30*c
os(d*x+c)-15)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*b^4*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1
/2))+(4*cos(d*x+c)^2+8*cos(d*x+c)+4)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*EllipticF(cot(d*x+c)-csc(
d*x+c),((a-b)/(a+b))^(1/2))+(-2*cos(d*x+c)^2-4*cos(d*x+c)-2)*(cos(d*x+c...
```

Fricas [F]

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\cos\left(dx+c\right)^2}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^2/(b^2*sec(d*x + c)^2 + 2*a
*b*sec(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos^2{(c+dx)}}{(a+b\sec{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)**2/(a + b*sec(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos{(dx+c)}^2}{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos{(dx+c)}^2}{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^2/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^2/(a + b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)b+a}\,\cos\left(dx+c\right)^2}{\sec\left(dx+c\right)^2b^2+2\sec\left(dx+c\right)ab+a^2}dx$$

input
```
int(cos(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*cos(c + d*x)**2)/(sec(c + d*x)**2*b**2 + 2*s
ec(c + d*x)*a*b + a**2),x)
```

3.570 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 427

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{8a(4a^4-7a^2b^2+2b^4)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c}{a+b}}}{3(a-b)b^5(a+b)^{3/2}d}$$

$$+\frac{2(16a^4+12a^3b-16a^2b^2-9ab^3-b^4)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{3(a-b)b^4(a+b)^{3/2}d}$$

$$-\frac{2a^2\sec^2(c+dx)\tan(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^{3/2}}+\frac{4a^3(3a^2-5b^2)\tan(c+dx)}{3b^3(a^2-b^2)^2d\sqrt{a+b\sec(c+dx)}}$$

$$+\frac{2(2a^2-b^2)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3b^3(a^2-b^2)d}$$

output
```
8/3*a*(4*a^4-7*a^2*b^2+2*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/
(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec
(d*x+c))/(a-b))^(1/2)/(a-b)/b^5/(a+b)^(3/2)/d+2/3*(16*a^4+12*a^3*b-16*a^2*
b^2-9*a*b^3-b^4)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(
(a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b
))^(1/2)/(a-b)/b^4/(a+b)^(3/2)/d-2/3*a^2*sec(d*x+c)^2*tan(d*x+c)/b/(a^2-b^
2)/d/(a+b*sec(d*x+c))^(3/2)+4/3*a^3*(3*a^2-5*b^2)*tan(d*x+c)/b^3/(a^2-b^2)
^2/d/(a+b*sec(d*x+c))^(1/2)+2/3*(2*a^2-b^2)*(a+b*sec(d*x+c))^(1/2)*tan(d*x
+c)/b^3/(a^2-b^2)/d
```

Mathematica [A] (warning: unable to verify)

Time = 13.93 (sec) , antiderivative size = 578, normalized size of antiderivative = 1.35

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{4(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\Big(4a(4a^5+}{}$$

$$+\frac{(b+a\cos(c+dx))^3\sec^3(c+dx)\left(-\frac{8a(4a^4-7a^2b^2+2b^4)\sin(c+dx)}{3b^4(-a^2+b^2)^2}-\frac{2a^3\sin(c+dx)}{3b^2(-a^2+b^2)(b+a\cos(c+dx))^2}-\frac{2(-7a^5\sin(c+dx)+1}{3b^3(-a^2+b^2)^2(b+}\right.}{d(a+b\sec(c+dx))^{5/2}}$$

input
```
Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(4*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
 + d*x]]*(4*a*(4*a^5 + 4*a^4*b - 7*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4 + 2*b^5)*
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] +
 b*(-16*a^5 - 4*a^4*b + 28*a^3*b^2 + 7*a^2*b^3 - 8*a*b^4 + b^5)*Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*a*(4*a^
4 - 7*a^2*b^2 + 2*b^4)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^
2*Tan[(c + d*x)/2]))/(3*b^4*(a^2 - b^2)^2*d*Sqrt[Sec[(c + d*x)/2]^2]*(a +
b*Sec[c + d*x])^(5/2)) + ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((-8*a*(4*
a^4 - 7*a^2*b^2 + 2*b^4)*Sin[c + d*x])/(3*b^4*(-a^2 + b^2)^2) - (2*a^3*Sin
[c + d*x])/(3*b^2*(-a^2 + b^2)*(b + a*Cos[c + d*x])^2) - (2*(-7*a^5*Sin[c
+ d*x] + 11*a^3*b^2*Sin[c + d*x]))/(3*b^3*(-a^2 + b^2)^2*(b + a*Cos[c + d*
x])) + (2*Tan[c + d*x])/(3*b^3)))/(d*(a + b*Sec[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 2.00 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4332, 27, 3042, 4578, 27, 3042, 4570, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4332}$$

$$-\frac{2\int \frac{\sec^2(c+dx)\left(4a^2-3b\sec(c+dx)a-3\left(2a^2-b^2\right)\sec^2(c+dx)\right)}{2(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sec^2(c+dx)\left(4a^2-3b\sec(c+dx)a-3\left(2a^2-b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(4a^2-3b\csc\left(c+dx+\frac{\pi}{2}\right)a-3\left(2a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4578}$$

$$-\frac{\frac{2\int \frac{\sec(c+dx)\left(2b\left(3a^2-5b^2\right)a^2+2\left(6a^4-11b^2a^2+3b^4\right)\sec(c+dx)a-3b\left(a^2-b^2\right)\left(2a^2-b^2\right)\sec^2(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{b^2\left(a^2-b^2\right)}-\frac{4a^3\left(3a^2-5b^2\right)\tan(c+dx)}{b^2d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{\sec(c+dx)\left(2b\left(3a^2-5b^2\right)a^2+2\left(6a^4-11b^2a^2+3b^4\right)\sec(c+dx)a-3b\left(a^2-b^2\right)\left(2a^2-b^2\right)\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{b^2\left(a^2-b^2\right)}-\frac{4a^3\left(3a^2-5b^2\right)\tan(c+dx)}{b^2d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2b\left(3a^2-5b^2\right)a^2+2\left(6a^4-11b^2a^2+3b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)a-3b\left(a^2-b^2\right)\left(2a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2(a^2-b^2)}-\frac{4a^3\left(3a^2-5b^2\right)\tan(c+dx)}{b^2d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \; 4570$$

$$-\frac{\frac{\frac{2\int \frac{3\sec(c+dx)\left(\left(4a^4-7b^2a^2-b^4\right)b^2+4a\left(4a^4-7b^2a^2+2b^4\right)\sec(c+dx)b\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{3b}-\frac{2\left(a^2-b^2\right)\left(2a^2-b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{d}}{b^2(a^2-b^2)}-\frac{4a^3\left(3a^2-5b^2\right)\tan(c+}{b^2d(a^2-b^2)\sqrt{a+b\sec(c}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{\int \frac{\sec(c+dx)\left(\left(4a^4-7b^2a^2-b^4\right)b^2+4a\left(4a^4-7b^2a^2+2b^4\right)\sec(c+dx)b\right)}{\sqrt{a+b\sec(c+dx)}}dx}{b}-\frac{2\left(a^2-b^2\right)\left(2a^2-b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{d}}{b^2(a^2-b^2)}-\frac{4a^3\left(3a^2-5b^2\right)\tan(c+dx}{b^2d(a^2-b^2)\sqrt{a+b\sec(c+a}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\left(4a^4-7b^2a^2-b^4\right)b^2+4a\left(4a^4-7b^2a^2+2b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)b\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(a^2-b^2\right)\left(2a^2-b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{d}}{b^2(a^2-b^2)}-\frac{4a^3\left(3a^2-5b^2\right)t}{b^2d(a^2-b^2)\sqrt{a+l}}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \; 4493$$

$$-\frac{\frac{\frac{4ab\left(4a^4-7a^2b^2+2b^4\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx-b(a-b)\left(16a^4+12a^3b-16a^2b^2-9ab^3-b^4\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{b}-\frac{2\left(a^2-b^2\right)\left(2a^2-b^2\right)\tan(c+dx)\sqrt{a+b}}{d}}{b^2(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{4ab\left(4a^4-7a^2b^2+2b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b(a-b)\left(16a^4+12a^3b-16a^2b^2-9ab^3-b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(a^2-b^2\right)\left(2a^2-b^2\right)\ \mathrm{ta}}{}}{b^2\left(a^2-b^2\right)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$\downarrow$ 4319

$$-\frac{\frac{\frac{4ab\left(4a^4-7a^2b^2+2b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\left(16a^4+12a^3b-16a^2b^2-9ab^3-b^4\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)}{a-b}}}{d}}{b}}{b^2\left(a^2-b^2\right)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$\downarrow$ 4492

$$-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}-$$

$$\frac{\frac{-\frac{8a(a-b)\sqrt{a+b}\left(4a^4-7a^2b^2+2b^4\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}-\frac{2(a-b)\sqrt{a+b}\left(16a^4+12a^3b-16a^2b^2-\right.}{}}{b}}{b^2\left(a^2-\right.}$$

input
```
Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*a^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x
])^(3/2)) - ((-4*a^3*(3*a^2 - 5*b^2)*Tan[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt
[a + b*Sec[c + d*x]]) + (((-8*a*(a - b)*Sqrt[a + b]*(4*a^4 - 7*a^2*b^2 + 2
*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))])/(b*d) - (2*(a - b)*Sqrt[a + b]*(16*a^4 + 12*a^3*b -
16*a^2*b^2 - 9*a*b^3 - b^4)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d)/b - (2*(a^2 - b^2)*(2*a^2 -
 b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/d)/(b^2*(a^2 - b^2)))/(3*b*(a
^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

rule 4578
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x
])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - Simp[1/(b^2*(m + 1)*(a^2 - b^
2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*((-a)*(b
*B - a*C) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
 b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1854 vs. $2(393) = 786$.

Time = 18.12 (sec) , antiderivative size = 1855, normalized size of antiderivative = 4.34

method	result	size
default	Expression too large to display	1855

input
```
int(sec(d*x+c)^5/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d/(a-b)^2/(a+b)^2/b^4*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)/(cos(d*x+
c)^2*a^2+2*a*b*cos(d*x+c)+b^2)*(16*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^7*EllipticE(cot(d*x+c)-csc(d*
x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)^3+2*cos(d*x+c)^2+cos(d*x+c))+16*(cos
(d*x+c)^3+3*cos(d*x+c)^2+3*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^6*b*EllipticE(cot(d*x+c
)-csc(d*x+c),((a-b)/(a+b))^(1/2))+4*(4-7*cos(d*x+c)^3-10*cos(d*x+c)^2+cos(
d*x+c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*a^5*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2
))+28*(-cos(d*x+c)^3-3*cos(d*x+c)^2-3*cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b^3*Ellipt
icE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+4*(-7+2*cos(d*x+c)^3-3*cos(
d*x+c)^2-12*cos(d*x+c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^4*EllipticE(cot(d*x+c)-csc(d*x+c),((
a-b)/(a+b))^(1/2))+8*(cos(d*x+c)^3+3*cos(d*x+c)^2+3*cos(d*x+c)+1)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
a^2*b^5*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+8*(cos(d*x+c)
^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*a*b^6*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a
+b))^(1/2))+16*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+...
```

Fricas [F]

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^5}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^5/(b^3*sec(d*x + c)^3 + 3*a
*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^5{(c+dx)}}{(a+b\sec{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**5/(a + b*sec(c + d*x))**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec{(dx+c)}^5}{(b\sec{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^5/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^5\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^5}{\sec\left(dx+c\right)^3 b^3+3\sec\left(dx+c\right)^2 a\,b^2+3\sec\left(dx+c\right)a^2b+a^3}dx$$

input
```
int(sec(d*x+c)^5/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**5)/(sec(c + d*x)**3*b**3 + 3*s
ec(c + d*x)**2*a*b**2 + 3*sec(c + d*x)*a**2*b + a**3),x)
```

3.571 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 362

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2(8a^4-15a^2b^2+3b^4)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3(a-b)b^4(a+b)^{3/2}d} - \frac{2(8a^3+6a^2b-9ab^2-3b^3)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3(a-b)b^3(a+b)^{3/2}d} - \frac{2a^2\sec(c+dx)\tan(c+dx)}{3b(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{8a^2(a^2-2b^2)\tan(c+dx)}{3b^2(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}$$

output
```
-2/3*(8*a^4-15*a^2*b^2+3*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/
(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec
(d*x+c))/(a-b))^(1/2)/(a-b)/b^4/(a+b)^(3/2)/d-2/3*(8*a^3+6*a^2*b-9*a*b^2-3
*b^3)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b)
)^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/(a
-b)/b^3/(a+b)^(3/2)/d-2/3*a^2*sec(d*x+c)*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec
(d*x+c))^(3/2)-8/3*a^2*(a^2-2*b^2)*tan(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(d
*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 14.04 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.54

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{(b+a\cos(c+dx))^3\sec^3(c+dx)\left(\frac{2(8a^4-15a^2b^2+3b^4)\sin(c+dx)}{3b^3(-a^2+b^2)^2}+\frac{2a^2\sin(c+d}{3b(-a^2+b^2)(b+a\cos}\right.}{d(a+b\sec(c+dx))^{5/2}}$$

$$-\frac{2(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2(8a^5+8a^4b-15a^3b^2-15a^2b^3+3ab\right.}{}$$

input
```
Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*(8*a^4 - 15*a^2*b^2 + 3*b^4)*Si
n[c + d*x])/(3*b^3*(-a^2 + b^2)^2) + (2*a^2*Sin[c + d*x])/(3*b*(-a^2 + b^2
)*(b + a*Cos[c + d*x])^2) + (8*(-(a^4*Sin[c + d*x]) + 2*a^2*b^2*Sin[c + d*
x]))/(3*b^2*(-a^2 + b^2)^2*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])
^(5/2)) - (2*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/
2]^2*Sec[c + d*x]]*(2*(8*a^5 + 8*a^4*b - 15*a^3*b^2 - 15*a^2*b^3 + 3*a*b^4
 + 3*b^5)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/
(a + b)] - 2*b*(8*a^4 + 2*a^3*b - 15*a^2*b^2 - 6*a*b^3 + 3*b^4)*Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (8*a^4 -
15*a^2*b^2 + 3*b^4)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2]))/(3*b^3*(a^2 - b^2)^2*d*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*S
ec[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4332, 27, 3042, 4568, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx$$

$\Big\downarrow$ 3042

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$\Big\downarrow$ 4332

$$-\frac{2\int \frac{\sec(c+dx)\left(2a^2-3b\sec(c+dx)a-\left(4a^2-3b^2\right)\sec^2(c+dx)\right)}{2(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 27

$$-\frac{\int \frac{\sec(c+dx)\left(2a^2-3b\sec(c+dx)a-\left(4a^2-3b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2a^2-3b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(3b^2-4a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 4568

$$-\frac{\frac{8a^2\left(a^2-2b^2\right)\tan(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\int \frac{\sec(c+dx)\left(2ab\left(a^2-3b^2\right)+\left(8a^4-15b^2a^2+3b^4\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 27

$$-\frac{\frac{8a^2\left(a^2-2b^2\right)\tan(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{\sec(c+dx)\left(2ab\left(a^2-3b^2\right)+\left(8a^4-15b^2a^2+3b^4\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{8a^2(a^2-2b^2)\tan(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2ab\left(a^2-3b^2\right)+\left(8a^4-15b^2a^2+3b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \textbf{4493}$$

$$-\frac{\frac{8a^2(a^2-2b^2)\tan(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{(8a^4-15a^2b^2+3b^4)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx-(a-b)(8a^3+6a^2b-9ab^2-3b^3)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{\frac{8a^2(a^2-2b^2)\tan(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{(8a^4-15a^2b^2+3b^4)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)(8a^3+6a^2b-9ab^2-3b^3)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \textbf{4319}$$

$$-\frac{\frac{8a^2(a^2-2b^2)\tan(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{(8a^4-15a^2b^2+3b^4)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\left(8a^3+6a^2b-9ab^2-3b^3\right)\cot(c+dx)\sqrt{\frac{b(1-\text{se}}{a}}}{}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \textbf{4492}$$

$$-\frac{2a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}-$$

$$\frac{\frac{8a^2(a^2-2b^2)\tan(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{2(a-b)\sqrt{a+b}\left(8a^4-15a^2b^2+3b^4\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2}{}}{b(}}{3b\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*a^2*Sec[c + d*x]*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])
^(3/2)) - (-(((-2*(a - b)*Sqrt[a + b]*(8*a^4 - 15*a^2*b^2 + 3*b^4)*Cot[c +
 d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/(b^2*d) - (2*(a - b)*Sqrt[a + b]*(8*a^3 + 6*a^2*b - 9*a*b^2 - 3*b
^3)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
+ d*x]))/(a - b))])/(b*d))/(b*(a^2 - b^2))) + (8*a^2*(a^2 - 2*b^2)*Tan[c +
 d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))/(3*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

rule 4568
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cot[e + f*x]*((a + b*Csc[e + f*x]
)^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))
 Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m
+ 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x],
x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^
2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1661 vs. $2(332) = 664$.

Time = 13.09 (sec) , antiderivative size = 1662, normalized size of antiderivative = 4.59

method	result	size
default	Expression too large to display	1662

input
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d/(a-b)^2/(a+b)^2/b^3*(8*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^6*EllipticE(cot(d*x+c)-csc(d*x+c),
((a-b)/(a+b))^(1/2))*(-cos(d*x+c)^3-2*cos(d*x+c)^2-cos(d*x+c))+8*(-cos(d*x
+c)^3-3*cos(d*x+c)^2-3*cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5*b*EllipticE(cot(d*x+c)-cs
c(d*x+c),((a-b)/(a+b))^(1/2))+(15*cos(d*x+c)^3+22*cos(d*x+c)^2-cos(d*x+c)-
8)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*a^4*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+15
*(cos(d*x+c)^3+3*cos(d*x+c)^2+3*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^3*EllipticE(co
t(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+3*(-cos(d*x+c)^3+3*cos(d*x+c)^2+9
*cos(d*x+c)+5)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*a^2*b^4*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b
))^(1/2))+3*(-cos(d*x+c)^3-3*cos(d*x+c)^2-3*cos(d*x+c)-1)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^5*El
lipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+3*(-cos(d*x+c)^2-2*cos(
d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*b^6*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))
+8*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*a^5*b*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(...
```

Fricas [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^4}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^4/(b^3*sec(d*x + c)^3 + 3*a
*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^4{(c+dx)}}{(a+b\sec{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec{(dx+c)}^4}{(b\sec{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^4\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^4}{\sec\left(dx+c\right)^3b^3+3\sec\left(dx+c\right)^2a\,b^2+3\sec\left(dx+c\right)a^2b+a^3}dx$$

input
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**4)/(sec(c + d*x)**3*b**3 + 3*s
ec(c + d*x)**2*a*b**2 + 3*sec(c + d*x)*a**2*b + a**3),x)
```

3.572 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 337

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{4a(a^2-3b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-}}{3(a-b)b^3(a+b)^{3/2}d} + \frac{2(2a^2+3ab-3b^2)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3(a-b)b^2(a+b)^{3/2}d} - \frac{2a^2\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{4a(a^2-3b^2)\tan(c+dx)}{3b\,(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}$$

output
```
4/3*a*(a^2-3*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),
((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-
b))^(1/2)/(a-b)/b^3/(a+b)^(3/2)/d+2/3*(2*a^2+3*a*b-3*b^2)*cot(d*x+c)*Ellip
ticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x
+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/(a-b)/b^2/(a+b)^(3/2)/d-
2/3*a^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)+4/3*a*(a^2-3*b^2)*
tan(d*x+c)/b/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 11.72 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.49

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{(b+a\cos(c+dx))^3\sec^3(c+dx)\left(\frac{4a(-a^2+3b^2)\sin(c+dx)}{3b^2(-a^2+b^2)^2} - \frac{2a\sin(c+dx)}{3(-a^2+b^2)(b+a\cos(c+dx)}\right.}{d(a+b\sec(c+dx))^{5/2}}$$
$$+\frac{4(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2a(a^3+a^2b-3ab^2-3b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx}}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((4*a*(-a^2 + 3*b^2)*Sin[c + d*x])/
(3*b^2*(-a^2 + b^2)^2) - (2*a*Sin[c + d*x])/(3*(-a^2 + b^2)*(b + a*Cos[c +
 d*x])^2) - (2*(-(a^3*Sin[c + d*x]) + 5*a*b^2*Sin[c + d*x]))/(3*b*(-a^2 +
b^2)^2*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(5/2)) + (4*(b + a*
Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(
2*a*(a^3 + a^2*b - 3*a*b^2 - 3*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*
Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[T
an[(c + d*x)/2]], (a - b)/(a + b)] + b*(-2*a^3 + a^2*b + 6*a*b^2 + 3*b^3)*
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] +
 a*(a^2 - 3*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[
(c + d*x)/2]))/(3*(-(a^2*b) + b^3)^2*d*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*Sec
[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4326, 27, 3042, 4491, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4326}$$

$$-\frac{2\int -\frac{\sec(c+dx)\left(3ab+\left(2a^2-3b^2\right)\sec(c+dx)\right)}{2(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sec(c+dx)\left(3ab+\left(2a^2-3b^2\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3ab+\left(2a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4491}$$

$$\frac{\frac{4a\left(a^2-3b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{\sec(c+dx)\left(b\left(a^2+3b^2\right)-2a\left(a^2-3b^2\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\sec(c+dx)\left(b\left(a^2+3b^2\right)-2a\left(a^2-3b^2\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}+\frac{4a\left(a^2-3b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(b\left(a^2+3b^2\right)-2a\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}+\frac{4a\left(a^2-3b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4493

$$\frac{\frac{(a-b)(2a^2+3ab-3b^2)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-2a(a^2-3b^2)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}+\frac{4a(a^2-3b^2)\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b(a^2-b^2)}-\frac{2a^2\tan(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{(a-b)(2a^2+3ab-3b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-2a(a^2-3b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}+\frac{4a(a^2-3b^2)\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b(a^2-b^2)}-\frac{2a^2\tan(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4319

$$\frac{\frac{\frac{2(a-b)\sqrt{a+b}(2a^2+3ab-3b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-2a(a^2-3b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)(\csc}{\sqrt{a+b\csc(c}}}{a^2-b^2}}{3b(a^2-b^2)}$$

$$\frac{2a^2\tan(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4492

$$\frac{\frac{\frac{2(a-b)\sqrt{a+b}(2a^2+3ab-3b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}+\frac{4a(a-b)\sqrt{a+b}(a^2-3b^2)\cot(c+dx)}{}}{a^2-b^2}}{3b(a^2-b^2)}$$

$$\frac{2a^2\tan(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

input
```
Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*a^2*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (((
4*a*(a - b)*Sqrt[a + b]*(a^2 - 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) + (2*(a - b)
*Sqrt[a + b]*(2*a^2 + 3*a*b - 3*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d))/(a^2 - b^2) +
(4*a*(a^2 - 3*b^2)*Tan[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))
/(3*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4326
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-a^2)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m
 + 1)*(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]
*(a + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e
+ f*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

rule 4491
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1
/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp
[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; Free
Q[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m
, -1]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1291 vs. $2(307) = 614$.

Time = 11.13 (sec) , antiderivative size = 1292, normalized size of antiderivative = 3.83

method	result	size
default	Expression too large to display	1292

input
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d/(a-b)^2/(a+b)^2/b^2*((1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^5*EllipticE(cot(d*x+c)-csc(d*x+c),((
a-b)/(a+b))^(1/2))*(2*cos(d*x+c)^3+4*cos(d*x+c)^2+2*cos(d*x+c))+(2*cos(d*x
+c)^3+6*cos(d*x+c)^2+6*cos(d*x+c)+2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^4*b*EllipticE(cot(d*x+c)-cs
c(d*x+c),((a-b)/(a+b))^(1/2))+(-6*cos(d*x+c)^3-10*cos(d*x+c)^2-2*cos(d*x+c
)+2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*a^3*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+
(-6*cos(d*x+c)^3-18*cos(d*x+c)^2-18*cos(d*x+c)-6)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b^3*Elliptic
E(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(-6*cos(d*x+c)^2-12*cos(d*x+c
)-6)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*a*b^4*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*a^4*b*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(-2*cos(d
*x+c)^3-4*cos(d*x+c)^2-2*cos(d*x+c))+(cos(d*x+c)^3-3*cos(d*x+c)-2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*a^3*b^2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(6*cos(d*x+c
)^3+13*cos(d*x+c)^2+8*cos(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b^3*EllipticF(cot(d*x+c...
```

Fricas [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{\sec\left(dx+c\right)^3}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3/(b^3*sec(d*x + c)^3 + 3*a
*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^3{(c+dx)}}{(a+b\sec{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec{(dx+c)}^3}{(b\sec{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^3\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^3}{\sec\left(dx+c\right)^3 b^3+3\sec\left(dx+c\right)^2 a\,b^2+3\sec\left(dx+c\right)a^2b+a^3}dx$$

input
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**3)/(sec(c + d*x)**3*b**3 + 3*s
ec(c + d*x)**2*a*b**2 + 3*sec(c + d*x)*a**2*b + a**3),x)
```

3.573 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 317

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2(a^2+3b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(}{}}}{3(a-b)b^2(a+b)^{3/2}d}$$
$$+\frac{2(a-3b)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3(a-b)b(a+b)^{3/2}d}$$
$$+\frac{2a\tan(c+dx)}{3(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}}+\frac{2(a^2+3b^2)\tan(c+dx)}{3(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}$$

output
```
2/3*(a^2+3*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((
a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b)
)^(1/2)/(a-b)/b^2/(a+b)^(3/2)/d+2/3*(a-3*b)*cot(d*x+c)*EllipticF((a+b*sec(
d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1
/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/(a-b)/b/(a+b)^(3/2)/d+2/3*a*tan(d*x+c)
/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)+2/3*(a^2+3*b^2)*tan(d*x+c)/(a^2-b^2)^2
/d/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 11.06 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.53

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{(b+a\cos(c+dx))^3\sec^3(c+dx)\left(-\frac{2(a^2+3b^2)\sin(c+dx)}{3b(-a^2+b^2)^2}+\frac{2b\sin(c+dx)}{3(-a^2+b^2)(b+a\cos(c+dx))}\right.}{d(a+b\sec(c+dx))^{5/2}}$$
$$+\frac{2(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2(a^3+a^2b+3ab^2+3b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((-2*(a^2 + 3*b^2)*Sin[c + d*x])/(3
*b*(-a^2 + b^2)^2) + (2*b*Sin[c + d*x])/(3*(-a^2 + b^2)*(b + a*Cos[c + d*x
])^2) + (4*(a^2*Sin[c + d*x] + b^2*Sin[c + d*x]))/(3*(-a^2 + b^2)^2*(b + a
*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(5/2)) + (2*(b + a*Cos[c + d*x])
^2*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a^3 + a^2*
b + 3*a*b^2 + 3*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]
], (a - b)/(a + b)] - 2*b*(a^2 + 4*a*b + 3*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipt
icF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (a^2 + 3*b^2)*Cos[c + d*x
]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b*(a^2 - b
^2)^2*d*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*Sec[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4323, 27, 3042, 4491, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \ 4323$$

$$\frac{2\int -\frac{\sec(c+dx)(3b-a\sec(c+dx))}{2(a+b\sec(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}+\frac{2a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{2a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\int \frac{\sec(c+dx)(3b-a\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 3042$$

$$\frac{2a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3b-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 4491$$

$$\frac{2a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{-\frac{2\int -\frac{\sec(c+dx)\left(4ab+\left(a^2+3b^2\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}-\frac{2\left(a^2+3b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{2a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\int \frac{\sec(c+dx)\left(4ab+\left(a^2+3b^2\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}-\frac{2\left(a^2+3b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 3042$$

$$\frac{2a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(4ab+\left(a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2\left(a^2+3b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 4493$$

$$\frac{2a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\left(a^2+3b^2\right)\int\frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx-(a-3b)(a-b)\int\frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}-\frac{2\left(a^2+3b^2\right)\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\left(a^2+3b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-3b)(a-b)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2\left(a^2+3b^2\right)\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 4319}$$

$$\frac{2a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\left(a^2+3b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-3b)(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\right.}{bd}}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 4492}$$

$$\frac{2a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{-\frac{2(a-b)\sqrt{a+b}\left(a^2+3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2(a-3b)(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx}{a+b}}}{}}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

input

```
Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(5/2),x]
```

output

```
(2*a*Tan[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (((-2*(a
 - b)*Sqrt[a + b]*(a^2 + 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*S
ec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*(a - 3*b)*(a
- b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/(b*d))/(a^2 - b^2) - (2*(a^2 + 3*b^2)*Tan
[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))/(3*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4323
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[a*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(
a^2 - b^2))), x] - Simp[1/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

rule 4491
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1
/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp
[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; Free
Q[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m
, -1]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1170 vs. $2(287) = 574$.

Time = 7.15 (sec) , antiderivative size = 1171, normalized size of antiderivative = 3.69

method	result	size
default	Expression too large to display	1171

input
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d/(a-b)^2/(a+b)^2/b*((cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*EllipticE(cot(d*x+c)-csc(d*x+c),((a-
b)/(a+b))^(1/2))*(cos(d*x+c)^3+2*cos(d*x+c)^2+cos(d*x+c))+(cos(d*x+c)^3+3*
cos(d*x+c)^2+3*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b*EllipticE(cot(d*x+c)-csc(d*x+c)
,((a-b)/(a+b))^(1/2))+(3*cos(d*x+c)^3+7*cos(d*x+c)^2+5*cos(d*x+c)+1)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*a^2*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(3*cos(d*x
+c)^3+9*cos(d*x+c)^2+9*cos(d*x+c)+3)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3*EllipticE(cot(d*x+c)-cs
c(d*x+c),((a-b)/(a+b))^(1/2))+(3*cos(d*x+c)^2+6*cos(d*x+c)+3)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4*
EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b*Ellipti
cF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(-cos(d*x+c)^3-2*cos(d*x+c)^
2-cos(d*x+c))+(-4*cos(d*x+c)^3-9*cos(d*x+c)^2-6*cos(d*x+c)-1)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*
b^2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(-3*cos(d*x+c)^3-
10*cos(d*x+c)^2-11*cos(d*x+c)-4)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3*EllipticF(cot(d*x+c)-csc...
```

Fricas [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2/(b^3*sec(d*x + c)^3 + 3*a
*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{\sec(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{\cos(c+d\,x)^2\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{\sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)^2}{\sec(dx+c)^3\,b^3+3\sec(dx+c)^2\,a\,b^2+3\sec(dx+c)\,a^2b+a^3}dx$$

input
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**2)/(sec(c + d*x)**3*b**3 + 3*s
ec(c + d*x)**2*a*b**2 + 3*sec(c + d*x)*a**2*b + a**3),x)
```

3.574 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 304

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{8a\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3(a-b)b(a+b)^{3/2}d}$$
$$+\frac{2(3a-b)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3(a-b)b(a+b)^{3/2}d}$$
$$-\frac{2b\tan(c+dx)}{3(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}}-\frac{8ab\tan(c+dx)}{3(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}$$

output
```
-8/3*a*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b
))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/(
a-b)/b/(a+b)^(3/2)/d+2/3*(3*a-b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/
2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+
sec(d*x+c))/(a-b))^(1/2)/(a-b)/b/(a+b)^(3/2)/d-2/3*b*tan(d*x+c)/(a^2-b^2)/
d/(a+b*sec(d*x+c))^(3/2)-8/3*a*b*tan(d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
^(1/2)
```

Mathematica [A] (verified)

Time = 5.86 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.18

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx =$$

$$-\frac{2(b+a\cos(c+dx))\sec^3(c+dx)\left(b^2(-a^2+b^2)\sin(c+dx)-b(-5a^2+b^2)\left(b+a\cos(c+dx)\right)\sin(c+d\right.}{}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*(b + a*Cos[c + d*x])*Sec[c + d*x]^3*(b^2*(-a^2 + b^2)*Sin[c + d*x] - b
*(-5*a^2 + b^2)*(b + a*Cos[c + d*x])*Sin[c + d*x] - 4*a^2*(b + a*Cos[c + d
*x])^2*Sin[c + d*x] + 2*a*Cos[(c + d*x)/2]^2*(b + a*Cos[c + d*x])*(4*a*(a
+ b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)] - (3*a^2 + 4*a*b + b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
+ a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)] + 2*a*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c
+ d*x)/2]^2*Tan[(c + d*x)/2])))/(3*a*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^
(5/2))
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {3042, 4320, 27, 3042, 4491, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4320}$$

$$-\frac{2\int -\frac{\sec(c+dx)(3a-b\sec(c+dx))}{2(a+b\sec(c+dx))^{3/2}}dx}{3(a^2-b^2)}-\frac{2b\tan(c+dx)}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sec(c+dx)(3a-b\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}dx}{3(a^2-b^2)}-\frac{2b\tan(c+dx)}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3a-b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3(a^2-b^2)}-\frac{2b\tan(c+dx)}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4491}$$

$$\frac{-\frac{2\int -\frac{\sec(c+dx)\left(3a^2+4b\sec(c+dx)a+b^2\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}-\frac{8ab\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3(a^2-b^2)}-\frac{2b\tan(c+dx)}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\sec(c+dx)\left(3a^2+4b\sec(c+dx)a+b^2\right)}{\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}-\frac{8ab\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3(a^2-b^2)}-\frac{2b\tan(c+dx)}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3a^2+4b\csc\left(c+dx+\frac{\pi}{2}\right)a+b^2\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{8ab\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3(a^2-b^2)}-\frac{2b\tan(c+dx)}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4493}$$

$$\frac{\frac{(a-b)(3a-b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+4ab\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}-\frac{8ab\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3(a^2-b^2)}-\frac{2b\tan(c+dx)}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(a-b)(3a-b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+4ab\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{8ab\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\big\downarrow$ 4319

$$\frac{\frac{4ab\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)(3a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$$\frac{2b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\big\downarrow$ 4492

$$\frac{\frac{\frac{2(a-b)(3a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-\frac{8a(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{}}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$$\frac{2b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

input
```
Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*b*Tan[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (((-8*a
*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (2*(a - b)*(3*a - b)*Sqrt[a +
 b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
+ d*x]))/(a - b))])/(b*d))/(a^2 - b^2) - (8*a*b*Tan[c + d*x])/((a^2 - b^2)
*d*Sqrt[a + b*Sec[c + d*x]]))/(3*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4320
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*
Csc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ
[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 4491
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1
/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp
[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; Free
Q[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m
, -1]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(274) = 548$.

Time = 8.26 (sec) , antiderivative size = 925, normalized size of antiderivative = 3.04

method	result	size
default	Expression too large to display	925

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/(a-b)^2/(a+b)^2*((1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/
(a+b))^(1/2))*(4*cos(d*x+c)^3+8*cos(d*x+c)^2+4*cos(d*x+c))+(4*cos(d*x+c)^3
+12*cos(d*x+c)^2+12*cos(d*x+c)+4)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b*EllipticE(cot(d*x+c)-csc(d
*x+c),((a-b)/(a+b))^(1/2))+(4*cos(d*x+c)^2+8*cos(d*x+c)+4)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2*E
llipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3*EllipticF(
cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(-3*cos(d*x+c)^3-6*cos(d*x+c)^2
-3*cos(d*x+c))+(-4*cos(d*x+c)^3-11*cos(d*x+c)^2-10*cos(d*x+c)-3)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a
^2*b*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(-4-cos(d*x+c)^3
-6*cos(d*x+c)^2-9*cos(d*x+c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2*EllipticF(cot(d*x+c)-csc(d*x+c
),((a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^3*Elliptic
F(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+4*a^3*cos(d*x+c)^2*sin(d*x+c)
+sin(d*x+c)*cos(d*x+c)*(-5*cos(d*x+c)+3)*a^2*b-4*a*b^2*cos(d*x+c)*sin(d*x+
c)+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)+1)*b^3)*(a+b*sec(d*x+c))^(1/2)/(co...
```

Fricas [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)/(b^3*sec(d*x + c)^3 + 3*a*b
^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos(c+d\,x)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)}{\sec(dx+c)^{3}\,b^{3}+3\sec(dx+c)^{2}\,a\,b^{2}+3\sec(dx+c)\,a^{2}b+a^{3}}dx$$

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sec(c + d*x))/(sec(c + d*x)**3*b**3 + 3*sec(
c + d*x)**2*a*b**2 + 3*sec(c + d*x)*a**2*b + a**3),x)
```

3.575 $\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 448

$$\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2(7a^2-3b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\ldots}}{3a^2(a-b)(a+b)^{3/2}d}$$
$$-\frac{2(6a^2-ab-3b^2)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3a^2(a-b)(a+b)^{3/2}d}$$
$$-\frac{2\sqrt{a+b}\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a^3d}$$
$$+\frac{2b^2\tan(c+dx)}{3a(a^2-b^2)d(a+b\sec(c+dx))^{3/2}}+\frac{2b^2(7a^2-3b^2)\tan(c+dx)}{3a^2(a^2-b^2)^2d\sqrt{a+b\sec(c+dx)}}$$

output
```
2/3*(7*a^2-3*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),
((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-
b))^(1/2)/a^2/(a-b)/(a+b)^(3/2)/d-2/3*(6*a^2-a*b-3*b^2)*cot(d*x+c)*Ellipti
cF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c
))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/(a-b)/(a+b)^(3/2)/d-2*
(a+b)^(1/2)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)
/a,((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/
(a-b))^(1/2)/a^3/d+2/3*b^2*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)
+2/3*b^2*(7*a^2-3*b^2)*tan(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1382 vs. $2(448) = 896$.

Time = 12.56 (sec) , antiderivative size = 1382, normalized size of antiderivative = 3.08

$$\int \frac{1}{(a + b\sec(c + dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(-5/2),x]
```

output
```
((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*b*(-7*a^2 + 3*b^2)*Sin[c + d*x]
)/(3*a^2*(-a^2 + b^2)^2) - (2*b^3*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*(b + a*
Cos[c + d*x])^2) - (8*(-2*a^2*b^2*Sin[c + d*x] + b^4*Sin[c + d*x]))/(3*a^2
*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(5/2)) + (2
*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)*Sqrt[(a + b - a*Tan[(c + d*
x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(7*a^3*b*Tan[(c
+ d*x)/2] + 7*a^2*b^2*Tan[(c + d*x)/2] - 3*a*b^3*Tan[(c + d*x)/2] - 3*b^4*
Tan[(c + d*x)/2] - 14*a^3*b*Tan[(c + d*x)/2]^3 + 6*a*b^3*Tan[(c + d*x)/2]^
3 + 7*a^3*b*Tan[(c + d*x)/2]^5 - 7*a^2*b^2*Tan[(c + d*x)/2]^5 - 3*a*b^3*Ta
n[(c + d*x)/2]^5 + 3*b^4*Tan[(c + d*x)/2]^5 + 6*a^4*EllipticPi[-1, ArcSin[
Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
 b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 12*a^2*b^2*El
lipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c +
d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
 b)] + 6*b^4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqr
t[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c +
d*x)/2]^2)/(a + b)] + 6*a^4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a -
b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b -
a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 12*a^2*b^2*Ellipti
cPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*S...
```

Rubi [A] (verified)

Time = 1.93 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4272, 27, 3042, 4548, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \ 4272$$

$$\frac{2b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2\int -\frac{b^2\sec^2(c+dx)-3ab\sec(c+dx)+3\left(a^2-b^2\right)}{2(a+b\sec(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{b^2\sec^2(c+dx)-3ab\sec(c+dx)+3\left(a^2-b^2\right)}{(a+b\sec(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2-3ab\csc\left(c+dx+\frac{\pi}{2}\right)+3\left(a^2-b^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 4548$$

$$\frac{\frac{2b^2\left(7a^2-3b^2\right)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{3\left(a^2-b^2\right)^2-b^2\left(7a^2-3b^2\right)\sec^2(c+dx)-2ab\left(3a^2-b^2\right)\sec(c+dx)}{2\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}}{3a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{3\left(a^2-b^2\right)^2-b^2\left(7a^2-3b^2\right)\sec^2(c+dx)-2ab\left(3a^2-b^2\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}+\frac{2b^2\left(7a^2-3b^2\right)\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{3\left(a^2-b^2\right)^2-b^2\left(7a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-2ab\left(3a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{2b^2\left(7a^2-3b^2\right)\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4546

$$\frac{\frac{\int \frac{3\left(a^2-b^2\right)^2+\left(b^2\left(7a^2-3b^2\right)-2ab\left(3a^2-b^2\right)\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-b^2\left(7a^2-3b^2\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}+\frac{2b^2\left(7a^2-3b^2\right)\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{3\left(a^2-b^2\right)^2+\left(b^2\left(7a^2-3b^2\right)-2ab\left(3a^2-b^2\right)\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b^2\left(7a^2-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{2b^2\left(7a^2-3b^2\right)\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4409

$$\frac{\frac{-\left(b^2\left(7a^2-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-b(a-b)\left(6a^2-ab-3b^2\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+3\left(a^2-b^2\right)^2\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}+\frac{2b^2(7}{ad(a^2-}}{3a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{-\left(b^2(7a^2-3b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-b(a-b)(6a^2-ab-3b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+3(a^2-b^2)^2\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+}{3a(a^2-b^2)}$$

$$\frac{2b^2\tan(c+dx)}{3ad(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4271

$$\frac{\frac{-\left(b^2(7a^2-3b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-b(a-b)(6a^2-ab-3b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6\sqrt{a+b}(a^2-b^2)^2\cot(c+dx)\sqrt{\frac{b(1-\sec(c+}{a+b}}}{}}{a(a^2-b^2)}}{3a(a^2-b^2)}$$

$$\frac{2b^2\tan(c+dx)}{3ad(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4319

$$\frac{\frac{-\left(b^2(7a^2-3b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-\frac{2(a-b)\sqrt{a+b}(6a^2-ab-3b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\right.\right.}{d}}{a(a^2-b^2)}}{}$$

$$\frac{2b^2\tan(c+dx)}{3ad(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4492

$$\frac{\frac{-\frac{2(a-b)\sqrt{a+b}(6a^2-ab-3b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}+\frac{2(a-b)\sqrt{a+b}(7a^2-3b^2)\cot(c+dx}{}}{}}{}$$

$$\frac{2b^2\tan(c+dx)}{3ad(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

input
```
Int[(a + b*Sec[c + d*x])^(-5/2),x]
```

output
```
(2*b^2*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (((2
*(a - b)*Sqrt[a + b]*(7*a^2 - 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d - (2*(a - b)*Sqrt[a
 + b]*(6*a^2 - a*b - 3*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d - (6*Sqrt[a + b]*(a^2 - b^2)
^2*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt
[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(a*d))/(a*(a^2 - b^2)) + (2*b^2*(7*a^2 - 3*
b^2)*Tan[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))/(3*a*(a^2 -
 b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1934 vs. $2(409) = 818$.

Time = 8.91 (sec) , antiderivative size = 1935, normalized size of antiderivative = 4.32

method	result	size
default	Expression too large to display	1935

input
```
int(1/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d/(a-b)^2/(a+b)^2/a^2*((cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5*EllipticPi(cot(d*x+c)-csc(d*x+c),-
1,((a-b)/(a+b))^(1/2))*(6*cos(d*x+c)^3+12*cos(d*x+c)^2+6*cos(d*x+c))+(6*co
s(d*x+c)^2+12*cos(d*x+c)+6)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b*EllipticPi(cot(d*x+c)-csc(d*x+c)
,-1,((a-b)/(a+b))^(1/2))+(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2*EllipticPi(cot(d*x+c)-csc(d*x+c),
-1,((a-b)/(a+b))^(1/2))*(-12*cos(d*x+c)^3-24*cos(d*x+c)^2-12*cos(d*x+c))+(
-12*cos(d*x+c)^2-24*cos(d*x+c)-12)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3*EllipticPi(cot(d*x+c)-c
sc(d*x+c),-1,((a-b)/(a+b))^(1/2))+(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^4*EllipticPi(cot(d*x+c)-csc(
d*x+c),-1,((a-b)/(a+b))^(1/2))*(6*cos(d*x+c)^3+12*cos(d*x+c)^2+6*cos(d*x+c
))+(6*cos(d*x+c)^2+12*cos(d*x+c)+6)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^5*EllipticPi(cot(d*x+c)-csc(
d*x+c),-1,((a-b)/(a+b))^(1/2))+(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b*EllipticE(cot(d*x+c)-csc(d*x+
c),((a-b)/(a+b))^(1/2))*(7*cos(d*x+c)^3+14*cos(d*x+c)^2+7*cos(d*x+c))+(7*c
os(d*x+c)^3+21*cos(d*x+c)^2+21*cos(d*x+c)+7)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2*EllipticE(...
```

Fricas [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x +
c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int(1/(a + b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^3\,b^3+3\sec(dx+c)^2\,a\,b^2+3\sec(dx+c)\,a^2b+a^3}dx$$

input
```
int(1/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)/(sec(c + d*x)**3*b**3 + 3*sec(c + d*x)**2*a*b
**2 + 3*sec(c + d*x)*a**2*b + a**3),x)
```

3.576 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 510

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{(3a^4-26a^2b^2+15b^4)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c}{a+b}}\cdots}{3a^3(a-b)b(a+b)^{3/2}d}$$

$$+\frac{(3a^3+21a^2b-5ab^2-15b^3)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{se}}{a}}\cdots}{3a^3(a-b)(a+b)^{3/2}d}$$

$$+\frac{5b\sqrt{a+b}\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a^4d}$$

$$+\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}+\frac{b(3a^2-5b^2)\tan(c+dx)}{3a^2(a^2-b^2)d(a+b\sec(c+dx))^{3/2}}$$

$$+\frac{b(3a^4-26a^2b^2+15b^4)\tan(c+dx)}{3a^3(a^2-b^2)^2d\sqrt{a+b\sec(c+dx)}}$$

output
```
1/3*(3*a^4-26*a^2*b^2+15*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/
(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec
(d*x+c))/(a-b))^(1/2)/a^3/(a-b)/b/(a+b)^(3/2)/d+1/3*(3*a^3+21*a^2*b-5*a*b^
2-15*b^3)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(
a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2
)/a^3/(a-b)/(a+b)^(3/2)/d+5*b*(a+b)^(1/2)*cot(d*x+c)*EllipticPi((a+b*sec(d
*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a
+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d+sin(d*x+c)/a/d/(a+b*sec(d
*x+c))^(3/2)+1/3*b*(3*a^2-5*b^2)*tan(d*x+c)/a^2/(a^2-b^2)/d/(a+b*sec(d*x+c
))^(3/2)+1/3*b*(3*a^4-26*a^2*b^2+15*b^4)*tan(d*x+c)/a^3/(a^2-b^2)^2/d/(a+b
*sec(d*x+c))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1481 vs. $2(510) = 1020$.

Time = 14.83 (sec) , antiderivative size = 1481, normalized size of antiderivative = 2.90

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((-4*b^2*(-5*a^2 + 3*b^2)*Sin[c + d
*x])/(3*a^3*(-a^2 + b^2)^2) + (2*b^4*Sin[c + d*x])/(3*a^3*(a^2 - b^2)*(b +
 a*Cos[c + d*x])^2) + (2*(-11*a^2*b^3*Sin[c + d*x] + 7*b^5*Sin[c + d*x]))/
(3*a^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(5/2)
) - ((b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)*Sqrt[(1 - Tan[(c + d*x)
/2]^2)^(-1)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1
 + Tan[(c + d*x)/2]^2)]*(3*a^5*Tan[(c + d*x)/2] + 3*a^4*b*Tan[(c + d*x)/2]
 - 26*a^3*b^2*Tan[(c + d*x)/2] - 26*a^2*b^3*Tan[(c + d*x)/2] + 15*a*b^4*Ta
n[(c + d*x)/2] + 15*b^5*Tan[(c + d*x)/2] - 6*a^5*Tan[(c + d*x)/2]^3 + 52*a
^3*b^2*Tan[(c + d*x)/2]^3 - 30*a*b^4*Tan[(c + d*x)/2]^3 + 3*a^5*Tan[(c + d
*x)/2]^5 - 3*a^4*b*Tan[(c + d*x)/2]^5 - 26*a^3*b^2*Tan[(c + d*x)/2]^5 + 26
*a^2*b^3*Tan[(c + d*x)/2]^5 + 15*a*b^4*Tan[(c + d*x)/2]^5 - 15*b^5*Tan[(c
+ d*x)/2]^5 - 30*a^4*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
 + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b
*Tan[(c + d*x)/2]^2)/(a + b)] + 60*a^2*b^3*EllipticPi[-1, ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Ta
n[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*b^5*EllipticPi[-1,
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sq
rt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*a^4
*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d...
```

Rubi [A] (verified)

Time = 2.30 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$, Rules used = {3042, 4333, 27, 3042, 4549, 27, 3042, 4548, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4333}$$

$$\frac{\int -\frac{5b-3b\sec^2(c+dx)}{2(a+b\sec(c+dx))^{5/2}}dx}{a}+\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\int \frac{5b-3b\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\int \frac{5b-3b\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{2a}$$

$$\Big\downarrow \text{4549}$$

$$\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{-\frac{2\int -\frac{-6a\sec(c+dx)b^2-\left(3a^2-5b^2\right)\sec^2(c+dx)b+15\left(a^2-b^2\right)b}{2(a+b\sec(c+dx))^{3/2}}dx}{3a(a^2-b^2)}-\frac{2b(3a^2-5b^2)\tan(c+dx)}{3ad(a^2-b^2)(a+b\sec(c+dx))^{3/2}}}{2a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\int \frac{-6a\sec(c+dx)b^2-\left(3a^2-5b^2\right)\sec^2(c+dx)b+15\left(a^2-b^2\right)b}{(a+b\sec(c+dx))^{3/2}}dx}{3a(a^2-b^2)}-\frac{2b(3a^2-5b^2)\tan(c+dx)}{3ad(a^2-b^2)(a+b\sec(c+dx))^{3/2}}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\int \frac{-6a\csc\left(c+dx+\frac{\pi}{2}\right)b^2-\left(3a^2-5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2b+15\left(a^2-b^2\right)b}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a(a^2-b^2)}-\frac{2b(3a^2-5b^2)\tan(c+dx)}{3ad(a^2-b^2)(a+b\sec(c+dx))^{3/2}}}{2a}$$

$$\Big\downarrow \text{4548}$$

$$\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{-\frac{2\int -\frac{-2a\left(9a^2-5b^2\right)\sec(c+dx)b^2+15\left(a^2-b^2\right)^2b+\left(3a^4-26b^2a^2+15b^4\right)\sec^2(c+dx)b}{2\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{2b\left(3a^4-26a^2b^2+15b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3a(a^2-b^2)}-\frac{2b(3a^2-5b^2)\tan(c+}{3ad(a^2-b^2)(a+b\sec(c+}}{2a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\int \frac{-2a\left(9a^2-5b^2\right)\sec(c+dx)b^2+15\left(a^2-b^2\right)^2b+\left(3a^4-26b^2a^2+15b^4\right)\sec^2(c+dx)b}{\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{2b\left(3a^4-26a^2b^2+15b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3a(a^2-b^2)}-\frac{2b(3a^2-5b^2)\tan(c+dx)}{3ad(a^2-b^2)(a+b\sec(c+dx))^{3}}}{2a}$$

$\downarrow$ 3042

$$\frac{\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\int \frac{-2a\left(9a^2-5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)b^2+15\left(a^2-b^2\right)^2b+\left(3a^4-26b^2a^2+15b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2b}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}-\frac{2b\left(3a^4-26a^2b^2+15b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3a(a^2-b^2)}-\frac{2b(3a^2-5b^2)\tan(}{3ad(a^2-b^2)(a+b\sec(}}{2a}$$

$\downarrow$ 4546

$$\frac{\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(3a^4-26a^2b^2+15b^4\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx+\int \frac{15b\left(a^2-b^2\right)^2+\left(-2a\left(9a^2-5b^2\right)b^2-\left(3a^4-26b^2a^2+15b^4\right)b\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{2b\left(3a^4-26a^2b^2+15b^4\right)\tan}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+}}}{3a(a^2-b^2)}}{2a}$$

$\downarrow$ 3042

$$\frac{\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(3a^4-26a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{15b\left(a^2-b^2\right)^2+\left(-2a\left(9a^2-5b^2\right)b^2-\left(3a^4-26b^2a^2+15b^4\right)b\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}-\frac{2b\left(3a^4-26a\right.}{ad\left(a^2-b\right.}}{3a(a^2-b^2)}}{2a}$$

$\downarrow$ 4409

$$\frac{\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{15b\left(a^2-b^2\right)^2\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx+b\left(3a^4-26a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b(a-b)\left(3a^3+21a^2b-5ab^2-15b^3\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}-}{3a(a^2-b^2)}}{2a}$$

$\downarrow$ 3042

$$\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-$$

$$15b\left(a^2-b^2\right)^2\int\frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\left(3a^4-26a^2b^2+15b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b(a-b)\left(3a^3+21a^2b-5ab^2-15b^3\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\right.}}$$

$$a\left(a^2-b^2\right)$$

$$3a(a^2-b^2)$$

$$2a$$

$\downarrow$ 4271

$$\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-$$

$$b\left(3a^4-26a^2b^2+15b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b(a-b)\left(3a^3+21a^2b-5ab^2-15b^3\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{30b\sqrt{a+b}\left(a^2-b^2\right)^2\cot(c+dx)\sqrt{b}}{}$$

$$a\left(a^2-b^2\right)$$

$$3a(a^2-b^2)$$

$\downarrow$ 4319

$$\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-$$

$$b\left(3a^4-26a^2b^2+15b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{30b\sqrt{a+b}\left(a^2-b^2\right)^2\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\sqrt{}\right.\right.}{ad}$$

$$a\left(a^2-\right.$$

$\downarrow$ 4492

$$\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-$$

$$-\frac{30b\sqrt{a+b}\left(a^2-b^2\right)^2\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}-\frac{2(a-b)\sqrt{a+b}\left(3a^4-26a^2b^2+15b^4\right)\text{cc}}{}$$

input

```
Int[Cos[c + d*x]/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Sin[c + d*x]/(a*d*(a + b*Sec[c + d*x])^(3/2)) - ((-2*b*(3*a^2 - 5*b^2)*Tan
[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (((-2*(a - b)*
Sqrt[a + b]*(3*a^4 - 26*a^2*b^2 + 15*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sq
rt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) - (2*(a -
b)*Sqrt[a + b]*(3*a^3 + 21*a^2*b - 5*a*b^2 - 15*b^3)*Cot[c + d*x]*Elliptic
F[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d - (
30*b*Sqrt[a + b]*(a^2 - b^2)^2*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[S
qrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d))/(a*(a^2
- b^2)) - (2*b*(3*a^4 - 26*a^2*b^2 + 15*b^4)*Tan[c + d*x])/(a*(a^2 - b^2)*
d*Sqrt[a + b*Sec[c + d*x]]))/(3*a*(a^2 - b^2)))/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4333
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*
Csc[e + f*x])^n/(a*f*n)), x] - Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[b*(m + n + 1) - a*(n + 1)*Csc[e + f*x] - b*(m
 + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m + 1/2, 0] && ILtQ[n, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

rule 4549
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 + a^2*C)*Cot[e + f*x]*((a + b*Csc[
e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 -
 b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*
(A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m
] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2241 vs. $2(469) = 938$.

Time = 11.64 (sec) , antiderivative size = 2242, normalized size of antiderivative = 4.40

method	result	size
default	Expression too large to display	2242

input
```
int(cos(d*x+c)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d/(a-b)^2/(a+b)^2/a^3*((-3*cos(d*x+c)^3-9*cos(d*x+c)^2-9*cos(d*x+c)-3)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*a^5*b*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+sin(d*
x+c)*cos(d*x+c)*(-6*cos(d*x+c)^2+20*cos(d*x+c)+3)*a^4*b^2+sin(d*x+c)*cos(d
*x+c)*(-34*cos(d*x+c)+18)*a^3*b^3+sin(d*x+c)*cos(d*x+c)*(3*cos(d*x+c)^2-12
*cos(d*x+c)-26)*a^2*b^4+sin(d*x+c)*cos(d*x+c)*(20*cos(d*x+c)-10)*a*b^5+(30
*cos(d*x+c)^2+60*cos(d*x+c)+30)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^6*EllipticPi(cot(d*x+c)-csc(d*x+
c),-1,((a-b)/(a+b))^(1/2))+(-15*cos(d*x+c)^2-30*cos(d*x+c)-15)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^6
*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^6*Elliptic
E(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(-3*cos(d*x+c)^3-6*cos(d*x+c)
^2-3*cos(d*x+c))+6*a^5*b*cos(d*x+c)^2*sin(d*x+c)+3*a^6*cos(d*x+c)^3*sin(d*
x+c)+15*b^6*cos(d*x+c)*sin(d*x+c)+(26*cos(d*x+c)^3+49*cos(d*x+c)^2+20*cos(
d*x+c)-3)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*a^4*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1
/2))+(26*cos(d*x+c)^3+78*cos(d*x+c)^2+78*cos(d*x+c)+26)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3*b^3*El
lipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+(-15*cos(d*x+c)^3-4*...
```

Fricas [F]

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos{(dx+c)}}{(b\sec{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)/(b^3*sec(d*x + c)^3 + 3*a*b
^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos{(c+dx)}}{(a+b\sec{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(cos(c + d*x)/(a + b*sec(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos{(dx+c)}}{(b\sec{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos{(dx+c)}}{(b\sec{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)/(a + b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}\,\cos(dx+c)}{\sec(dx+c)^3\,b^3+3\sec(dx+c)^2\,a\,b^2+3\sec(dx+c)\,a^2b+a^3}dx$$

input
```
int(cos(d*x+c)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*cos(c + d*x))/(sec(c + d*x)**3*b**3 + 3*sec(
c + d*x)**2*a*b**2 + 3*sec(c + d*x)*a**2*b + a**3),x)
```

3.577 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 562

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx =$$

$$-\frac{(33a^4-170a^2b^2+105b^4)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{12a^4(a-b)(a+b)^{3/2}d}$$

$$+\frac{(a+3b)\left(6a^3-45a^2b+35b^3\right)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+}{}}}{12a^4(a-b)(a+b)^{3/2}d}$$

$$-\frac{\sqrt{a+b}(4a^2+35b^2)\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c-}{a-b}}}{4a^5d}$$

$$-\frac{7b\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}}+\frac{\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}$$

$$-\frac{b^2(27a^2-35b^2)\tan(c+dx)}{12a^3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}}-\frac{b^2(33a^4-170a^2b^2+105b^4)\tan(c+dx)}{12a^4\left(a^2-b^2\right)^2d\sqrt{a+b\sec(c+dx)}}$$

output
```
-1/12*(33*a^4-170*a^2*b^2+105*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(
1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(
1+sec(d*x+c))/(a-b))^(1/2)/a^4/(a-b)/(a+b)^(3/2)/d+1/12*(a+3*b)*(6*a^3-45*
a^2*b+35*b^3)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+
b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^
(1/2)/a^4/(a-b)/(a+b)^(3/2)/d-1/4*(a+b)^(1/2)*(4*a^2+35*b^2)*cot(d*x+c)*El
lipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(
b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^5/d-7/4*b*
sin(d*x+c)/a^2/d/(a+b*sec(d*x+c))^(3/2)+1/2*cos(d*x+c)*sin(d*x+c)/a/d/(a+b
*sec(d*x+c))^(3/2)-1/12*b^2*(27*a^2-35*b^2)*tan(d*x+c)/a^3/(a^2-b^2)/d/(a+
b*sec(d*x+c))^(3/2)-1/12*b^2*(33*a^4-170*a^2*b^2+105*b^4)*tan(d*x+c)/a^4/(
a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1730 vs. $2(562) = 1124$.

Time = 13.59 (sec) , antiderivative size = 1730, normalized size of antiderivative = 3.08

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*b^3*(-13*a^2 + 9*b^2)*Sin[c + d
*x])/(3*a^4*(-a^2 + b^2)^2) - (2*b^5*Sin[c + d*x])/(3*a^4*(a^2 - b^2)*(b +
 a*Cos[c + d*x])^2) - (4*(-7*a^2*b^4*Sin[c + d*x] + 5*b^6*Sin[c + d*x]))/(
3*a^4*(a^2 - b^2)^2*(b + a*Cos[c + d*x])) + Sin[2*(c + d*x)]/(4*a^3)))/(d*
(a + b*Sec[c + d*x])^(5/2)) + ((b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/
2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c
+ d*x)/2]^2)]*(-33*a^5*b*Tan[(c + d*x)/2] - 33*a^4*b^2*Tan[(c + d*x)/2] +
170*a^3*b^3*Tan[(c + d*x)/2] + 170*a^2*b^4*Tan[(c + d*x)/2] - 105*a*b^5*Ta
n[(c + d*x)/2] - 105*b^6*Tan[(c + d*x)/2] + 66*a^5*b*Tan[(c + d*x)/2]^3 -
340*a^3*b^3*Tan[(c + d*x)/2]^3 + 210*a*b^5*Tan[(c + d*x)/2]^3 - 33*a^5*b*T
an[(c + d*x)/2]^5 + 33*a^4*b^2*Tan[(c + d*x)/2]^5 + 170*a^3*b^3*Tan[(c + d
*x)/2]^5 - 170*a^2*b^4*Tan[(c + d*x)/2]^5 - 105*a*b^5*Tan[(c + d*x)/2]^5 +
 105*b^6*Tan[(c + d*x)/2]^5 + 24*a^6*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c +
 d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 162*a^4*b^2*EllipticPi[-1, A
rcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 396*a^2
*b^4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^
2)/(a + b)] + 210*b^6*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/...
```

Rubi [A] (verified)

Time = 2.89 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.07, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4333, 27, 3042, 4592, 27, 3042, 4548, 27, 3042, 4548, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4333}$$

$$\frac{\int -\frac{\cos(c+dx)\left(-5b\sec^2(c+dx)-2a\sec(c+dx)+7b\right)}{2(a+b\sec(c+dx))^{5/2}}dx}{2a}+\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\int \frac{\cos(c+dx)\left(-5b\sec^2(c+dx)-2a\sec(c+dx)+7b\right)}{(a+b\sec(c+dx))^{5/2}}dx}{4a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\int \frac{-5b\csc\left(c+dx+\frac{\pi}{2}\right)^2-2a\csc\left(c+dx+\frac{\pi}{2}\right)+7b}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{4a}$$

$$\Big\downarrow \text{4592}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\int \frac{4a^2+10b\sec(c+dx)a+35b^2-21b^2\sec^2(c+dx)}{2(a+b\sec(c+dx))^{5/2}}dx}{a}}{4a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\int \frac{4a^2+10b\sec(c+dx)a+35b^2-21b^2\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx}{2a}}{4a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\int \frac{4a^2+10b\csc\left(c+dx+\frac{\pi}{2}\right)a+35b^2-21b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{2a}}{4a}$$

$$\Big\downarrow \text{4548}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{-\frac{2\int -\frac{-b^2\left(27a^2-35b^2\right)\sec^2(c+dx)+6ab\left(3a^2-7b^2\right)\sec(c+dx)+3\left(a^2-b^2\right)\left(4a^2+35b^2\right)}{2(a+b\sec(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}-\frac{2b^2\left(27a^2-35b^2\right)\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}}{2a}}{4a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\int\frac{-b^2(27a^2-35b^2)\sec^2(c+dx)+6ab(3a^2-7b^2)\sec(c+dx)+3(a^2-b^2)(4a^2+35b^2)}{(a+b\sec(c+dx))^{3/2}}dx}{3a(a^2-b^2)}-\frac{2b^2(27a^2-35b^2)\tan(c+dx)}{3ad(a^2-b^2)(a+b\sec(c+dx))^{3/2}}}{2a}}{4a}$$

$\Big\downarrow$ 3042

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\int\frac{-b^2(27a^2-35b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2+6ab(3a^2-7b^2)\csc\left(c+dx+\frac{\pi}{2}\right)+3(a^2-b^2)(4a^2+35b^2)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a(a^2-b^2)}-\frac{2b^2(27a^2-35b^2)\tan(c+dx)}{3ad(a^2-b^2)(a+b\sec(c+dx))^{3/2}}}{2a}}{4a}$$

$\Big\downarrow$ 4548

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{-\frac{2\int-\frac{3(4a^2+35b^2)(a^2-b^2)^2+b^2(33a^4-170b^2a^2+105b^4)\sec^2(c+dx)+2ab(3a^4-54b^2a^2+35b^4)\sec(c+dx)}{2\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}-\frac{2b^2(33a^4-170a\cdots}{ad(a^2-b^2\cdots}}{3a(a^2-b^2)}\cdots}{2a}}{4a}$$

$\Big\downarrow$ 27

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\frac{\int\frac{3(4a^2+35b^2)(a^2-b^2)^2+b^2(33a^4-170b^2a^2+105b^4)\sec^2(c+dx)+2ab(3a^4-54b^2a^2+35b^4)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}-\frac{2b^2(33a^4-170a^2b^2+\cdots}{ad(a^2-b^2)\sqrt{a\cdots}}}{3a(a^2-b^2)}\cdots}{2a}}{4a}$$

$\Big\downarrow$ 3042

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\frac{\int\frac{3(4a^2+35b^2)(a^2-b^2)^2+b^2(33a^4-170b^2a^2+105b^4)\csc\left(c+dx+\frac{\pi}{2}\right)^2+2ab(3a^4-54b^2a^2+35b^4)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}-\frac{2b^2(33a^4-\cdots}{ad(a^2\cdots}}{3a(a^2-b^2)}\cdots}{2a}}{4a}$$

$\Big\downarrow$ 4546

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\frac{b^2\left(33a^4-170a^2b^2+105b^4\right)\int\frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx+\int\frac{3\left(4a^2+35b^2\right)\left(a^2-b^2\right)^2+\left(2ab\left(3a^4-54b^2a^2+35b^4\right)-b^2\left(33a^4-170b^2\cdots\right.\right.}{\sqrt{a+b\sec(c+dx)}}\cdots}{a\left(a^2-b^2\right)}\cdots}{3a\left(a^2-b^2\right)}\cdots}{2a}\cdots}{4a}$$

$\big\downarrow$ 3042

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\frac{b^2\left(33a^4-170a^2b^2+105b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int\frac{3\left(4a^2+35b^2\right)\left(a^2-b^2\right)^2+\left(2ab\left(3a^4-54b^2a^2+35b^4\right)-b^2\left(3\cdots\right.\right.}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}\cdots}{a\left(a^2-b^2\right)}\cdots}{3a\left(a^2-b^2\right)}\cdots}{2a}\cdots}{4a}$$

$\big\downarrow$ 4409

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\frac{3\left(a^2-b^2\right)^2\left(4a^2+35b^2\right)\int\frac{1}{\sqrt{a+b\sec(c+dx)}}dx+b^2\left(33a^4-170a^2b^2+105b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(a-b)(a+3b)\cdots}{a\left(a^2-b^2\right)}\cdots}{3a\left(a^2-b^2\right)}\cdots}{2a}\cdots}{4a}$$

$\big\downarrow$ 3042

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\frac{3\left(a^2-b^2\right)^2\left(4a^2+35b^2\right)\int\frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b^2\left(33a^4-170a^2b^2+105b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(a-b)(\cdots}{a\left(a^2-b^2\right)}\cdots}{3a\left(a^2-b^2\right)}\cdots}{2a\cdots}\cdots}{4a}$$

$\big\downarrow$ 4271

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} - \frac{\frac{b^2\left(33a^4-170a^2b^2+105b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx + b(a-b)(a+3b)\left(6a^3-45a^2b+35b^3\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx \cdots}{a\left(a^2 \cdots\right.}}{\cdots}}{\cdots}$$

$\downarrow$ 4319

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} - \frac{b^2\left(33a^4-170a^2b^2+105b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx - \frac{6\sqrt{a+b}\left(a^2-b^2\right)^2\left(4a^2+35b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{\cdots}}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 4492

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} - \frac{-\frac{6\sqrt{a+b}\left(4a^2+35b^2\right)\left(a^2-b^2\right)^2\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\cdots\right)}{ad}\cdots}{\cdots}}{\cdots}$$

input
```
Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(Cos[c + d*x]*Sin[c + d*x])/(2*a*d*(a + b*Sec[c + d*x])^(3/2)) - ((7*b*Sin
[c + d*x])/(a*d*(a + b*Sec[c + d*x])^(3/2)) - ((-2*b^2*(27*a^2 - 35*b^2)*T
an[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (((-2*(a - b
)*Sqrt[a + b]*(33*a^4 - 170*a^2*b^2 + 105*b^4)*Cot[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*(a -
 b)*Sqrt[a + b]*(a + 3*b)*(6*a^3 - 45*a^2*b + 35*b^3)*Cot[c + d*x]*Ellipti
cF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d -
(6*Sqrt[a + b]*(a^2 - b^2)^2*(4*a^2 + 35*b^2)*Cot[c + d*x]*EllipticPi[(a +
 b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt
[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/
(a*d))/(a*(a^2 - b^2)) - (2*b^2*(33*a^4 - 170*a^2*b^2 + 105*b^4)*Tan[c + d
*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))/(3*a*(a^2 - b^2)))/(2*a))
/(4*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4333
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*
Csc[e + f*x])^n/(a*f*n)), x] - Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[b*(m + n + 1) - a*(n + 1)*Csc[e + f*x] - b*(m
 + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m + 1/2, 0] && ILtQ[n, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2746 vs. $2(513) = 1026$.

Time = 13.27 (sec) , antiderivative size = 2747, normalized size of antiderivative = 4.89

method	result	size
default	Expression too large to display	2747

input
```
int(cos(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/d/(a-b)^2/(a+b)^2/a^4*(33*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^6*b*EllipticE(cot(d*x+c)-csc(d*x+
c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)^3+2*cos(d*x+c)^2+cos(d*x+c))+6*(8*cos(
d*x+c)^3+15*cos(d*x+c)^2+6*cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5*b^2*EllipticF(cot(d*x
+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+12*(4+9*cos(d*x+c)^3+22*cos(d*x+c)^2+1
7*cos(d*x+c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*a^4*b^3*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b)
)^(1/2))+4*(27-7*cos(d*x+c)^3+13*cos(d*x+c)^2+47*cos(d*x+c))*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b
^4*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+14*(-5*cos(d*x+c)^
3-12*cos(d*x+c)^2-9*cos(d*x+c)-2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^5*EllipticF(cot(d*x+c)-csc
(d*x+c),((a-b)/(a+b))^(1/2))+70*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b
^6*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))+24*(-cos(d*x+c)^2-
2*cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*a^6*b*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(
a+b))^(1/2))+162*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b^3*Ellipti...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(cos(c + d*x)**2/(a + b*sec(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{\cos\left(c+d\,x\right)^2}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^2/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^2/(a + b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)b+a}\,\cos\left(dx+c\right)^2}{\sec\left(dx+c\right)^3b^3+3\sec\left(dx+c\right)^2a\,b^2+3\sec\left(dx+c\right)a^2b+a^3}dx$$

input
```
int(cos(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*cos(c + d*x)**2)/(sec(c + d*x)**3*b**3 + 3*s
ec(c + d*x)**2*a*b**2 + 3*sec(c + d*x)*a**2*b + a**3),x)
```

3.578 $\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 535

$$\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx = \frac{2(58a^4-41a^2b^2+15b^4)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{se}}{}}}{15a^3(a-b)^2(a+b)^{5/2}d}$$

$$-\frac{2(45a^4-13a^3b-36a^2b^2+5ab^3+15b^4)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+}{a+b}}}{15a^3(a-b)^2(a+b)^{5/2}d}$$

$$-\frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a^4d}$$

$$+\frac{2b^2\tan(c+dx)}{5a(a^2-b^2)\,d(a+b\sec(c+dx))^{5/2}}+\frac{2b^2(13a^2-5b^2)\tan(c+dx)}{15a^2(a^2-b^2)^2\,d(a+b\sec(c+dx))^{3/2}}$$

$$+\frac{2b^2(58a^4-41a^2b^2+15b^4)\tan(c+dx)}{15a^3(a^2-b^2)^3\,d\sqrt{a+b\sec(c+dx)}}$$

output
```
2/15*(58*a^4-41*a^2*b^2+15*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2
)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+s
ec(d*x+c))/(a-b))^(1/2)/a^3/(a-b)^2/(a+b)^(5/2)/d-2/15*(45*a^4-13*a^3*b-36
*a^2*b^2+5*a*b^3+15*b^4)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)
^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+
c))/(a-b))^(1/2)/a^3/(a-b)^2/(a+b)^(5/2)/d-2*(a+b)^(1/2)*cot(d*x+c)*Ellipt
icPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(b*(1
-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d+2/5*b^2*ta
n(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(5/2)+2/15*b^2*(13*a^2-5*b^2)*tan(
d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(3/2)+2/15*b^2*(58*a^4-41*a^2*b^
2+15*b^4)*tan(d*x+c)/a^3/(a^2-b^2)^3/d/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1790 vs. $2(535) = 1070$.

Time = 13.15 (sec) , antiderivative size = 1790, normalized size of antiderivative = 3.35

$$\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(-7/2),x]
```

output
```
((b + a*Cos[c + d*x])^4*Sec[c + d*x]^4*((2*b*(58*a^4 - 41*a^2*b^2 + 15*b^4
)*Sin[c + d*x])/(15*a^3*(-a^2 + b^2)^3) + (2*b^4*Sin[c + d*x])/(5*a^3*(a^2
 - b^2)*(b + a*Cos[c + d*x])^3) + (2*(-19*a^2*b^3*Sin[c + d*x] + 11*b^5*Si
n[c + d*x]))/(15*a^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + (2*(74*a^4*b^
2*Sin[c + d*x] - 65*a^2*b^4*Sin[c + d*x] + 23*b^6*Sin[c + d*x]))/(15*a^3*(
a^2 - b^2)^3*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(7/2)) + (2*(
b + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/2)*Sqrt[(a + b - a*Tan[(c + d*x)
/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(58*a^5*b*Tan[(c +
 d*x)/2] + 58*a^4*b^2*Tan[(c + d*x)/2] - 41*a^3*b^3*Tan[(c + d*x)/2] - 41*
a^2*b^4*Tan[(c + d*x)/2] + 15*a*b^5*Tan[(c + d*x)/2] + 15*b^6*Tan[(c + d*x
)/2] - 116*a^5*b*Tan[(c + d*x)/2]^3 + 82*a^3*b^3*Tan[(c + d*x)/2]^3 - 30*a
*b^5*Tan[(c + d*x)/2]^3 + 58*a^5*b*Tan[(c + d*x)/2]^5 - 58*a^4*b^2*Tan[(c
+ d*x)/2]^5 - 41*a^3*b^3*Tan[(c + d*x)/2]^5 + 41*a^2*b^4*Tan[(c + d*x)/2]^
5 + 15*a*b^5*Tan[(c + d*x)/2]^5 - 15*b^6*Tan[(c + d*x)/2]^5 + 30*a^6*Ellip
ticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)
] - 90*a^4*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*S
qrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
+ d*x)/2]^2)/(a + b)] + 90*a^2*b^4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c ...
```

Rubi [A] (verified)

Time = 2.48 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.09, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.214$, Rules used = {3042, 4272, 27, 3042, 4548, 27, 3042, 4548, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{4272}$$

$$\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{5/2}}-\frac{2\int -\frac{3b^2\sec^2(c+dx)-5ab\sec(c+dx)+5\left(a^2-b^2\right)}{2(a+b\sec(c+dx))^{5/2}}dx}{5a\left(a^2-b^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{3b^2\sec^2(c+dx)-5ab\sec(c+dx)+5\left(a^2-b^2\right)}{(a+b\sec(c+dx))^{5/2}}dx}{5a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{3b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2-5ab\csc\left(c+dx+\frac{\pi}{2}\right)+5\left(a^2-b^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{5a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{5/2}}$$

$\Big\downarrow$ 4548

$$\frac{\frac{2b^2\left(13a^2-5b^2\right)\tan(c+dx)}{3ad(a^2-b^2)(a+b\sec(c+dx))^{3/2}}-\frac{2\int -\frac{15\left(a^2-b^2\right)^2+b^2\left(13a^2-5b^2\right)\sec^2(c+dx)-6ab\left(5a^2-b^2\right)\sec(c+dx)}{2(a+b\sec(c+dx))^{3/2}}dx}{3a(a^2-b^2)}}{5a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{5/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{15\left(a^2-b^2\right)^2+b^2\left(13a^2-5b^2\right)\sec^2(c+dx)-6ab\left(5a^2-b^2\right)\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx}{3a(a^2-b^2)}+\frac{2b^2\left(13a^2-5b^2\right)\tan(c+dx)}{3ad(a^2-b^2)(a+b\sec(c+dx))^{3/2}}}{5a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{15\left(a^2-b^2\right)^2+b^2\left(13a^2-5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-6ab\left(5a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a(a^2-b^2)}+\frac{2b^2\left(13a^2-5b^2\right)\tan(c+dx)}{3ad(a^2-b^2)(a+b\sec(c+dx))^{3/2}}}{5a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{5/2}}$$

$\Big\downarrow$ 4548

$$\frac{\frac{\frac{2b^2\left(58a^4-41a^2b^2+15b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{15\left(a^2-b^2\right)^3-b^2\left(58a^4-41b^2a^2+15b^4\right)\sec^2(c+dx)-ab\left(45a^4-23b^2a^2+10b^4\right)\sec(c+dx)}{2\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}}{3a(a^2-b^2)}+\frac{2b^2\left(13a^2-5b^2\right.}{3ad(a^2-b^2)(a+b}}{5a\left(a^2-b^2\right)}$$

$$\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{5/2}}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{15\left(a^2-b^2\right)^3-b^2\left(58a^4-41b^2a^2+15b^4\right)\sec^2(c+dx)-ab\left(45a^4-23b^2a^2+10b^4\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\left(58a^4-41a^2b^2+15b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3a(a^2-b^2)}+\frac{2b^2(13a^2-5b^2)\,\mathrm{ta}}{3ad(a^2-b^2)(a+b\sec}}{5a\left(a^2-b^2\right)}$$

$$\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{5/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{15\left(a^2-b^2\right)^3-b^2\left(58a^4-41b^2a^2+15b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-ab\left(45a^4-23b^2a^2+10b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\left(58a^4-41a^2b^2+15b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3a(a^2-b^2)}+\frac{2b^2\left(13a^2\right.}{3ad(a^2-b^2)}}{5a\left(a^2-b^2\right)}$$

$$\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{5/2}}$$

$\downarrow$ 4546

$$\frac{\frac{\frac{\int \frac{15\left(a^2-b^2\right)^3+\left(b^2\left(58a^4-41b^2a^2+15b^4\right)-ab\left(45a^4-23b^2a^2+10b^4\right)\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-b^2\left(58a^4-41a^2b^2+15b^4\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\left(58a^4-41a^2b\right.}{ad\left(a^2-b^2\right)}}{3a(a^2-b^2)}}{5a\left(a^2-b^2\right)}$$

$$\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{5/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{15\left(a^2-b^2\right)^3+\left(b^2\left(58a^4-41b^2a^2+15b^4\right)-ab\left(45a^4-23b^2a^2+10b^4\right)\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b^2\left(58a^4-41a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b}{}}{3a(a^2-b^2)}}{5a\left(a^2-b^2\right)}$$

$$\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{5/2}}$$

$\downarrow$ 4409

$$\frac{\frac{\frac{15\left(a^2-b^2\right)^3\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx-b^2\left(58a^4-41a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b(a-b)\left(45a^4-13a^3b-36a^2b^2+5ab^3+15b^4\right)\int \frac{\sec(c+d}{\sqrt{a+b\sec(}}}{a\left(a^2-b^2\right)}}{3a(a^2-b^2)}}{5a\left(a^2-b^2\right)}$$

$$\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{5/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{15\left(a^2-b^2\right)^3\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b^2\left(58a^4-41a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b(a-b)\left(45a^4-13a^3b-36a^2b^2+5ab^3+15b^4\right)\int \frac{\csc}{\sqrt{a+b}}}{a\left(a^2-b^2\right)}}{3a(a^2-b^2)}}{5a\left(a^2-b^2\right)}$$

$$\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{5/2}}$$

$\downarrow$ 4271

$$\frac{\frac{-b^2\left(58a^4-41a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b(a-b)\left(45a^4-13a^3b-36a^2b^2+5ab^3+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{30\sqrt{a+b}\left(a^2-b^2\right)^3}{}}{a\left(a^2-b^2\right)}}{3a(a^2-b^2)}$$

$$\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{5/2}}$$

$\downarrow$ 4319

$$\frac{-b^2\left(58a^4-41a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{30\sqrt{a+b}\left(a^2-b^2\right)^3\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\right.}{ad}}{}$$

$$\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{5/2}}$$

$$\Big\downarrow \text{4492}$$

$$\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{5/2}}+$$

$$\frac{2b^2\left(13a^2-5b^2\right)\tan(c+dx)}{3ad(a^2-b^2)(a+b\sec(c+dx))^{3/2}}+\frac{\frac{2b^2\left(58a^4-41a^2b^2+15b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{-\frac{30\sqrt{a+b}\left(a^2-b^2\right)^3\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\right.}{ad}}{}}{}$$

input

```
Int[(a + b*Sec[c + d*x])^(-7/2),x]
```

output

```
(2*b^2*Tan[c + d*x])/(5*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(5/2)) + ((2*
b^2*(13*a^2 - 5*b^2)*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])
^(3/2)) + (((2*(a - b)*Sqrt[a + b]*(58*a^4 - 41*a^2*b^2 + 15*b^4)*Cot[c +
d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
- b))])/d - (2*(a - b)*Sqrt[a + b]*(45*a^4 - 13*a^3*b - 36*a^2*b^2 + 5*a*b
^3 + 15*b^4)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
 + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/d - (30*Sqrt[a + b]*(a^2 - b^2)^3*Cot[c + d*x
]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
 b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d
*x]))/(a - b))])/(a*d))/(a*(a^2 - b^2)) + (2*b^2*(58*a^4 - 41*a^2*b^2 + 15
*b^4)*Tan[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))/(3*a*(a^2
- b^2)))/(5*a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3258 vs. $2(492) = 984$.

Time = 11.30 (sec) , antiderivative size = 3259, normalized size of antiderivative = 6.09

method	result	size
default	Expression too large to display	3259

input
```
int(1/(a+b*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15/d/(a-b)^3/(a+b)^3/a^3*((-90*cos(d*x+c)^4-180*cos(d*x+c)^3-60*cos(d*x
+c)^2+60*cos(d*x+c)+30)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^6*b^2*EllipticPi(cot(d*x+c)-csc(d*x+c),-
1,((a-b)/(a+b))^(1/2))+(-30*cos(d*x+c)^2-60*cos(d*x+c)-30)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^8*Ell
ipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))+sin(d*x+c)*cos(d*x+c
)^2*(-74*cos(d*x+c)+100)*a^6*b^2+sin(d*x+c)*cos(d*x+c)*(-41*cos(d*x+c)^2-1
29*cos(d*x+c)+45)*a^5*b^3+sin(d*x+c)*cos(d*x+c)*(65*cos(d*x+c)^2-58*cos(d*
x+c)-58)*a^4*b^4+sin(d*x+c)*cos(d*x+c)*(15*cos(d*x+c)^2+100*cos(d*x+c)-23)
*a^3*b^5+sin(d*x+c)*cos(d*x+c)*(-23*cos(d*x+c)^2+22*cos(d*x+c)+41)*a^2*b^6
+sin(d*x+c)*cos(d*x+c)*(-35*cos(d*x+c)+10)*a*b^7+(15*cos(d*x+c)^2+30*cos(d
*x+c)+15)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*b^8*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))
+(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*a^8*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*(30*
cos(d*x+c)^4+60*cos(d*x+c)^3+30*cos(d*x+c)^2)+(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^8*EllipticF(cot(d*
x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(-15*cos(d*x+c)^4-30*cos(d*x+c)^3-15*
cos(d*x+c)^2)+(-10*cos(d*x+c)^4-28*cos(d*x+c)^3-3*cos(d*x+c)^2+38*cos(d*x+
c)+23)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos...
```

Fricas [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx=\int \frac{1}{(b\sec(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)/(b^4*sec(d*x + c)^4 + 4*a*b^3*sec(d*x +
c)^3 + 6*a^2*b^2*sec(d*x + c)^2 + 4*a^3*b*sec(d*x + c) + a^4), x)
```

Sympy [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx = \int \frac{1}{(a+b\sec{(c+dx)})^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**(7/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(-7/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\sec{(dx+c)}+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\sec{(dx+c)}+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int(1/(a + b/cos(c + d*x))^(7/2),x)
```

output
```
int(1/(a + b/cos(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx=\int \frac{\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^4\,b^4+4\sec(dx+c)^3\,a\,b^3+6\sec(dx+c)^2\,a^2b^2+4\sec(dx+c}$$

input
```
int(1/(a+b*sec(d*x+c))^(7/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)/(sec(c + d*x)**4*b**4 + 4*sec(c + d*x)**3*a*b
**3 + 6*sec(c + d*x)**2*a**2*b**2 + 4*sec(c + d*x)*a**3*b + a**4),x)
```

3.579 $\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 151

$$\begin{aligned}\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx &= -\frac{6b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &\quad+\frac{2a\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\ &\quad+\frac{6b\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} \\ &\quad+\frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2b\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}\end{aligned}$$

output
```
-6/5*b*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(
1/2)/d+2/3*a*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d
*x+c)^(1/2)/d+6/5*b*sec(d*x+c)^(1/2)*sin(d*x+c)/d+2/3*a*sec(d*x+c)^(3/2)*s
in(d*x+c)/d+2/5*b*sec(d*x+c)^(5/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.64

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{\sec^{\frac{5}{2}}(c+dx)\left(-36b\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+20a\cos^{\frac{5}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+21b\sin(\right.}{30d}$$

input
```
Integrate[Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x]),x]
```

output
```
(Sec[c + d*x]^(5/2)*(-36*b*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] +
20*a*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 21*b*Sin[c + d*x] + 10
*a*Sin[2*(c + d*x)] + 9*b*Sin[3*(c + d*x)]))/(30*d)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {3042, 4274, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$
$$\Big\downarrow \text{4274}$$
$$a\int \sec^{\frac{5}{2}}(c+dx)dx+b\int \sec^{\frac{7}{2}}(c+dx)dx$$
$$\Big\downarrow \text{3042}$$
$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+b\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}dx$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+b\left(\frac{3}{5}\int \sec^{\frac{3}{2}}(c+dx)dx+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+b\left(\frac{3}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+b\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+b\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+b\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+b\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+$$
$$b\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+$$
$$b\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)$$

input
```
Int[Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x]),x]
```

output
```
a*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)) + b*((2*Sec[c + d*x]^(5/2)
*Sin[c + d*x])/(5*d) + (3*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/5)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(130) = 260$.

Time = 6.85 (sec) , antiderivative size = 502, normalized size of antiderivative = 3.32

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2a\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{6\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^2}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cdots}}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cdots}}\cdots\right.\right.}{\cdots}$
parts	$-\frac{2a\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(\cdots}{\cdots}}\cdots\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cdots\right.}$

input
```
int(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a*(-1/6*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*b/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2
*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*sin(1/2*d
*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/
2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*sin(1/2*d*x+1/2*c)^2+8*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.25

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$

$$= \frac{-5i\sqrt{2}a\cos(dx+c)^2 \text{ weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5i\sqrt{2}a\cos(dx+c)^2 \text{ w}}{}$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/15*(-5*I*sqrt(2)*a*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) + 5*I*sqrt(2)*a*cos(d*x + c)^2*weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*b*cos(d*x + c)^2*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
+ 9*I*sqrt(2)*b*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*b*cos(d*x + c)^2 + 5*a*cos(d
*x + c) + 3*b)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)*(a+b*sec(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int (b\sec(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int (b\sec(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,dx$$

input
```
int((a + b/cos(c + d*x))*(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + b/cos(c + d*x))*(1/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \left(\int \sqrt{\sec\left(dx+c\right)}\,\sec\left(dx+c\right)^{3}dx\right)b + \left(\int \sqrt{\sec\left(dx+c\right)}\,\sec\left(dx+c\right)^{2}dx\right)a$$

input
```
int(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x)
```

output
```
int(sqrt(sec(c + d*x))*sec(c + d*x)**3,x)*b + int(sqrt(sec(c + d*x))*sec(c
 + d*x)**2,x)*a
```

3.580 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx = -\frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2b\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2b\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}$$

output
```
-2*a*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/
2)/d+2/3*b*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x
+c)^(1/2)/d+2*a*sec(d*x+c)^(1/2)*sin(d*x+c)/d+2/3*b*sec(d*x+c)^(3/2)*sin(d
*x+c)/d
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.69

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$
$$=\frac{\sec^{\frac{3}{2}}(c+dx)\left(-6a\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2b\cos^{\frac{3}{2}}(c+dx)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2(b+3a\,\mathrm{c}\right.}{3d}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x]),x]
```

output
```
(Sec[c + d*x]^(3/2)*(-6*a*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 2
*b*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 2*(b + 3*a*Cos[c + d*x])
*Sin[c + d*x]))/(3*d)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$
$$\Big\downarrow \text{4274}$$
$$a\int \sec^{\frac{3}{2}}(c+dx)dx+b\int \sec^{\frac{5}{2}}(c+dx)dx$$
$$\Big\downarrow \text{3042}$$
$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+b\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx$$

$$\Big\downarrow \; 4255$$

$$a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)+b\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \; 3042$$

$$a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+b\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \; 4258$$

$$a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)+b\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \; 3042$$

$$a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+b\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \; 3119$$

$$b\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \; 3120$$

$$a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+b\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

input
```
Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x]),x]
```

output
```
a*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d) + b*((2*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[
c + d*x])/(3*d))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(110) = 220$.

Time = 2.98 (sec) , antiderivative size = 396, normalized size of antiderivative = 3.22

method	result
default	$-\frac{2\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a-2\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\mathrm{EllipticF}\left(\mathrm{co}\right.\right.}{}$
parts	$-\frac{2a\left(-2\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+}\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$

input
```
int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*
d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(12*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/
2*c)^2*b-6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2*a-6*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^2*a-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b+
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*b+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.36

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$

$$=\frac{-i\sqrt{2}b\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}b\cos(dx+c)\,\mathrm{weiers}}{}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*b*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) + I*sqrt(2)*b*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*a*cos(d*x + c)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(
2)*a*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))) + 2*(3*a*cos(d*x + c) + b)*sin(d*x + c)/sqrt(cos
(d*x + c)))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*sec(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int (b\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int (b\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

input
```
int((a + b/cos(c + d*x))*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))*(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^2\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)\,dx\right)a$$

input
```
int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x)
```

output
```
int(sqrt(sec(c + d*x))*sec(c + d*x)**2,x)*b + int(sqrt(sec(c + d*x))*sec(c
 + d*x),x)*a
```

3.581 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\begin{aligned}\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))\,dx &= -\frac{2b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} \\ &\quad+\frac{2a\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d} \\ &\quad+\frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{d}\end{aligned}$$

output
```
-2*b*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/
2)/d+2*a*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c
)^(1/2)/d+2*b*sec(d*x+c)^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.73

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))\,dx$$
$$=\frac{2\sqrt{\sec(c+dx)}\Big(-b\sqrt{\cos(c+dx)}E\big(\tfrac{1}{2}(c+dx)\big|2\big)+a\sqrt{\cos(c+dx)}\,\text{EllipticF}\big(\tfrac{1}{2}(c+dx),2\big)+b\sin(c+}{d}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x]),x]
```

output
```
(2*Sqrt[Sec[c + d*x]]*(-(b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) +
 a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*Sin[c + d*x]))/d
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$
$$\Big\downarrow \text{4274}$$
$$a\int \sqrt{\sec(c+dx)}dx+b\int \sec^{\frac{3}{2}}(c+dx)dx$$
$$\Big\downarrow \text{3042}$$
$$a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+b\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx$$

$$\Big\downarrow \text{4255}$$

$$a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+b\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+b\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{4258}$$

$$a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+ b\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+ b\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3119}$$

$$a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+ b\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{3120}$$

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+ b\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

input

```
Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x]),x]
```

output
```
(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
b*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.55

method	result
default	$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\right.}}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$
parts	$-\frac{2a\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}-\frac{2b\left(-2\sqrt{-2\sin\right.}{}$

input
```
int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b-(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a
-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*b)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.28

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))\,dx$$
$$=\frac{-i\sqrt{2}a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}a\mathrm{weierstrassPInverse}(-4,0,\cos(}{}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) +
I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - I*
sqrt(2)*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) + I*sqrt(2)*b*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*b*sin(d*x + c)/sqrt(cos(d*x +
c)))/d
```

Sympy [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\sqrt{\sec(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*sqrt(sec(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))\,dx = \int (b\sec(dx+c)+a)\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))\,dx=\int (b\sec(dx+c)+a)\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))\,dx=\int \left(a+\frac{b}{\cos(c+d\,x)}\right)\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

input
```
int((a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))\,dx=\left(\int \sqrt{\sec(dx+c)}dx\right)a+\left(\int \sqrt{\sec(dx+c)}\sec(dx+c)\,dx\right)b$$

input
```
int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x)
```

output
```
int(sqrt(sec(c + d*x)),x)*a + int(sqrt(sec(c + d*x))*sec(c + d*x),x)*b
```

3.582 $\int \frac{a+b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 75

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = \frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d}$$

output
```
2*a*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2
)/d+2*b*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)
^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.69

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = \frac{2\sqrt{\cos(c+dx)}\left(aE\left(\frac{1}{2}(c+dx)\middle|2\right)+b\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right)\sqrt{\sec(c+dx)}}{d}$$

input
```
Integrate[(a + b*Sec[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*(a*EllipticE[(c + d*x)/2, 2] + b*EllipticF[(c + d*x)
/2, 2])*Sqrt[Sec[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \frac{1}{\sqrt{\sec(c+dx)}}dx+b\int \sqrt{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4258}$$

$$a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3119}$$

$$b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx + \frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

input
```
Int[(a + b*Sec[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(70) = 140$.

Time = 2.14 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.03

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\left(b\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-a\,\mathrm{EllipticE}\left(\cos\left(\right.\right.\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$
parts	$\frac{2a\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}-\frac{2b\sqrt{\left(2\cos\left(\frac{dx}{2}+\right.\right.}}{}$
risch	$-\frac{ia\sqrt{2}}{d\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{i\left(\frac{ib\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(dx+c)}-i\right)}\,\sqrt{ie^{i(dx+c)}}\,\mathrm{EllipticF}\left(\sqrt{-i\left(e^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)}{\sqrt{e^{3i(dx+c)}+e^{i(dx+c)}}}+a\left(-\frac{2\left(e^{2i(dx+c)}\right.}{\sqrt{e^{i(dx+c)}\left(e^{2i(c\right.}}}\right.\right.}{}$

input
```
int((a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(b*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.43

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{-i\sqrt{2}b\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}b\text{weierstrassPInverse}(-4,0,\cos(}{}$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) +
I*sqrt(2)*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + I*
sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) - I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F]

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{a+b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*sec(c + d*x))/sqrt(sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{b\sec\left(dx+c\right)+a}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{b\sec\left(dx+c\right)+a}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{a+\frac{b}{\cos(c+d\,x)}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{a + b\sec(c + dx)}{\sqrt{\sec(c + dx)}}\,dx = \left(\int \frac{\sqrt{\sec(dx + c)}}{\sec(dx + c)}dx\right)a + \left(\int \sqrt{\sec(dx + c)}dx\right)b$$

input
```
int((a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x))/sec(c + d*x),x)*a + int(sqrt(sec(c + d*x)),x)*b
```

3.583 $\int \frac{a+b\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
2*b*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2
)/d+2/3*a*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+
c)^(1/2)/d+2/3*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.75

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{\sec(c+dx)}\left(6b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+a\left(2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c-\right.\right.}{3d}$$

input
```
Integrate[(a + b*Sec[c + d*x])/Sec[c + d*x]^(3/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(6*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + a*
(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)])))/(3*d
)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+b\int \frac{1}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+b\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+b\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+b\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+ b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+ b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+ \frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+ \frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

input
```
Int[(a + b*Sec[c + d*x])/Sec[c + d*x]^(3/2),x]
```

output
```
(2*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
a*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(90) = 180$.

Time = 4.40 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.26

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\right.}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}\right.}$
parts	$-\frac{2a\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}\right.}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$

input
```
int((a+b*sec(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*
a+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*a-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.24

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2\,a\sqrt{\cos(dx+c)}\sin(dx+c) - i\sqrt{2}a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)) + i\sqrt{2}a\text{w}\cdots}{}$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(2*a*sqrt(cos(d*x + c))*sin(d*x + c) - I*sqrt(2)*a*weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*a*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*b*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)
*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))))/d
```

Sympy [F]

$$\int \frac{a + b\sec(c + dx)}{\sec^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{a + b\sec{(c + dx)}}{\sec^{\frac{3}{2}}{(c + dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))/sec(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{a + b\sec(c + dx)}{\sec^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{b\sec{(dx + c)} + a}{\sec{(dx + c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{b\sec(dx+c)+a}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{a+\frac{b}{\cos(c+d\,x)}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx\right)a+\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)b$$

input
```
int((a+b*sec(d*x+c))/sec(d*x+c)^(3/2),x)
```

output
```
int(sqrt(sec(c + d*x))/sec(c + d*x)**2,x)*a + int(sqrt(sec(c + d*x))/sec(c
 + d*x),x)*b
```

3.584 $\int \frac{a+b\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 127

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{6a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
6/5*a*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1
/2)/d+2/3*b*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*
x+c)^(1/2)/d+2/5*a*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/3*b*sin(d*x+c)/d/sec(d*
x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.69

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{\sec(c+dx)}\left(18a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+10b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(5b+3a\right.}{15d}$$

input
```
Integrate[(a + b*Sec[c + d*x])/Sec[c + d*x]^(5/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(18*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 1
0*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*b + 3*a*Cos[c + d*x]
)*Sin[2*(c + d*x)]))/(15*d)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \; 4274$$

$$a\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx+b\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+b\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+b\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+ b\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+ b\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+ b\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+ a\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)+ b\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

input
```
Int[(a + b*Sec[c + d*x])/Sec[c + d*x]^(5/2),x]
```

output
```
a*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*
d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))) + b*((2*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3
*d*Sqrt[Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(110) = 220$.

Time = 7.33 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.06

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-24a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6+(24a+20b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-6a-10b)\sin\right.}{15\sqrt{-2\sin\left(\frac{dx}{2}+\right.}}$
parts	$-\frac{2a\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\right.\right.}{5\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\right.}}$

input
```
int((a+b*sec(d*x+c))/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*a+20*b)*sin(1/2*d*x+1/2*c)^4*cos(1/
2*d*x+1/2*c)+(-6*a-10*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*b-9*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.14

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{-5i\sqrt{2}b\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}b\text{weierstrassPInverse}(-4,0,\text{co}}{}$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/15*(-5*I*sqrt(2)*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
 c)) + 5*I*sqrt(2)*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c)) + 9*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c))) - 9*I*sqrt(2)*a*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*a*cos(d*x + c)
^2 + 5*b*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{a+b\sec{(c+dx)}}{\sec^{\frac{5}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

output
```
Integral((a + b*sec(c + d*x))/sec(c + d*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{b\sec{(dx+c)}+a}{\sec{(dx+c)}^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{a + b\sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)}\,dx = \int \frac{b\sec(dx + c) + a}{\sec(dx + c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)}\,dx = \int \frac{a + \frac{b}{\cos(c+d\,x)}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{a + b\sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)}\,dx = \left(\int \frac{\sqrt{\sec(dx + c)}}{\sec(dx + c)^3}dx\right)a + \left(\int \frac{\sqrt{\sec(dx + c)}}{\sec(dx + c)^2}dx\right)b$$

input
```
int((a+b*sec(d*x+c))/sec(d*x+c)^(5/2),x)
```

output
```
int(sqrt(sec(c + d*x))/sec(c + d*x)**3,x)*a + int(sqrt(sec(c + d*x))/sec(c
 + d*x)**2,x)*b
```

3.585 $\int \frac{a+b\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 151

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{6b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{10a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} + \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{10a\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}$$

output
```
6/5*b*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1
/2)/d+10/21*a*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(
d*x+c)^(1/2)/d+2/7*a*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/5*b*sin(d*x+c)/d/sec(
d*x+c)^(3/2)+10/21*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.66

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{\sec(c+dx)}\Big(252b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+100a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(65a}{210d}$$

input
```
Integrate[(a + b*Sec[c + d*x])/Sec[c + d*x]^(7/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(252*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] +
100*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (65*a + 42*b*Cos[c +
d*x] + 15*a*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(210*d)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4274, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}dx+b\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+b\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{5}{7}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+b\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{7}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+b\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+b\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+b\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+b\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+b\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$\downarrow$ 3119

$$a\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+b\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$\downarrow$ 3120

$$a\left(\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{5}{7}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)\right)+b\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

input
```
Int[(a + b*Sec[c + d*x])/Sec[c + d*x]^(7/2),x]
```

output
```
b*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*
d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))) + a*((2*Sin[c + d*x])/(7*d
*Sec[c + d*x]^(5/2)) + (5*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/7
)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(130) = 260$.

Time = 10.83 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.92

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\ (240a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+(-360a-168b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(280a+16}{}$
parts	$-\frac{2a\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\ (48\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}-120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+128\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-72\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+5\sqrt{\frac{1}{2}-\cos}}{21\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\ \sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos(\frac{dx}{2}}}$

input
```
int((a+b*sec(d*x+c))/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*a-168*b)*sin(1/2*d*x+1/2*c)^6*co
s(1/2*d*x+1/2*c)+(280*a+168*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-8
0*a-42*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+25*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*a-63*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.03

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{-25i\sqrt{2}a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+25i\sqrt{2}a\text{weierstrassPInverse}(-4,0,}{}$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
1/105*(-25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c)) + 25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)) + 63*I*sqrt(2)*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*b*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(15*a*cos(d*
x + c)^3 + 21*b*cos(d*x + c)^2 + 25*a*cos(d*x + c))*sin(d*x + c)/sqrt(cos(
d*x + c)))/d
```

Sympy [F]

$$\int \frac{a + b\sec(c + dx)}{\sec^{\frac{7}{2}}(c + dx)}\,dx = \int \frac{a + b\sec{\left(c + dx \right)}}{\sec^{\frac{7}{2}}{\left(c + dx \right)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)**(7/2),x)
```

output
```
Integral((a + b*sec(c + d*x))/sec(c + d*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{a + b\sec(c + dx)}{\sec^{\frac{7}{2}}(c + dx)}\,dx = \int \frac{b\sec\left(dx + c\right) + a}{\sec\left(dx + c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \frac{a + b\sec(c + dx)}{\sec^{\frac{7}{2}}(c + dx)}\,dx = \int \frac{b\sec\left(dx + c\right) + a}{\sec\left(dx + c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{a+\frac{b}{\cos(c+d\,x)}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(7/2),x)
```

output
```
int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^4}dx\right)a+\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3}dx\right)b$$

input
```
int((a+b*sec(d*x+c))/sec(d*x+c)^(7/2),x)
```

output
```
int(sqrt(sec(c + d*x))/sec(c + d*x)**4,x)*a + int(sqrt(sec(c + d*x))/sec(c
 + d*x)**3,x)*b
```

3.586 $\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 200

$$\begin{aligned}\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx &= -\frac{12ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}\\ &+\frac{2(7a^2+5b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d}\\ &+\frac{12ab\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2(7a^2+5b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d}\\ &+\frac{4ab\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2b^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}\end{aligned}$$

output
```
-12/5*a*b*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c
)^(1/2)/d+2/21*(7*a^2+5*b^2)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*
c,2^(1/2))*sec(d*x+c)^(1/2)/d+12/5*a*b*sec(d*x+c)^(1/2)*sin(d*x+c)/d+2/21*
(7*a^2+5*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+4/5*a*b*sec(d*x+c)^(5/2)*sin(d
*x+c)/d+2/7*b^2*sec(d*x+c)^(7/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.70

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$=\frac{\sec^{\frac{7}{2}}(c+dx)\left(-504ab\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+20(7a^2+5b^2)\cos^{\frac{7}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right.\right.}{2}$$

input
```
Integrate[Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2,x]
```

output
```
(Sec[c + d*x]^(7/2)*(-504*a*b*Cos[c + d*x]^(7/2)*EllipticE[(c + d*x)/2, 2]
 + 20*(7*a^2 + 5*b^2)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)/2, 2] + 2*(35
*a^2 + 55*b^2 + 273*a*b*Cos[c + d*x] + 5*(7*a^2 + 5*b^2)*Cos[2*(c + d*x)]
+ 63*a*b*Cos[3*(c + d*x)])*Sin[c + d*x]))/(210*d)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.97, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.739$, Rules used = {3042, 4275, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119, 4534, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$\int \sec^{\frac{5}{2}}(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int \sec^{\frac{7}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}dx$$

$$\Big\downarrow \text{4255}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{5}\int \sec^{\frac{3}{2}}(c+dx)dx+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{4255}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{4258}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3119}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+$$
$$2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right.}{d}\right.\right.$$

$\downarrow$ 4534

$$\frac{1}{7}\left(7a^2+5b^2\right)\int \sec^{\frac{5}{2}}(c+dx)dx+$$
$$2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right.}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(7a^2+5b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+$$
$$2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right.}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}$$

$\downarrow$ 4255

$$\frac{1}{7}\left(7a^2+5b^2\right)\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+$$
$$2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right.}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(7a^2+5b^2\right)\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+$$
$$2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right.}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}$$

$\downarrow$ 4258

$$\frac{1}{7}(7a^2+5b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right|2\right.}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}(7a^2+5b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right|2\right.}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{7}(7a^2+5b^2)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right|2\right.}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}$$

input
```
Int[Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2,x]
```

output
```
(2*b^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d) + ((7*a^2 + 5*b^2)*((2*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)))/7 + 2*a*b*((2*Sec[c + d*x]^(5/2)*Sin
[c + d*x])/(5*d) + (3*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/5)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 688 vs. 2(175) = 350.

Time = 22.23 (sec) , antiderivative size = 689, normalized size of antiderivative = 3.44

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2a^2\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{6\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^2}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4}}\right.\right.}{}$
parts	Expression too large to display

input
```
int(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*(-1/6*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^2*(-1/56*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^
4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+4/5*a*b/sin(1/2*d*x+1/2*c)^2
/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)
*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.18

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{-126i\sqrt{2}ab\cos(dx+c)^3 \operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)}{}$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/105*(-126*I*sqrt(2)*a*b*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 126*I*sqrt(2)*a*b*cos(d
*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))) - 5*sqrt(2)*(7*I*a^2 + 5*I*b^2)*cos(d*x + c)^3*weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*sqrt(2)*(-7*I*a^2 - 5
*I*b^2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
 + c)) + 2*(126*a*b*cos(d*x + c)^3 + 42*a*b*cos(d*x + c) + 5*(7*a^2 + 5*b^
2)*cos(d*x + c)^2 + 15*b^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x +
c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)*(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^2\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

input
```
int((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^4\,dx\right)b^2 + 2\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^3\,dx\right)ab + \left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^2\,dx\right)a^2$$

input

```
int(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x)
```

output

```
int(sqrt(sec(c + d*x))*sec(c + d*x)**4,x)*b**2 + 2*int(sqrt(sec(c + d*x))*
sec(c + d*x)**3,x)*a*b + int(sqrt(sec(c + d*x))*sec(c + d*x)**2,x)*a**2
```

3.587 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 175

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx &= -\frac{2(5a^2+3b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}\\ &\quad+\frac{4ab\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}\\ &\quad+\frac{2(5a^2+3b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}\\ &\quad+\frac{4ab\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2b^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}\end{aligned}$$

output

```
-2/5*(5*a^2+3*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*
sec(d*x+c)^(1/2)/d+4/3*a*b*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,
2^(1/2))*sec(d*x+c)^(1/2)/d+2/5*(5*a^2+3*b^2)*sec(d*x+c)^(1/2)*sin(d*x+c)/
d+4/3*a*b*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*b^2*sec(d*x+c)^(5/2)*sin(d*x+c
)/d
```

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.72

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{\sec^{\frac{5}{2}}(c+dx)\left(-12(5a^2+3b^2)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+40ab\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right.}{30d}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2,x]
```

output
```
(Sec[c + d*x]^(5/2)*(-12*(5*a^2 + 3*b^2)*Cos[c + d*x]^(5/2)*EllipticE[(c +
 d*x)/2, 2] + 40*a*b*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 2*(15*
(a^2 + b^2) + 20*a*b*Cos[c + d*x] + 3*(5*a^2 + 3*b^2)*Cos[2*(c + d*x)])*Si
n[c + d*x]))/(30*d)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.95, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4275, 3042, 4255, 3042, 4258, 3042, 3120, 4534, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$\int \sec^{\frac{3}{2}}(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int \sec^{\frac{5}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx$$

$$\Big\downarrow \text{4255}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+ 2ab\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+ 2ab\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{4258}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+ 2ab\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+ 2ab\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{3120}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+ 2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \text{4534}$$

$$\frac{1}{5}\left(5a^2+3b^2\right)\int \sec^{\frac{3}{2}}(c+dx)dx+ 2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+ \frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(5a^2+3b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{5}\left(5a^2+3b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(5a^2+3b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{5}\left(5a^2+3b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(5a^2+3b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \; 3119$$

$$\frac{1}{5}\left(5a^2+3b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

input
```
Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2,x]
```

output
```
(2*b^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + ((5*a^2 + 3*b^2)*((-2*Sqrt
[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/d))/5 + 2*a*b*((2*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[c +
d*x])/(3*d))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(154) = 308$.

Time = 6.14 (sec) , antiderivative size = 633, normalized size of antiderivative = 3.62

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\frac{2a^2\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \text{Ellipti}\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}\right.}{}$
parts	Expression too large to display

input
```
int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2/sin(1/2*
d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2))+2/5*b^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*s
in(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1
/2*d*x+1/2*c)-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+
1/2*c)^2+8*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+4*a*b*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(
1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.27

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$= \frac{-10i\sqrt{2}ab\cos{(dx+c)}^2\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+10i\sqrt{2}ab\cos{(dx+c)}}{}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/15*(-10*I*sqrt(2)*a*b*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) + 10*I*sqrt(2)*a*b*cos(d*x + c)^2*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(5*I*a^2 + 3*I*b^2)*c
os(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) - 3*sqrt(2)*(-5*I*a^2 - 3*I*b^2)*cos(d*x + c)^2*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c
))) + 2*(10*a*b*cos(d*x + c) + 3*(5*a^2 + 3*b^2)*cos(d*x + c)^2 + 3*b^2)*s
in(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^2\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx &= \left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^3\,dx\right)b^2\\&\quad+2\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^2\,dx\right)ab\\&\quad+\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)\,dx\right)a^2\end{aligned}$$

input
```
int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x)
```

output
```
int(sqrt(sec(c + d*x))*sec(c + d*x)**3,x)*b**2 + 2*int(sqrt(sec(c + d*x))*
sec(c + d*x)**2,x)*a*b + int(sqrt(sec(c + d*x))*sec(c + d*x),x)*a**2
```

3.588 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 135

$$\begin{aligned}\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\,dx &= -\frac{4ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}\\ &\quad+\frac{2(3a^2+b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}\\ &\quad+\frac{4ab\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{2b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}\end{aligned}$$

output
```
-4*a*b*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(
1/2)/d+2/3*(3*a^2+b^2)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1
/2))*sec(d*x+c)^(1/2)/d+4*a*b*sec(d*x+c)^(1/2)*sin(d*x+c)/d+2/3*b^2*sec(d*
x+c)^(3/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.69

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\,dx$$
$$=\frac{2\sec^{\frac{3}{2}}(c+dx)\left(-6ab\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+(3a^2+b^2)\cos^{\frac{3}{2}}(c+dx)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\right.}{3d}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2,x]
```

output
```
(2*Sec[c + d*x]^(3/2)*(-6*a*b*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2]
 + (3*a^2 + b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + b*(b + 6*a
*Cos[c + d*x])*Sin[c + d*x]))/(3*d)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {3042, 4275, 3042, 4255, 3042, 4258, 3042, 3119, 4534, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{ 4275}$$

$$\int \sqrt{\sec(c+dx)}\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int \sec^{\frac{3}{2}}(c+dx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx$$

$$\Big\downarrow \text{4255}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{4258}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3119}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{4534}$$

$$\frac{1}{3}(3a^2+b^2)\int \sqrt{\sec(c+dx)}dx+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(3a^2+b^2\right)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{3}\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

input
```
Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2,x]
```

output
```
(2*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(3*d) + (2*b^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + 2*a*b*((-2
*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(122) = 244$.

Time = 4.51 (sec) , antiderivative size = 513, normalized size of antiderivative = 3.80

method	result
default	$-\frac{2\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4ab-6\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\cdots}\right.}{\cdots}$
parts	$-\frac{2a^2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}-\frac{2b^2\left(-2\sqrt{\frac{1}{2}-\cdots}\right.}{\cdots}$

input
```
int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*
d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(24*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+
1/2*c)^2*a^2-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2*b^2-12*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2*a*b-12*cos(1/2*d*x+1/2*c)*sin(1/
2*d*x+1/2*c)^2*a*b-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^2+3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*a^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^2+6*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*a*b)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.41

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\,dx$$
$$= \frac{-6i\sqrt{2}ab\cos(dx+c)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))\,-}{}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3*(-6*I*sqrt(2)*a*b*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 6*I*sqrt(2)*a*b*cos(d*x + c)*
weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
 + c))) + sqrt(2)*(-3*I*a^2 - I*b^2)*cos(d*x + c)*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(3*I*a^2 + I*b^2)*cos(d*x + c)
*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(6*a*b*cos(
d*x + c) + b^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2\sqrt{\sec(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*sqrt(sec(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^2\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

input
```
int((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\,dx = \left(\int \sqrt{\sec(dx+c)}dx\right)a^2 + \left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^2\,dx\right)b^2 + 2\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)\,dx\right)ab$$

input
```
int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x)
```

output
```
int(sqrt(sec(c + d*x)),x)*a**2 + int(sqrt(sec(c + d*x))*sec(c + d*x)**2,x)
*b**2 + 2*int(sqrt(sec(c + d*x))*sec(c + d*x),x)*a*b
```

3.589 $\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \frac{2(a^2-b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{4ab\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
2*(a^2-b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x
+c)^(1/2)/d+4*a*b*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*
sec(d*x+c)^(1/2)/d+2*b^2*sec(d*x+c)^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.76

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \frac{2\sqrt{\sec(c+dx)}\left((a^2-b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\left(2a\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\right.\right.}{d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*Sqrt[Sec[c + d*x]]*((a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/
2, 2] + b*(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*Sin[c + d*
x])))/d
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4275, 3042, 4258, 3042, 3120, 4534, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4275}$$

$$\int \frac{a^2+b^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx+2ab\int \sqrt{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2ab\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4258}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$\Big\downarrow$ 3120

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

$\Big\downarrow$ 4534

$$\left(a^2-b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

$\Big\downarrow$ 3042

$$\left(a^2-b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

$\Big\downarrow$ 4258

$$\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

$\Big\downarrow$ 3042

$$\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

$\Big\downarrow$ 3119

$$\frac{2\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

input

```
Int[(a + b*Sec[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*(a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (4*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/d + (2*b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 4.39 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.87

method	result
default	$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b^{2}-4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)ab+2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}\right.}}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right.}}$
parts	$\frac{2a^{2}\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}-\frac{2b^{2}\left(-2\sqrt{-2\sin\right.}}{}$

input
```
int((a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^2-2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*a*b+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.35

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$$
$$=\frac{-2i\sqrt{2}ab\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+2i\sqrt{2}ab\mathrm{weierstrassPInverse}(-4,0,}{}$$

input
```
integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
(-2*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)
) + 2*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
c)) + 2*b^2*sin(d*x + c)/sqrt(cos(d*x + c)) + sqrt(2)*(I*a^2 - I*b^2)*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c
))) + sqrt(2)*(-I*a^2 + I*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**2/sqrt(sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)a^2+2\left(\int \sqrt{\sec(dx+c)}dx\right)ab + \left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)\,dx\right)b^2$$

input
```
int((a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x))/sec(c + d*x),x)*a**2 + 2*int(sqrt(sec(c + d*x)),x)*
a*b + int(sqrt(sec(c + d*x))*sec(c + d*x),x)*b**2
```

3.590 $\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 112

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{4ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2(a^2+3b^2)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output

```
4*a*b*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1
/2)/d+2/3*(a^2+3*b^2)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/
2))*sec(d*x+c)^(1/2)/d+2/3*a^2*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{\sec(c+dx)}\left(12ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(a^2+3b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right.}{3d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(12*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] +
 2*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + a^2*Sin[2*
(c + d*x)]))/(3*d)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4275, 3042, 4258, 3042, 3119, 4533, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4275}$$

$$\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx+2ab\int \frac{1}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+2ab\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4258}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3119}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{ 4533}$$

$$\frac{1}{3}\left(a^2+3b^2\right)\int \sqrt{\sec(c+dx)}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}\left(a^2+3b^2\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{1}{3}\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{2(a^2+3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

input
```
Int[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

output
```
(4*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (2*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(3*d) + (2*a^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(101) = 202$.

Time = 4.81 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.53

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(4a^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-2a^{2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \text{Ellipti}\right.}{3\sqrt{-}}$
parts	$-\frac{2a^{2}\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{2\sin\left(\right.}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\ \sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$

input
```
int((a+b*sec(d*x+c))^2/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a^2*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2*a^2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1
/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^2-6*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.31

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\,a^2\sqrt{\cos(dx+c)}\sin(dx+c) + 6i\sqrt{2}ab\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)$$

input
```
integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(2*a^2*sqrt(cos(d*x + c))*sin(d*x + c) + 6*I*sqrt(2)*a*b*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 6*I
*sqrt(2)*a*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))) + sqrt(2)*(-I*a^2 - 3*I*b^2)*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*a^2 + 3*I*b^2)*weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**2/sec(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx\right)a^2 + 2\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)ab + \left(\int \sqrt{\sec(dx+c)}dx\right)b^2$$

input
```
int((a+b*sec(d*x+c))^2/sec(d*x+c)^(3/2),x)
```

output
```
int(sqrt(sec(c + d*x))/sec(c + d*x)**2,x)*a**2 + 2*int(sqrt(sec(c + d*x))/
sec(c + d*x),x)*a*b + int(sqrt(sec(c + d*x)),x)*b**2
```

3.591 $\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 141

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2(3a^2+5b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4ab\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
2/5*(3*a^2+5*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*s
ec(d*x+c)^(1/2)/d+4/3*a*b*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2
^(1/2))*sec(d*x+c)^(1/2)/d+2/5*a^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)+4/3*a*b*s
in(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.71

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$=\frac{\sqrt{\sec(c+dx)}\Big(6(3a^2+5b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+20ab\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{15d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(5/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(6*(3*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d
*x)/2, 2] + 20*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + a*(10*b
+ 3*a*Cos[c + d*x])*Sin[2*(c + d*x)]))/(15*d)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {3042, 4275, 3042, 4256, 3042, 4258, 3042, 3120, 4533, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \; 4275$$

$$\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx+2ab\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2ab\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4256}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2ab\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2ab\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{ 4258}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+$$
$$2ab\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+$$
$$2ab\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{ 3120}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+$$
$$2ab\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \text{ 4533}$$

$$\frac{1}{5}\left(3a^2+5b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+$$
$$2ab\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\left(3a^2+5b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+ 2ab\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{5}\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+ 2ab\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+ 2ab\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+ 2ab\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

input

```
Int[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(5/2),x]
```

output

```
(2*(3*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (2*a^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + 2*a*b*((
2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) +
 (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(124) = 248.

Time = 7.79 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.53

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-24a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6+24a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\right.}{}$
parts	$-\frac{2a^2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\right.\right.}{5\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\right.}}$

input
```
int((a+b*sec(d*x+c))^2/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a^2*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+24*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^4+40*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b-6*a^2*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)-20*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a*
b+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-15*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.21

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$=\frac{-10i\sqrt{2}ab\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+10i\sqrt{2}ab\text{weierstrassPInverse}(-4,}{}$$

input
```
integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/15*(-10*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) + 10*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c)) - 3*sqrt(2)*(-3*I*a^2 - 5*I*b^2)*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(3*I*a^2 +
 5*I*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))) + 2*(3*a^2*cos(d*x + c)^2 + 10*a*b*cos(d*x + c))*sin(d*x
 + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**2/sec(d*x+c)**(5/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**2/sec(c + d*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3}dx\right)a^2+2\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx\right)ab+\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)b^2$$

input
```
int((a+b*sec(d*x+c))^2/sec(d*x+c)^(5/2),x)
```

output
```
int(sqrt(sec(c + d*x))/sec(c + d*x)**3,x)*a**2 + 2*int(sqrt(sec(c + d*x))/
sec(c + d*x)**2,x)*a*b + int(sqrt(sec(c + d*x))/sec(c + d*x),x)*b**2
```

3.592 $\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 175

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{12ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2(5a^2+7b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} + \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(5a^2+7b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}$$

output
```
12/5*a*b*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)
^(1/2)/d+2/21*(5*a^2+7*b^2)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c
,2^(1/2))*sec(d*x+c)^(1/2)/d+2/7*a^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)+4/5*a*b
*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/21*(5*a^2+7*b^2)*sin(d*x+c)/d/sec(d*x+c)^
(1/2)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.69

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{\sec(c+dx)}\left(504ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+20(5a^2+7b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\right.\right.}{210d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(7/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(504*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
+ 20*(5*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (65*a^
2 + 70*b^2 + 84*a*b*Cos[c + d*x] + 15*a^2*Cos[2*(c + d*x)])*Sin[2*(c + d*x
)]))/(210*d)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4275, 3042, 4256, 3042, 4258, 3042, 3119, 4533, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$
$$\Big\downarrow \text{4275}$$
$$\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}dx+2ab\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2ab\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2ab\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2ab\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{4258}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+$$
$$2ab\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+$$
$$2ab\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3119}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+$$
$$2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$$\Big\downarrow \text{4533}$$

$$\frac{1}{7}\left(5a^2+7b^2\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+$$
$$2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}(5a^2+7b^2)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+ 2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$\Big\downarrow$ 4256

$$\frac{1}{7}(5a^2+7b^2)\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+ 2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$\Big\downarrow$ 3042

$$\frac{1}{7}(5a^2+7b^2)\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+ 2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$\Big\downarrow$ 4258

$$\frac{1}{7}(5a^2+7b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+ \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$\Big\downarrow$ 3042

$$\frac{1}{7}(5a^2+7b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+ \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$\Big\downarrow$ 3120

$$\frac{1}{7}(5a^2+7b^2)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+ \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

input
```
Int[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(7/2),x]
```

output
```
(2*a^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + 2*a*b*((6*Sqrt[Cos[c + d*x
]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*Sin[c + d*x])/
(5*d*Sec[c + d*x]^(3/2))) + ((5*a^2 + 7*b^2)*((2*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt
[Sec[c + d*x]])))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(154) = 308$.

Time = 11.59 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.07

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(240a^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+\left(-360a^{2}-336ab\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(280a^{2}\right.\right.}{}$
parts	$-\frac{2a^{2}\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(48\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}-120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+128\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-72\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+5\sqrt{\frac{1}{2}-\right.}{21\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}\right.}}$

input
```
int((a+b*sec(d*x+c))^2/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a^2*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*a^2-336*a*b)*sin(1/2*d*x+1/2*c
)^6*cos(1/2*d*x+1/2*c)+(280*a^2+336*a*b+140*b^2)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-80*a^2-84*a*b-70*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/
2*c)+25*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2+35*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^2-126
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{126i\sqrt{2}ab\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-126i\sqrt{2}ab}{}$$

input
```
integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
1/105*(126*I*sqrt(2)*a*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c))) - 126*I*sqrt(2)*a*b*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 5*sqrt(2)*(
5*I*a^2 + 7*I*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c
)) - 5*sqrt(2)*(-5*I*a^2 - 7*I*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c)) + 2*(15*a^2*cos(d*x + c)^3 + 42*a*b*cos(d*x + c)^2 + 5
*(5*a^2 + 7*b^2)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**2/sec(d*x+c)**(7/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**2/sec(c + d*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(7/2),x)
```

output
```
int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^4}dx\right)a^2 + 2\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3}dx\right)ab + \left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx\right)b^2$$

input
```
int((a+b*sec(d*x+c))^2/sec(d*x+c)^(7/2),x)
```

output
```
int(sqrt(sec(c + d*x))/sec(c + d*x)**4,x)*a**2 + 2*int(sqrt(sec(c + d*x))/
sec(c + d*x)**3,x)*a*b + int(sqrt(sec(c + d*x))/sec(c + d*x)**2,x)*b**2
```

3.593 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 234

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx &= -\frac{2a(5a^2+9b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}\\ &\quad+\frac{2b(21a^2+5b^2)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d}\\ &\quad+\frac{2a(5a^2+9b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2b(21a^2+5b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d}\\ &\quad+\frac{32ab^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d}+\frac{2b^2\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{7d}\end{aligned}$$

output

```
-2/5*a*(5*a^2+9*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)*sec(d*x+c)^(1/2)/d+2/21*b*(21*a^2+5*b^2)*cos(d*x+c)^(1/2)*InverseJacobiA
M(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+2/5*a*(5*a^2+9*b^2)*sec(d*x+c)
^(1/2)*sin(d*x+c)/d+2/21*b*(21*a^2+5*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+32
/35*a*b^2*sec(d*x+c)^(5/2)*sin(d*x+c)/d+2/7*b^2*sec(d*x+c)^(5/2)*(a+b*sec(
d*x+c))*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 2.93 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.76

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$=\frac{\sec^{\frac{7}{2}}(c+dx)\left(-168a(5a^2+9b^2)\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+40b(21a^2+5b^2)\cos^{\frac{7}{2}}(c+dx)\operatorname{EllipticF}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]^(7/2)*(-168*a*(5*a^2 + 9*b^2)*Cos[c + d*x]^(7/2)*EllipticE[(
c + d*x)/2, 2] + 40*b*(21*a^2 + 5*b^2)*Cos[c + d*x]^(7/2)*EllipticF[(c + d
*x)/2, 2] + 2*(210*a^2*b + 110*b^3 + 63*a*(5*a^2 + 13*b^2)*Cos[c + d*x] +
10*(21*a^2*b + 5*b^3)*Cos[2*(c + d*x)] + 105*a^3*Cos[3*(c + d*x)] + 189*a*
b^2*Cos[3*(c + d*x)])*Sin[c + d*x]))/(420*d)
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.93, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4329, 27, 3042, 4535, 3042, 4255, 3042, 4258, 3042, 3120, 4534, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \ 4329$$

$$\frac{2}{7}\int \frac{1}{2}\sec^{\frac{3}{2}}(c+dx)\left(16ab^2\sec^2(c+dx)+b\left(21a^2+5b^2\right)\sec(c+dx)+a\left(7a^2+3b^2\right)\right)dx+\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$\Big\downarrow$ 27

$$\frac{1}{7}\int \sec^{\frac{3}{2}}(c+dx)\left(16ab^2\sec^2(c+dx)+b\left(21a^2+5b^2\right)\sec(c+dx)+a\left(7a^2+3b^2\right)\right)dx+\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\left(21a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+a\left(7a^2+3b^2\right)\right)dx+\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$\Big\downarrow$ 4535

$$\frac{1}{7}\left(b\left(21a^2+5b^2\right)\int \sec^{\frac{5}{2}}(c+dx)dx+\int \sec^{\frac{3}{2}}(c+dx)\left(16ab^2\sec^2(c+dx)+a\left(7a^2+3b^2\right)\right)dx\right)+\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(b\left(21a^2+5b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(7a^2+3b^2\right)\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$\Big\downarrow$ 4255

$$\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(7a^2+3b^2\right)\right)dx+b\left(21a^2+5b^2\right)\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a(7a^2+3b^2)\right)dx+b(21a^2+5b^2)\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx\right.}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a(7a^2+3b^2)\right)dx+b(21a^2+5b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a(7a^2+3b^2)\right)dx+b(21a^2+5b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\mathrm{s}}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a(7a^2+3b^2)\right)dx+b(21a^2+5b^2)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c}{3d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\Big\downarrow \text{4534}$$

$$\frac{1}{7}\left(\frac{7}{5}a(5a^2+9b^2)\int\sec^{\frac{3}{2}}(c+dx)dx+b(21a^2+5b^2)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{3d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{7}{5}a(5a^2+9b^2)\int\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+b(21a^2+5b^2)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}}}{}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$\Big\downarrow$ 4255

$$\frac{1}{7}\left(\frac{7}{5}a(5a^2+9b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)+b(21a^2+5b^2)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{7}{5}a(5a^2+9b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+b(21a^2+5b^2)\left(\frac{2\sin(c+dx)\sec}{3d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$\Big\downarrow$ 4258

$$\frac{1}{7}\left(\frac{7}{5}a(5a^2+9b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)+b(21a^2+5\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{7}{5}a(5a^2+9b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+b(21\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$\Big\downarrow$ 3119

$$\frac{1}{7}\left(b(21a^2+5b^2)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+\frac{7}{5}a(5\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

input

```
Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3,x]
```

output
```
(2*b^2*Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(7*d) + ((32*
a*b^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + (7*a*(5*a^2 + 9*b^2)*((-2*S
qrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/d))/5 + b*(21*a^2 + 5*b^2)*((2*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x
]^(3/2)*Sin[c + d*x])/(3*d)))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 819 vs. $2(209) = 418$.

Time = 8.43 (sec) , antiderivative size = 820, normalized size of antiderivative = 3.50

method	result	size
default	Expression too large to display	820
parts	Expression too large to display	1008

input
```
int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^3/sin(1/2*
d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2))+2*b^3*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/
2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2
*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2)))+6/5*a*b^2/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2
*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(24*sin(1/2*d*x+1/
2*c)^6*cos(1/2*d*x+1/2*c)-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^
4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
sin(1/2*d*x+1/2*c)^2+8*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+6*a
^2*b*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.15

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx =$$

$$-\frac{5\sqrt{2}(21i\,a^2b+5i\,b^3)\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}(-\cdots}{\cdots}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/105*(5*sqrt(2)*(21*I*a^2*b + 5*I*b^3)*cos(d*x + c)^3*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-21*I*a^2*b - 5*I*b^3
)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
 + 21*sqrt(2)*(5*I*a^3 + 9*I*a*b^2)*cos(d*x + c)^3*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*sqrt(2)*(-
5*I*a^3 - 9*I*a*b^2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(63*a*b^2*cos(d*x + c) + 2
1*(5*a^3 + 9*a*b^2)*cos(d*x + c)^3 + 15*b^3 + 5*(21*a^2*b + 5*b^3)*cos(d*x
 + c)^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \int (b\sec(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^3\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

input
```
int((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx &= \left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^4\,dx\right)b^3\\&\quad+3\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^3\,dx\right)a\,b^2\\&\quad+3\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^2\,dx\right)a^2b\\&\quad+\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)\,dx\right)a^3\end{aligned}$$

input
```
int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x)
```

output
```
int(sqrt(sec(c + d*x))*sec(c + d*x)**4,x)*b**3 + 3*int(sqrt(sec(c + d*x))*
sec(c + d*x)**3,x)*a*b**2 + 3*int(sqrt(sec(c + d*x))*sec(c + d*x)**2,x)*a*
*2*b + int(sqrt(sec(c + d*x))*sec(c + d*x),x)*a**3
```

3.594 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 189

$$\begin{aligned}\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3\,dx &= -\frac{6b(5a^2+b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}\\ &+\frac{2a(a^2+b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d}\\ &+\frac{6b(5a^2+b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{8ab^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}\\ &+\frac{2b^2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{5d}\end{aligned}$$

output
```
-6/5*b*(5*a^2+b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*
sec(d*x+c)^(1/2)/d+2*a*(a^2+b^2)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+
1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+6/5*b*(5*a^2+b^2)*sec(d*x+c)^(1/2)*sin(d
*x+c)/d+8/5*a*b^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*b^2*sec(d*x+c)^(3/2)*(
a+b*sec(d*x+c))*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.71

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3\,dx = \frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-3b(5a^2+b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+5a(a^2+b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{b}{}\right.}{5d}$$

input

```
Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3,x]
```

output

```
(2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-3*b*(5*a^2 + b^2)*EllipticE[(c
+ d*x)/2, 2] + 5*a*(a^2 + b^2)*EllipticF[(c + d*x)/2, 2] + (b*(5*(3*a^2 +
b^2) + 10*a*b*Cos[c + d*x] + 3*(5*a^2 + b^2)*Cos[2*(c + d*x)])*Sin[c + d*x
])/(2*Cos[c + d*x]^(5/2))))/(5*d)
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.96, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 4329, 27, 3042, 4535, 3042, 4255, 3042, 4258, 3042, 3119, 4534, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{ 4329}$$

$$\frac{2}{5}\int \frac{1}{2}\sqrt{\sec(c+dx)}\left(12ab^2\sec^2(c+dx)+3b\left(5a^2+b^2\right)\sec(c+dx)+a\left(5a^2+b^2\right)\right)dx+\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\big\downarrow \text{27}$$

$$\frac{1}{5}\int \sqrt{\sec(c+dx)}\left(12ab^2\sec^2(c+dx)+3b\left(5a^2+b^2\right)\sec(c+dx)+a\left(5a^2+b^2\right)\right)dx+\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\big\downarrow \text{3042}$$

$$\frac{1}{5}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+3b(5a^2+b^2)\csc\left(c+dx+\frac{\pi}{2}\right)+a\left(5a^2+b^2\right)\right)dx+\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\big\downarrow \text{4535}$$

$$\frac{1}{5}\left(3b(5a^2+b^2)\int \sec^{\frac{3}{2}}(c+dx)dx+\int \sqrt{\sec(c+dx)}\left(12ab^2\sec^2(c+dx)+a\left(5a^2+b^2\right)\right)dx\right)+\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\big\downarrow \text{3042}$$

$$\frac{1}{5}\left(3b(5a^2+b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(5a^2+b^2\right)\right)d\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\big\downarrow \text{4255}$$

$$\frac{1}{5}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(5a^2+b^2\right)\right)dx+3b\left(5a^2+b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+}}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(5a^2+b^2\right)\right)dx+3b\left(5a^2+b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+}}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\big\downarrow \text{4258}$$

$$\frac{1}{5}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(5a^2+b^2\right)\right)dx+3b\left(5a^2+b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+}}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(5a^2+b^2\right)\right)dx+3b\left(5a^2+b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+}}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

$\downarrow$ 3119

$$\frac{1}{5}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(5a^2+b^2\right)\right)dx+3b\left(5a^2+b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+}}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

$\downarrow$ 4534

$$\frac{1}{5}\left(5a\left(a^2+b^2\right)\int\sqrt{\sec(c+dx)}dx+3b\left(5a^2+b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(5a\left(a^2+b^2\right)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+3b\left(5a^2+b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c-}}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

$\downarrow$ 4258

$$\frac{1}{5}\left(5a\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+3b\left(5a^2+b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2}{}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(5a(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b(5a^2+b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx}}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

$\downarrow$ 3120

$$\frac{1}{5}\left(\frac{10a(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+3b(5a^2+b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx}}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

input
```
Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3,x]
```

output
```
(2*b^2*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(5*d) + ((10*
a*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (8*a*b^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/d + 3*b*(5*a^2 + b^2)*(
(-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. $2(170) = 340$.

Time = 6.39 (sec) , antiderivative size = 711, normalized size of antiderivative = 3.76

method	result	size
default	Expression too large to display	711
parts	Expression too large to display	898

input
```
int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2/
5*b^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)
^2-1)/sin(1/2*d*x+1/2*c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+6*a*b^2*(-1/6*cos(1/2*d*x+1/2*c)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^
2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2)))+6*a^2*b/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)))/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.29

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3\,dx =$$

$$-\frac{5\sqrt{2}(i\,a^3+i\,ab^2)\cos{(dx+c)}^2\,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+5\sqrt{2}(-i\,a^3}{}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/5*(5*sqrt(2)*(I*a^3 + I*a*b^2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-I*a^3 - I*a*b^2)*cos(d*x +
c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)
*(5*I*a^2*b + I*b^3)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(-5*I*a^2*b - I*b^
3)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))) - 2*(5*a*b^2*cos(d*x + c) + b^3 + 3*(5*a^2*b + b
^3)*cos(d*x + c)^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3\,dx = \int (b\sec(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3\,dx = \int (b\sec(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^3\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

input
```
int((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3\,dx = \left(\int \sqrt{\sec(dx+c)}dx\right)a^3 + \left(\int \sqrt{\sec(dx+c)}\sec(dx+c)^3\,dx\right)b^3 + 3\left(\int \sqrt{\sec(dx+c)}\sec(dx+c)^2\,dx\right)a\,b^2 + 3\left(\int \sqrt{\sec(dx+c)}\sec(dx+c)\,dx\right)a^2b$$

input
```
int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x)
```

output
```
int(sqrt(sec(c + d*x)),x)*a**3 + int(sqrt(sec(c + d*x))*sec(c + d*x)**3,x)
*b**3 + 3*int(sqrt(sec(c + d*x))*sec(c + d*x)**2,x)*a*b**2 + 3*int(sqrt(se
c(c + d*x))*sec(c + d*x),x)*a**2*b
```

3.595 $\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 158

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx &= \frac{2a(a^2-3b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}\\ &+\frac{2b(9a^2+b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}\\ &+\frac{16ab^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d}+\frac{2b^2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))\sin(c+dx)}{3d}\end{aligned}$$

output
```
2*a*(a^2-3*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec
(d*x+c)^(1/2)/d+2/3*b*(9*a^2+b^2)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x
+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+16/3*a*b^2*sec(d*x+c)^(1/2)*sin(d*x+c)/
d+2/3*b^2*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.67

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx = \frac{\sec^{\frac{3}{2}}(c+dx)\left(6a(a^2-3b^2)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\Big(2(9a^2+b^2)\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+\right.\right.}{3d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```

output
```
(Sec[c + d*x]^(3/2)*(6*a*(a^2 - 3*b^2)*Cos[c + d*x]^(3/2)*EllipticE[(c + d
*x)/2, 2] + b*(2*(9*a^2 + b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2
] + 2*b*(b + 9*a*Cos[c + d*x])*Sin[c + d*x])))/(3*d)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4329, 27, 3042, 4535, 3042, 4258, 3042, 3120, 4534, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4329}$$

$$\frac{2}{3}\int \frac{8ab^2\sec^2(c+dx)+b(9a^2+b^2)\sec(c+dx)+a(3a^2-b^2)}{2\sqrt{\sec(c+dx)}}dx+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}{3d}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{3}\int \frac{8ab^2\sec^2(c+dx)+b(9a^2+b^2)\sec(c+dx)+a(3a^2-b^2)}{\sqrt{\sec(c+dx)}}dx+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}{3d}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{1}{3}\int \frac{8ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+b(9a^2+b^2)\csc\left(c+dx+\frac{\pi}{2}\right)+a(3a^2-b^2)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}{3d}$$

$$\Big\downarrow \textbf{4535}$$

$$\frac{1}{3}\left(\int \frac{8ab^2\sec^2(c+dx)+a(3a^2-b^2)}{\sqrt{\sec(c+dx)}}dx+b(9a^2+b^2)\int \sqrt{\sec(c+dx)}dx\right)+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}{3d}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{1}{3}\left(b(9a^2+b^2)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\int \frac{8ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a(3a^2-b^2)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}{3d}$$

$$\Big\downarrow \textbf{4258}$$

$$\frac{1}{3}\left(\int \frac{8ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a(3a^2-b^2)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(9a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx\right)+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}{3d}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{1}{3}\left(\int \frac{8ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(3a^2-b^2\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}d$$

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}{3d}$$

$\big\downarrow$ 3120

$$\frac{1}{3}\left(\int \frac{8ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(3a^2-b^2\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\right.}{d}\right.$$

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}{3d}$$

$\big\downarrow$ 4534

$$\frac{1}{3}\left(3a\left(a^2-3b^2\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{16ab}{}\right.$$

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}{3d}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\left(3a\left(a^2-3b^2\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\right.$$

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}{3d}$$

$\big\downarrow$ 4258

$$\frac{1}{3}\left(3a\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{Ellip}}{d}\right.$$

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}{3d}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\left(3a\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+da}}{d}\right.$$

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}{3d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{3}\left(\frac{2b(9a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a(a^2-3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.$$

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}{3d}$$

input
```
Int[(a + b*Sec[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*b^2*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])*Sin[c + d*x])/(3*d) + ((6*a
*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (2*b*(9*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/d + (16*a*b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(143) = 286$.

Time = 5.55 (sec) , antiderivative size = 630, normalized size of antiderivative = 3.99

method	result
default	$-\frac{2\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(36\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a\,b^2-18\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{\cdots}\right.\right.}{\cdots}$
parts	$\frac{2a^3\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\ \text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\ \sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}-\frac{2b^3\left(-2\sqrt{\frac{1}{2}-\frac{\cos\cdots}{\cdots}}\right.}{\cdots}$

input
```
int((a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*
d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(36*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b^2-18*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d
*x+1/2*c)^2*a^2*b-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2*b^3+6*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2*a^3-18*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*sin(1/2*d*x+1/2*c)^2*a*b^2-18*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2
*a*b^2-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^3+9*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*a^2*b+(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3
+9*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.35

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx = \frac{\sqrt{2}(-9i\,a^2b-i\,b^3)\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}(9i\,a^2b+i}{}$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*(-9*I*a^2*b - I*b^3)*cos(d*x + c)*weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(9*I*a^2*b + I*b^3)*cos(d*x + c)*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(-I*
a^3 + 3*I*a*b^2)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(I*a^3 - 3*I*a*b^2)*cos(
d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*(9*a*b^2*cos(d*x + c) + b^3)*sin(d*x + c)/sqrt(cos(d*
x + c)))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(a+b\sec{(c+dx)})^3}{\sqrt{\sec{(c+dx)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**3/sqrt(sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(b\sec{(dx+c)}+a)^3}{\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx &= \left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)a^3 + 3\left(\int \sqrt{\sec(dx+c)}dx\right)a^2b \\ &\quad + \left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^2\,dx\right)b^3 \\ &\quad + 3\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)\,dx\right)a\,b^2\end{aligned}$$

input
```
int((a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x))/sec(c + d*x),x)*a**3 + 3*int(sqrt(sec(c + d*x)),x)*
a**2*b + int(sqrt(sec(c + d*x))*sec(c + d*x)**2,x)*b**3 + 3*int(sqrt(sec(c
 + d*x))*sec(c + d*x),x)*a*b**2
```

3.596 $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 166

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= \frac{2b(3a^2-b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} \\ &+ \frac{2a(a^2+9b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\ &- \frac{2b(a^2-3b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
2*b*(3*a^2-b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec
(d*x+c)^(1/2)/d+2/3*a*(a^2+9*b^2)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x
+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d-2/3*b*(a^2-3*b^2)*sec(d*x+c)^(1/2)*sin(
d*x+c)/d+2/3*a^2*(a+b*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.65

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{\sec(c+dx)}\left(-6b(-3a^2+b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+2a(a^2+9b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\right.\right.}{3d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(-6*b*(-3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2] + 2*a*(a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2] + 2*(3*b^3 + a^3*Cos[c + d*x])*Sin[c + d*x]))/(3*d)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {3042, 4328, 27, 3042, 4535, 3042, 4258, 3042, 3120, 4534, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4328}$$

$$\frac{2}{3}\int \frac{8ba^2+\left(a^2+9b^2\right)\sec(c+dx)a-b\left(a^2-3b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\int \frac{8ba^2+\left(a^2+9b^2\right)\sec(c+dx)a-b\left(a^2-3b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}\int \frac{8ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a-b\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 4535}$$

$$\frac{1}{3}\left(\int \frac{8a^2b-b\left(a^2-3b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx+a\left(a^2+9b^2\right)\int \sqrt{\sec(c+dx)}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}\left(a\left(a^2+9b^2\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\int \frac{8a^2b-b\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{1}{3}\left(\int \frac{8a^2b-b\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}\left(\int \frac{8a^2b-b(a^2-3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a(a^2+9b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}d$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3120

$$\frac{1}{3}\left(\int \frac{8a^2b-b(a^2-3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a(a^2+9b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\right.}{d}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 4534

$$\frac{1}{3}\left(3b(3a^2-b^2)\int \frac{1}{\sqrt{\sec(c+dx)}}dx-\frac{2b(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a(a^2+9b^2)\sqrt{\cos(c+dx)}\sqrt{se}}{}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(3b(3a^2-b^2)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a(a^2+9b^2)\sqrt{\cos(c+dx}}{}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 4258

$$\frac{1}{3}\left(3b(3a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-\frac{2b(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a(}{}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(3b(3a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{2b(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\sec(c+dx)}}$$

$\big\downarrow$ 3119

$$\frac{1}{3}\left(-\frac{2b(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a(a^2+9b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\sec(c+dx)}}$$

input
```
Int[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

output
```
(2*a^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + ((6*b
*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (2*a*(a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/d - (2*b*(a^2 - 3*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/d)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Maple [A] (verified)

Time = 6.08 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.83

method	result
default	$-\frac{2\left(4a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2a^3-6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2b^3+a^3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}\right.}\right.}{}$
parts	$-\frac{2a^3\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\right.}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$

input
```
int((a+b*sec(d*x+c))^3/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(4*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2*a^3-6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^3+a^3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))+9*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*a^2*b*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3)/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{2}(-i\,a^3-9i\,ab^2)\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+\sqrt{2}(i\,a^3+9i\,ab^2)\text{weierstra}$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*(-I*a^3 - 9*I*a*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + sqrt(2)*(I*a^3 + 9*I*a*b^2)*weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(-3*I*a^2*b + I*b^3)*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) -
 3*sqrt(2)*(3*I*a^2*b - I*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(a^3*cos(d*x + c) + 3*b^3)*sin(
d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+b\sec{(c+dx)})^3}{\sec^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**3/sec(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec{(dx+c)}+a)^3}{\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec{(dx+c)}+a)^3}{\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx\right)a^3 + 3\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)a^2b + 3\left(\int \sqrt{\sec(dx+c)}dx\right)a\,b^2 + \left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)\,dx\right)b^3$$

input
```
int((a+b*sec(d*x+c))^3/sec(d*x+c)^(3/2),x)
```

output
```
int(sqrt(sec(c + d*x))/sec(c + d*x)**2,x)*a**3 + 3*int(sqrt(sec(c + d*x))/
sec(c + d*x),x)*a**2*b + 3*int(sqrt(sec(c + d*x)),x)*a*b**2 + int(sqrt(sec
(c + d*x))*sec(c + d*x),x)*b**3
```

3.597 $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 156

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx &= \frac{6a(a^2+5b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &+ \frac{2b(a^2+b^2)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d} \\ &+ \frac{8a^2b\sin(c+dx)}{5d\sqrt{\sec(c+dx)}} + \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\end{aligned}$$

output
```
6/5*a*(a^2+5*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*s
ec(d*x+c)^(1/2)/d+2*b*(a^2+b^2)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1
/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+8/5*a^2*b*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2
/5*a^2*(a+b*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.68

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{\sec(c+dx)}\Big(6a(a^2+5b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+10b(a^2+b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+\right.}{5d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(5/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(6*a*(a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d
*x)/2, 2] + 10*b*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
+ a^2*(5*b + a*Cos[c + d*x])*Sin[2*(c + d*x)]))/(5*d)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4328, 27, 3042, 4535, 3042, 4258, 3042, 3119, 4533, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4328}$$

$$\frac{2}{5}\int \frac{12ba^2+3\left(a^2+5b^2\right)\sec(c+dx)a+b\left(a^2+5b^2\right)\sec^2(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{12ba^2+3\left(a^2+5b^2\right)\sec(c+dx)a+b\left(a^2+5b^2\right)\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{12ba^2+3\left(a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4535}$$

$$\frac{1}{5}\left(\int \frac{12ba^2+b\left(a^2+5b^2\right)\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx+3a\left(a^2+5b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(3a\left(a^2+5b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{12ba^2+b\left(a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{5}\left(\int \frac{12ba^2+b\left(a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+3a\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\int \frac{12ba^2+b\left(a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+3a\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right.}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{5}\left(\int \frac{12ba^2+b\left(a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{6a\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4533}$$

$$\frac{1}{5}\left(5b\left(a^2+b^2\right)\int\sqrt{\sec(c+dx)}dx+\frac{6a\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{8a^2b\sin(c+d}{d\sqrt{\sec(c+dx}}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(5b\left(a^2+b^2\right)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{6a\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{8a^2b\sin}{d\sqrt{\sec(}}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{5}\left(5b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{6a\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}\right.}{d}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(5b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{5}\left(\frac{10b(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a(a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.$$

$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{5d\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(5/2),x]
```

output
```
(2*a^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + ((6*a
*(a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (10*b*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/d + (8*a^2*b*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(141) = 282$.

Time = 7.99 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.64

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-8a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6+8a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+20a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\mathrm{co}\right.}{}$
parts	$-\frac{2a^3\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\right.\right.}{5\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\right.}}$

input
```
int((a+b*sec(d*x+c))^3/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*a^3*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^4+20*a^2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)*b-2*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2*a^3-10*a^2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
*b+5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b+5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3-3*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^3-15*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b^2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.24

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{5\sqrt{2}(i\,a^2b+i\,b^3)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}(-i\,a^2b-i\,b^3)\text{weiers}}{}$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/5*(5*sqrt(2)*(I*a^2*b + I*b^3)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + 5*sqrt(2)*(-I*a^2*b - I*b^3)*weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(-I*a^3 - 5*I*a*b^2)*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
+ 3*sqrt(2)*(I*a^3 + 5*I*a*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(a^3*cos(d*x + c)^2 + 5*a^2*b*
cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**3/sec(d*x+c)**(5/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**3/sec(c + d*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3}dx\right)a^3+3\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx\right)a^2b+3\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)a\,b^2+\left(\int \sqrt{\sec(dx+c)}dx\right)b^3$$

input
```
int((a+b*sec(d*x+c))^3/sec(d*x+c)^(5/2),x)
```

output
```
int(sqrt(sec(c + d*x))/sec(c + d*x)**3,x)*a**3 + 3*int(sqrt(sec(c + d*x))/
sec(c + d*x)**2,x)*a**2*b + 3*int(sqrt(sec(c + d*x))/sec(c + d*x),x)*a*b**
2 + int(sqrt(sec(c + d*x)),x)*b**3
```

3.598 $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 199

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx &= \frac{2b(9a^2+5b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}\\&\quad+\frac{2a(5a^2+21b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d}\\&\quad+\frac{32a^2b\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a(5a^2+21b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}\\&\quad+\frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\end{aligned}$$

output
```
2/5*b*(9*a^2+5*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))
*sec(d*x+c)^(1/2)/d+2/21*a*(5*a^2+21*b^2)*cos(d*x+c)^(1/2)*InverseJacobiAM
(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+32/35*a^2*b*sin(d*x+c)/d/sec(d*
x+c)^(3/2)+2/21*a*(5*a^2+21*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/7*a^2*(a+
b*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(5/2)
```

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.66

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{\sec(c+dx)}\Big(84b(9a^2+5b^2)\sqrt{\cos(c+dx)}E\big(\tfrac{1}{2}(c+dx)\big|2\big)+20a(5a^2+21b^2)\sqrt{\cos(c+dx)}\,\mathrm{EllipticF}}{210d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(7/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(84*b*(9*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, 2] + 20*a*(5*a^2 + 21*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2] + a*(65*a^2 + 210*b^2 + 126*a*b*Cos[c + d*x] + 15*a^2*Cos[2*(c + d
*x)])*Sin[2*(c + d*x)]))/(210*d)
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.97, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 4328, 27, 3042, 4535, 3042, 4256, 3042, 4258, 3042, 3120, 4533, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \; 4328$$

$$\frac{2}{7}\int \frac{16ba^2+\left(5a^2+21b^2\right)\sec(c+dx)a+b\left(3a^2+7b^2\right)\sec^2(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \frac{16ba^2+\left(5a^2+21b^2\right)\sec(c+dx)a+b\left(3a^2+7b^2\right)\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \frac{16ba^2+\left(5a^2+21b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(3a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4535}$$

$$\frac{1}{7}\left(a\left(5a^2+21b^2\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\int \frac{16ba^2+b\left(3a^2+7b^2\right)\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(a\left(5a^2+21b^2\right)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\int \frac{16ba^2+b\left(3a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4256}$$

$$\frac{1}{7}\left(\int \frac{16ba^2+b\left(3a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+a\left(5a^2+21b^2\right)\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\int \frac{16ba^2+b(3a^2+7b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+a(5a^2+21b^2)\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+d}}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 4258

$$\frac{1}{7}\left(\int \frac{16ba^2+b(3a^2+7b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+a(5a^2+21b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+d}}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\int \frac{16ba^2+b(3a^2+7b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+a(5a^2+21b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin(c+c}}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3120

$$\frac{1}{7}\left(\int \frac{16ba^2+b(3a^2+7b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+a(5a^2+21b^2)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 4533

$$\frac{1}{7}\left(\frac{7}{5}b(9a^2+5b^2)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+a(5a^2+21b^2)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{Ellipti}}{3d}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{7}{5}b(9a^2+5b^2)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a(5a^2+21b^2)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{E}}{3d}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 4258

$$\frac{1}{7}\left(\frac{7}{5}b(9a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+a(5a^2+21b^2)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(}}{}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{7}{5}b(9a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+a(5a^2+21b^2)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2}{}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3119

$$\frac{1}{7}\left(\frac{14b(9a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+a(5a^2+21b^2)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c-}}{}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Int[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(7/2),x]
```

output
```
(2*a^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + ((14*
b*(9*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (32*a^2*b*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + a*(5*a
^2 + 21*b^2)*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(178) = 356$.

Time = 12.10 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.12

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(240a^{3}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+\left(-360a^{3}-504a^{2}b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(280a\right.}{}$
parts	Expression too large to display

input
```
int((a+b*sec(d*x+c))^3/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a^3*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*a^3-504*a^2*b)*sin(1/2*d*x+1/2
*c)^6*cos(1/2*d*x+1/2*c)+(280*a^3+504*a^2*b+420*a*b^2)*sin(1/2*d*x+1/2*c)^
4*cos(1/2*d*x+1/2*c)+(-80*a^3-126*a^2*b-210*a*b^2)*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)+25*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+105*a*b^2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-189*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-105*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx =$$

$$-\frac{5\sqrt{2}(5i\,a^3+21i\,ab^2)\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+5\sqrt{2}(-5i\,a^3-21i\,ab^2}{}$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-1/105*(5*sqrt(2)*(5*I*a^3 + 21*I*a*b^2)*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-5*I*a^3 - 21*I*a*b^2)*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*sqrt(2)*(-9*I*a^2*b - 5*
I*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c))) + 21*sqrt(2)*(9*I*a^2*b + 5*I*b^3)*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(15*a^3*cos(
d*x + c)^3 + 63*a^2*b*cos(d*x + c)^2 + 5*(5*a^3 + 21*a*b^2)*cos(d*x + c))*
sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(a+b\sec{(c+dx)})^3}{\sec^{\frac{7}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**3/sec(d*x+c)**(7/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**3/sec(c + d*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+dx)}\right)^3}{\left(\frac{1}{\cos(c+dx)}\right)^{7/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(7/2),x)
```

output
```
int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^4}dx\right)a^3 + 3\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3}dx\right)a^2b + 3\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx\right)a\,b^2 + \left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)b^3$$

input
```
int((a+b*sec(d*x+c))^3/sec(d*x+c)^(7/2),x)
```

output
```
int(sqrt(sec(c + d*x))/sec(c + d*x)**4,x)*a**3 + 3*int(sqrt(sec(c + d*x))/
sec(c + d*x)**3,x)*a**2*b + 3*int(sqrt(sec(c + d*x))/sec(c + d*x)**2,x)*a*
b**2 + int(sqrt(sec(c + d*x))/sec(c + d*x),x)*b**3
```

3.599 $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 234

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx &= \frac{2a(7a^2+27b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} \\ &+ \frac{2b(15a^2+7b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\ &+ \frac{40a^2b\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a(7a^2+27b^2)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} \\ &+ \frac{2b(15a^2+7b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}\end{aligned}$$

output
```
2/15*a*(7*a^2+27*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2
))*sec(d*x+c)^(1/2)/d+2/21*b*(15*a^2+7*b^2)*cos(d*x+c)^(1/2)*InverseJacobi
AM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+40/63*a^2*b*sin(d*x+c)/d/sec(
d*x+c)^(5/2)+2/45*a*(7*a^2+27*b^2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/21*b*(1
5*a^2+7*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/9*a^2*(a+b*sec(d*x+c))*sin(d*
x+c)/d/sec(d*x+c)^(7/2)
```

Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.68

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{\sec(c+dx)}\Big(168a(7a^2+27b^2)\sqrt{\cos(c+dx)}E\big(\tfrac{1}{2}(c+dx)\big|\,2\big)+120b(15a^2+7b^2)\sqrt{\cos(c+dx)}\,\text{Ellipt}}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(9/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(168*a*(7*a^2 + 27*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2] + 120*b*(15*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2] + (7*a*(43*a^2 + 108*b^2)*Cos[c + d*x] + 5*(234*a^2*b + 84*b^3
+ 54*a^2*b*Cos[2*(c + d*x)] + 7*a^3*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)]))/
(1260*d)
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.95, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4328, 27, 3042, 4535, 3042, 4256, 3042, 4258, 3042, 3119, 4533, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{4328}$$

$$\frac{2}{9}\int \frac{20ba^2+(7a^2+27b^2)\sec(c+dx)a+b(5a^2+9b^2)\sec^2(c+dx)}{2\sec^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{9}\int \frac{20ba^2+(7a^2+27b^2)\sec(c+dx)a+b(5a^2+9b^2)\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{9}\int \frac{20ba^2+(7a^2+27b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a+b(5a^2+9b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 4535}$$

$$\frac{1}{9}\left(a(7a^2+27b^2)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx+\int \frac{20ba^2+b(5a^2+9b^2)\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{9}\left(a(7a^2+27b^2)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\int \frac{20ba^2+b(5a^2+9b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 4256}$$

$$\frac{1}{9}\left(\int \frac{20ba^2+b(5a^2+9b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+a(7a^2+27b^2)\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{9}\left(\int \frac{20ba^2+b(5a^2+9b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+a(7a^2+27b^2)\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 4258

$$\frac{1}{9}\left(\int \frac{20ba^2+b(5a^2+9b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+a(7a^2+27b^2)\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}\left(\int \frac{20ba^2+b(5a^2+9b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+a(7a^2+27b^2)\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+d\right.}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 3119

$$\frac{1}{9}\left(\int \frac{20ba^2+b(5a^2+9b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+a(7a^2+27b^2)\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{5d}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 4533

$$\frac{1}{9}\left(\frac{9}{7}b(15a^2+7b^2)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+a(7a^2+27b^2)\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{5d}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}\left(\frac{9}{7}b(15a^2+7b^2)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+a(7a^2+27b^2)\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{5d}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4256}$$

$$\frac{1}{9}\left(\frac{9}{7}b(15a^2+7b^2)\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+a(7a^2+27b^2)\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c-}}{}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{9}{7}b(15a^2+7b^2)\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+a(7a^2+27b^2)\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{}}{}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{9}\left(\frac{9}{7}b(15a^2+7b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+a(7a^2+27b^2)\left(\frac{}{5}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{9}{7}b(15a^2+7b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+a(7a^2+27b\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{9}\left(\frac{9}{7}b(15a^2+7b^2)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+a(7a^2+27b^2\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

input
```
Int[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(9/2),x]
```

output
```
(2*a^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + ((40*
a^2*b*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + a*(7*a^2 + 27*b^2)*((6*Sqrt
[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*Si
n[c + d*x])/(5*d*Sec[c + d*x]^(3/2))) + (9*b*(15*a^2 + 7*b^2)*((2*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c
+ d*x])/(3*d*Sqrt[Sec[c + d*x]])))/7)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(209) = 418$.

Time = 15.17 (sec) , antiderivative size = 470, normalized size of antiderivative = 2.01

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-1120a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+\left(2240a^3+2160a^2b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(\right.}{}$
parts	Expression too large to display

input
```
int((a+b*sec(d*x+c))^3/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*a^3*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*a^3+2160*a^2*b)*sin(1/2*d*x
+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-2072*a^3-3240*a^2*b-1512*a*b^2)*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)+(952*a^3+2520*a^2*b+1512*a*b^2+420*b^3)*sin(1
/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-168*a^3-720*a^2*b-378*a*b^2-210*b^3)*
sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+225*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*
b+105*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3-147*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3-567*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*a*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.02

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx =$$

$$-\frac{15\sqrt{2}(15i\,a^2b+7i\,b^3)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+15\sqrt{2}(-15i\,a^2b-7i}{}$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
-1/315*(15*sqrt(2)*(15*I*a^2*b + 7*I*b^3)*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + 15*sqrt(2)*(-15*I*a^2*b - 7*I*b^3)*weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*sqrt(2)*(-7*I*a^3 - 27
*I*a*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) + 21*sqrt(2)*(7*I*a^3 + 27*I*a*b^2)*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(35*a^3*
cos(d*x + c)^4 + 135*a^2*b*cos(d*x + c)^3 + 7*(7*a^3 + 27*a*b^2)*cos(d*x +
 c)^2 + 15*(15*a^2*b + 7*b^3)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)
))/d
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**3/sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(9/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(9/2),x)
```

output
```
int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(9/2), x)
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^5}dx\right)a^3+3\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^4}dx\right)a^2b +3\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3}dx\right)a\,b^2+\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx\right)b^3$$

input
```
int((a+b*sec(d*x+c))^3/sec(d*x+c)^(9/2),x)
```

output
```
int(sqrt(sec(c + d*x))/sec(c + d*x)**5,x)*a**3 + 3*int(sqrt(sec(c + d*x))/
sec(c + d*x)**4,x)*a**2*b + 3*int(sqrt(sec(c + d*x))/sec(c + d*x)**3,x)*a*
b**2 + int(sqrt(sec(c + d*x))/sec(c + d*x)**2,x)*b**3
```

3.600 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal result

Integrand size = 23, antiderivative size = 287

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx &= -\frac{2(15a^4+54a^2b^2+7b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} \\ &+\frac{8ab(7a^2+5b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\ &+\frac{2(15a^4+54a^2b^2+7b^4)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} \\ &+\frac{8ab(7a^2+5b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d}+\frac{14b^2(7a^2+b^2)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{45d} \\ &+\frac{44ab^3\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d}+\frac{2b^2\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{9d}\end{aligned}$$

output
```
-2/15*(15*a^4+54*a^2*b^2+7*b^4)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))*sec(d*x+c)^(1/2)/d+8/21*a*b*(7*a^2+5*b^2)*cos(d*x+c)^(1/2)*In
verseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+2/15*(15*a^4+54*a^
2*b^2+7*b^4)*sec(d*x+c)^(1/2)*sin(d*x+c)/d+8/21*a*b*(7*a^2+5*b^2)*sec(d*x+
c)^(3/2)*sin(d*x+c)/d+14/45*b^2*(7*a^2+b^2)*sec(d*x+c)^(5/2)*sin(d*x+c)/d+
44/63*a*b^3*sec(d*x+c)^(7/2)*sin(d*x+c)/d+2/9*b^2*sec(d*x+c)^(5/2)*(a+b*se
c(d*x+c))^2*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 2.63 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.89

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx =$$

$$-\frac{2(a+b\sec(c+dx))^4\left(21(15a^4+54a^2b^2+7b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)-60ab(7a^2+5b^2)\sqrt{\text{co}}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^4,x]
```

output
```
(-2*(a + b*Sec[c + d*x])^4*(21*(15*a^4 + 54*a^2*b^2 + 7*b^4)*Sqrt[Cos[c +
d*x]]*EllipticE[(c + d*x)/2, 2] - 60*a*b*(7*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2] - 315*a^4*Sin[c + d*x] - 1134*a^2*b^2*Sin[c +
d*x] - 147*b^4*Sin[c + d*x] - 420*a^3*b*Tan[c + d*x] - 300*a*b^3*Tan[c + d
*x] - 378*a^2*b^2*Sec[c + d*x]*Tan[c + d*x] - 49*b^4*Sec[c + d*x]*Tan[c +
d*x] - 180*a*b^3*Sec[c + d*x]^2*Tan[c + d*x] - 35*b^4*Sec[c + d*x]^3*Tan[c
 + d*x]))/(315*d*(b + a*Cos[c + d*x])^4*Sec[c + d*x]^(7/2))
```

Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.94, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.913$, Rules used = {3042, 4329, 27, 3042, 4564, 27, 3042, 4535, 3042, 4255, 3042, 4258, 3042, 3120, 4534, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4dx$$

$$\Big\downarrow \text{ 4329}$$

$$\frac{2}{9}\int \frac{1}{2}\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\left(22ab^2\sec^2(c+dx)+b\left(27a^2+7b^2\right)\sec(c+dx)+3a\left(3a^2+b^2\right)\right)dx+\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$\downarrow$ 27

$$\frac{1}{9}\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\left(22ab^2\sec^2(c+dx)+b\left(27a^2+7b^2\right)\sec(c+dx)+3a\left(3a^2+b^2\right)\right)dx+\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$\downarrow$ 3042

$$\frac{1}{9}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(22ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\left(27a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$\downarrow$ 4564

$$\frac{1}{9}\left(\frac{2}{7}\int \frac{1}{2}\sec^{\frac{3}{2}}(c+dx)\left(21\left(3a^2+b^2\right)a^2+36b\left(7a^2+5b^2\right)\sec(c+dx)a+49b^2\left(7a^2+b^2\right)\sec^2(c+dx)\right)dx+\frac{44a\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$\downarrow$ 27

$$\frac{1}{9}\left(\frac{1}{7}\int \sec^{\frac{3}{2}}(c+dx)\left(21\left(3a^2+b^2\right)a^2+36b\left(7a^2+5b^2\right)\sec(c+dx)a+49b^2\left(7a^2+b^2\right)\sec^2(c+dx)\right)dx+\frac{44ab^3\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(21\left(3a^2+b^2\right)a^2+36b\left(7a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+49b^2\left(7a^2+b^2\right)\csc\left(c+d\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$\downarrow$ 4535

$$\frac{1}{9}\left(\frac{1}{7}\left(36ab(7a^2+5b^2)\int \sec^{\frac{5}{2}}(c+dx)dx+\int \sec^{\frac{3}{2}}(c+dx)\left(21\left(3a^2+b^2\right)a^2+49b^2\left(7a^2+b^2\right)\sec^2(c+dx)\right)dx\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\left(36ab(7a^2+5b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(21\left(3a^2+b^2\right)a^2+49b^2\left(7a^2+b^2\right)\mathrm{c}\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$\downarrow$ 4255

$$\frac{1}{9}\left(\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(21\left(3a^2+b^2\right)a^2+49b^2\left(7a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+36ab(7a^2+5b^2)\left(\frac{1}{3}\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(21\left(3a^2+b^2\right)a^2+49b^2\left(7a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+36ab(7a^2+5b^2)\left(\frac{1}{3}\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$\downarrow$ 4258

$$\frac{1}{9}\left(\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(21\left(3a^2+b^2\right)a^2+49b^2\left(7a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+36ab(7a^2+5b^2)\left(\frac{1}{3}\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(21\left(3a^2+b^2\right)a^2+49b^2\left(7a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+36ab(7a^2+5b^2)\left(\frac{1}{3}\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(21\left(3a^2+b^2\right)a^2+49b^2\left(7a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+36ab\left(7a^2+5b^2\right)\left(\frac{2}{-}\right.\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$$\Big\downarrow \text{4534}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{21}{5}\left(15a^4+54a^2b^2+7b^4\right)\int \sec^{\frac{3}{2}}(c+dx)dx+\frac{98b^2\left(7a^2+b^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+36ab\left(7a^2+5b^2\right)\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{21}{5}\left(15a^4+54a^2b^2+7b^4\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\frac{98b^2\left(7a^2+b^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+36ab(7a\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{21}{5}\left(15a^4+54a^2b^2+7b^4\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{98b^2\left(7a^2+b^2\right)\sin(c+}{5d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{21}{5}\left(15a^4+54a^2b^2+7b^4\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{98b^2\left(7a^2+b^2\right)\sin}{}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{21}{5}\left(15a^4+54a^2b^2+7b^4\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{21}{5}\left(15a^4+54a^2b^2+7b^4\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\right.}\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{98b^2(7a^2+b^2)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+36ab\left(7a^2+5b^2\right)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx}}{}\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

input
```
Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^4,x]
```

output
```
(2*b^2*Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(9*d) + ((4
4*a*b^3*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d) + ((98*b^2*(7*a^2 + b^2)*Se
c[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + (21*(15*a^4 + 54*a^2*b^2 + 7*b^4)*(
(-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/5 + 36*a*b*(7*a^2 + 5*b^2)*((2*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Se
c[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)))/7)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4564
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Simp[1/(n + 2)   Int[(d*Csc[e + f*x])^n*Simp[A*a*(n +
2) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
&&  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1146 vs. $2(258) = 516$.

Time = 10.62 (sec) , antiderivative size = 1147, normalized size of antiderivative = 4.00

method	result	size
default	Expression too large to display	1147
parts	Expression too large to display	1396

input
```
int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^4/sin(1/2*
d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2))+2*b^4*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1
/2*c)^2-1/2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2
*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))))+8*a*b^3*(-1/56*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^
4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+12/5*a^2*b^2/(8*sin(1/2*d*x+
1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.11

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx =$$

$$-\frac{60\sqrt{2}(7i\,a^3b+5i\,ab^3)\cos{(dx+c)}^4\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+60\sqrt{2}(}{}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/315*(60*sqrt(2)*(7*I*a^3*b + 5*I*a*b^3)*cos(d*x + c)^4*weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 60*sqrt(2)*(-7*I*a^3*b - 5*I*a
*b^3)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c)) + 21*sqrt(2)*(15*I*a^4 + 54*I*a^2*b^2 + 7*I*b^4)*cos(d*x + c)^4*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)
)) + 21*sqrt(2)*(-15*I*a^4 - 54*I*a^2*b^2 - 7*I*b^4)*cos(d*x + c)^4*weiers
trassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
) - 2*(180*a*b^3*cos(d*x + c) + 21*(15*a^4 + 54*a^2*b^2 + 7*b^4)*cos(d*x +
 c)^4 + 35*b^4 + 60*(7*a^3*b + 5*a*b^3)*cos(d*x + c)^3 + 7*(54*a^2*b^2 + 7
*b^4)*cos(d*x + c)^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**4,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx=\int (b\sec(dx+c)+a)^4\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^4*sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx=\int \left(a+\frac{b}{\cos(c+d\,x)}\right)^4\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

input
```
int((a + b/cos(c + d*x))^4*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^4*(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx=&\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^5\,dx\right)b^4\\&+4\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^4\,dx\right)a\,b^3\\&+6\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^3\,dx\right)a^2b^2\\&+4\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^2\,dx\right)a^3b\\&+\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)\,dx\right)a^4\end{aligned}$$

input
```
int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^4,x)
```

output
```
int(sqrt(sec(c + d*x))*sec(c + d*x)**5,x)*b**4 + 4*int(sqrt(sec(c + d*x))*
sec(c + d*x)**4,x)*a*b**3 + 6*int(sqrt(sec(c + d*x))*sec(c + d*x)**3,x)*a*
*2*b**2 + 4*int(sqrt(sec(c + d*x))*sec(c + d*x)**2,x)*a**3*b + int(sqrt(se
c(c + d*x))*sec(c + d*x),x)*a**4
```

3.601 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^4\,dx$

Optimal result

Integrand size = 23, antiderivative size = 247

$$\begin{aligned}\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^4\,dx &= -\frac{8ab(5a^2+3b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &\quad+\frac{2(21a^4+42a^2b^2+5b^4)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\ &\quad+\frac{8ab(5a^2+3b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} \\ &\quad+\frac{2b^2(39a^2+5b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d}+\frac{36ab^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} \\ &\quad+\frac{2b^2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{7d}\end{aligned}$$

output
```
-8/5*a*b*(5*a^2+3*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/
2))*sec(d*x+c)^(1/2)/d+2/21*(21*a^4+42*a^2*b^2+5*b^4)*cos(d*x+c)^(1/2)*Inv
erseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+8/5*a*b*(5*a^2+3*b^
2)*sec(d*x+c)^(1/2)*sin(d*x+c)/d+2/21*b^2*(39*a^2+5*b^2)*sec(d*x+c)^(3/2)*
sin(d*x+c)/d+36/35*a*b^3*sec(d*x+c)^(5/2)*sin(d*x+c)/d+2/7*b^2*sec(d*x+c)^
(3/2)*(a+b*sec(d*x+c))^2*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.96 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.68

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^4\,dx$$
$$= \frac{2\sec^{\frac{7}{2}}(c+dx)\left(-84ab(5a^2+3b^2)\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5(21a^4+42a^2b^2+5b^4)\cos^{\frac{7}{2}}(c+dx)\right.}{}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^4,x]
```

output
```
(2*Sec[c + d*x]^(7/2)*(-84*a*b*(5*a^2 + 3*b^2)*Cos[c + d*x]^(7/2)*Elliptic
E[(c + d*x)/2, 2] + 5*(21*a^4 + 42*a^2*b^2 + 5*b^4)*Cos[c + d*x]^(7/2)*Ell
ipticF[(c + d*x)/2, 2] + b*(15*b^3 + 5*b*(42*a^2 + 5*b^2)*Cos[c + d*x]^2 +
 84*a*(5*a^2 + 3*b^2)*Cos[c + d*x]^3)*Sin[c + d*x] + 42*a*b^3*Sin[2*(c + d
*x)]))/(105*d)
```

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.98, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.826$, Rules used = {3042, 4329, 27, 3042, 4564, 27, 3042, 4535, 3042, 4255, 3042, 4258, 3042, 3119, 4534, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4dx$$

$$\Big\downarrow \text{4329}$$

$$\frac{2}{7}\int \frac{1}{2}\sqrt{\sec(c+dx)}(a+b\sec(c+dx))\left(18ab^2\sec^2(c+dx)+b\left(21a^2+5b^2\right)\sec(c+dx)+a\left(7a^2+b^2\right)\right)dx+\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$\Big\downarrow$ 27

$$\frac{1}{7}\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))\left(18ab^2\sec^2(c+dx)+b\left(21a^2+5b^2\right)\sec(c+dx)+a\left(7a^2+b^2\right)\right)dx+\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(18ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\left(21a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$\Big\downarrow$ 4564

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{1}{2}\sqrt{\sec(c+dx)}\left(5\left(7a^2+b^2\right)a^2+28b\left(5a^2+3b^2\right)\sec(c+dx)a+5b^2\left(39a^2+5b^2\right)\sec^2(c+dx)\right)dx+\frac{36a}{}\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$\Big\downarrow$ 27

$$\frac{1}{7}\left(\frac{1}{5}\int \sqrt{\sec(c+dx)}\left(5\left(7a^2+b^2\right)a^2+28b\left(5a^2+3b^2\right)\sec(c+dx)a+5b^2\left(39a^2+5b^2\right)\sec^2(c+dx)\right)dx+\frac{36ab^{}}{}\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5\left(7a^2+b^2\right)a^2+28b\left(5a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+5b^2\left(39a^2+5b^2\right)\csc\left(c+dx\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$\Big\downarrow$ 4535

$$\frac{1}{7}\left(\frac{1}{5}\left(28ab(5a^2+3b^2)\int \sec^{\frac{3}{2}}(c+dx)dx+\int \sqrt{\sec(c+dx)}\left(5\left(7a^2+b^2\right)a^2+5b^2\left(39a^2+5b^2\right)\sec^2(c+dx)\right)dx\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(28ab(5a^2+3b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5\left(7a^2+b^2\right)a^2+5b^2\left(39a^2+5b^2\right)\text{cs}\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5\left(7a^2+b^2\right)a^2+5b^2\left(39a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+28ab(5a^2+3b^2)\left(\frac{2\,\text{s}}{}\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5\left(7a^2+b^2\right)a^2+5b^2\left(39a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+28ab(5a^2+3b^2)\left(\frac{2}{}\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5\left(7a^2+b^2\right)a^2+5b^2\left(39a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+28ab(5a^2+3b^2)\left(\frac{2\,\text{s}}{}\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5\left(7a^2+b^2\right)a^2+5b^2\left(39a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+28ab(5a^2+3b^2)\left(\frac{2\,\text{s}}{}\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5\left(7a^2+b^2\right)a^2+5b^2\left(39a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+28ab\left(5a^2+3b^2\right)\left(\frac{2s\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$$\Big\downarrow \text{4534}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{5}{3}\left(21a^4+42a^2b^2+5b^4\right)\int \sqrt{\sec(c+dx)}dx+\frac{10b^2\left(39a^2+5b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+28ab\left(5a^2+3\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{5}{3}\left(21a^4+42a^2b^2+5b^4\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{10b^2\left(39a^2+5b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+28ab(5a\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{5}{3}\left(21a^4+42a^2b^2+5b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{10b^2\left(39a^2+5b^2\right)\sin(c+dx)s}{3d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{5}{3}\left(21a^4+42a^2b^2+5b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{10b^2\left(39a^2+5b^2\right)\sin(c+}{3d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{10b^2(39a^2+5b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+28ab(5a^2+3b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+\right.\right.\right.$$

$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

input
```
Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^4,x]
```

output
```
(2*b^2*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(7*d) + ((3
6*a*b^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + ((10*(21*a^4 + 42*a^2*b^2
 + 5*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(3*d) + (10*b^2*(39*a^2 + 5*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) +
 28*a*b*(5*a^2 + 3*b^2)*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/5)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4564
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Simp[1/(n + 2)   Int[(d*Csc[e + f*x])^n*Simp[A*a*(n +
2) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
&&  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 897 vs. $2(222) = 444$.

Time = 8.61 (sec) , antiderivative size = 898, normalized size of antiderivative = 3.64

method	result	size
default	Expression too large to display	898
parts	Expression too large to display	1146

input
```
int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^4*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*
b^4*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2)))+8/5*a*b^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/
2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*
x+1/2*c)-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+
1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c
)^2+8*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+12*a^2*b^2*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(
1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.17

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^4\,dx =$$

$$-\frac{5\sqrt{2}(21i\,a^4+42i\,a^2b^2+5i\,b^4)\cos{(dx+c)}^3\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})}{}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/105*(5*sqrt(2)*(21*I*a^4 + 42*I*a^2*b^2 + 5*I*b^4)*cos(d*x + c)^3*weier
strassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-21*I*a^
4 - 42*I*a^2*b^2 - 5*I*b^4)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(
d*x + c) - I*sin(d*x + c)) + 84*sqrt(2)*(5*I*a^3*b + 3*I*a*b^3)*cos(d*x +
c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c))) + 84*sqrt(2)*(-5*I*a^3*b - 3*I*a*b^3)*cos(d*x + c)^3*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))
- 2*(84*a*b^3*cos(d*x + c) + 15*b^4 + 84*(5*a^3*b + 3*a*b^3)*cos(d*x + c)^
3 + 5*(42*a^2*b^2 + 5*b^4)*cos(d*x + c)^2)*sin(d*x + c)/sqrt(cos(d*x + c))
)/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^4\,dx = \int (b\sec(dx+c)+a)^4\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^4*sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^4\,dx = \int (b\sec(dx+c)+a)^4\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^4*sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^4\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^4\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

input
```
int((a + b/cos(c + d*x))^4*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^4*(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^4\,dx = \left(\int \sqrt{\sec(dx+c)}dx\right)a^4 + \left(\int \sqrt{\sec(dx+c)}\sec(dx+c)^4\,dx\right)b^4 + 4\left(\int \sqrt{\sec(dx+c)}\sec(dx+c)^3\,dx\right)a\,b^3 + 6\left(\int \sqrt{\sec(dx+c)}\sec(dx+c)^2\,dx\right)a^2b^2 + 4\left(\int \sqrt{\sec(dx+c)}\sec(dx+c)\,dx\right)a^3b$$

input
```
int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^4,x)
```

output
```
int(sqrt(sec(c + d*x)),x)*a**4 + int(sqrt(sec(c + d*x))*sec(c + d*x)**4,x)
*b**4 + 4*int(sqrt(sec(c + d*x))*sec(c + d*x)**3,x)*a*b**3 + 6*int(sqrt(se
c(c + d*x))*sec(c + d*x)**2,x)*a**2*b**2 + 4*int(sqrt(sec(c + d*x))*sec(c
+ d*x),x)*a**3*b
```

3.602 $\int \frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 209

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx &= \frac{2(5a^4-30a^2b^2-3b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}\\ &+\frac{8ab(3a^2+b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}\\ &+\frac{2b^2(29a^2+3b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{28ab^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d}\\ &+\frac{2b^2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\sin(c+dx)}{5d}\end{aligned}$$

output
```
2/5*(5*a^4-30*a^2*b^2-3*b^4)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c)
,2^(1/2))*sec(d*x+c)^(1/2)/d+8/3*a*b*(3*a^2+b^2)*cos(d*x+c)^(1/2)*InverseJ
acobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+2/5*b^2*(29*a^2+3*b^2)*s
ec(d*x+c)^(1/2)*sin(d*x+c)/d+28/15*a*b^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5
*b^2*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 2.48 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.70

$$\int \frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{\sec^{\frac{5}{2}}(c+dx)\left(12(5a^4-30a^2b^2-3b^4)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\left(80a(3a^2+b^2)\cos^{\frac{5}{2}}(c+dx)\,\text{Elli}\right.\right.}{30}$$

input
```
Integrate[(a + b*Sec[c + d*x])^4/Sqrt[Sec[c + d*x]],x]
```

output
```
(Sec[c + d*x]^(5/2)*(12*(5*a^4 - 30*a^2*b^2 - 3*b^4)*Cos[c + d*x]^(5/2)*El
lipticE[(c + d*x)/2, 2] + b*(80*a*(3*a^2 + b^2)*Cos[c + d*x]^(5/2)*Ellipti
cF[(c + d*x)/2, 2] + 2*b*(15*(6*a^2 + b^2) + 40*a*b*Cos[c + d*x] + 9*(10*a
^2 + b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])))/(30*d)
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.02, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4329, 27, 3042, 4564, 27, 3042, 4535, 3042, 4258, 3042, 3120, 4534, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4329$$

$$\frac{2}{5}\int \frac{(a+b\sec(c+dx))\left(14ab^2\sec^2(c+dx)+3b\left(5a^2+b^2\right)\sec(c+dx)+a\left(5a^2-b^2\right)\right)}{2\sqrt{\sec(c+dx)}}dx+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

$\Big\downarrow$ 27

$$\frac{1}{5}\int \frac{(a+b\sec(c+dx))\left(14ab^2\sec^2(c+dx)+3b\left(5a^2+b^2\right)\sec(c+dx)+a\left(5a^2-b^2\right)\right)}{\sqrt{\sec(c+dx)}}dx+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(14ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+3b\left(5a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+a\left(5a^2-b^2\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

$\Big\downarrow$ 4564

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{3\left(5a^2-b^2\right)a^2+20b\left(3a^2+b^2\right)\sec(c+dx)a+3b^2\left(29a^2+3b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}}dx+\frac{28ab^3\sin(c+dx)\sec^{\frac{3}{2}}}{3d}\right.$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

$\Big\downarrow$ 27

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{3\left(5a^2-b^2\right)a^2+20b\left(3a^2+b^2\right)\sec(c+dx)a+3b^2\left(29a^2+3b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx+\frac{28ab^3\sin(c+dx)\sec^{\frac{3}{2}}}{3d}\right.$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{3\left(5a^2-b^2\right)a^2+20b\left(3a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+3b^2\left(29a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{28ab^3\sin(c\right.$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

$\Big\downarrow$ 4535

$$\frac{1}{5}\left(\frac{1}{3}\left(\int \frac{3(5a^2-b^2)a^2+3b^2(29a^2+3b^2)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx+20ab(3a^2+b^2)\int \sqrt{\sec(c+dx)}dx\right)+\frac{28ab^3\sin(c\right.$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(20ab(3a^2+b^2)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\int \frac{3(5a^2-b^2)a^2+3b^2(29a^2+3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\right.$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

$\Big\downarrow$ 4258

$$\frac{1}{5}\left(\frac{1}{3}\left(\int \frac{3(5a^2-b^2)a^2+3b^2(29a^2+3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+20ab(3a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(\int \frac{3(5a^2-b^2)a^2+3b^2(29a^2+3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+20ab(3a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

$\Big\downarrow$ 3120

$$\frac{1}{5}\left(\frac{1}{3}\left(\int \frac{3(5a^2-b^2)a^2+3b^2(29a^2+3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{40ab(3a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{E}}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

$\Big\downarrow$ 4534

$$\frac{1}{5}\left(\frac{1}{3}\left(3(5a^4-30a^2b^2-3b^4)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{6b^2(29a^2+3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{40ab(3a^2+b^2)}{}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(3(5a^4-30a^2b^2-3b^4)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6b^2(29a^2+3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{40ab(3a^2\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(3(5a^4-30a^2b^2-3b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{6b^2(29a^2+3b^2)\sin(c+dx)\sqrt{\mathrm{se}}}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(3(5a^4-30a^2b^2-3b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{6b^2(29a^2+3b^2)\sin(c+dx}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{6b^2(29a^2+3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{40ab(3a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+\right.}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

input
```
Int[(a + b*Sec[c + d*x])^4/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*b^2*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(5*d) + ((2
8*a*b^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + ((6*(5*a^4 - 30*a^2*b^2 -
 3*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
 + (40*a*b*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/d + (6*b^2*(29*a^2 + 3*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/d)/3)/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4564
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Simp[1/(n + 2)   Int[(d*Csc[e + f*x])^n*Simp[A*a*(n +
2) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
&&  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 879 vs. 2(188) = 376.

Time = 7.38 (sec) , antiderivative size = 880, normalized size of antiderivative = 4.21

method	result	size
default	Expression too large to display	880
parts	Expression too large to display	1036

input
```
int((a+b*sec(d*x+c))^4/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/5*b^4/(8*sin
(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/
2*d*x+1/2*c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^2-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2)))+8*b*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*a*b^3*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2
+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.26

$$\int \frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{20\sqrt{2}(3i\,a^3b+i\,ab^3)\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+20\sqrt{2}(-\cdots}{\cdots}$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*(20*sqrt(2)*(3*I*a^3*b + I*a*b^3)*cos(d*x + c)^2*weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 20*sqrt(2)*(-3*I*a^3*b - I*a*b^3)
*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
+ 3*sqrt(2)*(-5*I*a^4 + 30*I*a^2*b^2 + 3*I*b^4)*cos(d*x + c)^2*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3
*sqrt(2)*(5*I*a^4 - 30*I*a^2*b^2 - 3*I*b^4)*cos(d*x + c)^2*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(20
*a*b^3*cos(d*x + c) + 3*b^4 + 9*(10*a^2*b^2 + b^4)*cos(d*x + c)^2)*sin(d*x
 + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^4}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^4}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^4}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx &= \left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)a^4+4\left(\int \sqrt{\sec(dx+c)}dx\right)a^3b \\ &\quad+\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^3\,dx\right)b^4 \\ &\quad+4\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^2\,dx\right)a\,b^3 \\ &\quad+6\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)\,dx\right)a^2b^2\end{aligned}$$

input
```
int((a+b*sec(d*x+c))^4/sec(d*x+c)^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x))/sec(c + d*x),x)*a**4 + 4*int(sqrt(sec(c + d*x)),x)*
a**3*b + int(sqrt(sec(c + d*x))*sec(c + d*x)**3,x)*b**4 + 4*int(sqrt(sec(c
 + d*x))*sec(c + d*x)**2,x)*a*b**3 + 6*int(sqrt(sec(c + d*x))*sec(c + d*x)
,x)*a**2*b**2
```

3.603 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 208

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{8ab(a^2-b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2(a^4+18a^2b^2+b^4)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} - \frac{4ab(a^2-6b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} - \frac{2b^2(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
8*a*b*(a^2-b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec
(d*x+c)^(1/2)/d+2/3*(a^4+18*a^2*b^2+b^4)*cos(d*x+c)^(1/2)*InverseJacobiAM(
1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d-4/3*a*b*(a^2-6*b^2)*sec(d*x+c)^(
1/2)*sin(d*x+c)/d-2/3*b^2*(a^2-b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/3*a^2*
(a+b*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.62

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Big(24ab(a^2-b^2)\,E\big(\tfrac{1}{2}(c+dx)\big|\,2\big)+2(a^4+18a^2b^2+b^4)\,\mathrm{EllipticF}\big(\tfrac{1}{2}(c+dx),}{3d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(3/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(24*a*b*(a^2 - b^2)*EllipticE[(c +
d*x)/2, 2] + 2*(a^4 + 18*a^2*b^2 + b^4)*EllipticF[(c + d*x)/2, 2] + ((a^4
+ 2*b^4 + 24*a*b^3*Cos[c + d*x] + a^4*Cos[2*(c + d*x)])*Sin[c + d*x])/Cos[
c + d*x]^(3/2)))/(3*d)
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4328, 27, 3042, 4564, 27, 3042, 4535, 3042, 4258, 3042, 3120, 4534, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \; 4328$$

$$\frac{2}{3}\int \frac{(a+b\sec(c+dx))\left(10ba^2+\left(a^2+9b^2\right)\sec(c+dx)a-3b\left(a^2-b^2\right)\sec^2(c+dx)\right)}{2\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{(a+b\sec(c+dx))\left(10ba^2+\left(a^2+9b^2\right)\sec(c+dx)a-3b\left(a^2-b^2\right)\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(10ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a-3b\left(a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4564}$$

$$\frac{1}{3}\left(\frac{2}{3}\int \frac{3\left(10ba^3-2b\left(a^2-6b^2\right)\sec^2(c+dx)a+\left(a^4+18b^2a^2+b^4\right)\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)}}dx-\frac{2b^2\left(a^2-b^2\right)\sin(c+dx)\text{s}}{d}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\int \frac{10ba^3-2b\left(a^2-6b^2\right)\sec^2(c+dx)a+\left(a^4+18b^2a^2+b^4\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx-\frac{2b^2\left(a^2-b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c}{d}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\int \frac{10ba^3-2b\left(a^2-6b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2a+\left(a^4+18b^2a^2+b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b^2\left(a^2-b^2\right)\sin(c+}{d}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4535}$$

$$\frac{1}{3}\left(\left(a^4+18a^2b^2+b^4\right)\int\sqrt{\sec(c+dx)}dx+\int\frac{10a^3b-2ab\left(a^2-6b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx-\frac{2b^2\left(a^2-b^2\right)\sin(c+dx}{d}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(\left(a^4+18a^2b^2+b^4\right)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\int\frac{10a^3b-2ab\left(a^2-6b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b^2\left(a^2-b^2\right.}{}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 4258

$$\frac{1}{3}\left(\left(a^4+18a^2b^2+b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\int\frac{10a^3b-2ab\left(a^2-6b^2\right)\csc\left(c+dx+\right.}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(\left(a^4+18a^2b^2+b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\int\frac{10a^3b-2ab\left(a^2-6b^2\right)\csc\left(c+\right.}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3120

$$\frac{1}{3}\left(\int\frac{10a^3b-2ab\left(a^2-6b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b^2\left(a^2-b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d}+\frac{2\left(a^4+18a^2b^2+b^4\right.}{}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 4534

$$\frac{1}{3}\left(12ab(a^2-b^2)\int \frac{1}{\sqrt{\sec(c+dx)}}dx-\frac{2b^2(a^2-b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d}-\frac{4ab(a^2-6b^2)\sin(c+dx)\sqrt{\sec}}{d}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(12ab(a^2-b^2)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b^2(a^2-b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d}-\frac{4ab(a^2-6b^2)\sin(c+dx)}{d}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 4258

$$\frac{1}{3}\left(12ab(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-\frac{2b^2(a^2-b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d}-\frac{4ab}{}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(12ab(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{2b^2(a^2-b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3119

$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}+$$
$$\frac{1}{3}\left(-\frac{2b^2(a^2-b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d}-\frac{4ab(a^2-6b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{24ab(a^2-b^2)\sqrt{\cos(c+}}{}\right.$$

input
```
Int[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(3/2),x]
```

output
```
(2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + ((2
4*a*b*(a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/d + (2*(a^4 + 18*a^2*b^2 + b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (4*a*b*(a^2 - 6*b^2)*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/d - (2*b^2*(a^2 - b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/d)
/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4564
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Simp[1/(n + 2)   Int[(d*Csc[e + f*x])^n*Simp[A*a*(n +
2) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
&&  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 776 vs. $2(189) = 378$.

Time = 7.05 (sec) , antiderivative size = 777, normalized size of antiderivative = 3.74

method	result	size
default	Expression too large to display	777
parts	Expression too large to display	857

input
```
int((a+b*sec(d*x+c))^4/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-8*a^4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*a*(a^3+6*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(a^4+12*a*b^3+b^4)*sin(1/2
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^4+18*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*a^2*b^2+EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-12*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*a*b^3)*sin(1/2*d*x+1/2*c)^2+a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+18*a^2*b^2*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-12*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^
3*b+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.16

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{\sqrt{2}(-i\,a^4-18i\,a^2b^2-i\,b^4)\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}(i}{}$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*(-I*a^4 - 18*I*a^2*b^2 - I*b^4)*cos(d*x + c)*weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*a^4 + 18*I*a^2*b^2
+ I*b^4)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
+ c)) - 12*sqrt(2)*(-I*a^3*b + I*a*b^3)*cos(d*x + c)*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 12*sqrt(2)*
(I*a^3*b - I*a*b^3)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(a^4*cos(d*x + c)^2 + 12*a*b^
3*cos(d*x + c) + b^4)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+b\sec{(c+dx)})^4}{\sec^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**4/sec(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec{(dx+c)}+a)^4}{\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx = {} & \left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx\right)a^4 + 4\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)a^3b \\ & + 6\left(\int \sqrt{\sec(dx+c)}dx\right)a^2b^2 \\ & + \left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^2\,dx\right)b^4 \\ & + 4\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)\,dx\right)a\,b^3\end{aligned}$$

input
```
int((a+b*sec(d*x+c))^4/sec(d*x+c)^(3/2),x)
```

output
```
int(sqrt(sec(c + d*x))/sec(c + d*x)**2,x)*a**4 + 4*int(sqrt(sec(c + d*x))/
sec(c + d*x),x)*a**3*b + 6*int(sqrt(sec(c + d*x)),x)*a**2*b**2 + int(sqrt(
sec(c + d*x))*sec(c + d*x)**2,x)*b**4 + 4*int(sqrt(sec(c + d*x))*sec(c + d
*x),x)*a*b**3
```

3.604 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 207

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2(3a^4+30a^2b^2-5b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{8ab(a^2+3b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{28a^3b\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} - \frac{2b^2(a^2-5b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

output
```
2/5*(3*a^4+30*a^2*b^2-5*b^4)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c)
,2^(1/2))*sec(d*x+c)^(1/2)/d+8/3*a*b*(a^2+3*b^2)*cos(d*x+c)^(1/2)*InverseJ
acobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+28/15*a^3*b*sin(d*x+c)/d
/sec(d*x+c)^(1/2)-2/5*b^2*(a^2-5*b^2)*sec(d*x+c)^(1/2)*sin(d*x+c)/d+2/5*a^
2*(a+b*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.67

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{\sec(c+dx)}\Big(12(3a^4+30a^2b^2-5b^4)\sqrt{\cos(c+dx)}E\big(\frac{1}{2}(c+dx)\big|\,2\big)+80ab(a^2+3b^2)\sqrt{\cos(c+dx)}\,\mathrm{El}}{30d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(5/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(12*(3*a^4 + 30*a^2*b^2 - 5*b^4)*Sqrt[Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2] + 80*a*b*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2] + 2*(3*a^4 + 30*b^4 + 40*a^3*b*Cos[c + d*x] + 3*a^4*Cos[
2*(c + d*x)])*Sin[c + d*x]))/(30*d)
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.02, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4328, 27, 3042, 4562, 27, 3042, 4535, 3042, 4258, 3042, 3120, 4534, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4328}$$

$$\frac{2}{5}\int \frac{(a+b\sec(c+dx))\left(14ba^2+3\left(a^2+5b^2\right)\sec(c+dx)a-b\left(a^2-5b^2\right)\sec^2(c+dx)\right)}{2\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}\int \frac{(a+b\sec(c+dx))\left(14ba^2+3\left(a^2+5b^2\right)\sec(c+dx)a-b\left(a^2-5b^2\right)\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(14ba^2+3\left(a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a-b\left(a^2-5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 4562$$

$$\frac{1}{5}\left(\frac{28a^3b\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}-\frac{2}{3}\int -\frac{3\left(3a^2+29b^2\right)a^2+20b\left(a^2+3b^2\right)\sec(c+dx)a-3b^2\left(a^2-5b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}}dx\right)$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{3\left(3a^2+29b^2\right)a^2+20b\left(a^2+3b^2\right)\sec(c+dx)a-3b^2\left(a^2-5b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx+\frac{28a^3b\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{3\left(3a^2+29b^2\right)a^2+20b\left(a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a-3b^2\left(a^2-5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{28a^3b\sin(c}{3d\sqrt{\sec(c}}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 4535$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\int \frac{3a^2(3a^2+29b^2)-3b^2(a^2-5b^2)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx+20ab(a^2+3b^2)\int \sqrt{\sec(c+dx)}dx\right)+\frac{28a^3b\sin(c}{3d\sqrt{\sec(c}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(20ab(a^2+3b^2)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\int \frac{3a^2(3a^2+29b^2)-3b^2(a^2-5b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 4258

$$\frac{1}{5}\left(\frac{1}{3}\left(\int \frac{3a^2(3a^2+29b^2)-3b^2(a^2-5b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+20ab(a^2+3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(\int \frac{3a^2(3a^2+29b^2)-3b^2(a^2-5b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+20ab(a^2+3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3120

$$\frac{1}{5}\left(\frac{1}{3}\left(\int \frac{3a^2(3a^2+29b^2)-3b^2(a^2-5b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{40ab(a^2+3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{E}}{d}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 4534

$$\frac{1}{5}\left(\frac{1}{3}\left(3\left(3a^4+30a^2b^2-5b^4\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx-\frac{6b^2\left(a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{40ab\left(a^2+3b^2\right)\sqrt{}}{}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(3\left(3a^4+30a^2b^2-5b^4\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6b^2\left(a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{40ab\left(a^2+3\right.}{}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(3\left(3a^4+30a^2b^2-5b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-\frac{6b^2\left(a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c}}{d}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(3\left(3a^4+30a^2b^2-5b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{6b^2\left(a^2-5b^2\right)\sin(c+dx)\sqrt{}}{d}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}+$$
$$\frac{1}{5}\left(\frac{28a^3b\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{1}{3}\left(-\frac{6b^2\left(a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{40ab\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{d}\right.\right.$$

input
```
Int[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(5/2),x]
```

output
```
(2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + ((2
8*a^3*b*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + ((6*(3*a^4 + 30*a^2*b^2 -
 5*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
 + (40*a*b*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/d - (6*b^2*(a^2 - 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
d)/3)/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4562
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Si
mp[1/(d*n)   Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*
b) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(186) = 372$.

Time = 8.70 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.09

method	result
default	$\frac{\frac{16a^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{5}-\frac{16a^4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}-\frac{32a^3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)b}{3}+\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2a^4}{5}+\frac{16\cos(}{}}{}$
parts	Expression too large to display

input
```
int((a+b*sec(d*x+c))^4/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(24*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-24*a^4*sin(1/2*d*x+1/
2*c)^4*cos(1/2*d*x+1/2*c)-80*a^3*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)*b
+6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^4+40*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^2*a^3*b+30*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^4-20*b*
a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-60*a*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*a^4*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))+90*a^2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*b^4)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.04

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{20\sqrt{2}(i\,a^3b+3i\,ab^3)\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+20\sqrt{2}(-i\,a^3b-3i\,ab^3}{}$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/15*(20*sqrt(2)*(I*a^3*b + 3*I*a*b^3)*weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c)) + 20*sqrt(2)*(-I*a^3*b - 3*I*a*b^3)*weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(-3*I*a^4 - 30*I*a
^2*b^2 + 5*I*b^4)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(3*I*a^4 + 30*I*a^2*b^2 - 5*I*b^4)*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
+ c))) - 2*(3*a^4*cos(d*x + c)^2 + 20*a^3*b*cos(d*x + c) + 15*b^4)*sin(d*x
 + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(5/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**4/sec(c + d*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3}dx\right)a^4 + 4\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx\right)a^3b + 6\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)a^2b^2 + 4\left(\int \sqrt{\sec(dx+c)}dx\right)a\,b^3 + \left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)\,dx\right)b^4$$

input
```
int((a+b*sec(d*x+c))^4/sec(d*x+c)^(5/2),x)
```

output
```
int(sqrt(sec(c + d*x))/sec(c + d*x)**3,x)*a**4 + 4*int(sqrt(sec(c + d*x))/
sec(c + d*x)**2,x)*a**3*b + 6*int(sqrt(sec(c + d*x))/sec(c + d*x),x)*a**2*
b**2 + 4*int(sqrt(sec(c + d*x)),x)*a*b**3 + int(sqrt(sec(c + d*x))*sec(c +
 d*x),x)*b**4
```

3.605 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 211

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{8ab(3a^2+5b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2(5a^4+42a^2b^2+21b^4)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} + \frac{36a^3b\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^2(5a^2+39b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

output
```
8/5*a*b*(3*a^2+5*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2
))*sec(d*x+c)^(1/2)/d+2/21*(5*a^4+42*a^2*b^2+21*b^4)*cos(d*x+c)^(1/2)*Inve
rseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+36/35*a^3*b*sin(d*x+
c)/d/sec(d*x+c)^(3/2)+2/21*a^2*(5*a^2+39*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2
)+2/7*a^2*(a+b*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)
```

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.67

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{\sec(c+dx)}\Big(336ab(3a^2+5b^2)\sqrt{\cos(c+dx)}E\big(\tfrac{1}{2}(c+dx)\big|\,2\big)+20(5a^4+42a^2b^2+21b^4)\sqrt{\cos(c+dx)}}{21}$$

input
```
Integrate[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(7/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(336*a*b*(3*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[
(c + d*x)/2, 2] + 20*(5*a^4 + 42*a^2*b^2 + 21*b^4)*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2] + a^2*(65*a^2 + 420*b^2 + 168*a*b*Cos[c + d*x] + 15*
a^2*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(210*d)
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.04, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4328, 27, 3042, 4562, 27, 3042, 4535, 3042, 4258, 3042, 3119, 4533, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$
$$\Big\downarrow \; 4328$$

$$\frac{2}{7}\int \frac{(a+b\sec(c+dx))\left(18ba^2+\left(5a^2+21b^2\right)\sec(c+dx)a+b\left(a^2+7b^2\right)\sec^2(c+dx)\right)}{2\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{7}\int \frac{(a+b\sec(c+dx))\left(18ba^2+\left(5a^2+21b^2\right)\sec(c+dx)a+b\left(a^2+7b^2\right)\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(18ba^2+\left(5a^2+21b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 4562}$$

$$\frac{1}{7}\left(\frac{36a^3b\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}-\frac{2}{5}\int -\frac{5\left(5a^2+39b^2\right)a^2+28b\left(3a^2+5b^2\right)\sec(c+dx)a+5b^2\left(a^2+7b^2\right)\sec^2(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{7}\left(\frac{1}{5}\int \frac{5\left(5a^2+39b^2\right)a^2+28b\left(3a^2+5b^2\right)\sec(c+dx)a+5b^2\left(a^2+7b^2\right)\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{36a^3b\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\int \frac{5\left(5a^2+39b^2\right)a^2+28b\left(3a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+5b^2\left(a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{36a^3b\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 4535}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\int \frac{5\left(5a^2+39b^2\right)a^2+5b^2\left(a^2+7b^2\right)\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx+28ab\left(3a^2+5b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{36a^3b\sin}{5d\sec^{\frac{3}{2}}(}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(28ab\left(3a^2+5b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{5\left(5a^2+39b^2\right)a^2+5b^2\left(a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx\right)\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\int \frac{5\left(5a^2+39b^2\right)a^2+5b^2\left(a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+28ab\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\int \frac{5\left(5a^2+39b^2\right)a^2+5b^2\left(a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+28ab\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\int \frac{5\left(5a^2+39b^2\right)a^2+5b^2\left(a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{56ab\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E}{d}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4533}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{5}{3}\left(5a^4+42a^2b^2+21b^4\right)\int \sqrt{\sec(c+dx)}dx+\frac{10a^2\left(5a^2+39b^2\right)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{56ab\left(3a^2+5b^2\right)\sqrt{\cos(c+}}{}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{5}{3}(5a^4+42a^2b^2+21b^4)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{10a^2(5a^2+39b^2)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{56ab(3a^2+5b^2)\sqrt{c}}{}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 4258

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{5}{3}(5a^4+42a^2b^2+21b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{10a^2(5a^2+39b^2)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{5}{3}(5a^4+42a^2b^2+21b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{10a^2(5a^2+39b^2)\sin(c+}{3d\sqrt{\sec(c+dx)}}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3120

$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}+$$
$$\frac{1}{7}\left(\frac{36a^3b\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{1}{5}\left(\frac{10a^2(5a^2+39b^2)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{56ab(3a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{d}\right.\right.$$

input
```
Int[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(7/2),x]
```

output
```
(2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + ((3
6*a^3*b*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + ((56*a*b*(3*a^2 + 5*b^2)*
Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (10*(
5*a^4 + 42*a^2*b^2 + 21*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/(3*d) + (10*a^2*(5*a^2 + 39*b^2)*Sin[c + d*x])/(3*d*Sq
rt[Sec[c + d*x]]))/5)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4562
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Si
mp[1/(d*n)   Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*
b) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(190) = 380$.

Time = 11.44 (sec) , antiderivative size = 476, normalized size of antiderivative = 2.26

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(240a^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+\left(-360a^{4}-672b\,a^{3}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(280}{}$
parts	Expression too large to display

input
```
int((a+b*sec(d*x+c))^4/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a^4*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*a^4-672*a^3*b)*sin(1/2*d*x+1/2
*c)^6*cos(1/2*d*x+1/2*c)+(280*a^4+672*a^3*b+840*a^2*b^2)*sin(1/2*d*x+1/2*c
)^4*cos(1/2*d*x+1/2*c)+(-80*a^4-168*a^3*b-420*a^2*b^2)*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c)+25*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+210*a^2*b^2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+105*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-252*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*a^3*b-420*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx =$$

$$-\frac{5\sqrt{2}(5i\,a^4+42i\,a^2b^2+21i\,b^4)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}(-5i\,a^4}{}$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-1/105*(5*sqrt(2)*(5*I*a^4 + 42*I*a^2*b^2 + 21*I*b^4)*weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-5*I*a^4 - 42*I*a^2*b^2
 - 21*I*b^4)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 8
4*sqrt(2)*(-3*I*a^3*b - 5*I*a*b^3)*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 84*sqrt(2)*(3*I*a^3*b + 5*I*a
*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c))) - 2*(15*a^4*cos(d*x + c)^3 + 84*a^3*b*cos(d*x + c)^2 + 5*(5*
a^4 + 42*a^2*b^2)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(a+b\sec{(c+dx)})^4}{\sec^{\frac{7}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(7/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**4/sec(c + d*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(b\sec{(dx+c)}+a)^4}{\sec{(dx+c)}^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(b\sec{(dx+c)}+a)^4}{\sec{(dx+c)}^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(7/2),x)
```

output
```
int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^4}dx\right)a^4+4\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3}dx\right)a^3b+6\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx\right)a^2b^2+4\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)a\,b^3+\left(\int \sqrt{\sec(dx+c)}dx\right)b^4$$

input
```
int((a+b*sec(d*x+c))^4/sec(d*x+c)^(7/2),x)
```

output
```
int(sqrt(sec(c + d*x))/sec(c + d*x)**4,x)*a**4 + 4*int(sqrt(sec(c + d*x))/
sec(c + d*x)**3,x)*a**3*b + 6*int(sqrt(sec(c + d*x))/sec(c + d*x)**2,x)*a*
*2*b**2 + 4*int(sqrt(sec(c + d*x))/sec(c + d*x),x)*a*b**3 + int(sqrt(sec(c
 + d*x)),x)*b**4
```

3.606 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 245

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx &= \frac{2(7a^4+54a^2b^2+15b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d}\\
&+\frac{8ab(5a^2+7b^2)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d}\\
&+\frac{44a^3b\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)}+\frac{14a^2(a^2+7b^2)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}\\
&+\frac{8ab(5a^2+7b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}\end{aligned}$$

output
```
2/15*(7*a^4+54*a^2*b^2+15*b^4)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2))*sec(d*x+c)^(1/2)/d+8/21*a*b*(5*a^2+7*b^2)*cos(d*x+c)^(1/2)*Inv
erseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+44/63*a^3*b*sin(d*x
+c)/d/sec(d*x+c)^(5/2)+14/45*a^2*(a^2+7*b^2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)
+8/21*a*b*(5*a^2+7*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/9*a^2*(a+b*sec(d*x
+c))^2*sin(d*x+c)/d/sec(d*x+c)^(7/2)
```

Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.69

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{\sec(c+dx)}\left(168(7a^4+54a^2b^2+15b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+480ab(5a^2+7b^2)\sqrt{\cos(c+dx)}\right.}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(9/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(168*(7*a^4 + 54*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]]*
EllipticE[(c + d*x)/2, 2] + 480*a*b*(5*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2] + a*(7*a*(43*a^2 + 216*b^2)*Cos[c + d*x] + 5*(312*a
^2*b + 336*b^3 + 72*a^2*b*Cos[2*(c + d*x)] + 7*a^3*Cos[3*(c + d*x)]))*Sin[
2*(c + d*x)]))/(1260*d)
```

Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.826$, Rules used = {3042, 4328, 27, 3042, 4562, 27, 3042, 4535, 3042, 4256, 3042, 4258, 3042, 3120, 4533, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{4328}$$

$$\frac{2}{9}\int \frac{(a+b\sec(c+dx))\left(22ba^2+\left(7a^2+27b^2\right)\sec(c+dx)a+3b\left(a^2+3b^2\right)\sec^2(c+dx)\right)}{2\sec^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\int \frac{(a+b\sec(c+dx))\left(22ba^2+\left(7a^2+27b^2\right)\sec(c+dx)a+3b\left(a^2+3b^2\right)\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(22ba^2+\left(7a^2+27b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+3b\left(a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4562}$$

$$\frac{1}{9}\left(\frac{44a^3b\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}-\frac{2}{7}\int -\frac{49\left(a^2+7b^2\right)a^2+36b\left(5a^2+7b^2\right)\sec(c+dx)a+21b^2\left(a^2+3b^2\right)\sec^2(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\left(\frac{1}{7}\int \frac{49\left(a^2+7b^2\right)a^2+36b\left(5a^2+7b^2\right)\sec(c+dx)a+21b^2\left(a^2+3b^2\right)\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{44a^3b\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{1}{7}\int \frac{49\left(a^2+7b^2\right)a^2+36b\left(5a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+21b^2\left(a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{44a^3b\sin(}{7d\sec^{\frac{5}{2}}(c}\right.$$

$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4535}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(36ab(5a^2+7b^2)\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\int\frac{49(a^2+7b^2)a^2+21b^2(a^2+3b^2)\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx\right)+\frac{44a^3b\sin(}{7d\sec^{\frac{5}{2}}(c}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\left(36ab(5a^2+7b^2)\int\frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\int\frac{49(a^2+7b^2)a^2+21b^2(a^2+3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx\right)\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 4256

$$\frac{1}{9}\left(\frac{1}{7}\left(\int\frac{49(a^2+7b^2)a^2+21b^2(a^2+3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+36ab(5a^2+7b^2)\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2}{3d}\right.\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\left(\int\frac{49(a^2+7b^2)a^2+21b^2(a^2+3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+36ab(5a^2+7b^2)\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx\right.\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 4258

$$\frac{1}{9}\left(\frac{1}{7}\left(\int\frac{49(a^2+7b^2)a^2+21b^2(a^2+3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+36ab(5a^2+7b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\left(\int\frac{49(a^2+7b^2)a^2+21b^2(a^2+3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+36ab(5a^2+7b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3120

$$\frac{1}{9}\left(\frac{1}{7}\left(\int \frac{49\left(a^2+7b^2\right)a^2+21b^2\left(a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+36ab\left(5a^2+7b^2\right)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos}}{}\right.\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 4533

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{21}{5}\left(7a^4+54a^2b^2+15b^4\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{98a^2\left(a^2+7b^2\right)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+36ab\left(5a^2+7b^2\right)\left(\frac{2\sin(c}{3d\sqrt{\sec(}}\right.\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{21}{5}\left(7a^4+54a^2b^2+15b^4\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{98a^2\left(a^2+7b^2\right)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+36ab\left(5a^2+7b^2\right)\left(\frac{2}{3d}\right.\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 4258

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{21}{5}\left(7a^4+54a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{98a^2\left(a^2+7b^2\right)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{21}{5}\left(7a^4+54a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{98a^2\left(a^2+7b^2\right)\sin(c+}{5d\sec^{\frac{3}{2}}(c+dx)}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3119

$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}+$$
$$\frac{1}{9}\left(\frac{44a^3b\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{1}{7}\left(\frac{98a^2(a^2+7b^2)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+36ab(5a^2+7b^2)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{}}{}\right.\right.\right.$$

input
```
Int[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(9/2),x]
```

output
```
(2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + ((4
4*a^3*b*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + ((42*(7*a^4 + 54*a^2*b^2
+ 15*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(5*d) + (98*a^2*(a^2 + 7*b^2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + 36
*a*b*(5*a^2 + 7*b^2)*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/7)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
 + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4562
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Si
mp[1/(d*n)   Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*
b) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(220) = 440$.

Time = 17.10 (sec) , antiderivative size = 529, normalized size of antiderivative = 2.16

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(-1120a^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+\left(2240a^{4}+2880b\,a^{3}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}{}$
parts	Expression too large to display

input

```
int((a+b*sec(d*x+c))^4/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output

```
-2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*a^4*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*a^4+2880*a^3*b)*sin(1/2*d*x
+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-2072*a^4-4320*a^3*b-3024*a^2*b^2)*sin(1/2*d
*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(952*a^4+3360*a^3*b+3024*a^2*b^2+1680*a*b^3
)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-168*a^4-960*a^3*b-756*a^2*b^2-
840*a*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+300*b*a^3*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+420*a*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*a^4*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))-1134*a^2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-315*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*b^4)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.05

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx =$$

$$-\frac{60\sqrt{2}(5i\,a^3b+7i\,ab^3)\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+60\sqrt{2}(-5i\,a^3b-7i\,a}{}$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
-1/315*(60*sqrt(2)*(5*I*a^3*b + 7*I*a*b^3)*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + 60*sqrt(2)*(-5*I*a^3*b - 7*I*a*b^3)*weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*sqrt(2)*(-7*I*a^4 -
54*I*a^2*b^2 - 15*I*b^4)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c))) + 21*sqrt(2)*(7*I*a^4 + 54*I*a^2*b^2 + 15
*I*b^4)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c))) - 2*(35*a^4*cos(d*x + c)^4 + 180*a^3*b*cos(d*x + c)^3 + 7*
(7*a^4 + 54*a^2*b^2)*cos(d*x + c)^2 + 60*(5*a^3*b + 7*a*b^3)*cos(d*x + c))
*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(9/2),x)
```

output
```
int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(9/2), x)
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^5}dx\right)a^4 + 4\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^4}dx\right)a^3b + 6\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3}dx\right)a^2b^2 + 4\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx\right)a\,b^3 + \left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)b^4$$

input
```
int((a+b*sec(d*x+c))^4/sec(d*x+c)^(9/2),x)
```

output
```
int(sqrt(sec(c + d*x))/sec(c + d*x)**5,x)*a**4 + 4*int(sqrt(sec(c + d*x))/
sec(c + d*x)**4,x)*a**3*b + 6*int(sqrt(sec(c + d*x))/sec(c + d*x)**3,x)*a*
*2*b**2 + 4*int(sqrt(sec(c + d*x))/sec(c + d*x)**2,x)*a*b**3 + int(sqrt(se
c(c + d*x))/sec(c + d*x),x)*b**4
```

3.607 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 289

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx &= \frac{8ab(7a^2+9b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} \\ &+ \frac{2(45a^4+330a^2b^2+77b^4)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{231d} \\ &+ \frac{52a^3b\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2a^2(9a^2+59b^2)\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)} + \frac{8ab(7a^2+9b^2)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} \\ &+ \frac{2(45a^4+330a^2b^2+77b^4)\sin(c+dx)}{231d\sqrt{\sec(c+dx)}} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)}\end{aligned}$$

output
```
8/15*a*b*(7*a^2+9*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/
2))*sec(d*x+c)^(1/2)/d+2/231*(45*a^4+330*a^2*b^2+77*b^4)*cos(d*x+c)^(1/2)*
InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+52/99*a^3*b*sin(
d*x+c)/d/sec(d*x+c)^(7/2)+2/77*a^2*(9*a^2+59*b^2)*sin(d*x+c)/d/sec(d*x+c)^
(5/2)+8/45*a*b*(7*a^2+9*b^2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/231*(45*a^4+3
30*a^2*b^2+77*b^4)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/11*a^2*(a+b*sec(d*x+c))
^2*sin(d*x+c)/d/sec(d*x+c)^(9/2)
```

Mathematica [A] (verified)

Time = 2.28 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.69

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{\sec(c+dx)}\Big(14784ab(7a^2+9b^2)\sqrt{\cos(c+dx)}E\big(\frac{1}{2}(c+dx)\big|2\big)+240(45a^4+330a^2b^2+77b^4)\sqrt{\cos(c}}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(11/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(14784*a*b*(7*a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2] + 240*(45*a^4 + 330*a^2*b^2 + 77*b^4)*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2] + (616*a*b*(43*a^2 + 36*b^2)*Cos[c + d*x] + 5*(
1593*a^4 + 10296*a^2*b^2 + 1848*b^4 + 72*(8*a^4 + 33*a^2*b^2)*Cos[2*(c + d
*x)] + 616*a^3*b*Cos[3*(c + d*x)] + 63*a^4*Cos[4*(c + d*x)]))*Sin[2*(c + d
*x)]))/(27720*d)
```

Rubi [A] (verified)

Time = 1.90 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.95, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.913$, Rules used = {3042, 4328, 27, 3042, 4562, 27, 3042, 4535, 3042, 4256, 3042, 4258, 3042, 3119, 4533, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^{11/2}}dx$$

$$\Big\downarrow \; 4328$$

$$\frac{2}{11}\int \frac{(a+b\sec(c+dx))\left(26ba^2+3\left(3a^2+11b^2\right)\sec(c+dx)a+b\left(5a^2+11b^2\right)\sec^2(c+dx)\right)}{2\sec^{\frac{9}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{11}\int \frac{(a+b\sec(c+dx))\left(26ba^2+3\left(3a^2+11b^2\right)\sec(c+dx)a+b\left(5a^2+11b^2\right)\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{11}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(26ba^2+3\left(3a^2+11b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(5a^2+11b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 4562

$$\frac{1}{11}\left(\frac{52a^3b\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}-\frac{2}{9}\int -\frac{9\left(9a^2+59b^2\right)a^2+44b\left(7a^2+9b^2\right)\sec(c+dx)a+9b^2\left(5a^2+11b^2\right)\sec^2(c+dx)}{2\sec^{\frac{7}{2}}(c+dx)}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{11}\left(\frac{1}{9}\int \frac{9\left(9a^2+59b^2\right)a^2+44b\left(7a^2+9b^2\right)\sec(c+dx)a+9b^2\left(5a^2+11b^2\right)\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}dx+\frac{52a^3b\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{11}\left(\frac{1}{9}\int \frac{9\left(9a^2+59b^2\right)a^2+44b\left(7a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+9b^2\left(5a^2+11b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{52a^3b\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{4535}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(44ab(7a^2+9b^2)\int\frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx+\int\frac{9(9a^2+59b^2)a^2+9b^2(5a^2+11b^2)\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}dx\right)+\frac{52a^3b\,\mathrm{s}}{9d\,\mathrm{sec}}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(44ab(7a^2+9b^2)\int\frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\int\frac{9(9a^2+59b^2)a^2+9b^2(5a^2+11b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{4256}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\int\frac{9(9a^2+59b^2)a^2+9b^2(5a^2+11b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+44ab(7a^2+9b^2)\left(\frac{3}{5}\int\frac{1}{\sqrt{\sec(c+dx)}}dx+\right.\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\int\frac{9(9a^2+59b^2)a^2+9b^2(5a^2+11b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+44ab(7a^2+9b^2)\left(\frac{3}{5}\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right.}}\right.\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\int\frac{9(9a^2+59b^2)a^2+9b^2(5a^2+11b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+44ab(7a^2+9b^2)\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c}\right.\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\int \frac{9\left(9a^2+59b^2\right)a^2+9b^2\left(5a^2+11b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+44ab\left(7a^2+9b^2\right)\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\int \frac{9\left(9a^2+59b^2\right)a^2+9b^2\left(5a^2+11b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+44ab\left(7a^2+9b^2\right)\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{}}{}\right.\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{4533}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{9}{7}\left(45a^4+330a^2b^2+77b^4\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{18a^2\left(9a^2+59b^2\right)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+44ab\left(7a^2+9b^2\right)\left(\frac{2\,\mathrm{si}}{5d\,\mathrm{se}}\right.\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{9}{7}\left(45a^4+330a^2b^2+77b^4\right)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{18a^2\left(9a^2+59b^2\right)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+44ab\left(7a^2+9b^2\right)\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{4256}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{9}{7}\left(45a^4+330a^2b^2+77b^4\right)\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{18a^2\left(9a^2+59b^2\right)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{9}{7}\left(45a^4+330a^2b^2+77b^4\right)\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{18a^2\left(9a^2+59b^2\right)\sin(c}{7d\sec^{\frac{5}{2}}(c+dx)}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{9}{7}(45a^4+330a^2b^2+77b^4)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{18}{}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{9}{7}(45a^4+330a^2b^2+77b^4)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}+$$
$$\frac{1}{11}\left(\frac{52a^3b\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{1}{9}\left(\frac{18a^2(9a^2+59b^2)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+44ab(7a^2+9b^2)\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}}{}\right.\right.\right.$$

input
```
Int[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(11/2),x]
```

output
```
(2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9/2)) + ((
52*a^3*b*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + ((18*a^2*(9*a^2 + 59*b^2
)*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + 44*a*b*(7*a^2 + 9*b^2)*((6*Sqrt
[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*Si
n[c + d*x])/(5*d*Sec[c + d*x]^(3/2))) + (9*(45*a^4 + 330*a^2*b^2 + 77*b^4)
*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d
) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/7)/9)/11
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4562
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Si
mp[1/(d*n)   Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*
b) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(260) = 520$.

Time = 20.93 (sec) , antiderivative size = 586, normalized size of antiderivative = 2.03

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(20160a^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}+\left(-50400a^{4}-49280b\,a^{3}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}\cos\left(\frac{dx}{2}+\right.\right.}{}$
parts	Expression too large to display

input
```
int((a+b*sec(d*x+c))^4/sec(d*x+c)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*a^4
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-50400*a^4-49280*a^3*b)*sin(1/2
*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(56880*a^4+98560*a^3*b+47520*a^2*b^2)*si
n(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-34920*a^4-91168*a^3*b-71280*a^2*b^
2-22176*a*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(13860*a^4+41888*a^
3*b+55440*a^2*b^2+22176*a*b^3+4620*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+(-2790*a^4-7392*a^3*b-15840*a^2*b^2-5544*a*b^3-2310*b^4)*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)+675*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4950*a^2*b^
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))+1155*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6468*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*a^3*b-8316*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.99

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx =$$

$$-\frac{15\sqrt{2}(45i\,a^4+330i\,a^2b^2+77i\,b^4)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+15\sqrt{2}(-$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="fricas")
```

output
```
-1/3465*(15*sqrt(2)*(45*I*a^4 + 330*I*a^2*b^2 + 77*I*b^4)*weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 15*sqrt(2)*(-45*I*a^4 - 330*I*
a^2*b^2 - 77*I*b^4)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
c)) + 924*sqrt(2)*(-7*I*a^3*b - 9*I*a*b^3)*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 924*sqrt(2)*(7*I*a^3*
b + 9*I*a*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c))) - 2*(315*a^4*cos(d*x + c)^5 + 1540*a^3*b*cos(d*x +
c)^4 + 135*(3*a^4 + 22*a^2*b^2)*cos(d*x + c)^3 + 308*(7*a^3*b + 9*a*b^3)*c
os(d*x + c)^2 + 15*(45*a^4 + 330*a^2*b^2 + 77*b^4)*cos(d*x + c))*sin(d*x +
 c)/sqrt(cos(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(11/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{11/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(11/2),x)
```

output
```
int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(11/2), x)
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx=\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^6}dx\right)a^4+4\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^5}dx\right)a^3b+6\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^4}dx\right)a^2b^2+4\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3}dx\right)a\,b^3+\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx\right)b^4$$

input
```
int((a+b*sec(d*x+c))^4/sec(d*x+c)^(11/2),x)
```

output

```
int(sqrt(sec(c + d*x))/sec(c + d*x)**6,x)*a**4 + 4*int(sqrt(sec(c + d*x))/
sec(c + d*x)**5,x)*a**3*b + 6*int(sqrt(sec(c + d*x))/sec(c + d*x)**4,x)*a*
*2*b**2 + 4*int(sqrt(sec(c + d*x))/sec(c + d*x)**3,x)*a*b**3 + int(sqrt(se
c(c + d*x))/sec(c + d*x)**2,x)*b**4
```

3.608 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 188

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2d} + \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3bd} + \frac{2a^2\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{b^2(a+b)d} - \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2d} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3bd}$$

output
```
2*a*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2
)/b^2/d+2/3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*
x+c)^(1/2)/b/d+2*a^2*cos(d*x+c)^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a
+b),2^(1/2))*sec(d*x+c)^(1/2)/b^2/(a+b)/d-2*a*sec(d*x+c)^(1/2)*sin(d*x+c)/
b^2/d+2/3*sec(d*x+c)^(3/2)*sin(d*x+c)/b/d
```

Mathematica [A] (warning: unable to verify)

Time = 35.37 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.88

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx =$$
$$-\frac{\cot(c+dx)\left(-b^2\sec^{\frac{5}{2}}(c+dx)+b^2\cos(2(c+dx))\sec^{\frac{5}{2}}(c+dx)+6abE\left(\arcsin\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x]),x]
```

output
```
-1/3*(Cot[c + d*x]*(-(b^2*Sec[c + d*x]^(5/2)) + b^2*Cos[2*(c + d*x)]*Sec[c
 + d*x]^(5/2) + 6*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[
c + d*x]^2] - 2*(3*a^2 + 3*a*b + b^2)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]]
, -1]*Sqrt[-Tan[c + d*x]^2] + 6*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c +
 d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(b^3*d)
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.07, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4338, 27, 3042, 4590, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4338}$$

$$\frac{2\int \frac{\sqrt{\sec(c+dx)}\left(-3a\sec^2(c+dx)+b\sec(c+dx)+a\right)}{2(a+b\sec(c+dx))}dx}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{\sqrt{\sec(c+dx)}\left(-3a\sec^2(c+dx)+b\sec(c+dx)+a\right)}{a+b\sec(c+dx)}dx}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(-3a\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$\Big\downarrow$ 4590

$$\frac{\frac{2\int \frac{3a^2+4b\sec(c+dx)a+\left(3a^2+b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{3a^2+4b\sec(c+dx)a+\left(3a^2+b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{3a^2+4b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(3a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$\Big\downarrow$ 4594

$$\frac{\frac{3a^2\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int \frac{3a^3+b\sec(c+dx)a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{3a^3+b\csc\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$\Big\downarrow$ 4274

$$\frac{\frac{3a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^3\int \frac{1}{\sqrt{\sec(c+dx)}}dx+a^2b\int \sqrt{\sec(c+dx)}dx}{a^2}}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^3\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a^2b\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$$\Big\downarrow \; 4258$$

$$\frac{\frac{3a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{a^2}}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{3a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$$\Big\downarrow \; 3120$$

$$\frac{\frac{3a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+$$

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$\Big\downarrow$ 4336

$$\frac{\frac{3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx+\frac{\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+$$

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+$$

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$\Big\downarrow$ 3284

$$\frac{\frac{\frac{6a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+$$

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

input
```
Int[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x]),x]
```

output
```
(2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*b*d) + ((((6*a^3*Sqrt[Cos[c + d*x]]
*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a^2*b*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (6*a^2*Sqrt[Co
s[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
((a + b)*d))/b - (6*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d))/(3*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4338
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 3)/(b*f*
(n - 2))), x] + Simp[d^3/(b*(n - 2))   Int[(d*Csc[e + f*x])^(n - 3)*(Simp[a
*(n - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x]/(a + b*Csc
[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && Gt
Q[n, 3]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(173) = 346$.

Time = 10.10 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.25

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(\frac{-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{3\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{1}{2}\right)^{2}}+\frac{2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \mathrm{E}}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin(}}}{b}\right.}{}$

input
```
int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b*(-1/6*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2*a^3/b^2/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-2*a
/b^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2
-(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{7}{2}}}{b\sec\left(dx+c\right)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{7}{2}}}{b\sec\left(dx+c\right)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^3}{\sec(dx+c)\,b+a}dx$$

input
```
int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x)
```

output
```
int((sqrt(sec(c + d*x))*sec(c + d*x)**3)/(sec(c + d*x)*b + a),x)
```

3.609 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{bd} - \frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{b(a+b)d} + \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{bd}$$

output

```
-2*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)
/b/d-2*a*cos(d*x+c)^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))
*sec(d*x+c)^(1/2)/b/(a+b)/d+2*sec(d*x+c)^(1/2)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 15.61 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.71

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{2\cot(c+dx)\left(bE\left(\arcsin\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-(a+b)\,\text{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)+a}{b^2d}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x]),x]
```

output
```
(2*Cot[c + d*x]*(b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1] - (a + b)*Ell
ipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + a*EllipticPi[-(b/a), ArcSin[Sqrt[
Sec[c + d*x]]], -1])*Sqrt[-Tan[c + d*x]^2])/(b^2*d)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4337, 3042, 4255, 3042, 4258, 3042, 3119, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4337}$$

$$\frac{\int \sec^{\frac{3}{2}}(c+dx)dx}{b}-\frac{a\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx}{b}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \; 4255$$

$$\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{b}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \; 4258$$

$$\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{b}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \; 4336$$

$$\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{b}-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{b}-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}$$

$$\Big\downarrow \; 3284$$

$$\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{b}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{bd(a+b)}$$

input
```
Int[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x]),x]
```

output
```
(-2*a*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(b*(a + b)*d) + ((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d)/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4337
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(5/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d/b   Int[(d*Csc[e + f*x])^(3/2), x], x] - Simp[a*
(d/b)   Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(111) = 222$.

Time = 8.86 (sec) , antiderivative size = 353, normalized size of antiderivative = 3.02

method	result
default	$-\frac{2\left(-2\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,(a-b)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\cdots}\right.}{\cdots}$

input
```
int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(a-b)*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b+a*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b
),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/b/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(a-b)/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**(5/2)/(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec(dx+c)^{\frac{5}{2}}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{b\sec\left(dx+c\right)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}\,\sec\left(dx+c\right)^{2}}{\sec\left(dx+c\right)b+a}dx$$

input
```
int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x)
```

output
```
int((sqrt(sec(c + d*x))*sec(c + d*x)**2)/(sec(c + d*x)*b + a),x)
```

3.610 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 49

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx=\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{(a+b)d}$$

output
```
2*cos(d*x+c)^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*sec(d*
x+c)^(1/2)/(a+b)/d
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.29

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx =\frac{2\cot(c+dx)\left(\text{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)-\text{EllipticPi}\left(-\frac{b}{a},\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)\right)}{bd}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x]),x]
```

output
```
(2*Cot[c + d*x]*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(
b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sqrt[-Tan[c + d*x]^2])/(b*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4336}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3284}$$

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}$$

input
```
Int[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x]),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/((a + b)*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(47) = 94$.

Time = 0.84 (sec) , antiderivative size = 150, normalized size of antiderivative = 3.06

method	result	size
default	$\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\operatorname{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\frac{2a}{a-b},\sqrt{2}\right)}{(a-b)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$	150

input
```
int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),2*a/(a-b),2^(1/2))/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**(3/2)/(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{b\sec\left(dx+c\right)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}\,\sec\left(dx+c\right)}{\sec\left(dx+c\right)b+a}dx$$

input
```
int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x)
```

output
```
int((sqrt(sec(c + d*x))*sec(c + d*x))/(sec(c + d*x)*b + a),x)
```

3.611 $\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 93

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{2b\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a(a+b)d}$$

output
```
2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)
/a/d-2*b*cos(d*x+c)^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))
*sec(d*x+c)^(1/2)/a/(a+b)/d
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \frac{2\cot(c+dx)\,\text{EllipticPi}\left(-\frac{b}{a},\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)\sqrt{-\tan^2(c+dx)}}{ad}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

output
```
(2*Cot[c + d*x]*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-T
an[c + d*x]^2])/(a*d)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.80, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 4335, 3042, 3282, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4335}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)}}{b+a\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3282}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{a}-\frac{b\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{b\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}\right)$$

$$\Big\downarrow \text{3120}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{b\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}\right)$$

$$\Big\downarrow \text{3284}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{2b\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{ad(a+b)}\right)$$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

output
```
Sqrt[Cos[c + d*x]]*((2*EllipticF[(c + d*x)/2, 2])/(a*d) - (2*b*EllipticPi[
(2*a)/(a + b), (c + d*x)/2, 2])/(a*(a + b)*d))*Sqrt[Sec[c + d*x]]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3282
```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int[1/Sqrt[c + d*Sin[e + f*x]], x], x
] + Simp[(b*c - a*d)/b   Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4335
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[Sqrt[d*Sin[e + f*x]]*(Sqrt[d*Csc[e + f*x]]/d)   Int[
Sqrt[d*Sin[e + f*x]]/(b + a*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f},
 x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(88) = 176$.

Time = 1.83 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.01

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\left(\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a-b\,\text{EllipticF}\left(\cos\left(\right.\right.\right.}{a(a-b)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$

input
```
int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*a-b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+b*EllipticPi(cos(1/
2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/a/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sqrt(sec(c + d*x))/(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)\,b+a}dx$$

input
```
int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x)
```

output
```
int(sqrt(sec(c + d*x))/(sec(c + d*x)*b + a),x)
```

3.612 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx = \frac{2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{2b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a^2(a+b)d}$$

output
```
2*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)/
a/d-2*b*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)
^(1/2)/a^2/d+2*b^2*cos(d*x+c)^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b
),2^(1/2))*sec(d*x+c)^(1/2)/a^2/(a+b)/d
```

Mathematica [A] (warning: unable to verify)

Time = 19.73 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx = \frac{\cot(c+dx)\left(-a\sec^{\frac{3}{2}}(c+dx) - a\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx) + a\sec^{\frac{7}{2}}(c+dx) + a\cos(2(c+dx))\sec^{\frac{7}{2}}(\right.}{}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])),x]
```

output
```
(Cot[c + d*x]*(-(a*Sec[c + d*x]^(3/2)) - a*Cos[2*(c + d*x)]*Sec[c + d*x]^(
3/2) + a*Sec[c + d*x]^(7/2) + a*Cos[2*(c + d*x)]*Sec[c + d*x]^(7/2) - 2*a*
EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2*a*Elli
pticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 2*b*Elliptic
Pi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(a^2*d)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4339, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4339}$$

$$\frac{b^2\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}+\frac{\int \frac{a-b\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\int \frac{a-b\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}$$

$$\Big\downarrow \text{4274}$$

$$\frac{b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\int \frac{1}{\sqrt{\sec(c+dx)}}dx-b\int \sqrt{\sec(c+dx)}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}$$

$$\Big\downarrow 4336$$

$$\frac{b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx}{a^2}+\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}$$

$$\Big\downarrow 3042$$

$$\frac{b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2}+\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}$$

$$\Big\downarrow 3284$$

$$\frac{2b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a^2d(a+b)}+\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])),x]
```

output
```
((2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d -
 (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/
a^2 + (2*b^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4339
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_))), x_Symbol] :> Simp[b^2/(a^2*d^2)   Int[(d*Csc[e + f*x])^(3/2)/(a +
b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a - b*Csc[e + f*x])/Sqrt[d*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 3.56 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.67

method	result
default	$\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\left(\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)ab-\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}\right.\right.\right.}{a^{2}(a-b)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin}$

input

```
int(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output

```
2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*a*b-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*a^2-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+b^2*El
lipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/a^2/(a-b)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx=\text{Timed out}$$

input

```
integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output

```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx = \int \frac{1}{(a+b\sec(c+dx))\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))*sqrt(sec(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx=\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)}}{\sec\left(dx+c\right)^{2}b+\sec\left(dx+c\right)a}dx$$

input
```
int(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x)
```

output
```
int(sqrt(sec(c + d*x))/(sec(c + d*x)**2*b + sec(c + d*x)*a),x)
```

3.613 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 172

$$\begin{aligned}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx &= -\frac{2b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} \\ &\quad+\frac{2(a^2+3b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^3d} \\ &\quad-\frac{2b^3\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a^3(a+b)d}+\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
-2*b*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/
2)/a^2/d+2/3*(a^2+3*b^2)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^
(1/2))*sec(d*x+c)^(1/2)/a^3/d-2*b^3*cos(d*x+c)^(1/2)*EllipticPi(sin(1/2*d*
x+1/2*c),2*a/(a+b),2^(1/2))*sec(d*x+c)^(1/2)/a^3/(a+b)/d+2/3*sin(d*x+c)/a/
d/sec(d*x+c)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 21.60 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.13

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = -\frac{\cot(c+dx)\left(-a^2\sqrt{\sec(c+dx)}+6ab\sec^{\frac{3}{2}}(c+dx)-6ab\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx)+a^2\cos(3(c+dx))\right.}{}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]
```

output
```
-1/6*(Cot[c + d*x]*(-(a^2*Sqrt[Sec[c + d*x]]) + 6*a*b*Sec[c + d*x]^(3/2) -
 6*a*b*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) + a^2*Cos[3*(c + d*x)]*Sec[c +
d*x]^(3/2) - 12*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c
+ d*x]^2] - 4*a*(a - 3*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-
Tan[c + d*x]^2] - 12*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1
]*Sqrt[-Tan[c + d*x]^2]))/(a^3*d)
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4340, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{ 4340}$$

$$\frac{2\int -\frac{-b\sec^2(c+dx)-a\sec(c+dx)+3b}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{3a}+\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{-b\sec^2(c+dx)-a\sec(c+dx)+3b}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{-b\csc\left(c+dx+\frac{\pi}{2}\right)^2-a\csc\left(c+dx+\frac{\pi}{2}\right)+3b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3a}$$

$$\Big\downarrow \text{4594}$$

$$\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}+\frac{\int \frac{3ab-\left(a^2+3b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\int \frac{3ab+\left(-a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{3a}$$

$$\Big\downarrow \text{4274}$$

$$\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\int \frac{1}{\sqrt{\sec(c+dx)}}dx-(a^2+3b^2)\int \sqrt{\sec(c+dx)}dx}{a^2}}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-(a^2+3b^2)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{3a}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-(a^2+3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{a^2}}{3a}$$

$\Big\downarrow$ 3042

$$\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{3a}$$

$\Big\downarrow$ 3119

$$\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{6ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{3a}$$

$\Big\downarrow$ 3120

$$\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{6ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}}{3a}$$

$\Big\downarrow$ 4336

$$\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx}{a^2}+\frac{\frac{6ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{Elliptic}}{d}}{a^2}}{3a}$$

$\Big\downarrow$ 3042

$$\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2}+\frac{\frac{6ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{d}}{a^2}}{3a}$$

$\Big\downarrow$ 3284

$$\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{6b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a^2d(a+b)}+\frac{\frac{6ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}(}{d}}{a^2}}{3a}$$

input
```
Int[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]
```

output
```
-1/3*(((6*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d - (2*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/d)/a^2 + (6*b^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a +
b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d))/a + (2*Sin[c + d*
x])/(3*a*d*Sqrt[Sec[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4340
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Sim
p[1/(a*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n
- a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(159) = 318$.

Time = 3.86 (sec) , antiderivative size = 552, normalized size of antiderivative = 3.21

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(4a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-4a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)b-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right.\right.}{}$

input
```
int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a^3*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-4*a^2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)*b-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^3+2*a^2*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c)*b+a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*a^2*b+3*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*b^3+3*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b^
2+3*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/a^3/(a-b)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{(a+b\sec(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))*sec(c + d*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)}}{\sec\left(dx+c\right)^{3}b+\sec\left(dx+c\right)^{2}a}dx$$

input
```
int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x)
```

output
```
int(sqrt(sec(c + d*x))/(sec(c + d*x)**3*b + sec(c + d*x)**2*a),x)
```

3.614 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 342

$$\begin{aligned}
\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx
&= \frac{a(5a^2-4b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^3(a^2-b^2)\,d} \\
&\quad + \frac{(5a^2-2b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3b^2(a^2-b^2)\,d} \\
&\quad + \frac{a^2(5a^2-7b^2)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{(a-b)b^3(a+b)^2d} \\
&\quad - \frac{a(5a^2-4b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^3(a^2-b^2)\,d} \\
&\quad + \frac{(5a^2-2b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2(a^2-b^2)\,d} - \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{b(a^2-b^2)\,d(a+b\sec(c+dx))}
\end{aligned}$$

output
```
a*(5*a^2-4*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec
(d*x+c)^(1/2)/b^3/(a^2-b^2)/d+1/3*(5*a^2-2*b^2)*cos(d*x+c)^(1/2)*InverseJa
cobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d+a^2*(5*a^2-
7*b^2)*cos(d*x+c)^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*s
ec(d*x+c)^(1/2)/(a-b)/b^3/(a+b)^2/d-a*(5*a^2-4*b^2)*sec(d*x+c)^(1/2)*sin(d
*x+c)/b^3/(a^2-b^2)/d+1/3*(5*a^2-2*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/b^2/(a
^2-b^2)/d-a^2*sec(d*x+c)^(5/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 5.09 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.86

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{2b\left(-\frac{3a^2\left(5a^2-4b^2\right)\sin(c+dx)}{a^2-b^2}+2b(-5a+b\sec(c+dx))\tan(c+dx)\right)}{(b+a\cos(c+dx))\sqrt{\sec(c+dx)}}+\frac{\cot(c+dx)\left(-6ab(5a^2-4b^2)E\left(\arcsin\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+d}\right.}{}}{}$$

input
```
Integrate[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^2,x]
```

output
```
((2*b*((-3*a^2*(5*a^2 - 4*b^2)*Sin[c + d*x])/(a^2 - b^2) + 2*b*(-5*a + b*S
ec[c + d*x])*Tan[c + d*x]))/((b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]) + (C
ot[c + d*x]*(-6*a*b*(5*a^2 - 4*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]],
-1]*Sqrt[-Tan[c + d*x]^2] + 2*(15*a^4 + 15*a^3*b - 16*a^2*b^2 - 12*a*b^3 -
 2*b^4)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] -
6*a*(b*(-5*a^2 + 4*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x]^2 + a*(5*a^2 - 7*b
^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^
2])))/((a - b)*(a + b)))/(6*b^4*d)
```

Rubi [A] (verified)

Time = 2.50 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.95, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.913, Rules used = {3042, 4332, 27, 3042, 4590, 27, 3042, 4590, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4332}$$

$$-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(3a^2-2b\sec(c+dx)a-\left(5a^2-2b^2\right)\sec^2(c+dx)\right)}{2(a+b\sec(c+dx))}dx}{b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(3a^2-2b\sec(c+dx)a-\left(5a^2-2b^2\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3a^2-2b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(2b^2-5a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{4590}$$

$$-\frac{\frac{2\int -\frac{\sqrt{\sec(c+dx)}\left(-3a\left(5a^2-4b^2\right)\sec^2(c+dx)-2b\left(2a^2+b^2\right)\sec(c+dx)+a\left(5a^2-2b^2\right)\right)}{2(a+b\sec(c+dx))}dx}{3b}-\frac{2\left(5a^2-2b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(-3a\left(5a^2-4b^2\right)\sec^2(c+dx)-2b\left(2a^2+b^2\right)\sec(c+dx)+a\left(5a^2-2b^2\right)\right)}{a+b\sec(c+dx)}dx}{3b}-\frac{2\left(5a^2-2b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$\Big\downarrow$ 3042

$$-\frac{-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(-3a\left(5a^2-4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-2b\left(2a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+a\left(5a^2-2b^2\right)\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3b}-\frac{2\left(5a^2-2b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$\Big\downarrow$ 4590

$$-\frac{-\frac{\frac{2\int \frac{3\left(5a^2-4b^2\right)a^2+2b\left(10a^2-7b^2\right)\sec(c+dx)a+\left(15a^4-16b^2a^2-2b^4\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}\left(a+b\sec(c+dx)\right)}dx}{b}-\frac{6a\left(5a^2-4b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}-\frac{2\left(5a^2-2b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{\frac{\int \frac{3\left(5a^2-4b^2\right)a^2+2b\left(10a^2-7b^2\right)\sec(c+dx)a+\left(15a^4-16b^2a^2-2b^4\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\left(a+b\sec(c+dx)\right)}dx}{b}-\frac{6a\left(5a^2-4b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}-\frac{2\left(5a^2-2b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$\Big\downarrow$ 3042

$$-\frac{-\frac{\frac{\int \frac{3\left(5a^2-4b^2\right)a^2+2b\left(10a^2-7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(15a^4-16b^2a^2-2b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{6a\left(5a^2-4b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}-\frac{2\left(5a^2-2b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \; 4594$$

$$-\frac{-\frac{\frac{3a^2(5a^2-7b^2)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int \frac{3(5a^2-4b^2)a^3+b(5a^2-2b^2)\sec(c+dx)a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{b}-\frac{6a(5a^2-4b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}-\frac{2(5a^2-2b^2)\sin(c+dx)s}{3bd}}{2b(a^2-b^2)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{\frac{3a^2(5a^2-7b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{3(5a^2-4b^2)a^3+b(5a^2-2b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{b}-\frac{6a(5a^2-4b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}-\frac{2(5a^2-2b^2)\sin}{}}{2b(a^2-b^2)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 4274$$

$$-\frac{-\frac{\frac{3a^2(5a^2-7b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b(5a^2-2b^2)\int \sqrt{\sec(c+dx)}dx+3a^3(5a^2-4b^2)\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{a^2}}{b}-\frac{6a(5a^2-4b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}-\frac{2(}{}}{2b(a^2-b^2)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{\frac{3a^2(5a^2-7b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b(5a^2-2b^2)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+3a^3(5a^2-4b^2)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{b}-\frac{6a(5a^2-4b^2)\sin(c+dx)\sqrt{\sec(c+d}}{bd}}{3b}}{2b(a^2-b^2)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 4258$$

$$-\frac{-\frac{3a^2(5a^2-7b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b(5a^2-2b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+3a^3(5a^2-4b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}d}{a^2}}{b}}{3b}}{2b(a^2-b^2)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$\downarrow$ 3042

$$-\frac{-\frac{3a^2(5a^2-7b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b(5a^2-2b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^3(5a^2-4b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin(c+}}{a^2}}{b}}{3b}}{2b(a^2-b^2)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$\downarrow$ 3119

$$-\frac{-\frac{3a^2(5a^2-7b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b(5a^2-2b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^3(5a^2-4b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+d\right.}{d}}{a^2}}{b}}{3b}}{2b(a^2-b^2)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$\downarrow$ 3120

$$-\frac{-\frac{3a^2(5a^2-7b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{2a^2b(5a^2-2b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a^3(5a^2-4b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{d}}{a^2}}{b}}{3b}}{2b(a^2-b^2)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$\downarrow$ 4336

$$-\frac{-\frac{3a^2(5a^2-7b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx+\frac{\frac{2a^2b(5a^2-2b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a^3(5a^2-4b^2}{}}{a^2}}{b}}{3b}}{2b(a^2-b^2)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$\big\downarrow$ 3042

$$-\frac{-\frac{\frac{3a^2(5a^2-7b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{\frac{2a^2b(5a^2-2b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+6a^3(}{a^2}}{b}}{3b}}{2b(a^2-b^2)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$\big\downarrow$ 3284

$$-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-$$

$$\frac{-\frac{2(5a^2-2b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}-\frac{\frac{6a^2(5a^2-7b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{\frac{2a^2b(5a^2-2b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{E}}{d}}{}}{b}}{2b(a^2-b^2)}$$

input
```
Int[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^2,x]
```

output
```
-((a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*
x]))) - ((-2*(5*a^2 - 2*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*b*d) - ((
((6*a^3*(5*a^2 - 4*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/d + (2*a^2*b*(5*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (6*a^2*(5*a^2 - 7*b^2)*Sqrt[Co
s[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
((a + b)*d))/b - (6*a*(5*a^2 - 4*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*
d))/(3*b))/(2*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 974 vs. $2(325) = 650$.

Time = 35.18 (sec) , antiderivative size = 975, normalized size of antiderivative = 2.85

method	result	size
default	Expression too large to display	975

input
```
int(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^2*(-1/6*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*a^2/b^2*(a^2/b/(a^2-b^2)*cos(1/2*d*x
+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*
d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b
^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(
a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),2*a/(a-b),2^(1/2)))-4/b^3*a/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1
/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*cos(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(9/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{9}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(9/2)/(b*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int((1/cos(c + d*x))^(9/2)/(a + b/cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(9/2)/(a + b/cos(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^4}{\sec(dx+c)^2 b^2+2\sec(dx+c)\,ab+a^2}dx$$

input
```
int(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^2,x)
```

output
```
int((sqrt(sec(c + d*x))*sec(c + d*x)**4)/(sec(c + d*x)**2*b**2 + 2*sec(c +
 d*x)*a*b + a**2),x)
```

3.615 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 279

$$\begin{aligned}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= -\frac{(3a^2-2b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2(a^2-b^2)\,d}\\ &\quad-\frac{a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{b(a^2-b^2)\,d}\\ &\quad-\frac{a(3a^2-5b^2)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{(a-b)b^2(a+b)^2d}\\ &\quad+\frac{(3a^2-2b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2(a^2-b^2)\,d}-\frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b(a^2-b^2)\,d(a+b\sec(c+dx))}\end{aligned}$$

output
```
-(3*a^2-2*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(
d*x+c)^(1/2)/b^2/(a^2-b^2)/d-a*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/
2*c,2^(1/2))*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d-a*(3*a^2-5*b^2)*cos(d*x+c)^(1/
2)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*sec(d*x+c)^(1/2)/(a-b)
/b^2/(a+b)^2/d+(3*a^2-2*b^2)*sec(d*x+c)^(1/2)*sin(d*x+c)/b^2/(a^2-b^2)/d-a
^2*sec(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 3.95 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.26

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{2b(-3a^3+2ab^2+2b(-a^2+b^2)\sec(c+dx))\sin(c+dx)}{(-a^2+b^2)(b+a\cos(c+dx))\sqrt{\sec(c+dx)}}+\frac{\cot(c+dx)\left(-3a^2b\sec^{\frac{3}{2}}(c+dx)+2b^3\sec^{\frac{3}{2}}(c+dx)+3a^2b\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx)-2b^3\right.}{}}{}$$

input
```
Integrate[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^2,x]
```

output
```
((2*b*(-3*a^3 + 2*a*b^2 + 2*b*(-a^2 + b^2)*Sec[c + d*x])*Sin[c + d*x])/((-
a^2 + b^2)*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]) + (Cot[c + d*x]*(-3*a^
2*b*Sec[c + d*x]^(3/2) + 2*b^3*Sec[c + d*x]^(3/2) + 3*a^2*b*Cos[2*(c + d*x
)]*Sec[c + d*x]^(3/2) - 2*b^3*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) + 2*b*(3
*a^2 - 2*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]
^2] - 2*(3*a^3 + 3*a^2*b - 4*a*b^2 - 2*b^3)*EllipticF[ArcSin[Sqrt[Sec[c +
d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 6*a^3*EllipticPi[-(b/a), ArcSin[Sqrt[S
ec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 10*a*b^2*EllipticPi[-(b/a), Arc
Sin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/((a - b)*(a + b)))/(2
*b^3*d)
```

Rubi [A] (verified)

Time = 1.95 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.95, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4332, 27, 3042, 4590, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4332}$$

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-2b\sec(c+dx)a-\left(3a^2-2b^2\right)\sec^2(c+dx)\right)}{2(a+b\sec(c+dx))}dx}{b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-2b\sec(c+dx)a-\left(3a^2-2b^2\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2-2b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(2b^2-3a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{4590}$$

$$-\frac{\frac{2\int \frac{a\left(3a^2-4b^2\right)\sec^2(c+dx)+2b\left(2a^2-b^2\right)\sec(c+dx)+a\left(3a^2-2b^2\right)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b}-\frac{2\left(3a^2-2b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{a\left(3a^2-4b^2\right)\sec^2(c+dx)+2b\left(2a^2-b^2\right)\sec(c+dx)+a\left(3a^2-2b^2\right)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b}-\frac{2\left(3a^2-2b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{a\left(3a^2-4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(2a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+a\left(3a^2-2b^2\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2\left(3a^2-2b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{4594}$$

$$-\frac{\frac{a(3a^2-5b^2)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int \frac{b\sec(c+dx)a^3+(3a^2-2b^2)a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{b}-\frac{2(3a^2-2b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{2b(a^2-b^2)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{a(3a^2-5b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{b\csc\left(c+dx+\frac{\pi}{2}\right)a^3+(3a^2-2b^2)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{b}-\frac{2(3a^2-2b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{2b(a^2-b^2)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{ 4274}$$

$$-\frac{\frac{a(3a^2-5b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3b\int \sqrt{\sec(c+dx)}dx+a^2(3a^2-2b^2)\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{a^2}}{b}-\frac{2(3a^2-2b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{2b(a^2-b^2)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{a(3a^2-5b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3b\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+a^2(3a^2-2b^2)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{b}-\frac{2(3a^2-2b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{2b(a^2-b^2)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{ 4258}$$

$$-\frac{\frac{a(3a^2-5b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+a^2(3a^2-2b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx}{a^2}}{b}-2(3a^2-}{2b(a^2-b^2)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{a(3a^2-5b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a^2\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{b} - \ldots}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$\downarrow$ 3119

$$-\frac{\frac{a(3a^2-5b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}}{b} - \frac{2(3\ldots}{\ldots}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$\downarrow$ 3120

$$-\frac{\frac{a(3a^2-5b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{2a^3b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}}{b} - \frac{2(\ldots}{\ldots}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$\downarrow$ 4336

$$-\frac{\frac{a(3a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx+\frac{\frac{2a^3b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2\left(3a^2-2b^2\right)\sqrt{\cos(c+\ldots}}{\ldots}}{a^2}}{b}\ldots}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$\downarrow$ 3042

$$-\frac{\frac{a(3a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{\frac{2a^3b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2\left(3a^2-2b^2\right)\ldots}{\ldots}}{a^2}}{b}\ldots}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \textbf{3284}$$

$$-\frac{a^2 \sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\,(a^2-b^2)\,(a+b\sec(c+dx))}-\frac{\frac{\frac{2a\left(3a^2-5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{\frac{2a^3b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}{d}}{a^2}}{b}}{2b\,(a^2-b^2)}$$

input

```
Int[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^2,x]
```

output

```
-((a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*
x]))) - ((((2*a^2*(3*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/d + (2*a^3*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (2*a*(3*a^2 - 5*b^2)*Sqrt[Cos[c + d*
x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a + b)
*d))/b - (2*(3*a^2 - 2*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d))/(2*b*(
a^2 - b^2))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 840 vs. $2(268) = 536$.

Time = 34.05 (sec) , antiderivative size = 841, normalized size of antiderivative = 3.01

method	result	size
default	Expression too large to display	841

input
```
int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^2/sin(1/2*
d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2))-2/b*a*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+
b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)
/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2
)))+2*a^2/b^2/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{7}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^3}{\sec(dx+c)^2 b^2+2\sec(dx+c)\,ab+a^2}dx$$

input
```
int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x)
```

output
```
int((sqrt(sec(c + d*x))*sec(c + d*x)**3)/(sec(c + d*x)**2*b**2 + 2*sec(c +
 d*x)*a*b + a**2),x)
```

3.616 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 214

$$\begin{aligned}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= \frac{a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b(a^2-b^2)d} \\ &+ \frac{\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{(a^2-b^2)d} \\ &+ \frac{(a^2-3b^2)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{(a-b)b(a+b)^2d} \\ &- \frac{a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{b(a^2-b^2)d(a+b\sec(c+dx))}\end{aligned}$$

output
```
a*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)/
b/(a^2-b^2)/d+cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(
d*x+c)^(1/2)/(a^2-b^2)/d+(a^2-3*b^2)*cos(d*x+c)^(1/2)*EllipticPi(sin(1/2*d
*x+1/2*c),2*a/(a+b),2^(1/2))*sec(d*x+c)^(1/2)/(a-b)/b/(a+b)^2/d-a^2*sec(d*
x+c)^(1/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 582 vs. $2(214) = 428$.

Time = 6.45 (sec) , antiderivative size = 582, normalized size of antiderivative = 2.72

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)}{b(-a^2+b^2)}+\frac{a\sin(c+dx)}{(a^2-b^2)(b+a\cos(c+dx))}\right)}{d}$$
$$+ \frac{\frac{2(3a^2-4b^2)\cos^2(c+dx)\left(\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)-\operatorname{EllipticPi}\left(-\frac{b}{a},\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)\right)(a+b\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\dots}{b(b+a\cos(c+dx))(1-\cos^2(c+dx))}\dots}{\dots}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^2,x]
```

output
```
(Sqrt[Sec[c + d*x]]*((a*Sin[c + d*x])/(b*(-a^2 + b^2)) + (a*Sin[c + d*x])/
((a^2 - b^2)*(b + a*Cos[c + d*x]))))/d + ((2*(3*a^2 - 4*b^2)*Cos[c + d*x]^
2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[S
qrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin
[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (8*b*Cos[c + d*
x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x
])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/((b + a*Cos[c + d*x])*(1 - Cos[c
 + d*x]^2)) + (Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c
 + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d
*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c
+ d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2*Elliptic
Pi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec
[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sq
rt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(b*(b + a*Cos[c +
 d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(4*(
a - b)*b*(a + b)*d)
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.99, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {3042, 4332, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4332}$$

$$-\frac{\int -\frac{a^2+2b\sec(c+dx)a+(a^2-2b^2)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b(a^2-b^2)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a^2+2b\sec(c+dx)a+(a^2-2b^2)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2b(a^2-b^2)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a+(a^2-2b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2b(a^2-b^2)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{4594}$$

$$\frac{(a^2-3b^2)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int \frac{a^3+b\sec(c+dx)a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{2b(a^2-b^2)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2-3b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{a^3+b\csc\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{2b(a^2-b^2)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{4274}$$

$$\frac{\left(a^2-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3\int \frac{1}{\sqrt{\sec(c+dx)}}dx+a^2b\int \sqrt{\sec(c+dx)}dx}{a^2}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a^2b\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\left(a^2-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{a^2}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\left(a^2-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\left(a^2-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{2a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 4336$$

$$\frac{\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx+\frac{\frac{2a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2b\sqrt{\cos(c+\cdots}{\cdots}}{a^2}}{2b\left(a^2-b^2\right)}$$
$$\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{\frac{2a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2b\cdots}{\cdots}}{a^2}}{2b\left(a^2-b^2\right)}$$
$$\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 3284$$

$$\frac{\frac{2\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{\frac{2a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{Elliptic}\cdots}{d}}{a^2}}{2b\left(a^2-b^2\right)}$$
$$\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

input
```
Int[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^2,x]
```

output
```
(((2*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
d + (2*a^2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d)/a^2 + (2*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b),
 (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a + b)*d))/(2*b*(a^2 - b^2)) - (a^2
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 607 vs. 2(205) = 410.

Time = 31.97 (sec) , antiderivative size = 608, normalized size of antiderivative = 2.84

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(\frac{2a^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{b\left(a^{2}-b^{2}\right)\left(2a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-a+b\right)}-\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}}{(a+b)b\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}}\right.}{}$

input
```
int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2/b/(a^2-b
^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+a/b/(a^2-b^2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-a/b/(
a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-1/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3*b/(a^2-b^2)/
(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**(5/2)/(a + b*sec(c + d*x))**2, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx=\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^2}{\sec(dx+c)^2\,b^2+2\sec(dx+c)\,ab+a^2}dx$$

input
```
int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x)
```

output
```
int((sqrt(sec(c + d*x))*sec(c + d*x)**2)/(sec(c + d*x)**2*b**2 + 2*sec(c +
 d*x)*a*b + a**2),x)
```

3.617 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 208

$$\begin{aligned}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= -\frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{(a^2-b^2)\,d} \\ &\quad-\frac{b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a\,(a^2-b^2)\,d} \\ &\quad+\frac{(a^2+b^2)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a(a-b)(a+b)^2d} \\ &\quad+\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))}\end{aligned}$$

output
```
-cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)/(
a^2-b^2)/d-b*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d
*x+c)^(1/2)/a/(a^2-b^2)/d+(a^2+b^2)*cos(d*x+c)^(1/2)*EllipticPi(sin(1/2*d*
x+1/2*c),2*a/(a+b),2^(1/2))*sec(d*x+c)^(1/2)/a/(a-b)/(a+b)^2/d+a*sec(d*x+c
)^(1/2)*sin(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 5.64 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.39

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{4a\sin(c+dx)}{(a^2-b^2)(b+a\cos(c+dx))\sqrt{\sec(c+dx)}} + \frac{2\cot(c+dx)\left(ab\sec^{\frac{3}{2}}(c+dx)+ab\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx)-ab\sec^{\frac{7}{2}}(c+dx)-ab\cos(2(c+dx))\sec^{\frac{7}{2}}(c\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

output
```
((4*a*Sin[c + d*x])/((a^2 - b^2)*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
+ (2*Cot[c + d*x]*(a*b*Sec[c + d*x]^(3/2) + a*b*Cos[2*(c + d*x)]*Sec[c + d
*x]^(3/2) - a*b*Sec[c + d*x]^(7/2) - a*b*Cos[2*(c + d*x)]*Sec[c + d*x]^(7/
2) + 2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]
 + 2*a*(a - b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x
]^2] - 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[
c + d*x]^2] - 2*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqr
t[-Tan[c + d*x]^2]))/(a*(a - b)*b*(a + b)))/(4*d)
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4331, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4331}$$

$$\frac{\int -\frac{-a\sec^2(c+dx)+2b\sec(c+dx)+a}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{a^2-b^2}+\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\,(a^2-b^2)\,(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\,(a^2-b^2)\,(a+b\sec(c+dx))}-\frac{\int \frac{-a\sec^2(c+dx)+2b\sec(c+dx)+a}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2\,(a^2-b^2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\,(a^2-b^2)\,(a+b\sec(c+dx))}-\frac{\int \frac{-a\csc\left(c+dx+\frac{\pi}{2}\right)^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2\,(a^2-b^2)}$$

$$\Big\downarrow \text{4594}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\,(a^2-b^2)\,(a+b\sec(c+dx))}-\frac{\frac{\int \frac{a^2+b\sec(c+dx)a}{\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{(a^2+b^2)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a}}{2\,(a^2-b^2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\,(a^2-b^2)\,(a+b\sec(c+dx))}-\frac{\frac{\int \frac{a^2+b\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{(a^2+b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2\,(a^2-b^2)}$$

$$\Big\downarrow \text{4274}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\,(a^2-b^2)\,(a+b\sec(c+dx))}-\frac{\frac{a^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx+ab\int \sqrt{\sec(c+dx)}dx}{a^2}-\frac{(a^2+b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2\,(a^2-b^2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\,(a^2-b^2)\,(a+b\sec(c+dx))}-\frac{\frac{a^2\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+ab\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{(a^2+b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2\,(a^2-b^2)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}-\frac{\frac{a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{a^2}-\frac{\left(a^2+b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}-\frac{\frac{a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{\left(a^2+b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3119$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}-\frac{\frac{ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}-\frac{\left(a^2+b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3120$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}-\frac{\frac{\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}-\frac{\left(a^2+b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 4336$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}-\frac{\frac{\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}-\frac{\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a}}{a}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}-\frac{\frac{\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}-\frac{\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}(}}{a}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3284}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}-\frac{\frac{\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}-\frac{2\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}\right.}{ad(a+b)}}{2\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

output
```
-1/2*(((2*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (2*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/d)/a^2 - (2*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b)
, (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a + b)*d))/(a^2 - b^2) + (a*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4331
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 2)/(f*(m + 1)*(a^2 - b^2))), x] - Simp[d^2/((m + 1
)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*
(a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /;
FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2
] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(199) = 398$.

Time = 4.15 (sec) , antiderivative size = 707, normalized size of antiderivative = 3.40

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-\frac{2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\frac{2a}{a-b},\sqrt{2}\right)}{\left(a^{2}-ab\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}-\frac{2b\left(\frac{a^{2}\cos\left(\frac{d}{2}\right.}{b\left(\right.}\right.}{}\right.}{}$

input
```
int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/(a^2-a*b)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a
/(a-b),2^(1/2))-2/a*b*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a
+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2
)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/
2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**(3/2)/(a + b*sec(c + d*x))**2, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)}{\sec(dx+c)^2\,b^2+2\sec(dx+c)\,ab+a^2}dx$$

input
```
int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x)
```

output
```
int((sqrt(sec(c + d*x))*sec(c + d*x))/(sec(c + d*x)**2*b**2 + 2*sec(c + d*
x)*a*b + a**2),x)
```

3.618 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 227

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \frac{b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a(a^2-b^2)\,d} + \frac{(2a^2-b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a^2(a^2-b^2)\,d} - \frac{b(3a^2-b^2)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a^2(a-b)(a+b)^2d} - \frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))}$$

output
```
b*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)/
a/(a^2-b^2)/d+(2*a^2-b^2)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2
^(1/2))*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d-b*(3*a^2-b^2)*cos(d*x+c)^(1/2)*El
lipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*sec(d*x+c)^(1/2)/a^2/(a-b)/
(a+b)^2/d-b*sec(d*x+c)^(1/2)*sin(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 3.00 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\cos(2(c+dx))\csc(c+dx)\sqrt{\sec(c+dx)}\left(a(a-b)(b+a\cos(c+dx))\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right)\right.\right.}{}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```

output
```
(Cos[2*(c + d*x)]*Csc[c + d*x]*Sqrt[Sec[c + d*x]]*(a*(a - b)*(b + a*Cos[c
+ d*x])*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[
-Tan[c + d*x]^2] - (3*a^2 - b^2)*(b + a*Cos[c + d*x])*EllipticPi[-(b/a), A
rcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] +
a*b*(-(b*Tan[c + d*x]^2) + (b + a*Cos[c + d*x])*EllipticE[ArcSin[Sqrt[Sec[
c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2])))/(a^2*(a - b)*(
a + b)*d*(b + a*Cos[c + d*x])*(-2 + Sec[c + d*x]^2))
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.95, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4330, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{ 4330}$$

$$-\frac{\int -\frac{-b\sec^2(c+dx)+2a\sec(c+dx)+b}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{a^2-b^2}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{-b\sec^2(c+dx)+2a\sec(c+dx)+b}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2(a^2-b^2)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{-b\csc\left(c+dx+\frac{\pi}{2}\right)^2+2a\csc\left(c+dx+\frac{\pi}{2}\right)+b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2(a^2-b^2)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{ 4594}$$

$$\frac{\frac{\int \frac{ab+\left(2a^2-b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{b(3a^2-b^2)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}}{2(a^2-b^2)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int \frac{ab+\left(2a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{b(3a^2-b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2(a^2-b^2)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{ 4274}$$

$$\frac{\frac{(2a^2-b^2)\int \sqrt{\sec(c+dx)}dx+ab\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{b(3a^2-b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2(a^2-b^2)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{(2a^2-b^2)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+ab\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{b(3a^2-b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2(a^2-b^2)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{\frac{(2a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx}{a^2}-\frac{b(3a^2-b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2(a^2-b^2)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}$$

$\downarrow$ 3042

$$\frac{\frac{(2a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{b(3a^2-b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2(a^2-b^2)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}$$

$\downarrow$ 3119

$$\frac{\frac{(2a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{a^2}-\frac{b(3a^2-b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2(a^2-b^2)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}$$

$\downarrow$ 3120

$$\frac{\frac{\frac{2(2a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{a^2}-\frac{b(3a^2-b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2(a^2-b^2)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}$$

$\downarrow$ 4336

$$\frac{\frac{\frac{2(2a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{a^2}-\frac{b(3a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{}{\sqrt{\cos(c}}}{a^2}}{2(a^2-b^2)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2(2a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{a^2}-\frac{b(3a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{}{\sqrt{\sin(c}}}{a^2}}{2(a^2-b^2)}$$
$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}$$

$\downarrow$ 3284

$$\frac{\frac{\frac{2(2a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{a^2}-\frac{2b(3a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{Elliptic}}{a^2d(a+b)}}{2(a^2-b^2)}$$
$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}$$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```

output
```
(((2*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
d + (2*(2*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/d)/a^2 - (2*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a
)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d))/(2*(a^2 -
b^2)) - (b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c +
d*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4330
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)
*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*S
imp[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 787 vs. $2(218) = 436$.

Time = 4.73 (sec) , antiderivative size = 788, normalized size of antiderivative = 3.47

method	result	size
default	Expression too large to display	788

input
```
int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2/
a^2*b^2*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/
2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+4*b/a/(a^
2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/
2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(sqrt(sec(c + d*x))/(a + b*sec(c + d*x))**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{(b\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}}{\left(b\sec\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}}{\sec\left(dx+c\right)^2 b^2+2\sec\left(dx+c\right)ab+a^2}dx$$

input
```
int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x)
```

output
```
int(sqrt(sec(c + d*x))/(sec(c + d*x)**2*b**2 + 2*sec(c + d*x)*a*b + a**2),
x)
```

3.619 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 244

$$\begin{aligned}\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx &= \frac{(2a^2-3b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2(a^2-b^2)\,d} \\ &\quad - \frac{b(4a^2-3b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a^3(a^2-b^2)\,d} \\ &\quad + \frac{b^2(5a^2-3b^2)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a^3(a-b)(a+b)^2d} \\ &\quad + \frac{b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{a(a^2-b^2)\,d(a+b\sec(c+dx))}\end{aligned}$$

output
```
(2*a^2-3*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d
*x+c)^(1/2)/a^2/(a^2-b^2)/d-b*(4*a^2-3*b^2)*cos(d*x+c)^(1/2)*InverseJacobi
AM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)/d+b^2*(5*a^2-3*b^
2)*cos(d*x+c)^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*sec(d
*x+c)^(1/2)/a^3/(a-b)/(a+b)^2/d+b^2*sec(d*x+c)^(1/2)*sin(d*x+c)/a/(a^2-b^2
)/d/(a+b*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 4.25 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.31

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{4b^2\sin(c+dx)}{a(a^2-b^2)(b+a\cos(c+dx))\sqrt{\sec(c+dx)}} + \frac{2\cot(c+dx)\left(2a^3\sec^{\frac{3}{2}}(c+dx)-3ab^2\sec^{\frac{3}{2}}(c+dx)-2a^3\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx)+3ab^2\cos(2(c+dx))\right.}{}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2),x]
```

output
```
((4*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x
]]) + (2*Cot[c + d*x]*(2*a^3*Sec[c + d*x]^(3/2) - 3*a*b^2*Sec[c + d*x]^(3/
2) - 2*a^3*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) + 3*a*b^2*Cos[2*(c + d*x)]*
Sec[c + d*x]^(3/2) - 2*a*(2*a^2 - 3*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x
]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2*a*(2*a^2 + a*b - 3*b^2)*EllipticF[ArcSi
n[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 10*a^2*b*EllipticPi[-(b
/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 6*b^3*Ellipti
cPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(a^3*(
a - b)*(a + b)))/(4*d)
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.95, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4334, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4334}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}-\frac{\int -\frac{2a^2-2b\sec(c+dx)a-3b^2+b^2\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2a^2-2b\sec(c+dx)a-3b^2+b^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2a^2-2b\csc\left(c+dx+\frac{\pi}{2}\right)a-3b^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{4594}$$

$$\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int \frac{a\left(2a^2-3b^2\right)-b\left(4a^2-3b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{a\left(2a^2-3b^2\right)-b\left(4a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{4274}$$

$$\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\left(2a^2-3b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx-b\left(4a^2-3b^2\right)\int \sqrt{\sec(c+dx)}dx}{a^2}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a(2a^2-3b^2)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b(4a^2-3b^2)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$\downarrow$ 4258

$$\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a(2a^2-3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-b(4a^2-3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{}{\sqrt{}}}{a^2}}{2a\left(a^2-b^2\right)}$$
$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$\downarrow$ 3042

$$\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a(2a^2-3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-b(4a^2-3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^2}}{2a\left(a^2-b^2\right)}$$
$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$\downarrow$ 3119

$$\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{2a\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-b(4a^2-3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+\right)}}}{a^2}}{2a\left(a^2-b^2\right)}$$
$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$\downarrow$ 3120

$$\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{2a\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2b\left(4a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+\right)}{d}}{a^2}}{2a\left(a^2-b^2\right)}$$
$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$\downarrow$ 4336

$$\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx+\frac{\frac{2a\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2b}{}}{a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 3042

$$\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{2a\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+da\right.}{d}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 3284

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}+$$

$$\frac{\frac{2b^2\left(5-\frac{3b^2}{a^2}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{\frac{2a\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2b\left(4a^2-3b^2\right)\sqrt{\cos(c+da}}{}}{a^2}}{2a\left(a^2-b^2\right)}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2),x]
```

output
```
(((2*a*(2*a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/d - (2*b*(4*a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (2*b^2*(5 - (3*b^2)/a^2)*Sqrt[Cos[c
 + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a
 + b)*d))/(2*a*(a^2 - b^2)) + (b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^
2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(235) = 470$.

Time = 6.05 (sec) , antiderivative size = 809, normalized size of antiderivative = 3.32

method	result	size
default	Expression too large to display	809

input
```
int(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^3/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))+a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-6*b^2/a^2/(a^2-a*b)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^
(1/2))-2/a^3*b^3*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^
2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{(a+b\sec{(c+dx)})^2\sqrt{\sec{(c+dx)}}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**2*sqrt(sec(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{(b\sec{(dx+c)}+a)^2\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{(b\sec{(dx+c)}+a)^2\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3b^2+2\sec(dx+c)^2ab+\sec(dx+c)\,a^2}dx$$

input
```
int(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x)
```

output
```
int(sqrt(sec(c + d*x))/(sec(c + d*x)**3*b**2 + 2*sec(c + d*x)**2*a*b + sec
(c + d*x)*a**2),x)
```

3.620 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 304

$$\begin{aligned}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx &= -\frac{b(4a^2-5b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^3(a^2-b^2)d} \\ &\quad+\frac{(2a^4+16a^2b^2-15b^4)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^4(a^2-b^2)d} \\ &\quad-\frac{b^3(7a^2-5b^2)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a^4(a-b)(a+b)^2d} \\ &\quad+\frac{(2a^2-5b^2)\sin(c+dx)}{3a^2(a^2-b^2)d\sqrt{\sec(c+dx)}}+\frac{b^2\sin(c+dx)}{a(a^2-b^2)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\end{aligned}$$

output
```
-b*(4*a^2-5*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*se
c(d*x+c)^(1/2)/a^3/(a^2-b^2)/d+1/3*(2*a^4+16*a^2*b^2-15*b^4)*cos(d*x+c)^(1
/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/a^4/(a^2-b^2)/
d-b^3*(7*a^2-5*b^2)*cos(d*x+c)^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+
b),2^(1/2))*sec(d*x+c)^(1/2)/a^4/(a-b)/(a+b)^2/d+1/3*(2*a^2-5*b^2)*sin(d*x
+c)/a^2/(a^2-b^2)/d/sec(d*x+c)^(1/2)+b^2*sin(d*x+c)/a/(a^2-b^2)/d/sec(d*x+
c)^(1/2)/(a+b*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 4.93 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{a^2(2a^2b-5b^3+2a(a^2-b^2)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)(b+a\cos(c+dx))\sqrt{\sec(c+dx)}} + \frac{\cot(c+dx)\left(3ab(4a^2-5b^2)E\left(\arcsin\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}+a(2a^3-12a^2b-\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2),x]
```

output
```
((a^2*(2*a^2*b - 5*b^3 + 2*a*(a^2 - b^2)*Cos[c + d*x])*Sin[c + d*x])/((a^2
 - b^2)*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]) + (Cot[c + d*x]*(3*a*b*(4
*a^2 - 5*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]
^2] + a*(2*a^3 - 12*a^2*b - 5*a*b^2 + 15*b^3)*EllipticF[ArcSin[Sqrt[Sec[c
+ d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 3*b*(a*(4*a^2 - 5*b^2)*Sec[c + d*x]^
(3/2)*Sin[c + d*x]^2 + b*(-7*a^2 + 5*b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[S
ec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2])))/((a - b)*(a + b)))/(3*a^4*d)
```

Rubi [A] (verified)

Time = 2.01 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.95, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4334, 27, 3042, 4592, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \text{4334}$$

$$\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}-\frac{\int -\frac{2a^2-2b\sec(c+dx)a-5b^2+3b^2\sec^2(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}dx}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int \frac{2a^2-2b\sec(c+dx)a-5b^2+3b^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\int \frac{2a^2-2b\csc\left(c+dx+\frac{\pi}{2}\right)a-5b^2+3b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$$\Big\downarrow \textbf{4592}$$

$$\frac{\frac{2\left(2a^2-5b^2\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{2\int \frac{-b\left(2a^2-5b^2\right)\sec^2(c+dx)-2a\left(a^2+2b^2\right)\sec(c+dx)+3b\left(4a^2-5b^2\right)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{3a}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{2\left(2a^2-5b^2\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{-b\left(2a^2-5b^2\right)\sec^2(c+dx)-2a\left(a^2+2b^2\right)\sec(c+dx)+3b\left(4a^2-5b^2\right)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{3a}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{2\left(2a^2-5b^2\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{-b\left(2a^2-5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-2a\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+3b\left(4a^2-5b^2\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3a}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$$\Big\downarrow \textbf{4594}$$

$$\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\int\frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}+\frac{\int\frac{3ab(4a^2-5b^2)-(2a^4+16b^2a^2-15b^4)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}}{3a}}{2a(a^2-b^2)}+\frac{b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$\downarrow$ 3042

$$\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\int\frac{3ab(4a^2-5b^2)+(-2a^4-16b^2a^2+15b^4)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{3a}}{2a(a^2-b^2)}+\frac{b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$\downarrow$ 4274

$$\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab(4a^2-5b^2)\int\frac{1}{\sqrt{\sec(c+dx)}}dx-(2a^4+16a^2b^2-15b^4)\int\sqrt{\sec(c+dx)}dx}{a^2}}{3a}}{2a(a^2-b^2)}+\frac{b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$\downarrow$ 3042

$$\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab(4a^2-5b^2)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-(2a^4+16a^2b^2-15b^4)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{3a}}{2a(a^2-b^2)}+\frac{b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$\downarrow$ 4258

$$\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab(4a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-(2a^4+16a^2b^2-15b^4)\sqrt{\cos(c+}}{a^2}}{3a}}{2a(a^2-b^2)}+\frac{b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{\frac{3b^3(7a^2-5b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab(4a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\left(2a^4+16a^2b^2-15b^4\right)\sqrt{co}}{a^2}}{3a}}{2a(a^2-b^2)}$$

$$\frac{b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{\frac{3b^3(7a^2-5b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{6ab(4a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\left(2a^4+16a^2b^2-15b^4\right)\sqrt{\cos(c+}}{a^2}}{3a}}{2a(a^2-b^2)}$$

$$\frac{b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{\frac{3b^3(7a^2-5b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{6ab(4a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2\left(2a^4+16a^2b^2-15b^4\right)\sqrt{\cos(c-}}{}}{a^2}}{3a}}{2a(a^2-b^2)}$$

$$\frac{b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{4336}$$

$$\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{\frac{3b^3(7a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx}{a^2}+\frac{\frac{6ab(4a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right.}{d}}{}}{3a}}{2a(a^2-b^2)}$$

$$\frac{b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2}+\frac{\frac{6ab(4a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}\right.}{d}}{3a}}{2a(a^2-b^2)}$$

$$\frac{b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$\Big\downarrow$ 3284

$$\frac{b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}+$$

$$\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{6b^3(7a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a^2d(a+b)}+\frac{\frac{6ab(4a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2(}{}}{3a}}{2a(a^2-b^2)}$$

input
```
Int[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2),x]
```

output
```
(b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x]
)) + (-1/3*(((6*a*b*(4*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/d - (2*(2*a^4 + 16*a^2*b^2 - 15*b^4)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (6*b^3*(7*
a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d))/a + (2*(2*a^2 - 5*b^2)*Sin[c + d*x])/
(3*a*d*Sqrt[Sec[c + d*x]]))/(2*a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1063 vs. $2(289) = 578$.

Time = 6.00 (sec) , antiderivative size = 1064, normalized size of antiderivative = 3.50

method	result	size
default	Expression too large to display	1064

input
```
int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(a^2+2*a*b+3
*b^2)/a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+2/a^4*b^4*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/
2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2
-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2
^(1/2)))+4/3/a^2*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{(a+b\sec(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**2*sec(c + d*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^4 b^2+2\sec(dx+c)^3 ab+\sec(dx+c)^2 a^2}dx$$

input
```
int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x)
```

output
```
int(sqrt(sec(c + d*x))/(sec(c + d*x)**4*b**2 + 2*sec(c + d*x)**3*a*b + sec
(c + d*x)**2*a**2),x)
```

3.621 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 388

$$\begin{aligned}
\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx
&= -\frac{(15a^4-29a^2b^2+8b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b^3(a^2-b^2)^2d} \\
&\quad - \frac{a(5a^2-11b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4b^2(a^2-b^2)^2d} \\
&\quad - \frac{a(15a^4-38a^2b^2+35b^4)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4(a-b)^2b^3(a+b)^3d} \\
&\quad + \frac{(15a^4-29a^2b^2+8b^4)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b^3(a^2-b^2)^2d} \\
&\quad - \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b(a^2-b^2)d(a+b\sec(c+dx))^2} - \frac{a^2(5a^2-11b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4b^2(a^2-b^2)^2d(a+b\sec(c+dx))}
\end{aligned}$$

output
```
-1/4*(15*a^4-29*a^2*b^2+8*b^4)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2))*sec(d*x+c)^(1/2)/b^3/(a^2-b^2)^2/d-1/4*a*(5*a^2-11*b^2)*cos(d*
x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/b^2/(a^
2-b^2)^2/d-1/4*a*(15*a^4-38*a^2*b^2+35*b^4)*cos(d*x+c)^(1/2)*EllipticPi(si
n(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*sec(d*x+c)^(1/2)/(a-b)^2/b^3/(a+b)^3/d
+1/4*(15*a^4-29*a^2*b^2+8*b^4)*sec(d*x+c)^(1/2)*sin(d*x+c)/b^3/(a^2-b^2)^2
/d-1/2*a^2*sec(d*x+c)^(5/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^2-1/
4*a^2*(5*a^2-11*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*se
c(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 6.54 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.37

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$= \frac{\frac{2b(15a^6-13a^4b^2-24a^2b^4+16b^6+(50a^5b-94a^3b^3+32ab^5)\cos(c+dx)+(15a^6-29a^4b^2+8a^2b^4)\cos(2(c+dx)))\tan(c+dx)}{(a^2-b^2)^2} - \frac{4\cos(c+dx)(b+a}{}}{}$$

input
```
Integrate[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^3,x]
```

output
```
((2*b*(15*a^6 - 13*a^4*b^2 - 24*a^2*b^4 + 16*b^6 + (50*a^5*b - 94*a^3*b^3
+ 32*a*b^5)*Cos[c + d*x] + (15*a^6 - 29*a^4*b^2 + 8*a^2*b^4)*Cos[2*(c + d*
x)])*Tan[c + d*x])/(a^2 - b^2)^2 - (4*Cos[c + d*x]*(b + a*Cos[c + d*x])*Co
t[c + d*x]*(a + b*Sec[c + d*x])*(-15*a^4*b + 29*a^2*b^3 - 8*b^5 + 15*a^4*b
*Sec[c + d*x]^2 - 29*a^2*b^3*Sec[c + d*x]^2 + 8*b^5*Sec[c + d*x]^2 - b*(15
*a^4 - 29*a^2*b^2 + 8*b^4)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[
Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] + (15*a^5 + 15*a^4*b - 33*a^3*b^2 - 29
*a^2*b^3 + 24*a*b^4 + 8*b^5)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqr
t[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] - 15*a^5*EllipticPi[-(b/a), ArcSin[S
qrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] + 38*a^3*
b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*
Sqrt[-Tan[c + d*x]^2] - 35*a*b^4*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*
x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2]))/((a - b)^2*(a + b)^2)
)/(16*b^4*d*(b + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 2.86 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.98, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.913, Rules used = {3042, 4332, 27, 3042, 4586, 27, 3042, 4590, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4332}$$

$$-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(3a^2-4b\sec(c+dx)a-\left(5a^2-4b^2\right)\sec^2(c+dx)\right)}{2(a+b\sec(c+dx))^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(3a^2-4b\sec(c+dx)a-\left(5a^2-4b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3a^2-4b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(4b^2-5a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{4586}$$

$$-\frac{\frac{a^2\left(5a^2-11b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int -\frac{\sqrt{\sec(c+dx)}\left(\left(5a^2-11b^2\right)a^2-4b\left(a^2-4b^2\right)\sec(c+dx)a-\left(15a^4-29b^2a^2+8b^4\right)\sec^2(c+dx)\right)}{2(a+b\sec(c+dx))}dx}{b(a^2-b^2)}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\int \frac{\sqrt{\sec(c+dx)}\left(\left(5a^2-11b^2\right)a^2-4b\left(a^2-4b^2\right)\sec(c+dx)a-\left(15a^4-29b^2a^2+8b^4\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{2b(a^2-b^2)}+\frac{a^2(5a^2-11b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\left(5a^2-11b^2\right)a^2-4b\left(a^2-4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(-15a^4+29b^2a^2-8b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2b(a^2-b^2)}+\frac{a^2(5a^2-11b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx}{bd(a^2-b^2)(a+b\sec(c+dx))}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$\Big\downarrow$ 4590

$$-\frac{\frac{\frac{2\int \frac{3a\left(5a^4-11b^2a^2+8b^4\right)\sec^2(c+dx)+4b\left(5a^4-10b^2a^2+2b^4\right)\sec(c+dx)+a\left(15a^4-29b^2a^2+8b^4\right)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b}-\frac{2\left(15a^4-29a^2b^2+8b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{2b(a^2-b^2)}+\frac{a^2(5}{}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\frac{\int \frac{3a\left(5a^4-11b^2a^2+8b^4\right)\sec^2(c+dx)+4b\left(5a^4-10b^2a^2+2b^4\right)\sec(c+dx)+a\left(15a^4-29b^2a^2+8b^4\right)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b}-\frac{2\left(15a^4-29a^2b^2+8b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{2b(a^2-b^2)}+\frac{a^2(5a}{b}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{\int \frac{3a\left(5a^4-11b^2a^2+8b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+4b\left(5a^4-10b^2a^2+2b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)+a\left(15a^4-29b^2a^2+8b^4\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2\left(15a^4-29a^2b^2+8b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$\Big\downarrow$ 4594

$$-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int \frac{b\left(5a^2-11b^2\right)\sec(c+dx)a^3+\left(15a^4-29b^2a^2+8b^4\right)a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{b}-\frac{2\left(15a^4-29a^2b^2+8b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{2b(a^2-b^2)}+\frac{a}{}}{4b\left(a^2-b^2\right)}$$
$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{b\left(5a^2-11b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^3+\left(15a^4-29b^2a^2+8b^4\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{b}-\frac{2\left(15a^4-29a^2b^2+8b^4\right)\sin(c+dx)\sqrt{\sec(c+}}{bd}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$
$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$\Big\downarrow$ 4274

$$-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2\left(15a^4-29a^2b^2+8b^4\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+a^3b\left(5a^2-11b^2\right)\int \sqrt{\sec(c+dx)}dx}{a^2}}{b}-\frac{2\left(15a^4-29a^2b^2+8b^4\right)\sin(c}{bd}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$
$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2\left(15a^4-29a^2b^2+8b^4\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a^3b\left(5a^2-11b^2\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{b}-\frac{2\left(15a^4-29a^2b^2+8\right.}{}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$
$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$\Big\downarrow$ 4258

$$-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+a^3b\left(5a^2-11b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^2}}{b}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+a^3b\left(5a^2-11b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{a^2}}{b}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3119

$$-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3b\left(5a^2-11b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec}}{d}}{a^2}}{b}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3120

$$-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{2a^2\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}+\frac{2a^3b\left(5a^2-11b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{d}}{a^2}}{b}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 4336

$$-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx+\frac{2a^2\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3b(}{a^2}}{b}}{2b(a^2-b^2)}}{4b\,(a^2-}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\,(a^2-b^2)\,(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{2a^2\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right.}{d}}{b}}{2b(a^2-b^2)}}{4b\,(}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\,(a^2-b^2)\,(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3284

$$-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\,(a^2-b^2)\,(a+b\sec(c+dx))^2}-$$

$$\frac{\frac{a^2(5a^2-11b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}+\frac{\frac{2a\left(15a^4-38a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{2a^2\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\cos(c}}{}}{b}}{4b\,(a^2-b^2}$$

input
```
Int[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^3,x]
```

output
```
-1/2*(a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c +
 d*x])^2) - ((a^2*(5*a^2 - 11*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^
2 - b^2)*d*(a + b*Sec[c + d*x])) + ((((2*a^2*(15*a^4 - 29*a^2*b^2 + 8*b^4)
*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a
^3*b*(5*a^2 - 11*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/d)/a^2 + (2*a*(15*a^4 - 38*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x
]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a + b)*
d))/b - (2*(15*a^4 - 29*a^2*b^2 + 8*b^4)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(b*d))/(2*b*(a^2 - b^2)))/(4*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4586
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Simp[d/(b*(a^2 - b^2)*(m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
 - b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n
+ b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C
}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1986 vs. $2(363) = 726$.

Time = 154.42 (sec) , antiderivative size = 1987, normalized size of antiderivative = 5.12

method	result	size
default	Expression too large to display	1987

input
```
int(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^3/sin(1/2*
d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2))-2/b*a*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3
/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/
(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(9/2)/(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{9}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(9/2)/(b*sec(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int((1/cos(c + d*x))^(9/2)/(a + b/cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(9/2)/(a + b/cos(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^4}{\sec(dx+c)^3\,b^3+3\sec(dx+c)^2\,a\,b^2+3\sec(dx+c)\,a^2b+a^3}dx$$

input
```
int(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x)
```

output
```
int((sqrt(sec(c + d*x))*sec(c + d*x)**4)/(sec(c + d*x)**3*b**3 + 3*sec(c +
d*x)**2*a*b**2 + 3*sec(c + d*x)*a**2*b + a**3),x)
```

3.622 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 315

$$\begin{aligned}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx &= \frac{3a(a^2-3b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b^2(a^2-b^2)^2d} \\ &+ \frac{(a^2-7b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4b(a^2-b^2)^2d} \\ &+ \frac{3(a^4-2a^2b^2+5b^4)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4(a-b)^2b^2(a+b)^3d} \\ &- \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b(a^2-b^2)d(a+b\sec(c+dx))^2} - \frac{3a^2(a^2-3b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b^2(a^2-b^2)^2d(a+b\sec(c+dx))}\end{aligned}$$

output
```
3/4*a*(a^2-3*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*s
ec(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d+1/4*(a^2-7*b^2)*cos(d*x+c)^(1/2)*Inverse
JacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/b/(a^2-b^2)^2/d+3/4*(a^4-
2*a^2*b^2+5*b^4)*cos(d*x+c)^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),
2^(1/2))*sec(d*x+c)^(1/2)/(a-b)^2/b^2/(a+b)^3/d-1/2*a^2*sec(d*x+c)^(3/2)*s
in(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^2-3/4*a^2*(a^2-3*b^2)*sec(d*x+c)^
(1/2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 5.60 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.06

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{\frac{4a^2b(-5a^2b+11b^3-3a(a^2-3b^2)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2} + \frac{4\cos(c+dx)(b+a\cos(c+dx))\cot(c+dx)(a+b\sec(c+dx))\left(3ab(a^2-3b^2)\tan^2(c+dx)\right.}{}}{}$$

input
```
Integrate[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^3,x]
```

output
```
((4*a^2*b*(-5*a^2*b + 11*b^3 - 3*a*(a^2 - 3*b^2)*Cos[c + d*x])*Sin[c + d*x
])/(a^2 - b^2)^2 + (4*Cos[c + d*x]*(b + a*Cos[c + d*x])*Cot[c + d*x]*(a +
b*Sec[c + d*x])*(3*a*b*(a^2 - 3*b^2)*Tan[c + d*x]^2 - 3*a*b*(a^2 - 3*b^2)*
EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c +
 d*x]^2] + (3*a^4 + 3*a^3*b - 5*a^2*b^2 - 9*a*b^3 + 8*b^4)*EllipticF[ArcSi
n[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] - 3*(a
^4 - 2*a^2*b^2 + 5*b^4)*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]
*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*b^3
*d*(b + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 2.18 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.01, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4332, 27, 3042, 4586, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$\Big\downarrow$ 4332

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-4b\sec(c+dx)a-\left(3a^2-4b^2\right)\sec^2(c+dx)\right)}{2(a+b\sec(c+dx))^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 27

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-4b\sec(c+dx)a-\left(3a^2-4b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

$$-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2-4b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(4b^2-3a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 4586

$$-\frac{\frac{3a^2\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int \frac{3\left(a^2-3b^2\right)a^2+4b\left(a^2-4b^2\right)\sec(c+dx)a+\left(3a^4-5b^2a^2+8b^4\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b\left(a^2-b^2\right)}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 27

$$-\frac{\frac{3a^2\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int \frac{3\left(a^2-3b^2\right)a^2+4b\left(a^2-4b^2\right)\sec(c+dx)a+\left(3a^4-5b^2a^2+8b^4\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2b\left(a^2-b^2\right)}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{3a^2\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int \frac{3\left(a^2-3b^2\right)a^2+4b\left(a^2-4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(3a^4-5b^2a^2+8b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2b\left(a^2-b^2\right)}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 4594$$

$$-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int \frac{3(a^2-3b^2)a^3+b(a^2-7b^2)\sec(c+dx)a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{2b(a^2-b^2)}}{4b(a^2-b^2)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{3(a^2-3b^2)a^3+b(a^2-7b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{2b(a^2-b^2)}}{4b(a^2-b^2)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 4274$$

$$-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b(a^2-7b^2)\int \sqrt{\sec(c+dx)}dx+3a^3(a^2-3b^2)\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{a^2}}{2b(a^2-b^2)}}{4b(a^2-b^2)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b(a^2-7b^2)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+3a^3(a^2-3b^2)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{2b(a^2-b^2)}}{4b(a^2-b^2)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 4258$$

$$-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{a^2b(a^2-7b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+3a^3(a^2}{a^2}}{2b(a^2-b^2)}}{4b(a^2-b^2)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3042

$$-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{a^2b(a^2-7b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a}{a^2}}{2b(a^2-b^2)}}{4b(a^2-b^2)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3119

$$-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{a^2b(a^2-7b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a}{}}{a^2}}{2b(a^2-b^2)}}{4b(a^2-b^2)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3120

$$-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{2a^2b(a^2-7b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+}{a^2}}{2b(a^2-b^2)}}{4b(a^2-b^2)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}$$

$\downarrow$ 4336

$$-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx+\frac{2a^2b(a^2-7b^2)\sqrt{\cos(c+dx}}{}}{2b(a^2-b^2)}}{4b(a^2-b^2)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{2a^2b\left(a^2-7b^2\right)\sqrt{}}{}}{2b(a^2-b^2}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{3284}$$

$$-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-$$

$$\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\frac{6\left(a^4-2a^2b^2+5b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{\frac{2a^2b\left(a^2-7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}}{}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^3,x]
```

output
```
-1/2*(a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c +
 d*x])^2) - (-1/2*(((6*a^3*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a^2*b*(a^2 - 7*b^2)*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (6*(a^4 - 2*a^2
*b^2 + 5*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/((a + b)*d))/(b*(a^2 - b^2)) + (3*a^2*(a^2 - 3*b^2)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])))/(4
*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4586
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Simp[d/(b*(a^2 - b^2)*(m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
 - b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n
+ b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C
}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1202 vs. $2(294) = 588$.

Time = 148.67 (sec) , antiderivative size = 1203, normalized size of antiderivative = 3.82

method	result	size
default	Expression too large to display	1203

input
```
int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(a^2/b/(a^2-b^2
)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3/2*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*co
s(1/2*d*x+1/2*c)^2-a+b)-3/4/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/2/(a+b)/(a^2-b
^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*a+7/4/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/4*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/4*
a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-3/4*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/4*a/(a^2-b^2)^2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{7}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^3}{\sec(dx+c)^3 b^3+3\sec(dx+c)^2 a\,b^2+3\sec(dx+c)\,a^2b+a^3}dx$$

input
```
int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x)
```

output
```
int((sqrt(sec(c + d*x))*sec(c + d*x)**3)/(sec(c + d*x)**3*b**3 + 3*sec(c +
 d*x)**2*a*b**2 + 3*sec(c + d*x)*a**2*b + a**3),x)
```

3.623 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 313

$$\begin{aligned}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx &= \frac{(a^2+5b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b(a^2-b^2)^2d} \\ &\quad + \frac{3(a^2+b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4a(a^2-b^2)^2d} \\ &\quad + \frac{(a^4-10a^2b^2-3b^4)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4a(a-b)^2b(a+b)^3d} \\ &\quad - \frac{a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{2b(a^2-b^2)d(a+b\sec(c+dx))^2} + \frac{a(a^2-7b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b(a^2-b^2)^2d(a+b\sec(c+dx))}\end{aligned}$$

output
```
1/4*(a^2+5*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec
(d*x+c)^(1/2)/b/(a^2-b^2)^2/d+3/4*(a^2+b^2)*cos(d*x+c)^(1/2)*InverseJacobi
AM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/a/(a^2-b^2)^2/d+1/4*(a^4-10*a^2
*b^2-3*b^4)*cos(d*x+c)^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/
2))*sec(d*x+c)^(1/2)/a/(a-b)^2/b/(a+b)^3/d-1/2*a^2*sec(d*x+c)^(1/2)*sin(d*
x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^2+1/4*a*(a^2-7*b^2)*sec(d*x+c)^(1/2)*s
in(d*x+c)/b/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 5.64 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.37

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$= \frac{-\frac{4ab(-a^2b+7b^3+a(a^2+5b^2)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2}+\frac{4\cos(c+dx)(b+a\cos(c+dx))\cot(c+dx)(a+b\sec(c+dx))\left(-a^3b-5ab^3+a^3b\sec^2(c+dx\right.}{}}{}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^3,x]
```

output
```
((-4*a*b*(-(a^2*b) + 7*b^3 + a*(a^2 + 5*b^2)*Cos[c + d*x])*Sin[c + d*x])/(
a^2 - b^2)^2 + (4*Cos[c + d*x]*(b + a*Cos[c + d*x])*Cot[c + d*x]*(a + b*Se
c[c + d*x])*(-(a^3*b) - 5*a*b^3 + a^3*b*Sec[c + d*x]^2 + 5*a*b^3*Sec[c + d
*x]^2 - a*b*(a^2 + 5*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[S
ec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] + a*(a^3 + a^2*b - 7*a*b^2 + 5*b^3)*Ell
ipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*
x]^2] - a^4*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c
+ d*x]]*Sqrt[-Tan[c + d*x]^2] + 10*a^2*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[
Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] + 3*b^4*Ellip
ticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan
[c + d*x]^2]))/(a*(a - b)^2*(a + b)^2))/(16*b^2*d*(b + a*Cos[c + d*x])^2*S
qrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 2.14 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.99, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4332, 27, 3042, 4588, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4332}$$

$$-\frac{\int -\frac{a^2+4b\sec(c+dx)a+\left(a^2-4b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a^2+4b\sec(c+dx)a+\left(a^2-4b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a^2+4b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(a^2-4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{4588}$$

$$\frac{\frac{a\left(a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int -\frac{\left(a^2-7b^2\right)\sec^2(c+dx)a^2+\left(a^2+5b^2\right)a^2+4b\left(a^2+2b^2\right)\sec(c+dx)a}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{a(a^2-b^2)}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\left(a^2-7b^2\right)\sec^2(c+dx)a^2+\left(a^2+5b^2\right)a^2+4b\left(a^2+2b^2\right)\sec(c+dx)a}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2a(a^2-b^2)}+\frac{a\left(a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\left(a^2-7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+\left(a^2+5b^2\right)a^2+4b\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}+\frac{a\left(a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{4594}$$

$$\frac{\frac{(a^4-10a^2b^2-3b^4)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int \frac{(a^2+5b^2)a^3+3b(a^2+b^2)\sec(c+dx)a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{2a(a^2-b^2)}+\frac{a(a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(a^4-10a^2b^2-3b^4)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{(a^2+5b^2)a^3+3b(a^2+b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{2a(a^2-b^2)}+\frac{a(a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 4274$$

$$\frac{\frac{(a^4-10a^2b^2-3b^4)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^2b(a^2+b^2)\int \sqrt{\sec(c+dx)}dx+a^3(a^2+5b^2)\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{a^2}}{2a(a^2-b^2)}+\frac{a(a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(a^4-10a^2b^2-3b^4)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^2b(a^2+b^2)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+a^3(a^2+5b^2)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{2a(a^2-b^2)}+\frac{a(a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 4258$$

$$\frac{\frac{(a^4-10a^2b^2-3b^4)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^2b(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+a^3(a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx}{a^2}}{2a(a^2-b^2)}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(a^4-10a^2b^2-3b^4)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^2b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a^3\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx\right.}\cdots}{a^2}}{2a(a^2-b^2)}\cdots}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$\downarrow$ 3119

$$\frac{\frac{(a^4-10a^2b^2-3b^4)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^2b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^3\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx\right.\cdots}{d}}{a^2}}{2a(a^2-b^2)}\cdots}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$\downarrow$ 3120

$$\frac{\frac{(a^4-10a^2b^2-3b^4)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{6a^2b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^3\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+d\right.\cdots}{d}}{a^2}}{2a(a^2-b^2)}\cdots}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$\downarrow$ 4336

$$\frac{\frac{(a^4-10a^2b^2-3b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx+\frac{\frac{6a^2b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+2a^3\left(a^2+5b\right.\cdots}{a^2}}{2a(a^2-b^2)}\cdots}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$\downarrow$ 3042

$$\frac{\frac{(a^4-10a^2b^2-3b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{\frac{6a^2b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\cdots}{a^2}}{2a(a^2-b^2)}\cdots}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$$\Big\downarrow \text{3284}$$

$$\frac{\frac{a(a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}+\frac{\frac{2\left(a^4-10a^2b^2-3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{6a^2b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{El}}{d}}{2a(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

input
```
Int[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^3,x]
```

output
```
-1/2*(a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c +
 d*x])^2) + ((((2*a^3*(a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/d + (6*a^2*b*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (2*(a^4 - 10*a^2*b^2 -
 3*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/((a + b)*d))/(2*a*(a^2 - b^2)) + (a*(a^2 - 7*b^2)*Sqrt[Sec[
c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x])))/(4*b*(a^2 -
b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1759 vs. $2(292) = 584$.

Time = 151.49 (sec) , antiderivative size = 1760, normalized size of antiderivative = 5.62

method	result	size
default	Expression too large to display	1760

input
```
int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a*(a^2/b/(a^
2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))
+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2/a*b*(1/2*a^2/b/(a^2-b^2)*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2
*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2
*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(
1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**(5/2)/(a + b*sec(c + d*x))**3, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sec(dx+c)^{\frac{5}{2}}}{(b\sec(dx+c)+a)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^2}{\sec(dx+c)^3 b^3+3\sec(dx+c)^2 a\,b^2+3\sec(dx+c)\,a^2b+a^3}dx$$

input
```
int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x)
```

output
```
int((sqrt(sec(c + d*x))*sec(c + d*x)**2)/(sec(c + d*x)**3*b**3 + 3*sec(c +
 d*x)**2*a*b**2 + 3*sec(c + d*x)*a**2*b + a**3),x)
```

3.624 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 306

$$\begin{aligned}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx &= -\frac{(5a^2+b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a(a^2-b^2)^2 d}\\&\quad-\frac{b(7a^2-b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4a^2(a^2-b^2)^2 d}\\&\quad+\frac{(3a^4+10a^2b^2-b^4)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4a^2(a-b)^2(a+b)^3 d}\\&\quad+\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(a+b\sec(c+dx))^2}+\frac{3(a^2+b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{4(a^2-b^2)^2 d(a+b\sec(c+dx))}\end{aligned}$$

output

```
-1/4*(5*a^2+b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*se
c(d*x+c)^(1/2)/a/(a^2-b^2)^2/d-1/4*b*(7*a^2-b^2)*cos(d*x+c)^(1/2)*InverseJ
acobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d+1/4*(3*a
^4+10*a^2*b^2-b^4)*cos(d*x+c)^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b
),2^(1/2))*sec(d*x+c)^(1/2)/a^2/(a-b)^2/(a+b)^3/d+1/2*a*sec(d*x+c)^(1/2)*s
in(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^2+3/4*(a^2+b^2)*sec(d*x+c)^(1/2)*si
n(d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 5.44 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.40

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{\frac{4a^2(3b(a^2+b^2)+a(5a^2+b^2)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2} + \frac{4\cos(c+dx)(b+a\cos(c+dx))\cot(c+dx)(a+b\sec(c+dx))\left(5a^3b+ab^3-5a^3b\sec^2(c+dx)-\right.}{}}{}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^3,x]
```

output
```
((4*a^2*(3*b*(a^2 + b^2) + a*(5*a^2 + b^2)*Cos[c + d*x])*Sin[c + d*x])/(a^
2 - b^2)^2 + (4*Cos[c + d*x]*(b + a*Cos[c + d*x])*Cot[c + d*x]*(a + b*Sec[
c + d*x])*(5*a^3*b + a*b^3 - 5*a^3*b*Sec[c + d*x]^2 - a*b^3*Sec[c + d*x]^2
 + a*b*(5*a^2 + b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c
+ d*x]]*Sqrt[-Tan[c + d*x]^2] + a*(3*a^3 - 5*a^2*b + 3*a*b^2 - b^3)*Ellipt
icF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^
2] - 3*a^4*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c +
 d*x]]*Sqrt[-Tan[c + d*x]^2] - 10*a^2*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[S
ec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] + b^4*Elliptic
Pi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c
+ d*x]^2]))/((a - b)^2*b*(a + b)^2))/(16*a^2*d*(b + a*Cos[c + d*x])^2*Sqrt
[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 2.03 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4331, 27, 3042, 4588, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4331}$$

$$\frac{\int -\frac{-3a\sec^2(c+dx)+4b\sec(c+dx)+a}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{2\left(a^2-b^2\right)}+\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\int \frac{-3a\sec^2(c+dx)+4b\sec(c+dx)+a}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{4\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\int \frac{-3a\csc\left(c+dx+\frac{\pi}{2}\right)^2+4b\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{4588}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{-\frac{\int -\frac{12b\sec(c+dx)a^2-3\left(a^2+b^2\right)\sec^2(c+dx)a+\left(5a^2+b^2\right)a}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{a\left(a^2-b^2\right)}-\frac{3\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{4\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\int \frac{12b\sec(c+dx)a^2-3\left(a^2+b^2\right)\sec^2(c+dx)a+\left(5a^2+b^2\right)a}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2a\left(a^2-b^2\right)}-\frac{3\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{4\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\int \frac{12b\csc\left(c+dx+\frac{\pi}{2}\right)a^2-3\left(a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2a+\left(5a^2+b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a\left(a^2-b^2\right)}-\frac{3\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{4\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{4594}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}-\frac{\frac{\frac{\int\frac{\left(5a^2+b^2\right)a^2+b\left(7a^2-b^2\right)\sec(c+dx)a}{\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{\left(3a^4+10a^2b^2-b^4\right)\int\frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a}}{2a(a^2-b^2)}-\frac{3(a^2+b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}{4\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}-\frac{\frac{\frac{\int\frac{\left(5a^2+b^2\right)a^2+b\left(7a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{\left(3a^4+10a^2b^2-b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2a(a^2-b^2)}-\frac{3(a^2+b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}{4\left(a^2-b^2\right)}$$

$\Big\downarrow$ 4274

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}-\frac{\frac{\frac{a^2\left(5a^2+b^2\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx+ab\left(7a^2-b^2\right)\int\sqrt{\sec(c+dx)}dx}{a^2}-\frac{\left(3a^4+10a^2b^2-b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2a(a^2-b^2)}-\frac{3(a^2+b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}{4\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}-\frac{\frac{\frac{a^2\left(5a^2+b^2\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+ab\left(7a^2-b^2\right)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{\left(3a^4+10a^2b^2-b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2a(a^2-b^2)}-\frac{3(a^2+b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}{4\left(a^2-b^2\right)}$$

$\Big\downarrow$ 4258

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}-\frac{\frac{\frac{a^2\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+ab\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{a^2}-\frac{\left(3a^4+10a^2b^2-b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2a(a^2-b^2)}-}{4\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{a^2(5a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+ab(7a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{(3a^4+10a^2b^2-b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{a}}{2a(a^2-b^2)}}{4(a^2-b^2)}$$

↓ 3119

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{ab(7a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2(5a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{a^2}-\frac{(3a^4+10a^2b^2-b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}d}{a}}{2a(a^2-b^2)}}{4(a^2-b^2)}$$

↓ 3120

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{2ab(7a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2(5a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{a^2}-\frac{(3a^4+10a^2b^2-b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{a}}{2a(a^2-b^2)}}{4(a^2-b^2)}$$

↓ 4336

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{2ab(7a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2(5a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{a^2}-\frac{(3a^4+10a^2b^2-b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{a}}{2a(a^2-b^2)}}{4(a^2-b^2)}$$

↓ 3042

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{2ab(7a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2(5a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{a^2}-\frac{(3a^4+10a^2b^2-b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{}}{2a(a^2-b^2)}}{4(a^2-b^2)}$$

↓ 3284

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\,(a^2-b^2)\,(a+b\sec(c+dx))^2}-\frac{\frac{\frac{\frac{2ab\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}-\frac{2\left(3a^4+10a^2b^2-b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c-}{ad(a+b}}{2a(a^2-b^2)}}{4\,(a^2-b^2)}$$

input
```
Int[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^3,x]
```

output
```
(a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^
2) - ((((2*a^2*(5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/d + (2*a*b*(7*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 - (2*(3*a^4 + 10*a^2*b^2 - b^4)
*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(a*(a + b)*d))/(2*a*(a^2 - b^2)) - (3*(a^2 + b^2)*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x])))/(4*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4331
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 2)/(f*(m + 1)*(a^2 - b^2))), x] - Simp[d^2/((m + 1
)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*
(a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /;
FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2
] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1857 vs. $2(285) = 570$.

Time = 6.53 (sec) , antiderivative size = 1858, normalized size of antiderivative = 6.07

method	result	size
default	Expression too large to display	1858

input
```
int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a/(a^2-a*b)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2
*a/(a-b),2^(1/2))+2/a^2*b^2*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a
+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-3/8
/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^
2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3
/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**(3/2)/(a + b*sec(c + d*x))**3, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)}{\sec(dx+c)^3 b^3+3\sec(dx+c)^2 a\,b^2+3\sec(dx+c)\,a^2b+a^3}dx$$

input
```
int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x)
```

output
```
int((sqrt(sec(c + d*x))*sec(c + d*x))/(sec(c + d*x)**3*b**3 + 3*sec(c + d*
x)**2*a*b**2 + 3*sec(c + d*x)*a**2*b + a**3),x)
```

3.625 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 323

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx &= \frac{3b(3a^2-b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^2(a^2-b^2)^2 d} \\ &\quad + \frac{(8a^4-5a^2b^2+3b^4)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4a^3(a^2-b^2)^2 d} \\ &\quad - \frac{3b(5a^4-2a^2b^2+b^4)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4a^3(a-b)^2(a+b)^3 d} \\ &\quad - \frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(a+b\sec(c+dx))^2} - \frac{b(7a^2-b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a(a^2-b^2)^2 d(a+b\sec(c+dx))}\end{aligned}$$

output

```
3/4*b*(3*a^2-b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*s
ec(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d+1/4*(8*a^4-5*a^2*b^2+3*b^4)*cos(d*x+c)^(
1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)
^2/d-3/4*b*(5*a^4-2*a^2*b^2+b^4)*cos(d*x+c)^(1/2)*EllipticPi(sin(1/2*d*x+1
/2*c),2*a/(a+b),2^(1/2))*sec(d*x+c)^(1/2)/a^3/(a-b)^2/(a+b)^3/d-1/2*b*sec(
d*x+c)^(1/2)*sin(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^2-1/4*b*(7*a^2-b^2)*s
ec(d*x+c)^(1/2)*sin(d*x+c)/a/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 4.82 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{\frac{4b(-7a^2b+b^3+(-9a^3+3ab^2)\cos(c+dx))\sin(c+dx)}{a(a^2-b^2)^2(b+a\cos(c+dx))^2\sqrt{\sec(c+dx)}} + \frac{2\cot(c+dx)\left(6ab(3a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sin^2(c+dx)-6ab(3a^2-b^2)E\left(\arcsin\left(\sqrt{\sec(c+\cdots}\right.\right.\right.}{}}{}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^3,x]
```

output
```
((4*b*(-7*a^2*b + b^3 + (-9*a^3 + 3*a*b^2)*Cos[c + d*x])*Sin[c + d*x])/(a*
(a^2 - b^2)^2*(b + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]) + (2*Cot[c + d*x]
*(6*a*b*(3*a^2 - b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x]^2 - 6*a*b*(3*a^2 - b
^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 2*a*
(7*a^3 - 9*a^2*b - a*b^2 + 3*b^3)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1
]*Sqrt[-Tan[c + d*x]^2] + 6*(5*a^4 - 2*a^2*b^2 + b^4)*EllipticPi[-(b/a), A
rcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(a^3*(a - b)^2*(a +
 b)^2))/(16*d)
```

Rubi [A] (verified)

Time = 2.11 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.99, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4330, 27, 3042, 4588, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \; 4330$$

$$-\frac{\int -\frac{-3b\sec^2(c+dx)+4a\sec(c+dx)+b}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{2(a^2-b^2)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{-3b\sec^2(c+dx)+4a\sec(c+dx)+b}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{4(a^2-b^2)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{-3b\csc\left(c+dx+\frac{\pi}{2}\right)^2+4a\csc\left(c+dx+\frac{\pi}{2}\right)+b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4(a^2-b^2)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 4588$$

$$\frac{-\frac{\int -\frac{-b(7a^2-b^2)\sec^2(c+dx)+4a(2a^2+b^2)\sec(c+dx)+3b(3a^2-b^2)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{a(a^2-b^2)}-\frac{b(7a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}}{4(a^2-b^2)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{-b(7a^2-b^2)\sec^2(c+dx)+4a(2a^2+b^2)\sec(c+dx)+3b(3a^2-b^2)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2a(a^2-b^2)}-\frac{b(7a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}}{4(a^2-b^2)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{-b(7a^2-b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2+4a(2a^2+b^2)\csc\left(c+dx+\frac{\pi}{2}\right)+3b(3a^2-b^2)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}-\frac{b(7a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}}{4(a^2-b^2)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 4594$$

$$\frac{\frac{\frac{\int \frac{3ab\left(3a^2-b^2\right)+\left(8a^4-5b^2a^2+3b^4\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{3b\left(5a^4-2a^2b^2+b^4\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}}{2a(a^2-b^2)}-\frac{b(7a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{3ab\left(3a^2-b^2\right)+\left(8a^4-5b^2a^2+3b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{3b\left(5a^4-2a^2b^2+b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2a(a^2-b^2)}-\frac{b(7a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$\downarrow$ 4274

$$\frac{\frac{\frac{3ab\left(3a^2-b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\left(8a^4-5a^2b^2+3b^4\right)\int \sqrt{\sec(c+dx)}dx}{a^2}-\frac{3b\left(5a^4-2a^2b^2+b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2a(a^2-b^2)}-\frac{b(7a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{3ab\left(3a^2-b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(8a^4-5a^2b^2+3b^4\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{3b\left(5a^4-2a^2b^2+b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2a(a^2-b^2)}-\frac{b(7a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$\downarrow$ 4258

$$\frac{\frac{\frac{3ab\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\left(8a^4-5a^2b^2+3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{a^2}-\frac{3b\left(5a^4-2a^2b^2+b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2a(a^2-b^2)}-\frac{b(7a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$\big\downarrow$ 3042

$$\frac{\frac{\frac{3ab(3a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+(8a^4-5a^2b^2+3b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{3b(5a^4-2a^2b^2+b^4)\int\frac{\csc(c+\cdots}{a+b\csc(\cdots}}{a^2}}{2a(a^2-b^2)}}{4(a^2-b^2)}$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

$\big\downarrow$ 3119

$$\frac{\frac{\frac{(8a^4-5a^2b^2+3b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6ab(3a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{a^2}-\frac{3b(5a^4-2a^2b^2+b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\cdots\right)}{a+b\csc(c+d\cdots}}{a^2}}{2a(a^2-b^2)}}{4(a^2-b^2)}$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

$\big\downarrow$ 3120

$$\frac{\frac{\frac{\frac{6ab(3a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}+\frac{2(8a^4-5a^2b^2+3b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}-\frac{3b(5a^4-2a^2b^2+b^4)\int\frac{\csc(c+dx+\cdots}{a+b\csc(c+\cdots}}{a^2}}{2a(a^2-b^2)}}{4(a^2-b^2)}$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

$\big\downarrow$ 4336

$$\frac{\frac{\frac{\frac{6ab(3a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}+\frac{2(8a^4-5a^2b^2+3b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}-\frac{3b(5a^4-2a^2b^2+b^4)\sqrt{\cos(c+dx)}\cdots}{\cdots}}{2a(a^2-b^2)}}{4(a^2-b^2)}$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

$\big\downarrow$ 3042

$$\frac{\frac{\frac{\frac{6ab(3a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}+\frac{2(8a^4-5a^2b^2+3b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}-\frac{3b(5a^4-2a^2b^2+b^4)\sqrt{\cos(c+dx)}\cdots}{\cdots}}{2a(a^2-b^2)}}{4(a^2-b^2)}$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3284$$

$$\frac{\frac{\frac{\frac{6ab(3a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(8a^4-5a^2b^2+3b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}-\frac{6b(5a^4-2a^2b^2+b^4)\sqrt{\cos(c+dx)}\sqrt{\cdots}}{a^2\cdots}}{2a(a^2-b^2)}\cdots}{4\left(a^2-b^2\right)}$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^3,x]
```

output
```
-1/2*(b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x
])^2) + ((((6*a*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/d + (2*(8*a^4 - 5*a^2*b^2 + 3*b^4)*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 - (6*b*(5*a^4 - 2*
a^2*b^2 + b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d))/(2*a*(a^2 - b^2)) - (b*(7*a^2 - b^2
)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])))
/(4*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4330
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)
*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*S
imp[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1935 vs. $2(302) = 604$.

Time = 6.88 (sec) , antiderivative size = 1936, normalized size of antiderivative = 5.99

method	result	size
default	Expression too large to display	1936

input
```
int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6/
a^2*b/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+6/a^3*b^2*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/
2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x
+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2
-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),2*a/(a-b),2^(1/2)))-2/a^3*b^3*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\sec{(c+dx)}}}{(a+b\sec{(c+dx)})^3}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(sqrt(sec(c + d*x))/(a + b*sec(c + d*x))**3, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\sec{(dx+c)}}}{(b\sec{(dx+c)}+a)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3 b^3+3\sec(dx+c)^2 a\,b^2+3\sec(dx+c)\,a^2b+a^3}dx$$

input
```
int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x)
```

output
```
int(sqrt(sec(c + d*x))/(sec(c + d*x)**3*b**3 + 3*sec(c + d*x)**2*a*b**2 +
3*sec(c + d*x)*a**2*b + a**3),x)
```

3.626 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 342

$$\begin{aligned}\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx &= \frac{(8a^4-29a^2b^2+15b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^3(a^2-b^2)^2d}\\ &- \frac{3b(8a^4-11a^2b^2+5b^4)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4a^4(a^2-b^2)^2d}\\ &+ \frac{b^2(35a^4-38a^2b^2+15b^4)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4a^4(a-b)^2(a+b)^3d}\\ &+ \frac{b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{2a(a^2-b^2)d(a+b\sec(c+dx))^2} + \frac{b^2(11a^2-5b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a^2(a^2-b^2)^2d(a+b\sec(c+dx))}\end{aligned}$$

output
```
1/4*(8*a^4-29*a^2*b^2+15*b^4)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c
),2^(1/2))*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)^2/d-3/4*b*(8*a^4-11*a^2*b^2+5*b^
4)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2
)/a^4/(a^2-b^2)^2/d+1/4*b^2*(35*a^4-38*a^2*b^2+15*b^4)*cos(d*x+c)^(1/2)*El
lipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*sec(d*x+c)^(1/2)/a^4/(a-b)^
2/(a+b)^3/d+1/2*b^2*sec(d*x+c)^(1/2)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x
+c))^2+1/4*b^2*(11*a^2-5*b^2)*sec(d*x+c)^(1/2)*sin(d*x+c)/a^2/(a^2-b^2)^2/
d/(a+b*sec(d*x+c))
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 707 vs. $2(342) = 684$.

Time = 6.63 (sec) , antiderivative size = 707, normalized size of antiderivative = 2.07

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{\frac{2(8a^4-7a^2b^2+5b^4)\cos^2(c+dx)\left(\text{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)-\text{EllipticPi}\left(-\frac{b}{a},\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)\right)(a+b\sec(c+dx))\sqrt{1-\sec^2(c+dx)}}{b(b+a\cos(c+dx))(1-\cos^2(c+dx))}}{}$$
$$+\frac{\sqrt{\sec(c+dx)}\left(-\frac{b^2(-13a^2+7b^2)\sin(c+dx)}{4a^3(-a^2+b^2)^2}+\frac{b^4\sin(c+dx)}{2a^3(a^2-b^2)(b+a\cos(c+dx))^2}+\frac{3(-5a^2b^3\sin(c+dx)+3b^5\sin(c+dx))}{4a^3(a^2-b^2)^2(b+a\cos(c+dx))}\right)}{d}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3),x]
```

output
```
((2*(8*a^4 - 7*a^2*b^2 + 5*b^4)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[
c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a +
 b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c +
d*x])*(1 - Cos[c + d*x]^2)) + (2*(-32*a^3*b + 8*a*b^3)*Cos[c + d*x]^2*Elli
pticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1
 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]
^2)) + ((8*a^4 - 29*a^2*b^2 + 15*b^4)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x]
)*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x
]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*Ellip
ticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d
*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec
[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt
[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c +
d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*
(2 - Sec[c + d*x]^2)))/(16*a^2*(a - b)^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]
]*(-1/4*(b^2*(-13*a^2 + 7*b^2)*Sin[c + d*x])/(a^3*(-a^2 + b^2)^2) + (b^4*S
in[c + d*x])/(2*a^3*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (3*(-5*a^2*b^3*S
in[c + d*x] + 3*b^5*Sin[c + d*x]))/(4*a^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x
]))))/d
```

Rubi [A] (verified)

Time = 2.29 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4334, 27, 3042, 4588, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4334}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\int -\frac{4a^2-4b\sec(c+dx)a-5b^2+3b^2\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4a^2-4b\sec(c+dx)a-5b^2+3b^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{4a^2-4b\csc\left(c+dx+\frac{\pi}{2}\right)a-5b^2+3b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{4588}$$

$$\frac{\frac{b^2\left(11a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int -\frac{8a^4-29b^2a^2-4b\left(4a^2-b^2\right)\sec(c+dx)a+15b^4+b^2\left(11a^2-5b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{a\left(a^2-b^2\right)}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{8a^4-29b^2a^2-4b(4a^2-b^2)\sec(c+dx)a+15b^4+b^2(11a^2-5b^2)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2a(a^2-b^2)}+\frac{b^2(11a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}}{4a(a^2-b^2)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{8a^4-29b^2a^2-4b(4a^2-b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a+15b^4+b^2(11a^2-5b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}+\frac{b^2(11a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}}{4a(a^2-b^2)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 4594

$$\frac{\frac{\frac{b^2(35a^4-38a^2b^2+15b^4)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}+\frac{\int \frac{a(8a^4-29b^2a^2+15b^4)-3b(8a^4-11b^2a^2+5b^4)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}}{2a(a^2-b^2)}+\frac{b^2(11a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}}{4a(a^2-b^2)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{b^2(35a^4-38a^2b^2+15b^4)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\int \frac{a(8a^4-29b^2a^2+15b^4)-3b(8a^4-11b^2a^2+5b^4)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{2a(a^2-b^2)}+\frac{b^2(11a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}}{4a(a^2-b^2)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 4274

$$\frac{\frac{\frac{b^2(35a^4-38a^2b^2+15b^4)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a(8a^4-29a^2b^2+15b^4)\int \frac{1}{\sqrt{\sec(c+dx)}}dx-3b(8a^4-11a^2b^2+5b^4)\int \sqrt{\sec(c+dx)}dx}{a^2}}{2a(a^2-b^2)}+\frac{b^2(11a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}}{4a(a^2-b^2)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\left(8a^4-29a^2b^2+15b^4\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-3b\left(8a^4-11a^2b^2+5b^4\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2a(a^2-b^2)}+\frac{b^2(11a^2-5}{ad(a^2}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$\downarrow$ 4258

$$\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-3b\left(8a^4-11a^2b^2+5b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c-}}{a^2}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-3b\left(8a^4-11a^2b^2+5b^4\right)\sqrt{\cos(c+dx)}\sqrt{se}}{a^2}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$\downarrow$ 3119

$$\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{2a\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-3b\left(8a^4-11a^2b^2+5b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{a^2}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$\downarrow$ 3120

$$\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{2a\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{6b\left(8a^4-11a^2b^2+5b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{d}}{a^2}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}$$

$$\Big\downarrow \ 4336$$

$$\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx}{a^2}+\frac{\frac{2a\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{6b\left(8a^4-\right.}{a^2}}{}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2}+\frac{2a\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \ 3284$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+$$

$$\frac{\frac{b^2\left(11a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}+\frac{\frac{2b^2\left(35a^4-38a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a^2d(a+b)}+\frac{2a\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c-}}{}}{2a(a^2-b^2}}{4a\left(a^2-b^2\right)}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3),x]
```

output
```
(b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*
x])^2) + ((((2*a*(8*a^4 - 29*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (6*b*(8*a^4 - 11*a^2*b^2 + 5*b^4
)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2
+ (2*b^2*(35*a^4 - 38*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a
)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d))/(2*a*(a^2
- b^2)) + (b^2*(11*a^2 - 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 -
 b^2)*d*(a + b*Sec[c + d*x])))/(4*a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1956 vs. 2(321) = 642.

Time = 8.01 (sec) , antiderivative size = 1957, normalized size of antiderivative = 5.72

method	result	size
default	Expression too large to display	1957

input
```
int(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^4/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))+a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2/a^4*b^4*(1/2*a^2/b/(a^2-b^2)
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
2*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos
(1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^
2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a
/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{(a+b\sec(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**3*sqrt(sec(c + d*x))), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^4b^3+3\sec(dx+c)^3ab^2+3\sec(dx+c)^2a^2b+\sec(dx+c)a^3}dx$$

input
```
int(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x)
```

output
```
int(sqrt(sec(c + d*x))/(sec(c + d*x)**4*b**3 + 3*sec(c + d*x)**3*a*b**2 +
3*sec(c + d*x)**2*a**2*b + sec(c + d*x)*a**3),x)
```

3.627 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 406

$$\begin{aligned}
&\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx \\
&= -\frac{b(24a^4-65a^2b^2+35b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^4(a^2-b^2)^2d} \\
&\quad+\frac{(8a^6+128a^4b^2-223a^2b^4+105b^6)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{12a^5(a^2-b^2)^2d} \\
&\quad-\frac{b^3(63a^4-86a^2b^2+35b^4)\sqrt{\cos(c+dx)}\operatorname{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4a^5(a-b)^2(a+b)^3d} \\
&\quad+\frac{(8a^4-61a^2b^2+35b^4)\sin(c+dx)}{12a^3(a^2-b^2)^2d\sqrt{\sec(c+dx)}}+\frac{b^2\sin(c+dx)}{2a(a^2-b^2)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2} \\
&\quad+\frac{b^2(13a^2-7b^2)\sin(c+dx)}{4a^2(a^2-b^2)^2d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}
\end{aligned}$$

output
```
-1/4*b*(24*a^4-65*a^2*b^2+35*b^4)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))*sec(d*x+c)^(1/2)/a^4/(a^2-b^2)^2/d+1/12*(8*a^6+128*a^4*b^2-
223*a^2*b^4+105*b^6)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2
))*sec(d*x+c)^(1/2)/a^5/(a^2-b^2)^2/d-1/4*b^3*(63*a^4-86*a^2*b^2+35*b^4)*c
os(d*x+c)^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*sec(d*x+c
)^(1/2)/a^5/(a-b)^2/(a+b)^3/d+1/12*(8*a^4-61*a^2*b^2+35*b^4)*sin(d*x+c)/a^
3/(a^2-b^2)^2/d/sec(d*x+c)^(1/2)+1/2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/sec(d*x+
c)^(1/2)/(a+b*sec(d*x+c))^2+1/4*b^2*(13*a^2-7*b^2)*sin(d*x+c)/a^2/(a^2-b^2
)^2/d/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 6.75 (sec) , antiderivative size = 731, normalized size of antiderivative = 1.80

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{\frac{2(-56a^4b+73a^2b^3-35b^5)\cos^2(c+dx)\left(\text{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)-\text{EllipticPi}\left(-\frac{b}{a},\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)\right)(a+b\sec(c+dx))\sqrt{1-}}{b(b+a\cos(c+dx))(1-\cos^2(c+dx))}}{}$$
$$+\frac{\sqrt{\sec(c+dx)}\left(\frac{b^3(-17a^2+11b^2)\sin(c+dx)}{4a^4(-a^2+b^2)^2}-\frac{b^5\sin(c+dx)}{2a^4(a^2-b^2)(b+a\cos(c+dx))^2}+\frac{19a^2b^4\sin(c+dx)-13b^6\sin(c+dx)}{4a^4(a^2-b^2)^2(b+a\cos(c+dx))}+\frac{\sin(2(c+dx))}{3a^3}\right.}{d}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3),x]
```

output
```
((2*(-56*a^4*b + 73*a^2*b^3 - 35*b^5)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqr
t[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]
)*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Co
s[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(16*a^5 + 112*a^3*b^2 - 56*a*b^4)*C
os[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Se
c[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])
*(1 - Cos[c + d*x]^2)) + ((-72*a^4*b + 195*a^2*b^3 - 105*b^5)*Cos[2*(c + d
*x)]*(a + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE
[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^
2] - 2*a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c +
d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[
c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*Ellipt
icPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - S
ec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x
]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(48*a^3*(a - b)^2*(a + b)^2
*d) + (Sqrt[Sec[c + d*x]]*((b^3*(-17*a^2 + 11*b^2)*Sin[c + d*x])/(4*a^4*(-
a^2 + b^2)^2) - (b^5*Sin[c + d*x])/(2*a^4*(a^2 - b^2)*(b + a*Cos[c + d*x])
^2) + (19*a^2*b^4*Sin[c + d*x] - 13*b^6*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2
*(b + a*Cos[c + d*x])) + Sin[2*(c + d*x)]/(3*a^3)))/d
```

Rubi [A] (verified)

Time = 2.87 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.99, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.913$, Rules used = {3042, 4334, 27, 3042, 4588, 27, 3042, 4592, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{ 4334}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}-\frac{\int -\frac{4a^2-4b\sec(c+dx)a-7b^2+5b^2\sec^2(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}dx}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{4a^2-4b\sec(c+dx)a-7b^2+5b^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}dx}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{4a^2-4b\csc\left(c+dx+\frac{\pi}{2}\right)a-7b^2+5b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{ 4588}$$

$$\frac{\frac{b^2\left(13a^2-7b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}-\frac{\int -\frac{8a^4-61b^2a^2-4b\left(4a^2-b^2\right)\sec(c+dx)a+35b^4+3b^2\left(13a^2-7b^2\right)\sec^2(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}dx}{a(a^2-b^2)}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{8a^4-61b^2a^2-4b\left(4a^2-b^2\right)\sec(c+dx)a+35b^4+3b^2\left(13a^2-7b^2\right)\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}dx}{2a(a^2-b^2)}+\frac{b^2\left(13a^2-7b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int \frac{8a^4-61b^2a^2-4b\left(4a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+35b^4+3b^2\left(13a^2-7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}+\frac{b^2\left(13a^2-7b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{ 4592}$$

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{2\int \frac{-b\left(8a^4-61b^2a^2+35b^4\right)\sec^2(c+dx)-4a\left(2a^4+14b^2a^2-7b^4\right)\sec(c+dx)+3b\left(24a^4-65b^2a^2+35b^4\right)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{3a}}{2a(a^2-b^2)}+\frac{b^2\left(13a \ldots\right.}{ad(a^2-b^2)\sqrt{s \ldots}}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$\downarrow$ 27

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{-b\left(8a^4-61b^2a^2+35b^4\right)\sec^2(c+dx)-4a\left(2a^4+14b^2a^2-7b^4\right)\sec(c+dx)+3b\left(24a^4-65b^2a^2+35b^4\right)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{3a}}{2a(a^2-b^2)}+\frac{b^2\left(13a^2 \ldots\right.}{ad(a^2-b^2)\sqrt{\sec \ldots}}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{-b\left(8a^4-61b^2a^2+35b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-4a\left(2a^4+14b^2a^2-7b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)+3b\left(24a^4-65b^2a^2+35b^4\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3a}}{2a(a^2-b^2)}+\frac{\ldots}{ad(a^2- \ldots}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$\downarrow$ 4594

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}+\frac{\int \frac{3ab\left(24a^4-65b^2a^2+35b^4\right)-\left(8a^6+128b^2a^4-223b^4a^2+105b^6\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}d \ldots}{a^2}}{3a}}{2a(a^2-b^2)} \ldots}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\int\frac{3ab\left(24a^4-65b^2a^2+35b^4\right)+\left(-8a^6-128b^2a^4+223b^4a^2-105b^6\right)\csc(c}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}}{a^2}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$\downarrow$ 4274

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\left(24a^4-65a^2b^2+35b^4\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx-\left(8a^6+128a^4b^2-223a^2b^4+\right.}{a^2}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\left(24a^4-65a^2b^2+35b^4\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\left(8a^6+128a^4b^2-223a\right.}{a^2}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$\downarrow$ 4258

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-\left(8\right.}{}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3119

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{6ab\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\left(8a\right.}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3120

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{6ab\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-2\left(\right.}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 4336

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx}{a^2}+\frac{\frac{6ab\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\cos(c+dx}}{d}}{3a}}{2a(a^2-b^2)}}{4a\left(a\right.}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2}+\frac{6ab\left(24a^4-65a^2b^2+35b^4\right)\sqrt{}}{3a}}{2a(a^2-b^2)}$$

$$4a$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{3284}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}+$$

$$\frac{\frac{b^2(13a^2-7b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}+\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{6b^3\left(63a^4-86a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx\right.}{a^2d(a+b)}}{}}{4a\left(a^2\right.}$$

input
```
Int[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3),x]
```

output
```
(b^2*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*
x])^2) + ((b^2*(13*a^2 - 7*b^2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Sec[c
+ d*x]]*(a + b*Sec[c + d*x])) + (-1/3*(((6*a*b*(24*a^4 - 65*a^2*b^2 + 35*b
^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (
2*(8*a^6 + 128*a^4*b^2 - 223*a^2*b^4 + 105*b^6)*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (6*b^3*(63*a^4 - 86*a^2*b^
2 + 35*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(a^2*(a + b)*d))/a + (2*(8*a^4 - 61*a^2*b^2 + 35*b^4)*S
in[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]))/(2*a*(a^2 - b^2)))/(4*a*(a^2 - b^
2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2215 vs. $2(381) = 762$.

Time = 8.24 (sec) , antiderivative size = 2216, normalized size of antiderivative = 5.46

method	result	size
default	Expression too large to display	2216

input
```
int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(a^2+3*a*b+6
*b^2)/a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+4/3/a^3*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-2/a^4
*(2*a+3*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+20*b^3/a^4/(a^2-a
*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),2*a/(a-b),2^(1/2))+10/a^5*b^4*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a
+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/...
```

Fricas [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(sqrt(sec(d*x + c))/(b^3*sec(d*x + c)^5 + 3*a*b^2*sec(d*x + c)^4 +
 3*a^2*b*sec(d*x + c)^3 + a^3*sec(d*x + c)^2), x)
```

Sympy [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{(a+b\sec{(c+dx)})^3\sec^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**3*sec(c + d*x)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{(b\sec{(dx+c)}+a)^3\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^5 b^3+3\sec(dx+c)^4 a\,b^2+3\sec(dx+c)^3 a^2b+\sec(dx+c)^2 a^3}dx$$

input
```
int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x)
```

output
```
int(sqrt(sec(c + d*x))/(sec(c + d*x)**5*b**3 + 3*sec(c + d*x)**4*a*b**2 +
3*sec(c + d*x)**3*a**2*b + sec(c + d*x)**2*a**3),x)
```

3.628 $\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 237

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx &= \frac{b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} \\ &\quad+\frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} \\ &\quad-\frac{E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} \\ &\quad+\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d}\end{aligned}$$

output
```
b*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(a/
(a+b))^(1/2))*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+a*((b+a*cos(d*x+c)
)/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*se
c(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)-EllipticE(sin(1/2*d*x+1/2*c),2^(1/
2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2
)/sec(d*x+c)^(1/2)+sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.86 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.35

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \frac{\sqrt{a+b\sec(c+dx)}\left(\frac{2a\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{(a+b)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}-\frac{2i\sqrt{-\frac{a(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{a(1+\cos(c+dx))}{a-b}}\csc(c+dx)\left(-2b(a+b)E\left(i\text{arcsinh}\left(\right.\right.\right.}{}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(Sqrt[a + b*Sec[c + d*x]]*((2*a*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])
/((a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - ((2*I)*Sqrt[-((a*(-1 + Cos
[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-
2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]
]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt
[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh
[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(a*Sqr
t[(a - b)^(-1)]*b*Sqrt[b + a*Cos[c + d*x]]) + 4*Tan[c + d*x]))/(4*d*Sqrt[S
ec[c + d*x]])
```

Rubi [A] (verified)

Time = 2.33 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.07, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.920$, Rules used = {3042, 4342, 25, 3042, 4597, 3042, 4346, 3042, 3286, 3042, 3284, 4349, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 4342$$

$$\frac{1}{2}\int -\frac{a-a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \ 25$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{1}{2}\int \frac{a-a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \ 3042$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{1}{2}\int \frac{a-a\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \ 4597$$

$$\frac{1}{2}\left(a\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-a\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{2}\left(a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \ 4346$$

$$\frac{1}{2}\left(\frac{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}-a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{1}{2}\left(\frac{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}-a\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+\right.}}\right.}{\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}}$$

$\downarrow$ 3286

$$\frac{\frac{1}{2}\left(\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-a\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+}{\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}}$$

$\downarrow$ 3042

$$\frac{\frac{1}{2}\left(\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-a\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+d\right.}}\right.}{\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}}$$

$\downarrow$ 3284

$$\frac{\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\right.}}\right.}{\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}}$$

$\downarrow$ 4349

$$\frac{\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{b\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}\right.\right.}{\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{b\int\frac{\sqrt{\csc(c+dx+}}{\sqrt{a+b\csc(c+dx}}}{a}\right.\right.$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 4343

$$\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}\right.\right.$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx\right.}}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}\right.\right.$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3134

$$\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}d}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.\right.$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin(c+dx+}{a+b}}}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.\right.$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3132}$$

$$\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\right.\right.$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4345}$$

$$\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\right.\right.$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\right.\right.$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3142}$$

$$\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\right.\right.$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\right.\right.$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3140}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}+$$

$$\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\right.\right.$$

input
```
Int[Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
((2*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/
(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) - a*((-2*b*Sqrt[
(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[S
ec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*EllipticE[(c + d*x)/2, (
2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a
+ b)]*Sqrt[Sec[c + d*x]])))/2 + (Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x
]]*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4342
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d*Cos[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*
Csc[e + f*x])^(n - 1)/(f*(2*n - 1))), x] + Simp[d^2/(2*n - 1)   Int[(d*Csc[
e + f*x])^(n - 2)*(Simp[2*a*(n - 2) + b*(2*n - 3)*Csc[e + f*x] + a*Csc[e +
f*x]^2, x]/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4343

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4349

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Cs
c[e + f*x]], x], x] - Simp[b/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Cs
c[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4597

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[C/d^
2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[A
Int[1/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.89 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.03

method	result
default	$-\frac{\left(\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\,a\,\mathrm{EllipticPi}\left(\sqrt{\frac{a-b}{a+b}}\,(-\cot(dx+c)+\csc(dx+c)),\frac{a+b}{a-b},\frac{i}{\sqrt{\frac{a-b}{a+b}}}\right)\left(-2\cos(dx+c)^3-4\cos(dx\right.\right.}{}$

input
```
int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d/((a-b)/(a+b))^(1/2)*((1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*a*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc
(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(-2*cos(d*x+c)^3-4*cos(d*x+c)^
2-2*cos(d*x+c))+(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*a*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-
(a+b)/(a-b))^(1/2))*(cos(d*x+c)^3+2*cos(d*x+c)^2+cos(d*x+c))+(1/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b*EllipticE(((
a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-cos(d*x
+c)^3-2*cos(d*x+c)^2-cos(d*x+c))-((a-b)/(a+b))^(1/2)*a*cos(d*x+c)*sin(d*x+
c)-((a-b)/(a+b))^(1/2)*b*sin(d*x+c))*cos(d*x+c)*sec(d*x+c)^(3/2)*(a+b*sec(
d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+b*cos(d*x+c)+b)
```

Fricas [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+b\sec{(c+dx)}}\sec^{\frac{3}{2}}{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec{(dx+c)}+a}\sec{(dx+c)}^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec{(dx+c)}+a}\sec{(dx+c)}^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx=\int \sqrt{a+\frac{b}{\cos(c+d\,x)}}\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx=\int \sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)\,dx$$

input
```
int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a)*sec(c + d*x),x)
```

3.629 $\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 138

$$\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \frac{2a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}$$

output
```
2*a*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(
a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+2*b*((b+a*cos(d*
x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2)
)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 12.63 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.70

$$\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \frac{2\left(a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+b\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\right)\sqrt{a+b\sec(c+dx)}}{(a+b)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*(a*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + b*EllipticPi[2, (c + d*x)/2,
 (2*a)/(a + b)])*Sqrt[a + b*Sec[c + d*x]])/((a + b)*d*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3042, 4341, 3042, 4345, 3042, 3142, 3042, 3140, 4346, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4341}$$

$$b\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+a\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{\sqrt{\csc \left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc \left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\csc \left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc \left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4345}$$

$$b\int \frac{\csc \left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc \left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$b\int \frac{\csc \left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc \left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3142}$$

$$b\int \frac{\csc \left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc \left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$b\int \frac{\csc \left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc \left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3140}$$

$$b\int \frac{\csc \left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc \left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{4346}$$

$$\frac{b\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3286}$$

$$\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3284}$$

$$\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

input
```
Int[Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c
+ d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4341
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[a   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e +
f*x]], x], x] + Simp[b/d   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*
x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.44 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.82

method	result
default	$\frac{2\left(\operatorname{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(-\cot(dx+c)+\csc(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)a-\operatorname{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(-\cot(dx+c)+\csc(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)b+2\,\operatorname{EllipticPi}\right.}{d\sqrt{\frac{a-b}{a+b}}\,(b+a\cos(dx+c))\sqrt{}}$

input
```
int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/((a-b)/(a+b))^(1/2)*(EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*
x+c)),(-(a+b)/(a-b))^(1/2))*a-EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+c
sc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b+2*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(
d*x+c)+csc(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b)*sec(d*x+c)^(1/2)*
(a+b*sec(d*x+c))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos
(d*x+c)/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+1))^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+b\sec{(c+dx)}}\sqrt{\sec{(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))*sqrt(sec(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec{(dx+c)}+a}\sqrt{\sec{(dx+c)}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx=\int \sqrt{b\sec\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx=\int \sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx=\int \sqrt{\sec\left(dx+c\right)}\,\sqrt{\sec\left(dx+c\right)b+a}\,dx$$

input
```
int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a),x)
```

3.630 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 67

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \frac{2E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$

output
```
2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(
1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \frac{2E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4343, 3042, 3134, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4343}$$

$$\frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}$$

$$\Big\downarrow \text{3134}$$

$$\frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$$\Big\downarrow \text{3132}$$

$$\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 537 vs. $2(66) = 132$.

Time = 3.35 (sec) , antiderivative size = 538, normalized size of antiderivative = 8.03

method	result
default	$\frac{2\left(\left(-\cos(dx+c)^2-2\cos(dx+c)-1\right)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,a\,\text{EllipticE}\left(\sqrt{\frac{a-b}{a+b}}\,(\cot(dx+c)-\csc(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)+\left(\right.}{}$
risch	Expression too large to display

input
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/((a-b)/(a+b))^(1/2)*((-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*EllipticE(((a-b)/
(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))+(cos(d*x+c)^2+2
*cos(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*b*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a
+b)/(a-b))^(1/2))+(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*EllipticF(((a-b)/(a+b))^(
1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*
x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*b*EllipticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-
b))^(1/2))+((a-b)/(a+b))^(1/2)*a*cos(d*x+c)*sin(d*x+c)+((a-b)/(a+b))^(1/2)
*b*sin(d*x+c))*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+b*cos(d
*x+c)+b)/sec(d*x+c)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 355, normalized size of antiderivative = 5.30

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \frac{-i\sqrt{2}\sqrt{a}b\,\text{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)+i\sqrt{2}\sqrt{a}b\,\text{weierstra}}{}$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*sqrt(a)*b*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/
27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b
)/a) + I*sqrt(2)*sqrt(a)*b*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*
b)/a) + 3*I*sqrt(2)*a^(3/2)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27
*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27
*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/
a)) - 3*I*sqrt(2)*a^(3/2)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(
9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(
9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)
))/(a*d)
```

Sympy [F]

$$\int \frac{\sqrt{a + b\sec(c + dx)}}{\sqrt{\sec(c + dx)}}\,dx = \int \frac{\sqrt{a + b\sec{(c + dx)}}}{\sqrt{\sec{(c + dx)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))/sqrt(sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a + b\sec(c + dx)}}{\sqrt{\sec(c + dx)}}\,dx = \int \frac{\sqrt{b\sec{(dx + c)} + a}}{\sqrt{\sec{(dx + c)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sqrt{b\sec(dx+c)+a}}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)}dx$$

input
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/sec(c + d*x),x)
```

3.631 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 192

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3ad\sqrt{a+b\sec(c+dx)}} + \frac{2bE\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
2/3*(a^2-b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c
,2^(1/2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/a/d/(a+b*sec(d*x+c))^(1/2)+2/3*
b*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(
1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+2/3*(a+b*sec(d*x+
c))^(1/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2\sqrt{a+b\sec(c+dx)}\left(b(a+b)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+\right.\right.}{3ad(b+a\cos(c+dx))\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

output
```
(2*Sqrt[a + b*Sec[c + d*x]]*(b*(a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*
EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a^2 - b^2)*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + a*(b + a*Cos[c + d*x]
)*Sin[c + d*x]))/(3*a*d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4344, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4344}$$

$$\frac{1}{3}\int \frac{b+a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\int \frac{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\big\downarrow$ 4523

$$\frac{1}{3}\left(\frac{(a^2-b^2)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}+\frac{b\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\big\downarrow$ 4343

$$\frac{1}{3}\left(\frac{(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\big\downarrow$ 3134

$$\frac{1}{3}\left(\frac{(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3132

$$\frac{1}{3}\left(\frac{(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 4345

$$\frac{1}{3}\left(\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3142

$$\frac{1}{3}\left(\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{1}{3}\left(\frac{2(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

output
```
((2*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*E
llipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/3 + (2*Sqrt[a + b*Sec[c
+ d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4344
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*Csc[e
 + f*x])^n/(f*n)), x] - Simp[1/(2*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*(Simp
[b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x]/Sqrt[a + b*C
sc[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] &&
LeQ[n, -1] && IntegerQ[2*n]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(179) = 358$.

Time = 6.28 (sec) , antiderivative size = 572, normalized size of antiderivative = 2.98

method	result
default	$\frac{2\sqrt{a+b\sec(dx+c)}\left(\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\sqrt{\frac{1}{\cos(dx+c)+1}}\,ab\,\text{EllipticE}\left(\sqrt{\frac{a-b}{a+b}}\,(\cot(dx+c)-\csc(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)(-\cos(dx+c)-2-\right.}{}$

input
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/a/((a-b)/(a+b))^(1/2)*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d
*x+c)+b*cos(d*x+c)+b)/sec(d*x+c)^(3/2)*((1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b*EllipticE(((a-b)/(a+b))^(1/2)*(
cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-cos(d*x+c)-2-sec(d*x+c))+(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^2
*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2
))*(2+cos(d*x+c)+sec(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*EllipticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-
csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-cos(d*x+c)-2-sec(d*x+c))+(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b*EllipticF(
((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(2+cos(d
*x+c)+sec(d*x+c))+sin(d*x+c)*(cos(d*x+c)+1)*((a-b)/(a+b))^(1/2)*a^2+((a-b)
/(a+b))^(1/2)*a*b*(2*sin(d*x+c)+tan(d*x+c))+((a-b)/(a+b))^(1/2)*b^2*tan(d*
x+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.16

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{6\,a^2\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+3i\,\sqrt{2}a^{\frac{3}{2}}b\text{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\text{weiers}\right.}{}$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/9*(6*a^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c) + 3*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) +
2*b)/a)) - 3*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) +
2*b)/a)) + sqrt(2)*(-3*I*a^2 + 2*I*b^2)*sqrt(a)*weierstrassPInverse(-4/3*(
3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*
I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(3*I*a^2 - 2*I*b^2)*sqrt(a)*weierstra
ssPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*
cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a))/(a^2*d)
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sqrt{a+b\sec{\left(c+dx \right)}}}{\sec^{\frac{3}{2}}{\left(c+dx \right)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(c+dx)}}}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{2}}dx$$

input
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/sec(c + d*x)**2,x)
```

3.632 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 244

$$\begin{aligned}\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx &= -\frac{4b(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{15a^2d\sqrt{a+b\sec(c+dx)}}\\ &\quad+\frac{2(9a^2-2b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{15a^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}\\ &\quad+\frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15ad\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
-4/15*b*(a^2-b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1
/2*c,2^(1/2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/a^2/d/(a+b*sec(d*x+c))^(1/2
)+2/15*(9*a^2-2*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))
*(a+b*sec(d*x+c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1
/2)+2/5*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/15*b*(a+b*s
ec(d*x+c))^(1/2)*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{a+b\sec(c+dx)}\Big(4(9a^3+9a^2b-2ab^2-2b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+8b(-a^2+b^2)\sqrt{\frac{b+a\,\mathrm{co}}{a}}}{30a^2d(b+a\cos(c+}$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/2),x]
```

output
```
(Sqrt[a + b*Sec[c + d*x]]*(4*(9*a^3 + 9*a^2*b - 2*a*b^2 - 2*b^3)*Sqrt[(b +
 a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 8*b*(-a^
2 + b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(
a + b)] + 2*a*(3*a^2 + 2*b^2 + 8*a*b*Cos[c + d*x] + 3*a^2*Cos[2*(c + d*x)]
)*Sin[c + d*x]))/(30*a^2*d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 1.91 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.04, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 4344, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4344}$$

$$\frac{1}{5}\int \frac{2b\sec^2(c+dx)+3a\sec(c+dx)+b}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)^2+3a\csc\left(c+dx+\frac{\pi}{2}\right)+b}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 4592

$$\frac{1}{5}\left(\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{2\int -\frac{9a^2+7b\sec(c+dx)a-2b^2}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{5}\left(\frac{\int \frac{9a^2+7b\sec(c+dx)a-2b^2}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{\int \frac{9a^2+7b\csc\left(c+dx+\frac{\pi}{2}\right)a-2b^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 4523

$$\frac{1}{5}\left(\frac{\frac{(9a^2-2b^2)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{2b(a^2-b^2)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{\frac{(9a^2-2b^2)\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{2b(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 4343

$$\frac{1}{5}\left(\frac{\frac{(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{2b(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{\frac{(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{2b(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3134

$$\frac{1}{5}\left(\frac{\frac{(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{\frac{(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3132

$$\frac{1}{5}\left(\frac{\frac{2(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}\right)+$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 4345

$$\frac{1}{5}\left(\frac{\frac{2(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{a\sqrt{a+b\sec(c+dx)}}}{3a}+\frac{2b\sin(c+dx)\sqrt{\right.}{3ad\sqrt{\sec}}$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{\frac{2(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\sqrt{a+b\sec(c+dx)}}}{3a}+\frac{2b\sin(c+dx)}{3ad\sqrt{}}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3142

$$\frac{1}{5}\left(\frac{\frac{2(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}}{3a}+\frac{2b\sin(c+dx)}{3ad\sqrt{s}}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{\frac{2(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}}{3a}+\frac{2b\sin(c+d}{3ad}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3140

$$\frac{1}{5}\left(\frac{\frac{2(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{4b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}}{3a}+\frac{2b\sin(c+dx)}{3ad\sqrt{}}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/2),x]
```

output
```
(2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (((-4
*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(9*a
^2 - 2*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]]
)/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(3*a) + (2*
b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4344
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*Csc[e
 + f*x])^n/(f*n)), x] - Simp[1/(2*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*(Simp
[b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x]/Sqrt[a + b*C
sc[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] &&
LeQ[n, -1] && IntegerQ[2*n]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 983 vs. 2(225) = 450.

Time = 8.43 (sec) , antiderivative size = 984, normalized size of antiderivative = 4.03

method	result	size
default	Expression too large to display	984

input
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d/a^2/((a-b)/(a+b))^(1/2)*((-9*cos(d*x+c)^2-18*cos(d*x+c)-9)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3*Ellip
ticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))+(9*
cos(d*x+c)^2+18*cos(d*x+c)+9)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+
c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))+(2*cos(d*x+c)^2+4*cos(d*x+c)+2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2*
EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2)
)+(-2*cos(d*x+c)^2-4*cos(d*x+c)-2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^3*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d
*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))+(9*cos(d*x+c)^2+18*cos(d*x+c)+9)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^
3*EllipticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/
2))+(-7*cos(d*x+c)^2-14*cos(d*x+c)-7)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b*EllipticF(((a-b)/(a+b))^(1/2)*(
cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-2*cos(d*x+c)^2-4*cos(d*x+c)
-2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*a*b^2*EllipticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-
b))^(1/2))+sin(d*x+c)*cos(d*x+c)*(3*cos(d*x+c)^2+3*cos(d*x+c)+9)*((a-b)/(a
+b))^(1/2)*a^3+(4*cos(d*x+c)^2+4*cos(d*x+c)+9)*sin(d*x+c)*((a-b)/(a+b))...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.89

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{\sqrt{2}(-3i\,a^2b-4i\,b^3)\sqrt{a}\text{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)+\sqrt{2}(3}{}$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/45*(sqrt(2)*(-3*I*a^2*b - 4*I*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(3*a
^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a
*sin(d*x + c) + 2*b)/a) + sqrt(2)*(3*I*a^2*b + 4*I*b^3)*sqrt(a)*weierstras
sPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*c
os(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(-9*I*a^3 + 2*I*a*b
^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^
3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^
3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*
(9*I*a^3 - 2*I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/
27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/
27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b
)/a)) + 6*(3*a^3*cos(d*x + c)^2 + a^2*b*cos(d*x + c))*sqrt((a*cos(d*x + c)
 + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d)
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\sec{(c+dx)}}}{\sec^{\frac{5}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(5/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{3}}dx$$

input
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/sec(c + d*x)**3,x)
```

3.633 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 305

$$\begin{aligned}\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx &= \frac{2(25a^4-17a^2b^2-8b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{105a^3d\sqrt{a+b\sec(c+dx)}}\\ &\quad+\frac{2b(19a^2+8b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{105a^3d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}\\ &\quad+\frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35ad\sec^{\frac{3}{2}}(c+dx)}\\ &\quad+\frac{2(25a^2-4b^2)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{105a^2d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
2/105*(25*a^4-17*a^2*b^2-8*b^4)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJaco
biAM(1/2*d*x+1/2*c,2^(1/2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/a^3/d/(a+b*se
c(d*x+c))^(1/2)+2/105*b*(19*a^2+8*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2
)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^3/d/((b+a*cos(d*x+c))/(a+b))^(
1/2)/sec(d*x+c)^(1/2)+2/7*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d/sec(d*x+c)^(
5/2)+2/35*b*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/a/d/sec(d*x+c)^(3/2)+2/105*(
25*a^2-4*b^2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/a^2/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{a+b\sec(c+dx)}\left(8b(19a^3+19a^2b+8ab^2+8b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)+8(25a^4-17a^2b^2-\right.}{}$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/2),x]
```

output
```
(Sqrt[a + b*Sec[c + d*x]]*(8*b*(19*a^3 + 19*a^2*b + 8*a*b^2 + 8*b^3)*Sqrt[
(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 8*(2
5*a^4 - 17*a^2*b^2 - 8*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)] + 2*a*(136*a^2*b - 16*b^3 + a*(145*a^2 - 4*b^2)
*Cos[c + d*x] + 36*a^2*b*Cos[2*(c + d*x)] + 15*a^3*Cos[3*(c + d*x)])*Sin[c
 + d*x]))/(420*a^3*d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 2.46 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.06, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 4344, 3042, 4592, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4344}$$

$$\frac{1}{7}\int \frac{4b\sec^2(c+dx)+5a\sec(c+dx)+b}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \frac{4b\csc\left(c+dx+\frac{\pi}{2}\right)^2+5a\csc\left(c+dx+\frac{\pi}{2}\right)+b}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4592}$$

$$\frac{1}{7}\left(\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{2\int -\frac{25a^2+23b\sec(c+dx)a-4b^2+2b^2\sec^2(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(\frac{\int \frac{25a^2+23b\sec(c+dx)a-4b^2+2b^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{\int \frac{25a^2+23b\csc\left(c+dx+\frac{\pi}{2}\right)a-4b^2+2b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4592}$$

$$\frac{1}{7}\left(\frac{\frac{2(25a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{2\int -\frac{b(19a^2+8b^2)+a(25a^2+2b^2)\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{5a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{7}\left(\frac{\frac{\int \frac{b(19a^2+8b^2)+a(25a^2+2b^2)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}+\frac{2(25a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}}{5a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}\right)+$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{\frac{\int \frac{b(19a^2+8b^2)+a(25a^2+2b^2)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2(25a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}}{5a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 4523

$$\frac{1}{7}\left(\frac{\frac{\frac{b(19a^2+8b^2)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}+\frac{(25a^4-17a^2b^2-8b^4)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}}{3a}+\frac{2(25a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}}{5a}+\frac{2b\sin(c+d}{5ad}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{\frac{\frac{b(19a^2+8b^2)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{(25a^4-17a^2b^2-8b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2(25a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}}{5a}+\frac{2b\sin\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 4343

$$\frac{1}{7}\left(\frac{\frac{\frac{b(19a^2+8b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{(25a^4-17a^2b^2-8b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2(25a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}}{5a}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{\frac{\frac{b(19a^2+8b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{(25a^4-17a^2b^2-8b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2(25a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+d}}{3ad\sqrt{\sec(c+dx)}}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3134

$$\frac{1}{7}\left(\frac{\frac{\frac{b(19a^2+8b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{(25a^4-17a^2b^2-8b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2(25a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+d}}{3ad\sqrt{\sec(c+dx)}}}{5a}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{\frac{\frac{b(19a^2+8b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{(25a^4-17a^2b^2-8b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2(25a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(}}{3ad\sqrt{\sec(c+dx)}}}{5a}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3132}$$

$$\frac{1}{7}\left(\frac{\frac{\frac{(25a^4-17a^2b^2-8b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2b(19a^2+8b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2(25a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}}{5a}+\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4345}$$

$$\frac{1}{7}\left(\frac{\frac{\frac{\left(25a^4-17a^2b^2-8b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\left(19a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2\left(25a^2-4b^2\right)\sin(c+dx}{3ad\sqrt{\sec(}}}{5a}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{\frac{\frac{\left(25a^4-17a^2b^2-8b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\left(19a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2\left(25a^2-4b^2\right)\sin(c}{3ad\sqrt{}}}{5a}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3142$$

$$\frac{1}{7}\left(\frac{\frac{\frac{\left(25a^4-17a^2b^2-8b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\left(19a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2\left(25a^2-4b^2\right)\sin(}{3ad\sqrt{}}}{5a}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{\frac{\frac{\left(25a^4-17a^2b^2-8b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\left(19a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2(25a^2-4b^2)}{3}}{5a}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3140

$$\frac{1}{7}\left(\frac{\frac{2(25a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}+\frac{\frac{2b\left(19a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\left(25a^4-17a^2b^2-8b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{ad\sqrt{a+b\sec(c+dx)}}}{3a}}{5a}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/2),x]
```

output
```
(2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + ((2*b
*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) + (((2*
(25*a^4 - 17*a^2*b^2 - 8*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d
*x]]) + (2*b*(19*a^2 + 8*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
 + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d
*x]]))/(3*a) + (2*(25*a^2 - 4*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/
(3*a*d*Sqrt[Sec[c + d*x]]))/(5*a))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4344
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*Csc[e
 + f*x])^n/(f*n)), x] - Simp[1/(2*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*(Simp
[b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x]/Sqrt[a + b*C
sc[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] &&
LeQ[n, -1] && IntegerQ[2*n]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1163 vs. $2(280) = 560$.

Time = 10.73 (sec) , antiderivative size = 1164, normalized size of antiderivative = 3.82

method	result	size
default	Expression too large to display	1164

input
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/105/d/a^3/((a-b)/(a+b))^(1/2)*((19*cos(d*x+c)^2+38*cos(d*x+c)+19)*(1/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b*E
llipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2)
)+(-19*cos(d*x+c)^2-38*cos(d*x+c)-19)*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2*EllipticE(((a-b)/(a+b))^(1/2)
*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))+(8*cos(d*x+c)^2+16*cos(d*x
+c)+8)*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*a*b^3*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)
/(a-b))^(1/2))+(-8*cos(d*x+c)^2-16*cos(d*x+c)-8)*(1/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4*EllipticE(((a-b)/(a+b)
)^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))+(25*cos(d*x+c)^2+50
*cos(d*x+c)+25)*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*a^4*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),
(-(a+b)/(a-b))^(1/2))+(-19*cos(d*x+c)^2-38*cos(d*x+c)-19)*(1/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b*EllipticF((
(a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))+(2*cos(d
*x+c)^2+4*cos(d*x+c)+2)*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*a^2*b^2*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+
csc(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-8*cos(d*x+c)^2-16*cos(d*x+c)-8)*(1/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
1/315*(sqrt(2)*(-75*I*a^4 + 32*I*a^2*b^2 + 16*I*b^4)*sqrt(a)*weierstrassPI
nverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(
d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(75*I*a^4 - 32*I*a^2*b^2
 - 16*I*b^4)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)
- 3*sqrt(2)*(-19*I*a^3*b - 8*I*a*b^3)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2
- 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2
- 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*si
n(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(19*I*a^3*b + 8*I*a*b^3)*sqrt(a)*weierst
rassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrass
PInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*co
s(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)) + 6*(15*a^4*cos(d*x + c)^3 + 3*
a^3*b*cos(d*x + c)^2 + (25*a^4 - 4*a^2*b^2)*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(7/2),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{\sec\left(dx+c\right)b+a}}{\sec\left(dx+c\right)^{4}}dx$$

input
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/sec(c + d*x)**4,x)
```

3.634 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 299

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx &= \frac{7ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a+b\sec(c+dx)}}\\
&+\frac{(3a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a+b\sec(c+dx)}}\\
&-\frac{5aE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{4d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}\\
&+\frac{5a\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d}\\
&+\frac{b\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d}\end{aligned}$$

output
```
7/4*a*b*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/
2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+1/4*(3*a^2+4
*b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/
2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)-5/4*a*Ellipt
icE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/(
(b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+5/4*a*sec(d*x+c)^(1/2)*(a+b
*sec(d*x+c))^(1/2)*sin(d*x+c)/d+1/2*b*sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1
/2)*sin(d*x+c)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.22 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.37

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{(a+b\sec(c+dx))^{3/2}\left(\frac{4ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{(b+a\cos(c+dx))^2}+\frac{(a^2+8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c\right.}{(b+a\cos(c+dx))^2}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
((a + b*Sec[c + d*x])^(3/2)*((4*a*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ell
ipticF[(c + d*x)/2, (2*a)/(a + b)])/(b + a*Cos[c + d*x])^2 + ((a^2 + 8*b^2
)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
 b)])/(b + a*Cos[c + d*x])^2 - ((5*I)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a +
b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2*b*(a + b)*Ellip
ticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a +
 b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*
x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1
)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(Sqrt[(a - b)^(-1)]*b*(
b + a*Cos[c + d*x])^(3/2)) + (2*(5*a + 2*b*Sec[c + d*x])*Tan[c + d*x])/(b
+ a*Cos[c + d*x])))/(8*d*Sec[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 3.25 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.03, number of steps used = 26, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.040, Rules used = {3042, 4353, 27, 3042, 4590, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4353}$$

$$\frac{1}{2}\int \frac{\sqrt{\sec(c+dx)}\left(5ab\sec^2(c+dx)+2\left(2a^2+b^2\right)\sec(c+dx)+ab\right)}{2\sqrt{a+b\sec(c+dx)}}dx+\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{\sqrt{\sec(c+dx)}\left(5ab\sec^2(c+dx)+2\left(2a^2+b^2\right)\sec(c+dx)+ab\right)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5ab\csc\left(c+dx+\frac{\pi}{2}\right)^2+2\left(2a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+ab\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{4590}$$

$$\frac{1}{4}\left(\frac{\int -\frac{5ba^2-2b^2\sec(c+dx)a-b(3a^2+4b^2)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b}+\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}\right)+\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 27

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int \frac{5ba^2-2b^2\sec(c+dx)a-b(3a^2+4b^2)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}\right)+\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int \frac{5ba^2-2b^2\csc\left(c+dx+\frac{\pi}{2}\right)a-b(3a^2+4b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}\right)+\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 4596

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int \frac{5a^2b-2ab^2\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx-b(3a^2+4b^2)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+d}}}{2b}\right.$$
$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int \frac{5a^2b-2ab^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b(3a^2+4b^2)\int \frac{\csc(c}{\sqrt{a+b}}}{2b}\right.$$
$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 4346

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5a^2b-2ab^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b(3a^2+4b^2)\sqrt{\sec(c+dx)}\sqrt{}}{\sqrt{a}}}{2b}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5a^2b-2ab^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b(3a^2+4b^2)\sqrt{\sec(c+dx)}\sqrt{}}{}}{2b}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3286

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5a^2b-2ab^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b(3a^2+4b^2)\sqrt{\sec(c+dx)}\sqrt{}}{\sqrt{}}}{2b}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5a^2b-2ab^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b(3a^2+4b^2)\sqrt{\sec(c+dx)}\sqrt{}}{}}{2b}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3284

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5a^2b-2ab^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b(3a^2+4b^2)\sqrt{\sec(c+dx)}}{}}{2b}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{4523}$$

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-7ab^2\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+5ab\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx-\frac{2b(3a^2}{}}{2b}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-7ab^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5ab\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-}{2b}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{4343}$$

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-7ab^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{5ab\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)}}}{}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-7ab^2\int\frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{5ab\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin(c}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx}}}{}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3134

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-7ab^2\int\frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{5ab\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\,c}{}}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)}{a+b}}}}{}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-7ab^2\int\frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{5ab\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\,\mathrm{si}}{}}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+d}{a+b}}}}{}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3132

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-7ab^2\int\frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx-\frac{2b(3a^2+4b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c}{a}}}{d\sqrt{a+b}}}{}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 4345

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{7ab^2\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}-\frac{2b(3a^2+4b^2)}{}}{}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{7ab^2\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}-\frac{2b(3a^2+}{}}{}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{3142}$$

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{7ab^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-\frac{2b(3a^2+4}{}}{}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{7ab^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-\frac{2b(3a}{}}{}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{3140}$$

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{2b(3a^2+4b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}\cdots}{\cdots}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

input
```
Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(b*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (-1/2
*((-14*a*b^2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*
a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) - (2*b*(3*a^2
 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (10*a*b*El
lipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b +
 a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/b + (5*a*Sqrt[Sec[c + d*x]]
*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4353
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + n - 1))), x] + Simp[d/(m + n - 1)   In
t[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n - 1)*Simp[a*b*(n - 1) +
(b^2*(m + n - 2) + a^2*(m + n - 1))*Csc[e + f*x] + a*b*(2*m + n - 2)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
tQ[0, m, 2] && LtQ[0, n, 3] && NeQ[m + n - 1, 0] && (IntegerQ[m] || Integer
sQ[2*m, 2*n])
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.43 (sec) , antiderivative size = 1030, normalized size of antiderivative = 3.44

method	result	size
default	Expression too large to display	1030

input
```
int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/((a-b)/(a+b))^(1/2)*((1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*a^2*EllipticPi(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-
csc(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(6*cos(d*x+c)^4+12*cos(d*x+
c)^3+6*cos(d*x+c)^2)+(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*b^2*EllipticPi(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x
+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(8*cos(d*x+c)^4+16*cos(d*x+c)^3+8*
cos(d*x+c)^2)+(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*a^2*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(
a+b)/(a-b))^(1/2))*(-5*cos(d*x+c)^4-10*cos(d*x+c)^3-5*cos(d*x+c)^2)+(1/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b*Ell
ipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(
5*cos(d*x+c)^4+10*cos(d*x+c)^3+5*cos(d*x+c)^2)+(1/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*EllipticF(((a-b)/(a+b))^
(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(2*cos(d*x+c)^4+4*cos(
d*x+c)^3+2*cos(d*x+c)^2)+(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*a*b*EllipticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(
d*x+c)),(-(a+b)/(a-b))^(1/2))*(2*cos(d*x+c)^4+4*cos(d*x+c)^3+2*cos(d*x+c)^
2)+(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*b^2*EllipticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))
^(1/2))*(-4*cos(d*x+c)^4-8*cos(d*x+c)^3-4*cos(d*x+c)^2)-5*((a-b)/(a+b))...
```

Fricas [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)^2\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)\,dx\right)a$$

input
```
int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**2,x)*b + int
(sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a)*sec(c + d*x),x)*a
```

3.635 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 249

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \frac{(2a^2+b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{3ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} - \frac{bE\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{b\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
(2*a^2+b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2
^(1/2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+3*a*b*((
b+a*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a
+b))^(1/2))*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)-b*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x
+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+b*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/
2)*sin(d*x+c)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.15 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.58

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \frac{(a+b\sec(c+dx))^{3/2}\left(\frac{8a^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{(b+a\cos(c+dx))^2}+\frac{10ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx)\right.}{(b+a\cos(c+dx))^2}\right.}{}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
((a + b*Sec[c + d*x])^(3/2)*((8*a^2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ell
ipticF[(c + d*x)/2, (2*a)/(a + b)])/(b + a*Cos[c + d*x])^2 + (10*a*b*Sqrt[
(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(
b + a*Cos[c + d*x])^2 - ((2*I)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sq
rt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*
ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] +
a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (
-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt
[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(a*Sqrt[(a - b)^(-1)]*(b + a*C
os[c + d*x])^(3/2)) + (4*b*Tan[c + d*x])/(b + a*Cos[c + d*x])))/(4*d*Sec[c
 + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 2.49 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.02, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.920$, Rules used = {3042, 4353, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \; 4353$$

$$\int -\frac{-2\sec(c+dx)a^2-3b\sec^2(c+dx)a+ba}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{1}{2}\int \frac{-2\sec(c+dx)a^2-3b\sec^2(c+dx)a+ba}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{1}{2}\int \frac{-2\csc\left(c+dx+\frac{\pi}{2}\right)a^2-3b\csc\left(c+dx+\frac{\pi}{2}\right)^2a+ba}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4596$$

$$\frac{1}{2}\left(3ab\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-\int \frac{ab-2a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\left(3ab\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\int \frac{ab-2a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \; 4346$$

$$\frac{1}{2}\left(\frac{3ab\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}-\int\frac{ab-2a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right.$$
$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{3ab\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}-\int\frac{ab-2a^2\csc\left(c+dx+\frac{\pi}{2}\right.}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c\right.}}\right.$$
$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3286

$$\frac{1}{2}\left(\frac{3ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-\int\frac{ab-2a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+$$
$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{3ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-\int\frac{ab-2a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+\right.}}\right.$$
$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3284

$$\frac{1}{2}\left(\frac{6ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-\int\frac{ab-2a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\right.}}\right.$$
$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 4523

$$\frac{1}{2}\left((2a^2+b^2)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx - b\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx + \frac{6ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}}{d\sqrt{a+b\sec(c+dx}}\right.$$
$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left((2a^2+b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx - b\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx + \frac{6ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)-}{a+b}}}{d\sqrt{a+b}}\right.$$
$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 4343

$$\frac{1}{2}\left((2a^2+b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx - \frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}} + \frac{6ab\sqrt{\sec(c+dx}}{}\right.$$
$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left((2a^2+b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx - \frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}} + \frac{6ab\sqrt{\sec(c}}{}\right.$$
$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3134

$$\frac{1}{2}\left((2a^2+b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx - \frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}} + \frac{6ab\sqrt{\sec(c+dx)}}{}\right.$$
$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\left(2a^2+b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6ab\sqrt{\sec(c+\right.$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\big\downarrow$ 3132

$$\frac{1}{2}\left(\left(2a^2+b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6ab\sqrt{\sec(c+dx)}\sqrt{a\right.$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\big\downarrow$ 4345

$$\frac{1}{2}\left(\frac{\left(2a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}-\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\right.$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\big\downarrow$ 3042

$$\frac{1}{2}\left(\frac{\left(2a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}-\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\big\downarrow$ 3142

$$\frac{1}{2}\left(\frac{\left(2a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6a}{}\right.$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\big\downarrow$ 3042

$$\frac{1}{2}\left(\frac{(2a^2+b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\right.$$
$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3140}$$

$$\frac{1}{2}\left(\frac{2(2a^2+b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.$$
$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

input
```
Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
((2*(2*a^2 + b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (6*a*b
*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) - (2*b*EllipticE[(c +
 d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d
*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/2 + (b*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Se
c[c + d*x]]*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4353
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + n - 1))), x] + Simp[d/(m + n - 1)   In
t[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n - 1)*Simp[a*b*(n - 1) +
(b^2*(m + n - 2) + a^2*(m + n - 1))*Csc[e + f*x] + a*b*(2*m + n - 2)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
tQ[0, m, 2] && LtQ[0, n, 3] && NeQ[m + n - 1, 0] && (IntegerQ[m] || Integer
sQ[2*m, 2*n])
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.54 (sec) , antiderivative size = 711, normalized size of antiderivative = 2.86

method	result
default	$\frac{\left(\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\,ab\,\mathrm{EllipticPi}\left(\sqrt{\frac{a-b}{a+b}}\,(-\cot(dx+c)+\csc(dx+c)),\frac{a+b}{a-b},\frac{i}{\sqrt{\frac{a-b}{a+b}}}\right)\left(6\cos(dx+c)^3+12\cos(dx+c\right.\right.}{}$

input
```
int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/((a-b)/(a+b))^(1/2)*((1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*a*b*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+cs
c(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(6*cos(d*x+c)^3+12*cos(d*x+c)
^2+6*cos(d*x+c))+(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*a*b*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c))
,(-(a+b)/(a-b))^(1/2))*(-cos(d*x+c)^3-2*cos(d*x+c)^2-cos(d*x+c))+(1/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^2*Ellipt
icE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(co
s(d*x+c)^3+2*cos(d*x+c)^2+cos(d*x+c))+(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*EllipticF(((a-b)/(a+b))^(1/2)*(-c
ot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(2*cos(d*x+c)^3+4*cos(d*x+c)^2
+2*cos(d*x+c))+(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*a*b*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(
-(a+b)/(a-b))^(1/2))*(-2*cos(d*x+c)^3-4*cos(d*x+c)^2-2*cos(d*x+c))+((a-b)/
(a+b))^(1/2)*a*b*cos(d*x+c)*sin(d*x+c)+((a-b)/(a+b))^(1/2)*b^2*sin(d*x+c))
*(a+b*sec(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+b*co
s(d*x+c)+b)
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \int (a+b\sec{(c+dx)})^{\frac{3}{2}}\sqrt{\sec{(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)*sqrt(sec(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec{(dx+c)}+a)^{\frac{3}{2}}\sqrt{\sec{(dx+c)}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\begin{aligned}\int \sqrt{\sec(c+dx)}(a & \\ &+b\sec(c+dx))^{3/2}\,dx = \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)\,dx\right)b \\ &+\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}dx\right)a\end{aligned}$$

input
```
int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a)*sec(c + d*x),x)*b + int(sq
rt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a),x)*a
```

3.636 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 209

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{2ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{2b^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{2aE\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$

output
```
2*a*b*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)
*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+2*b^2*((b+a*co
s(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(
1/2))*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+2*a*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))
/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 26.79 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.62

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\left(a(a+b)E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)+b\left(a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\right.\right.}{d(b+a\cos(c+dx))^2\sec^{\frac{3}{2}}(}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*(a*(a + b)*EllipticE[(c + d*x)/2, (2
*a)/(a + b)] + b*(a*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + b*EllipticPi[2
, (c + d*x)/2, (2*a)/(a + b)]))*(a + b*Sec[c + d*x])^(3/2))/(d*(b + a*Cos[
c + d*x])^2*Sec[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 2.06 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 4355, 3042, 4341, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140, 4346, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4355$$

$$a\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+b\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$a\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4341}$$

$$a\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\left(b\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+a\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+a\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4343}$$

$$b\left(a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{a\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}$$

$$\Big\downarrow \text{3042}$$

$$b\left(a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{a\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}$$

$$\Big\downarrow \text{3134}$$

$$b\left(a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{a\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$$\Big\downarrow \text{3042}$$

$$b\left(a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{a\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$\downarrow$ 3132

$$b\left(a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$\downarrow$ 4345

$$b\left(b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}\right)+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$\downarrow$ 3042

$$b\left(b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}\right)+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$\downarrow$ 3142

$$b\left(b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}\right)+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$\downarrow$ 3042

$$b\left(b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}\right)+$$
$$\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$\Big\downarrow$ 3140

$$b\left(b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}\right)+$$
$$\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$\Big\downarrow$ 4346

$$b\left(\frac{b\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\right.}{d\sqrt{a+b\sec(c+dx)}}\right.$$
$$\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$\Big\downarrow$ 3042

$$b\left(\frac{b\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{Elliptic}}{d\sqrt{a+b\sec(c+dx)}}\right.$$
$$\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$\Big\downarrow$ 3286

$$b\left(\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}\right)$$

$$\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\right.}{d\sqrt{a+b\sec(c+dx)}}\right)$$

$$\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$$\Big\downarrow \text{3284}$$

$$\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+$$

$$b\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c\right.}{d\sqrt{a+b\sec(c+dx)}}\right)$$

input
```
Int[(a + b*Sec[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*a*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + b*((2*a*Sqrt[(b + a
*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c +
 d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*Sqrt[(b + a*Cos[c + d*x])/(a +
 b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt
[a + b*Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4341
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[a   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e +
f*x]], x], x] + Simp[b/d   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*
x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4355
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_
)]*(d_.)], x_Symbol] :> Simp[a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e
+ f*x]], x], x] + Simp[b/d   Int[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*
x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.49 (sec) , antiderivative size = 745, normalized size of antiderivative = 3.56

method	result
default	$-\frac{2\left(\left((1-\cos(dx+c))^3\csc(dx+c)^3+\csc(dx+c)-\cot(dx+c)\right)ba\sqrt{\frac{a-b}{a+b}}+\left(-(1-\cos(dx+c))^3\csc(dx+c)^3+\csc(dx+c)-\cot(dx+c)\right)a\right.}{}$

input
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/((a-b)/(a+b))^(1/2)*(((1-cos(d*x+c))^3*csc(d*x+c)^3+csc(d*x+c)-cot(d*
x+c))*b*a*((a-b)/(a+b))^(1/2)+(-(1-cos(d*x+c))^3*csc(d*x+c)^3+csc(d*x+c)-c
ot(d*x+c))*a^2*((a-b)/(a+b))^(1/2)+2*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*
x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2-2*EllipticE(((a-b)/(a+b))^(1/2)*
(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b+4*EllipticPi(((a-b)/(a+b)
)^(1/2)*(csc(d*x+c)-cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(1/(a+b
)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^2-2*(1
/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ell
ipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*a
^2+4*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(
1/2))*a*b-2*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(
a-b))^(1/2))*b^2)*(a+b*sec(d*x+c))^(1/2)/(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-
b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/sec(d*x+c)^(1/2)
```

Fricas [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a+b\sec{(c+dx)})^{\frac{3}{2}}}{\sqrt{\sec{(c+dx)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)/sqrt(sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}{\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}{\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx=\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)}dx\right)a+\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}dx\right)b$$

input
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/sec(c + d*x),x)*a + int(
sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a),x)*b
```

3.637 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 187

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3d\sqrt{a+b\sec(c+dx)}} + \frac{8bE\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{2a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output

```
2/3*(a^2-b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c
,2^(1/2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+8/3*b*
EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/
2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+2/3*a*(a+b*sec(d*x+c)
)^(1/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.83

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{(a+b\sec(c+dx))^{3/2}\left(8b(a+b)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+2(a^2-b\right.}{3d(b+a\cos(c+}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

output
```
((a + b*Sec[c + d*x])^(3/2)*(8*b*(a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 2*(a^2 - b^2)*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + 2*a*(b + a*Cos[c
+ d*x])*Sin[c + d*x]))/(3*d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.01, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3042, 4351, 25, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \; 4351$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}-\frac{1}{3}\int -\frac{4ab+\left(a^2+3b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \; 25$$

$$\frac{1}{3}\int \frac{4ab+\left(a^2+3b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\int \frac{4ab+\left(a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 4523

$$\frac{1}{3}\left(\left(a^2-b^2\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+4b\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+4b\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 4343

$$\frac{1}{3}\left(\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4b\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4b\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3134

$$\frac{1}{3}\left(\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4b\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4b\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3132

$$\frac{1}{3}\left(\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{8b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 4345

$$\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3142

$$\frac{1}{3}\left(\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{1}{3}\left(\frac{2(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{8b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

input
```
Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

output
```
((2*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (8*b*Ell
ipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/3 + (2*a*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4351
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*C
sc[e + f*x])^n/(f*n)), x] + Simp[1/(2*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/
Sqrt[a + b*Csc[e + f*x]])*Simp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[
e + f*x] + a*b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f
}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 679 vs. $2(174) = 348$.

Time = 4.98 (sec) , antiderivative size = 680, normalized size of antiderivative = 3.64

method	result
default	$\frac{2\sqrt{a+b\sec(dx+c)}\left(\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\sqrt{\frac{1}{1+\cos(dx+c)}}\,ab\,\text{EllipticE}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)(4\cos(dx+c)+8+}{}$

input
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/((a-b)/(a+b))^(1/2)*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x
+c)+cos(d*x+c)*b+b)/sec(d*x+c)^(3/2)*((1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b*EllipticE(((a-b)/(a+b))^(1/2)*(cs
c(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(4*cos(d*x+c)+8+4*sec(d*x+c))+(
1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^
2*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/
2))*(-4*cos(d*x+c)-8-4*sec(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d
*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(cos(d*x+c)+2+sec(d*x+c))+(1/(a+b)
*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b*Ellip
ticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-4
*cos(d*x+c)-8-4*sec(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*b^2*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-c
ot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(3*cos(d*x+c)+6+3*sec(d*x+c))+sin(d*x+c)*
(1+cos(d*x+c))*((a-b)/(a+b))^(1/2)*a^2+((a-b)/(a+b))^(1/2)*a*b*(5*sin(d*x+
c)+tan(d*x+c))+4*((a-b)/(a+b))^(1/2)*b^2*tan(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.22

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{6\,a^2\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos{(dx+c)}}\sin{(dx+c)}+12i\,\sqrt{2}a^{\frac{3}{2}}b\text{weierstrassZeta}\Big(-$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/9*(6*a^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c) + 12*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2
, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2
, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) +
 2*b)/a)) - 12*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^
2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^
2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c)
+ 2*b)/a)) + sqrt(2)*(-3*I*a^2 - I*b^2)*sqrt(a)*weierstrassPInverse(-4/3*(
3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*
I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(3*I*a^2 + I*b^2)*sqrt(a)*weierstrass
PInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*co
s(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a))/(a*d)
```

Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a+b\sec{(c+dx)})^{\frac{3}{2}}}{\sec^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)/sec(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}{\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{2}}dx\right)a+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)}dx\right)b$$

input
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/sec(c + d*x)**2,x)*a + i
nt((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/sec(c + d*x),x)*b
```

3.638 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 240

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2b(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{5ad\sqrt{a+b\sec(c+dx)}} + \frac{2(3a^2+b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{5ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{2a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sqrt{\sec(c+dx)}}$$

output
```
2/5*b*(a^2-b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2
*c,2^(1/2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/a/d/(a+b*sec(d*x+c))^(1/2)+2/
5*(3*a^2+b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*s
ec(d*x+c))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+2/5*a
*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+4/5*b*(a+b*sec(d*x+c
))^(1/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.82

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{(a+b\sec(c+dx))^{3/2}\left(4(3a^3+3a^2b+ab^2+b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)\right|\right.}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/2),x]
```

output
```
((a + b*Sec[c + d*x])^(3/2)*(4*(3*a^3 + 3*a^2*b + a*b^2 + b^3)*Sqrt[(b + a
*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 4*b*(a^2 -
 b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)] + 2*a*(a^2 + 4*b^2 + 6*a*b*Cos[c + d*x] + a^2*Cos[2*(c + d*x)])*Sin[c
 + d*x]))/(10*a*d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 1.87 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.99, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {3042, 4351, 25, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4351}$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}-\frac{1}{5}\int -\frac{2ab\sec^2(c+dx)+\left(3a^2+5b^2\right)\sec(c+dx)+6ab}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{5}\int \frac{2ab\sec^2(c+dx)+(3a^2+5b^2)\sec(c+dx)+6ab}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\int \frac{2ab\csc\left(c+dx+\frac{\pi}{2}\right)^2+(3a^2+5b^2)\csc\left(c+dx+\frac{\pi}{2}\right)+6ab}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 4592}$$

$$\frac{1}{5}\left(\frac{4b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}-\frac{2\int -\frac{3(4b\sec(c+dx)a^2+(3a^2+b^2)a)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5}\left(\frac{\int \frac{4b\sec(c+dx)a^2+(3a^2+b^2)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a}+\frac{4b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\left(\frac{\int \frac{4b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+(3a^2+b^2)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{4b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 4523}$$

$$\frac{1}{5}\left(\frac{b(a^2-b^2)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+(3a^2+b^2)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}+\frac{4b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(3a^2+b^2\right)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{4b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}\right)+$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4343}$$

$$\frac{1}{5}\left(\frac{b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{(3a^2+b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a}+\frac{4b\sin(c+dx)\sqrt{a+b\sec(c+dx}}{d\sqrt{\sec(c+dx)}}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{(3a^2+b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a}+\frac{4b\sin(c+dx)\sqrt{a+b\sec(c+}}{d\sqrt{\sec(c+dx)}}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3134}$$

$$\frac{1}{5}\left(\frac{b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{(3a^2+b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{4b\sin(c+dx)\sqrt{a+b\sec(c+}}{d\sqrt{\sec(c+dx)}}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{b(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{(3a^2+b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{4b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3132

$$\frac{1}{5}\left(\frac{b(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(3a^2+b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{4b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 4345

$$\frac{1}{5}\left(\frac{\frac{b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^2+b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{4b\sin(c+dx)\sqrt{a-}}{d\sqrt{\sec(c}}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{\frac{b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^2+b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{4b\sin(c+dx)\sqrt{}}{d\sqrt{\sec}}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3142

$$\frac{1}{5}\left(\frac{\frac{b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^2+b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{4b\sin(c+dx)\sqrt{}}{d\sqrt{\sec(}}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{\frac{b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^2+b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{4b\sin(c+dx)}{d\sqrt{s}}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3140}$$

$$\frac{1}{5}\left(\frac{\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^2+b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{4b\sin(c+dx)}{d\sqrt{se}}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/2),x]
```

output
```
(2*a*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (((
2*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^
2 + b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(
d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/a + (4*b*Sqrt[a
+ b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]))/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4351
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*C
sc[e + f*x])^n/(f*n)), x] + Simp[1/(2*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/
Sqrt[a + b*Csc[e + f*x]])*Simp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[
e + f*x] + a*b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f
}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 976 vs. $2(221) = 442$.

Time = 4.97 (sec) , antiderivative size = 977, normalized size of antiderivative = 4.07

method	result	size
default	Expression too large to display	977

input
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/d/((a-b)/(a+b))^(1/2)/a*((3*cos(d*x+c)^2+6*cos(d*x+c)+3)*(1/(a+b)*(b+a
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*EllipticE(
((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-3*cos(
d*x+c)^2-6*cos(d*x+c)-3)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-co
t(d*x+c)),(-(a+b)/(a-b))^(1/2))+(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b+
a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2*Ellipti
cE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-cos
(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)*b^3*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot
(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-3*cos(d*x+c)^2-6*cos(d*x+c)-3)*(1/(a+b)*(
b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*Ellipti
cF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(4*co
s(d*x+c)^2+8*cos(d*x+c)+4)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-
cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*
(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2*Elli
pticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+si
n(d*x+c)*cos(d*x+c)*(cos(d*x+c)^2+cos(d*x+c)+3)*((a-b)/(a+b))^(1/2)*a^3+(3
*cos(d*x+c)^2+3*cos(d*x+c)+3)*sin(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b+(2+3...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.93

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{2\sqrt{2}(3i\,a^2b-i\,b^3)\sqrt{a}\text{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\,\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\,\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)+2\sqrt{2}(-}{}$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/15*(2*sqrt(2)*(3*I*a^2*b - I*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(3*a
^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a
*sin(d*x + c) + 2*b)/a) + 2*sqrt(2)*(-3*I*a^2*b + I*b^3)*sqrt(a)*weierstra
ssPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*
cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) + 3*sqrt(2)*(-3*I*a^3 - I*a*b^
2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3
)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3
)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) + 3*sqrt(2)*(
3*I*a^3 + I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*
(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*
(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a
)) - 6*(a^3*cos(d*x + c)^2 + 2*a^2*b*cos(d*x + c))*sqrt((a*cos(d*x + c) +
b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d)
```

Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(a+b\sec{(c+dx)})^{\frac{3}{2}}}{\sec^{\frac{5}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(5/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)/sec(c + d*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}{\sec{(dx+c)}^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^3}dx\right)a + \left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^2}dx\right)b$$

input
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/sec(c + d*x)**3,x)*a + i
nt((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/sec(c + d*x)**2,x)*b
```

3.639 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 303

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2(25a^4-31a^2b^2+6b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+d}}{105a^2d\sqrt{a+b\sec(c+dx)}}$$
$$+\frac{4b(41a^2-3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{105a^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$
$$+\frac{2a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{16b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}$$
$$+\frac{2(25a^2+3b^2)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{105ad\sqrt{\sec(c+dx)}}$$

output
```
2/105*(25*a^4-31*a^2*b^2+6*b^4)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJaco
biAM(1/2*d*x+1/2*c,2^(1/2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/a^2/d/(a+b*se
c(d*x+c))^(1/2)+4/105*b*(41*a^2-3*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2
)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b))^(
1/2)/sec(d*x+c)^(1/2)+2/7*a*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d/sec(d*x+c)
^(5/2)+16/35*b*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/105*
(25*a^2+3*b^2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.78

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{(a+b\sec(c+dx))^{3/2}\left(16b(41a^3+41a^2b-3ab^2-3b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c\right.\right.}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/2),x]
```

output
```
((a + b*Sec[c + d*x])^(3/2)*(16*b*(41*a^3 + 41*a^2*b - 3*a*b^2 - 3*b^3)*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 8
*(25*a^4 - 31*a^2*b^2 + 6*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*a)/(a + b)] + 2*a*(178*a^2*b + 12*b^3 + a*(145*a^2 + 108
*b^2)*Cos[c + d*x] + 78*a^2*b*Cos[2*(c + d*x)] + 15*a^3*Cos[3*(c + d*x)])*
Sin[c + d*x]))/(420*a^2*d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 2.50 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.03, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.880$, Rules used = {3042, 4351, 25, 3042, 4592, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \; 4351$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}-\frac{1}{7}\int -\frac{4ab\sec^2(c+dx)+\left(5a^2+7b^2\right)\sec(c+dx)+8ab}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \; 25$$

$$\frac{1}{7}\int \frac{4ab\sec^2(c+dx)+\left(5a^2+7b^2\right)\sec(c+dx)+8ab}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\int \frac{4ab\csc\left(c+dx+\frac{\pi}{2}\right)^2+\left(5a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+8ab}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 4592$$

$$\frac{1}{7}\left(\frac{16b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}-\frac{2\int -\frac{44b\sec(c+dx)a^2+16b^2\sec^2(c+dx)a+\left(25a^2+3b^2\right)a}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\left(\frac{\int \frac{44b\sec(c+dx)a^2+16b^2\sec^2(c+dx)a+\left(25a^2+3b^2\right)a}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}+\frac{16b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{\int \frac{44b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+16b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a+\left(25a^2+3b^2\right)a}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}+\frac{16b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 4592$$

$$\frac{1}{7}\left(\frac{\frac{2(25a^2+3b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}-\frac{2\int -\frac{(25a^2+51b^2)\sec(c+dx)a^2+2b(41a^2-3b^2)a}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{5a}+\frac{16b\sin(c+dx)\sqrt{a+b\sec(c+d}{5d\sec^{\frac{3}{2}}(c+dx)}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\left(\frac{\frac{\int \frac{(25a^2+51b^2)\sec(c+dx)a^2+2b(41a^2-3b^2)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}+\frac{2(25a^2+3b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{5a}+\frac{16b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{\frac{\int \frac{(25a^2+51b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a^2+2b(41a^2-3b^2)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2(25a^2+3b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{5a}+\frac{16b\sin(c+dx)\sqrt{a+b\sec(c+}{5d\sec^{\frac{3}{2}}(c+dx)}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 4523$$

$$\frac{1}{7}\left(\frac{\frac{2b(41a^2-3b^2)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+(25a^4-31a^2b^2+6b^4)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{3a}+\frac{2(25a^2+3b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{5a}+16b\sin(c\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{\frac{2b(41a^2-3b^2)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+(25a^4-31a^2b^2+6b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2(25a^2+3b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{5a}+16\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 4343

$$\frac{1}{7}\left(\frac{\frac{\frac{2b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+(25a^4-31a^2b^2+6b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2(25a^2+3b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx}}{3d\sqrt{\sec(c+dx)}}}{5a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{\frac{\frac{2b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+(25a^4-31a^2b^2+6b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2(25a^2+3b^2)\sin(c+dx)\sqrt{a+b\sec(c}}{3d\sqrt{\sec(c+dx)}}}{5a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3134

$$\frac{1}{7}\left(\frac{\frac{\frac{2b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+(25a^4-31a^2b^2+6b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2(25a^2+3b^2)\sin(c+dx)\sqrt{a+b\sec(c}}{3d\sqrt{\sec(c+dx)}}}{5a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{\frac{\frac{2b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\left(25a^4-31a^2b^2+6b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\sec}}{3d\sqrt{\sec(c+dx)}}}{5a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3132

$$\frac{1}{7}\left(\frac{\frac{\left(25a^4-31a^2b^2+6b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{5a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 4345

$$\frac{1}{7}\left(\frac{\frac{\frac{\left(25a^4-31a^2b^2+6b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2\left(25a^2+3b^2\right)\sin(c+dx}{3d\sqrt{\sec(}}}{5a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{\frac{\frac{\left(25a^4-31a^2b^2+6b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2\left(25a^2+3b^2\right)\sin(\cdots}{3d\sqrt{\cdots}}}{5a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3142

$$\frac{1}{7}\left(\frac{\frac{\frac{\left(25a^4-31a^2b^2+6b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2\left(25a^2+3b^2\right)\sin(\cdots}{3d\sqrt{\cdots}}}{5a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{\frac{\frac{\left(25a^4-31a^2b^2+6b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2\left(25a^2+3b^2\right)\cdots}{3\cdots}}{5a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3140

$$\frac{1}{7}\left(\frac{\frac{2(25a^2+3b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}+\frac{\frac{4b(41a^2-3b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2(25a^4-31a^2b^2+6b^4)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{d\sqrt{a+b\sec(c+dx)}}}{3a}}{5a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/2),x]
```

output
```
(2*a*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + ((1
6*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (((2
*(25*a^4 - 31*a^2*b^2 + 6*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*
x]]) + (4*b*(41*a^2 - 3*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
+ b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]
]))/(3*a) + (2*(25*a^2 + 3*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*
d*Sqrt[Sec[c + d*x]]))/(5*a))/7
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4351
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*C
sc[e + f*x])^n/(f*n)), x] + Simp[1/(2*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/
Sqrt[a + b*Csc[e + f*x]])*Simp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[
e + f*x] + a*b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f
}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1163 vs. $2(278) = 556$.

Time = 6.22 (sec) , antiderivative size = 1164, normalized size of antiderivative = 3.84

method	result	size
default	Expression too large to display	1164

input
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/105/d/((a-b)/(a+b))^(1/2)/a^2*((82*cos(d*x+c)^2+164*cos(d*x+c)+82)*(1/(a
+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b*
EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2)
)+(-82*cos(d*x+c)^2-164*cos(d*x+c)-82)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b^2*EllipticE(((a-b)/(a+b))^(1/2
)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-6*cos(d*x+c)^2-12*cos(d*
x+c)-6)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*a*b^3*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)
/(a-b))^(1/2))+(6*cos(d*x+c)^2+12*cos(d*x+c)+6)*(1/(a+b)*(b+a*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^4*EllipticE(((a-b)/(a+b))
^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(25*cos(d*x+c)^2+50*c
os(d*x+c)+25)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*a^4*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(
a+b)/(a-b))^(1/2))+(-82*cos(d*x+c)^2-164*cos(d*x+c)-82)*(1/(a+b)*(b+a*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b*EllipticF(((a
-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(51*cos(d*x
+c)^2+102*cos(d*x+c)+51)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*a^2*b^2*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-
cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(6*cos(d*x+c)^2+12*cos(d*x+c)+6)*(1/(a+b
)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^3...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.65

$$\int \frac{(a + b\sec(c + dx))^{3/2}}{\sec^{\frac{7}{2}}(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
1/315*(sqrt(2)*(-75*I*a^4 + 11*I*a^2*b^2 - 12*I*b^4)*sqrt(a)*weierstrassPI
nverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(
d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(75*I*a^4 - 11*I*a^2*b^2
 + 12*I*b^4)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)
- 6*sqrt(2)*(-41*I*a^3*b + 3*I*a*b^3)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2
- 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2
- 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*si
n(d*x + c) + 2*b)/a)) - 6*sqrt(2)*(41*I*a^3*b - 3*I*a*b^3)*sqrt(a)*weierst
rassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrass
PInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*co
s(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)) + 6*(15*a^4*cos(d*x + c)^3 + 24
*a^3*b*cos(d*x + c)^2 + (25*a^4 + 3*a^2*b^2)*cos(d*x + c))*sqrt((a*cos(d*x
 + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{(b\sec\left(dx+c\right)+a)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(7/2),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\left(\int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{\sec\left(dx+c\right)b+a}}{\sec\left(dx+c\right)^{4}}dx\right)a+\left(\int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{\sec\left(dx+c\right)b+a}}{\sec\left(dx+c\right)^{3}}dx\right)b$$

input
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/sec(c + d*x)**4,x)*a + i
nt((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/sec(c + d*x)**3,x)*b
```

3.640 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 369

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{b(59a^2+16b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{24d\sqrt{a+b\sec(c+dx)}} + \frac{5a(a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{8d\sqrt{a+b\sec(c+dx)}} - \frac{(33a^2+16b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{24d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{(33a^2+16b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d} + \frac{13ab\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{12d} + \frac{b^2\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d}$$

output
```
1/24*b*(59*a^2+16*b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*
d*x+1/2*c,2^(1/2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/
2)+5/8*a*(a^2+4*b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x
+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/
2)-1/24*(33*a^2+16*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/
2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/
2)+1/24*(33*a^2+16*b^2)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)
/d+13/12*a*b*sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d+1/3*b^2*
sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.53 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.63

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx =$$

$$-\frac{a(a+b\sec(c+dx))^{5/2}\left(-\frac{104ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2(3a^2-104b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\right.}{\sqrt{b+a\cos(c+dx)}}\right.}{}$$

$$+\frac{(a+b\sec(c+dx))^{5/2}\left(\frac{1}{24}\sec(c+dx)\left(33a^2\sin(c+dx)+16b^2\sin(c+dx)\right)+\frac{13}{12}ab\sec(c+dx)\tan(c+dx\right.}{d(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
-1/96*(a*(a + b*Sec[c + d*x])^(5/2)*((-104*a*b*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] +
(2*(3*a^2 - 104*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
 d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(33*a^2 + 16*b^
2)*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*C
os[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[
b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[
(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[
1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/
(a + b)]))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sq
rt[(a^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]
) + 2*(b + a*Cos[c + d*x])^2))))/(d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x
]^(5/2)) + ((a + b*Sec[c + d*x])^(5/2)*((Sec[c + d*x]*(33*a^2*Sin[c + d*x]
 + 16*b^2*Sin[c + d*x]))/24 + (13*a*b*Sec[c + d*x]*Tan[c + d*x])/12 + (b^2
*Sec[c + d*x]^2*Tan[c + d*x])/3))/(d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(
5/2))
```

Rubi [A] (verified)

Time = 4.14 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.06, number of steps used = 29, number of rules used = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 1.160$, Rules used = {3042, 4329, 27, 3042, 4590, 27, 3042, 4590, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{ 4329}$$

$$\frac{1}{3}\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(13ab^2\sec^2(c+dx)+2b\left(9a^2+2b^2\right)\sec(c+dx)+3a\left(2a^2+b^2\right)\right)}{2\sqrt{a+b\sec(c+dx)}}dx+$$
$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(13ab^2\sec^2(c+dx)+2b\left(9a^2+2b^2\right)\sec(c+dx)+3a\left(2a^2+b^2\right)\right)}{\sqrt{a+b\sec(c+dx)}}dx+$$
$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(13ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(9a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+3a\left(2a^2+b^2\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+$$
$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{4590}$$

$$\frac{1}{6}\left(\frac{\int \frac{\sqrt{\sec(c+dx)}\left(13a^2b^2+\left(33a^2+16b^2\right)\sec^2(c+dx)b^2+2a\left(12a^2+19b^2\right)\sec(c+dx)b\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{2b}+\frac{13ab\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\,\text{s}}}{2d}\right.$$
$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(\frac{\int \frac{\sqrt{\sec(c+dx)}\left(13a^2b^2+\left(33a^2+16b^2\right)\sec^2(c+dx)b^2+2a\left(12a^2+19b^2\right)\sec(c+dx)b\right)}{\sqrt{a+b\sec(c+dx)}}dx}{4b}+\frac{13ab\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\,\text{s}}}{2d}\right.$$
$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(13a^2b^2+\left(33a^2+16b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2b^2+2a\left(12a^2+19b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)b\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{4b}+\frac{13ab\sin(c+dx)\sec^{\frac{3}{2}}(c+dx\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\downarrow$ 4590

$$\frac{1}{6}\left(\frac{\frac{\int -\frac{-26a^2\sec(c+dx)b^3-15a\left(a^2+4b^2\right)\sec^2(c+dx)b^2+a\left(33a^2+16b^2\right)b^2}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b}+\frac{b\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}}{4b}+\frac{13ab\,\mathrm{s}}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\downarrow$ 27

$$\frac{1}{6}\left(\frac{\frac{b\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int \frac{-26a^2\sec(c+dx)b^3-15a\left(a^2+4b^2\right)\sec^2(c+dx)b^2+a\left(33a^2+16b^2\right)b^2}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}}{4b}+\frac{13ab\sin}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{\frac{b\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int \frac{-26a^2\csc\left(c+dx+\frac{\pi}{2}\right)b^3-15a\left(a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2b^2+a\left(33a^2+16b^2\right)b^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}}{4b}+\frac{1}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\downarrow$ 4596

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{ab^2(33a^2+16b^2)-26a^2b^3\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx-15ab^2(a^2+4b^2)\int\frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{2b}}{4b}+\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{ab^2(33a^2+16b^2)-26a^2b^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-15ab^2(a^2+4b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}}{2b}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{4346}$$

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{ab^2(33a^2+16b^2)-26a^2b^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{15ab^2(a^2+4b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}}{2b}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{ab^2(33a^2+16b^2)-26a^2b^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{15ab^2(a^2+4b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)}}{\sqrt{a+b}}}{2b}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{3286}$$

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{ab^2\left(33a^2+16b^2\right)-26a^2b^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{15ab^2\left(a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+d}{a+b}}}{\sqrt{a+b\sec(c+}}}{2b}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\big\downarrow$ 3042

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{ab^2\left(33a^2+16b^2\right)-26a^2b^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{15ab^2\left(a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+d}{a+b}}}{\sqrt{a}}}{2b}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\big\downarrow$ 3284

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{ab^2\left(33a^2+16b^2\right)-26a^2b^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{30ab^2\left(a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+d}{a+b}}}{d\sqrt{a+b\sec(}}}{2b}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\big\downarrow$ 4523

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{b^2(33a^2+16b^2)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx-\left(b^3(59a^2+16b^2)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx\right)-\frac{3}{}}{2b}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\big\downarrow$ 3042

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{b^2(33a^2+16b^2)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\left(b^3(59a^2+16b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}\right.}{2b}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\Big\downarrow$ 4343

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\frac{b^2(33a^2+16b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\left(b^3(59a^2+16b^2)\int\frac{\sqrt{\csc(c+}}{\sqrt{a+b\csc(}}\right.}{}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\frac{b^2(33a^2+16b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\left(b^3(59a^2+16b^2)\int\frac{\sqrt{\mathrm{cs}}}{\sqrt{a+b}}\right.}{}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\Big\downarrow$ 3134

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\frac{b^2(33a^2+16b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\left(b^3(59a^2+16b^2)\int\frac{\sqrt{\mathrm{cs}}}{\sqrt{a+b}}\right.}{}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{6}\left(\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d} - \frac{\frac{b^2(33a^2+16b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}} - \left(b^3(59a^2+16b^2)\int \cdots \right.}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\downarrow$ 3132

$$\frac{1}{6}\left(\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d} - \frac{-\left(b^3(59a^2+16b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right) + \frac{2b^2(33a^2+16b^2)\sqrt{a+b\sec(c+dx)}E(\cdots}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+d\cdots}{a+b}}}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\downarrow$ 4345

$$\frac{1}{6}\left(\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d} - \frac{-\frac{b^3(59a^2+16b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}} + \frac{2b^2(33a^2+16b^2)\cdots}{d\sqrt{\sec(\cdots}}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d} - \frac{-\frac{b^3(59a^2+16b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}} + \frac{2b^2(33a^2+16\cdots}{d\sqrt{\cdots}}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\downarrow$ 3142

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{b^3(59a^2+16b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2b^2(33a^2+1}{d}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{b^3(59a^2+16b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2b^2(33a}{}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\Big\downarrow$ 3140

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\frac{2b^2(33a^2+16b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{30ab^2(a^2+4b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\,\mathrm{co}}{}}}{d\sqrt{a+}}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

input
```
Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(b^2*Sec[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d) + ((1
3*a*b*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (-
1/2*((-2*b^3*(59*a^2 + 16*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*
x]]) - (30*a*b^2*(a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elliptic
Pi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c
+ d*x]]) + (2*b^2*(33*a^2 + 16*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c
 + d*x]]))/b + (b*(33*a^2 + 16*b^2)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/d)/(4*b))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 15.95 (sec) , antiderivative size = 1331, normalized size of antiderivative = 3.61

method	result	size
default	Expression too large to display	1331

input
```
int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/d/((a-b)/(a+b))^(1/2)*(a+b*sec(d*x+c))^(1/2)*sec(d*x+c)^(3/2)/(cos(d*
x+c)^2*a+a*cos(d*x+c)+cos(d*x+c)*b+b)*((1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*EllipticPi(((a-b)/(a+b))^(1/2)*(
csc(d*x+c)-cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(30*cos(d*x+c)^4
+60*cos(d*x+c)^3+30*cos(d*x+c)^2)+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2*EllipticPi(((a-b)/(a+b))^(1/2)*(csc
(d*x+c)-cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(120*cos(d*x+c)^4+2
40*cos(d*x+c)^3+120*cos(d*x+c)^2)+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*
x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-33*cos(d*x+c)^4-66*cos(d*x+c)^3-3
3*cos(d*x+c)^2)+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*a^2*b*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c))
,(-(a+b)/(a-b))^(1/2))*(33*cos(d*x+c)^4+66*cos(d*x+c)^3+33*cos(d*x+c)^2)+(
1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*
b^2*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(
1/2))*(-16*cos(d*x+c)^4-32*cos(d*x+c)^3-16*cos(d*x+c)^2)+(1/(a+b)*(b+a*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^3*EllipticE(((a-
b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(16*cos(d*x+
c)^4+32*cos(d*x+c)^3+16*cos(d*x+c)^2)+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*EllipticF(((a-b)/(a+b))^(1/2)*...
```

Fricas [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)^3\,dx\right)b^2 + 2\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)^2\,dx\right)ab + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)\,dx\right)a^2$$

input
```
int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**3,x)*b**2 +
2*int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**2,x)*a*b +
 int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a)*sec(c + d*x),x)*a**2
```

3.641 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 314

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \frac{a(8a^2+11b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a+b\sec(c+dx)}} + \frac{b(15a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a+b\sec(c+dx)}} - \frac{9abE\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{4d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{9ab\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{b^2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d}$$

output
```
1/4*a*(8*a^2+11*b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*
x+1/2*c,2^(1/2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)
+1/4*b*(15*a^2+4*b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*
x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1
/2)-9/4*a*b*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec
(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+9/4*a*b*s
ec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d+1/2*b^2*sec(d*x+c)^(3/
2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.44 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.78

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \frac{(a+b\sec(c+dx))^{5/2}\left(\frac{2(16a^3+4ab^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2(21a^2b+8b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{E}\cdots}{\sqrt{b+a\cos(\cdots}}\cdots\right.}{\cdots} + \frac{(a+b\sec(c+dx))^{5/2}\left(\frac{9}{4}ab\tan(c+dx)+\frac{1}{2}b^2\sec(c+dx)\tan(c+dx)\right)}{d(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
((a + b*Sec[c + d*x])^(5/2)*((2*(16*a^3 + 4*a*b^2)*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]
] + (2*(21*a^2*b + 8*b^3)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2,
 (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] - ((18*I)*a^2*Sqrt[
(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c
+ d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Co
s[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^
(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b,
 I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]
))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 -
a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b +
 a*Cos[c + d*x])^2))))/(16*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)
) + ((a + b*Sec[c + d*x])^(5/2)*((9*a*b*Tan[c + d*x])/4 + (b^2*Sec[c + d*x
]*Tan[c + d*x])/2))/(d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 3.44 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.03, number of steps used = 26, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 1.040$, Rules used = {3042, 4329, 27, 3042, 4590, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \; 4329$$

$$\frac{1}{2}\int \frac{\sqrt{\sec(c+dx)}\left(9ab^2\sec^2(c+dx)+2b\left(6a^2+b^2\right)\sec(c+dx)+a\left(4a^2+b^2\right)\right)}{2\sqrt{a+b\sec(c+dx)}}dx+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}\int \frac{\sqrt{\sec(c+dx)}\left(9ab^2\sec^2(c+dx)+2b\left(6a^2+b^2\right)\sec(c+dx)+a\left(4a^2+b^2\right)\right)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(9ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(6a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+a\left(4a^2+b^2\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\Big\downarrow$ 4590

$$\frac{1}{4}\left(\frac{\int -\frac{9a^2b^2-\left(15a^2+4b^2\right)\sec^2(c+dx)b^2-2a\left(4a^2+b^2\right)\sec(c+dx)b}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b}+\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}\right)+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\Big\downarrow$ 27

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int \frac{9a^2b^2-\left(15a^2+4b^2\right)\sec^2(c+dx)b^2-2a\left(4a^2+b^2\right)\sec(c+dx)b}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}\right)+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int \frac{9a^2b^2-\left(15a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2b^2-2a\left(4a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}\right)+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\Big\downarrow$ 4596

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx-b^2\left(15a^2+4b^2\right)\int\frac{\sec^{\frac{3}{2}}(c}{\sqrt{a+b\sec}}}{2b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b^2\left(15a^2+4b^2\right)\int\frac{c}{\sqrt{}}}{2b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 4346

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b^2\left(15a^2+4b^2\right)\sqrt{\sec(c+d}}{\sqrt{}}}{2b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b^2\left(15a^2+4b^2\right)\sqrt{\sec(c+d}}{}}{2b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3286

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b^2(15a^2+4b^2)\sqrt{\sec(c+d}}{}}{2b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b^2(15a^2+4b^2)\sqrt{\sec(c+d}}{}}{2b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3284

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b^2(15a^2+4b^2)\sqrt{\sec(c+}}{}}{2b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 4523

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-ab(8a^2+11b^2)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+9ab^2\int\frac{\sqrt{a+b\sec(c+}}{\sqrt{\sec(c+dx)}}}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-ab\left(8a^2+11b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+9ab^2\int\frac{\sqrt{a+b\csc(}}{\sqrt{\csc(c-}}}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 4343

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-ab\left(8a^2+11b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{9ab^2\sqrt{a+b\sec(c+d}}{\sqrt{\sec(c+dx)}}}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-ab\left(8a^2+11b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{9ab^2\sqrt{a+b\sec(c+d}}{\sqrt{\sec(c+d}}}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3134

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-ab\left(8a^2+11b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{9ab^2\sqrt{a+b\sec(c+d}}{\sqrt{\sec(c+d}}}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-ab\left(8a^2+11b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{9ab^2\sqrt{a+b\sec(c+d\cdots}}{\sqrt{\sec(c+\cdots}}}{\cdots}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\Big\downarrow$ 3132

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-ab\left(8a^2+11b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b^2\left(15a^2+4b^2\right)\sqrt{\mathrm{se}\cdots}}{\cdots}}{\cdots}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\Big\downarrow$ 4345

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{ab\left(8a^2+11b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}-\frac{2b\cdots}{\cdots}}{\cdots}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{ab\left(8a^2+11b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}-\cdots}{\cdots}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\Big\downarrow$ 3142

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{ab(8a^2+11b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{ab(8a^2+11b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\Big\downarrow$ 3140

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{2ab(8a^2+11b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

input
```
Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(b^2*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (-1
/2*((-2*a*b*(8*a^2 + 11*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[
(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]
]) - (2*b^2*(15*a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi
[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c +
d*x]]) + (18*a*b^2*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c
+ d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/b + (9
*a*b*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.70 (sec) , antiderivative size = 1136, normalized size of antiderivative = 3.62

method	result	size
default	Expression too large to display	1136

input
```
int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/((a-b)/(a+b))^(1/2)*(a+b*sec(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/(cos(d*x
+c)^2*a+a*cos(d*x+c)+cos(d*x+c)*b+b)*((1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticPi(((a-b)/(a+b))^(1/2)*
(csc(d*x+c)-cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(30*cos(d*x+c)^
3+60*cos(d*x+c)^2+30*cos(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^3*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(d*
x+c)-cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(8*cos(d*x+c)^3+16*cos
(d*x+c)^2+8*cos(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot
(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-9*cos(d*x+c)^3-18*cos(d*x+c)^2-9*cos(d*x+
c))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*a*b^2*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-
b))^(1/2))*(9*cos(d*x+c)^3+18*cos(d*x+c)^2+9*cos(d*x+c))+(1/(a+b)*(b+a*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*EllipticF(((a-
b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(8*cos(d*x+c
)^3+16*cos(d*x+c)^2+8*cos(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticF(((a-b)/(a+b))^(1/2)*(csc(
d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-6*cos(d*x+c)^3-12*cos(d*x+c)^2-
6*cos(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*a*b^2*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)...
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{5/2}\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \left(\int \sqrt{\sec\left(dx+c\right)}\,\sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^{2}dx\right)b^{2} + 2\left(\int \sqrt{\sec\left(dx+c\right)}\,\sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)dx\right)ab + \left(\int \sqrt{\sec\left(dx+c\right)}\,\sqrt{\sec\left(dx+c\right)b+a}dx\right)a^{2}$$

input
```
int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**2,x)*b**2 +
2*int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a)*sec(c + d*x),x)*a*b + in
t(sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a),x)*a**2
```

3.642 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 263

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{b(4a^2+b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{5ab^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{(2a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{b^2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d}$$

output

```
b*(4*a^2+b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c
,2^(1/2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+5*a*b^
2*((b+a*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(
a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+(2*a^2-b^2)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d
/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+b^2*sec(d*x+c)^(1/2)*(a+b
*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.52 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.60

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{(a+b\sec(c+dx))^{5/2}\left(\frac{24a^2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{(b+a\cos(c+dx))^3}+\frac{2a(2a^2+9b^2)\sqrt{\frac{b+}{}}}{}\right.}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

output
```
((a + b*Sec[c + d*x])^(5/2)*((24*a^2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*
EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(b + a*Cos[c + d*x])^3 + (2*a*(2*a^
2 + 9*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (
2*a)/(a + b)])/(b + a*Cos[c + d*x])^3 + ((2*I)*(2*a^2 - b^2)*Sqrt[-((a*(-1
 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d
*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c
+ d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)
]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*A
rcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/
(a*Sqrt[(a - b)^(-1)]*b*(b + a*Cos[c + d*x])^(5/2)) + (4*b^2*Tan[c + d*x])
/(b + a*Cos[c + d*x])^2))/(4*d*Sec[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 2.71 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.03, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.920$, Rules used = {3042, 4329, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4329}$$

$$\int \frac{6b\sec(c+dx)a^2+5b^2\sec^2(c+dx)a+\left(2a^2-b^2\right)a}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\int \frac{6b\sec(c+dx)a^2+5b^2\sec^2(c+dx)a+\left(2a^2-b^2\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\int \frac{6b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+5b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a+\left(2a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{ 4596}$$

$$\frac{1}{2}\left(\int \frac{6b\sec(c+dx)a^2+\left(2a^2-b^2\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+5ab^2\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\left(\int \frac{6b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(2a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5ab^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{ 4346}$$

$$\frac{1}{2}\left(\int \frac{6b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(2a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{5ab^2\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}\right.$$

$$+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \frac{6b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(2a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{5ab^2\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a}}}{\sqrt{a+b\sec(c+dx)}}\right.$$
$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3286}$$

$$\frac{1}{2}\left(\int \frac{6b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(2a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{5ab^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}\right)-$$
$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \frac{6b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(2a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{5ab^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin}{}}}}{\sqrt{a+b\sec(c+dx)}}\right.$$
$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3284}$$

$$\frac{1}{2}\left(\int \frac{6b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(2a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{10ab^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx\right.}{d\sqrt{a+b\sec(c+dx)}}\right.$$
$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4523}$$

$$\frac{1}{2}\left(b\left(4a^2+b^2\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+\left(2a^2-b^2\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\frac{10ab^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+d}{a+b}}}{d\sqrt{a+}}\right.$$
$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\left(b\left(4a^2+b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(2a^2-b^2\right)\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{10ab^2\sqrt{\sec(c+dx)}}{\quad}\right.$$
$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \; 4343$$

$$\frac{1}{2}\left(b\left(4a^2+b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(2a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{10ab^2}{\quad}\right.$$
$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\left(b\left(4a^2+b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(2a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\right.$$
$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \; 3134$$

$$\frac{1}{2}\left(b\left(4a^2+b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(2a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{10ab^2}{\quad}\right.$$
$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\left(b\left(4a^2+b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(2a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{10a}{\quad}\right.$$
$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \; 3132$$

$$\frac{1}{2}\left(b\left(4a^2+b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(2a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{10ab^2\sqrt{\mathrm{se}}}{}\right.$$
$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 4345

$$\frac{1}{2}\left(\frac{b\left(4a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(2a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.$$
$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{b\left(4a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(2a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+}{a+b}}}\right.$$
$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3142

$$\frac{1}{2}\left(\frac{b\left(4a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(2a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.$$
$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{b\left(4a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(2a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.$$
$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3140

$$\frac{1}{2}\left(\frac{2b(4a^2+b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2(2a^2-b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx}{a+b}}}\right.$$
$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

input
```
Int[(a + b*Sec[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

output
```
((2*b*(4*a^2 + b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (10*
a*b^2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/
(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*(2*a^2 - b^
2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/2 + (b^2*Sqrt[Sec[c +
d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.42 (sec) , antiderivative size = 1051, normalized size of antiderivative = 4.00

method	result	size
default	Expression too large to display	1051

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/((a-b)/(a+b))^(1/2)*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c
)+cos(d*x+c)*b+b)/sec(d*x+c)^(1/2)*((10*cos(d*x+c)^2+20*cos(d*x+c)+10)*(1/
(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^
2*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(a+b)/(a-b),I/((a
-b)/(a+b))^(1/2))+(2*cos(d*x+c)^2+4*cos(d*x+c)+2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*EllipticE(((a-b)/(a+b
))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-2*cos(d*x+c)^2-4*
cos(d*x+c)-2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*a^2*b*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(
-(a+b)/(a-b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2*EllipticE(((a-b)/
(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(cos(d*x+c)^2+2
*cos(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*b^3*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-
(a+b)/(a-b))^(1/2))+(-2*cos(d*x+c)^2-4*cos(d*x+c)-2)*(1/(a+b)*(b+a*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*EllipticF(((a-b)/(
a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(6*cos(d*x+c)^2+
12*cos(d*x+c)+6)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*a^2*b*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)
),(-(a+b)/(a-b))^(1/2))+(-4*cos(d*x+c)^2-8*cos(d*x+c)-4)*(1/(a+b)*(b+a*...
```

Fricas [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x +
c) + a)/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx=\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)}dx\right)a^2+\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)\,dx\right)b^2+2\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}dx\right)ab$$

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/sec(c + d*x),x)*a**2 + i
nt(sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a)*sec(c + d*x),x)*b**2 + 2*in
t(sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a),x)*a*b
```

3.643 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 262

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= \frac{2a(a^2+2b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3d\sqrt{a+b\sec(c+dx)}}\\
&+\frac{2b^3\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}\\
&+\frac{14abE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}+\frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
2/3*a*(a^2+2*b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1
/2*c,2^(1/2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+2*
b^3*((b+a*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)
*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+14/3*a*b*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/
((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+2/3*a^2*(a+b*sec(d*x+c))^(
1/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.89 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.56

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{(a+b\sec(c+dx))^{5/2}\left(\frac{2a(a^2+9b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{(b+a\cos(c+dx))^3}+\frac{b(7a^2+6b^2)}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

output
```
((a + b*Sec[c + d*x])^(5/2)*((2*a*(a^2 + 9*b^2)*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(b + a*Cos[c + d*x])^3 + (
b*(7*a^2 + 6*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*
x)/2, (2*a)/(a + b)])/(b + a*Cos[c + d*x])^3 + ((7*I)*Sqrt[-((a*(-1 + Cos[
c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2
*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]
], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[
b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[
Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(Sqrt[(
a - b)^(-1)]*(b + a*Cos[c + d*x])^(5/2)) + (2*a^2*Sin[c + d*x])/(b + a*Cos
[c + d*x])^2))/(3*d*Sec[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 2.74 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.920$, Rules used = {3042, 4328, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$\Big\downarrow$ 4328

$$\frac{2}{3}\int \frac{3\sec^2(c+dx)b^3+7a^2b+a\left(a^2+9b^2\right)\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 27

$$\frac{1}{3}\int \frac{3\sec^2(c+dx)b^3+7a^2b+a\left(a^2+9b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\int \frac{3\csc\left(c+dx+\frac{\pi}{2}\right)^2b^3+7a^2b+a\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 4596

$$\frac{1}{3}\left(\int \frac{7ba^2+\left(a^2+9b^2\right)\sec(c+dx)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+3b^3\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\int \frac{7ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 4346

$$\frac{1}{3}\left(\int \frac{7ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3b^3\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\int \frac{7ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3b^3\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin}}}{\sqrt{a+b\sec(c+dx)}}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3286

$$\frac{1}{3}\left(\int \frac{7ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}\right)+$$
$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\int \frac{7ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin(}{}}}}{\sqrt{a+b\sec(c+dx)}}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3284

$$\frac{1}{3}\left(\int \frac{7ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\right.}{d\sqrt{a+b\sec(c+dx)}}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 4523

$$\frac{1}{3}\left(a\left(a^2+2b^2\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+7ab\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\frac{6b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{Ellip}}{d\sqrt{a+b\sec(c+}}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(a\left(a^2+2b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+7ab\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+}{a+}}}{d\sqrt{a}}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4343}$$

$$\frac{1}{3}\left(a\left(a^2+2b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{7ab\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{6b^3\sqrt{\sec(c+}}{}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(a\left(a^2+2b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{7ab\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{6b^3\sqrt{se}}{}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3134}$$

$$\frac{1}{3}\left(a\left(a^2+2b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{7ab\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6b^3\sqrt{\sec(c+}}{}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(a\left(a^2+2b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{7ab\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6b^3\sqrt{\sec(c}}{}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3132}$$

$$\frac{1}{3}\left(a(a^2+2b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4345}$$

$$\frac{1}{3}\left(\frac{a(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{6b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticP}}{d\sqrt{a+b\sec(c+dx)}}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{a(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{6b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{Ellipti}}{d\sqrt{a+b\sec(c+}}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3142}$$

$$\frac{1}{3}\left(\frac{a(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{6b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2\right.}{d\sqrt{a+b\sec(c+dx)}}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{a(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{6b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticP}}{d\sqrt{a+b\sec(c+dx)}}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{1}{3}\left(\frac{2a(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{6b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{Ellip}}{d\sqrt{a+b\sec(c+}}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

input
```
Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

output
```
((2*a*(a^2 + 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (6*b
^3*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (14*a*b*EllipticE
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[
c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/3 + (2*a^2*Sqrt[a + b*Sec[c + d*x]
]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.88 (sec) , antiderivative size = 914, normalized size of antiderivative = 3.49

method	result	size
default	Expression too large to display	914

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/((a-b)/(a+b))^(1/2)*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x
+c)+cos(d*x+c)*b+b)/sec(d*x+c)^(3/2)*((1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^3*EllipticPi(((a-b)/(a+b))^(1/2)*(c
sc(d*x+c)-cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(6*cos(d*x+c)+12+
6*sec(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*a^2*b*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(
-(a+b)/(a-b))^(1/2))*(7*cos(d*x+c)+14+7*sec(d*x+c))+(1/(a+b)*(b+a*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2*EllipticE(((a-b)/
(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-7*cos(d*x+c)-
14-7*sec(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*a^3*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),
(-(a+b)/(a-b))^(1/2))*(cos(d*x+c)+2+sec(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticF(((a-b)/(a+b
))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-7*cos(d*x+c)-14-7
*sec(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*a*b^2*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-
(a+b)/(a-b))^(1/2))*(9*cos(d*x+c)+18+9*sec(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^3*EllipticF(((a-b)/(a+
b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-3*cos(d*x+c)-6-3
*sec(d*x+c))+sin(d*x+c)*(1+cos(d*x+c))*((a-b)/(a+b))^(1/2)*a^3+((a-b)/(...
```

Fricas [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x +
c) + a)/sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{2}}dx\right)a^{2}+2\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)}dx\right)ab+\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}dx\right)b^{2}$$

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x)
```

output

```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/sec(c + d*x)**2,x)*a**2
+ 2*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/sec(c + d*x),x)*a*b
+ int(sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a),x)*b**2
```

3.644 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 239

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{16b(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{15d\sqrt{a+b\sec(c+dx)}} + \frac{2(9a^2+23b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{15d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{22ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}$$

output
```
16/15*b*(a^2-b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1
/2*c,2^(1/2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+2/
15*(9*a^2+23*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a
+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+2/5
*a^2*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+22/15*a*b*(a+b*s
ec(d*x+c))^(1/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{(a+b\sec(c+dx))^{5/2}\left(4(9a^3+9a^2b+23ab^2+23b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+\right.\right.}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/2),x]
```

output
```
((a + b*Sec[c + d*x])^(5/2)*(4*(9*a^3 + 9*a^2*b + 23*a*b^2 + 23*b^3)*Sqrt[
(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 32*b
*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*
a)/(a + b)] + 2*a*(3*a^2 + 22*b^2 + 28*a*b*Cos[c + d*x] + 3*a^2*Cos[2*(c +
 d*x)])*Sin[c + d*x]))/(30*d*(b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 1.98 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.05, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {3042, 4328, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \; 4328$$

$$\frac{2}{5}\int \frac{11ba^2+3\left(a^2+5b^2\right)\sec(c+dx)a+b\left(2a^2+5b^2\right)\sec^2(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{5}\int \frac{11ba^2+3\left(a^2+5b^2\right)\sec(c+dx)a+b\left(2a^2+5b^2\right)\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\int \frac{11ba^2+3\left(a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(2a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 4592

$$\frac{1}{5}\left(\frac{22ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}-\frac{2\int -\frac{\left(9a^2+23b^2\right)a^2+b\left(17a^2+15b^2\right)\sec(c+dx)a}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{5}\left(\frac{\int \frac{\left(9a^2+23b^2\right)a^2+b\left(17a^2+15b^2\right)\sec(c+dx)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}+\frac{22ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{\int \frac{\left(9a^2+23b^2\right)a^2+b\left(17a^2+15b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{22ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 4523

$$\frac{1}{5}\left(\frac{8ab\left(a^2-b^2\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+a\left(9a^2+23b^2\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{3a}+\frac{22ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{8ab\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(9a^2+23b^2\right)\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{22ab\sin(c+dx)\sqrt{a+b\sec(c+}}{3d\sqrt{\sec(c+dx)}}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 4343

$$\frac{1}{5}\left(\frac{8ab\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(9a^2+23b^2\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{3a}+\frac{22ab\sin(c+dx)\sqrt{a+b\,\mathrm{se}}}{3d\sqrt{\sec(c+dx)}}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{8ab\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(9a^2+23b^2\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{3a}+\frac{22ab\sin(c+dx)\sqrt{a+}}{3d\sqrt{\sec(c+}}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 3134

$$\frac{1}{5}\left(\frac{8ab\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(9a^2+23b^2\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{22ab\sin(c+dx)\sqrt{a+b}}{3d\sqrt{\sec(c+d}}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{8ab\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(9a^2+23b^2\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{22ab\sin(c+dx)\sqrt{a}}{3d\sqrt{\sec(c}}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3132

$$\frac{1}{5}\left(\frac{8ab\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\left(9a^2+23b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{22ab\sin(c+dx)\sqrt{a+b\sec(}}{3d\sqrt{\sec(c+dx)}}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 4345

$$\frac{1}{5}\left(\frac{\frac{8ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(9a^2+23b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{22ab\sin(c+d}{3d\sqrt{}}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{\frac{8ab(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a(9a^2+23b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{22ab\sin(c+\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3142

$$\frac{1}{5}\left(\frac{\frac{8ab(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a(9a^2+23b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{22ab\sin(c+}{3a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{\frac{8ab(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a(9a^2+23b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{22ab\sin(}{}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3140

$$\frac{1}{5}\left(\frac{\frac{16ab(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2a(9a^2+23b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{22ab\sin(}{}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/2),x]
```

output
```
(2*a^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (
((16*a*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x
)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*
a*(9*a^2 + 23*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c
+ d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(3*a)
+ (22*a*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]))
/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1088 vs. $2(220) = 440$.

Time = 5.06 (sec) , antiderivative size = 1089, normalized size of antiderivative = 4.56

method	result	size
default	Expression too large to display	1089

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d/((a-b)/(a+b))^(1/2)*((9*cos(d*x+c)^2+18*cos(d*x+c)+9)*(1/(a+b)*(b+a
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*EllipticE(
((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-9*cos(
d*x+c)^2-18*cos(d*x+c)-9)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-c
ot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(23*cos(d*x+c)^2+46*cos(d*x+c)+23)*(1/(a+
b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2*E
llipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))
+(-23*cos(d*x+c)^2-46*cos(d*x+c)-23)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^3*EllipticE(((a-b)/(a+b))^(1/2)*(csc
(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-9*cos(d*x+c)^2-18*cos(d*x+c)-9
)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*a^3*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^
(1/2))+(17*cos(d*x+c)^2+34*cos(d*x+c)+17)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticF(((a-b)/(a+b))^(1/
2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-23*cos(d*x+c)^2-46*cos(
d*x+c)-23)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*a*b^2*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a
+b)/(a-b))^(1/2))+(15*cos(d*x+c)^2+30*cos(d*x+c)+15)*(1/(a+b)*(b+a*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^3*EllipticF(((a-b...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.93

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{2}(-33i\,a^2b+i\,b^3)\sqrt{a}\text{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(d}{}\right.}{}$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/45*(sqrt(2)*(-33*I*a^2*b + I*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^
2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*
sin(d*x + c) + 2*b)/a) + sqrt(2)*(33*I*a^2*b - I*b^3)*sqrt(a)*weierstrassP
Inverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos
(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(-9*I*a^3 - 23*I*a*b^
2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3
)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3
)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(
9*I*a^3 + 23*I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/
27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/
27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b
)/a)) + 6*(3*a^3*cos(d*x + c)^2 + 11*a^2*b*cos(d*x + c))*sqrt((a*cos(d*x +
 c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{3}}dx\right)a^{2}+2\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{2}}dx\right)ab+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)}dx\right)b^{2}$$

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/sec(c + d*x)**3,x)*a**2
+ 2*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/sec(c + d*x)**2,x)*a
*b + int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/sec(c + d*x),x)*b**
2
```

3.645 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 303

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2(5a^4-2a^2b^2-3b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{21ad\sqrt{a+b\sec(c+dx)}}$$
$$+\frac{2b(29a^2+3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{21ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$
$$+\frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{6ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{3}{2}}(c+dx)}$$
$$+\frac{2(5a^2+9b^2)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}$$

output
```
2/21*(5*a^4-2*a^2*b^2-3*b^4)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiA
M(1/2*d*x+1/2*c,2^(1/2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/a/d/(a+b*sec(d*x
+c))^(1/2)+2/21*b*(29*a^2+3*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(
a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec
(d*x+c)^(1/2)+2/7*a^2*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d/sec(d*x+c)^(5/2)
+6/7*a*b*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/21*(5*a^2+
9*b^2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.78

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{(a+b\sec(c+dx))^{5/2}\left(8b(29a^3+29a^2b+3ab^2+3b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c\right.\right.}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/2),x]
```

output
```
((a + b*Sec[c + d*x])^(5/2)*(8*b*(29*a^3 + 29*a^2*b + 3*a*b^2 + 3*b^3)*Sqr
t[(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 8*
(5*a^4 - 2*a^2*b^2 - 3*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)] + 2*a*(44*a^2*b + 36*b^3 + a*(29*a^2 + 72*b^2)*
Cos[c + d*x] + 24*a^2*b*Cos[2*(c + d*x)] + 3*a^3*Cos[3*(c + d*x)])*Sin[c +
 d*x]))/(84*a*d*(b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 2.48 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.03, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.880, Rules used = {3042, 4328, 27, 3042, 4592, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4328}$$

$$\frac{2}{7}\int \frac{15ba^2+\left(5a^2+21b^2\right)\sec(c+dx)a+b\left(4a^2+7b^2\right)\sec^2(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{7}\int \frac{15ba^2+\left(5a^2+21b^2\right)\sec(c+dx)a+b\left(4a^2+7b^2\right)\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}\int \frac{15ba^2+\left(5a^2+21b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(4a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 4592}$$

$$\frac{1}{7}\left(\frac{6ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sec^{\frac{3}{2}}(c+dx)}-\frac{2\int -\frac{5\left(6b^2\sec^2(c+dx)a^2+\left(5a^2+9b^2\right)a^2+b\left(13a^2+7b^2\right)\sec(c+dx)a\right)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{7}\left(\frac{\int \frac{6b^2\sec^2(c+dx)a^2+\left(5a^2+9b^2\right)a^2+b\left(13a^2+7b^2\right)\sec(c+dx)a}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{a}+\frac{6ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}\left(\frac{\int \frac{6b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+\left(5a^2+9b^2\right)a^2+b\left(13a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{6ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 4592

$$\frac{1}{7}\left(\frac{\frac{2a\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}-\frac{2\int -\frac{\left(5a^2+27b^2\right)\sec(c+dx)a^3+b\left(29a^2+3b^2\right)a^2}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{a}+\frac{6ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{7}\left(\frac{\frac{\int \frac{\left(5a^2+27b^2\right)\sec(c+dx)a^3+b\left(29a^2+3b^2\right)a^2}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}+\frac{2a\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{a}+\frac{6ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{\frac{\int \frac{\left(5a^2+27b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^3+b\left(29a^2+3b^2\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2a\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{a}+\frac{6ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 4523

$$\frac{1}{7}\left(\frac{\frac{ab(29a^2+3b^2)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+a(5a^4-2a^2b^2-3b^4)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{3a}+\frac{2a(5a^2+9b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{a}+\frac{6ab\sin(c\ldots\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{\frac{ab(29a^2+3b^2)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a(5a^4-2a^2b^2-3b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2a(5a^2+9b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{a}+\frac{6ab\ldots\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\big\downarrow$ 4343

$$\frac{1}{7}\left(\frac{\frac{\frac{ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+a(5a^4-2a^2b^2-3b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2a(5a^2+9b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{a}\ldots\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{\frac{\frac{ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+a(5a^4-2a^2b^2-3b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2a(5a^2+9b^2)\sin(c+dx)\sqrt{a+b\sec(c+\ldots}}{3d\sqrt{\sec(c+dx)}}}{a}\ldots\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\big\downarrow$ 3134

$$\frac{1}{7}\left(\frac{\frac{\frac{ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+a\left(5a^4-2a^2b^2-3b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2a\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+}}{3d\sqrt{\sec(c+dx)}}}{a}\right)$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{7}\left(\frac{\frac{\frac{ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+a\left(5a^4-2a^2b^2-3b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2a\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\sec}}{3d\sqrt{\sec(c+dx)}}}{a}\right)$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \ 3132$$

$$\frac{1}{7}\left(\frac{\frac{a\left(5a^4-2a^2b^2-3b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2a\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{a}\right)$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \ 4345$$

$$\frac{1}{7}\left(\frac{\frac{\frac{a\left(5a^4-2a^2b^2-3b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2a\left(5a^2+9b^2\right)\sin(c+d}{3d\sqrt{\sec(}}}{a}\right)$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{\frac{\frac{a\left(5a^4-2a^2b^2-3b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2a\left(5a^2+9b^2\right)\sin(\cdots}{3d\sqrt{\cdots}}}{a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3142

$$\frac{1}{7}\left(\frac{\frac{\frac{a\left(5a^4-2a^2b^2-3b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2a\left(5a^2+9b^2\right)\sin(\cdots}{3d\sqrt{\cdots}}}{a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{\frac{\frac{a\left(5a^4-2a^2b^2-3b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2a\left(5a^2+9b^2\right)\cdots}{\cdots}}{a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3140

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}+$$

$$\frac{1}{7}\left(\frac{\frac{2a(5a^2+9b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}+\frac{\frac{2ab(29a^2+3b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2a(5a^4-2a^2b^2-3b^4)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{d\sqrt{a+b\sec(c+dx)}}}{3a}}{a}\right.$$

input
```
Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/2),x]
```

output
```
(2*a^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (
(6*a*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sec[c + d*x]^(3/2)) + (((
2*a*(5*a^4 - 2*a^2*b^2 - 3*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipti
cF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d
*x]]) + (2*a*b*(29*a^2 + 3*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt
[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d
*x]]))/(3*a) + (2*a*(5*a^2 + 9*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])
/(3*d*Sqrt[Sec[c + d*x]]))/a)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1163 vs. $2(278) = 556$.

Time = 5.20 (sec) , antiderivative size = 1164, normalized size of antiderivative = 3.84

method	result	size
default	Expression too large to display	1164

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/d/((a-b)/(a+b))^(1/2)/a*((29*cos(d*x+c)^2+58*cos(d*x+c)+29)*(1/(a+b)*
(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b*Elli
pticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-
29*cos(d*x+c)^2-58*cos(d*x+c)-29)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b^2*EllipticE(((a-b)/(a+b))^(1/2)*(cs
c(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(3*cos(d*x+c)^2+6*cos(d*x+c)+3)
*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
a*b^3*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))
^(1/2))+(-3*cos(d*x+c)^2-6*cos(d*x+c)-3)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^4*EllipticE(((a-b)/(a+b))^(1/2)*
(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(5*cos(d*x+c)^2+10*cos(d*x+c
)+5)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*a^4*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b
))^(1/2))+(-29*cos(d*x+c)^2-58*cos(d*x+c)-29)*(1/(a+b)*(b+a*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b*EllipticF(((a-b)/(a+b))
^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(27*cos(d*x+c)^2+54*c
os(d*x+c)+27)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*a^2*b^2*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c))
,(-(a+b)/(a-b))^(1/2))+(-3*cos(d*x+c)^2-6*cos(d*x+c)-3)*(1/(a+b)*(b+a*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^3*EllipticF(...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.65

$$\int \frac{(a + b\sec(c + dx))^{5/2}}{\sec^{\frac{7}{2}}(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
1/63*(sqrt(2)*(-15*I*a^4 - 23*I*a^2*b^2 + 6*I*b^4)*sqrt(a)*weierstrassPInv
erse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*
x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(15*I*a^4 + 23*I*a^2*b^2 -
 6*I*b^4)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^
2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3
*sqrt(2)*(-29*I*a^3*b - 3*I*a*b^3)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4
*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4
*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d
*x + c) + 2*b)/a)) - 3*sqrt(2)*(29*I*a^3*b + 3*I*a*b^3)*sqrt(a)*weierstras
sZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPIn
verse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d
*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)) + 6*(3*a^4*cos(d*x + c)^3 + 9*a^3*
b*cos(d*x + c)^2 + (5*a^4 + 9*a^2*b^2)*cos(d*x + c))*sqrt((a*cos(d*x + c)
+ b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(7/2),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{4}}dx\right)a^{2}+2\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{3}}dx\right)ab+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{2}}dx\right)b^{2}$$

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x)
```

output

```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/sec(c + d*x)**4,x)*a**2
+ 2*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/sec(c + d*x)**3,x)*a
*b + int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/sec(c + d*x)**2,x)*
b**2
```

3.646 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 363

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{4b(57a^4-62a^2b^2+5b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+}{315a^2d\sqrt{a+b\sec(c+dx)}}$$

$$+\frac{2(147a^4+279a^2b^2-10b^4)\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{315a^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$

$$+\frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{38ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)}$$

$$+\frac{2(49a^2+75b^2)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)}$$

$$+\frac{2b(163a^2+5b^2)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{315ad\sqrt{\sec(c+dx)}}$$

output
```
4/315*b*(57*a^4-62*a^2*b^2+5*b^4)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJa
cobiAM(1/2*d*x+1/2*c,2^(1/2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/a^2/d/(a+b*
sec(d*x+c))^(1/2)+2/315*(147*a^4+279*a^2*b^2-10*b^4)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^2/d/((b+a*cos(d*
x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+2/9*a^2*(a+b*sec(d*x+c))^(1/2)*sin(d*x
+c)/d/sec(d*x+c)^(7/2)+38/63*a*b*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d/sec(d
*x+c)^(5/2)+2/315*(49*a^2+75*b^2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d/sec(
d*x+c)^(3/2)+2/315*b*(163*a^2+5*b^2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/a/d
/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 2.06 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.79

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{(a+b\sec(c+dx))^{5/2}\left(16(147a^5+147a^4b+279a^3b^2+279a^2b^3-10ab^4-1\right.}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(9/2),x]
```

output
```
((a + b*Sec[c + d*x])^(5/2)*(16*(147*a^5 + 147*a^4*b + 279*a^3*b^2 + 279*a
^2*b^3 - 10*a*b^4 - 10*b^5)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(
c + d*x)/2, (2*a)/(a + b)] + 32*b*(57*a^4 - 62*a^2*b^2 + 5*b^4)*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + 2*a*(301*
a^4 + 1984*a^2*b^2 + 40*b^4 + 4*a*b*(619*a^2 + 160*b^2)*Cos[c + d*x] + 8*(
42*a^4 + 85*a^2*b^2)*Cos[2*(c + d*x)] + 260*a^3*b*Cos[3*(c + d*x)] + 35*a^
4*Cos[4*(c + d*x)])*Sin[c + d*x]))/(2520*a^2*d*(b + a*Cos[c + d*x])^3*Sec[
c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 3.23 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.04, number of steps used = 25, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4328, 27, 3042, 4592, 27, 3042, 4592, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{4328}$$

$$\frac{2}{9}\int \frac{19ba^2+\left(7a^2+27b^2\right)\sec(c+dx)a+3b\left(2a^2+3b^2\right)\sec^2(c+dx)}{2\sec^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\int \frac{19ba^2+\left(7a^2+27b^2\right)\sec(c+dx)a+3b\left(2a^2+3b^2\right)\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\int \frac{19ba^2+\left(7a^2+27b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+3b\left(2a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4592}$$

$$\frac{1}{9}\left(\frac{38ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}-\frac{2\int-\frac{76b^2\sec^2(c+dx)a^2+(49a^2+75b^2)a^2+b(137a^2+63b^2)\sec(c+dx)a}{2\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{7a}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{9}\left(\frac{\int\frac{76b^2\sec^2(c+dx)a^2+(49a^2+75b^2)a^2+b(137a^2+63b^2)\sec(c+dx)a}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{7a}+\frac{38ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}\left(\frac{\int\frac{76b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+(49a^2+75b^2)a^2+b(137a^2+63b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{7a}+\frac{38ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 4592

$$\frac{1}{9}\left(\frac{\frac{2a(49a^2+75b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}-\frac{2\int-\frac{(147a^2+605b^2)\sec(c+dx)a^3+2b(49a^2+75b^2)\sec^2(c+dx)a^2+3b(163a^2+5b^2)a^2}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}}{7a}+\frac{38ab\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{9}\left(\frac{\frac{\int \frac{\left(147a^2+605b^2\right)\sec(c+dx)a^3+2b\left(49a^2+75b^2\right)\sec^2(c+dx)a^2+3b\left(163a^2+5b^2\right)a^2}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}+\frac{2a\left(49a^2+75b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}}{7a}+\frac{38ab\sin\cdots}{\cdots}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}\left(\frac{\frac{\int \frac{\left(147a^2+605b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^3+2b\left(49a^2+75b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+3b\left(163a^2+5b^2\right)a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}+\frac{2a\left(49a^2+75b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}}{7a}+\cdots\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 4592

$$\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}-\frac{2\int -\frac{3\left(b\left(261a^2+155b^2\right)\sec(c+dx)a^3+\left(147a^4+279b^2a^2-10b^4\right)a^2\right)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{5a}+\frac{2a\left(49a^2+75b^2\right)\sin(c+dx)\sqrt{\cdots}}{5d\sec^{\frac{3}{2}}(c+dx)\cdots}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{9}\left(\frac{\frac{\frac{\int \frac{b\left(261a^2+155b^2\right)\sec(c+dx)a^3+\left(147a^4+279b^2a^2-10b^4\right)a^2}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a}+\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}}{5a}+\frac{2a\left(49a^2+75b^2\right)\sin(c+dx)\sqrt{a+b\,\mathrm{se}\cdots}}{5d\sec^{\frac{3}{2}}(c+dx)}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{\frac{\frac{\int \frac{b\left(261a^2+155b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^3+\left(147a^4+279b^2a^2-10b^4\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}}{5a}+\frac{2a\left(49a^2+75b^2\right)\sin(c+dx)\sqrt{a-}}{5d\sec^{\frac{3}{2}}(c+dx)}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 4523

$$\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+a\left(147a^4+279a^2b^2-10b^4\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}+\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}}{5a}+\frac{2a(49}{}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(147a^4+279a^2b^2-10b^4\right)\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}}{5a}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 4343

$$\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a}+\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\,\mathrm{s}}}{d\sqrt{\sec(c+dx)}}}{5a}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a}+\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{}}{d\sqrt{\sec(c+dx)}}}{5a}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3134

$$\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{}}{d\sqrt{\sec(c+dx)}}}{5a}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{\frac{\frac{2ab(57a^4-62a^2b^2+5b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a(147a^4+279a^2b^2-10b^4)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{2ab(163a^2+5b^2)\sin(c+d\cdots}{d\sqrt{\sec(c+\cdots}}}{5a}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3132

$$\frac{1}{9}\left(\frac{\frac{\frac{2ab(57a^4-62a^2b^2+5b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a(147a^4+279a^2b^2-10b^4)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{2ab(163a^2+5b^2)\sin(c+dx)\sqrt{a+b\,\mathrm{se}\cdots}}{d\sqrt{\sec(c+dx)}}}{5a}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 4345

$$\frac{1}{9}\left(\frac{\frac{\frac{\frac{2ab(57a^4-62a^2b^2+5b^4)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a(147a^4+279a^2b^2-10b^4)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{2ab(163a^2\cdots}{\cdots}}{5a}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{2ab\left(16\cdots\right.}{\cdots}}{5a}\cdots}{7a}\cdots\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 3142

$$\frac{1}{9}\left(\frac{\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{2ab\left(1\cdots\right.}{\cdots}}{5a}\cdots}{7a}\cdots\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}\left(\frac{\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{2\cdots}{\cdots}}{5a}\cdots}{7a}\cdots\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 3140

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}+$$

$$\frac{1}{9}\left(\frac{\frac{2a(49a^2+75b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{\frac{2ab(163a^2+5b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}+\frac{\frac{4ab(57a^4-62a^2b^2+5b^4)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{d\sqrt{a+b\sec(c+dx)}}}{}}{5a}}{7a}\right.$$

input
```
Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(9/2),x]
```

output
```
(2*a^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (
(38*a*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) +
((2*a*(49*a^2 + 75*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c
+ d*x]^(3/2)) + (((4*a*b*(57*a^4 - 62*a^2*b^2 + 5*b^4)*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[a + b*Sec[c + d*x]]) + (2*a*(147*a^4 + 279*a^2*b^2 - 10*b^4)*Ellip
ticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/a + (2*a*b*(163*a^2 + 5*b^2)*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]))/(5*a))/(7*a)
)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1569 vs. $2(332) = 664$.

Time = 7.55 (sec) , antiderivative size = 1570, normalized size of antiderivative = 4.33

method	result	size
default	Expression too large to display	1570

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315/d/((a-b)/(a+b))^(1/2)/a^2*((-147*cos(d*x+c)^2-294*cos(d*x+c)-147)*(
1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^
5*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/
2))+(147*cos(d*x+c)^2+294*cos(d*x+c)+147)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4*b*EllipticE(((a-b)/(a+b))^(1/
2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-279*cos(d*x+c)^2-558*co
s(d*x+c)-279)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*a^3*b^2*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c))
,(-(a+b)/(a-b))^(1/2))+(279*cos(d*x+c)^2+558*cos(d*x+c)+279)*(1/(a+b)*(b+a
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b^3*Ellipt
icE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(10*
cos(d*x+c)^2+20*cos(d*x+c)+10)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^4*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x
+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-10*cos(d*x+c)^2-20*cos(d*x+c)-10)*
(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b
^5*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1
/2))+(147*cos(d*x+c)^2+294*cos(d*x+c)+147)*(1/(a+b)*(b+a*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^5*EllipticF(((a-b)/(a+b))^(1/2
)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-261*cos(d*x+c)^2-522*cos
(d*x+c)-261)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.49

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
1/945*(sqrt(2)*(-489*I*a^4*b + 93*I*a^2*b^3 - 20*I*b^5)*sqrt(a)*weierstras
sPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*c
os(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(489*I*a^4*b - 93*I*a
^2*b^3 + 20*I*b^5)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*
b)/a) - 3*sqrt(2)*(-147*I*a^5 - 279*I*a^3*b^2 + 10*I*a*b^4)*sqrt(a)*weiers
trassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstras
sPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*c
os(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(147*I*a^5 + 279*I
*a^3*b^2 - 10*I*a*b^4)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*
b)/a)) + 6*(35*a^5*cos(d*x + c)^4 + 95*a^4*b*cos(d*x + c)^3 + (49*a^5 + 75
*a^3*b^2)*cos(d*x + c)^2 + (163*a^4*b + 5*a^2*b^3)*cos(d*x + c))*sqrt((a*c
os(d*x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(9/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(9/2),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(9/2), x)
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^5}dx\right)a^2 + 2\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^4}dx\right)ab + \left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^3}dx\right)b^2$$

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/sec(c + d*x)**5,x)*a**2
+ 2*int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/sec(c + d*x)**4,x)*a
*b + int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/sec(c + d*x)**3,x)*
b**2
```

3.647 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 312

$$\begin{aligned}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx &= -\frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4bd\sqrt{a+b\sec(c+dx)}}\\
&\quad+\frac{(3a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4b^2d\sqrt{a+b\sec(c+dx)}}\\
&\quad+\frac{3aE\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{4b^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}\\
&\quad-\frac{3a\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4b^2d}\\
&\quad+\frac{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2bd}\end{aligned}$$

output
```
-1/4*a*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2
)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/b/d/(a+b*sec(d*x+c))^(1/2)+1/4*(3*a^2+
4*b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1
/2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/b^2/d/(a+b*sec(d*x+c))^(1/2)+3/4*a*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2
)/b^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)-3/4*a*sec(d*x+c)^(
1/2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/b^2/d+1/2*sec(d*x+c)^(3/2)*(a+b*sec
(d*x+c))^(1/2)*sin(d*x+c)/b/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.32 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.27

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\sqrt{\sec(c+dx)}\left(4ab^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+b(9a^2+8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(7/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(Sqrt[Sec[c + d*x]]*(4*a*b^2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[
(c + d*x)/2, (2*a)/(a + b)] + b*(9*a^2 + 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)] + ((3*I)*Sqrt[-((a*(-1
+ Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Sqrt[b + a
*Cos[c + d*x]]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)
^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*A
rcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a
*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]
], (-a + b)/(a + b)])))/Sqrt[(a - b)^(-1)] + 2*b*(2*b - 3*a*Cos[c + d*x])*
(b + a*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x]))/(8*b^3*d*Sqrt[a + b*Sec[c
 + d*x]])
```

Rubi [A] (verified)

Time = 3.04 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4347, 3042, 4590, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4347}$$

$$\frac{\int \frac{\sqrt{\sec(c+dx)}\left(-3a\sec^2(c+dx)+2b\sec(c+dx)+a\right)}{\sqrt{a+b\sec(c+dx)}}dx}{4b}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(-3a\csc\left(c+dx+\frac{\pi}{2}\right)^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{4b}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$$\Big\downarrow \text{4590}$$

$$\frac{\frac{\int \frac{3a^2+2b\sec(c+dx)a+\left(3a^2+4b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{4b}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{3a^2+2b\sec(c+dx)a+\left(3a^2+4b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{4b}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$$\downarrow 3042$$

$$\frac{\frac{\int \frac{3a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(3a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{4b}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$$\downarrow 4596$$

$$\frac{\frac{\left(3a^2+4b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\int \frac{3a^2+2b\sec(c+dx)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{4b}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$$\downarrow 3042$$

$$\frac{\frac{\left(3a^2+4b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{3a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{4b}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$$\downarrow 4346$$

$$\frac{\frac{\frac{\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\int \frac{3a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{4b}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$$\downarrow 3042$$

$$\frac{\frac{\frac{\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\int \frac{3a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{4b}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$$\downarrow 3286$$

$$\frac{\frac{\frac{\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\int \frac{3a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c}{bd}}{4b}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\int \frac{3a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec(c}}{b}}{4b}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$\downarrow$ 3284

$$\frac{\frac{\int \frac{3a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec}}{bd}}{4b}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$\downarrow$ 4523

$$\frac{\frac{-ab\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+3a\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx}}{bd}}{4b}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$\downarrow$ 3042

$$\frac{\frac{-ab\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+3a\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}-\frac{3a\sin(c+dx)\sqrt{\mathrm{se}}}{}}{4b}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$\downarrow$ 4343

$$\frac{\frac{-ab\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}-\frac{3a\sin(}{}}{4b}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$\downarrow$ 3042

$$\frac{\frac{-ab\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}-\frac{3a}{}}{4b}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$\downarrow$ 3134

$$\frac{\frac{-ab\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}-\frac{3a}{}}{4b}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$\downarrow$ 3042

$$\frac{\frac{-ab\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}-}{4b}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$\downarrow$ 3132

$$\frac{\frac{-ab\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{6a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{2b}-\frac{3a\sin(c+}{}}{4b}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$\downarrow$ 4345

$$\frac{\frac{-\frac{ab\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{6a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+d\cdots\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+d\cdots}{a+b}}}}{2b}}{4b}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{-\frac{ab\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{6a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(\cdots\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(\cdots}{\cdots}}}}{2b}}{4b}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$$\Big\downarrow \; 3142$$

$$\frac{\frac{-\frac{ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{6a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}\cdots\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos\cdots}{\cdots}}}}{2b}}{4b}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{-\frac{ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{6a\sqrt{a+b\sec(c+dx)}\cdots}{d\sqrt{\sec(c+dx)}\sqrt{\cdots}}}{2b}}{4b}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$$\Big\downarrow \; 3140$$

$$\frac{\frac{\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-\frac{2ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{6a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(\cdots\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(\cdots}{\cdots}}}}{2b}}{4b}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

input
```
Int[Sec[c + d*x]^(7/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*b*d) + (((-2
*a*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^2 + 4*b^2)*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b
)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (6*a*EllipticE[(c +
d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(2*b) - (3*a*Sqrt[Sec[c + d*x]]*Sqrt[a +
 b*Sec[c + d*x]]*Sin[c + d*x])/(b*d))/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4347
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*(S
qrt[a + b*Csc[e + f*x]]/(b*f*(2*n - 3))), x] + Simp[d^3/(b*(2*n - 3))   Int
[((d*Csc[e + f*x])^(n - 3)/Sqrt[a + b*Csc[e + f*x]])*Simp[2*a*(n - 3) + b*(
2*n - 5)*Csc[e + f*x] - 2*a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.68 (sec) , antiderivative size = 1043, normalized size of antiderivative = 3.34

method	result	size
default	Expression too large to display	1043

input
```
int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/b^2/((a-b)/(a+b))^(1/2)*sec(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(1/2)/(co
s(d*x+c)^2*a+a*cos(d*x+c)+cos(d*x+c)*b+b)*((1/(a+b)*(b+a*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*EllipticPi(((a-b)/(a+b))^(1/
2)*(csc(d*x+c)-cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(-6*cos(d*x+
c)^6-12*cos(d*x+c)^5-6*cos(d*x+c)^4)+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^2*EllipticPi(((a-b)/(a+b))^(1/2)*(cs
c(d*x+c)-cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(-8*cos(d*x+c)^6-1
6*cos(d*x+c)^5-8*cos(d*x+c)^4)+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c
)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-3*cos(d*x+c)^6-6*cos(d*x+c)^5-3*cos(
d*x+c)^4)+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*a*b*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)
/(a-b))^(1/2))*(3*cos(d*x+c)^6+6*cos(d*x+c)^5+3*cos(d*x+c)^4)+(1/(a+b)*(b+
a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*EllipticF
(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(6*cos(
d*x+c)^6+12*cos(d*x+c)^5+6*cos(d*x+c)^4)+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b*EllipticF(((a-b)/(a+b))^(1/2)*
(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-2*cos(d*x+c)^6-4*cos(d*x+c
)^5-2*cos(d*x+c)^4)+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*b^2*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{7}{2}}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(7/2)/sqrt(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{7}{2}}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(7/2)/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^{3}}{\sec\left(dx+c\right)b+a}dx$$

input
```
int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**3)/(sec(c +
 d*x)*b + a),x)
```

3.648 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 246

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} - \frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}} - \frac{E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{bd\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd}$$

output
```
((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(a/(a
+b))^(1/2))*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)-a*((b+a*cos(d*x+c))/
(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*sec(
d*x+c)^(1/2)/b/d/(a+b*sec(d*x+c))^(1/2)-EllipticE(sin(1/2*d*x+1/2*c),2^(1/
2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/b/d/((b+a*cos(d*x+c))/(a+b))^(1
/2)/sec(d*x+c)^(1/2)+sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/b/
d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 15.38 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.34

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{\sqrt{\sec(c+dx)}\left(-6a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)-\frac{2i\sqrt{-\frac{a(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{a(1+\cos(c+dx))}{a-b}}\sqrt{b+a\cos(c}}{}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(Sqrt[Sec[c + d*x]]*(-6*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2,
 (c + d*x)/2, (2*a)/(a + b)] - ((2*I)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a +
b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Sqrt[b + a*Cos[c + d*x]]*Csc[c +
 d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[
c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-
1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I
*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]))
)/(a*Sqrt[(a - b)^(-1)]*b) + 4*(b + a*Cos[c + d*x])*Tan[c + d*x]))/(4*b*d*
Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 2.33 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.05, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.920$, Rules used = {3042, 4347, 25, 3042, 4597, 3042, 4346, 3042, 3286, 3042, 3284, 4349, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4347}$$

$$\frac{\int -\frac{a\sec^2(c+dx)+a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{\int \frac{a\sec^2(c+dx)+a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)^2+a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}$$

$$\Big\downarrow \text{4597}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+a\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}$$

$$\Big\downarrow \text{4346}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}}{2b}$$

$$\Big\downarrow \text{3286}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}}{2b}$$

$$\Big\downarrow \text{3284}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}$$

$$\Big\downarrow \text{4349}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{b\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}\right)+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{b\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}\right)+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}$$

$$\downarrow \text{4343}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{b\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}\right)+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}$$

$$\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{b\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}\right)+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}$$

$$\downarrow \text{3134}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}\right)+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}$$

$$\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}\right)+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+}\right.}{d\sqrt{a+b\sec(c+dx)}}}{2b}$$

$\Big\downarrow$ 3132

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}\right)+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}$$

$\Big\downarrow$ 4345

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{a\sqrt{a+b\sec(c+dx)}}\right)+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}$$

$\Big\downarrow$ 3042

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\sqrt{a+b\sec(c+dx)}}\right)+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}$$

$\Big\downarrow$ 3142

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}\right)+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}$$

$\Big\downarrow$ 3042

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}\right)+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}$$

$\Big\downarrow$ 3140

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd} - \frac{\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}} + a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}} - \frac{2b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{Elli}}{ad\sqrt{a+b\sec(c+}}\right.}{2b}$$

input
```
Int[Sec[c + d*x]^(5/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
-1/2*((2*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (
2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + a*((-2*b*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*S
qrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*EllipticE[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x]
)/(a + b)]*Sqrt[Sec[c + d*x]])))/b + (Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c
+ d*x]]*Sin[c + d*x])/(b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4347
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*(S
qrt[a + b*Csc[e + f*x]]/(b*f*(2*n - 3))), x] + Simp[d^3/(b*(2*n - 3))   Int
[((d*Csc[e + f*x])^(n - 3)/Sqrt[a + b*Csc[e + f*x]])*Simp[2*a*(n - 3) + b*(
2*n - 5)*Csc[e + f*x] - 2*a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

rule 4349
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Cs
c[e + f*x]], x], x] - Simp[b/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Cs
c[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4597
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[C/d^
2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[A
Int[1/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.42 (sec) , antiderivative size = 597, normalized size of antiderivative = 2.43

method	result
default	$\frac{\left(\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,a\,\mathrm{EllipticPi}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\frac{a+b}{a-b},\frac{i}{\sqrt{\frac{a-b}{a+b}}}\right)\left(-2\cos(dx+c)^3-4\cos(dx+c)^2\right.\right.}{}$

input
```
int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/((a-b)/(a+b))^(1/2)/b*((1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*a*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot
(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(-2*cos(d*x+c)^3-4*cos(d*x+c)^
2-2*cos(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*a*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(
a+b)/(a-b))^(1/2))*(-cos(d*x+c)^3-2*cos(d*x+c)^2-cos(d*x+c))+(1/(a+b)*(b+a
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b*EllipticE(((
a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(cos(d*x+c
)^3+2*cos(d*x+c)^2+cos(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-
cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(2*cos(d*x+c)^3+4*cos(d*x+c)^2+2*cos(d*x
+c))+((a-b)/(a+b))^(1/2)*a*cos(d*x+c)*sin(d*x+c)+((a-b)/(a+b))^(1/2)*b*sin
(d*x+c))*cos(d*x+c)^2*sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^
2*a+a*cos(d*x+c)+cos(d*x+c)*b+b)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)^2}{\sec(dx+c)\,b+a}dx$$

input
```
int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**2)/(sec(c +
 d*x)*b + a),x)
```

3.649 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 68

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}$$

output
```
2*((b+a*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(
a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 4346, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4346}$$

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3286}$$

$$\frac{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3284}$$

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^(3/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.46 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.99

method	result
default	$-\frac{2\left(\operatorname{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)-2\operatorname{EllipticPi}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\frac{a+b}{a-b},\frac{i}{\sqrt{\frac{a-b}{a+b}}}\right)\right)\sec(dx+}{d\sqrt{\frac{a-b}{a+b}}\,(b+a\cos(dx+c))\sqrt{\frac{1}{1+\cos(dx+c)}}}$

input
```
int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/((a-b)/(a+b))^(1/2)*(EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*
x+c)),(-(a+b)/(a-b))^(1/2))-2*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-c
ot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2)))*sec(d*x+c)^(3/2)*(a+b*sec(d
*x+c))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2/
(b+a*cos(d*x+c))/(1/(1+cos(d*x+c)))^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**(3/2)/sqrt(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)}{\sec(dx+c)\,b+a}dx$$

input
```
int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a)*sec(c + d*x))/(sec(c + d*
x)*b + a),x)
```

3.650 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 67

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}$$

output
```
2*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(a/
(a+b))^(1/2))*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)
]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4345}$$

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3142}$$

$$\frac{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)
]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(65) = 130$.

Time = 2.98 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.10

method	result	size
default	$\frac{2\sqrt{\sec(dx+c)}\,\sqrt{a+b\sec(dx+c)}\,\operatorname{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\,\cos(dx+c)}{d\sqrt{\frac{a-b}{a+b}}\,(b+a\cos(dx+c))\sqrt{\frac{1}{1+\cos(dx+c)}}}$	141

input
```
int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/((a-b)/(a+b))^(1/2)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)*EllipticF(
((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)
*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)/(b+a*cos(d*x+c))/(1/(1+
cos(d*x+c)))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.18

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{-i\sqrt{2}\sqrt{a}\text{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)+i\sqrt{2}\sqrt{a}\text{weierstrass}}{ad}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*
a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) +
 I*sqrt(2)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a
^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a))/(
a*d)
```

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/sqrt(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)\,b+a}dx$$

input
```
int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/(sec(c + d*x)*b + a),x)
```

3.651 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 142

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{ad\sqrt{a+b\sec(c+dx)}} + \frac{2E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$

output

```
-2*b*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*
(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/a/d/(a+b*sec(d*x+c))^(1/2)+2*EllipticE(s
in(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a/d/((b+
a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 2.95 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)-b\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\right)\sqrt{\sec(c+dx)}}{ad\sqrt{a+b\sec(c+dx)}}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
(2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*((a + b)*EllipticE[(c + d*x)/2, (2*a
)/(a + b)] - b*EllipticF[(c + d*x)/2, (2*a)/(a + b)])*Sqrt[Sec[c + d*x]])/
(a*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4349, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4349}$$

$$\frac{\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{b\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{b\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}$$

$$\Big\downarrow \text{ 4343}$$

$$\frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{b\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{b\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}$$

$$\Big\downarrow \text{ 3134}$$

$$\frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}$$

$$\Big\downarrow \text{ 3132}$$

$$\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}$$

$$\Big\downarrow \text{ 4345}$$

$$\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{a\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3142}$$

$$\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3140}$$

$$\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
(-2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*EllipticE[(c
+ d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)]*Sqrt[Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4349
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Cs
c[e + f*x]], x], x] - Simp[b/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Cs
c[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(139) = 278$.

Time = 4.42 (sec) , antiderivative size = 437, normalized size of antiderivative = 3.08

method	result
default	$-\frac{2\left(\left(-\cos(dx+c)^2-2\cos(dx+c)-1\right)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,a\,\mathrm{EllipticE}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)\cdots\right.}{\cdots}$
risch	$-\frac{i\left(e^{2i(dx+c)}a+2b\,e^{i(dx+c)}+a\right)\sqrt{2}}{ad\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{e^{2i(dx+c)}a+2b\,e^{i(dx+c)}+a}{e^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(e^{2i(dx+c)}a+2b\,e^{i(dx+c)}+a\right)}{a\sqrt{e^{i(dx+c)}\left(e^{2i(dx+c)}a+2b\,e^{i(dx+c)}+a\right)}}+\frac{2\left(b+\sqrt{-a^2+b^2}\right)\sqrt{\cdots}}{\cdots}\cdots\right)}{\cdots}$

input
```
int(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/((a-b)/(a+b))^(1/2)/a*((-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*EllipticE(((a-
b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(cos(d*x+c)^
2+2*cos(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*b*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(
-(a+b)/(a-b))^(1/2))+(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*EllipticF(((a-b)/(a+b)
)^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))-((a-b)/(a+b))^(1/2)*
a*cos(d*x+c)*sin(d*x+c)-((a-b)/(a+b))^(1/2)*b*sin(d*x+c))*(a+b*sec(d*x+c))
^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+cos(d*x+c)*b+b)/sec(d*x+c)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.50

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2i\sqrt{2}\sqrt{a}b\text{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right) - 2i\sqrt{2}\sqrt{a}b\text{weierstr}}{}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*I*sqrt(2)*sqrt(a)*b*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*
b)/a) - 2*I*sqrt(2)*sqrt(a)*b*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2
, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) +
 2*b)/a) + 3*I*sqrt(2)*a^(3/2)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*
b)/a)) - 3*I*sqrt(2)*a^(3/2)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/2
7*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/2
7*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)
/a)))/(a^2*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*sec(c + d*x))*sqrt(sec(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{\sec\left(dx+c\right)b+a}}{\sec\left(dx+c\right)^{2}b+\sec\left(dx+c\right)a}dx$$

input
```
int(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/(sec(c + d*x)**2*b + sec
(c + d*x)*a),x)
```

3.652 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 195

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2(a^2+2b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^2d\sqrt{a+b\sec(c+dx)}} - \frac{4bE\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3a^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}$$

output

```
2/3*(a^2+2*b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2
*c,2^(1/2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/a^2/d/(a+b*sec(d*x+c))^(1/2)-
4/3*b*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c
))^(1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+2/3*(a+b*se
c(d*x+c))^(1/2)*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{\sqrt{\sec(c+dx)}\left(-4b(a+b)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+2(a^2+2b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+\right.\right.}{3a^2d\sqrt{a+b\sec(c+dx)}}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(-4*b*(a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellip
ticE[(c + d*x)/2, (2*a)/(a + b)] + 2*(a^2 + 2*b^2)*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + 2*a*(b + a*Cos[c + d*x
])*Sin[c + d*x]))/(3*a^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 4350, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4350}$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{2b-a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}$$

$\Big\downarrow$ 3042

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{2b-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}$$

$\Big\downarrow$ 4523

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{(a^2+2b^2)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}}{3a}$$

$\Big\downarrow$ 3042

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{(a^2+2b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}$$

$\Big\downarrow$ 4343

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{(a^2+2b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}$$

$\Big\downarrow$ 3042

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{(a^2+2b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}$$

$\Big\downarrow$ 3134

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^2+2b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^2+2b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}$$

$$\Big\downarrow \text{3132}$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^2+2b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}$$

$$\Big\downarrow \text{4345}$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{a\sqrt{a+b\sec(c+dx)}}}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\sqrt{a+b\sec(c+dx)}}}{3a}$$

$$\Big\downarrow \text{3142}$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}}{3a}$$

$\big\downarrow$ 3042

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}}{3a}$$

$\big\downarrow$ 3140

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}}{3a}$$

input
```
Int[1/(Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
-1/3*((-2*(a^2 + 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c +
d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]])
+ (4*b*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*
d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/a + (2*Sqrt[a +
b*Sec[c + d*x]]*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4350
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1)*(Sqrt[a +
 b*Csc[e + f*x]]/(a*d*f*n)), x] + Simp[1/(2*a*d*n)   Int[((d*Csc[e + f*x])^
(n + 1)/Sqrt[a + b*Csc[e + f*x]])*Simp[(-b)*(2*n + 1) + 2*a*(n + 1)*Csc[e +
 f*x] + b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 4523

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 582 vs. 2(182) = 364.

Time = 5.44 (sec) , antiderivative size = 583, normalized size of antiderivative = 2.99

method	result
default	$-\,2\sqrt{a+b\sec(dx+c)}\left(\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,ab\,\mathrm{EllipticE}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)(2\cos(dx+c)+4$

input

```
int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-2/3/d/a^2/((a-b)/(a+b))^(1/2)*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*co
s(d*x+c)+cos(d*x+c)*b+b)/sec(d*x+c)^(3/2)*((1/(a+b)*(b+a*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b*EllipticE(((a-b)/(a+b))^(1/2
)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(2*cos(d*x+c)+4+2*sec(d*x+
c))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*b^2*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b)
)^(1/2))*(-2*cos(d*x+c)-4-2*sec(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*EllipticF(((a-b)/(a+b))^(1/2)*(
csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-cos(d*x+c)-2-sec(d*x+c))+(1
/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b
*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2
))*(-2*cos(d*x+c)-4-2*sec(d*x+c))+sin(d*x+c)*(-cos(d*x+c)-1)*((a-b)/(a+b))
^(1/2)*a^2+((a-b)/(a+b))^(1/2)*a*b*(-tan(d*x+c)+sin(d*x+c))+2*((a-b)/(a+b)
)^(1/2)*b^2*tan(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.13

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \frac{6\,a^2\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c) - 6i\sqrt{2}a^{\frac{3}{2}}b\text{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2}, \frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3}, \text{weiers}\right.}{}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/9*(6*a^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c) - 6*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) +
2*b)/a)) + 6*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) +
2*b)/a)) + sqrt(2)*(-3*I*a^2 - 4*I*b^2)*sqrt(a)*weierstrassPInverse(-4/3*(
3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*
I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(3*I*a^2 + 4*I*b^2)*sqrt(a)*weierstra
ssPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*
cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a))/(a^3*d)
```

Sympy [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^3\,b+\sec(dx+c)^2\,a}dx$$

input
```
int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/(sec(c + d*x)**3*b + sec
(c + d*x)**2*a),x)
```

3.653 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 249

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2b(7a^2+8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{15a^3d\sqrt{a+b\sec(c+dx)}} + \frac{2(9a^2+8b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{15a^3d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{8b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^2d\sqrt{\sec(c+dx)}}$$

output
```
-2/15*b*(7*a^2+8*b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d
*x+1/2*c,2^(1/2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/a^3/d/(a+b*sec(d*x+c))^
(1/2)+2/15*(9*a^2+8*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1
/2))*(a+b*sec(d*x+c))^(1/2)/a^3/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c
)^(1/2)+2/5*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/a/d/sec(d*x+c)^(3/2)-8/15*b*
(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/a^2/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{\sqrt{\sec(c+dx)}\left(4(9a^3+9a^2b+8ab^2+8b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)-4b(7a^2+8b^2)\sqrt{\frac{b+a\cos(c+dx}{a+b}}\right.}{30a^3d\sqrt{a+b\,\mathrm{se}}}$$

input
```
Integrate[1/(Sec[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(4*(9*a^3 + 9*a^2*b + 8*a*b^2 + 8*b^3)*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - 4*b*(7*a^2 + 8
*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)] + 2*a*(3*a^2 - 8*b^2 - 2*a*b*Cos[c + d*x] + 3*a^2*Cos[2*(c + d*x)])*S
in[c + d*x]))/(30*a^3*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 1.97 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.05, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 4350, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{4350}$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\int\frac{-2b\sec^2(c+dx)-3a\sec(c+dx)+4b}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}$$

$\Big\downarrow$ 3042

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\int\frac{-2b\csc\left(c+dx+\frac{\pi}{2}\right)^2-3a\csc\left(c+dx+\frac{\pi}{2}\right)+4b}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}$$

$\Big\downarrow$ 4592

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{2\int\frac{9a^2+2b\sec(c+dx)a+8b^2}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{5a}$$

$\Big\downarrow$ 27

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{9a^2+2b\sec(c+dx)a+8b^2}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{5a}$$

$\Big\downarrow$ 3042

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{9a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a+8b^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}}{5a}$$

$\Big\downarrow$ 4523

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{(9a^2+8b^2)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{b(7a^2+8b^2)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}}{3a}}{5a}$$

$\Big\downarrow$ 3042

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{(9a^2+8b^2)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{b(7a^2+8b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}}{5a}$$

$\Big\downarrow$ 4343

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{(9a^2+8b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{b(7a^2+8b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}}{5a}$$

$\Big\downarrow$ 3042

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{(9a^2+8b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{b(7a^2+8b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}}{5a}$$

$\Big\downarrow$ 3134

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{(9a^2+8b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b(7a^2+8b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}}{5a}$$

$\Big\downarrow$ 3042

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{(9a^2+8b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b(7a^2+8b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}}{5a}$$

$\Big\downarrow$ 3132

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2(9a^2+8b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b(7a^2+8b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}}{5a}$$

$\Big\downarrow$ 4345

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(7a^2+8b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{a\sqrt{a+b\sec(c+dx)}}}{3a}}{5a}$$

$\Big\downarrow$ 3042

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(7a^2+8b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\sqrt{a+b\sec(c+dx)}}}{3a}}{5a}$$

$\Big\downarrow$ 3142

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(7a^2+8b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}}{3a}}{5a}$$

$\Big\downarrow$ 3042

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(7a^2+8b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}d}{a\sqrt{a+b\sec(c+dx)}}}{3a}}{5a}$$

$\Big\downarrow$ 3140

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b\left(7a^2+8b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}}{3a}}{5a}$$

input
```
Int[1/(Sec[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
(2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) - (-1
/3*((-2*b*(7*a^2 + 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c
+ d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]
) + (2*(9*a^2 + 8*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Se
c[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/
a + (8*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]])
)/(5*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4350
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1)*(Sqrt[a +
 b*Csc[e + f*x]]/(a*d*f*n)), x] + Simp[1/(2*a*d*n)   Int[((d*Csc[e + f*x])^
(n + 1)/Sqrt[a + b*Csc[e + f*x]])*Simp[(-b)*(2*n + 1) + 2*a*(n + 1)*Csc[e +
 f*x] + b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 983 vs. 2(230) = 460.

Time = 6.72 (sec) , antiderivative size = 984, normalized size of antiderivative = 3.95

method	result	size
default	Expression too large to display	984

input
```
int(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d/a^3/((a-b)/(a+b))^(1/2)*((9*cos(d*x+c)^2+18*cos(d*x+c)+9)*(1/(a+b)*
(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*Ellipt
icE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-9*
cos(d*x+c)^2-18*cos(d*x+c)-9)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+
c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(8*cos(d*x+c)^2+16*cos(d*x+c)+8)*(1/(
a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2
*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2
))+(-8*cos(d*x+c)^2-16*cos(d*x+c)-8)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^3*EllipticE(((a-b)/(a+b))^(1/2)*(csc
(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-9*cos(d*x+c)^2-18*cos(d*x+c)-9
)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*a^3*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^
(1/2))+(2*cos(d*x+c)^2+4*cos(d*x+c)+2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticF(((a-b)/(a+b))^(1/2)*
(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-8*cos(d*x+c)^2-16*cos(d*x+
c)-8)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*a*b^2*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(
a-b))^(1/2))+sin(d*x+c)*cos(d*x+c)*(3*cos(d*x+c)^2+3*cos(d*x+c)+9)*((a-b)/
(a+b))^(1/2)*a^3+(-cos(d*x+c)^2-cos(d*x+c)+9)*sin(d*x+c)*((a-b)/(a+b))^...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.86

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx =$$

$$-\frac{4\sqrt{2}(-3i\,a^2b-4i\,b^3)\sqrt{a}\text{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)+4}{}$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/45*(4*sqrt(2)*(-3*I*a^2*b - 4*I*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(
3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*
I*a*sin(d*x + c) + 2*b)/a) + 4*sqrt(2)*(3*I*a^2*b + 4*I*b^3)*sqrt(a)*weier
strassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(
3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) + 3*sqrt(2)*(-9*I*a^3 - 8*
I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b -
 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b -
 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) + 3*sqr
t(2)*(9*I*a^3 + 8*I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^
2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^
2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c)
+ 2*b)/a)) - 6*(3*a^3*cos(d*x + c)^2 - 4*a^2*b*cos(d*x + c))*sqrt((a*cos(d
*x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d)
```

Sympy [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^4\,b+\sec(dx+c)^3\,a}dx$$

input
```
int(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/(sec(c + d*x)**4*b + sec
(c + d*x)**3*a),x)
```

3.654 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 345

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}} - \frac{3a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{b^2d\sqrt{a+b\sec(c+dx)}} - \frac{(3a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{b^2(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} - \frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{(3a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{b^2(a^2-b^2)\,d}$$

output
```
((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(a/(a
+b))^(1/2))*sec(d*x+c)^(1/2)/b/d/(a+b*sec(d*x+c))^(1/2)-3*a*((b+a*cos(d*x+
c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*
sec(d*x+c)^(1/2)/b^2/d/(a+b*sec(d*x+c))^(1/2)-(3*a^2-b^2)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/b^2/(a^2-b^2)
/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)-2*a^2*sec(d*x+c)^(3/2)*
sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+(3*a^2-b^2)*sec(d*x+c)^(1/
2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/b^2/(a^2-b^2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.98 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.39

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{\sec^{\frac{3}{2}}(c+dx)\left(-\frac{a(b+a\cos(c+dx))^{3/2}\left(\frac{8ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2\left(9a^2-7b^2\right)\sqrt{\cdots}}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(Sec[c + d*x]^(3/2)*(-((a*(b + a*Cos[c + d*x])^(3/2)*((8*a*b*Sqrt[(b + a*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Co
s[c + d*x]] + (2*(9*a^2 - 7*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipt
icPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(3*
a^2 - b^2)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d
*x]))/(a - b)]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)
^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*A
rcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a
*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]
], (-a + b)/(a + b)])))/(a^2*Sqrt[(a - b)^(-1)]*b)))/((a - b)*b^2*(a + b))
) + (4*(b + a*Cos[c + d*x])*(-(a^2*b) + b^3 + (-3*a^3 + a*b^2)*Cos[c + d*x
])*Tan[c + d*x])/(-(a^2*b^2) + b^4)))/(4*d*(a + b*Sec[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 3.66 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.04, number of steps used = 26, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 1.040$, Rules used = {3042, 4332, 27, 3042, 4590, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx$$

$\Big\downarrow$ 3042

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$\Big\downarrow$ 4332

$$-\frac{2\int \frac{\sqrt{\sec(c+dx)}\left(a^2-b\sec(c+dx)a-(3a^2-b^2)\sec^2(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 27

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-b\sec(c+dx)a-(3a^2-b^2)\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a+(b^2-3a^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 4590

$$-\frac{\frac{\int \frac{2b\sec(c+dx)a^2+3\left(a^2-b^2\right)\sec^2(c+dx)a+\left(3a^2-b^2\right)a}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b}-\frac{(3a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\int \frac{2b\sec(c+dx)a^2+3\left(a^2-b^2\right)\sec^2(c+dx)a+\left(3a^2-b^2\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}-\frac{(3a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+3\left(a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2a+\left(3a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{(3a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{4596}$$

$$-\frac{\frac{3a(a^2-b^2)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\int \frac{2b\sec(c+dx)a^2+\left(3a^2-b^2\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}-\frac{(3a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{3a(a^2-b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{(3a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{4346}$$

$$-\frac{\frac{\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}}{2b}-\frac{(3a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b}}{bd}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}}{2b}-(3a^2-b^2)\sin(c+dx)\sqrt{}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3286}$$

$$-\frac{\frac{\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}}{2b}-\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{}}{bd}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}}{2b}-\left(3a^2-b^2\right)\sin(c+d}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3284

$$-\frac{\frac{\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}-\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 4523

$$-\frac{\frac{-b\left(a^2-b^2\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+\left(3a^2-b^2\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}-\left(3a^2-b^2\right.}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{-b\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(3a^2-b^2\right)\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}-}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 4343$$

$$-\frac{-b(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\right.}{d\sqrt{a+b\sec(c+dx)}}}{2b} \over b\left(a^2-b^2\right)$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-b(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+\right.}{d\sqrt{a+b\sec(c+dx)}}}{2b} \over b\left(a^2-b^2\right)$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 3134$$

$$-\frac{-b(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+\right.}{d\sqrt{a+b\sec(c+dx)}}}{2b} \over b\left(a^2-b^2\right)$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-b(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}\right.}{d\sqrt{a+b\sec(c+dx)}}}{2b} \over b\left(a^2-b^2\right)$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 3132$$

$$-\frac{\frac{-b(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2}{a-}\right.}{d\sqrt{a+b\sec(c+dx)}}}{2b}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 4345

$$-\frac{\frac{-\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{E}}{d\sqrt{a+b\sec(c+da}}}{2b}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$-\frac{\frac{-\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)}{a+b}}}{d\sqrt{a+b\sec(}}}{2b}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3142

$$-\frac{\frac{-\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx}{a+b}}}{d\sqrt{a+b\sec(}}}{2b}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$-\frac{-\frac{-\frac{b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^2-b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6a(a^2-b^2)\sqrt{\sec(c}\sqrt{\frac{a\cos(c}{a}}}{d\sqrt{a+b}}}{2b}}{b(a^2-b^2)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3140}$$

$$-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-$$

$$\frac{-\frac{-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^2-b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6a(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+}{a+b}}}{d\sqrt{a+b\sec(c-}}}{2b}}{b(a^2-b^2)}$$

input
```
Int[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c
 + d*x]]) - (((-2*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipti
cF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d
*x]]) + (6*a*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2,
(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]
]) + (2*(3*a^2 - b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec
[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(2*
b) - ((3*a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*
x])/(b*d))/(b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.12 (sec) , antiderivative size = 898, normalized size of antiderivative = 2.60

method	result	size
default	Expression too large to display	898

input
```
int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d/(a+b)/((a-b)/(a+b))^(1/2)/b^2*((1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*EllipticPi(((a-b)/(a+b))^(1/2)*(csc
(d*x+c)-cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(6*cos(d*x+c)^3+12*
cos(d*x+c)^2+6*cos(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*a*b*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-c
ot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(6*cos(d*x+c)^3+12*cos(d*x+c
)^2+6*cos(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*a^2*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c))
,(-(a+b)/(a-b))^(1/2))*(3*cos(d*x+c)^3+6*cos(d*x+c)^2+3*cos(d*x+c))+(1/(a+
b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^2*Ell
ipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(
-cos(d*x+c)^3-2*cos(d*x+c)^2-cos(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*EllipticF(((a-b)/(a+b))^(1/2)*
(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-6*cos(d*x+c)^3-12*cos(d*x+
c)^2-6*cos(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*a*b*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)
),(-(a+b)/(a-b))^(1/2))*(-4*cos(d*x+c)^3-8*cos(d*x+c)^2-4*cos(d*x+c))-3*((
a-b)/(a+b))^(1/2)*a^2*cos(d*x+c)*sin(d*x+c)+(-cos(d*x+c)-1)*sin(d*x+c)*((a
-b)/(a+b))^(1/2)*a*b-((a-b)/(a+b))^(1/2)*b^2*sin(d*x+c))*cos(d*x+c)^3*sec(
d*x+c)^(7/2)*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+cos(d*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{7}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{7}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^{3}}{\sec\left(dx+c\right)^{2}b^{2}+2\sec\left(dx+c\right)ab+a^{2}}dx$$

input
```
int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**3)/(sec(c +
 d*x)**2*b**2 + 2*sec(c + d*x)*a*b + a**2),x)
```

3.655 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 206

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}} + \frac{2aE\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{b(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} - \frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{b(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}$$

output
```
2*((b+a*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(
a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/b/d/(a+b*sec(d*x+c))^(1/2)+2*a*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/b/(a^2-
b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)-2*a^2*sec(d*x+c)^(1
/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.31 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.11

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{(b+a\cos(c+dx))^{3/2}\left(\frac{4ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2\left(3a^2-2b^2\right)\sqrt{\frac{b+}{}}}{}\right.}{}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(Sec[c + d*x]^(3/2)*(((b + a*Cos[c + d*x])^(3/2)*((4*a*b*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c
+ d*x]] + (2*(3*a^2 - 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi
[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*Sqrt[-(
(a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc
[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*
Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b
)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/
b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b
)])))/(Sqrt[(a - b)^(-1)]*b)))/((a - b)*(a + b)) + (4*a^2*(b + a*Cos[c + d
*x])*Sin[c + d*x])/(-a^2 + b^2)))/(2*b*d*(a + b*Sec[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.04, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3042, 4332, 27, 3042, 4596, 2011, 3042, 4343, 3042, 3134, 3042, 3132, 4346, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4332}$$

$$-\frac{2\int -\frac{a^2+b\sec(c+dx)a+\left(a^2-b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a^2+b\sec(c+dx)a+\left(a^2-b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a^2+b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{4596}$$

$$\frac{\left(a^2-b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\int \frac{a^2+b\sec(c+dx)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{2011}$$

$$\frac{\left(a^2-b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+a\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{4343}$$

$$\frac{\left(a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\left(a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \textbf{3134}$$

$$\frac{\left(a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\left(a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \textbf{3132}$$

$$\frac{\left(a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \textbf{4346}$$

$$\frac{\frac{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3286}$$

$$\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3284}$$

$$\frac{\frac{2(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
((2*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)
/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*a
*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(b*(a^2 - b^2)) - (2*a^2
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]
])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.68 (sec) , antiderivative size = 661, normalized size of antiderivative = 3.21

method	result
default	$-\frac{2((2\cos(dx+c)^2+4\cos(dx+c)+2)\,\text{EllipticF}(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\sqrt{-\frac{a+b}{a-b}})\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,a}{}$

input
```
int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/(a+b)/((a-b)/(a+b))^(1/2)/b*((2*cos(d*x+c)^2+4*cos(d*x+c)+2)*Elliptic
F(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+
b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a+(cos(
d*x+c)^2+2*cos(d*x+c)+1)*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x
+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*b+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*EllipticE(((a-
b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-2*cos(d*x+
c)^2-4*cos(d*x+c)-2)*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)
),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a+(-2*cos(d*x+c)^2-4*cos(d*x+c)-2)*Ell
ipticPi(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(
a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*b+((a-b)/(a+b))^(1/2)*a*sin(d*x+c))*cos(d*x+c)^3*sec(d*x+c)^(5
/2)*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+cos(d*x+c)*b+b)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{\sec\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^{2}}{\sec\left(dx+c\right)^{2}b^{2}+2\sec\left(dx+c\right)ab+a^{2}}dx$$

input
```
int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**2)/(sec(c +
 d*x)**2*b**2 + 2*sec(c + d*x)*a*b + a**2),x)
```

3.656 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 126

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}$$

output
```
-2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^
(1/2)/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+2*a*sec(
d*x+c)^(1/2)*sin(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.82

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\left((a+b)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)-a\sin(c+dx)\right)}{(a-b)(a+b)d(a+b\sec(c+dx))^{3/2}}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*((a + b)*Sqrt[(b + a*Cos[c + d
*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - a*Sin[c + d*x]))/((a
 - b)*(a + b)*d*(a + b*Sec[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4331, 27, 3042, 4343, 3042, 3134, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \ 4331$$

$$\frac{2\int -\frac{\sqrt{a+b\sec(c+dx)}}{2\sqrt{\sec(c+dx)}}dx}{a^2-b^2}+\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \ 27$$

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a^2-b^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}$$

$$\Big\downarrow \ 4343$$

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}$$

$$\Big\downarrow \textbf{3134}$$

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$$\Big\downarrow \textbf{3132}$$

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

input

```
Int[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

output

```
(-2*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/((a^2
- b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 4331
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 2)/(f*(m + 1)*(a^2 - b^2))), x] - Simp[d^2/((m + 1
)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*
(a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /;
FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2
] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(121) = 242$.

Time = 0.31 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.47

method	result
default	$\frac{2\left(\sqrt{\frac{a-b}{a+b}}\,\sin(dx+c)+\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\operatorname{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\,\sqrt{}}{d(a+}$

input
```
int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/(a+b)/((a-b)/(a+b))^(1/2)*(((a-b)/(a+b))^(1/2)*sin(d*x+c)+(cos(d*x+c)^
2+2*cos(d*x+c)+1)*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-
(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE(((a-b)/(a+b))^(
1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2)))*cos(d*x+c)^2*sec(d*x+c
)^(3/2)*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+cos(d*x+c)*b+b
)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 488, normalized size of antiderivative = 3.87

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{6\,a^2\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\sqrt{2}(-i\,ab\cos\left(dx+c\right)-i\,b^2)}{}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(6*a^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c) - sqrt(2)*(-I*a*b*cos(d*x + c) - I*b^2)*sqrt(a)*weierstrassPInver
se(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x
+ c) + 3*I*a*sin(d*x + c) + 2*b)/a) - sqrt(2)*(I*a*b*cos(d*x + c) + I*b^2)
*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b
^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) + 3*sqrt(2)*
(-I*a^2*cos(d*x + c) - I*a*b)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)
/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)
/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x +
c) + 2*b)/a)) + 3*sqrt(2)*(I*a^2*cos(d*x + c) + I*a*b)*sqrt(a)*weierstrass
Zeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInv
erse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*
x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/((a^4 - a^2*b^2)*d*cos(d*x + c) +
(a^3*b - a*b^3)*d)
```

Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**(3/2)/(a + b*sec(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)}{\sec(dx+c)^2\,b^2+2\sec(dx+c)\,ab+a^2}dx$$

input
```
int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a)*sec(c + d*x))/(sec(c + d*
x)**2*b**2 + 2*sec(c + d*x)*a*b + a**2),x)
```

3.657 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 200

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{ad\sqrt{a+b\sec(c+dx)}} + \frac{2bE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{a\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} - \frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}$$

output
```
2*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(a/
(a+b))^(1/2))*sec(d*x+c)^(1/2)/a/d/(a+b*sec(d*x+c))^(1/2)+2*b*EllipticE(si
n(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a/(a^2-b^
2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)-2*b*sec(d*x+c)^(1/2)*
sin(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\left(b(a+b)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right.}{a(a-b)(a+b)d(a+}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*(b*(a + b)*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a^2 - b^2)*Sqrt[(b
 + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] - a*b*Si
n[c + d*x]))/(a*(a - b)*(a + b)*d*(a + b*Sec[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4330, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4330}$$

$$-\frac{2\int -\frac{b+a\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b+a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 4523

$$\frac{\frac{\left(a^2-b^2\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}+\frac{b\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 4343

$$\frac{\frac{\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3134

$$\frac{\frac{\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 3132$$

$$\frac{\frac{(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 4345$$

$$\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 3142$$

$$\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{\frac{2(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
((2*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*E
llipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(a^2 - b^2) - (2*b*Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4330
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)
*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*S
imp[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 4.15 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.58

method	result
default	$-\frac{2\left(\sqrt{\frac{a-b}{a+b}}\, b\sin(dx+c)+\left(-\cos(dx+c)^2-2\cos(dx+c)-1\right)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\ \text{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot\right.}{}$

input
```
int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/(a+b)/((a-b)/(a+b))^(1/2)/a*(((a-b)/(a+b))^(1/2)*b*sin(d*x+c)+(-cos(d
*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c
)),(-(a+b)/(a-b))^(1/2))*a+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE(((a-b)/
(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*b)*cos(d*x+c)*s
ec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+cos(d*
x+c)*b+b)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 525, normalized size of antiderivative = 2.62

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\, dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(6*a^2*b*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c) + sqrt(2)*(3*I*a^2*b - 2*I*b^3 + (3*I*a^3 - 2*I*a*b^2)*cos(d*x
 + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b
 - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt
(2)*(-3*I*a^2*b + 2*I*b^3 + (-3*I*a^3 + 2*I*a*b^2)*cos(d*x + c))*sqrt(a)*w
eierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1
/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(I*a^2*b*c
os(d*x + c) + I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*
b)/a)) - 3*sqrt(2)*(-I*a^2*b*cos(d*x + c) - I*a*b^2)*sqrt(a)*weierstrassZe
ta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInver
se(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x
+ c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/((a^5 - a^3*b^2)*d*cos(d*x + c) + (a
^4*b - a^2*b^3)*d)
```

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec\left(c+dx\right)}}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/(a + b*sec(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{\sec\left(dx+c\right)b+a}}{\sec\left(dx+c\right)^{2}b^{2}+2\sec\left(dx+c\right)ab+a^{2}}dx$$

input
```
int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/(sec(c + d*x)**2*b**2 +
2*sec(c + d*x)*a*b + a**2),x)
```

3.658 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 214

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{4b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{a^2d\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2-2b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{a^2(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{2b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{a(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}$$

output
```
-4*b*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*
(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/a^2/d/(a+b*sec(d*x+c))^(1/2)+2*(a^2-2*b^
2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^
(1/2)/a^2/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+2*b^
2*sec(d*x+c)^(1/2)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.77

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\sqrt{\sec(c+dx)}\left(\left(a^3+a^2b-2ab^2-2b^3\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c\right.\right.}{a^2(a}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
(2*Sqrt[Sec[c + d*x]]*((a^3 + a^2*b - 2*a*b^2 - 2*b^3)*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + b*(-2*(a^2 - b^2)*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] +
 a*b*Sin[c + d*x])))/(a^2*(a - b)*(a + b)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3042, 4334, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4334}$$

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{a^2-b\sec(c+dx)a-2b^2}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a^2-b\sec(c+dx)a-2b^2}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a-2b^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 4523

$$\frac{\frac{\left(a^2-2b^2\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{2b\left(a^2-b^2\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\left(a^2-2b^2\right)\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{2b\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 4343

$$\frac{\frac{\left(a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{2b\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\left(a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{2b\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3134

$$\frac{\frac{(a^2-2b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(a^2-2b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3132

$$\frac{\frac{2(a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 4345

$$\frac{\frac{2(a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{a\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2(a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3142

$$\frac{\frac{2(a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2(a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{\frac{2(a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{4b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
((-4*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*
(a^2 - 2*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x
]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(a*(a^2 -
 b^2)) + (2*b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a +
 b*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(207) = 414$.

Time = 5.47 (sec) , antiderivative size = 593, normalized size of antiderivative = 2.77

method	result
default	$-\frac{2\left(\left(-\cos(dx+c)^2-2\cos(dx+c)-1\right)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,a^2\,\mathrm{EllipticE}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)\right.}{}$

input
```
int(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/((a-b)/(a+b))^(1/2)/(a+b)/a^2*((-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b
)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*Elli
pticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(2
*cos(d*x+c)^2+4*cos(d*x+c)+2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*b^2*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)
-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*
(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*Ellipt
icF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(2*c
os(d*x+c)^2+4*cos(d*x+c)+2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*a*b*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-c
ot(d*x+c)),(-(a+b)/(a-b))^(1/2))-((a-b)/(a+b))^(1/2)*a^2*cos(d*x+c)*sin(d*
x+c)+(-cos(d*x+c)-1)*sin(d*x+c)*((a-b)/(a+b))^(1/2)*a*b-2*((a-b)/(a+b))^(1
/2)*b^2*sin(d*x+c))*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+co
s(d*x+c)*b+b)/sec(d*x+c)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 565, normalized size of antiderivative = 2.64

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(6*a^2*b^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c) - sqrt(2)*(-5*I*a^2*b^2 + 4*I*b^4 + (-5*I*a^3*b + 4*I*a*b^3)*
cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(
9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)
 - sqrt(2)*(5*I*a^2*b^2 - 4*I*b^4 + (5*I*a^3*b - 4*I*a*b^3)*cos(d*x + c))*
sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^
3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) + 3*sqrt(2)*(
I*a^3*b - 2*I*a*b^3 + (I*a^4 - 2*I*a^2*b^2)*cos(d*x + c))*sqrt(a)*weierstr
assZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassP
Inverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos
(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) + 3*sqrt(2)*(-I*a^3*b + 2*I*a*b^
3 + (-I*a^4 + 2*I*a^2*b^2)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a
^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a
^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a
*sin(d*x + c) + 2*b)/a)))/((a^6 - a^4*b^2)*d*cos(d*x + c) + (a^5*b - a^3*b
^3)*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+b\sec(c+dx))^{\frac{3}{2}}\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**(3/2)*sqrt(sec(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^3\,b^2+2\sec(dx+c)^2\,ab+\sec(dx+c)\,a^2}dx$$

input
```
int(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/(sec(c + d*x)**3*b**2 +
2*sec(c + d*x)**2*a*b + sec(c + d*x)*a**2),x)
```

3.659 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 289

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2(a^2+8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+}{3a^3d\sqrt{a+b\sec(c+dx)}}$$
$$-\frac{2b(5a^2-8b^2)\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3a^3(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$
$$+\frac{2b^2\sin(c+dx)}{a(a^2-b^2)\,d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$
$$+\frac{2(a^2-4b^2)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3a^2(a^2-b^2)\,d\sqrt{\sec(c+dx)}}$$

output
```
2/3*(a^2+8*b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2
*c,2^(1/2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/a^3/d/(a+b*sec(d*x+c))^(1/2)-
2/3*b*(5*a^2-8*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*
(a+b*sec(d*x+c))^(1/2)/a^3/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(
d*x+c)^(1/2)+2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/sec(d*x+c)^(1/2)/(a+b*sec(d*x+
c))^(1/2)+2/3*(a^2-4*b^2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/a^2/(a^2-b^2)/
d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\sqrt{\sec(c+dx)}\Big(b(-5a^3-5a^2b+8ab^2+8b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E(}{}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
(2*Sqrt[Sec[c + d*x]]*(b*(-5*a^3 - 5*a^2*b + 8*a*b^2 + 8*b^3)*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a^4 + 7*a^
2*b^2 - 8*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*a)/(a + b)] + a*(b*(a^2 - 4*b^2) + a*(a^2 - b^2)*Cos[c + d*x])*Sin[c + d
*x]))/(3*a^3*(a - b)*(a + b)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 2.16 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.02, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {3042, 4334, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4334}$$

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{a^2-b\sec(c+dx)a-4b^2+2b^2\sec^2(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{a^2-b\sec(c+dx)a-4b^2+2b^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a-4b^2+2b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \ 4592$$

$$\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{2\int \frac{b\left(5a^2-8b^2\right)-a\left(a^2+2b^2\right)\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{b\left(5a^2-8b^2\right)-a\left(a^2+2b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{b\left(5a^2-8b^2\right)-a\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \ 4523$$

$$\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{b\left(5a^2-8b^2\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{\left(a^4+7a^2b^2-8b^4\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}}{3a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{b(5a^2-8b^2)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{(a^4+7a^2b^2-8b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 4343

$$\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{b(5a^2-8b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{(a^4+7a^2b^2-8b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{b(5a^2-8b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{(a^4+7a^2b^2-8b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3134

$$\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{b(5a^2-8b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^4+7a^2b^2-8b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{b\left(5a^2-8b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^4+7a^2b^2-8b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3132

$$\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\left(5a^2-8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^4+7a^2b^2-8b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 4345

$$\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\left(5a^2-8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^4+7a^2b^2-8b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c\cdots}}}{a\sqrt{a+b\sec(c+dx)}}}{3a}}{a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\left(5a^2-8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^4+7a^2b^2-8b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c\cdots\right.}}}{a\sqrt{a+b\sec(c+dx)}}}{3a}}{a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3142

$$\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b(5a^2-8b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^4+7a^2b^2-8b^4)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a}{\cdots}}}}{a\sqrt{a+b\sec(c+dx)}}}{3a}}{a(a^2-b^2)}$$
$$\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b(5a^2-8b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^4+7a^2b^2-8b^4)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{}{\sqrt{\frac{b}{a+b}+\frac{a}{\cdots}}}}{a\sqrt{a+b\sec(c+dx)}}}{3a}}{a(a^2-b^2)}$$
$$\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3140

$$\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}+$$
$$\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b(5a^2-8b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2(a^4+7a^2b^2-8b^4)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}\cdots\right.}{ad\sqrt{a+b\sec(c+dx)}}}{3a}}{a(a^2-b^2)}$$

input
```
Int[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
(2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c
+ d*x]]) + (-1/3*((-2*(a^4 + 7*a^2*b^2 - 8*b^4)*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sq
rt[a + b*Sec[c + d*x]]) + (2*b*(5*a^2 - 8*b^2)*EllipticE[(c + d*x)/2, (2*a
)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b
)]*Sqrt[Sec[c + d*x]]))/a + (2*(a^2 - 4*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[
c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]))/(a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 738 vs. $2(272) = 544$.

Time = 6.89 (sec) , antiderivative size = 739, normalized size of antiderivative = 2.56

method	result
default	$\frac{2\sqrt{a+b\sec(dx+c)}\left(\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\sqrt{\frac{1}{1+\cos(dx+c)}}\,a^2b\,\text{EllipticE}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)(-5\cos(dx+c)-\right.}{}$

input
```
int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/a^3/((a-b)/(a+b))^(1/2)/(a+b)*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a
+a*cos(d*x+c)+cos(d*x+c)*b+b)/sec(d*x+c)^(3/2)*((1/(a+b)*(b+a*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticE(((a-b)/(a+b
))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-5*cos(d*x+c)-10-5
*sec(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*b^3*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a
+b)/(a-b))^(1/2))*(8*cos(d*x+c)+16+8*sec(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*EllipticF(((a-b)/(a+b)
)^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(cos(d*x+c)+2+sec(d*
x+c))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*a^2*b*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(
a-b))^(1/2))*(6*cos(d*x+c)+12+6*sec(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2*EllipticF(((a-b)/(a+b))^(
1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(8*cos(d*x+c)+16+8*sec(
d*x+c))+sin(d*x+c)*(1+cos(d*x+c))*((a-b)/(a+b))^(1/2)*a^3+(cos(d*x+c)^2-3*
cos(d*x+c)+1)*((a-b)/(a+b))^(1/2)*a^2*b*tan(d*x+c)+((a-b)/(a+b))^(1/2)*a*b
^2*(-4*sin(d*x+c)-4*tan(d*x+c))-8*((a-b)/(a+b))^(1/2)*b^3*tan(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 632, normalized size of antiderivative = 2.19

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/9*(sqrt(2)*(3*I*a^4*b + 16*I*a^2*b^3 - 16*I*b^5 + (3*I*a^5 + 16*I*a^3*b
^2 - 16*I*a*b^4)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4
*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d
*x + c) + 2*b)/a) + sqrt(2)*(-3*I*a^4*b - 16*I*a^2*b^3 + 16*I*b^5 + (-3*I*
a^5 - 16*I*a^3*b^2 + 16*I*a*b^4)*cos(d*x + c))*sqrt(a)*weierstrassPInverse
(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x +
c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(-5*I*a^3*b^2 + 8*I*a*b^4 +
(-5*I*a^4*b + 8*I*a^2*b^3)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a
^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a
^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a
*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(5*I*a^3*b^2 - 8*I*a*b^4 + (5*I*a^4*b
 - 8*I*a^2*b^3)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)
/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)
/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x +
c) + 2*b)/a)) - 6*((a^5 - a^3*b^2)*cos(d*x + c)^2 + (a^4*b - 4*a^2*b^3)*co
s(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(
d*x + c)))/((a^7 - a^5*b^2)*d*cos(d*x + c) + (a^6*b - a^4*b^3)*d)
```

Sympy [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^4\,b^2+2\sec(dx+c)^3\,ab+\sec(dx+c)^2\,a^2}dx$$

input
```
int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/(sec(c + d*x)**4*b**2 +
2*sec(c + d*x)**3*a*b + sec(c + d*x)**2*a**2),x)
```

3.660 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 360

$$\begin{aligned}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = & -\frac{8b(a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{5a^4d\sqrt{a+b\sec(c+dx)}} \\ & +\frac{2(3a^4+8a^2b^2-16b^4)\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{5a^4(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} \\ & +\frac{2b^2\sin(c+dx)}{a(a^2-b^2)\,d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} \\ & +\frac{2(a^2-6b^2)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5a^2(a^2-b^2)\,d\sec^{\frac{3}{2}}(c+dx)} \\ & -\frac{2b(3a^2-8b^2)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5a^3(a^2-b^2)\,d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
-8/5*b*(a^2+4*b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+
1/2*c,2^(1/2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/a^4/d/(a+b*sec(d*x+c))^(1/
2)+2/5*(3*a^4+8*a^2*b^2-16*b^4)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a
+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^4/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b)
)^(1/2)/sec(d*x+c)^(1/2)+2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/sec(d*x+c)^(3/2)/(
a+b*sec(d*x+c))^(1/2)+2/5*(a^2-6*b^2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/a^
2/(a^2-b^2)/d/sec(d*x+c)^(3/2)-2/5*b*(3*a^2-8*b^2)*(a+b*sec(d*x+c))^(1/2)*
sin(d*x+c)/a^3/(a^2-b^2)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{\sec(c+dx)}\left(4(3a^5+3a^4b+8a^3b^2+8a^2b^3-16ab^4-16b^5)\,\sqrt{}\right.}{}$$

input
```
Integrate[1/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(4*(3*a^5 + 3*a^4*b + 8*a^3*b^2 + 8*a^2*b^3 - 16*a*b^4
 - 16*b^5)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)
/(a + b)] - 16*b*(a^4 + 3*a^2*b^2 - 4*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a +
b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + 2*a*(a^4 - 7*a^2*b^2 + 16*b^4
- 4*a*b*(a^2 - b^2)*Cos[c + d*x] + (a^4 - a^2*b^2)*Cos[2*(c + d*x)])*Sin[c
 + d*x]))/(10*a^4*(a - b)*(a + b)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 2.84 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.880$, Rules used = {3042, 4334, 27, 3042, 4592, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$$

$\Big\downarrow$ 3042

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$\Big\downarrow$ 4334

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{a^2-b\sec(c+dx)a-6b^2+4b^2\sec^2(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{a^2-b\sec(c+dx)a-6b^2+4b^2\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a-6b^2+4b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 4592

$$\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{2\int \frac{-2b\left(a^2-6b^2\right)\sec^2(c+dx)-a\left(3a^2+2b^2\right)\sec(c+dx)+3b\left(3a^2-8b^2\right)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 27

$$\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-2b\left(a^2-6b^2\right)\sec^2(c+dx)-a\left(3a^2+2b^2\right)\sec(c+dx)+3b\left(3a^2-8b^2\right)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-2b\left(a^2-6b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-a\left(3a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+3b\left(3a^2-8b^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 4592$$

$$\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{2\int \frac{3\left(3a^4+8b^2a^2-b\left(a^2+4b^2\right)\sec(c+dx)a-16b^4\right)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{5a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{3a^4+8b^2a^2-b\left(a^2+4b^2\right)\sec(c+dx)a-16b^4}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a}}{5a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{3a^4+8b^2a^2-b\left(a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a-16b^4}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{5a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 4523$$

$$\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{\left(3a^4+8a^2b^2-16b^4\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{4b\left(a^4+3a^2b^2-4b^4\right)\int \cdots}{a}}{a}}{5a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{(3a^4+8a^2b^2-16b^4)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{4b(a^4+3a^2b^2-4b^4)}{a}}{5a}}{a(a^2-b^2)}$$
$$\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{4343}$$

$$\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{(3a^4+8a^2b^2-16b^4)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{4b(a^4+3}{}}{a}}{5a}}{a(a^2-b^2)}$$
$$\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{(3a^4+8a^2b^2-16b^4)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{4b(}{}}{a}}{5a}}{a(a^2-b^2)}$$
$$\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3134}$$

$$\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{(3a^4+8a^2b^2-16b^4)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{4b(}{}}{a}}{5a}}{a(a^2-b^2)}$$
$$\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{\left(3a^4+8a^2b^2-16b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\cdots}{a}}{5a}}{a\left(a^2-b^2\right)}\cdots$$

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3132

$$\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(3a^4+8a^2b^2-16b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{4b\left(a^4+3a\cdots\right.}{\cdots}}{a}}{5a}}{a\left(a^2-b^2\right)}\cdots$$

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 4345

$$\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(3a^4+8a^2b^2-16b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{4b\left(a^4+3a\cdots\right.}{\cdots}}{a}}{5a}}{a\left(a^2-b^2\right)}\cdots$$

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(3a^4+8a^2b^2-16b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{4b\left(a^4+3a\cdots\right.}{\cdots}}{a}}{5a}}{a\left(a^2-b^2\right)}\cdots$$

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3142

$$\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{2(3a^4+8a^2b^2-16b^4)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{4b(a^4+3a\cdots}{a\cdots}}{\cdots}}{5a}}{a(a^2-b^2)}$$

$$\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{2(3a^4+8a^2b^2-16b^4)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{4b(a^4+3a\cdots}{\cdots}}{\cdots}}{5a}}{a(a^2-b^2)}$$

$$\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3140

$$\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}+$$

$$\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{2(3a^4+8a^2b^2-16b^4)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{8b(a^4+3a\cdots}{a\cdots}}{\cdots}}{5a}}{a(a^2-b^2)}$$

input
```
Int[1/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
(2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c
+ d*x]]) + ((2*(a^2 - 6*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a*d
*Sec[c + d*x]^(3/2)) - (-(((-8*b*(a^4 + 3*a^2*b^2 - 4*b^4)*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x
]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^4 + 8*a^2*b^2 - 16*b^4)*Ellip
ticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/a) + (2*b*(3*a^2 - 8*b^2)*Sq
rt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(a*d*Sqrt[Sec[c + d*x]]))/(5*a))/(a*(
a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1044 vs. $2(337) = 674$.

Time = 7.96 (sec) , antiderivative size = 1045, normalized size of antiderivative = 2.90

method	result	size
default	Expression too large to display	1045

input
```
int(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/d/a^4/((a-b)/(a+b))^(1/2)/(a+b)*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a
+a*cos(d*x+c)+cos(d*x+c)*b+b)/sec(d*x+c)^(5/2)*((1/(a+b)*(b+a*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4*EllipticE(((a-b)/(a+b))
^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(3+6*sec(d*x+c)+3*sec
(d*x+c)^2)+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*a^2*b^2*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-
(a+b)/(a-b))^(1/2))*(8+16*sec(d*x+c)+8*sec(d*x+c)^2)+(1/(a+b)*(b+a*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^4*EllipticE(((a-b)/(
a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-16-32*sec(d*x+
c)-16*sec(d*x+c)^2)+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*a^4*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c
)),(-(a+b)/(a-b))^(1/2))*(-3-6*sec(d*x+c)-3*sec(d*x+c)^2)+(1/(a+b)*(b+a*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b*EllipticF((
(a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-4-8*sec
(d*x+c)-4*sec(d*x+c)^2)+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*a^2*b^2*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-c
ot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-12-24*sec(d*x+c)-12*sec(d*x+c)^2)+(1/(a
+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^3*
EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2)
)*(-16-32*sec(d*x+c)-16*sec(d*x+c)^2)+(cos(d*x+c)^2+cos(d*x+c)+3)*((a-b...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 692, normalized size of antiderivative = 1.92

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/15*(sqrt(2)*(-9*I*a^4*b^2 - 28*I*a^2*b^4 + 32*I*b^6 + (-9*I*a^5*b - 28*
I*a^3*b^3 + 32*I*a*b^5)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*
a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*
a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(9*I*a^4*b^2 + 28*I*a^2*b^4 - 32*I*b^6
+ (9*I*a^5*b + 28*I*a^3*b^3 - 32*I*a*b^5)*cos(d*x + c))*sqrt(a)*weierstras
sPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*c
os(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(3*I*a^5*b + 8*I*a^
3*b^3 - 16*I*a*b^5 + (3*I*a^6 + 8*I*a^4*b^2 - 16*I*a^2*b^4)*cos(d*x + c))*
sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a
^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a
^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(-3*
I*a^5*b - 8*I*a^3*b^3 + 16*I*a*b^5 + (-3*I*a^6 - 8*I*a^4*b^2 + 16*I*a^2*b^
4)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a))
 - 6*((a^6 - a^4*b^2)*cos(d*x + c)^3 - 2*(a^5*b - a^3*b^3)*cos(d*x + c)^2
- (3*a^4*b^2 - 8*a^2*b^4)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x
+ c))*sin(d*x + c)/sqrt(cos(d*x + c)))/((a^8 - a^6*b^2)*d*cos(d*x + c) + (
a^7*b - a^5*b^3)*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec{(dx+c)}+a)^{\frac{3}{2}}\sec{(dx+c)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec{(dx+c)}+a)^{\frac{3}{2}}\sec{(dx+c)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^5\,b^2+2\sec(dx+c)^4\,ab+\sec(dx+c)^3\,a^2}dx$$

input

```
int(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x)
```

output

```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/(sec(c + d*x)**5*b**2 +
2*sec(c + d*x)**4*a*b + sec(c + d*x)**3*a**2),x)
```

3.661 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 458

$$\begin{aligned}\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx &= \frac{(5a^2-3b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3b^2(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}\\
&-\frac{5a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{b^3d\sqrt{a+b\sec(c+dx)}}\\
&-\frac{(15a^4-26a^2b^2+3b^4)\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3b^3(a^2-b^2)^2\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}\\
&-\frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}}-\frac{2a^2(5a^2-9b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}\\
&+\frac{(15a^4-26a^2b^2+3b^4)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3b^3(a^2-b^2)^2\,d}\end{aligned}$$

output
```
1/3*(5*a^2-3*b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1
/2*c,2^(1/2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d/(a+b*sec(d*
x+c))^(1/2)-5*a*((b+a*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*
c),2,2^(1/2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/b^3/d/(a+b*sec(d*x+c))^(1/2
)-1/3*(15*a^4-26*a^2*b^2+3*b^4)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a
+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/b^3/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+
b))^(1/2)/sec(d*x+c)^(1/2)-2/3*a^2*sec(d*x+c)^(5/2)*sin(d*x+c)/b/(a^2-b^2)
/d/(a+b*sec(d*x+c))^(3/2)-2/3*a^2*(5*a^2-9*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c
)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)+1/3*(15*a^4-26*a^2*b^2+3*b^4)*s
ec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/b^3/(a^2-b^2)^2/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.97 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.22

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{\sec^{\frac{5}{2}}(c+dx)\left(-\frac{a(b+a\cos(c+dx))^{5/2}\left(\frac{8ab\left(5a^2-9b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2(\cdots}{\cdots}\right)}{\cdots}\cdots\right)}{\cdots}$$

input
```
Integrate[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(Sec[c + d*x]^(5/2)*(-((a*(b + a*Cos[c + d*x])^(5/2)*((8*a*b*(5*a^2 - 9*b^
2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)
])/Sqrt[b + a*Cos[c + d*x]] + (2*(45*a^4 - 86*a^2*b^2 + 33*b^4)*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b
 + a*Cos[c + d*x]] + ((2*I)*(15*a^4 - 26*a^2*b^2 + 3*b^4)*Sqrt[-((a*(-1 +
Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]
*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d
*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*S
qrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcS
inh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(a^
2*Sqrt[(a - b)^(-1)]*b)))/((a - b)^2*(a + b)^2)) + (2*(b + a*Cos[c + d*x])
*(15*a^6 - 20*a^4*b^2 - 9*a^2*b^4 + 6*b^6 + 4*a*b*(10*a^4 - 17*a^2*b^2 + 3
*b^4)*Cos[c + d*x] + (15*a^6 - 26*a^4*b^2 + 3*a^2*b^4)*Cos[2*(c + d*x)])*T
an[c + d*x])/(a^2 - b^2)^2))/(12*b^3*d*(a + b*Sec[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 4.75 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.03, number of steps used = 29, number of rules used = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 1.160$, Rules used = {3042, 4332, 27, 3042, 4586, 27, 3042, 4590, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4332}$$

$$-\frac{2\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(3a^2-3b\sec(c+dx)a-\left(5a^2-3b^2\right)\sec^2(c+dx)\right)}{2(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(3a^2-3b\sec(c+dx)a-\left(5a^2-3b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3a^2-3b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(3b^2-5a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 4586

$$-\frac{\frac{2a^2\left(5a^2-9b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{\sqrt{\sec(c+dx)}\left(\left(5a^2-9b^2\right)a^2-2b\left(a^2-3b^2\right)\sec(c+dx)a-\left(15a^4-26b^2a^2+3b^4\right)\sec^2(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\int \frac{\sqrt{\sec(c+dx)}\left(\left(5a^2-9b^2\right)a^2-2b\left(a^2-3b^2\right)\sec(c+dx)a-\left(15a^4-26b^2a^2+3b^4\right)\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}+\frac{2a^2\left(5a^2-9b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\left(5a^2-9b^2\right)a^2-2b\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(-15a^4+26b^2a^2-3b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\left(a^2-b^2\right)}+\frac{2a^2\left(5a^2-9b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 4590

$$-\frac{\frac{\frac{\int \frac{2b\left(5a^2-9b^2\right)\sec(c+dx)a^2+15\left(a^2-b^2\right)^2\sec^2(c+dx)a+\left(15a^4-26b^2a^2+3b^4\right)a}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b}-\frac{\left(15a^4-26a^2b^2+3b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{b(a^2-b^2)}+\frac{2a^2(5}{}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{\int \frac{2b\left(5a^2-9b^2\right)\sec(c+dx)a^2+15\left(a^2-b^2\right)^2\sec^2(c+dx)a+\left(15a^4-26b^2a^2+3b^4\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}-\frac{\left(15a^4-26a^2b^2+3b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{b(a^2-b^2)}+\frac{2a^2(5}{}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{\int \frac{2b\left(5a^2-9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^2+15\left(a^2-b^2\right)^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a+\left(15a^4-26b^2a^2+3b^4\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{\left(15a^4-26a^2b^2+3b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4596

$$-\frac{\frac{\frac{15a\left(a^2-b^2\right)^2\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\int \frac{2b\left(5a^2-9b^2\right)\sec(c+dx)a^2+\left(15a^4-26b^2a^2+3b^4\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}-\frac{\left(15a^4-26a^2b^2+3b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{15a\left(a^2-b^2\right)^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{2b\left(5a^2-9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(15a^4-26b^2a^2+3b^4\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{\left(15a^4-26a^2b^2+3b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\,\mathrm{se}}}{bd}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 4346

$$-\frac{\frac{\frac{\frac{15a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\int\frac{2b\left(5a^2-9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(15a^4-26b^2a^2+3b^4\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{\left(15a^4-26a^2b^2+3b^4\right)\sin(c\cdots}{}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{\frac{15a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\int\frac{2b\left(5a^2-9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(15a^4-26b^2a^2+3b^4\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{\left(15a^4-\cdots\right.}{}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3286

$$-\frac{\frac{\frac{\frac{15a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\int\frac{2b\left(5a^2-9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(15a^4-26b^2a^2+3b^4\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{\left(15a^4-26a^2b^2+3b^4\right)\cdots}{}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{\frac{15a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\int\frac{2b\left(5a^2-9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(15a^4-26b^2a^2+3b^4\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{(15\cdots}{}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3284

$$-\frac{\frac{\frac{\int \frac{2b\left(5a^2-9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(15a^4-26b^2a^2+3b^4\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{30a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}-\left(15a^4-26a^2b^2+3b^4\cdots\right.}{b(a^2-b^2)}\cdots}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \; 4523$$

$$-\frac{\frac{\frac{-b\left(5a^4-8a^2b^2+3b^4\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+\left(15a^4-26a^2b^2+3b^4\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\frac{30a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\cdots\right.}{d\sqrt{a+b\sec(c+dx)}}}{2b}\cdots}{b(a^2-b^2)}\cdots}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{-b\left(5a^4-8a^2b^2+3b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(15a^4-26a^2b^2+3b^4\right)\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{30a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\cdots\right.}{d\sqrt{a+b\sec(c+dx)}}}{2b}\cdots}{b(a^2-b^2)}\cdots}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \; 4343$$

$$-\frac{\frac{\frac{-b\left(5a^4-8a^2b^2+3b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(15a^4-26a^2b^2+3b^4\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{30a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{E}\cdots}{d\sqrt{a+b\sec(c+dx)}}}{2b}\cdots}{b(a^2-b^2)}\cdots}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{-b\left(5a^4-8a^2b^2+3b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(15a^4-26a^2b^2+3b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{30a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)\cdots}{a+b}}}{d\sqrt{a+b\sec(c+\cdots}}}{2b}}{b(a^2-b^2)} \cdots}{3b\left(a^2-b^2\right)\cdots}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3134

$$-\frac{\frac{-b\left(5a^4-8a^2b^2+3b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(15a^4-26a^2b^2+3b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{30a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)\cdots}{a+b}}}{d\sqrt{a+b\sec(c+\cdots}}}{2b}}{b(a^2-b^2)} \cdots}{3b\left(a^2-b^2\right)\cdots}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{-b\left(5a^4-8a^2b^2+3b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(15a^4-26a^2b^2+3b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{30a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+\cdots}{a+\cdots}}}{d\sqrt{a+b\,\mathrm{se}\cdots}}}{2b}}{b(a^2-b^2)} \cdots}{3b\left(a^2-b^{\cdots}\right.}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3132

$$-\frac{\frac{-b\left(5a^4-8a^2b^2+3b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{30a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(15a^4-26a^2b^2+3b^4\right)\sqrt{a+b\sec\cdots}}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\,\mathrm{c}\cdots}{}}}}{2b}}{b(a^2-b^2)} \cdots}{3b\left(a^2-b^2\right)\cdots}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4345

$$-\frac{\frac{\frac{-\frac{b\left(5a^4-8a^2b^2+3b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{30a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(15a^4 \cdots\right.}{\cdots}}{2b}}{b(a^2-b^2)}}{3b\left(a \cdots\right.}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\downarrow 3042$$

$$-\frac{\frac{\frac{-\frac{b\left(5a^4-8a^2b^2+3b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{30a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(1 \cdots\right.}{\cdots}}{2b}}{b(a^2-b^2)}}{3b\left(\cdots\right.}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\downarrow 3142$$

$$-\frac{\frac{\frac{-\frac{b\left(5a^4-8a^2b^2+3b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{30a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(\cdots\right.}{\cdots}}{2b}}{b(a^2-b^2)}}{3b\left(\cdots\right.}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\downarrow 3042$$

$$-\frac{\frac{\frac{-\frac{b\left(5a^4-8a^2b^2+3b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{30a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}} \cdots}{2b}}{b(a^2-b^2)}}{3 \cdots}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\downarrow 3140$$

$$-\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{2a^2\left(5a^2-9b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{\frac{30a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-\frac{2b\left(5a^4-8a^2b^2+3b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\,c\cdots}{}}}{d\sqrt{a+b\,\text{se}\cdots}}}{2b}}{3b\left(a^2\cdots\right.}$$

input
```
Int[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c +
 d*x])^(3/2)) - ((2*a^2*(5*a^2 - 9*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (((-2*b*(5*a^4 - 8*a^2*b^2 + 3
*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (30*a*(a^2 - b^2)^
2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
 b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*(15*a^4 - 26*a^
2*b^2 + 3*b^4)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*
x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(2*b) - ((
15*a^4 - 26*a^2*b^2 + 3*b^4)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*S
in[c + d*x])/(b*d))/(b*(a^2 - b^2)))/(3*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4586
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Simp[d/(b*(a^2 - b^2)*(m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
 - b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n
+ b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C
}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.53 (sec) , antiderivative size = 2354, normalized size of antiderivative = 5.14

method	result	size
default	Expression too large to display	2354

input
```
int(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d/b^3/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)*((1/(a+b)*(b+a*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^5*EllipticPi(((a-b)/(a+b))^(
1/2)*(csc(d*x+c)-cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(-30*cos(d
*x+c)^4-60*cos(d*x+c)^3-30*cos(d*x+c)^2)+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4*b*EllipticPi(((a-b)/(a+b))^(1/
2)*(csc(d*x+c)-cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(-30*cos(d*x
+c)^4-90*cos(d*x+c)^3-90*cos(d*x+c)^2-30*cos(d*x+c))+cos(d*x+c)*(-30*cos(d
*x+c)-30)*sin(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*a^3*b^2*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-c
ot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))+(1/(a+b)*(b+a*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b^3*EllipticPi(((a-b)/(a+
b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(30*c
os(d*x+c)^4+90*cos(d*x+c)^3+90*cos(d*x+c)^2+30*cos(d*x+c))+(1/(a+b)*(b+a*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^4*EllipticPi
(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(
1/2))*(30*cos(d*x+c)^3+60*cos(d*x+c)^2+30*cos(d*x+c))+(1/(a+b)*(b+a*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^5*EllipticE(((a-b)/
(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-15*cos(d*x+c)
^4-30*cos(d*x+c)^3-15*cos(d*x+c)^2)+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4*b*EllipticE(((a-b)/(a+b))^(1/2)*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(9/2)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{9}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(9/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{9}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(9/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(9/2)/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(9/2)/(a + b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)^4}{\sec(dx+c)^3\,b^3+3\sec(dx+c)^2\,a\,b^2+3\sec(dx+c)\,a^2b+a^3}dx$$

input
```
int(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**4)/(sec(c +
 d*x)**3*b**3 + 3*sec(c + d*x)**2*a*b**2 + 3*sec(c + d*x)*a**2*b + a**3),x
)
```

3.662 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 370

$$\begin{aligned}
&\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \\
&-\frac{2a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3b\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} \\
&+\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{b^2d\sqrt{a+b\sec(c+dx)}} \\
&+\frac{2a(3a^2-7b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3b^2\,(a^2-b^2)^2\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} \\
&-\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}}-\frac{2a^2(3a^2-7b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b^2\,(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}
\end{aligned}$$

output
```
-2/3*a*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2
)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+2
*((b+a*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a
/(a+b))^(1/2))*sec(d*x+c)^(1/2)/b^2/d/(a+b*sec(d*x+c))^(1/2)+2/3*a*(3*a^2-
7*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+
c))^(1/2)/b^2/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2
)-2/3*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)
-2/3*a^2*(3*a^2-7*b^2)*sec(d*x+c)^(1/2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*
sec(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.71 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.32

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{4ab^2(a^2-3b^2)\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{5/2}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{(a-b)^2}+\frac{b(9a^4-19a^2b^2+6b}{}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(Sec[c + d*x]^(5/2)*((4*a*b^2*(a^2 - 3*b^2)*((b + a*Cos[c + d*x])/(a + b))
^(5/2)*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(a - b)^2 + (b*(9*a^4 - 19*a
^2*b^2 + 6*b^4)*((b + a*Cos[c + d*x])/(a + b))^(5/2)*EllipticPi[2, (c + d*
x)/2, (2*a)/(a + b)])/(a - b)^2 + (I*((a - b)^(-1))^(3/2)*(3*a^2 - 7*b^2)*
Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a -
b)]*(b + a*Cos[c + d*x])^(5/2)*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcS
inh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2
*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a +
 b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b +
 a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(a + b)^2 + (2*a^2*b^2*(b + a*Cos[
c + d*x])*Sin[c + d*x])/(-a^2 + b^2) + (2*a^2*b*(-3*a^2 + 7*b^2)*(b + a*Co
s[c + d*x])^2*Sin[c + d*x])/(a^2 - b^2)^2))/(3*b^3*d*(a + b*Sec[c + d*x])^
(5/2))
```

Rubi [A] (verified)

Time = 3.94 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.05, number of steps used = 26, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.040, Rules used = {3042, 4332, 27, 3042, 4586, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4332}$$

$$-\frac{2\int \frac{\sqrt{\sec(c+dx)}\left(a^2-3b\sec(c+dx)a-3\left(a^2-b^2\right)\sec^2(c+dx)\right)}{2(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-3b\sec(c+dx)a-3\left(a^2-b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2-3b\csc\left(c+dx+\frac{\pi}{2}\right)a-3\left(a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4586}$$

$$-\frac{\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{2\int \frac{\left(3a^2-7b^2\right)a^2+2b\left(a^2-3b^2\right)\sec(c+dx)a+3\left(a^2-b^2\right)^2\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\big\downarrow$ 27

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{\left(3a^2-7b^2\right)a^2+2b\left(a^2-3b^2\right)\sec(c+dx)a+3\left(a^2-b^2\right)^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$\big\downarrow$ 3042

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{\left(3a^2-7b^2\right)a^2+2b\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+3\left(a^2-b^2\right)^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$\big\downarrow$ 4596

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{3(a^2-b^2)^2\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\int \frac{\left(3a^2-7b^2\right)a^2+2b\left(a^2-3b^2\right)\sec(c+dx)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$\big\downarrow$ 3042

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{3(a^2-b^2)^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{\left(3a^2-7b^2\right)a^2+2b\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$\big\downarrow$ 4346

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{\left(3a^2-7b^2\right)a^2+2b\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}}{\sqrt{a+b\sec(c+dx)}}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$\big\downarrow$ 3042

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3(a^2-b^2)^2\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{}}}{\sqrt{a+b\sec(c+dx)}}}{b(a^2-b^2)}}{3b(a^2-b^2)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3286

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3(a^2-b^2)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{\sec(c+dx}{\sqrt{\frac{b}{a+b}+\frac{a\cos(}{a-}}}}{\sqrt{a+b\sec(c+dx)}}}{b(a^2-b^2)}}{3b(a^2-b^2)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3(a^2-b^2)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{}{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sec(c+dx)}}}{b(a^2-b^2)}}{3b(a^2-b^2)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3284

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6(a^2-b^2)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c-\right.}{d\sqrt{a+b\sec(c+dx)}}}{b(a^2-b^2)}}{3b(a^2-b^2)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4523

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-ab(a^2-b^2)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+a(3a^2-7b^2)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\frac{6(a^2-b^2)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c\cdots}{a+\cdots}}}{d\sqrt{a+b\cdots}}}{b(a^2-b^2)}}{3b(a^2-b^2)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-ab(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a(3a^2-7b^2)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6(a^2-b^2)^2\sqrt{\sec(c+dx)}\cdots}{\cdots}}{b(a^2-b^2)}}{3b(a^2-b^2)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4343

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-ab(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a(3a^2-7b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{6(a^2-b^2)^2\sqrt{\cdots}}{\cdots}}{b(a^2-b^2)}}{3b(a^2-b^2)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-ab(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a(3a^2-7b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{6(a^2-b^2\cdots}{\cdots}}{b(a^2-b^2)}}{3b(a^2-b^2)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3134

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-ab(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a(3a^2-7b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6(a^2-b^2\cdots}{\cdots}}{b(a^2-b^2)}}{3b(a^2-b^2)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-ab(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a(3a^2-7b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6(a^2}{}}{b(a^2-b^2)}}{3b(a^2-b^2)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3132}$$

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-ab(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a(3a^2-7b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6(a^2-b^2)^2\sqrt{\text{se}}}{}}{b(a^2-b^2)}}{3b(a^2-b^2)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4345}$$

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{ab(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a(3a^2-7b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b(a^2-b^2)}}{3b(a^2-b^2)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{ab(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a(3a^2-7b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b(a^2-b^2)}}{3b(a^2-b^2)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3142}$$

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{ab(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a(3a^2-7b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+}{a+b}}}}{b(a^2-b^2)}}{3b(a^2-b^2)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\big\downarrow$ 3042

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{ab(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a(3a^2-7b^2)\sqrt{a+b\sec(c+dx)}E\left(\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+}{a+b}}}}{b(a^2-b^2)}}{3b(a^2-b^2)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\big\downarrow$ 3140

$$-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}-$$

$$\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{2ab(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2a(3a^2-7b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+d\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b(a^2-b^2)}}{3b(a^2-b^2)}$$

input

```
Int[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

output

```
(-2*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c +
 d*x])^(3/2)) - (-(((-2*a*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*S
ec[c + d*x]]) + (6*(a^2 - b^2)^2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipt
icPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[
c + d*x]]) + (2*a*(3*a^2 - 7*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sq
rt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c +
 d*x]]))/(b*(a^2 - b^2))) + (2*a^2*(3*a^2 - 7*b^2)*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))/(3*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4586
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Simp[d/(b*(a^2 - b^2)*(m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
 - b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n
+ b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C
}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.73 (sec) , antiderivative size = 1916, normalized size of antiderivative = 5.18

method	result	size
default	Expression too large to display	1916

input
```
int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/((a-b)/(a+b))^(1/2)/(a+b)^2/(a-b)/b^2*((1/(a+b)*(b+a*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4*EllipticPi(((a-b)/(a+b))^(
1/2)*(csc(d*x+c)-cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(6*cos(d*x
+c)^3+12*cos(d*x+c)^2+6*cos(d*x+c))+(6*cos(d*x+c)^3+18*cos(d*x+c)^2+18*cos
(d*x+c)+6)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*a^3*b*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(a+
b)/(a-b),I/((a-b)/(a+b))^(1/2))+(6*cos(d*x+c)+6)*sin(d*x+c)^2*(1/(a+b)*(b+
a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b^2*Ellip
ticPi(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+
b))^(1/2))+(-6*cos(d*x+c)^3-18*cos(d*x+c)^2-18*cos(d*x+c)-6)*(1/(a+b)*(b+a
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^3*Elliptic
Pi(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))
^(1/2))+(-6*cos(d*x+c)^2-12*cos(d*x+c)-6)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^4*EllipticPi(((a-b)/(a+b))^(1/2
)*(csc(d*x+c)-cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))+(1/(a+b)*(b+a
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4*EllipticE(
((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(3*cos(d
*x+c)^3+6*cos(d*x+c)^2+3*cos(d*x+c))+(3*cos(d*x+c)^2+6*cos(d*x+c)+3)*(1/(a
+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b*
EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{7}{2}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{7}{2}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)^3}{\sec(dx+c)^3\,b^3+3\sec(dx+c)^2\,a\,b^2+3\sec(dx+c)\,a^2b+a^3}dx$$

input
```
int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**3)/(sec(c +
 d*x)**3*b**3 + 3*sec(c + d*x)**2*a*b**2 + 3*sec(c + d*x)*a**2*b + a**3),x
)
```

3.663 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 277

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{8bE\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3(a^2-b^2)^2\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} - \frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3b(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{2a(a^2-5b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}$$

output
```
2/3*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(
a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+8/3*b*
EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/
2)/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)-2/3*a^2*s
ec(d*x+c)^(1/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)+2/3*a*(a^2
-5*b^2)*sec(d*x+c)^(1/2)*sin(d*x+c)/b/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.61

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx =$$
$$-\frac{2\sec^{\frac{3}{2}}(c+dx)\left(-4b(a+b)^2\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{3/2}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)-(a-b)(a+b)^2\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{3/2}\text{Ellip}\right.}{3(a-b)^2(a+b)^2d(a+b\sec(c+dx))}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*Sec[c + d*x]^(3/2)*(-4*b*(a + b)^2*((b + a*Cos[c + d*x])/(a + b))^(3/2
)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - (a - b)*(a + b)^2*((b + a*Cos[c
+ d*x])/(a + b))^(3/2)*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + a*(-a^2 + 5
*b^2 + 4*a*b*Cos[c + d*x])*Sin[c + d*x]))/(3*(a - b)^2*(a + b)^2*d*(a + b*
Sec[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 2.09 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.06, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.760, Rules used = {3042, 4332, 27, 3042, 4588, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4332}$$

$$-\frac{2\int -\frac{a^2+3b\sec(c+dx)a+(a^2-3b^2)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{a^2+3b\sec(c+dx)a+(a^2-3b^2)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{a^2+3b\csc\left(c+dx+\frac{\pi}{2}\right)a+(a^2-3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4588$$

$$\frac{\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{4a^2b^2+a\left(a^2+3b^2\right)\sec(c+dx)b}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{4a^2b^2+a\left(a^2+3b^2\right)\sec(c+dx)b}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{4a^2b^2+a\left(a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4523$$

$$\frac{\frac{ab(a^2-b^2)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+4ab^2\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{ab(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+4ab^2\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \textbf{4343}$$

$$\frac{\frac{ab(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4ab^2\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{ab(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4ab^2\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \textbf{3134}$$

$$\frac{\frac{ab(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4ab^2\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{ab(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4ab^2\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \textbf{3132}$$

$$\frac{\frac{\frac{ab(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{8ab^2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 4345

$$\frac{\frac{\frac{ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8ab^2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8ab^2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3142

$$\frac{\frac{\frac{ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8ab^2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{ab(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8ab^2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b(a^2-b^2)}$$
$$\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{\frac{2ab(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{8ab^2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}}{3b(a^2-b^2)}-$$
$$\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

input
```
Int[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c +
 d*x])^(3/2)) + (((2*a*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*El
lipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[
c + d*x]]) + (8*a*b^2*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec
[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(a*
(a^2 - b^2)) + (2*a*(a^2 - 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 -
 b^2)*d*Sqrt[a + b*Sec[c + d*x]]))/(3*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(258) = 516$.

Time = 3.98 (sec) , antiderivative size = 763, normalized size of antiderivative = 2.75

method	result
default	$\frac{2\left(\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\sqrt{\frac{1}{1+\cos(dx+c)}}\,ab\,\text{EllipticE}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)\left(4\cos(dx+c)^3+8\cos(dx+c)^2+4\cos\right.\right.}{}$

input
```
int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)*((1/(a+b)*(b+a*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b*EllipticE(((a-b)/(a+b))^(1/2)*
(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(4*cos(d*x+c)^3+8*cos(d*x+c)
^2+4*cos(d*x+c))+(4*cos(d*x+c)^2+8*cos(d*x+c)+4)*(1/(a+b)*(b+a*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^2*EllipticE(((a-b)/(a+b)
)^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(1/(a+b)*(b+a*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*EllipticF(((a-b)/
(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(cos(d*x+c)^3+2
*cos(d*x+c)^2+cos(d*x+c))+(-3*cos(d*x+c)^3-5*cos(d*x+c)^2-cos(d*x+c)+1)*(1
/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b
*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2
))+(-3*cos(d*x+c)^2-6*cos(d*x+c)-3)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^2*EllipticF(((a-b)/(a+b))^(1/2)*(csc(
d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+sin(d*x+c)*(1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)*a^2+(-3*cos(d*x+c)+1)*sin(d*x+c)*((a-b)/(a+b))^(1/2)*a*b-4*((
a-b)/(a+b))^(1/2)*b^2*sin(d*x+c))*cos(d*x+c)^3*sec(d*x+c)^(5/2)*(a+b*sec(d
*x+c))^(1/2)/(cos(d*x+c)^2*(1+cos(d*x+c))*a^2+cos(d*x+c)*(2*cos(d*x+c)+2)*
a*b+(1+cos(d*x+c))*b^2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 688, normalized size of antiderivative = 2.48

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/9*(sqrt(2)*(-3*I*a^2*b^2 - I*b^4 + (-3*I*a^4 - I*a^2*b^2)*cos(d*x + c)^2
 - 2*(3*I*a^3*b + I*a*b^3)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*
(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3
*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(3*I*a^2*b^2 + I*b^4 + (3*I*a^4 + I*
a^2*b^2)*cos(d*x + c)^2 - 2*(-3*I*a^3*b - I*a*b^3)*cos(d*x + c))*sqrt(a)*w
eierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1
/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 12*sqrt(2)*(-I*a^3*b
*cos(d*x + c)^2 - 2*I*a^2*b^2*cos(d*x + c) - I*a*b^3)*sqrt(a)*weierstrassZ
eta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInve
rse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x
 + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 12*sqrt(2)*(I*a^3*b*cos(d*x + c)^2
 + 2*I*a^2*b^2*cos(d*x + c) + I*a*b^3)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2
 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2
 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*s
in(d*x + c) + 2*b)/a)) - 6*(4*a^3*b*cos(d*x + c)^2 - (a^4 - 5*a^2*b^2)*cos
(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d
*x + c)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c)^2 + 2*(a^6*b - 2*a^4
*b^3 + a^2*b^5)*d*cos(d*x + c) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)^{2}}{\sec(dx+c)^{3}\,b^{3}+3\sec(dx+c)^{2}\,a\,b^{2}+3\sec(dx+c)\,a^{2}b+a^{3}}dx$$

input
```
int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**2)/(sec(c +
 d*x)**3*b**3 + 3*sec(c + d*x)**2*a*b**2 + 3*sec(c + d*x)*a**2*b + a**3),x
)
```

3.664 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 281

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2(3a^2+b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3a\left(a^2-b^2\right)^2 d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4(a^2+b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}}$$

output
```
-2/3*b*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2
)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)-2
/3*(3*a^2+b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*
sec(d*x+c))^(1/2)/a/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c
)^(1/2)+2/3*a*sec(d*x+c)^(1/2)*sin(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/
2)+4/3*(a^2+b^2)*sec(d*x+c)^(1/2)*sin(d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c)
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.63

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{\sec^{\frac{5}{2}}(c+dx)\left(-\frac{2(a+b)\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{5/2}\left((3a^2+b^2)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+(a-b)b\,\text{EllipticF}\left(\frac{1}{2}(c+\cdots\right.\right.}{a(a-b)^2}\cdots\right.}{3d(a+b\sec(c+dx\cdots}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(Sec[c + d*x]^(5/2)*((-2*(a + b)*((b + a*Cos[c + d*x])/(a + b))^(5/2)*((3*
a^2 + b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a - b)*b*EllipticF[(c
+ d*x)/2, (2*a)/(a + b)]))/(a*(a - b)^2) + (2*(b + a*Cos[c + d*x])*(2*b*(a
^2 + b^2) + a*(3*a^2 + b^2)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2))/(3
*d*(a + b*Sec[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 2.02 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.02, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.760, Rules used = {3042, 4331, 27, 3042, 4588, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4331}$$

$$\frac{2\int -\frac{-2a\sec^2(c+dx)+3b\sec(c+dx)+a}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}+\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\int\frac{-2a\sec^2(c+dx)+3b\sec(c+dx)+a}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 3042$$

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\int\frac{-2a\csc\left(c+dx+\frac{\pi}{2}\right)^2+3b\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 4588$$

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{-\frac{2\int-\frac{4b\sec(c+dx)a^2+\left(3a^2+b^2\right)a}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\int\frac{4b\sec(c+dx)a^2+\left(3a^2+b^2\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 3042$$

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\int\frac{4b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2+b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}-\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 4523$$

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+\left(3a^2+b^2\right)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(3a^2+b^2\right)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}-\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$\downarrow$ 4343

$$\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a\left(a^2-b^2\right)}-\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$\downarrow$ 3042

$$\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a\left(a^2-b^2\right)}-\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$\downarrow$ 3134

$$\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a\left(a^2-b^2\right)}-\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$\downarrow$ 3042

$$\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a\left(a^2-b^2\right)}-\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$\downarrow$ 3132

$$\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}-\frac{4(a^2+b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$\downarrow$ 4345

$$\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}-\frac{4(a^2+b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$\downarrow$ 3042

$$\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}-\frac{4(a^2+b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$\downarrow$ 3142

$$\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}-\frac{4(a^2+b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$\downarrow$ 3042

$$\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}-\frac{4(a^2+b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$\downarrow$ 3140

$$\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^2+b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}-\frac{4(a^2+b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3(a^2-b^2)}}{}$$

input
```
Int[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sec[c + d*x]
)^(3/2)) - (((2*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x
]]) + (2*(3*a^2 + b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Se
c[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(a
*(a^2 - b^2)) - (4*(a^2 + b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^
2)*d*Sqrt[a + b*Sec[c + d*x]]))/(3*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4331
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 2)/(f*(m + 1)*(a^2 - b^2))), x] - Simp[d^2/((m + 1
)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*
(a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /;
FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2
] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1023 vs. $2(262) = 524$.

Time = 2.77 (sec) , antiderivative size = 1024, normalized size of antiderivative = 3.64

method	result	size
default	Expression too large to display	1024

input
```
int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/(a-b)/(a+b)^2/a/((a-b)/(a+b))^(1/2)*((1/(a+b)*(b+a*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*EllipticE(((a-b)/(a+b))^(1/2
)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-3*cos(d*x+c)^3-6*cos(d*x
+c)^2-3*cos(d*x+c))+(-3*cos(d*x+c)^2-6*cos(d*x+c)-3)*(1/(a+b)*(b+a*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticE(((a-b)
/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(1/(a+b)*(b+a*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2*EllipticE
(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-cos(d
*x+c)^3-2*cos(d*x+c)^2-cos(d*x+c))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)
*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^3*Ellip
ticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(1/
(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*
EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2)
)*(3*cos(d*x+c)^3+6*cos(d*x+c)^2+3*cos(d*x+c))+(-cos(d*x+c)^3+cos(d*x+c)^2
+5*cos(d*x+c)+3)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*a^2*b*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)
),(-(a+b)/(a-b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2*EllipticF(((a-
b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+3*((a-b)/(a+
b))^(1/2)*a^3*cos(d*x+c)*sin(d*x+c)+(-cos(d*x+c)+2)*sin(d*x+c)*((a-b)/(...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 759, normalized size of antiderivative = 2.70

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/9*(2*sqrt(2)*(-3*I*a^2*b^3 + I*b^5 + (-3*I*a^4*b + I*a^2*b^3)*cos(d*x +
 c)^2 + 2*(-3*I*a^3*b^2 + I*a*b^4)*cos(d*x + c))*sqrt(a)*weierstrassPInver
se(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x
+ c) + 3*I*a*sin(d*x + c) + 2*b)/a) + 2*sqrt(2)*(3*I*a^2*b^3 - I*b^5 + (3*
I*a^4*b - I*a^2*b^3)*cos(d*x + c)^2 + 2*(3*I*a^3*b^2 - I*a*b^4)*cos(d*x +
c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b -
8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) + 3*sqrt(
2)*(3*I*a^3*b^2 + I*a*b^4 + (3*I*a^5 + I*a^3*b^2)*cos(d*x + c)^2 + 2*(3*I*
a^4*b + I*a^2*b^3)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b
^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b
^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x
 + c) + 2*b)/a)) + 3*sqrt(2)*(-3*I*a^3*b^2 - I*a*b^4 + (-3*I*a^5 - I*a^3*b
^2)*cos(d*x + c)^2 + 2*(-3*I*a^4*b - I*a^2*b^3)*cos(d*x + c))*sqrt(a)*weie
rstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstr
assPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a
*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)) - 6*((3*a^5 + a^3*b^2)*cos(d
*x + c)^2 + 2*(a^4*b + a^2*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/co
s(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/((a^8 - 2*a^6*b^2 + a^4*b^4)*
d*cos(d*x + c)^2 + 2*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*x + c) + (a^6*b
^2 - 2*a^4*b^4 + a^2*b^6)*d)
```

Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{\sec^{\frac{3}{2}}\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**(3/2)/(a + b*sec(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}\,\sec(dx+c)}{\sec(dx+c)^{3}\,b^{3}+3\sec(dx+c)^{2}\,a\,b^{2}+3\sec(dx+c)\,a^{2}b+a^{3}}dx$$

input
```
int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a)*sec(c + d*x))/(sec(c + d*
x)**3*b**3 + 3*sec(c + d*x)**2*a*b**2 + 3*sec(c + d*x)*a**2*b + a**3),x)
```

3.665 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 302

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2(3a^2-2b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^2(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{4b(3a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3a^2(a^2-b^2)^2\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} - \frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{3(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{2b(5a^2-b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}$$

output
```
2/3*(3*a^2-2*b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1
/2*c,2^(1/2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d/(a+b*sec(d*
x+c))^(1/2)+4/3*b*(3*a^2-b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b
))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^2/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b)
)^(1/2)/sec(d*x+c)^(1/2)-2/3*b*sec(d*x+c)^(1/2)*sin(d*x+c)/(a^2-b^2)/d/(a+
b*sec(d*x+c))^(3/2)-2/3*b*(5*a^2-b^2)*sec(d*x+c)^(1/2)*sin(d*x+c)/a/(a^2-b
^2)^2/d/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2(b+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\left(\frac{\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{3/2}\left((6a^2b-2b^3)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+(3\right.}{(a-b)}\right.}{3a^2d(a+b}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*((((b + a*Cos[c + d*x])/(a + b)
)^(3/2)*((6*a^2*b - 2*b^3)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (3*a^3
- 3*a^2*b - 2*a*b^2 + 2*b^3)*EllipticF[(c + d*x)/2, (2*a)/(a + b)]))/(a -
b)^2 + (a*b*(-5*a^2*b + b^3 + (-6*a^3 + 2*a*b^2)*Cos[c + d*x])*Sin[c + d*x
])/(a^2 - b^2)^2))/(3*a^2*d*(a + b*Sec[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 2.18 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.04, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {3042, 4330, 27, 3042, 4588, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4330}$$

$$-\frac{2\int -\frac{-2b\sec^2(c+dx)+3a\sec(c+dx)+b}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{-2b\sec^2(c+dx)+3a\sec(c+dx)+b}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3(a^2-b^2)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{-2b\csc\left(c+dx+\frac{\pi}{2}\right)^2+3a\csc\left(c+dx+\frac{\pi}{2}\right)+b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3(a^2-b^2)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 4588$$

$$\frac{-\frac{2\int -\frac{2b(3a^2-b^2)+a(3a^2+b^2)\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3(a^2-b^2)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{2b(3a^2-b^2)+a(3a^2+b^2)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3(a^2-b^2)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{2b(3a^2-b^2)+a(3a^2+b^2)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3(a^2-b^2)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 4523$$

$$\frac{\frac{\frac{2b(3a^2-b^2)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}+\frac{(3a^4-5a^2b^2+2b^4)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3(a^2-b^2)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{2b(3a^2-b^2)\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{(3a^4-5a^2b^2+2b^4)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3(a^2-b^2)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 4343

$$\frac{\frac{\frac{2b(3a^2-b^2)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{(3a^4-5a^2b^2+2b^4)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3(a^2-b^2)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{2b(3a^2-b^2)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{(3a^4-5a^2b^2+2b^4)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3(a^2-b^2)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3134

$$\frac{\frac{\frac{2b(3a^2-b^2)\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{(3a^4-5a^2b^2+2b^4)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3(a^2-b^2)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{2b(3a^2-b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{(3a^4-5a^2b^2+2b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3(a^2-b^2)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\big\downarrow$ 3132

$$\frac{\frac{\frac{(3a^4-5a^2b^2+2b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{4b(3a^2-b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3(a^2-b^2)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\big\downarrow$ 4345

$$\frac{\frac{\frac{(3a^4-5a^2b^2+2b^4)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{4b(3a^2-b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3(a^2-b^2)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{\frac{\frac{(3a^4-5a^2b^2+2b^4)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{4b(3a^2-b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3(a^2-b^2)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\big\downarrow$ 3142

$$\frac{\frac{\frac{\left(3a^4-5a^2b^2+2b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)\sqrt{\text{se}}}{ad(a^2-b^2)\sqrt{a+b\sec(c}}}{3\left(a^2-b^2\right)}$$
$$\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\left(3a^4-5a^2b^2+2b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\,\text{se}}}}{3\left(a^2-b^2\right)}$$
$$\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{\frac{\frac{4b\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\left(3a^4-5a^2b^2+2b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)\sqrt{\text{s}}}{ad(a^2-b^2)\sqrt{a+b\sec(}}}{3\left(a^2-b^2\right)}$$
$$\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sec[c + d*x
])^(3/2)) + (((2*(3*a^4 - 5*a^2*b^2 + 2*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a
+ b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[
a + b*Sec[c + d*x]]) + (4*b*(3*a^2 - b^2)*EllipticE[(c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sq
rt[Sec[c + d*x]]))/(a*(a^2 - b^2)) - (2*b*(5*a^2 - b^2)*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))/(3*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4330
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)
*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*S
imp[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1148 vs. $2(283) = 566$.

Time = 5.30 (sec) , antiderivative size = 1149, normalized size of antiderivative = 3.80

method	result	size
default	Expression too large to display	1149

input
```
int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)/a^2*((1/(a+b)*(b+a*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b*EllipticE(((a-b)/(a+b))
^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-6*cos(d*x+c)^3-12*c
os(d*x+c)^2-6*cos(d*x+c))+(-6*cos(d*x+c)^2-12*cos(d*x+c)-6)*(1/(a+b)*(b+a*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b^2*Ellipti
cE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(1/(a
+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^3*
EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2)
)*(2*cos(d*x+c)^3+4*cos(d*x+c)^2+2*cos(d*x+c))+(2*cos(d*x+c)^2+4*cos(d*x+c
)+2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*b^4*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b
))^(1/2))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*a^4*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)
/(a-b))^(1/2))*(-3*cos(d*x+c)^3-6*cos(d*x+c)^2-3*cos(d*x+c))+(-3*cos(d*x+c
)-3)*sin(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*a^3*b*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c
)),(-(a+b)/(a-b))^(1/2))+(2*cos(d*x+c)^3+7*cos(d*x+c)^2+8*cos(d*x+c)+3)*(1
/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2
*b^2*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^
(1/2))+(2*cos(d*x+c)^2+4*cos(d*x+c)+2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 811, normalized size of antiderivative = 2.69

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/9*(sqrt(2)*(-9*I*a^4*b^2 + 9*I*a^2*b^4 - 4*I*b^6 + (-9*I*a^6 + 9*I*a^4*b
^2 - 4*I*a^2*b^4)*cos(d*x + c)^2 - 2*(9*I*a^5*b - 9*I*a^3*b^3 + 4*I*a*b^5)
*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*
(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a
) + sqrt(2)*(9*I*a^4*b^2 - 9*I*a^2*b^4 + 4*I*b^6 + (9*I*a^6 - 9*I*a^4*b^2
+ 4*I*a^2*b^4)*cos(d*x + c)^2 - 2*(-9*I*a^5*b + 9*I*a^3*b^3 - 4*I*a*b^5)*c
os(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)
- 6*sqrt(2)*(-3*I*a^3*b^3 + I*a*b^5 + (-3*I*a^5*b + I*a^3*b^3)*cos(d*x + c
)^2 + 2*(-3*I*a^4*b^2 + I*a^2*b^4)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-
4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-
4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c)
 + 3*I*a*sin(d*x + c) + 2*b)/a)) - 6*sqrt(2)*(3*I*a^3*b^3 - I*a*b^5 + (3*I
*a^5*b - I*a^3*b^3)*cos(d*x + c)^2 + 2*(3*I*a^4*b^2 - I*a^2*b^4)*cos(d*x +
 c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b
^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b
^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)) - 6*(2*(3*a
^5*b - a^3*b^3)*cos(d*x + c)^2 + (5*a^4*b^2 - a^2*b^4)*cos(d*x + c))*sqrt(
(a*cos(d*x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/((a^9
- 2*a^7*b^2 + a^5*b^4)*d*cos(d*x + c)^2 + 2*(a^8*b - 2*a^6*b^3 + a^4*b^...
```

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec{(c+dx)}}}{(a+b\sec{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/(a + b*sec(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^3\,b^3+3\sec(dx+c)^2\,a\,b^2+3\sec(dx+c)\,a^2b+a^3}dx$$

input
```
int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/(sec(c + d*x)**3*b**3 +
3*sec(c + d*x)**2*a*b**2 + 3*sec(c + d*x)*a**2*b + a**3),x)
```

3.666 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 317

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2b(9a^2-8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^3(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2(3a^4-15a^2b^2+8b^4)\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3a^3(a^2-b^2)^2\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{2b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{8b^2(2a^2-b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}$$

output
```
-2/3*b*(9*a^2-8*b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*
x+1/2*c,2^(1/2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)/d/(a+b*sec
(d*x+c))^(1/2)+2/3*(3*a^4-15*a^2*b^2+8*b^4)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^3/(a^2-b^2)^2/d/((b+a*cos
(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+2/3*b^2*sec(d*x+c)^(1/2)*sin(d*x+c)
/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)+8/3*b^2*(2*a^2-b^2)*sec(d*x+c)^(1/2)
*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.66

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2(b+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\left(\frac{\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{3/2}\left((3a^4-15a^2b^2+\right.}{}\right.}{}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
(2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*((((b + a*Cos[c + d*x])/(a + b)
)^(3/2)*((3*a^4 - 15*a^2*b^2 + 8*b^4)*EllipticE[(c + d*x)/2, (2*a)/(a + b)
] + b*(-9*a^3 + 9*a^2*b + 8*a*b^2 - 8*b^3)*EllipticF[(c + d*x)/2, (2*a)/(a
 + b)]))/(a - b)^2 + (a*b^2*(8*a^2*b - 4*b^3 + a*(9*a^2 - 5*b^2)*Cos[c + d
*x])*Sin[c + d*x])/(a^2 - b^2)^2))/(3*a^3*d*(a + b*Sec[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 2.35 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.05, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {3042, 4334, 27, 3042, 4588, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4334}$$

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2\int -\frac{3a^2-3b\sec(c+dx)a-4b^2+2b^2\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{3a^2-3b\sec(c+dx)a-4b^2+2b^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{3a^2-3b\csc\left(c+dx+\frac{\pi}{2}\right)a-4b^2+2b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 4588$$

$$\frac{\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{3a^4-15b^2a^2-2b\left(3a^2-b^2\right)\sec(c+dx)a+8b^4}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{3a^4-15b^2a^2-2b\left(3a^2-b^2\right)\sec(c+dx)a+8b^4}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{3a^4-15b^2a^2-2b\left(3a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+8b^4}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 4523$$

$$\frac{\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}}{a\left(a^2-b^2\right)}+\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{8b^2(2a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \text{4343}$$

$$\frac{\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{8b^2(2a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{8b^2(2a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \text{3134}$$

$$\frac{\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{8b^2(2a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{8b^2(2a^2-b^2)\sin(c+dx)\sqrt{\sec(c+}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \text{3132}$$

$$\frac{\frac{\frac{2\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{8b^2(2a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \text{4345}$$

$$\frac{\frac{\frac{2\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{a\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}+\frac{8b^2(2a^2-b^2)\sin(c+d}{ad(a^2-b^2)\sqrt{a+b}}}{3a\left(a^2-b^2\right)}$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{2\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}+\frac{8b^2(2a^2-b^2)\sin(}{ad(a^2-b^2)\sqrt{}}}{3a\left(a^2-b^2\right)}$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \text{3142}$$

$$\frac{\frac{\frac{2\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}+\frac{8b^2(2a^2-b^2)\sin}{ad(a^2-b^2)\sqrt{}}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}+\frac{8b^2(2a^2-b^2}{ad(a^2-b}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3140

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+$$

$$\frac{\frac{8b^2(2a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{\frac{2\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b\left(9a^4-17a^2b^2+8b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{Elli}}{ad\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
(2*b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c +
d*x])^(3/2)) + (((-2*b*(9*a^4 - 17*a^2*b^2 + 8*b^4)*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*
d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^4 - 15*a^2*b^2 + 8*b^4)*EllipticE[(c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(a*(a^2 - b^2)) + (8*b^2*(2*a^2 - b
^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d
*x]]))/(3*a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1599 vs. $2(298) = 596$.

Time = 6.78 (sec) , antiderivative size = 1600, normalized size of antiderivative = 5.05

method	result	size
default	Expression too large to display	1600

input
```
int(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/a^3/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)*((1/(a+b)*(b+a*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^5*EllipticE(((a-b)/(a+b))^(1
/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(3*cos(d*x+c)^3+6*cos(d*
x+c)^2+3*cos(d*x+c))+(3*cos(d*x+c)^2+6*cos(d*x+c)+3)*(1/(a+b)*(b+a*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4*b*EllipticE(((a-b)
/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(1/(a+b)*(b+a*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b^2*Ellipti
cE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-15*
cos(d*x+c)^3-30*cos(d*x+c)^2-15*cos(d*x+c))+(-15*cos(d*x+c)^2-30*cos(d*x+c
)-15)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*a^2*b^3*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)
/(a-b))^(1/2))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*a*b^4*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),
(-(a+b)/(a-b))^(1/2))*(8*cos(d*x+c)^3+16*cos(d*x+c)^2+8*cos(d*x+c))+(8*cos
(d*x+c)^2+16*cos(d*x+c)+8)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*b^5*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-co
t(d*x+c)),(-(a+b)/(a-b))^(1/2))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^5*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+
c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-3*cos(d*x+c)^3-6*cos(d*x+c)^2-3*cos
(d*x+c))+(-9*cos(d*x+c)^3-21*cos(d*x+c)^2-15*cos(d*x+c)-3)*(1/(a+b)*(b+...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 863, normalized size of antiderivative = 2.72

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/9*(4*sqrt(2)*(-6*I*a^4*b^3 + 9*I*a^2*b^5 - 4*I*b^7 + (-6*I*a^6*b + 9*I*
a^4*b^3 - 4*I*a^2*b^5)*cos(d*x + c)^2 + 2*(-6*I*a^5*b^2 + 9*I*a^3*b^4 - 4*
I*a*b^6)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^
2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c)
+ 2*b)/a) + 4*sqrt(2)*(6*I*a^4*b^3 - 9*I*a^2*b^5 + 4*I*b^7 + (6*I*a^6*b -
9*I*a^4*b^3 + 4*I*a^2*b^5)*cos(d*x + c)^2 + 2*(6*I*a^5*b^2 - 9*I*a^3*b^4 +
 4*I*a*b^6)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)
/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x +
c) + 2*b)/a) + 3*sqrt(2)*(-3*I*a^5*b^2 + 15*I*a^3*b^4 - 8*I*a*b^6 + (-3*I*
a^7 + 15*I*a^5*b^2 - 8*I*a^3*b^4)*cos(d*x + c)^2 + 2*(-3*I*a^6*b + 15*I*a^
4*b^3 - 8*I*a^2*b^5)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4
*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4
*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d
*x + c) + 2*b)/a)) + 3*sqrt(2)*(3*I*a^5*b^2 - 15*I*a^3*b^4 + 8*I*a*b^6 + (
3*I*a^7 - 15*I*a^5*b^2 + 8*I*a^3*b^4)*cos(d*x + c)^2 + 2*(3*I*a^6*b - 15*I
*a^4*b^3 + 8*I*a^2*b^5)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2
- 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2
- 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*si
n(d*x + c) + 2*b)/a)) - 6*((9*a^5*b^2 - 5*a^3*b^4)*cos(d*x + c)^2 + 4*(2*a
^4*b^3 - a^2*b^5)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))...
```

Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{(a+b\sec{(c+dx)})^{\frac{5}{2}}\sqrt{\sec{(c+dx)}}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**(5/2)*sqrt(sec(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^4\,b^3+3\sec(dx+c)^3\,a\,b^2+3\sec(dx+c)^2\,a^2b+}$$

input
```
int(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/(sec(c + d*x)**4*b**3 +
3*sec(c + d*x)**3*a*b**2 + 3*sec(c + d*x)**2*a**2*b + sec(c + d*x)*a**3),x
)
```

3.667 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 391

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2(a^4+16a^2b^2-16b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^4(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{8b(2a^4-7a^2b^2+4b^4)\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3a^4(a^2-b^2)^2\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{2b^2\sin(c+dx)}{3a(a^2-b^2)\,d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{4b^2(5a^2-3b^2)\sin(c+dx)}{3a^2(a^2-b^2)^2\,d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2(a^4-13a^2b^2+8b^4)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3a^3(a^2-b^2)^2\,d\sqrt{\sec(c+dx)}}$$

output
```
2/3*(a^4+16*a^2*b^2-16*b^4)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM
(1/2*d*x+1/2*c,2^(1/2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/a^4/(a^2-b^2)/d/(
a+b*sec(d*x+c))^(1/2)-8/3*b*(2*a^4-7*a^2*b^2+4*b^4)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^4/(a^2-b^2)^2/d/(
(b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+2/3*b^2*sin(d*x+c)/a/(a^2-b
^2)/d/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2)+4/3*b^2*(5*a^2-3*b^2)*sin(d*
x+c)/a^2/(a^2-b^2)^2/d/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*(a^4-13
*a^2*b^2+8*b^4)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/a^3/(a^2-b^2)^2/d/sec(d*
x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.66

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2(b+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\left(\frac{\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{3/2}\left(-4(2a^4b-7a^2b^3\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
(2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*((((b + a*Cos[c + d*x])/(a + b)
)^(3/2)*(-4*(2*a^4*b - 7*a^2*b^3 + 4*b^5)*EllipticE[(c + d*x)/2, (2*a)/(a
+ b)] + (a^5 - a^4*b + 16*a^3*b^2 - 16*a^2*b^3 - 16*a*b^4 + 16*b^5)*Ellipt
icF[(c + d*x)/2, (2*a)/(a + b)]))/(a - b)^2 + (a*(a^6 - 25*a^2*b^4 + 16*b^
6 + 4*a*b*(a^4 - 8*a^2*b^2 + 5*b^4)*Cos[c + d*x] + (a^3 - a*b^2)^2*Cos[2*(
c + d*x)])*Sin[c + d*x])/(2*(a^2 - b^2)^2)))/(3*a^4*d*(a + b*Sec[c + d*x])
^(5/2))
```

Rubi [A] (verified)

Time = 3.07 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.02, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.880$, Rules used = {3042, 4334, 27, 3042, 4588, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4334}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}-\frac{2\int -\frac{4b^2\sec^2(c+dx)-3ab\sec(c+dx)+3\left(a^2-2b^2\right)}{2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4b^2\sec^2(c+dx)-3ab\sec(c+dx)+3\left(a^2-2b^2\right)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{4b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2-3ab\csc\left(c+dx+\frac{\pi}{2}\right)+3\left(a^2-2b^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4588}$$

$$\frac{\frac{4b^2\left(5a^2-3b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{4b^2\left(5a^2-3b^2\right)\sec^2(c+dx)-2ab\left(3a^2-b^2\right)\sec(c+dx)+3\left(a^4-13b^2a^2+8b^4\right)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{4b^2\left(5a^2-3b^2\right)\sec^2(c+dx)-2ab\left(3a^2-b^2\right)\sec(c+dx)+3\left(a^4-13b^2a^2+8b^4\right)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}+\frac{4b^2\left(5a^2-3b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{4b^2(5a^2-3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2-2ab(3a^2-b^2)\csc\left(c+dx+\frac{\pi}{2}\right)+3(a^4-13b^2a^2+8b^4)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{4b^2(5a^2-3b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}}{3a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{3ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4592}$$

$$\frac{\frac{\frac{2(a^4-13a^2b^2+8b^4)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{2\int \frac{3(4b(2a^4-7b^2a^2+4b^4)-a(a^4+7b^2a^2-4b^4)\sec(c+dx))}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{a(a^2-b^2)}+\frac{4b^2(5a^2-3b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}}{3a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{3ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{2(a^4-13a^2b^2+8b^4)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{4b(2a^4-7b^2a^2+4b^4)-a(a^4+7b^2a^2-4b^4)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a}}{a(a^2-b^2)}+\frac{4b^2(5a^2-3b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}}{3a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{3ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{2(a^4-13a^2b^2+8b^4)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{4b(2a^4-7b^2a^2+4b^4)-a(a^4+7b^2a^2-4b^4)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{4b^2(5a^2-3b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}}{3a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{3ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4523}$$

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{\left(a^6+15a^4b^2-32a^2b^4+16b^6\right)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}}{a}}{a(a^2-b^2)}+\frac{\ldots}{ad(a^2-\ldots}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{\left(a^6+15a^4b^2-32a^2b^4+16b^6\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a}}{a(a^2-b^2)}+\ldots}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4343

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{\left(a^6+15a^4b^2-32a^2b^4+16b^6\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx\ldots\right.}}\ldots}{a}}{a}}{a(a^2-b^2)}\ldots}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{\left(a^6+15a^4b^2-32a^2b^4+16b^6\right)\int\frac{\sqrt{\csc\left(c+d\ldots\right.}}{\sqrt{a+b\csc\left(c\ldots\right.}}\ldots}{a}}{a}}{a(a^2-b^2)}\ldots}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3134

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^6+15a^4b^2-32a^2b^4+16b^6\right)\int\frac{\sqrt{\csc\left(c+\cdots\right.}}{\sqrt{a+b\csc\left(c\cdots\right.}}\cdots}{a}}{a}}{a(a^2-b^2)}\cdots}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^6+15a^4b^2-32a^2b^4+16b^6\right)\int\frac{\sqrt{\csc\left(\cdots\right.}}{\sqrt{a+b\,c\cdots}}\cdots}{a}}{a}}{a(a^2-b^2)}\cdots}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3132

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{8b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^6+15a^4b^2-32a^2b^4+16b^6\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\cdots\right.}}\cdots}{a}}{a}}{a(a^2-b^2)}\cdots}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 4345

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{8b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^6+15a^4b^2-32a^2b^4+16b^6\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+\cdots}\cdots}{a\sqrt{a+b\sec(c+dx)}}\cdots}{a}}{a(a^2-b^2)}\cdots}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{8b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^6+15a^4b^2-32a^2b^4+16b^6\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+}}{a\sqrt{a+b\sec(c+dx)}}}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3142$$

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{8b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^6+15a^4b^2-32a^2b^4+16b^6\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c-}{a-}}}{a\sqrt{a+b\sec(c+dx)}}}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{8b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^6+15a^4b^2-32a^2b^4+16b^6\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c-}{a-}}}{a\sqrt{a+b\sec(c+}}}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3140$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}+$$

$$\frac{\frac{4b^2(5a^2-3b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{8b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-2}{}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

input

```
Int[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
(2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c +
d*x])^(3/2)) + ((4*b^2*(5*a^2 - 3*b^2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt
[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (-(((-2*(a^6 + 15*a^4*b^2 - 32*
a^2*b^4 + 16*b^6)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (8*b
*(2*a^4 - 7*a^2*b^2 + 4*b^4)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
+ b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*
x]]))/a) + (2*(a^4 - 13*a^2*b^2 + 8*b^4)*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(a*d*Sqrt[Sec[c + d*x]]))/(a*(a^2 - b^2)))/(3*a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1742 vs. $2(366) = 732$.

Time = 8.22 (sec) , antiderivative size = 1743, normalized size of antiderivative = 4.46

method	result	size
default	Expression too large to display	1743

input
```
int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/a^4/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)*(a+b*sec(d*x+c))^(1/2)/(cos(d*
x+c)^2*(1+cos(d*x+c))*a^2+cos(d*x+c)*(2*cos(d*x+c)+2)*a*b+(1+cos(d*x+c))*b
^2)/sec(d*x+c)^(3/2)*((-8*cos(d*x+c)^2-16*cos(d*x+c)-8)*(1/(a+b)*(b+a*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^5*b*EllipticE(((a
-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(1/(a+b)*(b
+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4*b^2*Elli
pticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-
8*cos(d*x+c)-16-8*sec(d*x+c))+(28*cos(d*x+c)^2+56*cos(d*x+c)+28)*(1/(a+b)*
(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b^3*El
lipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+
(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a
^2*b^4*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b)
)^(1/2))*(28*cos(d*x+c)+56+28*sec(d*x+c))+(-16*cos(d*x+c)^2-32*cos(d*x+c)-
16)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*a*b^5*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-
b))^(1/2))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*b^6*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b
)/(a-b))^(1/2))*(-16*cos(d*x+c)-32-16*sec(d*x+c))+(cos(d*x+c)^2+2*cos(d*x+
c)+1)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*a^6*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 949, normalized size of antiderivative = 2.43

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx=\text{Too large to display}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/9*(sqrt(2)*(-3*I*a^6*b^2 - 37*I*a^4*b^4 + 68*I*a^2*b^6 - 32*I*b^8 + (-3*
I*a^8 - 37*I*a^6*b^2 + 68*I*a^4*b^4 - 32*I*a^2*b^6)*cos(d*x + c)^2 - 2*(3*
I*a^7*b + 37*I*a^5*b^3 - 68*I*a^3*b^5 + 32*I*a*b^7)*cos(d*x + c))*sqrt(a)*
weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3,
1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(3*I*a^6*b^
2 + 37*I*a^4*b^4 - 68*I*a^2*b^6 + 32*I*b^8 + (3*I*a^8 + 37*I*a^6*b^2 - 68*
I*a^4*b^4 + 32*I*a^2*b^6)*cos(d*x + c)^2 - 2*(-3*I*a^7*b - 37*I*a^5*b^3 +
68*I*a^3*b^5 - 32*I*a*b^7)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*
(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3
*I*a*sin(d*x + c) + 2*b)/a) - 12*sqrt(2)*(2*I*a^5*b^3 - 7*I*a^3*b^5 + 4*I*
a*b^7 + (2*I*a^7*b - 7*I*a^5*b^3 + 4*I*a^3*b^5)*cos(d*x + c)^2 + 2*(2*I*a^
6*b^2 - 7*I*a^4*b^4 + 4*I*a^2*b^6)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-
4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-
4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c)
 + 3*I*a*sin(d*x + c) + 2*b)/a)) - 12*sqrt(2)*(-2*I*a^5*b^3 + 7*I*a^3*b^5
- 4*I*a*b^7 + (-2*I*a^7*b + 7*I*a^5*b^3 - 4*I*a^3*b^5)*cos(d*x + c)^2 + 2*
(-2*I*a^6*b^2 + 7*I*a^4*b^4 - 4*I*a^2*b^6)*cos(d*x + c))*sqrt(a)*weierstra
ssZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPI
nverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(
d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)) + 6*((a^8 - 2*a^6*b^2 + a^4*b^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^5\,b^3+3\sec(dx+c)^4\,a\,b^2+3\sec(dx+c)^3\,a^2b+$$

input
```
int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/(sec(c + d*x)**5*b**3 +
3*sec(c + d*x)**4*a*b**2 + 3*sec(c + d*x)**3*a**2*b + sec(c + d*x)**2*a**3
),x)
```

3.668 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 474

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2b(17a^4+116a^2b^2-128b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{15a^5(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2(9a^6+55a^4b^2-212a^2b^4+128b^6)\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{15a^5(a^2-b^2)^2\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{2b^2\sin(c+dx)}{3a(a^2-b^2)\,d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{8b^2(3a^2-2b^2)\sin(c+dx)}{3a^2(a^2-b^2)^2\,d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{2(3a^4-71a^2b^2+48b^4)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^3(a^2-b^2)^2\,d\sec^{\frac{3}{2}}(c+dx)} - \frac{4b(7a^4-49a^2b^2+32b^4)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^4(a^2-b^2)^2\,d\sqrt{\sec(c+dx)}}$$

output
```
-2/15*b*(17*a^4+116*a^2*b^2-128*b^4)*((b+a*cos(d*x+c))/(a+b))^(1/2)*Invers
eJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(a/(a+b))^(1/2))*sec(d*x+c)^(1/2)/a^5/(a^2
-b^2)/d/(a+b*sec(d*x+c))^(1/2)+2/15*(9*a^6+55*a^4*b^2-212*a^2*b^4+128*b^6)
*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1
/2)/a^5/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+2/3*
b^2*sin(d*x+c)/a/(a^2-b^2)/d/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2)+8/3*b
^2*(3*a^2-2*b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/sec(d*x+c)^(3/2)/(a+b*sec(d*
x+c))^(1/2)+2/15*(3*a^4-71*a^2*b^2+48*b^4)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+
c)/a^3/(a^2-b^2)^2/d/sec(d*x+c)^(3/2)-4/15*b*(7*a^4-49*a^2*b^2+32*b^4)*(a+
b*sec(d*x+c))^(1/2)*sin(d*x+c)/a^4/(a^2-b^2)^2/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \frac{(b+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\left(\frac{2\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{3/2}\left((9a^6+55a^4b^2-2\right.}{}\right.}{}$$

input
```
Integrate[1/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
((b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*((2*((b + a*Cos[c + d*x])/(a + b)
)^(3/2)*((9*a^6 + 55*a^4*b^2 - 212*a^2*b^4 + 128*b^6)*EllipticE[(c + d*x)/
2, (2*a)/(a + b)] + b*(-17*a^5 + 17*a^4*b - 116*a^3*b^2 + 116*a^2*b^3 + 12
8*a*b^4 - 128*b^5)*EllipticF[(c + d*x)/2, (2*a)/(a + b)]))/(a - b)^2 + a*(
(10*b^5*Sin[c + d*x])/(-a^2 + b^2) - (10*b^4*(-15*a^2 + 11*b^2)*(b + a*Cos
[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2 - 28*b*(b + a*Cos[c + d*x])^2*Sin[c
 + d*x] + 3*a*(b + a*Cos[c + d*x])^2*Sin[2*(c + d*x)])))/(15*a^5*d*(a + b*
Sec[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 3.84 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.01, number of steps used = 25, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4334, 27, 3042, 4588, 27, 3042, 4592, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4334}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}-\frac{2\int -\frac{3a^2-3b\sec(c+dx)a-8b^2+6b^2\sec^2(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3a^2-3b\sec(c+dx)a-8b^2+6b^2\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3a^2-3b\csc\left(c+dx+\frac{\pi}{2}\right)a-8b^2+6b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4588}$$

$$\frac{\frac{8b^2\left(3a^2-2b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{3a^4-71b^2a^2-2b\left(3a^2-b^2\right)\sec(c+dx)a+48b^4+16b^2\left(3a^2-2b^2\right)\sec^2(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{3a^4-71b^2a^2-2b\left(3a^2-b^2\right)\sec(c+dx)a+48b^4+16b^2\left(3a^2-2b^2\right)\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}+\frac{8b^2(3a^2-2b^2)\sin(c+dx)}{ad(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{3a^4-71b^2a^2-2b\left(3a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+48b^4+16b^2\left(3a^2-2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{8b^2(3a^2-2b^2)\sin(c+dx)}{ad(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 4592

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{2\int \frac{-2b\left(3a^4-71b^2a^2+48b^4\right)\sec^2(c+dx)-a\left(9a^4+27b^2a^2-16b^4\right)\sec(c+dx)+6b\left(7a^4-49b^2a^2+32b^4\right)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}}{a(a^2-b^2)}+}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-2b\left(3a^4-71b^2a^2+48b^4\right)\sec^2(c+dx)-a\left(9a^4+27b^2a^2-16b^4\right)\sec(c+dx)+6b\left(7a^4-49b^2a^2+32b^4\right)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}}{a(a^2-b^2)}+}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\int\frac{-2b\left(3a^4-71b^2a^2+48b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-a\left(9a^4+27b^2a^2-16b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)+6b\left(7a^4-49b^2a^2+32b^4\right)\cdots}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}\cdots}{5a}\cdots}{a(a^2-b^2)}\cdots}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 4592

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{4b\left(7a^4-49a^2b^2+32b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{2\int\frac{3\left(9a^6+55b^2a^4-212b^4a^2-4b\left(2a^4+11b^2a^2-8b^4\right)\sec\cdots\right.}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\cdots}{3a}}{5a}\cdots}{a(a^2-b^2)}\cdots}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{4b\left(7a^4-49a^2b^2+32b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{9a^6+55b^2a^4-212b^4a^2-4b\left(2a^4+11b^2a^2-8b^4\right)\sec(c\cdots}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\cdots}{a}}{5a}\cdots}{a(a^2-b^2)}\cdots}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{4b\left(7a^4-49a^2b^2+32b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{9a^6+55b^2a^4-212b^4a^2-4b\left(2a^4+11b^2a^2-8b^4\right)\csc\left(c\cdots\right.}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\cdots\right.}}\cdots}{a}}{5a}\cdots}{a(a^2-b^2)}\cdots}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 4523

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{4b\left(7a^4-49a^2b^2+32b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\left(9a^6+55a^4b^2-212a^2b^4+128b^6\right)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}}{5a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{4b\left(7a^4-49a^2b^2+32b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\left(9a^6+55a^4b^2-212a^2b^4+128b^6\right)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}}{a}}{5a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4343

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{4b\left(7a^4-49a^2b^2+32b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\left(9a^6+55a^4b^2-212a^2b^4+128b^6\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b}}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{5a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{4b\left(7a^4-49a^2b^2+32b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\left(9a^6+55a^4b^2-212a^2b^4+128b^6\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b}}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{5a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3134

$$\frac{\frac{\frac{2(3a^4-71a^2b^2+48b^4)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{4b(7a^4-49a^2b^2+32b^4)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{(9a^6+55a^4b^2-212a^2b^4+128b^6)\sqrt{a+b\sec(c+dx)}\int\sqrt{\cdots}}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\cdots}{5a}}{a(a^2-b^2)}\cdots}{3a(a^2-b^2)}$$

$$\frac{2b^2\sin(c+dx)}{3ad(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2(3a^4-71a^2b^2+48b^4)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{4b(7a^4-49a^2b^2+32b^4)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{(9a^6+55a^4b^2-212a^2b^4+128b^6)\sqrt{a+b\sec(c+dx)}\int\sqrt{\cdots}}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\cdots}{5a}}{a(a^2-b^2)}\cdots}{3a(a^2-b^2)}$$

$$\frac{2b^2\sin(c+dx)}{3ad(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3132

$$\frac{\frac{\frac{2(3a^4-71a^2b^2+48b^4)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{4b(7a^4-49a^2b^2+32b^4)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{2(9a^6+55a^4b^2-212a^2b^4+128b^6)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}\cdots\right.}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\cdots}{5a}}{a(a^2-b^2)}\cdots}{3a(a^2-b^2)}$$

$$\frac{2b^2\sin(c+dx)}{3ad(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4345

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{4b\left(7a^4-49a^2b^2+32b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(9a^6+55a^4b^2-212a^2b^4+128b^6\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}\cdots\right.}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\cdots}{5a\cdots}}{a(a^2-b^2)}\cdots}{3a\left(a^2-\cdots\right.}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{4b\left(7a^4-49a^2b^2+32b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(9a^6+55a^4b^2-212a^2b^4+128b^6\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}\cdots\right.}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\cdots}{5a\cdots}}{a(a^2-b^2)}\cdots}{3a\left(a^2\cdots\right.}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3142

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{4b\left(7a^4-49a^2b^2+32b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(9a^6+55a^4b^2-212a^2b^4+128b^6\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}\cdots\right.}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\cdots}{5\cdots}}{a(a^2-b^2)}\cdots}{3a\left(a^2\cdots\right.}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{4b\left(7a^4-49a^2b^2+32b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(9a^6+55a^4b^2-212a^2b^4+128b^6\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}\cdots\right.}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\cdots}{\cdots}}{a(a^2-b^2)}\cdots}{3a\left(a\cdots\right.}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3140

$$\frac{2b^2\sin(c+dx)}{3ad\,(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}+$$

$$\frac{\frac{8b^2(3a^2-2b^2)\sin(c+dx)}{ad(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}+\frac{\frac{2(3a^4-71a^2b^2+48b^4)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{4b(7a^4-49a^2b^2+32b^4)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{2(9a}{}}{}}{3a\,(a^2}$$

input
```
Int[1/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
(2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Sec[c + d*x]^(3/2)*(a + b*Sec[c +
d*x])^(3/2)) + ((8*b^2*(3*a^2 - 2*b^2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sec[
c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) + ((2*(3*a^4 - 71*a^2*b^2 + 48*b^
4)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) - (-(
((-2*b*(17*a^6 + 99*a^4*b^2 - 244*a^2*b^4 + 128*b^6)*Sqrt[(b + a*Cos[c + d
*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a
*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(9*a^6 + 55*a^4*b^2 - 212*a^2*b^4 + 128*
b^6)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/a) + (4*b*(7*a^4 -
 49*a^2*b^2 + 32*b^4)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(a*d*Sqrt[Sec
[c + d*x]]))/(5*a))/(a*(a^2 - b^2)))/(3*a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2209 vs. $2(443) = 886$.

Time = 9.46 (sec) , antiderivative size = 2210, normalized size of antiderivative = 4.66

method	result	size
default	Expression too large to display	2210

input
```
int(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d/a^5/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)*(a+b*sec(d*x+c))^(1/2)/(cos(d
*x+c)^2*(1+cos(d*x+c))*a^2+cos(d*x+c)*(2*cos(d*x+c)+2)*a*b+(1+cos(d*x+c))*
b^2)/sec(d*x+c)^(5/2)*((1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*a^7*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*
x+c)),(-(a+b)/(a-b))^(1/2))*(9*cos(d*x+c)+18+9*sec(d*x+c))+(1/(a+b)*(b+a*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^6*b*EllipticE(
((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(9+18*se
c(d*x+c)+9*sec(d*x+c)^2)+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*a^5*b^2*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-
cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(55*cos(d*x+c)+110+55*sec(d*x+c))+(1/(a+
b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4*b^3
*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2
))*(55+110*sec(d*x+c)+55*sec(d*x+c)^2)+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b^4*EllipticE(((a-b)/(a+b))^(1/2
)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-212*cos(d*x+c)-424-212*s
ec(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*a^2*b^5*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-
(a+b)/(a-b))^(1/2))*(-212-424*sec(d*x+c)-212*sec(d*x+c)^2)+(1/(a+b)*(b+a*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^6*EllipticE(
((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(128*...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 1036, normalized size of antiderivative = 2.19

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/45*(2*sqrt(2)*(-21*I*a^6*b^3 - 121*I*a^4*b^5 + 260*I*a^2*b^7 - 128*I*b^
9 + (-21*I*a^8*b - 121*I*a^6*b^3 + 260*I*a^4*b^5 - 128*I*a^2*b^7)*cos(d*x
+ c)^2 + 2*(-21*I*a^7*b^2 - 121*I*a^5*b^4 + 260*I*a^3*b^6 - 128*I*a*b^8)*c
os(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)
+ 2*sqrt(2)*(21*I*a^6*b^3 + 121*I*a^4*b^5 - 260*I*a^2*b^7 + 128*I*b^9 + (2
1*I*a^8*b + 121*I*a^6*b^3 - 260*I*a^4*b^5 + 128*I*a^2*b^7)*cos(d*x + c)^2
+ 2*(21*I*a^7*b^2 + 121*I*a^5*b^4 - 260*I*a^3*b^6 + 128*I*a*b^8)*cos(d*x +
 c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b -
 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) + 3*sqrt
(2)*(-9*I*a^7*b^2 - 55*I*a^5*b^4 + 212*I*a^3*b^6 - 128*I*a*b^8 + (-9*I*a^9
 - 55*I*a^7*b^2 + 212*I*a^5*b^4 - 128*I*a^3*b^6)*cos(d*x + c)^2 + 2*(-9*I*
a^8*b - 55*I*a^6*b^3 + 212*I*a^4*b^5 - 128*I*a^2*b^7)*cos(d*x + c))*sqrt(a
)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, we
ierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/
3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) + 3*sqrt(2)*(9*I*a^7*b
^2 + 55*I*a^5*b^4 - 212*I*a^3*b^6 + 128*I*a*b^8 + (9*I*a^9 + 55*I*a^7*b^2
- 212*I*a^5*b^4 + 128*I*a^3*b^6)*cos(d*x + c)^2 + 2*(9*I*a^8*b + 55*I*a^6*
b^3 - 212*I*a^4*b^5 + 128*I*a^2*b^7)*cos(d*x + c))*sqrt(a)*weierstrassZeta
(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInve...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{6}\,b^{3}+3\sec(dx+c)^{5}\,a\,b^{2}+3\sec(dx+c)^{4}\,a^{2}b+$$

input

```
int(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x)
```

output

```
int((sqrt(sec(c + d*x))*sqrt(sec(c + d*x)*b + a))/(sec(c + d*x)**6*b**3 +
3*sec(c + d*x)**5*a*b**2 + 3*sec(c + d*x)**4*a**2*b + sec(c + d*x)**3*a**3
),x)
```

3.669 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{2+3\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 122

$$\begin{aligned}\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{2+3\sec(c+dx)}}\,dx &= -\frac{3\sqrt{3+2\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{2+3\sec(c+dx)}}\\ &\quad+\frac{\sqrt{5}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)\sqrt{2+3\sec(c+dx)}}{d\sqrt{3+2\cos(c+dx)}\sqrt{\sec(c+dx)}}\end{aligned}$$

output

```
-3/5*(3+2*cos(d*x+c))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2/5*5^(1/2))*sec
(d*x+c)^(1/2)*5^(1/2)/d/(2+3*sec(d*x+c))^(1/2)+5^(1/2)*EllipticE(sin(1/2*d
*x+1/2*c),2/5*5^(1/2))*(2+3*sec(d*x+c))^(1/2)/d/(3+2*cos(d*x+c))^(1/2)/sec
(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{2+3\sec(c+dx)}}\,dx = \frac{\sqrt{3+2\cos(c+dx)}\left(5E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)-3\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{2+3\sec(c+dx)}}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[2 + 3*Sec[c + d*x]]),x]
```

output
```
(Sqrt[3 + 2*Cos[c + d*x]]*(5*EllipticE[(c + d*x)/2, 4/5] - 3*EllipticF[(c
+ d*x)/2, 4/5])*Sqrt[Sec[c + d*x]])/(Sqrt[5]*d*Sqrt[2 + 3*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 4349, 3042, 4343, 3042, 3132, 4345, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{3\sec(c+dx)+2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{3\csc\left(c+dx+\frac{\pi}{2}\right)+2}}dx$$

$$\Big\downarrow \text{ 4349}$$

$$\frac{1}{2}\int \frac{\sqrt{3\sec(c+dx)+2}}{\sqrt{\sec(c+dx)}}dx-\frac{3}{2}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3\sec(c+dx)+2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\int \frac{\sqrt{3\csc\left(c+dx+\frac{\pi}{2}\right)+2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3\csc\left(c+dx+\frac{\pi}{2}\right)+2}}dx$$

$$\Big\downarrow \text{4343}$$

$$\frac{\sqrt{3\sec(c+dx)+2}\int \sqrt{2\cos(c+dx)+3}dx}{2\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}-\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3\csc\left(c+dx+\frac{\pi}{2}\right)+2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{3\sec(c+dx)+2}\int \sqrt{2\sin\left(c+dx+\frac{\pi}{2}\right)+3}dx}{2\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}-\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3\csc\left(c+dx+\frac{\pi}{2}\right)+2}}dx$$

$$\Big\downarrow \text{3132}$$

$$\frac{\sqrt{5}\sqrt{3\sec(c+dx)+2}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}-\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3\csc\left(c+dx+\frac{\pi}{2}\right)+2}}dx$$

$$\Big\downarrow \text{4345}$$

$$\frac{\sqrt{5}\sqrt{3\sec(c+dx)+2}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}-\frac{3\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{2\cos(c+dx)+3}}dx}{2\sqrt{3\sec(c+dx)+2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{5}\sqrt{3\sec(c+dx)+2}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}-\frac{3\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{2\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx}{2\sqrt{3\sec(c+dx)+2}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{\sqrt{5}\sqrt{3\sec(c+dx)+2}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}-\frac{3\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)}{\sqrt{5}d\sqrt{3\sec(c+dx)+2}}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[2 + 3*Sec[c + d*x]]),x]
```

output
```
(-3*Sqrt[3 + 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]
])/(Sqrt[5]*d*Sqrt[2 + 3*Sec[c + d*x]]) + (Sqrt[5]*EllipticE[(c + d*x)/2,
4/5]*Sqrt[2 + 3*Sec[c + d*x]])/(d*Sqrt[3 + 2*Cos[c + d*x]]*Sqrt[Sec[c + d*
x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4349
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Cs
c[e + f*x]], x], x] - Simp[b/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Cs
c[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.85 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.92

method	result
default	$\frac{\sqrt{2+3\sec(dx+c)}\left(2i\sqrt{5}\ \text{EllipticF}\left(\frac{i\sqrt{5}\,(\csc(dx+c)-\cot(dx+c))}{5},\sqrt{5}\right)\sqrt{10}\,\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}\,\sqrt{\frac{1}{1+\cos(dx+c)}}+i\sqrt{5}\ \text{EllipticE}\left(\frac{i\sqrt{5}\,(\csc\right.\right.}{5d\left((1-\cos(dx+c))^2\csc(dx\right.}$
risch	$-\frac{i\left(e^{2i(dx+c)}+3\,e^{i(dx+c)}+1\right)}{d\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{e^{2i(dx+c)}+3\,e^{i(dx+c)}+1}{e^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(e^{2i(dx+c)}+3\,e^{i(dx+c)}+1\right)}{\sqrt{\left(e^{2i(dx+c)}+3\,e^{i(dx+c)}+1\right)e^{i(dx+c)}}}+\frac{2\left(\frac{3}{2}+\frac{\sqrt{5}}{2}\right)\sqrt{\frac{e^{i(dx+c)}+\frac{3}{2}}{\frac{3}{2}+\frac{\sqrt{5}}{2}}}}{\right.}$

input
```
int(1/sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/d*(2+3*sec(d*x+c))^(1/2)*(2*I*5^(1/2)*EllipticF(1/5*I*5^(1/2)*(csc(d*x
+c)-cot(d*x+c)),5^(1/2))*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)+I*5^(1/2)*EllipticE(1/5*I*5^(1/2)*(csc(d*
x+c)-cot(d*x+c)),5^(1/2))*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)+5*(1-cos(d*x+c))^3*csc(d*x+c)^3+25*csc(d
*x+c)-25*cot(d*x+c))/((1-cos(d*x+c))^2*csc(d*x+c)^2+5)/sec(d*x+c)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{2+3\sec(c+dx)}}\,dx$$
$$= \frac{i\,\text{weierstrassPInverse}(8,-4,\cos(dx+c)+i\,\sin(dx+c)+1)-i\,\text{weierstrassPInverse}(8,-4,\cos(dx+c}{}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(I*weierstrassPInverse(8, -4, cos(d*x + c) + I*sin(d*x + c) + 1) - I*weier
strassPInverse(8, -4, cos(d*x + c) - I*sin(d*x + c) + 1) + I*weierstrassZe
ta(8, -4, weierstrassPInverse(8, -4, cos(d*x + c) + I*sin(d*x + c) + 1)) -
 I*weierstrassZeta(8, -4, weierstrassPInverse(8, -4, cos(d*x + c) - I*sin(
d*x + c) + 1)))/d
```

Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{2+3\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{3\sec{(c+dx)}+2}\sqrt{\sec{(c+dx)}}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(2+3*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(3*sec(c + d*x) + 2)*sqrt(sec(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{2+3\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{3\sec{(dx+c)}+2}\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(3*sec(d*x + c) + 2)*sqrt(sec(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{2+3\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{3\sec\left(dx+c\right)+2}\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(3*sec(d*x + c) + 2)*sqrt(sec(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{2+3\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\frac{3}{\cos(c+d\,x)}+2}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((3/cos(c + d*x) + 2)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((3/cos(c + d*x) + 2)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{2+3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{3\sec\left(dx+c\right)+2}}{3\sec\left(dx+c\right)^{2}+2\sec\left(dx+c\right)}dx$$

input
```
int(1/sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(3*sec(c + d*x) + 2))/(3*sec(c + d*x)**2 + 2*s
ec(c + d*x)),x)
```

3.670 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-2+3\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 109

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-2+3\sec(c+dx)}}\,dx = \frac{3\sqrt{3-2\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)\sqrt{\sec(c+dx)}}{d\sqrt{-2+3\sec(c+dx)}} - \frac{E\left(\frac{1}{2}(c+dx)\middle|-4\right)\sqrt{-2+3\sec(c+dx)}}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

output
```
3*(3-2*cos(d*x+c))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2*I)*sec(d*x+c)^(1/
2)/d/(-2+3*sec(d*x+c))^(1/2)-EllipticE(sin(1/2*d*x+1/2*c),2*I)*(-2+3*sec(d
*x+c))^(1/2)/d/(3-2*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-2+3\sec(c+dx)}}\,dx = -\frac{\sqrt{3-2\cos(c+dx)}\left(E\left(\frac{1}{2}(c+dx)\middle|-4\right)-3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)\right)\sqrt{\sec(c+dx)}}{d\sqrt{-2+3\sec(c+dx)}}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[-2 + 3*Sec[c + d*x]]),x]
```

output
```
-((Sqrt[3 - 2*Cos[c + d*x]]*(EllipticE[(c + d*x)/2, -4] - 3*EllipticF[(c +
 d*x)/2, -4])*Sqrt[Sec[c + d*x]])/(d*Sqrt[-2 + 3*Sec[c + d*x]]))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4349, 3042, 4343, 3042, 3132, 4345, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{3\sec(c+dx)-2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}dx$$

$$\Big\downarrow \text{ 4349}$$

$$\frac{3}{2}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3\sec(c+dx)-2}}dx-\frac{1}{2}\int \frac{\sqrt{3\sec(c+dx)-2}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}dx-\frac{1}{2}\int \frac{\sqrt{3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$\downarrow$ 4343

$$\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}dx-\frac{\sqrt{3\sec(c+dx)-2}\int \sqrt{3-2\cos(c+dx)}dx}{2\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}dx-\frac{\sqrt{3\sec(c+dx)-2}\int \sqrt{3-2\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3132

$$\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}dx-\frac{\sqrt{3\sec(c+dx)-2}E\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$\downarrow$ 4345

$$\frac{3\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{3-2\cos(c+dx)}}dx}{2\sqrt{3\sec(c+dx)-2}}-\frac{\sqrt{3\sec(c+dx)-2}E\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{3\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{3-2\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2\sqrt{3\sec(c+dx)-2}}-\frac{\sqrt{3\sec(c+dx)-2}E\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3140

$$\frac{3\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)}{d\sqrt{3\sec(c+dx)-2}}-\frac{\sqrt{3\sec(c+dx)-2}E\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[-2 + 3*Sec[c + d*x]]),x]
```

output
```
(3*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, -4]*Sqrt[Sec[c + d*x]])
/(d*Sqrt[-2 + 3*Sec[c + d*x]]) - (EllipticE[(c + d*x)/2, -4]*Sqrt[-2 + 3*S
ec[c + d*x]])/(d*Sqrt[3 - 2*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4349
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Cs
c[e + f*x]], x], x] - Simp[b/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Cs
c[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(98) = 196.

Time = 3.10 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.99

method	result
default	$-\frac{\sqrt{-2+3\sec(dx+c)}\left(2i\,\mathrm{EllipticF}\left(i(\csc(dx+c)-\cot(dx+c)),\sqrt{5}\right)\sqrt{2}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{-\frac{2(2\cos(dx+c)-3)}{1+\cos(dx+c)}}+i\,\mathrm{EllipticE}\left(i(\csc(dx+c\right.\right.}{d\left(5(1-\cos(dx+c))^{2}\csc(dx+c)\right.}$
risch	$-\frac{i\left(\mathrm{e}^{2i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}+1\right)\sqrt{2}}{d\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{-\frac{2\left(\mathrm{e}^{2i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}+1\right)}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{i\left(\frac{-2\,\mathrm{e}^{2i(dx+c)}+6\,\mathrm{e}^{i(dx+c)}-2}{\sqrt{\mathrm{e}^{i(dx+c)}\left(-2\,\mathrm{e}^{2i(dx+c)}+6\,\mathrm{e}^{i(dx+c)}-2\right)}}+\frac{2\left(\frac{\sqrt{5}}{2}-\frac{3}{2}\right)\sqrt{-\frac{\mathrm{e}^{i(}}{}}}{}\right.}{}$

input
```
int(1/sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(-2+3*sec(d*x+c))^(1/2)*(2*I*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),5^(1
/2))*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*(-2*(2*cos(d*x+c)-3)/(1+cos(d*x+c)))
^(1/2)+I*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),5^(1/2))*2^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*(-2*(2*cos(d*x+c)-3)/(1+cos(d*x+c)))^(1/2)+5*(1-cos(d*x+c))^3
*csc(d*x+c)^3+csc(d*x+c)-cot(d*x+c))/(5*(1-cos(d*x+c))^2*csc(d*x+c)^2+1)/s
ec(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-2+3\sec(c+dx)}}\,dx = -\frac{\mathrm{weierstrassPInverse}(8,4,\cos(dx+c)+i\sin(dx+c)-1)+\mathrm{weierstrassPInverse}(8,4,\cos(dx+c)-}{}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="fricas"
)
```

output
```
-(weierstrassPInverse(8, 4, cos(d*x + c) + I*sin(d*x + c) - 1) + weierstra
ssPInverse(8, 4, cos(d*x + c) - I*sin(d*x + c) - 1) - weierstrassZeta(8, 4
, weierstrassPInverse(8, 4, cos(d*x + c) + I*sin(d*x + c) - 1)) - weierstr
assZeta(8, 4, weierstrassPInverse(8, 4, cos(d*x + c) - I*sin(d*x + c) - 1)
))/d
```

Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-2+3\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{3\sec{\left(c+dx\right)}-2}\sqrt{\sec{\left(c+dx\right)}}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(-2+3*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(3*sec(c + d*x) - 2)*sqrt(sec(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-2+3\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{3\sec\left(dx+c\right)-2}\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(1/(sqrt(3*sec(d*x + c) - 2)*sqrt(sec(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-2+3\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{3\sec(dx+c)-2}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(3*sec(d*x + c) - 2)*sqrt(sec(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-2+3\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\frac{3}{\cos(c+d\,x)}-2}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((3/cos(c + d*x) - 2)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((3/cos(c + d*x) - 2)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-2+3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{3\sec(dx+c)-2}}{3\sec(dx+c)^2-2\sec(dx+c)}dx$$

input
```
int(1/sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(3*sec(c + d*x) - 2))/(3*sec(c + d*x)**2 - 2*s
ec(c + d*x)),x)
```

3.671 $\int \frac{1}{\sqrt{2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 108

$$\int \frac{1}{\sqrt{2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \frac{E\left(\frac{1}{2}(c+dx)\middle|-4\right)\sqrt{2-3\sec(c+dx)}}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{3\sqrt{3-2\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)\sqrt{\sec(c+dx)}}{d\sqrt{2-3\sec(c+dx)}}$$

output
```
EllipticE(sin(1/2*d*x+1/2*c),2*I)*(2-3*sec(d*x+c))^(1/2)/d/(3-2*cos(d*x+c)
)^(1/2)/sec(d*x+c)^(1/2)+3*(3-2*cos(d*x+c))^(1/2)*InverseJacobiAM(1/2*d*x+
1/2*c,2*I)*sec(d*x+c)^(1/2)/d/(2-3*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = -\frac{\sqrt{3-2\cos(c+dx)}\left(E\left(\frac{1}{2}(c+dx)\middle|-4\right)-3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)\right)\sqrt{\sec(c+dx)}}{d\sqrt{2-3\sec(c+dx)}}$$

input
```
Integrate[1/(Sqrt[2 - 3*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

output
```
-((Sqrt[3 - 2*Cos[c + d*x]]*(EllipticE[(c + d*x)/2, -4] - 3*EllipticF[(c +
 d*x)/2, -4])*Sqrt[Sec[c + d*x]])/(d*Sqrt[2 - 3*Sec[c + d*x]]))
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4349, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{2-3\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4349}$$

$$\frac{1}{2}\int \frac{\sqrt{2-3\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\frac{3}{2}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\int \frac{\sqrt{2-3\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2-3\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4343}$$

$$\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2-3\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\sqrt{2-3\sec(c+dx)}\int \sqrt{2\cos(c+dx)-3}dx}{2\sqrt{2\cos(c+dx)-3}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2-3\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\sqrt{2-3\sec(c+dx)}\int \sqrt{2\sin\left(c+dx+\frac{\pi}{2}\right)-3}dx}{2\sqrt{2\cos(c+dx)-3}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3134}$$

$$\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2-3\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\sqrt{2-3\sec(c+dx)}\int \sqrt{3-2\cos(c+dx)}dx}{2\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2-3\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\sqrt{2-3\sec(c+dx)}\int \sqrt{3-2\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3132}$$

$$\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2-3\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\sqrt{2-3\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4345}$$

$$\frac{3\sqrt{2\cos(c+dx)-3}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{2\cos(c+dx)-3}}dx}{2\sqrt{2-3\sec(c+dx)}}+\frac{\sqrt{2-3\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\sqrt{2\cos(c+dx)-3}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{2\sin\left(c+dx+\frac{\pi}{2}\right)-3}}dx}{2\sqrt{2-3\sec(c+dx)}}+\frac{\sqrt{2-3\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3142}$$

$$\frac{3\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{3-2\cos(c+dx)}}dx}{2\sqrt{2-3\sec(c+dx)}}+\frac{\sqrt{2-3\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{3-2\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2\sqrt{2-3\sec(c+dx)}}+\frac{\sqrt{2-3\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{3\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)}{d\sqrt{2-3\sec(c+dx)}}+\frac{\sqrt{2-3\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[2 - 3*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

output
```
(EllipticE[(c + d*x)/2, -4]*Sqrt[2 - 3*Sec[c + d*x]])/(d*Sqrt[3 - 2*Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]]) + (3*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d
*x)/2, -4]*Sqrt[Sec[c + d*x]])/(d*Sqrt[2 - 3*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4349
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Cs
c[e + f*x]], x], x] - Simp[b/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Cs
c[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(97) = 194$.

Time = 3.00 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.18

method	result
default	$-\frac{\sqrt{2-3\sec(dx+c)}\left(2i\,\mathrm{EllipticF}\left(i\sqrt{5}\,(\csc(dx+c)-\cot(dx+c)),\frac{\sqrt{5}}{5}\right)\sqrt{5}\sqrt{-\frac{2(2\cos(dx+c)-3)}{1+\cos(dx+c)}}\,\sqrt{2}\sqrt{\frac{1}{1+\cos(dx+c)}}-5i\,\mathrm{EllipticE}\left(i\sqrt{5}\right.\right.}{5d\left(5(1-\cos(dx+c))^2\right.}$
risch	$-\frac{i\left(\mathrm{e}^{2i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}+1\right)}{d\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{\mathrm{e}^{2i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}+1}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(\mathrm{e}^{2i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}+1\right)}{\sqrt{\mathrm{e}^{i(dx+c)}\left(\mathrm{e}^{2i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}+1\right)}}+\frac{2\left(\frac{\sqrt{5}}{2}-\frac{3}{2}\right)\sqrt{-\frac{\mathrm{e}^{i(dx+c)}-}{\frac{3}{2}-}}}{}\right.}{}$

input
```
int(1/(2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/d*(2-3*sec(d*x+c))^(1/2)*(2*I*EllipticF(I*5^(1/2)*(csc(d*x+c)-cot(d*x
+c)),1/5*5^(1/2))*5^(1/2)*(-2*(2*cos(d*x+c)-3)/(1+cos(d*x+c)))^(1/2)*2^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)-5*I*EllipticE(I*5^(1/2)*(csc(d*x+c)-cot(d*x+c)
),1/5*5^(1/2))*5^(1/2)*(-2*(2*cos(d*x+c)-3)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)-25*(1-cos(d*x+c))^3*csc(d*x+c)^3-5*csc(d*x+c)+5*c
ot(d*x+c))/(5*(1-cos(d*x+c))^2*csc(d*x+c)^2+1)/sec(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{-i\,\mathrm{weierstrassPInverse}(8,4,\cos(dx+c)+i\,\sin(dx+c)-1)+i\,\mathrm{weierstrassPInverse}(8,4,\cos(dx+c)}{}$$

input
```
integrate(1/(2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
(-I*weierstrassPInverse(8, 4, cos(d*x + c) + I*sin(d*x + c) - 1) + I*weier
strassPInverse(8, 4, cos(d*x + c) - I*sin(d*x + c) - 1) + I*weierstrassZet
a(8, 4, weierstrassPInverse(8, 4, cos(d*x + c) + I*sin(d*x + c) - 1)) - I*
weierstrassZeta(8, 4, weierstrassPInverse(8, 4, cos(d*x + c) - I*sin(d*x +
 c) - 1)))/d
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{2-3\sec\left(c+dx\right)}\sqrt{\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(2-3*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(2 - 3*sec(c + d*x))*sqrt(sec(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{-3\,\sec\left(dx+c\right)+2}\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(1/(2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-3*sec(d*x + c) + 2)*sqrt(sec(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{-3\sec(dx+c)+2}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-3*sec(d*x + c) + 2)*sqrt(sec(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{2-\frac{3}{\cos(c+d\,x)}}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((2 - 3/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((2 - 3/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = -\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{-3\sec(dx+c)+2}}{3\sec(dx+c)^2-2\sec(dx+c)}dx\right)$$

input
```
int(1/(2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

output
```
- int((sqrt(sec(c + d*x))*sqrt( - 3*sec(c + d*x) + 2))/(3*sec(c + d*x)**2
- 2*sec(c + d*x)),x)
```

3.672 $\int \frac{1}{\sqrt{-2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 123

$$\int \frac{1}{\sqrt{-2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = -\frac{\sqrt{5}E\left(\frac{1}{2}(c+dx)\middle|\frac{4}{5}\right)\sqrt{-2-3\sec(c+dx)}}{d\sqrt{3+2\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{3\sqrt{3+2\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{-2-3\sec(c+dx)}}$$

output
```
-5^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2/5*5^(1/2))*(-2-3*sec(d*x+c))^(1/2)
/d/(3+2*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)-3/5*(3+2*cos(d*x+c))^(1/2)*Inve
rseJacobiAM(1/2*d*x+1/2*c,2/5*5^(1/2))*sec(d*x+c)^(1/2)*5^(1/2)/d/(-2-3*se
c(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{-2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \frac{\sqrt{3+2\cos(c+dx)}\left(5E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)-3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{-2-3\sec(c+dx)}}$$

input
```
Integrate[1/(Sqrt[-2 - 3*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

output
```
(Sqrt[3 + 2*Cos[c + d*x]]*(5*EllipticE[(c + d*x)/2, 4/5] - 3*EllipticF[(c
+ d*x)/2, 4/5])*Sqrt[Sec[c + d*x]])/(Sqrt[5]*d*Sqrt[-2 - 3*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3042, 4349, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3\sec(c+dx)-2}\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{-3\csc\left(c+dx+\frac{\pi}{2}\right)-2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4349}$$

$$-\frac{1}{2}\int \frac{\sqrt{-3\sec(c+dx)-2}}{\sqrt{\sec(c+dx)}}dx-\frac{3}{2}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3\sec(c+dx)-2}}dx$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}\int \frac{\sqrt{-3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}dx$$

$$\Big\downarrow \text{4343}$$

$$-\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}dx-\frac{\sqrt{-3\sec(c+dx)-2}\int \sqrt{-2\cos(c+dx)-3}dx}{2\sqrt{-2\cos(c+dx)-3}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}dx-\frac{\sqrt{-3\sec(c+dx)-2}\int \sqrt{-2\sin\left(c+dx+\frac{\pi}{2}\right)-3}dx}{2\sqrt{-2\cos(c+dx)-3}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3134}$$

$$-\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}dx-\frac{\sqrt{5}\sqrt{-3\sec(c+dx)-2}\int \sqrt{\frac{2}{5}\cos(c+dx)+\frac{3}{5}}dx}{2\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}dx-\frac{\sqrt{5}\sqrt{-3\sec(c+dx)-2}\int \sqrt{\frac{2}{5}\sin\left(c+dx+\frac{\pi}{2}\right)+\frac{3}{5}}dx}{2\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3132}$$

$$-\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}dx-\frac{\sqrt{5}\sqrt{-3\sec(c+dx)-2}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4345}$$

$$-\frac{3\sqrt{-2\cos(c+dx)-3}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{-2\cos(c+dx)-3}}dx}{2\sqrt{-3\sec(c+dx)-2}}-\frac{\sqrt{5}\sqrt{-3\sec(c+dx)-2}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sqrt{-2\cos(c+dx)-3}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{-2\sin\left(c+dx+\frac{\pi}{2}\right)-3}}dx}{2\sqrt{-3\sec(c+dx)-2}}-\frac{\sqrt{5}\sqrt{-3\sec(c+dx)-2}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3142}$$

$$-\frac{3\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\frac{2}{5}\cos(c+dx)+\frac{3}{5}}}dx}{2\sqrt{5}\sqrt{-3\sec(c+dx)-2}}-\frac{\sqrt{5}\sqrt{-3\sec(c+dx)-2}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\frac{2}{5}\sin\left(c+dx+\frac{\pi}{2}\right)+\frac{3}{5}}}dx}{2\sqrt{5}\sqrt{-3\sec(c+dx)-2}}-\frac{\sqrt{5}\sqrt{-3\sec(c+dx)-2}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3140}$$

$$-\frac{3\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)}{\sqrt{5}d\sqrt{-3\sec(c+dx)-2}}-\frac{\sqrt{5}\sqrt{-3\sec(c+dx)-2}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[-2 - 3*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

output
```
-((Sqrt[5]*EllipticE[(c + d*x)/2, 4/5]*Sqrt[-2 - 3*Sec[c + d*x]])/(d*Sqrt[
3 + 2*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])) - (3*Sqrt[3 + 2*Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]])/(Sqrt[5]*d*Sqrt[-2 - 3*Sec[c
+ d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4349
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Cs
c[e + f*x]], x], x] - Simp[b/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Cs
c[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.50 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.84

method	result
default	$\frac{\sqrt{-2-3\sec(dx+c)}\left(2i\,\mathrm{EllipticF}\left(i(\csc(dx+c)-\cot(dx+c)),\frac{\sqrt{5}}{5}\right)\sqrt{2}\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{10}\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}-5i\,\mathrm{EllipticE}\left(i(\csc(dx+c)\right.\right.}{5d\left((1-\cos(dx+c))^2\csc(dx+c\right.}$
risch	$-\frac{i\left(\mathrm{e}^{2i(dx+c)}+3\,\mathrm{e}^{i(dx+c)}+1\right)\sqrt{2}}{d\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{-\frac{2\left(\mathrm{e}^{2i(dx+c)}+3\,\mathrm{e}^{i(dx+c)}+1\right)}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{i\left(\frac{-2\,\mathrm{e}^{2i(dx+c)}-6\,\mathrm{e}^{i(dx+c)}-2}{\sqrt{\mathrm{e}^{i(dx+c)}\left(-2\,\mathrm{e}^{2i(dx+c)}-6\,\mathrm{e}^{i(dx+c)}-2\right)}}+\frac{2\left(\frac{3}{2}+\frac{\sqrt{5}}{2}\right)\sqrt{\frac{\mathrm{e}^{i(dx}}{}}}{}\right.}{}$

input
```
int(1/(-2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/d*(-2-3*sec(d*x+c))^(1/2)*(2*I*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),1/5
*5^(1/2))*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)-5*I*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),1/5*5^(1/2))*2^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)-5*(1-cos(d*x+c))^3*csc(d*x+c)^3-25*csc(d*x+c)+25*cot(d*x+c))/((1-cos(
d*x+c))^2*csc(d*x+c)^2+5)/sec(d*x+c)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{-2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$$
$$=\frac{\mathrm{weierstrassPInverse}(8,-4,\cos(dx+c)+i\,\sin(dx+c)+1)+\mathrm{weierstrassPInverse}(8,-4,\cos(dx+c)-}{}$$

input
```
integrate(1/(-2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas"
)
```

output
```
(weierstrassPInverse(8, -4, cos(d*x + c) + I*sin(d*x + c) + 1) + weierstra
ssPInverse(8, -4, cos(d*x + c) - I*sin(d*x + c) + 1) + weierstrassZeta(8,
-4, weierstrassPInverse(8, -4, cos(d*x + c) + I*sin(d*x + c) + 1)) + weier
strassZeta(8, -4, weierstrassPInverse(8, -4, cos(d*x + c) - I*sin(d*x + c)
 + 1)))/d
```

Sympy [F]

$$\int \frac{1}{\sqrt{-2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{-3\sec(c+dx)-2}\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate(1/(-2-3*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(-3*sec(c + d*x) - 2)*sqrt(sec(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{-3\sec(dx+c)-2}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(-2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(1/(sqrt(-3*sec(d*x + c) - 2)*sqrt(sec(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{-3\sec(dx+c)-2}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(-2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-3*sec(d*x + c) - 2)*sqrt(sec(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{-\frac{3}{\cos(c+d\,x)}-2}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((- 3/cos(c + d*x) - 2)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((- 3/cos(c + d*x) - 2)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = -\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{-3\sec(dx+c)-2}}{3\sec(dx+c)^2+2\sec(dx+c)}dx\right)$$

input
```
int(1/(-2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

output
```
- int((sqrt(sec(c + d*x))*sqrt( - 3*sec(c + d*x) - 2))/(3*sec(c + d*x)**2
+ 2*sec(c + d*x)),x)
```

3.673 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{3+2\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 127

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{3+2\sec(c+dx)}}\,dx = -\frac{4\sqrt{2+3\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{3\sqrt{5}d\sqrt{3+2\sec(c+dx)}} + \frac{2\sqrt{5}E\left(\frac{1}{2}(c+dx)\middle|\frac{6}{5}\right)\sqrt{3+2\sec(c+dx)}}{3d\sqrt{2+3\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

output
```
-4/15*(2+3*cos(d*x+c))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,1/5*30^(1/2))*s
ec(d*x+c)^(1/2)*5^(1/2)/d/(3+2*sec(d*x+c))^(1/2)+2/3*5^(1/2)*EllipticE(sin
(1/2*d*x+1/2*c),1/5*30^(1/2))*(3+2*sec(d*x+c))^(1/2)/d/(2+3*cos(d*x+c))^(1
/2)/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{3+2\sec(c+dx)}}\,dx = \frac{2\sqrt{2+3\cos(c+dx)}\left(5E\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)-2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{6}{5}\right)\right)\sqrt{\sec(c+dx)}}{3\sqrt{5}d\sqrt{3+2\sec(c+dx)}}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[3 + 2*Sec[c + d*x]]),x]
```

output
```
(2*Sqrt[2 + 3*Cos[c + d*x]]*(5*EllipticE[(c + d*x)/2, 6/5] - 2*EllipticF[(
c + d*x)/2, 6/5])*Sqrt[Sec[c + d*x]])/(3*Sqrt[5]*d*Sqrt[3 + 2*Sec[c + d*x]
])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4349, 3042, 4343, 3042, 3132, 4345, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{2\sec(c+dx)+3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{2\csc\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow \text{4349}$$

$$\frac{1}{3}\int \frac{\sqrt{2\sec(c+dx)+3}}{\sqrt{\sec(c+dx)}}dx-\frac{2}{3}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2\sec(c+dx)+3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\sqrt{2\csc\left(c+dx+\frac{\pi}{2}\right)+3}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2\csc\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow \text{ 4343}$$

$$\frac{\sqrt{2\sec(c+dx)+3}\int \sqrt{3\cos(c+dx)+2}dx}{3\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}}-\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2\csc\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{2\sec(c+dx)+3}\int \sqrt{3\sin\left(c+dx+\frac{\pi}{2}\right)+2}dx}{3\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}}-\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2\csc\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow \text{ 3132}$$

$$\frac{2\sqrt{5}\sqrt{2\sec(c+dx)+3}E\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)}{3d\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}}-\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2\csc\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow \text{ 4345}$$

$$\frac{2\sqrt{5}\sqrt{2\sec(c+dx)+3}E\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)}{3d\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}}-\frac{2\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{3\cos(c+dx)+2}}dx}{3\sqrt{2\sec(c+dx)+3}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\sqrt{5}\sqrt{2\sec(c+dx)+3}E\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)}{3d\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}}-\frac{2\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{3\sin\left(c+dx+\frac{\pi}{2}\right)+2}}dx}{3\sqrt{2\sec(c+dx)+3}}$$

$$\Big\downarrow \text{ 3140}$$

$$\frac{2\sqrt{5}\sqrt{2\sec(c+dx)+3}E\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)}{3d\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}}-\frac{4\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{6}{5}\right)}{3\sqrt{5}d\sqrt{2\sec(c+dx)+3}}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[3 + 2*Sec[c + d*x]]),x]
```

output
```
(-4*Sqrt[2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6/5]*Sqrt[Sec[c + d*x]
])/(3*Sqrt[5]*d*Sqrt[3 + 2*Sec[c + d*x]]) + (2*Sqrt[5]*EllipticE[(c + d*x)
/2, 6/5]*Sqrt[3 + 2*Sec[c + d*x]])/(3*d*Sqrt[2 + 3*Cos[c + d*x]]*Sqrt[Sec[
c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4349
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Cs
c[e + f*x]], x], x] - Simp[b/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Cs
c[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.80 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.86

method	result
default	$\frac{2\sqrt{3+2\sec(dx+c)}\left(3\,\text{EllipticF}\left(\frac{\sqrt{5}\,(\csc(dx+c)-\cot(dx+c))}{5},i\sqrt{5}\right)\sqrt{5}\,\sqrt{10}\,\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}\,\sqrt{\frac{1}{1+\cos(dx+c)}}-\text{EllipticE}\left(\frac{\sqrt{5}\,(\csc(dx+c}{}\right.\right.}{15d\left((1-\cos(dx+c))^2\csc(dx\right.}$
risch	$-\frac{i\left(3\,e^{2i(dx+c)}+4\,e^{i(dx+c)}+3\right)\sqrt{2}}{3d\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{3\,e^{2i(dx+c)}+4\,e^{i(dx+c)}+3}{e^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(3\,e^{2i(dx+c)}+4\,e^{i(dx+c)}+3\right)}{3\sqrt{e^{i(dx+c)}\left(3\,e^{2i(dx+c)}+4\,e^{i(dx+c)}+3\right)}}+\frac{\left(\frac{2}{3}+\frac{i\sqrt{5}}{3}\right)\sqrt{\frac{e^{i(dx+}}{\frac{2}{3}}}}{}\right.}{}$

input
```
int(1/sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d*(3+2*sec(d*x+c))^(1/2)*(3*EllipticF(1/5*5^(1/2)*(csc(d*x+c)-cot(d*x
+c)),I*5^(1/2))*5^(1/2)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)-EllipticE(1/5*5^(1/2)*(csc(d*x+c)-cot(d*x+
c)),I*5^(1/2))*5^(1/2)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^
(1/2)*(1/(1+cos(d*x+c)))^(1/2)+5*(1-cos(d*x+c))^3*csc(d*x+c)^3-25*csc(d*x+
c)+25*cot(d*x+c))/((1-cos(d*x+c))^2*csc(d*x+c)^2-5)/sec(d*x+c)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{3+2\sec(c+dx)}}\,dx$$
$$= \frac{4i\sqrt{6}\text{weierstrassPInverse}\left(-\frac{44}{27},\frac{784}{729},\cos(dx+c)+i\sin(dx+c)+\frac{4}{9}\right)-4i\sqrt{6}\text{weierstrassPInverse}\left(-\frac{44}{27},\right.}{}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/27*(4*I*sqrt(6)*weierstrassPInverse(-44/27, 784/729, cos(d*x + c) + I*si
n(d*x + c) + 4/9) - 4*I*sqrt(6)*weierstrassPInverse(-44/27, 784/729, cos(d
*x + c) - I*sin(d*x + c) + 4/9) + 9*I*sqrt(6)*weierstrassZeta(-44/27, 784/
729, weierstrassPInverse(-44/27, 784/729, cos(d*x + c) + I*sin(d*x + c) +
4/9)) - 9*I*sqrt(6)*weierstrassZeta(-44/27, 784/729, weierstrassPInverse(-
44/27, 784/729, cos(d*x + c) - I*sin(d*x + c) + 4/9)))/d
```

Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{3+2\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{2\sec(c+dx)+3}\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(3+2*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(2*sec(c + d*x) + 3)*sqrt(sec(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{3+2\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{2\sec(dx+c)+3}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(2*sec(d*x + c) + 3)*sqrt(sec(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{3+2\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{2\sec(dx+c)+3}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(2*sec(d*x + c) + 3)*sqrt(sec(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{3+2\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\frac{2}{\cos(c+d\,x)}+3}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((2/cos(c + d*x) + 3)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((2/cos(c + d*x) + 3)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{3+2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{2\sec(dx+c)+3}}{2\sec(dx+c)^2+3\sec(dx+c)}dx$$

input
```
int(1/sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(2*sec(c + d*x) + 3))/(2*sec(c + d*x)**2 + 3*s
ec(c + d*x)),x)
```

3.674 $\int \frac{1}{\sqrt{3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 113

$$\int \frac{1}{\sqrt{3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \frac{2E\left(\frac{1}{2}(c+dx)\middle|6\right)\sqrt{3-2\sec(c+dx)}}{3d\sqrt{-2+3\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{4\sqrt{-2+3\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),6\right)\sqrt{\sec(c+dx)}}{3d\sqrt{3-2\sec(c+dx)}}$$

output
```
2/3*EllipticE(sin(1/2*d*x+1/2*c),6^(1/2))*(3-2*sec(d*x+c))^(1/2)/d/(-2+3*c
os(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+4/3*(-2+3*cos(d*x+c))^(1/2)*InverseJacob
iAM(1/2*d*x+1/2*c,6^(1/2))*sec(d*x+c)^(1/2)/d/(3-2*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \frac{\sqrt{-2+3\cos(c+dx)}\left(2E\left(\frac{1}{2}(c+dx)\middle|6\right)+4\,\text{EllipticF}\left(\frac{1}{2}(c+dx),6\right)\right)\sqrt{\sec(c+dx)}}{3d\sqrt{3-2\sec(c+dx)}}$$

input
```
Integrate[1/(Sqrt[3 - 2*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

output
```
(Sqrt[-2 + 3*Cos[c + d*x]]*(2*EllipticE[(c + d*x)/2, 6] + 4*EllipticF[(c +
 d*x)/2, 6])*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[3 - 2*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4349, 3042, 4343, 3042, 3132, 4345, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sqrt{3-2\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4349$$

$$\frac{1}{3}\int \frac{\sqrt{3-2\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\frac{2}{3}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\int \frac{\sqrt{3-2\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3-2\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4343}$$

$$\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3-2\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\sqrt{3-2\sec(c+dx)}\int \sqrt{3\cos(c+dx)-2}dx}{3\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3-2\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\sqrt{3-2\sec(c+dx)}\int \sqrt{3\sin\left(c+dx+\frac{\pi}{2}\right)-2}dx}{3\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3132}$$

$$\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3-2\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{3-2\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|6\right)}{3d\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4345}$$

$$\frac{2\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{3\cos(c+dx)-2}}dx}{3\sqrt{3-2\sec(c+dx)}}+\frac{2\sqrt{3-2\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|6\right)}{3d\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{3\sin\left(c+dx+\frac{\pi}{2}\right)-2}}dx}{3\sqrt{3-2\sec(c+dx)}}+\frac{2\sqrt{3-2\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|6\right)}{3d\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{4\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),6\right)}{3d\sqrt{3-2\sec(c+dx)}}+\frac{2\sqrt{3-2\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|6\right)}{3d\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[3 - 2*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

output
```
(2*EllipticE[(c + d*x)/2, 6]*Sqrt[3 - 2*Sec[c + d*x]])/(3*d*Sqrt[-2 + 3*Co
s[c + d*x]]*Sqrt[Sec[c + d*x]]) + (4*Sqrt[-2 + 3*Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 6]*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[3 - 2*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4349
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Cs
c[e + f*x]], x], x] - Simp[b/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Cs
c[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.33 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.97

method	result
default	$\frac{2\sqrt{3-2\sec(dx+c)}\left(6\sqrt{5}\ \mathrm{EllipticF}\left(\sqrt{5}\left(\csc(dx+c)-\cot(dx+c)\right),\frac{i\sqrt{5}}{5}\right)\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\ \sqrt{\frac{1}{1+\cos(dx+c)}}-10\sqrt{5}\ \mathrm{EllipticE}\left(\sqrt{5}\left(\csc(\right.\right.}{15d\left(5(1-\cos(dx+c))^{2}\csc(dx+\right.}$
risch	$-\frac{i\left(3\,\mathrm{e}^{2i(dx+c)}-4\,\mathrm{e}^{i(dx+c)}+3\right)\sqrt{2}}{3d\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{3\,\mathrm{e}^{2i(dx+c)}-4\,\mathrm{e}^{i(dx+c)}+3}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(3\,\mathrm{e}^{2i(dx+c)}-4\,\mathrm{e}^{i(dx+c)}+3\right)}{3\sqrt{\mathrm{e}^{i(dx+c)}\left(3\,\mathrm{e}^{2i(dx+c)}-4\,\mathrm{e}^{i(dx+c)}+3\right)}}+\frac{\left(-\frac{2}{3}+\frac{i\sqrt{5}}{3}\right)\sqrt{\frac{\mathrm{e}^{i(dx}}{}}}{}\right.}{}$

input
```
int(1/(3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d*(3-2*sec(d*x+c))^(1/2)*(6*5^(1/2)*EllipticF(5^(1/2)*(csc(d*x+c)-cot
(d*x+c)),1/5*I*5^(1/2))*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)-10*5^(1/2)*EllipticE(5^(1/2)*(csc(d*x+c)-cot(d*x+c)),1/5*I
*5^(1/2))*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)+25*(1-cos(d*x+c))^3*csc(d*x+c)^3-5*csc(d*x+c)+5*cot(d*x+c))/(5*(1-cos(d*
x+c))^2*csc(d*x+c)^2-1)/sec(d*x+c)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{-4i\sqrt{6}\text{weierstrassPInverse}\left(-\frac{44}{27},-\frac{784}{729},\cos(dx+c)+i\sin(dx+c)-\frac{4}{9}\right)+4i\sqrt{6}\text{weierstrassPInverse}\left(-\right.}{}$$

input
```
integrate(1/(3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/27*(-4*I*sqrt(6)*weierstrassPInverse(-44/27, -784/729, cos(d*x + c) + I*
sin(d*x + c) - 4/9) + 4*I*sqrt(6)*weierstrassPInverse(-44/27, -784/729, co
s(d*x + c) - I*sin(d*x + c) - 4/9) + 9*I*sqrt(6)*weierstrassZeta(-44/27, -
784/729, weierstrassPInverse(-44/27, -784/729, cos(d*x + c) + I*sin(d*x +
c) - 4/9)) - 9*I*sqrt(6)*weierstrassZeta(-44/27, -784/729, weierstrassPInv
erse(-44/27, -784/729, cos(d*x + c) - I*sin(d*x + c) - 4/9)))/d
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate(1/(3-2*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(3 - 2*sec(c + d*x))*sqrt(sec(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{-2\sec(dx+c)+3}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-2*sec(d*x + c) + 3)*sqrt(sec(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{-2\sec(dx+c)+3}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-2*sec(d*x + c) + 3)*sqrt(sec(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{3-\frac{2}{\cos(c+d\,x)}}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((3 - 2/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((3 - 2/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$$
$$= -\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{-2\sec(dx+c)+3}}{2\sec(dx+c)^2-3\sec(dx+c)}dx\right)$$

input
```
int(1/(3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

output
```
- int((sqrt(sec(c + d*x))*sqrt( - 2*sec(c + d*x) + 3))/(2*sec(c + d*x)**2
- 3*sec(c + d*x)),x)
```

3.675 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-3+2\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 129

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-3+2\sec(c+dx)}}\,dx = \frac{4\sqrt{2-3\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+\pi+dx),\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{3\sqrt{5}d\sqrt{-3+2\sec(c+dx)}} - \frac{2\sqrt{5}E\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{6}{5}\right)\sqrt{-3+2\sec(c+dx)}}{3d\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

output
```
4/15*(2-3*cos(d*x+c))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*Pi+1/2*c,1/5*30^(1
/2))*sec(d*x+c)^(1/2)*5^(1/2)/d/(-3+2*sec(d*x+c))^(1/2)-2/3*5^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),1/5*30^(1/2))*(-3+2*sec(d*x+c))^(1/2)/d/(2-3*cos(d*
x+c))^(1/2)/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.56

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-3+2\sec(c+dx)}}\,dx = \frac{\sqrt{-2+3\cos(c+dx)}\left(2E\left(\frac{1}{2}(c+dx)\middle|6\right)+4\,\text{EllipticF}\left(\frac{1}{2}(c+dx),6\right)\right)\sqrt{\sec(c+dx)}}{3d\sqrt{-3+2\sec(c+dx)}}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[-3 + 2*Sec[c + d*x]]),x]
```

output
```
(Sqrt[-2 + 3*Cos[c + d*x]]*(2*EllipticE[(c + d*x)/2, 6] + 4*EllipticF[(c +
d*x)/2, 6])*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[-3 + 2*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4349, 3042, 4343, 3042, 3133, 4345, 3042, 3141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{2\sec(c+dx)-3}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{2\csc\left(c+dx+\frac{\pi}{2}\right)-3}}dx$$

$$\Big\downarrow \text{ 4349}$$

$$\frac{2}{3}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2\sec(c+dx)-3}}dx-\frac{1}{3}\int \frac{\sqrt{2\sec(c+dx)-3}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2\csc\left(c+dx+\frac{\pi}{2}\right)-3}}dx-\frac{1}{3}\int \frac{\sqrt{2\csc\left(c+dx+\frac{\pi}{2}\right)-3}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4343}$$

$$\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2\csc\left(c+dx+\frac{\pi}{2}\right)-3}}dx-\frac{\sqrt{2\sec(c+dx)-3}\int \sqrt{2-3\cos(c+dx)}dx}{3\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2\csc\left(c+dx+\frac{\pi}{2}\right)-3}}dx-\frac{\sqrt{2\sec(c+dx)-3}\int \sqrt{2-3\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{3\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3133}$$

$$\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2\csc\left(c+dx+\frac{\pi}{2}\right)-3}}dx-\frac{2\sqrt{5}\sqrt{2\sec(c+dx)-3}E\left(\frac{1}{2}(c+dx+\pi)|\frac{6}{5}\right)}{3d\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4345}$$

$$\frac{2\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{2-3\cos(c+dx)}}dx}{3\sqrt{2\sec(c+dx)-3}}-\frac{2\sqrt{5}\sqrt{2\sec(c+dx)-3}E\left(\frac{1}{2}(c+dx+\pi)|\frac{6}{5}\right)}{3d\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{2-3\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{2\sec(c+dx)-3}}-\frac{2\sqrt{5}\sqrt{2\sec(c+dx)-3}E\left(\frac{1}{2}(c+dx+\pi)|\frac{6}{5}\right)}{3d\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3141}$$

$$\frac{4\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi),\frac{6}{5}\right)}{3\sqrt{5}d\sqrt{2\sec(c+dx)-3}}-\frac{2\sqrt{5}\sqrt{2\sec(c+dx)-3}E\left(\frac{1}{2}(c+dx+\pi)|\frac{6}{5}\right)}{3d\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[-3 + 2*Sec[c + d*x]]),x]
```

output
```
(4*Sqrt[2 - 3*Cos[c + d*x]]*EllipticF[(c + Pi + d*x)/2, 6/5]*Sqrt[Sec[c +
d*x]])/(3*Sqrt[5]*d*Sqrt[-3 + 2*Sec[c + d*x]]) - (2*Sqrt[5]*EllipticE[(c +
 Pi + d*x)/2, 6/5]*Sqrt[-3 + 2*Sec[c + d*x]])/(3*d*Sqrt[2 - 3*Cos[c + d*x]
]*Sqrt[Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3133
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 - b]/d)*EllipticE[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 3141
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4349
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Cs
c[e + f*x]], x], x] - Simp[b/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Cs
c[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.98 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.67

method	result
default	$-\frac{2\sqrt{-3+2\sec(dx+c)}\left(6i\,\text{EllipticF}\left(i(\csc(dx+c)-\cot(dx+c)),i\sqrt{5}\right)\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}-2i\,\text{EllipticE}\left(i(\csc(dx+c)-\cdots\right.\right.}{3d\left(5(1-\cos(dx+c))^{2}\csc(dx+c)^{2}-\cdots\right.}$
risch	$-\frac{i\left(3\,\mathrm{e}^{2i(dx+c)}-4\,\mathrm{e}^{i(dx+c)}+3\right)\sqrt{2}}{3d\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{-\frac{3\,\mathrm{e}^{2i(dx+c)}-4\,\mathrm{e}^{i(dx+c)}+3}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{i\left(\frac{-2\,\mathrm{e}^{2i(dx+c)}+\frac{8\,\mathrm{e}^{i(dx+c)}}{3}-2}{\sqrt{\left(-3\,\mathrm{e}^{2i(dx+c)}+4\,\mathrm{e}^{i(dx+c)}-3\right)\mathrm{e}^{i(dx+c)}}}+\frac{\left(-\frac{2}{3}+\frac{i\sqrt{5}}{3}\right)\sqrt{\frac{\mathrm{e}^{i(d\cdots}}{\cdots}}\cdots}{\cdots}\right.}{\cdots}$

input
```
int(1/sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d*(-3+2*sec(d*x+c))^(1/2)*(6*I*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I*
5^(1/2))*(1/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
-2*I*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I*5^(1/2))*(1/(1+cos(d*x+c)))^(1/
2)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)+5*(1-cos(d*x+c))^3*csc(d*x+c)^
3-csc(d*x+c)+cot(d*x+c))/(5*(1-cos(d*x+c))^2*csc(d*x+c)^2-1)/sec(d*x+c)^(1
/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-3+2\sec(c+dx)}}\,dx = -\frac{4\sqrt{6}\text{weierstrassPInverse}\left(-\frac{44}{27},-\frac{784}{729},\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)-\frac{4}{9}\right)+4\sqrt{6}\text{weierstrassPInverse}\left(-\right.}{}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="fricas"
)
```

output
```
-1/27*(4*sqrt(6)*weierstrassPInverse(-44/27, -784/729, cos(d*x + c) + I*si
n(d*x + c) - 4/9) + 4*sqrt(6)*weierstrassPInverse(-44/27, -784/729, cos(d*
x + c) - I*sin(d*x + c) - 4/9) - 9*sqrt(6)*weierstrassZeta(-44/27, -784/72
9, weierstrassPInverse(-44/27, -784/729, cos(d*x + c) + I*sin(d*x + c) - 4
/9)) - 9*sqrt(6)*weierstrassZeta(-44/27, -784/729, weierstrassPInverse(-44
/27, -784/729, cos(d*x + c) - I*sin(d*x + c) - 4/9)))/d
```

Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-3+2\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{2\sec\left(c+dx\right)-3}\sqrt{\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(-3+2*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(2*sec(c + d*x) - 3)*sqrt(sec(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-3+2\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{2\,\sec\left(dx+c\right)-3}\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(1/(sqrt(2*sec(d*x + c) - 3)*sqrt(sec(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-3+2\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{2\,\sec\left(dx+c\right)-3}\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(2*sec(d*x + c) - 3)*sqrt(sec(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-3+2\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\frac{2}{\cos(c+d\,x)}-3}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((2/cos(c + d*x) - 3)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((2/cos(c + d*x) - 3)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-3+2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{2\sec(dx+c)-3}}{2\sec(dx+c)^2-3\sec(dx+c)}dx$$

input
```
int(1/sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(2*sec(c + d*x) - 3))/(2*sec(c + d*x)**2 - 3*s
ec(c + d*x)),x)
```

3.676 $\int \frac{1}{\sqrt{-3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{1}{\sqrt{-3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = -\frac{2E\left(\frac{1}{2}(c+\pi+dx)\middle|6\right)\sqrt{-3-2\sec(c+dx)}}{3d\sqrt{-2-3\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{4\sqrt{-2-3\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+\pi+dx),6\right)\sqrt{\sec(c+dx)}}{3d\sqrt{-3-2\sec(c+dx)}}$$

output
```
-2/3*EllipticE(cos(1/2*d*x+1/2*c),6^(1/2))*(-3-2*sec(d*x+c))^(1/2)/d/(-2-3
*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)-4/3*(-2-3*cos(d*x+c))^(1/2)*InverseJac
obiAM(1/2*d*x+1/2*Pi+1/2*c,6^(1/2))*sec(d*x+c)^(1/2)/d/(-3-2*sec(d*x+c))^(
1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{-3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \frac{2\sqrt{2+3\cos(c+dx)}\left(5E\left(\frac{1}{2}(c+dx)\middle|\frac{6}{5}\right)-2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{6}{5}\right)\right)\sqrt{\sec(c+dx)}}{3\sqrt{5}d\sqrt{-3-2\sec(c+dx)}}$$

input
```
Integrate[1/(Sqrt[-3 - 2*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

output
```
(2*Sqrt[2 + 3*Cos[c + d*x]]*(5*EllipticE[(c + d*x)/2, 6/5] - 2*EllipticF[(
c + d*x)/2, 6/5])*Sqrt[Sec[c + d*x]])/(3*Sqrt[5]*d*Sqrt[-3 - 2*Sec[c + d*x
]])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4349, 3042, 4343, 3042, 3133, 4345, 3042, 3141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2\sec(c+dx)-3}\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{-2\csc\left(c+dx+\frac{\pi}{2}\right)-3}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4349}$$

$$-\frac{1}{3}\int \frac{\sqrt{-2\sec(c+dx)-3}}{\sqrt{\sec(c+dx)}}dx-\frac{2}{3}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2\sec(c+dx)-3}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{1}{3}\int \frac{\sqrt{-2\csc\left(c+dx+\frac{\pi}{2}\right)-3}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-2\csc\left(c+dx+\frac{\pi}{2}\right)-3}}dx$$

$$\Big\downarrow \text{4343}$$

$$-\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-2\csc\left(c+dx+\frac{\pi}{2}\right)-3}}dx-\frac{\sqrt{-2\sec(c+dx)-3}\int \sqrt{-3\cos(c+dx)-2}dx}{3\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-2\csc\left(c+dx+\frac{\pi}{2}\right)-3}}dx-\frac{\sqrt{-2\sec(c+dx)-3}\int \sqrt{-3\sin\left(c+dx+\frac{\pi}{2}\right)-2}dx}{3\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3133}$$

$$-\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-2\csc\left(c+dx+\frac{\pi}{2}\right)-3}}dx-\frac{2\sqrt{-2\sec(c+dx)-3}E\left(\frac{1}{2}(c+dx+\pi)\middle|6\right)}{3d\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4345}$$

$$-\frac{2\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{-3\cos(c+dx)-2}}dx}{3\sqrt{-2\sec(c+dx)-3}}-\frac{2\sqrt{-2\sec(c+dx)-3}E\left(\frac{1}{2}(c+dx+\pi)\middle|6\right)}{3d\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{-3\sin\left(c+dx+\frac{\pi}{2}\right)-2}}dx}{3\sqrt{-2\sec(c+dx)-3}}-\frac{2\sqrt{-2\sec(c+dx)-3}E\left(\frac{1}{2}(c+dx+\pi)\middle|6\right)}{3d\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3141}$$

$$-\frac{4\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi),6\right)}{3d\sqrt{-2\sec(c+dx)-3}}-\frac{2\sqrt{-2\sec(c+dx)-3}E\left(\frac{1}{2}(c+dx+\pi)\middle|6\right)}{3d\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[-3 - 2*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

output
```
(-2*EllipticE[(c + Pi + d*x)/2, 6]*Sqrt[-3 - 2*Sec[c + d*x]])/(3*d*Sqrt[-2
 - 3*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (4*Sqrt[-2 - 3*Cos[c + d*x]]*Elli
pticF[(c + Pi + d*x)/2, 6]*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[-3 - 2*Sec[c + d*
x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3133
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 - b]/d)*EllipticE[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 3141
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4349
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Cs
c[e + f*x]], x], x] - Simp[b/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Cs
c[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.27 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.98

method	result
default	$-\frac{2\sqrt{-3-2\sec(dx+c)}\left(3i\,\text{EllipticF}\left(i(\csc(dx+c)-\cot(dx+c)),\frac{i\sqrt{5}}{5}\right)\sqrt{2}\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{10}\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}-5i\,\text{EllipticE}\left(i(\csc(d\right.\right.}{15d\left((1-\cos(dx+c))^2\csc(\right.}$
risch	$-\frac{i\left(3\,e^{2i(dx+c)}+4\,e^{i(dx+c)}+3\right)\sqrt{2}}{3d\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}+1\right)\sqrt{-\frac{3\,e^{2i(dx+c)}+4\,e^{i(dx+c)}+3}{e^{2i(dx+c)}+1}}}-\frac{i\left(\frac{-2\,e^{2i(dx+c)}-\frac{8\,e^{i(dx+c)}}{3}-2}{\sqrt{\left(-3\,e^{2i(dx+c)}-4\,e^{i(dx+c)}-3\right)e^{i(dx+c)}}}+\frac{\left(\frac{2}{3}+\frac{i\sqrt{5}}{3}\right)\sqrt{\frac{e^{i(dx+}}{}}}{}\right.}{}$

input
```
int(1/(-3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15/d*(-3-2*sec(d*x+c))^(1/2)*(3*I*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),1
/5*I*5^(1/2))*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((2+3*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)-5*I*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),1/5*I*5^(1/2
))*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)+5*(1-cos(d*x+c))^3*csc(d*x+c)^3-25*csc(d*x+c)+25*cot(d*x+c))/((
1-cos(d*x+c))^2*csc(d*x+c)^2-5)/sec(d*x+c)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{-3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{4\sqrt{6}\text{weierstrassPInverse}\left(-\frac{44}{27},\frac{784}{729},\cos\left(dx+c\right)+i\sin\left(dx+c\right)+\frac{4}{9}\right)+4\sqrt{6}\text{weierstrassPInverse}\left(-\frac{44}{27},\frac{7}{7}\right.}{}$$

input
```
integrate(1/(-3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas"
)
```

output
```
1/27*(4*sqrt(6)*weierstrassPInverse(-44/27, 784/729, cos(d*x + c) + I*sin(
d*x + c) + 4/9) + 4*sqrt(6)*weierstrassPInverse(-44/27, 784/729, cos(d*x +
 c) - I*sin(d*x + c) + 4/9) + 9*sqrt(6)*weierstrassZeta(-44/27, 784/729, w
eierstrassPInverse(-44/27, 784/729, cos(d*x + c) + I*sin(d*x + c) + 4/9))
+ 9*sqrt(6)*weierstrassZeta(-44/27, 784/729, weierstrassPInverse(-44/27, 7
84/729, cos(d*x + c) - I*sin(d*x + c) + 4/9)))/d
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{-2\sec\left(c+dx\right)-3}\sqrt{\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(-3-2*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(-2*sec(c + d*x) - 3)*sqrt(sec(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{-2\sec(dx+c)-3}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(-3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(1/(sqrt(-2*sec(d*x + c) - 3)*sqrt(sec(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{-2\sec(dx+c)-3}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(-3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-2*sec(d*x + c) - 3)*sqrt(sec(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{-\frac{2}{\cos(c+d\,x)}-3}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((- 2/cos(c + d*x) - 3)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((- 2/cos(c + d*x) - 3)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = -\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{-2\sec(dx+c)-3}}{2\sec(dx+c)^2+3\sec(dx+c)}dx\right)$$

input

```
int(1/(-3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

output

```
- int((sqrt(sec(c + d*x))*sqrt( - 2*sec(c + d*x) - 3))/(2*sec(c + d*x)**2
+ 3*sec(c + d*x)),x)
```

3.677 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2+3\sec(c+dx)}} dx$

Optimal result

Integrand size = 25, antiderivative size = 61

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2+3\sec(c+dx)}} dx = \frac{2\sqrt{3+2\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{2+3\sec(c+dx)}}$$

output
```
2/5*(3+2*cos(d*x+c))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2/5*5^(1/2))*sec(
d*x+c)^(1/2)*5^(1/2)/d/(2+3*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2+3\sec(c+dx)}} dx = \frac{2\sqrt{3+2\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{2+3\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/Sqrt[2 + 3*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[3 + 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]]
)/(Sqrt[5]*d*Sqrt[2 + 3*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 4345, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3\sec(c+dx)+2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3\csc\left(c+dx+\frac{\pi}{2}\right)+2}}dx$$

$$\Big\downarrow \text{4345}$$

$$\frac{\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{2\cos(c+dx)+3}}dx}{\sqrt{3\sec(c+dx)+2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{2\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx}{\sqrt{3\sec(c+dx)+2}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)}{\sqrt{5}d\sqrt{3\sec(c+dx)+2}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/Sqrt[2 + 3*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[3 + 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]]
)/(Sqrt[5]*d*Sqrt[2 + 3*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.02

method	result
default	$-\frac{i\sqrt{5}\,\sqrt{2+3\sec(dx+c)}\,\sqrt{\sec(dx+c)}\,\sqrt{10}\,\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,\operatorname{EllipticF}\left(\frac{i\sqrt{5}\,(\csc(dx+c)-\cot(dx+c))}{5},\sqrt{5}\right)\left(\cos(dx+c\right.}{5d(3+2\cos(dx+c))}$

input
```
int(sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*I/d*5^(1/2)*(2+3*sec(d*x+c))^(1/2)*sec(d*x+c)^(1/2)*10^(1/2)*((3+2*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF
(1/5*I*5^(1/2)*(csc(d*x+c)-cot(d*x+c)),5^(1/2))/(3+2*cos(d*x+c))*(cos(d*x+
c)^2+cos(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2+3\sec(c+dx)}}\,dx = \frac{-i\,\text{weierstrassPInverse}(8,-4,\cos(dx+c)+i\,\sin(dx+c)+1)+i\,\text{weierstrassPInverse}(8,-4,\cos(dx+$$

input
```
integrate(sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(-I*weierstrassPInverse(8, -4, cos(d*x + c) + I*sin(d*x + c) + 1) + I*weie
rstrassPInverse(8, -4, cos(d*x + c) - I*sin(d*x + c) + 1))/d
```

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2+3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3\sec(c+dx)+2}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(2+3*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/sqrt(3*sec(c + d*x) + 2), x)
```

Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2+3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sqrt{3\sec(dx+c)+2}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(3*sec(d*x + c) + 2), x)
```

Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2+3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sqrt{3\sec(dx+c)+2}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(3*sec(d*x + c) + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2+3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{\frac{3}{\cos(c+d\,x)}+2}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(3/cos(c + d*x) + 2)^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(3/cos(c + d*x) + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2+3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{3\sec(dx+c)+2}}{3\sec(dx+c)+2}dx$$

input
```
int(sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(3*sec(c + d*x) + 2))/(3*sec(c + d*x) + 2),x)
```

3.678 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2+3\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 54

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2+3\sec(c+dx)}}\,dx = \frac{2\sqrt{3-2\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)\sqrt{\sec(c+dx)}}{d\sqrt{-2+3\sec(c+dx)}}$$

output
```
2*(3-2*cos(d*x+c))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2*I)*sec(d*x+c)^(1/
2)/d/(-2+3*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2+3\sec(c+dx)}}\,dx = \frac{2\sqrt{3-2\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)\sqrt{\sec(c+dx)}}{d\sqrt{-2+3\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/Sqrt[-2 + 3*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, -4]*Sqrt[Sec[c + d*x]])
/(d*Sqrt[-2 + 3*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 4345, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3\sec(c+dx)-2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}dx$$

$$\Big\downarrow \text{4345}$$

$$\frac{\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{3-2\cos(c+dx)}}dx}{\sqrt{3\sec(c+dx)-2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{3-2\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{3\sec(c+dx)-2}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)}{d\sqrt{3\sec(c+dx)-2}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/Sqrt[-2 + 3*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, -4]*Sqrt[Sec[c + d*x]])
/(d*Sqrt[-2 + 3*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(48) = 96$.

Time = 0.99 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.13

method	result
default	$\frac{i\sqrt{-2+3\sec(dx+c)}\,\sqrt{2}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{-\frac{2(2\cos(dx+c)-3)}{1+\cos(dx+c)}}\ \mathrm{EllipticF}\left(i(\csc(dx+c)-\cot(dx+c)),\sqrt{5}\right)\sqrt{\sec(dx+c)}\,\left(\cos(dx+c)^2+\mathrm{c}\right.}{d(2\cos(dx+c)-3)}$

input
```
int(sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I/d*(-2+3*sec(d*x+c))^(1/2)*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*(-2*(2*cos(d*
x+c)-3)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),5^(1/2))
*sec(d*x+c)^(1/2)/(2*cos(d*x+c)-3)*(cos(d*x+c)^2+cos(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2+3\sec(c+dx)}}\,dx = -\frac{\text{weierstrassPInverse}(8,4,\cos(dx+c)+i\sin(dx+c)-1)+\text{weierstrassPInverse}(8,4,\cos(dx+c)-}{d}$$

input
```
integrate(sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-(weierstrassPInverse(8, 4, cos(d*x + c) + I*sin(d*x + c) - 1) + weierstra
ssPInverse(8, 4, cos(d*x + c) - I*sin(d*x + c) - 1))/d
```

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2+3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3\sec(c+dx)-2}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(-2+3*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/sqrt(3*sec(c + d*x) - 2), x)
```

Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2+3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sqrt{3\sec(dx+c)-2}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(3*sec(d*x + c) - 2), x)
```

Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2+3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{3\sec\left(dx+c\right)-2}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(3*sec(d*x + c) - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2+3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{\frac{3}{\cos(c+d\,x)}-2}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(3/cos(c + d*x) - 2)^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(3/cos(c + d*x) - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2+3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{3\sec\left(dx+c\right)-2}}{3\sec\left(dx+c\right)-2}dx$$

input
```
int(sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(3*sec(c + d*x) - 2))/(3*sec(c + d*x) - 2),x)
```

3.679 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 54

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx = \frac{2\sqrt{3-2\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)\sqrt{\sec(c+dx)}}{d\sqrt{2-3\sec(c+dx)}}$$

output
```
2*(3-2*cos(d*x+c))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2*I)*sec(d*x+c)^(1/
2)/d/(2-3*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx = \frac{2\sqrt{3-2\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)\sqrt{\sec(c+dx)}}{d\sqrt{2-3\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/Sqrt[2 - 3*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, -4]*Sqrt[Sec[c + d*x]])
/(d*Sqrt[2 - 3*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2-3\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4345}$$

$$\frac{\sqrt{2\cos(c+dx)-3}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{2\cos(c+dx)-3}}dx}{\sqrt{2-3\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{2\cos(c+dx)-3}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{2\sin\left(c+dx+\frac{\pi}{2}\right)-3}}dx}{\sqrt{2-3\sec(c+dx)}}$$

$$\Big\downarrow \text{3142}$$

$$\frac{\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{3-2\cos(c+dx)}}dx}{\sqrt{2-3\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{3-2\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{2-3\sec(c+dx)}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)}{d\sqrt{2-3\sec(c+dx)}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/Sqrt[2 - 3*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, -4]*Sqrt[Sec[c + d*x]])
/(d*Sqrt[2 - 3*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(48) = 96.

Time = 1.00 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.28

method	result
default	$-\frac{i\sqrt{5}\,\sqrt{2}\,\sqrt{\sec(dx+c)}\,\sqrt{2-3\sec(dx+c)}\,\sqrt{-\frac{2(2\cos(dx+c)-3)}{1+\cos(dx+c)}}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,\text{EllipticF}\left(i\sqrt{5}\,(\csc(dx+c)-\cot(dx+c)),\frac{\sqrt{5}}{5}\right)\left(\cos(dx\right.}{5d(2\cos(dx+c)-3)}$

input
```
int(sec(d*x+c)^(1/2)/(2-3*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*I/d*5^(1/2)*2^(1/2)*sec(d*x+c)^(1/2)*(2-3*sec(d*x+c))^(1/2)*(-2*(2*co
s(d*x+c)-3)/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF(I*5^(
1/2)*(csc(d*x+c)-cot(d*x+c)),1/5*5^(1/2))/(2*cos(d*x+c)-3)*(cos(d*x+c)^2+c
os(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx = \frac{-i\,\text{weierstrassPInverse}(8,4,\cos(dx+c)+i\,\sin(dx+c)-1)+i\,\text{weierstrassPInverse}(8,4,\cos(dx+c)}{d}$$

input
```
integrate(sec(d*x+c)^(1/2)/(2-3*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(-I*weierstrassPInverse(8, 4, cos(d*x + c) + I*sin(d*x + c) - 1) + I*weier
strassPInverse(8, 4, cos(d*x + c) - I*sin(d*x + c) - 1))/d
```

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(2-3*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/sqrt(2 - 3*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sqrt{-3\sec(dx+c)+2}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(2-3*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(-3*sec(d*x + c) + 2), x)
```

Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sqrt{-3\sec(dx+c)+2}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(2-3*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(-3*sec(d*x + c) + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{2-\frac{3}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(2 - 3/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(2 - 3/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx = -\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{-3\sec(dx+c)+2}}{3\sec(dx+c)-2}dx\right)$$

input
```
int(sec(d*x+c)^(1/2)/(2-3*sec(d*x+c))^(1/2),x)
```

output
```
 - int((sqrt(sec(c + d*x))*sqrt( - 3*sec(c + d*x) + 2))/(3*sec(c + d*x) -
2),x)
```

3.680 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2-3\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 61

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2-3\sec(c+dx)}}\,dx = \frac{2\sqrt{3+2\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{-2-3\sec(c+dx)}}$$

output
```
2/5*(3+2*cos(d*x+c))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2/5*5^(1/2))*sec(
d*x+c)^(1/2)*5^(1/2)/d/(-2-3*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2-3\sec(c+dx)}}\,dx = \frac{2\sqrt{3+2\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{-2-3\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/Sqrt[-2 - 3*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[3 + 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]]
)/(Sqrt[5]*d*Sqrt[-2 - 3*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3\sec(c+dx)-2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}dx$$

$$\Big\downarrow \text{4345}$$

$$\frac{\sqrt{-2\cos(c+dx)-3}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{-2\cos(c+dx)-3}}dx}{\sqrt{-3\sec(c+dx)-2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{-2\cos(c+dx)-3}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{-2\sin\left(c+dx+\frac{\pi}{2}\right)-3}}dx}{\sqrt{-3\sec(c+dx)-2}}$$

$$\Big\downarrow \text{3142}$$

$$\frac{\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\frac{2}{5}\cos(c+dx)+\frac{3}{5}}}dx}{\sqrt{5}\sqrt{-3\sec(c+dx)-2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\frac{2}{5}\sin\left(c+dx+\frac{\pi}{2}\right)+\frac{3}{5}}}dx}{\sqrt{5}\sqrt{-3\sec(c+dx)-2}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)}{\sqrt{5}d\sqrt{-3\sec(c+dx)-2}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/Sqrt[-2 - 3*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[3 + 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]]
)/(Sqrt[5]*d*Sqrt[-2 - 3*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.16 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.95

method	result
default	$\frac{i\sqrt{-2-3\sec(dx+c)}\,\sqrt{\sec(dx+c)}\,\sqrt{2}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{10}\,\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\,\text{EllipticF}\left(i(\csc(dx+c)-\cot(dx+c)),\frac{\sqrt{5}}{5}\right)\left(\cos(dx+c)^2+c\right.}{5d(3+2\cos(dx+c))}$

input
```
int(sec(d*x+c)^(1/2)/(-2-3*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*I/d*(-2-3*sec(d*x+c))^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)*(1/(1+cos(d*x+c))
)^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(csc(
d*x+c)-cot(d*x+c)),1/5*5^(1/2))/(3+2*cos(d*x+c))*(cos(d*x+c)^2+cos(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2-3\sec(c+dx)}}\,dx = -\frac{\text{weierstrassPInverse}(8,-4,\cos(dx+c)+i\sin(dx+c)+1)+\text{weierstrassPInverse}(8,-4,\cos(dx+c)}{d}$$

input
```
integrate(sec(d*x+c)^(1/2)/(-2-3*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-(weierstrassPInverse(8, -4, cos(d*x + c) + I*sin(d*x + c) + 1) + weierstr
assPInverse(8, -4, cos(d*x + c) - I*sin(d*x + c) + 1))/d
```

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2-3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec{(c+dx)}}}{\sqrt{-3\sec{(c+dx)}-2}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(-2-3*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/sqrt(-3*sec(c + d*x) - 2), x)
```

Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2-3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec{(dx+c)}}}{\sqrt{-3\sec{(dx+c)}-2}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(-2-3*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(-3*sec(d*x + c) - 2), x)
```

Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2-3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec{(dx+c)}}}{\sqrt{-3\sec{(dx+c)}-2}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(-2-3*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(-3*sec(d*x + c) - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2-3\sec(c+dx)}}\,dx=\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{-\frac{3}{\cos(c+d\,x)}-2}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(- 3/cos(c + d*x) - 2)^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(- 3/cos(c + d*x) - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2-3\sec(c+dx)}}\,dx=-\left(\int \frac{\sqrt{\sec (dx+c)}\,\sqrt{-3\sec (dx+c)-2}}{3\sec (dx+c)+2}dx\right)$$

input
```
int(sec(d*x+c)^(1/2)/(-2-3*sec(d*x+c))^(1/2),x)
```

output
```
- int((sqrt(sec(c + d*x))*sqrt( - 3*sec(c + d*x) - 2))/(3*sec(c + d*x) +
2),x)
```

3.681 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3+2\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 61

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3+2\sec(c+dx)}}\,dx=\frac{2\sqrt{2+3\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{3+2\sec(c+dx)}}$$

output
```
2/5*(2+3*cos(d*x+c))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,1/5*30^(1/2))*sec
(d*x+c)^(1/2)*5^(1/2)/d/(3+2*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3+2\sec(c+dx)}}\,dx=\frac{2\sqrt{2+3\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{3+2\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/Sqrt[3 + 2*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6/5]*Sqrt[Sec[c + d*x]]
)/(Sqrt[5]*d*Sqrt[3 + 2*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 4345, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2\sec(c+dx)+3}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2\csc\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow \ 4345$$

$$\frac{\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{3\cos(c+dx)+2}}dx}{\sqrt{2\sec(c+dx)+3}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{3\sin\left(c+dx+\frac{\pi}{2}\right)+2}}dx}{\sqrt{2\sec(c+dx)+3}}$$

$$\Big\downarrow \ 3140$$

$$\frac{2\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{6}{5}\right)}{\sqrt{5}d\sqrt{2\sec(c+dx)+3}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/Sqrt[3 + 2*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6/5]*Sqrt[Sec[c + d*x]]
)/(Sqrt[5]*d*Sqrt[3 + 2*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.03

method	result
default	$\frac{\sqrt{5}\,\sqrt{2}\,\sqrt{3+2\sec(dx+c)}\,\sqrt{\sec(dx+c)}\,\sqrt{10}\,\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,\operatorname{EllipticF}\left(\frac{\sqrt{5}\,(\csc(dx+c)-\cot(dx+c))}{5},i\sqrt{5}\right)\left(\cos(dx+c)^2+\right.}{5d(2+3\cos(dx+c))}$

input
```
int(sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/d*5^(1/2)*2^(1/2)*(3+2*sec(d*x+c))^(1/2)*sec(d*x+c)^(1/2)*10^(1/2)*((2
+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF(1/
5*5^(1/2)*(csc(d*x+c)-cot(d*x+c)),I*5^(1/2))/(2+3*cos(d*x+c))*(cos(d*x+c)^
2+cos(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3+2\sec(c+dx)}}\,dx = \frac{-i\sqrt{6}\text{weierstrassPInverse}\left(-\frac{44}{27},\frac{784}{729},\cos(dx+c)+i\sin(dx+c)+\frac{4}{9}\right)+i\sqrt{6}\text{weierstrassPInverse}\left(-\frac{44}{27},\right.}{3d}$$

input
```
integrate(sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(6)*weierstrassPInverse(-44/27, 784/729, cos(d*x + c) + I*sin(
d*x + c) + 4/9) + I*sqrt(6)*weierstrassPInverse(-44/27, 784/729, cos(d*x +
 c) - I*sin(d*x + c) + 4/9))/d
```

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3+2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2\sec(c+dx)+3}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(3+2*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/sqrt(2*sec(c + d*x) + 3), x)
```

Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3+2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sqrt{2\sec(dx+c)+3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(2*sec(d*x + c) + 3), x)
```

Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3+2\sec(c+dx)}}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{2\,\sec\left(dx+c\right)+3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(2*sec(d*x + c) + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3+2\sec(c+dx)}}\,dx=\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{\frac{2}{\cos(c+d\,x)}+3}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(2/cos(c + d*x) + 3)^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(2/cos(c + d*x) + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3+2\sec(c+dx)}}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{2\sec\left(dx+c\right)+3}}{2\sec\left(dx+c\right)+3}dx$$

input
```
int(sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(2*sec(c + d*x) + 3))/(2*sec(c + d*x) + 3),x)
```

3.682 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 54

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx = \frac{2\sqrt{-2+3\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),6\right)\sqrt{\sec(c+dx)}}{d\sqrt{3-2\sec(c+dx)}}$$

output
```
2*(-2+3*cos(d*x+c))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,6^(1/2))*sec(d*x+c
)^(1/2)/d/(3-2*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx = \frac{2\sqrt{-2+3\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),6\right)\sqrt{\sec(c+dx)}}{d\sqrt{3-2\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/Sqrt[3 - 2*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[-2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6]*Sqrt[Sec[c + d*x]])
/(d*Sqrt[3 - 2*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 4345, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3-2\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4345}$$

$$\frac{\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{3\cos(c+dx)-2}}dx}{\sqrt{3-2\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{3\sin\left(c+dx+\frac{\pi}{2}\right)-2}}dx}{\sqrt{3-2\sec(c+dx)}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),6\right)}{d\sqrt{3-2\sec(c+dx)}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/Sqrt[3 - 2*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[-2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6]*Sqrt[Sec[c + d*x]])
/(d*Sqrt[3 - 2*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.17

method	result
default	$\frac{2\sqrt{5}\,\sqrt{3-2\sec(dx+c)}\,\sqrt{\sec(dx+c)}\,\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,\operatorname{EllipticF}\left(\sqrt{5}\,(\csc(dx+c)-\cot(dx+c)),\frac{i\sqrt{5}}{5}\right)\left(\cos(dx+c)^2+\cos(dx+c)\right)}{5d(-2+3\cos(dx+c))}$

input
```
int(sec(d*x+c)^(1/2)/(3-2*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/d*5^(1/2)*(3-2*sec(d*x+c))^(1/2)*sec(d*x+c)^(1/2)*((-2+3*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF(5^(1/2)*(csc(d*x+c
)-cot(d*x+c)),1/5*I*5^(1/2))/(-2+3*cos(d*x+c))*(cos(d*x+c)^2+cos(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx = \frac{-i\sqrt{6}\text{weierstrassPInverse}\left(-\frac{44}{27},-\frac{784}{729},\cos(dx+c)+i\sin(dx+c)-\frac{4}{9}\right)+i\sqrt{6}\text{weierstrassPInverse}\left(-\frac{44}{27}\right.}{3d}$$

input
```
integrate(sec(d*x+c)^(1/2)/(3-2*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(6)*weierstrassPInverse(-44/27, -784/729, cos(d*x + c) + I*sin
(d*x + c) - 4/9) + I*sqrt(6)*weierstrassPInverse(-44/27, -784/729, cos(d*x
 + c) - I*sin(d*x + c) - 4/9))/d
```

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(3-2*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/sqrt(3 - 2*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sqrt{-2\sec(dx+c)+3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(3-2*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(-2*sec(d*x + c) + 3), x)
```

Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{-2\sec\left(dx+c\right)+3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(3-2*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(-2*sec(d*x + c) + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{3-\frac{2}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(3 - 2/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(3 - 2/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx = -\left(\int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{-2\sec\left(dx+c\right)+3}}{2\sec\left(dx+c\right)-3}dx\right)$$

input
```
int(sec(d*x+c)^(1/2)/(3-2*sec(d*x+c))^(1/2),x)
```

output
```
 - int((sqrt(sec(c + d*x))*sqrt( - 2*sec(c + d*x) + 3))/(2*sec(c + d*x) -
3),x)
```

3.683 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3+2\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 62

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3+2\sec(c+dx)}}\,dx = \frac{2\sqrt{2-3\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+\pi+dx),\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{-3+2\sec(c+dx)}}$$

output
```
2/5*(2-3*cos(d*x+c))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*Pi+1/2*c,1/5*30^(1/
2))*sec(d*x+c)^(1/2)*5^(1/2)/d/(-3+2*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3+2\sec(c+dx)}}\,dx = \frac{2\sqrt{-2+3\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),6\right)\sqrt{\sec(c+dx)}}{d\sqrt{-3+2\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/Sqrt[-3 + 2*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[-2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6]*Sqrt[Sec[c + d*x]])
/(d*Sqrt[-3 + 2*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 4345, 3042, 3141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2\sec(c+dx)-3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2\csc\left(c+dx+\frac{\pi}{2}\right)-3}}dx$$

$$\Big\downarrow \text{4345}$$

$$\frac{\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{2-3\cos(c+dx)}}dx}{\sqrt{2\sec(c+dx)-3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{2-3\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{2\sec(c+dx)-3}}$$

$$\Big\downarrow \text{3141}$$

$$\frac{2\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi),\frac{6}{5}\right)}{\sqrt{5}d\sqrt{2\sec(c+dx)-3}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/Sqrt[-3 + 2*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[2 - 3*Cos[c + d*x]]*EllipticF[(c + Pi + d*x)/2, 6/5]*Sqrt[Sec[c +
d*x]])/(Sqrt[5]*d*Sqrt[-3 + 2*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3141
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.84

method	result
default	$\frac{2i\sqrt{\sec(dx+c)}\,\sqrt{-3+2\sec(dx+c)}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{EllipticF}\left(i(\csc(dx+c)-\cot(dx+c)),i\sqrt{5}\right)\left(\cos(dx+c)^2+\cos(d\right.}{d(-2+3\cos(dx+c))}$

input
```
int(sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d*sec(d*x+c)^(1/2)*(-3+2*sec(d*x+c))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*((
-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c))
,I*5^(1/2))/(-2+3*cos(d*x+c))*(cos(d*x+c)^2+cos(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3+2\sec(c+dx)}}\,dx = -\frac{\sqrt{6}\text{weierstrassPInverse}\left(-\frac{44}{27},-\frac{784}{729},\cos(dx+c)+i\sin(dx+c)-\frac{4}{9}\right)+\sqrt{6}\text{weierstrassPInverse}\left(-\frac{44}{27},\right.}{3d}$$

input
```
integrate(sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(sqrt(6)*weierstrassPInverse(-44/27, -784/729, cos(d*x + c) + I*sin(d
*x + c) - 4/9) + sqrt(6)*weierstrassPInverse(-44/27, -784/729, cos(d*x + c
) - I*sin(d*x + c) - 4/9))/d
```

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3+2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2\sec(c+dx)-3}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(-3+2*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/sqrt(2*sec(c + d*x) - 3), x)
```

Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3+2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sqrt{2\sec(dx+c)-3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(2*sec(d*x + c) - 3), x)
```

Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3+2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{2\,\sec\left(dx+c\right)-3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(2*sec(d*x + c) - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3+2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{\frac{2}{\cos(c+d\,x)}-3}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(2/cos(c + d*x) - 3)^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(2/cos(c + d*x) - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3+2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{2\sec\left(dx+c\right)-3}}{2\sec\left(dx+c\right)-3}dx$$

input
```
int(sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(2*sec(c + d*x) - 3))/(2*sec(c + d*x) - 3),x)
```

3.684 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3-2\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 55

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3-2\sec(c+dx)}}\,dx = \frac{2\sqrt{-2-3\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+\pi+dx),6\right)\sqrt{\sec(c+dx)}}{d\sqrt{-3-2\sec(c+dx)}}$$

output
```
2*(-2-3*cos(d*x+c))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*Pi+1/2*c,6^(1/2))*se
c(d*x+c)^(1/2)/d/(-3-2*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3-2\sec(c+dx)}}\,dx = \frac{2\sqrt{2+3\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{-3-2\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/Sqrt[-3 - 2*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6/5]*Sqrt[Sec[c + d*x]]
)/(Sqrt[5]*d*Sqrt[-3 - 2*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 4345, 3042, 3141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2\sec(c+dx)-3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-2\csc\left(c+dx+\frac{\pi}{2}\right)-3}}dx$$

$$\Big\downarrow \text{4345}$$

$$\frac{\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{-3\cos(c+dx)-2}}dx}{\sqrt{-2\sec(c+dx)-3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{-3\sin\left(c+dx+\frac{\pi}{2}\right)-2}}dx}{\sqrt{-2\sec(c+dx)-3}}$$

$$\Big\downarrow \text{3141}$$

$$\frac{2\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi),6\right)}{d\sqrt{-2\sec(c+dx)-3}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/Sqrt[-3 - 2*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[-2 - 3*Cos[c + d*x]]*EllipticF[(c + Pi + d*x)/2, 6]*Sqrt[Sec[c + d
*x]])/(d*Sqrt[-3 - 2*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3141
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.16 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.18

method	result
default	$\frac{i\sqrt{2}\,\sqrt{-3-2\sec(dx+c)}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{10}\,\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{EllipticF}\left(i(\csc(dx+c)-\cot(dx+c)),\frac{i\sqrt{5}}{5}\right)\sqrt{\sec(dx+c)}\,\left(\cos(dx+c)^2+\cdots\right.}{5d(2+3\cos(dx+c))}$

input
```
int(sec(d*x+c)^(1/2)/(-3-2*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*I/d*2^(1/2)*(-3-2*sec(d*x+c))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*10^(1/2)*
((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)
),1/5*I*5^(1/2))*sec(d*x+c)^(1/2)/(2+3*cos(d*x+c))*(cos(d*x+c)^2+cos(d*x+c
))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3-2\sec(c+dx)}}\,dx = -\frac{\sqrt{6}\text{weierstrassPInverse}\left(-\frac{44}{27},\frac{784}{729},\cos(dx+c)+i\sin(dx+c)+\frac{4}{9}\right)+\sqrt{6}\text{weierstrassPInverse}\left(-\frac{44}{27},\frac{78}{72}\right.}{3\,d}$$

input
```
integrate(sec(d*x+c)^(1/2)/(-3-2*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(sqrt(6)*weierstrassPInverse(-44/27, 784/729, cos(d*x + c) + I*sin(d*
x + c) + 4/9) + sqrt(6)*weierstrassPInverse(-44/27, 784/729, cos(d*x + c)
- I*sin(d*x + c) + 4/9))/d
```

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3-2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2\sec(c+dx)-3}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(-3-2*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/sqrt(-2*sec(c + d*x) - 3), x)
```

Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3-2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sqrt{-2\sec(dx+c)-3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(-3-2*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(-2*sec(d*x + c) - 3), x)
```

Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3-2\sec(c+dx)}}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{-2\,\sec\left(dx+c\right)-3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(-3-2*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(-2*sec(d*x + c) - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3-2\sec(c+dx)}}\,dx=\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{-\frac{2}{\cos(c+d\,x)}-3}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(- 2/cos(c + d*x) - 3)^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(- 2/cos(c + d*x) - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3-2\sec(c+dx)}}\,dx=-\left(\int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{-2\sec\left(dx+c\right)-3}}{2\sec\left(dx+c\right)+3}dx\right)$$

input
```
int(sec(d*x+c)^(1/2)/(-3-2*sec(d*x+c))^(1/2),x)
```

output
```
- int((sqrt(sec(c + d*x))*sqrt( - 2*sec(c + d*x) - 3))/(2*sec(c + d*x) +
3),x)
```

3.685 $\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx = \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+b\sec(c+dx)}\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

output
```
2^(1/2)*AppellF1(1/2,-1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c
))*(a+b*sec(d*x+c))^(1/3)*tan(d*x+c)/d/(1+sec(d*x+c))^(1/2)/((a+b*sec(d*x+
c))/(a+b))^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7160 vs. $2(105) = 210$.

Time = 48.27 (sec) , antiderivative size = 7160, normalized size of antiderivative = 68.19

$$\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(1/3),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4321}$$

$$-\frac{\tan(c+dx)\int \frac{\sqrt[3]{a+b\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{156}$$

$$-\frac{\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}\int \frac{\sqrt[3]{\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{155}$$

$$\frac{\sqrt{2}\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

input
```
Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(1/3),x]
```

output
```
(Sqrt[2]*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
 + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*Sqrt[1 + Se
c[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3))
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
 - e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Maple [F]

$$\int \sec (dx+c) (a+b\sec (dx+c))^{\frac{1}{3}} dx$$

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx=\int (b\sec (dx+c)+a)^{\frac{1}{3}}\sec (dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c), x)
```

Sympy [F]

$$\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx=\int \sqrt[3]{a+b\sec (c+dx)}\sec (c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(1/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(1/3)*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx=\int (b\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c), x)
```

Giac [F]

$$\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx=\int (b\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{1/3}}{\cos(c+d\,x)}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/3)/cos(c + d*x),x)
```

output
```
int((a + b/cos(c + d*x))^(1/3)/cos(c + d*x), x)
```

Reduce [F]

$$\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx = \int (\sec(dx+c)\,b+a)^{\frac{1}{3}}\sec(dx+c)\,dx$$

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(1/3),x)
```

output
```
int((sec(c + d*x)*b + a)**(1/3)*sec(c + d*x),x)
```

3.686 $\int \sqrt[3]{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \sqrt[3]{a+b\sec(c+dx)}\,dx=\text{Int}\left(\sqrt[3]{a+b\sec(c+dx)},x\right)$$

output
```
Defer(Int)((a+b*sec(d*x+c))^(1/3),x)
```

Mathematica [N/A]

Not integrable

Time = 17.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \sqrt[3]{a+b\sec(c+dx)}\,dx=\int \sqrt[3]{a+b\sec(c+dx)}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(1/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(1/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4273}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4273}$$

$$\int \sqrt[3]{a+b\sec(c+dx)}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(1/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4273
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Unintegrable[
(a + b*Csc[c + d*x])^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0
] && !IntegerQ[2*n]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (a + b\sec(dx + c))^{\frac{1}{3}}\, dx$$

input
```
int((a+b*sec(d*x+c))^(1/3),x)
```

output
```
int((a+b*sec(d*x+c))^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt[3]{a + b\sec(c + dx)}\, dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{a + b\sec(c + dx)}\, dx = \int \sqrt[3]{a + b\sec(c + dx)}\, dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(1/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{a+b\sec(c+dx)}\,dx = \int (b\sec(dx+c)+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(1/3), x)
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{a+b\sec(c+dx)}\,dx = \int (b\sec(dx+c)+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(1/3), x)
```

Mupad [N/A]

Not integrable

Time = 10.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \sqrt[3]{a+b\sec(c+dx)}\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{1/3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/3),x)
```

output
```
int((a + b/cos(c + d*x))^(1/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{a+b\sec(c+dx)}\,dx=\int (\sec (dx+c)\,b+a)^{\frac{1}{3}}\,dx$$

input
```
int((a+b*sec(d*x+c))^(1/3),x)
```

output
```
int((sec(c + d*x)*b + a)**(1/3),x)
```

3.687 $\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 362

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \frac{3(9a^2+32b^2)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{220b^2d} - \frac{9a(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{44b^2d} + \frac{3\sec(c+dx)(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{11bd} + \frac{a(18a^2+49b^2)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{110\sqrt{2}b^3d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}} - \frac{(9a^4+23a^2b^2-32b^4)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\tan(c+dx)}{55\sqrt{2}b^3d\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}$$

output

```
3/220*(9*a^2+32*b^2)*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b^2/d-9/44*a*(a+b*s
ec(d*x+c))^(5/3)*tan(d*x+c)/b^2/d+3/11*sec(d*x+c)*(a+b*sec(d*x+c))^(5/3)*t
an(d*x+c)/b/d+1/220*a*(18*a^2+49*b^2)*AppellF1(1/2,-2/3,1/2,3/2,b*(1-sec(d
*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)*2^(1/2)
/b^3/d/(1+sec(d*x+c))^(1/2)/((a+b*sec(d*x+c))/(a+b))^(2/3)-1/110*(9*a^4+23
*a^2*b^2-32*b^4)*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*s
ec(d*x+c))*((a+b*sec(d*x+c))/(a+b))^(1/3)*tan(d*x+c)*2^(1/2)/b^3/d/(1+sec(
d*x+c))^(1/2)/(a+b*sec(d*x+c))^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 21877 vs. $2(362) = 724$.

Time = 45.11 (sec) , antiderivative size = 21877, normalized size of antiderivative = 60.43

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(2/3),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4352, 27, 3042, 4570, 27, 3042, 4490, 27, 3042, 4495, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx$$

$$\Big\downarrow \text{4352}$$

$$\frac{3\int \frac{1}{3}\sec(c+dx)(a+b\sec(c+dx))^{2/3}\left(-6a\sec^2(c+dx)+8b\sec(c+dx)+3a\right)dx}{11b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\left(-6a\sec^2(c+dx)+8b\sec(c+dx)+3a\right)dx}{11b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(-6a\csc\left(c+dx+\frac{\pi}{2}\right)^2+8b\csc\left(c+dx+\frac{\pi}{2}\right)+3a\right)dx}{11b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

$$\Big\downarrow \; 4570$$

$$\frac{\frac{3\int -\frac{2}{3}\sec(c+dx)(a+b\sec(c+dx))^{2/3}\left(3ab-\left(9a^2+32b^2\right)\sec(c+dx)\right)dx}{8b}-\frac{9a\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{4bd}}{11b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\left(3ab-\left(9a^2+32b^2\right)\sec(c+dx)\right)dx}{4b}-\frac{9a\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{4bd}}{11b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(3ab+\left(-9a^2-32b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{4b}-\frac{9a\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{4bd}}{11b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

$$\Big\downarrow \; 4490$$

$$\frac{-\frac{\frac{3}{5}\int -\frac{\sec(c+dx)\left(b\left(3a^2+64b^2\right)+a\left(18a^2+49b^2\right)\sec(c+dx)\right)}{3\sqrt[3]{a+b\sec(c+dx)}}dx-\frac{3\left(9a^2+32b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}}{4b}-\frac{9a\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{4bd}}{11b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{-\frac{1}{5}\int \frac{\sec(c+dx)\left(b\left(3a^2+64b^2\right)+a\left(18a^2+49b^2\right)\sec(c+dx)\right)}{\sqrt[3]{a+b\sec(c+dx)}}dx-\frac{3\left(9a^2+32b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}}{4b}-\frac{9a\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{4bd}}{11b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

$\big\downarrow$ 3042

$$-\frac{\frac{-\frac{1}{5}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(b\left(3a^2+64b^2\right)+a\left(18a^2+49b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{3\left(9a^2+32b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}}{4b}-\frac{9a\tan(c+dx)(a+b\sec(c+dx))}{4bd}}{11b}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

$\big\downarrow$ 4495

$$-\frac{\frac{\frac{1}{5}\left(\frac{2\left(9a^4+23a^2b^2-32b^4\right)\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}dx}{b}-\frac{a\left(18a^2+49b^2\right)\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}dx}{b}\right)-\frac{3\left(9a^2+32b^2\right)\tan(c+dx)(a+b\sec(c+dx))}{5d}}{4b}}{11b}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

$\big\downarrow$ 3042

$$-\frac{\frac{\frac{1}{5}\left(\frac{2\left(9a^4+23a^2b^2-32b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\left(18a^2+49b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b}\right)-3\left(9a^2+32b^2\right)\tan(c+}{4b}}{11b}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

$\big\downarrow$ 4321

$$-\frac{\frac{1}{5}\left(\frac{a\left(18a^2+49b^2\right)\tan(c+dx)\int\frac{(a+b\sec(c+dx))^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{2\left(9a^4+23a^2b^2-32b^4\right)\tan(c+dx)\int\frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}\right.}{4b}}{11b}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

$\downarrow$ 156

$$-\frac{\frac{1}{5}\left(\frac{a\left(18a^2+49b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}\int\frac{\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{2\left(9a^4+23a^2b^2-32b^4\right)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}{bd\sqrt{1-\sec(c+dx)}\sqrt{s}}\right.}{4b}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

$\downarrow$ 155

$$-\frac{\frac{1}{5}\left(\frac{2\sqrt{2}\left(9a^4+23a^2b^2-32b^4\right)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}-\frac{\sqrt{2}a\left(18a^2+49b^2\right)\tan(c+dx)}{}\right.}{4b}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

input
```
Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(2/3),x]
```

output
```
(3*Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(11*b*d) + ((-9*a
*(a + b*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(4*b*d) - ((-3*(9*a^2 + 32*b^2)*
(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*d) + (-((Sqrt[2]*a*(18*a^2 + 4
9*b^2)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
 d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Se
c[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3))) + (2*Sqrt[2]*(9*a^4 + 2
3*a^2*b^2 - 32*b^4)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*
(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c +
d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3)))/5)/(4*b))/(
11*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

rule 4352
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + n - 1))), x] + Simp[d^3/(b*(m + n -
1))   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 3)*Simp[a*(n - 3) +
b*(m + n - 2)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 3] && (IntegerQ[n] ||
 IntegersQ[2*m, 2*n]) &&  !IGtQ[m, 2]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

rule 4495
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)/b   Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Simp[B/b   Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && N
eQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \sec(dx+c)^4 (a+b\sec(dx+c))^{\frac{2}{3}} dx$$

input
```
int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(2/3),x)
```

output
```
int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3} dx = \int (b\sec(dx+c)+a)^{\frac{2}{3}} \sec(dx+c)^4 dx$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^4, x)
```

Sympy [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3} dx = \int (a+b\sec(c+dx))^{\frac{2}{3}} \sec^4(c+dx) dx$$

input
```
integrate(sec(d*x+c)**4*(a+b*sec(d*x+c))**(2/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(2/3)*sec(c + d*x)**4, x)
```

Maxima [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3}\,dx=\int (b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^4, x)
```

Giac [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3}\,dx=\int (b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{2/3}}{\cos(c+d\,x)^4}\,dx$$

input
```
int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x)^4,x)
```

output
```
int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x)^4, x)
```

Reduce [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int (\sec(dx+c)\,b+a)^{\frac{2}{3}}\sec(dx+c)^4\,dx$$

input
```
int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(2/3),x)
```

output
```
int((sec(c + d*x)*b + a)**(2/3)*sec(c + d*x)**4,x)
```

3.688 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 305

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{40bd}+\frac{3(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{8bd}-\frac{(6a^2-25b^2)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{20\sqrt{2}b^2d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}+\frac{3a(a^2-b^2)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\tan(c+dx)}{10\sqrt{2}b^2d\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}$$

output
```
-9/40*a*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b/d+3/8*(a+b*sec(d*x+c))^(5/3)*t
an(d*x+c)/b/d-1/40*(6*a^2-25*b^2)*AppellF1(1/2,-2/3,1/2,3/2,b*(1-sec(d*x+c
))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)*2^(1/2)/b^2
/d/(1+sec(d*x+c))^(1/2)/((a+b*sec(d*x+c))/(a+b))^(2/3)+3/20*a*(a^2-b^2)*Ap
pellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*se
c(d*x+c))/(a+b))^(1/3)*tan(d*x+c)*2^(1/2)/b^2/d/(1+sec(d*x+c))^(1/2)/(a+b*
sec(d*x+c))^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7783 vs. $2(305) = 610$.

Time = 44.28 (sec) , antiderivative size = 7783, normalized size of antiderivative = 25.52

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(2/3),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4327, 27, 3042, 4490, 27, 3042, 4495, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx$$

$$\Big\downarrow \text{4327}$$

$$\frac{3\int \frac{1}{3}\sec(c+dx)(5b-3a\sec(c+dx))(a+b\sec(c+dx))^{2/3}dx}{8b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sec(c+dx)(5b-3a\sec(c+dx))(a+b\sec(c+dx))^{2/3}dx}{8b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(5b-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{8b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \text{4490}$$

$$\frac{\frac{3}{5}\int \frac{\sec(c+dx)\left(19ab-\left(6a^2-25b^2\right)\sec(c+dx)\right)}{3\sqrt[3]{a+b\sec(c+dx)}}dx-\frac{9a\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}}{8b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{5}\int \frac{\sec(c+dx)\left(19ab-\left(6a^2-25b^2\right)\sec(c+dx)\right)}{\sqrt[3]{a+b\sec(c+dx)}}dx-\frac{9a\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}}{8b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{5}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(19ab+\left(25b^2-6a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{9a\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}}{8b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \text{4495}$$

$$\frac{\frac{1}{5}\left(\frac{6a\left(a^2-b^2\right)\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}dx}{b}-\frac{\left(6a^2-25b^2\right)\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}dx}{b}\right)-\frac{9a\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}}{8b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{5}\left(\frac{6a(a^2-b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{(6a^2-25b^2)\int \csc(c+dx+\frac{\pi}{2})\left(a+b\csc(c+dx+\frac{\pi}{2})\right)^{2/3}dx}{b}\right)-\frac{9a\tan(c+dx)(a+b\sec(}{5d}}{8b}$$

$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8bd}$$

$\Big\downarrow$ 4321

$$\frac{\frac{1}{5}\left(\frac{(6a^2-25b^2)\tan(c+dx)\int \frac{(a+b\sec(c+dx))^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{6a(a^2-b^2)\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c-}}}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}\right.}{8b}$$

$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8bd}$$

$\Big\downarrow$ 156

$$\frac{\frac{1}{5}\left(\frac{(6a^2-25b^2)\tan(c+dx)(a+b\sec(c+dx))^{2/3}\int \frac{\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{6a(a^2-b^2)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\int}{bd\sqrt{1-\sec(c+dx)}\sqrt{}}\right.}{8b}$$

$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8bd}$$

$\Big\downarrow$ 155

$$\frac{\frac{1}{5}\left(\frac{6\sqrt{2}a(a^2-b^2)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}-\frac{\sqrt{2}(6a^2-25b^2)\tan(c+dx)(a+}{}\right.}{8b}$$

$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8bd}$$

input
```
Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(2/3),x]
```

output
```
(3*(a + b*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(8*b*d) + ((-9*a*(a + b*Sec[c
+ d*x])^(2/3)*Tan[c + d*x])/(5*d) + (-((Sqrt[2]*(6*a^2 - 25*b^2)*AppellF1[
1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]
*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*((a
+ b*Sec[c + d*x])/(a + b))^(2/3))) + (6*Sqrt[2]*a*(a^2 - b^2)*AppellF1[1/2
, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a
 + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]
]*(a + b*Sec[c + d*x])^(1/3)))/5)/(8*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

rule 4327
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 -
b^2, 0] &&  !LtQ[m, -1]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

rule 4495
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)/b   Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Simp[B/b   Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && N
eQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sec(dx+c)^3 (a+b\sec(dx+c))^{\frac{2}{3}} dx$$

input
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(2/3),x)
```

output
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int (a+b\sec(c+dx))^{\frac{2}{3}}\sec^3(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(2/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(2/3)*sec(c + d*x)**3, x)
```

Maxima [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

Giac [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{2/3}}{\cos(c+d\,x)^3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x)^3,x)
```

output
```
int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x)^3, x)
```

Reduce [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int (\sec(dx+c)\,b+a)^{\frac{2}{3}}\sec(dx+c)^3\,dx$$

input
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(2/3),x)
```

output
```
int((sec(c + d*x)*b + a)**(2/3)*sec(c + d*x)**3,x)
```

3.689 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 260

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{2\sqrt{2}a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5bd\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}} - \frac{2\sqrt{2}(a^2-b^2)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\tan(c+dx)}{5bd\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}$$

output
```
3/5*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/d+2/5*2^(1/2)*a*AppellF1(1/2,-2/3,1/
2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*ta
n(d*x+c)/b/d/(1+sec(d*x+c))^(1/2)/((a+b*sec(d*x+c))/(a+b))^(2/3)-2/5*2^(1/
2)*(a^2-b^2)*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d
*x+c))*((a+b*sec(d*x+c))/(a+b))^(1/3)*tan(d*x+c)/b/d/(1+sec(d*x+c))^(1/2)/
(a+b*sec(d*x+c))^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 10909 vs. 2(260) = 520.

Time = 41.58 (sec) , antiderivative size = 10909, normalized size of antiderivative = 41.96

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(2/3),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4322, 3042, 4495, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx$$

$$\Big\downarrow \ 4322$$

$$\frac{2}{5}\int \frac{\sec(c+dx)(b+a\sec(c+dx))}{\sqrt[3]{a+b\sec(c+dx)}}dx+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}$$

$$\Big\downarrow \ 3042$$

$$\frac{2}{5}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}$$

$$\Big\downarrow \ 4495$$

$$\frac{2}{5}\left(\frac{a\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}dx}{b}-\frac{(a^2-b^2)\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}dx}{b}\right)+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}$$

$\Big\downarrow$ 3042

$$\frac{2}{5}\left(\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b}-\frac{(a^2-b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}$$

$\Big\downarrow$ 4321

$$\frac{2}{5}\left(\frac{(a^2-b^2)\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{a\tan(c+dx)\int \frac{(a+}{\sqrt{1-\sec(c}}}{bd\sqrt{1-\sec(c+}}\right.\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}$$

$\Big\downarrow$ 156

$$\frac{2}{5}\left(\frac{(a^2-b^2)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}-\right.\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}$$

$\Big\downarrow$ 155

$$\frac{2}{5}\left(\frac{\sqrt{2}a\tan(c+dx)(a+b\sec(c+dx))^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{\sqrt{2}(a^2-}{}\right.\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}$$

input
```
Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(2/3),x]
```

output
```
(3*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*d) + (2*((Sqrt[2]*a*AppellF
1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b
)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*((
a + b*Sec[c + d*x])/(a + b))^(2/3)) - (Sqrt[2]*(a^2 - b^2)*AppellF1[1/2, 1
/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a +
b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*(
a + b*Sec[c + d*x])^(1/3))))/5
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

rule 4322
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[m/(m + 1)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc
[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0]
```

rule 4495
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)/b   Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Simp[B/b   Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && N
eQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sec(dx+c)^2 (a+b\sec(dx+c))^{\frac{2}{3}} dx$$

input
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(2/3),x)
```

output
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{2/3} dx = \int (b\sec(dx+c)+a)^{\frac{2}{3}} \sec(dx+c)^2 dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int (a+b\sec{(c+dx)})^{\frac{2}{3}}\sec^2{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(2/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(2/3)*sec(c + d*x)**2, x)
```

Maxima [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int (b\sec{(dx+c)}+a)^{\frac{2}{3}}\sec{(dx+c)}^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

Giac [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int (b\sec{(dx+c)}+a)^{\frac{2}{3}}\sec{(dx+c)}^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{2/3}}{\cos\left(c+d\,x\right)^2}\,dx$$

input
```
int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x)^2,x)
```

output
```
int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int \left(\sec\left(dx+c\right)b+a\right)^{\frac{2}{3}}\sec\left(dx+c\right)^2dx$$

input
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(2/3),x)
```

output
```
int((sec(c + d*x)*b + a)**(2/3)*sec(c + d*x)**2,x)
```

3.690 $\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan(c+d}{d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

output
```
2^(1/2)*AppellF1(1/2,-2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c
))*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/d/(1+sec(d*x+c))^(1/2)/((a+b*sec(d*x+
c))/(a+b))^(2/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7142 vs. $2(105) = 210$.

Time = 44.81 (sec) , antiderivative size = 7142, normalized size of antiderivative = 68.02

$$\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(2/3),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx$$

$$\Big\downarrow \text{4321}$$

$$-\frac{\tan(c+dx)\int \frac{(a+b\sec(c+dx))^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{156}$$

$$-\frac{\tan(c+dx)(a+b\sec(c+dx))^{2/3}\int \frac{\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

$$\Big\downarrow \text{155}$$

$$\frac{\sqrt{2}\tan(c+dx)(a+b\sec(c+dx))^{2/3}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

input
```
Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(2/3),x]
```

output
```
(Sqrt[2]*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
 + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(d*Sqrt[1 + Se
c[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3))
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Maple [F]

$$\int \sec (dx+c) (a+b \sec (dx+c))^{\frac{2}{3}} dx$$

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(2/3),x)
```

output
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{2/3} dx = \int (b \sec (dx+c)+a)^{\frac{2}{3}} \sec (dx+c) \, dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

Sympy [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{2/3} dx = \int (a+b \sec (c+dx))^{\frac{2}{3}} \sec (c+dx) dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(2/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(2/3)*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx=\int (b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

Giac [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx=\int (b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{2/3}}{\cos(c+d\,x)}\,dx$$

input
```
int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x),x)
```

output
```
int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x), x)
```

Reduce [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx=\int (\sec(dx+c)\,b+a)^{\frac{2}{3}}\sec(dx+c)\,dx$$

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(2/3),x)
```

output
```
int((sec(c + d*x)*b + a)**(2/3)*sec(c + d*x),x)
```

3.691 $\int (a+b\sec(c+dx))^{2/3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (a+b\sec(c+dx))^{2/3}\,dx = \operatorname{Int}\left((a+b\sec(c+dx))^{2/3},x\right)$$

output
```
Defer(Int)((a+b*sec(d*x+c))^(2/3),x)
```

Mathematica [N/A]

Not integrable

Time = 18.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (a+b\sec(c+dx))^{2/3}\,dx = \int (a+b\sec(c+dx))^{2/3}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(2/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(2/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4273}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(c+dx))^{2/3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx$$
$$\Big\downarrow \text{4273}$$
$$\int (a+b\sec(c+dx))^{2/3}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(2/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4273
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Unintegrable[
(a + b*Csc[c + d*x])^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0
] && !IntegerQ[2*n]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (a+b\sec(dx+c))^{\frac{2}{3}}\,dx$$

input
```
int((a+b*sec(d*x+c))^(2/3),x)
```

output
```
int((a+b*sec(d*x+c))^(2/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^{2/3}\,dx=\text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{2/3}\,dx=\int (a+b\sec(c+dx))^{\frac{2}{3}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(2/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(2/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{2/3}\,dx=\int (b\sec(dx+c)+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(2/3), x)
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{2/3}\,dx=\int (b\sec(dx+c)+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(2/3), x)
```

Mupad [N/A]

Not integrable

Time = 9.85 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (a+b\sec(c+dx))^{2/3}\,dx=\int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{2/3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(2/3),x)
```

output
```
int((a + b/cos(c + d*x))^(2/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{2/3}\,dx=\int (\sec (dx+c)\,b+a)^{\frac{2}{3}}\,dx$$

input
```
int((a+b*sec(d*x+c))^(2/3),x)
```

output
```
int((sec(c + d*x)*b + a)**(2/3),x)
```

3.692 $\int \sec(c+dx)(a+b\sec(c+dx))^{4/3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \sec(c+dx)(a+b\sec(c+dx))^{4/3}\,dx=\frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{4/3}\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{4/3}}$$

output
```
2^(1/2)*AppellF1(1/2,-4/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c
))*(a+b*sec(d*x+c))^(4/3)*tan(d*x+c)/d/(1+sec(d*x+c))^(1/2)/((a+b*sec(d*x+
c))/(a+b))^(4/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7313 vs. 2(105) = 210.

Time = 47.90 (sec) , antiderivative size = 7313, normalized size of antiderivative = 69.65

$$\int \sec(c+dx)(a+b\sec(c+dx))^{4/3}\,dx=\text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(4/3),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\sec(c+dx))^{4/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}dx$$

$$\Big\downarrow \text{4321}$$

$$-\frac{\tan(c+dx)\int \frac{(a+b\sec(c+dx))^{4/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{156}$$

$$-\frac{(a+b)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}\int \frac{\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{4/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{155}$$

$$\frac{\sqrt{2}(a+b)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

input
```
Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(4/3),x]
```

output
```
(Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1
 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*Sqr
t[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3))
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Maple [F]

$$\int \sec(dx+c)\,(a+b\sec(dx+c))^{\frac{4}{3}}\,dx$$

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(4/3),x)
```

output
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{4/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{4}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c)^2 + a*sec(d*x + c))*(b*sec(d*x + c) + a)^(1/3), x
)
```

Sympy [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{4/3}\,dx = \int (a+b\sec(c+dx))^{\frac{4}{3}}\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(4/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(4/3)*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{4/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{4}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(4/3)*sec(d*x + c), x)
```

Giac [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{4/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{4}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(4/3)*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)(a+b\sec(c+dx))^{4/3}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{4/3}}{\cos(c+d\,x)}\,dx$$

input
```
int((a + b/cos(c + d*x))^(4/3)/cos(c + d*x),x)
```

output
```
int((a + b/cos(c + d*x))^(4/3)/cos(c + d*x), x)
```

Reduce [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{4/3}\,dx = \left(\int (\sec (dx+c)\, b+a)^{\frac{1}{3}} \sec (dx+c)^2\, dx\right) b + \left(\int (\sec (dx+c)\, b+a)^{\frac{1}{3}} \sec (dx+c)\, dx\right) a$$

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(4/3),x)
```

output
```
int((sec(c + d*x)*b + a)**(1/3)*sec(c + d*x)**2,x)*b + int((sec(c + d*x)*b
 + a)**(1/3)*sec(c + d*x),x)*a
```

3.693 $\int (a+b\sec(c+dx))^{4/3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (a+b\sec(c+dx))^{4/3}\,dx=\text{Int}\left((a+b\sec(c+dx))^{4/3},x\right)$$

output
```
Defer(Int)((a+b*sec(d*x+c))^(4/3),x)
```

Mathematica [N/A]

Not integrable

Time = 99.33 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (a+b\sec(c+dx))^{4/3}\,dx=\int (a+b\sec(c+dx))^{4/3}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(4/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(4/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4273}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(c+dx))^{4/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}dx$$

$$\Big\downarrow \text{4273}$$

$$\int (a+b\sec(c+dx))^{4/3}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(4/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4273
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Unintegrable[
(a + b*Csc[c + d*x])^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0
] &&  !IntegerQ[2*n]
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (a+b\sec (dx+c))^{\frac{4}{3}} dx$$

input
```
int((a+b*sec(d*x+c))^(4/3),x)
```

output
```
int((a+b*sec(d*x+c))^(4/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^{4/3} dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [N/A]

Not integrable

Time = 13.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{4/3} dx = \int (a+b\sec (c+dx))^{\frac{4}{3}} dx$$

input
```
integrate((a+b*sec(d*x+c))**(4/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(4/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{4/3}\,dx=\int (b\sec(dx+c)+a)^{\frac{4}{3}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(4/3), x)
```

Giac [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{4/3}\,dx=\int (b\sec(dx+c)+a)^{\frac{4}{3}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(4/3), x)
```

Mupad [N/A]

Not integrable

Time = 10.64 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (a+b\sec(c+dx))^{4/3}\,dx=\int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{4/3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(4/3),x)
```

output
```
int((a + b/cos(c + d*x))^(4/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.86

$$\int (a+b\sec(c+dx))^{4/3}\,dx = \left(\int (\sec(dx+c)\,b+a)^{\frac{1}{3}}\,dx\right)a + \left(\int (\sec(dx+c)\,b+a)^{\frac{1}{3}}\sec(dx+c)\,dx\right)b$$

input
```
int((a+b*sec(d*x+c))^(4/3),x)
```

output
```
int((sec(c + d*x)*b + a)**(1/3),x)*a + int((sec(c + d*x)*b + a)**(1/3)*sec
(c + d*x),x)*b
```

3.694 $\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 412

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \frac{3a(18a^2+97b^2)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{1232b^2d} + \frac{3(18a^2+121b^2)(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{1232b^2d} - \frac{9a(a+b\sec(c+dx))^{8/3}\tan(c+dx)}{77b^2d} + \frac{3\sec(c+dx)(a+b\sec(c+dx))^{8/3}\tan(c+dx)}{14bd} + \frac{(36a^4+164a^2b^2+605b^4)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}}{616\sqrt{2}b^3d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}} - \frac{a(18a^4+79a^2b^2-97b^4)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\tan(}{308\sqrt{2}b^3d\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}$$

output
```
3/1232*a*(18*a^2+97*b^2)*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b^2/d+3/1232*(1
8*a^2+121*b^2)*(a+b*sec(d*x+c))^(5/3)*tan(d*x+c)/b^2/d-9/77*a*(a+b*sec(d*x
+c))^(8/3)*tan(d*x+c)/b^2/d+3/14*sec(d*x+c)*(a+b*sec(d*x+c))^(8/3)*tan(d*x
+c)/b/d+1/1232*(36*a^4+164*a^2*b^2+605*b^4)*AppellF1(1/2,-2/3,1/2,3/2,b*(1
-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)*2
^(1/2)/b^3/d/(1+sec(d*x+c))^(1/2)/((a+b*sec(d*x+c))/(a+b))^(2/3)-1/616*a*(
18*a^4+79*a^2*b^2-97*b^4)*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),
1/2-1/2*sec(d*x+c))*((a+b*sec(d*x+c))/(a+b))^(1/3)*tan(d*x+c)*2^(1/2)/b^3/
d/(1+sec(d*x+c))^(1/2)/(a+b*sec(d*x+c))^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 28057 vs. $2(412) = 824$.

Time = 45.68 (sec) , antiderivative size = 28057, normalized size of antiderivative = 68.10

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.67 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.04, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4352, 27, 3042, 4570, 27, 3042, 4490, 27, 3042, 4490, 27, 3042, 4495, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4 \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx$$

$$\Big\downarrow \; 4352$$

$$\frac{3\int \frac{1}{3}\sec(c+dx)(a+b\sec(c+dx))^{5/3}\left(-6a\sec^2(c+dx)+11b\sec(c+dx)+3a\right)dx}{14b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \sec(c+dx)(a+b\sec(c+dx))^{5/3}\left(-6a\sec^2(c+dx)+11b\sec(c+dx)+3a\right)dx}{14b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}\left(-6a\csc\left(c+dx+\frac{\pi}{2}\right)^2+11b\csc\left(c+dx+\frac{\pi}{2}\right)+3a\right)dx}{14b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$$\Big\downarrow \; 4570$$

$$\frac{\frac{3\int -\frac{1}{3}\sec(c+dx)(a+b\sec(c+dx))^{5/3}\left(15ab-\left(18a^2+121b^2\right)\sec(c+dx)\right)dx}{11b}-\frac{18a\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}}{14b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \sec(c+dx)(a+b\sec(c+dx))^{5/3}\left(15ab-\left(18a^2+121b^2\right)\sec(c+dx)\right)dx}{11b}-\frac{18a\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}}{14b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}\left(15ab+\left(-18a^2-121b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{11b}-\frac{18a\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}}{14b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$$\Big\downarrow \; 4490$$

$$-\frac{\frac{-\frac{3}{8}\int \frac{5}{3}\sec(c+dx)(a+b\sec(c+dx))^{2/3}\left(b\left(6a^2-121b^2\right)-a\left(18a^2+97b^2\right)\sec(c+dx)\right)dx-\frac{3\left(18a^2+121b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}}{11b}-\frac{18a\tan(c+}{}}{14b}$$
$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$\downarrow$ 27

$$-\frac{\frac{-\frac{5}{8}\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\left(b\left(6a^2-121b^2\right)-a\left(18a^2+97b^2\right)\sec(c+dx)\right)dx-\frac{3\left(18a^2+121b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}}{11b}-\frac{18a\tan(c+dx}{}}{14b}$$
$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$\downarrow$ 3042

$$-\frac{\frac{-\frac{5}{8}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(b\left(6a^2-121b^2\right)-a\left(18a^2+97b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{3\left(18a^2+121b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}}{11b}-}{14b}$$
$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$\downarrow$ 4490

$$-\frac{\frac{\frac{5}{8}\left(\frac{3}{5}\int -\frac{\sec(c+dx)\left(ab\left(6a^2+799b^2\right)+\left(36a^4+164b^2a^2+605b^4\right)\sec(c+dx)\right)}{3\sqrt[3]{a+b\sec(c+dx)}}dx-\frac{3a\left(18a^2+97b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}\right)-\frac{3\left(18a^2+121b^2\right)\tan(c+}{8d}}{11b}}{14b}$$
$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{5}{8}\left(-\frac{1}{5}\int \frac{\sec(c+dx)\left(ab\left(6a^2+799b^2\right)+\left(36a^4+164b^2a^2+605b^4\right)\sec(c+dx)\right)}{\sqrt[3]{a+b\sec(c+dx)}}dx-\frac{3a\left(18a^2+97b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}\right)-\frac{3\left(18a^2+121b^2\right)\tan(c+}{8d}}{11b}}{14b}$$
$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$\downarrow$ 3042

$$-\frac{\frac{5}{8}\left(-\frac{1}{5}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(ab\left(6a^2+799b^2\right)+\left(36a^4+164b^2a^2+605b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{3a\left(18a^2+97b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}\right)-\frac{3\left(18a^2+121b^2\right.}{11b}}{14b}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$\downarrow$ 4495

$$-\frac{\frac{5}{8}\left(\frac{1}{5}\left(\frac{2a\left(18a^4+79a^2b^2-97b^4\right)\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}dx}{b}-\frac{\left(36a^4+164a^2b^2+605b^4\right)\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}dx}{b}\right)-\frac{3a\left(18a^2+97b^2\right)\tan(c+\right.}{5d}}{11b}}{14b}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$\downarrow$ 3042

$$-\frac{\frac{5}{8}\left(\frac{1}{5}\left(\frac{2a\left(18a^4+79a^2b^2-97b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(36a^4+164a^2b^2+605b^4\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b}\right)-\frac{3a\left(1\right.}{}}{11b}}{14b}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$\downarrow$ 4321

$$-\frac{\frac{5}{8}\left(\frac{1}{5}\left(\frac{\left(36a^4+164a^2b^2+605b^4\right)\tan(c+dx)\int \frac{(a+b\sec(c+dx))^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{2a\left(18a^4+79a^2b^2-97b^4\right)\tan(c+dx)\int \frac{}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+}}}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx}}\right.\right.}{11b}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$\downarrow$ 156

$$-\frac{\frac{5}{8}\left(\frac{1}{5}\left(\frac{\left(36a^4+164a^2b^2+605b^4\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}\int\frac{\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{2a\left(18a^4+79a^2b^2-97b^4\right)\tan(c+dx)\sqrt[3]{\frac{a+b}{}}}{bd\sqrt{}}\right.\right.}{}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$$\Big\downarrow \textbf{155}$$

$$-\frac{\frac{5}{8}\left(\frac{1}{5}\left(\frac{2\sqrt{2}a\left(18a^4+79a^2b^2-97b^4\right)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}-\frac{\sqrt{2}\left(36a^4+164a^2b^2+6\right.}{}\right.\right.}{}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

input
```
Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
(3*Sec[c + d*x]*(a + b*Sec[c + d*x])^(8/3)*Tan[c + d*x])/(14*b*d) + ((-18*
a*(a + b*Sec[c + d*x])^(8/3)*Tan[c + d*x])/(11*b*d) - ((-3*(18*a^2 + 121*b
^2)*(a + b*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(8*d) + (5*((-3*a*(18*a^2 + 9
7*b^2)*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*d) + (-((Sqrt[2]*(36*a^
4 + 164*a^2*b^2 + 605*b^4)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x]
)/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*
x])/(b*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3))) + (
2*Sqrt[2]*a*(18*a^4 + 79*a^2*b^2 - 97*b^4)*AppellF1[1/2, 1/2, 1/3, 3/2, (1
 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/
(a + b))^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*
x])^(1/3)))/5))/8)/(11*b))/(14*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

rule 4352
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + n - 1))), x] + Simp[d^3/(b*(m + n -
1))   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 3)*Simp[a*(n - 3) +
b*(m + n - 2)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 3] && (IntegerQ[n] ||
 IntegersQ[2*m, 2*n]) &&  !IGtQ[m, 2]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

rule 4495
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)/b   Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Simp[B/b   Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && N
eQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \sec (dx+c)^{4} (a+b\sec (dx+c))^{\frac{5}{3}} dx$$

input
```
int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/3),x)
```

Fricas [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx=\int (b\sec{(dx+c)}+a)^{\frac{5}{3}}\sec{(dx+c)}^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c)^5 + a*sec(d*x + c)^4)*(b*sec(d*x + c) + a)^(2/3),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**4*(a+b*sec(d*x+c))**(5/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx=\int (b\sec{(dx+c)}+a)^{\frac{5}{3}}\sec{(dx+c)}^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^4, x)
```

Giac [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx=\int (b\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/3}}{\cos(c+d\,x)^4}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x)^4,x)
```

output
```
int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x)^4, x)
```

Reduce [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx=\left(\int (\sec(dx+c)\,b+a)^{\frac{2}{3}}\sec(dx+c)^5\,dx\right)b+\left(\int (\sec(dx+c)\,b+a)^{\frac{2}{3}}\sec(dx+c)^4\,dx\right)a$$

input
```
int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/3),x)
```

output
```
int((sec(c + d*x)*b + a)**(2/3)*sec(c + d*x)**5,x)*b + int((sec(c + d*x)*b
 + a)**(2/3)*sec(c + d*x)**4,x)*a
```

3.695 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 356

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = -\frac{3(15a^2-64b^2)\,(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{440bd} - \frac{9a(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{88bd} + \frac{3(a+b\sec(c+dx))^{8/3}\tan(c+dx)}{11bd} - \frac{a(30a^2-373b^2)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan(c+}{220\sqrt{2}b^2d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}} + \frac{(15a^4-79a^2b^2+64b^4)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\tan(c}{110\sqrt{2}b^2d\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}$$

output

```
-3/440*(15*a^2-64*b^2)*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b/d-9/88*a*(a+b*s
ec(d*x+c))^(5/3)*tan(d*x+c)/b/d+3/11*(a+b*sec(d*x+c))^(8/3)*tan(d*x+c)/b/d
-1/440*a*(30*a^2-373*b^2)*AppellF1(1/2,-2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b)
,1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)*2^(1/2)/b^2/d/(1+se
c(d*x+c))^(1/2)/((a+b*sec(d*x+c))/(a+b))^(2/3)+1/220*(15*a^4-79*a^2*b^2+64
*b^4)*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*
((a+b*sec(d*x+c))/(a+b))^(1/3)*tan(d*x+c)*2^(1/2)/b^2/d/(1+sec(d*x+c))^(1/
2)/(a+b*sec(d*x+c))^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 21890 vs. 2(356) = 712.

Time = 45.26 (sec) , antiderivative size = 21890, normalized size of antiderivative = 61.49

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.02, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4327, 27, 3042, 4490, 27, 3042, 4490, 27, 3042, 4495, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx$$

$$\Big\downarrow \text{4327}$$

$$\frac{3\int \frac{1}{3}\sec(c+dx)(8b-3a\sec(c+dx))(a+b\sec(c+dx))^{5/3}dx}{11b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sec(c+dx)(8b-3a\sec(c+dx))(a+b\sec(c+dx))^{5/3}dx}{11b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}$$

$\downarrow$ 3042

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(8b-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx}{11b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}$$

$\downarrow$ 4490

$$\frac{\frac{3}{8}\int \frac{1}{3}\sec(c+dx)(a+b\sec(c+dx))^{2/3}\left(49ab-\left(15a^2-64b^2\right)\sec(c+dx)\right)dx-\frac{9a\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}}{11b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}$$

$\downarrow$ 27

$$\frac{\frac{1}{8}\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\left(49ab-\left(15a^2-64b^2\right)\sec(c+dx)\right)dx-\frac{9a\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}}{11b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}$$

$\downarrow$ 3042

$$\frac{\frac{1}{8}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(49ab+\left(64b^2-15a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{9a\tan(c+dx)(a+b\sec}{8d}}{11b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}$$

$\downarrow$ 4490

$$\frac{\frac{1}{8}\left(\frac{3}{5}\int \frac{\sec(c+dx)\left(b\left(215a^2+128b^2\right)-a\left(30a^2-373b^2\right)\sec(c+dx)\right)}{3\sqrt[3]{a+b\sec(c+dx)}}dx-\frac{3\left(15a^2-64b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}\right)-\frac{9a\tan(c+dx)(a+b}{8d}}{11b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}$$

$\downarrow$ 27

$$\frac{\frac{1}{8}\left(\frac{1}{5}\int \frac{\sec(c+dx)\left(b\left(215a^2+128b^2\right)-a\left(30a^2-373b^2\right)\sec(c+dx)\right)}{\sqrt[3]{a+b\sec(c+dx)}}dx-\frac{3\left(15a^2-64b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}\right)-\frac{9a\tan(c+dx)(a+b}{8d}}{11b}$$

$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}$$

$\downarrow$ 3042

$$\frac{\frac{1}{8}\left(\frac{1}{5}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(b\left(215a^2+128b^2\right)-a\left(30a^2-373b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{3\left(15a^2-64b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}\right)-\frac{9a\tan(c+}{}}{11b}$$

$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}$$

$\downarrow$ 4495

$$\frac{\frac{1}{8}\left(\frac{1}{5}\left(\frac{2\left(15a^4-79a^2b^2+64b^4\right)\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}dx}{b}-\frac{a\left(30a^2-373b^2\right)\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}dx}{b}\right)-\frac{3\left(15a^2-64b^2\right)\tan(}{}}{11b}$$

$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}$$

$\downarrow$ 3042

$$\frac{\frac{1}{8}\left(\frac{1}{5}\left(\frac{2\left(15a^4-79a^2b^2+64b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\left(30a^2-373b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b}\right)-\frac{3(}{}}{11b}$$

$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}$$

$\downarrow$ 4321

$$\frac{\frac{1}{8}\left(\frac{1}{5}\left(\frac{a\left(30a^2-373b^2\right)\tan(c+dx)\int \frac{(a+b\sec(c+dx))^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{2\left(15a^4-79a^2b^2+64b^4\right)\tan(c+dx)\int \frac{}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+}}}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+d}}\right.\right.}{11b}$$

$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}$$

$\downarrow$ 156

$$\frac{1}{8}\left(\frac{1}{5}\left(\frac{a\left(30a^2-373b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}\int\frac{\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{2\left(15a^4-79a^2b^2+64b^4\right)\tan(c+dx)\sqrt[3]{\frac{a+}{}}}{bd\sqrt{}}\right.\right.$$

$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}$$

$\downarrow$ 155

$$\frac{1}{8}\left(\frac{1}{5}\left(\frac{2\sqrt{2}\left(15a^4-79a^2b^2+64b^4\right)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}-\frac{\sqrt{2}a\left(30a^2-3\right.}{}\right.\right.$$

$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}$$

input
```
Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
(3*(a + b*Sec[c + d*x])^(8/3)*Tan[c + d*x])/(11*b*d) + ((-9*a*(a + b*Sec[c
 + d*x])^(5/3)*Tan[c + d*x])/(8*d) + ((-3*(15*a^2 - 64*b^2)*(a + b*Sec[c +
 d*x])^(2/3)*Tan[c + d*x])/(5*d) + (-((Sqrt[2]*a*(30*a^2 - 373*b^2)*Appell
F1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a +
b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*(
(a + b*Sec[c + d*x])/(a + b))^(2/3))) + (2*Sqrt[2]*(15*a^4 - 79*a^2*b^2 +
64*b^4)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
 d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(b*d*S
qrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3)))/5)/8)/(11*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

rule 4327
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 -
b^2, 0] &&  !LtQ[m, -1]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

rule 4495
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)/b   Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Simp[B/b   Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && N
eQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sec(dx+c)^3 (a+b\sec(dx+c))^{\frac{5}{3}}\,dx$$

input
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/3),x)
```

Fricas [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c)^4 + a*sec(d*x + c)^3)*(b*sec(d*x + c) + a)^(2/3),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(5/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^3, x)
```

Giac [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/3}}{\cos\left(c+d\,x\right)^3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x)^3,x)
```

output
```
int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x)^3, x)
```

Reduce [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx=\left(\int (\sec(dx+c)\,b+a)^{\frac{2}{3}}\sec(dx+c)^4\,dx\right)b+\left(\int (\sec(dx+c)\,b+a)^{\frac{2}{3}}\sec(dx+c)^3\,dx\right)a$$

input
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/3),x)
```

output
```
int((sec(c + d*x)*b + a)**(2/3)*sec(c + d*x)**4,x)*b + int((sec(c + d*x)*b
 + a)**(2/3)*sec(c + d*x)**3,x)*a
```

3.696 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 299

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \frac{3a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8d} + \frac{3(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{8d} + \frac{(2a^2+5b^2)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{4\sqrt{2}bd\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}} - \frac{a(a^2-b^2)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\tan(c+dx)}{2\sqrt{2}bd\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}$$

output
```
3/8*a*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/d+3/8*(a+b*sec(d*x+c))^(5/3)*tan(d
*x+c)/d+1/8*(2*a^2+5*b^2)*AppellF1(1/2,-2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b)
,1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)*2^(1/2)/b/d/(1+sec(
d*x+c))^(1/2)/((a+b*sec(d*x+c))/(a+b))^(2/3)-1/4*a*(a^2-b^2)*AppellF1(1/2,
1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*sec(d*x+c))/(
a+b))^(1/3)*tan(d*x+c)*2^(1/2)/b/d/(1+sec(d*x+c))^(1/2)/(a+b*sec(d*x+c))^(
1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7809 vs. $2(299) = 598$.

Time = 44.43 (sec) , antiderivative size = 7809, normalized size of antiderivative = 26.12

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4322, 3042, 4490, 27, 3042, 4495, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx$$

$$\Big\downarrow \text{4322}$$

$$\frac{5}{8}\int \sec(c+dx)(b+a\sec(c+dx))(a+b\sec(c+dx))^{2/3}dx+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{8}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}$$

$$\Big\downarrow \text{4490}$$

$$\frac{5}{8}\left(\frac{3}{5}\int \frac{\sec(c+dx)\left(7ab+\left(2a^2+5b^2\right)\sec(c+dx)\right)}{3\sqrt[3]{a+b\sec(c+dx)}}dx+\frac{3a\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}\right)+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}$$

$\big\downarrow$ 27

$$\frac{5}{8}\left(\frac{1}{5}\int \frac{\sec(c+dx)\left(7ab+\left(2a^2+5b^2\right)\sec(c+dx)\right)}{\sqrt[3]{a+b\sec(c+dx)}}dx+\frac{3a\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}\right)+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}$$

$\big\downarrow$ 3042

$$\frac{5}{8}\left(\frac{1}{5}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(7ab+\left(2a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}\right)+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}$$

$\big\downarrow$ 4495

$$\frac{5}{8}\left(\frac{1}{5}\left(\frac{\left(2a^2+5b^2\right)\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}dx}{b}-\frac{2a\left(a^2-b^2\right)\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}dx}{b}\right)+\frac{3a\tan(c-\right.$$

$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}$$

$\big\downarrow$ 3042

$$\frac{5}{8}\left(\frac{1}{5}\left(\frac{\left(2a^2+5b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b}-\frac{2a\left(a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}}{b}\right.\right.$$

$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}$$

$\big\downarrow$ 4321

$$\frac{5}{8}\left(\frac{1}{5}\left(\frac{2a\left(a^2-b^2\right)\tan(c+dx)\int\frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{\left(2a^2+5b^2\right)\tan(c}{bd\sqrt{}}\right.\right.$$

$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}$$

$$\Big\downarrow 156$$

$$\frac{5}{8}\left(\frac{1}{5}\left(\frac{2a\left(a^2-b^2\right)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\int\frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}}}d\sec(c+}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}\right.\right.$$

$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}$$

$$\Big\downarrow 155$$

$$\frac{5}{8}\left(\frac{1}{5}\left(\frac{\sqrt{2}\left(2a^2+5b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right.}{bd\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}\right.\right.$$

$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}$$

input
```
Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
(3*(a + b*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(8*d) + (5*((3*a*(a + b*Sec[c
 + d*x])^(2/3)*Tan[c + d*x])/(5*d) + ((Sqrt[2]*(2*a^2 + 5*b^2)*AppellF1[1/2
, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a
 + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*((a + b
*Sec[c + d*x])/(a + b))^(2/3)) - (2*Sqrt[2]*a*(a^2 - b^2)*AppellF1[1/2, 1/
2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b
*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*(a
 + b*Sec[c + d*x])^(1/3)))/5))/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

rule 4322
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[m/(m + 1)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc
[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

rule 4495
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)/b   Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Simp[B/b   Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && N
eQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sec\left(dx+c\right)^{2}\left(a+b\sec\left(dx+c\right)\right)^{\frac{5}{3}}dx$$

input
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/3),x)
```

Fricas [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx=\int \left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{3}}\sec\left(dx+c\right)^{2}\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c)^3 + a*sec(d*x + c)^2)*(b*sec(d*x + c) + a)^(2/3),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(5/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^2, x)
```

Giac [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/3}}{\cos\left(c+d\,x\right)^2}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x)^2,x)
```

output
```
int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx=\left(\int \left(\sec\left(dx+c\right)b+a\right)^{\frac{2}{3}}\sec\left(dx+c\right)^3dx\right)b+\left(\int \left(\sec\left(dx+c\right)b+a\right)^{\frac{2}{3}}\sec\left(dx+c\right)^2dx\right)a$$

input
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/3),x)
```

output
```
int((sec(c + d*x)*b + a)**(2/3)*sec(c + d*x)**3,x)*b + int((sec(c + d*x)*b
 + a)**(2/3)*sec(c + d*x)**2,x)*a
```

3.697 $\int \sec(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{5}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{5/3}}$$

output
```
2^(1/2)*AppellF1(1/2,-5/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c
))*(a+b*sec(d*x+c))^(5/3)*tan(d*x+c)/d/(1+sec(d*x+c))^(1/2)/((a+b*sec(d*x+
c))/(a+b))^(5/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7321 vs. 2(105) = 210.

Time = 44.49 (sec) , antiderivative size = 7321, normalized size of antiderivative = 69.72

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx$$

$$\Big\downarrow \text{ 4321}$$

$$-\frac{\tan(c+dx)\int \frac{(a+b\sec(c+dx))^{5/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{ 156}$$

$$-\frac{(a+b)\tan(c+dx)(a+b\sec(c+dx))^{2/3}\int \frac{\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{5/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

$$\Big\downarrow \text{ 155}$$

$$\frac{\sqrt{2}(a+b)\tan(c+dx)(a+b\sec(c+dx))^{2/3}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{5}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

input
```
Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
(Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1
 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(d*Sqr
t[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3))
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Maple [F]

$$\int \sec (dx+c) (a+b\sec (dx+c))^{\frac{5}{3}} dx$$

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(5/3),x)
```

Fricas [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/3} dx = \int (b\sec (dx+c)+a)^{\frac{5}{3}} \sec (dx+c) \, dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c)^2 + a*sec(d*x + c))*(b*sec(d*x + c) + a)^(2/3), x
)
```

Sympy [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/3} dx = \int (a+b\sec (c+dx))^{\frac{5}{3}} \sec (c+dx) dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(5/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(5/3)*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c), x)
```

Giac [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/3}}{\cos(c+d\,x)}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x),x)
```

output
```
int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x), x)
```

Reduce [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \left(\int (\sec(dx+c)\,b+a)^{\frac{2}{3}}\sec(dx+c)^2\,dx\right)b + \left(\int (\sec(dx+c)\,b+a)^{\frac{2}{3}}\sec(dx+c)\,dx\right)a$$

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(5/3),x)
```

output
```
int((sec(c + d*x)*b + a)**(2/3)*sec(c + d*x)**2,x)*b + int((sec(c + d*x)*b
 + a)**(2/3)*sec(c + d*x),x)*a
```

3.698 $\int (a+b\sec(c+dx))^{5/3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (a+b\sec(c+dx))^{5/3}\,dx=\text{Int}\left((a+b\sec(c+dx))^{5/3},x\right)$$

output
```
Defer(Int)((a+b*sec(d*x+c))^(5/3),x)
```

Mathematica [N/A]

Not integrable

Time = 97.43 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (a+b\sec(c+dx))^{5/3}\,dx=\int (a+b\sec(c+dx))^{5/3}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(5/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4273}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec (c+dx))^{5/3} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b \csc \left(c+dx+\frac{\pi }{2}\right)\right)^{5/3}dx$$

$$\Big\downarrow \text{4273}$$

$$\int (a+b\sec (c+dx))^{5/3}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4273
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Unintegrable[
(a + b*Csc[c + d*x])^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0
] &&  !IntegerQ[2*n]
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (a+b\sec (dx+c))^{\frac{5}{3}}\, dx$$

input
```
int((a+b*sec(d*x+c))^(5/3),x)
```

output
```
int((a+b*sec(d*x+c))^(5/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^{5/3}\, dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [N/A]

Not integrable

Time = 43.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{5/3}\, dx = \int (a+b\sec (c+dx))^{\frac{5}{3}}\, dx$$

input
```
integrate((a+b*sec(d*x+c))**(5/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(5/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{5/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{3}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/3), x)
```

Giac [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{5/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{3}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/3), x)
```

Mupad [N/A]

Not integrable

Time = 11.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (a+b\sec(c+dx))^{5/3}\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/3),x)
```

output
```
int((a + b/cos(c + d*x))^(5/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.86

$$\int (a+b\sec(c+dx))^{5/3}\,dx = \left(\int (\sec(dx+c)\,b+a)^{\frac{2}{3}}\,dx\right)a + \left(\int (\sec(dx+c)\,b+a)^{\frac{2}{3}}\sec(dx+c)\,dx\right)b$$

input
```
int((a+b*sec(d*x+c))^(5/3),x)
```

output
```
int((sec(c + d*x)*b + a)**(2/3),x)*a + int((sec(c + d*x)*b + a)**(2/3)*sec
(c + d*x),x)*b
```

3.699 $\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 313

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{20b^2d} + \frac{3\sec(c+dx)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8bd} + \frac{(18a^2+25b^2)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{20\sqrt{2}b^3d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}} - \frac{a(9a^2+11b^2)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\tan(c+dx)}{10\sqrt{2}b^3d\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}$$

output
```
-9/20*a*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b^2/d+3/8*sec(d*x+c)*(a+b*sec(d*
x+c))^(2/3)*tan(d*x+c)/b/d+1/40*(18*a^2+25*b^2)*AppellF1(1/2,-2/3,1/2,3/2,
b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*tan(d*x+
c)*2^(1/2)/b^3/d/(1+sec(d*x+c))^(1/2)/((a+b*sec(d*x+c))/(a+b))^(2/3)-1/20*
a*(9*a^2+11*b^2)*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*s
ec(d*x+c))*((a+b*sec(d*x+c))/(a+b))^(1/3)*tan(d*x+c)*2^(1/2)/b^3/d/(1+sec(
d*x+c))^(1/2)/(a+b*sec(d*x+c))^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7796 vs. $2(313) = 626$.

Time = 44.46 (sec) , antiderivative size = 7796, normalized size of antiderivative = 24.91

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(1/3),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4352, 27, 3042, 4570, 27, 3042, 4495, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \ 4352$$

$$\frac{3\int \frac{\sec(c+dx)\left(-6a\sec^2(c+dx)+5b\sec(c+dx)+3a\right)}{3\sqrt[3]{a+b\sec(c+dx)}}dx}{8b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{2/3}}{8bd}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\sec(c+dx)\left(-6a\sec^2(c+dx)+5b\sec(c+dx)+3a\right)}{\sqrt[3]{a+b\sec(c+dx)}}dx}{8b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{2/3}}{8bd}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(-6a\csc\left(c+dx+\frac{\pi}{2}\right)^2+5b\csc\left(c+dx+\frac{\pi}{2}\right)+3a\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{8b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{2/3}}{8bd}}{}$$

$$\Big\downarrow \ 4570$$

$$\frac{\frac{3\int \frac{\sec(c+dx)\left(3ab+\left(18a^2+25b^2\right)\sec(c+dx)\right)}{3\sqrt[3]{a+b\sec(c+dx)}}dx}{5b}-\frac{18a\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5bd}}{8b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{2/3}}{8bd}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{\sec(c+dx)\left(3ab+\left(18a^2+25b^2\right)\sec(c+dx)\right)}{\sqrt[3]{a+b\sec(c+dx)}}dx}{5b}-\frac{18a\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5bd}}{8b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{2/3}}{8bd}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3ab+\left(18a^2+25b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b}-\frac{18a\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5bd}}{8b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{2/3}}{8bd}$$

$$\Big\downarrow \ 4495$$

$$\frac{\frac{\frac{\left(18a^2+25b^2\right)\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}dx}{b}-\frac{2a\left(9a^2+11b^2\right)\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}dx}{b}}{5b}-\frac{18a\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5bd}}{8b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{2/3}}{8bd}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{\left(18a^2+25b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b}-\frac{2a\left(9a^2+11b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{5b}-\frac{18a\tan(c+dx)(a+b\sec(c+dx))^{2/}}{5bd}}{8b}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{2/3}}{8bd}$$

$$\Big\downarrow \text{4321}$$

$$\frac{\frac{\frac{2a\left(9a^2+11b^2\right)\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{\left(18a^2+25b^2\right)\tan(c+dx)\int \frac{(a+b\sec(c+dx))^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\,s}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}}{5b}}{8b}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{2/3}}{8bd}$$

$$\Big\downarrow \text{156}$$

$$\frac{\frac{\frac{2a\left(9a^2+11b^2\right)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}-\frac{\left(18a^2+25b^2\right)\tan(c+dx}{bd\sqrt{1-}}}{5b}}{8b}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{2/3}}{8bd}$$

$$\Big\downarrow \text{155}$$

$$\frac{\frac{\frac{\sqrt{2}\left(18a^2+25b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{2\sqrt{2}a\left(9a^2+11b^2\right)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+}{a+b}}}{bd\sqrt{\sec(c+dx)+1}}}{5b}}{8b}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{2/3}}{8bd}$$

input
```
Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(1/3),x]
```

output
```
(3*Sec[c + d*x]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(8*b*d) + ((-18*a
*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*b*d) + ((Sqrt[2]*(18*a^2 + 25
*b^2)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec
[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) - (2*Sqrt[2]*a*(9*a^2 + 1
1*b^2)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(b*d*Sq
rt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3)))/(5*b))/(8*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

rule 4352
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + n - 1))), x] + Simp[d^3/(b*(m + n -
1))   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 3)*Simp[a*(n - 3) +
b*(m + n - 2)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 3] && (IntegerQ[n] ||
 IntegersQ[2*m, 2*n]) &&  !IGtQ[m, 2]
```

rule 4495
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)/b   Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Simp[B/b   Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && N
eQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{\sec (dx+c)^4}{(a+b\sec (dx+c))^{\frac{1}{3}}}dx$$

input
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx=\int \frac{\sec\left(dx+c\right)^4}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(1/3), x)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx=\int \frac{\sec^4\left(c+dx\right)}{\sqrt[3]{a+b\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**(1/3),x)
```

output
```
Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx=\int \frac{\sec\left(dx+c\right)^4}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(1/3), x)
```

Giac [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^4}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)^4\left(a+\frac{b}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(1/3)),x)
```

output
```
int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(1/3)), x)
```

Reduce [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^4}{(\sec(dx+c)\,b+a)^{\frac{1}{3}}}\,dx$$

input
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(c + d*x)**4/(sec(c + d*x)*b + a)**(1/3),x)
```

3.700 $\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 265

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5bd} - \frac{3\sqrt{2}a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5b^2d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}} + \frac{\sqrt{2}(3a^2+2b^2)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\tan(c+dx)}{5b^2d\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}$$

output
```
3/5*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b/d-3/5*2^(1/2)*a*AppellF1(1/2,-2/3,
1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*
tan(d*x+c)/b^2/d/(1+sec(d*x+c))^(1/2)/((a+b*sec(d*x+c))/(a+b))^(2/3)+1/5*2
^(1/2)*(3*a^2+2*b^2)*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1
/2*sec(d*x+c))*((a+b*sec(d*x+c))/(a+b))^(1/3)*tan(d*x+c)/b^2/d/(1+sec(d*x+
c))^(1/2)/(a+b*sec(d*x+c))^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7195 vs. $2(265) = 530$.

Time = 43.06 (sec) , antiderivative size = 7195, normalized size of antiderivative = 27.15

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(1/3),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4327, 27, 3042, 4495, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4327}$$

$$\frac{3\int \frac{\sec(c+dx)(2b-3a\sec(c+dx))}{3\sqrt[3]{a+b\sec(c+dx)}}dx}{5b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sec(c+dx)(2b-3a\sec(c+dx))}{\sqrt[3]{a+b\sec(c+dx)}}dx}{5b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2b-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5bd}$$

$$\Big\downarrow \text{4495}$$

$$\frac{\frac{(3a^2+2b^2)\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}dx}{b}-\frac{3a\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}dx}{b}}{5b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(3a^2+2b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{3a\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b}}{5b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5bd}$$

$$\Big\downarrow \text{4321}$$

$$\frac{\frac{3a\tan(c+dx)\int \frac{(a+b\sec(c+dx))^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{(3a^2+2b^2)\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}}{5b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5bd}$$

$$\Big\downarrow \text{156}$$

$$\frac{\frac{3a\tan(c+dx)(a+b\sec(c+dx))^{2/3}\int \frac{\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{(3a^2+2b^2)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\int \frac{}{\sqrt{1-\sec(c+dx)}\sqrt{}}}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}}{5b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5bd}$$

$$\Big\downarrow \text{155}$$

$$\frac{\frac{\sqrt{2}(3a^2+2b^2)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}-\frac{3\sqrt{2}a\tan(c+dx)(a+b\sec(c+dx))^{2}}{bd\sqrt{\sec}}}{5b}$$

$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5bd}$$

input
```
Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(1/3),x]
```

output
```
(3*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*b*d) + ((-3*Sqrt[2]*a*Appel
lF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a +
 b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*
((a + b*Sec[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(3*a^2 + 2*b^2)*AppellF1[
1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*
((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d
*x]]*(a + b*Sec[c + d*x])^(1/3)))/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
 - e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

rule 4327
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 -
b^2, 0] &&  !LtQ[m, -1]
```

rule 4495
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)/b   Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Simp[B/b   Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && N
eQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{\sec (dx+c)^3}{(a+b\sec (dx+c))^{\frac{1}{3}}}dx$$

input
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^3}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(1/3), x)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^3\left(c+dx\right)}{\sqrt[3]{a+b\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**(1/3),x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^3}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(1/3), x)
```

Giac [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^3}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^3\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{1/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(1/3)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(1/3)), x)
```

Reduce [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^3}{\left(\sec\left(dx+c\right)b+a\right)^{\frac{1}{3}}}\,dx$$

input
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(c + d*x)**3/(sec(c + d*x)*b + a)**(1/3),x)
```

3.701 $\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 219

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{bd\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}} - \frac{\sqrt{2}a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\tan(c+dx)}{bd\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}$$

output
```
2^(1/2)*AppellF1(1/2,-2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c
))*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b/d/(1+sec(d*x+c))^(1/2)/((a+b*sec(d*
x+c))/(a+b))^(2/3)-2^(1/2)*a*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+
b),1/2-1/2*sec(d*x+c))*((a+b*sec(d*x+c))/(a+b))^(1/3)*tan(d*x+c)/b/d/(1+se
c(d*x+c))^(1/2)/(a+b*sec(d*x+c))^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1748 vs. $2(219) = 438$.

Time = 23.46 (sec) , antiderivative size = 1748, normalized size of antiderivative = 7.98

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \text{Too large to display}$$

input

```
Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(1/3),x]
```

output

```
(3*(b + a*Cos[c + d*x])*Tan[c + d*x])/(2*b*d*(a + b*Sec[c + d*x])^(1/3)) -
 (3*(b + 3*a*Cos[c + d*x])*(a + b*Sec[c + d*x])^(2/3)*(5*(a^2 - b^2) + 3*b
*AppellF1[5/3, 3/2, 3/2, 8/3, (a + b*Sec[c + d*x])/(a - b), (a + b*Sec[c +
 d*x])/(a + b)]*Sec[c + d*x]*Sqrt[(b*(1 + Sec[c + d*x]))/(-a + b)]*Sqrt[(b
 - b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x])))/(10*b*(-a^2 + b^2)*d*(b
 + a*Cos[c + d*x])^(1/3)*Sqrt[1 - Cos[c + d*x]^2]*Sec[c + d*x]^(7/3)*((3*b
*(5*(a^2 - b^2) + 3*b*AppellF1[5/3, 3/2, 3/2, 8/3, (a + b*Sec[c + d*x])/(a
 - b), (a + b*Sec[c + d*x])/(a + b)]*Sec[c + d*x]*Sqrt[(b*(1 + Sec[c + d*x
]))/(-a + b)]*Sqrt[(b - b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x]))*Sin
[c + d*x])/(5*(-a^2 + b^2)*(b + a*Cos[c + d*x])^(1/3)*Sqrt[1 - Cos[c + d*x
]^2]*Sec[c + d*x]^(1/3)) - (3*(a + b*Sec[c + d*x])*(5*(a^2 - b^2) + 3*b*Ap
pellF1[5/3, 3/2, 3/2, 8/3, (a + b*Sec[c + d*x])/(a - b), (a + b*Sec[c + d*
x])/(a + b)]*Sec[c + d*x]*Sqrt[(b*(1 + Sec[c + d*x]))/(-a + b)]*Sqrt[(b -
b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x]))*Sin[c + d*x])/(5*(-a^2 + b^
2)*(b + a*Cos[c + d*x])^(1/3)*(1 - Cos[c + d*x]^2)^(3/2)*Sec[c + d*x]^(10/
3)) + (a*(a + b*Sec[c + d*x])*(5*(a^2 - b^2) + 3*b*AppellF1[5/3, 3/2, 3/2,
 8/3, (a + b*Sec[c + d*x])/(a - b), (a + b*Sec[c + d*x])/(a + b)]*Sec[c +
d*x]*Sqrt[(b*(1 + Sec[c + d*x]))/(-a + b)]*Sqrt[(b - b*Sec[c + d*x])/(a +
b)]*(a + b*Sec[c + d*x]))*Sin[c + d*x])/(5*(-a^2 + b^2)*(b + a*Cos[c + d*x
])^(4/3)*Sqrt[1 - Cos[c + d*x]^2]*Sec[c + d*x]^(7/3)) - (7*(a + b*Sec[c...
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 4325, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4325}$$

$$\frac{\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}dx}{b}-\frac{a\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{4321}$$

$$\frac{a\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{\tan(c+dx)\int \frac{(a+b\sec(c+dx))^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{156}$$

$$\frac{a\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\int\frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}-\frac{\tan(c+dx)(a+b\sec(c+dx))^{2/3}\int\frac{\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

$$\Big\downarrow \text{155}$$

$$\frac{\sqrt{2}\tan(c+dx)(a+b\sec(c+dx))^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{\sqrt{2}a\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(1/3),x]
```

output
```
(Sqrt[2]*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
 + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 +
Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) - (Sqrt[2]*a*AppellF1[
1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*
((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d
*x]]*(a + b*Sec[c + d*x])^(1/3))
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

rule 4325
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[-a/b   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] +
Simp[1/b   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{
a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{\sec (dx+c)^2}{(a+b\sec (dx+c))^{\frac{1}{3}}}dx$$

input
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{\sec (dx+c)^2}{(b\sec (dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(1/3), x)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^2 (c+dx)}{\sqrt[3]{a+b\sec (c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**(1/3),x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(1/3), x)
```

Giac [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)^2\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(1/3)),x)
```

output
```
int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(1/3)), x)
```

Reduce [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^2}{(\sec(dx+c)\,b+a)^{\frac{1}{3}}}dx$$

input
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(c + d*x)**2/(sec(c + d*x)*b + a)**(1/3),x)
```

3.702 $\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}$$

output
```
2^(1/2)*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c)
)*((a+b*sec(d*x+c))/(a+b))^(1/3)*tan(d*x+c)/d/(1+sec(d*x+c))^(1/2)/(a+b*se
c(d*x+c))^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 310 vs. 2(105) = 210.

Time = 14.22 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.95

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

$$= \frac{15(a-b)^2(a+b)\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},\frac{a+b\sec(c+dx)}{a-b},\frac{a+b\sec}{a+}\right.}{b^2(-a+b)d\left(3(a-b)\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{2},\frac{3}{2},\frac{8}{3},\frac{a+b\sec(c+dx)}{a-b},\frac{a+b\sec(c+dx)}{a+b}\right)(b+a\cos(c+dx))+(a+b)\left(10(a\right.\right.}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(1/3),x]
```

output
```
(15*(a - b)^2*(a + b)*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Sec[c + d*x])/(a
 - b), (a + b*Sec[c + d*x])/(a + b)]*Cos[c + d*x]*Cot[c + d*x]^3*(1 + Sec[
c + d*x])*(b - b*Sec[c + d*x])*(a + b*Sec[c + d*x])^(2/3))/(b^2*(-a + b)*d
*(3*(a - b)*AppellF1[5/3, 1/2, 3/2, 8/3, (a + b*Sec[c + d*x])/(a - b), (a
+ b*Sec[c + d*x])/(a + b)]*(b + a*Cos[c + d*x]) + (a + b)*(10*(a - b)*Appe
llF1[2/3, 1/2, 1/2, 5/3, (a + b*Sec[c + d*x])/(a - b), (a + b*Sec[c + d*x]
)/(a + b)]*Cos[c + d*x] + 3*AppellF1[5/3, 3/2, 1/2, 8/3, (a + b*Sec[c + d*
x])/(a - b), (a + b*Sec[c + d*x])/(a + b)]*(b + a*Cos[c + d*x]))))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4321}$$

$$-\frac{\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{156}$$

$$-\frac{\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{155}$$

$$\frac{\sqrt{2}\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(1/3),x]
```

output
```
(Sqrt[2]*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
+ d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(d*Sq
rt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3))
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Maple [F]

$$\int \frac{\sec(dx+c)}{(a+b\sec(dx+c))^{\frac{1}{3}}}dx$$

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)/(b*sec(d*x + c) + a)^(1/3), x)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx=\int \frac{\sec{(c+dx)}}{\sqrt[3]{a+b\sec{(c+dx)}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(1/3),x)
```

output
```
Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx=\int \frac{\sec{(dx+c)}}{(b\sec{(dx+c)}+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(1/3), x)
```

Giac [F]

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx=\int \frac{\sec{(dx+c)}}{(b\sec{(dx+c)}+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(1/3)),x)
```

output
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(1/3)), x)
```

Reduce [F]

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)}{\left(\sec\left(dx+c\right)b+a\right)^{\frac{1}{3}}}dx$$

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(c + d*x)/(sec(c + d*x)*b + a)**(1/3),x)
```

3.703 $\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx=\text{Int}\left(\frac{1}{\sqrt[3]{a+b\sec(c+dx)}},x\right)$$

output
```
Defer(Int)(1/(a+b*sec(d*x+c))^(1/3),x)
```

Mathematica [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(-1/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(-1/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4273}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{ 4273}$$
$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(-1/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4273
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Unintegrable[
(a + b*Csc[c + d*x])^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0
] &&  !IntegerQ[2*n]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(a+b*sec(d*x+c))^(1/3),x)
```

output
```
int(1/(a+b*sec(d*x+c))^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt[3]{a+b\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**(1/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(-1/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(-1/3), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(-1/3), x)
```

Mupad [N/A]

Not integrable

Time = 10.40 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int(1/(a + b/cos(c + d*x))^(1/3),x)
```

output
```
int(1/(a + b/cos(c + d*x))^(1/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\left(\sec\left(dx+c\right)b+a\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(a+b*sec(d*x+c))^(1/3),x)
```

output
```
int(1/(sec(c + d*x)*b + a)**(1/3),x)
```

3.704 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx = \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}{d\sqrt{1+\sec(c+dx)}(a+b\sec(c+dx))^{2/3}}$$

output
```
2^(1/2)*AppellF1(1/2,2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c)
)*((a+b*sec(d*x+c))/(a+b))^(2/3)*tan(d*x+c)/d/(1+sec(d*x+c))^(1/2)/(a+b*se
c(d*x+c))^(2/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 310 vs. 2(105) = 210.

Time = 15.11 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.95

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx = \frac{24(a-b)^2(a+b)\,\text{AppellF1}\,(}{b^2(-a+b)d\left(3(a-b)\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{3}{2},\frac{7}{3},\frac{a+b\sec(c+dx)}{a-b},\frac{a+b\sec(c+dx)}{a+b}\right)(b+a\right.}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(2/3),x]
```

output
```
(24*(a - b)^2*(a + b)*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Sec[c + d*x])/(a
 - b), (a + b*Sec[c + d*x])/(a + b)]*Cos[c + d*x]*Cot[c + d*x]^3*(1 + Sec[
c + d*x])*(b - b*Sec[c + d*x])*(a + b*Sec[c + d*x])^(1/3))/(b^2*(-a + b)*d
*(3*(a - b)*AppellF1[4/3, 1/2, 3/2, 7/3, (a + b*Sec[c + d*x])/(a - b), (a
+ b*Sec[c + d*x])/(a + b)]*(b + a*Cos[c + d*x]) + (a + b)*(8*(a - b)*Appel
lF1[1/3, 1/2, 1/2, 4/3, (a + b*Sec[c + d*x])/(a - b), (a + b*Sec[c + d*x])
/(a + b)]*Cos[c + d*x] + 3*AppellF1[4/3, 3/2, 1/2, 7/3, (a + b*Sec[c + d*x
])/(a - b), (a + b*Sec[c + d*x])/(a + b)]*(b + a*Cos[c + d*x]))))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{4321}$$

$$-\frac{\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{156}$$

$$-\frac{\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{2/3}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}$$

$$\Big\downarrow \text{155}$$

$$\frac{\sqrt{2}\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}$$

input
```
Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(2/3),x]
```

output
```
(Sqrt[2]*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
+ d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/(d*Sq
rt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(2/3))
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Maple [F]

$$\int \frac{\sec (dx+c)}{(a+b\sec (dx+c))^{\frac{2}{3}}}dx$$

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(2/3),x)
```

output
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx = \int \frac{\sec (dx+c)}{(b\sec (dx+c)+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)/(b*sec(d*x + c) + a)^(2/3), x)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx = \int \frac{\sec (c+dx)}{(a+b\sec (c+dx))^{\frac{2}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(2/3),x)
```

output
```
Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(2/3), x)
```

Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx = \int \frac{\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(2/3), x)
```

Giac [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx = \int \frac{\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{2/3}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(2/3)),x)
```

output
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(2/3)), x)
```

Reduce [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx = \int \frac{\sec(dx+c)}{(\sec(dx+c)\,b+a)^{\frac{2}{3}}}dx$$

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(2/3),x)
```

output
```
int(sec(c + d*x)/(sec(c + d*x)*b + a)**(2/3),x)
```

3.705 $\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx = \text{Int}\left(\frac{1}{(a+b\sec(c+dx))^{2/3}},x\right)$$

output
```
Defer(Int)(1/(a+b*sec(d*x+c))^(2/3),x)
```

Mathematica [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx = \int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(-2/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(-2/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4273}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec(c+dx))^{2/3}} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$
$$\Big\downarrow \text{4273}$$
$$\int \frac{1}{(a+b\sec(c+dx))^{2/3}}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(-2/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4273
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Unintegrable[
(a + b*Csc[c + d*x])^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0
] &&  !IntegerQ[2*n]
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+b\sec \left(dx+c\right)\right)^{\frac{2}{3}}}dx$$

input
```
int(1/(a+b*sec(d*x+c))^(2/3),x)
```

output
```
int(1/(a+b*sec(d*x+c))^(2/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx=\int \frac{1}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**(2/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(-2/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(-2/3), x)
```

Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(-2/3), x)
```

Mupad [N/A]

Not integrable

Time = 10.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

input
```
int(1/(a + b/cos(c + d*x))^(2/3),x)
```

output
```
int(1/(a + b/cos(c + d*x))^(2/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx=\int \frac{1}{(\sec(dx+c)\,b+a)^{\frac{2}{3}}}dx$$

input
```
int(1/(a+b*sec(d*x+c))^(2/3),x)
```

output
```
int(1/(sec(c + d*x)*b + a)**(2/3),x)
```

3.706 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{4/3}}\,dx = \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{4/3} \cdots}{d\sqrt{1+\sec(c+dx)}(a+b\sec(c+dx))^{4/3}}$$

output
```
2^(1/2)*AppellF1(1/2,4/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c)
)*((a+b*sec(d*x+c))/(a+b))^(4/3)*tan(d*x+c)/d/(1+sec(d*x+c))^(1/2)/(a+b*se
c(d*x+c))^(4/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 10343 vs. 2(105) = 210.

Time = 46.39 (sec) , antiderivative size = 10343, normalized size of antiderivative = 98.50

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{4/3}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(4/3),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$

$$\Big\downarrow \text{ 4321}$$

$$-\frac{\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{4/3}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{ 156}$$

$$-\frac{\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{4/3}}d\sec(c+dx)}{d(a+b)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 155}$$

$$\frac{\sqrt{2}\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d(a+b)\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(4/3),x]
```

output
```
(Sqrt[2]*AppellF1[1/2, 1/2, 4/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
+ d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/((a +
 b)*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3))
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Maple [F]

$$\int \frac{\sec(dx+c)}{(a+b\sec(dx+c))^{\frac{4}{3}}}dx$$

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(4/3),x)
```

output
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{4/3}}\,dx = \int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)/(b^2*sec(d*x + c)^2 + 2*a
*b*sec(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{4/3}}\,dx = \int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(4/3),x)
```

output
```
Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(4/3), x)
```

Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{4/3}}\,dx = \int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(4/3), x)
```

Giac [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{4/3}}\,dx = \int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{4/3}}\,dx = \int \frac{1}{\cos(c+d\,x)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{4/3}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(4/3)),x)
```

output
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(4/3)), x)
```

Reduce [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{4/3}}\,dx = \int \frac{\sec (dx+c)}{(\sec (dx+c)\,b+a)^{\frac{1}{3}}\sec (dx+c)\,b+(\sec (dx+c)\,b+a)^{\frac{1}{3}}\,a}dx$$

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(4/3),x)
```

output
```
int(sec(c + d*x)/((sec(c + d*x)*b + a)**(1/3)*sec(c + d*x)*b + (sec(c + d*
x)*b + a)**(1/3)*a),x)
```

3.707 $\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx = \operatorname{Int}\left(\frac{1}{(a+b\sec(c+dx))^{4/3}},x\right)$$

output
```
Defer(Int)(1/(a+b*sec(d*x+c))^(4/3),x)
```

Mathematica [N/A]

Not integrable

Time = 88.93 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx = \int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(-4/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(-4/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4273}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$
$$\Big\downarrow \ 4273$$
$$\int \frac{1}{(a+b\sec(c+dx))^{4/3}}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(-4/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4273
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Unintegrable[
(a + b*Csc[c + d*x])^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0
] && !IntegerQ[2*n]
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+b\sec\left(dx+c\right)\right)^{\frac{4}{3}}}dx$$

input
```
int(1/(a+b*sec(d*x+c))^(4/3),x)
```

output
```
int(1/(a+b*sec(d*x+c))^(4/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx = \int \frac{1}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**(4/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(-4/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx=\int \frac{1}{(b\sec{(dx+c)}+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(-4/3), x)
```

Giac [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx=\int \frac{1}{(b\sec{(dx+c)}+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(-4/3), x)
```

Mupad [N/A]

Not integrable

Time = 11.93 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx=\int \frac{1}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{4/3}}\,dx$$

input
```
int(1/(a + b/cos(c + d*x))^(4/3),x)
```

output
```
int(1/(a + b/cos(c + d*x))^(4/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.79

$$\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx = \int \frac{1}{(\sec(dx+c)\,b+a)^{\frac{1}{3}}\sec(dx+c)\,b+(\sec(dx+c)\,b+a)^{\frac{1}{3}}\,a}dx$$

input
```
int(1/(a+b*sec(d*x+c))^(4/3),x)
```

output
```
int(1/((sec(c + d*x)*b + a)**(1/3)*sec(c + d*x)*b + (sec(c + d*x)*b + a)**
(1/3)*a),x)
```

3.708 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 378

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx &= -\frac{3a^2\sec(c+dx)\tan(c+dx)}{2b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{2/3}} \\ &+ \frac{3(3a^2-b^2)\sqrt[3]{a+b\sec(c+dx)}\tan(c+dx)}{4b^2\,(a^2-b^2)\,d} \\ &- \frac{a(9a^2-7b^2)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+b\sec(c+dx)}\tan(c+dx)}{2\sqrt{2}b^3\,(a^2-b^2)\,d\sqrt{1+\sec(c+dx)}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}} \\ &+ \frac{(9a^4-10a^2b^2-b^4)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\tan(c+dx)}{2\sqrt{2}b^3\,(a^2-b^2)\,d\sqrt{1+\sec(c+dx)}(a+b\sec(c+dx))^{2/3}}\end{aligned}$$

output

```
-3/2*a^2*sec(d*x+c)*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(2/3)+3/4*(3
*a^2-b^2)*(a+b*sec(d*x+c))^(1/3)*tan(d*x+c)/b^2/(a^2-b^2)/d-1/4*a*(9*a^2-7
*b^2)*AppellF1(1/2,-1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))
*(a+b*sec(d*x+c))^(1/3)*tan(d*x+c)*2^(1/2)/b^3/(a^2-b^2)/d/(1+sec(d*x+c))^
(1/2)/((a+b*sec(d*x+c))/(a+b))^(1/3)+1/4*(9*a^4-10*a^2*b^2-b^4)*AppellF1(1
/2,2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*sec(d*x+c)
)/(a+b))^(2/3)*tan(d*x+c)*2^(1/2)/b^3/(a^2-b^2)/d/(1+sec(d*x+c))^(1/2)/(a+
b*sec(d*x+c))^(2/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 8160 vs. $2(378) = 756$.

Time = 43.59 (sec) , antiderivative size = 8160, normalized size of antiderivative = 21.59

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.97, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4332, 27, 3042, 4570, 27, 3042, 4495, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}}dx$$

$$\Big\downarrow \text{4332}$$

$$-\frac{3\int \frac{\sec(c+dx)\left(3a^2-2b\sec(c+dx)a-2\left(3a^2-b^2\right)\sec^2(c+dx)\right)}{3(a+b\sec(c+dx))^{2/3}}dx}{2b\left(a^2-b^2\right)}-\frac{3a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sec(c+dx)\left(3a^2-2b\sec(c+dx)a-2\left(3a^2-b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{2/3}}dx}{2b\left(a^2-b^2\right)}-\frac{3a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3a^2-2b\csc\left(c+dx+\frac{\pi}{2}\right)a-2\left(3a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{2b\left(a^2-b^2\right)}-\frac{3a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{2/3}}$$

$$\Big\downarrow \text{4570}$$

$$-\frac{\frac{3\int \frac{2\sec(c+dx)\left(b\left(3a^2+b^2\right)+a\left(9a^2-7b^2\right)\sec(c+dx)\right)}{3(a+b\sec(c+dx))^{2/3}}dx}{4b}-\frac{3\left(3a^2-b^2\right)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}}{2bd}}{2b\left(a^2-b^2\right)}-\frac{3a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{2/3}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{\sec(c+dx)\left(b\left(3a^2+b^2\right)+a\left(9a^2-7b^2\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^{2/3}}dx}{2b}-\frac{3\left(3a^2-b^2\right)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}}{2bd}}{2b\left(a^2-b^2\right)}-\frac{3a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(b\left(3a^2+b^2\right)+a\left(9a^2-7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{2b}-\frac{3\left(3a^2-b^2\right)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}}{2bd}}{2b\left(a^2-b^2\right)}-\frac{3a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{2/3}}$$

$$\Big\downarrow \text{4495}$$

$$-\frac{\frac{\frac{a\left(9a^2-7b^2\right)\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}dx}{b}-\frac{\left(9a^4-10a^2b^2-b^4\right)\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}dx}{b}}{2b}-\frac{3\left(3a^2-b^2\right)\tan(c+dx)\sqrt[3]{a+b\sec(c+d}}{2bd}}{2b\left(a^2-b^2\right)}$$

$$\frac{3a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{a(9a^2-7b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{(9a^4-10a^2b^2-b^4)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{b}}{2b}-\frac{3(3a^2-b^2)\tan(c+dx)\sqrt[3]{a}}{2bd}}{2b(a^2-b^2)}$$

$$\frac{3a^2\tan(c+dx)\sec(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^{2/3}}$$

$\downarrow$ 4321

$$-\frac{\frac{\frac{(9a^4-10a^2b^2-b^4)\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{a(9a^2-7b^2)\tan(c+dx)\int \frac{\sqrt[3]{a+b\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}}{2b}}{2b(a^2-b^2)}$$

$$\frac{3a^2\tan(c+dx)\sec(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^{2/3}}$$

$\downarrow$ 156

$$-\frac{\frac{\frac{(9a^4-10a^2b^2-b^4)\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{2/3}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}-\frac{a(9a^2-7b^2)\tan(c+dx)\sqrt[3]{a+}}{bd\sqrt{1-\sec(c+}}}{2b}}{2b(a^2-b^2)}$$

$$\frac{3a^2\tan(c+dx)\sec(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^{2/3}}$$

$\downarrow$ 155

$$-\frac{3a^2\tan(c+dx)\sec(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^{2/3}}-$$

$$\frac{\frac{\frac{\sqrt{2}a(9a^2-7b^2)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}-\frac{\sqrt{2}(9a^4-10a^2b^2-b^4)\tan(c+dx)\left(\frac{a+b\sec(c}{a+b}\right.}{bd\sqrt{\sec(}}}{2b}}{2b(a^2-b^2)}$$

input
```
Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
(-3*a^2*Sec[c + d*x]*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])
^(2/3)) - ((-3*(3*a^2 - b^2)*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(2*b
*d) + ((Sqrt[2]*a*(9*a^2 - 7*b^2)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c
 + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan
[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3
)) - (Sqrt[2]*(9*a^4 - 10*a^2*b^2 - b^4)*AppellF1[1/2, 1/2, 2/3, 3/2, (1 -
 Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a
 + b))^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x]
)^(2/3)))/(2*b))/(2*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4495
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)/b   Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Simp[B/b   Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && N
eQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{\sec (dx+c)^4}{(a+b\sec (dx+c))^{\frac{5}{3}}}dx$$

input
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/3),x)
```

Fricas [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec\left(dx+c\right)^4}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^4/(b^2*sec(d*x + c)^2 + 2
*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec^4\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**(5/3),x)
```

output
```
Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**(5/3), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)^4}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{1}{\cos(c+d\,x)^4\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(5/3)),x)
```

output
```
int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(5/3)), x)
```

Reduce [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)^4}{(\sec(dx+c)\,b+a)^{\frac{2}{3}}\sec(dx+c)\,b+(\sec(dx+c)\,b+a)^{\frac{2}{3}}\,a}dx$$

input
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(c + d*x)**4/((sec(c + d*x)*b + a)**(2/3)*sec(c + d*x)*b + (sec(c +
 d*x)*b + a)**(2/3)*a),x)
```

3.709 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 307

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = -\frac{3a^2\tan(c+dx)}{2b(a^2-b^2)\,d(a+b\sec(c+dx))^{2/3}} + \frac{(3a^2-2b^2)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+b\sec(c+dx)}\tan(c+dx)}{\sqrt{2}b^2(a^2-b^2)\,d\sqrt{1+\sec(c+dx)}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}} - \frac{a(3a^2-4b^2)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\tan(c+dx)}{\sqrt{2}b^2(a^2-b^2)\,d\sqrt{1+\sec(c+dx)}(a+b\sec(c+dx))^{2/3}}$$

output
```
-3/2*a^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(2/3)+1/2*(3*a^2-2*b^2)
*AppellF1(1/2,-1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b
*sec(d*x+c))^(1/3)*tan(d*x+c)*2^(1/2)/b^2/(a^2-b^2)/d/(1+sec(d*x+c))^(1/2)
/((a+b*sec(d*x+c))/(a+b))^(1/3)-1/2*a*(3*a^2-4*b^2)*AppellF1(1/2,2/3,1/2,3
/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*sec(d*x+c))/(a+b))^(2/
3)*tan(d*x+c)*2^(1/2)/b^2/(a^2-b^2)/d/(1+sec(d*x+c))^(1/2)/(a+b*sec(d*x+c)
)^(2/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7918 vs. $2(307) = 614$.

Time = 43.09 (sec) , antiderivative size = 7918, normalized size of antiderivative = 25.79

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4326, 27, 3042, 4495, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/3}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}}dx$$

$$\Big\downarrow \text{ 4326}$$

$$-\frac{3\int -\frac{\sec(c+dx)\left(2ab+\left(3a^2-2b^2\right)\sec(c+dx)\right)}{3(a+b\sec(c+dx))^{2/3}}dx}{2b\left(a^2-b^2\right)}-\frac{3a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sec(c+dx)\left(2ab+\left(3a^2-2b^2\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^{2/3}}dx}{2b\left(a^2-b^2\right)}-\frac{3a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2ab+\left(3a^2-2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{2b\left(a^2-b^2\right)}-\frac{3a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}$$

$$\Big\downarrow \ 4495$$

$$\frac{\frac{\left(3a^2-2b^2\right)\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}dx}{b}-\frac{a\left(3a^2-4b^2\right)\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}dx}{b}}{2b\left(a^2-b^2\right)}-\frac{3a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\left(3a^2-2b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{a\left(3a^2-4b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{b}}{2b\left(a^2-b^2\right)}-\frac{3a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}$$

$$\Big\downarrow \ 4321$$

$$\frac{\frac{a\left(3a^2-4b^2\right)\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{\left(3a^2-2b^2\right)\tan(c+dx)\int \frac{\sqrt[3]{a+b\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}}{2b\left(a^2-b^2\right)}-\frac{3a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}$$

$$\Big\downarrow \ 156$$

$$\frac{\frac{a\left(3a^2-4b^2\right)\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{2/3}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}-\frac{\left(3a^2-2b^2\right)\tan(c+dx)\sqrt[3]{a+b}}{bd\sqrt{1-\sec(c+a}}}{2b\left(a^2-b^2\right)}-\frac{3a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}$$

$$\Big\downarrow \ 155$$

$$\frac{\frac{\sqrt{2}(3a^2-2b^2)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}-\frac{\sqrt{2}a(3a^2-4b^2)\tan(c+dx)\left(\frac{a+b\sec}{a}\right.}{bd}}{2b\left(a^2-b^2\right)}$$

$$\frac{3a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{2/3}}$$

input
```
Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
(-3*a^2*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(2/3)) + ((S
qrt[2]*(3*a^2 - 2*b^2)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2,
 (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/
(b*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3)) - (Sqrt[
2]*a*(3*a^2 - 4*b^2)*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Sec[c + d*x])/2, (b
*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c +
 d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(2/3)))/(2*b*(a^2
- b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

rule 4326
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-a^2)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m
 + 1)*(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]
*(a + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e
+ f*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

rule 4495
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)/b   Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Simp[B/b   Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && N
eQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{\sec(dx+c)^3}{(a+b\sec(dx+c))^{\frac{5}{3}}}dx$$

input
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/3),x)
```

Fricas [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)^3}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^3/(b^2*sec(d*x + c)^2 + 2
*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**(5/3),x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**(5/3), x)
```

Maxima [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)^3}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(5/3), x)
```

Giac [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)^3}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{1}{\cos(c+d\,x)^3\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(5/3)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(5/3)), x)
```

Reduce [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)^3}{(\sec(dx+c)\,b+a)^{\frac{2}{3}}\sec(dx+c)\,b+(\sec(dx+c)\,b+a)^{\frac{2}{3}}\,a}dx$$

input
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(c + d*x)**3/((sec(c + d*x)*b + a)**(2/3)*sec(c + d*x)*b + (sec(c +
 d*x)*b + a)**(2/3)*a),x)
```

3.710 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 289

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \frac{3a\tan(c+dx)}{2(a^2-b^2)\,d(a+b\sec(c+dx))^{2/3}} - \frac{a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+b\sec(c+dx)}\tan(c+dx)}{\sqrt{2}b(a^2-b^2)\,d\sqrt{1+\sec(c+dx)}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}} + \frac{(a^2-2b^2)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\tan(c+dx)}{\sqrt{2}b(a^2-b^2)\,d\sqrt{1+\sec(c+dx)}(a+b\sec(c+dx))^{2/3}}$$

output
```
3/2*a*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(2/3)-1/2*a*AppellF1(1/2,-1/
3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(1/3
)*tan(d*x+c)*2^(1/2)/b/(a^2-b^2)/d/(1+sec(d*x+c))^(1/2)/((a+b*sec(d*x+c))/
(a+b))^(1/3)+1/2*(a^2-2*b^2)*AppellF1(1/2,2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+
b),1/2-1/2*sec(d*x+c))*((a+b*sec(d*x+c))/(a+b))^(2/3)*tan(d*x+c)*2^(1/2)/b
/(a^2-b^2)/d/(1+sec(d*x+c))^(1/2)/(a+b*sec(d*x+c))^(2/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7325 vs. $2(289) = 578$.

Time = 46.11 (sec) , antiderivative size = 7325, normalized size of antiderivative = 25.35

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4323, 27, 3042, 4495, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}}dx$$

$$\Big\downarrow \ 4323$$

$$\frac{3\int -\frac{\sec(c+dx)(2b+a\sec(c+dx))}{3(a+b\sec(c+dx))^{2/3}}dx}{2\left(a^2-b^2\right)}+\frac{3a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}$$

$$\Big\downarrow \ 27$$

$$\frac{3a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}-\frac{\int \frac{\sec(c+dx)(2b+a\sec(c+dx))}{(a+b\sec(c+dx))^{2/3}}dx}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 3042$$

$$\frac{3a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 4495$$

$$\frac{3a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}-\frac{\frac{a\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}dx}{b}-\frac{\left(a^2-2b^2\right)\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}dx}{b}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 3042$$

$$\frac{3a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}-\frac{\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(a^2-2b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{b}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 4321$$

$$\frac{3a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}-\frac{\frac{\left(a^2-2b^2\right)\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{a\tan(c+dx)\int \frac{\sqrt[3]{a+b\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 156$$

$$\frac{3a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}-\frac{\frac{\left(a^2-2b^2\right)\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{2/3}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}-\frac{a\tan(c+dx)\sqrt[3]{a+b\sec(c+d}}{bd\sqrt{1-\sec(c+dx)}\sqrt{se}}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 155$$

$$\frac{3a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}-\frac{\frac{\sqrt{2}a\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}-\frac{\sqrt{2}\left(a^2-2b^2\right)\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}{bd\sqrt{\sec(c+d}}}{2\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
(3*a*Tan[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(2/3)) - ((Sqrt[2
]*a*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*
x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c
 + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3)) - (Sqrt[2]*(a^2 - 2*b^2)*Ap
pellF1[1/2, 1/2, 2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a
 + b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Se
c[c + d*x]]*(a + b*Sec[c + d*x])^(2/3)))/(2*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
 - e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

rule 4323
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[a*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(
a^2 - b^2))), x] - Simp[1/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

rule 4495
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)/b   Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Simp[B/b   Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && N
eQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{\sec (dx+c)^2}{(a+b\sec (dx+c))^{\frac{5}{3}}}dx$$

input
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/3),x)
```

Fricas [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^2/(b^2*sec(d*x + c)^2 + 2
*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**(5/3),x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**(5/3), x)
```

Maxima [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(5/3), x)
```

Giac [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx=\int \frac{\sec\left(dx+c\right)^2}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^2\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(5/3)),x)
```

output
```
int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(5/3)), x)
```

Reduce [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx=\int \frac{\sec\left(dx+c\right)^2}{\left(\sec\left(dx+c\right)b+a\right)^{\frac{2}{3}}\sec\left(dx+c\right)b+\left(\sec\left(dx+c\right)b+a\right)^{\frac{2}{3}}a}dx$$

input
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(c + d*x)**2/((sec(c + d*x)*b + a)**(2/3)*sec(c + d*x)*b + (sec(c +
 d*x)*b + a)**(2/3)*a),x)
```

3.711 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/3}} dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/3}} dx = \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{5}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{5/3}}{d\sqrt{1+\sec(c+dx)}(a+b\sec(c+dx))^{5/3}}$$

output
```
2^(1/2)*AppellF1(1/2,5/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c)
)*((a+b*sec(d*x+c))/(a+b))^(5/3)*tan(d*x+c)/d/(1+sec(d*x+c))^(1/2)/(a+b*se
c(d*x+c))^(5/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 10363 vs. 2(105) = 210.

Time = 49.08 (sec) , antiderivative size = 10363, normalized size of antiderivative = 98.70

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/3}} dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}}dx$$

$$\Big\downarrow \text{4321}$$

$$-\frac{\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{5/3}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{156}$$

$$-\frac{\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{5/3}}d\sec(c+dx)}{d(a+b)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}$$

$$\Big\downarrow \text{155}$$

$$\frac{\sqrt{2}\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{5}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d(a+b)\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}$$

input
```
Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
(Sqrt[2]*AppellF1[1/2, 1/2, 5/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
+ d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/((a +
 b)*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(2/3))
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Maple [F]

$$\int \frac{\sec(dx+c)}{(a+b\sec(dx+c))^{\frac{5}{3}}}dx$$

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(5/3),x)
```

Fricas [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c)/(b^2*sec(d*x + c)^2 + 2*a
*b*sec(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(5/3),x)
```

output
```
Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(5/3), x)
```

Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(5/3), x)
```

Giac [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{1}{\cos(c+d\,x)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(5/3)),x)
```

output
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(5/3)), x)
```

Reduce [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)}{(\sec(dx+c)\,b+a)^{\frac{2}{3}}\sec(dx+c)\,b+(\sec(dx+c)\,b+a)^{\frac{2}{3}}\,a}dx$$

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(c + d*x)/((sec(c + d*x)*b + a)**(2/3)*sec(c + d*x)*b + (sec(c + d*
x)*b + a)**(2/3)*a),x)
```

3.712 $\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx=\text{Int}\left(\frac{1}{(a+b\sec(c+dx))^{5/3}},x\right)$$

output
```
Defer(Int)(1/(a+b*sec(d*x+c))^(5/3),x)
```

Mathematica [N/A]

Not integrable

Time = 84.39 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx=\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(-5/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(-5/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4273}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}}dx$$
$$\Big\downarrow \; 4273$$
$$\int \frac{1}{(a+b\sec(c+dx))^{5/3}}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(-5/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4273
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Unintegrable[
(a + b*Csc[c + d*x])^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0
] && !IntegerQ[2*n]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+b\sec \left(dx+c\right)\right)^{\frac{5}{3}}}dx$$

input
```
int(1/(a+b*sec(d*x+c))^(5/3),x)
```

output
```
int(1/(a+b*sec(d*x+c))^(5/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx=\int \frac{1}{\left(a+b\sec \left(c+dx\right)\right)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**(5/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(-5/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(-5/3), x)
```

Giac [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(-5/3), x)
```

Mupad [N/A]

Not integrable

Time = 11.61 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

input
```
int(1/(a + b/cos(c + d*x))^(5/3),x)
```

output
```
int(1/(a + b/cos(c + d*x))^(5/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.79

$$\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{1}{(\sec(dx+c)\,b+a)^{\frac{2}{3}}\sec(dx+c)\,b+(\sec(dx+c)\,b+a)^{\frac{2}{3}}\,a}dx$$

input
```
int(1/(a+b*sec(d*x+c))^(5/3),x)
```

output
```
int(1/((sec(c + d*x)*b + a)**(2/3)*sec(c + d*x)*b + (sec(c + d*x)*b + a)**
(2/3)*a),x)
```

3.713 $\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 174

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{a\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{6},1,\frac{3}{2},\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin(c+dx)}{(a^2-b^2)\,d\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}} - \frac{b\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sqrt[3]{\cos^2(c+dx)}\sec^{\frac{2}{3}}(c+dx)\sin(c+dx)}{(a^2-b^2)\,d}$$

output
```
a*AppellF1(1/2,-1/6,1,3/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*sin(d*x
+c)/(a^2-b^2)/d/(cos(d*x+c)^2)^(1/6)/sec(d*x+c)^(1/3)-b*AppellF1(1/2,1/3,1
,3/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(1/3)*sec(d*x
+c)^(2/3)*sin(d*x+c)/(a^2-b^2)/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 4543 vs. 2(174) = 348.

Time = 36.23 (sec) , antiderivative size = 4543, normalized size of antiderivative = 26.11

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x]),x]
```

output
```
(9*(a^2 - b^2)*Sec[c + d*x]^(5/3)*Sin[c + d*x]*((b*AppellF1[1/2, 1/6, 1, 3
/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2
])/(9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c +
 d*x]^2)/(a^2 - b^2)] + (6*b^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2,
 (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/
2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) + (
a*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 -
b^2)])/(-9*(a^2 - b^2)*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Ta
n[c + d*x]^2)/(a^2 - b^2)] - 2*(3*b^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c +
d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(-a^2 + b^2)*AppellF1[3/2, 5
/3, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*
x]^2)))/(d*(Sec[c + d*x]^2)^(2/3)*(a + b*Sec[c + d*x])*(-a^2 + b^2*Sec[c +
 d*x]^2)*((9*(a^2 - b^2)*(Sec[c + d*x]^2)^(1/3)*((b*AppellF1[1/2, 1/6, 1,
3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^
2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c
+ d*x]^2)/(a^2 - b^2)] + (6*b^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2
, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5
/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) +
(a*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 -
 b^2)])/(-9*(a^2 - b^2)*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, (b^...
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4356, 3042, 3302, 3042, 3668, 25, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{2/3}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4356$$

$$\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\int \frac{\sqrt[3]{\cos(c+dx)}}{b+a\cos(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\int \frac{\sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)}}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3302$$

$$\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\left(b\int \frac{\sqrt[3]{\cos(c+dx)}}{b^2-a^2\cos^2(c+dx)}dx-a\int \frac{\cos^{\frac{4}{3}}(c+dx)}{b^2-a^2\cos^2(c+dx)}dx\right)$$

$$\Big\downarrow \; 3042$$

$$\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\left(b\int \frac{\sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)}}{b^2-a^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx-a\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{4/3}}{b^2-a^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx\right)$$

$$\Big\downarrow \; 3668$$

$$\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\left(\frac{b\sqrt[3]{\cos^2(c+dx)}\int -\frac{1}{\sqrt[3]{1-\sin^2(c+dx)}\left(-\sin^2(c+dx)a^2+a^2-b^2\right)}d\sin(c+dx)}{d\cos^{\frac{2}{3}}(c+dx)}-\frac{a\sqrt[3]{\cos(c+dx)}\int -\frac{\sqrt[6]{1-\sin^2}}{-\sin^2(c+dx)}}{d\sqrt[6]{\cos^2(c+}}\right.$$

$$\Big\downarrow \; 25$$

$$\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\left(\frac{a\sqrt[3]{\cos(c+dx)}\int \frac{\sqrt[6]{1-\sin^2(c+dx)}}{-\sin^2(c+dx)a^2+a^2-b^2}d\sin(c+dx)}{d\sqrt[6]{\cos^2(c+dx)}}-\frac{b\sqrt[3]{\cos^2(c+dx)}\int \frac{1}{\sqrt[3]{1-\sin^2(c+dx)}\left(-\sin^2(c+dx)a^2+a\right.}}{d\cos^{\frac{2}{3}}(c+dx)}\right.$$

$$\Big\downarrow \; 333$$

$$\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\left(\frac{a\sin(c+dx)\sqrt[3]{\cos(c+dx)}\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{6},1,\frac{3}{2},\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\sqrt[6]{\cos^2(c+dx)}}-\frac{b\sin(c+dx)\sqrt[3]{\cos^2(c+dx}}{d}\right.$$

input
```
Int[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x]),x]
```

output
```
Cos[c + d*x]^(2/3)*Sec[c + d*x]^(2/3)*((a*AppellF1[1/2, -1/6, 1, 3/2, Sin[
c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Cos[c + d*x]^(1/3)*Sin[c + d
*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(1/6)) - (b*AppellF1[1/2, 1/3, 1, 3/2
, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*(Cos[c + d*x]^2)^(1/3)
*Sin[c + d*x])/((a^2 - b^2)*d*Cos[c + d*x]^(2/3)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3302
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[a   Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]
^2), x], x] - Simp[b/d   Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*
x]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 3668
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]))   Subst[Int[(1 - ff^2*x^2)^((m -
 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
 d, e, f, m, p}, x] && !IntegerQ[m]
```

rule 4356
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Simp[Sin[e + f*x]^n*(d*Csc[e + f*x])^n   Int[(b +
 a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n}
, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{\sec\left(dx+c\right)^{\frac{2}{3}}}{a+b\sec\left(dx+c\right)}dx$$

input
```
int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x)
```

output
```
int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec^{\frac{2}{3}}\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

input
```
integrate(sec(d*x+c)**(2/3)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**(2/3)/(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{2}{3}}}{b\sec\left(dx+c\right)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(2/3)/(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{2}{3}}}{b\sec\left(dx+c\right)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(2/3)/(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{2}{3}}}{\sec\left(dx+c\right)b+a}dx$$

input
```
int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x)
```

output
```
int(sec(c + d*x)**(2/3)/(sec(c + d*x)*b + a),x)
```

3.714 $\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 174

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \frac{a\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{3},1,\frac{3}{2},\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin(c+dx)}{(a^2-b^2)\,d\sqrt[3]{\cos^2(c+dx)}\sec^{\frac{2}{3}}(c+dx)} - \frac{b\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{6},1,\frac{3}{2},\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d}$$

output
```
a*AppellF1(1/2,-1/3,1,3/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*sin(d*x
+c)/(a^2-b^2)/d/(cos(d*x+c)^2)^(1/3)/sec(d*x+c)^(2/3)-b*AppellF1(1/2,1/6,1
,3/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(1/6)*sec(d*x
+c)^(1/3)*sin(d*x+c)/(a^2-b^2)/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 4544 vs. $2(174) = 348$.

Time = 36.11 (sec) , antiderivative size = 4544, normalized size of antiderivative = 26.11

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x]),x]
```

output
```
(9*(a^2 - b^2)*Sec[c + d*x]^(4/3)*Sin[c + d*x]*((b*AppellF1[1/2, 1/3, 1, 3
/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2
])/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c +
 d*x]^2)/(a^2 - b^2)] + 2*(3*b^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^
2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 4/3, 1,
5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) +
 (a*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2
- b^2)])/(-9*(a^2 - b^2)*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*
Tan[c + d*x]^2)/(a^2 - b^2)] + (-6*b^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c +
 d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 5*(a^2 - b^2)*AppellF1[3/2, 1
1/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d
*x]^2)))/(d*(Sec[c + d*x]^2)^(5/6)*(a + b*Sec[c + d*x])*(-a^2 + b^2*Sec[c
+ d*x]^2)*((9*(a^2 - b^2)*(Sec[c + d*x]^2)^(1/6)*((b*AppellF1[1/2, 1/3, 1,
 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]
^2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c
 + d*x]^2)/(a^2 - b^2)] + 2*(3*b^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x
]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 4/3, 1
, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)
 + (a*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^
2 - b^2)])/(-9*(a^2 - b^2)*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, ...
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 4356, 3042, 3302, 3042, 3668, 25, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4356}$$

$$\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\int \frac{\cos^{\frac{2}{3}}(c+dx)}{b+a\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{2/3}}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3302}$$

$$\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\left(b\int \frac{\cos^{\frac{2}{3}}(c+dx)}{b^2-a^2\cos^2(c+dx)}dx-a\int \frac{\cos^{\frac{5}{3}}(c+dx)}{b^2-a^2\cos^2(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\left(b\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{2/3}}{b^2-a^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx-a\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/3}}{b^2-a^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx\right)$$

$$\Big\downarrow \text{3668}$$

$$\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\left(\frac{b\sqrt[6]{\cos^2(c+dx)}\int -\frac{1}{\sqrt[6]{1-\sin^2(c+dx)}\left(-\sin^2(c+dx)a^2+a^2-b^2\right)}d\sin(c+dx)}{d\sqrt[3]{\cos(c+dx)}}-\frac{a\cos^{\frac{2}{3}}\right.$$

$$\Big\downarrow 25$$

$$\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\left(\frac{a\cos^{\frac{2}{3}}(c+dx)\int\frac{\sqrt[3]{1-\sin^2(c+dx)}}{-\sin^2(c+dx)a^2+a^2-b^2}d\sin(c+dx)}{d\sqrt[3]{\cos^2(c+dx)}}-\frac{b\sqrt[6]{\cos^2(c+dx)}\int\frac{}{\sqrt[6]{1-\sin^2(}}}{d}\right.$$

$$\Big\downarrow 333$$

$$\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\left(\frac{a\sin(c+dx)\cos^{\frac{2}{3}}(c+dx)\text{AppellF1}\left(\frac{1}{2},-\frac{1}{3},1,\frac{3}{2},\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d(a^2-b^2)\sqrt[3]{\cos^2(c+dx)}}-\frac{b\sin}{}\right.$$

input
```
Int[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x]),x]
```

output
```
Cos[c + d*x]^(1/3)*Sec[c + d*x]^(1/3)*((a*AppellF1[1/2, -1/3, 1, 3/2, Sin[
c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Cos[c + d*x]^(2/3)*Sin[c + d
*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(1/3)) - (b*AppellF1[1/2, 1/6, 1, 3/2
, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*(Cos[c + d*x]^2)^(1/6)
*Sin[c + d*x])/((a^2 - b^2)*d*Cos[c + d*x]^(1/3)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3302
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[a   Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]
^2), x], x] - Simp[b/d   Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*
x]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 3668
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]))   Subst[Int[(1 - ff^2*x^2)^((m -
 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
 d, e, f, m, p}, x] &&  !IntegerQ[m]
```

rule 4356
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Simp[Sin[e + f*x]^n*(d*Csc[e + f*x])^n   Int[(b +
 a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n}
, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{\sec (dx+c)^{\frac{1}{3}}}{a+b\sec (dx+c)}dx$$

input
```
int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x)
```

output
```
int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt[3]{\sec\left(c+dx\right)}}{a+b\sec\left(c+dx\right)}\,dx$$

input
```
integrate(sec(d*x+c)**(1/3)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**(1/3)/(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{1}{3}}}{b\sec\left(dx+c\right)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(1/3)/(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{1}{3}}}{b\sec\left(dx+c\right)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(1/3)/(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{1}{3}}}{\sec\left(dx+c\right)b+a}dx$$

input
```
int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x)
```

output
```
int(sec(c + d*x)**(1/3)/(sec(c + d*x)*b + a),x)
```

3.715 $\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 174

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))}\,dx = -\frac{b\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{6},1,\frac{3}{2},\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin(c+dx)}{(a^2-b^2)\,d\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}} + \frac{a\,\text{AppellF1}\left(\frac{1}{2},-\frac{2}{3},1,\frac{3}{2},\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sqrt[3]{\cos^2(c+dx)}\sec^{\frac{2}{3}}(c+dx)\sin(c+dx)}{(a^2-b^2)\,d}$$

output
```
-b*AppellF1(1/2,-1/6,1,3/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*sin(d*
x+c)/(a^2-b^2)/d/(cos(d*x+c)^2)^(1/6)/sec(d*x+c)^(1/3)+a*AppellF1(1/2,-2/3
,1,3/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(1/3)*sec(d
*x+c)^(2/3)*sin(d*x+c)/(a^2-b^2)/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7430 vs. $2(174) = 348$.

Time = 77.99 (sec) , antiderivative size = 7430, normalized size of antiderivative = 42.70

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4356, 3042, 3302, 3042, 3668, 25, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4356}$$

$$\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\int \frac{\cos^{\frac{4}{3}}(c+dx)}{b+a\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{4/3}}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3302}$$

$$\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\left(b\int \frac{\cos^{\frac{4}{3}}(c+dx)}{b^2-a^2\cos^2(c+dx)}dx-a\int \frac{\cos^{\frac{7}{3}}(c+dx)}{b^2-a^2\cos^2(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\left(b\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{4/3}}{b^2-a^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx-a\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/3}}{b^2-a^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx\right)$$

$$\Big\downarrow \text{3668}$$

$$\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\left(\frac{b\sqrt[3]{\cos(c+dx)}\int -\frac{\sqrt[6]{1-\sin^2(c+dx)}}{-\sin^2(c+dx)a^2+a^2-b^2}d\sin(c+dx)}{d\sqrt[6]{\cos^2(c+dx)}}-\frac{a\cos^{\frac{4}{3}}(c+dx)\int -\frac{(1-\sin^2(c+dx))^{2/3}}{-\sin^2(c+dx)a^2+a^2-b^2}d\sin(c+dx)}{d\cos^2(c+dx)^{2/3}}\right.$$

$$\Big\downarrow \text{25}$$

$$\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\left(\frac{a\cos^{\frac{4}{3}}(c+dx)\int \frac{(1-\sin^2(c+dx))^{2/3}}{-\sin^2(c+dx)a^2+a^2-b^2}d\sin(c+dx)}{d\cos^2(c+dx)^{2/3}}-\frac{b\sqrt[3]{\cos(c+dx)}\int \frac{\sqrt[6]{1-\sin^2(c+dx)}}{-\sin^2(c+dx)a^2+a^2-b^2}d\sin(c+dx)}{d\sqrt[6]{\cos^2(c+dx)}}\right)$$

$$\Big\downarrow \text{333}$$

$$\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\left(\frac{a\sin(c+dx)\cos^{\frac{4}{3}}(c+dx)\operatorname{AppellF1}\left(\frac{1}{2},-\frac{2}{3},1,\frac{3}{2},\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d(a^2-b^2)\cos^2(c+dx)^{2/3}}-\frac{b\sin(c+dx)\sqrt[3]{\cos(c+dx)}\ldots}{d(\ldots}\right.$$

input
```
Int[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])),x]
```

output
```
Cos[c + d*x]^(2/3)*Sec[c + d*x]^(2/3)*((a*AppellF1[1/2, -2/3, 1, 3/2, Sin[
c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Cos[c + d*x]^(4/3)*Sin[c + d
*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(2/3)) - (b*AppellF1[1/2, -1/6, 1, 3/
2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Cos[c + d*x]^(1/3)*Si
n[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(1/6)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3302
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[a   Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]
^2), x], x] - Simp[b/d   Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*
x]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 3668
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]))   Subst[Int[(1 - ff^2*x^2)^((m -
 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
 d, e, f, m, p}, x] &&  !IntegerQ[m]
```

rule 4356
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Simp[Sin[e + f*x]^n*(d*Csc[e + f*x])^n   Int[(b +
 a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n}
, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{\sec (dx+c)^{\frac{1}{3}} (a+b\sec (dx+c))}dx$$

input
```
int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x)
```

output
```
int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))}\,dx=\text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))}\,dx=\int \frac{1}{(a+b\sec (c+dx))\sqrt[3]{\sec (c+dx)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/3)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))*sec(c + d*x)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\sec(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\sec(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)\,\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/3)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))}\,dx = \int \frac{1}{\sec(dx+c)^{\frac{4}{3}}b+\sec(dx+c)^{\frac{1}{3}}a}dx$$

input
```
int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x)
```

output
```
int(1/(sec(c + d*x)**(1/3)*sec(c + d*x)*b + sec(c + d*x)**(1/3)*a),x)
```

3.716 $\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 174

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))}\,dx = -\frac{b\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{3},1,\frac{3}{2},\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin(c+dx)}{(a^2-b^2)\,d\sqrt[3]{\cos^2(c+dx)}\sec^{\frac{2}{3}}(c+dx)} + \frac{a\,\text{AppellF1}\left(\frac{1}{2},-\frac{5}{6},1,\frac{3}{2},\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d}$$

output
```
-b*AppellF1(1/2,-1/3,1,3/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*sin(d*
x+c)/(a^2-b^2)/d/(cos(d*x+c)^2)^(1/3)/sec(d*x+c)^(2/3)+a*AppellF1(1/2,-5/6
,1,3/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(1/6)*sec(d
*x+c)^(1/3)*sin(d*x+c)/(a^2-b^2)/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7588 vs. $2(174) = 348$.

Time = 78.57 (sec) , antiderivative size = 7588, normalized size of antiderivative = 43.61

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4356, 3042, 3302, 3042, 3668, 25, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{2/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \ 4356$$

$$\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\int \frac{\cos^{\frac{5}{3}}(c+dx)}{b+a\cos(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/3}}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 3302$$

$$\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\left(b\int \frac{\cos^{\frac{5}{3}}(c+dx)}{b^2-a^2\cos^2(c+dx)}dx-a\int \frac{\cos^{\frac{8}{3}}(c+dx)}{b^2-a^2\cos^2(c+dx)}dx\right)$$

$\Big\downarrow$ 3042

$$\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\left(b\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/3}}{b^2-a^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx-a\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{8/3}}{b^2-a^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx\right)$$

$\Big\downarrow$ 3668

$$\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\left(\frac{b\cos^{\frac{2}{3}}(c+dx)\int -\frac{\sqrt[3]{1-\sin^2(c+dx)}}{-\sin^2(c+dx)a^2+a^2-b^2}d\sin(c+dx)}{d\sqrt[3]{\cos^2(c+dx)}}-\frac{a\cos^{\frac{5}{3}}(c+dx)\int -\frac{(1-\sin^2(c}{-\sin^2(c+dx}}{d\cos^2(c+c}\right.$$

$\Big\downarrow$ 25

$$\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\left(\frac{a\cos^{\frac{5}{3}}(c+dx)\int \frac{(1-\sin^2(c+dx))^{5/6}}{-\sin^2(c+dx)a^2+a^2-b^2}d\sin(c+dx)}{d\cos^2(c+dx)^{5/6}}-\frac{b\cos^{\frac{2}{3}}(c+dx)\int \frac{\sqrt[3]{1-\sin^2(c}}{-\sin^2(c+dx)a^2-}}{d\sqrt[3]{\cos^2(c+c}}\right.$$

$\Big\downarrow$ 333

$$\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\left(\frac{a\sin(c+dx)\cos^{\frac{5}{3}}(c+dx)\,\text{AppellF1}\left(\frac{1}{2},-\frac{5}{6},1,\frac{3}{2},\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\cos^2(c+dx)^{5/6}}-\frac{b\sin}{}\right.$$

input
```
Int[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])),x]
```

output
```
Cos[c + d*x]^(1/3)*Sec[c + d*x]^(1/3)*((a*AppellF1[1/2, -5/6, 1, 3/2, Sin[
c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Cos[c + d*x]^(5/3)*Sin[c + d
*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(5/6)) - (b*AppellF1[1/2, -1/3, 1, 3/
2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Cos[c + d*x]^(2/3)*Si
n[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(1/3)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3302
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[a   Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]
^2), x], x] - Simp[b/d   Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*
x]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 3668
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]))   Subst[Int[(1 - ff^2*x^2)^((m -
 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
 d, e, f, m, p}, x] &&  !IntegerQ[m]
```

rule 4356
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Simp[Sin[e + f*x]^n*(d*Csc[e + f*x])^n   Int[(b +
 a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n}
, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{\sec(dx+c)^{\frac{2}{3}}(a+b\sec(dx+c))}dx$$

input
```
int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x)
```

output
```
int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{(a+b\sec(c+dx))\sec^{\frac{2}{3}}(c+dx)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(2/3)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))*sec(c + d*x)**(2/3)), x)
```

Maxima [F]

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\sec(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)), x)
```

Giac [F]

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\sec(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(2/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(2/3)), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{\sec(dx+c)^{\frac{5}{3}}b+\sec(dx+c)^{\frac{2}{3}}a}dx$$

input
```
int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x)
```

output
```
int(1/(sec(c + d*x)**(2/3)*sec(c + d*x)*b + sec(c + d*x)**(2/3)*a),x)
```

3.717 $\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \text{Int}\left(\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Mathematica [F(-1)]

Timed out.

$$\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \$\text{Aborted}$$

input
```
Integrate[Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
$Aborted
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/3}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 4357}$$

$$\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}dx$$

input
```
Int[Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec(dx+c)^{\frac{7}{3}} \sqrt{a+b\sec(dx+c)}dx$$

input
```
int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{7}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/3)*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{7}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3), x)
```

Giac [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{7}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3), x)
```

Mupad [N/A]

Not integrable

Time = 11.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+\frac{b}{\cos(c+d\,x)}}\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(7/3),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(7/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sec\left(dx+c\right)^{\frac{7}{3}}\sqrt{\sec\left(dx+c\right)b+a}dx$$

input
```
int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**2,x)
```

3.718 $\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \operatorname{Int}\left(\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 173.73 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

input
```
Integrate[Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
Integrate[Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/3}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}dx$$

input
```
Int[Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec (dx + c)^{\frac{5}{3}} \sqrt{a + b \sec (dx + c)}dx$$

input
```
int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{5}{3}}(c + dx)\sqrt{a + b\sec(c + dx)}\, dx = \int \sqrt{b \sec (dx + c) + a} \sec (dx + c)^{\frac{5}{3}} \, dx$$

input
```
integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{3}}(c + dx)\sqrt{a + b\sec(c + dx)}\, dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/3)*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{5}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3), x)
```

Giac [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{5}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3), x)
```

Mupad [N/A]

Not integrable

Time = 10.80 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+\frac{b}{\cos(c+d\,x)}}\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/3),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sec\left(dx+c\right)^{\frac{5}{3}}\sqrt{\sec\left(dx+c\right)b+a}dx$$

input
```
int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sec(c + d*x)**(2/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x),x)
```

3.719 $\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \text{Int}\left(\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 88.81 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

input
```
Integrate[Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
Integrate[Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}dx$$

input
```
Int[Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec(dx+c)^{\frac{4}{3}} \sqrt{a+b\sec(dx+c)}dx$$

input
```
int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{4}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(4/3)*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{4}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3), x)
```

Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{4}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3), x)
```

Mupad [N/A]

Not integrable

Time = 10.75 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{4/3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(4/3),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(4/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sec\left(dx+c\right)^{\frac{4}{3}}\sqrt{\sec\left(dx+c\right)b+a}dx$$

input
```
int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x),x)
```

3.720 $\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \text{Int}\left(\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 118.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

input
```
Integrate[Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
Integrate[Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{2/3}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}dx$$

input
```
Int[Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec(dx+c)^{\frac{2}{3}} \sqrt{a+b\sec(dx+c)}dx$$

input
```
int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{2}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3), x)
```

Sympy [N/A]

Not integrable

Time = 9.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+b\sec(c+dx)}\sec^{\frac{2}{3}}(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**(2/3)*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(2/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{2}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3), x)
```

Giac [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{2}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3), x)
```

Mupad [N/A]

Not integrable

Time = 10.74 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+\frac{b}{\cos(c+d\,x)}}\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(2/3),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(2/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx=\int \sec(dx+c)^{\frac{2}{3}}\sqrt{\sec(dx+c)\,b+a}dx$$

input
```
int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sec(c + d*x)**(2/3)*sqrt(sec(c + d*x)*b + a),x)
```

3.721 $\int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \text{Int}\left(\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 18.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$$

input
```
Integrate[Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
Integrate[Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 4357}$$

$$\int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}dx$$

input
```
Int[Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec (dx+c)^{\frac{1}{3}} \sqrt{a+b\sec (dx+c)}dx$$

input
```
int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec (dx+c)+a}\sec (dx+c)^{\frac{1}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3), x)
```

Sympy [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+b\sec (c+dx)}\sqrt[3]{\sec (c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**(1/3)*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(1/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{1}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3), x)
```

Giac [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{1}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3), x)
```

Mupad [N/A]

Not integrable

Time = 10.70 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+\frac{b}{\cos(c+d\,x)}}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/3),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx=\int \sec\left(dx+c\right)^{\frac{1}{3}}\sqrt{\sec\left(dx+c\right)b+a}dx$$

input
```
int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a),x)
```

3.722 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx = \text{Int}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}},x\right)$$

output
```
Defer(Int)((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x)
```

Mathematica [N/A]

Not integrable

Time = 71.66 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx = \int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(1/3),x]
```

output
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(1/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}dx$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(1/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+b\sec(dx+c)}}{\sec(dx+c)^{\frac{1}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x)
```

output
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x)
```

Fricas [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx = \int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(1/3), x)
```

Sympy [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx = \int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/3),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(1/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx=\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(1/3), x)
```

Giac [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx=\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(1/3), x)
```

Mupad [N/A]

Not integrable

Time = 11.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx=\int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/3),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx=\int \frac{\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{\frac{1}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)/sec(c + d*x)**(1/3),x)
```

3.723 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \text{Int}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)},x\right)$$

output
```
Defer(Int)((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x)
```

Mathematica [N/A]

Not integrable

Time = 79.51 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(2/3),x]
```

output
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(2/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{2/3}}dx$$

$$\Big\downarrow \; 4357$$

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}dx$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(2/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+b\sec \left(dx+c\right)}}{\sec \left(dx+c\right)^{\frac{2}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x)
```

output
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x)
```

Fricas [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx=\int \frac{\sqrt{b\sec \left(dx+c\right)+a}}{\sec \left(dx+c\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(2/3), x)
```

Sympy [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx=\int \frac{\sqrt{a+b\sec \left(c+dx\right)}}{\sec^{\frac{2}{3}}\left(c+dx\right)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(2/3),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(2/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx=\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(2/3), x)
```

Giac [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx=\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(2/3), x)
```

Mupad [N/A]

Not integrable

Time = 10.72 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(2/3),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(2/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{\frac{2}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)/sec(c + d*x)**(2/3),x)
```

3.724 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \text{Int}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)},x\right)$$

output
```
Defer(Int)((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x)
```

Mathematica [N/A]

Not integrable

Time = 71.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(4/3),x]
```

output
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(4/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}}dx$$
$$\Big\downarrow \text{4357}$$
$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}dx$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(4/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+b\sec(dx+c)}}{\sec(dx+c)^{\frac{4}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x)
```

output
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x)
```

Fricas [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(4/3), x)
```

Sympy [N/A]

Not integrable

Time = 4.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(4/3),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(4/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx=\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(4/3), x)
```

Giac [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx=\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(4/3), x)
```

Mupad [N/A]

Not integrable

Time = 11.88 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx=\int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{4/3}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(4/3),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(4/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)b+a}}{\sec\left(dx+c\right)^{\frac{4}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)/(sec(c + d*x)**(1/3)*sec(c + d*x)),x)
```

3.725 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx=\mathrm{Int}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)},x\right)$$

output
```
Defer(Int)((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x)
```

Mathematica [N/A]

Not integrable

Time = 129.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx=\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/3),x]
```

output
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/3}}dx$$

$$\Big\downarrow \ 4357$$

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}dx$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+b\sec(dx+c)}}{\sec(dx+c)^{\frac{5}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x)
```

output
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x)
```

Fricas [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/3), x)
```

Sympy [N/A]

Not integrable

Time = 11.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(5/3),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(5/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx=\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/3), x)
```

Giac [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx=\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/3), x)
```

Mupad [N/A]

Not integrable

Time = 10.62 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(5/3),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(5/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)b+a}}{\sec\left(dx+c\right)^{\frac{5}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)/(sec(c + d*x)**(2/3)*sec(c + d*x)),x)
```

3.726 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \text{Int}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)},x\right)$$

output
```
Defer(Int)((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x)
```

Mathematica [N/A]

Not integrable

Time = 149.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/3),x]
```

output
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/3}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}dx$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+b\sec(dx+c)}}{\sec(dx+c)^{\frac{7}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x)
```

output
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x)
```

Fricas [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(7/3),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/3), x)
```

Giac [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/3), x)
```

Mupad [N/A]

Not integrable

Time = 12.52 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{7/3}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(7/3),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(7/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{\frac{7}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)/(sec(c + d*x)**(1/3)*sec(c + d*x)**2),x)
```

3.727 $\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\text{Int}\left(\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 149.69 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

input
```
Integrate[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Integrate[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}dx$$

input
```
Int[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec \left(dx+c\right)^{\frac{7}{3}} \left(a+b\sec \left(dx+c\right)\right)^{\frac{3}{2}}dx$$

input
```
int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec \left(dx+c\right)+a)^{\frac{3}{2}}\sec \left(dx+c\right)^{\frac{7}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c)^3 + a*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a)*se
c(d*x + c)^(1/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/3)*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int (b\sec{(dx+c)}+a)^{\frac{3}{2}}\sec{(dx+c)}^{\frac{7}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/3), x)
```

Giac [N/A]

Not integrable

Time = 2.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int (b\sec{(dx+c)}+a)^{\frac{3}{2}}\sec{(dx+c)}^{\frac{7}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/3), x)
```

Mupad [N/A]

Not integrable

Time = 12.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int {\left(a+\frac{b}{\cos{(c+d\,x)}}\right)}^{3/2}\,{\left(\frac{1}{\cos{(c+d\,x)}}\right)}^{7/3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(7/3),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(7/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \left(\int \sec(dx+c)^{\frac{10}{3}}\sqrt{\sec(dx+c)\,b+a}dx\right)b + \left(\int \sec(dx+c)^{\frac{7}{3}}\sqrt{\sec(dx+c)\,b+a}dx\right)a$$

input
```
int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**3,x)*b + in
t(sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**2,x)*a
```

3.728 $\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\text{Int}\left(\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 124.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

input
```
Integrate[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Integrate[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}dx$$

input
```
Int[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec \left(d x+c\right)^{\frac{5}{3}} \left(a+b \sec \left(d x+c\right)\right)^{\frac{3}{2}}d x$$

input
```
int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int { {\left(b \sec \left(d x + c\right) + a\right)}^{\frac{3}{2}} \sec \left(d x + c\right)^{\frac{5}{3}} \,d x}$$

input
```
integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c)^2 + a*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sec(
d*x + c)^(2/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/3)*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/3), x)
```

Giac [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/3), x)
```

Mupad [N/A]

Not integrable

Time = 11.71 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/3),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \left(\int \sec(dx+c)^{\frac{8}{3}}\sqrt{\sec(dx+c)\,b+a}dx\right)b + \left(\int \sec(dx+c)^{\frac{5}{3}}\sqrt{\sec(dx+c)\,b+a}dx\right)a$$

input
```
int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sec(c + d*x)**(2/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**2,x)*b + in
t(sec(c + d*x)**(2/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x),x)*a
```

3.729 $\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \text{Int}\left(\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 91.86 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

input
```
Integrate[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Integrate[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}dx$$

input
```
Int[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec \left(dx+c\right)^{\frac{4}{3}} \left(a+b\sec \left(dx+c\right)\right)^{\frac{3}{2}}dx$$

input
```
int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int \left(b\sec \left(dx+c\right)+a\right)^{\frac{3}{2}}\sec \left(dx+c\right)^{\frac{4}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c)^2 + a*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sec(
d*x + c)^(1/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**(4/3)*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{4}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(4/3), x)
```

Giac [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{4}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(4/3), x)
```

Mupad [N/A]

Not integrable

Time = 11.65 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(4/3),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(4/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \left(\int \sec(dx+c)^{\frac{7}{3}}\sqrt{\sec(dx+c)\,b+a}dx\right)b + \left(\int \sec(dx+c)^{\frac{4}{3}}\sqrt{\sec(dx+c)\,b+a}dx\right)a$$

input
```
int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**2,x)*b + in
t(sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x),x)*a
```

3.730 $\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\operatorname{Int}\left(\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 119.67 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

input
```
Integrate[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Integrate[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{2/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}dx$$

input
```
Int[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec(dx+c)^{\frac{2}{3}} (a+b\sec(dx+c))^{\frac{3}{2}}\, dx$$

input
```
int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\, dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}} \sec(dx+c)^{\frac{2}{3}}\, dx$$

input
```
integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(2/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\, dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(2/3)*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{2}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(2/3), x)
```

Giac [N/A]

Not integrable

Time = 1.67 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{2}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(2/3), x)
```

Mupad [N/A]

Not integrable

Time = 11.47 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(2/3),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(2/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \left(\int \sec\left(dx+c\right)^{\frac{5}{3}}\sqrt{\sec\left(dx+c\right)b+a}dx\right)b + \left(\int \sec\left(dx+c\right)^{\frac{2}{3}}\sqrt{\sec\left(dx+c\right)b+a}dx\right)a$$

input
```
int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sec(c + d*x)**(2/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x),x)*b + int(s
ec(c + d*x)**(2/3)*sqrt(sec(c + d*x)*b + a),x)*a
```

3.731 $\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \text{Int}\left(\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 84.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$$

input
```
Integrate[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Integrate[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}dx$$

input
```
Int[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec\left(dx+c\right)^{\frac{1}{3}} \left(a+b\sec\left(dx+c\right)\right)^{\frac{3}{2}} dx$$

input
```
int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx=\int \left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{1}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(1/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**(1/3)*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{1}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(1/3), x)
```

Giac [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{1}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(1/3), x)
```

Mupad [N/A]

Not integrable

Time = 11.60 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/3),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \left(\int \sec(dx+c)^{\frac{4}{3}}\sqrt{\sec(dx+c)\,b+a}dx\right)b+\left(\int \sec(dx+c)^{\frac{1}{3}}\sqrt{\sec(dx+c)\,b+a}dx\right)a$$

input
```
int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x),x)*b + int(s
ec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a),x)*a
```

3.732 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx = \text{Int}\left(\frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}},x\right)$$

output
```
Defer(Int)((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x)
```

Mathematica [N/A]

Not integrable

Time = 151.76 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx = \int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{ 4357}$$
$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sec(dx+c))^{\frac{3}{2}}}{\sec(dx+c)^{\frac{1}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x)
```

output
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x)
```

Fricas [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(1/3), x)
```

Sympy [N/A]

Not integrable

Time = 22.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx = \int \frac{(a+b\sec(c+dx))^{\frac{3}{2}}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)/sec(c + d*x)**(1/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(1/3), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 12.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/3),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx=\left(\int \frac{\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{\frac{1}{3}}}dx\right)a+\left(\int \sec(dx+c)^{\frac{2}{3}}\sqrt{\sec(dx+c)\,b+a}dx\right)b$$

input
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)/sec(c + d*x)**(1/3),x)*a + int((sqrt(sec(c +
d*x)*b + a)*sec(c + d*x))/sec(c + d*x)**(1/3),x)*b
```

3.733 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \operatorname{Int}\left(\frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)},x\right)$$

output
```
Defer(Int)((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x)
```

Mathematica [N/A]

Not integrable

Time = 82.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(2/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(2/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{2/3}}dx$$

$$\Big\downarrow \ 4357$$

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(2/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sec (dx+c))^{\frac{3}{2}}}{\sec (dx+c)^{\frac{2}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x)
```

output
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x)
```

Fricas [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx=\int \frac{(b\sec (dx+c)+a)^{\frac{3}{2}}}{\sec (dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(2/3), x)
```

Sympy [N/A]

Not integrable

Time = 13.92 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx=\int \frac{(a+b\sec (c+dx))^{\frac{3}{2}}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(2/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)/sec(c + d*x)**(2/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(2/3), x)
```

Giac [N/A]

Not integrable

Time = 154.88 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(2/3), x)
```

Mupad [N/A]

Not integrable

Time = 10.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(2/3),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(2/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{\frac{2}{3}}}dx\right)a + \left(\int \sec(dx+c)^{\frac{1}{3}}\sqrt{\sec(dx+c)\,b+a}dx\right)b$$

input
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)/sec(c + d*x)**(2/3),x)*a + int((sqrt(sec(c +
d*x)*b + a)*sec(c + d*x))/sec(c + d*x)**(2/3),x)*b
```

3.734 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \text{Int}\left(\frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)},x\right)$$

output
```
Defer(Int)((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x)
```

Mathematica [N/A]

Not integrable

Time = 70.67 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(4/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(4/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}}dx$$
$$\Big\downarrow \; 4357$$
$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(4/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sec (dx+c))^{\frac{3}{2}}}{\sec (dx+c)^{\frac{4}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x)
```

output
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x)
```

Fricas [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx=\int \frac{(b\sec (dx+c)+a)^{\frac{3}{2}}}{\sec (dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(4/3), x)
```

Sympy [N/A]

Not integrable

Time = 33.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx=\int \frac{(a+b\sec (c+dx))^{\frac{3}{2}}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(4/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)/sec(c + d*x)**(4/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(4/3), x)
```

Giac [N/A]

Not integrable

Time = 175.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(4/3), x)
```

Mupad [N/A]

Not integrable

Time = 11.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(4/3),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(4/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{\frac{1}{3}}}dx\right)b + \left(\int \frac{\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{\frac{4}{3}}}dx\right)a$$

input
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)/sec(c + d*x)**(1/3),x)*b + int(sqrt(sec(c + d
*x)*b + a)/(sec(c + d*x)**(1/3)*sec(c + d*x)),x)*a
```

3.735 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \text{Int}\left(\frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)},x\right)$$

output
```
Defer(Int)((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x)
```

Mathematica [N/A]

Not integrable

Time = 147.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/3}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sec(dx+c))^{\frac{3}{2}}}{\sec(dx+c)^{\frac{5}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x)
```

output
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x)
```

Fricas [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/3), x)
```

Sympy [N/A]

Not integrable

Time = 82.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{\frac{3}{2}}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(5/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)/sec(c + d*x)**(5/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx=\int \frac{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}{\sec{(dx+c)}^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/3), x)
```

Giac [N/A]

Not integrable

Time = 145.38 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx=\int \frac{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}{\sec{(dx+c)}^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/3), x)
```

Mupad [N/A]

Not integrable

Time = 10.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(5/3),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(5/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx=\left(\int \frac{\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{\frac{2}{3}}}dx\right)b+\left(\int \frac{\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{\frac{5}{3}}}dx\right)a$$

input
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)/sec(c + d*x)**(2/3),x)*b + int(sqrt(sec(c + d
*x)*b + a)/(sec(c + d*x)**(2/3)*sec(c + d*x)),x)*a
```

3.736 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx=\text{Int}\left(\frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)},x\right)$$

output
```
Defer(Int)((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x)
```

Mathematica [N/A]

Not integrable

Time = 172.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx=\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/3}}dx$$

$$\Big\downarrow \text{ 4357}$$

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sec(dx+c))^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x)
```

output
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x)
```

Fricas [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(7/3),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/3), x)
```

Giac [N/A]

Not integrable

Time = 146.38 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/3), x)
```

Mupad [N/A]

Not integrable

Time = 11.84 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(7/3),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(7/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{\frac{7}{3}}}dx\right)a + \left(\int \frac{\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{\frac{4}{3}}}dx\right)b$$

input
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)/(sec(c + d*x)**(1/3)*sec(c + d*x)**2),x)*a +
int(sqrt(sec(c + d*x)*b + a)/(sec(c + d*x)**(1/3)*sec(c + d*x)),x)*b
```

3.737 $\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Int}\left(\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 143.63 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

input
```
Integrate[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Integrate[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}dx$$

input
```
Int[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec(dx+c)^{\frac{7}{3}} (a+b\sec(dx+c))^{\frac{5}{2}}\,dx$$

input
```
int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.40

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{7}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^4 + 2*a*b*sec(d*x + c)^3 + a^2*sec(d*x + c)^2)*
sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/3)*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 2.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{7}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/3), x)
```

Giac [N/A]

Not integrable

Time = 3.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{7}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/3), x)
```

Mupad [N/A]

Not integrable

Time = 13.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(7/3),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(7/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.16

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \left(\int \sec(dx+c)^{\frac{13}{3}}\sqrt{\sec(dx+c)\,b+a}dx\right)b^2 + 2\left(\int \sec(dx+c)^{\frac{10}{3}}\sqrt{\sec(dx+c)\,b+a}dx\right)ab + \left(\int \sec(dx+c)^{\frac{7}{3}}\sqrt{\sec(dx+c)\,b+a}dx\right)a^2$$

input
```
int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**4,x)*b**2 +
 2*int(sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**3,x)*a*b
 + int(sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**2,x)*a**
2
```

3.738 $\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx=\text{Int}\left(\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 118.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx=\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

input
```
Integrate[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Integrate[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$
$$\Big\downarrow \text{4357}$$
$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}dx$$

input
```
Int[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec(dx+c)^{\frac{5}{3}} (a+b\sec(dx+c))^{\frac{5}{2}} dx$$

input
```
int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.32

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2} dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}} \sec(dx+c)^{\frac{5}{3}} dx$$

input
```
integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^3 + 2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c))*sq
rt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2} dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/3)*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 2.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/3), x)
```

Giac [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/3), x)
```

Mupad [N/A]

Not integrable

Time = 12.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/3),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.16

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \left(\int \sec(dx+c)^{\frac{11}{3}}\sqrt{\sec(dx+c)\,b+a}dx\right)b^2 + 2\left(\int \sec(dx+c)^{\frac{8}{3}}\sqrt{\sec(dx+c)\,b+a}dx\right)ab + \left(\int \sec(dx+c)^{\frac{5}{3}}\sqrt{\sec(dx+c)\,b+a}dx\right)a^2$$

input
```
int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sec(c + d*x)**(2/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**3,x)*b**2 +
 2*int(sec(c + d*x)**(2/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**2,x)*a*b
 + int(sec(c + d*x)**(2/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x),x)*a**2
```

3.739 $\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Int}\left(\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 112.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

input
```
Integrate[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Integrate[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}dx$$

input
```
Int[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec(dx+c)^{\frac{4}{3}} (a+b\sec(dx+c))^{\frac{5}{2}} dx$$

input
```
int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.32

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{4}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^3 + 2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c))*sq
rt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(4/3)*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 2.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{4}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(4/3), x)
```

Giac [N/A]

Not integrable

Time = 2.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{4}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(4/3), x)
```

Mupad [N/A]

Not integrable

Time = 12.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int {\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{4/3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(4/3),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(4/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.16

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \left(\int \sec(dx+c)^{\frac{10}{3}}\sqrt{\sec(dx+c)\,b+a}\,dx\right)b^2 + 2\left(\int \sec(dx+c)^{\frac{7}{3}}\sqrt{\sec(dx+c)\,b+a}\,dx\right)ab + \left(\int \sec(dx+c)^{\frac{4}{3}}\sqrt{\sec(dx+c)\,b+a}\,dx\right)a^2$$

input
```
int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**3,x)*b**2 +
 2*int(sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**2,x)*a*b
 + int(sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x),x)*a**2
```

3.740 $\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Int}\left(\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 113.64 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

input
```
Integrate[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Integrate[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{2/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}dx$$

input
```
Int[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec (dx+c)^{\frac{2}{3}} (a+b\sec (dx+c))^{\frac{5}{2}} dx$$

input
```
int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec (dx+c)+a)^{\frac{5}{2}} \sec (dx+c)^{\frac{2}{3}} \,dx$$

input
```
integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x +
c) + a)*sec(d*x + c)^(2/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(2/3)*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 2.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{2}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(2/3), x)
```

Giac [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{2}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(2/3), x)
```

Mupad [N/A]

Not integrable

Time = 12.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int {\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{5/2}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{2/3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(2/3),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(2/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.16

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \left(\int \sec(dx+c)^{\frac{8}{3}}\sqrt{\sec(dx+c)\,b+a}dx\right)b^2 + 2\left(\int \sec(dx+c)^{\frac{5}{3}}\sqrt{\sec(dx+c)\,b+a}dx\right)ab + \left(\int \sec(dx+c)^{\frac{2}{3}}\sqrt{\sec(dx+c)\,b+a}dx\right)a^2$$

input
```
int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sec(c + d*x)**(2/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**2,x)*b**2 +
 2*int(sec(c + d*x)**(2/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x),x)*a*b +
int(sec(c + d*x)**(2/3)*sqrt(sec(c + d*x)*b + a),x)*a**2
```

3.741 $\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \text{Int}\left(\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 97.59 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$$

input
```
Integrate[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Integrate[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$
$$\Big\downarrow \text{4357}$$
$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}dx$$

input
```
Int[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec(dx+c)^{\frac{1}{3}} (a+b\sec(dx+c))^{\frac{5}{2}} dx$$

input
```
int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2} dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}} \sec(dx+c)^{\frac{1}{3}} dx$$

input
```
integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x +
c) + a)*sec(d*x + c)^(1/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2} dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(1/3)*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 1.87 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{1}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(1/3), x)
```

Giac [N/A]

Not integrable

Time = 1.72 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{1}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(1/3), x)
```

Mupad [N/A]

Not integrable

Time = 11.99 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/3),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.16

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \left(\int \sec(dx+c)^{\frac{7}{3}}\sqrt{\sec(dx+c)\,b+a}dx\right)b^2 + 2\left(\int \sec(dx+c)^{\frac{4}{3}}\sqrt{\sec(dx+c)\,b+a}dx\right)ab + \left(\int \sec(dx+c)^{\frac{1}{3}}\sqrt{\sec(dx+c)\,b+a}dx\right)a^2$$

input
```
int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**2,x)*b**2 +
 2*int(sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x),x)*a*b +
int(sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a),x)*a**2
```

3.742 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx = \text{Int}\left(\frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}},x\right)$$

output
```
Defer(Int)((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x)
```

Mathematica [N/A]

Not integrable

Time = 158.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx = \int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(1/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(1/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(1/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sec(dx+c))^{\frac{5}{2}}}{\sec(dx+c)^{\frac{1}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x)
```

output
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x)
```

Fricas [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x +
c) + a)/sec(d*x + c)^(1/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/3),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 1.85 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx=\int \frac{(b\sec{(dx+c)}+a)^{\frac{5}{2}}}{\sec{(dx+c)}^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(1/3), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 12.78 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/3),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.16

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx=\left(\int \frac{\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{\frac{1}{3}}}dx\right)a^2 + \left(\int \sec(dx+c)^{\frac{5}{3}}\sqrt{\sec(dx+c)\,b+a}dx\right)b^2 + 2\left(\int \sec(dx+c)^{\frac{2}{3}}\sqrt{\sec(dx+c)\,b+a}dx\right)ab$$

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)/sec(c + d*x)**(1/3),x)*a**2 + int((sqrt(sec(c
 + d*x)*b + a)*sec(c + d*x)**2)/sec(c + d*x)**(1/3),x)*b**2 + 2*int((sqrt(
sec(c + d*x)*b + a)*sec(c + d*x))/sec(c + d*x)**(1/3),x)*a*b
```

3.743 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \text{Int}\left(\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)},x\right)$$

output
```
Defer(Int)((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x)
```

Mathematica [N/A]

Not integrable

Time = 134.93 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(2/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(2/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{2/3}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(2/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sec(dx+c))^{\frac{5}{2}}}{\sec(dx+c)^{\frac{2}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x)
```

output
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x)
```

Fricas [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x +
c) + a)/sec(d*x + c)^(2/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(2/3),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 1.82 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(2/3), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 10.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(2/3),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(2/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.16

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx=\left(\int \frac{\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{\frac{2}{3}}}dx\right)a^2 +\left(\int \sec(dx+c)^{\frac{4}{3}}\sqrt{\sec(dx+c)\,b+a}dx\right)b^2 +2\left(\int \sec(dx+c)^{\frac{1}{3}}\sqrt{\sec(dx+c)\,b+a}dx\right)ab$$

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)/sec(c + d*x)**(2/3),x)*a**2 + int((sqrt(sec(c
 + d*x)*b + a)*sec(c + d*x)**2)/sec(c + d*x)**(2/3),x)*b**2 + 2*int((sqrt(
sec(c + d*x)*b + a)*sec(c + d*x))/sec(c + d*x)**(2/3),x)*a*b
```

3.744 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \text{Int}\left(\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)},x\right)$$

output
```
Defer(Int)((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x)
```

Mathematica [N/A]

Not integrable

Time = 149.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(4/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(4/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}}dx$$
$$\Big\downarrow \; 4357$$
$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(4/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sec (dx+c))^{\frac{5}{2}}}{\sec (dx+c)^{\frac{4}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x)
```

output
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x)
```

Fricas [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx=\int \frac{(b\sec (dx+c)+a)^{\frac{5}{2}}}{\sec (dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x +
c) + a)/sec(d*x + c)^(4/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(4/3),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 2.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(4/3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:int()   Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 12.72 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a + b\sec(c + dx))^{5/2}}{\sec^{\frac{4}{3}}(c + dx)}\, dx = \int \frac{\left(a + \frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}}\, dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(4/3),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(4/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.16

$$\int \frac{(a + b\sec(c + dx))^{5/2}}{\sec^{\frac{4}{3}}(c + dx)}\, dx = 2\left(\int \frac{\sqrt{\sec(dx + c)\, b + a}}{\sec(dx + c)^{\frac{1}{3}}}dx\right) ab + \left(\int \frac{\sqrt{\sec(dx + c)\, b + a}}{\sec(dx + c)^{\frac{4}{3}}}dx\right) a^2 + \left(\int \sec(dx + c)^{\frac{2}{3}} \sqrt{\sec(dx + c)\, b + a}dx\right) b^2$$

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x)
```

output
```
2*int(sqrt(sec(c + d*x)*b + a)/sec(c + d*x)**(1/3),x)*a*b + int(sqrt(sec(c
 + d*x)*b + a)/(sec(c + d*x)**(1/3)*sec(c + d*x)),x)*a**2 + int((sqrt(sec(
c + d*x)*b + a)*sec(c + d*x))/sec(c + d*x)**(1/3),x)*b**2
```

3.745 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx=\text{Int}\left(\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)},x\right)$$

output
```
Defer(Int)((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x)
```

Mathematica [N/A]

Not integrable

Time = 160.37 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx=\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/3}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sec(dx+c))^{\frac{5}{2}}}{\sec(dx+c)^{\frac{5}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x)
```

output
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x)
```

Fricas [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x +
c) + a)/sec(d*x + c)^(5/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(5/3),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 1.86 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5/3), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 10.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(5/3),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(5/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.16

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = 2\left(\int \frac{\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{\frac{2}{3}}}dx\right)ab + \left(\int \frac{\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{\frac{5}{3}}}dx\right)a^2 + \left(\int \sec(dx+c)^{\frac{1}{3}}\sqrt{\sec(dx+c)\,b+a}\,dx\right)b^2$$

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x)
```

output
```
2*int(sqrt(sec(c + d*x)*b + a)/sec(c + d*x)**(2/3),x)*a*b + int(sqrt(sec(c
 + d*x)*b + a)/(sec(c + d*x)**(2/3)*sec(c + d*x)),x)*a**2 + int((sqrt(sec(
c + d*x)*b + a)*sec(c + d*x))/sec(c + d*x)**(2/3),x)*b**2
```

3.746 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \text{Int}\left(\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)},x\right)$$

output
```
Defer(Int)((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x)
```

Mathematica [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \$\text{Aborted}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/3),x]
```

output
```
$Aborted
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/3}}dx$$
$$\Big\downarrow \text{4357}$$
$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sec(dx+c))^{\frac{5}{2}}}{\sec(dx+c)^{\frac{7}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x)
```

output
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x)
```

Fricas [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x +
c) + a)/sec(d*x + c)^(7/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(7/3),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 1.82 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \int \frac{(b\sec{(dx+c)}+a)^{\frac{5}{2}}}{\sec{(dx+c)}^{\frac{7}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7/3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:int()  Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 12.70 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a + b\sec(c + dx))^{5/2}}{\sec^{\frac{7}{3}}(c + dx)}\, dx = \int \frac{\left(a + \frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}}\, dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(7/3),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(7/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.16

$$\int \frac{(a + b\sec(c + dx))^{5/2}}{\sec^{\frac{7}{3}}(c + dx)}\, dx = \left(\int \frac{\sqrt{\sec (dx + c)\, b + a}}{\sec (dx + c)^{\frac{1}{3}}} dx\right) b^2 + \left(\int \frac{\sqrt{\sec (dx + c)\, b + a}}{\sec (dx + c)^{\frac{7}{3}}} dx\right) a^2 + 2\left(\int \frac{\sqrt{\sec (dx + c)\, b + a}}{\sec (dx + c)^{\frac{4}{3}}} dx\right) ab$$

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)/sec(c + d*x)**(1/3),x)*b**2 + int(sqrt(sec(c
+ d*x)*b + a)/(sec(c + d*x)**(1/3)*sec(c + d*x)**2),x)*a**2 + 2*int(sqrt(s
ec(c + d*x)*b + a)/(sec(c + d*x)**(1/3)*sec(c + d*x)),x)*a*b
```

3.747 $\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Int}\left(\frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 94.80 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(7/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
Integrate[Sec[c + d*x]^(7/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/3}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{4357}$$
$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

input
```
Int[Sec[c + d*x]^(7/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec\left(dx+c\right)^{\frac{7}{3}}}{\sqrt{a+b\sec\left(dx+c\right)}}dx$$

input
```
int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{7}{3}}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^(7/3)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{7}{3}}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(7/3)/sqrt(b*sec(d*x + c) + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 11.35 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/3}}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(7/3)/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(7/3)/(a + b/cos(c + d*x))^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{7}{3}}\sqrt{\sec\left(dx+c\right)b+a}}{\sec\left(dx+c\right)b+a}dx$$

input
```
int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**2)/(sec(c
+ d*x)*b + a),x)
```

3.748 $\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Int}\left(\frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 111.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(5/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
Integrate[Sec[c + d*x]^(5/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/3}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{ 4357}$$
$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

input
```
Int[Sec[c + d*x]^(5/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec (dx+c)^{\frac{5}{3}}}{\sqrt{a+b\sec (dx+c)}}dx$$

input
```
int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec (dx+c)^{\frac{5}{3}}}{\sqrt{b\sec (dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^(5/3)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^{\frac{5}{3}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(5/3)/sqrt(b*sec(d*x + c) + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 11.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/3)/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(5/3)/(a + b/cos(c + d*x))^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{3}}\sqrt{\sec\left(dx+c\right)b+a}}{\sec\left(dx+c\right)b+a}dx$$

input
```
int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((sec(c + d*x)**(2/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x))/(sec(c + d
*x)*b + a),x)
```

3.749 $\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\text{Int}\left(\frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 16.69 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(4/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
Integrate[Sec[c + d*x]^(4/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4357}$$

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

input
```
Int[Sec[c + d*x]^(4/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec\left(dx+c\right)^{\frac{4}{3}}}{\sqrt{a+b\sec\left(dx+c\right)}}dx$$

input
```
int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{4}{3}}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^(4/3)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{4}{3}}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(4/3)/sqrt(b*sec(d*x + c) + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 11.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(4/3)/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(4/3)/(a + b/cos(c + d*x))^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{4}{3}}\sqrt{\sec\left(dx+c\right)b+a}}{\sec\left(dx+c\right)b+a}dx$$

input
```
int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x))/(sec(c + d
*x)*b + a),x)
```

3.750 $\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Int}\left(\frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 16.66 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(2/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
Integrate[Sec[c + d*x]^(2/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{2/3}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{4357}$$
$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

input
```
Int[Sec[c + d*x]^(2/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec(dx+c)^{\frac{2}{3}}}{\sqrt{a+b\sec(dx+c)}}dx$$

input
```
int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^{\frac{2}{3}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^(2/3)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 3.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**(2/3)/sqrt(a + b*sec(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{2}{3}}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(2/3)/sqrt(b*sec(d*x + c) + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 11.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x))^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^{\frac{2}{3}}\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)\,b+a}dx$$

input
```
int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((sec(c + d*x)**(2/3)*sqrt(sec(c + d*x)*b + a))/(sec(c + d*x)*b + a),x)
```

3.751 $\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Int}\left(\frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(1/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
Integrate[Sec[c + d*x]^(1/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx$$

input
```
Int[Sec[c + d*x]^(1/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec(dx+c)^{\frac{1}{3}}}{\sqrt{a+b\sec(dx+c)}}dx$$

input
```
int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^{\frac{1}{3}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^(1/3)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**(1/3)/sqrt(a + b*sec(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{1}{3}}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(1/3)/sqrt(b*sec(d*x + c) + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 11.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x))^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\sec(dx+c)^{\frac{1}{3}}\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)\,b+a}dx$$

input
```
int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a))/(sec(c + d*x)*b + a),x)
```

3.752 $\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx=\text{Int}\left(\frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}},x\right)$$

output
```
Defer(Int)(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 2.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4357}$$

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx$$

input
```
Int[1/(Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec(dx+c)^{\frac{1}{3}}\sqrt{a+b\sec(dx+c)}}dx$$

input
```
int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b*sec(d*x + c)^2 + a
*sec(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\sec(c+dx)}\sqrt[3]{\sec(c+dx)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(1/3)), x)
```

Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 12.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{1/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/3)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{\frac{4}{3}}\,b+\sec(dx+c)^{\frac{1}{3}}\,a}dx$$

input
```
int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)/(sec(c + d*x)**(1/3)*sec(c + d*x)*b + sec(c +
 d*x)**(1/3)*a),x)
```

3.753 $\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \text{Int}\left(\frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}},x\right)$$

output
```
Defer(Int)(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 93.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{2/3}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{4357}$$
$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$

input
```
Int[1/(Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec(dx+c)^{\frac{2}{3}} \sqrt{a+b\sec(dx+c)}}dx$$

input
```
int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b*sec(d*x + c)^2 + a
*sec(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 1.64 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\sec(c+dx)}\sec^{\frac{2}{3}}(c+dx)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(2/3)), x)
```

Maxima [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 10.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{2/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(2/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(2/3)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.72

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^{\frac{1}{3}}\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^2 b+\sec(dx+c)\,a}dx$$

input
```
int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a))/(sec(c + d*x)**2*b + se
c(c + d*x)*a),x)
```

3.754 $\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \text{Int}\left(\frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}},x\right)$$

output
```
Defer(Int)(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 96.78 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{ 4357}$$
$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$

input
```
Int[1/(Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec (dx + c)^{\frac{4}{3}} \sqrt{a + b \sec (dx + c)}} dx$$

input
```
int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \frac{1}{\sec^{\frac{4}{3}}(c + dx)\sqrt{a + b\sec(c + dx)}} dx = \int \frac{1}{\sqrt{b \sec (dx + c) + a} \sec (dx + c)^{\frac{4}{3}}} dx$$

input
```
integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b*sec(d*x + c)^3 + a
*sec(d*x + c)^2), x)
```

Sympy [N/A]

Not integrable

Time = 12.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sec^{\frac{4}{3}}(c + dx)\sqrt{a + b\sec(c + dx)}} dx = \int \frac{1}{\sqrt{a + b \sec (c + dx)} \sec^{\frac{4}{3}} (c + dx)} dx$$

input
```
integrate(1/sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(4/3)), x)
```

Maxima [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 12.78 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{4/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(4/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(4/3)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{\frac{7}{3}}\,b+\sec(dx+c)^{\frac{4}{3}}\,a}dx$$

input
```
int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)/(sec(c + d*x)**(1/3)*sec(c + d*x)**2*b + sec(
c + d*x)**(1/3)*sec(c + d*x)*a),x)
```

3.755 $\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx=\text{Int}\left(\frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}},x\right)$$

output
```
Defer(Int)(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 112.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/3}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$

input
```
Int[1/(Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec (dx+c)^{\frac{5}{3}} \sqrt{a+b\sec (dx+c)}}dx$$

input
```
int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\, dx = \int \frac{1}{\sqrt{b\sec (dx+c)+a}\sec (dx+c)^{\frac{5}{3}}}\, dx$$

input
```
integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b*sec(d*x + c)^3 + a
*sec(d*x + c)^2), x)
```

Sympy [N/A]

Not integrable

Time = 29.68 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\, dx = \int \frac{1}{\sqrt{a+b\sec (c+dx)}\sec^{\frac{5}{3}}(c+dx)}\, dx$$

input
```
integrate(1/sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(5/3)), x)
```

Maxima [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 10.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/3)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^{\frac{1}{3}}\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^3\,b+\sec(dx+c)^2\,a}dx$$

input
```
int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a))/(sec(c + d*x)**3*b + se
c(c + d*x)**2*a),x)
```

3.756 $\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \text{Int}\left(\frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}},x\right)$$

output
```
Defer(Int)(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 125.51 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/3}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{4357}$$
$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$

input
```
Int[1/(Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec\left(dx+c\right)^{\frac{7}{3}}\sqrt{a+b\sec\left(dx+c\right)}}dx$$

input
```
int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b*sec(d*x + c)^4 + a
*sec(d*x + c)^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 13.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{7/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(7/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(7/3)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{\frac{10}{3}}\,b+\sec(dx+c)^{\frac{7}{3}}\,a}dx$$

input
```
int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)/(sec(c + d*x)**(1/3)*sec(c + d*x)**3*b + sec(
c + d*x)**(1/3)*sec(c + d*x)**2*a),x)
```

3.757 $\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Int}\left(\frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 121.98 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Integrate[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/3}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$
$$\Big\downarrow \text{4357}$$
$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx$$

input
```
Int[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec (dx+c)^{\frac{7}{3}}}{(a+b\sec (dx+c))^{\frac{3}{2}}}dx$$

input
```
int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec (dx+c)^{\frac{7}{3}}}{(b\sec (dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3)/(b^2*sec(d*x + c)^2 +
 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{7}{3}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(7/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 12.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(7/3)/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(7/3)/(a + b/cos(c + d*x))^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.00

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{7}{3}}\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^2\,b^2+2\sec(dx+c)\,ab+a^2}dx$$

input
```
int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int((sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**2)/(sec(c
+ d*x)**2*b**2 + 2*sec(c + d*x)*a*b + a**2),x)
```

3.758 $\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\text{Int}\left(\frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 128.82 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Integrate[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/3}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4357}$$

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx$$

input
```
Int[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec (dx+c)^{\frac{5}{3}}}{(a+b\sec (dx+c))^{\frac{3}{2}}}dx$$

input
```
int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec (dx+c)^{\frac{5}{3}}}{(b\sec (dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3)/(b^2*sec(d*x + c)^2 +
 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{3}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(5/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 12.66 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/3)/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(5/3)/(a + b/cos(c + d*x))^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.00

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{5}{3}}\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^2\,b^2+2\sec(dx+c)\,ab+a^2}dx$$

input
```
int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int((sec(c + d*x)**(2/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x))/(sec(c + d
*x)**2*b**2 + 2*sec(c + d*x)*a*b + a**2),x)
```

3.759 $\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Int}\left(\frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 112.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Integrate[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$
$$\Big\downarrow \text{ 4357}$$
$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx$$

input
```
Int[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec (dx+c)^{\frac{4}{3}}}{(a+b\sec (dx+c))^{\frac{3}{2}}}dx$$

input
```
int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\sec (dx+c)^{\frac{4}{3}}}{(b\sec (dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3)/(b^2*sec(d*x + c)^2 +
 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{4}{3}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(4/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 12.77 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(4/3)/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(4/3)/(a + b/cos(c + d*x))^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.00

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{4}{3}}\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^2\,b^2+2\sec(dx+c)\,ab+a^2}dx$$

input
```
int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int((sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x))/(sec(c + d
*x)**2*b**2 + 2*sec(c + d*x)*a*b + a**2),x)
```

3.760 $\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Int}\left(\frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 126.82 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Integrate[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{2/3}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$
$$\Big\downarrow \text{4357}$$
$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx$$

input
```
Int[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec \left(d x+c \right)^{\frac{2}{3}}}{\left(a +b \sec \left(d x+c \right)\right)^{\frac{3}{2}}}d x$$

input
```
int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\sec \left(dx+c\right)^{\frac{2}{3}}}{{\left(b \sec \left(dx+c\right)+a\right)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^2*sec(d*x + c)^2 +
 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 11.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\sec^{\frac{2}{3}}\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**(2/3)/(a + b*sec(c + d*x))**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{2}{3}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(2/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 12.96 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x))^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.00

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{2}{3}}\sqrt{\sec\left(dx+c\right)b+a}}{\sec\left(dx+c\right)^{2}b^{2}+2\sec\left(dx+c\right)ab+a^{2}}dx$$

input
```
int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int((sec(c + d*x)**(2/3)*sqrt(sec(c + d*x)*b + a))/(sec(c + d*x)**2*b**2 +
 2*sec(c + d*x)*a*b + a**2),x)
```

3.761 $\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Int}\left(\frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 109.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Integrate[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \ 4357$$

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}dx$$

input
```
Int[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec(dx+c)^{\frac{1}{3}}}{(a+b\sec(dx+c))^{\frac{3}{2}}}dx$$

input
```
int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{1}{3}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^2*sec(d*x + c)^2 +
 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 4.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**(1/3)/(a + b*sec(c + d*x))**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{1}{3}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(1/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 12.93 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x))^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{1}{3}}\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^2\,b^2+2\sec(dx+c)\,ab+a^2}dx$$

input
```
int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int((sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a))/(sec(c + d*x)**2*b**2 +
 2*sec(c + d*x)*a*b + a**2),x)
```

3.762 $\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx=\text{Int}\left(\frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}},x\right)$$

output
```
Defer(Int)(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 113.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$
$$\Big\downarrow \text{4357}$$
$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx$$

input
```
Int[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec (dx+c)^{\frac{1}{3}} (a+b\sec (dx+c))^{\frac{3}{2}}}dx$$

input
```
int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.40

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec (dx+c)+a)^{\frac{3}{2}} \sec (dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^2*sec(d*x + c)^3 +
 2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 5.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+b\sec (c+dx))^{\frac{3}{2}} \sqrt[3]{\sec (c+dx)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**(1/3)), x)
```

Maxima [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(1/3)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 13.72 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/3)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.12

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{\frac{7}{3}}\,b^2+2\sec(dx+c)^{\frac{4}{3}}\,ab+\sec(dx+c)^{\frac{1}{3}}\,a^2}dx$$

input
```
int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)/(sec(c + d*x)**(1/3)*sec(c + d*x)**2*b**2 + 2
*sec(c + d*x)**(1/3)*sec(c + d*x)*a*b + sec(c + d*x)**(1/3)*a**2),x)
```

3.763 $\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Int}\left(\frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}},x\right)$$

output
```
Defer(Int)(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 115.97 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{2/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$$

input
```
Int[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec(dx+c)^{\frac{2}{3}}\left(a+b\sec(dx+c)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.40

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(b\sec(dx+c)+a\right)^{\frac{3}{2}}\sec(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^2*sec(d*x + c)^3 +
 2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 13.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+b\sec(c+dx)\right)^{\frac{3}{2}}\sec^{\frac{2}{3}}(c+dx)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**(2/3)), x)
```

Maxima [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(2/3)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 10.94 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(2/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(2/3)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.36

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{1}{3}}\sqrt{\sec\left(dx+c\right)b+a}}{\sec\left(dx+c\right)^{3}b^{2}+2\sec\left(dx+c\right)^{2}ab+\sec\left(dx+c\right)a^{2}}dx$$

input
```
int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int((sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a))/(sec(c + d*x)**3*b**2 +
 2*sec(c + d*x)**2*a*b + sec(c + d*x)*a**2),x)
```

3.764 $\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx=\mathrm{Int}\left(\frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}},x\right)$$

output
```
Defer(Int)(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 127.64 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$
$$\Big\downarrow \text{4357}$$
$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$$

input
```
Int[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec\left(dx+c\right)^{\frac{4}{3}} \left(a+b\sec\left(dx+c\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.48

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^2*sec(d*x + c)^4 +
 2*a*b*sec(d*x + c)^3 + a^2*sec(d*x + c)^2), x)
```

Sympy [N/A]

Not integrable

Time = 74.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}\sec^{\frac{4}{3}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**(4/3)), x)
```

Maxima [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(4/3)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 13.80 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(4/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(4/3)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.12

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)b+a}}{\sec\left(dx+c\right)^{\frac{10}{3}}b^{2}+2\sec\left(dx+c\right)^{\frac{7}{3}}ab+\sec\left(dx+c\right)^{\frac{4}{3}}a^{2}}dx$$

input
```
int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)/(sec(c + d*x)**(1/3)*sec(c + d*x)**3*b**2 + 2
*sec(c + d*x)**(1/3)*sec(c + d*x)**2*a*b + sec(c + d*x)**(1/3)*sec(c + d*x
)*a**2),x)
```

3.765 $\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx=\text{Int}\left(\frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}},x\right)$$

output
```
Defer(Int)(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 103.94 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$
$$\Big\downarrow \text{4357}$$
$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$$

input
```
Int[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec(dx+c)^{\frac{5}{3}}\,(a+b\sec(dx+c))^{\frac{3}{2}}}dx$$

input
```
int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.48

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^2*sec(d*x + c)^4 +
 2*a*b*sec(d*x + c)^3 + a^2*sec(d*x + c)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/3)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 10.71 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/3)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.44

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{1}{3}}\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^4\,b^2+2\sec(dx+c)^3\,ab+\sec(dx+c)^2\,a^2}dx$$

input
```
int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int((sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a))/(sec(c + d*x)**4*b**2 +
 2*sec(c + d*x)**3*a*b + sec(c + d*x)**2*a**2),x)
```

3.766 $\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx=\text{Int}\left(\frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}},x\right)$$

output
```
Defer(Int)(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 133.74 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$
$$\Big\downarrow \text{ 4357}$$
$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$$

input
```
Int[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec(dx+c)^{\frac{7}{3}} (a+b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.48

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^2*sec(d*x + c)^5 +
 2*a*b*sec(d*x + c)^4 + a^2*sec(d*x + c)^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/3)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 14.97 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(7/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(7/3)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.12

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{\frac{13}{3}}\,b^2+2\sec(dx+c)^{\frac{10}{3}}\,ab+\sec(dx+c)^{\frac{7}{3}}\,a^2}dx$$

input
```
int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)/(sec(c + d*x)**(1/3)*sec(c + d*x)**4*b**2 + 2
*sec(c + d*x)**(1/3)*sec(c + d*x)**3*a*b + sec(c + d*x)**(1/3)*sec(c + d*x
)**2*a**2),x)
```

3.767 $\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Int}\left(\frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 118.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Integrate[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/3}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 4357$$

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx$$

input
```
Int[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec(dx+c)^{\frac{7}{3}}}{(a+b\sec(dx+c))^{\frac{5}{2}}}dx$$

input
```
int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.68

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{7}{3}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3)/(b^3*sec(d*x + c)^3 +
 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{7}{3}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(7/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 13.99 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(7/3)/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(7/3)/(a + b/cos(c + d*x))^(5/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.64

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{7}{3}}\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^3 b^3+3\sec(dx+c)^2 a\,b^2+3\sec(dx+c)\,a^2b+a^3}dx$$

input
```
int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int((sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x)**2)/(sec(c
+ d*x)**3*b**3 + 3*sec(c + d*x)**2*a*b**2 + 3*sec(c + d*x)*a**2*b + a**3),
x)
```

3.768 $\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx=\text{Int}\left(\frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 135.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Integrate[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/3}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx$$

input
```
Int[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec(dx+c)^{\frac{5}{3}}}{(a+b\sec(dx+c))^{\frac{5}{2}}}dx$$

input
```
int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.68

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{5}{3}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3)/(b^3*sec(d*x + c)^3 +
 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{3}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(5/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 13.98 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/3)/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(5/3)/(a + b/cos(c + d*x))^(5/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.64

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{\sec(dx+c)^{\frac{5}{3}}\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^3 b^3+3\sec(dx+c)^2 a\,b^2+3\sec(dx+c)\,a^2b+a^3}dx$$

input
```
int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int((sec(c + d*x)**(2/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x))/(sec(c + d
*x)**3*b**3 + 3*sec(c + d*x)**2*a*b**2 + 3*sec(c + d*x)*a**2*b + a**3),x)
```

3.769 $\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx=\text{Int}\left(\frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 115.63 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Integrate[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx$$

input
```
Int[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec (dx + c)^{\frac{4}{3}}}{(a + b\sec (dx + c))^{\frac{5}{2}}}dx$$

input
```
int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.68

$$\int \frac{\sec^{\frac{4}{3}}(c + dx)}{(a + b\sec(c + dx))^{5/2}}\, dx = \int \frac{\sec (dx + c)^{\frac{4}{3}}}{(b\sec (dx + c) + a)^{\frac{5}{2}}}\, dx$$

input
```
integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3)/(b^3*sec(d*x + c)^3 +
 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{4}{3}}(c + dx)}{(a + b\sec(c + dx))^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{4}{3}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(4/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 14.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(4/3)/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(4/3)/(a + b/cos(c + d*x))^(5/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.64

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{4}{3}}\sqrt{\sec\left(dx+c\right)b+a}}{\sec\left(dx+c\right)^{3}b^{3}+3\sec\left(dx+c\right)^{2}a\,b^{2}+3\sec\left(dx+c\right)a^{2}b+a^{3}}dx$$

input
```
int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int((sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a)*sec(c + d*x))/(sec(c + d
*x)**3*b**3 + 3*sec(c + d*x)**2*a*b**2 + 3*sec(c + d*x)*a**2*b + a**3),x)
```

3.770 $\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Int}\left(\frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 130.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Integrate[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{2/3}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 4357$$

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx$$

input
```
Int[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec(dx+c)^{\frac{2}{3}}}{(a+b\sec(dx+c))^{\frac{5}{2}}}dx$$

input
```
int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.68

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{2}{3}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^3*sec(d*x + c)^3 +
 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [N/A]

Not integrable

Time = 60.67 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**(2/3)/(a + b*sec(c + d*x))**(5/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{2}{3}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(2/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 15.51 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x))^(5/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.64

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{2}{3}}\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{3}b^{3}+3\sec(dx+c)^{2}a\,b^{2}+3\sec(dx+c)\,a^{2}b+a^{3}}dx$$

input
```
int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int((sec(c + d*x)**(2/3)*sqrt(sec(c + d*x)*b + a))/(sec(c + d*x)**3*b**3 +
 3*sec(c + d*x)**2*a*b**2 + 3*sec(c + d*x)*a**2*b + a**3),x)
```

3.771 $\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Int}\left(\frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}},x\right)$$

output
```
Defer(Int)(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 118.75 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Integrate[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$
$$\Big\downarrow \text{4357}$$
$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}dx$$

input
```
Int[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec (dx + c)^{\frac{1}{3}}}{(a + b\sec (dx + c))^{\frac{5}{2}}}dx$$

input
```
int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.68

$$\int \frac{\sqrt[3]{\sec(c + dx)}}{(a + b\sec(c + dx))^{5/2}}\,dx = \int \frac{\sec (dx + c)^{\frac{1}{3}}}{(b\sec (dx + c) + a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^3*sec(d*x + c)^3 +
 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [N/A]

Not integrable

Time = 25.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{\sec(c + dx)}}{(a + b\sec(c + dx))^{5/2}}\,dx = \int \frac{\sqrt[3]{\sec (c + dx)}}{(a + b\sec (c + dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**(1/3)/(a + b*sec(c + d*x))**(5/2), x)
```

Maxima [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{1}{3}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(1/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 15.85 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x))^(5/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.64

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{1}{3}}\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^3 b^3+3\sec(dx+c)^2 a\,b^2+3\sec(dx+c)\,a^2 b+a^3}dx$$

input
```
int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int((sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a))/(sec(c + d*x)**3*b**3 +
 3*sec(c + d*x)**2*a*b**2 + 3*sec(c + d*x)*a**2*b + a**3),x)
```

3.772 $\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \text{Int}\left(\frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}},x\right)$$

output
```
Defer(Int)(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 133.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}dx$$

input
```
Int[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec(dx+c)^{\frac{1}{3}} (a+b\sec(dx+c))^{\frac{5}{2}}}dx$$

input
```
int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.04

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^3*sec(d*x + c)^4 +
 3*a*b^2*sec(d*x + c)^3 + 3*a^2*b*sec(d*x + c)^2 + a^3*sec(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 75.90 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(a+b\sec(c+dx))^{\frac{5}{2}}\sqrt[3]{\sec(c+dx)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**(5/2)*sec(c + d*x)**(1/3)), x)
```

Maxima [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec{(dx+c)}+a)^{\frac{5}{2}}\sec{(dx+c)}^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(1/3)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 14.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/3)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.76

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{\frac{10}{3}}\,b^3+3\sec(dx+c)^{\frac{7}{3}}\,a\,b^2+3\sec(dx+c)^{\frac{4}{3}}\,a^2b}$$

input
```
int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)/(sec(c + d*x)**(1/3)*sec(c + d*x)**3*b**3 + 3
*sec(c + d*x)**(1/3)*sec(c + d*x)**2*a*b**2 + 3*sec(c + d*x)**(1/3)*sec(c
+ d*x)*a**2*b + sec(c + d*x)**(1/3)*a**3),x)
```

3.773 $\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Int}\left(\frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}},x\right)$$

output
```
Defer(Int)(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 122.88 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{2/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx$$

input
```
Int[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec\left(dx+c\right)^{\frac{2}{3}}\left(a+b\sec\left(dx+c\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.04

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^3*sec(d*x + c)^4 +
 3*a*b^2*sec(d*x + c)^3 + 3*a^2*b*sec(d*x + c)^2 + a^3*sec(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(2/3)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 11.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(2/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(2/3)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.00

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{1}{3}}\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^4 b^3+3\sec(dx+c)^3 a\,b^2+3\sec(dx+c)^2 a^2b+$$

input
```
int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int((sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a))/(sec(c + d*x)**4*b**3 +
 3*sec(c + d*x)**3*a*b**2 + 3*sec(c + d*x)**2*a**2*b + sec(c + d*x)*a**3),
x)
```

3.774 $\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Int}\left(\frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}},x\right)$$

output
```
Defer(Int)(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 136.78 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$
$$\Big\downarrow \text{4357}$$
$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx$$

input
```
Int[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec\left(dx+c\right)^{\frac{4}{3}} \left(a+b\sec\left(dx+c\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.12

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^3*sec(d*x + c)^5 +
 3*a*b^2*sec(d*x + c)^4 + 3*a^2*b*sec(d*x + c)^3 + a^3*sec(d*x + c)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(4/3)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 15.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(4/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(4/3)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.76

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{\frac{13}{3}}\,b^3+3\sec(dx+c)^{\frac{10}{3}}\,a\,b^2+3\sec(dx+c)^{\frac{7}{3}}\,a^2b}$$

input

```
int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output

```
int(sqrt(sec(c + d*x)*b + a)/(sec(c + d*x)**(1/3)*sec(c + d*x)**4*b**3 + 3
*sec(c + d*x)**(1/3)*sec(c + d*x)**3*a*b**2 + 3*sec(c + d*x)**(1/3)*sec(c
+ d*x)**2*a**2*b + sec(c + d*x)**(1/3)*sec(c + d*x)*a**3),x)
```

3.775 $\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \operatorname{Int}\left(\frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}},x\right)$$

output
```
Defer(Int)(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 108.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$
$$\Big\downarrow \text{4357}$$
$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx$$

input
```
Int[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec(dx+c)^{\frac{5}{3}} (a+b\sec(dx+c))^{\frac{5}{2}}}dx$$

input
```
int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 1.99 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.12

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^3*sec(d*x + c)^5 +
 3*a*b^2*sec(d*x + c)^4 + 3*a^2*b*sec(d*x + c)^3 + a^3*sec(d*x + c)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 2.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/3)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 11.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/3)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 3.08

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{1}{3}}\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{5}\,b^{3}+3\sec(dx+c)^{4}\,a\,b^{2}+3\sec(dx+c)^{3}\,a^{2}b+$$

input
```
int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int((sec(c + d*x)**(1/3)*sqrt(sec(c + d*x)*b + a))/(sec(c + d*x)**5*b**3 +
 3*sec(c + d*x)**4*a*b**2 + 3*sec(c + d*x)**3*a**2*b + sec(c + d*x)**2*a**
3),x)
```

3.776 $\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Int}\left(\frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}},x\right)$$

output
```
Defer(Int)(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 142.66 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$
$$\Big\downarrow \text{ 4357}$$
$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx$$

input
```
Int[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec\left(dx+c\right)^{\frac{7}{3}}\left(a+b\sec\left(dx+c\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.12

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^3*sec(d*x + c)^6 +
 3*a*b^2*sec(d*x + c)^5 + 3*a^2*b*sec(d*x + c)^4 + a^3*sec(d*x + c)^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 2.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/3)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 15.94 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(7/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(7/3)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.76

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}}{\sec(dx+c)^{\frac{16}{3}}\,b^3+3\sec(dx+c)^{\frac{13}{3}}\,a\,b^2+3\sec(dx+c)^{\frac{10}{3}}\,a^2b$$

input
```
int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)/(sec(c + d*x)**(1/3)*sec(c + d*x)**5*b**3 + 3
*sec(c + d*x)**(1/3)*sec(c + d*x)**4*a*b**2 + 3*sec(c + d*x)**(1/3)*sec(c
+ d*x)**3*a**2*b + sec(c + d*x)**(1/3)*sec(c + d*x)**2*a**3),x)
```

3.777 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 251

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^3\,dx = -\frac{ad(3b^2n+a^2(1+n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(d\sec(e+fx))^{-1+n}\sin(e+fx)}{f(1-n^2)\sqrt{\sin^2(e+fx)}} + \frac{b(b^2(1+n)+3a^2(2+n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)(d\sec(e+fx))^n\sin(e+fx)}{fn(2+n)\sqrt{\sin^2(e+fx)}} + \frac{ab^2(5+2n)(d\sec(e+fx))^n\tan(e+fx)}{f(1+n)(2+n)} + \frac{b^2(d\sec(e+fx))^n(a+b\sec(e+fx))\tan(e+fx)}{f(2+n)}$$

output
```
-a*d*(3*b^2*n+a^2*(1+n))*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)
^2)*(d*sec(f*x+e))^(-1+n)*sin(f*x+e)/f/(-n^2+1)/(sin(f*x+e)^2)^(1/2)+b*(b^
2*(1+n)+3*a^2*(2+n))*hypergeom([1/2, -1/2*n],[1-1/2*n],cos(f*x+e)^2)*(d*se
c(f*x+e))^n*sin(f*x+e)/f/n/(2+n)/(sin(f*x+e)^2)^(1/2)+a*b^2*(5+2*n)*(d*sec
(f*x+e))^n*tan(f*x+e)/f/(1+n)/(2+n)+b^2*(d*sec(f*x+e))^n*(a+b*sec(f*x+e))*
tan(f*x+e)/f/(2+n)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.92

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^3\,dx =$$
$$-\frac{\csc^3(e+fx)\left(a^3(6+11n+6n^2+n^3)\cos^3(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\sec^2(e+fx)\right)+bn\right.}{}$$

input
```
Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^3,x]
```

output
```
-((Csc[e + f*x]^3*(a^3*(6 + 11*n + 6*n^2 + n^3)*Cos[e + f*x]^3*Hypergeomet
ric2F1[1/2, n/2, (2 + n)/2, Sec[e + f*x]^2] + b*n*(3*a^2*(6 + 5*n + n^2)*C
os[e + f*x]^2*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sec[e + f*x]^2]
 + b*(1 + n)*(3*a*(3 + n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (
4 + n)/2, Sec[e + f*x]^2] + b*(2 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5
 + n)/2, Sec[e + f*x]^2])))*(d*Sec[e + f*x])^n*(-Tan[e + f*x]^2)^(3/2))/(f
*n*(1 + n)*(2 + n)*(3 + n)))
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4329, 3042, 4535, 3042, 4259, 3042, 3122, 4534, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))^3(d\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4329}$$

$$\frac{\int (d\sec(e+fx))^n \left(ab^2d(2n+5)\sec^2(e+fx)+bd\left(3(n+2)a^2+b^2(n+1)\right)\sec(e+fx)+ad\left((n+2)a^2+b^2n\right)\right)}{d(n+2)}$$
$$\frac{b^2\tan(e+fx)(a+b\sec(e+fx))(d\sec(e+fx))^n}{f(n+2)}$$

$\big\downarrow$ 3042

$$\frac{\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n \left(ab^2d(2n+5)\csc\left(e+fx+\frac{\pi}{2}\right)^2+bd\left(3(n+2)a^2+b^2(n+1)\right)\csc\left(e+fx+\frac{\pi}{2}\right)+ad((\right.}{d(n+2)}$$
$$\frac{b^2\tan(e+fx)(a+b\sec(e+fx))(d\sec(e+fx))^n}{f(n+2)}$$

$\big\downarrow$ 4535

$$\frac{\int (d\sec(e+fx))^n \left(ab^2d(2n+5)\sec^2(e+fx)+ad\left((n+2)a^2+b^2n\right)\right)dx+b\left(3a^2(n+2)+b^2(n+1)\right)\int (d\sec(e}{d(n+2)}$$
$$\frac{b^2\tan(e+fx)(a+b\sec(e+fx))(d\sec(e+fx))^n}{f(n+2)}$$

$\big\downarrow$ 3042

$$\frac{b\left(3a^2(n+2)+b^2(n+1)\right)\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{n+1}dx+\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n \left(ab^2d(2n+5)\csc\left(e+fx+\right.\right.}{d(n+2)}$$
$$\frac{b^2\tan(e+fx)(a+b\sec(e+fx))(d\sec(e+fx))^n}{f(n+2)}$$

$\big\downarrow$ 4259

$$\frac{\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n \left(ab^2d(2n+5)\csc\left(e+fx+\frac{\pi}{2}\right)^2+ad((n+2)a^2+b^2n)\right)dx+b(3a^2(n+2)+b^2(n+1}{d(n+2)}$$
$$\frac{b^2\tan(e+fx)(a+b\sec(e+fx))(d\sec(e+fx))^n}{f(n+2)}$$

$\big\downarrow$ 3042

$$\frac{\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n \left(ab^2d(2n+5)\csc\left(e+fx+\frac{\pi}{2}\right)^2+ad((n+2)a^2+b^2n)\right)dx+b(3a^2(n+2)+b^2(n+1}{d(n+2)}$$
$$\frac{b^2\tan(e+fx)(a+b\sec(e+fx))(d\sec(e+fx))^n}{f(n+2)}$$

$\big\downarrow$ 3122

$$\frac{\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n \left(ab^2d(2n+5)\csc\left(e+fx+\frac{\pi}{2}\right)^2+ad\left((n+2)a^2+b^2n\right)\right)dx+\frac{bd(3a^2(n+2)+b^2(n+1))\sin(e+}{}}{d(n+2)}$$

$$\frac{b^2\tan(e+fx)(a+b\sec(e+fx))(d\sec(e+fx))^n}{f(n+2)}$$

$\Big\downarrow$ 4534

$$\frac{\frac{ad(n+2)(a^2(n+1)+3b^2n)\int(d\sec(e+fx))^ndx}{n+1}+\frac{bd(3a^2(n+2)+b^2(n+1))\sin(e+fx)(d\sec(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+f\right.}{fn\sqrt{\sin^2(e+fx)}}}{d(n+2)}$$

$$\frac{b^2\tan(e+fx)(a+b\sec(e+fx))(d\sec(e+fx))^n}{f(n+2)}$$

$\Big\downarrow$ 3042

$$\frac{\frac{ad(n+2)(a^2(n+1)+3b^2n)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^ndx}{n+1}+\frac{bd(3a^2(n+2)+b^2(n+1))\sin(e+fx)(d\sec(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(\right.}{fn\sqrt{\sin^2(e+fx)}}}{d(n+2)}$$

$$\frac{b^2\tan(e+fx)(a+b\sec(e+fx))(d\sec(e+fx))^n}{f(n+2)}$$

$\Big\downarrow$ 4259

$$\frac{\frac{ad(n+2)(a^2(n+1)+3b^2n)\left(\frac{\cos(e+fx)}{d}\right)^n(d\sec(e+fx))^n\int\left(\frac{\cos(e+fx)}{d}\right)^{-n}dx}{n+1}+\frac{bd(3a^2(n+2)+b^2(n+1))\sin(e+fx)(d\sec(e+fx))^n\text{Hypergeome}}{fn\sqrt{\sin^2(e+fx)}}}{d(n+2)}$$

$$\frac{b^2\tan(e+fx)(a+b\sec(e+fx))(d\sec(e+fx))^n}{f(n+2)}$$

$\Big\downarrow$ 3042

$$\frac{\frac{ad(n+2)(a^2(n+1)+3b^2n)\left(\frac{\cos(e+fx)}{d}\right)^n(d\sec(e+fx))^n\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{-n}dx}{n+1}+\frac{bd(3a^2(n+2)+b^2(n+1))\sin(e+fx)(d\sec(e+fx))^n\text{Hyperge}}{fn\sqrt{\sin^2(e+fx)}}}{d(n+2)}$$

$$\frac{b^2\tan(e+fx)(a+b\sec(e+fx))(d\sec(e+fx))^n}{f(n+2)}$$

$\Big\downarrow$ 3122

$$\frac{-\frac{ad^2(n+2)(a^2(n+1)+3b^2n)\sin(e+fx)(d\sec(e+fx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)(n+1)\sqrt{\sin^2(e+fx)}}+\frac{bd(3a^2(n+2)+b^2(n+1))\sin(e+fx}{}}{d(n+2)}$$

$$\frac{b^2\tan(e+fx)(a+b\sec(e+fx))(d\sec(e+fx))^n}{f(n+2)}$$

input
```
Int[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^3,x]
```

output
```
(b^2*(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])*Tan[e + f*x])/(f*(2 + n)) + (
-((a*d^2*(2 + n)*(3*b^2*n + a^2*(1 + n))*Hypergeometric2F1[1/2, (1 - n)/2,
 (3 - n)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1
- n)*(1 + n)*Sqrt[Sin[e + f*x]^2])) + (b*d*(b^2*(1 + n) + 3*a^2*(2 + n))*H
ypergeometric2F1[1/2, -1/2*n, (2 - n)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^
n*Sin[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2]) + (a*b^2*d*(5 + 2*n)*(d*Sec[e +
 f*x])^n*Tan[e + f*x])/(f*(1 + n)))/(d*(2 + n))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Maple [F]

$$\int (d\sec(fx+e))^n (a+b\sec(fx+e))^3\,dx$$

input
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^3,x)
```

output
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^3,x)
```

Fricas [F]

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^3\,dx = \int (b\sec(fx+e)+a)^3(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
integral((b^3*sec(f*x + e)^3 + 3*a*b^2*sec(f*x + e)^2 + 3*a^2*b*sec(f*x +
e) + a^3)*(d*sec(f*x + e))^n, x)
```

Sympy [F]

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^3\,dx = \int (d\sec(e+fx))^n\,(a+b\sec(e+fx))^3\,dx$$

input
```
integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e))**3,x)
```

output
```
Integral((d*sec(e + f*x))**n*(a + b*sec(e + f*x))**3, x)
```

Maxima [F]

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^3\,dx = \int (b\sec(fx+e)+a)^3(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^3*(d*sec(f*x + e))^n, x)
```

Giac [F]

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^3\,dx = \int (b\sec(fx+e)+a)^3(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^3*(d*sec(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^3\,dx = \int \left(a+\frac{b}{\cos(e+f\,x)}\right)^3\left(\frac{d}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a + b/cos(e + f*x))^3*(d/cos(e + f*x))^n,x)
```

output
```
int((a + b/cos(e + f*x))^3*(d/cos(e + f*x))^n, x)
```

Reduce [F]

$$\begin{aligned}\int (d\sec(e+fx))^n(a+b\sec(e+fx))^3\,dx = d^n\Bigg(&\left(\left(\int \sec(fx+e)^n\,dx\right)a^3\right.\\&+\left(\int \sec(fx+e)^n\sec(fx+e)^3\,dx\right)b^3\\&+3\left(\int \sec(fx+e)^n\sec(fx+e)^2\,dx\right)a\,b^2\\&+3\left(\int \sec(fx+e)^n\sec(fx+e)\,dx\right)a^2b\Bigg)\end{aligned}$$

input
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^3,x)
```

output
```
d**n*(int(sec(e + f*x)**n,x)*a**3 + int(sec(e + f*x)**n*sec(e + f*x)**3,x)
*b**3 + 3*int(sec(e + f*x)**n*sec(e + f*x)**2,x)*a*b**2 + 3*int(sec(e + f*
x)**n*sec(e + f*x),x)*a**2*b)
```

3.778 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 181

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^2\,dx = -\frac{d(b^2n+a^2(1+n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(d\sec(e+fx))^{-1+n}\sin(e+fx)}{f(1-n^2)\sqrt{\sin^2(e+fx)}} + \frac{2ab\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)(d\sec(e+fx))^n\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}} + \frac{b^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+n)}$$

output
```
-d*(b^2*n+a^2*(1+n))*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*
(d*sec(f*x+e))^(-1+n)*sin(f*x+e)/f/(-n^2+1)/(sin(f*x+e)^2)^(1/2)+2*a*b*hyp
ergeom([1/2, -1/2*n],[1-1/2*n],cos(f*x+e)^2)*(d*sec(f*x+e))^n*sin(f*x+e)/f
/n/(sin(f*x+e)^2)^(1/2)+b^2*(d*sec(f*x+e))^n*tan(f*x+e)/f/(1+n)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.94

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^2\,dx$$
$$= \frac{\csc(e+fx)\left(a^2(2+3n+n^2)\cos^2(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\sec^2(e+fx)\right)+bn\big(2a(2+n)\right.}{}$$

input
```
Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^2,x]
```

output
```
(Csc[e + f*x]*(a^2*(2 + 3*n + n^2)*Cos[e + f*x]^2*Hypergeometric2F1[1/2, n
/2, (2 + n)/2, Sec[e + f*x]^2] + b*n*(2*a*(2 + n)*Cos[e + f*x]*Hypergeomet
ric2F1[1/2, (1 + n)/2, (3 + n)/2, Sec[e + f*x]^2] + b*(1 + n)*Hypergeometr
ic2F1[1/2, (2 + n)/2, (4 + n)/2, Sec[e + f*x]^2]))*Sec[e + f*x]*(d*Sec[e +
 f*x])^n*Sqrt[-Tan[e + f*x]^2])/(f*n*(1 + n)*(2 + n))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4275, 3042, 4259, 3042, 3122, 4534, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))^2(d\sec(e+fx))^n\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$
$$\Big\downarrow \text{4275}$$
$$\int (d\sec(e+fx))^n\left(a^2+b^2\sec^2(e+fx)\right)dx+\frac{2ab\int (d\sec(e+fx))^{n+1}dx}{d}$$
$$\Big\downarrow \text{3042}$$

$$\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n \left(a^2+b^2\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx+\frac{2ab\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{n+1}dx}{d}$$

$$\Big\downarrow \text{ 4259}$$

$$\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n \left(a^2+b^2\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx+\frac{2ab\left(\frac{\cos(e+fx)}{d}\right)^n (d\sec(e+fx))^n \int \left(\frac{\cos(e+fx)}{d}\right)^{-n-1}dx}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n \left(a^2+b^2\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx+\frac{2ab\left(\frac{\cos(e+fx)}{d}\right)^n (d\sec(e+fx))^n \int \left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{-n-1}dx}{d}$$

$$\Big\downarrow \text{ 3122}$$

$$\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n \left(a^2+b^2\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx+\frac{2ab\sin(e+fx)(d\sec(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}$$

$$\Big\downarrow \text{ 4534}$$

$$\left(a^2+\frac{b^2n}{n+1}\right)\int (d\sec(e+fx))^n dx+\frac{2ab\sin(e+fx)(d\sec(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}+\frac{b^2\tan(e+fx)(d\sec(e+fx))^n}{f(n+1)}$$

$$\Big\downarrow \text{ 3042}$$

$$\left(a^2+\frac{b^2n}{n+1}\right)\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx+\frac{2ab\sin(e+fx)(d\sec(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}+\frac{b^2\tan(e+fx)(d\sec(e+fx))^n}{f(n+1)}$$

$$\Big\downarrow \text{ 4259}$$

$$\left(a^2+\frac{b^2n}{n+1}\right)\left(\frac{\cos(e+fx)}{d}\right)^n (d\sec(e+fx))^n\int\left(\frac{\cos(e+fx)}{d}\right)^{-n}dx+\frac{2ab\sin(e+fx)(d\sec(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}+\frac{b^2\tan(e+fx)(d\sec(e+fx))^n}{f(n+1)}$$

$$\Big\downarrow \text{3042}$$

$$\left(a^2+\frac{b^2n}{n+1}\right)\left(\frac{\cos(e+fx)}{d}\right)^n (d\sec(e+fx))^n\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{-n}dx+\frac{2ab\sin(e+fx)(d\sec(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}+\frac{b^2\tan(e+fx)(d\sec(e+fx))^n}{f(n+1)}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{d\left(a^2+\frac{b^2n}{n+1}\right)\sin(e+fx)(d\sec(e+fx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}+\frac{2ab\sin(e+fx)(d\sec(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}+\frac{b^2\tan(e+fx)(d\sec(e+fx))^n}{f(n+1)}$$

input
```
Int[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^2,x]
```

output
```
-((d*(a^2 + (b^2*n)/(1 + n))*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2,
Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Si
n[e + f*x]^2])) + (2*a*b*Hypergeometric2F1[1/2, -1/2*n, (2 - n)/2, Cos[e +
 f*x]^2]*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2]) + (b^
2*(d*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + n))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Maple [F]

$$\int (d\sec(fx+e))^n (a+b\sec(fx+e))^2 \, dx$$

input
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^2,x)
```

output
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^2,x)
```

Fricas [F]

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^2\,dx=\int (b\sec(fx+e)+a)^2(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)*(d*sec(f*x + e))^
n, x)
```

Sympy [F]

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^2\,dx=\int (d\sec(e+fx))^n\,(a+b\sec(e+fx))^2\,dx$$

input
```
integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e))**2,x)
```

output
```
Integral((d*sec(e + f*x))**n*(a + b*sec(e + f*x))**2, x)
```

Maxima [F]

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^2\,dx=\int (b\sec(fx+e)+a)^2(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^2*(d*sec(f*x + e))^n, x)
```

Giac [F]

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^2\,dx=\int (b\sec(fx+e)+a)^2(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^2*(d*sec(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^2\,dx=\int \left(a+\frac{b}{\cos(e+f\,x)}\right)^2\left(\frac{d}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a + b/cos(e + f*x))^2*(d/cos(e + f*x))^n,x)
```

output
```
int((a + b/cos(e + f*x))^2*(d/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^2\,dx=d^n\left(\left(\int \sec(fx+e)^n\,dx\right)a^2+\left(\int \sec(fx+e)^n\sec(fx+e)^2\,dx\right)b^2+2\left(\int \sec(fx+e)^n\sec(fx+e)\,dx\right)ab\right)$$

input
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^2,x)
```

output
```
d**n*(int(sec(e + f*x)**n,x)*a**2 + int(sec(e + f*x)**n*sec(e + f*x)**2,x)
*b**2 + 2*int(sec(e + f*x)**n*sec(e + f*x),x)*a*b)
```

3.779 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 137

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))\,dx = -\frac{ad\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(d\sec(e+fx))^{-1+n}\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}} + \frac{b\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)(d\sec(e+fx))^n\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}}$$

output
```
-a*d*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(d*sec(f*x+e))^(
-1+n)*sin(f*x+e)/f/(1-n)/(sin(f*x+e)^2)^(1/2)+b*hypergeom([1/2, -1/2*n],[1
-1/2*n],cos(f*x+e)^2)*(d*sec(f*x+e))^n*sin(f*x+e)/f/n/(sin(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.78

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))\,dx = \frac{\csc(e+fx)\left(a(1+n)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\sec^2(e+fx)\right)+bn\,\text{Hypergeometric2F}\right.}{fn(1+n)}$$

input
```
Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x]),x]
```

output
```
(Csc[e + f*x]*(a*(1 + n)*Cos[e + f*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/
2, Sec[e + f*x]^2] + b*n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sec[
e + f*x]^2])*(d*Sec[e + f*x])^n*Sqrt[-Tan[e + f*x]^2])/(f*n*(1 + n))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))(d\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4274}$$

$$a\int (d\sec(e+fx))^n dx+\frac{b\int (d\sec(e+fx))^{n+1}dx}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx+\frac{b\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{n+1}dx}{d}$$

$$\Big\downarrow \text{4259}$$

$$a\left(\frac{\cos(e+fx)}{d}\right)^n(d\sec(e+fx))^n\int \left(\frac{\cos(e+fx)}{d}\right)^{-n}dx+\frac{b\left(\frac{\cos(e+fx)}{d}\right)^n(d\sec(e+fx))^n\int \left(\frac{\cos(e+fx)}{d}\right)^{-n-1}dx}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{\cos(e+fx)}{d}\right)^n(d\sec(e+fx))^n\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{-n}dx+\frac{b\left(\frac{\cos(e+fx)}{d}\right)^n(d\sec(e+fx))^n\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{-n-1}dx}{d}$$

$$\Big\downarrow \text{ 3122}$$

$$\frac{b\sin(e+fx)(d\sec(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}-\frac{ad\sin(e+fx)(d\sec(e+fx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x]),x]
```

output
```
-((a*d*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(d*Sec
[e + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2])) + (b*H
ypergeometric2F1[1/2, -1/2*n, (2 - n)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^
n*Sin[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Maple [F]

$$\int (d\sec (fx+e))^n (a+b\sec (fx+e))\,dx$$

input
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e)),x)
```

output
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e)),x)
```

Fricas [F]

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))\,dx=\int (b\sec (fx+e)+a)(d\sec (fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Sympy [F]

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))\,dx=\int (d\sec (e+fx))^n\,(a+b\sec (e+fx))\,dx$$

input
```
integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e)),x)
```

output
```
Integral((d*sec(e + f*x))**n*(a + b*sec(e + f*x)), x)
```

Maxima [F]

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))\,dx = \int (b\sec(fx+e)+a)(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Giac [F]

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))\,dx = \int (b\sec(fx+e)+a)(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))\,dx = \int \left(a+\frac{b}{\cos(e+f\,x)}\right)\left(\frac{d}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a + b/cos(e + f*x))*(d/cos(e + f*x))^n,x)
```

output
```
int((a + b/cos(e + f*x))*(d/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))\,dx = d^n\left(\left(\left(\int \sec(fx+e)^n\,dx\right)a + \left(\int \sec(fx+e)^n\sec(fx+e)\,dx\right)b\right)\right)$$

input
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e)),x)
```

output
```
d**n*(int(sec(e + f*x)**n,x)*a + int(sec(e + f*x)**n*sec(e + f*x),x)*b)
```

3.780 $\int \frac{(d\sec(e+fx))^n}{a+b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 192

$$\int \frac{(d\sec(e+fx))^n}{a+b\sec(e+fx)}\,dx = \frac{a\,\mathrm{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-1+n),1,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-1+n)}(d\sec(e+fx}{(a^2-b^2)\,f} - \frac{b\,\mathrm{AppellF1}\left(\frac{1}{2},\frac{n}{2},1,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos^2(e+fx)^{n/2}(d\sec(e+fx))^n\sin(e+fx)}{(a^2-b^2)\,f}$$

output
```
a*AppellF1(1/2,-1/2+1/2*n,1,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*c
os(f*x+e)*(cos(f*x+e)^2)^(-1/2+1/2*n)*(d*sec(f*x+e))^n*sin(f*x+e)/(a^2-b^2
)/f-b*AppellF1(1/2,1/2*n,1,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*(c
os(f*x+e)^2)^(1/2*n)*(d*sec(f*x+e))^n*sin(f*x+e)/(a^2-b^2)/f
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 5280 vs. $2(192) = 384$.

Time = 30.89 (sec) , antiderivative size = 5280, normalized size of antiderivative = 27.50

$$\int \frac{(d\sec(e+fx))^n}{a+b\sec(e+fx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4356, 3042, 3302, 3042, 3668, 25, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^n}{a+b\sec(e+fx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 4356}$$

$$\cos^n(e+fx)(d\sec(e+fx))^n\int \frac{\cos^{1-n}(e+fx)}{b+a\cos(e+fx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\cos^n(e+fx)(d\sec(e+fx))^n\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)^{1-n}}{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3302}$$

$$\cos^n(e+fx)(d\sec(e+fx))^n\left(b\int\frac{\cos^{1-n}(e+fx)}{b^2-a^2\cos^2(e+fx)}dx-a\int\frac{\cos^{2-n}(e+fx)}{b^2-a^2\cos^2(e+fx)}dx\right)$$

$\big\downarrow$ 3042

$$\cos^n(e+fx)(d\sec(e+fx))^n\left(b\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)^{1-n}}{b^2-a^2\sin\left(e+fx+\frac{\pi}{2}\right)^2}dx-a\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)^{2-n}}{b^2-a^2\sin\left(e+fx+\frac{\pi}{2}\right)^2}dx\right)$$

$\big\downarrow$ 3668

$$\cos^n(e+fx)(d\sec(e+fx))^n\left(\frac{b\cos^{-n}(e+fx)\cos^2(e+fx)^{n/2}\int-\frac{\left(1-\sin^2(e+fx)\right)^{-n/2}}{-\sin^2(e+fx)a^2+a^2-b^2}d\sin(e+fx)}{f}-\frac{a\cos^{1-n}(e+fx)\cos^2(e+fx)^{\frac{n-1}{2}}\ldots}{\ldots}\right.$$

$\big\downarrow$ 25

$$\cos^n(e+fx)(d\sec(e+fx))^n\left(\frac{a\cos^{1-n}(e+fx)\cos^2(e+fx)^{\frac{n-1}{2}}\int\frac{\left(1-\sin^2(e+fx)\right)^{\frac{1-n}{2}}}{-\sin^2(e+fx)a^2+a^2-b^2}d\sin(e+fx)}{f}-\frac{b\cos^{-n}(e+fx)\cos^2(e+fx)^{n/2}\ldots}{\ldots}\right.$$

$\big\downarrow$ 333

$$\cos^n(e+fx)(d\sec(e+fx))^n\left(\frac{a\sin(e+fx)\cos^{1-n}(e+fx)\cos^2(e+fx)^{\frac{n-1}{2}}\operatorname{AppellF1}\left(\frac{1}{2},\frac{n-1}{2},1,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}-\frac{b\sin\ldots}{\ldots}\right.$$

input
```
Int[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x]),x]
```

output
```
Cos[e + f*x]^n*(d*Sec[e + f*x])^n*((a*AppellF1[1/2, (-1 + n)/2, 1, 3/2, Si
n[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]^(1 - n)*(Cos[
e + f*x]^2)^((-1 + n)/2)*Sin[e + f*x])/((a^2 - b^2)*f) - (b*AppellF1[1/2,
n/2, 1, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*(Cos[e + f*
x]^2)^(n/2)*Sin[e + f*x])/((a^2 - b^2)*f*Cos[e + f*x]^n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3302
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[a   Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]
^2), x], x] - Simp[b/d   Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*
x]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 3668
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]))   Subst[Int[(1 - ff^2*x^2)^((m -
 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
 d, e, f, m, p}, x] &&  !IntegerQ[m]
```

rule 4356
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Simp[Sin[e + f*x]^n*(d*Csc[e + f*x])^n   Int[(b +
 a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n}
, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{(d \sec (fx+e))^n}{a+b \sec (fx+e)} dx$$

input
```
int((d*sec(f*x+e))^n/(a+b*sec(f*x+e)),x)
```

output
```
int((d*sec(f*x+e))^n/(a+b*sec(f*x+e)),x)
```

Fricas [F]

$$\int \frac{(d \sec(e+fx))^n}{a+b \sec(e+fx)} dx = \int \frac{(d \sec (fx+e))^n}{b \sec (fx+e)+a} dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{(d \sec(e+fx))^n}{a+b \sec(e+fx)} dx = \int \frac{(d \sec (e+fx))^n}{a+b \sec (e+fx)} dx$$

input
```
integrate((d*sec(f*x+e))**n/(a+b*sec(f*x+e)),x)
```

output
```
Integral((d*sec(e + f*x))**n/(a + b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(d\sec(e+fx))^n}{a+b\sec(e+fx)}\,dx=\int \frac{(d\sec{(fx+e)})^n}{b\sec{(fx+e)}+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^n}{a+b\sec(e+fx)}\,dx=\int \frac{(d\sec{(fx+e)})^n}{b\sec{(fx+e)}+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^n}{a+b\sec(e+fx)}\,dx=\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^n}{a+\frac{b}{\cos(e+f\,x)}}\,dx$$

input
```
int((d/cos(e + f*x))^n/(a + b/cos(e + f*x)),x)
```

output
```
int((d/cos(e + f*x))^n/(a + b/cos(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(d\sec(e+fx))^n}{a+b\sec(e+fx)}\,dx = d^n\left(\int \frac{\sec(fx+e)^n}{\sec(fx+e)\,b+a}dx\right)$$

input
```
int((d*sec(f*x+e))^n/(a+b*sec(f*x+e)),x)
```

output
```
d**n*int(sec(e + f*x)**n/(sec(e + f*x)*b + a),x)
```

3.781 $\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 299

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = \frac{a^2 \operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-3+n),2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-1+n)}(d\sec(e+fx))^n \sin(e+fx)}{(a^2-b^2)^2 f} + \frac{b^2 \operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-1+n),2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-1+n)}(d\sec(e+fx))^n \sin(e+fx)}{(a^2-b^2)^2 f} - \frac{2ab \operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-2+n),2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos^2(e+fx)^{n/2}(d\sec(e+fx))^n \sin(e+fx)}{(a^2-b^2)^2 f}$$

output
```
a^2*AppellF1(1/2,-3/2+1/2*n,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))
*cos(f*x+e)*(cos(f*x+e)^2)^(-1/2+1/2*n)*(d*sec(f*x+e))^n*sin(f*x+e)/(a^2-b
^2)^2/f+b^2*AppellF1(1/2,-1/2+1/2*n,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a
^2-b^2))*cos(f*x+e)*(cos(f*x+e)^2)^(-1/2+1/2*n)*(d*sec(f*x+e))^n*sin(f*x+e
)/(a^2-b^2)^2/f-2*a*b*AppellF1(1/2,-1+1/2*n,2,3/2,sin(f*x+e)^2,a^2*sin(f*x
+e)^2/(a^2-b^2))*(cos(f*x+e)^2)^(1/2*n)*(d*sec(f*x+e))^n*sin(f*x+e)/(a^2-b
^2)^2/f
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 13816 vs. 2(299) = 598.

Time = 45.80 (sec) , antiderivative size = 13816, normalized size of antiderivative = 46.21

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^2,x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4356, 3042, 3303, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4356}$$

$$\cos^n(e+fx)(d\sec(e+fx))^n\int \frac{\cos^{2-n}(e+fx)}{(b+a\cos(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^n(e+fx)(d\sec(e+fx))^n\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)^{2-n}}{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3303}$$

$$\cos^n(e+fx)(d\sec(e+fx))^n\int\left(\frac{b^2\cos^{2-n}(e+fx)}{(b^2-a^2\cos^2(e+fx))^2}-\frac{2ab\cos^{3-n}(e+fx)}{(b^2-a^2\cos^2(e+fx))^2}+\frac{a^2\cos^{4-n}(e+fx)}{(a^2\cos^2(e+fx)-b^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\cos^n(e+fx)(d\sec(e+fx))^n\left(\frac{a^2\sin(e+fx)\cos^2(e+fx)^{\frac{n-1}{2}}\cos^{1-n}(e+fx)\,\text{AppellF1}\left(\frac{1}{2},\frac{n-3}{2},2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f(a^2-b^2)^2}+\frac{b^2 s}{}\right.$$

input
```
Int[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^2,x]
```

output
```
Cos[e + f*x]^n*(d*Sec[e + f*x])^n*((a^2*AppellF1[1/2, (-3 + n)/2, 2, 3/2,
Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]^(1 - n)*(Co
s[e + f*x]^2)^((-1 + n)/2)*Sin[e + f*x])/((a^2 - b^2)^2*f) + (b^2*AppellF1
[1/2, (-1 + n)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)
]*Cos[e + f*x]^(1 - n)*(Cos[e + f*x]^2)^((-1 + n)/2)*Sin[e + f*x])/((a^2 -
 b^2)^2*f) - (2*a*b*AppellF1[1/2, (-2 + n)/2, 2, 3/2, Sin[e + f*x]^2, (a^2
*Sin[e + f*x]^2)/(a^2 - b^2)]*(Cos[e + f*x]^2)^(n/2)*Sin[e + f*x])/((a^2 -
 b^2)^2*f*Cos[e + f*x]^n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3303
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n*(1/((a - b*sin[
e + f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m)), x], x] /; FreeQ[{a, b, d, e, f,
 n}, x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

rule 4356
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Simp[Sin[e + f*x]^n*(d*Csc[e + f*x])^n   Int[(b +
 a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n}
, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{(d\sec(fx+e))^n}{(a+b\sec(fx+e))^2}dx$$

input
```
int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^2,x)
```

output
```
int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^n}{(b\sec(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^n/(b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2
), x)
```

Sympy [F]

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx$$

input
```
integrate((d*sec(f*x+e))**n/(a+b*sec(f*x+e))**2,x)
```

output
```
Integral((d*sec(e + f*x))**n/(a + b*sec(e + f*x))**2, x)
```

Maxima [F]

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^n}{(b\sec(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a)^2, x)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^n}{(b\sec(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^n}{\left(a+\frac{b}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int((d/cos(e + f*x))^n/(a + b/cos(e + f*x))^2,x)
```

output
```
int((d/cos(e + f*x))^n/(a + b/cos(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = d^n\left(\int \frac{\sec(fx+e)^n}{\sec(fx+e)^2 b^2 + 2\sec(fx+e)ab + a^2}dx\right)$$

input
```
int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^2,x)
```

output
```
d**n*int(sec(e + f*x)**n/(sec(e + f*x)**2*b**2 + 2*sec(e + f*x)*a*b + a**2
),x)
```

3.782 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx = \text{Int}\left((d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2},x\right)$$

output
```
Defer(Int)((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 65.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx = \int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx$$

input
```
Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^(3/2),x]
```

output
```
Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))^{3/2}(d\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\,dx$$

$$\Big\downarrow \text{4357}$$

$$\int (a+b\sec(e+fx))^{3/2}(d\sec(e+fx))^n dx$$

input
```
Int[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (d\sec (fx+e))^n (a+b\sec (fx+e))^{\frac{3}{2}} dx$$

input
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x)
```

output
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx = \int (b\sec (fx+e)+a)^{\frac{3}{2}}(d\sec (fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

Sympy [N/A]

Not integrable

Time = 26.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx = \int (d\sec (e+fx))^n (a+b\sec (e+fx))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e))**(3/2),x)
```

output
```
Integral((d*sec(e + f*x))**n*(a + b*sec(e + f*x))**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx=\int (b\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

Giac [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx=\int (b\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

Mupad [N/A]

Not integrable

Time = 12.72 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx=\int \left(a+\frac{b}{\cos(e+f\,x)}\right)^{3/2}\left(\frac{d}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((a + b/cos(e + f*x))^(3/2)*(d/cos(e + f*x))^n,x)
```

output
```
int((a + b/cos(e + f*x))^(3/2)*(d/cos(e + f*x))^n, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.36

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx = d^n\left(\left(\int \sec(fx+e)^n\sqrt{\sec(fx+e)\,b+a}\,\sec(fx+e)\,dx\right)b+\left(\int \sec(fx+e)^n\sqrt{\sec(fx+e)\,b+a}dx\right)a\right)$$

input
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x)
```

output
```
d**n*(int(sec(e + f*x)**n*sqrt(sec(e + f*x)*b + a)*sec(e + f*x),x)*b + int
(sec(e + f*x)**n*sqrt(sec(e + f*x)*b + a),x)*a)
```

3.783 $\int (d\sec(e+fx))^n\sqrt{a+b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (d\sec(e+fx))^n\sqrt{a+b\sec(e+fx)}\,dx = \text{Int}\Big((d\sec(e+fx))^n\sqrt{a+b\sec(e+fx)},x\Big)$$

output
```
Defer(Int)((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (d\sec(e+fx))^n\sqrt{a+b\sec(e+fx)}\,dx = \int (d\sec(e+fx))^n\sqrt{a+b\sec(e+fx)}\,dx$$

input
```
Integrate[(d*Sec[e + f*x])^n*Sqrt[a + b*Sec[e + f*x]],x]
```

output
```
Integrate[(d*Sec[e + f*x])^n*Sqrt[a + b*Sec[e + f*x]], x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sec(e+fx)}(d\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4357}$$

$$\int \sqrt{a+b\sec(e+fx)}(d\sec(e+fx))^n dx$$

input
```
Int[(d*Sec[e + f*x])^n*Sqrt[a + b*Sec[e + f*x]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (d\sec (fx+e))^n \sqrt{a+b\sec (fx+e)}dx$$

input
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x)
```

output
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (d\sec(e+fx))^n\sqrt{a+b\sec(e+fx)}\,dx=\int \sqrt{b\sec (fx+e)+a}(d\sec (fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Sympy [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int (d\sec(e+fx))^n\sqrt{a+b\sec(e+fx)}\,dx=\int (d\sec (e+fx))^n\sqrt{a+b\sec (e+fx)}\,dx$$

input
```
integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e))**(1/2),x)
```

output
```
Integral((d*sec(e + f*x))**n*sqrt(a + b*sec(e + f*x)), x)
```

Maxima [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (d\sec(e+fx))^n\sqrt{a+b\sec(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)+a}(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (d\sec(e+fx))^n\sqrt{a+b\sec(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)+a}(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Mupad [N/A]

Not integrable

Time = 11.82 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int (d\sec(e+fx))^n\sqrt{a+b\sec(e+fx)}\,dx = \int \sqrt{a+\frac{b}{\cos(e+f\,x)}}\left(\frac{d}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a + b/cos(e + f*x))^(1/2)*(d/cos(e + f*x))^n,x)
```

output
```
int((a + b/cos(e + f*x))^(1/2)*(d/cos(e + f*x))^n, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int (d\sec(e+fx))^n\sqrt{a+b\sec(e+fx)}\,dx = d^n\left(\int \sec(fx+e)^n\sqrt{\sec(fx+e)\,b+a}dx\right)$$

input
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x)
```

output
```
d**n*int(sec(e + f*x)**n*sqrt(sec(e + f*x)*b + a),x)
```

3.784 $\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx = \text{Int}\left(\frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}},x\right)$$

output
```
Defer(Int)((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 17.93 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx$$

input
```
Integrate[(d*Sec[e + f*x])^n/Sqrt[a + b*Sec[e + f*x]],x]
```

output
```
Integrate[(d*Sec[e + f*x])^n/Sqrt[a + b*Sec[e + f*x]], x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{4357}$$
$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}dx$$

input
```
Int[(d*Sec[e + f*x])^n/Sqrt[a + b*Sec[e + f*x]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(d \sec (fx+e))^n}{\sqrt{a+b \sec (fx+e)}}dx$$

input
```
int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x)
```

output
```
int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(d \sec(e+fx))^n}{\sqrt{a+b \sec(e+fx)}}\, dx = \int \frac{(d \sec (fx+e))^n}{\sqrt{b \sec (fx+e)+a}}\, dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^n/sqrt(b*sec(f*x + e) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(d \sec(e+fx))^n}{\sqrt{a+b \sec(e+fx)}}\, dx = \int \frac{(d \sec (e+fx))^n}{\sqrt{a+b \sec (e+fx)}}\, dx$$

input
```
integrate((d*sec(f*x+e))**n/(a+b*sec(f*x+e))**(1/2),x)
```

output
```
Integral((d*sec(e + f*x))**n/sqrt(a + b*sec(e + f*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^n}{\sqrt{b\sec(fx+e)+a}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^n/sqrt(b*sec(f*x + e) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^n}{\sqrt{b\sec(fx+e)+a}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^n/sqrt(b*sec(f*x + e) + a), x)
```

Mupad [N/A]

Not integrable

Time = 12.50 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^n}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}\,dx$$

input
```
int((d/cos(e + f*x))^n/(a + b/cos(e + f*x))^(1/2),x)
```

output
```
int((d/cos(e + f*x))^n/(a + b/cos(e + f*x))^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx = d^n\left(\int \frac{\sec(fx+e)^n\sqrt{\sec(fx+e)\,b+a}}{\sec(fx+e)\,b+a}dx\right)$$

input
```
int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x)
```

output
```
d**n*int((sec(e + f*x)**n*sqrt(sec(e + f*x)*b + a))/(sec(e + f*x)*b + a),x
)
```

3.785 $\int \frac{(d \sec(e+fx))^n}{(a+b \sec(e+fx))^{3/2}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(d \sec(e+fx))^n}{(a+b \sec(e+fx))^{3/2}} \, dx = \text{Int}\left(\frac{(d \sec(e+fx))^n}{(a+b \sec(e+fx))^{3/2}}, x\right)$$

output
```
Defer(Int)((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 17.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(d \sec(e+fx))^n}{(a+b \sec(e+fx))^{3/2}} \, dx = \int \frac{(d \sec(e+fx))^n}{(a+b \sec(e+fx))^{3/2}} \, dx$$

input
```
Integrate[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^(3/2),x]
```

output
```
Integrate[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4357}$$

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}dx$$

input
```
Int[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(d\sec(fx+e))^n}{(a+b\sec(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x)
```

output
```
int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.12

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(fx+e))^n}{(b\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e))^n/(b^2*sec(f*x + e)^2 +
 2*a*b*sec(f*x + e) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))**n/(a+b*sec(f*x+e))**(3/2),x)
```

output
```
Integral((d*sec(e + f*x))**n/(a + b*sec(e + f*x))**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx=\int \frac{(d\sec(fx+e))^n}{(b\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx=\int \frac{(d\sec(fx+e))^n}{(b\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 14.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^n}{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((d/cos(e + f*x))^n/(a + b/cos(e + f*x))^(3/2),x)
```

output
```
int((d/cos(e + f*x))^n/(a + b/cos(e + f*x))^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.16

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx = d^n\left(\int \frac{\sec(fx+e)^n\sqrt{\sec(fx+e)\,b+a}}{\sec(fx+e)^2\,b^2+2\sec(fx+e)\,ab+a^2}dx\right)$$

input
```
int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x)
```

output
```
d**n*int((sec(e + f*x)**n*sqrt(sec(e + f*x)*b + a))/(sec(e + f*x)**2*b**2
+ 2*sec(e + f*x)*a*b + a**2),x)
```

3.786 $\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx = \text{Int}(\sec^n(e+fx)(a+b\sec(e+fx))^m,x)$$

output
```
Defer(Int)(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x)
```

Mathematica [N/A]

Not integrable

Time = 5.41 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx = \int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx$$

input
```
Integrate[Sec[e + f*x]^n*(a + b*Sec[e + f*x])^m,x]
```

output
```
Integrate[Sec[e + f*x]^n*(a + b*Sec[e + f*x])^m, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$
$$\Big\downarrow \text{4357}$$
$$\int \sec^n(e+fx)(a+b\sec(e+fx))^m dx$$

input
```
Int[Sec[e + f*x]^n*(a + b*Sec[e + f*x])^m,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \sec\left(fx+e\right)^{n}\left(a+b\sec\left(fx+e\right)\right)^{m}dx$$

input
```
int(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x)
```

output
```
int(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx=\int \left(b\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{n}\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

Sympy [N/A]

Not integrable

Time = 6.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx=\int \left(a+b\sec\left(e+fx\right)\right)^{m}\sec^{n}\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)**n*(a+b*sec(f*x+e))**m,x)
```

output
```
Integral((a + b*sec(e + f*x))**m*sec(e + f*x)**n, x)
```

Maxima [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx = \int (b\sec(fx+e)+a)^m \sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx = \int (b\sec(fx+e)+a)^m \sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

Mupad [N/A]

Not integrable

Time = 11.84 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx = \int \left(a+\frac{b}{\cos(e+f\,x)}\right)^m \left(\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a + b/cos(e + f*x))^m*(1/cos(e + f*x))^n,x)
```

output
```
int((a + b/cos(e + f*x))^m*(1/cos(e + f*x))^n, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx=\int \sec\left(fx+e\right)^n\left(\sec\left(fx+e\right)b+a\right)^m dx$$

input
```
int(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x)
```

output
```
int(sec(e + f*x)**n*(sec(e + f*x)*b + a)**m,x)
```

3.787 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx = \text{Int}((d\sec(e+fx))^n(a+b\sec(e+fx))^m,x)$$

output
```
Defer(Int)((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x)
```

Mathematica [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx = \int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx$$

input
```
Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m,x]
```

output
```
Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{4357}$$

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m dx$$

input

```
Int[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m,x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (d\sec (fx+e))^n (a+b\sec (fx+e))^m dx$$

input
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x)
```

output
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m dx = \int (b\sec (fx+e)+a)^m(d\sec (fx+e))^n dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

Sympy [N/A]

Not integrable

Time = 6.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m dx = \int (d\sec (e+fx))^n (a+b\sec (e+fx))^m dx$$

input
```
integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e))**m,x)
```

output
```
Integral((d*sec(e + f*x))**n*(a + b*sec(e + f*x))**m, x)
```

Maxima [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx = \int (b\sec(fx+e)+a)^m(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx = \int (b\sec(fx+e)+a)^m(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

Mupad [N/A]

Not integrable

Time = 11.68 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx = \int \left(a+\frac{b}{\cos(e+f\,x)}\right)^m\left(\frac{d}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a + b/cos(e + f*x))^m*(d/cos(e + f*x))^n,x)
```

output
```
int((a + b/cos(e + f*x))^m*(d/cos(e + f*x))^n, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx = d^n\left(\int \sec(fx+e)^n\,(\sec(fx+e)\,b+a)^m\,dx\right)$$

input
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x)
```

output
```
d**n*int(sec(e + f*x)**n*(sec(e + f*x)*b + a)**m,x)
```

3.788 $\int \sec^3(e+fx)(a+b\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 21, antiderivative size = 274

$$\int \sec^3(e+fx)(a+b\sec(e+fx))^m\,dx = \frac{(a+b\sec(e+fx))^{1+m}\tan(e+fx)}{bf(2+m)} - \frac{\sqrt{2}a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-1-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)(a+b\sec(e+fx))^{1+m}\left(\frac{a+b\sec(e+fx)}{a+b}\right.}{b^2f(2+m)\sqrt{1+\sec(e+fx)}} + \frac{\sqrt{2}(a^2+b^2(1+m))\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)(a+b\sec(e+fx))^m\left(\right.}{b^2f(2+m)\sqrt{1+\sec(e+fx)}}$$

output
```
(a+b*sec(f*x+e))^(1+m)*tan(f*x+e)/b/f/(2+m)-2^(1/2)*a*AppellF1(1/2,-1-m,1/
2,3/2,b*(1-sec(f*x+e))/(a+b),1/2-1/2*sec(f*x+e))*(a+b*sec(f*x+e))^(1+m)*((
a+b*sec(f*x+e))/(a+b))^(-1-m)*tan(f*x+e)/b^2/f/(2+m)/(1+sec(f*x+e))^(1/2)+
2^(1/2)*(a^2+b^2*(1+m))*AppellF1(1/2,-m,1/2,3/2,b*(1-sec(f*x+e))/(a+b),1/2
-1/2*sec(f*x+e))*(a+b*sec(f*x+e))^m*tan(f*x+e)/b^2/f/(2+m)/(1+sec(f*x+e))^
(1/2)/(((a+b*sec(f*x+e))/(a+b))^m)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 8899 vs. $2(274) = 548$.

Time = 26.20 (sec) , antiderivative size = 8899, normalized size of antiderivative = 32.48

$$\int \sec^3(e + fx)(a + b\sec(e + fx))^m\, dx = \text{Result too large to show}$$

input
```
Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x])^m,x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4327, 3042, 4495, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(e + fx)(a + b\sec(e + fx))^m\, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e + fx + \frac{\pi}{2}\right)^3 \left(a + b\csc\left(e + fx + \frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{4327}$$

$$\frac{\int \sec(e + fx)(b(m + 1) - a\sec(e + fx))(a + b\sec(e + fx))^m dx}{b(m + 2)} + \frac{\tan(e + fx)(a + b\sec(e + fx))^{m+1}}{bf(m + 2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(b(m+1)-a\csc\left(e+fx+\frac{\pi}{2}\right)\right)\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx}{b(m+2)}+\frac{\tan(e+fx)(a+b\sec(e+fx))^{m+1}}{bf(m+2)}$$

$\Big\downarrow$ 4495

$$\frac{\frac{(a^2+b^2(m+1))\int \sec(e+fx)(a+b\sec(e+fx))^m dx}{b}-\frac{a\int \sec(e+fx)(a+b\sec(e+fx))^{m+1}dx}{b}}{b(m+2)}+\frac{\tan(e+fx)(a+b\sec(e+fx))^{m+1}}{bf(m+2)}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(a^2+b^2(m+1))\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx}{b}-\frac{a\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m+1}dx}{b}}{b(m+2)}+\frac{\tan(e+fx)(a+b\sec(e+fx))^{m+1}}{bf(m+2)}$$

$\Big\downarrow$ 4321

$$\frac{\frac{a\tan(e+fx)\int \frac{(a+b\sec(e+fx))^{m+1}}{\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}d\sec(e+fx)}{bf\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}-\frac{(a^2+b^2(m+1))\tan(e+fx)\int \frac{(a+b\sec(e+fx))^m}{\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}d\sec(e+fx)}{bf\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}}{b(m+2)}+\frac{\tan(e+fx)(a+b\sec(e+fx))^{m+1}}{bf(m+2)}$$

$\Big\downarrow$ 156

$$\frac{\frac{a(a+b)\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}\int \frac{\left(\frac{a}{a+b}+\frac{b\sec(e+fx)}{a+b}\right)^{m+1}}{\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}d\sec(e+fx)}{bf\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}-\frac{(a^2+b^2(m+1))\tan(e+fx)(a+b\sec(e+f}{bf}}{b(m+2)}$$

$$\frac{\tan(e+fx)(a+b\sec(e+fx))^{m+1}}{bf(m+2)}$$

$\Big\downarrow$ 155

$$\frac{\frac{\sqrt{2}(a^2+b^2(m+1))\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)}{bf\sqrt{\sec(e+fx)+1}}-\frac{\sqrt{2}a(a+b)\tan}{}}{b(m+2)}$$

$$\frac{\tan(e+fx)(a+b\sec(e+fx))^{m+1}}{bf(m+2)}$$

input
```
Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x])^m,x]
```

output
```
((a + b*Sec[e + f*x])^(1 + m)*Tan[e + f*x])/(b*f*(2 + m)) + (-((Sqrt[2]*a*
(a + b)*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Sec[e + f*x])/2, (b*(1 - Sec[
e + f*x]))/(a + b)]*(a + b*Sec[e + f*x])^m*Tan[e + f*x])/(b*f*Sqrt[1 + Sec
[e + f*x]]*((a + b*Sec[e + f*x])/(a + b))^m)) + (Sqrt[2]*(a^2 + b^2*(1 + m
))*AppellF1[1/2, 1/2, -m, 3/2, (1 - Sec[e + f*x])/2, (b*(1 - Sec[e + f*x])
)/(a + b)]*(a + b*Sec[e + f*x])^m*Tan[e + f*x])/(b*f*Sqrt[1 + Sec[e + f*x]
]*((a + b*Sec[e + f*x])/(a + b))^m))/(b*(2 + m))
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

rule 4327
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 -
b^2, 0] &&  !LtQ[m, -1]
```

rule 4495
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)/b   Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Simp[B/b   Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && N
eQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sec (fx+e)^3 (a+b\sec (fx+e))^m \, dx$$

input
```
int(sec(f*x+e)^3*(a+b*sec(f*x+e))^m,x)
```

output
```
int(sec(f*x+e)^3*(a+b*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int \sec^3(e+fx)(a+b\sec(e+fx))^m \, dx = \int (b\sec (fx+e)+a)^m \sec (fx+e)^3 \, dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

Sympy [F]

$$\int \sec^3(e+fx)(a+b\sec(e+fx))^m\,dx=\int (a+b\sec{(e+fx)})^m\sec^3{(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)**3*(a+b*sec(f*x+e))**m,x)
```

output
```
Integral((a + b*sec(e + f*x))**m*sec(e + f*x)**3, x)
```

Maxima [F]

$$\int \sec^3(e+fx)(a+b\sec(e+fx))^m\,dx=\int (b\sec{(fx+e)}+a)^m\sec{(fx+e)}^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

Giac [F]

$$\int \sec^3(e+fx)(a+b\sec(e+fx))^m\,dx=\int (b\sec{(fx+e)}+a)^m\sec{(fx+e)}^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(e+fx)(a+b\sec(e+fx))^m\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^m}{\cos\left(e+f\,x\right)^3}\,dx$$

input
```
int((a + b/cos(e + f*x))^m/cos(e + f*x)^3,x)
```

output
```
int((a + b/cos(e + f*x))^m/cos(e + f*x)^3, x)
```

Reduce [F]

$$\int \sec^3(e+fx)(a+b\sec(e+fx))^m\,dx=\int \left(\sec\left(fx+e\right)b+a\right)^m\sec\left(fx+e\right)^3dx$$

input
```
int(sec(f*x+e)^3*(a+b*sec(f*x+e))^m,x)
```

output
```
int((sec(e + f*x)*b + a)**m*sec(e + f*x)**3,x)
```

3.789 $\int \sec^2(e+fx)(a+b\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 21, antiderivative size = 221

$$\int \sec^2(e+fx)(a+b\sec(e+fx))^m\,dx = \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-1-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)(a+b\sec(e+fx))^{1+m}\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-}}{bf\sqrt{1+\sec(e+fx)}} - \frac{\sqrt{2}a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)(a+b\sec(e+fx))^{m}\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}}{bf\sqrt{1+\sec(e+fx)}}$$

output
```
2^(1/2)*AppellF1(1/2,-1-m,1/2,3/2,b*(1-sec(f*x+e))/(a+b),1/2-1/2*sec(f*x+e
))*(a+b*sec(f*x+e))^(1+m)*((a+b*sec(f*x+e))/(a+b))^(-1-m)*tan(f*x+e)/b/f/(
1+sec(f*x+e))^(1/2)-2^(1/2)*a*AppellF1(1/2,-m,1/2,3/2,b*(1-sec(f*x+e))/(a+
b),1/2-1/2*sec(f*x+e))*(a+b*sec(f*x+e))^m*tan(f*x+e)/b/f/(1+sec(f*x+e))^(1
/2)/(((a+b*sec(f*x+e))/(a+b))^m)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 5564 vs. $2(221) = 442$.

Time = 20.73 (sec) , antiderivative size = 5564, normalized size of antiderivative = 25.18

$$\int \sec^2(e+fx)(a+b\sec(e+fx))^m\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x])^m,x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4325, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)(a+b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{4325}$$

$$\frac{\int \sec(e+fx)(a+b\sec(e+fx))^{m+1}dx}{b}-\frac{a\int \sec(e+fx)(a+b\sec(e+fx))^m dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m+1}dx}{b}-\frac{a\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx}{b}$$

$$\Big\downarrow \text{4321}$$

$$\frac{a\tan(e+fx)\int \frac{(a+b\sec(e+fx))^m}{\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}d\sec(e+fx)}{bf\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}-\frac{\tan(e+fx)\int \frac{(a+b\sec(e+fx))^{m+1}}{\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}d\sec(e+fx)}{bf\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow 156$$

$$\frac{a\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}\int \frac{\left(\frac{a}{a+b}+\frac{b\sec(e+fx)}{a+b}\right)^m}{\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}d\sec(e+fx)}{bf\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}-\frac{(a+b)\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}\int \frac{\left(\frac{a}{a+b}+\frac{b\sec(e+fx)}{a+b}\right)^{m+1}}{\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}d\sec(e+fx)}{bf\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow 155$$

$$\frac{\sqrt{2}(a+b)\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m-1,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx)),\frac{b(1}{}\right.}{bf\sqrt{\sec(e+fx)+1}}$$
$$\frac{\sqrt{2}a\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx)}{a+b}\right.}{bf\sqrt{\sec(e+fx)+1}}$$

input
```
Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x])^m,x]
```

output
```
(Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Sec[e + f*x])/2, (b*
(1 - Sec[e + f*x]))/(a + b)]*(a + b*Sec[e + f*x])^m*Tan[e + f*x])/(b*f*Sqr
t[1 + Sec[e + f*x]]*((a + b*Sec[e + f*x])/(a + b))^m) - (Sqrt[2]*a*AppellF
1[1/2, 1/2, -m, 3/2, (1 - Sec[e + f*x])/2, (b*(1 - Sec[e + f*x]))/(a + b)]
*(a + b*Sec[e + f*x])^m*Tan[e + f*x])/(b*f*Sqrt[1 + Sec[e + f*x]]*((a + b*
Sec[e + f*x])/(a + b))^m)
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

rule 4325
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[-a/b   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] +
Simp[1/b   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{
a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sec (fx+e)^2 (a+b\sec (fx+e))^m \,dx$$

input
```
int(sec(f*x+e)^2*(a+b*sec(f*x+e))^m,x)
```

output
```
int(sec(f*x+e)^2*(a+b*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int \sec^2(e+fx)(a+b\sec(e+fx))^m \,dx = \int (b\sec (fx+e)+a)^m \sec (fx+e)^2 \,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

Sympy [F]

$$\int \sec^2(e+fx)(a+b\sec(e+fx))^m \,dx = \int (a+b\sec (e+fx))^m \sec^2 (e+fx) \,dx$$

input
```
integrate(sec(f*x+e)**2*(a+b*sec(f*x+e))**m,x)
```

output
```
Integral((a + b*sec(e + f*x))**m*sec(e + f*x)**2, x)
```

Maxima [F]

$$\int \sec^2(e+fx)(a+b\sec(e+fx))^m\,dx=\int (b\sec(fx+e)+a)^m\sec(fx+e)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

Giac [F]

$$\int \sec^2(e+fx)(a+b\sec(e+fx))^m\,dx=\int (b\sec(fx+e)+a)^m\sec(fx+e)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(e+fx)(a+b\sec(e+fx))^m\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^m}{\cos(e+f\,x)^2}\,dx$$

input
```
int((a + b/cos(e + f*x))^m/cos(e + f*x)^2,x)
```

output
```
int((a + b/cos(e + f*x))^m/cos(e + f*x)^2, x)
```

Reduce [F]

$$\int \sec^2(e+fx)(a+b\sec(e+fx))^m\,dx = \int (\sec(fx+e)\,b+a)^m \sec(fx+e)^2\,dx$$

input
```
int(sec(f*x+e)^2*(a+b*sec(f*x+e))^m,x)
```

output
```
int((sec(e + f*x)*b + a)**m*sec(e + f*x)**2,x)
```

3.790 $\int \sec(e+fx)(a+b\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 103

$$\int \sec(e+fx)(a+b\sec(e+fx))^m\,dx$$
$$= \frac{\sqrt{2}\,\mathrm{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}\tan}{f\sqrt{1+\sec(e+fx)}}$$

output
```
2^(1/2)*AppellF1(1/2,-m,1/2,3/2,b*(1-sec(f*x+e))/(a+b),1/2-1/2*sec(f*x+e))
*(a+b*sec(f*x+e))^m*tan(f*x+e)/f/(1+sec(f*x+e))^(1/2)/(((a+b*sec(f*x+e))/(
a+b))^m)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2828 vs. 2(103) = 206.

Time = 14.36 (sec) , antiderivative size = 2828, normalized size of antiderivative = 27.46

$$\int \sec(e+fx)(a+b\sec(e+fx))^m\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x])^m,x]
```

output
```
(-6*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan
[(e + f*x)/2]^2)/(a + b)]*(b + a*Cos[e + f*x])^m*Sec[e + f*x]^(1 + m)*(a +
 b*Sec[e + f*x])^m*Tan[(e + f*x)/2])/(f*(-1 + Tan[(e + f*x)/2]^2)*(3*(a +
b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x
)/2]^2)/(a + b)] + 2*(-((a - b)*m*AppellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e
+ f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]) + (a + b)*(1 + m)*Appe
llF1[3/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)
/(a + b)])*Tan[(e + f*x)/2]^2)*((6*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, T
an[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]*(b + a*Cos[e + f*
x])^m*Sec[(e + f*x)/2]^2*Sec[e + f*x]^m*Tan[(e + f*x)/2]^2)/((-1 + Tan[(e
+ f*x)/2]^2)^2*(3*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]^2
, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)] + 2*(-((a - b)*m*AppellF1[3/2, 1 +
 m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)])
 + (a + b)*(1 + m)*AppellF1[3/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2]^2, ((a -
 b)*Tan[(e + f*x)/2]^2)/(a + b)])*Tan[(e + f*x)/2]^2)) - (3*(a + b)*Appell
F1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(
a + b)]*(b + a*Cos[e + f*x])^m*Sec[(e + f*x)/2]^2*Sec[e + f*x]^m)/((-1 + T
an[(e + f*x)/2]^2)*(3*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/
2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)] + 2*(-((a - b)*m*AppellF1[3/2,
 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a...
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a+b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{4321}$$

$$-\frac{\tan(e+fx)\int \frac{(a+b\sec(e+fx))^m}{\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{156}$$

$$-\frac{\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}\int \frac{\left(\frac{a}{a+b}+\frac{b\sec(e+fx)}{a+b}\right)^m}{\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{155}$$

$$\frac{\sqrt{2}\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right.}{f\sqrt{\sec(e+fx)+1}}$$

input
```
Int[Sec[e + f*x]*(a + b*Sec[e + f*x])^m,x]
```

output
```
(Sqrt[2]*AppellF1[1/2, 1/2, -m, 3/2, (1 - Sec[e + f*x])/2, (b*(1 - Sec[e +
 f*x]))/(a + b)]*(a + b*Sec[e + f*x])^m*Tan[e + f*x])/(f*Sqrt[1 + Sec[e +
f*x]]*((a + b*Sec[e + f*x])/(a + b))^m)
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
 - e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Maple [F]

$$\int \sec(fx+e)\,(a+b\sec(fx+e))^m\,dx$$

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e))^m,x)
```

output
```
int(sec(f*x+e)*(a+b*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int \sec(e+fx)(a+b\sec(e+fx))^m\,dx = \int (b\sec(fx+e)+a)^m\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Sympy [F]

$$\int \sec(e+fx)(a+b\sec(e+fx))^m\,dx = \int (a+b\sec(e+fx))^m \sec(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))**m,x)
```

output
```
Integral((a + b*sec(e + f*x))**m*sec(e + f*x), x)
```

Maxima [F]

$$\int \sec(e+fx)(a+b\sec(e+fx))^m\,dx = \int (b\sec(fx+e)+a)^m \sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Giac [F]

$$\int \sec(e+fx)(a+b\sec(e+fx))^m\,dx = \int (b\sec(fx+e)+a)^m \sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+b\sec(e+fx))^m\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^m}{\cos\left(e+f\,x\right)}\,dx$$

input
```
int((a + b/cos(e + f*x))^m/cos(e + f*x),x)
```

output
```
int((a + b/cos(e + f*x))^m/cos(e + f*x), x)
```

Reduce [F]

$$\int \sec(e+fx)(a+b\sec(e+fx))^m\,dx=\int \left(\sec\left(fx+e\right)b+a\right)^m\sec\left(fx+e\right)dx$$

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e))^m,x)
```

output
```
int((sec(e + f*x)*b + a)**m*sec(e + f*x),x)
```

3.791 $\int (a+b\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int (a+b\sec(e+fx))^m\,dx=\text{Int}((a+b\sec(e+fx))^m,x)$$

output
```
Defer(Int)((a+b*sec(f*x+e))^m,x)
```

Mathematica [N/A]

Not integrable

Time = 4.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (a+b\sec(e+fx))^m\,dx=\int (a+b\sec(e+fx))^m\,dx$$

input
```
Integrate[(a + b*Sec[e + f*x])^m,x]
```

output
```
Integrate[(a + b*Sec[e + f*x])^m, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4273}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \ 4273$$

$$\int (a+b\sec(e+fx))^m dx$$

input
```
Int[(a + b*Sec[e + f*x])^m,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4273
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Unintegrable[
(a + b*Csc[c + d*x])^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0
] && !IntegerQ[2*n]
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a+b\sec(fx+e))^m\,dx$$

input
```
int((a+b*sec(f*x+e))^m,x)
```

output
```
int((a+b*sec(f*x+e))^m,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (a+b\sec(e+fx))^m\,dx=\int (b\sec(fx+e)+a)^m\,dx$$

input
```
integrate((a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e) + a)^m, x)
```

Sympy [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a+b\sec(e+fx))^m\,dx=\int (a+b\sec(e+fx))^m\,dx$$

input
```
integrate((a+b*sec(f*x+e))**m,x)
```

output
```
Integral((a + b*sec(e + f*x))**m, x)
```

Maxima [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (a+b\sec(e+fx))^m\,dx=\int (b\sec{(fx+e)}+a)^m\,dx$$

input
```
integrate((a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^m, x)
```

Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (a+b\sec(e+fx))^m\,dx=\int (b\sec{(fx+e)}+a)^m\,dx$$

input
```
integrate((a+b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^m, x)
```

Mupad [N/A]

Not integrable

Time = 10.82 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int (a+b\sec(e+fx))^m\,dx=\int \left(a+\frac{b}{\cos{(e+f\,x)}}\right)^m\,dx$$

input
```
int((a + b/cos(e + f*x))^m,x)
```

output
```
int((a + b/cos(e + f*x))^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (a+b\sec(e+fx))^m\,dx=\int \left(\sec\left(fx+e\right)b+a\right)^m dx$$

input
```
int((a+b*sec(f*x+e))^m,x)
```

output
```
int((sec(e + f*x)*b + a)**m,x)
```

3.792 $\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx = \text{Int}(\cos(e+fx)(a+b\sec(e+fx))^m,x)$$

output
```
Defer(Int)(cos(f*x+e)*(a+b*sec(f*x+e))^m,x)
```

Mathematica [N/A]

Not integrable

Time = 9.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx = \int \cos(e+fx)(a+b\sec(e+fx))^m\,dx$$

input
```
Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x])^m,x]
```

output
```
Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x])^m, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m}{\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \cos(e+fx)(a+b\sec(e+fx))^m dx$$

input
```
Int[Cos[e + f*x]*(a + b*Sec[e + f*x])^m,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cos(fx+e)\,(a+b\sec(fx+e))^m\,dx$$

input
```
int(cos(f*x+e)*(a+b*sec(f*x+e))^m,x)
```

output
```
int(cos(f*x+e)*(a+b*sec(f*x+e))^m,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx=\int (b\sec(fx+e)+a)^m\cos(fx+e)\;dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

Sympy [N/A]

Not integrable

Time = 8.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx=\int (a+b\sec(e+fx))^m\cos(e+fx)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e))**m,x)
```

output
```
Integral((a + b*sec(e + f*x))**m*cos(e + f*x), x)
```

Maxima [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx = \int (b\sec(fx+e)+a)^m\cos(fx+e)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx = \int (b\sec(fx+e)+a)^m\cos(fx+e)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

Mupad [N/A]

Not integrable

Time = 10.93 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx = \int \cos(e+f\,x)\,\left(a+\frac{b}{\cos(e+f\,x)}\right)^m\,dx$$

input
```
int(cos(e + f*x)*(a + b/cos(e + f*x))^m,x)
```

output
```
int(cos(e + f*x)*(a + b/cos(e + f*x))^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx=\int \left(\sec\left(fx+e\right)b+a\right)^m\cos\left(fx+e\right)dx$$

input
```
int(cos(f*x+e)*(a+b*sec(f*x+e))^m,x)
```

output
```
int((sec(e + f*x)*b + a)**m*cos(e + f*x),x)
```

3.793 $\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx = \text{Int}\left(\cos^2(e+fx)(a+b\sec(e+fx))^m,x\right)$$

output
```
Defer(Int)(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x)
```

Mathematica [N/A]

Not integrable

Time = 11.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx = \int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx$$

input
```
Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x])^m,x]
```

output
```
Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x])^m, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m}{\csc\left(e+fx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \cos^2(e+fx)(a+b\sec(e+fx))^m dx$$

input
```
Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x])^m,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \cos\left(fx+e\right)^2 \left(a+b\sec\left(fx+e\right)\right)^m dx$$

input
```
int(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x)
```

output
```
int(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx = \int \left(b\sec\left(fx+e\right)+a\right)^m \cos\left(fx+e\right)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

Sympy [N/A]

Not integrable

Time = 39.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx = \int \left(a+b\sec\left(e+fx\right)\right)^m \cos^2\left(e+fx\right)dx$$

input
```
integrate(cos(f*x+e)**2*(a+b*sec(f*x+e))**m,x)
```

output
```
Integral((a + b*sec(e + f*x))**m*cos(e + f*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.94 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx=\int (b\sec(fx+e)+a)^m\cos(fx+e)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx=\int (b\sec(fx+e)+a)^m\cos(fx+e)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 11.72 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx=\int \cos(e+f\,x)^2\,\left(a+\frac{b}{\cos(e+f\,x)}\right)^m\,dx$$

input
```
int(cos(e + f*x)^2*(a + b/cos(e + f*x))^m,x)
```

output
```
int(cos(e + f*x)^2*(a + b/cos(e + f*x))^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx = \int (\sec(fx+e)\,b+a)^m\cos(fx+e)^2\,dx$$

input
```
int(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x)
```

output
```
int((sec(e + f*x)*b + a)**m*cos(e + f*x)**2,x)
```

3.794 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 135

$$\begin{aligned}\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\,dx &= \frac{14aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}\\ &+\frac{10b\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}\\ &+\frac{10b\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}\\ &+\frac{14a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d}\\ &+\frac{2b\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}\\ &+\frac{2a\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d}\end{aligned}$$

output
```
14/15*a*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+10/21*b*InverseJacobiAM(1/
2*d*x+1/2*c,2^(1/2))/d+10/21*b*cos(d*x+c)^(1/2)*sin(d*x+c)/d+14/45*a*cos(d
*x+c)^(3/2)*sin(d*x+c)/d+2/7*b*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*a*cos(d*x
+c)^(7/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.67

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$
$$=\frac{1176aE\left(\frac{1}{2}(c+dx)\middle|2\right)+600b\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(690b\sin(c+dx)+266a\sin(2(c}{1260d}$$

input
```
Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x]),x]
```

output
```
(1176*a*EllipticE[(c + d*x)/2, 2] + 600*b*EllipticF[(c + d*x)/2, 2] + Sqrt
[Cos[c + d*x]]*(690*b*Sin[c + d*x] + 266*a*Sin[2*(c + d*x)] + 90*b*Sin[3*(
c + d*x)] + 35*a*Sin[4*(c + d*x)]))/(1260*d)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {3042, 4713, 3042, 3227, 3042, 3115, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$
$$\Big\downarrow \text{4713}$$
$$\int \cos^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+b)dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \cos^{\frac{9}{2}}(c+dx)dx+b\int \cos^{\frac{7}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}dx+b\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}dx$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{7}{9}\int \cos^{\frac{5}{2}}(c+dx)dx+\frac{2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}\right)+b\left(\frac{5}{7}\int \cos^{\frac{3}{2}}(c+dx)dx+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{7}{9}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+\frac{2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}\right)+b\left(\frac{5}{7}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{7}{9}\left(\frac{3}{5}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+\frac{2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}\right)+b\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{7}{9}\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+\frac{2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}\right)+b\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)$$

$$\Big\downarrow \text{3119}$$

$$b\left(\frac{5}{7}\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)+a\left(\frac{2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{7}{9}\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)\right)$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{7}{9}\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)\right)+b\left(\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{5}{7}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)$$

input

```
Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x]),x]
```

output

```
b*((2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (5*((2*EllipticF[(c + d*x)/
2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/7) + a*((2*Cos[
c + d*x]^(7/2)*Sin[c + d*x])/(9*d) + (7*((6*EllipticE[(c + d*x)/2, 2])/(5*
d) + (2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)))/9)
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(118) = 236$.

Time = 11.86 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.36

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-1120a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(2240a+720b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-2072\right.}{}$

input
```
int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*a*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*a+720*b)*sin(1/2*d*x+1/2*c)^8
*cos(1/2*d*x+1/2*c)+(-2072*a-1080*b)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*
c)+(952*a+840*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-168*a-240*b)*si
n(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+75*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.18

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{2\left(35\,a\cos\left(dx+c\right)^{3}+45\,b\cos\left(dx+c\right)^{2}+49\,a\cos\left(dx+c\right)+75\,b\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-75i\,\sqrt{2}b}{}$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/315*(2*(35*a*cos(d*x + c)^3 + 45*b*cos(d*x + c)^2 + 49*a*cos(d*x + c) +
75*b)*sqrt(cos(d*x + c))*sin(d*x + c) - 75*I*sqrt(2)*b*weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 75*I*sqrt(2)*b*weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 147*I*sqrt(2)*a*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 147*I*
sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\,dx=\int (b\sec(dx+c)+a)\cos(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(9/2), x)
```

Giac [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\,dx=\int (b\sec(dx+c)+a)\cos(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(9/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.98 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.64

$$\begin{aligned}\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\,dx &= -\frac{2\,a\cos(c+d\,x)^{11/2}\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos(c+d\,x)^2\right)}{11\,d\,\sqrt{\sin(c+d\,x)^2}}\\ &\quad-\frac{2\,b\cos(c+d\,x)^{9/2}\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos(c+d\,x)^2\right)}{9\,d\,\sqrt{\sin(c+d\,x)^2}}\end{aligned}$$

input
```
int(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x)),x)
```

output
```
- (2*a*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos(c
 + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (2*b*cos(c + d*x)^(9/2)*sin(c
+ d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^
(1/2))
```

Reduce [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^4\sec(dx+c)\,dx\right)b + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^4\,dx\right)a$$

input
```
int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))*cos(c + d*x)**4*sec(c + d*x),x)*b + int(sqrt(cos(c
+ d*x))*cos(c + d*x)**4,x)*a
```

3.795 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{10a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} + \frac{10a\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2b\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}$$

output
```
6/5*b*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+10/21*a*InverseJacobiAM(1/2*
d*x+1/2*c,2^(1/2))/d+10/21*a*cos(d*x+c)^(1/2)*sin(d*x+c)/d+2/5*b*cos(d*x+c
)^(3/2)*sin(d*x+c)/d+2/7*a*cos(d*x+c)^(5/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.69

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \frac{126bE\left(\frac{1}{2}(c+dx)\middle|2\right)+50a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(65a+42b\cos(c+dx)+15a\cos(2(c+dx)))\sin(c+dx)}{105d}$$

input

```
Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x]),x]
```

output

```
(126*b*EllipticE[(c + d*x)/2, 2] + 50*a*EllipticF[(c + d*x)/2, 2] + Sqrt[C
os[c + d*x]]*(65*a + 42*b*Cos[c + d*x] + 15*a*Cos[2*(c + d*x)])*Sin[c + d*
x])/(105*d)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4713, 3042, 3227, 3042, 3115, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4713}$$

$$\int \cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+b)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \cos^{\frac{7}{2}}(c+dx)dx+b\int \cos^{\frac{5}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}dx+b\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{5}{7}\int \cos^{\frac{3}{2}}(c+dx)dx+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)+ b\left(\frac{3}{5}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{7}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)+ b\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)+ b\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)+ b\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)+ b\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{5}{7}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)+ b\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

input
```
Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x]),x]
```

output
```
b*((6*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*Cos[c + d*x]^(3/2)*Sin[c + d*x
])/(5*d)) + a*((2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (5*((2*Elliptic
F[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/7)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(98) = 196$.

Time = 7.26 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.61

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(240a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8+(-360a-168b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(280a+16\right.}{}$

input
```
int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*a-168*b)*sin(1/2*d*x+1/2*c)^6*co
s(1/2*d*x+1/2*c)+(280*a+168*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-8
0*a-42*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+25*a*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-63*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.33

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{2\left(15\,a\cos\left(dx+c\right)^2+21\,b\cos\left(dx+c\right)+25\,a\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-25i\,\sqrt{2}a\text{weierstrassPInverse}}{}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/105*(2*(15*a*cos(d*x + c)^2 + 21*b*cos(d*x + c) + 25*a)*sqrt(cos(d*x + c
))*sin(d*x + c) - 25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + 25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c)) + 63*I*sqrt(2)*b*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*b*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int (b\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int (b\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.90 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\,dx = -\frac{2\,a\cos(c+d\,x)^{9/2}\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos(c+d\,x)^2\right)}{9\,d\,\sqrt{\sin(c+d\,x)^2}} - \frac{2\,b\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+d\,x)^2\right)}{7\,d\,\sqrt{\sin(c+d\,x)^2}}$$

input
```
int(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x)),x)
```

output
```
- (2*a*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c +
 d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (2*b*cos(c + d*x)^(7/2)*sin(c + d
*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/
2))
```

Reduce [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)dx\right)b + \left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^{3}dx\right)a$$

input
```
int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x),x)*b + int(sqrt(cos(c
+ d*x))*cos(c + d*x)**3,x)*a
```

3.796 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx=\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2b\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}$$

output
```
6/5*a*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*b*InverseJacobiAM(1/2*d*
x+1/2*c,2^(1/2))/d+2/3*b*cos(d*x+c)^(1/2)*sin(d*x+c)/d+2/5*a*cos(d*x+c)^(3
/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.76

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx=\frac{2\left(9aE\left(\frac{1}{2}(c+dx)\middle|2\right)+5b\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(5b+3a\cos(c+dx))\sin(c+dx)\right)}{15d}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x]),x]
```

output
```
(2*(9*a*EllipticE[(c + d*x)/2, 2] + 5*b*EllipticF[(c + d*x)/2, 2] + Sqrt[C
os[c + d*x]]*(5*b + 3*a*Cos[c + d*x])*Sin[c + d*x]))/(15*d)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4713, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4713}$$

$$\int \cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \cos^{\frac{5}{2}}(c+dx)dx+b\int \cos^{\frac{3}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+b\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{3}{5}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+b\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+b\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{3119}$$

$$b\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+a\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+b\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

input
```
Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x]),x]
```

output
```
b*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(3*d)) + a*((6*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*Cos[c + d*x]^(3/2)
*Sin[c + d*x])/(5*d))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(78) = 156$.

Time = 5.67 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.01

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-24a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+(24a+20b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-6a-10b)\sin\cdots\right.}{15\sqrt{-2\sin\left(\frac{dx}{2}+\cdots\right.}}$

input
```
int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*a+20*b)*sin(1/2*d*x+1/2*c)^4*cos(1/
2*d*x+1/2*c)+(-6*a-10*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*b*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.57

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{2\,(3\,a\cos{(dx+c)}+5\,b)\sqrt{\cos{(dx+c)}}\sin{(dx+c)}-5i\,\sqrt{2}b\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/15*(2*(3*a*cos(d*x + c) + 5*b)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*I*sqr
t(2)*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqr
t(2)*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*I*sqr
t(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c))) - 9*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int (b\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int (b\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.76 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \frac{2\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,b\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d} - \frac{2\,a\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

input
```
int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x)),x)
```

output
```
(2*b*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*b*cos(c + d*x)^(1/2)*sin(c +
d*x))/(3*d) - (2*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 1
1/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \left(\int \sqrt{\cos\left(dx+c\right)}\,{\cos\left(dx+c\right)}^{2}\sec\left(dx+c\right)dx\right)b + \left(\int \sqrt{\cos\left(dx+c\right)}\,{\cos\left(dx+c\right)}^{2}dx\right)a$$

input
```
int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))*cos(c + d*x)**2*sec(c + d*x),x)*b + int(sqrt(cos(c
+ d*x))*cos(c + d*x)**2,x)*a
```

3.797 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx=\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
2*b*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*InverseJacobiAM(1/2*d*x+
1/2*c,2^(1/2))/d+2/3*a*cos(d*x+c)^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx=\frac{2\left(3bE\left(\frac{1}{2}(c+dx)\middle|2\right)+a\left(\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}\sin(c+dx)\right)\right)}{3d}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x]),x]
```

output
```
(2*(3*b*EllipticE[(c + d*x)/2, 2] + a*(EllipticF[(c + d*x)/2, 2] + Sqrt[Co
s[c + d*x]]*Sin[c + d*x])))/(3*d)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4713, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4713}$$

$$\int \sqrt{\cos(c+dx)}(a\cos(c+dx)+b)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \cos^{\frac{3}{2}}(c+dx)dx+b\int \sqrt{\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+b\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+b\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+b\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x]),x]
```

output
```
(2*b*EllipticE[(c + d*x)/2, 2])/d + a*((2*EllipticF[(c + d*x)/2, 2])/(3*d)
 + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(58) = 116$.

Time = 2.45 (sec) , antiderivative size = 228, normalized size of antiderivative = 3.74

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(4a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-2a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+a\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\right.}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}\right.}$

input
```
int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2*a*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c
)+a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.05

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{2\,a\sqrt{\cos(dx+c)}\sin(dx+c) - i\,\sqrt{2}a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)) + i\,\sqrt{2}a\text{w}}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/3*(2*a*sqrt(cos(d*x + c))*sin(d*x + c) - I*sqrt(2)*a*weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*a*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*b*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)
*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int (b\sec{(dx+c)}+a)\cos{(dx+c)}^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int (b\sec{(dx+c)}+a)\cos{(dx+c)}^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \frac{2\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{2\,a\,\sqrt{\cos{(c+d\,x)}}\,\sin{(c+d\,x)}}{3\,d}$$

input
```
int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x)),x)
```

output
```
(2*a*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*b*ellipticE(c/2 + (d*x)/2, 2)
)/d + (2*a*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d)
```

Reduce [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)\,dx\right)b + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a$$

input
```
int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x),x)*b + int(sqrt(cos(c + d
*x))*cos(c + d*x),x)*a
```

3.798 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 35

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))\,dx = \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

output
```
2*a*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*b*InverseJacobiAM(1/2*d*x+1/
2*c,2^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))\,dx = \frac{2\left(aE\left(\frac{1}{2}(c+dx)\middle|2\right)+b\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right)}{d}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x]),x]
```

output
```
(2*(a*EllipticE[(c + d*x)/2, 2] + b*EllipticF[(c + d*x)/2, 2]))/d
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4713, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4713}$$

$$\int \frac{a\cos(c+dx)+b}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+b}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \sqrt{\cos(c+dx)}dx+b\int \frac{1}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+b\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3119}$$

$$b\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x]),x]
```

output
```
(2*a*EllipticE[(c + d*x)/2, 2])/d + (2*b*EllipticF[(c + d*x)/2, 2])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(38) = 76$.

Time = 0.87 (sec) , antiderivative size = 152, normalized size of antiderivative = 4.34

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{1-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(b\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-a\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}\cdots\right.\right.\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$
risch	$-\frac{ia\sqrt{2}\,\sqrt{\left(e^{2i(dx+c)}+1\right)e^{-i(dx+c)}}}{d}-\frac{i\left(\frac{ib\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(dx+c)}-i\right)}\,\sqrt{ie^{i(dx+c)}}\,\mathrm{EllipticF}\left(\sqrt{-i\left(e^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)}{\sqrt{e^{3i(dx+c)}+e^{i(dx+c)}}}+a\left(\cdots\right.\right.}{\cdots}$

input
```
int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)*(b*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.06

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))\,dx = \frac{-i\sqrt{2}b\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}b\mathrm{weierstrassPInverse}(-4,0,\cos(\cdots}{\cdots}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) +
I*sqrt(2)*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + I*
sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) - I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\sqrt{\cos(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*sqrt(cos(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))\,dx = \int (b\sec(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))\,dx = \int (b\sec(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 10.66 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))\,dx = \frac{2\,a\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{2\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}$$

input
```
int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x)),x)
```

output
```
(2*a*ellipticE(c/2 + (d*x)/2, 2))/d + (2*b*ellipticF(c/2 + (d*x)/2, 2))/d
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))\,dx = \left(\int \sqrt{\cos(dx+c)}dx\right)a + \left(\int \sqrt{\cos(dx+c)}\sec(dx+c)\,dx\right)b$$

input
```
int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x)),x)*a + int(sqrt(cos(c + d*x))*sec(c + d*x),x)*b
```

3.799 $\int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = -\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
-2*b*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a*InverseJacobiAM(1/2*d*x+1
/2*c,2^(1/2))/d+2*b*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = \frac{2\left(-bE\left(\frac{1}{2}(c+dx)\middle|2\right)+a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{b\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{d}$$

input
```
Integrate[(a + b*Sec[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

output
```
(2*(-(b*EllipticE[(c + d*x)/2, 2]) + a*EllipticF[(c + d*x)/2, 2] + (b*Sin[
c + d*x])/Sqrt[Cos[c + d*x]]))/d
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4713, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4713}$$

$$\int \frac{a\cos(c+dx)+b}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+b}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3227}$$

$$a\int \frac{1}{\sqrt{\cos(c+dx)}}dx+b\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3116}$$

$$a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+b\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+b\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3119}$$

$$a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+b\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{3120}$$

$$\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+b\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

input
```
Int[(a + b*Sec[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

output
```
(2*a*EllipticF[(c + d*x)/2, 2])/d + b*((-2*EllipticE[(c + d*x)/2, 2])/d +
(2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(58) = 116$.

Time = 1.11 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.63

method	result
default	$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b-2a\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\right.}}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$

input
```
int((a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b-a*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*b)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.74

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = \frac{-i\sqrt{2}a\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}a\cos(dx+c)\,\text{weiers}}{}$$

input
```
integrate((a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c)) + I*sqrt(2)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) - I*sqrt(2)*b*cos(d*x + c)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*b*cos
(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))) + 2*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*sec(c + d*x))/sqrt(cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{b\sec\left(dx+c\right)+a}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{b\sec\left(dx+c\right)+a}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 10.90 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = \frac{2\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

input
```
int((a + b/cos(c + d*x))/cos(c + d*x)^(1/2),x)
```

output
```
(2*a*ellipticF(c/2 + (d*x)/2, 2))/d + (2*b*sin(c + d*x)*hypergeom([-1/4, 1
/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a + \left(\int \frac{\sqrt{\cos(dx+c)}\sec(dx+c)}{\cos(dx+c)}dx\right)b$$

input
```
int((a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x)
```

output
```
int(sqrt(cos(c + d*x))/cos(c + d*x),x)*a + int((sqrt(cos(c + d*x))*sec(c +
 d*x))/cos(c + d*x),x)*b
```

3.800 $\int \frac{a+b\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2b\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
-2*a*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*b*InverseJacobiAM(1/2*d*x
+1/2*c,2^(1/2))/d+2/3*b*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2*a*sin(d*x+c)/d/cos
(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{-6aE\left(\frac{1}{2}(c+dx)\middle|2\right)+2b\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{2(b+3a\cos(c+dx))\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}}{3d}$$

input
```
Integrate[(a + b*Sec[c + d*x])/Cos[c + d*x]^(3/2),x]
```

output
```
(-6*a*EllipticE[(c + d*x)/2, 2] + 2*b*EllipticF[(c + d*x)/2, 2] + (2*(b +
3*a*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2))/(3*d)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4713, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4713}$$

$$\int \frac{a\cos(c+dx)+b}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+b}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx+b\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+b\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int\sqrt{\cos(c+dx)}dx\right)+b\left(\frac{1}{3}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \ 3042$$

$$a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+b\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \ 3119$$

$$b\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \ 3120$$

$$a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+b\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

input
```
Int[(a + b*Sec[c + d*x])/Cos[c + d*x]^(3/2),x]
```

output
```
b*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x
]^(3/2))) + a*((-2*EllipticE[(c + d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt
[Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(78) = 156$.

Time = 2.55 (sec) , antiderivative size = 396, normalized size of antiderivative = 4.77

method	result
default	$-\frac{2\sqrt{-\left(1-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(12a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-2\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \mathrm{EllipticF}\left(\cos\left(\right.\right.}{}$

input
```
int((a+b*sec(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-(1-2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d
*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(12*a*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2
*c)^2*b-6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2*a-6*a*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b+b
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.11

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{-i\sqrt{2}b\cos{(dx+c)}^2\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+i\sqrt{2}b\cos{(dx+c)}^2\,\text{weie}}{}$$

input
```
integrate((a+b*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*b*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + I*sqrt(2)*b*cos(d*x + c)^2*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*a*cos(d*x + c)^2*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I
*sqrt(2)*a*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*a*cos(d*x + c) + b)*sqrt(cos(d*x
+ c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{a+b\sec\left(c+dx\right)}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/cos(d*x+c)**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))/cos(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{b\sec\left(dx+c\right)+a}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{b\sec\left(dx+c\right)+a}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 11.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}} + \frac{2\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int((a + b/cos(c + d*x))/cos(c + d*x)^(3/2),x)
```

output
```
(2*a*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c +
d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*b*sin(c + d*x)*hypergeom([-3/4, 1/
2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^2}dx\right)a + \left(\int \frac{\sqrt{\cos\left(dx+c\right)}\,\sec\left(dx+c\right)}{\cos\left(dx+c\right)^2}dx\right)b$$

input
```
int((a+b*sec(d*x+c))/cos(d*x+c)^(3/2),x)
```

output
```
int(sqrt(cos(c + d*x))/cos(c + d*x)**2,x)*a + int((sqrt(cos(c + d*x))*sec(
c + d*x))/cos(c + d*x)**2,x)*b
```

3.801 $\int \frac{a+b\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = -\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2b\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{6b\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output
```
-6/5*b*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*InverseJacobiAM(1/2*d
*x+1/2*c,2^(1/2))/d+2/5*b*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/3*a*sin(d*x+c)/d
/cos(d*x+c)^(3/2)+6/5*b*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{-18b\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10a\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+10a\sin(c+dx)+9b\,\text{si}}{15d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(a + b*Sec[c + d*x])/Cos[c + d*x]^(5/2),x]
```

output
```
(-18*b*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*a*Cos[c + d*x]^(3
/2)*EllipticF[(c + d*x)/2, 2] + 10*a*Sin[c + d*x] + 9*b*Sin[2*(c + d*x)] +
 6*b*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4713, 3042, 3227, 3042, 3116, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4713}$$

$$\int \frac{a\cos(c+dx)+b}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+b}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx+b\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+b\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$a\left(\frac{1}{3}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+b\left(\frac{3}{5}\int\frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+ b\left(\frac{3}{5}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3116}$$

$$a\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+ b\left(\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int\sqrt{\cos(c+dx)}dx\right)+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+ b\left(\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+ b\left(\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+ b\left(\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)$$

input
```
Int[(a + b*Sec[c + d*x])/Cos[c + d*x]^(5/2),x]
```

output
```
a*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x
]^(3/2))) + b*((2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (3*((-2*Ellipti
cE[(c + d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])))/5)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. 2(98) = 196.

Time = 3.92 (sec) , antiderivative size = 502, normalized size of antiderivative = 4.52

method	result
default	$-\frac{\sqrt{-\left(1-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2a\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{6\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^2}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{1-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin}}\right.\right.}{}$

input
```
int((a+b*sec(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(1-2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a*(-1/6*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/
2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/
2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*b/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d
*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*
c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))
/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.69

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{-5i\sqrt{2}a\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}a\cos(dx+c)^3\,\text{w}}{}$$

input
```
integrate((a+b*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/15*(-5*I*sqrt(2)*a*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) + 5*I*sqrt(2)*a*cos(d*x + c)^3*weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*b*cos(d*x + c)^3*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
+ 9*I*sqrt(2)*b*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*b*cos(d*x + c)^2 + 5*a*cos(d
*x + c) + 3*b)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\sec(c + dx)}{\cos^{\frac{5}{2}}(c + dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a + b\sec(c + dx)}{\cos^{\frac{5}{2}}(c + dx)}\,dx = \int \frac{b\sec(dx + c) + a}{\cos(dx + c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{b\sec{(dx+c)}+a}{\cos{(dx+c)}^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 11.77 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\,a\,\sin{(c+d\,x)}\;{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\;\frac{1}{4};\;\cos{(c+d\,x)}^2\right)}{3\,d\cos{(c+d\,x)}^{3/2}\,\sqrt{\sin{(c+d\,x)}^2}} + \frac{2\,b\,\sin{(c+d\,x)}\;{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\;-\frac{1}{4};\;\cos{(c+d\,x)}^2\right)}{5\,d\cos{(c+d\,x)}^{5/2}\,\sqrt{\sin{(c+d\,x)}^2}}$$

input
```
int((a + b/cos(c + d*x))/cos(c + d*x)^(5/2),x)
```

output
```
(2*a*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c
+ d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*b*sin(c + d*x)*hypergeom([-5/4,
1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1/2
))
```

Reduce [F]

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)a + \left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)}{\cos(dx+c)^3}dx\right)b$$

input
```
int((a+b*sec(d*x+c))/cos(d*x+c)^(5/2),x)
```

output
```
int(sqrt(cos(c + d*x))/cos(c + d*x)**3,x)*a + int((sqrt(cos(c + d*x))*sec(
c + d*x))/cos(c + d*x)**3,x)*b
```

3.802 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 160

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{2(7a^2+9b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15d} + \frac{20ab\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} + \frac{20ab\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2(7a^2+9b^2)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{4ab\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2a^2\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d}$$

output
```
2/15*(7*a^2+9*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+20/21*a*b*Inver
seJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+20/21*a*b*cos(d*x+c)^(1/2)*sin(d*x+c)/
d+2/45*(7*a^2+9*b^2)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+4/7*a*b*cos(d*x+c)^(5/2
)*sin(d*x+c)/d+2/9*a^2*cos(d*x+c)^(7/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.71

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$=\frac{84(7a^2+9b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+600ab\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(7(43a^2+36b^2)\cos(c+}{630d}$$

input
```
Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^2,x]
```

output
```
(84*(7*a^2 + 9*b^2)*EllipticE[(c + d*x)/2, 2] + 600*a*b*EllipticF[(c + d*x
)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(43*a^2 + 36*b^2)*Cos[c + d*x] + 5*a*(156*
b + 36*b*Cos[2*(c + d*x)] + 7*a*Cos[3*(c + d*x)]))*Sin[c + d*x])/(630*d)
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.37, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.826, Rules used = {3042, 4752, 3042, 4275, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120, 4533, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{4275}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{9}{2}}(c+dx)}dx+2ab\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2ab\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx\right)$$

$$\Big\downarrow \text{4256}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2ab\left(\frac{5}{7}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2ab\left(\frac{5}{7}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(c+dx}{7d\sec^{\frac{5}{2}}(c+d}\right.\right.$$

$$\Big\downarrow \text{4256}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2ab\left(\frac{5}{7}\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2ab\left(\frac{5}{7}\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+}{3d\sqrt{\sec(c}}\right.\right.\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2ab\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c}}\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2ab\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{}{\sqrt{\sin\left(\right.}}\right.\right.\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2ab\left(\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{5}{7}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2}{}\right.\right.\right.$$

$\downarrow$ 4533

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(7a^2+9b^2\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+2ab\left(\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{5}{7}\left(\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(7a^2+9b^2\right)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+2ab\left(\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right.\right.$$

$\downarrow$ 4256

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(7a^2+9b^2\right)\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+2ab\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(7a^2+9b^2\right)\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(7a^2+9b^2\right)\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(7a^2+9b^2\right)\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+}{5d\sec^{\frac{3}{2}}(c}\right.\right.$$

$$\Big\downarrow \text{3119}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(7a^2+9b^2\right)\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{5d}\right)\right.$$

input
```
Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^2,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sin[c + d*x])/(9*d*Sec[c + d
*x]^(7/2)) + ((7*a^2 + 9*b^2)*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
))/9 + 2*a*b*((2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (5*((2*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c +
 d*x])/(3*d*Sqrt[Sec[c + d*x]])))/7))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(143) = 286$.

Time = 30.68 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.49

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(-1120a^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+\left(2240a^{2}+1440ab\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(\right.}{}$

input
```
int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*a^2*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*a^2+1440*a*b)*sin(1/2*d*x+1
/2*c)^8*cos(1/2*d*x+1/2*c)+(-2072*a^2-2160*a*b-504*b^2)*sin(1/2*d*x+1/2*c)
^6*cos(1/2*d*x+1/2*c)+(952*a^2+1680*a*b+504*b^2)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-168*a^2-480*a*b-126*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c)+150*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^
2-189*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.22

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{-150i\sqrt{2}ab\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+150i\sqrt{2}ab\text{weierstrassPInverse}(-$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/315*(-150*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) + 150*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*
sin(d*x + c)) + 2*(35*a^2*cos(d*x + c)^3 + 90*a*b*cos(d*x + c)^2 + 150*a*b
 + 7*(7*a^2 + 9*b^2)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 21*sq
rt(2)*(-7*I*a^2 - 9*I*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) - 21*sqrt(2)*(7*I*a^2 + 9*I*b^2)*weiers
trassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(9/2), x)
```

Giac [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(9/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.94 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.84

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{2\,a^2\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right)^2\right)}{11\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} - \frac{2\,b^2\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} - \frac{4\,a\,b\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x))^2,x)
```

output
```
- (2*a^2*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos
(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (2*b^2*cos(c + d*x)^(7/2)*si
n(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)
^2)^(1/2)) - (4*a*b*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4],
13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^4\sec\left(dx+c\right)^2dx\right)b^2+2\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^4\sec\left(dx+c\right)dx\right)ab+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^4dx\right)a^2$$

input
```
int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^2,x)
```

output
```
int(sqrt(cos(c + d*x))*cos(c + d*x)**4*sec(c + d*x)**2,x)*b**2 + 2*int(sqr
t(cos(c + d*x))*cos(c + d*x)**4*sec(c + d*x),x)*a*b + int(sqrt(cos(c + d*x
))*cos(c + d*x)**4,x)*a**2
```

3.803 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{12abE\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} + \frac{2(5a^2+7b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} + \frac{2(5a^2+7b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{4ab\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}$$

output
```
12/5*a*b*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*(5*a^2+7*b^2)*Invers
eJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+2/21*(5*a^2+7*b^2)*cos(d*x+c)^(1/2)*sin
(d*x+c)/d+4/5*a*b*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*a^2*cos(d*x+c)^(5/2)*s
in(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.73

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{252abE\left(\frac{1}{2}(c+dx)\middle|2\right)+10(5a^2+7b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(65a^2+70b^2+84ab\cos}{105d}$$

input
```
Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2,x]
```

output
```
(252*a*b*EllipticE[(c + d*x)/2, 2] + 10*(5*a^2 + 7*b^2)*EllipticF[(c + d*x
)/2, 2] + Sqrt[Cos[c + d*x]]*(65*a^2 + 70*b^2 + 84*a*b*Cos[c + d*x] + 15*a
^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*d)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.41, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4752, 3042, 4275, 3042, 4256, 3042, 4258, 3042, 3119, 4533, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$\big\downarrow$ 4275

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}dx+2ab\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx\right)$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2ab\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx\right)$$

$\big\downarrow$ 4256

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2ab\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)\right)$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2ab\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx}{5d\sec^{\frac{3}{2}}(c+d}\right.\right.$$

$\big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2ab\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+d}\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2ab\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+\right.}\right.\right.$$

$\big\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{5d}\right.\right.$$

$\big\downarrow$ 4533

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(5a^2+7b^2)\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{c}}{}\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(5a^2+7b^2)\int\frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right.\right.$$

$\big\downarrow$ 4256

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(5a^2+7b^2)\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2ab\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(5a^2+7b^2)\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right.$$

$\big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(5a^2+7b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(5a^2+7b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+}{3d\sqrt{\sec(c}}\right.\right.$$

$\big\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(5a^2+7b^2)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+d\right.}{3d}\right.\right.$$

input
```
Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sin[c + d*x])/(7*d*Sec[c + d
*x]^(5/2)) + 2*a*b*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(5*d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))) + ((5*a^2
 + 7*b^2)*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/7)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. 2(122) = 244.

Time = 29.04 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.68

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(240a^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+\left(-360a^{2}-336ab\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(280a^{2}}{}$

input
```
int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a^2*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*a^2-336*a*b)*sin(1/2*d*x+1/2*c
)^6*cos(1/2*d*x+1/2*c)+(280*a^2+336*a*b+140*b^2)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-80*a^2-84*a*b-70*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/
2*c)+25*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+35*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-126
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.33

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{126i\sqrt{2}ab\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-126i\sqrt{2}ab}{}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/105*(126*I*sqrt(2)*a*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c))) - 126*I*sqrt(2)*a*b*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(15*a^2*c
os(d*x + c)^2 + 42*a*b*cos(d*x + c) + 25*a^2 + 35*b^2)*sqrt(cos(d*x + c))*
sin(d*x + c) - 5*sqrt(2)*(5*I*a^2 + 7*I*b^2)*weierstrassPInverse(-4, 0, co
s(d*x + c) + I*sin(d*x + c)) - 5*sqrt(2)*(-5*I*a^2 - 7*I*b^2)*weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.77 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.95

$$\begin{aligned}\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx &= \frac{2\left(b^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+b^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{3\,d}\\&\quad-\frac{2\,a^2\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}\\&\quad-\frac{4\,a\,b\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}\end{aligned}$$

input
```
int(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^2,x)
```

output
```
(2*(b^2*ellipticF(c/2 + (d*x)/2, 2) + b^2*cos(c + d*x)^(1/2)*sin(c + d*x))
)/(3*d) - (2*a^2*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/
4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (4*a*b*cos(c + d*x)^(7/
2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c +
 d*x)^2)^(1/2))
```

Reduce [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\sec(dx+c)^2\,dx\right)b^2 + 2\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\sec(dx+c)\,dx\right)ab + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)a^2$$

input
```
int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2,x)
```

output
```
int(sqrt(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x)**2,x)*b**2 + 2*int(sqr
t(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x),x)*a*b + int(sqrt(cos(c + d*x
))*cos(c + d*x)**3,x)*a**2
```

3.804 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 101

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{2(3a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4ab\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{4ab\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}$$

output
```
2/5*(3*a^2+5*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a*b*InverseJ
acobiAM(1/2*d*x+1/2*c,2^(1/2))/d+4/3*a*b*cos(d*x+c)^(1/2)*sin(d*x+c)/d+2/5
*a^2*cos(d*x+c)^(3/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.78

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$=\frac{6(3a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+20ab\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2a\sqrt{\cos(c+dx)}(10b+3a\cos(c+dx))\,\text{s}}{15d}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2,x]
```

output
```
(6*(3*a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2] + 20*a*b*EllipticF[(c + d*x)/
2, 2] + 2*a*Sqrt[Cos[c + d*x]]*(10*b + 3*a*Cos[c + d*x])*Sin[c + d*x])/(15
*d)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.61, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {3042, 4752, 3042, 4275, 3042, 4256, 3042, 4258, 3042, 3120, 4533, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$\downarrow$ 4275

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx+2ab\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2ab\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx\right)$$

$\downarrow$ 4256

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2ab\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2ab\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx}{3d\sqrt{\sec(c+d}}\right.\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2ab\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+d}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2ab\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin(c+}}\right.\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2ab\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{}\right.\right.$$

$\downarrow$ 4533

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(3a^2+5b^2\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+2ab\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{}}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(3a^2+5b^2\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+2ab\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right.\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+d}{5d\sec^{\frac{3}{2}}(c+d}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+2ab\right.$$

input
```
Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(3*a^2 + 5*b^2)*Sqrt[Cos[c + d*x
]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*a^2*Sin[c + d*
x])/(5*d*Sec[c + d*x]^(3/2)) + 2*a*b*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c +
 d*x]])))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(92) = 184$.

Time = 27.18 (sec) , antiderivative size = 357, normalized size of antiderivative = 3.53

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-24a^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+24a^{2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+40a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\mathrm{c}\right.}{}$

input
```
int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a^2*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+24*a^2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d
*x+1/2*c)+40*a*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)*b-6*a^2*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)-20*a*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)*
b+10*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-15*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.60

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{-10i\sqrt{2}ab\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+10i\sqrt{2}ab\text{weierstrassPInverse}(-4,}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/15*(-10*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) + 10*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c)) + 2*(3*a^2*cos(d*x + c) + 10*a*b)*sqrt(cos(d*x + c))*sin(d*x +
c) - 3*sqrt(2)*(-3*I*a^2 - 5*I*b^2)*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(3*I*a^2 + 5*I*b^2
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 11.16 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{2\,b^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{4\,a\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d} + \frac{4\,a\,b\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3\,d} - \frac{2\,a^2\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+d\,x)^2\right)}{7\,d\,\sqrt{\sin(c+d\,x)^2}}$$

input
```
int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^2,x)
```

output
```
(2*b^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a*b*ellipticF(c/2 + (d*x)/2, 2)
)/(3*d) + (4*a*b*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) - (2*a^2*cos(c + d
*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(
sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^2\,dx\right)b^2 + 2\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)\,dx\right)ab + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)a^2$$

input
```
int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x)
```

output
```
int(sqrt(cos(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**2,x)*b**2 + 2*int(sqr
t(cos(c + d*x))*cos(c + d*x)**2*sec(c + d*x),x)*a*b + int(sqrt(cos(c + d*x
))*cos(c + d*x)**2,x)*a**2
```

3.805 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2(a^2+3b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
4*a*b*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(a^2+3*b^2)*InverseJacob
iAM(1/2*d*x+1/2*c,2^(1/2))/d+2/3*a^2*cos(d*x+c)^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{2\left(6abE\left(\frac{1}{2}(c+dx)\middle|2\right)+(a^2+3b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+a^2\sqrt{\cos(c+dx)}\sin(c+dx)\right)}{3d}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2,x]
```

output
```
(2*(6*a*b*EllipticE[(c + d*x)/2, 2] + (a^2 + 3*b^2)*EllipticF[(c + d*x)/2,
 2] + a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.85, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4752, 3042, 4275, 3042, 4258, 3042, 3119, 4533, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4275}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx+2ab\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+2ab\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}d\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx\right.}\right.$$

$$\Big\downarrow \text{3119}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{4533}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(a^2+3b^2\right)\int \sqrt{\sec(c+dx)}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{d}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(a^2+3b^2\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec}}{}\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right.$$

$$\Big\downarrow \text{3120}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right.$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((4*a*b*Sqrt[Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a^2*Sin[c + d*
x])/(3*d*Sqrt[Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(69) = 138$.

Time = 2.66 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.93

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(4a^{2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2a^{2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+a^{2}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\,\mathrm{s}}\right.}{3\sqrt{-}}$

input
```
int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a^2*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*a^2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1
/2*c)+a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.04

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{2}\,dx$$
$$=\frac{2\,a^{2}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+6i\,\sqrt{2}ab\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3*(2*a^2*sqrt(cos(d*x + c))*sin(d*x + c) + 6*I*sqrt(2)*a*b*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 6*I
*sqrt(2)*a*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))) + sqrt(2)*(-I*a^2 - 3*I*b^2)*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*a^2 + 3*I*b^2)*weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 11.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{2\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,b^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}+\frac{4\,a\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}$$

input
```
int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^2,x)
```

output
```
(2*a^2*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*b^2*ellipticF(c/2 + (d*x)/2
, 2))/d + (2*a^2*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) + (4*a*b*ellipticE
(c/2 + (d*x)/2, 2))/d
```

Reduce [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)^2\,dx\right)b^2 + 2\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)\,dx\right)ab + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a^2$$

input
```
int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x)
```

output
```
int(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x)**2,x)*b**2 + 2*int(sqrt(c
os(c + d*x))*cos(c + d*x)*sec(c + d*x),x)*a*b + int(sqrt(cos(c + d*x))*cos
(c + d*x),x)*a**2
```

3.806 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 68

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2\,dx = \frac{2(a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d} + \frac{4ab\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d} + \frac{2b^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
2*(a^2-b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4*a*b*InverseJacobiAM(
1/2*d*x+1/2*c,2^(1/2))/d+2*b^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2\,dx = \frac{2\left((a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+b\left(2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{b\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)\right)}{d}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2,x]
```

output
```
(2*((a^2 - b^2)*EllipticE[(c + d*x)/2, 2] + b*(2*a*EllipticF[(c + d*x)/2,
2] + (b*Sin[c + d*x])/Sqrt[Cos[c + d*x]])))/d
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.90, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4752, 3042, 4275, 3042, 4258, 3042, 3120, 4534, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4275}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx+2ab\int \sqrt{\sec(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2ab\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx\right.}}\right.$$

$\big\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+\right.}{d}\right.$$

$\big\downarrow$ 4534

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\left(a^2-b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx\right.}{d}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\left(a^2-b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c\right.}{d}\right.$$

$\big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(}}{}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{4ab\sqrt{\cos(c+dx)}}{}\right.$$

$\big\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{}\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(a^2 - b^2)*Sqrt[Cos[c + d*x]]*E
llipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (4*a*b*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*b^2*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(69) = 138$.

Time = 2.88 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.97

method	result
default	$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b^{2}-4ab\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}\right.}}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right.}}$

input
```
int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^2-2*a*b*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.62

$$\int\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2\,dx$$
$$=\frac{-2i\sqrt{2}ab\cos\left(dx+c\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+2i\sqrt{2}ab\cos\left(dx+c\right)\mathrm{w}}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
(-2*I*sqrt(2)*a*b*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) + 2*I*sqrt(2)*a*b*cos(d*x + c)*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) + 2*b^2*sqrt(cos(d*x + c))*sin(d*x + c) + sq
rt(2)*(I*a^2 - I*b^2)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(-I*a^2 + I*b^2)*cos(
d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2\sqrt{\cos(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*sqrt(cos(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 11.45 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.19

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2\,dx = \frac{2a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{d} + \frac{4ab\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{d} + \frac{2b^2\sin(c+dx)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+dx)^2\right)}{d\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)^2}}$$

input
```
int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^2,x)
```

output
```
(2*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a*b*ellipticF(c/2 + (d*x)/2, 2)
)/d + (2*b^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*
cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2\,dx = \left(\int \sqrt{\cos(dx+c)}dx\right)a^2 + \left(\int \sqrt{\cos(dx+c)}\,\sec(dx+c)^2\,dx\right)b^2 + 2\left(\int \sqrt{\cos(dx+c)}\,\sec(dx+c)\,dx\right)ab$$

input
```
int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x)
```

output
```
int(sqrt(cos(c + d*x)),x)*a**2 + int(sqrt(cos(c + d*x))*sec(c + d*x)**2,x)
*b**2 + 2*int(sqrt(cos(c + d*x))*sec(c + d*x),x)*a*b
```

3.807 $\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 95

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = -\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2(3a^2+b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2b^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
-4*a*b*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(3*a^2+b^2)*InverseJaco
biAM(1/2*d*x+1/2*c,2^(1/2))/d+2/3*b^2*sin(d*x+c)/d/cos(d*x+c)^(3/2)+4*a*b*
sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.77

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = \frac{2\left(-6abE\left(\frac{1}{2}(c+dx)\middle|2\right)+(3a^2+b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{b(b+6a\cos(c+dx))\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\right)}{3d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

output
```
(2*(-6*a*b*EllipticE[(c + d*x)/2, 2] + (3*a^2 + b^2)*EllipticF[(c + d*x)/2
, 2] + (b*(b + 6*a*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.65, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4752, 3042, 4275, 3042, 4255, 3042, 4258, 3042, 3119, 4534, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \sqrt{\sec(c+dx)}\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int \sec^{\frac{3}{2}}(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\right.$$

$$\Big\downarrow \text{4255}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(}}{d}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(}}{d}\right.\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(}}{d}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(}}{d}\right.\right.$$

$$\Big\downarrow \text{3119}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(}}{d}\right.\right.$$

$$\Big\downarrow \text{4534}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(3a^2+b^2\right)\int \sqrt{\sec(c+dx)}dx+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+d}}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(3a^2+b^2\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos}}{}\right.\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}(3a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+2ab\left(\frac{2\sin(c+dx)\sqrt{}}{d}\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}(3a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+2ab\left(\frac{2\sin(c+}{}\right.\right.$$

$\big\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2(3a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+2ab\left(\frac{2\sin(c+d}{}\right.\right.$$

input
```
Int[(a + b*Sec[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b^2*Sec[c + d*x]
^(3/2)*Sin[c + d*x])/(3*d) + 2*a*b*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(90) = 180$.

Time = 3.59 (sec) , antiderivative size = 513, normalized size of antiderivative = 5.40

method	result
default	$-\frac{2\sqrt{-\left(1-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(24a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)b-6\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\cdots}\right.}{\cdots}$

input
```
int((a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-(1-2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d
*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(24*a*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)*b-6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1
/2*c)^2*a^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2*b^2-12*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2*a*b-12*a*sin(1/2*d*x+1/2*c)^2*cos
(1/2*d*x+1/2*c)*b-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^2+3*a^2*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))+b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*a*b)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.08

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{-6i\sqrt{2}ab\cos(dx+c)^2\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))}{}$$

input
```
integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(-6*I*sqrt(2)*a*b*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 6*I*sqrt(2)*a*b*cos(d*x + c
)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c))) + sqrt(2)*(-3*I*a^2 - I*b^2)*cos(d*x + c)^2*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(3*I*a^2 + I*b^2)*cos(d*
x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(6*
a*b*cos(d*x + c) + b^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2
)
```

Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(a+b\sec{(c+dx)})^2}{\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**2/cos(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**2/sqrt(cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(b\sec{(dx+c)}+a)^2}{\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 11.54 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.14

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = \frac{2a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{d} + \frac{2b^2\sin(c+dx)\;{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\;\frac{1}{4};\;\cos(c+dx)^2\right)}{3d\cos(c+dx)^{3/2}\sqrt{\sin(c+dx)^2}} + \frac{4ab\sin(c+dx)\;{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\;\frac{3}{4};\;\cos(c+dx)^2\right)}{d\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)^2}}$$

input
```
int((a + b/cos(c + d*x))^2/cos(c + d*x)^(1/2),x)
```

output
```
(2*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*b^2*sin(c + d*x)*hypergeom([-3/
4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1
/2)) + (4*a*b*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d
*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = \left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a^2 + \left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)^2}{\cos(dx+c)}dx\right)b^2 + 2\left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)}{\cos(dx+c)}dx\right)ab$$

input
```
int((a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x)
```

output
```
int(sqrt(cos(c + d*x))/cos(c + d*x),x)*a**2 + int((sqrt(cos(c + d*x))*sec(
c + d*x)**2)/cos(c + d*x),x)*b**2 + 2*int((sqrt(cos(c + d*x))*sec(c + d*x)
)/cos(c + d*x),x)*a*b
```

3.808 $\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2(5a^2+3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} + \frac{4ab\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2b^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(5a^2+3b^2)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output
```
-2/5*(5*a^2+3*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a*b*Inverse
JacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+2/5*b^2*sin(d*x+c)/d/cos(d*x+c)^(5/2)+4/
3*a*b*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/5*(5*a^2+3*b^2)*sin(d*x+c)/d/cos(d*x
+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{-6(5a^2+3b^2)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+20ab\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+20ab\sin(c+}{15d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(a + b*Sec[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

output
```
(-6*(5*a^2 + 3*b^2)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 20*a*b*
Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 20*a*b*Sin[c + d*x] + 15*a^
2*Sin[2*(c + d*x)] + 9*b^2*Sin[2*(c + d*x)] + 6*b^2*Tan[c + d*x])/(15*d*Co
s[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.39, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4752, 3042, 4275, 3042, 4255, 3042, 4258, 3042, 3120, 4534, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$
$$\Big\downarrow \text{4752}$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \; 4275$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \sec^{\frac{3}{2}}(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int \sec^{\frac{5}{2}}(c+dx)dx\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)\right.$$

$$\Big\downarrow \; 4255$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx\right.}\right.\right.$$

$$\Big\downarrow \; 4258$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{s}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{s}\right.\right.$$

$$\Big\downarrow \; 3120$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(}{3d}\right.\right.$$

$$\Big\downarrow \; 4534$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(5a^2+3b^2\right)\int \sec^{\frac{3}{2}}(c+dx)dx+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(5a^2+3b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\mathrm{co}}\right.\right.$$

$\downarrow$ 4255

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(5a^2+3b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)+2ab\left(\frac{2\sin(c-}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(5a^2+3b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+2ab\left(\frac{2}{}\right.\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(5a^2+3b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(5a^2+3b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(\right.}\right.\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(5a^2+3b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{d}\right.\right.$$

input

```
Int[(a + b*Sec[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b^2*Sec[c + d*x]^(5/2)*Sin[c + d
*x])/(5*d) + ((5*a^2 + 3*b^2)*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/5 +
2*a*b*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(122) = 244$.

Time = 4.79 (sec) , antiderivative size = 633, normalized size of antiderivative = 4.69

method	result
default	$-\frac{\sqrt{-\left(1-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\frac{2a^2\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\sqrt{\frac{1}{2}-\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}\right.}{}$

input
```
int((a+b*sec(d*x+c))^2/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(1-2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2/sin(1/2*d
*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))+2/5*b^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*si
n(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1/
2*d*x+1/2*c)-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1
/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+4*a*b*(-1/6*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1
/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.65

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{-10i\,\sqrt{2}ab\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+10i\,\sqrt{2}ab\cos(dx+c}{}$$

input
```
integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/15*(-10*I*sqrt(2)*a*b*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) + 10*I*sqrt(2)*a*b*cos(d*x + c)^3*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(5*I*a^2 + 3*I*b^2)*c
os(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) - 3*sqrt(2)*(-5*I*a^2 - 3*I*b^2)*cos(d*x + c)^3*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c
))) + 2*(10*a*b*cos(d*x + c) + 3*(5*a^2 + 3*b^2)*cos(d*x + c)^2 + 3*b^2)*s
qrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+b\sec{(c+dx)})^2}{\cos^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**2/cos(d*x+c)**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**2/cos(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec{(dx+c)}+a)^2}{\cos{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 11.57 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$=\frac{6\,b^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\,-\frac{1}{4};\,\cos(c+d\,x)^2\right)+30\,a^2\cos(c+d\,x)^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\,\frac{3}{4};\,\cos(\right.}{15\,d\cos(c+d\,x)^{5/2}\,\sqrt{1-\cos(c+}}$$

input
```
int((a + b/cos(c + d*x))^2/cos(c + d*x)^(3/2),x)
```

output
```
(6*b^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 30*a^2*
cos(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2) +
20*a*b*cos(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^
2))/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)a^2 + \left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)^2}{\cos(dx+c)^2}dx\right)b^2 + 2\left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)}{\cos(dx+c)^2}dx\right)ab$$

input

```
int((a+b*sec(d*x+c))^2/cos(d*x+c)^(3/2),x)
```

output

```
int(sqrt(cos(c + d*x))/cos(c + d*x)**2,x)*a**2 + int((sqrt(cos(c + d*x))*s
ec(c + d*x)**2)/cos(c + d*x)**2,x)*b**2 + 2*int((sqrt(cos(c + d*x))*sec(c
+ d*x))/cos(c + d*x)**2,x)*a*b
```

3.809 $\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 160

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx =& -\frac{12abE\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} \\ &+ \frac{2(7a^2+5b^2)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} \\ &+ \frac{2b^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} \\ &+ \frac{2(7a^2+5b^2)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{12ab\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}\end{aligned}$$

output
```
-12/5*a*b*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*(7*a^2+5*b^2)*Inver
seJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+2/7*b^2*sin(d*x+c)/d/cos(d*x+c)^(7/2)+
4/5*a*b*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/21*(7*a^2+5*b^2)*sin(d*x+c)/d/cos(
d*x+c)^(3/2)+12/5*a*b*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.89

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{-252ab\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10(7a^2+5b^2)\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+84ab\sin(c}{105d\cos^{\frac{5}{2}}}$$

input
```
Integrate[(a + b*Sec[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

output
```
(-252*a*b*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 10*(7*a^2 + 5*b^2
)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 84*a*b*Sin[c + d*x] + 252
*a*b*Cos[c + d*x]^2*Sin[c + d*x] + 35*a^2*Sin[2*(c + d*x)] + 25*b^2*Sin[2*
(c + d*x)] + 30*b^2*Tan[c + d*x])/(105*d*Cos[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.34, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.826$, Rules used = {3042, 4752, 3042, 4275, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119, 4534, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \sec^{\frac{5}{2}}(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int \sec^{\frac{7}{2}}(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)\right.$$

$$\Big\downarrow \text{4255}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{5}\int \sec^{\frac{3}{2}}(c+dx)d\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{5}\int \csc\left(c+dx+\right.\right.\right.$$

$$\Big\downarrow \text{4255}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{}}{d}\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{}}{d}\right.\right.\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{}}{d}\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{}}{d}\right.\right.\right.$$

$\Big\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(}{5d}\right.\right.$$

$\Big\downarrow$ 4534

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(7a^2+5b^2)\int \sec^{\frac{5}{2}}(c+dx)dx+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+}{}\right.\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(7a^2+5b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2}{}\right.\right.\right.$$

$\Big\downarrow$ 4255

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(7a^2+5b^2)\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+2ab\left(\frac{2\sin(c}{}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(7a^2+5b^2)\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+2ab\left(\right.\right.$$

$\Big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(7a^2+5b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\text{se}}{3d}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(7a^2+5b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+}{}\right.\right.$$

$\Big\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(7a^2+5b^2)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{EllipticF}}{3d}\right.\right.$$

input
```
Int[(a + b*Sec[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b^2*Sec[c + d*x]^(7/2)*Sin[c + d
*x])/(7*d) + ((7*a^2 + 5*b^2)*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
))/7 + 2*a*b*((2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + (3*((-2*Sqrt[Cos
[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c
 + d*x]]*Sin[c + d*x])/d))/5))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 688 vs. $2(143) = 286$.

Time = 6.89 (sec) , antiderivative size = 689, normalized size of antiderivative = 4.31

method	result
default	$-\frac{\sqrt{-\left(1-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\Big(2a^{2}\Big(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{6\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{1}{2}\right)^{2}}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{1-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cdots}}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\cdots}}\cdots}{\cdots}$

input
```
int((a+b*sec(d*x+c))^2/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(1-2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(
1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+
1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^2*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-
5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos
(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+4/5*a*b/(8*sin(1/2*d*x+1/2*c)^6
-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(2
4*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin
(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/
2*c)-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.47

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{-126i\sqrt{2}ab\cos(dx+c)^4\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)}{}$$

input
```
integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/105*(-126*I*sqrt(2)*a*b*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 126*I*sqrt(2)*a*b*cos(d
*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))) - 5*sqrt(2)*(7*I*a^2 + 5*I*b^2)*cos(d*x + c)^4*weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*sqrt(2)*(-7*I*a^2 - 5
*I*b^2)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
 + c)) + 2*(126*a*b*cos(d*x + c)^3 + 42*a*b*cos(d*x + c) + 5*(7*a^2 + 5*b^
2)*cos(d*x + c)^2 + 15*b^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**2/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 11.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.71

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{30\,b^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\ -\frac{3}{4};\ \cos(c+d\,x)^2\right)+70\,a^2\cos(c+d\,x)^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\right.}{105\,d\cos(c+d\,x)^{7/2}\sqrt{1-\cos(c}}$$

input
```
int((a + b/cos(c + d*x))^2/cos(c + d*x)^(5/2),x)
```

output
```
(30*b^2*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2) + 70*a^2
*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2) +
 84*a*b*cos(c + d*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x
)^2))/(105*d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)a^2 + \left(\int \frac{\sqrt{\cos(dx+c)}\sec(dx+c)^2}{\cos(dx+c)^3}dx\right)b^2 + 2\left(\int \frac{\sqrt{\cos(dx+c)}\sec(dx+c)}{\cos(dx+c)^3}dx\right)ab$$

input
```
int((a+b*sec(d*x+c))^2/cos(d*x+c)^(5/2),x)
```

output
```
int(sqrt(cos(c + d*x))/cos(c + d*x)**3,x)*a**2 + int((sqrt(cos(c + d*x))*s
ec(c + d*x)**2)/cos(c + d*x)**3,x)*b**2 + 2*int((sqrt(cos(c + d*x))*sec(c
+ d*x))/cos(c + d*x)**3,x)*a*b
```

3.810 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 194

$$\begin{aligned}
&\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx \\
&= \frac{2a(7a^2+27b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15d}+\frac{2b(15a^2+7b^2)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} \\
&\quad+\frac{2b(15a^2+7b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} \\
&\quad+\frac{2a(7a^2+27b^2)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d}+\frac{40a^2b\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d} \\
&\quad+\frac{2a^2\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{9d}
\end{aligned}$$

output
```
2/15*a*(7*a^2+27*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*b*(15*a
^2+7*b^2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+2/21*b*(15*a^2+7*b^2)*c
os(d*x+c)^(1/2)*sin(d*x+c)/d+2/45*a*(7*a^2+27*b^2)*cos(d*x+c)^(3/2)*sin(d*
x+c)/d+40/63*a^2*b*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*a^2*cos(d*x+c)^(7/2)*
(a+b*sec(d*x+c))*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.71

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$=\frac{84(7a^3+27ab^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+60(15a^2b+7b^3)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(7a(43a^2+\ldots}{630d}$$

input
```
Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^3,x]
```

output
```
(84*(7*a^3 + 27*a*b^2)*EllipticE[(c + d*x)/2, 2] + 60*(15*a^2*b + 7*b^3)*E
llipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*a*(43*a^2 + 108*b^2)*Cos[
c + d*x] + 5*(234*a^2*b + 84*b^3 + 54*a^2*b*Cos[2*(c + d*x)] + 7*a^3*Cos[3
*(c + d*x)]))*Sin[c + d*x])/(630*d)
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.25, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4752, 3042, 4328, 27, 3042, 4535, 3042, 4256, 3042, 4258, 3042, 3119, 4533, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{ 4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$\downarrow$ 4328

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{9}\int \frac{20ba^2+\left(7a^2+27b^2\right)\sec(c+dx)a+b\left(5a^2+9b^2\right)\sec^2(c+dx)}{2\sec^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\int \frac{20ba^2+\left(7a^2+27b^2\right)\sec(c+dx)a+b\left(5a^2+9b^2\right)\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\int \frac{20ba^2+\left(7a^2+27b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(5a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+$$

$\downarrow$ 4535

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(a\left(7a^2+27b^2\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx+\int \frac{20ba^2+b\left(5a^2+9b^2\right)\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}dx\right)+$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(a\left(7a^2+27b^2\right)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\int \frac{20ba^2+b\left(5a^2+9b^2\right)\csc\left(c+dx\right.}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}$$

$\downarrow$ 4256

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\int \frac{20ba^2+b\left(5a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+a\left(7a^2+27b^2\right)\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+}}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\int \frac{20ba^2+b\left(5a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+a\left(7a^2+27b^2\right)\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+\right.}}\right.\right.\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\int \frac{20ba^2+b(5a^2+9b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+a(7a^2+27b^2)\left(\frac{3}{5}\sqrt{\cos(c+dx)}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\int \frac{20ba^2+b(5a^2+9b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+a(7a^2+27b^2)\left(\frac{3}{5}\sqrt{\cos(c+dx)}\right.\right.\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\int \frac{20ba^2+b(5a^2+9b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+a(7a^2+27b^2)\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx}\right.\right.\right.$$

$\downarrow$ 4533

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{9}{7}b(15a^2+7b^2)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+a(7a^2+27b^2)\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos}}{}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{9}{7}b(15a^2+7b^2)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+a(7a^2+27b^2)\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\right.\right.\right.$$

$\downarrow$ 4256

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{9}{7}b(15a^2+7b^2)\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+a(7a^2+27b^2)\left(\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{9}{7}b(15a^2+7b^2)\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+a(7a^2+2\right.\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{9}{7}b(15a^2+7b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c-}{3d\sqrt{\sec(c}}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{9}{7}b(15a^2+7b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\,s}{3d\sqrt{}}\right.\right.\right.$$

$$\Big\downarrow \text{3120}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{9}{7}b(15a^2+7b^2)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(\right.}{3d}\right.\right.\right.$$

input
```
Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^3,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*(a + b*Sec[c + d*x])*Sin[c +
 d*x])/(9*d*Sec[c + d*x]^(7/2)) + ((40*a^2*b*Sin[c + d*x])/(7*d*Sec[c + d*
x]^(5/2)) + a*(7*a^2 + 27*b^2)*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)
)) + (9*b*(15*a^2 + 7*b^2)*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/
7)/9)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(177) = 354$.

Time = 204.16 (sec) , antiderivative size = 470, normalized size of antiderivative = 2.42

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-1120a^{3}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+\left(2240a^{3}+2160a^{2}b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(\right.\right.}{}$

input
```
int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*a^3*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*a^3+2160*a^2*b)*sin(1/2*d*x
+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-2072*a^3-3240*a^2*b-1512*a*b^2)*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)+(952*a^3+2520*a^2*b+1512*a*b^2+420*b^3)*sin(1
/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-168*a^3-720*a^2*b-378*a*b^2-210*b^3)*
sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+225*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+105*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-567*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*a*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.17

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{3}\,dx$$
$$= \frac{2\left(35\,a^{3}\cos\left(dx+c\right)^{3}+135\,a^{2}b\cos\left(dx+c\right)^{2}+225\,a^{2}b+105\,b^{3}+7\left(7\,a^{3}+27\,ab^{2}\right)\cos\left(dx+c\right)\right)\sqrt{\cos\left(d\right.}}{}$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/315*(2*(35*a^3*cos(d*x + c)^3 + 135*a^2*b*cos(d*x + c)^2 + 225*a^2*b + 1
05*b^3 + 7*(7*a^3 + 27*a*b^2)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c
) - 15*sqrt(2)*(15*I*a^2*b + 7*I*b^3)*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) - 15*sqrt(2)*(-15*I*a^2*b - 7*I*b^3)*weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 21*sqrt(2)*(-7*I*a^3 - 27*I*a
*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*s
in(d*x + c))) - 21*sqrt(2)*(7*I*a^3 + 27*I*a*b^2)*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \int (b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2), x)
```

Giac [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \int (b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.82 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.92

$$\begin{aligned}
&\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx \\
&= \frac{2\,b^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d} + \frac{2\,b^3\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3\,d} \\
&\quad - \frac{2\,a^3\cos(c+d\,x)^{11/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos(c+d\,x)^2\right)}{11\,d\,\sqrt{\sin(c+d\,x)^2}} \\
&\quad - \frac{6\,a\,b^2\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+d\,x)^2\right)}{7\,d\,\sqrt{\sin(c+d\,x)^2}} \\
&\quad - \frac{2\,a^2\,b\cos(c+d\,x)^{9/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos(c+d\,x)^2\right)}{3\,d\,\sqrt{\sin(c+d\,x)^2}}
\end{aligned}$$

input
```
int(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x))^3,x)
```

output
```
(2*b^3*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*b^3*cos(c + d*x)^(1/2)*sin(
c + d*x))/(3*d) - (2*a^3*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2,
11/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (6*a*b^2*cos
(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/
(7*d*(sin(c + d*x)^2)^(1/2)) - (2*a^2*b*cos(c + d*x)^(9/2)*sin(c + d*x)*hy
pergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^4\sec(dx+c)^3\,dx\right)b^3 + 3\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^4\sec(dx+c)^2\,dx\right)a\,b^2 + 3\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^4\sec(dx+c)\,dx\right)a^2b + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^4\,dx\right)a^3$$

input
```
int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^3,x)
```

output
```
int(sqrt(cos(c + d*x))*cos(c + d*x)**4*sec(c + d*x)**3,x)*b**3 + 3*int(sqr
t(cos(c + d*x))*cos(c + d*x)**4*sec(c + d*x)**2,x)*a*b**2 + 3*int(sqrt(cos
(c + d*x))*cos(c + d*x)**4*sec(c + d*x),x)*a**2*b + int(sqrt(cos(c + d*x))
*cos(c + d*x)**4,x)*a**3
```

3.811 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 159

$$\begin{aligned}\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx &= \frac{2b(9a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(5a^2+21b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} \\ &+ \frac{2a(5a^2+21b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{32a^2b\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} \\ &+ \frac{2a^2\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{7d}\end{aligned}$$

output
```
2/5*b*(9*a^2+5*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*a*(5*a^2+
21*b^2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+2/21*a*(5*a^2+21*b^2)*cos
(d*x+c)^(1/2)*sin(d*x+c)/d+32/35*a^2*b*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*a
^2*cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.69

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$=\frac{42(9a^2b+5b^3)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+10(5a^3+21ab^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+a\sqrt{\cos(c+dx)}(65a^2+21\ldots}{105d}$$

input
```
Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^3,x]
```

output
```
(42*(9*a^2*b + 5*b^3)*EllipticE[(c + d*x)/2, 2] + 10*(5*a^3 + 21*a*b^2)*El
lipticF[(c + d*x)/2, 2] + a*Sqrt[Cos[c + d*x]]*(65*a^2 + 210*b^2 + 126*a*b
*Cos[c + d*x] + 15*a^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*d)
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.35, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.783$, Rules used = {3042, 4752, 3042, 4328, 27, 3042, 4535, 3042, 4256, 3042, 4258, 3042, 3120, 4533, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{ 4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 4328}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{7}\int \frac{16ba^2+(5a^2+21b^2)\sec(c+dx)a+b(3a^2+7b^2)\sec^2(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c\right.$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int \frac{16ba^2+(5a^2+21b^2)\sec(c+dx)a+b(3a^2+7b^2)\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int \frac{16ba^2+(5a^2+21b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a+b(3a^2+7b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\right.$$

$$\Big\downarrow \text{ 4535}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(a(5a^2+21b^2)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\int \frac{16ba^2+b(3a^2+7b^2)\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx\right)+\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(a(5a^2+21b^2)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\int \frac{16ba^2+b(3a^2+7b^2)\csc\left(c+dx\right.}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}\right.\right.$$

$$\Big\downarrow \text{ 4256}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\int \frac{16ba^2+b(3a^2+7b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+a(5a^2+21b^2)\left(\frac{1}{3}\int\sqrt{\sec(c+\right.\right.\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\int \frac{16ba^2+b(3a^2+7b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+a(5a^2+21b^2)\left(\frac{1}{3}\int\sqrt{\csc\left(c+\right.}\right.\right.\right.$$

$$\Big\downarrow \text{ 4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\int \frac{16ba^2+b(3a^2+7b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+a(5a^2+21b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\int \frac{16ba^2+b(3a^2+7b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+a(5a^2+21b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx}\right.\right.\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\int \frac{16ba^2+b(3a^2+7b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+a(5a^2+21b^2)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx}}\right.\right.\right.$$

$\downarrow$ 4533

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{7}{5}b(9a^2+5b^2)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+a(5a^2+21b^2)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos}}{}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{7}{5}b(9a^2+5b^2)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a(5a^2+21b^2)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\right.\right.\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{7}{5}b(9a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+a(5a^2+21b^2)\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{7}{5}b(9a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+a(5a^2+\right.\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{14b(9a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+a(5a^2+21b^2)\left(\right.\right.\right.$$

input
```
Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^3,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*(a + b*Sec[c + d*x])*Sin[c +
 d*x])/(7*d*Sec[c + d*x]^(5/2)) + ((14*b*(9*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]
]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (32*a^2*b*Sin[c +
d*x])/(5*d*Sec[c + d*x]^(3/2)) + a*(5*a^2 + 21*b^2)*((2*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3
*d*Sqrt[Sec[c + d*x]])))/7)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(146) = 292$.

Time = 202.75 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.65

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(240a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8+\left(-360a^3-504a^2b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(280a\right.}{}$

input
```
int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a^3*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*a^3-504*a^2*b)*sin(1/2*d*x+1/2
*c)^6*cos(1/2*d*x+1/2*c)+(280*a^3+504*a^2*b+420*a*b^2)*sin(1/2*d*x+1/2*c)^
4*cos(1/2*d*x+1/2*c)+(-80*a^3-126*a^2*b-210*a*b^2)*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)+25*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+105*a*b^2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-189*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-105*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.29

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{2\left(15\,a^3\cos\left(dx+c\right)^2+63\,a^2b\cos\left(dx+c\right)+25\,a^3+105\,ab^2\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-5\sqrt{2}(5i\,a^3+}{}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/105*(2*(15*a^3*cos(d*x + c)^2 + 63*a^2*b*cos(d*x + c) + 25*a^3 + 105*a*b
^2)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*sqrt(2)*(5*I*a^3 + 21*I*a*b^2)*wei
erstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*sqrt(2)*(-5*I*a
^3 - 21*I*a*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
 - 21*sqrt(2)*(-9*I*a^2*b - 5*I*b^3)*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*sqrt(2)*(9*I*a^2*b + 5*I
*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \int (b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \int (b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.76 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.92

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{2\left(b^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a\,b^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a\,b^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{d}$$
$$-\frac{2\,a^3\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\;\frac{13}{4};\;\cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$-\frac{6\,a^2\,b\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\;\frac{11}{4};\;\cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^3,x)
```

output
```
(2*(b^3*ellipticE(c/2 + (d*x)/2, 2) + a*b^2*ellipticF(c/2 + (d*x)/2, 2) +
a*b^2*cos(c + d*x)^(1/2)*sin(c + d*x)))/d - (2*a^3*cos(c + d*x)^(9/2)*sin(
c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2
)^(1/2)) - (6*a^2*b*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4],
11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^3\sec\left(dx+c\right)^3dx\right)b^3+3\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^3\sec\left(dx+c\right)^2dx\right)a\,b^2+3\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^3\sec\left(dx+c\right)dx\right)a^2b+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^3dx\right)a^3$$

input
```
int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^3,x)
```

output
```
int(sqrt(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x)**3,x)*b**3 + 3*int(sqr
t(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x)**2,x)*a*b**2 + 3*int(sqrt(cos
(c + d*x))*cos(c + d*x)**3*sec(c + d*x),x)*a**2*b + int(sqrt(cos(c + d*x))
*cos(c + d*x)**3,x)*a**3
```

3.812 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 116

$$\begin{aligned}\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx &= \frac{6a(a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b(a^2+b^2)\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}\\&\quad+\frac{8a^2b\sqrt{\cos(c+dx)}\sin(c+dx)}{5d}+\frac{2a^2\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{5d}\end{aligned}$$

output
```
6/5*a*(a^2+5*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*b*(a^2+b^2)*In
verseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+8/5*a^2*b*cos(d*x+c)^(1/2)*sin(d*x+
c)/d+2/5*a^2*cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.72

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{2\Big(3(a^3+5ab^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+5b(a^2+b^2)\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+a^2\sqrt{\cos(c+dx)}(5b+a\cos(c+}{5d}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^3,x]
```

output
```
(2*(3*(a^3 + 5*a*b^2)*EllipticE[(c + d*x)/2, 2] + 5*b*(a^2 + b^2)*Elliptic
F[(c + d*x)/2, 2] + a^2*Sqrt[Cos[c + d*x]]*(5*b + a*Cos[c + d*x])*Sin[c +
d*x]))/(5*d)
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.53, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 4752, 3042, 4328, 27, 3042, 4535, 3042, 4258, 3042, 3119, 4533, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{ 4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4328}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{5}\int \frac{12ba^2+3(a^2+5b^2)\sec(c+dx)a+b(a^2+5b^2)\sec^2(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2\sin(c+}{5d}\right.$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{12ba^2+3\left(a^2+5b^2\right)\sec(c+dx)a+b\left(a^2+5b^2\right)\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2\sin(c+}{5d}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{12ba^2+3\left(a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\right.$$

$\Big\downarrow$ 4535

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\int \frac{12ba^2+b\left(a^2+5b^2\right)\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx+3a\left(a^2+5b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(3a\left(a^2+5b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{12ba^2+b\left(a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}\right.\right.$$

$\Big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\int \frac{12ba^2+b\left(a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+3a\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\int \frac{12ba^2+b\left(a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+3a\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec\right.\right.$$

$\Big\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\int \frac{12ba^2+b\left(a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{6a\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec}}{d}\right.\right.$$

$\Big\downarrow$ 4533

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(5b\left(a^2+b^2\right)\int \sqrt{\sec(c+dx)}dx+\frac{6a\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}\right.}{d}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(5b(a^2+b^2)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{6a(a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}\right.\right.$$

$\Big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(5b(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{6a(a^2+5b^2)\sqrt{c}}{}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(5b(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a(a^2+5b^2)}{}\right.\right.$$

$\Big\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{10b(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a(a^2+5b^2)}{}\right.\right.$$

input
```
Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^3,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*(a + b*Sec[c + d*x])*Sin[c +
 d*x])/(5*d*Sec[c + d*x]^(3/2)) + ((6*a*(a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*E
llipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (10*b*(a^2 + b^2)*Sqrt[Co
s[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (8*a^2*b*Sin
[c + d*x])/(d*Sqrt[Sec[c + d*x]]))/5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(109) = 218$.

Time = 200.97 (sec) , antiderivative size = 412, normalized size of antiderivative = 3.55

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-8a^{3}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+8a^{3}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+20a^{2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\mathrm{co}\right.}{}$

input
```
int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*a^3*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*a^3*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+
1/2*c)+20*a^2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)*b-2*a^3*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c)-10*a^2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
*b+5*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*a^3-15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.59

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{2\,(a^3\cos\left(dx+c\right)+5\,a^2b)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-5\,\sqrt{2}(i\,a^2b+i\,b^3)\mathrm{weierstrassPInverse}(-4,0,\cos(}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/5*(2*(a^3*cos(d*x + c) + 5*a^2*b)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*sq
rt(2)*(I*a^2*b + I*b^3)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) - 5*sqrt(2)*(-I*a^2*b - I*b^3)*weierstrassPInverse(-4, 0, cos(d*x
+ c) - I*sin(d*x + c)) - 3*sqrt(2)*(-I*a^3 - 5*I*a*b^2)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2
)*(I*a^3 + 5*I*a*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \int (b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \int (b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.71 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.08

$$\begin{aligned}
&\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx \\
&= \frac{2\,b^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{6\,a\,b^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} \\
&\quad + \frac{2\,a^2\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{2\,a^2\,b\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{d} \\
&\quad - \frac{2\,a^3\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}
\end{aligned}$$

input
```
int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^3,x)
```

output
```
(2*b^3*ellipticF(c/2 + (d*x)/2, 2))/d + (6*a*b^2*ellipticE(c/2 + (d*x)/2,
2))/d + (2*a^2*b*ellipticF(c/2 + (d*x)/2, 2))/d + (2*a^2*b*cos(c + d*x)^(1
/2)*sin(c + d*x))/d - (2*a^3*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/
2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^3\,dx\right)b^3 + 3\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^2\,dx\right)a\,b^2 + 3\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)\,dx\right)a^2b + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)a^3$$

input
```
int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^3,x)
```

output
```
int(sqrt(cos(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**3,x)*b**3 + 3*int(sqr
t(cos(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**2,x)*a*b**2 + 3*int(sqrt(cos
(c + d*x))*cos(c + d*x)**2*sec(c + d*x),x)*a**2*b + int(sqrt(cos(c + d*x))
*cos(c + d*x)**2,x)*a**3
```

3.813 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 126

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{2b(3a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d} + \frac{2a(a^2+9b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} - \frac{2b(a^2-3b^2)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2a^2\sqrt{\cos(c+dx)}(a+b\sec(c+dx))\sin(c+dx)}{3d}$$

output
```
2*b*(3*a^2-b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*(a^2+9*b^2)*
InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d-2/3*b*(a^2-3*b^2)*sin(d*x+c)/d/co
s(d*x+c)^(1/2)+2/3*a^2*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.69

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{2\left((9a^2b-3b^3)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+(a^3+9ab^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{(3b^3+a^3\cos(c+dx))\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{3d}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3,x]
```

output
```
(2*((9*a^2*b - 3*b^3)*EllipticE[(c + d*x)/2, 2] + (a^3 + 9*a*b^2)*Elliptic
F[(c + d*x)/2, 2] + ((3*b^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/Sqrt[Cos[c +
 d*x]]))/(3*d)
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.49, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 4752, 3042, 4328, 27, 3042, 4535, 3042, 4258, 3042, 3120, 4534, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4328}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{3}\int \frac{8ba^2+(a^2+9b^2)\sec(c+dx)a-b(a^2-3b^2)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx}{3d\sqrt{}}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int\frac{8ba^2+\left(a^2+9b^2\right)\sec(c+dx)a-b\left(a^2-3b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx}{3d\sqrt{}}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int\frac{8ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a-b\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2}{}\right.$$

$\big\downarrow$ 4535

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int\frac{8a^2b-b\left(a^2-3b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx+a\left(a^2+9b^2\right)\int\sqrt{\sec(c+dx)}dx\right)+\frac{2a^2}{}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(a\left(a^2+9b^2\right)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\int\frac{8a^2b-b\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}\right.\right.$$

$\big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int\frac{8a^2b-b\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c}\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int\frac{8a^2b-b\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c}\right.\right.$$

$\big\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int\frac{8a^2b-b\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\left(a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(}}{d}\right.\right.$$

$\big\downarrow$ 4534

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(3b\left(3a^2-b^2\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx-\frac{2b\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a}{}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(3b(3a^2-b^2)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}\right.\right.$$

$\Big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(3b(3a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-\frac{2b(a^2-3b^2)\sin\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(3b(3a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{2b(a^2-3b\right.\right.$$

$\Big\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(-\frac{2b(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a(a^2+9b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c-}}{d}\right.\right.$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*(a + b*Sec[c + d*x])*Sin[c +
 d*x])/(3*d*Sqrt[Sec[c + d*x]]) + ((6*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*E
llipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a*(a^2 + 9*b^2)*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (2*b*(a^2 -
 3*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d)/3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(119) = 238.

Time = 3.80 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.40

method	result
default	$-\frac{2\left(4a^3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2a^3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2b^3+a^3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}\right.}}{}$

input
```
int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(4*a^3*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*a^3*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)-6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^3+a^3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))+9*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*a^2*b+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3)/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.70

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$=\frac{\sqrt{2}(-i\,a^3-9i\,ab^2)\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}(i\,a^3+9i}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*(-I*a^3 - 9*I*a*b^2)*cos(d*x + c)*weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*a^3 + 9*I*a*b^2)*cos(d*x + c)*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(-3*
I*a^2*b + I*b^3)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(3*I*a^2*b - I*b^3)*cos(
d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*(a^3*cos(d*x + c) + 3*b^3)*sqrt(cos(d*x + c))*sin(d*x
 + c))/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \int (b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx=\int (b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.75 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98

$$\begin{aligned}&\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx\\&=\frac{2\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{6\,a^2\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,a\,b^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}\\&\quad+\frac{2\,a^3\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}+\frac{2\,b^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}\end{aligned}$$

input
```
int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^3,x)
```

output
```
(2*a^3*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (6*a^2*b*ellipticE(c/2 + (d*x)
/2, 2))/d + (6*a*b^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*a^3*cos(c + d*x)^
(1/2)*sin(c + d*x))/(3*d) + (2*b^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4
, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)^3\,dx\right)b^3 + 3\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)^2\,dx\right)a\,b^2 + 3\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)\,dx\right)a^2b + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a^3$$

input
```
int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x)
```

output
```
int(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x)**3,x)*b**3 + 3*int(sqrt(c
os(c + d*x))*cos(c + d*x)*sec(c + d*x)**2,x)*a*b**2 + 3*int(sqrt(cos(c + d
*x))*cos(c + d*x)*sec(c + d*x),x)*a**2*b + int(sqrt(cos(c + d*x))*cos(c +
d*x),x)*a**3
```

3.814 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 118

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3\,dx = \frac{2a(a^2-3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d} + \frac{2b(9a^2+b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{16ab^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

output
```
2*a*(a^2-3*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*b*(9*a^2+b^2)*
InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+16/3*a*b^2*sin(d*x+c)/d/cos(d*x+c
)^(1/2)+2/3*b^2*(a+b*sec(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.71

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3\,dx$$
$$=\frac{2\left(3(a^3-3ab^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\left((9a^2+b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{b(b+9a\cos(c+dx))\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\right)\right)}{3d}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^3,x]
```

output
```
(2*(3*(a^3 - 3*a*b^2)*EllipticE[(c + d*x)/2, 2] + b*((9*a^2 + b^2)*Ellipti
cF[(c + d*x)/2, 2] + (b*(b + 9*a*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^
(3/2))))/(3*d)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.53, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.696, Rules used = {3042, 4752, 3042, 4329, 27, 3042, 4535, 3042, 4258, 3042, 3120, 4534, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$\downarrow$ 4329

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{3}\int \frac{8ab^2\sec^2(c+dx)+b(9a^2+b^2)\sec(c+dx)+a(3a^2-b^2)}{2\sqrt{\sec(c+dx)}}dx+\frac{2b^2\sin(c+dx}{}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{8ab^2\sec^2(c+dx)+b(9a^2+b^2)\sec(c+dx)+a(3a^2-b^2)}{\sqrt{\sec(c+dx)}}dx+\frac{2b^2\sin(c+dx}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{8ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+b(9a^2+b^2)\csc\left(c+dx+\frac{\pi}{2}\right)+a(3a^2-b^2)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b^2}{}\right.$$

$\downarrow$ 4535

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int \frac{8ab^2\sec^2(c+dx)+a(3a^2-b^2)}{\sqrt{\sec(c+dx)}}dx+b(9a^2+b^2)\int \sqrt{\sec(c+dx)}dx\right)+\frac{2b^2}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(b(9a^2+b^2)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\int \frac{8ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a(3a^2-b^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}\right.\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int \frac{8ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a(3a^2-b^2)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(9a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int \frac{8ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a(3a^2-b^2)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(9a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.\right.$$

$$\Big\downarrow \text{3120}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int \frac{8ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a(3a^2-b^2)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b(9a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(}{d}\right.\right.$$

$$\Big\downarrow \text{4534}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(3a(a^2-3b^2)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2b(9a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{El}}{d}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(3a(a^2-3b^2)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b(9a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{d}\right.\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(3a(a^2-3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2b(9a^2+b^2)\sqrt{}}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(3a(a^2-3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b(9a^2+}{}\right.\right.$$

$$\Big\downarrow \text{3119}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{2b(9a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a(a^2-3b}{}\right.\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^3,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b^2*Sqrt[Sec[c + d*x]]*(a + b*Se
c[c + d*x])*Sin[c + d*x])/(3*d) + ((6*a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*E
llipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*b*(9*a^2 + b^2)*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (16*a*b^2*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/d)/3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(111) = 222$.

Time = 3.79 (sec) , antiderivative size = 630, normalized size of antiderivative = 5.34

method	result
default	$-\frac{2\sqrt{-\left(1-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(36\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a b^2-18\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2}\right.}{}$

input
```
int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-(1-2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d
*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(36*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b^2-18*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*
x+1/2*c)^2*a^2*b-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2*b^3+6*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2*a^3-18*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*sin(1/2*d*x+1/2*c)^2*a*b^2-18*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*
a*b^2-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^3+9*a^2*b*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3+
9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.88

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3\,dx$$
$$= \frac{\sqrt{2}(-9i\,a^2b-i\,b^3)\cos{(dx+c)}^2\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+\sqrt{2}(9i\,a^2b+}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*(-9*I*a^2*b - I*b^3)*cos(d*x + c)^2*weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(9*I*a^2*b + I*b^3)*cos(d*x + c
)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*
(-I*a^3 + 3*I*a*b^2)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(I*a^3 - 3*I*a*b^2
)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c))) + 2*(9*a*b^2*cos(d*x + c) + b^3)*sqrt(cos(d*x + c
))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3\,dx = \int (a+b\sec{(c+dx)})^3\sqrt{\cos{(c+dx)}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**3,x)
```

output
```
Integral((a + b*sec(c + d*x))**3*sqrt(cos(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3\,dx = \int (b\sec{(dx+c)}+a)^3\sqrt{\cos{(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3\,dx = \int (b\sec(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 11.57 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.08

$$\begin{aligned}&\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3\,dx\\&=\frac{2\left(\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\,a^3+3\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\,a^2\right)}{d}\\&\quad+\frac{2\,b^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\;\frac{1}{4};\;\cos\left(c+d\,x\right)^2\right)}{3\,d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}\\&\quad+\frac{6\,a\,b^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\;\frac{3}{4};\;\cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}\end{aligned}$$

input
```
int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^3,x)
```

output
```
(2*(a^3*ellipticE(c/2 + (d*x)/2, 2) + 3*a^2*b*ellipticF(c/2 + (d*x)/2, 2))
)/d + (2*b^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*
d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (6*a*b^2*sin(c + d*x)*hyper
geom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x
)^2)^(1/2))
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3\,dx = \left(\int \sqrt{\cos(dx+c)}dx\right)a^3 + \left(\int \sqrt{\cos(dx+c)}\sec(dx+c)^3\,dx\right)b^3 + 3\left(\int \sqrt{\cos(dx+c)}\sec(dx+c)^2\,dx\right)a\,b^2 + 3\left(\int \sqrt{\cos(dx+c)}\sec(dx+c)\,dx\right)a^2b$$

input
```
int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x)
```

output
```
int(sqrt(cos(c + d*x)),x)*a**3 + int(sqrt(cos(c + d*x))*sec(c + d*x)**3,x)
*b**3 + 3*int(sqrt(cos(c + d*x))*sec(c + d*x)**2,x)*a*b**2 + 3*int(sqrt(co
s(c + d*x))*sec(c + d*x),x)*a**2*b
```

3.815 $\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 149

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = -\frac{6b(5a^2+b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(a^2+b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d} + \frac{8ab^2\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} + \frac{6b(5a^2+b^2)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
-6/5*b*(5*a^2+b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a*(a^2+b^2)*I
nverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+8/5*a*b^2*sin(d*x+c)/d/cos(d*x+c)^
(3/2)+6/5*b*(5*a^2+b^2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+2/5*b^2*(a+b*sec(d*x
+c))*sin(d*x+c)/d/cos(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{-6b(5a^2+b^2)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10a(a^2+b^2)\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+10ab}{5d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(a + b*Sec[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

output
```
(-6*b*(5*a^2 + b^2)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*a*(a
^2 + b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*a*b^2*Sin[c +
d*x] + 3*(5*a^2*b + b^3)*Sin[2*(c + d*x)] + 2*b^3*Tan[c + d*x])/(5*d*Cos[c
 + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.36, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.783, Rules used = {3042, 4752, 3042, 4329, 27, 3042, 4535, 3042, 4255, 3042, 4258, 3042, 3119, 4534, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3dx$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$\downarrow$ 4329

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{5}\int\frac{1}{2}\sqrt{\sec(c+dx)}\left(12ab^2\sec^2(c+dx)+3b\left(5a^2+b^2\right)\sec(c+dx)+a\left(5a^2+b^2\right)\right)\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int\sqrt{\sec(c+dx)}\left(12ab^2\sec^2(c+dx)+3b\left(5a^2+b^2\right)\sec(c+dx)+a\left(5a^2+b^2\right)\right)d\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+3b\left(5a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right.\right.\right.$$

$\downarrow$ 4535

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(3b\left(5a^2+b^2\right)\int\sec^{\frac{3}{2}}(c+dx)dx+\int\sqrt{\sec(c+dx)}\left(12ab^2\sec^2(c+dx)+a\left(5a^2\right.\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(3b\left(5a^2+b^2\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2\csc\left(c\right.\right.\right.\right.$$

$\downarrow$ 4255

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(5a^2+b^2\right)\right)dx+3b\left(5a^2\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(5a^2+b^2\right)\right)dx+3b\left(5a^2\right.\right.\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(5a^2+b^2\right)\right)dx+3b\left(5a^2+\right.\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(5a^2+b^2\right)\right)dx+3b\left(5a^2+\right.\right.\right.$$

$\big\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(5a^2+b^2\right)\right)dx+3b\left(5a^2+\right.\right.\right.$$

$\big\downarrow$ 4534

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(5a\left(a^2+b^2\right)\int\sqrt{\sec(c+dx)}dx+3b\left(5a^2+b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\right.\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(5a\left(a^2+b^2\right)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+3b\left(5a^2+b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+}}{d}\right.\right.\right.$$

$\big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(5a\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+3b\left(5a^2+b^2\right)\left(\frac{2}{}\right.\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(5a\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b\left(5a^2+b^2\right.\right.\right.$$

$\big\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{10a\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+3b\left(5a^2+b^2\right.\right.\right.$$

input
```
Int[(a + b*Sec[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b^2*Sec[c + d*x]^(3/2)*(a + b*Se
c[c + d*x])*Sin[c + d*x])/(5*d) + ((10*a*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (8*a*b^2*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/d + 3*b*(5*a^2 + b^2)*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
d))/5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. $2(138) = 276$.

Time = 4.69 (sec) , antiderivative size = 711, normalized size of antiderivative = 4.77

method	result	size
default	Expression too large to display	711

input
```
int((a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(1-2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2/5*
b^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2
-1)/sin(1/2*d*x+1/2*c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1
/2*d*x+1/2*c)+12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*
d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+6*a*b^2*(-1/6*cos(1/2*d*x+1/2*c)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-
1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2)))+6*a^2*b/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.64

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx =$$
$$-\frac{5\sqrt{2}(i\,a^3+i\,ab^2)\cos{(dx+c)}^3\,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+5\sqrt{2}(-i\,a^3$$

input
```
integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/5*(5*sqrt(2)*(I*a^3 + I*a*b^2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-I*a^3 - I*a*b^2)*cos(d*x +
c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)
*(5*I*a^2*b + I*b^3)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(-5*I*a^2*b - I*b^
3)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))) - 2*(5*a*b^2*cos(d*x + c) + b^3 + 3*(5*a^2*b + b
^3)*cos(d*x + c)^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(a+b\sec{(c+dx)})^3}{\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**3/sqrt(cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(b\sec{(dx+c)}+a)^3}{\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(b\sec(dx+c)+a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 11.61 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.05

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx=\frac{2a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{d}+\frac{2b^3\sin(c+dx)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos(c+dx)^2\right)}{5d\cos(c+dx)^{5/2}\sqrt{\sin(c+dx)^2}}+\frac{6a^2b\sin(c+dx)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+dx)^2\right)}{d\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)^2}}+\frac{2ab^2\sin(c+dx)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+dx)^2\right)}{d\cos(c+dx)^{3/2}\sqrt{\sin(c+dx)^2}}$$

input
```
int((a + b/cos(c + d*x))^3/cos(c + d*x)^(1/2),x)
```

output
```
(2*a^3*ellipticF(c/2 + (d*x)/2, 2))/d + (2*b^3*sin(c + d*x)*hypergeom([-5/
4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(
1/2)) + (6*a^2*b*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))
/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*a*b^2*sin(c + d*x)*hyp
ergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(3/2)*(sin(c + d
*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = \left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a^3 + \left(\int \frac{\sqrt{\cos(dx+c)}\sec(dx+c)^3}{\cos(dx+c)}dx\right)b^3 + 3\left(\int \frac{\sqrt{\cos(dx+c)}\sec(dx+c)^2}{\cos(dx+c)}dx\right)a\,b^2 + 3\left(\int \frac{\sqrt{\cos(dx+c)}\sec(dx+c)}{\cos(dx+c)}dx\right)a^2b$$

input
```
int((a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x)
```

output
```
int(sqrt(cos(c + d*x))/cos(c + d*x),x)*a**3 + int((sqrt(cos(c + d*x))*sec(
c + d*x)**3)/cos(c + d*x),x)*b**3 + 3*int((sqrt(cos(c + d*x))*sec(c + d*x)
**2)/cos(c + d*x),x)*a*b**2 + 3*int((sqrt(cos(c + d*x))*sec(c + d*x))/cos(
c + d*x),x)*a**2*b
```

3.816 $\int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 194

$$\int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2a(5a^2+9b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} + \frac{2b(21a^2+5b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} + \frac{32ab^2\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2b(21a^2+5b^2)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(5a^2+9b^2)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)}$$

output

```
-2/5*a*(5*a^2+9*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*b*(21*a^
2+5*b^2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+32/35*a*b^2*sin(d*x+c)/d
/cos(d*x+c)^(5/2)+2/21*b*(21*a^2+5*b^2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/5*
a*(5*a^2+9*b^2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+2/7*b^2*(a+b*sec(d*x+c))*sin
(d*x+c)/d/cos(d*x+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{-42a(5a^2+9b^2)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10b(21a^2+5b^2)\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

output
```
(-42*a*(5*a^2 + 9*b^2)*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 10*b
*(21*a^2 + 5*b^2)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 126*a*b^2
*Sin[c + d*x] + 210*a^3*Cos[c + d*x]^2*Sin[c + d*x] + 378*a*b^2*Cos[c + d*
x]^2*Sin[c + d*x] + 105*a^2*b*Sin[2*(c + d*x)] + 25*b^3*Sin[2*(c + d*x)] +
 30*b^3*Tan[c + d*x])/(105*d*Cos[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.23, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4752, 3042, 4329, 27, 3042, 4535, 3042, 4255, 3042, 4258, 3042, 3120, 4534, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$
$$\Big\downarrow \; 4752$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4329}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{7}\int \frac{1}{2}\sec^{\frac{3}{2}}(c+dx)\left(16ab^2\sec^2(c+dx)+b(21a^2+5b^2)\sec(c+dx)+a(7a^2+3b^2\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int \sec^{\frac{3}{2}}(c+dx)\left(16ab^2\sec^2(c+dx)+b(21a^2+5b^2)\sec(c+dx)+a(7a^2+3b^2)\right)\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+b(21a^2+5b^2)\csc\left(c+dx+\right.\right.\right.$$

$$\Big\downarrow \text{4535}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(b(21a^2+5b^2)\int \sec^{\frac{5}{2}}(c+dx)dx+\int \sec^{\frac{3}{2}}(c+dx)\left(16ab^2\sec^2(c+dx)+a(7a^2\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(b(21a^2+5b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16ab^2\csc\left(\right.\right.\right.\right.$$

$$\Big\downarrow \text{4255}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a(7a^2+3b^2)\right)dx+b(21a^2\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a(7a^2+3b^2)\right)dx+b(21a^2\right.\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(7a^2+3b^2\right)\right)dx+b(21a^2$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(7a^2+3b^2\right)\right)dx+b(21a$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(7a^2+3b^2\right)\right)dx+b(21a^2$$

$\downarrow$ 4534

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{7}{5}a\left(5a^2+9b^2\right)\int \sec^{\frac{3}{2}}(c+dx)dx+b\left(21a^2+5b^2\right)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{7}{5}a\left(5a^2+9b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+b\left(21a^2+5b^2\right)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c}{3d}\right.\right.\right.$$

$\downarrow$ 4255

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{7}{5}a\left(5a^2+9b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)+b(21a^2+\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{7}{5}a\left(5a^2+9b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+b(\right.\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{7}{5}a\left(5a^2+9b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{7}{5}a\left(5a^2+9b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\right.\right.\right.$$

$$\Big\downarrow \text{3119}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(b\left(21a^2+5b^2\right)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{Ellip}}{3d}\right.\right.\right.$$

input
```
Int[(a + b*Sec[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b^2*Sec[c + d*x]^(5/2)*(a + b*Se
c[c + d*x])*Sin[c + d*x])/(7*d) + ((32*a*b^2*Sec[c + d*x]^(5/2)*Sin[c + d*
x])/(5*d) + (7*a*(5*a^2 + 9*b^2)*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/5
 + b*(21*a^2 + 5*b^2)*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)))/7)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 819 vs. $2(177) = 354$.

Time = 5.96 (sec) , antiderivative size = 820, normalized size of antiderivative = 4.23

method	result	size
default	Expression too large to display	820

input
```
int((a+b*sec(d*x+c))^3/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(1-2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^3/sin(1/2*d
*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))+2*b^3*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2
*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*
c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2)))+6/5*a*b^2/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c
)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(24*sin(1/2*d*x+1/2*
c)^6*cos(1/2*d*x+1/2*c)-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-
24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*si
n(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+6*a^2
*b*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.39

$$\int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx =$$
$$-\frac{5\sqrt{2}(21i\,a^2b+5i\,b^3)\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}(-\ldots}{\ldots}$$

input
```
integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/105*(5*sqrt(2)*(21*I*a^2*b + 5*I*b^3)*cos(d*x + c)^4*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-21*I*a^2*b - 5*I*b^3
)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
 + 21*sqrt(2)*(5*I*a^3 + 9*I*a*b^2)*cos(d*x + c)^4*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*sqrt(2)*(-
5*I*a^3 - 9*I*a*b^2)*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(63*a*b^2*cos(d*x + c) + 2
1*(5*a^3 + 9*a*b^2)*cos(d*x + c)^3 + 15*b^3 + 5*(21*a^2*b + 5*b^3)*cos(d*x
 + c)^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**3/cos(d*x+c)**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**3/cos(c + d*x)**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^3}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 11.81 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.76

$$\int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\frac{2b^3\sin(c+dx)\,{}_2F_1\left(-\frac{7}{4},\frac{1}{2};\,-\frac{3}{4};\,\cos(c+dx)^2\right)}{7} + 2a^3\cos(c+dx)^3\sin(c+dx)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\,\frac{3}{4};\,\cos(c+dx)^2\right) + \frac{6a}{}}{d\cos(c+dx)^{7/2}}$$

input
```
int((a + b/cos(c + d*x))^3/cos(c + d*x)^(3/2),x)
```

output
```
((2*b^3*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2))/7 + 2*a
^3*cos(c + d*x)^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2)
 + (6*a*b^2*cos(c + d*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c +
 d*x)^2))/5 + 2*a^2*b*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1
/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)a^3 + \left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)^3}{\cos(dx+c)^2}dx\right)b^3 + 3\left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)^2}{\cos(dx+c)^2}dx\right)a\,b^2 + 3\left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)}{\cos(dx+c)^2}dx\right)a^2b$$

input
```
int((a+b*sec(d*x+c))^3/cos(d*x+c)^(3/2),x)
```

output
```
int(sqrt(cos(c + d*x))/cos(c + d*x)**2,x)*a**3 + int((sqrt(cos(c + d*x))*s
ec(c + d*x)**3)/cos(c + d*x)**2,x)*b**3 + 3*int((sqrt(cos(c + d*x))*sec(c
+ d*x)**2)/cos(c + d*x)**2,x)*a*b**2 + 3*int((sqrt(cos(c + d*x))*sec(c + d
*x))/cos(c + d*x)**2,x)*a**2*b
```

3.817 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 152

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{2(3a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5a^3d} - \frac{2b(a^2+3b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^4d} + \frac{2b^4\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a^4(a+b)d} - \frac{2b\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad}$$

output
```
2/5*(3*a^2+5*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-2/3*b*(a^2+3
*b^2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/a^4/d+2*b^4*EllipticPi(sin(1/
2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a^4/(a+b)/d-2/3*b*cos(d*x+c)^(1/2)*sin(d*x
+c)/a^2/d+2/5*cos(d*x+c)^(3/2)*sin(d*x+c)/a/d
```

Mathematica [A] (warning: unable to verify)

Time = 11.34 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.49

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{2(9a^2+5b^2)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+8b\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right)+4\sqrt{\cos(c+}{}$$

input

```
Integrate[Cos[c + d*x]^(5/2)/(a + b*Sec[c + d*x]),x]
```

output

```
((2*(9*a^2 + 5*b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + 8
*b*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)
/2, 2])/(a + b)) + 4*Sqrt[Cos[c + d*x]]*(-5*b + 3*a*Cos[c + d*x])*Sin[c +
d*x] + (6*(3*a^2 + 5*b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1
] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*
EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*S
qrt[Sin[c + d*x]^2]))/(30*a^2*d)
```

Rubi [A] (verified)

Time = 1.85 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.61, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4752, 3042, 4340, 27, 3042, 4592, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4340}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int -\frac{-3b\sec^2(c+dx)-3a\sec(c+dx)+5b}{2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}dx}{5a}+\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-3b\sec^2(c+dx)-3a\sec(c+dx)+5b}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}dx}{5a}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-3b\csc\left(c+dx+\frac{\pi}{2}\right)^2-3a\csc\left(c+dx+\frac{\pi}{2}\right)+5b}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{5a}\right)$$

$$\Big\downarrow \text{4592}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{10b\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{2\int \frac{-5b^2\sec^2(c+dx)+4ab\sec(c+dx)+3\left(3a^2+5b^2\right)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{3a}}{5a}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{10b\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{-5b^2\sec^2(c+dx)+4ab\sec(c+dx)+3\left(3a^2+5b^2\right)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{3a}}{5a}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{10b\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{-5b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+4ab\csc\left(c+dx+\frac{\pi}{2}\right)+3\left(3a^2+5b^2\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3a}}{5a}\right)$$

$\downarrow$ 4594

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{10b\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{15b^4\int\frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}+\frac{\int\frac{3a\left(3a^2+5b^2\right)-5b\left(a^2+3b^2\right)\sec(c+d}{\sqrt{\sec(c+dx)}}}{a^2}}{3a}}{5a}$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{10b\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{15b^4\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\int\frac{3a\left(3a^2+5b^2\right)-5b\left(a^2+3b^2\right)\csc}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}}{a^2}}{3a}}{5a}$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{10b\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{15b^4\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3a\left(3a^2+5b^2\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx-5}{a^2}}{3a}}{5a}$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{10b\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{15b^4\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3a\left(3a^2+5b^2\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}}{}}{3a}}{5a}$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{10b\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{15b^4\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3a\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{}}{}}{5a}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{10b\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{15b^4\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3a\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{}}{}}{5a}\right.$$

$$\Big\downarrow \text{3119}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{10b\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{15b^4\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{6a\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c-}}{d}}{}}{}}{5a}\right.$$

$$\Big\downarrow \text{3120}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{10b\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{15b^4\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{6a\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c-}}{d}}{}}{5a}\right.$$

$$\Big\downarrow \text{4336}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{10b\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{15b^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx}{a^2}+\cdots}{\cdots}}{\cdots}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{10b\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{15b^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\cdots\right.\right.}}{a^2}\cdots}{\cdots}}{\cdots}\right.$$

$\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{10b\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{30b^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a^2d(a+b)}+\frac{6a\left(3\cdots\right.}{\cdots}}{\cdots}}{\cdots}\right.$$

input
```
Int[Cos[c + d*x]^(5/2)/(a + b*Sec[c + d*x]),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*Sin[c + d*x])/(5*a*d*Sec[c + d*x
]^(3/2)) - (-1/3*(((6*a*(3*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (10*b*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (30*b^4*Sqrt[Cos[c
+ d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2
*(a + b)*d))/a + (10*b*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]))/(5*a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4340
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Sim
p[1/(a*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n
- a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(145) = 290$.

Time = 4.93 (sec) , antiderivative size = 668, normalized size of antiderivative = 4.39

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\left(-24a^4+24a^3b\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6+\left(24a^4-44a^3b+20a^2b^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\right.\cdots}{\cdots}$

input
```
int(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((-24*a^4+24
*a^3*b)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*a^4-44*a^3*b+20*a^2*b^
2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-6*a^4+16*a^3*b-10*a^2*b^2)*si
n(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-15*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2
^(1/2))-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*
b^2-15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3+15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-9*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*a^4+9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-15*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*a^2*b^2+15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3)/a^4/(a-b)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+b*sec(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{b\sec\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{b\sec\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cos\left(c+d\,x\right)^{5/2}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a + b/cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + b/cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^{2}}{\sec\left(dx+c\right)b+a}dx$$

input
```
int(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x)
```

output
```
int((sqrt(cos(c + d*x))*cos(c + d*x)**2)/(sec(c + d*x)*b + a),x)
```

3.818 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 112

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2(a^2+3b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^3d} - \frac{2b^3\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a^3(a+b)d} + \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad}$$

output
```
-2*b*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+2/3*(a^2+3*b^2)*InverseJa
cobiAM(1/2*d*x+1/2*c,2^(1/2))/a^3/d-2*b^3*EllipticPi(sin(1/2*d*x+1/2*c),2*
a/(a+b),2^(1/2))/a^3/(a+b)/d+2/3*cos(d*x+c)^(1/2)*sin(d*x+c)/a/d
```

Mathematica [A] (warning: unable to verify)

Time = 1.16 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.41

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{4\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{6b\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+4\sqrt{\cos(c+dx)}\sin(c+dx)-\frac{6\left(-2abE\left(\arcsin\left(\sqrt{\cos(c+dx)}\right)\ldots\right.\right.}{\ldots}}{6ad}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x]),x]
```

output
```
(4*EllipticF[(c + d*x)/2, 2] - (6*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2])/(a + b) + 4*Sqrt[Cos[c + d*x]]*Sin[c + d*x] - (6*(-2*a*b*EllipticE[Ar
cSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c +
d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]],
 -1])*Sin[c + d*x])/(a^2*Sqrt[Sin[c + d*x]^2]))/(6*a*d)
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.78, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.739, Rules used = {3042, 4752, 3042, 4340, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{ 4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$\Big\downarrow$ 4340

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int -\frac{-b\sec^2(c+dx)-a\sec(c+dx)+3b}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{3a}+\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{-b\sec^2(c+dx)-a\sec(c+dx)+3b}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{3a}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{-b\csc\left(c+dx+\frac{\pi}{2}\right)^2-a\csc\left(c+dx+\frac{\pi}{2}\right)+3b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3a}\right)$$

$\Big\downarrow$ 4594

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}+\frac{\int \frac{3ab-\left(a^2+3b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}}{3a}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\int \frac{3ab+\left(-a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{3a}\right)$$

$\Big\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\int\frac{1}{\sqrt{\sec(c+dx)}}dx-(a^2+3b^2)\int\sqrt{\sec(c+dx)}dx}{a^2}}{3a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-(a^2+3b^2)\int\sqrt{\csc(c+dx}}{a^2}}{3a}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-(}{}}{3a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}d}{}}{3a}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{6ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-(a^2+3b}{a^2}}{3a}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{6ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2\left(a^2+3\right.}{a^2}}{3a}\right.$$

$\downarrow$ 4336

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx}{a^2}+\frac{6ab\sqrt{\cos(c+dx)}}{}}{3}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2}+\frac{6ab\sqrt{c}}{}}{}\right.$$

$\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{6b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a^2d(a+b)}+\frac{6ab\sqrt{\cos(c+dx)}\sqrt{se}}{}}{3a}\right.$$

input
```
Int[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x]),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/3*(((6*a*b*Sqrt[Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (2*(a^2 + 3*b^2)*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (6*b^3*Sq
rt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a^2*(a + b)*d))/a + (2*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4340
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Sim
p[1/(a*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n
- a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(111) = 222$.

Time = 2.67 (sec) , antiderivative size = 552, normalized size of antiderivative = 4.93

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(4a^3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-4a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)b-2a^3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\right.\right.}{}$

input
```
int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a^3*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-4*a^2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)*b-2*a^3*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+2*a^2*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c)*b+a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-a^2*b*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+3*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*b^3*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^
2+3*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/a^3/(a-b)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)**(3/2)/(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{b\sec\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{b\sec\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cos\left(c+d\,x\right)^{3/2}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)}{\sec(dx+c)\,b+a}dx$$

input
```
int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x)
```

output
```
int((sqrt(cos(c + d*x))*cos(c + d*x))/(sec(c + d*x)*b + a),x)
```

3.819 $\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx = \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{2b\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a^2d}+\frac{2b^2\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a^2(a+b)d}$$

output
```
2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d-2*b*InverseJacobiAM(1/2*d*x+1/
2*c,2^(1/2))/a^2/d+2*b^2*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/
a^2/(a+b)/d
```

Mathematica [A] (verified)

Time = 10.45 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx = -\frac{2\left(aE\left(\arcsin\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-(a+b)\,\text{EllipticF}\left(\arcsin\left(\sqrt{\cos(c+dx)}\right),-1\right)+b\,\text{EllipticPi}\ldots\right.}{a^2d\sqrt{\sin^2(c+dx)}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

output
```
(-2*(a*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a + b)*EllipticF[ArcSi
n[Sqrt[Cos[c + d*x]]], -1] + b*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]
]], -1])*Sin[c + d*x])/(a^2*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 156 vs. $2(75) = 150$.

Time = 1.06 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.08, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4752, 3042, 4339, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4339}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}+\frac{\int \frac{a-b\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\int \frac{a-b\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}\right)$$

$$\Big\downarrow \text{4274}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\int \frac{1}{\sqrt{\sec(c+dx)}}dx-b\int \sqrt{\sec(c+dx)}dx}{a^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}\right)$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-b\sqrt{\cos}}{a^2}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-b}{a^2}\right.$$

$$\Big\downarrow \text{3119}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-b\sqrt{\cos(c+dx)}\sqrt{\sec}}{a^2}\right.$$

$$\Big\downarrow \text{3120}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}}{a^2}\right.$$

$$\Big\downarrow \text{4336}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx}{a^2}+\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}}}{}\right.$$

$$\Big\downarrow \text{3284}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a^2d(a+b)}+\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c-}}{d}}{}\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

output
```
Sqrt[Cos[c + d*x]]*(((2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/d - (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/d)/a^2 + (2*b^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a +
b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d))*Sqrt[Sec[c + d*x]
]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4339
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_))), x_Symbol] :> Simp[b^2/(a^2*d^2)   Int[(d*Csc[e + f*x])^(3/2)/(a +
b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a - b*Csc[e + f*x])/Sqrt[d*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(80) = 160$.

Time = 2.20 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.01

method	result
default	$\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{1-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)ab-\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{a^{2}(a-b)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin(}$

input
```
int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*a*b-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*a^2-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+b^2*Ell
ipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/a^2/(a-b)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{b\sec\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{\cos\left(c+dx\right)}}{a+b\sec\left(c+dx\right)}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sqrt(cos(c + d*x))/(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{b\sec\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{b\sec\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\sec\left(dx+c\right)b+a}dx$$

input
```
int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))/(sec(c + d*x)*b + a),x)
```

3.820 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad} - \frac{2b\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a(a+b)d}$$

output
```
2*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/a/d-2*b*EllipticPi(sin(1/2*d*x+1/
2*c),2*a/(a+b),2^(1/2))/a/(a+b)/d
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) - \frac{2b\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}}{ad}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])),x]
```

output
```
(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2])/(a + b))/(a*d)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4752, 3042, 4335, 3042, 3282, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{ 4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 4335}$$

$$\int \frac{\sqrt{\cos(c+dx)}}{a\cos(c+dx)+b}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{a\sin\left(c+dx+\frac{\pi}{2}\right)+b}dx$$

$$\Big\downarrow \text{ 3282}$$

$$\frac{\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{a}-\frac{b\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{b\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{b\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}$$

$$\Big\downarrow \text{3284}$$

$$\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{2b\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{ad(a+b)}$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])),x]
```

output
```
(2*EllipticF[(c + d*x)/2, 2])/(a*d) - (2*b*EllipticPi[(2*a)/(a + b), (c +
d*x)/2, 2])/(a*(a + b)*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3282
```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int[1/Sqrt[c + d*Sin[e + f*x]], x], x
] + Simp[(b*c - a*d)/b   Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4335
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[Sqrt[d*Sin[e + f*x]]*(Sqrt[d*Csc[e + f*x]]/d)   Int[
Sqrt[d*Sin[e + f*x]]/(b + a*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f},
 x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(56) = 112$.

Time = 1.12 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.53

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{1-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\left(\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a-b\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2} \cdots\right.\right.\right.}{a(a-b)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$

input
```
int(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*a-b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+b*EllipticPi(cos(1/2
*d*x+1/2*c),2*a/(a-b),2^(1/2)))/a/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\,dx = \int \frac{1}{(a+b\sec{(c+dx)})\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))*sqrt(cos(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec{(dx+c)}+a)\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\,dx = \int \frac{1}{\sqrt{\cos(c+d\,x)}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)\sec(dx+c)\,b+\cos(dx+c)\,a}dx$$

input
```
int(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)*sec(c + d*x)*b + cos(c + d*x)*a),x)
```

3.821 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 29

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \frac{2\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{(a+b)d}$$

output
```
2*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/(a+b)/d
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \frac{2\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{(a+b)d}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]
```

output
```
(2*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/((a + b)*d)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4752, 3042, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4336}$$

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a\cos(c+dx)+b)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)}dx$$

$$\Big\downarrow \text{3284}$$

$$\frac{2\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]
```

output
```
(2*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/((a + b)*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(31) = 62$.

Time = 0.11 (sec) , antiderivative size = 150, normalized size of antiderivative = 5.17

method	result	size
default	$\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{1-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\operatorname{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\frac{2a}{a-b},\sqrt{2}\right)}{(a-b)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$	150

input
```
int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),2*a/(a-b),2^(1/2))/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{(a+b\sec(c+dx))\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))*cos(c + d*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{\cos(c+d\,x)^{3/2}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2\sec(dx+c)\,b+\cos(dx+c)^2\,a}dx$$

input
```
int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**2*sec(c + d*x)*b + cos(c + d*x)**2*a
),x)
```

3.822 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = -\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} - \frac{2a\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{b(a+b)d} + \frac{2\sin(c+dx)}{bd\sqrt{\cos(c+dx)}}$$

output
```
-2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b/d-2*a*EllipticPi(sin(1/2*d*x+1/
2*c),2*a/(a+b),2^(1/2))/b/(a+b)/d+2*sin(d*x+c)/b/d/cos(d*x+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 195 vs. 2(77) = 154.

Time = 1.99 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.53

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx =$$

$$-\frac{\frac{6a\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+\frac{2b\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right)}{a}-\frac{4\sin(c+dx)}{\sqrt{\cos(c+dx)}}+\frac{2\left(-2abE\left(\arcsin\left(\sqrt{c\right.\right.\right.}{2bd}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])),x]
```

output
```
-1/2*((6*a*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (2*b*(2*El
lipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/
(a + b)))/a - (4*Sin[c + d*x])/Sqrt[Cos[c + d*x]] + (2*(-2*a*b*EllipticE[A
rcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c +
 d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]]
, -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/(b*d)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.78, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {3042, 4752, 3042, 4337, 3042, 4255, 3042, 4258, 3042, 3119, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 4337$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \sec^{\frac{3}{2}}(c+dx)dx}{b}-\frac{a\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{b}\right)$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx}{b}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)$$

$$\Big\downarrow \ 4255$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{b}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)$$

$$\Big\downarrow \ 4258$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b}-\frac{a\int \frac{\csc(c+d}{a+b\csc(}}{b}\right.$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{a\int \frac{\text{cs}}{a+}}{}\right.$$

$$\Big\downarrow \ 3119$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{b}-\frac{a\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)$$

$\downarrow$ 4336

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{b}-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{b}-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{}\right.$$

$\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{b}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{}\right.$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*a*Sqrt[Cos[c + d*x]]*EllipticPi
[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*(a + b)*d) + ((-2*S
qrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/d)/b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4337
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(5/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d/b   Int[(d*Csc[e + f*x])^(3/2), x], x] - Simp[a*
(d/b)   Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(79) = 158.

Time = 1.97 (sec) , antiderivative size = 353, normalized size of antiderivative = 4.58

method	result
default	$-\frac{2\left(-2\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,(a-b)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\cdots}\right.}{\cdots}$

input
```
int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(a-b)*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b+a*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b
),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/b/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(a-b)/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)b+\cos\left(dx+c\right)^{3}a}dx$$

input
```
int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**3*sec(c + d*x)*b + cos(c + d*x)**3*a
),x)
```

3.823 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 128

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd}+\frac{2a^2\operatorname{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{b^2(a+b)d}+\frac{2\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)}-\frac{2a\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}}$$

output
```
2*a*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/d+2/3*InverseJacobiAM(1/2*d*
x+1/2*c,2^(1/2))/b/d+2*a^2*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2)
)/b^2/(a+b)/d+2/3*sin(d*x+c)/b/d/cos(d*x+c)^(3/2)-2*a*sin(d*x+c)/b^2/d/cos
(d*x+c)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.71 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.64

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \frac{\frac{2(9a^2+2b^2)\operatorname{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+8b\left(2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\operatorname{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right)+\frac{4(b-3a\cos(c+\cdots}{\cos^{\frac{3}{2}}(\cdots}}{\cdots}$$

input
```
Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])),x]
```

output
```
((2*(9*a^2 + 2*b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + 8
*b*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)
/2, 2])/(a + b)) + (4*(b - 3*a*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3
/2) + (6*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*E
llipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b)
, ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b*Sqrt[Sin[c + d*x]^2]))
/(6*b^2*d)
```

Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.73, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4752, 3042, 4338, 27, 3042, 4590, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 4338}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int \frac{\sqrt{\sec(c+dx)}\left(-3a\sec^2(c+dx)+b\sec(c+dx)+a\right)}{2(a+b\sec(c+dx))}dx}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sec(c+dx)}\left(-3a\sec^2(c+dx)+b\sec(c+dx)+a\right)}{a+b\sec(c+dx)}dx}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(-3a\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}\right)$$

$$\Big\downarrow \text{ 4590}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\int \frac{3a^2+4b\sec(c+dx)a+\left(3a^2+b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c}{3bd}\right.$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{3a^2+4b\sec(c+dx)a+\left(3a^2+b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+}{3bd}\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{3a^2+4b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(3a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin(c+dx)\operatorname{se}}{3bd}\right.$$

$\downarrow$ 4594

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3a^2\int\frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int\frac{3a^3+b\sec(c+dx)a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c}{3bd}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3a^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int\frac{3a^3+b\csc\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin(c+dx)}{3l}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3a^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^3\int\frac{1}{\sqrt{\sec(c+dx)}}dx+a^2b\int\sqrt{\sec(c+dx)}dx}{a^2}}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\operatorname{si}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3a^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^3\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a^2b\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3a^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+d}}}{a^2}}{b}}{3b}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3a^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{}{\sqrt{\sin}}}{a^2}}{b}}{3b}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3a^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{d}}{a^2}}{b}}{3b}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3a^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\right.}{d}}{a^2}}{b}}{3b}\right.$$

$\downarrow$ 4336

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx+\frac{\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a}{}}{a}}{b}}{3b}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{d}}{b}}{3b}\right.$$

$\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{6a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2b\sqrt{\cos(c\cdot}}{}}{a^2}}{b}}{3b}\right.$$

input
```
Int[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*Sec[c + d*x]^(3/2)*Sin[c + d*x])
/(3*b*d) + ((((6*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/d + (2*a^2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/d)/a^2 + (6*a^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b
), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a + b)*d))/b - (6*a*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(b*d))/(3*b))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4338
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 3)/(b*f*
(n - 2))), x] + Simp[d^3/(b*(n - 2))   Int[(d*Csc[e + f*x])^(n - 3)*(Simp[a
*(n - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x]/(a + b*Csc
[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && Gt
Q[n, 3]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(125) = 250$.

Time = 2.88 (sec) , antiderivative size = 423, normalized size of antiderivative = 3.30

method	result
default	$-\frac{\sqrt{-\left(1-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(\frac{-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{3\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{1}{2}\right)^{2}}+\frac{2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{1-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\mathrm{Ellip}}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}\right.}}}{b}\right.}{}$

input
```
int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-(-(1-2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b*(-1/6*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/
2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/
2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2)))-2*a^3/b^2/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-2*a/b^
2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{\cos(c+d\,x)^{7/2}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^4\sec(dx+c)\,b+\cos(dx+c)^4\,a}dx$$

input
```
int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**4*sec(c + d*x)*b + cos(c + d*x)**4*a
),x)
```

3.824 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 244

$$\begin{aligned}\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = & -\frac{b(4a^2-5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{a^3\,(a^2-b^2)\,d} \\ & + \frac{(2a^4+16a^2b^2-15b^4)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^4\,(a^2-b^2)\,d} \\ & - \frac{b^3(7a^2-5b^2)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a^4(a-b)(a+b)^2d} \\ & + \frac{(2a^2-5b^2)\,\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2\,(a^2-b^2)\,d} \\ & + \frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{a\,(a^2-b^2)\,d(a+b\sec(c+dx))}\end{aligned}$$

output
```
-b*(4*a^2-5*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/(a^2-b^2)/d+1/3
*(2*a^4+16*a^2*b^2-15*b^4)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/a^4/(a^2
-b^2)/d-b^3*(7*a^2-5*b^2)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))
/a^4/(a-b)/(a+b)^2/d+1/3*(2*a^2-5*b^2)*cos(d*x+c)^(1/2)*sin(d*x+c)/a^2/(a^
2-b^2)/d+b^2*cos(d*x+c)^(1/2)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 1.37 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.09

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{4\sqrt{\cos(c+dx)}\left(2+\frac{3b^3}{(-a^2+b^2)(b+a\cos(c+dx))}\right)\sin(c+dx)-\frac{\frac{2(-8a^2b+5b^3)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+\frac{8(a^2+2b^2)((a+b)\,\text{Ellipti}}{}}{}}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

output
```
(4*Sqrt[Cos[c + d*x]]*(2 + (3*b^3)/((-a^2 + b^2)*(b + a*Cos[c + d*x])))*Si
n[c + d*x] - ((2*(-8*a^2*b + 5*b^3)*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2])/(a + b) + (8*(a^2 + 2*b^2)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*Ell
ipticPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a + b) - (6*(4*a^2 - 5*b^2)*(-2*
a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcS
in[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt
[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*Sqrt[Sin[c + d*x]^2]))/((-a + b)*
(a + b)))/(12*a^2*d)
```

Rubi [A] (verified)

Time = 2.18 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.27, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4752, 3042, 4334, 27, 3042, 4592, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 4334$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}-\frac{\int -\frac{2a^2-2b\sec(c+dx)a-5b^2+3b^2\sec^2(c+a}{2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}}{a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{2a^2-2b\sec(c+dx)a-5b^2+3b^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+d}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{2a^2-2b\csc\left(c+dx+\frac{\pi}{2}\right)a-5b^2+3b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec}\right.$$

$$\Big\downarrow \; 4592$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(2a^2-5b^2\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{2\int \frac{-b\left(2a^2-5b^2\right)\sec^2(c+dx)-2a\left(a^2+2b^2\right)\sec(c+dx)+3b\left(4a^2-5b^2\right)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{3a}}{2a\left(a^2-b^2\right)}+\frac{}{ad\left(a^2\right.}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(2a^2-5b^2\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{-b\left(2a^2-5b^2\right)\sec^2(c+dx)-2a\left(a^2+2b^2\right)\sec(c+dx)+3b\left(4a^2-5b^2\right)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{3a}}{2a\left(a^2-b^2\right)}+\frac{}{ad\left(a^2-\right.}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{-b(2a^2-5b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2-2a(a^2+2b^2)\csc\left(c+dx+\frac{\pi}{2}\right)+3b(4a^2-5b^2)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3a}}{2a(a^2-b^2)}+\frac{\quad}{a\quad}\right.$$

$\downarrow$ 4594

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}+\frac{\int \frac{3ab(4a^2-5b^2)-(2a^4+16b^2a^2-15b^4)\sec(c+dx)}{\sqrt{\sec(c+dx)}}}{a^2}}{3a}}{2a(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\int \frac{3ab(4a^2-5b^2)+(-2a^4-16b^2a^2+15b^4)\csc}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}}{a^2}}{3a}}{2a(a^2-b^2)}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab(4a^2-5b^2)\int \frac{1}{\sqrt{\sec(c+dx)}}dx-(2a^4+16a^2}{a^2}}{3a}}{2a(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab(4a^2-5b^2)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\left(2a^4+\right.}{a^2}}{3a}}{2a(a^2-b^2)}\right.$$

$$\Big\downarrow \; 4258$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+3ab(4a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos}}{3a}}{2a(a^2-b^2)}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+3ab(4a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin}}{3a}}{2a(a^2-b^2)}\right.$$

$$\Big\downarrow \; 3119$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{6ab(4a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}\right.}{d}}{3a}}{2a(a^2-b^2)}\right.$$

$$\Big\downarrow \; 3120$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{6ab(4a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(\right.}{d}}{3a}}{2a(a^2-b^2)}\right.$$

$\downarrow$ 4336

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx}{a^2}+\frac{6ab(4a^2-5}{}}{}}{2a(a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2}+\frac{6a}{}}{}}{2a(}\right.$$

$\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}+\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{6b^3(7a^2-5b^2)\sqrt{c}}{}}{}\right.$$

input
```
Int[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d
*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])) + (-1/3*(((6*a*b*(4*a^2 - 5*b^2)
*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (2*(
2*a^4 + 16*a^2*b^2 - 15*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/d)/a^2 + (6*b^3*(7*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*Ell
ipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d)
)/a + (2*(2*a^2 - 5*b^2)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]))/(2*a*(a
^2 - b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1063 vs. $2(241) = 482$.

Time = 5.14 (sec) , antiderivative size = 1064, normalized size of antiderivative = 4.36

method	result	size
default	Expression too large to display	1064

input
```
int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(1-2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(a^2+2*a*b+3*
b^2)/a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))+2/a^4*b^4*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/
(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d
*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)
/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2
*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))
)+4/3/a^2*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1/2*d*x+1/2*c)^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(cos(c + d*x)**(3/2)/(a + b*sec(c + d*x))**2, x)
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^2, x)
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos\left(c+d\,x\right)^{3/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^2,x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)}{\sec\left(dx+c\right)^2 b^2+2\sec\left(dx+c\right)ab+a^2}dx$$

input
```
int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x)
```

output
```
int((sqrt(cos(c + d*x))*cos(c + d*x))/(sec(c + d*x)**2*b**2 + 2*sec(c + d*
x)*a*b + a**2),x)
```

3.825 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 184

$$\begin{aligned}\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2} \, dx =& \frac{(2a^2-3b^2)\, E\left(\frac{1}{2}(c+dx)\middle|\, 2\right)}{a^2\,(a^2-b^2)\, d} \\ &- \frac{b(4a^2-3b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a^3\,(a^2-b^2)\, d} \\ &+ \frac{b^2(5a^2-3b^2)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a^3(a-b)(a+b)^2 d} \\ &+ \frac{b^2\sin(c+dx)}{a\,(a^2-b^2)\, d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\end{aligned}$$

output

```
(2*a^2-3*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2)/d-b*(4*a
^2-3*b^2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/a^3/(a^2-b^2)/d+b^2*(5*a^
2-3*b^2)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a^3/(a-b)/(a+b)^
2/d+b^2*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 1.34 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{4b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{(a^2-b^2)(b+a\cos(c+dx))}+\frac{\frac{2(2a^2-b^2)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+8b\left(-\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{b\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right)+\frac{2(2a^2-3b^2)}{\cdots}}{\cdots}}{4ad}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```

output
```
((4*b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(b + a*Cos[c + d*x])
) + ((2*(2*a^2 - b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) +
 8*b*(-EllipticF[(c + d*x)/2, 2] + (b*EllipticPi[(2*a)/(a + b), (c + d*x)/
2, 2])/(a + b)) + (2*(2*a^2 - 3*b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c +
 d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^
2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x
])/(a^2*b*Sqrt[Sin[c + d*x]^2]))/((a - b)*(a + b)))/(4*a*d)
```

Rubi [A] (verified)

Time = 1.67 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.38, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4752, 3042, 4334, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4334}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int -\frac{2a^2-2b\sec(c+dx)a-3b^2+b^2\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{a(a^2-b^2)}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{2a^2-2b\sec(c+dx)a-3b^2+b^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2a(a^2-b^2)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{2a^2-2b\csc\left(c+dx+\frac{\pi}{2}\right)a-3b^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}\right)$$

$$\Big\downarrow \text{4594}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int \frac{a(2a^2-3b^2)-b(4a^2-3b^2)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}}{2a(a^2-b^2)}+\frac{b^2\sin(c+dx)\sqrt{}}{ad(a^2-b^2)(a+}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{a(2a^2-3b^2)-b(4a^2-3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{2a(a^2-b^2)}+\frac{b^2\sin(c-}{ad(a^2-b^}\right.$$

$\Big\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a(2a^2-3b^2)\int\frac{1}{\sqrt{\sec(c+dx)}}dx-b(4a^2-3b^2)\int\sqrt{\sec(c+dx)}dx}{a^2}}{2a\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a(2a^2-3b^2)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b(4a^2-3b^2)\int\sqrt{\csc(c+dx}}{a^2}}{2a\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a(2a^2-3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-b(4a}{a^2}}{2a\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a(2a^2-3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-}{a^2}}{2a\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{2a\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-b(4a^2-3b^2}{a^2}}{2a\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{2a\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2b\left(4a^2-3b\right.}{}}{a^2}}{2a\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 4336

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx+\frac{2a\left(2a^2-3b^2\right)\sqrt{}}{}}{2a\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{2a\left(2a^2\right.}{}}{2a\left(a^2-\right.}\right.$$

$\Big\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}+\frac{2b^2\left(5-\frac{3b^2}{a^2}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+d\right.}{d(a+b)}\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((((2*a*(2*a^2 - 3*b^2)*Sqrt[Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (2*b*(4*a^2 - 3*b
^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^
2 + (2*b^2*(5 - (3*b^2)/a^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b),
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a + b)*d))/(2*a*(a^2 - b^2)) + (b^2*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(187) = 374$.

Time = 3.73 (sec) , antiderivative size = 809, normalized size of antiderivative = 4.40

method	result	size
default	Expression too large to display	809

input
```
int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(1-2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^3/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-6*b^2/a^2/(a^2-a*b)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1
/2))-2/a^3*b^3*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)
^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b
)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin...
```

Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(b\sec\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(cos(d*x + c))/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2
), x)
```

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\cos{(c+dx)}}}{(a+b\sec{(c+dx)})^2}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(sqrt(cos(c + d*x))/(a + b*sec(c + d*x))**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\cos{(dx+c)}}}{(b\sec{(dx+c)}+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\cos{(dx+c)}}}{(b\sec{(dx+c)}+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\cos(c+d\,x)}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^2,x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{\sec(dx+c)^2 b^2+2\sec(dx+c)\,ab+a^2}dx$$

input
```
int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x)
```

output
```
int(sqrt(cos(c + d*x))/(sec(c + d*x)**2*b**2 + 2*sec(c + d*x)*a*b + a**2),
x)
```

3.826 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 167

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx = \frac{bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a(a^2-b^2)d} + \frac{(2a^2-b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a^2(a^2-b^2)d} - \frac{b(3a^2-b^2)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a^2(a-b)(a+b)^2d} - \frac{b\sin(c+dx)}{(a^2-b^2)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}$$

output
```
b*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/(a^2-b^2)/d+(2*a^2-b^2)*InverseJ
acobiAM(1/2*d*x+1/2*c,2^(1/2))/a^2/(a^2-b^2)/d-b*(3*a^2-b^2)*EllipticPi(si
n(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a^2/(a-b)/(a+b)^2/d-b*sin(d*x+c)/(a^2-
b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 2.50 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{4b\sqrt{\cos(c+dx)}\sin(c+dx)}{(-a^2+b^2)(b+a\cos(c+dx))} - \frac{8\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) - \frac{10b\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b} + \frac{2\left(-2abE\left(\arcsin\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)\,\text{EllipticF}\left(\text{arcs}\right.\right.}{}}{(-a+b)(a+b)}}{4d}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2),x]
```

output
```
((4*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((-a^2 + b^2)*(b + a*Cos[c + d*x]))
 - (8*EllipticF[(c + d*x)/2, 2] - (10*b*EllipticPi[(2*a)/(a + b), (c + d*x
)/2, 2])/(a + b) + (2*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] +
2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*Elli
pticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*Sqrt[Si
n[c + d*x]^2]))/((-a + b)*(a + b)))/(4*d)
```

Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.42, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4752, 3042, 4330, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \text{ 4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4330}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{-b\sec^2(c+dx)+2a\sec(c+dx)+b}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{a^2-b^2}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{-b\sec^2(c+dx)+2a\sec(c+dx)+b}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{-b\csc\left(c+dx+\frac{\pi}{2}\right)^2+2a\csc\left(c+dx+\frac{\pi}{2}\right)+b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}\right)$$

$$\Big\downarrow \text{4594}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{ab+\left(2a^2-b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{b\left(3a^2-b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{ab+\left(2a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{b\left(3a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}\right)$$

$$\Big\downarrow \text{4274}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(2a^2-b^2)\int\sqrt{\sec(c+dx)}dx+ab\int\frac{1}{\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{b(3a^2-b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2(a^2-b^2)}-\frac{b\sin(c+d}{d(a^2-b^2)(}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(2a^2-b^2)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+ab\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{b(3a^2-b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2(a^2-b^2)}-\frac{b\sin}{d(a^2}\right.$$

$$\Big\downarrow \; 4258$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(2a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx}{a^2}-\frac{b(3}{}}{2(a^2-b^2)}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(2a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2(a^2-b^2)}\right.$$

$$\Big\downarrow \; 3119$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(2a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}-\frac{b(3a}{}}{2(a^2-b^2)}\right.$$

$$\Big\downarrow \; 3120$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}-\frac{b(3a^2-}{}}{2\left(a^2-b^2\right)}\right.$$

$\big\downarrow$ 4336

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}-\frac{b(3a^2-}{}}{2\left(a^2-b^2\right)}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}-\frac{b(3a^2-}{}}{2\left(a^2-b^2\right)}\right.$$

$\big\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}-\frac{2b(3a^2-}{}}{2\left(a^2-b^2\right)}\right.$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((((2*a*b*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*(2*a^2 - b^2)*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 - (2*b*(3*a^2 -
b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(a^2*(a + b)*d))/(2*(a^2 - b^2)) - (b*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x])))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4330
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)
*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*S
imp[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 787 vs. $2(170) = 340$.

Time = 3.15 (sec) , antiderivative size = 788, normalized size of antiderivative = 4.72

method	result	size
default	Expression too large to display	788

input
```
int(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(1-2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2/a^
2*b^2*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a
/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(
1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a
-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2
*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+4/a*b/(a^2-a*b)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a
/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{\left(a+b\sec\left(c+dx\right)\right)^2\sqrt{\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**2*sqrt(cos(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^2\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^2\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^2),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx$$
$$= \int \frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)\sec\left(dx+c\right)^2 b^2+2\cos\left(dx+c\right)\sec\left(dx+c\right)ab+\cos\left(dx+c\right)a^2}dx$$

input
```
int(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)*sec(c + d*x)**2*b**2 + 2*cos(c + d*x)
*sec(c + d*x)*a*b + cos(c + d*x)*a**2),x)
```

3.827 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = -\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{(a^2-b^2)\,d} - \frac{b\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a\,(a^2-b^2)\,d} + \frac{(a^2+b^2)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a(a-b)(a+b)^2d} + \frac{a\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}$$

output

```
-EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/(a^2-b^2)/d-b*InverseJacobiAM(1/2*d
*x+1/2*c,2^(1/2))/a/(a^2-b^2)/d+(a^2+b^2)*EllipticPi(sin(1/2*d*x+1/2*c),2*
a/(a+b),2^(1/2))/a/(a-b)/(a+b)^2/d+a*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(1/
2)/(a+b*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 2.30 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.55

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{4a\sqrt{\cos(c+dx)}\sin(c+dx)}{(a^2-b^2)(b+a\cos(c+dx))} - \frac{2\left(-\frac{a^2 \operatorname{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+2b\left(2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\operatorname{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right)+\frac{(-2abE(\arcsin(}{}}{a(a}}{4d}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2),x]
```

output
```
((4*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(b + a*Cos[c + d*x]))
- (2*(-((a^2*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)) + 2*b*(2*
EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]
)/(a + b)) + ((-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a +
 b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-
(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b*Sqrt[Sin[c + d*x]
^2])))/(a*(a - b)*(a + b)))/(4*d)
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.51, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4752, 3042, 4331, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$\Big\downarrow$ 4331

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int -\frac{-a\sec^2(c+dx)+2b\sec(c+dx)+a}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{a^2-b^2}+\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int \frac{-a\sec^2(c+dx)+2b\sec(c+dx)+a}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2\left(a^2-b^2\right)}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int \frac{-a\csc\left(c+dx+\frac{\pi}{2}\right)^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2\left(a^2-b^2\right)}\right)$$

$\Big\downarrow$ 4594

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\frac{\int \frac{a^2+b\sec(c+dx)a}{\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{\left(a^2+b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a}}{2\left(a^2-b^2\right)}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\frac{\int \frac{a^2+b\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{\left(a^2+b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2\left(a^2-b^2\right)}\right)$$

$\Big\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\frac{a^2\int\frac{1}{\sqrt{\sec(c+dx)}}dx+ab\int\sqrt{\sec(c+dx)}dx}{a^2}-\frac{\left(a^2+b^2\right)\int\frac{\csc\left(c+\cdots\right.}{a+b\csc\left(\cdots\right.}}{a}}{2\left(a^2-b^2\right)}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\frac{a^2\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+ab\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{\left(a^2+b^2\right)\int\cdots}{\cdots}}{2\left(a^2-b^2\right)}\right.$$

$\big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\frac{a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+ab\sqrt{\cos(c+dx)}\cdots}{a^2}\cdots}{2\left(a^2\cdots\right.}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\frac{a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+ab\sqrt{\cos(c+\cdots}}{a^2}\cdots}{2\left(\cdots\right.}\right.$$

$\big\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\frac{ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sqrt{\cos(c+dx\cdots}}{\cdots}}{a^2}\cdots}{2\left(a^2\cdots\right.}\right.$$

$\big\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}-\frac{\frac{\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{}}{a^2}}{2\left(a^2-\right.}\right.$$

$$\Big\downarrow \text{4336}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}-\frac{\frac{\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{}}{a^2}}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}-\frac{\frac{\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{}}{a^2}}{}\right.$$

$$\Big\downarrow \text{3284}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}-\frac{\frac{\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{}}{a^2}}{}\right.$$

input

```
Int[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2),x]
```

output

```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*(((2*a^2*Sqrt[Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a*b*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 - (2*(a^2 + b^2)*Sqrt
[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a*(a + b)*d))/(a^2 - b^2) + (a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2
- b^2)*d*(a + b*Sec[c + d*x])))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4331
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 2)/(f*(m + 1)*(a^2 - b^2))), x] - Simp[d^2/((m + 1
)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*
(a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /;
FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2
] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(151) = 302$.

Time = 2.77 (sec) , antiderivative size = 707, normalized size of antiderivative = 4.78

method	result
default	$-\frac{\sqrt{-\left(1-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-\frac{2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{1-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\frac{2a}{a-b},\sqrt{2}\right)}{\left(a^{2}-ab\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}-\frac{2b\left(\frac{a^{2}\cos\left(\frac{dx}{2}+\right.}{b\left(a^{2}-\right.}\right.}{}\right.}{}$

input
```
int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(1-2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/(a^2-a*b)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(
a-b),2^(1/2))-2*b/a*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b
)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1
/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^
2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx=\text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{(a+b\sec{(c+dx)})^2\cos^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**2*cos(c + d*x)**(3/2)), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{(b\sec{(dx+c)}+a)^2\cos{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^2),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^2), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$$
$$= \int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2\sec(dx+c)^2b^2+2\cos(dx+c)^2\sec(dx+c)\,ab+\cos(dx+c)^2a^2}dx$$

input
```
int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**2*sec(c + d*x)**2*b**2 + 2*cos(c + d
*x)**2*sec(c + d*x)*a*b + cos(c + d*x)**2*a**2),x)
```

3.828 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 154

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \frac{aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b(a^2-b^2)\,d} + \frac{\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{(a^2-b^2)\,d} + \frac{(a^2-3b^2)\operatorname{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{(a-b)b(a+b)^2d} - \frac{a^2\sin(c+dx)}{b(a^2-b^2)\,d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}$$

output

```
a*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b/(a^2-b^2)/d+InverseJacobiAM(1/2*
d*x+1/2*c,2^(1/2))/(a^2-b^2)/d+(a^2-3*b^2)*EllipticPi(sin(1/2*d*x+1/2*c),2
*a/(a+b),2^(1/2))/(a-b)/b/(a+b)^2/d-a^2*sin(d*x+c)/b/(a^2-b^2)/d/cos(d*x+c
)^(1/2)/(a+b*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 2.38 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.55

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{4a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{(-a^2+b^2)(b+a\cos(c+dx))} + \frac{\frac{2(3a^2-4b^2)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+4b\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right)+\frac{2\left(-2abE(\right.}{}}{}}{4bd}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2),x]
```

output
```
((4*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((-a^2 + b^2)*(b + a*Cos[c + d*x]
)) + ((2*(3*a^2 - 4*b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b
) + 4*b*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c +
 d*x)/2, 2])/(a + b)) + (2*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -
1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)
*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b*Sqrt
[Sin[c + d*x]^2]))/((a - b)*(a + b)))/(4*b*d)
```

Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.51, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4752, 3042, 4332, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$\Big\downarrow$ 4332

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{a^2+2b\sec(c+dx)a+(a^2-2b^2)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b(a^2-b^2)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2+2b\sec(c+dx)a+(a^2-2b^2)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2b(a^2-b^2)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a+(a^2-2b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2b(a^2-b^2)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}\right)$$

$\Big\downarrow$ 4594

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(a^2-3b^2)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int \frac{a^3+b\sec(c+dx)a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{2b(a^2-b^2)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(a^2-3b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{a^3+b\csc\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{2b(a^2-b^2)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}\right)$$

$\Big\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(a^2-3b^2)\int\frac{\csc(c+dx+\frac{\pi}{2})^{3/2}}{a+b\csc(c+dx+\frac{\pi}{2})}dx+\frac{a^3\int\frac{1}{\sqrt{\sec(c+dx)}}dx+a^2b\int\sqrt{\sec(c+dx)}dx}{a^2}}{2b(a^2-b^2)}-\frac{a^2\sin(c+a}{bd(a^2-b^2)(}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(a^2-3b^2)\int\frac{\csc(c+dx+\frac{\pi}{2})^{3/2}}{a+b\csc(c+dx+\frac{\pi}{2})}dx+\frac{a^3\int\frac{1}{\sqrt{\csc(c+dx+\frac{\pi}{2})}}dx+a^2b\int\sqrt{\csc(c+dx+\frac{\pi}{2})}dx}{a^2}}{2b(a^2-b^2)}-\frac{a^2\sin}{bd(a^2}\right.$$

$\big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(a^2-3b^2)\int\frac{\csc(c+dx+\frac{\pi}{2})^{3/2}}{a+b\csc(c+dx+\frac{\pi}{2})}dx+\frac{a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+a^2b\sqrt{\cos(c+dx)}}{a^2}}{2b(a^2-b^2)}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(a^2-3b^2)\int\frac{\csc(c+dx+\frac{\pi}{2})^{3/2}}{a+b\csc(c+dx+\frac{\pi}{2})}dx+\frac{a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin(c+dx+\frac{\pi}{2})}dx+a^2b\sqrt{\cos(c-}}{a^2}}{2b(a^2-b^2)}\right.$$

$\big\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(a^2-3b^2)\int\frac{\csc(c+dx+\frac{\pi}{2})^{3/2}}{a+b\csc(c+dx+\frac{\pi}{2})}dx+\frac{a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx+\frac{\pi}{2})}}dx+\frac{2a^3\sqrt{\cos(c+}}{}}{a^2}}{2b(a^2-b^2)}\right.$$

$\big\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(a^2-3b^2)\int\frac{\csc(c+dx+\frac{\pi}{2})^{3/2}}{a+b\csc(c+dx+\frac{\pi}{2})}dx+\frac{\frac{2a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(}}{}}{a^2}}{2b(a^2-b^2)}\right.$$

$$\Big\downarrow \text{4336}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx+\frac{2a^3\sqrt{\cos(c+dx)}\sqrt{}}{}}{2b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{2a^3\sqrt{\cos}}{}}{2b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{3284}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{2a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{d}}{2b\left(a^2-b^2\right)}\right.$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((((2*a^3*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a^2*b*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (2*(a^2 - 3*b^2)*Sqrt[
Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/((a + b)*d))/(2*b*(a^2 - b^2)) - (a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 607 vs. $2(157) = 314$.

Time = 1.95 (sec) , antiderivative size = 608, normalized size of antiderivative = 3.95

method	result
default	$-\frac{\sqrt{-\left(1-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\frac{2a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{b\left(a^2-b^2\right)\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-a+b\right)}-\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{1-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\ \mathrm{Elli}}{(a+b)b\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\mathrm{si}}}$

input
```
int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(1-2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2/b/(a^2-b^
2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-
2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+a/b/(a^2-b^2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-a/b/(a^2
-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))-1/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*c
os(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3*b/(a^2-b^2)/(a^2-
a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^2),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^2), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$$
$$= \int \frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^3\sec\left(dx+c\right)^2b^2+2\cos\left(dx+c\right)^3\sec\left(dx+c\right)ab+\cos\left(dx+c\right)^3a^2}dx$$

input
```
int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**3*sec(c + d*x)**2*b**2 + 2*cos(c + d
*x)**3*sec(c + d*x)*a*b + cos(c + d*x)**3*a**2),x)
```

3.829 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 219

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = -\frac{(3a^2-2b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2(a^2-b^2)\,d} - \frac{a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{b(a^2-b^2)\,d} - \frac{a(3a^2-5b^2)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{(a-b)b^2(a+b)^2d} + \frac{(3a^2-2b^2)\sin(c+dx)}{b^2(a^2-b^2)\,d\sqrt{\cos(c+dx)}} - \frac{a^2\sin(c+dx)}{b(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}$$

output
```
-(3*a^2-2*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^2-b^2)/d-a*Inv
erseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/b/(a^2-b^2)/d-a*(3*a^2-5*b^2)*Elliptic
Pi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/(a-b)/b^2/(a+b)^2/d+(3*a^2-2*b^2)
*sin(d*x+c)/b^2/(a^2-b^2)/d/cos(d*x+c)^(1/2)-a^2*sin(d*x+c)/b/(a^2-b^2)/d/
cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 1.96 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.27

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{-\frac{\frac{2\left(9a^3-10ab^2\right)\operatorname{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+\frac{\left(8a^2b-4b^3\right)\left(2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\operatorname{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right)}{a}+\frac{2\left(3a^2-2b^2\right)\left(-2abE\left(\arcsin\left(\sqrt{\cos \cdots\right.\right.\right.}{(a-b)(a+b)}}{\cdots}$$

input
```
Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2),x]
```

output
```
(-((((2*(9*a^3 - 10*a*b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a +
b) + ((8*a^2*b - 4*b^3)*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*
a)/(a + b), (c + d*x)/2, 2])/(a + b)))/a + (2*(3*a^2 - 2*b^2)*(-2*a*b*Elli
pticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[
Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c +
 d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)*(a + b)))
 + 4*Sqrt[Cos[c + d*x]]*((a^3*Sin[c + d*x])/((a^2 - b^2)*(b + a*Cos[c + d*
x])) + 2*Tan[c + d*x]))/(4*b^2*d)
```

Rubi [A] (verified)

Time = 2.11 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.31, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4752, 3042, 4332, 27, 3042, 4590, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$\Big\downarrow$ 4752

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$\Big\downarrow$ 4332

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-2b\sec(c+dx)a-\left(3a^2-2b^2\right)\sec^2(c+dx)\right)}{2(a+b\sec(c+dx))}dx}{b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx}{bd\left(a^2-b^2\right)(a+b\sec(c+d}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-2b\sec(c+dx)a-\left(3a^2-2b^2\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx}{bd\left(a^2-b^2\right)(a+b\sec(c+d}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2-2b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(2b^2-3a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\,\mathrm{se}}{bd\left(a^2-b^2\right)(a+}\right.$$

$\Big\downarrow$ 4590

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2\int \frac{a\left(3a^2-4b^2\right)\sec^2(c+dx)+2b\left(2a^2-b^2\right)\sec(c+dx)+a\left(3a^2-2b^2\right)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b}-\frac{2\left(3a^2-2b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{2b\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{a\left(3a^2-4b^2\right)\sec^2(c+dx)+2b\left(2a^2-b^2\right)\sec(c+dx)+a\left(3a^2-2b^2\right)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b}-\frac{2\left(3a^2-2b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{2b\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{a\left(3a^2-4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(2a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+a\left(3a^2-2b^2\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2(3a^2-2b^2)\sin(c+dx)\sqrt{\sec(}}{bd}}{2b\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 4594

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a(3a^2-5b^2)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int \frac{b\sec(c+dx)a^3+\left(3a^2-2b^2\right)a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{b}-\frac{2(3a^2-2b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{2b\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a(3a^2-5b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{b\csc\left(c+dx+\frac{\pi}{2}\right)a^3+\left(3a^2-2b^2\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{b}-\frac{2(3a^2-2b^2)\sin(c+dx)\sqrt{se}}{bd}}{2b\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a(3a^2-5b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3b\int \sqrt{\sec(c+dx)}dx+a^2\left(3a^2-2b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{a^2}}{b}-\frac{2(3a^2-2b^2)\,\mathrm{si}}{}}{2b\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a(3a^2-5b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3b\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+a^2\left(3a^2-2b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{b}-\frac{2(3a^2}{}}{2b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a(3a^2-5b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+a^2\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{}}{a^2}}{b}}{2b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a(3a^2-5b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a^2\left(3a^2-2b^2\right)\sqrt{\cos(c+}}{a^2}}{b}}{2b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{3119}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a(3a^2-5b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\left(3a^2-2b^2\right)\sqrt{\cos(c}}{}}{a^2}}{b}}{2b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{3120}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a(3a^2-5b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{2a^3b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2\left(3a^2-2b^2\right)\sqrt{\cos}}{}}{a^2}}{b}}{2b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{4336}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a(3a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx+\frac{\frac{2a^3b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{Elliptic}}{d}}{}}{b}}{2b\left(a^{2}\right.}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a(3a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{2a^3b\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}{d}}{b}}{2b}\right.$$

$\Big\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\frac{2a\left(3a^2-5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}}{}\right.$$

input
```
Int[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-((a^2*Sec[c + d*x]^(3/2)*Sin[c + d
*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))) - ((((2*a^2*(3*a^2 - 2*b^2)*S
qrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a^3
*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2
 + (2*a*(3*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a + b)*d))/b - (2*(3*a^2 - 2*b^2)*Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(b*d))/(2*b*(a^2 - b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 840 vs. 2(220) = 440.

Time = 3.37 (sec) , antiderivative size = 841, normalized size of antiderivative = 3.84

method	result	size
default	Expression too large to display	841

input
```
int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(1-2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^2/sin(1/2*d
*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))-2*a/b*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b
)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1
/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^
2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2*
a^2/b^2/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{\cos(c+d\,x)^{7/2}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^2),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^2), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^4\sec(dx+c)^2 b^2+2\cos(dx+c)^4\sec(dx+c)\,ab+\cos(dx+c)^4 a^2}dx$$

input
```
int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**4*sec(c + d*x)**2*b**2 + 2*cos(c + d
*x)**4*sec(c + d*x)*a*b + cos(c + d*x)**4*a**2),x)
```

3.830 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 346

$$\begin{aligned}\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx &= -\frac{b(24a^4-65a^2b^2+35b^4)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4(a^2-b^2)^2\,d} \\ &+ \frac{(8a^6+128a^4b^2-223a^2b^4+105b^6)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{12a^5(a^2-b^2)^2\,d} \\ &- \frac{b^3(63a^4-86a^2b^2+35b^4)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{4a^5(a-b)^2(a+b)^3d} \\ &+ \frac{(8a^4-61a^2b^2+35b^4)\sqrt{\cos(c+dx)}\sin(c+dx)}{12a^3(a^2-b^2)^2\,d} \\ &+ \frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{2a(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{b^2(13a^2-7b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{4a^2(a^2-b^2)^2\,d(a+b\sec(c+dx))}\end{aligned}$$

output
```
-1/4*b*(24*a^4-65*a^2*b^2+35*b^4)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^
4/(a^2-b^2)^2/d+1/12*(8*a^6+128*a^4*b^2-223*a^2*b^4+105*b^6)*InverseJacobi
AM(1/2*d*x+1/2*c,2^(1/2))/a^5/(a^2-b^2)^2/d-1/4*b^3*(63*a^4-86*a^2*b^2+35*
b^4)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a^5/(a-b)^2/(a+b)^3/
d+1/12*(8*a^4-61*a^2*b^2+35*b^4)*cos(d*x+c)^(1/2)*sin(d*x+c)/a^3/(a^2-b^2)
^2/d+1/2*b^2*cos(d*x+c)^(1/2)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^2+
1/4*b^2*(13*a^2-7*b^2)*cos(d*x+c)^(1/2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*
sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 2.66 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.02

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$= \frac{\frac{4\sqrt{\cos(c+dx)}\left(4a^6-57a^2b^4+35b^6+ab\left(16a^4-83a^2b^2+49b^4\right)\cos(c+dx)+4\left(a^3-ab^2\right)^2\cos(2(c+dx))\right)\sin(c+dx)}{\left(a^2-b^2\right)^2(b+a\cos(c+dx))^2}+\frac{-\frac{2\left(56a^4b-73a^2b^3+35b^5\right)\mathrm{Ell}}{a+}}{}}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^3,x]
```

output
```
((4*Sqrt[Cos[c + d*x]]*(4*a^6 - 57*a^2*b^4 + 35*b^6 + a*b*(16*a^4 - 83*a^2
*b^2 + 49*b^4)*Cos[c + d*x] + 4*(a^3 - a*b^2)^2*Cos[2*(c + d*x)])*Sin[c +
d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + ((-2*(56*a^4*b - 73*a^2*b^3
 + 35*b^5)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*(2*a^4
 + 14*a^2*b^2 - 7*b^4)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(
2*a)/(a + b), (c + d*x)/2, 2]))/(a + b) - (6*(24*a^4 - 65*a^2*b^2 + 35*b^4
)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*Elliptic
F[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSi
n[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*Sqrt[Sin[c + d*x]^2]))/((a
- b)^2*(a + b)^2))/(48*a^3*d)
```

Rubi [A] (verified)

Time = 3.01 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.23, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4752, 3042, 4334, 27, 3042, 4588, 27, 3042, 4592, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4334}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}-\frac{\int -\frac{4a^2-4b\sec(c+dx)a-7b^2+5b^2\sec^2(c}{2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}}{2a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{4a^2-4b\sec(c+dx)a-7b^2+5b^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}dx}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+\right.}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{4a^2-4b\csc\left(c+dx+\frac{\pi}{2}\right)a-7b^2+5b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\,\mathrm{se}\right.$$

$\downarrow$ 4588

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{b^2(13a^2-7b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}-\frac{\int -\frac{8a^4-61b^2a^2-4b\left(4a^2-b^2\right)\sec(c+dx)a+35b^4+3b^2\left(13a^2-7b^2\right)\,\mathrm{se}}{2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}}{a(a^2-b^2)}}{4a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{8a^4-61b^2a^2-4b\left(4a^2-b^2\right)\sec(c+dx)a+35b^4+3b^2\left(13a^2-7b^2\right)\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}dx}{2a(a^2-b^2)}+\frac{b^2(13a^2-7b^2)\sin(c+d}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\,\mathrm{se}}}{4a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{8a^4-61b^2a^2-4b\left(4a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+35b^4+3b^2\left(13a^2-7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}+\frac{b^2(13a^2-7b^2)}{ad(a^2-b^2)\sqrt{\sec(c+d}}}{4a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4592

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{2\int \frac{-b\left(8a^4-61b^2a^2+35b^4\right)\sec^2(c+dx)-4a\left(2a^4+14b^2a^2-7b^4\right)\sec(c+dx)+3b\left(24\right.}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{-b\left(8a^4-61b^2a^2+35b^4\right)\sec^2(c+dx)-4a\left(2a^4+14b^2a^2-7b^4\right)\sec(c+dx)+3b\left(24a^4\right.}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{-b\left(8a^4-61b^2a^2+35b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-4a\left(2a^4+14b^2a^2-7b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4594

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}+\int\frac{3ab\left(24a^4-65b^2a^2+35b^4\right)-\left(8a\right.}{\sqrt{}}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\int\frac{3ab\left(24a^4-65b^2a^2+35b^4\right)}{}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\left(24a^4-65a^2b^2+35b^4\right)\int\cdots}{3a}}{2a(a^2-b^2)}}{4a\left(a^2\cdots\right.}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\left(24a^4-65a^2b^2+35b^4\right)\int\cdots}{3a}}{2a(a^2-b^2)}}{4a\cdots}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\left(24a^4-65a^2b^2+35b^4\right)\sqrt{c\cdots}}{\cdots}}{2a(a^2\cdots}}{\cdots}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\left(24a^4-65a^2b^2+35b^4\right)\sqrt{c\cdots}}{\cdots}}{2a\cdots}}{\cdots}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{6ab\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\cdots}}{\cdots}}{2a(a\cdots}\cdots\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{6ab\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\cdots}}{\cdots}}{2a(\cdots}\cdots\right.$$

$\downarrow$ 4336

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}\cdots}{a^2}\cdots}{\cdots}\cdots\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+\cdots\right.\right.}\cdots}{a^2}\cdots}{\cdots}\cdots\right.$$

$\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\sin(c+dx)}{2ad\,(a^2-b^2)\,\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}+\frac{\frac{b^2(13a^2-7b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}}{\right.$$

input
```
Int[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^3,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b^2*Sin[c + d*x])/(2*a*(a^2 - b^2)
*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2) + ((b^2*(13*a^2 - 7*b^2)*Sin
[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])) + (-1
/3*(((6*a*b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (2*(8*a^6 + 128*a^4*b^2 - 223*a^2*b^
4 + 105*b^6)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d)/a^2 + (6*b^3*(63*a^4 - 86*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x]]*Elli
pticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d))
/a + (2*(8*a^4 - 61*a^2*b^2 + 35*b^4)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*
x]]))/(2*a*(a^2 - b^2)))/(4*a*(a^2 - b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2215 vs. $2(333) = 666$.

Time = 6.33 (sec) , antiderivative size = 2216, normalized size of antiderivative = 6.40

method	result	size
default	Expression too large to display	2216

input
```
int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(1-2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(a^2+3*a*b+6*
b^2)/a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))+4/3/a^3*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-sin(1/2*d
*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-2/a^4*(
2*a+3*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+20*b^3/a^4/(a^2-a*b)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*
a/(a-b),2^(1/2))+10/a^5*b^4*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-
1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1
/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+...
```

Fricas [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(cos(d*x + c)^(3/2)/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 +
 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^3, x)
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\cos\left(c+d\,x\right)^{3/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^3,x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \int \frac{\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)}{\sec\left(dx+c\right)^3 b^3+3\sec\left(dx+c\right)^2 a\,b^2+3\sec\left(dx+c\right)a^2b+a^3}dx$$

input
```
int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x)
```

output
```
int((sqrt(cos(c + d*x))*cos(c + d*x))/(sec(c + d*x)**3*b**3 + 3*sec(c + d*
x)**2*a*b**2 + 3*sec(c + d*x)*a**2*b + a**3),x)
```

3.831 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 282

$$\begin{aligned}\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx &= \frac{(8a^4-29a^2b^2+15b^4)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4a^3\,(a^2-b^2)^2\,d} \\ &\quad-\frac{3b(8a^4-11a^2b^2+5b^4)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4a^4\,(a^2-b^2)^2\,d} \\ &\quad+\frac{b^2(35a^4-38a^2b^2+15b^4)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{4a^4(a-b)^2(a+b)^3d} \\ &\quad+\frac{b^2\sin(c+dx)}{2a\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2} \\ &\quad+\frac{b^2(11a^2-5b^2)\sin(c+dx)}{4a^2\,(a^2-b^2)^2\,d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\end{aligned}$$

output
```
1/4*(8*a^4-29*a^2*b^2+15*b^4)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/(a
^2-b^2)^2/d-3/4*b*(8*a^4-11*a^2*b^2+5*b^4)*InverseJacobiAM(1/2*d*x+1/2*c,2
^(1/2))/a^4/(a^2-b^2)^2/d+1/4*b^2*(35*a^4-38*a^2*b^2+15*b^4)*EllipticPi(si
n(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a^4/(a-b)^2/(a+b)^3/d+1/2*b^2*sin(d*x+
c)/a/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2+1/4*b^2*(11*a^2-5*b^2
)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 2.04 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{\frac{2b^2\sqrt{\cos(c+dx)}\left(11a^2b-5b^3+a\left(13a^2-7b^2\right)\cos(c+dx)\right)\sin(c+dx)}{(a^2-b^2)^2(b+a\cos(c+dx))^2}+\frac{\frac{\left(8a^4-7a^2b^2+5b^4\right)\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}-\frac{8\left(4a^2b-b^3\right)\left((a+b)\,\text{EllipticF}\left(\right.\right.}{}}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^3,x]
```

output
```
((2*b^2*Sqrt[Cos[c + d*x]]*(11*a^2*b - 5*b^3 + a*(13*a^2 - 7*b^2)*Cos[c +
d*x])*Sin[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + (((8*a^4 - 7*
a^2*b^2 + 5*b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) - (8*(
4*a^2*b - b^3)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a
+ b), (c + d*x)/2, 2]))/(a + b) + ((8*a^4 - 29*a^2*b^2 + 15*b^4)*(-2*a*b*E
llipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sq
rt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[
c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a
 + b)^2))/(8*a^2*d)
```

Rubi [A] (verified)

Time = 2.45 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.29, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4752, 3042, 4334, 27, 3042, 4588, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$\Big\downarrow$ 4752

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$\Big\downarrow$ 4334

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\int -\frac{4a^2-4b\sec(c+dx)a-5b^2+3b^2\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{2a\left(a^2-b^2\right)}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{4a^2-4b\sec(c+dx)a-5b^2+3b^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{4a^2-4b\csc\left(c+dx+\frac{\pi}{2}\right)a-5b^2+3b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}\right)$$

$\Big\downarrow$ 4588

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{b^2\left(11a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int -\frac{8a^4-29b^2a^2-4b\left(4a^2-b^2\right)\sec(c+dx)a+15b^4+b^2\left(11a^2-5b^2\right)\sec^2(}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}}{a\left(a^2-b^2\right)}}{4a\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{8a^4-29b^2a^2-4b(4a^2-b^2)\sec(c+dx)a+15b^4+b^2(11a^2-5b^2)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2a(a^2-b^2)}+\frac{b^2(11a^2-5b^2)\sin(c+dx)\sqrt{\sec(}}{ad(a^2-b^2)(a+b\sec(c+dx)}}{4a(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{8a^4-29b^2a^2-4b(4a^2-b^2)\csc(c+dx+\frac{\pi}{2})a+15b^4+b^2(11a^2-5b^2)\csc(c+dx+\frac{\pi}{2})^2}{\sqrt{\csc(c+dx+\frac{\pi}{2})}(a+b\csc(c+dx+\frac{\pi}{2}))}dx}{2a(a^2-b^2)}+\frac{b^2(11a^2-5b^2)\sin(c+d}{ad(a^2-b^2)(a+b\,\mathrm{s}}}{4a(a^2-b^2)}\right.$$

$\downarrow$ 4594

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{b^2(35a^4-38a^2b^2+15b^4)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}+\frac{\int \frac{a(8a^4-29b^2a^2+15b^4)-3b(8a^4-11b^2a^2+5b^4)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}}{2a(a^2-b^2)}+\frac{b^2}{}}{4a(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{b^2(35a^4-38a^2b^2+15b^4)\int \frac{\csc(c+dx+\frac{\pi}{2})^{3/2}}{a+b\csc(c+dx+\frac{\pi}{2})}dx}{a^2}+\frac{\int \frac{a(8a^4-29b^2a^2+15b^4)-3b(8a^4-11b^2a^2+5b^4)\csc(c+dx+\frac{\pi}{2})}{\sqrt{\csc(c+dx+\frac{\pi}{2})}}d}{a^2}}{2a(a^2-b^2)}}{4a(a^2-b^2)}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{b^2(35a^4-38a^2b^2+15b^4)\int \frac{\csc(c+dx+\frac{\pi}{2})^{3/2}}{a+b\csc(c+dx+\frac{\pi}{2})}dx}{a^2}+\frac{a(8a^4-29a^2b^2+15b^4)\int \frac{1}{\sqrt{\sec(c+dx)}}dx-3b(8a^4-11a^2b^2+5b^4)\int}{a^2}}{2a(a^2-b^2)}}{4a(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\left(8a^4-29a^2b^2+15b^4\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-3b\left(8a^4-11a^2b^2+5\right.}{a^2}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-}{a^2}}{2a(a^2-b^2)}}{4a\left(a^2-\right.}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{}}{2a(a^2-b^2)}}{4a\left(a\right.}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{2a\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-}{}}{2a(a^2-b^2)}}{4a\left(a^2\right.}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{2a\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-}{}}{2a(a^2-b^2)}}{4a\left(a^2\right.}\right.$$

$\downarrow$ 4336

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx}{a^2}+\frac{2a\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c-}{}}{2a(a^2-b^2)}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2}+\frac{2a\left(8a^4-29a^2b^2+15b^4\right.}{}}{2a(a^2-}}{}\right.$$

$\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{\frac{b^2(11a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}+\frac{2b^2\left(35a^4-38a^2b^2\right.}{}}{}\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^3,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((((2*a*(8*a^4 - 29*a^2*b^
2 + 15*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/d - (6*b*(8*a^4 - 11*a^2*b^2 + 5*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (2*b^2*(35*a^4 - 38*a^2*b^2 + 15*
b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(a^2*(a + b)*d))/(2*a*(a^2 - b^2)) + (b^2*(11*a^2 - 5*b^2)*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])))/(4*a
*(a^2 - b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1956 vs. $2(273) = 546$.

Time = 5.11 (sec) , antiderivative size = 1957, normalized size of antiderivative = 6.94

method	result	size
default	Expression too large to display	1957

input
```
int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(1-2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^4/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2/a^4*b^4*(1/2*a^2/b/(a^2-b^2)*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2
*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2
*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/
2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)
/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*
x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2
-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\cos{(c+dx)}}}{(a+b\sec{(c+dx)})^3}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(sqrt(cos(c + d*x))/(a + b*sec(c + d*x))**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\cos{(dx+c)}}}{(b\sec{(dx+c)}+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^3, x)
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{(b\sec(dx+c)+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\cos(c+d\,x)}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^3,x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{\sec(dx+c)^3 b^3+3\sec(dx+c)^2 a\,b^2+3\sec(dx+c)\,a^2b+a^3}dx$$

input
```
int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x)
```

output
```
int(sqrt(cos(c + d*x))/(sec(c + d*x)**3*b**3 + 3*sec(c + d*x)**2*a*b**2 +
3*sec(c + d*x)*a**2*b + a**3),x)
```

3.832 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 263

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx = \frac{3b(3a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4a^2\left(a^2-b^2\right)^2 d} + \frac{\left(8a^4-5a^2b^2+3b^4\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4a^3\left(a^2-b^2\right)^2 d} - \frac{3b\left(5a^4-2a^2b^2+b^4\right)\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{4a^3(a-b)^2(a+b)^3 d} - \frac{b\sin(c+dx)}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2} - \frac{b(7a^2-b^2)\sin(c+dx)}{4a\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}$$

output
```
3/4*b*(3*a^2-b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2)^2/d+
1/4*(8*a^4-5*a^2*b^2+3*b^4)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/a^3/(a^
2-b^2)^2/d-3/4*b*(5*a^4-2*a^2*b^2+b^4)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(
a+b),2^(1/2))/a^3/(a-b)^2/(a+b)^3/d-1/2*b*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c
)^(1/2)/(a+b*sec(d*x+c))^2-1/4*b*(7*a^2-b^2)*sin(d*x+c)/a/(a^2-b^2)^2/d/co
s(d*x+c)^(1/2)/(a+b*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 2.12 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx = \frac{\frac{4b\sqrt{\cos(c+dx)}(-7a^2b+b^3+(-9a^3+3ab^2)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2(b+a\cos(c+dx))^2}+\frac{-\frac{2(5a^2b+b^3)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+\frac{16(2a^2+b^2)\left((a+b)\,\text{EllipticF}\left(\frac{1}{2}(c+\right.\right.}{}}{}}{}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^3),x]
```

output
```
((4*b*Sqrt[Cos[c + d*x]]*(-7*a^2*b + b^3 + (-9*a^3 + 3*a*b^2)*Cos[c + d*x]
)*Sin[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + ((-2*(5*a^2*b + b
^3)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*(2*a^2 + b^2)
*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b), (c + d*x
)/2, 2]))/(a + b) + (6*(3*a^2 - b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c +
 d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^
2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x
])/(a^2*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*a*d)
```

Rubi [A] (verified)

Time = 2.30 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.30, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4752, 3042, 4330, 27, 3042, 4588, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4330}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{-3b\sec^2(c+dx)+4a\sec(c+dx)+b}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{-3b\sec^2(c+dx)+4a\sec(c+dx)+b}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{-3b\csc\left(c+dx+\frac{\pi}{2}\right)^2+4a\csc\left(c+dx+\frac{\pi}{2}\right)+b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}\right)$$

$$\Big\downarrow \text{4588}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{\int -\frac{-b\left(7a^2-b^2\right)\sec^2(c+dx)+4a\left(2a^2+b^2\right)\sec(c+dx)+3b\left(3a^2-b^2\right)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{a(a^2-b^2)}-\frac{b(7a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}}{4\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{-b\left(7a^2-b^2\right)\sec^2(c+dx)+4a\left(2a^2+b^2\right)\sec(c+dx)+3b\left(3a^2-b^2\right)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2a(a^2-b^2)}-\frac{b(7a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}}{4\left(a^2-b^2\right)}-\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{-b\left(7a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+4a\left(2a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+3b\left(3a^2-b^2\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}-\frac{b(7a^2-b^2)\sin(c+dx)\sqrt{\sec(c+}}{ad(a^2-b^2)(a+b\sec(c+dx))}}{4\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 4594

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int \frac{3ab\left(3a^2-b^2\right)+\left(8a^4-5b^2a^2+3b^4\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{3b\left(5a^4-2a^2b^2+b^4\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}}{2a(a^2-b^2)}-\frac{b(7a^2-b^2)\sin(c+}{ad(a^2-b^2)(a+}}{4\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int \frac{3ab\left(3a^2-b^2\right)+\left(8a^4-5b^2a^2+3b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{3b\left(5a^4-2a^2b^2+b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2a(a^2-b^2)}-\frac{b(7a^2-b^2}{ad(a^2}}{4\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3ab\left(3a^2-b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\left(8a^4-5a^2b^2+3b^4\right)\int \sqrt{\sec(c+dx)}dx}{a^2}-\frac{3b\left(5a^4-2a^2b^2+b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2a(a^2-b^2)}}{4\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3ab\left(3a^2-b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(8a^4-5a^2b^2+3b^4\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{3b\left(5a^4-2a^2b^2+b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}}{a+b\csc\left(c+dx\right.}}{a^2}}{2a(a^2-b^2)}}{4\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3ab\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\left(8a^4-5a^2b^2+3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{a^2}}{2a(a^2-b^2)}}{4\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3ab\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\left(8a^4-5a^2b^2+3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx\right.}}}{a^2}}{2a(a^2-b^2)}}{4\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(8a^4-5a^2b^2+3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6ab\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{d}}{a^2}}{2a(a^2-b^2)}}{4\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\frac{6ab\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2\left(8a^4-5a^2b^2+3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+\right.}{d}}{a^2}}{2a(a^2-b^2)}}{4\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4336

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\frac{6ab\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2\left(8a^4-5a^2b^2+3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+\right.}{d}}{a^2}}{2a(a^2-b^2)}}{4}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{6ab(3a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}+\frac{2(8a^4-5a^2b^2+3b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+\right.}{d}}{a^2}}{2a(a^2-b^2)}\right.$$

$\Big\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\frac{6ab(3a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}+\frac{2(8a^4-5a^2b^2+3b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+\right.}{d}}{a^2}}{2a(a^2-b^2)}}{4(}\right.$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^3),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*(b*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((((6*a*b*(3*a^2 - b^2)*Sqr
t[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*(8*a^
4 - 5*a^2*b^2 + 3*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/d)/a^2 - (6*b*(5*a^4 - 2*a^2*b^2 + b^4)*Sqrt[Cos[c + d*x]]*E
llipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*
d))/(2*a*(a^2 - b^2)) - (b*(7*a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])))/(4*(a^2 - b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4330
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)
*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*S
imp[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1935 vs. $2(254) = 508$.

Time = 4.33 (sec) , antiderivative size = 1936, normalized size of antiderivative = 7.36

method	result	size
default	Expression too large to display	1936

input
```
int(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(1-2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6/a^
2*b/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*
d*x+1/2*c),2*a/(a-b),2^(1/2))+6/a^3*b^2*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*
c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*
c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d
*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*
a/(a-b),2^(1/2)))-2*b^3/a^3*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{(a+b\sec{(c+dx)})^3\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**3*sqrt(cos(c + d*x))), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{(b\sec{(dx+c)}+a)^3\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx=\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^3}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^3),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx$$
$$=\int \frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)\sec\left(dx+c\right)^{3}b^{3}+3\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}a\,b^{2}+3\cos\left(dx+c\right)\sec\left(dx+c\right)a^{2}b+\cos\left(\right.}$$

input
```
int(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)*sec(c + d*x)**3*b**3 + 3*cos(c + d*x)
*sec(c + d*x)**2*a*b**2 + 3*cos(c + d*x)*sec(c + d*x)*a**2*b + cos(c + d*x
)*a**3),x)
```

3.833 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 246

$$\begin{aligned} \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3} dx &= -\frac{(5a^2+b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4a\,(a^2-b^2)^2\,d} - \frac{b(7a^2-b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4a^2\,(a^2-b^2)^2\,d} \\ &+ \frac{(3a^4+10a^2b^2-b^4)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{4a^2(a-b)^2(a+b)^3d} \\ &+ \frac{a\sin(c+dx)}{2\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2} \\ &+ \frac{3(a^2+b^2)\sin(c+dx)}{4\,(a^2-b^2)^2\,d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))} \end{aligned}$$

output
```
-1/4*(5*a^2+b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/(a^2-b^2)^2/d-1/4
*b*(7*a^2-b^2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/a^2/(a^2-b^2)^2/d+1/
4*(3*a^4+10*a^2*b^2-b^4)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/
a^2/(a-b)^2/(a+b)^3/d+1/2*a*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*s
ec(d*x+c))^2+3/4*(a^2+b^2)*sin(d*x+c)/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*
sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 1.42 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.11

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$$

$$= \frac{\frac{4\sqrt{\cos(c+dx)}\left(3b(a^2+b^2)+a(5a^2+b^2)\cos(c+dx)\right)\sin(c+dx)}{(a^2-b^2)^2(b+a\cos(c+dx))^2} - \frac{-\frac{2\left(a^2+5b^2\right)\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+24b\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\,\text{Elli}}{}\right.}{}}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3),x]
```

output
```
((4*Sqrt[Cos[c + d*x]]*(3*b*(a^2 + b^2) + a*(5*a^2 + b^2)*Cos[c + d*x])*Si
n[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) - ((-2*(a^2 + 5*b^2)*El
lipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + 24*b*(2*EllipticF[(c +
d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)) + (2
*(5*a^2 + b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a
+ b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[
-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c +
 d*x]^2]))/((a - b)^2*(a + b)^2))/(16*d)
```

Rubi [A] (verified)

Time = 2.24 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.33, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4752, 3042, 4331, 27, 3042, 4588, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \; 4331$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int -\frac{-3a\sec^2(c+dx)+4b\sec(c+dx)+a}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{2(a^2-b^2)}+\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\int \frac{-3a\sec^2(c+dx)+4b\sec(c+dx)+a}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{4(a^2-b^2)}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\int \frac{-3a\csc\left(c+dx+\frac{\pi}{2}\right)^2+4b\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4(a^2-b^2)}\right)$$

$$\Big\downarrow \; 4588$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{-\frac{\int -\frac{12b\sec(c+dx)a^2-3\left(a^2+b^2\right)\sec^2(c+dx)a+\left(5a^2+b^2\right)a}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{a(a^2-b^2)} - \cdots}{4(a^2-b^2)}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\frac{\int \frac{12b\sec(c+dx)a^2-3\left(a^2+b^2\right)\sec^2(c+dx)a+\left(5a^2+b^2\right)a}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2a(a^2-b^2)} - \frac{3(\cdots}{\cdots}}{4(a^2-b^2)}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\int\frac{12b\csc\left(c+dx+\frac{\pi}{2}\right)a^2-3\left(a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2a+\left(5a^2+b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}d\cdots}{2a(a^2-b^2)}}{4\left(a^2-b^2\right)\cdots}\right.$$

$$\Big\downarrow \text{4594}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{\int\frac{\left(5a^2+b^2\right)a^2+b\left(7a^2-b^2\right)\sec(c+dx)a}{\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{\left(3a^4+10a^2b^2-b^4\right)\int\cdots}{a\cdots}}{2a(a^2-b^2)}}{4\left(a^2-\cdots\right.}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{\int\frac{\left(5a^2+b^2\right)a^2+b\left(7a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{\left(3a^4+10a^2b^2-b\cdots\right.}{\cdots}}{2a(a^2-b^2)}}{4\left(a^2\cdots\right.}\right.$$

$$\Big\downarrow \text{4274}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{a^2\left(5a^2+b^2\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx+ab\left(7a^2-b^2\right)\int\sqrt{\sec(c+dx)}dx}{a^2}-\frac{\left(3\cdots\right.}{\cdots}}{2a(a^2-b^2)}}{\cdots}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{a^2\left(5a^2+b^2\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+ab\left(7a^2-b^2\right)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\cdots\right.}}{a^2}\cdots}{2a(a^2-b^2)}}{\cdots}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\frac{a^2(5a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+ab(7a^2-b^2\cdots}{a^2}}{\cdots}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\frac{a^2(5a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+ab(7a^2\cdots}{a^2}}{\cdots}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\frac{ab(7a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2(5\cdots}{\cdots}}{a^2}}{\cdots}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{2ab(7a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2\cdots}{\cdots}}{a^2}}{\cdots}\right.$$

$\downarrow$ 4336

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{2ab(7a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2\cdots}{\cdots}}{a^2}}{\cdots}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\,(a^2-b^2)\,(a+b\sec(c+dx))^2}-\frac{\frac{\frac{2ab(7a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2}{}}{a^2}}{}\right.$$

$$\Big\downarrow \text{3284}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\,(a^2-b^2)\,(a+b\sec(c+dx))^2}-\frac{\frac{\frac{2ab(7a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2}{}}{a^2}}{}\right.$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) - ((((2*a^2*(5*a^2 + b^2)*Sqrt[C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a*b*(7*a
^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/d)/a^2 - (2*(3*a^4 + 10*a^2*b^2 - b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a
)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a + b)*d))/(2*a*(a^2 -
b^2)) - (3*(a^2 + b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(a
+ b*Sec[c + d*x])))/(4*(a^2 - b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4331
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 2)/(f*(m + 1)*(a^2 - b^2))), x] - Simp[d^2/((m + 1
)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*
(a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /;
FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2
] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1857 vs. $2(237) = 474$.

Time = 4.22 (sec) , antiderivative size = 1858, normalized size of antiderivative = 7.55

method	result	size
default	Expression too large to display	1858

input
```
int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(1-2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a/(a^2-a*b)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a
/(a-b),2^(1/2))+2/a^2*b^2*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b
)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(
a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*
d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a
^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^3\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{\cos(c+d\,x)^{3/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^3),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^3), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$$
$$= \int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2\sec(dx+c)^3b^3+3\cos(dx+c)^2\sec(dx+c)^2a\,b^2+3\cos(dx+c)^2\sec(dx+c)\,a^2b+\mathrm{co}}$$

input
```
int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**2*sec(c + d*x)**3*b**3 + 3*cos(c + d
*x)**2*sec(c + d*x)**2*a*b**2 + 3*cos(c + d*x)**2*sec(c + d*x)*a**2*b + co
s(c + d*x)**2*a**3),x)
```

3.834 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 253

$$\begin{aligned}\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx &= \frac{(a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b\,(a^2-b^2)^2\,d}+\frac{3(a^2+b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4a\,(a^2-b^2)^2\,d}\\&+\frac{(a^4-10a^2b^2-3b^4)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{4a(a-b)^2b(a+b)^3d}\\&-\frac{a^2\sin(c+dx)}{2b\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\\&+\frac{a(a^2-7b^2)\sin(c+dx)}{4b\,(a^2-b^2)^2\,d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\end{aligned}$$

output
```
1/4*(a^2+5*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b/(a^2-b^2)^2/d+3/4*
(a^2+b^2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/a/(a^2-b^2)^2/d+1/4*(a^4-
10*a^2*b^2-3*b^4)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a/(a-b)
^2/b/(a+b)^3/d-1/2*a^2*sin(d*x+c)/b/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(
d*x+c))^2+1/4*a*(a^2-7*b^2)*sin(d*x+c)/b/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a
+b*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 2.02 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.14

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$$

$$= \frac{-\frac{4a\sqrt{\cos(c+dx)}\left(-a^2b+7b^3+a\left(a^2+5b^2\right)\cos(c+dx)\right)\sin(c+dx)}{(a^2-b^2)^2(b+a\cos(c+dx))^2} + \frac{\frac{6\left(a^3-3ab^2\right)\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b} + \frac{8b\left(a^2+2b^2\right)\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right.}{a}}{}}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^3),x]
```

output
```
((-4*a*Sqrt[Cos[c + d*x]]*(-(a^2*b) + 7*b^3 + a*(a^2 + 5*b^2)*Cos[c + d*x]
)*Sin[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + ((6*(a^3 - 3*a*b^
2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*b*(a^2 + 2*b^2)
*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2
, 2])/(a + b)))/a + (2*(a^2 + 5*b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c +
 d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^
2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x
])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*b*d)
```

Rubi [A] (verified)

Time = 2.32 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.31, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4752, 3042, 4332, 27, 3042, 4588, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4332}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{a^2+4b\sec(c+dx)a+\left(a^2-4b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2+4b\sec(c+dx)a+\left(a^2-4b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2+4b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(a^2-4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}\right)$$

$$\Big\downarrow \text{4588}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{a\left(a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int -\frac{\left(a^2-7b^2\right)\sec^2(c+dx)a^2+\left(a^2+5b^2\right)a^2+4b\left(a^2+2b^2\right)\sec(c+dx)a}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{a(a^2-b^2)}}{4b\left(a^2-b^2\right)} -\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{\left(a^2-7b^2\right)\sec^2(c+dx)a^2+\left(a^2+5b^2\right)a^2+4b\left(a^2+2b^2\right)\sec(c+dx)a}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2a(a^2-b^2)}+\frac{a(a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}{4b\left(a^2-b^2\right)}-\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{\left(a^2-7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+\left(a^2+5b^2\right)a^2+4b\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}+\frac{a(a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+}}{d(a^2-b^2)(a+b\sec(c+dx))}}{4b\left(a^2-b^2\right)}\right.$$

$\big\downarrow$ 4594

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^4-10a^2b^2-3b^4)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int \frac{\left(a^2+5b^2\right)a^3+3b\left(a^2+b^2\right)\sec(c+dx)a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{2a(a^2-b^2)}+\frac{a\left(a^2-7b^2\right)\sin(c+dx)\sqrt{}}{d(a^2-b^2)(a+b\sec(}}{4b\left(a^2-b^2\right)}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^4-10a^2b^2-3b^4)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{\left(a^2+5b^2\right)a^3+3b\left(a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{2a(a^2-b^2)}+\frac{a\left(a^2-7b^2\right)\sin}{d(a^2-b^2)(}}{4b\left(a^2-b^2\right)}\right.$$

$\big\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^4-10a^2b^2-3b^4)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^2b\left(a^2+b^2\right)\int \sqrt{\sec(c+dx)}dx+a^3\left(a^2+5b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{a^2}}{2a(a^2-b^2)}+\frac{a}{}}{4b\left(a^2-b^2\right)}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^4-10a^2b^2-3b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^2b(a^2+b^2)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+a^3(a^2+5b^2)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}}{a^2}}{2a(a^2-b^2)}}{4b(a^2-b^2)}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^4-10a^2b^2-3b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^2b(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+a^3(a^2+5b^2)}{a^2}}{2a(a^2-b^2)}}{4b(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^4-10a^2b^2-3b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^2b(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a^3(a^2+}{a^2}}{2a(a^2-b^2)}}{4b(a^2-b^2)}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^4-10a^2b^2-3b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^2b(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^3(a^2}{}}{a^2}}{2a(a^2-b^2)}}{4b(a^2-b^2)}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^4-10a^2b^2-3b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{6a^2b(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^3(a}{}}{a^2}}{2a(a^2-b^2)}}{4b(a^2-b^2)}\right.$$

$$\Big\downarrow \; 4336$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^4-10a^2b^2-3b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx+\frac{6a^2b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(}}{d}}{2a(a^2-b^2)}}{4b}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^4-10a^2b^2-3b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{6a^2b\left(a^2+b^2\right)\sqrt{\cos(c+}}{}}{2a(a^2-b^2)}}{}\right.$$

$$\Big\downarrow \; 3284$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{a(a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}+\frac{\frac{2\left(a^4-10a^2b^2-3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+}{}}{4b\,(a^2}\right.$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^3),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*(a^2*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((((2*a^3*(a^2 + 5*b^2)
*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (6*a
^2*b*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/d)/a^2 + (2*(a^4 - 10*a^2*b^2 - 3*b^4)*Sqrt[Cos[c + d*x]]*Elliptic
Pi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a + b)*d))/(2*a*(a
^2 - b^2)) + (a*(a^2 - 7*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2
)*d*(a + b*Sec[c + d*x])))/(4*b*(a^2 - b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1759 vs. $2(244) = 488$.

Time = 3.54 (sec) , antiderivative size = 1760, normalized size of antiderivative = 6.96

method	result	size
default	Expression too large to display	1760

input
```
int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(1-2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a*(a^2/b/(a^2
-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*
b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*
c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2*b/a*(1/2*a^2/b/(a^2-b^2)*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1
/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+
1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*
cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{\cos(c+d\,x)^{5/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^3),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^3), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$$
$$= \int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3\sec(dx+c)^3b^3+3\cos(dx+c)^3\sec(dx+c)^2ab^2+3\cos(dx+c)^3\sec(dx+c)a^2b+\mathrm{co}$$

input
```
int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**3*sec(c + d*x)**3*b**3 + 3*cos(c + d
*x)**3*sec(c + d*x)**2*a*b**2 + 3*cos(c + d*x)**3*sec(c + d*x)*a**2*b + co
s(c + d*x)**3*a**3),x)
```

3.835 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 255

$$\begin{aligned}
&\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx \\
&= \frac{3a(a^2-3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)^2 d} + \frac{(a^2-7b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4b\left(a^2-b^2\right)^2 d} \\
&\quad + \frac{3(a^4-2a^2b^2+5b^4)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{4(a-b)^2b^2(a+b)^3d} \\
&\quad - \frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2} \\
&\quad - \frac{3a^2(a^2-3b^2)\sin(c+dx)}{4b^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}
\end{aligned}$$

output

```
3/4*a*(a^2-3*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^2-b^2)^2/d+
1/4*(a^2-7*b^2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/b/(a^2-b^2)^2/d+3/4
*(a^4-2*a^2*b^2+5*b^4)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/(a
-b)^2/b^2/(a+b)^3/d-1/2*a^2*sin(d*x+c)/b/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b
*sec(d*x+c))^2-3/4*a^2*(a^2-3*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/cos(d*x+c)
^(1/2)/(a+b*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 1.97 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.16

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{-\frac{4a^2\sqrt{\cos(c+dx)}(5a^2b-11b^3+3a(a^2-3b^2)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2(b+a\cos(c+dx))^2} + \frac{\frac{2(9a^4-19a^2b^2+16b^4)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b} + \frac{16b(a^2-4b^2)((a+b)\,\text{Elli}}{}}{}}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^3),x]
```

output
```
((-4*a^2*Sqrt[Cos[c + d*x]]*(5*a^2*b - 11*b^3 + 3*a*(a^2 - 3*b^2)*Cos[c +
d*x])*Sin[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + ((2*(9*a^4 -
19*a^2*b^2 + 16*b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) +
(16*b*(a^2 - 4*b^2)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a
)/(a + b), (c + d*x)/2, 2]))/(a + b) + (6*(a^2 - 3*b^2)*(-2*a*b*EllipticE[
ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c
+ d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]
], -1])*Sin[c + d*x])/(b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16
*b^2*d)
```

Rubi [A] (verified)

Time = 2.35 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.33, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4752, 3042, 4332, 27, 3042, 4586, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$\Big\downarrow$ 4752

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$\Big\downarrow$ 4332

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-4b\sec(c+dx)a-\left(3a^2-4b^2\right)\sec^2(c+dx)\right)}{2(a+b\sec(c+dx))^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+d}{2bd\left(a^2-b^2\right)(a+b\sec(c+}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-4b\sec(c+dx)a-\left(3a^2-4b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+d}{2bd\left(a^2-b^2\right)(a+b\sec(c+}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2-4b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(4b^2-3a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)}{2bd\left(a^2-b^2\right)(a+}\right.$$

$\Big\downarrow$ 4586

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a^2\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int \frac{3\left(a^2-3b^2\right)a^2+4b\left(a^2-4b^2\right)\sec(c+dx)a+\left(3a^4-5b^2a^2+8b^4\right)\sec^2(c}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}}{b\left(a^2-b^2\right)}}{4b\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int\frac{3(a^2-3b^2)a^2+4b(a^2-4b^2)\sec(c+dx)a+(3a^4-5b^2a^2+8b^4)\sec^2(c}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}}{2b(a^2-b^2)}}{4b(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int\frac{3(a^2-3b^2)a^2+4b(a^2-4b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a+(3a^4-5b^2a^2+8b^4)\mathrm{cs}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}}{2b(a^2-b^2)}}{4b(a^2-b^2)}\right.$$

$\downarrow$ 4594

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\int\frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int\frac{3(a^2-3b^2)a^3+b(a^2-7b}{\sqrt{\sec(c+d}}}{a^2}}{2b(a^2-b^2)}}{4b(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int\frac{3(a^2-3b^2)a^3+b(a}{\sqrt{\csc}}}{}}{2b(a^2-b^2)}}{4b(a^2-b^2)}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b(a^2-7b^2)\int\sqrt{\sec(}}{}}{2b(a^2-b^2)}}{4b(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a^2\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3\left(a^4-2a^2b^2+5b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b\left(a^2-7b^2\right)\int\sqrt{\csc(\cdots}}{\cdots}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a^2\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3\left(a^4-2a^2b^2+5b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b\left(a^2-7b^2\right)\sqrt{\cos(c\cdots}}{\cdots}}{\cdots}}{4b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a^2\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3\left(a^4-2a^2b^2+5b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b\left(a^2-7b^2\right)\sqrt{\cos(c\cdots}}{\cdots}}{\cdots}}{4b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a^2\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3\left(a^4-2a^2b^2+5b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b\left(a^2-7b^2\right)\sqrt{\cos(c\cdots}}{\cdots}}{\cdots}}{4b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a^2\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3\left(a^4-2a^2b^2+5b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2b\left(a^2-7b^2\right)\sqrt{\cos(\cdots}}{\cdots}}{\cdots}}{4b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4336

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}\ldots}{}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}(\ldots}}{}}{}\right.$$

$\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{6(a^4-2a^2b^2+5b^4\ldots}{}}{}\right.$$

input
```
Int[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^3),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*(a^2*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) - (-1/2*(((6*a^3*(a^2 - 3
*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (2*a^2*b*(a^2 - 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/d)/a^2 + (6*(a^4 - 2*a^2*b^2 + 5*b^4)*Sqrt[Cos[c + d*x]]*El
lipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a + b)*d))/(
b*(a^2 - b^2)) + (3*a^2*(a^2 - 3*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*
(a^2 - b^2)*d*(a + b*Sec[c + d*x])))/(4*b*(a^2 - b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4332

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4586

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Simp[d/(b*(a^2 - b^2)*(m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
 - b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n
+ b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C
}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

rule 4594

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1202 vs. 2(246) = 492.

Time = 2.48 (sec) , antiderivative size = 1203, normalized size of antiderivative = 4.72

method	result	size
default	Expression too large to display	1203

input
```
int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(1-2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(a^2/b/(a^2-b^2)
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/2*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1
/2*d*x+1/2*c)^2*a-a+b)-3/4/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/2/(a+b)/(a^2-b^2
)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*a+7/4/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d
*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/4*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/4*a/(a^
2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-3/4*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*co
s(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/4*a/(a^2-b^2)^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{\cos(c+d\,x)^{7/2}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^3),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^3), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$$
$$= \int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^4\sec(dx+c)^3b^3+3\cos(dx+c)^4\sec(dx+c)^2ab^2+3\cos(dx+c)^4\sec(dx+c)a^2b+\mathrm{co}}$$

input
```
int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**4*sec(c + d*x)**3*b**3 + 3*cos(c + d
*x)**4*sec(c + d*x)**2*a*b**2 + 3*cos(c + d*x)**4*sec(c + d*x)*a**2*b + co
s(c + d*x)**4*a**3),x)
```

3.836 $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 328

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = -\frac{(15a^4-29a^2b^2+8b^4)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4b^3(a^2-b^2)^2d} - \frac{a(5a^2-11b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4b^2(a^2-b^2)^2d}$$
$$-\frac{a(15a^4-38a^2b^2+35b^4)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{4(a-b)^2b^3(a+b)^3d}$$
$$+\frac{(15a^4-29a^2b^2+8b^4)\sin(c+dx)}{4b^3(a^2-b^2)^2d\sqrt{\cos(c+dx)}}$$
$$-\frac{a^2\sin(c+dx)}{2b(a^2-b^2)\,d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}$$
$$-\frac{a^2(5a^2-11b^2)\sin(c+dx)}{4b^2(a^2-b^2)^2\,d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}$$

output
```
-1/4*(15*a^4-29*a^2*b^2+8*b^4)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/(
a^2-b^2)^2/d-1/4*a*(5*a^2-11*b^2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/b
^2/(a^2-b^2)^2/d-1/4*a*(15*a^4-38*a^2*b^2+35*b^4)*EllipticPi(sin(1/2*d*x+1
/2*c),2*a/(a+b),2^(1/2))/(a-b)^2/b^3/(a+b)^3/d+1/4*(15*a^4-29*a^2*b^2+8*b^
4)*sin(d*x+c)/b^3/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)-1/2*a^2*sin(d*x+c)/b/(a^2
-b^2)/d/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2-1/4*a^2*(5*a^2-11*b^2)*sin(d*x
+c)/b^2/(a^2-b^2)^2/d/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 2.48 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.02

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{-\frac{\frac{2\left(45a^5-95a^3b^2+56ab^4\right)\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+\frac{8b\left(5a^4-10a^2b^2+2b^4\right)\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right)}{a}+\frac{2\left(15a^4-29a^2\right.}{}}{(a-b)^2(}}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^3),x]
```

output
```
(-(((2*(45*a^5 - 95*a^3*b^2 + 56*a*b^4)*EllipticPi[(2*a)/(a + b), (c + d*x
)/2, 2])/(a + b) + (8*b*(5*a^4 - 10*a^2*b^2 + 2*b^4)*(2*EllipticF[(c + d*x
)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)))/a + (2
*(15*a^4 - 29*a^2*b^2 + 8*b^4)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]
], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*
b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*
b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2)) + 4*Sqrt[Cos[c + d*x]]*((a
^3*(9*a^2*b - 15*b^3 + a*(7*a^2 - 13*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a^
2 - b^2)^2*(b + a*Cos[c + d*x])^2) + 8*Tan[c + d*x]))/(16*b^3*d)
```

Rubi [A] (verified)

Time = 3.07 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.23, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4752, 3042, 4332, 27, 3042, 4586, 27, 3042, 4590, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4332}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)(3a^2-4b\sec(c+dx)a-(5a^2-4b^2)\sec^2(c+dx))}{2(a+b\sec(c+dx))^2}dx}{2b(a^2-b^2)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+}{2bd(a^2-b^2)(a+b\sec(c+}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)(3a^2-4b\sec(c+dx)a-(5a^2-4b^2)\sec^2(c+dx))}{(a+b\sec(c+dx))^2}dx}{4b(a^2-b^2)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+}{2bd(a^2-b^2)(a+b\sec(c+}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3a^2-4b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(4b^2-5a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx}{2bd\left(a^2-b^2\right)(a}\right.$$

$\downarrow$ 4586

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a^2\left(5a^2-11b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int -\frac{\sqrt{\sec(c+dx)}\left(\left(5a^2-11b^2\right)a^2-4b\left(a^2-4b^2\right)\sec(c+dx)a-\left(15a^4-\right.\right.}{2(a+b\sec(c+dx))}}{b(a^2-b^2)}}{4b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{\sqrt{\sec(c+dx)}\left(\left(5a^2-11b^2\right)a^2-4b\left(a^2-4b^2\right)\sec(c+dx)a-\left(15a^4-29b^2a^2+8b^4\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{2b(a^2-b^2)}+\frac{a^2\left(5a^2-11b\right.}{bd(a^2-}}{4b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\left(5a^2-11b^2\right)a^2-4b\left(a^2-4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(-15a^4+29b^2a^2-8b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2b(a^2-b^2)}+}{4b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4590

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{2\int \frac{3a\left(5a^4-11b^2a^2+8b^4\right)\sec^2(c+dx)+4b\left(5a^4-10b^2a^2+2b^4\right)\sec(c+dx)+a\left(15a^4-29b^2a^2+8b^4\right)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b}-\frac{2\left(15a^4-29a\right.}{}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{\int \frac{3a\left(5a^4-11b^2a^2+8b^4\right)\sec^2(c+dx)+4b\left(5a^4-10b^2a^2+2b^4\right)\sec(c+dx)+a\left(15a^4-29b^2a^2+8b^4\right)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b}-\frac{2\left(15a^4-29a^2\right.}{}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{\int \frac{3a\left(5a^4-11b^2a^2+8b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+4b\left(5a^4-10b^2a^2+2b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)+a\left(15a^4-29b^2a^2+8b^4\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2\left(15\cdots\right.}{\cdots}}{2b(a^2-b^2)}\cdots}{4b\left(a^2-b^2\right)\cdots}\cdots\right.$$

$$\Big\downarrow \ 4594$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int \frac{b\left(5a^2-11b^2\right)\sec(c+dx)a^3+\left(15a^4-29b^2a^2+8b^4\right)a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{b}-\frac{2\left(15a^4-\cdots\right.}{\cdots}}{2b(a^2-b^2)}\cdots}{4b\left(a^2-b^2\right)\cdots}\cdots\right.$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{b\left(5a^2-11b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^3+\left(15a^4-29b^2a^2+8b^4\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{b}-\cdots}{2b(a^2-b^2)}\cdots}{4b\left(a^2-b^2\right)\cdots}\cdots\right.$$

$$\Big\downarrow \ 4274$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2\left(15a^4-29a^2b^2+8b^4\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+a^3b\left(5a^2-11b^2\right)\int \sqrt{\mathrm{s}\cdots}}{a^2}}{b}\cdots}{2b(a^2-b^2)}\cdots}{4b\left(a^2-\cdots\right.}\cdots\right.$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2\left(15a^4-29a^2b^2+8b^4\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a^3b\left(5a^2-11b^2\right)}{a^2}}{b}}{2b(a^2-b^2)}}{4b\,(a^2}\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx}{a^2}}{b}}{2b(a^2-b}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+a^2\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{b}}{2b(a}\right.$$

$$\Big\downarrow \text{3119}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3b\left(5a^2-11b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+}{a}}{b}}{2b(a^2-}\right.$$

$$\Big\downarrow \text{3120}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{b}}{2b(a^2}\right.$$

$\downarrow$ 4336

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx+\frac{2a^2\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\cos}}{}}{b}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{2a^2\left(15a^4-29a^2b^2+8\right.}{}}{b}}{}\right.$$

$\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{a^2(5a^2-11b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}+\frac{2a\left(15a^4-38a^2b\right.}{}}{}\right.$$

input

```
Int[1/(Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^3),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*(a^2*Sec[c + d*x]^(5/2)*Sin[c
+ d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) - ((a^2*(5*a^2 - 11*b^2)*
Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])) +
((((2*a^2*(15*a^4 - 29*a^2*b^2 + 8*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a^3*b*(5*a^2 - 11*b^2)*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (2*a*(15*a^4
- 38*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a + b)*d))/b - (2*(15*a^4 - 29*a^2*b^2 + 8
*b^4)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d))/(2*b*(a^2 - b^2)))/(4*b*(a^2
 - b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4586
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Simp[d/(b*(a^2 - b^2)*(m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
 - b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n
+ b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C
}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1986 vs. $2(315) = 630$.

Time = 4.57 (sec) , antiderivative size = 1987, normalized size of antiderivative = 6.06

method	result	size
default	Expression too large to display	1987

input
```
int(1/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(1-2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^3/sin(1/2*d
*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))-2*a/b*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/
4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(
a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-2*cos(1/2*d*x+1/
2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(1-2*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(9/2)/(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{9/2}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int(1/(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x))^3),x)
```

output
```
int(1/(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x))^3), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$$
$$= \int \frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^5\sec\left(dx+c\right)^3b^3+3\cos\left(dx+c\right)^5\sec\left(dx+c\right)^2a\,b^2+3\cos\left(dx+c\right)^5\sec\left(dx+c\right)a^2b+\mathrm{co}}$$

input
```
int(1/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**5*sec(c + d*x)**3*b**3 + 3*cos(c + d
*x)**5*sec(c + d*x)**2*a*b**2 + 3*cos(c + d*x)**5*sec(c + d*x)*a**2*b + co
s(c + d*x)**5*a**3),x)
```

3.837 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 244

$$\begin{aligned}\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx &= -\frac{4b(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\\ &\quad+\frac{2(9a^2-2b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{15a^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}\\ &\quad+\frac{2b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15ad}\\ &\quad+\frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d}\end{aligned}$$

output
```
-4/15*b*(a^2-b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1
/2*c,2^(1/2)*(a/(a+b))^(1/2))/a^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2
)+2/15*(9*a^2-2*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b))^(1
/2)+2/15*b*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/a/d+2/5*cos(
d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.63 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.39

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$= \frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\left(a(b+3a\cos(c+dx))\sin(c+dx)-\frac{\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(-i(9a^3+\right.}{}\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(a*(b + 3*a*Cos[c + d*x])*S
in[c + d*x] - ((Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((-I)*(9*a^3 + 9*a^
2*b - 2*a*b^2 - 2*b^3)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a
 + b)] + I*a*(9*a^2 + 7*a*b - 2*b^2)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]]
, (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2)/(a + b)] - (9*a^2 - 2*b^2)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)
/2]^2)^(3/2)*Tan[(c + d*x)/2]))/((b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)))
)/(15*a^2*d)
```

Rubi [A] (verified)

Time = 2.12 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.12, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 4752, 3042, 4344, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4344}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{2b\sec^2(c+dx)+3a\sec(c+dx)+b}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)^2+3a\csc\left(c+dx+\frac{\pi}{2}\right)+b}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\sec(c}{5d\sec^{\frac{3}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{4592}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{2\int -\frac{9a^2+7b\sec(c+dx)a-2b^2}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}\right)+\frac{2\sin}{}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\int \frac{9a^2+7b\sec(c+dx)a-2b^2}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+}{5}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\int \frac{9a^2+7b\csc\left(c+dx+\frac{\pi}{2}\right)a-2b^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}\right)+2\,\mathrm{s}\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{(9a^2-2b^2)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\,\mathrm{se}}}{3ad\sqrt{\sec(c+d}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{(9a^2-2b^2)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2b\sin(c+dx)\sqrt{a}}{3ad\sqrt{\mathrm{sec}}}\right.\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2b\sin}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2b}{}\right.\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2b\,\mathrm{s}}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\right.\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{2(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2b\sin(}{}\right.\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{2(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{}{\sqrt{b+a}}}{a\sqrt{a+b\sec(c+dx)}}}{3a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{2(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{}{\sqrt{b+a}}}{a\sqrt{a+b\sec(c+dx)}}}{3a}\right.\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{2(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{}{\sqrt{\frac{b}{a+b}+}}}{a\sqrt{a+b\sec(c+dx)}}}{3a}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{2(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{}{\sqrt{\frac{b}{a+b}+}}}{a\sqrt{a+b\sec(c+dx)}}}{3a}\right.\right.$$

$\Big\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{2(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{4b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}}{ad\sqrt{a+b\sec(c+dx)}}}{3a}\right.\right.$$

input
```
Int[Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
 d*x])/(5*d*Sec[c + d*x]^(3/2)) + (((-4*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(
a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(9*a^2 - 2*b^2)*EllipticE[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*Sqrt[Sec[c + d*x]]))/(3*a) + (2*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(3*a*d*Sqrt[Sec[c + d*x]]))/5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4344
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*Csc[e
 + f*x])^n/(f*n)), x] - Simp[1/(2*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*(Simp
[b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x]/Sqrt[a + b*C
sc[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] &&
LeQ[n, -1] && IntegerQ[2*n]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 983 vs. $2(225) = 450$.

Time = 8.05 (sec) , antiderivative size = 984, normalized size of antiderivative = 4.03

method	result	size
default	Expression too large to display	984

input
```
int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d/((a-b)/(a+b))^(1/2)/a^2*((9*cos(d*x+c)^2+18*cos(d*x+c)+9)*(1/(a+b)*
(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*Ellipt
icE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-9*
cos(d*x+c)^2-18*cos(d*x+c)-9)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+
c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-2*cos(d*x+c)^2-4*cos(d*x+c)-2)*(1/(
a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2
*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2
))+(2*cos(d*x+c)^2+4*cos(d*x+c)+2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^3*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d
*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-9*cos(d*x+c)^2-18*cos(d*x+c)-9)*
(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a
^3*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1
/2))+(7*cos(d*x+c)^2+14*cos(d*x+c)+7)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticF(((a-b)/(a+b))^(1/2)*(
csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(2*cos(d*x+c)^2+4*cos(d*x+c)+
2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*a*b^2*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b
))^(1/2))+sin(d*x+c)*cos(d*x+c)*(3*cos(d*x+c)^2+3*cos(d*x+c)+9)*((a-b)/(a+
b))^(1/2)*a^3+(4*cos(d*x+c)^2+4*cos(d*x+c)+9)*sin(d*x+c)*((a-b)/(a+b))^...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.86

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$
$$= \frac{2\left(3\left(3a^3\cos(dx+c)+a^2b\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-\sqrt{\frac{1}{2}}(3i\,a^2b+4i\,b^3)\sqrt{a}\text{weierstras}\right.}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/45*(3*(3*a^3*cos(d*x + c) + a^2*b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c) - sqrt(1/2)*(3*I*a^2*b + 4*I*b^3)*sqrt(
a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^
3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) - sqrt(1/2)*(-3*I*
a^2*b - 4*I*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/2
7*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)
/a) - 3*sqrt(1/2)*(-9*I*a^3 + 2*I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a
^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a
^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a
*sin(d*x + c) + 2*b)/a)) - 3*sqrt(1/2)*(9*I*a^3 - 2*I*a*b^2)*sqrt(a)*weier
strassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstra
ssPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*
cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx=\int \sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx=\int \cos(c+d\,x)^{5/2}\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx=\int \sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^{2}\,dx$$

input
```
int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x)**2,x)
```

3.838 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 192

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \frac{2(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d}$$

output
```
2/3*(a^2-b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c
,2^(1/2)*(a/(a+b))^(1/2))/a/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*
b*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(
a+b*sec(d*x+c))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)+2/3*cos(d*x+c)^(1
/2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.30 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.42

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$
$$= \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\left(ib(a+b)\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(i\operatorname{arcsinh}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sqrt{\sec \cdots}\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*(I*b*(a + b)*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Tan[(c + d*x
)/2]], (-a + b)/(a + b)]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]] - I*a*(
a + b)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I
*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[Sec[c + d*x]]*Sqrt[1 +
Sec[c + d*x]] + a^2*Sin[c + d*x] + a*b*Tan[(c + d*x)/2] + b^2*Sec[c + d*x]
*Tan[(c + d*x)/2] + a*b*Tan[c + d*x]))/(3*a*d*(b + a*Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.11, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3042, 4752, 3042, 4344, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4344}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{b+a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+}}{3d\sqrt{\sec(c+dx)}}\right.$$

$$\Big\downarrow \text{4523}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{(a^2-b^2)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}+\frac{b\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}\right)+\frac{2\sin(c+dx)\sqrt{a+b\,\mathrm{se}}}{3d\sqrt{\sec(c+d}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}\right)+\frac{2\sin(c+dx)\sqrt{a}}{3d\sqrt{\sec}}\right.$$

$$\Big\downarrow \text{4343}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{(a^2-b^2)\int\frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\right.}}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}\right.\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{(a^2-b^2)\int\frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{(a^2-b^2)\int\frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin(c+dx+\frac{\pi}{2})}{a+b}}}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{(a^2-b^2)\int\frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx}{a}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(}}{ad\sqrt{\sec(}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin(c+dx+\frac{\pi}{2})}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\,\mathrm{s}}}{ad\sqrt{\mathrm{s}}}\right.\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+}}{ad\sqrt{\sec(c+}}\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(}}{ad\sqrt{\sec(}}\right.\right.$$

$\big\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{2(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b}}{ad\sqrt{}}\right.\right.$$

input
```
Int[Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*(a^2 - b^2)*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/
(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*EllipticE[(c + d*x)/2, (2*a)/(a + b)
]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[S
ec[c + d*x]]))/3 + (2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec
[c + d*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4344
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*Csc[e
 + f*x])^n/(f*n)), x] - Simp[1/(2*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*(Simp
[b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x]/Sqrt[a + b*C
sc[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] &&
LeQ[n, -1] && IntegerQ[2*n]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(179) = 358$.

Time = 2.75 (sec) , antiderivative size = 591, normalized size of antiderivative = 3.08

method	result
default	$\frac{2\left(\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\sqrt{\frac{1}{1+\cos(dx+c)}}\,ab\,\mathrm{EllipticE}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)+\left(-\right.}{}$

input
```
int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/((a-b)/(a+b))^(1/2)/a*((cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b+a*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b*EllipticE(((
a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-cos(d*x+
c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*b^2*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+
c)),(-(a+b)/(a-b))^(1/2))+(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b+a*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*EllipticF(((a-b
)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-cos(d*x+c)^
2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*a*b*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c))
,(-(a+b)/(a-b))^(1/2))+sin(d*x+c)*cos(d*x+c)*(1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)*a^2+(2*cos(d*x+c)+1)*sin(d*x+c)*((a-b)/(a+b))^(1/2)*a*b+((a-b)/(a+b)
)^(1/2)*b^2*sin(d*x+c))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c
)^2*a+a*cos(d*x+c)+cos(d*x+c)*b+b)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.17

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$
$$= \frac{2\left(3\,a^2\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+3i\,\sqrt{\frac{1}{2}}a^{\frac{3}{2}}b\text{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\text{we}\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/9*(3*a^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c) + 3*I*sqrt(1/2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^
2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^
2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c)
+ 2*b)/a)) - 3*I*sqrt(1/2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/
a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/
a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c
) + 2*b)/a)) - sqrt(1/2)*(3*I*a^2 - 2*I*b^2)*sqrt(a)*weierstrassPInverse(-
4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c)
 + 3*I*a*sin(d*x + c) + 2*b)/a) - sqrt(1/2)*(-3*I*a^2 + 2*I*b^2)*sqrt(a)*w
eierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1
/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a))/(a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx=\int \sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx=\int \cos(c+d\,x)^{3/2}\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx=\int \sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx$$

input
```
int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x),x)
```

3.839 $\int \sqrt{\cos (c+d x)} \sqrt{a+b \sec (c+d x)} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 67

$$\int \sqrt{\cos (c+d x)} \sqrt{a+b \sec (c+d x)} \, dx = \frac{2 \sqrt{\cos (c+d x)} E\left(\frac{1}{2} (c+d x)|\frac{2 a}{a+b}\right) \sqrt{a+b \sec (c+d x)}}{d \sqrt{\frac{b+a \cos (c+d x)}{a+b}}}$$

output
```
2*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(
a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.30 (sec) , antiderivative size = 244, normalized size of antiderivative = 3.64

$$\int \sqrt{\cos (c+d x)} \sqrt{a+b \sec (c+d x)} \, dx = \frac{2 \sqrt{\cos (c+d x)} \sqrt{\cos ^2\left(\frac{1}{2} (c+d x)\right) \sec (c+d x)} \sqrt{a+b \sec (c+d x)} \left(i (a+b) E\left(i \text{arcsinh}\left(\tan \left(\frac{1}{2} (c+d x)\right)\right)\right.\right.}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[a + b*Sec
[c + d*x]]*(I*(a + b)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*(a + b)*E
llipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((b + a*Cos[c
 + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (b + a*Cos[c + d*x])*Sqrt[Sec[(c +
 d*x)/2]^2]*Tan[(c + d*x)/2]))/(d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4752, 3042, 4343, 3042, 3134, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4343}$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\sqrt{a\cos(c+dx)+b}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{a\cos(c+dx)+b}}$$

$$\Big\downarrow \text{3134}$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$$\Big\downarrow \text{3132}$$

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

input
```
Int[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec
[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(66) = 132$.

Time = 0.69 (sec) , antiderivative size = 540, normalized size of antiderivative = 8.06

method	result
default	$-\frac{2\left(\left(-\cos(dx+c)^2-2\cos(dx+c)-1\right)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,a\,\mathrm{EllipticE}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)\right.}{}$
risch	Expression too large to display

input
```
int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/((a-b)/(a+b))^(1/2)*((-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*EllipticE(((a-b)
/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(cos(d*x+c)^2+
2*cos(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*b*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(
a+b)/(a-b))^(1/2))+(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*EllipticF(((a-b)/(a+b))^
(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-cos(d*x+c)^2-2*cos(d
*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c))
)^(1/2)*b*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a
-b))^(1/2))-((a-b)/(a+b))^(1/2)*a*cos(d*x+c)*sin(d*x+c)-((a-b)/(a+b))^(1/2
)*b*sin(d*x+c))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*
cos(d*x+c)+cos(d*x+c)*b+b)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 355, normalized size of antiderivative = 5.30

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = -\frac{2\left(i\sqrt{\frac{1}{2}}\sqrt{a}b\text{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)-i\sqrt{\frac{1}{2}}\sqrt{a}b\text{weie}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(I*sqrt(1/2)*sqrt(a)*b*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2,
8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2
*b)/a) - I*sqrt(1/2)*sqrt(a)*b*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^
2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c)
+ 2*b)/a) - 3*I*sqrt(1/2)*a^(3/2)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2
, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2
, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) +
 2*b)/a)) + 3*I*sqrt(1/2)*a^(3/2)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2
, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2
, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) +
 2*b)/a))/(a*d)
```

Sympy [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+b\sec{(c+dx)}}\sqrt{\cos{(c+dx)}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))*sqrt(cos(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec{(dx+c)}+a}\sqrt{\cos{(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec{(dx+c)}+a}\sqrt{\cos{(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx=\int \sqrt{\cos\left(c+d\,x\right)}\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx=\int \sqrt{\sec\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}dx$$

input
```
int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x)),x)
```

3.840 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 138

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \frac{2a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

output
```
2*a*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(
a/(a+b))^(1/2))/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*b*((b+a*cos(d*
x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2)
)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 29.06 (sec) , antiderivative size = 14885, normalized size of antiderivative = 107.86

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.15, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4752, 3042, 4341, 3042, 4345, 3042, 3142, 3042, 3140, 4346, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 4341}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+a\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)$$

$\Big\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+}}}{\sqrt{a+b\sec(c+dx)}}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin(c+}}}{\sqrt{a+b\sec(c+dx)}}\right.$$

$\Big\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}}{\sqrt{a+b\sec(c+dx)}}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\right.}{a+b}}}}{\sqrt{a+b\sec(c+dx)}}\right.$$

$\Big\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+\right.}{d\sqrt{a+b\sec(c+dx)}}\right.$$

$\Big\downarrow$ 4346

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+d}{a+b}}}{d\sqrt{a+}}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+}}{}\right.$$

$\Big\downarrow$ 3286

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+}{a+b}}}{d\sqrt{a+b\,\mathrm{se}}}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+dx}}{}\right.$$

$\Big\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{\sec(c+dx)}\sqrt{\frac{a\,\mathrm{co}}{}}}{d}\right.$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*Sqrt[(b + a*Cos[c + d*x])/(a +
 b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a +
 b*Sec[c + d*x]]) + (2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2,
(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]
]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4341
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[a   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e +
f*x]], x], x] + Simp[b/d   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*
x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.32 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.78

method	result
default	$\frac{2\left(\operatorname{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)a-\operatorname{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)b+2\operatorname{EllipticPi}\left(\sqrt{\cdots}\right.\right.}{d\sqrt{\frac{a-b}{a+b}}\,(b+a\cos(dx+c))\sqrt{\frac{1}{1+\cos(dx}}}$

input
```
int((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/((a-b)/(a+b))^(1/2)*(EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x
+c)),(-(a+b)/(a-b))^(1/2))*a-EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot
(d*x+c)),(-(a+b)/(a-b))^(1/2))*b+2*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(d*x
+c)-cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b)*(a+b*sec(d*x+c))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)/(b+a*c
os(d*x+c))/(1/(1+cos(d*x+c)))^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{\sqrt{a+b\sec\left(c+dx\right)}}{\sqrt{\cos\left(c+dx\right)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))/sqrt(cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)}dx$$

input
```
int((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x),x)
```

3.841 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 237

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
b*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(a/
(a+b))^(1/2))/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+a*((b+a*cos(d*x+c)
)/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))/d/
cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-cos(d*x+c)^(1/2)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*
x+c))/(a+b))^(1/2)+(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 31.42 (sec) , antiderivative size = 23549, normalized size of antiderivative = 99.36

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 2.40 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.16, number of steps used = 25, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4752, 3042, 4342, 25, 3042, 4597, 3042, 4346, 3042, 3286, 3042, 3284, 4349, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4342}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int -\frac{a-a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}\right.$$

$$\Big\downarrow \text{25}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{1}{2}\int \frac{a-a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+a}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{1}{2}\int \frac{a-a\csc\left(c+dx+\right.}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc}}\right.$$

$$\Big\downarrow \text{4597}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(a\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-a\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{\sin(c}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\right.}}\right.\right.$$

$$\Big\downarrow \text{4346}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}-a\int \frac{}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right.}}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}-a\int \frac{}{\sqrt{\mathrm{cs}}}\right.\right.$$

$$\Big\downarrow \text{3286}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-a\int\frac{}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-a\int\frac{}{\sqrt{\csc\left(\right.}}\right.\right.$$

$\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\int\frac{}{\sqrt{\csc\left(c+\right.}}\right.\right.$$

$\downarrow$ 4349

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{\int\frac{\sqrt{a+b\sec(c}}{\sqrt{\sec(c+}}}{a}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{\int\frac{\sqrt{a+b\csc(c}}{\sqrt{\csc(c+}}}{a}\right.\right.\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{\sqrt{a+b\sec(}}{a\sqrt{\sec}}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{\sqrt{a+b\sec(}}{a\sqrt{s}}\right.\right.\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{\sqrt{a+b\sec(}}{a\sqrt{s}}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{\sqrt{a+b\sec(}}{a\sqrt{}}\right.\right.\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{2\sqrt{a+b\sec}}{ad\sqrt{sec}}\right.\right.\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{2\sqrt{a+b\sec}}{ad\sqrt{sec}}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{2\sqrt{a+b\sec}}{ad\sqrt{sec}}\right.\right.\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{2\sqrt{a+b\sec}}{ad\sqrt{\sec}}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{2\sqrt{a+b\sec}}{ad\sqrt{\sec}}\right.\right.\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}+\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{d\sqrt{a+b\sec}}\right.\right.$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*a*Sqrt[(b + a*Cos[c + d*x])/(a
+ b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqr
t[a + b*Sec[c + d*x]]) - a*((-2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellip
ticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c
 + d*x]]) + (2*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*
x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]])))/2 + (Sq
rt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4342
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d*Cos[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*
Csc[e + f*x])^(n - 1)/(f*(2*n - 1))), x] + Simp[d^2/(2*n - 1)   Int[(d*Csc[
e + f*x])^(n - 2)*(Simp[2*a*(n - 2) + b*(2*n - 3)*Csc[e + f*x] + a*Csc[e +
f*x]^2, x]/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4349
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Cs
c[e + f*x]], x], x] - Simp[b/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Cs
c[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4597
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[C/d^
2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[A
Int[1/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.04 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.99

method	result
default	$\frac{\left(\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,a\,\mathrm{EllipticPi}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\frac{a+b}{a-b},\frac{i}{\sqrt{\frac{a-b}{a+b}}}\right)\left(2\cos(dx+c)^3+4\cos(dx+c)^2+2\right.}{}$

input
```
int((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/((a-b)/(a+b))^(1/2)*((1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*a*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d
*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(2*cos(d*x+c)^3+4*cos(d*x+c)^2+2
*cos(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*a*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b
)/(a-b))^(1/2))*(-cos(d*x+c)^3-2*cos(d*x+c)^2-cos(d*x+c))+(1/(a+b)*(b+a*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b*EllipticE(((a-b
)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(cos(d*x+c)^3
+2*cos(d*x+c)^2+cos(d*x+c))+((a-b)/(a+b))^(1/2)*a*cos(d*x+c)*sin(d*x+c)+((
a-b)/(a+b))^(1/2)*b*sin(d*x+c))*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2)/(c
os(d*x+c)^2*a+a*cos(d*x+c)+cos(d*x+c)*b+b)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))/cos(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\sec(dx+c)+a}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{2}}dx$$

input
```
int((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**2,x)
```

3.842 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 303

$$\begin{aligned}\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c&+dx))^{3/2}\,dx = \frac{2(25a^4-31a^2b^2+6b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{105a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\\
&+\frac{4b(41a^2-3b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{105a^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}\\
&+\frac{2(25a^2+3b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{105ad}\\
&+\frac{16b\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35d}\\
&+\frac{2a\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d}\end{aligned}$$

output
```
2/105*(25*a^4-31*a^2*b^2+6*b^4)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJaco
biAM(1/2*d*x+1/2*c,2^(1/2)*(a/(a+b))^(1/2))/a^2/d/cos(d*x+c)^(1/2)/(a+b*se
c(d*x+c))^(1/2)+4/105*b*(41*a^2-3*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^2/d/((b+a*cos
(d*x+c))/(a+b))^(1/2)+2/105*(25*a^2+3*b^2)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c
))^(1/2)*sin(d*x+c)/a/d+16/35*b*cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2)*si
n(d*x+c)/d+2/7*a*cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.14 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.26

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\Big(a(b+a\cos(c+dx))\left(65a^2+6b^2+48ab\cos(c+dx)+1\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(a*(b + a*Cos[c + d*x])*(65
*a^2 + 6*b^2 + 48*a*b*Cos[c + d*x] + 15*a^2*Cos[2*(c + d*x)])*Sin[c + d*x]
 - (2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((2*I)*b*(-41*a^3 - 41*a^2*b
 + 3*a*b^2 + 3*b^3)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b
)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a +
b)] + I*a*(25*a^3 + 82*a^2*b + 51*a*b^2 - 6*b^3)*EllipticF[I*ArcSinh[Tan[(
c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*
x])*Sec[(c + d*x)/2]^2)/(a + b)] + 2*b*(-41*a^2 + 3*b^2)*(b + a*Cos[c + d*
x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/Sec[c + d*x]^(3/2)))/(10
5*a^2*d*(b + a*Cos[c + d*x])^2)
```

Rubi [A] (verified)

Time = 2.72 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.10, number of steps used = 24, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.960$, Rules used = {3042, 4752, 3042, 4351, 25, 3042, 4592, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{ 4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 4351}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}-\frac{1}{7}\int -\frac{4ab\sec^2(c+dx)+\left(5a^2+7b^2\right)\sec(c+ \cdots}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx \cdots}}\right.$$

$$\Big\downarrow \text{ 25}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int \frac{4ab\sec^2(c+dx)+\left(5a^2+7b^2\right)\sec(c+dx)+8ab}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\,\mathrm{s}\cdots}}{7d\sec^{\frac{5}{2}}(c+d\cdots}\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int \frac{4ab\csc\left(c+dx+\frac{\pi}{2}\right)^2+\left(5a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+8ab}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+d\cdots}{7d\,\mathrm{s}\cdots}\right.$$

$\Big\downarrow$ 4592

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{16b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}-\frac{2\int -\frac{44b\sec(c+dx)a^2+16b^2\sec^2(c+dx)a+(25a^2+3}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}}{5a}\right.\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\int \frac{44b\sec(c+dx)a^2+16b^2\sec^2(c+dx)a+(25a^2+3b^2)a}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}+\frac{16b\sin(c+dx)\sqrt{a+b\sec(c+}}{5d\sec^{\frac{3}{2}}(c+dx)}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\int \frac{44b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+16b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a+(25a^2+3b^2)a}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}+\frac{16b\sin(c+dx)\sqrt{a+b\,\mathrm{se}}}{5d\sec^{\frac{3}{2}}(c+dx)}\right.\right.$$

$\Big\downarrow$ 4592

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{2(25a^2+3b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}-\frac{2\int -\frac{\left(25a^2+51b^2\right)\sec(c+dx)a^2+2b\left(41a^2-3b^2\right)a}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{5a}+\frac{16}{}\right.\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\int \frac{\left(25a^2+51b^2\right)\sec(c+dx)a^2+2b\left(41a^2-3b^2\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}+\frac{2(25a^2+3b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{5a}+\frac{16b\,\mathrm{s}}{}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\int \frac{\left(25a^2+51b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^2+2b\left(41a^2-3b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2(25a^2+3b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{5a}+\frac{1\mathrm{6}}{}\right.\right.$$

$\Big\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{2b(41a^2-3b^2)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+(25a^4-31a^2b^2+6b^4)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{3a}+\frac{2(25a^2+3b^2)\sin(c}{3d\sqrt{se}}}{5a}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{2b(41a^2-3b^2)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+(25a^4-31a^2b^2+6b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2(25a^2+3b}{}}{5a}\right.\right.$$

$\Big\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\frac{2b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+(25a^4-31a^2b^2+6b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2(}{}}{5a}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\frac{2b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+(25a^4-31a^2b^2+6b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}-}{5a}\right.\right.$$

$\Big\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\frac{2b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+(25a^4-31a^2b^2+6b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}-}{5a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\frac{2b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\left(25a^4-31a^2b^2+6b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}d}{3a}}{5a}\right.\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\left(25a^4-31a^2b^2+6b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2(25}{}}{5a}\right.\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\frac{\left(25a^4-31a^2b^2+6b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+}{a+b}}}}{3a}}{5a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\frac{\left(25a^4-31a^2b^2+6b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}E}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+}{a+}}}}{3a}}{5a}\right.\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\frac{\left(25a^4-31a^2b^2+6b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}E}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c}{a}}}}{3a}}{5a}\right.\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\frac{\left(25a^4-31a^2b^2+6b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+}}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\,c}{}}}}{3a}}{5a}\right.\right.$$

$$\Big\downarrow \text{ 3140}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{2(25a^2+3b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}+\frac{\frac{4b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\left(25a^4-\right.}{}}{}}{5a}\right.\right.$$

input
```
Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*Sqrt[a + b*Sec[c + d*x]]*Sin[c
 + d*x])/(7*d*Sec[c + d*x]^(5/2)) + ((16*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c
+ d*x])/(5*d*Sec[c + d*x]^(3/2)) + (((2*(25*a^4 - 31*a^2*b^2 + 6*b^4)*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[
Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (4*b*(41*a^2 - 3*b^2)*Ellipt
icE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*C
os[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(3*a) + (2*(25*a^2 + 3*b^2)*Sqr
t[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]))/(5*a))/7)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4351
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*C
sc[e + f*x])^n/(f*n)), x] + Simp[1/(2*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/
Sqrt[a + b*Csc[e + f*x]])*Simp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[
e + f*x] + a*b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f
}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1163 vs. 2(278) = 556.

Time = 7.68 (sec) , antiderivative size = 1164, normalized size of antiderivative = 3.84

method	result	size
default	Expression too large to display	1164

input
```
int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/105/d/a^2/((a-b)/(a+b))^(1/2)*((82*cos(d*x+c)^2+164*cos(d*x+c)+82)*(1/(a
+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b*
EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2)
)+(-82*cos(d*x+c)^2-164*cos(d*x+c)-82)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b^2*EllipticE(((a-b)/(a+b))^(1/2
)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-6*cos(d*x+c)^2-12*cos(d*
x+c)-6)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*a*b^3*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)
/(a-b))^(1/2))+(6*cos(d*x+c)^2+12*cos(d*x+c)+6)*(1/(a+b)*(b+a*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^4*EllipticE(((a-b)/(a+b))
^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(25*cos(d*x+c)^2+50*c
os(d*x+c)+25)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*a^4*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(
a+b)/(a-b))^(1/2))+(-82*cos(d*x+c)^2-164*cos(d*x+c)-82)*(1/(a+b)*(b+a*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b*EllipticF(((a
-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(51*cos(d*x
+c)^2+102*cos(d*x+c)+51)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*a^2*b^2*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-
cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(6*cos(d*x+c)^2+12*cos(d*x+c)+6)*(1/(a+b
)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^3...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.63

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/315*(3*(15*a^4*cos(d*x + c)^2 + 24*a^3*b*cos(d*x + c) + 25*a^4 + 3*a^2*b
^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c
) - sqrt(1/2)*(75*I*a^4 - 11*I*a^2*b^2 + 12*I*b^4)*sqrt(a)*weierstrassPInv
erse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*
x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) - sqrt(1/2)*(-75*I*a^4 + 11*I*a^2*b^
2 - 12*I*b^4)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(
9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)
 - 6*sqrt(1/2)*(-41*I*a^3*b + 3*I*a*b^3)*sqrt(a)*weierstrassZeta(-4/3*(3*a
^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a
^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a
*sin(d*x + c) + 2*b)/a)) - 6*sqrt(1/2)*(41*I*a^3*b - 3*I*a*b^3)*sqrt(a)*we
ierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weiers
trassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3
*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \cos(c+d\,x)^{7/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

input
```
int(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \left(\int \sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^{3}\sec(dx+c)\,dx\right)b + \left(\int \sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^{3}\,dx\right)a$$

input
```
int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*
x),x)*b + int(sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x)**3,
x)*a
```

3.843 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 240

$$\begin{aligned}\int \cos^{\frac{5}{2}}(c+dx)(a&+b\sec(c+dx))^{3/2}\,dx = \frac{2b(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\\&+\frac{2(3a^2+b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{5ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}\\&+\frac{4b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d}\\&+\frac{2a\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d}\end{aligned}$$

output

```
2/5*b*(a^2-b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2
*c,2^(1/2)*(a/(a+b))^(1/2))/a/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/
5*(3*a^2+b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+
b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)+4/5*b
*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d+2/5*a*cos(d*x+c)^(3/
2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.88 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.43

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\left(2(b+a\cos(c+dx))(2b+a\cos(c+dx))\sin(c+dx)-\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(2*(b + a*Cos[c + d*x])*(2*
b + a*Cos[c + d*x])*Sin[c + d*x] - (2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3
/2)*((-I)*(3*a^3 + 3*a^2*b + a*b^2 + b^3)*EllipticE[I*ArcSinh[Tan[(c + d*x
)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec
[(c + d*x)/2]^2)/(a + b)] + I*a*(3*a^2 + 4*a*b + b^2)*EllipticF[I*ArcSinh[
Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c
 + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (3*a^2 + b^2)*(b + a*Cos[c + d*x])
*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(a*Sec[c + d*x]^(3/2))))/(5
*d*(b + a*Cos[c + d*x])^2)
```

Rubi [A] (verified)

Time = 2.03 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.08, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 4752, 3042, 4351, 25, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4351}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}-\frac{1}{5}\int -\frac{2ab\sec^2(c+dx)+\left(3a^2+5b^2\right)\sec(c+\right.}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx}}$$

$$\Big\downarrow \text{25}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{2ab\sec^2(c+dx)+\left(3a^2+5b^2\right)\sec(c+dx)+6ab}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\,\mathrm{se}}}{5d\sec^{\frac{3}{2}}(c+dx}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{2ab\csc\left(c+dx+\frac{\pi}{2}\right)^2+\left(3a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+6ab}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx}{5d\,\mathrm{s}}\right.$$

$$\Big\downarrow \text{4592}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{4b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}-\frac{2\int -\frac{3\left(4b\sec(c+dx)a^2+\left(3a^2+b^2\right)a\right)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}\right)+\frac{2a\,\mathrm{si}}{}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\int \frac{4b\sec(c+dx)a^2+\left(3a^2+b^2\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a}+\frac{4b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin(c}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\int \frac{4b\csc(c+dx+\frac{\pi}{2})a^2+(3a^2+b^2)a}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx}{a}+\frac{4b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}\right)+\frac{2a}{}\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{b(a^2-b^2)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+(3a^2+b^2)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}+\frac{4b\sin(c+dx)}{d\sqrt{\text{se}}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{b(a^2-b^2)\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+(3a^2+b^2)\int \frac{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}{\sqrt{\csc(c+dx+\frac{\pi}{2})}}dx}{a}+4b\sin(c\right.\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{b(a^2-b^2)\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{(3a^2+b^2)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a}+4b\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{b(a^2-b^2)\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{(3a^2+b^2)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin(c+dx+\frac{\pi}{2})}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a}+\right.\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{b(a^2-b^2)\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{(3a^2+b^2)\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{b\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}\right.\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{b\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{4b\,s}{}\right.\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+}{a+b}}}}{a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+}{a+b}}}}{a}\right.\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx}{a+b}}}}{a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^2+b^2)\sqrt{a+b\sec(c+dx)}E}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(}{}}}}{a}\right.\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^2+b^2)\sqrt{a+b\sec(c+dx)}E\left(\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c}{a}}}}{a}\right.\right.$$

input
```
Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*Sqrt[a + b*Sec[c + d*x]]*Sin[c
 + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (((2*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^2 + b^2)*EllipticE[(c + d*x)/2, (2*
a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]*Sqrt[Sec[c + d*x]]))/a + (4*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(d*
Sqrt[Sec[c + d*x]]))/5)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4351
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*C
sc[e + f*x])^n/(f*n)), x] + Simp[1/(2*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/
Sqrt[a + b*Csc[e + f*x]])*Simp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[
e + f*x] + a*b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f
}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 976 vs. $2(221) = 442$.

Time = 3.95 (sec) , antiderivative size = 977, normalized size of antiderivative = 4.07

method	result	size
default	Expression too large to display	977

input
```
int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/d/a/((a-b)/(a+b))^(1/2)*((3*cos(d*x+c)^2+6*cos(d*x+c)+3)*(1/(a+b)*(b+a
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*EllipticE(
((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-3*cos(
d*x+c)^2-6*cos(d*x+c)-3)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-co
t(d*x+c)),(-(a+b)/(a-b))^(1/2))+(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b+
a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2*Ellipti
cE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-cos
(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)*b^3*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot
(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-3*cos(d*x+c)^2-6*cos(d*x+c)-3)*(1/(a+b)*(
b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*Ellipti
cF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(4*co
s(d*x+c)^2+8*cos(d*x+c)+4)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-
cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*
(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2*Elli
pticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+si
n(d*x+c)*cos(d*x+c)*(cos(d*x+c)^2+cos(d*x+c)+3)*((a-b)/(a+b))^(1/2)*a^3+(3
*cos(d*x+c)^2+3*cos(d*x+c)+3)*sin(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b+(2+3...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.90

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{2\left(3\left(a^3\cos(dx+c)+2a^2b\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2\sqrt{\frac{1}{2}}(3i\,a^2b-i\,b^3)\sqrt{\cdots}\right.}{\cdots}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/15*(3*(a^3*cos(d*x + c) + 2*a^2*b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*sqrt(1/2)*(3*I*a^2*b - I*b^3)*sqrt(
a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^
3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) - 2*sqrt(1/2)*(-3*
I*a^2*b + I*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/2
7*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)
/a) - 3*sqrt(1/2)*(-3*I*a^3 - I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2
 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2
 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*s
in(d*x + c) + 2*b)/a)) - 3*sqrt(1/2)*(3*I*a^3 + I*a*b^2)*sqrt(a)*weierstra
ssZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPI
nverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(
d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/(a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \cos(c+d\,x)^{5/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \left(\int \sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)\,dx\right)b + \left(\int \sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)a$$

input
```
int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x)**2*sec(c + d*
x),x)*b + int(sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x)**2,
x)*a
```

3.844 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 187

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{2(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{8b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{2a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d}$$

output
```
2/3*(a^2-b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c
,2^(1/2)*(a/(a+b))^(1/2))/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+8/3*b*
cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+
b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)+2/3*a*cos(d*x+c)^(1/2
)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.65 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.52

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{2\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\left(\frac{1}{2}a(b+a\cos(c+dx))\sin(2(c+dx))+\frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\ldots}}{\ldots}\right.}{\ldots}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*((a*(b + a*Cos[c + d*x])*
Sin[2*(c + d*x)])/2 + (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((4*I)*b*(a +
 b)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((b + a*
Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*(a^2 + 4*a*b + 3*b^2)*Ellip
ticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((b + a*Cos[c + d
*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 4*b*(b + a*Cos[c + d*x])*Sqrt[Sec[(c +
 d*x)/2]^2]*Tan[(c + d*x)/2]))/Sec[c + d*x]^(3/2)))/(3*d*(b + a*Cos[c + d*
x])^2)
```

Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.12, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 4752, 3042, 4351, 25, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4351}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}-\frac{1}{3}\int -\frac{4ab+\left(a^2+3b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx\right)$$

$$\Big\downarrow \text{25}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{4ab+\left(a^2+3b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{4ab+\left(a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{4523}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\left(a^2-b^2\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+4b\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+4b\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{4343}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4b\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)}}\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4b\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(\right.}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+d}}\right.\right.$$

$\big\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos}{}}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+}{a+b}}}\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin}{}}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)}{a+b}}}\right.\right.$$

$\big\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{8b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.\right.$$

$\big\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8b\sqrt{a+b\sec(}}{d\sqrt{\sec(c}}\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8b\sqrt{a+b\,\mathrm{s}}}{d\sqrt{\mathrm{se}}}\right.\right.$$

$\big\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8b\sqrt{a+b\sec(c+}}{d\sqrt{\sec(c+}}\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8b\sqrt{a+b\sec(}}{d\sqrt{\sec(c}}\right.\right.$$

$\big\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{2(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{8b\sqrt{a+b}}{d\sqrt{}}\right.\right.$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*(a^2 - b^2)*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[a + b*Sec[c + d*x]]) + (8*b*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c
 + d*x]]))/3 + (2*a*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c
 + d*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4351
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*C
sc[e + f*x])^n/(f*n)), x] + Simp[1/(2*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/
Sqrt[a + b*Csc[e + f*x]])*Simp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[
e + f*x] + a*b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f
}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 698 vs. 2(174) = 348.

Time = 2.75 (sec) , antiderivative size = 699, normalized size of antiderivative = 3.74

method	result
default	$\frac{2\left(\left(4\cos(dx+c)^2+8\cos(dx+c)+4\right)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\sqrt{\frac{1}{1+\cos(dx+c)}}\,ab\,\text{EllipticE}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)+\left(\right.}{}$

input
```
int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/((a-b)/(a+b))^(1/2)*((4*cos(d*x+c)^2+8*cos(d*x+c)+4)*(1/(a+b)*(b+a*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b*EllipticE(((
a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-4*cos(d*
x+c)^2-8*cos(d*x+c)-4)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*b^2*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*
x+c)),(-(a+b)/(a-b))^(1/2))+(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b+a*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*EllipticF(((a
-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-4*cos(d*x
+c)^2-8*cos(d*x+c)-4)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(
1+cos(d*x+c)))^(1/2)*a*b*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x
+c)),(-(a+b)/(a-b))^(1/2))+(3*cos(d*x+c)^2+6*cos(d*x+c)+3)*(1/(a+b)*(b+a*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^2*EllipticF(((
a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+sin(d*x+c)
*cos(d*x+c)*(1+cos(d*x+c))*((a-b)/(a+b))^(1/2)*a^2+(5*cos(d*x+c)+1)*sin(d*
x+c)*((a-b)/(a+b))^(1/2)*a*b+4*((a-b)/(a+b))^(1/2)*b^2*sin(d*x+c))*cos(d*x
+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+cos(d*x+c)*b
+b)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.23

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{2\left(3\,a^2\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+12i\,\sqrt{\frac{1}{2}}a^{\frac{3}{2}}b\text{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\ \right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/9*(3*a^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c) + 12*I*sqrt(1/2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a
^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a
^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c)
 + 2*b)/a)) - 12*I*sqrt(1/2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2
)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2
)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x +
 c) + 2*b)/a)) - sqrt(1/2)*(3*I*a^2 + I*b^2)*sqrt(a)*weierstrassPInverse(-
4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c)
 + 3*I*a*sin(d*x + c) + 2*b)/a) - sqrt(1/2)*(-3*I*a^2 - I*b^2)*sqrt(a)*wei
erstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3
*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a))/(a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \cos(c+d\,x)^{3/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

input
```
int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\begin{aligned}&\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c \\ &+dx))^{3/2}\,dx = \left(\int \sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)\,dx\right)b \\ &+\left(\int \sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a\end{aligned}$$

input
```
int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x),
x)*b + int(sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x),x)*a
```

3.845 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 209

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \frac{2ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2b^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

output
```
2*a*b*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)
*(a/(a+b))^(1/2))/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*b^2*((b+a*co
s(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(
1/2))/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*a*cos(d*x+c)^(1/2)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/
((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 30.68 (sec) , antiderivative size = 25369, normalized size of antiderivative = 121.38

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 2.22 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.10, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.880$, Rules used = {3042, 4752, 3042, 4355, 3042, 4341, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140, 4346, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4355}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(a\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+b\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(a\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right.}\right.$$

$$\Big\downarrow \text{4341}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(a\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\left(b\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+a\int \frac{\sqrt{\sec(c}}{\sqrt{a+b\sec}}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+a\int \frac{\sqrt{a}}{\sqrt{}}\right.$$

$$\Big\downarrow \text{4343}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{a\sqrt{a+}}{\sqrt{}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{a\sqrt{a+}}{}\right.$$

$$\Big\downarrow \text{3134}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{a\sqrt{a+}}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{a\sqrt{a+}}{}\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2a\sqrt{a+}}{d\sqrt{}}\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos}}}{\sqrt{a+b\sec(c+dx)}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{}{\sqrt{b+a\sin(}}}{\sqrt{a+b\sec(c+dx)}}\right.\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+}{a+b}}}}{\sqrt{a+b\sec(c+dx)}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+\right.}{a+}}}}{\sqrt{a+b\sec(c+dx)}}\right.\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c\right.}{d\sqrt{a+b\sec(c+dx)}}\right.\right.$$

$\Big\downarrow$ 4346

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(\frac{b\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(}{}}}{d\sqrt{a}}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(\frac{b\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(}}{}\right.\right.$$

$\Big\downarrow$ 3286

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+d}{a+b}}}{d\sqrt{a+}}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+}}{}\right.\right.$$

$\Big\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+b\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{Ellip}}{d\sqrt{a+b\sec(c+}}\right.\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*EllipticE[(c + d*x)/2, (2*a)/(
a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sq
rt[Sec[c + d*x]]) + b*((2*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]
) + (2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*
a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4341
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[a   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e +
f*x]], x], x] + Simp[b/d   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*
x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4355
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_
)]*(d_.)], x_Symbol] :> Simp[a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e
+ f*x]], x], x] + Simp[b/d   Int[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*
x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.59 (sec) , antiderivative size = 745, normalized size of antiderivative = 3.56

method	result
default	$\frac{2\sqrt{\cos(dx+c)}\,\sqrt{a+b\sec(dx+c)}\left(\left((1-\cos(dx+c))^{3}\csc(dx+c)^{3}-\csc(dx+c)+\cot(dx+c)\right)a^{2}\sqrt{\frac{a-b}{a+b}}+\left(-(1-\cos(dx+c))^{3}\csc(dx+c)\right.\right.}{}$

input
```
int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/((a-b)/(a+b))^(1/2)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)*(((1-cos(d
*x+c))^3*csc(d*x+c)^3-csc(d*x+c)+cot(d*x+c))*a^2*((a-b)/(a+b))^(1/2)+(-(1-
cos(d*x+c))^3*csc(d*x+c)^3-csc(d*x+c)+cot(d*x+c))*b*a*((a-b)/(a+b))^(1/2)+
2*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2
))*a^2-4*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c))
)^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b
))^(1/2))*a*b+2*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+
b)/(a-b))^(1/2))*b^2-2*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c
)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*a^2+2*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-co
t(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b-4*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(d
*x+c)-cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^2)/(a*(1-cos(d*x+c))
^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \int \sqrt{\cos(c+d\,x)}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \left(\int \sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\sec(dx+c)\,dx\right)b + \left(\int \sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}dx\right)a$$

input
```
int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*sec(c + d*x),x)*b + int(sq
rt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x)),x)*a
```

3.846 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 249

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{(2a^2+b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{3ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
(2*a^2+b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2
^(1/2)*(a/(a+b))^(1/2))/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+3*a*b*((
b+a*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a
+b))^(1/2))/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-b*cos(d*x+c)^(1/2)*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2
)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)+b*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d/c
os(d*x+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 31.42 (sec) , antiderivative size = 24604, normalized size of antiderivative = 98.81

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 2.82 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.11, number of steps used = 25, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4752, 3042, 4353, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$\downarrow$ 4353

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int -\frac{-2\sec(c+dx)a^2-3b\sec^2(c+dx)a+ba}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+}}{d}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{1}{2}\int\frac{-2\sec(c+dx)a^2-3b\sec^2(c}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{1}{2}\int\frac{-2\csc\left(c+dx+\frac{\pi}{2}\right)a^2-3b\,c}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+}}\right.$$

$\downarrow$ 4596

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(3ab\int\frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-\int\frac{ab-2a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{b\sin}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(3ab\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\int\frac{ab-2a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx\right.}}\right.\right.$$

$\downarrow$ 4346

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{3ab\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}-\int\frac{ab-2a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right.}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{3ab\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}-\int\frac{}{\sqrt{c}}\right.\right.$$

$\downarrow$ 3286

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{3ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-\int\frac{ab-2a^2\csc}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{}}\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{3ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-\int\frac{}{\sqrt{\csc}}\right.\right.$$

$\big\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{6ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-\int\frac{ab}{\sqrt{\csc\left(c+\right.}}\right.\right.$$

$\big\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\left(2a^2+b^2\right)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx-b\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\frac{6ab\sqrt{\sec(}}{}\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\left(2a^2+b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\right.\right.$$

$\big\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\left(2a^2+b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)}}\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\left(2a^2+b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(\right.}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+}}\right.\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\left(2a^2+b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos}{}}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+}{a+b}}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\left(2a^2+b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin}{}}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)}{a+b}}}\right.\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\left(2a^2+b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{\left(2a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}-\frac{2b\sqrt{a+b\sec}}{d\sqrt{\sec(}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{\left(2a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}-\frac{2b\sqrt{a+b}}{d\sqrt{s}}\right.\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{\left(2a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-\frac{2b\sqrt{a+b\sec(c}}{d\sqrt{\sec(c+}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{(2a^2+b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-\frac{2b\sqrt{a+b\sec}}{d\sqrt{\sec(}}\right.\right.$$

$$\Big\downarrow \text{3140}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2(2a^2+b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-\frac{2b\sqrt{a+}}{d\sqrt{}}\right.\right.$$

input
```
Int[(a + b*Sec[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*(2*a^2 + b^2)*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]]
)/(d*Sqrt[a + b*Sec[c + d*x]]) + (6*a*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a +
 b*Sec[c + d*x]]) - (2*b*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*
Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/
2 + (b*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4353
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + n - 1))), x] + Simp[d/(m + n - 1)   In
t[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n - 1)*Simp[a*b*(n - 1) +
(b^2*(m + n - 2) + a^2*(m + n - 1))*Csc[e + f*x] + a*b*(2*m + n - 2)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
tQ[0, m, 2] && LtQ[0, n, 3] && NeQ[m + n - 1, 0] && (IntegerQ[m] || Integer
sQ[2*m, 2*n])
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.37 (sec) , antiderivative size = 711, normalized size of antiderivative = 2.86

method	result
default	$\frac{\left(\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,ab\,\text{EllipticPi}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\frac{a+b}{a-b},\frac{i}{\sqrt{\frac{a-b}{a+b}}}\right)\left(6\cos(dx+c)^3+12\cos(dx+c)^2\right.\right.}{}$

input
```
int((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/((a-b)/(a+b))^(1/2)*((1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*a*b*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot
(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(6*cos(d*x+c)^3+12*cos(d*x+c)^
2+6*cos(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*a*b*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(
-(a+b)/(a-b))^(1/2))*(-cos(d*x+c)^3-2*cos(d*x+c)^2-cos(d*x+c))+(1/(a+b)*(b
+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^2*Elliptic
E(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(cos(d
*x+c)^3+2*cos(d*x+c)^2+cos(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d
*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(2*cos(d*x+c)^3+4*cos(d*x+c)^2+2*c
os(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*a*b*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b
)/(a-b))^(1/2))*(-2*cos(d*x+c)^3-4*cos(d*x+c)^2-2*cos(d*x+c))+((a-b)/(a+b)
)^(1/2)*a*b*cos(d*x+c)*sin(d*x+c)+((a-b)/(a+b))^(1/2)*b^2*sin(d*x+c))*(a+b
*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+cos(d*x+c)*b+b)/cos(d*x+c)
^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(a+b\sec{(c+dx)})^{\frac{3}{2}}}{\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)/sqrt(cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}{\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{\cos(c+d\,x)}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \left(\int \frac{\sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\sec(dx+c)}{\cos(dx+c)}dx\right)b + \left(\int \frac{\sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a$$

input
```
int((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*sec(c + d*x))/cos(c + d*x
),x)*b + int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x),x)
*a
```

3.847 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 299

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \frac{7ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\\ &+\frac{(3a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\\ &-\frac{5a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{4d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}\\ &+\frac{b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)}+\frac{5a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}}\end{aligned}$$

output

```
7/4*a*b*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/
2)*(a/(a+b))^(1/2))/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+1/4*(3*a^2+4
*b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/
2)*(a/(a+b))^(1/2))/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-5/4*a*cos(d*
x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(
d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)+1/2*b*(a+b*sec(d*x+c))^(1/2
)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+5/4*a*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d/
cos(d*x+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 32.43 (sec) , antiderivative size = 51904, normalized size of antiderivative = 173.59

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 3.42 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.10, number of steps used = 28, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 1.120$, Rules used = {3042, 4752, 3042, 4353, 27, 3042, 4590, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4353}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int \frac{\sqrt{\sec(c+dx)}\left(5ab\sec^2(c+dx)+2\left(2a^2+b^2\right)\sec(c+dx)+ab\right)}{2\sqrt{a+b\sec(c+dx)}}dx+\frac{b\sin(c+$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\int \frac{\sqrt{\sec(c+dx)}\left(5ab\sec^2(c+dx)+2\left(2a^2+b^2\right)\sec(c+dx)+ab\right)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{b\sin(c+$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5ab\csc\left(c+dx+\frac{\pi}{2}\right)^2+2\left(2a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+a\right.}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}$$

$$\Big\downarrow \text{4590}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{\int -\frac{5ba^2-2b^2\sec(c+dx)a-b\left(3a^2+4b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b}+\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a}}{d}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int \frac{5ba^2-2b^2\sec(c+dx)a-b\left(3a^2+4b^2\right.}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c}}}{2b}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int \frac{5ba^2-2b^2\csc\left(c+dx+\frac{\pi}{2}\right)a-b\left(3a^2\right.}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\,cs}}}{2b}\right.\right.$$

$$\Big\downarrow \text{4596}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int \frac{5a^2b-2ab^2\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{\ldots}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5a^2b-2ab^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc(c+dx-}}}{}\right.\right.$$

$\Big\downarrow$ 4346

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5a^2b-2ab^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc(c+dx-}}}{}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5a^2b-2ab^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc(c+dx-}}}{}\right.\right.$$

$\Big\downarrow$ 3286

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5a^2b-2ab^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc(c+dx-}}}{}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5a^2b-2ab^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc(c+dx-}}}{}\right.\right.$$

$\Big\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5a^2b-2ab^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx\right.}}}{}\right.\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-7ab^2\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+5}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-7ab^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{}\right.\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-7ab^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-7ab^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{}\right.\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-7ab^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-7ab^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{\quad}\right.\right.$$

$$\Big\downarrow \text{3132}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-7ab^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{\quad}\right.\right.$$

$$\Big\downarrow \text{4345}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{7ab^2\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}{\sqrt{a+b\sec(c+d}}}{\quad}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{7ab^2\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}{\sqrt{a+b\sec(c+}}}{\quad}\right.\right.$$

$$\Big\downarrow \text{3142}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{7ab^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{\sqrt{a+b\sec(c+}}}{\quad}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{7ab^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{\sqrt{a+b\sec(c}}}{}\right.\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{2b(3a^2+4b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(}{a}}}{d\sqrt{a+b}}}{}\right.\right.$$

input
```
Int[(a + b*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b*Sec[c + d*x]^(3/2)*Sqrt[a + b*Se
c[c + d*x]]*Sin[c + d*x])/(2*d) + (-1/2*((-14*a*b^2*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*
Sqrt[a + b*Sec[c + d*x]]) - (2*b*(3*a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])
/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d
*Sqrt[a + b*Sec[c + d*x]]) + (10*a*b*EllipticE[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[
c + d*x]]))/b + (5*a*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d
*x])/d)/4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4353
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + n - 1))), x] + Simp[d/(m + n - 1)   In
t[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n - 1)*Simp[a*b*(n - 1) +
(b^2*(m + n - 2) + a^2*(m + n - 1))*Csc[e + f*x] + a*b*(2*m + n - 2)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
tQ[0, m, 2] && LtQ[0, n, 3] && NeQ[m + n - 1, 0] && (IntegerQ[m] || Integer
sQ[2*m, 2*n])
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.60 (sec) , antiderivative size = 1030, normalized size of antiderivative = 3.44

method	result	size
default	Expression too large to display	1030

input
```
int((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/((a-b)/(a+b))^(1/2)*((1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*a^2*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-c
ot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(6*cos(d*x+c)^4+12*cos(d*x+c
)^3+6*cos(d*x+c)^2)+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*b^2*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+
c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(8*cos(d*x+c)^4+16*cos(d*x+c)^3+8*c
os(d*x+c)^2)+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*a^2*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a
+b)/(a-b))^(1/2))*(-5*cos(d*x+c)^4-10*cos(d*x+c)^3-5*cos(d*x+c)^2)+(1/(a+b
)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b*Elli
pticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(5
*cos(d*x+c)^4+10*cos(d*x+c)^3+5*cos(d*x+c)^2)+(1/(a+b)*(b+a*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*EllipticF(((a-b)/(a+b))^(
1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(2*cos(d*x+c)^4+4*cos(d
*x+c)^3+2*cos(d*x+c)^2)+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*a*b*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d
*x+c)),(-(a+b)/(a-b))^(1/2))*(2*cos(d*x+c)^4+4*cos(d*x+c)^3+2*cos(d*x+c)^2
)+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*b^2*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^
(1/2))*(-4*cos(d*x+c)^4-8*cos(d*x+c)^3-4*cos(d*x+c)^2)+5*((a-b)/(a+b))^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{\frac{3}{2}}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(3/2)/cos(d*x+c)**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)/cos(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\sec\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}\,\sec\left(dx+c\right)}{\cos\left(dx+c\right)^{2}}dx\right)b + \left(\int \frac{\sqrt{\sec\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{2}}dx\right)a$$

input
```
int((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*sec(c + d*x))/cos(c + d*x
)**2,x)*b + int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)
**2,x)*a
```

3.848 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 363

$$\begin{aligned}\int \cos^{\frac{9}{2}}(c+dx)(a&+b\sec(c+dx))^{5/2}\,dx = \frac{4b(57a^4-62a^2b^2+5b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{315a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\\
&+\frac{2(147a^4+279a^2b^2-10b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{315a^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}\\
&+\frac{2b(163a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{315ad}\\
&+\frac{2(49a^2+75b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{315d}\\
&+\frac{38ab\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{63d}\\
&+\frac{2a^2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{9d}\end{aligned}$$

output
```
4/315*b*(57*a^4-62*a^2*b^2+5*b^4)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJa
cobiAM(1/2*d*x+1/2*c,2^(1/2)*(a/(a+b))^(1/2))/a^2/d/cos(d*x+c)^(1/2)/(a+b*
sec(d*x+c))^(1/2)+2/315*(147*a^4+279*a^2*b^2-10*b^4)*cos(d*x+c)^(1/2)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a
^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)+2/315*b*(163*a^2+5*b^2)*cos(d*x+c)^(1/
2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/a/d+2/315*(49*a^2+75*b^2)*cos(d*x+c)^
(3/2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d+38/63*a*b*cos(d*x+c)^(5/2)*(a+b*
sec(d*x+c))^(1/2)*sin(d*x+c)/d+2/9*a^2*cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(
1/2)*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.75 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.31

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{b(747a^2+20b^2)\sin(c+dx)}{630a}+\frac{1}{630}(133a^2+150b^2)\sin(2(c+dx))\right.}{d(b+a\cos(c+dx))^2}$$
$$-\frac{2\cos^{\frac{3}{2}}(c+dx)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}(a+b\sec(c+dx))^{5/2}\left(-i(147a^5+147a^4b+279a^3b^2+\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*((b*(747*a^2 + 20*b^2)*Sin[
c + d*x])/(630*a) + ((133*a^2 + 150*b^2)*Sin[2*(c + d*x)])/630 + (19*a*b*S
in[3*(c + d*x)])/126 + (a^2*Sin[4*(c + d*x)])/36))/(d*(b + a*Cos[c + d*x])
^2) - (2*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(a + b
*Sec[c + d*x])^(5/2)*((-I)*(147*a^5 + 147*a^4*b + 279*a^3*b^2 + 279*a^2*b^
3 - 10*a*b^4 - 10*b^5)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a
 + b)] + I*a*(147*a^4 + 261*a^3*b + 279*a^2*b^2 + 155*a*b^3 - 10*b^4)*Elli
pticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sq
rt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (147*a^4 + 279*a^2
*b^2 - 10*b^4)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*
x)/2]))/(315*a^2*d*(b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 3.57 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.10, number of steps used = 27, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 1.080$, Rules used = {3042, 4752, 3042, 4328, 27, 3042, 4592, 27, 3042, 4592, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{4328}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{9}\int\frac{19ba^2+(7a^2+27b^2)\sec(c+dx)a+3b(2a^2+3b^2)\sec^2(c+dx)}{2\sec^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(\right.$$

$\big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\int\frac{19ba^2+(7a^2+27b^2)\sec(c+dx)a+3b(2a^2+3b^2)\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\int\frac{19ba^2+(7a^2+27b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a+3b(2a^2+3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right.$$

$\big\downarrow$ 4592

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{38ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}-\frac{2\int-\frac{76b^2\sec^2(c+dx)a^2+(49a^2+75b^2)a^2+b(137}{2\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+}}}{7a}\right.\right.$$

$\big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\int\frac{76b^2\sec^2(c+dx)a^2+(49a^2+75b^2)a^2+b(137a^2+63b^2)\sec(c+dx)a}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{7a}+\frac{38ab\sin(c+dx)\sqrt{a}}{7d\sec^{\frac{5}{2}}(c}\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\int\frac{76b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+(49a^2+75b^2)a^2+b(137a^2+63b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{7a}+\frac{38ab\sin(c+}{7d\,s}\right.\right.$$

$\big\downarrow$ 4592

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{2a(49a^2+75b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}-\frac{2\int-\frac{(147a^2+605b^2)\sec(c+dx)a^3+2b(49a^2+75b^2)\sec^2(c}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+d}}}{5a}}{7a}\right.\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\int\frac{(147a^2+605b^2)\sec(c+dx)a^3+2b(49a^2+75b^2)\sec^2(c+dx)a^2+3b(163a^2+5b^2)a^2}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}+\frac{2a(49a^2+75b^2)\sin}{5d\sec}}{7a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\int\frac{(147a^2+605b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a^3+2b(49a^2+75b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+3b(163a^2+5b^2)a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}+\frac{2a(49a^2+7}{}}{7a}\right.\right.$$

$\downarrow$ 4592

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{2ab(163a^2+5b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}-\frac{2\int-\frac{3\left(b(261a^2+155b^2)\sec(c+dx)a^3+(147a^4+279b^2a^2-10b^4)a\right.}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}}{3a}}{5a}}{7a}\right.\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{\int\frac{b(261a^2+155b^2)\sec(c+dx)a^3+(147a^4+279b^2a^2-10b^4)a^2}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a}+\frac{2ab(163a^2+5b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}}{5a}}{7a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{\int \frac{b\left(261a^2+155b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^3+\left(147a^4+279b^2a^2-10b^4\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+\ldots}}{d\sqrt{\sec(c+dx)}}\ldots}{5a}\ldots}{7a}\ldots\right.\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+a\left(147a^4+279a^2b^2-10b^4\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}+\frac{2ab\left(163a^2+5\ldots\right.}{\ldots}\ldots}{5a}\ldots}{7a}\ldots\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(147a^4+279a^2b^2-10b^4\right)\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2ab\ldots}{\ldots}\ldots}{5a}\ldots}{7a}\ldots\right.\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+\ldots}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}\ldots}{a}\ldots}{5a}\ldots}{7a}\ldots\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+\cdots\right.}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a}}{5a}}{7a\cdots}\right.\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(\cdots}{a\cdots}}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}}{5a}}{7a\cdots}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin(\cdots}{\cdots}}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}}{5a}}{7\cdots}\right.\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2\cdots}{a-\cdots}\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}}{5a}}{7a\cdots}\right.\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a+b\cdots}}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\,c\cdots}{}}}}{a}}{5a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{\cdots}}{d\sqrt{\sec(c+dx)}\cdots}}{a}}{5a}\right.\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{\cdots}}{d\sqrt{\sec(c+dx)}\cdots}}{a}}{5a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(147a^4+279a^2b^2-10b\cdots\right.}{d\sqrt{\sec(c\cdots}}}{a}}{5a}\right.\right.$$

$$\Big\downarrow \text{3140}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{1}{9}\left(\frac{\frac{2a(49a^2+75b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2ab(}{}}{}\right.\right.$$

input
```
Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sqrt[a + b*Sec[c + d*x]]*Sin
[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + ((38*a*b*Sqrt[a + b*Sec[c + d*x]]*Si
n[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + ((2*a*(49*a^2 + 75*b^2)*Sqrt[a + b*
Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (((4*a*b*(57*a^4 -
62*a^2*b^2 + 5*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*a
*(147*a^4 + 279*a^2*b^2 - 10*b^4)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sq
rt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c +
 d*x]]))/a + (2*a*b*(163*a^2 + 5*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x
])/(d*Sqrt[Sec[c + d*x]]))/(5*a))/(7*a))/9)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1569 vs. $2(332) = 664$.

Time = 1208.69 (sec) , antiderivative size = 1570, normalized size of antiderivative = 4.33

method	result	size
default	Expression too large to display	1570

input
```
int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/315/d/a^2/((a-b)/(a+b))^(1/2)*((147*cos(d*x+c)^2+294*cos(d*x+c)+147)*(1/
(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^5*
EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2)
)+(-147*cos(d*x+c)^2-294*cos(d*x+c)-147)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4*b*EllipticE(((a-b)/(a+b))^(1/2
)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(279*cos(d*x+c)^2+558*cos(
d*x+c)+279)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*a^3*b^2*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(
-(a+b)/(a-b))^(1/2))+(-279*cos(d*x+c)^2-558*cos(d*x+c)-279)*(1/(a+b)*(b+a*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b^3*Ellipti
cE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-10*
cos(d*x+c)^2-20*cos(d*x+c)-10)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^4*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x
+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(10*cos(d*x+c)^2+20*cos(d*x+c)+10)*(
1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^
5*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/
2))+(-147*cos(d*x+c)^2-294*cos(d*x+c)-147)*(1/(a+b)*(b+a*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^5*EllipticF(((a-b)/(a+b))^(1/2
)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(261*cos(d*x+c)^2+522*cos(
d*x+c)+261)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.47

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/945*(3*(35*a^5*cos(d*x + c)^3 + 95*a^4*b*cos(d*x + c)^2 + 163*a^4*b + 5*
a^2*b^3 + (49*a^5 + 75*a^3*b^2)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/co
s(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - sqrt(1/2)*(489*I*a^4*b - 93*
I*a^2*b^3 + 20*I*b^5)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2
, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) +
 2*b)/a) - sqrt(1/2)*(-489*I*a^4*b + 93*I*a^2*b^3 - 20*I*b^5)*sqrt(a)*weie
rstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*
(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(1/2)*(-147*I*a^5
 - 279*I*a^3*b^2 + 10*I*a*b^4)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2
)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2
)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x +
 c) + 2*b)/a)) - 3*sqrt(1/2)*(147*I*a^5 + 279*I*a^3*b^2 - 10*I*a*b^4)*sqrt
(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3,
weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3,
1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(9/2), x)
```

Giac [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \cos(c+d\,x)^{9/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}dx$$

input
```
int(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\begin{aligned}&\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c\\&+dx))^{5/2}\,dx = \left(\int \sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^{4}\sec(dx+c)^{2}dx\right)b^{2}\\&+2\left(\int \sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^{4}\sec(dx+c)\,dx\right)ab\\&+\left(\int \sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^{4}dx\right)a^{2}\end{aligned}$$

input
```
int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x)**4*sec(c + d*
x)**2,x)*b**2 + 2*int(sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c +
d*x)**4*sec(c + d*x),x)*a*b + int(sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*
x))*cos(c + d*x)**4,x)*a**2
```

3.849 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 303

$$\begin{aligned}\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c&+dx))^{5/2}\,dx = \frac{2(5a^4-2a^2b^2-3b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{21ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\\
&+\frac{2b(29a^2+3b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{21ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}\\
&+\frac{2(5a^2+9b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{21d}\\
&+\frac{6ab\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d}\\
&+\frac{2a^2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d}\end{aligned}$$

output
```
2/21*(5*a^4-2*a^2*b^2-3*b^4)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiA
M(1/2*d*x+1/2*c,2^(1/2)*(a/(a+b))^(1/2))/a/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x
+c))^(1/2)+2/21*b*(29*a^2+3*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a/d/((b+a*cos(d*x+c))
/(a+b))^(1/2)+2/21*(5*a^2+9*b^2)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)*s
in(d*x+c)/d+6/7*a*b*cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d+2
/7*a^2*cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.25 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.27

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{1}{2}(b+a\cos(c+dx))\left(13a^2+18b^2+18ab\cos(c+dx)+\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)*(((b + a*Cos[c + d*x])*(13*
a^2 + 18*b^2 + 18*a*b*Cos[c + d*x] + 3*a^2*Cos[2*(c + d*x)])*Sin[2*(c + d*
x)])/2 + (2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(I*b*(29*a^3 + 29*a^2*
b + 3*a*b^2 + 3*b^3)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a +
b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a +
 b)] - I*a*(5*a^3 + 29*a^2*b + 27*a*b^2 + 3*b^3)*EllipticF[I*ArcSinh[Tan[(
c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*
x])*Sec[(c + d*x)/2]^2)/(a + b)] + b*(29*a^2 + 3*b^2)*(b + a*Cos[c + d*x])
*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(a*Sec[c + d*x]^(5/2))))/(2
1*d*(b + a*Cos[c + d*x])^3)
```

Rubi [A] (verified)

Time = 2.78 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.10, number of steps used = 24, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.960, Rules used = {3042, 4752, 3042, 4328, 27, 3042, 4592, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4328}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{7}\int \frac{15ba^2+(5a^2+21b^2)\sec(c+dx)a+b(4a^2+7b^2)\sec^2(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int \frac{15ba^2+(5a^2+21b^2)\sec(c+dx)a+b(4a^2+7b^2)\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int \frac{15ba^2+(5a^2+21b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a+b(4a^2+7b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-$$

$$\Big\downarrow \text{4592}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{6ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sec^{\frac{3}{2}}(c+dx)}-\frac{2\int -\frac{5(6b^2\sec^2(c+dx)a^2+(5a^2+9b^2)a^2+b(13a^2+}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx}}}{5a}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\int \frac{6b^2\sec^2(c+dx)a^2+(5a^2+9b^2)a^2+b(13a^2+7b^2)\sec(c+dx)a}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{a}+\frac{6ab\sin(c+dx)\sqrt{a+b\,\mathrm{se}}}{d\sec^{\frac{3}{2}}(c+dx)}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\int \frac{6b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+(5a^2+9b^2)a^2+b(13a^2+7b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{6ab\sin(c+dx)\sqrt{}}{d\sec^{\frac{3}{2}}(c}\right.\right.$$

$$\Big\downarrow \text{4592}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{2a(5a^2+9b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}-\frac{2\int -\frac{\left(5a^2+27b^2\right)\sec(c+dx)a^3+b\left(29a^2+3b^2\right)a^2}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{a}+\frac{6a}{}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\int \frac{\left(5a^2+27b^2\right)\sec(c+dx)a^3+b\left(29a^2+3b^2\right)a^2}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}+\frac{2a(5a^2+9b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{a}+\frac{6ab\,\mathrm{si}}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\int \frac{\left(5a^2+27b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^3+b\left(29a^2+3b^2\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2a(5a^2+9b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{a}+\frac{6a}{}\right.\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{ab(29a^2+3b^2)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+a(5a^4-2a^2b^2-3b^4)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{3a}+\frac{2a(5a^2+9b^2)\sin(c+}{3d\sqrt{\sec}}}{a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{ab(29a^2+3b^2)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a(5a^4-2a^2b^2-3b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2a(5a^2+9b^2}{}}{a}\right.\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\frac{ab(29a^2+3b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+a(5a^4-2a^2b^2-3b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2a}{}}{a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\frac{ab(29a^2+3b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+a(5a^4-2a^2b^2-3b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+}{a}\right.\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\frac{ab(29a^2+3b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+a(5a^4-2a^2b^2-3b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+}{a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\frac{ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+a\left(5a^4-2a^2b^2-3b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}}{a}\right.\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{a\left(5a^4-2a^2b^2-3b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2a(5}{}}{a}\right.\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\frac{a\left(5a^4-2a^2b^2-3b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)-}{a+b}}}}{3a}}{a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\frac{a\left(5a^4-2a^2b^2-3b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}E}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+}{a+}}}}{3a}}{a}\right.\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\frac{a\left(5a^4-2a^2b^2-3b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}E}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c-}{a-}}}}{3a}}{a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\frac{a\left(5a^4-2a^2b^2-3b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+}}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\,c}{}}}}{3a}}{a}\right.\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{1}{7}\left(\frac{\frac{2a(5a^2+9b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}+\frac{\frac{2ab\left(29a\right.}{}}{}}{}\right.\right.$$

input
```
Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sqrt[a + b*Sec[c + d*x]]*Sin
[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + ((6*a*b*Sqrt[a + b*Sec[c + d*x]]*Sin
[c + d*x])/(d*Sec[c + d*x]^(3/2)) + (((2*a*(5*a^4 - 2*a^2*b^2 - 3*b^4)*Sqr
t[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt
[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*a*b*(29*a^2 + 3*b^2)*Ell
ipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(3*a) + (2*a*(5*a^2 + 9*b^2)
*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]))/a)/7)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1163 vs. 2(278) = 556.

Time = 1298.17 (sec) , antiderivative size = 1164, normalized size of antiderivative = 3.84

method	result	size
default	Expression too large to display	1164

input
```
int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/d/a/((a-b)/(a+b))^(1/2)*((29*cos(d*x+c)^2+58*cos(d*x+c)+29)*(1/(a+b)*
(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b*Elli
pticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-
29*cos(d*x+c)^2-58*cos(d*x+c)-29)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b^2*EllipticE(((a-b)/(a+b))^(1/2)*(cs
c(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(3*cos(d*x+c)^2+6*cos(d*x+c)+3)
*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
a*b^3*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))
^(1/2))+(-3*cos(d*x+c)^2-6*cos(d*x+c)-3)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^4*EllipticE(((a-b)/(a+b))^(1/2)*
(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(5*cos(d*x+c)^2+10*cos(d*x+c
)+5)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*a^4*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b
))^(1/2))+(-29*cos(d*x+c)^2-58*cos(d*x+c)-29)*(1/(a+b)*(b+a*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b*EllipticF(((a-b)/(a+b))
^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(27*cos(d*x+c)^2+54*c
os(d*x+c)+27)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*a^2*b^2*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c))
,(-(a+b)/(a-b))^(1/2))+(-3*cos(d*x+c)^2-6*cos(d*x+c)-3)*(1/(a+b)*(b+a*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^3*EllipticF(...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.63

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/63*(3*(3*a^4*cos(d*x + c)^2 + 9*a^3*b*cos(d*x + c) + 5*a^4 + 9*a^2*b^2)*
sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) -
sqrt(1/2)*(15*I*a^4 + 23*I*a^2*b^2 - 6*I*b^4)*sqrt(a)*weierstrassPInverse(
-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c
) + 3*I*a*sin(d*x + c) + 2*b)/a) - sqrt(1/2)*(-15*I*a^4 - 23*I*a^2*b^2 + 6
*I*b^4)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*
b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*s
qrt(1/2)*(-29*I*a^3*b - 3*I*a*b^3)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4
*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4
*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d
*x + c) + 2*b)/a)) - 3*sqrt(1/2)*(29*I*a^3*b + 3*I*a*b^3)*sqrt(a)*weierstr
assZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassP
Inverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos
(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/(a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \cos(c+d\,x)^{7/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$
\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \left(\int \sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^3\sec(dx+c)^2\,dx\right)b^2 + 2\left(\int \sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^3\sec(dx+c)\,dx\right)ab + \left(\int \sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)a^2
$$

input

```
int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2),x)
```

output

```
int(sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*
x)**2,x)*b**2 + 2*int(sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c +
d*x)**3*sec(c + d*x),x)*a*b + int(sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*
x))*cos(c + d*x)**3,x)*a**2
```

3.850 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 239

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{16b(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2(9a^2+23b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{15d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{22ab\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d}$$

output
```
16/15*b*(a^2-b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1
/2*c,2^(1/2)*(a/(a+b))^(1/2))/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/
15*(9*a^2+23*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a
/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)+22/
15*a*b*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d+2/5*a^2*cos(d*
x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.52 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.50

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\left(a(b+a\cos(c+dx))(11b+3a\cos(c+dx))\sin(2(c+dx\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)*(a*(b + a*Cos[c + d*x])*(11
*b + 3*a*Cos[c + d*x])*Sin[2*(c + d*x)] - (2*(Cos[(c + d*x)/2]^2*Sec[c + d
*x])^(3/2)*((-I)*(9*a^3 + 9*a^2*b + 23*a*b^2 + 23*b^3)*EllipticE[I*ArcSinh
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[
c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*(9*a^3 + 17*a^2*b + 23*a*b^2 +
15*b^3)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c +
d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (9*a^2
 + 23*b^2)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2
]))/Sec[c + d*x]^(5/2)))/(15*d*(b + a*Cos[c + d*x])^3)
```

Rubi [A] (verified)

Time = 2.20 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.13, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 4752, 3042, 4328, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4328}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{5}\int\frac{11ba^2+3(a^2+5b^2)\sec(c+dx)a+b(2a^2+5b^2)\sec^2(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c+}{5}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int\frac{11ba^2+3(a^2+5b^2)\sec(c+dx)a+b(2a^2+5b^2)\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c+}{5}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int\frac{11ba^2+3(a^2+5b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a+b(2a^2+5b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\right.$$

$$\Big\downarrow \text{4592}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{22ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}-\frac{2\int-\frac{(9a^2+23b^2)a^2+b(17a^2+15b^2)\sec(c+dx)a}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}d}{3a}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\int\frac{(9a^2+23b^2)a^2+b(17a^2+15b^2)\sec(c+dx)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}+\frac{22ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}\right)\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\int \frac{(9a^2+23b^2)a^2+b(17a^2+15b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{22ab\sin(c+dx)\sqrt{a+b\sec(c+dx}}{3d\sqrt{\sec(c+dx)}}\right.\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{8ab(a^2-b^2)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+a(9a^2+23b^2)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{3a}+\frac{22ab\sin}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{8ab(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a(9a^2+23b^2)\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{22}{}\right.\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{8ab(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a(9a^2+23b^2)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{3a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{8ab(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a(9a^2+23b^2)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right.}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{3a}\right.\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{8ab(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a(9a^2+23b^2)\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{8ab\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(9a^2+23b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\right.}{a+b}}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}\right.\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{8ab\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\left(9a^2+23b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\right.\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{8ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(9a^2+23b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+}{a+b}}}}{3a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{8ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(9a^2+23b^2\right)\sqrt{a+b\sec(c+dx}}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos}{}}}}{3a}\right.\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{8ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(9a^2+23b^2\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(}{}}}}{3a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{8ab(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a(9a^2+23b^2)\sqrt{a+b\sec(c+}}{d\sqrt{\sec(c+dx)}\sqrt{a}}}{3a}\right.\right.$$

$$\Big\downarrow \text{3140}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{16ab(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2a(9a^2+23b^2)\sqrt{a+b\sec(c+}}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\,c}{}}}}{3a}\right.\right.$$

input
```
Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sqrt[a + b*Sec[c + d*x]]*Sin
[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (((16*a*b*(a^2 - b^2)*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*
x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*a*(9*a^2 + 23*b^2)*EllipticE[(c + d
*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]*Sqrt[Sec[c + d*x]]))/(3*a) + (22*a*b*Sqrt[a + b*Sec[c + d*x]]*
Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]))/5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1088 vs. 2(220) = 440.

Time = 1237.28 (sec) , antiderivative size = 1089, normalized size of antiderivative = 4.56

method	result	size
default	Expression too large to display	1089

input
```
int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d/((a-b)/(a+b))^(1/2)*((9*cos(d*x+c)^2+18*cos(d*x+c)+9)*(1/(a+b)*(b+a
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*EllipticE(
((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-9*cos(
d*x+c)^2-18*cos(d*x+c)-9)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-c
ot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(23*cos(d*x+c)^2+46*cos(d*x+c)+23)*(1/(a+
b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2*E
llipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))
+(-23*cos(d*x+c)^2-46*cos(d*x+c)-23)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^3*EllipticE(((a-b)/(a+b))^(1/2)*(csc
(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-9*cos(d*x+c)^2-18*cos(d*x+c)-9
)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*a^3*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^
(1/2))+(17*cos(d*x+c)^2+34*cos(d*x+c)+17)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticF(((a-b)/(a+b))^(1/
2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-23*cos(d*x+c)^2-46*cos(
d*x+c)-23)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*a*b^2*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a
+b)/(a-b))^(1/2))+(15*cos(d*x+c)^2+30*cos(d*x+c)+15)*(1/(a+b)*(b+a*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^3*EllipticF(((a-b...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.91

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{2\left(3\left(3a^3\cos(dx+c)+11a^2b\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-\sqrt{\frac{1}{2}}(33i\,a^2b-i\,b^3$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/45*(3*(3*a^3*cos(d*x + c) + 11*a^2*b)*sqrt((a*cos(d*x + c) + b)/cos(d*x
+ c))*sqrt(cos(d*x + c))*sin(d*x + c) - sqrt(1/2)*(33*I*a^2*b - I*b^3)*sqr
t(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/
a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) - sqrt(1/2)*(-33
*I*a^2*b + I*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/
27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b
)/a) - 3*sqrt(1/2)*(-9*I*a^3 - 23*I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3
*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3
*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I
*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(1/2)*(9*I*a^3 + 23*I*a*b^2)*sqrt(a)*we
ierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weiers
trassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3
*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/(a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \cos(c+d\,x)^{5/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}dx$$

input
```
int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\begin{aligned}\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c&\\ +dx))^{5/2}\,dx &= \left(\int \sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^2\,dx\right)b^2\\ &+2\left(\int \sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)\,dx\right)ab\\ &+\left(\int \sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)a^2\end{aligned}$$

input
```
int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x)**2*sec(c + d*
x)**2,x)*b**2 + 2*int(sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c +
d*x)**2*sec(c + d*x),x)*a*b + int(sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*
x))*cos(c + d*x)**2,x)*a**2
```

3.851 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 262

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{2a(a^2+2b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2b^3\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{14ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{2a^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d}$$

output
```
2/3*a*(a^2+2*b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1
/2*c,2^(1/2)*(a/(a+b))^(1/2))/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*
b^3*((b+a*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)
*(a/(a+b))^(1/2))/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+14/3*a*b*cos(d
*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec
(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)+2/3*a^2*cos(d*x+c)^(1/2)*(
a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 33.92 (sec) , antiderivative size = 36889, normalized size of antiderivative = 140.80

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Result too large to show}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 2.93 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.08, number of steps used = 25, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4752, 3042, 4328, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4328}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{3}\int \frac{3\sec^2(c+dx)b^3+7a^2b+a\left(a^2+9b^2\right)\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b}}{3d\sqrt{\sec(c+}}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{3\sec^2(c+dx)b^3+7a^2b+a\left(a^2+9b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b}}{3d\sqrt{\sec(c+}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{3\csc\left(c+dx+\frac{\pi}{2}\right)^2b^3+7a^2b+a\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+}{3a}\right.$$

$$\Big\downarrow \text{4596}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int \frac{7ba^2+\left(a^2+9b^2\right)\sec(c+dx)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+3b^3\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{2a^2}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int \frac{7ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/}}{\sqrt{a+b\csc\left(c+dx-\right.}}\right.\right.$$

$$\Big\downarrow \text{4346}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int \frac{7ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3b^3\sqrt{\sec(c+dx)}\sqrt{a\cos(c}}{\sqrt{a+b\,s}}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int \frac{7ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3b^3\sqrt{\sec(c+dx)}\sqrt{a\cos(c}}{\sqrt{}}\right.\right.$$

$$\Big\downarrow \text{3286}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int\frac{7ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+}{a+b}}}{\sqrt{a+b\sec}}\right.\right.$$

↓ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int\frac{7ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+}{a+b}}}{\sqrt{a}}\right.\right.$$

↓ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int\frac{7ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+}{a+b}}}{d\sqrt{a-}}\right.\right.$$

↓ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(a\left(a^2+2b^2\right)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+7ab\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\frac{6b^3\sqrt{se}}{}\right.\right.$$

↓ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(a\left(a^2+2b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+7ab\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right.\right.$$

↓ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(a\left(a^2+2b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{7ab\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\,c}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+d}}\right.\right.$$

↓ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(a\left(a^2+2b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{7ab\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\,s}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c-}}\right.\right.$$

$\Big\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(a\left(a^2+2b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{7ab\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+d}{a+b}}}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(a\left(a^2+2b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{7ab\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+}{a+}}}\right.\right.$$

$\Big\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(a\left(a^2+2b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\mathrm{E}}{d\sqrt{a+b\sec(}}\right.\right.$$

$\Big\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{a\left(a^2+2b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{6b^3\sqrt{\sec(c}}{}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{a\left(a^2+2b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{6b^3\sqrt{\sec}}{}\right.\right.$$

$\Big\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{a\left(a^2+2b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{6b^3\sqrt{\sec(c+d}}{}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{a(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{6b^3\sqrt{\sec(c}{}\right.\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{2a(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{6b^3\sqrt{\mathrm{s}}}{}\right.\right.$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*a*(a^2 + 2*b^2)*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x
]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (6*b^3*Sqrt[(b + a*Cos[c + d*x])/(a + b
)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a
 + b*Sec[c + d*x]]) + (14*a*b*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
 + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x
]]))/3 + (2*a^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d
*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.14 (sec) , antiderivative size = 937, normalized size of antiderivative = 3.58

method	result	size
default	Expression too large to display	937

input
```
int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/((a-b)/(a+b))^(1/2)*((6*cos(d*x+c)^2+12*cos(d*x+c)+6)*(1/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^3*EllipticPi(
((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1
/2))+(7*cos(d*x+c)^2+14*cos(d*x+c)+7)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticE(((a-b)/(a+b))^(1/2)*(
csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-7*cos(d*x+c)^2-14*cos(d*x+c
)-7)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*a*b^2*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a
-b))^(1/2))+(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*EllipticF(((a-b)/(a+b))^(1/2)
*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-7*cos(d*x+c)^2-14*cos(d*x
+c)-7)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*a^2*b*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/
(a-b))^(1/2))+(9*cos(d*x+c)^2+18*cos(d*x+c)+9)*(1/(a+b)*(b+a*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2*EllipticF(((a-b)/(a+b)
)^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-3*cos(d*x+c)^2-6*c
os(d*x+c)-3)*(1/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*b^3*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a
+b)/(a-b))^(1/2))+sin(d*x+c)*cos(d*x+c)*(1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
*a^3+(8*cos(d*x+c)+1)*sin(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b+7*((a-b)/(a+...
```

Fricas [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*cos(d*x + c)*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)*sec(d*x + c
) + a^2*cos(d*x + c))*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \cos(c+d\,x)^{3/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}dx$$

input
```
int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \left(\int \sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)^{2}dx\right)b^{2} + 2\left(\int \sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)\,dx\right)ab + \left(\int \sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a^{2}$$

input
```
int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x)*
*2,x)*b**2 + 2*int(sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x
)*sec(c + d*x),x)*a*b + int(sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*co
s(c + d*x),x)*a**2
```

3.852 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 263

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \frac{b(4a^2+b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{5ab^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{(2a^2-b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{b^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
b*(4*a^2+b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c
,2^(1/2)*(a/(a+b))^(1/2))/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+5*a*b^
2*((b+a*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(
a/(a+b))^(1/2))/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+(2*a^2-b^2)*cos(
d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*se
c(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)+b^2*(a+b*sec(d*x+c))^(1/2
)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 32.79 (sec) , antiderivative size = 44895, normalized size of antiderivative = 170.70

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 2.97 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.11, number of steps used = 25, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4752, 3042, 4329, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4329}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{6b\sec(c+dx)a^2+5b^2\sec^2(c+dx)a+\left(2a^2-b^2\right)a}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{b^2\sin(c+dx)\sqrt{\sec(c+\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int \frac{6b\sec(c+dx)a^2+5b^2\sec^2(c+dx)a+\left(2a^2-b^2\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{b^2\sin(c+dx)\sqrt{\sec(c\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int \frac{6b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+5b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a+\left(2a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{b^2\sin(c+\right.$$

$$\Big\downarrow \text{4596}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\int \frac{6b\sec(c+dx)a^2+\left(2a^2-b^2\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+5ab^2\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{b^2 s}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\int \frac{6b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(2a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5ab^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx\right.}}\right.\right.$$

$$\Big\downarrow \text{4346}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\int \frac{6b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(2a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{5ab^2\sqrt{\sec(c+dx)}\sqrt{a\cos(c}}{\sqrt{a+b}}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\int \frac{6b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(2a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{5ab^2\sqrt{\sec(c+dx)}\sqrt{a\cos(c}}{\sqrt{}}\right.\right.$$

$$\Big\downarrow \text{3286}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\int \frac{6b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(2a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{5ab^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c-}{a+}}}{\sqrt{a+b\sec}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\int \frac{6b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(2a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{5ab^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c-}{a+}}}{\sqrt{}}\right.\right.$$

$\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\int \frac{6b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(2a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{10ab^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(}{}}}{d\sqrt{a}}\right.\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(b\left(4a^2+b^2\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+\left(2a^2-b^2\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(b\left(4a^2+b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(2a^2-b^2\right)\int \frac{\sqrt{a+b\csc\left(c+dx\right.}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right.}}\right.\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(b\left(4a^2+b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(2a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(b\left(4a^2+b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(2a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int}{\sqrt{\sec(c+dx)}\sqrt{a\cos}}\right.\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(b\left(4a^2+b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(2a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(}{}}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(b\left(4a^2+b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(2a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos}{}}}\right.\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(b\left(4a^2+b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(2a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c-}{a-}}}\right.\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{b\left(4a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(2a^2-b^2\right)}{d}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{b\left(4a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(2a^2-\right.}{}\right.\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{b\left(4a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(2a^2-b^2\right)\sqrt{}}{d\sqrt{\mathrm{s}}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{b(4a^2+b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2(2a^2-b^2)}{d}\right.\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2b(4a^2+b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2(2a^2-}{}\right.\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*b*(4*a^2 + b^2)*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x
]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (10*a*b^2*Sqrt[(b + a*Cos[c + d*x])/(a
+ b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqr
t[a + b*Sec[c + d*x]]) + (2*(2*a^2 - b^2)*EllipticE[(c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt
[Sec[c + d*x]]))/2 + (b^2*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[
c + d*x])/d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.22 (sec) , antiderivative size = 1135, normalized size of antiderivative = 4.32

method	result	size
default	Expression too large to display	1135

input
```
int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/((a-b)/(a+b))^(1/2)*((1/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*a*b^2*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-c
ot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(10*cos(d*x+c)^3+20*cos(d*x+
c)^2+10*cos(d*x+c))+(1/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*a^3*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c
)),(-(a+b)/(a-b))^(1/2))*(2*cos(d*x+c)^3+4*cos(d*x+c)^2+2*cos(d*x+c))+(1/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b
*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2
))*(-2*cos(d*x+c)^3-4*cos(d*x+c)^2-2*cos(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2*EllipticE(((a-b)/(a+
b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-cos(d*x+c)^3-2*c
os(d*x+c)^2-cos(d*x+c))+(1/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*b^3*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d
*x+c)),(-(a+b)/(a-b))^(1/2))*(cos(d*x+c)^3+2*cos(d*x+c)^2+cos(d*x+c))+(1/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*E
llipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))
*(-2*cos(d*x+c)^3-4*cos(d*x+c)^2-2*cos(d*x+c))+(1/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticF(((a-b)/(a+b)
)^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(6*cos(d*x+c)^3+12*c
os(d*x+c)^2+6*cos(d*x+c))+(1/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*...
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \int \sqrt{\cos(c+d\,x)}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\begin{aligned}\int \sqrt{\cos(c+dx)}(a &+b\sec(c+dx))^{5/2}\,dx = \left(\int \sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\sec(dx+c)^2\,dx\right)b^2 \\ &+2\left(\int \sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\sec(dx+c)\,dx\right)ab \\ &+\left(\int \sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}dx\right)a^2\end{aligned}$$

input
```
int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*sec(c + d*x)**2,x)*b**2 +
2*int(sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*sec(c + d*x),x)*a*b + in
t(sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x)),x)*a**2
```

3.853 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 314

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx &= \frac{a(8a^2+11b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\\
&+\frac{b(15a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\\
&-\frac{9ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{4d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}\\
&+\frac{b^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)}+\frac{9ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}}\end{aligned}$$

output
```
1/4*a*(8*a^2+11*b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*
x+1/2*c,2^(1/2)*(a/(a+b))^(1/2))/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)
+1/4*b*(15*a^2+4*b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*
x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1
/2)-9/4*a*b*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b)
)^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)+1/2*b^2*(
a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+9/4*a*b*(a+b*sec(d*x+c
))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 32.10 (sec) , antiderivative size = 53538, normalized size of antiderivative = 170.50

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 3.69 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.10, number of steps used = 28, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 1.120$, Rules used = {3042, 4752, 3042, 4329, 27, 3042, 4590, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4329}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int\frac{\sqrt{\sec(c+dx)}\left(9ab^2\sec^2(c+dx)+2b\left(6a^2+b^2\right)\sec(c+dx)+a\left(4a^2+b^2\right)\right)}{2\sqrt{a+b\sec(c+dx)}}dx\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\int\frac{\sqrt{\sec(c+dx)}\left(9ab^2\sec^2(c+dx)+2b\left(6a^2+b^2\right)\sec(c+dx)+a\left(4a^2+b^2\right)\right)}{\sqrt{a+b\sec(c+dx)}}dx\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(9ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(6a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+\right.}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}\right.$$

$$\Big\downarrow \text{4590}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{\int-\frac{9a^2b^2-\left(15a^2+4b^2\right)\sec^2(c+dx)b^2-2a\left(4a^2+b^2\right)\sec(c+dx)b}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b}+\frac{9ab\sin(c+dx)\sqrt{\sec(c\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-\left(15a^2+4b^2\right)\sec^2(c+dx)b}{\sqrt{\sec(c+dx)}\sqrt{a+}}}{2b}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-\left(15a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right.}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{}}}{}\right.\right.$$

$$\Big\downarrow \text{4596}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-2ab\left(4a^2+b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}d}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\csc(c+dx-}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx}}}{}\right.\right.$$

$$\Big\downarrow \text{4346}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\csc(c+dx-}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx}}}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\csc(c+dx-}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx}}}{}\right.\right.$$

$$\Big\downarrow \text{3286}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\csc(c+dx-}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx}}}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\csc(c+dx-}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx}}}{}\right.\right.$$

$$\Big\downarrow \text{3284}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\csc(c+dx\cdots}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx\cdots}}}{\cdots}\right.\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-ab(8a^2+11b^2)\int\frac{\sqrt{\sec(c\cdots}}{\sqrt{a+b\sec\cdots}}}{\cdots}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-ab(8a^2+11b^2)\int\frac{\sqrt{\csc(c\cdots}}{\sqrt{a+b\csc\cdots}}}{\cdots}\right.\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-ab(8a^2+11b^2)\int\frac{\sqrt{\csc(c\cdots}}{\sqrt{a+b\csc\cdots}}}{\cdots}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-ab(8a^2+11b^2)\int\frac{\sqrt{\csc(c\cdots}}{\sqrt{a+b\csc\cdots}}}{\cdots}\right.\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-ab(8a^2+11b^2)\int\frac{\sqrt{\csc(c\cdots}}{\sqrt{a+b\csc\cdots}}}{\cdots}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-ab(8a^2+11b^2)\int\frac{\sqrt{\csc(c}}{\sqrt{a+b\csc}}}{}\right.\right.$$

$\Big\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-ab(8a^2+11b^2)\int\frac{\sqrt{\csc(c}}{\sqrt{a+b\csc}}}{}\right.\right.$$

$\Big\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{ab(8a^2+11b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos}}{\sqrt{a+b\sec}}}{}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{ab(8a^2+11b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos}}{\sqrt{a+b}}}{}\right.\right.$$

$\Big\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{ab(8a^2+11b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos}{}}}{\sqrt{a+b\,s}}}{}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{ab(8a^2+11b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos}{}}}{\sqrt{a+}}}{}\right.\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{2ab(8a^2+11b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a}{}}}{d\sqrt{a+}}}{}\right.\right.$$

input
```
Int[(a + b*Sec[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b^2*Sec[c + d*x]^(3/2)*Sqrt[a + b*
Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (-1/2*((-2*a*b*(8*a^2 + 11*b^2)*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Se
c[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) - (2*b^2*(15*a^2 + 4*b^2)*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqr
t[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (18*a*b^2*EllipticE[(c + d
*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]*Sqrt[Sec[c + d*x]]))/b + (9*a*b*Sqrt[Sec[c + d*x]]*Sqrt[a + b*
Sec[c + d*x]]*Sin[c + d*x])/d)/4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 13.56 (sec) , antiderivative size = 1159, normalized size of antiderivative = 3.69

method	result	size
default	Expression too large to display	1159

input
```
int((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/((a-b)/(a+b))^(1/2)*((1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(d*x+c)
-cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(30*cos(d*x+c)^4+60*cos(d*
x+c)^3+30*cos(d*x+c)^2)+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*b^3*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(
d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(8*cos(d*x+c)^4+16*cos(d*x+c)^3
+8*cos(d*x+c)^2)+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*a^2*b*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)
),(-(a+b)/(a-b))^(1/2))*(-9*cos(d*x+c)^4-18*cos(d*x+c)^3-9*cos(d*x+c)^2)+(
1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*
b^2*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(
1/2))*(9*cos(d*x+c)^4+18*cos(d*x+c)^3+9*cos(d*x+c)^2)+(1/(a+b)*(b+a*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*EllipticF(((a-b)/
(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(8*cos(d*x+c)^4
+16*cos(d*x+c)^3+8*cos(d*x+c)^2)+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d
*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-6*cos(d*x+c)^4-12*cos(d*x+c)^3-6
*cos(d*x+c)^2)+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*a*b^2*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),
(-(a+b)/(a-b))^(1/2))*(2*cos(d*x+c)^4+4*cos(d*x+c)^3+2*cos(d*x+c)^2)+(1...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx=\left(\int \frac{\sqrt{\sec\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}\,\sec\left(dx+c\right)^{2}}{\cos\left(dx+c\right)}dx\right)b^{2}+2\left(\int \frac{\sqrt{\sec\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}\,\sec\left(dx+c\right)}{\cos\left(dx+c\right)}dx\right)ab+\left(\int \frac{\sqrt{\sec\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)}dx\right)a^{2}$$

input
```
int((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*sec(c + d*x)**2)/cos(c +
d*x),x)*b**2 + 2*int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*sec(c +
d*x))/cos(c + d*x),x)*a*b + int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x
)))/cos(c + d*x),x)*a**2
```

3.854 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 369

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \frac{b(59a^2+16b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{24d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\
&+ \frac{5a(a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{8d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\
&- \frac{(33a^2+16b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{24d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} \\
&+ \frac{b^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{12d\cos^{\frac{3}{2}}(c+dx)} \\
&+ \frac{(33a^2+16b^2)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d\sqrt{\cos(c+dx)}}\end{aligned}$$

output
```
1/24*b*(59*a^2+16*b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*
d*x+1/2*c,2^(1/2)*(a/(a+b))^(1/2))/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/
2)+5/8*a*(a^2+4*b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x
+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/
2)-1/24*(33*a^2+16*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/
2)+1/3*b^2*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)+13/12*a*b*
(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+1/24*(33*a^2+16*b^2)*
(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 32.61 (sec) , antiderivative size = 62811, normalized size of antiderivative = 170.22

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$
$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4329}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(13ab^2\sec^2(c+dx)+2b(9a^2+2b^2)\sec(c+dx)+3a(2a^2+b^2)\right)}{2\sqrt{a+b\sec(c+dx)}}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(13ab^2\sec^2(c+dx)+2b(9a^2+2b^2)\sec(c+dx)+3a(2a^2+b^2)\right)}{\sqrt{a+b\sec(c+dx)}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(13ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+2b(9a^2+2b^2)\csc\left(c+dx+\frac{\pi}{2}\right.\right.}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}\right.$$

$$\Big\downarrow \text{4590}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\int \frac{\sqrt{\sec(c+dx)}\left(13a^2b^2+(33a^2+16b^2)\sec^2(c+dx)b^2+2a(12a^2+19b^2)\sec(c+dx)b\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{2b}+\frac{13ab\sin\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\int \frac{\sqrt{\sec(c+dx)}\left(13a^2b^2+(33a^2+16b^2)\sec^2(c+dx)b^2+2a(12a^2+19b^2)\sec(c+dx)b\right)}{\sqrt{a+b\sec(c+dx)}}dx}{4b}+\frac{13ab\sin\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(13a^2b^2+(33a^2+16b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2b^2+2a(12a^2+19b^2)\csc\left(c+dx+\frac{\pi}{2}\right)b\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{4b}\right.\right.$$

$\Big\downarrow$ 4590

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{\int -\frac{-26a^2\sec(c+dx)b^3-15a\left(a^2+4b^2\right)\sec^2(c+dx)b^2+a\left(33a^2+16b^2\right)b^2}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b}+\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{se}}{d}}{4b}\right.\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{-26a^2\sec(c+dx)b^3-15a\left(a^2+4b^2\right)\sec^2(c+d}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+}}}{2b}}{4b}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{-26a^2\csc\left(c+dx+\frac{\pi}{2}\right)b^3-15a\left(a^2+4b^2\right)\csc\left(\right.}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\,cs}}}{2b}}{4b}\right.\right.$$

$\Big\downarrow$ 4596

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{ab^2\left(33a^2+16b^2\right)-26a^2b^3\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx-15}{2b}}{4b}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{ab^2\left(33a^2+16b^2\right)-26a^2b^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2}}{4b}\right.\right.$$

$\Big\downarrow$ 4346

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{ab^2(33a^2+16b^2)-26a^2b^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{4b}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{ab^2(33a^2+16b^2)-26a^2b^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{4b}\right.\right.$$

$\downarrow$ 3286

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{ab^2(33a^2+16b^2)-26a^2b^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{4b}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{ab^2(33a^2+16b^2)-26a^2b^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{4b}\right.\right.$$

$\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{ab^2\left(33a^2+16b^2\right)-26a^2b^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{}}{4b}\right.\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{b^2\left(33a^2+16b^2\right)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx-\left(b^3\right.}{}}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{b^2\left(33a^2+16b^2\right)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-}{}}{}\right.\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\frac{b^2\left(33a^2+16b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{}}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\frac{b^2\left(33a^2+16b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c\right.}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{}}{}\right.\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\frac{b^2\left(33a^2+16b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos}{}}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{}}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\frac{b^2\left(33a^2+16b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin}{}}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{}}{}\right.\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\left(b^3\left(59a^2+16b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}d\right.}{}}{}\right.\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{b^3\left(59a^2+16b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}{\sqrt{a+b\sec(c+dx)}}}{}}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{b^3\left(59a^2+16b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}{\sqrt{a+b\sec(c+dx)}}}{}}{}\right.\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{b^3\left(59a^2+16b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{\sqrt{a+b\sec(c+dx)}}}{\cdots}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{b^3\left(59a^2+16b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{\sqrt{a+b\sec(c+dx)}}}{\cdots}\right.\right.$$

input
```
Int[(a + b*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 17.53 (sec) , antiderivative size = 1339, normalized size of antiderivative = 3.63

method	result	size
default	Expression too large to display	1339

input
```
int((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/d/((a-b)/(a+b))^(1/2)*((1/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*a^3*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-
cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(30*cos(d*x+c)^5+60*cos(d*x
+c)^4+30*cos(d*x+c)^3)+(1/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*a*b^2*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot
(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(120*cos(d*x+c)^5+240*cos(d*x+
c)^4+120*cos(d*x+c)^3)+(1/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*a^3*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*
x+c)),(-(a+b)/(a-b))^(1/2))*(-33*cos(d*x+c)^5-66*cos(d*x+c)^4-33*cos(d*x+c
)^3)+(1/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*a^2*b*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a
-b))^(1/2))*(33*cos(d*x+c)^5+66*cos(d*x+c)^4+33*cos(d*x+c)^3)+(1/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2*Ellipti
cE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-16*
cos(d*x+c)^5-32*cos(d*x+c)^4-16*cos(d*x+c)^3)+(1/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^3*EllipticE(((a-b)/(a+b))^(
1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(16*cos(d*x+c)^5+32*cos
(d*x+c)^4+16*cos(d*x+c)^3)+(1/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*a^3*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-co
t(d*x+c)),(-(a+b)/(a-b))^(1/2))*(18*cos(d*x+c)^5+36*cos(d*x+c)^4+18*cos...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(5/2)/cos(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\cos(c+d\,x)^{3/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\left(\int \frac{\sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\sec(dx+c)^2}{\cos(dx+c)^2}dx\right)b^2+2\left(\int \frac{\sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\sec(dx+c)}{\cos(dx+c)^2}dx\right)ab+\left(\int \frac{\sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)a^2$$

input
```
int((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*sec(c + d*x)**2)/cos(c +
d*x)**2,x)*b**2 + 2*int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*sec(c
 + d*x))/cos(c + d*x)**2,x)*a*b + int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c
 + d*x)))/cos(c + d*x)**2,x)*a**2
```

3.855 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 249

$$\begin{aligned}\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx &= -\frac{2b(7a^2+8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\ &\quad+\frac{2(9a^2+8b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{15a^3d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} \\ &\quad-\frac{8b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^2d} \\ &\quad+\frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5ad}\end{aligned}$$

output
```
-2/15*b*(7*a^2+8*b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d
*x+1/2*c,2^(1/2)*(a/(a+b))^(1/2))/a^3/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^
(1/2)+2/15*(9*a^2+8*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(
1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^3/d/((b+a*cos(d*x+c))/(a+b)
)^(1/2)-8/15*b*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/a^2/d+2/
5*cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/a/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.32 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.37

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{2a(b+a\cos(c+dx))(-4b+3a\cos(c+dx))\sin(c+dx)+\frac{2\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(i(9a^3+9a^2b+8ab^2+8b^3)E\right.}{}}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*a*(b + a*Cos[c + d*x])*(-4*b + 3*a*Cos[c + d*x])*Sin[c + d*x] + (2*(Cos
[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(I*(9*a^3 + 9*a^2*b + 8*a*b^2 + 8*b^3)
*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]
^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(9*a^2 +
2*a*b + 8*b^2)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Se
c[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] +
 (9*a^2 + 8*b^2)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c +
d*x)/2]))/Sec[c + d*x]^(3/2))/(15*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[
c + d*x]])
```

Rubi [A] (verified)

Time = 2.14 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.13, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 4752, 3042, 4350, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$\downarrow$ 3042

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$\downarrow$ 4752

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$\downarrow$ 4350

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-2b\sec^2(c+dx)-3a\sec(c+dx)+4b}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-2b\csc\left(c+dx+\frac{\pi}{2}\right)^2-3a\csc\left(c+dx+\frac{\pi}{2}\right)+4b}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}\right)$$

$\downarrow$ 4592

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{2\int \frac{9a^2+2b\sec(c+dx)a+8}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c}}}{3a}}{5a}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{9a^2+2b\sec(c+dx)a+8b^2}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+d}}}{3a}}{5a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{9a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc}}}{3a}}{5a}\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\left(9a^2+8b^2\right)\int\frac{\sqrt{a+b\sec(c+d}}{\sqrt{\sec(c+dx)}}}{a}}{5a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\left(9a^2+8b^2\right)\int\frac{\sqrt{a+b\csc\left(c+d\right.}}{\sqrt{\csc\left(c+dx+\right.}}}{a}}{5a}\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}}{a\sqrt{\sec(c+dx)}\sqrt{a\cos}}}{5a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}}{a\sqrt{\sec(c+dx)}\sqrt{a}}}{5}\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}}{a\sqrt{\sec(c+dx)}\sqrt{a}}}{5}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}}{a\sqrt{\sec(c+dx)}\sqrt{}}}{}\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{2\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos}{}}}}{5a}\right.$$

$$\Big\downarrow \text{4345}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos}{}}}}{}}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos}{}}}}{}}{}\right.$$

$$\Big\downarrow \text{3142}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos}{}}}}{}}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos}{}}}}{}}{}\right.$$

$$\Big\downarrow \text{3140}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos\ldots}{}}}}{}}{}\right.$$

input
```
Int[Cos[c + d*x]^(5/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
 d*x])/(5*a*d*Sec[c + d*x]^(3/2)) - (-1/3*((-2*b*(7*a^2 + 8*b^2)*Sqrt[(b +
 a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c
 + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(9*a^2 + 8*b^2)*EllipticE[(c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/a + (8*b*Sqrt[a + b*Sec[c + d*x]]*S
in[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]))/(5*a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4350
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1)*(Sqrt[a +
 b*Csc[e + f*x]]/(a*d*f*n)), x] + Simp[1/(2*a*d*n)   Int[((d*Csc[e + f*x])^
(n + 1)/Sqrt[a + b*Csc[e + f*x]])*Simp[(-b)*(2*n + 1) + 2*a*(n + 1)*Csc[e +
 f*x] + b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 983 vs. $2(230) = 460$.

Time = 12.29 (sec) , antiderivative size = 984, normalized size of antiderivative = 3.95

method	result	size
default	Expression too large to display	984

input
```
int(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d/((a-b)/(a+b))^(1/2)/a^3*((9*cos(d*x+c)^2+18*cos(d*x+c)+9)*(1/(a+b)*
(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*Ellipt
icE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-9*
cos(d*x+c)^2-18*cos(d*x+c)-9)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+
c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(8*cos(d*x+c)^2+16*cos(d*x+c)+8)*(1/(
a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2
*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2
))+(-8*cos(d*x+c)^2-16*cos(d*x+c)-8)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^3*EllipticE(((a-b)/(a+b))^(1/2)*(csc
(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-9*cos(d*x+c)^2-18*cos(d*x+c)-9
)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*a^3*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^
(1/2))+(2*cos(d*x+c)^2+4*cos(d*x+c)+2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticF(((a-b)/(a+b))^(1/2)*
(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-8*cos(d*x+c)^2-16*cos(d*x+
c)-8)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*a*b^2*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(
a-b))^(1/2))+sin(d*x+c)*cos(d*x+c)*(3*cos(d*x+c)^2+3*cos(d*x+c)+9)*((a-b)/
(a+b))^(1/2)*a^3+(-cos(d*x+c)^2-cos(d*x+c)+9)*sin(d*x+c)*((a-b)/(a+b))^...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.83

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{2\left(3\left(3\,a^3\cos\left(dx+c\right)-4\,a^2b\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-4\sqrt{\frac{1}{2}}(-3i\,a^2b-4i\,b^3)\sqrt{a}\mathrm{weier}\right.}{}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/45*(3*(3*a^3*cos(d*x + c) - 4*a^2*b)*sqrt((a*cos(d*x + c) + b)/cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c) - 4*sqrt(1/2)*(-3*I*a^2*b - 4*I*b^3)*
sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^
3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) - 4*sqrt(1/2)
*(3*I*a^2*b + 4*I*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^
2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c)
+ 2*b)/a) - 3*sqrt(1/2)*(-9*I*a^3 - 8*I*a*b^2)*sqrt(a)*weierstrassZeta(-4/
3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/
3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) +
 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(1/2)*(9*I*a^3 + 8*I*a*b^2)*sqrt(a)
*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, wei
erstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3
*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/(a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^{5/2}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + b/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^{2}}{\sec\left(dx+c\right)b+a}dx$$

input
```
int(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x)**2)/(sec(c +
 d*x)*b + a),x)
```

3.856 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 195

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2(a^2+2b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{4b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3a^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad}$$

output
```
2/3*(a^2+2*b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2
*c,2^(1/2)*(a/(a+b))^(1/2))/a^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-
4/3*b*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2
))*(a+b*sec(d*x+c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)+2/3*cos(d*x
+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/a/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.46 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.36

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sqrt{\cos(c+dx)}\left(-2ib(a+b)\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(i\text{arcsinh}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sqrt{\sec(c+dx)}\sqrt{1+\text{s}}\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*((-2*I)*b*(a + b)*Sqrt[(b + a*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]] - I*a*(a - 2*b)*Sqrt[(b + a
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh[Tan[(c +
d*x)/2]], (-a + b)/(a + b)]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]] + a^
2*Sin[c + d*x] - 2*a*b*Tan[(c + d*x)/2] - 2*b^2*Sec[c + d*x]*Tan[(c + d*x)
/2] + a*b*Tan[c + d*x]))/(3*a^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.13, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3042, 4752, 3042, 4350, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4350}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{2b-a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{2b-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}\right)$$

$$\Big\downarrow \text{ 4523}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{(a^2+2b^2)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}}{3a}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{(a^2+2b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}\right)$$

$$\Big\downarrow \text{ 4343}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{(a^2+2b^2)\int\ldots}{\ldots}}{3a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{(a^2+2b^2)\ldots}{\ldots}}{3a}\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^2+2b^2)\int\ldots}{\ldots}}{3a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^2+2b^2)\ldots}{\ldots}}{3a}\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^2+2b^2)\int\frac{\sqrt{\csc}}{\sqrt{a+b}}}{a}}{3a}\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^2+2b^2)\sqrt{\sec(c+}}{}}{3a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^2+2b^2)\sqrt{\sec(c+}}{}}{3a}\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^2+2b^2)\sqrt{\sec(c+}}{}}{3a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^2+2b^2)\sqrt{\sec(c+}}{}}{3a}\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2(a^2+2b^2)\sqrt{\sec(c}{}}{3a}\right.$$

input
```
Int[Cos[c + d*x]^(3/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/3*((-2*(a^2 + 2*b^2)*Sqrt[(b + a
*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c +
 d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (4*b*EllipticE[(c + d*x)/2, (2*a)
/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]*Sqrt[Sec[c + d*x]]))/a + (2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a*
d*Sqrt[Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4350
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1)*(Sqrt[a +
 b*Csc[e + f*x]]/(a*d*f*n)), x] + Simp[1/(2*a*d*n)   Int[((d*Csc[e + f*x])^
(n + 1)/Sqrt[a + b*Csc[e + f*x]])*Simp[(-b)*(2*n + 1) + 2*a*(n + 1)*Csc[e +
 f*x] + b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(182) = 364$.

Time = 9.77 (sec) , antiderivative size = 594, normalized size of antiderivative = 3.05

method	result
default	$\frac{2\left(\left(-2\cos(dx+c)^2-4\cos(dx+c)-2\right)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,ab\,\text{EllipticE}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)+\right.}{}$

input
```
int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/((a-b)/(a+b))^(1/2)/a^2*((-2*cos(d*x+c)^2-4*cos(d*x+c)-2)*(1/(a+b)*(
b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b*Ellipti
cE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(2*co
s(d*x+c)^2+4*cos(d*x+c)+2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*b^2*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-co
t(d*x+c)),(-(a+b)/(a-b))^(1/2))+(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b+
a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*EllipticF
(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(2*cos(
d*x+c)^2+4*cos(d*x+c)+2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*a*b*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(
d*x+c)),(-(a+b)/(a-b))^(1/2))+sin(d*x+c)*cos(d*x+c)*(1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)*a^2+(1-cos(d*x+c))*sin(d*x+c)*((a-b)/(a+b))^(1/2)*a*b-2*((a-b
)/(a+b))^(1/2)*b^2*sin(d*x+c))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/(co
s(d*x+c)^2*a+a*cos(d*x+c)+cos(d*x+c)*b+b)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.14

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\left(3\,a^2\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-6i\,\sqrt{\frac{1}{2}}a^{\frac{3}{2}}b\,\text{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\text{we}\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/9*(3*a^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c) - 6*I*sqrt(1/2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^
2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^
2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c)
+ 2*b)/a)) + 6*I*sqrt(1/2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/
a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/
a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c
) + 2*b)/a)) - sqrt(1/2)*(3*I*a^2 + 4*I*b^2)*sqrt(a)*weierstrassPInverse(-
4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c)
 + 3*I*a*sin(d*x + c) + 2*b)/a) - sqrt(1/2)*(-3*I*a^2 - 4*I*b^2)*sqrt(a)*w
eierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1
/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a))/(a^3*d)
```

Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)**(3/2)/sqrt(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos(dx+c)^{\frac{3}{2}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\cos\left(c+d\,x\right)^{3/2}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)}{\sec\left(dx+c\right)b+a}dx$$

input
```
int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x))/(sec(c + d*
x)*b + a),x)
```

3.857 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 142

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

output
```
-2*b*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*
(a/(a+b))^(1/2))/a/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*cos(d*x+c)^
(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c
))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.06 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.52

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}\left(i(a+b)\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(i\text{arcsinh}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\right.\right.}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*(I*(a + b)
*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSi
nh[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - I*a*Sqrt[(b + a*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b
)/(a + b)] + Sqrt[(1 + Cos[c + d*x])^(-1)]*(b + a*Cos[c + d*x])*Tan[(c + d
*x)/2]))/(a*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.15, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4752, 3042, 4349, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4349}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{b\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{b\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}\right)$$

$$\Big\downarrow \text{4343}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{b\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{b\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}\right)$$

$$\Big\downarrow \text{3134}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}\right)$$

$$\Big\downarrow \text{3132}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}\right)$$

$$\Big\downarrow \text{4345}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{}{\sqrt{b-\cdots}}}{a\sqrt{a+b\sec(c+dx)}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{}{\sqrt{b-\cdots}}}{a\sqrt{a+b\sec(c+dx)}}\right.$$

$$\Big\downarrow \text{3142}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cdots}{}}}}{a\sqrt{a+b\sec(c+dx)}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{}{\sqrt{\frac{b}{a+b}+\frac{a\cdots}{}}}}{a\sqrt{a+b\sec(c+dx)}}\right.$$

$$\Big\downarrow \text{3140}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}}{ad\sqrt{a+b\sec(c+dx)}}\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*b*Sqrt[(b + a*Cos[c + d*x])/(a
+ b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[
a + b*Sec[c + d*x]]) + (2*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b
*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]
))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4349
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Cs
c[e + f*x]], x], x] - Simp[b/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Cs
c[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(139) = 278$.

Time = 7.20 (sec) , antiderivative size = 437, normalized size of antiderivative = 3.08

method	result
default	$-\frac{2\left(\left(-\cos(dx+c)^2-2\cos(dx+c)-1\right)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\sqrt{\frac{1}{1+\cos(dx+c)}}\, a\, \mathrm{EllipticE}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right) \cdots}{\cdots}$
risch	$-\frac{i\left(e^{2i(dx+c)}a+2b\,e^{i(dx+c)}+a\right)\sqrt{2}\sqrt{\left(e^{2i(dx+c)}+1\right)e^{-i(dx+c)}}}{ad\sqrt{\frac{e^{2i(dx+c)}a+2b\,e^{i(dx+c)}+a}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}+1\right)}-\frac{i\left(-\frac{2\left(e^{2i(dx+c)}a+2b\,e^{i(dx+c)}+a\right)}{a\sqrt{e^{i(dx+c)}\left(e^{2i(dx+c)}a+2b\,e^{i(dx+c)}+a\right)}}+\frac{2\left(b+\sqrt{-a^2+b^2}\right)\sqrt{\cdots}}{\cdots}\cdots\right.}{\cdots}$

input
```
int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/a/((a-b)/(a+b))^(1/2)*((-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*EllipticE(((a-
b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(cos(d*x+c)^
2+2*cos(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*b*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(
-(a+b)/(a-b))^(1/2))+(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*EllipticF(((a-b)/(a+b)
)^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))-((a-b)/(a+b))^(1/2)*
a*cos(d*x+c)*sin(d*x+c)-((a-b)/(a+b))^(1/2)*b*sin(d*x+c))*cos(d*x+c)^(1/2)
*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+cos(d*x+c)*b+b)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.50

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2\left(-2i\sqrt{\frac{1}{2}}\sqrt{a}b\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2}, \frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3}, \frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)+2i\sqrt{\frac{1}{2}}\sqrt{a}b\cdots\right.}{\cdots}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(-2*I*sqrt(1/2)*sqrt(a)*b*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^
2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c)
+ 2*b)/a) + 2*I*sqrt(1/2)*sqrt(a)*b*weierstrassPInverse(-4/3*(3*a^2 - 4*b^
2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x
+ c) + 2*b)/a) - 3*I*sqrt(1/2)*a^(3/2)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2
)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2
)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x +
 c) + 2*b)/a)) + 3*I*sqrt(1/2)*a^(3/2)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2
)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2
)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x +
 c) + 2*b)/a)))/(a^2*d)
```

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\cos{(c+dx)}}}{\sqrt{a+b\sec{(c+dx)}}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(cos(c + d*x))/sqrt(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec (dx+c)\,b+a}\,\sqrt{\cos (dx+c)}}{\sec (dx+c)\,b+a}dx$$

input
```
int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x)))/(sec(c + d*x)*b + a),x)
```

3.858 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 67

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

output
```
2*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(a/
(a+b))^(1/2))/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.52

$$\begin{aligned}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx \\ = -\frac{2i\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{-a+b}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{\frac{1}{1+\cos(c+dx)}}\sqrt{a+b\sec(c+dx)}}\end{aligned}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
((-2*I)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[
I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt
[(1 + Cos[c + d*x])^(-1)]*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4752, 3042, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4345}$$

$$\frac{\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3142}$$

$$\frac{\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
(2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)
])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(65) = 130$.

Time = 4.86 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.01

method	result	size
default	$\frac{2\sqrt{a+b\sec(dx+c)}\,\operatorname{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\,\sqrt{\cos(dx+c)}}{d\sqrt{\frac{a-b}{a+b}}\,(b+a\cos(dx+c))\sqrt{\frac{1}{1+\cos(dx+c)}}}$	135

input
```
int(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/((a-b)/(a+b))^(1/2)*(a+b*sec(d*x+c))^(1/2)*EllipticF(((a-b)/(a+b))^(1/
2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))/(1/(1+cos(d*x+c))
)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.19

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2\left(i\sqrt{\frac{1}{2}}\sqrt{a}\text{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)-i\sqrt{\frac{1}{2}}\sqrt{a}\text{weiers}\right.}{ad}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-2*(I*sqrt(1/2)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27
*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/
a) - I*sqrt(1/2)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/2
7*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)
/a))/(a*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\sec{(c+dx)}}\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*sec(c + d*x))*sqrt(cos(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec{(dx+c)}+a}\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec{(dx+c)}+a}\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\cos{(c+d\,x)}}\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)\sec(dx+c)\,b+\cos(dx+c)\,a}dx$$

input
```
int(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x)))/(cos(c + d*x)*sec(c + d*
x)*b + cos(c + d*x)*a),x)
```

3.859 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 68

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx=\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

output
```
2*((b+a*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(
a/(a+b))^(1/2))/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 28.97 (sec) , antiderivative size = 14986, normalized size of antiderivative = 220.38

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx=\text{Result too large to show}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output `Result too large to show`

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4752, 3042, 4346, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4346}$$

$$\frac{\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3286}$$

$$\frac{\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3284}$$

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

input

```
Int[1/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output

```
(2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a
+ b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3284

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.54 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.87

method	result
default	$-\frac{2\left(\operatorname{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)-2\operatorname{EllipticPi}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\frac{a+b}{a-b},\frac{i}{\sqrt{\frac{a-b}{a+b}}}\right)\right)\sqrt{a+b\,\mathrm{se}}}{d\sqrt{\frac{a-b}{a+b}}\,(b+a\cos(dx+c))\sqrt{\frac{1}{1+\cos(dx+c)}}}$

input
```
int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/((a-b)/(a+b))^(1/2)*(EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*
x+c)),(-(a+b)/(a-b))^(1/2))-2*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-c
ot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2)))*(a+b*sec(d*x+c))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+
c))/(1/(1+cos(d*x+c)))^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{a+b\sec{(c+dx)}}\cos^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*sec(c + d*x))*cos(c + d*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\sec{(dx+c)}+a}\cos{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)^{3/2}\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^2\sec(dx+c)\,b+\cos(dx+c)^2\,a}\,dx$$

input
```
int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x)))/(cos(c + d*x)**2*sec(c +
 d*x)*b + cos(c + d*x)**2*a),x)
```

3.860 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 246

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{bd\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd\sqrt{\cos(c+dx)}}$$

output
```
((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(a/(a
+b))^(1/2))/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-a*((b+a*cos(d*x+c))/
(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))/b/d/
cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-cos(d*x+c)^(1/2)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/b/d/((b+a*cos(
d*x+c))/(a+b))^(1/2)+(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/b/d/cos(d*x+c)^(1/2
)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 30.69 (sec) , antiderivative size = 21698, normalized size of antiderivative = 88.20

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 2.55 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.14, number of steps used = 25, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4752, 3042, 4347, 25, 3042, 4597, 3042, 4346, 3042, 3286, 3042, 3284, 4349, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4347}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int -\frac{a\sec^2(c+dx)+a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{\int \frac{a\sec^2(c+dx)+a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)^2+a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}\right.$$

$$\Big\downarrow \text{ 4597}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+a\int \frac{}{\sqrt{\sec(c+d}}}{2b}\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}\right.$$

$\downarrow$ 4346

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{}\right.$$

$\downarrow$ 3286

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{}\right.$$

$\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}d\right.$$

$\downarrow$ 4349

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{b\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+d}}}{a}\right.}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{b\int\frac{\sqrt{\csc(c}}{\sqrt{a+b\csc}}}{}\right.}{}\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}\right.}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}\right.}{}\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}d}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.}{}\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\right.}{}\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\right.}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\right.}{}\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\right.}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\right.}{}\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\right.}{d\sqrt{a+b\sec(c+dx)}}}{}\right.$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*((2*a*Sqrt[(b + a*Cos[c + d*x]
)/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(
d*Sqrt[a + b*Sec[c + d*x]]) + a*((-2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*
EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*
Sec[c + d*x]]) + (2*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c
 + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]])))/b
+ (Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(b*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4347
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*(S
qrt[a + b*Csc[e + f*x]]/(b*f*(2*n - 3))), x] + Simp[d^3/(b*(2*n - 3))   Int
[((d*Csc[e + f*x])^(n - 3)/Sqrt[a + b*Csc[e + f*x]])*Simp[2*a*(n - 3) + b*(
2*n - 5)*Csc[e + f*x] - 2*a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

rule 4349
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Cs
c[e + f*x]], x], x] - Simp[b/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Cs
c[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4597
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[C/d^
2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[A
Int[1/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.65 (sec) , antiderivative size = 589, normalized size of antiderivative = 2.39

method	result
default	$\frac{\left(\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\sqrt{\frac{1}{1+\cos(dx+c)}}\,a\,\text{EllipticPi}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\frac{a+b}{a-b},\frac{i}{\sqrt{\frac{a-b}{a+b}}}\right)\left(-2\cos(dx+c)^{3}-4\cos(dx+c)^{2}\right.\right.}{}$

input
```
int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/((a-b)/(a+b))^(1/2)/b*((1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*a*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot
(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(-2*cos(d*x+c)^3-4*cos(d*x+c)^
2-2*cos(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*a*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(
a+b)/(a-b))^(1/2))*(-cos(d*x+c)^3-2*cos(d*x+c)^2-cos(d*x+c))+(1/(a+b)*(b+a
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b*EllipticE(((
a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(cos(d*x+c
)^3+2*cos(d*x+c)^2+cos(d*x+c))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-
cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(2*cos(d*x+c)^3+4*cos(d*x+c)^2+2*cos(d*x
+c))+((a-b)/(a+b))^(1/2)*a*cos(d*x+c)*sin(d*x+c)+((a-b)/(a+b))^(1/2)*b*sin
(d*x+c))*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x
+c)+cos(d*x+c)*b+b)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{{\cos\left(c+d\,x\right)}^{5/2}\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)b+\cos\left(dx+c\right)^{3}a}dx$$

input
```
int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x)))/(cos(c + d*x)**3*sec(c +
 d*x)*b + cos(c + d*x)**3*a),x)
```

3.861 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 312

$$
\begin{aligned}
\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx
&= -\frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\
&\quad+\frac{(3a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\
&\quad+\frac{3a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{4b^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} \\
&\quad+\frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)}-\frac{3a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4b^2d\sqrt{\cos(c+dx)}}
\end{aligned}
$$

output
```
-1/4*a*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2
)*(a/(a+b))^(1/2))/b/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+1/4*(3*a^2+
4*b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1
/2)*(a/(a+b))^(1/2))/b^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+3/4*a*c
os(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b
*sec(d*x+c))^(1/2)/b^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)+1/2*(a+b*sec(d*x+c
))^(1/2)*sin(d*x+c)/b/d/cos(d*x+c)^(3/2)-3/4*a*(a+b*sec(d*x+c))^(1/2)*sin(
d*x+c)/b^2/d/cos(d*x+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 32.49 (sec) , antiderivative size = 51912, normalized size of antiderivative = 166.38

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/(Cos[c + d*x]^(7/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 3.32 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.07, number of steps used = 27, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 1.080$, Rules used = {3042, 4752, 3042, 4347, 3042, 4590, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4347}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sec(c+dx)}(-3a\sec^2(c+dx)+2b\sec(c+dx)+a)}{\sqrt{a+b\sec(c+dx)}}dx}{4b}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(}{2bd}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(-3a\csc\left(c+dx+\frac{\pi}{2}\right)^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{4b}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{}}{2bd}\right.$$

$$\Big\downarrow \text{4590}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{3a^2+2b\sec(c+dx)a+\left(3a^2+4b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{4b}+\frac{\sin(c+}{}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{3a^2+2b\sec(c+dx)a+\left(3a^2+4b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{4b}+\frac{\sin(c+}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{3a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(3a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{4b}+\right.$$

$\downarrow$ 4596

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(3a^2+4b^2)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\int \frac{3a^2+2b\sec(c+dx)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c}{bd}}{4b}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(3a^2+4b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{3a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{4b}\right.$$

$\downarrow$ 4346

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\int \frac{3a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-3a\,\mathrm{si}}{4b}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\int \frac{3a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+d\right.}}}{2b}}{4b}\right.$$

$\downarrow$ 3286

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\int\frac{3a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-}{4b}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\int\frac{3a^2+2b\csc\left(c+dx+\right.}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc}}}{2b}}{4b}\right.$$

$\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int\frac{3a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}}{4b}\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{-ab\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+3a\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+\right.}{d\sqrt{a+b\sec(c+dx)}}}{2b}}{4b}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{-ab\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+3a\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}}{d\sqrt{a+b\sec(c+dx)}}}{2b}}{4b}\right.$$

$\Big\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{-ab\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+\cdots}{a+b}}}{d\sqrt{a+b\sec(c+\cdots}}}{2b}}{4b}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{-ab\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+d\cdots}{a+b}}}{d\sqrt{a+b\sec\cdots}}}{2b}}{4b}\right.$$

$\Big\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{-ab\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+d\cdots}{a+b}}}{d\sqrt{a+b\sec\cdots}}}{2b}}{4b}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{-ab\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos\cdots}{}}}{d\sqrt{a+\cdots}}}{2b}}{4b}\right.$$

$\Big\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{-ab\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{6a\sqrt{a+b\sec\cdots}}{d\sqrt{\sec(\cdots}}}{2b}}{4b}\right.$$

$$\Big\downarrow \text{4345}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{-\frac{ab\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+\right.}{d\sqrt{a+b\sec(c+dx)}}}{2b}}{4b}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{-\frac{ab\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\right.}{d\sqrt{a+b\sec(c+dx)}}}{2b}}{4b}\right.$$

$$\Big\downarrow \text{3142}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{-\frac{ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\right.}{d\sqrt{a+b\sec(c+dx)}}}{2b}}{4b}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{-\frac{ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{Elliptic}}{d\sqrt{a+b\sec(c+dx)}}}{2b}}{4b}\right.$$

$$\Big\downarrow \text{3140}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-\frac{2ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}\right.}{d\sqrt{a+b\sec(c+dx)}}}{2b}}{4b}\right.$$

input
```
Int[1/(Cos[c + d*x]^(7/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[
c + d*x]]*Sin[c + d*x])/(2*b*d) + (((-2*a*b*Sqrt[(b + a*Cos[c + d*x])/(a +
 b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a +
 b*Sec[c + d*x]]) + (2*(3*a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*
EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a +
b*Sec[c + d*x]]) + (6*a*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*S
ec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(
2*b) - (3*a*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(b*d
))/(4*b))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4347
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*(S
qrt[a + b*Csc[e + f*x]]/(b*f*(2*n - 3))), x] + Simp[d^3/(b*(2*n - 3))   Int
[((d*Csc[e + f*x])^(n - 3)/Sqrt[a + b*Csc[e + f*x]])*Simp[2*a*(n - 3) + b*(
2*n - 5)*Csc[e + f*x] - 2*a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 15.63 (sec) , antiderivative size = 1033, normalized size of antiderivative = 3.31

method	result	size
default	Expression too large to display	1033

input
```
int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/((a-b)/(a+b))^(1/2)/b^2*((1/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(d*x
+c)-cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(-6*cos(d*x+c)^4-12*cos
(d*x+c)^3-6*cos(d*x+c)^2)+(1/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*b^2*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-co
t(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(-8*cos(d*x+c)^4-16*cos(d*x+c
)^3-8*cos(d*x+c)^2)+(1/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*a^2*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c
)),(-(a+b)/(a-b))^(1/2))*(-3*cos(d*x+c)^4-6*cos(d*x+c)^3-3*cos(d*x+c)^2)+(
1/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*
b*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/
2))*(3*cos(d*x+c)^4+6*cos(d*x+c)^3+3*cos(d*x+c)^2)+(1/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*EllipticF(((a-b)/(a+
b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(6*cos(d*x+c)^4+12
*cos(d*x+c)^3+6*cos(d*x+c)^2)+(1/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*a*b*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)
-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-2*cos(d*x+c)^4-4*cos(d*x+c)^3-2*cos(d
*x+c)^2)+(1/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*b^2*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/
(a-b))^(1/2))*(4*cos(d*x+c)^4+8*cos(d*x+c)^3+4*cos(d*x+c)^2)+3*((a-b)/(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)b+\cos\left(dx+c\right)^{4}a}dx$$

input
```
int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x)))/(cos(c + d*x)**4*sec(c +
 d*x)*b + cos(c + d*x)**4*a),x)
```

3.862 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 360

$$\begin{aligned}
\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx =
&-\frac{8b(a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{5a^4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\
&+\frac{2(3a^4+8a^2b^2-16b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{5a^4(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} \\
&+\frac{2b^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} \\
&-\frac{2b(3a^2-8b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5a^3(a^2-b^2)\,d} \\
&+\frac{2(a^2-6b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5a^2(a^2-b^2)\,d}
\end{aligned}$$

output
```
-8/5*b*(a^2+4*b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+
1/2*c,2^(1/2)*(a/(a+b))^(1/2))/a^4/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/
2)+2/5*(3*a^4+8*a^2*b^2-16*b^4)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^4/(a^2-b^2)/d/((b+a*
cos(d*x+c))/(a+b))^(1/2)+2*b^2*cos(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(
a+b*sec(d*x+c))^(1/2)-2/5*b*(3*a^2-8*b^2)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c)
)^(1/2)*sin(d*x+c)/a^3/(a^2-b^2)/d+2/5*(a^2-6*b^2)*cos(d*x+c)^(3/2)*(a+b*s
ec(d*x+c))^(1/2)*sin(d*x+c)/a^2/(a^2-b^2)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.96 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.16

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{(b+a\cos(c+dx))\left(a\sec^{\frac{3}{2}}(c+dx)\left(10b^4\sin(c+dx)+6b(-a^2+b^2)(b+a\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
((b + a*Cos[c + d*x])*(a*Sec[c + d*x]^(3/2)*(10*b^4*Sin[c + d*x] + 6*b*(-a
^2 + b^2)*(b + a*Cos[c + d*x])*Sin[c + d*x] + a*(a^2 - b^2)*(b + a*Cos[c +
 d*x])*Sin[2*(c + d*x)]) + 2*(a^2 + 4*b^2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x
])^(3/2)*(I*(3*a^3 + 3*a^2*b - 4*a*b^2 - 4*b^3)*EllipticE[I*ArcSinh[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x
])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(3*a^2 - a*b - 4*b^2)*EllipticF[I*Ar
cSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a
*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (3*a^2 - 4*b^2)*(b + a*Cos[c
 + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2])))/(5*a^4*(a^2 - b^2)
*d*Cos[c + d*x]^(3/2)*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 2.98 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.06, number of steps used = 24, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.960, Rules used = {3042, 4752, 3042, 4334, 27, 3042, 4592, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4334}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{a^2-b\sec(c+dx)a-6b^2+4b^2\sec^2(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2-b\sec(c+dx)a-6b^2+4b^2\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a-6b^2+4b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(\right.}\right.$$

$\downarrow$ 4592

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{2\int \frac{-2b\left(a^2-6b^2\right)\sec^2(c+dx)-a\left(3a^2+2b^2\right)\sec(c+dx)+3b\left(3a^2-8b^2\right)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}}{5a}}{a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-2b\left(a^2-6b^2\right)\sec^2(c+dx)-a\left(3a^2+2b^2\right)\sec(c+dx)+3b\left(3a^2-8b^2\right)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}d}{5a}}{a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-2b\left(a^2-6b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-a\left(3a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+3b\left(3a\right.}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}}{5a}}{a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4592

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{2\int \frac{3\left(3a^4+8b^2a^2-b\left(a^2+4\right.\right.}{2\sqrt{\sec(c+dx)}\sqrt{}}}{3a}}{5a}}{a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{3a^4+8b^2a^2-b(a^2+4b^2)\ldots}{\sqrt{\sec(c+dx)}\sqrt{a+\ldots}}}{a}}{5a}}{a(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{3a^4+8b^2a^2-b(a^2+4b^2)\ldots}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a\ldots}}}{a\ldots}}{5a}}{a(a^2-b^2)}\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{(3a^4+8a^2b^2-16b^4)\int\frac{\sqrt{a\ldots}}{\ldots}}{a}}{5a}}{a(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{(3a^4+8a^2b^2-16b^4)\int\frac{\sqrt{a\ldots}}{\sqrt{\ldots}}}{a}}{5\ldots}}{a(a^2-b^2)}\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{(3a^4+8a^2b^2-16b^4)\sqrt{a+b}}{a\sqrt{\sec(c+dx}}}{}}{a(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{(3a^4+8a^2b^2-16b^4)\sqrt{a+b}}{a\sqrt{\sec(c+}}}{}}{a(a^2-b^2)}\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{(3a^4+8a^2b^2-16b^4)\sqrt{a+b}}{a\sqrt{\sec(c+}}}{}}{a(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{(3a^4+8a^2b^2-16b^4)\sqrt{a+b}}{a\sqrt{\sec(c}}}{}}{a(a^2-b^2)}\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{2(3a^4+8a^2b^2-16b^4)\sqrt{a+}}{ad\sqrt{\sec(c+dx)}}}{}}{a(a^2-b^2)}\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{2(3a^4+8a^2b^2-16b^4)\sqrt{a+}}{ad\sqrt{\sec(c+dx)}}}{}}{a(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(3a^4+8a^2b^2-16b^4\right)\sqrt{a+\cdots}}{ad\sqrt{\sec(c+dx)}\cdots}\cdots}{\cdots}}{\cdots}}{a\left(a^2-b\cdots\right.}\cdots\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(3a^4+8a^2b^2-16b^4\right)\sqrt{a+\cdots}}{ad\sqrt{\sec(c+dx)}\cdots}\cdots}{\cdots}}{\cdots}}{a\left(a^2-b\cdots\right.}\cdots\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(3a^4+8a^2b^2-16b^4\right)\sqrt{a+\cdots}}{ad\sqrt{\sec(c+dx)}\cdots}\cdots}{\cdots}}{\cdots}}{a\left(a^2-\cdots\right.}\cdots\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2b^2\sin(c+dx)}{ad\,(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}+\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\right.$$

input
```
Int[Cos[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)
*d*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) + ((2*(a^2 - 6*b^2)*Sqrt[a
 + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) - (-(((-8*b*(a
^4 + 3*a^2*b^2 - 4*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c +
d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]])
+ (2*(3*a^4 + 8*a^2*b^2 - 16*b^4)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sq
rt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c
 + d*x]]))/a) + (2*b*(3*a^2 - 8*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x]
)/(a*d*Sqrt[Sec[c + d*x]]))/(5*a))/(a*(a^2 - b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1042 vs. $2(337) = 674$.

Time = 15.26 (sec) , antiderivative size = 1043, normalized size of antiderivative = 2.90

method	result	size
default	Expression too large to display	1043

input
```
int(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/d/(a+b)/((a-b)/(a+b))^(1/2)/a^4*((-3*cos(d*x+c)^2-6*cos(d*x+c)-3)*(1/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4*
EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2)
)+(-8*cos(d*x+c)^2-16*cos(d*x+c)-8)*(1/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2*EllipticE(((a-b)/(a+b))^(1/2)*(
csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(16*cos(d*x+c)^2+32*cos(d*x+c
)+16)*(1/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*b^4*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-
b))^(1/2))+(3*cos(d*x+c)^2+6*cos(d*x+c)+3)*(1/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4*EllipticF(((a-b)/(a+b))^(1/2
)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(4*cos(d*x+c)^2+8*cos(d*x+
c)+4)*(1/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*a^3*b*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(
a-b))^(1/2))+(12*cos(d*x+c)^2+24*cos(d*x+c)+12)*(1/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2*EllipticF(((a-b)/(a
+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(16*cos(d*x+c)^2+
32*cos(d*x+c)+16)*(1/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*a*b^3*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c
)),(-(a+b)/(a-b))^(1/2))+sin(d*x+c)*cos(d*x+c)*(-cos(d*x+c)^2-cos(d*x+c)-3
)*((a-b)/(a+b))^(1/2)*a^4+(-cos(d*x+c)^3+cos(d*x+c)^2-cos(d*x+c)-3)*sin...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 684, normalized size of antiderivative = 1.90

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-2/15*(3*(3*a^4*b^2 - 8*a^2*b^4 - (a^6 - a^4*b^2)*cos(d*x + c)^2 + 2*(a^5*
b - a^3*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c) - sqrt(1/2)*(9*I*a^4*b^2 + 28*I*a^2*b^4 - 32*I*b^
6 + (9*I*a^5*b + 28*I*a^3*b^3 - 32*I*a*b^5)*cos(d*x + c))*sqrt(a)*weierstr
assPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a
*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) - sqrt(1/2)*(-9*I*a^4*b^2 - 2
8*I*a^2*b^4 + 32*I*b^6 + (-9*I*a^5*b - 28*I*a^3*b^3 + 32*I*a*b^5)*cos(d*x
+ c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b
- 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqr
t(1/2)*(3*I*a^5*b + 8*I*a^3*b^3 - 16*I*a*b^5 + (3*I*a^6 + 8*I*a^4*b^2 - 16
*I*a^2*b^4)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2
, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2
, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) +
 2*b)/a)) - 3*sqrt(1/2)*(-3*I*a^5*b - 8*I*a^3*b^3 + 16*I*a*b^5 + (-3*I*a^6
 - 8*I*a^4*b^2 + 16*I*a^2*b^4)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*
(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*
(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3
*I*a*sin(d*x + c) + 2*b)/a)))/((a^8 - a^6*b^2)*d*cos(d*x + c) + (a^7*b - a
^5*b^3)*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)^{\frac{5}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)^{\frac{5}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+d\,x)^{5/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2}{\sec(dx+c)^2\,b^2+2\sec(dx+c)\,ab+a^2}dx$$

input
```
int(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x)**2)/(sec(c +
 d*x)**2*b**2 + 2*sec(c + d*x)*a*b + a**2),x)
```

3.863 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 289

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2(a^2+8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2b(5a^2-8b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3a^3(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{2b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{a(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2-4b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3a^2(a^2-b^2)\,d}$$

output
```
2/3*(a^2+8*b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2
*c,2^(1/2)*(a/(a+b))^(1/2))/a^3/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-
2/3*b*(5*a^2-8*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*
(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^3/(a^2-b^2)/d/((b+a*cos(d*x+c))/
(a+b))^(1/2)+2*b^2*cos(d*x+c)^(1/2)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+
c))^(1/2)+2/3*(a^2-4*b^2)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+
c)/a^2/(a^2-b^2)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.22 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.32

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx =$$
$$-\frac{2(b+a\cos(c+dx))\left(a(3b^3-(a^2-b^2)(b+a\cos(c+dx)))\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)-\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*(b + a*Cos[c + d*x])*(a*(3*b^3 - (a^2 - b^2)*(b + a*Cos[c + d*x]))*Sec
[c + d*x]^(3/2)*Sin[c + d*x] - (Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(I*
b*(-5*a^3 - 5*a^2*b + 8*a*b^2 + 8*b^3)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2
]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c
 + d*x)/2]^2)/(a + b)] - I*a*(a^3 - 5*a^2*b + 2*a*b^2 + 8*b^3)*EllipticF[I
*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b
+ a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + b*(-5*a^2 + 8*b^2)*(b + a
*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2])))/(3*a^3*(a^2
- b^2)*d*Cos[c + d*x]^(3/2)*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 2.34 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.09, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 4752, 3042, 4334, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4334}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{a^2-b\sec(c+dx)a-4b^2+2b^2\sec^2(c+d}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}}{a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2-b\sec(c+dx)a-4b^2+2b^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+d}}\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a-4b^2+2b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec}}\right.$$

$$\Big\downarrow \text{ 4592}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{2\int \frac{b\left(5a^2-8b^2\right)-a\left(a^2+2b^2\right)\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{a\left(a^2-b^2\right)}+\frac{}{ad\left(a^2-b^2\right)\sqrt{}}\right.$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{b(5a^2-8b^2)-a(a^2+2b^2)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{a(a^2-b^2)}+\frac{2\cdots}{ad(a^2-b^2)\sqrt{\sec\cdots}}\cdots\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{b(5a^2-8b^2)-a(a^2+2b^2)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}}{a(a^2-b^2)}+\frac{\cdots}{ad(a^2-b^2)\cdots}\cdots\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{b(5a^2-8b^2)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{(a^4+7a^2b^2-8b^4)\int\frac{\sqrt{\sec(c+d\cdots}}{\sqrt{a+b\sec(c\cdots}}\cdots}{a}}{3a}}{a(a^2-b^2)}\cdots\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{b(5a^2-8b^2)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{(a^4+7a^2b^2-8b^4)\int\frac{\sqrt{\csc\cdots}}{\sqrt{a+b\,c\cdots}}\cdots}{a}}{3a}}{a(a^2-b^2)}\cdots\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{b(5a^2-8b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{(a^4+7a^2b^2-8b^4\cdots}{\cdots}}{3a}}{a(a^2-b^2)}\cdots\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{b(5a^2-8b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{(a^4+7a^2b^2-}{}}{3a}}{a(a^2-b^2)}\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{b(5a^2-8b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^4+7a^2b^2-}{}}{3a}}{a(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{b(5a^2-8b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^4+7a}{}}{3a}}{a(a^2-b^2)}\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b(5a^2-8b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^4+7a^2b^2-8b^4)}{}}{3a}}{a(a^2-b^2)}\right.$$

$\Big\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b(5a^2-8b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^4+7a^2b^2-8b^4)}{}}{3a}}{a(a^2-b^2)}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b(5a^2-8b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^4+7a^2b^2-8b^4)}{}}{3a}}{a(a^2-b^2)}\right.$$

$\Big\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b(5a^2-8b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^4+7a^2b^2-8b^4)}{}}{3a}}{a(a^2-b^2)}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b(5a^2-8b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^4+7a^2b^2-8b^4)}{}}{3a}}{a(a^2-b^2)}\right.$$

$\Big\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\right.$$

input
```
Int[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)
*d*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (-1/3*((-2*(a^4 + 7*a^2*
b^2 - 8*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*
a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*(5*a
^2 - 8*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]]
)/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/a + (2*(a^2
 - 4*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]
))/(a*(a^2 - b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 755 vs. $2(272) = 544$.

Time = 12.89 (sec) , antiderivative size = 756, normalized size of antiderivative = 2.62

method	result
default	$\frac{2\left(\left(-5\cos(dx+c)^2-10\cos(dx+c)-5\right)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,a^2 b\,\mathrm{EllipticE}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right.\right.}{}$

input
```
int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/(a+b)/((a-b)/(a+b))^(1/2)/a^3*((-5*cos(d*x+c)^2-10*cos(d*x+c)-5)*(1/
(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*
b*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/
2))+(8*cos(d*x+c)^2+16*cos(d*x+c)+8)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^3*EllipticE(((a-b)/(a+b))^(1/2)*(csc
(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1
/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3
*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2
))+(6*cos(d*x+c)^2+12*cos(d*x+c)+6)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticF(((a-b)/(a+b))^(1/2)*(cs
c(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(8*cos(d*x+c)^2+16*cos(d*x+c)+8
)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*a*b^2*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b)
)^(1/2))+sin(d*x+c)*cos(d*x+c)*(1+cos(d*x+c))*((a-b)/(a+b))^(1/2)*a^3+(cos
(d*x+c)^2-3*cos(d*x+c)+1)*sin(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b+(-4*cos(d*x
+c)-4)*sin(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2-8*((a-b)/(a+b))^(1/2)*b^3*sin(
d*x+c))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+
c)+cos(d*x+c)*b+b)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 622, normalized size of antiderivative = 2.15

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/9*(3*(a^4*b - 4*a^2*b^3 + (a^5 - a^3*b^2)*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(1/2)*(-3*I*
a^4*b - 16*I*a^2*b^3 + 16*I*b^5 + (-3*I*a^5 - 16*I*a^3*b^2 + 16*I*a*b^4)*c
os(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)
+ sqrt(1/2)*(3*I*a^4*b + 16*I*a^2*b^3 - 16*I*b^5 + (3*I*a^5 + 16*I*a^3*b^2
 - 16*I*a*b^4)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b
^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x
 + c) + 2*b)/a) + 3*sqrt(1/2)*(-5*I*a^3*b^2 + 8*I*a*b^4 + (-5*I*a^4*b + 8*
I*a^2*b^3)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) +
2*b)/a)) + 3*sqrt(1/2)*(5*I*a^3*b^2 - 8*I*a*b^4 + (5*I*a^4*b - 8*I*a^2*b^3
)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*
a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*
a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))
/((a^7 - a^5*b^2)*d*cos(d*x + c) + (a^6*b - a^4*b^3)*d)
```

Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)**(3/2)/(a + b*sec(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\cos\left(c+d\,x\right)^{3/2}}{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)}{\sec\left(dx+c\right)^{2}b^{2}+2\sec\left(dx+c\right)ab+a^{2}}dx$$

input
```
int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x)
```

output

```
int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x))/(sec(c + d*
x)**2*b**2 + 2*sec(c + d*x)*a*b + a**2),x)
```

3.864 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 214

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{4b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2-2b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{a^2(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{2b^2\sin(c+dx)}{a(a^2-b^2)\,d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

output
```
-4*b*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*
(a/(a+b))^(1/2))/a^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*(a^2-2*b^
2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*
(a+b*sec(d*x+c))^(1/2)/a^2/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)+2*b^
2*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.51 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2(b+a\cos(c+dx))\left(ab^2\sin(c+dx)+\frac{\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(i(a^3+a^2b-2ab\right.}{}\right.}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*(b + a*Cos[c + d*x])*(a*b^2*Sin[c + d*x] + ((Cos[(c + d*x)/2]^2*Sec[c +
 d*x])^(3/2)*(I*(a^3 + a^2*b - 2*a*b^2 - 2*b^3)*EllipticE[I*ArcSinh[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x
])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(a^2 - a*b - 2*b^2)*EllipticF[I*ArcS
inh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (a^2 - 2*b^2)*(b + a*Cos[c + d
*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/Sec[c + d*x]^(3/2)))/(a
^2*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.16, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 4752, 3042, 4334, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b \csc \left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\csc \left(c+dx+\frac{\pi}{2}\right)} \left(a+b \csc \left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4334}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{a^2-b\sec(c+dx)a-2b^2}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2-b\sec(c+dx)a-2b^2}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a-2b^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{4523}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\left(a^2-2b^2\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{2b\left(a^2-b^2\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+d}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-2b^2)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec}}{ad\left(a^2-b^2\right)\sqrt{a+b\,\mathrm{s}}}\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-2b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c}{ad\left(a^2-b\right.}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-2b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a\left(a^2-b^2\right)}+\frac{2b^2\,\mathrm{si}}{ad\left(a^2\right.}\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-2b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin}{ad\left(a^2-\right.}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-2b^2)\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a\left(a^2-b^2\right)}+\frac{2b^2}{ad\left(a\right.}$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+}{ad\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+}}}{a\sqrt{a+b\sec(c+dx)}}}{a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+\right.}}}{a\sqrt{a+b\sec(c+dx)}}}{a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(}{a}}}}{a\sqrt{a+b\sec(c+dx)}}}{a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin(}{}}}}{a\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{\frac{2(a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{4b(a^2-b^2}{}}{a(a^2-}\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((-4*b*(a^2 - b^2)*Sqrt[(b + a*Cos[
c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]
])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(a^2 - 2*b^2)*EllipticE[(c + d*x)/2
, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]*Sqrt[Sec[c + d*x]]))/(a*(a^2 - b^2)) + (2*b^2*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 589 vs. $2(207) = 414$.

Time = 9.72 (sec) , antiderivative size = 590, normalized size of antiderivative = 2.76

method	result
default	$\frac{2\left(\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\,a^2\,\mathrm{EllipticE}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)+\left(-\right.}{}$

input
```
int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/a^2/(a+b)/((a-b)/(a+b))^(1/2)*((cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*Ellipt
icE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-2*
cos(d*x+c)^2-4*cos(d*x+c)-2)*(1/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*b^2*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-
cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*Ellipt
icF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-2*
cos(d*x+c)^2-4*cos(d*x+c)-2)*(1/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*a*b*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-
cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+((a-b)/(a+b))^(1/2)*a^2*cos(d*x+c)*sin(d
*x+c)+(1+cos(d*x+c))*sin(d*x+c)*((a-b)/(a+b))^(1/2)*a*b+2*((a-b)/(a+b))^(1
/2)*b^2*sin(d*x+c))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*
a+a*cos(d*x+c)+cos(d*x+c)*b+b)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 563, normalized size of antiderivative = 2.63

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/3*(3*a^2*b^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c) + sqrt(1/2)*(5*I*a^2*b^2 - 4*I*b^4 + (5*I*a^3*b - 4*I*a*b^3)*
cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(
9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)
 + sqrt(1/2)*(-5*I*a^2*b^2 + 4*I*b^4 + (-5*I*a^3*b + 4*I*a*b^3)*cos(d*x +
c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b -
8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) + 3*sqrt(
1/2)*(I*a^3*b - 2*I*a*b^3 + (I*a^4 - 2*I*a^2*b^2)*cos(d*x + c))*sqrt(a)*we
ierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weiers
trassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3
*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) + 3*sqrt(1/2)*(-I*a^3*b +
2*I*a*b^3 + (-I*a^4 + 2*I*a^2*b^2)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-
4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-
4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c)
 - 3*I*a*sin(d*x + c) + 2*b)/a)))/((a^6 - a^4*b^2)*d*cos(d*x + c) + (a^5*b
 - a^3*b^3)*d)
```

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos{\left(c+dx \right)}}}{\left(a+b\sec{\left(c+dx \right)}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sqrt(cos(c + d*x))/(a + b*sec(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos(c+d\,x)}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\sec(dx+c)^2\,b^2+2\sec(dx+c)\,ab+a^2}dx$$

input
```
int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x)))/(sec(c + d*x)**2*b**2 +
2*sec(c + d*x)*a*b + a**2),x)
```

3.865 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 200

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{a\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} - \frac{2b\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

output
```
2*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(a/
(a+b))^(1/2))/a/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*b*cos(d*x+c)^(
1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c)
)^(1/2)/a/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)-2*b*sin(d*x+c)/(a^2-b
^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.58 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\sqrt{\cos(c+dx)}(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\left(ib(a+b)\sqrt{\frac{}{(a}}\right.}{}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*(I*b*(a + b)
*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSi
nh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 + Sec[c + d*x]] - I*a*(a +
b)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*Arc
Sinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 + Sec[c + d*x]] + b*(-a +
 b)*Sqrt[Sec[c + d*x]]*Tan[(c + d*x)/2]))/(a*(a^2 - b^2)*d*(a + b*Sec[c +
d*x])^(3/2))
```

Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.16, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 4752, 3042, 4330, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \ 4330$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{2\int -\frac{b+a\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}\right)$$

$$\Big\downarrow \ 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{b+a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}\right)$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}\right)$$

$$\Big\downarrow \ 4523$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\left(a^2-b^2\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}+\frac{b\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}\right)$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}\right)$$

$$\Big\downarrow \ 4343$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{s}}{d\left(a^2-b^2\right)\sqrt{a+b}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a^2-b^2}-\frac{2b\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a}}\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{}}{d\left(a^2-b^2\right)\sqrt{a+}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}-\frac{2b\sin(c+d}{d\left(a^2-b^2\right)\sqrt{}}\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec}}{d(a^2-b^2)\sqrt{a+b\,\mathrm{se}}}\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}-\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}\right.$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*(a^2 - b^2)*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/
(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*EllipticE[(c + d*x)/2, (2*a)/(a + b)
]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[S
ec[c + d*x]]))/(a^2 - b^2) - (2*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 -
 b^2)*d*Sqrt[a + b*Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4330
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)
*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*S
imp[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [A] (verified)

Time = 7.36 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.56

method	result
default	$-\frac{2\left(\sqrt{\frac{a-b}{a+b}}\, b\sin(dx+c)+\left(-\cos(dx+c)^{2}-2\cos(dx+c)-1\right)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,\text{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot\right.\right.}{d(a+}$

input
```
int(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/(a+b)/((a-b)/(a+b))^(1/2)/a*(((a-b)/(a+b))^(1/2)*b*sin(d*x+c)+(-cos(d
*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c
)),(-(a+b)/(a-b))^(1/2))*a+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE(((a-b)/
(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*b)*cos(d*x+c)^(
1/2)*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+cos(d*x+c)*b+b)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.64

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(3*a^2*b*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c) - sqrt(1/2)*(-3*I*a^2*b + 2*I*b^3 + (-3*I*a^3 + 2*I*a*b^2)*cos
(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a
^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) -
sqrt(1/2)*(3*I*a^2*b - 2*I*b^3 + (3*I*a^3 - 2*I*a*b^2)*cos(d*x + c))*sqrt(
a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^
3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(1/2)*(I*a
^2*b*cos(d*x + c) + I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/
a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/
a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c
) + 2*b)/a)) - 3*sqrt(1/2)*(-I*a^2*b*cos(d*x + c) - I*a*b^2)*sqrt(a)*weier
strassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstra
ssPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*
cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/((a^5 - a^3*b^2)*d*cos(d*x +
 c) + (a^4*b - a^2*b^3)*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+b\sec{(c+dx)})^{\frac{3}{2}}\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**(3/2)*sqrt(cos(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec{(dx+c)}+a)^{\frac{3}{2}}\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\sqrt{\cos(c+d\,x)}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)\sec(dx+c)^{2}b^{2}+2\cos(dx+c)\sec(dx+c)\,ab}$$

input
```
int(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x)))/(cos(c + d*x)*sec(c + d*
x)**2*b**2 + 2*cos(c + d*x)*sec(c + d*x)*a*b + cos(c + d*x)*a**2),x)
```

3.866 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 126

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{2a\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

output
```
-2*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*
(a+b*sec(d*x+c))^(1/2)/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)+2*a*sin(
d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.60 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.90

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\sqrt{\cos(c+dx)}(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\left(-i(a+b)\sqrt{\frac{}{(a}}\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*((-I)*(a + b
)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcS
inh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 + Sec[c + d*x]] + I*(a + b
)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcS
inh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 + Sec[c + d*x]] + (a - b)*
Sqrt[Sec[c + d*x]]*Tan[(c + d*x)/2]))/((a^2 - b^2)*d*(a + b*Sec[c + d*x])^
(3/2))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 4752, 3042, 4331, 27, 3042, 4343, 3042, 3134, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \textbf{4331}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int -\frac{\sqrt{a+b\sec(c+dx)}}{2\sqrt{\sec(c+dx)}}dx}{a^2-b^2}+\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}\right)$$

$$\Big\downarrow \textbf{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \textbf{4343}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}\right)$$

$$\Big\downarrow \textbf{3134}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)$$

$$\Big\downarrow \text{3132}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*EllipticE[(c + d*x)/2, (2*a)/(a
 + b)]*Sqrt[a + b*Sec[c + d*x]])/((a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2
 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 4331
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 2)/(f*(m + 1)*(a^2 - b^2))), x] - Simp[d^2/((m + 1
)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*
(a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /;
FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2
] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(121) = 242$.

Time = 2.48 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.40

method	result
default	$\frac{2\left(\sqrt{\frac{a-b}{a+b}}\,\sin(dx+c)+\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\operatorname{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\,\sqrt{}}{d(a+b)\sqrt{}}$

input
```
int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/(a+b)/((a-b)/(a+b))^(1/2)*(((a-b)/(a+b))^(1/2)*sin(d*x+c)+(cos(d*x+c)^
2+2*cos(d*x+c)+1)*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-
(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE(((a-b)/(a+b))^(
1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2)))*cos(d*x+c)^(1/2)*(a+b*
sec(d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+cos(d*x+c)*b+b)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 486, normalized size of antiderivative = 3.86

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/3*(3*a^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c) + sqrt(1/2)*(I*a*b*cos(d*x + c) + I*b^2)*sqrt(a)*weierstrassPInve
rse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x
 + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(1/2)*(-I*a*b*cos(d*x + c) - I*
b^2)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b -
 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) + 3*sqrt
(1/2)*(-I*a^2*cos(d*x + c) - I*a*b)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 -
4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 -
4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(
d*x + c) + 2*b)/a)) + 3*sqrt(1/2)*(I*a^2*cos(d*x + c) + I*a*b)*sqrt(a)*wei
erstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierst
rassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*
a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/((a^4 - a^2*b^2)*d*cos(d*x
 + c) + (a^3*b - a*b^3)*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^{3/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^2\sec(dx+c)^2\,b^2+2\cos(dx+c)^2\sec(dx+c)\,ab$$

input
```
int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x)))/(cos(c + d*x)**2*sec(c +
 d*x)**2*b**2 + 2*cos(c + d*x)**2*sec(c + d*x)*a*b + cos(c + d*x)**2*a**2)
,x)
```

3.867 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 206

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{b(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} - \frac{2a^2\sin(c+dx)}{b(a^2-b^2)\,d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

output

```
2*((b+a*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(
a/(a+b))^(1/2))/b/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*a*cos(d*x+c)
^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+
c))^(1/2)/b/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)-2*a^2*sin(d*x+c)/b/
(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 32.50 (sec) , antiderivative size = 48278, normalized size of antiderivative = 234.36

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.92 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.14, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {3042, 4752, 3042, 4332, 27, 3042, 4596, 2011, 3042, 4343, 3042, 3134, 3042, 3132, 4346, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \ 4332$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{2\int -\frac{a^2+b\sec(c+dx)a+(a^2-b^2)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b\,(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\,(a^2-b^2)\sqrt{a+b\sec(c+dx)}}\right)$$

$$\Big\downarrow \ 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2+b\sec(c+dx)a+(a^2-b^2)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b\,(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\,(a^2-b^2)\sqrt{a+b\sec(c+dx)}}\right)$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2+b\csc\left(c+dx+\frac{\pi}{2}\right)a+(a^2-b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\,(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\,(a^2-b^2)\sqrt{a+b\sec(c+dx)}}\right)$$

$$\Big\downarrow \ 4596$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(a^2-b^2)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\int \frac{a^2+b\sec(c+dx)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b\,(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\mathrm{se}}}{bd\,(a^2-b^2)\sqrt{a+b}}\right.$$

$$\Big\downarrow \ 2011$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(a^2-b^2)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+a\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{b\,(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+d}}{bd\,(a^2-b^2)\sqrt{a+b\sec(c+}}\right.$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(a^2-b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\,(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec}}{bd\,(a^2-b^2)\sqrt{a+b\,\mathrm{se}}}\right.$$

$$\Big\downarrow \ 4343$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(a^2-b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{b(a^2-b^2)}-\frac{2a^2\sin(c+}{bd(a^2-b^2)\sqrt{}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(a^2-b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{b(a^2-b^2)}-\frac{2a^2\sin(c}{bd(a^2-b^2}\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(a^2-b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b(a^2-b^2)}-\frac{2a^2\sin(c}{bd(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(a^2-b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b(a^2-b^2)}-\frac{2a^2\sin}{bd(a^2-}\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(a^2-b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b(a^2-b^2)}-\frac{2a^2\sin(c+dx}{bd(a^2-b^2)\sqrt{c}}\right.$$

$\downarrow$ 4346

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b(a^2-b^2)}-\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(}{}}}}{b\,(a^2-b^2)}\right.$$

$\Big\downarrow$ 3286

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b\,(a^2-b^2)}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\,c}{}}}}{b\,(a^2-b^2)}\right.$$

$\Big\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2}{a-}\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b\,(a^2-b^2)}\right.$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*(a^2 - b^2)*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x
]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*a*EllipticE[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[S
ec[c + d*x]]))/(b*(a^2 - b^2)) - (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.10 (sec) , antiderivative size = 653, normalized size of antiderivative = 3.17

method	result
default	$-\frac{2\left(\left(2\cos(dx+c)^2+4\cos(dx+c)+2\right)\text{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,a-\right.}{}$

input
```
int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/(a+b)/((a-b)/(a+b))^(1/2)/b*((2*cos(d*x+c)^2+4*cos(d*x+c)+2)*Elliptic
F(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+
b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a+(cos(
d*x+c)^2+2*cos(d*x+c)+1)*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x
+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*b+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*EllipticE(((a-
b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(-2*cos(d*x+
c)^2-4*cos(d*x+c)-2)*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)
),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a+(-2*cos(d*x+c)^2-4*cos(d*x+c)-2)*Ell
ipticPi(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(
a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*b+((a-b)/(a+b))^(1/2)*a*sin(d*x+c))*cos(d*x+c)^(1/2)*(a+b*sec(
d*x+c))^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+cos(d*x+c)*b+b)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^{5/2}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^3\sec(dx+c)^2 b^2+2\cos(dx+c)^3\sec(dx+c)\,ab}$$

input
```
int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x)))/(cos(c + d*x)**3*sec(c +
 d*x)**2*b**2 + 2*cos(c + d*x)**3*sec(c + d*x)*a*b + cos(c + d*x)**3*a**2)
,x)
```

3.868 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 345

$$\begin{aligned}\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\\
&- \frac{3a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\\
&- \frac{(3a^2-b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{b^2(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}\\
&- \frac{2a^2\sin(c+dx)}{b(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\\
&+ \frac{(3a^2-b^2)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{b^2(a^2-b^2)\,d\sqrt{\cos(c+dx)}}\end{aligned}$$

output
```
((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(a/(a
+b))^(1/2))/b/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-3*a*((b+a*cos(d*x+
c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))/
b^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-(3*a^2-b^2)*cos(d*x+c)^(1/2)
*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1
/2)/b^2/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)-2*a^2*sin(d*x+c)/b/(a^2
-b^2)/d/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2)+(3*a^2-b^2)*(a+b*sec(d*x+c
))^(1/2)*sin(d*x+c)/b^2/(a^2-b^2)/d/cos(d*x+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 32.57 (sec) , antiderivative size = 52199, normalized size of antiderivative = 151.30

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 3.73 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.10, number of steps used = 28, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 1.120$, Rules used = {3042, 4752, 3042, 4332, 27, 3042, 4590, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4332}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{2\int \frac{\sqrt{\sec(c+dx)}\left(a^2-b\sec(c+dx)a-\left(3a^2-b^2\right)\sec^2(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+d}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+}}\right.$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-b\sec(c+dx)a-\left(3a^2-b^2\right)\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+d}}\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(b^2-3a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec}{bd\left(a^2-b^2\right)\sqrt{a+b}}\right.$$

$$\Big\downarrow \text{ 4590}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{2b\sec(c+dx)a^2+3\left(a^2-b^2\right)\sec^2(c+dx)a+\left(3a^2-b^2\right)a}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b}-\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+}}{bd}}{b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{2b\sec(c+dx)a^2+3\left(a^2-b^2\right)\sec^2(c+dx)a+\left(3a^2-b^2\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}-\frac{(3a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+}}{bd}}{b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+3\left(a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2a+\left(3a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{(3a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+}}{bd}}{b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4596

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a(a^2-b^2)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\int \frac{2b\sec(c+dx)a^2+\left(3a^2-b^2\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}-\frac{(3a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{}}{bd}}{b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a(a^2-b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{(3a^2-b^2)\sin(c+dx)\sqrt{}}{}}{b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4346

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}}{2b}-(3}{b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c\right.}}}{\sqrt{a+b\sec(c+dx)}}}{2b}}{b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3286

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}}{2b}}{b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\,\mathrm{s}}{}}}}{\sqrt{a+b\sec(c+dx)}}}{2b}}{b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}}{b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{-b(a^2-b^2)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+(3a^2-b^2)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\mathrm{E}}{d\sqrt{a+b\sec(c+dx)}}}{2b}}{b\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{-b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+(3a^2-b^2)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c}{a}}}{d\sqrt{a+b}}}{2b}}{b\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{-b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx}}{}}{2b}}{b\left(a^2-\right.}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{-b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(}}{}}{2b}}{b\left(a^2-\right.}\right.$$

$\Big\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{-b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(}}{}}{2b}}{b\left(a^2-\right.}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{-b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6a\left(a^2-b^2\right)\sqrt{}}{}}{2b}}{b\left(a^2\right.}\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{-b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}}{}}{2b}}{b\left(a^2-b\right.}\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{-\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+}{2b}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{-\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{2b}\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{-\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+}\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{2b}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{-\frac{b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^2-b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{2b}\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2}{a-}\right.}{d\sqrt{a+b\sec(c+dx)}}}{}\right.$$

input
```
Int[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*a^2*Sec[c + d*x]^(3/2)*Sin[c +
d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) - (((-2*b*(a^2 - b^2)*Sqr
t[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt
[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (6*a*(a^2 - b^2)*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Se
c[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^2 - b^2)*EllipticE[(c
+ d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(2*b) - ((3*a^2 - b^2)*Sqrt[Sec[c + d*
x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(b*d))/(b*(a^2 - b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 13.31 (sec) , antiderivative size = 882, normalized size of antiderivative = 2.56

method	result	size
default	Expression too large to display	882

input
```
int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/(a+b)/((a-b)/(a+b))^(1/2)/b^2*((1/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(
d*x+c)-cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(-6*cos(d*x+c)^3-12*
cos(d*x+c)^2-6*cos(d*x+c))+(1/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*a*b*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-c
ot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(-6*cos(d*x+c)^3-12*cos(d*x+
c)^2-6*cos(d*x+c))+(1/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*a^2*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)
),(-(a+b)/(a-b))^(1/2))*(-3*cos(d*x+c)^3-6*cos(d*x+c)^2-3*cos(d*x+c))+(1/(
a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^2*E
llipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))
*(cos(d*x+c)^3+2*cos(d*x+c)^2+cos(d*x+c))+(1/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*EllipticF(((a-b)/(a+b))^(1/2)
*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(6*cos(d*x+c)^3+12*cos(d*x+
c)^2+6*cos(d*x+c))+(1/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*a*b*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)
),(-(a+b)/(a-b))^(1/2))*(4*cos(d*x+c)^3+8*cos(d*x+c)^2+4*cos(d*x+c))+3*((a
-b)/(a+b))^(1/2)*a^2*cos(d*x+c)*sin(d*x+c)+(1+cos(d*x+c))*sin(d*x+c)*((a-b
)/(a+b))^(1/2)*a*b+((a-b)/(a+b))^(1/2)*b^2*sin(d*x+c))*(a+b*sec(d*x+c))^(1
/2)/cos(d*x+c)^(1/2)/(cos(d*x+c)^2*a+a*cos(d*x+c)+cos(d*x+c)*b+b)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2)), x)
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)^{2}b^{2}+2\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)ab$$

input
```
int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x)))/(cos(c + d*x)**4*sec(c +
 d*x)**2*b**2 + 2*cos(c + d*x)**4*sec(c + d*x)*a*b + cos(c + d*x)**4*a**2)
,x)
```

3.869 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 391

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2(a^4+16a^2b^2-16b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^4(a^2-b^2)\,d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{8b(2a^4-7a^2b^2+4b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3a^4(a^2-b^2)^2\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{2b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{4b^2(5a^2-3b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}} + \frac{2(a^4-13a^2b^2+8b^4)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3a^3(a^2-b^2)^2\,d}$$

output

```
2/3*(a^4+16*a^2*b^2-16*b^4)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM
(1/2*d*x+1/2*c,2^(1/2)*(a/(a+b))^(1/2))/a^4/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(
a+b*sec(d*x+c))^(1/2)-8/3*b*(2*a^4-7*a^2*b^2+4*b^4)*cos(d*x+c)^(1/2)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^
4/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)+2/3*b^2*cos(d*x+c)^(1/2)*si
n(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)+4/3*b^2*(5*a^2-3*b^2)*cos(d*
x+c)^(1/2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)+2/3*(a^4-13
*a^2*b^2+8*b^4)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)*sin(d*x+c)/a^3/(a^
2-b^2)^2/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.99 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.35

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{(b+a\cos(c+dx))^3\left(\frac{2\sin(c+dx)}{3a^3}+\frac{2b^4\sin(c+dx)}{3a^3(a^2-b^2)(b+a\cos(c+dx))^2}+\frac{8(-3a^2b^3\sin(c+dx)+2\cdots}{3a^3(a^2-b^2)^2(b+a\,c\cdots}\right.}{d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}} + \frac{2\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(-4ib(2a^5+2a^4b-7a\cdots\right.}{\cdots}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
((b + a*Cos[c + d*x])^3*((2*Sin[c + d*x])/(3*a^3) + (2*b^4*Sin[c + d*x])/(
3*a^3*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (8*(-3*a^2*b^3*Sin[c + d*x] +
2*b^5*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*Cos[c
 + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)) + (2*Cos[c + d*x]^(3/2)*(b + a*C
os[c + d*x])^2*Sec[c + d*x]^(5/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*
((-4*I)*b*(2*a^5 + 2*a^4*b - 7*a^3*b^2 - 7*a^2*b^3 + 4*a*b^4 + 4*b^5)*Elli
pticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sq
rt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(a^5 - 8*a^4*b
 + 7*a^3*b^2 + 28*a^2*b^3 - 4*a*b^4 - 16*b^5)*EllipticF[I*ArcSinh[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])
*Sec[(c + d*x)/2]^2)/(a + b)] - 4*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*(b + a*Cos
[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(3*a^4*(a^2 - b^2
)^2*d*(a + b*Sec[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 3.31 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.07, number of steps used = 24, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.960$, Rules used = {3042, 4752, 3042, 4334, 27, 3042, 4588, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4334}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}-\frac{2\int -\frac{4b^2\sec^2(c+dx)-3ab\sec(c+dx)-}{2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+a}}{3a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{4b^2\sec^2(c+dx)-3ab\sec(c+dx)+3\left(a^2-2b^2\right)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{4b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2-3ab\csc\left(c+dx+\frac{\pi}{2}\right)+3\left(a^2-2b^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\right.$$

$$\Big\downarrow \text{ 4588}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{4b^2\left(5a^2-3b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{4b^2\left(5a^2-3b^2\right)\sec^2(c+dx)-2ab\left(3a^2-b^2\right)\sec(c+dx)+3\left(a^4-1\right.}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{4b^2\left(5a^2-3b^2\right)\sec^2(c+dx)-2ab\left(3a^2-b^2\right)\sec(c+dx)+3\left(a^4-13b^2a^2+8b^4\right)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}+\frac{4b^2(5a^2-3b^2)\sin(c+\cdots}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\cdots}}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{4b^2\left(5a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-2ab\left(3a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+3\left(a^4-13b^2a^2+8b^4\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{4b^2(5a^2-3b^2\cdots}{ad(a^2-b^2)\sqrt{\sec(c+d\cdots}}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4592

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{2\int \frac{3\left(4b\left(2a^4-7b^2a^2+4b^4\right)-a\left(a^4+7b^2a^2-4b^4\right)\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{a(a^2-b^2)}+\cdots}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{4b\left(2a^4-7b^2a^2+4b^4\right)-a\left(a^4+7b^2a^2-4b^4\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a}}{a(a^2-b^2)}+\frac{\cdots}{ad(a\cdots}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{4b\left(2a^4-7b^2a^2+4b^4\right)-a\left(a^4+7b^2a^2-4b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\cdots}{3a\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{\left(a^6+15a^4b^2-32a^2b^4\right.}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{\left(a^6+15a^4b^2-32a\right.}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{\left(a^6+\right.}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{\left(\right.}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right.}\right.$$

$\Big\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\cdots}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)\cdots}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\cdots}{a}}{a(a^2-b^2)}}{3a\left(a^2-\cdots\right.}\right.$$

$\Big\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{8b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^6+15a\cdots\right.}{\cdots}}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)\cdots}\right.$$

$\Big\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{8b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^6+15a\right.}{}}{}}{a(a^2-b^2)}}{3a\,(a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{8b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^6+15a\right.}{}}{}}{a(a^2-b^2)}}{3a}\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{8b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^6+15a\right.}{}}{}}{a(a^2-b^2)}}{3a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{8b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^6+15a\right.}{}}{}}{a(a^2-b^2)}}{3}\right.$$

$$\Big\downarrow \; 3140$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2b^2\sin(c+dx)}{3ad\,(a^2-b^2)\,\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}+\frac{\frac{4b^2\,(5a^2-3b^2)\,\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}}{\right.$$

input
```
Int[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^
2)*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) + ((4*b^2*(5*a^2 - 3*b
^2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d
*x]]) + (-(((-2*(a^6 + 15*a^4*b^2 - 32*a^2*b^4 + 16*b^6)*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]]
)/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (8*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Ellipt
icE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a
*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/a) + (2*(a^4 - 13*a^2*b^2 + 8
*b^4)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(a*d*Sqrt[Sec[c + d*x]]))/(a*
(a^2 - b^2)))/(3*a*(a^2 - b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4588

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4592

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4752

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1801 vs. $2(366) = 732$.

Time = 15.72 (sec) , antiderivative size = 1802, normalized size of antiderivative = 4.61

method	result	size
default	Expression too large to display	1802

input
```
int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/((a-b)/(a+b))^(1/2)/(a+b)^2/(a-b)/a^4*((1/(a+b)*(b+a*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^5*b*EllipticE(((a-b)/(a+b))^
(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-8*cos(d*x+c)^3-16*co
s(d*x+c)^2-8*cos(d*x+c))+(-8*cos(d*x+c)^2-16*cos(d*x+c)-8)*(1/(a+b)*(b+a*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4*b^2*Elliptic
E(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(1/(a+
b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b^3
*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2
))*(28*cos(d*x+c)^3+56*cos(d*x+c)^2+28*cos(d*x+c))+(28*cos(d*x+c)^2+56*cos
(d*x+c)+28)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*a^2*b^4*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(
-(a+b)/(a-b))^(1/2))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*a*b^5*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*
x+c)),(-(a+b)/(a-b))^(1/2))*(-16*cos(d*x+c)^3-32*cos(d*x+c)^2-16*cos(d*x+c
))+(-16*cos(d*x+c)^2-32*cos(d*x+c)-16)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^6*EllipticE(((a-b)/(a+b))^(1/2)*(c
sc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(1/(a+b)*(b+a*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^6*EllipticF(((a-b)/(a+b))^(1/
2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(cos(d*x+c)^3+2*cos(d*x+c
)^2+cos(d*x+c))+(9*cos(d*x+c)^3+19*cos(d*x+c)^2+11*cos(d*x+c)+1)*(1/(a+...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 941, normalized size of antiderivative = 2.41

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/9*(3*(a^6*b^2 - 13*a^4*b^4 + 8*a^2*b^6 + (a^8 - 2*a^6*b^2 + a^4*b^4)*cos
(d*x + c)^2 + 2*(a^7*b - 8*a^5*b^3 + 5*a^3*b^5)*cos(d*x + c))*sqrt((a*cos(
d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - sqrt(1/2)*(3
*I*a^6*b^2 + 37*I*a^4*b^4 - 68*I*a^2*b^6 + 32*I*b^8 + (3*I*a^8 + 37*I*a^6*
b^2 - 68*I*a^4*b^4 + 32*I*a^2*b^6)*cos(d*x + c)^2 + 2*(3*I*a^7*b + 37*I*a^
5*b^3 - 68*I*a^3*b^5 + 32*I*a*b^7)*cos(d*x + c))*sqrt(a)*weierstrassPInver
se(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x
+ c) + 3*I*a*sin(d*x + c) + 2*b)/a) - sqrt(1/2)*(-3*I*a^6*b^2 - 37*I*a^4*b
^4 + 68*I*a^2*b^6 - 32*I*b^8 + (-3*I*a^8 - 37*I*a^6*b^2 + 68*I*a^4*b^4 - 3
2*I*a^2*b^6)*cos(d*x + c)^2 + 2*(-3*I*a^7*b - 37*I*a^5*b^3 + 68*I*a^3*b^5
- 32*I*a*b^7)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^
2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x
+ c) + 2*b)/a) - 12*sqrt(1/2)*(2*I*a^5*b^3 - 7*I*a^3*b^5 + 4*I*a*b^7 + (2*
I*a^7*b - 7*I*a^5*b^3 + 4*I*a^3*b^5)*cos(d*x + c)^2 + 2*(2*I*a^6*b^2 - 7*I
*a^4*b^4 + 4*I*a^2*b^6)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2
- 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2
- 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*si
n(d*x + c) + 2*b)/a)) - 12*sqrt(1/2)*(-2*I*a^5*b^3 + 7*I*a^3*b^5 - 4*I*a*b
^7 + (-2*I*a^7*b + 7*I*a^5*b^3 - 4*I*a^3*b^5)*cos(d*x + c)^2 + 2*(-2*I*a^6
*b^2 + 7*I*a^4*b^4 - 4*I*a^2*b^6)*cos(d*x + c))*sqrt(a)*weierstrassZeta...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)^{3/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)}{\sec(dx+c)^3\,b^3+3\sec(dx+c)^2\,a\,b^2+3\sec(dx+c)\,a^2b+a^3}dx$$

input
```
int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x))/(sec(c + d*
x)**3*b**3 + 3*sec(c + d*x)**2*a*b**2 + 3*sec(c + d*x)*a**2*b + a**3),x)
```

3.870 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 317

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2b(9a^2-8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^3(a^2-b^2)\,d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2(3a^4-15a^2b^2+8b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3a^3(a^2-b^2)^2\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{2b^2\sin(c+dx)}{3a(a^2-b^2)\,d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{8b^2(2a^2-b^2)\sin(c+dx)}{3a^2(a^2-b^2)^2\,d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

output

```
-2/3*b*(9*a^2-8*b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*
x+1/2*c,2^(1/2)*(a/(a+b))^(1/2))/a^3/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec
(d*x+c))^(1/2)+2/3*(3*a^4-15*a^2*b^2+8*b^4)*cos(d*x+c)^(1/2)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^3/(a^2-b
^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)+2/3*b^2*sin(d*x+c)/a/(a^2-b^2)/d/co
s(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2)+8/3*b^2*(2*a^2-b^2)*sin(d*x+c)/a^2/(
a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.39 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.60

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{(b+a\cos(c+dx))^3\left(-\frac{2b^3\sin(c+dx)}{3a^2(a^2-b^2)(b+a\cos(c+dx))^2}-\frac{2(-9a^2b^2\sin(c+dx)+5b^4\sin(c+dx))}{3a^2(a^2-b^2)^2(b+a\cos(c+dx))}\right.}{d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}$$
$$-\frac{2\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(-i(3a^5+3a^4b-15a\right.}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
((b + a*Cos[c + d*x])^3*((-2*b^3*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*(b + a*C
os[c + d*x])^2) - (2*(-9*a^2*b^2*Sin[c + d*x] + 5*b^4*Sin[c + d*x]))/(3*a^
2*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*Cos[c + d*x]^(5/2)*(a + b*Sec[c
 + d*x])^(5/2)) - (2*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x])^2*Sec[c + d*x
]^(5/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((-I)*(3*a^5 + 3*a^4*b - 1
5*a^3*b^2 - 15*a^2*b^3 + 8*a*b^4 + 8*b^5)*EllipticE[I*ArcSinh[Tan[(c + d*x
)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec
[(c + d*x)/2]^2)/(a + b)] + I*a*(3*a^4 - 6*a^3*b - 15*a^2*b^2 + 2*a*b^3 +
8*b^4)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d
*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (3*a^4
- 15*a^2*b^2 + 8*b^4)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[
(c + d*x)/2]))/(3*a*(a^3 - a*b^2)^2*d*(a + b*Sec[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 2.60 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.11, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 4752, 3042, 4334, 27, 3042, 4588, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4334}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2\int -\frac{3a^2-3b\sec(c+dx)a-4b^2+2b^2\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{3a^2-3b\sec(c+dx)a-4b^2+2b^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{3a^2-3b\csc\left(c+dx+\frac{\pi}{2}\right)a-4b^2+2b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}\right.$$

$$\Big\downarrow \text{4588}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{3a^4-15b^2a^2-2b\left(3a^2-b^2\right)\sec(c+dx)a+8b^4}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin}{3ad\left(a^2-\right.}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{3a^4-15b^2a^2-2b\left(3a^2-b^2\right)\sec(c+dx)a+8b^4}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}+\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c}{3ad\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{3a^4-15b^2a^2-2b\left(3a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+8b^4}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin}{3ad\left(a^2-\right.}\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}}{a(a^2-b^2)}+\frac{8b^2\left(2a^2-b^2\right)\sin(c+d}{ad(a^2-b^2)\sqrt{a+b}}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{8b^2\left(2a^2-b^2\right)}{ad(a^2-b}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+8b^2}{3a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+}{3a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \ 3134$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+}{3a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \ 3132$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{8b^2(\cdots}{\cdots}}{3a\left(a^2-b^2\right)}\cdots\right.$$

$$\downarrow \text{4345}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{\cdots}{\sqrt{b+a\cdots}}}{a\sqrt{a+b\sec(c+dx)}}\cdots}{a(a^2-b^2)}\cdots}{3a\left(a^2-b^2\right)}\cdots\right.$$

$$\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{\cdots}{\sqrt{b+a\cdots}}}{a\sqrt{a+b\sec(c+dx)}}\cdots}{a(a^2-b^2)}\cdots}{3a\left(a^2-b^2\right)}\cdots\right.$$

$$\downarrow \text{3142}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\cdots}{\sqrt{\frac{b}{a+\cdots}\cdots}}}{a\sqrt{a+b\sec(c+dx)}}\cdots}{a(a^2-b^2)}\cdots}{3a\left(a^2-b^2\right)}\cdots\right.$$

$$\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int}{a\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}\right.$$

$\big\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}+\frac{\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^4-15a^2b^2+8\right.}{ad\sqrt{}}}{}\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b^2*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (((-2*b*(9*a^4 - 17*
a^2*b^2 + 8*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3
*a^4 - 15*a^2*b^2 + 8*b^4)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a +
b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]
]))/(a*(a^2 - b^2)) + (8*b^2*(2*a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))/(3*a*(a^2 - b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1599 vs. $2(298) = 596$.

Time = 13.16 (sec) , antiderivative size = 1600, normalized size of antiderivative = 5.05

method	result	size
default	Expression too large to display	1600

input
```
int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/((a-b)/(a+b))^(1/2)/(a+b)^2/(a-b)/a^3*((1/(a+b)*(b+a*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^5*EllipticE(((a-b)/(a+b))^(1
/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(3*cos(d*x+c)^3+6*cos(d*
x+c)^2+3*cos(d*x+c))+(3*cos(d*x+c)^2+6*cos(d*x+c)+3)*(1/(a+b)*(b+a*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4*b*EllipticE(((a-b)
/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(1/(a+b)*(b+a*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b^2*Ellipti
cE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-15*
cos(d*x+c)^3-30*cos(d*x+c)^2-15*cos(d*x+c))+(-15*cos(d*x+c)^2-30*cos(d*x+c
)-15)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*a^2*b^3*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)
/(a-b))^(1/2))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*a*b^4*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),
(-(a+b)/(a-b))^(1/2))*(8*cos(d*x+c)^3+16*cos(d*x+c)^2+8*cos(d*x+c))+(8*cos
(d*x+c)^2+16*cos(d*x+c)+8)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*b^5*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-co
t(d*x+c)),(-(a+b)/(a-b))^(1/2))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^5*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+
c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-3*cos(d*x+c)^3-6*cos(d*x+c)^2-3*cos
(d*x+c))+(-9*cos(d*x+c)^3-21*cos(d*x+c)^2-15*cos(d*x+c)-3)*(1/(a+b)*(b+...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 852, normalized size of antiderivative = 2.69

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/9*(3*(8*a^4*b^3 - 4*a^2*b^5 + (9*a^5*b^2 - 5*a^3*b^4)*cos(d*x + c))*sqrt
((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 4*sq
rt(1/2)*(-6*I*a^4*b^3 + 9*I*a^2*b^5 - 4*I*b^7 + (-6*I*a^6*b + 9*I*a^4*b^3
- 4*I*a^2*b^5)*cos(d*x + c)^2 + 2*(-6*I*a^5*b^2 + 9*I*a^3*b^4 - 4*I*a*b^6)
*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*
(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a
) - 4*sqrt(1/2)*(6*I*a^4*b^3 - 9*I*a^2*b^5 + 4*I*b^7 + (6*I*a^6*b - 9*I*a^
4*b^3 + 4*I*a^2*b^5)*cos(d*x + c)^2 + 2*(6*I*a^5*b^2 - 9*I*a^3*b^4 + 4*I*a
*b^6)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2,
8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2
*b)/a) - 3*sqrt(1/2)*(-3*I*a^5*b^2 + 15*I*a^3*b^4 - 8*I*a*b^6 + (-3*I*a^7
+ 15*I*a^5*b^2 - 8*I*a^3*b^4)*cos(d*x + c)^2 + 2*(-3*I*a^6*b + 15*I*a^4*b^
3 - 8*I*a^2*b^5)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2
)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2
)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x +
 c) + 2*b)/a)) - 3*sqrt(1/2)*(3*I*a^5*b^2 - 15*I*a^3*b^4 + 8*I*a*b^6 + (3*
I*a^7 - 15*I*a^5*b^2 + 8*I*a^3*b^4)*cos(d*x + c)^2 + 2*(3*I*a^6*b - 15*I*a
^4*b^3 + 8*I*a^2*b^5)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 -
4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 -
4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*s...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos(c+d\,x)}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\sec(dx+c)^3\,b^3+3\sec(dx+c)^2\,a\,b^2+3\sec(dx+c)\,a^2b+a^3}dx$$

input
```
int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x)))/(sec(c + d*x)**3*b**3 +
3*sec(c + d*x)**2*a*b**2 + 3*sec(c + d*x)*a**2*b + a**3),x)
```

3.871 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 302

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2(3a^2-2b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^2(a^2-b^2)\,d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$
$$+\frac{4b(3a^2-b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3a^2(a^2-b^2)^2\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$
$$-\frac{2b\sin(c+dx)}{3(a^2-b^2)\,d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}$$
$$-\frac{2b(5a^2-b^2)\sin(c+dx)}{3a(a^2-b^2)^2\,d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

output
```
2/3*(3*a^2-2*b^2)*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1
/2*c,2^(1/2)*(a/(a+b))^(1/2))/a^2/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*
x+c))^(1/2)+4/3*b*(3*a^2-b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c
),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^2/(a^2-b^2)^2/d/((b+a*
cos(d*x+c))/(a+b))^(1/2)-2/3*b*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+
b*sec(d*x+c))^(3/2)-2/3*b*(5*a^2-b^2)*sin(d*x+c)/a/(a^2-b^2)^2/d/cos(d*x+c
)^(1/2)/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.25 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.32

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \frac{(b+a\cos(c+dx))^2\left(\frac{2b(-5a^2b+b^3+(-6a^3+2ab^2)\cos(c+dx))\sin(c+dx)}{a(a^2-b^2)^2(b+a\cos(c+dx))} - \cdots\right.}{\cdots}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
((b + a*Cos[c + d*x])^2*((2*b*(-5*a^2*b + b^3 + (-6*a^3 + 2*a*b^2)*Cos[c +
 d*x])*Sin[c + d*x])/(a*(a^2 - b^2)^2*(b + a*Cos[c + d*x])) - (2*(Cos[(c +
 d*x)/2]^2*Sec[c + d*x])^(3/2)*((2*I)*b*(-3*a^3 - 3*a^2*b + a*b^2 + b^3)*E
llipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2
*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(3*a^3 + 6*
a^2*b + a*b^2 - 2*b^3)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a
 + b)] + 2*b*(-3*a^2 + b^2)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2
)*Tan[(c + d*x)/2]))/((a^3 - a*b^2)^2*Sec[c + d*x]^(3/2))))/(3*d*Cos[c + d
*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 2.43 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.11, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 4752, 3042, 4330, 27, 3042, 4588, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4330}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{2\int -\frac{-2b\sec^2(c+dx)+3a\sec(c+dx)+b}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{-2b\sec^2(c+dx)+3a\sec(c+dx)+b}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{-2b\csc\left(c+dx+\frac{\pi}{2}\right)^2+3a\csc\left(c+dx+\frac{\pi}{2}\right)+b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{4588}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{2\int -\frac{2b\left(3a^2-b^2\right)+a\left(3a^2+b^2\right)\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+a}{3d\left(a^2-b^2\right)(a}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int\frac{2b(3a^2-b^2)+a(3a^2+b^2)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3(a^2-b^2)}-\frac{2b\sin(c+dx)\sqrt{}}{3d(a^2-b^2)(a+b}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int\frac{2b(3a^2-b^2)+a(3a^2+b^2)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3(a^2-b^2)}-\frac{2b\sin(c+dx}{3d(a^2-b^2)(a}\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2b(3a^2-b^2)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}+\frac{(3a^4-5a^2b^2+2b^4)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)\sqrt{\sec(c}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx}}}{3(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2b(3a^2-b^2)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{(3a^4-5a^2b^2+2b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\,s}}}{3(a^2-b^2)}\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2b(3a^2-b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{(3a^4-5a^2b^2+2b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)}{ad(a^2-b}}{3(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2b(3a^2-b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{(3a^4-5a^2b^2+2b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}-\frac{2b(5a^2-}{ad(a}}{3(a^2-b^2)}\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2b(3a^2-b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{(3a^4-5a^2b^2+2b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}-\frac{2b(5a^2-}{ad(a}}{3(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2b(3a^2-b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{(3a^4-5a^2b^2+2b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}-\frac{2b(5}{a}}{3(a^2-b^2)}\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{(3a^4-5a^2b^2+2b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{4b(3a^2-b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin}{ad(a^2-b^2)}}{3(a^2-b^2)}\right.$$

$\Big\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^4-5a^2b^2+2b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+\cdots}\right.}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}}{3\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^4-5a^2b^2+2b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}}{3\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^4-5a^2b^2+2b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}}{3\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^4-5a^2b^2+2b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx}{a+b}}}}{a(a^2-b^2)}}{3\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{4b(3a^2-b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2(3a^4-5a^2b^2+2b^4)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx)\right.}{ad\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}}{3(a^2-b^2)}\right.$$

input

```
Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)),x]
```

output

```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*b*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(3*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (((2*(3*a^4 - 5*a^2*b^2
 + 2*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/
(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (4*b*(3*a^2
- b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*
d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(a*(a^2 - b^2))
- (2*b*(5*a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqr
t[a + b*Sec[c + d*x]]))/(3*(a^2 - b^2)))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4330
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)
*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*S
imp[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1142 vs. $2(283) = 566$.

Time = 10.59 (sec) , antiderivative size = 1143, normalized size of antiderivative = 3.78

method	result	size
default	Expression too large to display	1143

input
```
int(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)/a^2*((1/(a+b)*(b+a*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b*EllipticE(((a-b)/(a+b))
^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-6*cos(d*x+c)^3-12*c
os(d*x+c)^2-6*cos(d*x+c))+(-6*cos(d*x+c)^2-12*cos(d*x+c)-6)*(1/(a+b)*(b+a*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b^2*Ellipti
cE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(1/(a
+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^3*
EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2)
)*(2*cos(d*x+c)^3+4*cos(d*x+c)^2+2*cos(d*x+c))+(2*cos(d*x+c)^2+4*cos(d*x+c
)+2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*b^4*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b
))^(1/2))+(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*a^4*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)
/(a-b))^(1/2))*(-3*cos(d*x+c)^3-6*cos(d*x+c)^2-3*cos(d*x+c))+(-3*cos(d*x+c
)-3)*sin(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*a^3*b*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c
)),(-(a+b)/(a-b))^(1/2))+(2*cos(d*x+c)^3+7*cos(d*x+c)^2+8*cos(d*x+c)+3)*(1
/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2
*b^2*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^
(1/2))+(2*cos(d*x+c)^2+4*cos(d*x+c)+2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 801, normalized size of antiderivative = 2.65

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-2/9*(3*(5*a^4*b^2 - a^2*b^4 + 2*(3*a^5*b - a^3*b^3)*cos(d*x + c))*sqrt((a
*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(1/
2)*(9*I*a^4*b^2 - 9*I*a^2*b^4 + 4*I*b^6 + (9*I*a^6 - 9*I*a^4*b^2 + 4*I*a^2
*b^4)*cos(d*x + c)^2 + 2*(9*I*a^5*b - 9*I*a^3*b^3 + 4*I*a*b^5)*cos(d*x + c
))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8
*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(1/2
)*(-9*I*a^4*b^2 + 9*I*a^2*b^4 - 4*I*b^6 + (-9*I*a^6 + 9*I*a^4*b^2 - 4*I*a^
2*b^4)*cos(d*x + c)^2 + 2*(-9*I*a^5*b + 9*I*a^3*b^3 - 4*I*a*b^5)*cos(d*x +
 c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b -
 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) + 6*sqrt
(1/2)*(-3*I*a^3*b^3 + I*a*b^5 + (-3*I*a^5*b + I*a^3*b^3)*cos(d*x + c)^2 +
2*(-3*I*a^4*b^2 + I*a^2*b^4)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3
*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3
*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I
*a*sin(d*x + c) + 2*b)/a)) + 6*sqrt(1/2)*(3*I*a^3*b^3 - I*a*b^5 + (3*I*a^5
*b - I*a^3*b^3)*cos(d*x + c)^2 + 2*(3*I*a^4*b^2 - I*a^2*b^4)*cos(d*x + c))
*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/
a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/
a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/((a^9 - 2*a^7*
b^2 + a^5*b^4)*d*cos(d*x + c)^2 + 2*(a^8*b - 2*a^6*b^3 + a^4*b^5)*d*cos...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\sqrt{\cos(c+d\,x)}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}}{\cos(dx+c)\sec(dx+c)^3 b^3 + 3\cos(dx+c)\sec(dx+c)^2 a}$$

input

```
int(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x)
```

output

```
int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x)))/(cos(c + d*x)*sec(c + d*
x)**3*b**3 + 3*cos(c + d*x)*sec(c + d*x)**2*a*b**2 + 3*cos(c + d*x)*sec(c
+ d*x)*a**2*b + cos(c + d*x)*a**3),x)
```

3.872 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 281

$$\begin{aligned}\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = & -\frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\ & -\frac{2(3a^2+b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3a\left(a^2-b^2\right)^2 d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} \\ & +\frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} \\ & +\frac{4(a^2+b^2)\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\end{aligned}$$

output
```
-2/3*b*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2
)*(a/(a+b))^(1/2))/a/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-2
/3*(3*a^2+b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a
+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b)
)^(1/2)+2/3*a*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/
2)+4/3*(a^2+b^2)*sin(d*x+c)/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)
)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.97 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.59

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \frac{(b+a\cos(c+dx))^3\left(\frac{2b\sin(c+dx)}{3(-a^2+b^2)(b+a\cos(c+dx))^2}+\frac{2(3a^2\sin(c+dx)+b^2\sin\cdots}{3(-a^2+b^2)^2(b+a\cos\cdots}\right.}{d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}} + \frac{2\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(-i(3a^3+3a^2b+ab^2\cdots\right.}{\cdots}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
((b + a*Cos[c + d*x])^3*((2*b*Sin[c + d*x])/(3*(-a^2 + b^2)*(b + a*Cos[c +
 d*x])^2) + (2*(3*a^2*Sin[c + d*x] + b^2*Sin[c + d*x]))/(3*(-a^2 + b^2)^2*
(b + a*Cos[c + d*x]))))/(d*Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2))
+ (2*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*(Cos[(c
+ d*x)/2]^2*Sec[c + d*x])^(3/2)*((-I)*(3*a^3 + 3*a^2*b + a*b^2 + b^3)*Elli
pticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sq
rt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(3*a^2 + 4*a*b
 + b^2)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c +
d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (3*a^2
 + b^2)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))
/(3*a*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 2.25 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.10, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 4752, 3042, 4331, 27, 3042, 4588, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4331}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int -\frac{-2a\sec^2(c+dx)+3b\sec(c+dx)+a}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}+\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\int \frac{-2a\sec^2(c+dx)+3b\sec(c+dx)+a}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\int \frac{-2a\csc\left(c+dx+\frac{\pi}{2}\right)^2+3b\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \text{4588}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{-\frac{2\int -\frac{4b\sec(c+dx)a^2+\left(3a^2+b^2\right)a}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}-\frac{4(a^2+b^2)\sin(c+}{d(a^2-b^2)\sqrt{a}}}{3\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\int\frac{4b\sec(c+dx)a^2+\left(3a^2+b^2\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}-\frac{4(a^2+b^2)\sin(c+dx)\sqrt{s}}{d(a^2-b^2)\sqrt{a+b\sec(}}}{3\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\int\frac{4b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2+b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}-\frac{4(a^2+b^2)\sin(}{d(a^2-b^2)\sqrt{}}}{3\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b(a^2-b^2)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+(3a^2+b^2)\int\frac{\sqrt{a+b\sec(c+dx}}{\sqrt{\sec(c+dx)}}}{a(a^2-b^2)}}{3\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+(3a^2+b^2)\int\frac{\sqrt{a+b\csc(}}{\sqrt{\csc(c+}}}{a(a^2-b^2)}}{3\left(a^2-b\right.}\right.$$

$\Big\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2+b^2\right)\sqrt{a+b\sec(c+d}}{\sqrt{\sec(c+dx)}\sqrt{c}}}{a(a^2-b^2)}}{3\left(a^2\right.}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2+b^2\right)\sqrt{a+b\sec(c+d}}{\sqrt{\sec(c+dx)}}}{a\left(a^2-b^2\right)}}{3\left(a\right.}\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2+b^2\right)\sqrt{a+b\sec(c+d}}{\sqrt{\sec(c+dx)}\sqrt{}}}{a\left(a^2-b^2\right)}}{3\left(a\right.}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2+b^2\right)\sqrt{a+b\sec(c+d}}{\sqrt{\sec(c+dx}}}{a\left(a^2-b^2\right)}}{3\left(\right.}\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+}}{d\sqrt{\sec(c+dx)}\sqrt{}}}{a\left(a^2-b^2\right)}}{3\left(a^2\right.}\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+}{a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}d}{\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\right.}{a+b}}}}{\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2}\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{2b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+}\right.}{d\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}\right.$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/(3*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (((2*b*(a^2 - b^2)*Sqrt[
(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[S
ec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^2 + b^2)*EllipticE[(c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(a*(a^2 - b^2)) - (4*(a^2 + b^2)*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))/(3*
(a^2 - b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4331
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 2)/(f*(m + 1)*(a^2 - b^2))), x] - Simp[d^2/((m + 1
)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*
(a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /;
FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2
] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1015 vs. $2(262) = 524$.

Time = 8.00 (sec) , antiderivative size = 1016, normalized size of antiderivative = 3.62

method	result	size
default	Expression too large to display	1016

input
```
int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/(a-b)/(a+b)^2/a/((a-b)/(a+b))^(1/2)*((1/(a+b)*(b+a*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*EllipticE(((a-b)/(a+b))^(1/2
)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-3*cos(d*x+c)^3-6*cos(d*x
+c)^2-3*cos(d*x+c))+(-3*cos(d*x+c)^2-6*cos(d*x+c)-3)*(1/(a+b)*(b+a*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticE(((a-b)
/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(1/(a+b)*(b+a*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2*EllipticE
(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-cos(d
*x+c)^3-2*cos(d*x+c)^2-cos(d*x+c))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)
*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^3*Ellip
ticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(1/
(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*
EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2)
)*(3*cos(d*x+c)^3+6*cos(d*x+c)^2+3*cos(d*x+c))+(-cos(d*x+c)^3+cos(d*x+c)^2
+5*cos(d*x+c)+3)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*a^2*b*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)
),(-(a+b)/(a-b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(a+b)*(b+a*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2*EllipticF(((a-
b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+3*((a-b)/(a+
b))^(1/2)*a^3*cos(d*x+c)*sin(d*x+c)+(-cos(d*x+c)+2)*sin(d*x+c)*((a-b)/(...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 750, normalized size of antiderivative = 2.67

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/9*(3*(2*a^4*b + 2*a^2*b^3 + (3*a^5 + a^3*b^2)*cos(d*x + c))*sqrt((a*cos(
d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*sqrt(1/2)*
(-3*I*a^2*b^3 + I*b^5 + (-3*I*a^4*b + I*a^2*b^3)*cos(d*x + c)^2 + 2*(-3*I*
a^3*b^2 + I*a*b^4)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 -
 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin
(d*x + c) + 2*b)/a) - 2*sqrt(1/2)*(3*I*a^2*b^3 - I*b^5 + (3*I*a^4*b - I*a^
2*b^3)*cos(d*x + c)^2 + 2*(3*I*a^3*b^2 - I*a*b^4)*cos(d*x + c))*sqrt(a)*we
ierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/
3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(1/2)*(3*I*a^3*
b^2 + I*a*b^4 + (3*I*a^5 + I*a^3*b^2)*cos(d*x + c)^2 + 2*(3*I*a^4*b + I*a^
2*b^3)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/2
7*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/2
7*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)
/a)) - 3*sqrt(1/2)*(-3*I*a^3*b^2 - I*a*b^4 + (-3*I*a^5 - I*a^3*b^2)*cos(d*
x + c)^2 + 2*(-3*I*a^4*b - I*a^2*b^3)*cos(d*x + c))*sqrt(a)*weierstrassZet
a(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInvers
e(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x +
 c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/((a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*
x + c)^2 + 2*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*x + c) + (a^6*b^2 - 2*a
^4*b^4 + a^2*b^6)*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^{3/2}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}}{\cos(dx+c)^2\sec(dx+c)^3\,b^3+3\cos(dx+c)^2\sec(dx+c)^2\,a}$$

input
```
int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x)))/(cos(c + d*x)**2*sec(c +
 d*x)**3*b**3 + 3*cos(c + d*x)**2*sec(c + d*x)**2*a*b**2 + 3*cos(c + d*x)*
*2*sec(c + d*x)*a**2*b + cos(c + d*x)**2*a**3),x)
```

3.873 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 277

$$\begin{aligned}\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx &= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3(a^2-b^2)\,d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\\
&+\frac{8b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3(a^2-b^2)^2\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}\\
&-\frac{2a^2\sin(c+dx)}{3b(a^2-b^2)\,d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\\
&+\frac{2a(a^2-5b^2)\sin(c+dx)}{3b(a^2-b^2)^2\,d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\end{aligned}$$

output

```
2/3*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(
a/(a+b))^(1/2))/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+8/3*b*
cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+
b*sec(d*x+c))^(1/2)/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)-2/3*a^2*s
in(d*x+c)/b/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2)+2/3*a*(a^2
-5*b^2)*sin(d*x+c)/b/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.79 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.12

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2(b+a\cos(c+dx))^2\left(\frac{a(a^2-5b^2-4ab\cos(c+dx))\sin(c+dx)}{b+a\cos(c+dx)}+\frac{\sqrt{\cos^2\left(\frac{1}{2}(c\right.}}{}\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
(2*(b + a*Cos[c + d*x])^2*((a*(a^2 - 5*b^2 - 4*a*b*Cos[c + d*x])*Sin[c + d
*x])/(b + a*Cos[c + d*x]) + (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((4*I)*
b*(a + b)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((
b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*(a^2 + 4*a*b + 3*b^2)
*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((b + a*Cos
[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 4*b*(b + a*Cos[c + d*x])*Sqrt[Se
c[(c + d*x)/2]^2]*Tan[(c + d*x)/2]))/Sqrt[Sec[c + d*x]]))/(3*(a^2 - b^2)^2
*d*Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 2.36 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.14, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 4752, 3042, 4332, 27, 3042, 4588, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 4332$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{2\int -\frac{a^2+3b\sec(c+dx)a+\left(a^2-3b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2+3b\sec(c+dx)a+\left(a^2-3b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2+3b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 4588$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a\left(a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{4a^2b^2+a\left(a^2+3b^2\right)\sec(c+dx)b}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{}}{3bd\left(a^2-b^2\right)(a+b}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{4a^2b^2+a\left(a^2+3b^2\right)\sec(c+dx)b}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\mathrm{se}}}{3bd\left(a^2-b^2\right)\left(a+b\,\mathrm{sec}\right.}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{4a^2b^2+a\left(a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx}{3bd\left(a^2-b^2\right)(a+}\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{ab(a^2-b^2)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+4ab^2\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{}{3bc}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{ab(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+4ab^2\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{ab(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4ab^2\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{s}}{d(a^2-b^2)\sqrt{a+b\sec(c}}}{3b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{ab(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4ab^2\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx}{d(a^2-b^2)\sqrt{a+b\,s}}}{3b\,(a^2-b^2)}\right.$$

$\Big\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{ab(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4ab^2\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx}{d(a^2-b^2)\sqrt{a+b\,s}}}{3b\,(a^2-b^2)}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{ab(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4ab^2\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c}{d(a^2-b^2)\sqrt{a}}}{3b\,(a^2-b^2)}\right.$$

$\Big\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{ab(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{8ab^2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec}}{d(a^2-b^2)\sqrt{a+b\sec(c+}}}{3b\,(a^2-b^2)}\right.$$

$\Big\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8ab^2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}+\frac{2a(a^2-}{d(}}{3b\,(a^2-b^2)}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8ab^2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}+\frac{2a(}{}}{3b\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8ab^2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}+\frac{2a}{}}{3b\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8ab^2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}+}{3b\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{\frac{2ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{8a}{}}{a(a^2-b^2)}}{3b\left(a^2-b^2\right)}\right.$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*a^2*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (((2*a*b*(a^2 - b^2
)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (8*a*b^2*EllipticE[(c
+ d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(a*(a^2 - b^2)) + (2*a*(a^2 - 5*b^2)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))/
(3*b*(a^2 - b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 754 vs. $2(258) = 516$.

Time = 8.00 (sec) , antiderivative size = 755, normalized size of antiderivative = 2.73

method	result
default	$\frac{2\left(\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\sqrt{\frac{1}{1+\cos(dx+c)}}\,ab\,\text{EllipticE}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(dx+c)-\cot(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)\left(4\cos(dx+c)^3+8\cos(dx+c)^2+4\cos\right.\right.}{}$

input
```
int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)*((1/(a+b)*(b+a*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b*EllipticE(((a-b)/(a+b))^(1/2)*
(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(4*cos(d*x+c)^3+8*cos(d*x+c)
^2+4*cos(d*x+c))+(4*cos(d*x+c)^2+8*cos(d*x+c)+4)*(1/(a+b)*(b+a*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^2*EllipticE(((a-b)/(a+b)
)^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+(1/(a+b)*(b+a*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*EllipticF(((a-b)/
(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(cos(d*x+c)^3+2
*cos(d*x+c)^2+cos(d*x+c))+(-3*cos(d*x+c)^3-5*cos(d*x+c)^2-cos(d*x+c)+1)*(1
/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b
*EllipticF(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2
))+(-3*cos(d*x+c)^2-6*cos(d*x+c)-3)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^2*EllipticF(((a-b)/(a+b))^(1/2)*(csc(
d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))+sin(d*x+c)*(1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)*a^2+(-3*cos(d*x+c)+1)*sin(d*x+c)*((a-b)/(a+b))^(1/2)*a*b-4*((
a-b)/(a+b))^(1/2)*b^2*sin(d*x+c))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/
(cos(d*x+c)^2*(1+cos(d*x+c))*a^2+cos(d*x+c)*(2*cos(d*x+c)+2)*a*b+(1+cos(d*
x+c))*b^2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 679, normalized size of antiderivative = 2.45

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-2/9*(3*(4*a^3*b*cos(d*x + c) - a^4 + 5*a^2*b^2)*sqrt((a*cos(d*x + c) + b)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(1/2)*(3*I*a^2*b^2 +
I*b^4 + (3*I*a^4 + I*a^2*b^2)*cos(d*x + c)^2 + 2*(3*I*a^3*b + I*a*b^3)*cos
(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a
^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) +
sqrt(1/2)*(-3*I*a^2*b^2 - I*b^4 + (-3*I*a^4 - I*a^2*b^2)*cos(d*x + c)^2 +
2*(-3*I*a^3*b - I*a*b^3)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3
*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I
*a*sin(d*x + c) + 2*b)/a) + 12*sqrt(1/2)*(-I*a^3*b*cos(d*x + c)^2 - 2*I*a^
2*b^2*cos(d*x + c) - I*a*b^3)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)
/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)
/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x +
c) + 2*b)/a)) + 12*sqrt(1/2)*(I*a^3*b*cos(d*x + c)^2 + 2*I*a^2*b^2*cos(d*x
 + c) + I*a*b^3)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a))
)/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a
^2*b^5)*d*cos(d*x + c) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^{5/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)b+a}}{\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)^{3}b^{3}+3\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)^{2}a}$$

input
```
int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x)))/(cos(c + d*x)**3*sec(c +
 d*x)**3*b**3 + 3*cos(c + d*x)**3*sec(c + d*x)**2*a*b**2 + 3*cos(c + d*x)*
*3*sec(c + d*x)*a**2*b + cos(c + d*x)**3*a**3),x)
```

3.874 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 370

$$\begin{aligned}
\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx =
&-\frac{2a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\
&+\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\
&+\frac{2a(3a^2-7b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3b^2\left(a^2-b^2\right)^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} \\
&-\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} \\
&-\frac{2a^2(3a^2-7b^2)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}
\end{aligned}$$

output
```
-2/3*a*((b+a*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2
)*(a/(a+b))^(1/2))/b/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2
*((b+a*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a
/(a+b))^(1/2))/b^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*a*(3*a^2-
7*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/
2))*(a+b*sec(d*x+c))^(1/2)/b^2/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2
)-2/3*a^2*sin(d*x+c)/b/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2)
-2/3*a^2*(3*a^2-7*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*
sec(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 33.25 (sec) , antiderivative size = 93062, normalized size of antiderivative = 251.52

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 4.07 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.11, number of steps used = 28, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 1.120$, Rules used = {3042, 4752, 3042, 4332, 27, 3042, 4586, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4332}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{2\int \frac{\sqrt{\sec(c+dx)}\left(a^2-3b\sec(c+dx)a-3\left(a^2-b^2\right)\sec^2(c+dx)\right)}{2(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+}{3bd\left(a^2-b^2\right)(a+b\sec(c+}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-3b\sec(c+dx)a-3\left(a^2-b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+}{3bd\left(a^2-b^2\right)(a+b\sec(c+}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2-3b\csc\left(c+dx+\frac{\pi}{2}\right)a-3\left(a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+}\right.$$

$$\Big\downarrow \text{4586}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\int \frac{\left(3a^2-7b^2\right)a^2+2b\left(a^2-3b^2\right)\sec(c+dx)a+3\left(a^2-b^2\right)^2\sec^2(c+dx}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}}{b\left(a^2-b^2\right)}}{3b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int\frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)\sec(c+dx)a+3(a^2-b^2)^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}d}{b(a^2-b^2)}}{3b(a^2-b^2)}\right.$$

↓ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int\frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a+3(a^2-b^2)^2\csc\left(c+\right.}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}}{b(a^2-b^2)}}{3b(a^2-b^2)}\right.$$

↓ 4596

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{3(a^2-b^2)^2\int\frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\int\frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)s}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c}}}{b(a^2-b^2)}}{3b(a^2-b^2)}\right.$$

↓ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{3(a^2-b^2)^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int\frac{(3a^2-7b^2)a^2+2b(a^2-3}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+}}}{b(a^2-b^2)}}{3b(a^2-b^2)}\right.$$

↓ 4346

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int\frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3(a^2-b^2)^2\sqrt{\sec}}{}}{b(a^2-b^2)}}{3b(a^2-b^2)}\right.$$

↓ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int\frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3\left(a^2-b^2\right)^2\sqrt{\sec\cdots}}{\cdots}}{b(a^2-\cdots}}{3b\left(a^2-b^2\right)}\cdots\right.$$

$\big\downarrow$ 3286

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int\frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3\left(a^2-b^2\right)^2\sqrt{\sec\cdots}}{\cdots}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}\cdots\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int\frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3\left(a^2-b^2\right)^2\sqrt{\sec\cdots}}{\cdots}}{b(a\cdots}}{3b\left(a^2-b^2\right)}\cdots\right.$$

$\big\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int\frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6\left(a^2-b^2\right)^2\sqrt{\sec\cdots}}{\cdots}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}\cdots\right.$$

$\big\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-ab(a^2-b^2)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+a(3a^2-7b^2)\int\frac{\sqrt{a+b\sec(c+}}{\sqrt{\sec(c+dx}}}{3b(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-ab(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a(3a^2-7b^2)\int\frac{\sqrt{a+b\,\mathrm{cs}}}{\sqrt{\csc(}}}{3b(a^2-b^2)}\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-ab(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(3a^2-7b^2\right)\sqrt{a+b\sec(c-}}{\sqrt{\sec(c+dx)}\sqrt{}}}{3b(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-ab(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(3a^2-7b^2\right)\sqrt{a+b\sec(c-}}{\sqrt{\sec(c+dx)}}}{3b(a^2-b^2}\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-ab(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(3a^2-7b^2\right)\sqrt{a+b\sec(c-}}{\sqrt{\sec(c+dx)}}}{3b(a^2-b^2}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-ab(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(3a^2-7b^2\right)\sqrt{a+b\sec(c\cdots}}{\sqrt{\sec(c+d\cdots}}}{3b\left(a^2-\cdots\right.}\right.$$

$\Big\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-ab(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\left(3a^2-7b^2\right)\sqrt{a+b\sec(\cdots}}{d\sqrt{\sec(c+dx)}\sqrt{\cdots}}}{3b\left(a^2-b^2\right)\cdots}\right.$$

$\Big\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\cdots}{\cdots}}{3b\left(a\cdots\right.}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}\cdots}{3b\cdots}\right.$$

$\Big\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}d}{\sqrt{a+b\sec(c+dx)}}}{}}{3b}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}}{\sqrt{a+b\sec(c+dx)}}}{}}{3}\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{2ab\left(a^2-b^2\right.}{}}{}}{}\right.$$

input
```
Int[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*a^2*Sec[c + d*x]^(3/2)*Sin[c +
d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (-(((-2*a*b*(a^2 -
b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (6*(a^2 - b^2)^2*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*a*(3*a^2 - 7*b^2)*E
llipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b
+ a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(b*(a^2 - b^2))) + (2*a^2*
(3*a^2 - 7*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a +
 b*Sec[c + d*x]]))/(3*b*(a^2 - b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4586
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Simp[d/(b*(a^2 - b^2)*(m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
 - b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n
+ b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C
}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 11.49 (sec) , antiderivative size = 1908, normalized size of antiderivative = 5.16

method	result	size
default	Expression too large to display	1908

input
```
int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d/((a-b)/(a+b))^(1/2)/(a+b)^2/(a-b)/b^2*((1/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4*EllipticPi(((a-b)/(a+b))^
(1/2)*(csc(d*x+c)-cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(-6*cos(d
*x+c)^3-12*cos(d*x+c)^2-6*cos(d*x+c))+(-6*cos(d*x+c)^3-18*cos(d*x+c)^2-18*
cos(d*x+c)-6)*(1/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*a^3*b*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),
(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))+(-6*cos(d*x+c)-6)*sin(d*x+c)^2*(1/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2*E
llipticPi(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(a+b)/(a-b),I/((a-b)
/(a+b))^(1/2))+(6*cos(d*x+c)^3+18*cos(d*x+c)^2+18*cos(d*x+c)+6)*(1/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3*Ellip
ticPi(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+
b))^(1/2))+(6*cos(d*x+c)^2+12*cos(d*x+c)+6)*(1/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^4*EllipticPi(((a-b)/(a+b))^(1
/2)*(csc(d*x+c)-cot(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))+(1/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4*Elliptic
E(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b))^(1/2))*(-3*co
s(d*x+c)^3-6*cos(d*x+c)^2-3*cos(d*x+c))+(-3*cos(d*x+c)^2-6*cos(d*x+c)-3)*(
1/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^
3*b*EllipticE(((a-b)/(a+b))^(1/2)*(csc(d*x+c)-cot(d*x+c)),(-(a+b)/(a-b)...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2)), x)
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^{7/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)\,b+a}}{\cos(dx+c)^4\sec(dx+c)^3 b^3+3\cos(dx+c)^4\sec(dx+c)^2 a}$$

input
```
int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int((sqrt(sec(c + d*x)*b + a)*sqrt(cos(c + d*x)))/(cos(c + d*x)**4*sec(c +
 d*x)**3*b**3 + 3*cos(c + d*x)**4*sec(c + d*x)**2*a*b**2 + 3*cos(c + d*x)*
*4*sec(c + d*x)*a**2*b + cos(c + d*x)**4*a**3),x)
```

3.875 $\int (d\cos(e+fx))^n(a+b\sec(e+fx))^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 266

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^3\,dx = -\frac{b(b^2(1-n)+3a^2(2-n))\,(d\cos(e+fx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{f(2-n)n\sqrt{\sin^2(e+fx)}} - \frac{a(a^2(1-n)-3b^2n)\cos(e+fx)(d\cos(e+fx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(e+fx)\right)\sin(e}{f(1-n)(1+n)\sqrt{\sin^2(e+fx)}} + \frac{ab^2(5-2n)(d\cos(e+fx))^n\tan(e+fx)}{f(1-n)(2-n)} + \frac{b^2(d\cos(e+fx))^n(a+b\sec(e+fx))\tan(e+fx)}{f(2-n)}$$

output
```
-b*(b^2*(1-n)+3*a^2*(2-n))*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2*n],[1+1/2*
n],cos(f*x+e)^2)*sin(f*x+e)/f/(2-n)/n/(sin(f*x+e)^2)^(1/2)-a*(a^2*(1-n)-3*
b^2*n)*cos(f*x+e)*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],
cos(f*x+e)^2)*sin(f*x+e)/f/(1-n)/(1+n)/(sin(f*x+e)^2)^(1/2)+a*b^2*(5-2*n)*
(d*cos(f*x+e))^n*tan(f*x+e)/f/(1-n)/(2-n)+b^2*(d*cos(f*x+e))^n*(a+b*sec(f*
x+e))*tan(f*x+e)/f/(2-n)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.83

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^3\,dx =$$
$$-\frac{(d\cos(e+fx))^n\csc(e+fx)\left(b^3n(-1+n^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n),\frac{n}{2},\cos^2(e+fx)\right)+\frac{1}{2}a\right.}{}$$

input
```
Integrate[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x])^3,x]
```

output
```
-(((d*Cos[e + f*x])^n*Csc[e + f*x]*(b^3*n*(-1 + n^2)*Hypergeometric2F1[1/2
, (-2 + n)/2, n/2, Cos[e + f*x]^2] + (a*(-2 + n)*Cos[e + f*x]*(6*b^2*n*(1
+ n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[e + f*x]^2] + 2*a*(
-1 + n)*Cos[e + f*x]*(3*b*(1 + n)*Hypergeometric2F1[1/2, n/2, (2 + n)/2, C
os[e + f*x]^2] + a*n*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n
)/2, Cos[e + f*x]^2])))/2)*Sec[e + f*x]^2*Sqrt[Sin[e + f*x]^2])/(f*(-2 + n
)*(-1 + n)*n*(1 + n)))
```

Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.12, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4752, 3042, 4329, 3042, 4535, 3042, 4259, 3042, 3122, 4534, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))^3(d\cos(e+fx))^n\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3\left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \ 4752$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int(d\sec(e+fx))^{-n}(a+b\sec(e+fx))^3dx$$

$$\Big\downarrow \ 3042$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \ 4329$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\int(d\sec(e+fx))^{-n}\left(ab^2d(5-2n)\sec^2(e+fx)+bd(3(2-n)a^2+b^2(1-n))\sec(e+fx)+ad(a^2(2-n\right.}{d(2-n)}\right.$$

$$\Big\downarrow \ 3042$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(ab^2d(5-2n)\csc\left(e+fx+\frac{\pi}{2}\right)^2+bd(3(2-n)a^2+b^2(1-n))\csc\left(e+fx+\frac{\pi}{2}\right.\right.}{d(2-n)}\right.$$

$$\Big\downarrow \ 4535$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\int(d\sec(e+fx))^{-n}\left(ab^2d(5-2n)\sec^2(e+fx)+ad\left(a^2(2-n)-b^2n\right)\right)dx+b\left(3a^2(2-n)+b^2(1-n)\right)\int}{d(2-n)}\right.$$

$$\Big\downarrow \ 3042$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{b\left(3a^2(2-n)+b^2(1-n)\right)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{1-n}dx+\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(ab^2d(5-2n)\csc\left(e\right.\right.}{d(2-n)}\right.$$

$$\Big\downarrow \ 4259$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(ab^2d(5-2n)\csc\left(e+fx+\frac{\pi}{2}\right)^2+ad\left(a^2(2-n)-b^2n\right)\right)dx+b\left(3a^2(2-n)+\right.}{d(2-n)}\right.$$

$$\Big\downarrow \ 3042$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(ab^2d(5-2n)\csc\left(e+fx+\frac{\pi}{2}\right)^2+ad\left(a^2(2-n)-b^2n\right)\right)dx+b\left(3a^2(2-n)+\right.}{d(2-n)}\right.$$

$\Big\downarrow$ 3122

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(ab^2d(5-2n)\csc\left(e+fx+\frac{\pi}{2}\right)^2+ad\left(a^2(2-n)-b^2n\right)\right)dx-\frac{bd(3a^2(2-n)+b^2(1}{}}{d(2-n)}\right.$$

$\Big\downarrow$ 4534

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\frac{ad(2-n)(a^2(1-n)-3b^2n)\int(d\sec(e+fx))^{-n}dx}{1-n}-\frac{bd(3a^2(2-n)+b^2(1-n))\sin(e+fx)(d\sec(e+fx))^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+}{2}\right.}{fn\sqrt{\sin^2(e+fx)}}}{d(2-n)}\right.$$

$\Big\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\frac{ad(2-n)(a^2(1-n)-3b^2n)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}dx}{1-n}-\frac{bd(3a^2(2-n)+b^2(1-n))\sin(e+fx)(d\sec(e+fx))^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},\right.}{fn\sqrt{\sin^2(e+fx)}}}{d(2-n)}\right.$$

$\Big\downarrow$ 4259

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\frac{ad(2-n)(a^2(1-n)-3b^2n)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\cos(e+fx)}{d}\right)^n dx}{1-n}-\frac{bd(3a^2(2-n)+b^2(1-n))\sin(e+fx)(d\sec(e+fx))^{-n}}{fn\sqrt{\sin^2(e}}}{d(2-n)}\right.$$

$\Big\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\frac{ad(2-n)(a^2(1-n)-3b^2n)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^n dx}{1-n}-\frac{bd(3a^2(2-n)+b^2(1-n))\sin(e+fx)(d\sec(e+f}{fn\sqrt{\mathrm{si}}}}{d(2-n)}\right.$$

$\Big\downarrow$ 3122

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{-\frac{ad^2(2-n)(a^2(1-n)-3b^2n)\sin(e+fx)(d\sec(e+fx))^{-n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(e+fx)\right)}{f(1-n)(n+1)\sqrt{\sin^2(e+fx)}}-\frac{bd(3a^2(2-n)+b^2(1-n}{}}{d(2-n)}\right.$$

input
```
Int[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x])^3,x]
```

output
```
(d*Cos[e + f*x])^n*(d*Sec[e + f*x])^n*((b^2*(a + b*Sec[e + f*x])*Tan[e + f
*x])/(f*(2 - n)*(d*Sec[e + f*x])^n) + (-((a*d^2*(2 - n)*(a^2*(1 - n) - 3*b
^2*n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]*(d*Sec[
e + f*x])^(-1 - n)*Sin[e + f*x])/(f*(1 - n)*(1 + n)*Sqrt[Sin[e + f*x]^2]))
 - (b*d*(b^2*(1 - n) + 3*a^2*(2 - n))*Hypergeometric2F1[1/2, n/2, (2 + n)/
2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*n*(d*Sec[e + f*x])^n*Sqrt[Sin[e + f*x]
^2]) + (a*b^2*d*(5 - 2*n)*Tan[e + f*x])/(f*(1 - n)*(d*Sec[e + f*x])^n))/(d
*(2 - n)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 && !(IGtQ[n, 2] && !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [F]

$$\int (d\cos(fx+e))^n (a+b\sec(fx+e))^3 \, dx$$

input
```
int((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^3,x)
```

output
```
int((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^3,x)
```

Fricas [F]

$$\int (d\cos(e+fx))^n (a+b\sec(e+fx))^3 \, dx = \int (b\sec(fx+e)+a)^3 (d\cos(fx+e))^n \, dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
integral((b^3*sec(f*x + e)^3 + 3*a*b^2*sec(f*x + e)^2 + 3*a^2*b*sec(f*x +
e) + a^3)*(d*cos(f*x + e))^n, x)
```

Sympy [F]

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^3\,dx = \int (d\cos(e+fx))^n\,(a+b\sec(e+fx))^3\,dx$$

input
```
integrate((d*cos(f*x+e))**n*(a+b*sec(f*x+e))**3,x)
```

output
```
Integral((d*cos(e + f*x))**n*(a + b*sec(e + f*x))**3, x)
```

Maxima [F]

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^3\,dx = \int (b\sec(fx+e)+a)^3(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^3*(d*cos(f*x + e))^n, x)
```

Giac [F]

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^3\,dx = \int (b\sec(fx+e)+a)^3(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^3*(d*cos(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^3\,dx = \int (d\,\cos\left(e+f\,x\right))^n\,\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)^3\,dx$$

input
```
int((d*cos(e + f*x))^n*(a + b/cos(e + f*x))^3,x)
```

output
```
int((d*cos(e + f*x))^n*(a + b/cos(e + f*x))^3, x)
```

Reduce [F]

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^3\,dx = d^n\left(\left(\int \cos\left(fx+e\right)^n dx\right)a^3 + \left(\int \cos\left(fx+e\right)^n \sec\left(fx+e\right)^3 dx\right)b^3 + 3\left(\int \cos\left(fx+e\right)^n \sec\left(fx+e\right)^2 dx\right)a\,b^2 + 3\left(\int \cos\left(fx+e\right)^n \sec\left(fx+e\right) dx\right)a^2b\right)$$

input
```
int((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^3,x)
```

output
```
d**n*(int(cos(e + f*x)**n,x)*a**3 + int(cos(e + f*x)**n*sec(e + f*x)**3,x)
*b**3 + 3*int(cos(e + f*x)**n*sec(e + f*x)**2,x)*a*b**2 + 3*int(cos(e + f*
x)**n*sec(e + f*x),x)*a**2*b)
```

3.876 $\int (d\cos(e+fx))^n(a+b\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 186

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^2\,dx = -\frac{2ab(d\cos(e+fx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}} - \frac{(a^2(1-n)-b^2n)\cos(e+fx)(d\cos(e+fx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{f(1-n)(1+n)\sqrt{\sin^2(e+fx)}} + \frac{b^2(d\cos(e+fx))^n\tan(e+fx)}{f(1-n)}$$

output
```
-2*a*b*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(f*x+e)^2)*sin
(f*x+e)/f/n/(sin(f*x+e)^2)^(1/2)-(a^2*(1-n)-b^2*n)*cos(f*x+e)*(d*cos(f*x+e
))^n*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/f/(1-
n)/(1+n)/(sin(f*x+e)^2)^(1/2)+b^2*(d*cos(f*x+e))^n*tan(f*x+e)/f/(1-n)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.87

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^2\,dx =$$
$$-\frac{d(d\cos(e+fx))^{-1+n}\csc(e+fx)\left(b^2n(1+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(e+fx)\right)\right.}{}$$

input
```
Integrate[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x])^2,x]
```

output
```
-((d*(d*Cos[e + f*x])^(-1 + n)*Csc[e + f*x]*(b^2*n*(1 + n)*Hypergeometric2
F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[e + f*x]^2] + a*(-1 + n)*Cos[e + f*x]*(
2*b*(1 + n)*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e + f*x]^2] + a*n*C
os[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]))
*Sqrt[Sin[e + f*x]^2])/(f*(-1 + n)*n*(1 + n)))
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4752, 3042, 4275, 3042, 4259, 3042, 3122, 4534, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))^2(d\cos(e+fx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2\left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{ 4752}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int (d\sec(e+fx))^{-n}(a+b\sec(e+fx))^2dx$$

$$\Big\downarrow \text{ 3042}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$

$\downarrow$ 4275

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\int(d\sec(e+fx))^{-n}\left(a^2+b^2\sec^2(e+fx)\right)dx+\frac{2ab\int(d\sec(e+fx))^{1-n}dx}{d}\right)$$

$\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(a^2+b^2\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx+\frac{2ab\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{1-n}dx}{d}\right)$$

$\downarrow$ 4259

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(a^2+b^2\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx+\frac{2ab\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\ldots\right.}{d}\right.$$

$\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(a^2+b^2\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx+\frac{2ab\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\ldots\right.}{d}\right.$$

$\downarrow$ 3122

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(a^2+b^2\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx-\frac{2ab\sin(e+fx)(d\sec(e+fx))^{-n}\text{Hyperg}\ldots}{fn\sqrt{\sin^2(\ldots}}\right.$$

$\downarrow$ 4534

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\left(a^2-\frac{b^2n}{1-n}\right)\int(d\sec(e+fx))^{-n}dx-\frac{2ab\sin(e+fx)(d\sec(e+fx))^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+2}{2}\ldots\right.}{fn\sqrt{\sin^2(e+fx)}}\right.$$

$\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\left(a^2-\frac{b^2n}{1-n}\right)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}dx-\frac{2ab\sin(e+fx)(d\sec(e+fx))^{-n}\text{Hypergeometric2F1}\left(\ldots\right.}{fn\sqrt{\sin^2(e+fx)}}\right.$$

$$\Big\downarrow \text{4259}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\left(a^2-\frac{b^2n}{1-n}\right)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\cos(e+fx)}{d}\right)^n dx-\frac{2ab\sin(e+fx)(d\sec(e+f$$

$$\Big\downarrow \text{3042}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\left(a^2-\frac{b^2n}{1-n}\right)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^n dx-\frac{2ab\sin(e+fx)(d\sec($$

$$\Big\downarrow \text{3122}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(-\frac{d\left(a^2-\frac{b^2n}{1-n}\right)\sin(e+fx)(d\sec(e+fx))^{-n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(e+fx)\right)}{f(n+1)\sqrt{\sin^2(e+fx)}}-\frac{2ab\,\mathrm{si}$$

input
```
Int[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x])^2,x]
```

output
```
(d*Cos[e + f*x])^n*(d*Sec[e + f*x])^n*(-((d*(a^2 - (b^2*n)/(1 - n))*Hyperg
eometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-
1 - n)*Sin[e + f*x])/(f*(1 + n)*Sqrt[Sin[e + f*x]^2])) - (2*a*b*Hypergeome
tric2F1[1/2, n/2, (2 + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*n*(d*Sec[e +
 f*x])^n*Sqrt[Sin[e + f*x]^2]) + (b^2*Tan[e + f*x])/(f*(1 - n)*(d*Sec[e +
f*x])^n))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [F]

$$\int (d\cos (fx+e))^n (a+b\sec (fx+e))^2 \, dx$$

input
```
int((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^2,x)
```

output
```
int((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^2,x)
```

Fricas [F]

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^2\,dx=\int (b\sec(fx+e)+a)^2(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)*(d*cos(f*x + e))^
n, x)
```

Sympy [F]

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^2\,dx=\int (d\cos(e+fx))^n\,(a+b\sec(e+fx))^2\,dx$$

input
```
integrate((d*cos(f*x+e))**n*(a+b*sec(f*x+e))**2,x)
```

output
```
Integral((d*cos(e + f*x))**n*(a + b*sec(e + f*x))**2, x)
```

Maxima [F]

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^2\,dx=\int (b\sec(fx+e)+a)^2(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^2*(d*cos(f*x + e))^n, x)
```

Giac [F]

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^2\,dx=\int (b\sec(fx+e)+a)^2(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^2*(d*cos(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^2\,dx=\int (d\cos(e+fx))^n\left(a+\frac{b}{\cos(e+fx)}\right)^2\,dx$$

input
```
int((d*cos(e + f*x))^n*(a + b/cos(e + f*x))^2,x)
```

output
```
int((d*cos(e + f*x))^n*(a + b/cos(e + f*x))^2, x)
```

Reduce [F]

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^2\,dx=d^n\left(\left(\int \cos(fx+e)^n\,dx\right)a^2+\left(\int \cos(fx+e)^n\sec(fx+e)^2\,dx\right)b^2+2\left(\int \cos(fx+e)^n\sec(fx+e)\,dx\right)ab\right)$$

input
```
int((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^2,x)
```

output
```
d**n*(int(cos(e + f*x)**n,x)*a**2 + int(cos(e + f*x)**n*sec(e + f*x)**2,x)
*b**2 + 2*int(cos(e + f*x)**n*sec(e + f*x),x)*a*b)
```

3.877 $\int (d\cos(e+fx))^n(a+b\sec(e+fx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 132

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))\,dx = -\frac{b(d\cos(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}} - \frac{a(d\cos(e+fx))^{1+n} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{df(1+n)\sqrt{\sin^2(e+fx)}}$$

output
```
-b*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(f*x+e)^2)*sin(f*x
+e)/f/n/(sin(f*x+e)^2)^(1/2)-a*(d*cos(f*x+e))^(1+n)*hypergeom([1/2, 1/2+1/
2*n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/d/f/(1+n)/(sin(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.80

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))\,dx = -\frac{(d\cos(e+fx))^n\csc(e+fx)\left(b(1+n)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(e+fx)\right)+an\cos(e+fx\right.}{fn(1+n)}$$

input
```
Integrate[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x]),x]
```

output
```
-(((d*Cos[e + f*x])^n*Csc[e + f*x]*(b*(1 + n)*Hypergeometric2F1[1/2, n/2,
(2 + n)/2, Cos[e + f*x]^2] + a*n*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 +
n)/2, (3 + n)/2, Cos[e + f*x]^2])*Sqrt[Sin[e + f*x]^2])/(f*n*(1 + n)))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4713, 3042, 2030, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))(d\cos(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)\left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4713}$$

$$\int \sec(e+fx)(a\cos(e+fx)+b)(d\cos(e+fx))^n dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+b\right)\left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\sin\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$d\int \left(d\sin\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{n-1}\left(b+a\sin\left(\frac{1}{2}(2e+\pi)+fx\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$d\left(\frac{a\int(d\cos(e+fx))^n dx}{d}+b\int(d\cos(e+fx))^{n-1}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$d\left(\frac{a\int\left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n dx}{d}+b\int\left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{n-1}dx\right)$$

$$\Big\downarrow \text{3122}$$

$$d\left(-\frac{a\sin(e+fx)(d\cos(e+fx))^{n+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(e+fx)\right)}{d^2f(n+1)\sqrt{\sin^2(e+fx)}}-\frac{b\sin(e+fx)(d\cos(e+\right.$$

input
```
Int[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x]),x]
```

output
```
d*(-((b*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e +
f*x]^2]*Sin[e + f*x])/(d*f*n*Sqrt[Sin[e + f*x]^2])) - (a*(d*Cos[e + f*x])^
(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]*Sin[e
 + f*x])/(d^2*f*(1 + n)*Sqrt[Sin[e + f*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Maple [F]

$$\int (d\cos(fx+e))^n (a+b\sec(fx+e))\,dx$$

input
```
int((d*cos(f*x+e))^n*(a+b*sec(f*x+e)),x)
```

output
```
int((d*cos(f*x+e))^n*(a+b*sec(f*x+e)),x)
```

Fricas [F]

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))\,dx = \int (b\sec(fx+e)+a)(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

Sympy [F]

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))\,dx = \int (d\cos(e+fx))^n\,(a+b\sec(e+fx))\,dx$$

input
```
integrate((d*cos(f*x+e))**n*(a+b*sec(f*x+e)),x)
```

output
```
Integral((d*cos(e + f*x))**n*(a + b*sec(e + f*x)), x)
```

Maxima [F]

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))\,dx = \int (b\sec(fx+e)+a)(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

Giac [F]

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))\,dx = \int (b\sec(fx+e)+a)(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))\,dx = \int (d\,\cos(e+f\,x))^n\left(a+\frac{b}{\cos(e+f\,x)}\right)\,dx$$

input
```
int((d*cos(e + f*x))^n*(a + b/cos(e + f*x)),x)
```

output
```
int((d*cos(e + f*x))^n*(a + b/cos(e + f*x)), x)
```

Reduce [F]

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))\,dx = d^n\left(\left(\int \cos\left(fx+e\right)^n dx\right)a + \left(\int \cos\left(fx+e\right)^n \sec\left(fx+e\right)dx\right)b\right)$$

input
```
int((d*cos(f*x+e))^n*(a+b*sec(f*x+e)),x)
```

output
```
d**n*(int(cos(e + f*x)**n,x)*a + int(cos(e + f*x)**n*sec(e + f*x),x)*b)
```

3.878 $\int \frac{(d\cos(e+fx))^n}{a+b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 196

$$\int \frac{(d\cos(e+fx))^n}{a+b\sec(e+fx)}\,dx = \frac{a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-1-n),1,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\cos(e+fx))^n\cos^2(e+fx)^{\frac{1}{2}(-1-}}{(a^2-b^2)\,f} - \frac{b\,\text{AppellF1}\left(\frac{1}{2},-\frac{n}{2},1,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)(d\cos(e+fx))^n\cos^2(e+fx)^{-n/2}\sin(e+fx)}{(a^2-b^2)\,f}$$

output
```
a*AppellF1(1/2,-1/2-1/2*n,1,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*c
os(f*x+e)*(d*cos(f*x+e))^n*(cos(f*x+e)^2)^(-1/2-1/2*n)*sin(f*x+e)/(a^2-b^2
)/f-b*AppellF1(1/2,-1/2*n,1,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*(
d*cos(f*x+e))^n*sin(f*x+e)/(a^2-b^2)/f/((cos(f*x+e)^2)^(1/2*n))
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 5216 vs. $2(196) = 392$.

Time = 26.59 (sec) , antiderivative size = 5216, normalized size of antiderivative = 26.61

$$\int \frac{(d\cos(e+fx))^n}{a+b\sec(e+fx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Cos[e + f*x])^n/(a + b*Sec[e + f*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4752, 3042, 4356, 3042, 3302, 3042, 3668, 25, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\cos(e+fx))^n}{a+b\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n}{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4752}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int \frac{(d\sec(e+fx))^{-n}}{a+b\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}}{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4356}$$

$$\cos^{-n}(e+fx)(d\cos(e+fx))^n\int\frac{\cos^{n+1}(e+fx)}{b+a\cos(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(e+fx)(d\cos(e+fx))^n\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)^{n+1}}{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3302}$$

$$\cos^{-n}(e+fx)(d\cos(e+fx))^n\left(b\int\frac{\cos^{n+1}(e+fx)}{b^2-a^2\cos^2(e+fx)}dx-a\int\frac{\cos^{n+2}(e+fx)}{b^2-a^2\cos^2(e+fx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(e+fx)(d\cos(e+fx))^n\left(b\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)^{n+1}}{b^2-a^2\sin\left(e+fx+\frac{\pi}{2}\right)^2}dx-a\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)^{n+2}}{b^2-a^2\sin\left(e+fx+\frac{\pi}{2}\right)^2}dx\right)$$

$$\Big\downarrow \text{3668}$$

$$\cos^{-n}(e+fx)(d\cos(e+fx))^n\left(\frac{b\cos^n(e+fx)\cos^2(e+fx)^{-n/2}\int-\frac{\left(1-\sin^2(e+fx)\right)^{n/2}}{-\sin^2(e+fx)a^2+a^2-b^2}d\sin(e+fx)}{f}-\frac{a\cos^{n+1}(e+fx)\cos^2(e+fx)^{\frac{1}{2}(}}{}\right.$$

$$\Big\downarrow \text{25}$$

$$\cos^{-n}(e+fx)(d\cos(e+fx))^n\left(\frac{a\cos^{n+1}(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-n-1)}\int\frac{\left(1-\sin^2(e+fx)\right)^{\frac{n+1}{2}}}{-\sin^2(e+fx)a^2+a^2-b^2}d\sin(e+fx)}{f}-\frac{b\cos^n(e+fx)\cos^2(e+fx)}{}\right.$$

$$\Big\downarrow \text{333}$$

$$\cos^{-n}(e+fx)(d\cos(e+fx))^n\left(\frac{a\sin(e+fx)\cos^{n+1}(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-n-1)}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-n-1),1,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right.}{f\left(a^2-b^2\right)}\right.$$

input
```
Int[(d*Cos[e + f*x])^n/(a + b*Sec[e + f*x]),x]
```

output
```
((d*Cos[e + f*x])^n*((a*AppellF1[1/2, (-1 - n)/2, 1, 3/2, Sin[e + f*x]^2,
(a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]^(1 + n)*(Cos[e + f*x]^2)^((
-1 - n)/2)*Sin[e + f*x])/((a^2 - b^2)*f) - (b*AppellF1[1/2, -1/2*n, 1, 3/2
, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]^n*Sin[e +
 f*x])/((a^2 - b^2)*f*(Cos[e + f*x]^2)^(n/2))))/Cos[e + f*x]^n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3302
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[a   Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]
^2), x], x] - Simp[b/d   Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*
x]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 3668
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]))   Subst[Int[(1 - ff^2*x^2)^((m -
 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
 d, e, f, m, p}, x] &&  !IntegerQ[m]
```

rule 4356
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Simp[Sin[e + f*x]^n*(d*Csc[e + f*x])^n   Int[(b +
 a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n}
, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [F]

$$\int \frac{(d\cos(fx+e))^n}{a+b\sec(fx+e)}dx$$

input
```
int((d*cos(f*x+e))^n/(a+b*sec(f*x+e)),x)
```

output
```
int((d*cos(f*x+e))^n/(a+b*sec(f*x+e)),x)
```

Fricas [F]

$$\int \frac{(d\cos(e+fx))^n}{a+b\sec(e+fx)}\,dx = \int \frac{(d\cos(fx+e))^n}{b\sec(fx+e)+a}\,dx$$

input
```
integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral((d*cos(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{(d\cos(e+fx))^n}{a+b\sec(e+fx)}\,dx = \int \frac{(d\cos(e+fx))^n}{a+b\sec(e+fx)}\,dx$$

input
```
integrate((d*cos(f*x+e))**n/(a+b*sec(f*x+e)),x)
```

output
```
Integral((d*cos(e + f*x))**n/(a + b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(d\cos(e+fx))^n}{a+b\sec(e+fx)}\,dx = \int \frac{(d\cos(fx+e))^n}{b\sec(fx+e)+a}\,dx$$

input
```
integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*cos(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(d\cos(e+fx))^n}{a+b\sec(e+fx)}\,dx = \int \frac{(d\cos(fx+e))^n}{b\sec(fx+e)+a}\,dx$$

input
```
integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*cos(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\cos(e+fx))^n}{a+b\sec(e+fx)}\,dx = \int \frac{(d\,\cos\left(e+f\,x\right))^n}{a+\frac{b}{\cos(e+f\,x)}}\,dx$$

input
```
int((d*cos(e + f*x))^n/(a + b/cos(e + f*x)),x)
```

output
```
int((d*cos(e + f*x))^n/(a + b/cos(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(d\cos(e+fx))^n}{a+b\sec(e+fx)}\,dx = d^n\left(\int \frac{\cos\left(fx+e\right)^n}{\sec\left(fx+e\right)b+a}dx\right)$$

input
```
int((d*cos(f*x+e))^n/(a+b*sec(f*x+e)),x)
```

output
```
d**n*int(cos(e + f*x)**n/(sec(e + f*x)*b + a),x)
```

3.879 $\int \frac{(d\cos(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 309

$$\int \frac{(d\cos(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = \frac{a^2\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-3-n),2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\cos(e+fx))^n\cos^2(e+fx)^{\frac{1}{2}(-\cdots}}{(a^2-b^2)^2 f} + \frac{b^2\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-1-n),2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\cos(e+fx))^n\cos^2(e+fx)^{\frac{1}{2}\cdots}}{(a^2-b^2)^2 f} - \frac{2ab\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-2-n),2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)(d\cos(e+fx))^n\cos^2(e+fx)^{-n/2}\sin(e+\cdots}{(a^2-b^2)^2 f}$$

output
```
a^2*AppellF1(1/2,-3/2-1/2*n,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))
*cos(f*x+e)*(d*cos(f*x+e))^n*(cos(f*x+e)^2)^(-1/2-1/2*n)*sin(f*x+e)/(a^2-b
^2)^2/f+b^2*AppellF1(1/2,-1/2-1/2*n,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a
^2-b^2))*cos(f*x+e)*(d*cos(f*x+e))^n*(cos(f*x+e)^2)^(-1/2-1/2*n)*sin(f*x+e
)/(a^2-b^2)^2/f-2*a*b*AppellF1(1/2,-1-1/2*n,2,3/2,sin(f*x+e)^2,a^2*sin(f*x
+e)^2/(a^2-b^2))*(d*cos(f*x+e))^n*sin(f*x+e)/(a^2-b^2)^2/f/((cos(f*x+e)^2)
^(1/2*n))
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 13974 vs. $2(309) = 618$.

Time = 48.04 (sec) , antiderivative size = 13974, normalized size of antiderivative = 45.22

$$\int \frac{(d\cos(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Cos[e + f*x])^n/(a + b*Sec[e + f*x])^2,x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4752, 3042, 4356, 3042, 3303, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\cos(e+fx))^n}{(a+b\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4752}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int \frac{(d\sec(e+fx))^{-n}}{(a+b\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4356}$$

$$\cos^{-n}(e+fx)(d\cos(e+fx))^n\int\frac{\cos^{n+2}(e+fx)}{(b+a\cos(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(e+fx)(d\cos(e+fx))^n\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)^{n+2}}{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3303}$$

$$\cos^{-n}(e+fx)(d\cos(e+fx))^n\int\left(\frac{b^2\cos^{n+2}(e+fx)}{(b^2-a^2\cos^2(e+fx))^2}-\frac{2ab\cos^{n+3}(e+fx)}{(b^2-a^2\cos^2(e+fx))^2}+\frac{a^2\cos^{n+4}(e+fx)}{(a^2\cos^2(e+fx)-b^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\cos^{-n}(e+fx)(d\cos(e+fx))^n\left(\frac{a^2\sin(e+fx)\cos^{n+1}(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-n-1)}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-n-3),2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+\right.}{a^2-b^2}}{f\left(a^2-b^2\right)^2}\right.$$

input
```
Int[(d*Cos[e + f*x])^n/(a + b*Sec[e + f*x])^2,x]
```

output
```
((d*Cos[e + f*x])^n*((a^2*AppellF1[1/2, (-3 - n)/2, 2, 3/2, Sin[e + f*x]^2
, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]^(1 + n)*(Cos[e + f*x]^2)^
((-1 - n)/2)*Sin[e + f*x])/((a^2 - b^2)^2*f) + (b^2*AppellF1[1/2, (-1 - n)
/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]
^(1 + n)*(Cos[e + f*x]^2)^((-1 - n)/2)*Sin[e + f*x])/((a^2 - b^2)^2*f) - (
2*a*b*AppellF1[1/2, (-2 - n)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^
2)/(a^2 - b^2)]*Cos[e + f*x]^n*Sin[e + f*x])/((a^2 - b^2)^2*f*(Cos[e + f*x
]^2)^(n/2))))/Cos[e + f*x]^n
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3303
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n*(1/((a - b*sin[
e + f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m)), x], x] /; FreeQ[{a, b, d, e, f,
 n}, x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

rule 4356
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Simp[Sin[e + f*x]^n*(d*Csc[e + f*x])^n   Int[(b +
 a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n}
, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

Maple [F]

$$\int \frac{(d\cos(fx+e))^n}{(a+b\sec(fx+e))^2}dx$$

input
```
int((d*cos(f*x+e))^n/(a+b*sec(f*x+e))^2,x)
```

output
```
int((d*cos(f*x+e))^n/(a+b*sec(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{(d\cos(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(d\cos(fx+e))^n}{(b\sec(fx+e)+a)^2}\,dx$$

input
```
integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((d*cos(f*x + e))^n/(b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2
), x)
```

Sympy [F]

$$\int \frac{(d\cos(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(d\cos{(e+fx)})^n}{(a+b\sec{(e+fx)})^2}\,dx$$

input
```
integrate((d*cos(f*x+e))**n/(a+b*sec(f*x+e))**2,x)
```

output
```
Integral((d*cos(e + f*x))**n/(a + b*sec(e + f*x))**2, x)
```

Maxima [F]

$$\int \frac{(d\cos(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(d\cos{(fx+e)})^n}{(b\sec{(fx+e)}+a)^2}\,dx$$

input
```
integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*cos(f*x + e))^n/(b*sec(f*x + e) + a)^2, x)
```

Giac [F]

$$\int \frac{(d\cos(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(d\cos{(fx+e)})^n}{(b\sec{(fx+e)}+a)^2}\,dx$$

input
```
integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*cos(f*x + e))^n/(b*sec(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\cos(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx=\int \frac{(d\ \cos(e+f\,x))^n}{\left(a+\frac{b}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int((d*cos(e + f*x))^n/(a + b/cos(e + f*x))^2,x)
```

output
```
int((d*cos(e + f*x))^n/(a + b/cos(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(d\cos(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx=d^n\left(\int \frac{\cos(fx+e)^n}{\sec(fx+e)^2b^2+2\sec(fx+e)\,ab+a^2}dx\right)$$

input
```
int((d*cos(f*x+e))^n/(a+b*sec(f*x+e))^2,x)
```

output
```
d**n*int(cos(e + f*x)**n/(sec(e + f*x)**2*b**2 + 2*sec(e + f*x)*a*b + a**2
),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational):  #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#            Albert Rich to use with Sagemath. This is used to
#            grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#            'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#            issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:  #isinstance(expn,list):
        return max(map(expnType, expn))  #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:  #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file